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The implications of first-order risk aversion for asset market risk premiums

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Abstract

In an effort to explain simultaneously the excess return predictability observed in equity, bond and foreign exchange markets, we incorporate preferences exhibiting first-order risk aversion into a general equilibrium two-country monetary model. When we calibrate the model to US and Japanese data, we find that first-order risk aversion substantially increases excess return predictability. However, this increased predictability is insufficient to match the data. We conclude that the observed patterns of excess return predictability are unlikely to be explained purely by time-varying risk premiums generated by highly risk averse agents in a complete markets economy.

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1. Introduction

It is generally accepted that excess returns on a variety of assets are predictable. This is true for returns in the equity markets, bond markets and foreign exchange markets of various countries. One interpretation of this evidence is that equilibrium risk premiums are highly variable. Attempts to model highly variable risk premiums with traditional time-separable expected utility preferences and homoskedastic driving processes have failed. Researchers consequently have incorporated time-nonseparabilities into preferences while maintaining expected utility, and they have abandoned traditional preference specifications. They have also employed conditionally heteroskedastic driving processes in attempts to generate variability in agents' intertemporal marginal rates of substitution (IMRSs).

In this paper we maintain time-separability of consumption with homoskedastic driving processes, but abandon the expected utility hypothesis in favor of preferences that exhibit first-order risk aversion.[1] With these preferences, agents are substantively averse to even small gambles. Hence, a small degree of uncertainty in the exogenous environment of economic agents can potentially induce relatively large fluctuations in agents' IMRSs. This, in turn, implies large fluctuations in expected rates of return on a variety of assets. Our goal is to determine whether a general equilibrium model incorporating preferences that exhibit first-order risk aversion is quantitatively consistent with the predictability of returns and with other time series properties observed in the data from the foreign exchange market, the equity markets, and the bond markets of the US and Japan.

Other papers that propose first-order risk aversion as an explanation for asset pricing anomalies include Epstein and Zin (1990, 1991) and Bonomo and Garcia (1993). In particular, Epstein and Zin (1991) are unable to reject the overidentifying restrictions implied by a closed economy model, analogous to the model of Hansen and Singleton (1982), when first-order risk aversion is assumed. Their approach requires the researcher to choose a proxy for the unobservable rate of return on aggregate wealth, and their inference about the validity of the model depends on this choice. Epstein and Zin's (1991) proxy for the return on aggregate wealth is the return on a value-weighted equity portfolio. As noted by these authors, this choice is subject to Roll's (1977) critique, since it incorporates the leverage implicit in corporate debt and leaves out all non-equity claims to wealth. In an open economy setting, the problems noted by Roll (1977) are exacerbated by the more intensive use of bank financing in some non-US corporate capital structures. Furthermore, in the absence of purchasing power

[1] The concept of first-order risk aversion was introduced by Segal and Spivak (1990).

parity, it is difficult, as a practical matter, to aggregate returns from different countries. For these reasons, we do not follow Epstein and Zin's (1991) approach. Instead of testing the first-order conditions of the model, we explicitly solve a two-country monetary model for the endogenous moments of interest.

In our model, the exogenous processes are the endowments and the money supplies of two countries. The growth rates of these exogenous processes follow a discrete Markov chain that is estimated from US and Japanese data using the method of Tauchen and Hussey (1991). The equilibrium processes for returns and other endogenous variables are found by numerically solving a system of Euler equations. Having solved the model, we compare a variety of statistics that provide evidence on the predictability of the model's returns to the corresponding statistics in the data.

Our article is part of a vast literature modelling asset returns as the outcome of a dynamic, stochastic equilibrium.[2] While these papers differ in what is considered to be exogenous, in whether the economy is open or closed, in the particular way that preferences are modeled, and in the choice of moments of asset returns that are deemed to be important, none of them simultaneously explain the observed predictability of asset returns in equity, bond and foreign exchange markets while matching the volatility of interest rates, exchange rates and equity returns.

We find that increasing the amount of first-order risk aversion dramatically increases the variance of risk premiums (defined as expected excess returns) in all markets. However, this increased risk-premium volatility fails to imply a comparable increase in excess-return predictability. The reason is that excess-return predictability is also affected by the variability of expected asset-price changes. We find that an increased level of first-order risk aversion increases the variance of expected changes in asset prices in such a way that the net effect on excess-return predictability is small. We conclude that the predictability of excess returns in financial markets is unlikely to be explained simply by modifying preference assumptions.

The remainder of the paper is organized as follows. In Section 2, we present evidence on the predictability of excess rates of return in the dollar–yen foreign exchange market, in the dollar and yen discount bond markets, and in the equity markets. These stylized facts provide the set of statistics that we would like the model to match. Section 3 introduces the concept of first-order risk aversion and

[2] Examples of recent papers that model excess returns in foreign exchange markets using approaches related to the one used here include Backus et al. (1992), Bansal et al. (1995), Bekaert (1994, 1996), Canova and Marrinan (1993), and Macklem (1991). For equity markets, Benninga and Protopapadakis (1990), Campbell and Cochrane (1994), Cecchetti et al. (1993), Hung (1994), and Kandel and Stambaugh (1991) model excess returns with general equilibrium methods.

demonstrates how to incorporate these preferences into a formal dynamic model. It also derives the model's equilibrium conditions for endogenous financial variables. Section 4 describes our procedure for calibrating the model, and Section 5 presents our results. Section 6 compares our results with Epstein and Zin (1991), and Section 7 provides concluding comments.

2. Some stylized facts on excess return predictability

In this section we document the predictability of excess rates of return on discount bonds, equities and foreign money markets using regression analysis. Since US and Japanese data are the exogenous processes of the model, we report results only for these two countries. Nevertheless, the evidence is consistent across the markets of most developed countries as documented by the recent empirical studies of Harvey (1991), Bekaert and Hodrick (1992, 1993), and Solnik (1993), among others.

2.1. The foreign exchange market

Let s_t denote the log of the spot exchange rate at time t of dollars per yen, and let f_t denote the log of the forward exchange rate of dollars per yen quoted at time t for date $t+1$ transactions. Using interest rate parity, the continuously compounded excess dollar rate of return from an uncovered investment in the Japanese money market is $s_{t+1} - f_t$. A common way of testing the predictability of this excess rate of return is to regress it on the forward premium:

$$s_{t+1} - f_t = \alpha_{rs} + \beta_{rs}(f_t - s_t) + \varepsilon_{t+1}. \tag{1}$$

The null hypothesis of an unpredictable excess rate of return implies $\beta_{rs} = 0$.

Our empirical analysis uses a quarterly holding period since that is the frequency we use for the exogenous processes in simulating the model. We use monthly observations on the dollar–yen exchange rate from January 1976 to December 1989, and all returns are expressed in percentage points per annum. The data are described more completely in Appendix A.

The first row of Table 1, Panel A, displays the regression results for Eq. (1) using the three-month forward premium as the predictor. As is typical in the literature, the slope coefficient of -4.016 is significantly negative.[3] The R^2 for

[3] For the dollar values of other major foreign currencies, the estimated coefficients are also significantly below zero. For example, Bekaert and Hodrick (1993) report slope coefficients for monthly returns of -4.015 for the dollar–deutsche mark, -3.021 for the dollar–pound, and -3.098 for the dollar–yen. Similar results arise in regressions using non-dollar exchange rates as demonstrated in Bekaert (1995).

Table 1
The stylized facts

Panel A: Regression results

Dependent variable	Coef. on constant	Coef. on fp_t	Coef. on $fb_t^{\$}$	Coef. on $fb_t^{¥}$	R^2
$s_{t+1} - f_t$	16.271 (3.674)	− 4.016 (0.766)			0.220
$h_{t+1,2}^{\$} - i_t^{\$}$	0.038 (0.050)		− 0.450 (0.129)		0.028
$h_{t+1,2}^{¥} - i_t^{¥}$	0.075 (0.019)			− 0.448 (0.028)	0.086
$r_{t+1}^{w} - i_t^{\$}$	21.540 (4.864)	− 3.543 (0.816)			0.139
$r_{t+1}^{\$} - i_t^{\$}$	11.413 (4.971)	− 2.024 (0.900)			0.041
$r_{t+1}^{¥} - i_t^{¥}$	15.397 (4.807)	− 1.045 (0.954)			0.013

Panel B: Means and standard deviations

Variable	Mean	Standard deviation
Δs_{t+1}	5.119	25.019
fp_t	3.698	3.077
fitted $s_{t+1} - f_t$	1.421	12.355
$fb_t^{\$}$	0.124	0.707
$h_{t+1,2}^{\$} - i_t^{\$}$	0.094	1.892
fitted $h_{t+1,2}^{\$} - i_t^{\$}$	0.094	0.318
$fb_t^{¥}$	0.116	0.826
$h_{t+1,2}^{¥} - i_t^{¥}$	0.128	1.259
fitted $h_{t+1,2}^{¥} - i_t^{¥}$	0.128	0.370
$r_{t-1}^{w} - i_t^{\$}$	8.440	29.204
fitted $r_{t-1}^{w} - i_t^{\$}$	8.440	10.899

Notes: The data are monthly observations on quarterly rates. The sample period is from January 1976 to December 1989 for exchange rates and equities and from October 1975 to June 1990 for interest rates. All rates are measured as percentage points per annum. Time subscripts denote quarters. The logarithms of the dollar/yen spot and forward exchange rates are denoted s_t and f_t. The quarterly rate of depreciation is Δs_{t+1}; the three-month forward premium on the yen in terms of the dollar is denoted fp_t; the quarterly dollar excess return on the world equity market (an equally-weighted average of the dollar excess returns to US and Japanese equities, defined in Eq. (4)) is $r_{t+1}^{w} - i_t^{\$}$; the three-month dollar excess return to US equities is $r_{t+1}^{\$} - i_t^{\$}$; the three month yen excess return to Japanese equities is $r_{t+1}^{¥} - i_t^{¥}$, $h_{t+1,2}^{\$} - i_t^{\$}$ ($h_{t+1,2}^{¥} - i_t^{¥}$) is the quarterly excess dollar (yen) return from t to $t+1$ obtained by holding dollar (yen) discount bonds that mature at $t+2$; $fb_t^{\$}$($fb_t^{¥}$) is the one-quarter-ahead forward premium, defined in Eq. (2), in the dollar (yen) discount bond market. In Panel B, the variable 'fitted $s_{t+1} - f_t$' is the fitted value of regression (1); the variable 'fitted $h_{t+1,2}^{\$} - i_t^{\$}$' ('fitted $h_{t+1,2}^{¥} - i_t^{¥}$') is the fitted value of regression (3) using data from the dollar (yen) bond market; the variable 'fitted $r_{t+1}^{w} - i_t^{\$}$' is the fitted value of regression (5). The numbers in parentheses are standard errors, which are heteroskedasticity-consistent and are corrected for the serial correlation induced by the overlap in the data using the method of Newey and West (1987).

the regression is 0.22, and the standard deviation of the fitted value of the excess return, reported in Table 1, Panel B, is 12.355%. The above statistics indicate that these excess returns are quite predictable and that foreign exchange risk premiums are quite variable.[4]

2.2. The discount bond market

Similar evidence of predictable excess holding period rates of return arises in the discount bond market. Let i_t be the continuously-compounded, nominally risk-free interest rate at time t, and let $i_{t,2}$ be the continuously-compounded nominal yield to maturity on a two-period risk-free zero coupon bond. Let the one-period continuously compounded holding period return on a two-period bond realized at time $t+1$ be denoted as $h_{t+1,2}$. Note that $h_{t+1,2} = 2i_{t,2} - i_{t+1}$. In the empirical analysis we examine the excess holding period return, $h_{t+1,2} - i_t$, in a regression analogous to Eq. (1). For parallel structure with the foreign exchange market, we define the forward premium in the bond market, denoted fb_t, as the logarithm of the contractual price today for a one-period bond delivered one period from now minus the logarithm of the price today of a one-period bond. Using the definition of the yield to maturity, we obtain

$$fb_t = -2i_{t,2} + 2i_t. \tag{2}$$

The bond market analogue to Eq. (1) is

$$h_{t+1,2} - i_t = \alpha_{\mathrm{rb}} + \beta_{\mathrm{rb}} fb_t + \varepsilon_{t+1}. \tag{3}$$

If excess holding period returns are unpredictable, β_{rb} should be zero.

Table 1 (Panel A, second and third rows) reports estimates of Eq. (3) for the US dollar and Japanese yen discount bond markets. Since the period is one quarter, $h_{t+1,2}$ is the three-month return on a six-month bond and fb_t is the forward premium on a three-month bond to be purchased three months in the future. For the empirical analysis we have monthly observations on three-month and six-month Eurodollar and Euroyen interest rates from October 1975 to June 1990.

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[4] The conditional expectation of an excess return is often referred to as a risk premium, and we will use this terminology interchangeably with expected excess return. This terminology is somewhat imprecise. An excess rate of return is the nominal rate of return on an asset in excess of the short-term interest rate. If inflation is stochastic, conditional expectations of excess rates of return can be non-zero even if agents are risk neutral, which makes use of the term 'risk premium' for these conditional expectations somewhat problematic. Engel (1992) provides a recent discussion of this issue for the risk premium in the foreign exchange market.

For both the dollar and the yen markets, the estimate of β_{rb} is -0.45, and both are significantly negative.[5] While the estimated β_{rb}s are not as negative as the estimates from the foreign exchange market, there is strong evidence of predictability of the excess rates of return. The R^2 for the US market is 0.03, and the R^2 for the yen market is 0.09. The standard deviations of the fitted values of the excess returns in the two markets are 0.318% for the US and 0.370% for Japan.

2.3. The equity markets

A similar set of results emerges from examining excess rates of return in equity markets. Bekaert and Hodrick (1992) show that excess rates of return to US and foreign equities are predicted by the forward premium in the foreign exchange market. Consistent with our two-country framework, we construct a dollar world equity market excess rate of return as an equally-weighted average of the dollar excess rates of returns on the equity markets of the US and Japan:

$$r^{\text{w}}_{t+1} - i^{\$}_t = [(r^{\$}_{t+1} - i^{\$}_t) + (r^{¥}_{t+1} - i^{¥}_t) + (s_{t+1} - f_t)]\ (1/2) \tag{4}$$

where $r^{\$}_{t+1}$ ($r^{¥}_{t+1}$) denote the one-period dollar (yen) continuously-compounded return in the equity market of the US (Japan). We regress this excess return on the three-month forward premium in the dollar–yen foreign exchange market:

$$r^{\text{w}}_{t+1} - i^{\$}_t = \alpha_{\text{rw}} + \beta_{\text{rw}}(f_t - s_t) + \varepsilon_{t+1}. \tag{5}$$

Table 1 (Panel A, fourth row) reports a slope coefficient of -3.543, with a standard error of 0.816. As Eq. (4) indicates, there are three components to this world equity excess rate of return: the excess dollar rate of return in the US equity market, the excess yen rate of return in the Japanese equity market, and the excess rate of return in the foreign exchange market. The regression of the third component on the forward premium is discussed above. Regressions of the first two components on the forward premium are contained in rows five and six of Table 1, Panel A. Each of the components has a negative slope coefficient, and all but the Japanese equity coefficient are significantly negative. Panel B of Table 1 documents a standard deviation of the risk premium in the world equity market of 10.899%.

2.4. Implications for modeling

The patterns of predictability in excess-return regressions can, in principle, be explained by time variation in equilibrium risk premiums. To provide some

[5] These results are similar to those reported by Fama (1984) and Stambaugh (1988) for monthly US data.

intuition regarding the amount of time-variation in risk premiums required to match the data, consider the following decomposition of forward premiums introduced by Fama (1984). Define the logarithmic risk premium in the foreign exchange market as $rp_t \equiv E_t(s_{t+1}) - f_t$. The forward premium can be decomposed into the expected rate of depreciation of the dollar relative to the yen minus this risk premium:

$$fp_t \equiv f_t - s_t = E_t(\Delta s_{t+1}) - rp_t, \tag{6}$$

where Δ is the first difference operator. Using this decomposition, the slope coefficient β_{rs} in Eq. (1) can be written

$$\begin{aligned}\beta_{rs} &= \frac{\operatorname{cov}(s_{t+1} - f_t,\ fp_t)}{\operatorname{var}(fp_t)} \\ &= \frac{\operatorname{cov}(rp_t, E_t(\Delta s_{t+1})) - \operatorname{var}(rp_t)}{\operatorname{var}(E_t(\Delta s_{t+1})) + \operatorname{var}(rp_t) - 2\operatorname{cov}(rp_t,\ E_t(\Delta s_{t+1}))}.\end{aligned} \tag{7}$$

(A similar decomposition can be performed for the bond market.)

Our estimate of β_{rs} is substantially below -1. From Eq. (7), $\beta_{rs} < -1$ implies

$$\operatorname{var}(rp_t) > \operatorname{cov}(rp_t, E_t(\Delta s_{t+1})) > \operatorname{var}(E_t(\Delta s_{t+1})). \tag{8}$$

Hence, the results imply that the risk premium in the foreign exchange market is more variable than the expected rate of depreciation and that the risk premium covaries positively with the expected rate of depreciation. For the bond market regressions (3), the estimated slope coefficients are insignificantly different from -0.5. The bond-market analogue to Eq. (7) then implies

$$\operatorname{var}(E_t(\Delta i_{t+1})) \approx \operatorname{var}(rb_t). \tag{9}$$

That is, the variabilities of the risk premiums in the two bond markets are roughly equal to the variabilities of the expected rates of change of the one-period bond yields.

As is well known, substantial variability in risk premiums requires substantial volatility in the IMRS. One way of generating a highly volatile IMRS is to assume that agents have a high degree of risk aversion.[6] In effect, the extreme

[6] Alternatively, high volatility in the IMRS can be generated by directly assuming time-varying conditional heteroskedasticity in the exogenous processes, as in Bekaert (1996). Kandel and Stambaugh (1991) successfully match many moments of equity returns using preferences that separate the roles of risk aversion and intertemporal substitution with a conditionally heteroskedastic driving process for consumption growth. Campbell and Cochrane (1994) match moments of equity returns using habit persistence and a time-varying sensitivity of habit to past consumption growth, which is conditionally homoskedastic. While these approaches prove successful along some dimensions, the models are closed economy, non-monetary models. In this paper, incorporating time-varying conditional heteroskedasticity substantially increases the dimensionality of the state space, rendering the approach computationally intractable.

nonlinearity associated with high risk aversion transforms the uncertainty due to conditionally homoskedastic exogenous inputs into endogenous risky asset returns whose moments are conditionally quite variable. However, matching the patterns in the data requires more than a highly volatile IMRS. From Eq. (7), changes in the model specification that increase the variances of risk premiums may also increase the variance of $E_t(\Delta s_{t+1})$, and may change $\mathrm{cov}(rp_t, \Delta s_{t+1})$. Thus, while it is likely that extreme risk aversion will increase the variability of the IMRS, it is unclear whether this will induce the patterns of predictability in excess returns observed in the data. To explore the effects of increasing risk aversion we must solve the model explicitly.

3. A two-country monetary model

This section presents a two-country, competitive-equilibrium model in which asset prices and exchange rates are determined by the optimal choice of a representative agent. Our discussion of the model is organized in four sub-sections. Section 3.1 discusses the use of a representative agent in a two-country setting. Section 3.2 discusses the preference structure that incorporates first-order risk aversion and is the main innovation of this model. Section 3.3 describes the agent's budget constraint and the transaction cost technology that provides a role for money in equilibrium. Section 3.4 focuses on the equilibrium determination of exchange rates and asset returns.

3.1. The representative agent equilibrium

The use of a representative agent who maximizes utility defined over a home and foreign consumption bundle relies on the perfectly pooled equilibrium introduced in Lucas (1982). The equilibrium assumes that agents in both countries are identical and that purchasing power parity (PPP) holds. Under these assumptions the usual closed-economy aggregation theorems continue to hold and the use of one representative agent is valid.

Although the equilibrium concept is valid, it has a number of unrealistic features. First, the consumption predictions do not replicate the intricate trade patterns observed in the data nor do they match the low correlations of measured consumptions across countries. Second, in the data there are marked deviations from PPP, at least in the short run. These deviations make agents from different countries inherently different from each other because they face different relative prices for consumption bundles. Although we believe that these are important drawbacks of our model, we know of no two-country, monetary general equilibrium model incorporating PPP-deviations and non-trivial current account dynamics that has been solved with standard preferences. To highlight the effect of first-order risk aversion relative to the existing literature, we maintain the perfectly pooled equilibrium in this paper.[7] We also devote Sub-section 5.4 to exploring different specifications for the exogenous processes to provide a robustness check on the implications of the model.

3.2. *The preference structure*

Section 2 examined how substantial risk aversion may help generate the regression results described there. Most models using expected utility preferences have not fared well in this regard. Even models by Backus et al. (1993) and Bekaert (1996), incorporating time-nonseparabilities in the form of habit persistence, fail to imply sufficient predictability in excess rates of return in the foreign exchange market while simultaneously matching the time series properties of interest rates.

One possible explanation for this failure is that expected utility preferences display second-order risk aversion. The utility loss associated with a fair gamble (one whose cost equals its expected value), is approximately proportional to the variance of the gamble.[8] This is a problem for consumption-based asset pricing models. At any given date, the conditional variance of next period's aggregate consumption is small, so the maximum amount an expected-utility maximizer would pay to hedge consumption-risk using financial assets is also small. It is of interest, then, to consider a class of (non-expected-utility) preferences that imply first-order risk aversion. Under first-order risk aversion, the utility loss associated with a fair gamble is approximately proportional to the standard deviation of the gamble. For low-variance gambles (such as gambles that mimic aggregate consumption risk), the standard deviation is considerably larger than the variance. Other things equal, agents with first-order risk aversion preferences are willing to pay substantially more to avoid such low-risk gambles than agents with expected-utility preferences.

Epstein and Zin (1991) examine a variety of preferences that exhibit first-order risk aversion, including Gul's (1991) disappointment aversion preferences. Disappointment aversion was developed to accommodate the Allais paradox within a parsimonious extension of expected utility. Camerer (1989) suggests that expected utility cannot explain the experimental evidence on preference

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[7] We are skeptical that it is computationally feasible to incorporate PPP-deviations in the model explored in this paper.

[8] Let $\tilde{c}$ denote a random consumption pay-off, with cumulative distribution function F and mean $\bar{c}$, and let $F_{\bar{c}}$ denote the degenerate distribution at $\bar{c}$. Suppose an agent ranks pay-off distributions according to an expected utility functional $V(F) = \int \mathrm{U}(\tilde{c})\,\mathrm{d}F(\tilde{c})$, where U is a twice-differentiable, strictly concave Von Neumann–Morgenstern utility function. If the agent gives up $\bar{c}$ in exchange for random consumption $\tilde{c}$, the change in utility is approximately $V(F) - V(F_{\bar{c}}) \approx (\mathrm{U}''(\bar{c})/2)\,\mathrm{var}(\tilde{c}) < 0$.

orderings under uncertainty. Rather, what is required is a preference ordering in which outcomes are evaluated relative to some reference point. Disappointment aversion has this property.

As in Epstein and Zin (1991), we use the following model of disappointment aversion. A preference ordering over the space of probability distributions $\mathscr{P}$ (e.g., over alternative lotteries) can be represented by a certainty equivalent function $\mu\colon \mathscr{P} \to \mathscr{R}$. For $P \in \mathscr{P}$, $\mu(P)$ is implicitly defined by

$$\frac{\mu(P)^\alpha}{\alpha} \equiv \frac{1}{K}\left(\int_{(-\infty,\mu(P)]} \frac{z^\alpha}{\alpha} \mathrm{d}P(z) + A\int_{(\mu(P),+\infty)} \frac{z^\alpha}{\alpha} \mathrm{d}P(z)\right), \quad A \le 1,\ \alpha < 1, \tag{10}$$

where $K = A\,\mathrm{prob}(z > \mu) + \mathrm{prob}(z \le \mu)$. If $A = 1$, the preferences described by Eq. (10) correspond to expected utility with a coefficient of relative risk aversion equal to $1 - \alpha$. If A differs from unity, Eq. (10) can be interpreted as follows. Those outcomes below the certainty equivalent are disappointing, while those above the certainty equivalent are elating. If $A < 1$, the elation region is down-weighted relative to the disappointment region.

We want the representative agent's preferences over current and uncertain future consumption to incorporate disappointment aversion as in Eq. (10). Let c_t^x (c_t^y) denote the agent's consumption of the good produced in country x (country y) in period t, and let M_{t+1}^x (M_{t+1}^y) denote the amount of currency x (currency y) acquired by the agent in period t. We refer to currency x as the dollar, and currency y as the yen. In addition to currency, agents can hold n capital assets. Let $z_{i,t+1}$ be the value (in units of c^x) of the representative agent's investment in asset i, chosen at t, and which pays off at $t+1$. Let W_t denote the agent's wealth at the beginning of period t, and let J_t denote the vector of exogenous state variables which span the agent's information set in period t. Finally, let the utility of W_t in state J_t be $V(W_t, J_t)$, and define it recursively by

$$\begin{aligned} V(W_t, J_t) = \max_{c_t^x,\, c_t^y,\, M_{t+1}^x,\, M_{t+1}^y,\, z_{i,t+1}} & [([c_t^x]^\delta\, [c_t^y]^{1-\delta})^\rho \\ & + \beta(\mu[P_{V(W_{t+1}, J_{t+1})} \mid J_t])^\rho]^{1/\rho}, \quad 0 < \delta < 1,\ \rho < 1. \end{aligned} \tag{11}$$

The maximization of Eq. (11) is subject to the budget constraint and the wealth constraint, which are given below, and the expression uses the definition of μ from Eq. (10).

The expression $\mu[P_{V(W_{t+1}, J_{t+1})} \mid J_t]$ in Eq. (11) denotes the certainty equivalent of the conditional distribution of the value function at date $t+1$, given information at date t. When agents make their consumption and portfolio choices, they care about two distinct effects: how their choices affect current utility, and what happens to the probability distribution of their future utility. With expected utility, the latter effect is incorporated by taking the conditional expectation of next-period's value function. In Eq. (11), effects of the probability distribution of future utility on current utility are captured by the certainty equivalent function μ. In addition, the two effects are aggregated in Eq. (11) by a CES function, while in the expected-utility framework, the two effects are simply added.

The parameter ρ governs intertemporal substitution in the following, somewhat unconventional, sense: The elasticity of substitution between current utility $(c^x)^\delta (c^y)^{1-\delta}$ and the certainty-equivalent of future utility, $\mu[P_{V(W_{t+1}, J_{t+1})} | J_t]$, is given by $1/(1+\rho)$. Therefore, ρ determines the optimal trade-off between present and future utility. When ρ is near unity, there is an extremely high degree of substitutability between these two sources of utility. Extremely negative values of ρ imply almost no substitutability. Note that this elasticity of substitution does not directly correspond to the elasticity of substitution between current and future consumption (as studied, for example, by Hall (1988)). The more conventional notion of intertemporal substitution elasticity is a function of all the preference parameters of the model.

3.3. The budget constraint and the transaction cost technology

Monies are incorporated into the model using the transaction cost technologies of Marshall (1992) and Bekaert (1996). Money is demanded by agents because consumption transactions are costly, and increasing real balance holdings decreases these transaction costs. Consumption of c^x involves transaction costs measured by

$$\psi_t^x \equiv \psi^x(c_t^x, M_{t+1}^x/P_t^x) \equiv \lambda (c_t^x)^\nu (M_{t+1}^x/P_t^x)^{1-\nu}, \quad \nu > 1, \lambda > 0, \tag{12}$$

denominated in units of c^x, where P_t^x is the dollar price of c^x at date t. Consumption of c^y involves a transaction cost of

$$\psi_t^y \equiv \psi^y(c_t^y, M_{t+1}^y/P_t^y) \equiv \zeta (c_t^y)^\xi (M_{t+1}^y/P_t^y)^{1-\xi}, \quad \xi > 1, \zeta > 0, \tag{13}$$

denominated in units of c^y, where P_t^y is the yen price of c^y at date t.[9]

[9] The timing in this model differs from the transaction-cost models of Feenstra (1986) and Marshall (1992), in that money provides transaction services in the period when it is acquired. However, money must be held until the following period, so losses in purchasing power due to inflation accrue in period $t+1$. This timing is imposed for tractability. With our timing, the only endogenous state variable affecting an individual agent's decisions is the agent's stock of wealth. If money provided transaction services only if acquired one period earlier, the agent's stock of money would represent a second endogenous state variable. The optimality conditions would then involve the derivatives of the (unknown) value function with respect to the money-wealth ratio. To solve such a model numerically would be extremely burdensome computationally.

The gross real return to asset i (measured in units of good x received in $t+1$ per unit of good x invested at date t) is denoted $R_{i,t+1}$. If S_t denotes the exchange rate (dollars/yen), the budget constraint for the representative agent in units of consumption good x is

$$c_t^x + \psi_t^x + \frac{S_t P_t^y}{P_t^x}(c_t^y + \psi_t^y) + \sum_{i=1}^{n} z_{i,t+1} + \frac{M_{t+1}^x + S_t M_{t+1}^y}{P_t^x} \leq W_t, \tag{14}$$

where the representative agent's wealth W_t satisfies:

$$W_t \equiv \frac{M_t^x + S_t M_t^y}{P_t^x} + \sum_{i=1}^{n} z_{i,t} R_{i,t}. \tag{15}$$

3.4. The equilibrium determination of exchange rates and asset returns

In order to derive equilibrium asset prices and exchange rates, we must solve the representative agent's decision problem in Eq. (11) subject to the budget constraint (14) and the definition of wealth (15) (in which we use transaction cost functions (12) and (13)). In addition, we must impose market clearing. The agent's optimal behavior is characterized by a set of Euler equations that involve the real return on optimally-invested aggregate wealth, which we denote R_t. (An explicit characterization of R_t can be found in Appendix B.) These equations also involve the real returns, inclusive of marginal transaction cost savings, from holding dollars and yen, defined as follows

$$R_{x,t+1} \equiv \left(\frac{P_t^x}{P_{t+1}^x}\right)\left(\frac{1}{1+\psi_{2t}^x}\right); \qquad R_{y,t+1} \equiv \left(\frac{S_{t+1} P_t^x}{S_t P_{t+1}^x}\right)\left(\frac{1}{1+\psi_{2t}^y}\right), \tag{16}$$

where $R_{x,t+1}$ ($R_{y,t+1}$) denotes the real return from holding dollars (yen), and where ψ_{it}^x (ψ_{it}^y) denotes the period t partial derivative of ψ_t^x (ψ_t^y) with respect to its ith argument. Both $R_{x,t+1}$ and $R_{y,t+1}$ are measured in units of good x received at $t+1$ per unit of good x invested at t.

The first-order conditions for the representative agent's optimal consumption, money holdings, and portfolio choices are the following[10]

$$\mathrm{E}_t\{I_A(Z_{t+1})[Z_{t+1}^{\alpha} - 1]\} = 0, \tag{17}$$

$$\mathrm{E}_t[I_A(Z_{t+1})Z_{t+1}^{\alpha} R_{t+1}^{-1} R_{i,t+1}] = \mathrm{E}_t[I_A(Z_{t+1})], \quad \forall i = x, y, 1, \ldots, n, \tag{18}$$

where

$$Z_{t+1} \equiv \left[\beta\left(\frac{c_{t+1}^x}{c_x^t}\right)^{\rho\delta - 1}\left(\frac{c_{t+1}^y}{c_t^y}\right)^{\rho(1-\delta)}\left(\frac{1+\psi_{1t}^x}{1+\psi_{1t+1}^x}\right)R_{t+1}\right]^{1/\rho}, \tag{19}$$

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[10] The derivation is a modification of the arguments in Epstein and Zin (1989), and is available upon request.

and

$$I_A(Z) \equiv \begin{cases} A & \text{if } Z \geq 1 \\ 1 & \text{if } Z < 1 \end{cases}. \tag{20}$$

Let v_t^x and v_t^y denote the consumption-velocities in countries x and y:

$$v_t^x \equiv \frac{c_t^x P_t^x}{M_{t+1}^x}; \qquad v_t^y \equiv \frac{c_t^y P_t^y}{M_{t+1}^y}. \tag{21}$$

The nominally risk-free continuously-compounded dollar- and yen-interest rates (denoted $i_t^{\$}$ and $i_t^{¥}$) are functions of the marginal transaction costs with respect to real balances:

$$i_t^{\$} = \ln\left(\frac{1}{1 + \psi_{2t}^x}\right); \qquad i_t^{¥} = \ln\left(\frac{1}{1 + \psi_{2t}^y}\right). \tag{22}$$

The exchange rate S_t is given by

$$S_t = \frac{P_t^x}{P_t^y}\left(\frac{1 + \psi_{1t}^x}{1 + \psi_{1t}^y}\right)\left(\frac{c_t^x}{c_t^y}\right)\left(\frac{1 - \delta}{\delta}\right). \tag{23}$$

Given Eqs. (22) and (23), the forward rate F_t can then be computed using covered interest parity.

4. Calibration and solution of the model

The endowments and money supplies of the two countries are exogenous. In this section we describe how we choose the parameters of the exogenous processes. We calibrate the money supply processes of the two countries to money supply data from the US and Japan. Calibration of the endowment processes is more problematic, since, in a multi-country world, there are no data corresponding precisely to the endowment constructs of the model. In an effort to capture realistic dynamics for consumption, we use two (admittedly imperfect) calibration procedures for the endowments. For the benchmark model, we calibrate the endowments of the two countries to consumption data from the US and Japan, as described in Appendix A. In Section 5.4, we consider an alternative approach in which the growth rates of the endowments are calibrated to the growth rates of industrial production in the US and Japan.

The growth rates of the four exogenous processes are assumed to follow a vector autoregression, which we approximate as a discrete Markov chain. A first-order VAR with conditionally homoskedastic errors fits the data well. In particular, the Akaike and Schwarz criteria and sequential likelihood ratio tests support the first-order specification. We find no evidence against normality or conditional homoskedasticity in the residuals from the first-order VAR. Only the residuals for the growth rate of Japanese consumption show marginal evidence of serial correlation.[11]

The four exogenous processes are approximated by a first-order Markov chain in which each variable can take four possible values, implying a state space with 256 possible values. The Markov chain is calibrated to the estimated VAR using the Gaussian quadrature method of Tauchen and Hussey (1991). The parameters of the first-order VAR implied by this Markov chain approximation are virtually indistinguishable from those of the estimated VAR.[12] This is evidence that the discrete approximation is unlikely to distort the economic implications of the model.

Given this exogenous process, the three unknown endogenous processes R_t, v_t^x, and v_t^y are found by solving the three Euler equations (17) and (18) (for $i = x$ and y) simultaneously. Since the state space is discrete, the Euler equations can be solved exactly for the 256 values of each endogenous variable. The only approximation is in the initial discretization of the driving processes. A detailed description of the solution procedure is in Appendix B. Given R_t, v_t^x, and v_t^y, all other endogenous variables are calculated from definitions and equilibrium conditions.

5. Implications of the model

In this section, we report results obtained from solving the model for a variety of parameters governing preferences. The quarterly subjective discount parameter β is fixed at $(0.96)^{0.25}$. The choice of δ (the weight on c^x in the current-period utility) is irrelevant, since we examine rates of depreciation, rather than levels of exchange rates. The parameters of the transaction cost functions (12) and (13) are chosen by fitting Eq. (22) to US (for ψ^x) and Japanese (for ψ^y) data, as described in Appendix A. Specifically, we set

$$\psi^x(c,m) = 0.0008\,c^{4.351}m^{1-4.351}; \qquad \psi^y(c,m) = 0.0166\,c^{2.109}m^{1-2.109}. \tag{24}$$

The remaining parameters are varied over the following grid: $A \in \{1.0, 0.85, 0.70, 0.55, 0.40, 0.25\}$, $\rho \in \{0.50, -0.33, -4.0, -9.0\}$. We experimented initially with

[11] See Bekaert et al. (1994, Table 2) for a detailed discussion of our estimated VAR, and of the specification tests for lag length, normality, conditional homoskedasticity, and residual serial correlation.

[12] All parameters of the Markov process VAR (including the elements of the covariance matrix decomposition) are within one-tenth of one standard error of the corresponding parameters in the estimated VAR. See Bekaert et al. (1994, Table 3) for a detailed description of this accuracy test for the Markov-chain approximation.

values of α between 0.5 and -9 and found that the choice of α had virtually no effect on the moments of interest. Consequently, we only report results for $\alpha = -1$. This corresponds to a coefficient of relative risk aversion of 2 in an economy with expected-utility preferences over timeless gambles.[13]

5.1. Implications for excess return predictability

We first discuss the ability of the model to replicate the predictability of excess returns documented in Section 2. We focus on three measures of predictability: the slope coefficient in the excess return regressions analogous to Eqs. (1), (3) and (5); the R^2, measured as the ratio of the variance of the expected excess return to the variance of the realized excess return; and the standard deviation of the expected excess return. All three statistics can be computed exactly given the discrete Markov chain driving process.

Consider the model's implications for the slope coefficients in the excess return regressions analogous to Eqs. (1), (3) and (5). The results are displayed in Tables 2–4 for the foreign exchange market and the dollar and yen discount bond markets, respectively. Table 5 displays the slope coefficient when the excess return to the aggregate wealth portfolio (which we interpret as an analogue to an unlevered equity portfolio) is regressed on the foreign exchange forward premium.

It is clear from these tables that the model cannot match the slope coefficients estimated from observed data. For no combination of parameters do the regression coefficients implied by the model come close to the magnitudes reported in Table 1. For example, for the foreign exchange market regression, the estimated slope coefficient in Table 1 (Panel A) is -4.016, with an estimated standard error of 0.766. The most negative slope coefficient implied by the model is -0.191, which is approximately five standard errors away from the estimated value. Similarly, the slope coefficients implied by the model for the term structure regressions analogous to Eq. (3) (reported in Tables 3 and 4) and the equity return regressions (reported in Table 5) are extremely small, and they are all more than 3.4 standard errors away from the corresponding estimates reported in Table 1.

The second measure of predictability is the model's R^2 as defined above. This theoretical R^2 cannot be observed in the data, but a lower bound is provided by the estimated R^2s reported in Table 1, Panel A. Whereas the R^2s in the data are

[13] Intuition for why the moments of interest are not sensitive to α can be found in Epstein and Zin (1990, pp. 393–397). They note that indifference curves over timeless gambles are kinked at the certainty equivalent in the case of first-order risk aversion. Indifference curves for various αs are tangent coming into the kink. Hence, for small gambles the choice of α is irrelevant. Epstein and Zin consequently work with $\alpha = 1$.

Table 2
Implications of the model for the foreign exchange market regression

		$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
	β_{rs}	− 0.007	− 0.012	− 0.068	− 0.097	0.038	− 0.191
	R^2	1.0×10^{-7}	2.1×10^{-5}	8.8×10^{-5}	0.00016	0.0003	0.0013
$\rho = 0.5$	$\sigma[rp_t]$	0.003	0.042	0.085	0.116	0.159	0.332
	$\sigma[E_t(\Delta s_{t+1})]$	0.228	0.229	0.226	0.230	0.274	0.370
	$\text{cov}[rp_t, E_t(\Delta s_{t+1})]$	− 0.0004	0.001	0.004	0.0087	0.0279	0.1017
	β_{rs}	− 0.007	− 0.023	− 0.057	− 0.107	− 0.035	− 0.044
	R^2	1.0×10^{-7}	1.8×10^{-5}	7.7×10^{-5}	0.00015	0.00029	0.0012
$\rho = -0.33$	$\sigma[rp_t]$	0.003	0.039	0.080	0.113	0.155	0.309
	$\sigma[E_t(\Delta s_{t+1})]$	0.236	0.231	0.229	0.228	0.269	0.363
	$\text{cov}[rp_t, E_t(\Delta s_{t+1})]$	− 0.0004	0.0003	0.003	0.008	0.026	0.094
	β_{rs}	− 0.001	− 0.003	− 0.006	− 0.021	− 0.009	0.057
	R^2	1.0×10^{-7}	2.1×10^{-5}	8.6×10^{-5}	0.00017	0.00032	0.0013
$\rho = -3$	$\sigma[rp_t]$	0.003	0.042	0.085	0.118	0.164	0.334
	$\sigma[E_t(\Delta s_{t+1})]$	0.485	0.472	0.461	0.443	0.438	0.509
	$\text{cov}[rp_t, E_t(\Delta s_{t+1})]$	− 0.0001	0.001	0.006	0.010	0.026	0.119
	β_{rs}	0.001	0.003	0.005	0.007	0.008	0.009
	R^2	1.7×10^{-7}	2.1×10^{-5}	8.2×10^{-5}	0.00015	0.00032	0.0014
$\rho = -9$	$\sigma[rp_t]$	0.004	0.044	0.088	0.120	0.173	0.356
	$\sigma[E_t(\Delta s_{t+1})]$	2.284	2.200	2.100	1.973	1.807	1.575
	$\text{cov}[rp_t, E_t(\Delta s_{t+1})]$	0.005	0.016	0.031	0.042	0.057	0.147

Notes: The logarithms of the dollar/yen spot and forward exchange rates are denoted s_t and f_t, and $rp_t = E_t(s_{t+1} - f_t)$. β_{rs} denotes the slope coefficient in the regression $s_{t+1} - f_t = \alpha_{rs} + \beta_{rs}(f_t - s_t) + \varepsilon_{t+1}$. $E_t(x_{t+1})$ denotes the expectation of x_{t+1} conditional on time t information, $\sigma[x_t]$ denotes the unconditional standard deviation of x_t, and $\text{cov}[x_t, y_t]$ denotes the unconditional covariance. $R^2 = \text{var}(E_t(s_{t+1} - f_t))/\text{var}(s_{t+1} - f_t)$. All moments reported are the exact population moments implied by the model at the indicated parameter specifications, given the Markov transition matrix for the exogenous process y_t. This transition matrix was computed using Gaussian quadrature from the estimated VAR, as described in Appendix B.

Table 3
Implications of the model for the dollar discount bond market regression

		$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
	$\beta^\$_{rb}$	-0.00001	-0.00022	-0.00026	0.00013	-0.00196	0.00084
	R^2	1.0×10^{-8}	7.7×10^{-6}	2.6×10^{-5}	5.4×10^{-5}	0.00014	0.00053
$\rho = 0.5$	$\sigma(rb^\$_t)$	0.00003	0.00067	0.00121	0.00172	0.00272	0.00512
	$E_t[\Delta v^\$_{t+1}]$	0.220	0.218	0.215	0.211	0.206	0.199
	$\text{cov}[rb^\$_t, E_t(\Delta v^\$_{t+1})]$	-5.0×10^{-7}	-1.0×10^{-5}	-1.0×10^{-5}	8.6×10^{-6}	-7.6×10^{-5}	-7.2×10^{-6}
	$\beta^\$_{rb}$	0.00005	0.00042	-0.00031	0.00046	0.00048	-0.00205
	R^2	5.0×10^{-8}	1.4×10^{-5}	5.1×10^{-5}	0.00012	0.00024	0.00087
$\rho = -0.33$	$\sigma(rp^\$_t)$	0.00005	0.00078	0.00146	0.00219	0.00297	0.00528
	$E_t[\Delta v^\$_{t+1}]$	0.230	0.226	0.221	0.214	0.206	0.192
	$\text{cov}[rb^\$_t, E_t(\Delta v^\$_{t+1})]$	3.0×10^{-6}	-2.2×10^{-5}	0.00013	2.6×10^{-5}	2.9×10^{-5}	-4.8×10^{-5}
	$\beta^\$_{rb}$	-0.00006	-0.00031	-0.00113	-0.00128	-0.00122	-0.00491
	R^2	4.0×10^{-8}	4.7×10^{-6}	2.2×10^{-5}	4.9×10^{-5}	0.00010	0.00034
$\rho = -3$	$\sigma(rb^\$_t)$	0.0002	0.0021	0.0044	0.0062	0.0083	0.0135
	$E_t[\Delta v^\$_{t+1}]$	1.109	1.074	1.030	0.977	0.910	0.805
	$\text{cov}[rb^\$_t), E_t(\Delta v^\$_{t+1})]$	-0.00007	-0.00036	-0.00118	-0.00119	-0.00095	0.00303
	$\beta^\$_{rb}$	-0.0001	-0.0006	-0.0012	-0.0018	-0.0025	-0.0047
	R^2	1.1×10^{-6}	1.9×10^{-5}	7.4×10^{-5}	0.0009	0.00045	0.00112
$\rho = -9$	$\sigma(rb^\$_t)$	0.006	0.024	0.045	0.067	0.093	0.123
	$E_t[\Delta v^\$_{t+1}]$	5.995	5.725	5.406	5.016	4.516	3.798
	$\text{cov}[rb^\$_t), E_t(\Delta v^\$_{t+1})]$	-0.0047	-0.0176	-0.0318	-0.0421	-0.0342	-0.0528

Notes: $h^\$_{t+1,2}$ denotes the continuously compounded one-period holding period return on two-period dollar discount bonds; $i^\$_t$ denotes the continuously compounded dollar spot interest rate; $\Delta v^\$_{t+1}$ denotes the rate of change in the logarithm of the price of one-period dollar bonds; and $rb^\$_t = E_t(h^\$_{t+1,2} - i^\$_t)$. $\beta^\$_{rb}$ denotes the slope coefficient in the regression $h^\$_{t+1,2} - i^\$_t = \alpha^\$_{rb} + \beta^\$_{rb}(fb^\$_t) + \varepsilon_{t+1}$. $R^2 = \text{var}(E_t(h^\$_{t+1,2} - i^\$_t))/\text{var}(h^\$_{t+1,2} - i^\$_t)$. See also the note to Table 2.

Table 4

Implications of the model for the yen discount bond market regression

		$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
$\rho = 0.5$	$\beta^{¥}_{rb}$	-0.00013	0.00002	-0.00044	-0.00052	0.00214	0.00169
	R^2	2.0×10^{-7}	5.0×10^{-5}	0.00012	0.00024	0.00054	0.00195
	$\sigma(rb^{¥}_t)$	0.00007	0.00091	0.00176	0.00247	0.00352	0.00658
	$\sigma[E_t(\Delta v^{¥}_{t+1})]$	0.249	0.247	0.243	0.238	0.230	0.223
	$\text{cov}[rb^{¥}_t, E_t(\Delta v^{¥}_{t+1})]$	-8.0×10^{-6}	2.0×10^{-6}	-2.3×10^{-5}	-2.3×10^{-5}	0.00013	0.00013
$\rho = -0.33$	$\beta^{¥}_{rb}$	-0.00021	-0.00024	-0.00095	-0.00118	0.00059	-0.00036
	R^2	3.5×10^{-7}	3.1×10^{-5}	0.00013	0.00027	0.00059	0.00193
	$\sigma(rb^{¥}_t)$	0.00012	0.00113	0.00226	0.00313	0.00442	0.00757
	$\sigma[E_t(\Delta v^{¥}_{t+1})]$	0.347	0.340	0.333	0.322	0.306	0.288
	$\text{cov}[rb^{¥}_t) E_t(\Delta v^{¥}_{t+1})]$	-2.0×10^{6}	-3.0×10^{-6}	-0.00010	-0.00011	0.00007	0.00003
$\rho = -3$	$\beta^{¥}_{rb}$	-0.00027	-0.00081	-0.00176	-0.00256	-0.00276	-0.00532
	R^2	3.2×10^{-7}	1.2×10^{-5}	4.8×10^{-5}	0.00010	0.00020	0.00067
	$\sigma(rb^{¥}_t)$	0.0004	0.0023	0.0045	0.0062	0.0080	0.0131
	$\sigma[E_t(\Delta v^{¥}_{t+1})]$	0.950	0.919	0.883	0.836	0.774	0.690
	$\text{cov}[rb^{¥}_t, E_t(\Delta v^{¥}_{t+1})]$	-0.00024	-0.00068	-0.00136	-0.00176	-0.00160	-0.00239
$\rho = -9$	$\beta^{¥}_{rb}$	-0.0003	-0.0009	-0.0017	-0.0028	-0.0039	-0.0065
	R^2	7.2×10^{-7}	1.4×10^{-5}	5.3×10^{-5}	0.00013	0.00029	0.00073
	$\sigma(rb^{¥}_t)$	0.036	0.013	0.024	0.034	0.046	0.0628
	$\sigma[E_t(\Delta v^{¥}_{t+1})]$	4.062	3.872	3.648	3.374	3.025	2.541
	$\text{cov}[rb^{¥}, E_t(\Delta v^{¥}_{t+1})]$	0.00422	-0.01396	-0.02262	-0.03069	-0.03398	-0.03876

Notes: $h^{¥}_{t+1,2}$ denotes the continuously compounded one-period holding period return on two-period yen discount bonds; $i^{¥}_t$ denotes the continuously compounded yen spot interest rate: $\Delta v^{¥}_{t+1}$ denotes the rate of change in the logarithm of the price of one-period yen bonds; and $rb^{¥}_t = E_t(h^{¥}_{t+1,2} - i^{¥}_t)$. $\beta^{¥}_{rb}$ denotes the slope coefficient in the regression $h^{¥}_{t+1,2} - i^{¥}_t = \alpha^{¥}_{rb} + \beta^{¥}_{rb}(fb^{¥}_t) + \varepsilon_{t+1}$. $R^2 = \text{var}(E_t(h^{\$}_{t+1,2} - i^{\$}_t))/\text{var}(E_t(h^{\$}_{t+1,2} - i^{\$}_t)$. See also the note to Table 2.

Table 5

Implications of the model for the excess dollar return on aggregate wealth

Panel A: Predictability of excess dollar return on aggregate wealth

		$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
$\rho = 0.5$	β_{rw}	-0.001	0.003	-0.015	-0.039	-0.034	-0.085
	R^2	1.0×10^{-8}	4.2×10^{-6}	2.1×10^{-5}	4.6×10^{-5}	0.00012	0.00042
	$\sigma[\mathrm{E}_t(r_{t+1} - i_t)]$	0.001	0.013	0.028	0.042	0.068	0.128
$\rho = -0.33$	β_{rw}	-0.001	-0.001	-0.004	-0.049	-0.040	0.006
	R^2	1.0×10^{-8}	3.6×10^{-6}	2.0×10^{-5}	4.6×10^{-5}	0.00011	0.00039
	$\sigma[\mathrm{E}_t(r_{t+1} - i_t)]$	0.001	0.012	0.028	0.042	0.067	0.123
$\rho = -3$	β_{rw}	0.000	0.002	0.002	-0.008	-0.020	0.025
	R^2	1.0×10^{-8}	3.1×10^{-6}	1.4×10^{-5}	3.4×10^{-5}	0.00010	0.00033
	$\sigma[\mathrm{E}_t(r_{t+1} - i_t)]$	0.001	0.012	0.025	0.039	0.068	0.122
$\rho = -9$	β_{rw}	0.001	0.003	0.006	0.010	0.009	0.009
	R^2	3.7×10^{-7}	8.1×10^{-6}	2.8×10^{-5}	7.5×10^{-5}	0.00016	0.00040
	$\sigma[\mathrm{E}_t(r_{t+1} - i_t)]$	0.006	0.028	0.051	0.082	0.115	0.175

Panel B: Mean of $(r_{t+1} - i_t)$

	$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
$\rho = 0.5$	0.060	0.227	0.430	0.688	0.999	1.510
$\rho = -0.33$	0.062	0.238	0.447	0.718	1.047	1.563
$\rho = -3$	0.077	0.315	0.597	0.940	1.356	1.991
$\rho = -9$	0.168	0.655	1.205	1.843	2.591	3.566

Notes: r_t denotes the continuously compounded dollar return to the aggregate wealth portfolio; i_t denotes the continuously compounded dollar spot interest rate. β_{rw} denotes the slope coefficient in the regression $r_{t+1} - i_t = \alpha_{rw} + \beta_{rw}(f_t - s_t) + \varepsilon_{t+1}$. $R^2 = \mathrm{var}(\mathrm{E}_t(r_{t+1} - i_t))/\mathrm{var}(r_{t+1} - i_t)$. See also the note to Table 2.

substantive, ranging between 1% and 22%, the corresponding R^2s in the model are negligible, all being less than 0.2%.

The third measure of the predictability of excess returns is the variability of the explained component of excess returns. As with the R^2 discussed above, a lower bound for this measure in the data is provided by the standard deviation of the fitted value of the excess return regressions reported in Table 1, Panel B. As with the previous two measures, the model is unable to reproduce the variability observed in the data. For example, the standard deviation of the fitted value of $s_{t+1} - f_t$ in Table 1 is 12.4%. The largest value of the standard deviation of $\mathrm{E}_t(s_{t+1} - f_t)$ from the model, reported in Table 2, is 0.356%, which is over thirty times too small. Analogously, the standard deviation of the fitted value of the excess world equity return in Table 1 is 10.9%. The largest standard deviation of $\mathrm{E}_t(r_{t+1} - i_t)$ from the model, reported in Table 5, is 0.175%, which is over sixty times too small. The standard deviations of the fitted values of the excess returns in the discount bond markets are 0.318% and 0.370% for the dollar and the yen markets, respectively. The maximum value of the standard deviations of the expected excess returns, reported in Tables 3 and 4, are 0.123% and 0.063% respectively.

These results are somewhat disappointing to those who favor risk-based explanations for the predictability of excess returns. The implications of first-order risk aversion for the slope coefficients are particularly puzzling. In all cases, setting $A = 1$ results in extremely small values for the slope coefficients. However, it is not generally true that increasing the amount of risk aversion (decreasing A) implies more negative slope coefficients. Furthermore, a large degree of risk aversion is not systematically associated with a particular sign of the regression coefficient. For example, the coefficients corresponding to $A = 0.40$ and $A = 0.25$ in Tables 2–5 are as likely to be positive as to be negative. Thus, even if it were assumed that agents in the economy display extreme risk aversion, it is not at all clear whether this would improve the performance of the model along this dimension.

To see why the model fails to replicate the observed slope coefficients, it is useful to return to the discussion of Section 2.4. In that section, we argued that substantial time-variation in risk premiums is necessary if a model is to match the patterns found in the data. Examination of Tables 2 through 5 reveals that the variances of the ex ante risk premiums are unambiguously increasing as the degree of first-order risk aversion increases. For foreign exchange, the standard deviation of the risk premium increases by a factor of 100 when A moves from 1 to 0.25. For discount bonds and the aggregate wealth portfolio, the standard deviation of the risk premium increases at least twenty-fold when A moves from 1 to 0.25. Similarly, the R^2's in all markets increase dramatically as first-order risk aversion is increased.

The reason why these dramatic increases in risk-premium volatility do not imply comparable increases in the magnitude of the slope coefficients in the prediction regressions is that these coefficients are functions of moments in addition to the variances of the risk premiums. As shown in Eq. (7), the slope coefficients also depend on the variances of the expected asset price changes and on the covariances between the expected changes in asset prices and the risk premiums. These moments are also affected by changes in the parameter governing first-order risk aversion. In particular, Tables 3 and 4 show that the variances of the expected changes in the prices of one-period discount bonds actually decrease unambiguously as A decreases. The variance of the expected change in the spot foreign exchange rate is not monotonic in A. As shown in Table 2, decreasing A from unity initially reduces this variance, while further reductions in A increase it. Increased first-order risk aversion also affects the covariances between the ex ante risk premiums and the expected changes in asset prices. In the foreign exchange market, decreasing A unambiguously increases this covariance. In the discount bond markets, the response of this covariance to increased risk aversion is not monotonic, and depends on the value of ρ. Thus, increasing first-order risk aversion affects all of the moments that enter the right-hand side of equation (7), and the corresponding equation for bond returns. The resulting effect on β_{rs} and β_{rb} is non-monotonic in A, and (as it turns out) small.

5.2. Implications for unconditional moments of endogenous variables

Our model also has implications for the unconditional mean equity premium and the unconditional standard deviations of financial variables, which provide additional dimensions to assess the model's performance. In Table 5, increasing the amount of first-order risk aversion dramatically increases the unconditional mean excess equity return. As A is reduced from 1 to 0.25, the mean equity risk premium increases by a factor of approximately 20. This increase is not sufficient to match the data as the largest mean equity premium generated by our model simulations is 3.5%. While this is substantially below the value of 8.4% estimated from our data set, the equity return data correspond to a levered portfolio, while the equity return computed in our model is unlevered. The results are comparable to those of Bonomo and Garcia (1993) for homoskedastic driving processes. These authors are able to increase the mean equity risk premium significantly by employing a richer driving process that incorporates regime switching.

Table 6 displays standard deviations implied by the model. In comparing Table 6 with Table 1, Panel B, notice that the magnitudes of the standard deviations in the model are almost always smaller than the corresponding statistics in the data. In particular, the standard deviation of currency depreciation is approximately 2.5 times higher in the data than in the model, and the standard deviation of the equity risk premium is approximately three times higher in the data than in the model. When $\rho = -9$, the standard

Table 6
Implications of the model for unconditional standard deviations

	$A = 1.0$	$A = 0.85$	$A = 0.70$	$A = 0.55$	$A = 0.40$	$A = 0.25$
Panel A: $(s_{t+1} - s_t)$						
$\rho = 0.5$	9.118	9.121	9.122	9.125	9.110	9.093
$\rho = -0.33$	9.166	9.121	9.120	9.125	9.111	9.081
$\rho = -3$	9.184	9.186	9.188	9.193	9.175	9.167
$\rho = -9$	10.066	10.026	9.978	9.921	9.833	9.702
Panel B: $(f_t - s_t)$						
$\rho = 0.5$	0.230	0.227	0.225	0.221	0.215	0.209
$\rho = -0.33$	0.237	0.233	0.228	0.223	0.212	0.198
$\rho = -3$	0.486	0.472	0.456	0.436	0.410	0.364
$\rho = -9$	2.282	2.193	2.087	1.955	1.784	1.521
Panel C: $(r_{t+1} - i_t)$						
$\rho = 0.5$	6.152	6.157	6.165	6.170	6.178	6.188
$\rho = -0.33$	6.230	6.235	6.240	6.249	6.255	6.254
$\rho = -3$	6.802	6.791	6.782	6.765	6.741	6.714
$\rho = -9$	10.002	9.850	9.672	9.458	9.163	8.746
Panel D: $(h^{\$}_{t+1,2} - i^{\$}_t)$						
$\rho = 0.5$	0.244	0.242	0.239	0.235	0.230	0.222
$\rho = -0.33$	0.214	0.210	0.205	0.200	0.193	0.179
$\rho = -3$	0.998	0.966	0.928	0.882	0.822	0.728
$\rho = -9$	5.759	5.503	5.199	4.827	4.352	3.699
Panel E: $(h^{¥}_{t+1,2} - i^{¥}_t)$						
$\rho = 0.5$	0.166	0.164	0.162	0.159	0.154	0.149
$\rho = -0.33$	0.206	0.202	0.197	0.191	0.182	0.172
$\rho = -3$	0.697	0.674	0.647	0.612	0.567	0.505
$\rho = -9$	3.685	3.510	3.304	3.054	2.735	2.292

Notes: See Tables 2–5.

deviation of the forward premium in the model is only 50% lower than that in the data; for the other values of ρ, the variability of the forward premium is almost an order of magnitude too low.

Although the model underpredicts the variability of both expected and realized excess returns, the parameterizations of the model that generate the largest variances of expected rates of return tend to overpredict the variances of the forward premiums in the discount bond markets. For example, with $\rho = -9$ and $A = 0.25$, the standard deviations of the forward premiums in the dollar and yen discount bond market are 3.81% and 2.56%, compared to 0.71% and 0.83% in Table 1, Panel B.

The source of this problem is as follows. In order to generate high volatility in excess returns, the model must generate high volatility in the conditional second moments of the IMRSs. Unfortunately, parameterizations of the model which do this also imply highly volatile spot interest rates. A similar problem has been noted in a closed-economy model by Heaton (1995). Consequently, one challenge for this class of models is to accommodate highly variable expected and realized excess returns on risky assets while keeping short-term interest rates relatively non-volatile.

5.3. *Isolating the effects of real and monetary shocks*

This model incorporates both real and monetary exogenous shocks. To help disentangle the effects of these two types of disturbances, we re-solve the model, first with only shocks to output growths, and second with only shocks to the money supplies. In the first exercise (the 'real model'), we set the growth rate of the money supplies in the two countries equal to their sample means in the data. In the second exercise (the 'monetary model'), we set the endowment growth rates in the two countries equal to their sample means. We conduct these exercises only for the extreme values of the preference parameters: $\rho \in \{0.5, -9\}$, $A \in \{1.0, 0.25\}$. In the monetary model there is virtually no real uncertainty.[14] (Formally, the process $\{Z_t\}$, defined in Eq. (19), is virtually constant.) As a result, the implications of this model are invariant to the value of parameter A.[15]

Table 7, Panel A, gives the results of these exercises for predictability of excess returns, as measured by the slope coefficients in regressions (1), (3) and (5), and by the standard deviation of the risk premiums. (For convenience, we also display the results for the full model, previously displayed in Tables 2–5.) In the

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[14] The only effect of monetary uncertainty on real allocations is through the level of the transaction cost. While marginal transactions costs can fluctuate significantly, the level of the transaction cost is always small. Fluctuations in this level have negligible impact on the quantity of goods consumed.

[15] As with the full model, we set $\alpha = -1$ in both the real model and the monetary model, and we calibrate the transaction cost functions as in Eq. (24). In the real model, the exogenous driving process is a bivariate vector including the growth rates of outputs in the two countries. We calibrate this exogenous process analogously to the full model. That is, we first estimate a bivariate first-order VAR including the growth rates of aggregate consumption in the US and Japan. This VAR is then approximated as a first-order Markov chain using the Gaussian quadrature method of Tauchen and Hussey (1991). Four discrete states are assumed for each variable. The exogenous process for the monetary model is calibrated analogously, using a VAR that includes the money growth rates in the two countries.

Table 7
Implications of the model with only real or only monetary shocks

Panel A: Predictability of risk premiums

		Monetary model	Real model		Full model	
			$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Foreign exchange market						
$\rho = 0.5$	β_{rs}	−0.015	−0.000	−0.002	−0.007	−0.191
	$\sigma[rp_t]$	0.002	0.000	0.001	0.003	0.332
$\rho = -9$	β_{rs}	−0.005	0.000	−0.001	0.001	0.009
	$\sigma[rp_t]$	0.002	0.002	0.060	0.004	0.356
Dollar discount bond market						
$\rho = 0.5$	$\beta_{rb\$}$	−0.000	0.000	−0.001	−0.000	0.001
	$\sigma(rb_t^{\$})$	0.000	0.000	0.001	0.000	0.005
$\rho = -9$	$\beta_{rb\$}$	−0.000	0.000	0.001	−0.000	−0.005
	$\sigma(rb_t^{\$})$	0.000	0.003	0.083	0.006	0.123
Yen discount bond market						
	$\beta_{rb¥}$	0.000	0.000	−0.000	−0.000	0.002
$\rho = 0.5$	$\sigma(rb_t^{¥})$	0.000	0.000	0.001	0.000	0.007
	$\beta_{rb¥}$	0.000	0.000	0.000	−0.000	−0.007
$\rho = -9$	$\sigma(rb_t^{¥})$	0.000	0.001	0.042	0.036	0.063
Equity market						
$\rho = 0.5$	β_{rw}	0.000	−0.000	−0.005	−0.001	−0.085
	$\sigma[E_t(r_{t+1} - i_t)]$	0.000	0.000	0.002	0.001	0.128
$\rho = -9$	β_{rw}	0.000	0.000	−0.003	0.001	0.009
	$\sigma[E_t(r_{t+1} - i_t)]$	0.000	0.003	0.104	0.006	0.175

Panel B: Unconditional moments of endogenous variables of interest

	Monetary model	Real model		Full model	
		$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Mean of $(r_{t+1} - i_t)$					
$\rho = 0.5$	0.047	−0.000	−0.056	0.060	1.510
$\rho = -9$	0.044	0.057	2.038	0.168	3.566
Standard deviation of $(s_{t+1} - s_t)$					
$\rho = 0.5$	9.179	0.065	0.069	9.118	9.093
$\rho = -9$	9.182	3.624	2.517	10.066	9.702

Table 7 (continued)

		Real model		Full model	
Monetary model		$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Standard deviation of $(f_t - s_t)$					
$\rho = 0.5$	0.090	0.043	0.046	0.203	0.209
$\rho = -9$	0.321	2.447	1.690	2.282	1.521
Standard deviation of $(r_{t+1} - i_t)$					
$\rho = 0.5$	6.111	0.096	0.101	6.152	6.188
$\rho = -9$	5.902	4.943	3.498	10.002	8.746
Standard deviation of $(h^{\$}_{t+1,2} - i^{\$}_t)$					
$\rho = 0.5$	0.125	0.091	0.097	0.244	0.222
$\rho = -9$	0.377	6.048	4.074	5.759	3.699
Standard deviation of $(h^{¥}_{t+1,2} - i^{¥}_t)$					
$\rho = 0.5$	0.143	0.049	0.052	0.166	0.149
$\rho = -9$	0.512	3.662	2.517	3.685	2.292

Notes: All moments reported are the exact population moments implied by the model at the indicated parameter specifications, given the Markov transition matrix for the exogenous process g_t. The Monetary Model sets the growth rates of US output to 1.00446, and the growth rate of Japanese output equal to 1.00916. The law of motion for money growth in the two countries is the Markov transition matrix computed using Gaussian quadrature from a bivariate VAR estimated using money growths in the US and Japan, as described in Appendix B. The real model sets the growth rate of the US money supply to 1.01572, and the growth rate of the Japanese money supply to 1.01667. The law of motion for output growth is the Markov transition matrix estimated from US and Japanese consumption data, as described in Appendix B. The logarithms of the dollar/yen spot and forward exchange rates are denoted s_t and f_t, and $rp_t = \mathrm{E}_t(s_{t+1} - f_t)$. β_{rs} denotes the slope coefficient in the regression $s_{t+1} - f_t = \alpha_{rs} + \beta_{rs}(f_t - s_t) + \varepsilon_{t+1}$. $h^{\$}_{t+1,2}(h^{¥}_{t+1,2})$ denotes the continuously compounded one-period holding period return on two-period dollar (yen) discount bonds; $i^{\$}_t$ $(i^{¥}_t)$ denotes the continuously compounded dollar (yen) spot interest rate. $rb^{\$}_t = \mathrm{E}_t(h^{\$}_{t+1,2} - i^{\$}_t)$; $\beta_{rb\$}$ denotes the slope coefficient in the regression $h^{\$}_{t+1,2} - i^{\$}_t = \alpha^{\$}_{rb} + \beta^{\$}_{rb}(fb^{\$}_t) + \varepsilon_{t+1}$. Similarly, $rb^{¥}_t = \mathrm{E}_t(h^{¥}_{t+1,2} - i^{¥}_t)$. $\beta_{rb¥}$ denotes the slope coefficient in the regression $h^{¥}_{t+1,2} - i^{¥}_t = \alpha^{¥}_{rb} + \beta^{¥}_{rb}(fb^{¥}_t) + \varepsilon_{t+1}$. r_t denotes the continuously compounded dollar return to the aggregate wealth portfolio; i_t denotes the continuously compounded dollar spot interest rate. β_{rw} denotes the slope coefficient in the regression $r_{t+1} - i_t = \alpha_{rw} + \beta_{rw}(f_t - s_t) + \varepsilon_{t+1}$.

Table 8
Implications of the model with production data

Panel A: Predictability of risk premiums

		Production data		Consumption data	
		$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Foreign exchange market					
	β_{rs}	− 0.006	− 0.152	− 0.007	− 0.191
$\rho = 0.5$	$\sigma[rp_t]$	0.003	0.329	0.003	0.332
	β_{rs}	− 0.001	− 0.015	− 0.001	0.057
$\rho = -3$	$\sigma[rp_t]$	0.003	0.340	0.003	0.334
Dollar discount bond market					
	$\beta_{rb\$}$	− 0.000	0.001	− 0.000	0.001
$\rho = 0.5$	$\sigma(rb_t^{\$})$	0.000	0.007	0.000	0.005
	$\beta_{rb\$}$	0.000	0.015	− 0.000	− 0.005
$\rho = -3$	$\sigma(rb_t^{\$})$	0.002	0.031	0.000	0.014
Yen discount bond market					
	$\beta_{re¥}$	0.000	− 0.002	− 0.000	0.002
$\rho = 0.5$	$\sigma(rb_t^{¥})$	0.000	0.005	0.000	0.007
	$\beta_{rb¥}$	0.000	0.010	− 0.000	− 0.005
$\rho = -3$	$\sigma(rb_t^{¥})$	0.001	0.011	0.000	0.013
Equity market					
$\rho = 0.5$	β_{rw}	− 0.001	− 0.101	− 0.001	− 0.085
	$\sigma[E_t(r_{t+1} - i_t)]$	0.001	0.254	0.001	0.128
$\rho = -3$	β_{rw}	− 0.001	− 0.001	0.000	0.251
	$\sigma[E_t(r_{t+1} - i_t)]$	0.002	0.272	0.001	0.122

Panel B: Unconditional moments of endogenous variables of interest

	Production data		Consumption data	
	$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Mean of $(r_{t+1} - i_t)$				
$\rho = 0.5$	0.052	0.361	0.060	1.510
$\rho = -3$	0.086	0.929	0.077	1.991
Standard deviation of $(s_{t+1} - s_t)$				
$\rho = 0.5$	9.301	9.274	9.118	9.093
$\rho = -3$	9.463	9.413	9.184	9.167

Table 8 (continued)

	Production data		Consumption data	
	$A = 1.0$	$A = 0.25$	$A = 1.0$	$A = 0.25$
Standard deviation of $(r_{t+1} - i_t)$				
$\rho = 0.5$	6.022	5.992	6.152	6.188
$\rho = -3$	6.757	6.494	6.802	6.714
Standard deviation of $(h^{\$}_{t+1,2} - i^{\$}_t)$				
$\rho = 0.5$	0.222	0.245	0.244	0.222
$\rho = -3$	2.490	1.178	0.998	0.728
Standard deviation of $(h^{¥}_{t+1,2} - i^{¥}_t)$				
$\rho = 0.5$	0.200	0.209	0.166	0.149
$\rho = -3$	1.240	0.539	0.697	0.505

Notes: All moments reported are the exact population moments implied by the model at the indicated parameter specifications, given the Markov transition matrix for the exogenous process g_t. In the columns labelled 'Production data', the growth rates of the endowments in the two countries are calibrated to the growth rates of the Industrial Production index for nondurables plus services in the US and Japan, respectively. In the columns labelled 'Consumption data', the growth rates of the endowments are calibrated to the growth rates of aggregate consumption Industrial Production index for nondurables plus services in the US and Japan, respectively. The law of motion for g_t is the Markov transition matrix computed using Gaussian quadrature from a four-variable VAR, as described in Appendix B. The logarithms of the dollar/yen spot and forward exchange rates are denoted s_t and f_t, and $rp_t = \mathrm{E}_t(s_{t+1} - f_t)$. β_{rs} denotes the slope coefficient in the regression $s_{t+1} - f_t = \alpha_{\mathrm{rs}} + \beta_{\mathrm{rs}}(f_t - s_t) + \varepsilon_{t+1}$. $h^{\$}_{t+1,2}$ $(h^{¥}_{t+1,2})$ denotes the continuously compounded one-period holding period return on two-period dollar (yen) discount bonds; $i^{\$}_t$ $(i^{¥}_t)$ denotes the continuously compounded dollar (yen) spot interest rate. $rb^{\$}_t = \mathrm{E}_t(h^{\$}_{t+1,2} - i^{\$}_t)$; $\beta_{\mathrm{rb\$}}$ denotes the slope coefficient in the regression $h^{\$}_{t+1,2} - i^{\$}_t = \alpha^{\$}_{\mathrm{rb}} + \beta^{\$}_{\mathrm{rb}}(fb^{\$}_t) + \varepsilon_{t+1}$. Similarly, $rb^{¥}_t = \mathrm{E}^{¥}_t(h^{¥}_{t+1,2} - i^{¥}_t)$. $\beta_{\mathrm{rb¥}}$ denotes the slope coefficient in the regression $h^{¥}_{t+1,2} - i^{¥}_t = \alpha^{¥}_{\mathrm{rb}} + \beta^{¥}_{\mathrm{rb}}(fb^{¥}_t) + \varepsilon_{t+1}$. r_t denotes the continuously compounded dollar return to the aggregate wealth portfolio; i_t denotes the continuously compounded dollar spot interest rate. β_{rw} denotes the slope coefficient in the regression $r_{t+1} - i_t = \alpha_{\mathrm{rw}} + \beta_{\mathrm{rw}}(f_t - s_t) + \varepsilon_{t+1}$.

monetary model, risk premiums are virtually constant. This should come as no surprise. When the only source of uncertainty is the growth rates of money supplies, the consumption allocations are known with (almost) perfect certainty.[16] There is somewhat more variability of risk premiums in the real model. As with the full model, increasing risk aversion (by reducing A) or decreasing the

[16] Even if consumption allocations were known with perfect certainty, the variances of these nominal risk premiums would not identically equal zero because the (stochastic) inflation rate is incorporated into the nominal asset pricing operator. See Engel (1992).

elasticity of intertemporal substitution (by making ρ more negative) does increase the standard deviation of risk premiums. However, these standard deviations are generally smaller than in the full model, and fall well short of the standard deviations estimated from the data in Table 1, Panel B. Regressions (1), (3), and (5) do yield negative slope coefficients for most parameter values, but, again, the magnitudes are smaller than in the full model, and are trivial compared to the estimates from the data in Table 1, Panel A.

Table 7, Panel B, displays the mean equity premium and the standard deviations of the dollar depreciation rate, the forward premium in the foreign exchange market, and the excess return in bond and equity markets. (We also display the corresponding results for the full model, from Table 5, Panel B, and from Table 6.) Unlike the full model, increasing risk-aversion in the real model only increases the mean equity premium when the intertemporal substitution parameter ρ is low. When, $\rho = -9$, increasing risk aversion by reducing A from 1 to 0.25 increases the mean annual equity premium from 0.057% to over 2%. However, with $\rho = 0.5$, this mean equity premium is *negative* (although small) in the real model, becoming more negative as first-order risk-aversion is increased. To understand this surprising result, note that we are reporting nominal equity premiums. Negative equity premiums can arise if the IMRS is positively correlated with the equity return. When $\rho = 0.5$, the real model generates negative correlations between the real IMRS and the real equity return (implying a positive real equity premium) while generating positive correlations between the nominal IMRS and the nominal equity return (implying a negative nominal equity premium). This unusual state of affairs arises because the nominal equity return has strong negative correlation with the inflation rate.

The remaining rows of Table 7, Panel B, give unconditional standard deviations corresponding to those displayed in Table 1, Panel B, for the observed data. With $\rho = 0.5$, the standard deviations in the real model are always smaller than in the monetary model. In particular, the standard deviation of exchange rate changes and excess equity returns are more than an order of magnitude larger in the monetary model than in the real model. Increasing the elasticity-of-substitution parameter ρ to -9 dramatically increases the standard deviations implied by the real model for these variables. Notice that the excess returns in the bond markets are now more variable than the data. (See Table 1, Panel B.) The model still cannot generate sufficient variability in equity-market excess returns in the equity markets, forward premiums, and the yen/dollar exchange rate to match the data.

From these experiments, we conclude that most of the variation in exchange rates is due to monetary shocks, rather than real output shocks. However, risk premiums reflect the response of agents in the economy to real shocks. Not surprisingly, the main effect of increased curvature in the utility function (whether through ρ or A) is to magnify the impact of these real shocks. For these reasons, a model of risk premiums in foreign exchange markets must incorporate both monetary and real output uncertainty.

5.4. *Robustness to alternative measures of output*

The results presented thus far use aggregate consumption data from the US and Japan as the proxy for the exogenous endowment process in the two countries. While this follows common practice in the literature, (see, for example, Macklem (1991), Canova and Marrinan (1993), Bekaert (1996)), it is not entirely satisfactory, since consumption data include consumption imported from countries other than the US and Japan, and excludes goods produced in the home country but exported elsewhere. Ideally, we would like to measure the endowment for the US (Japan) as the consumption of US-produced (Japanese-produced) goods by American and Japanese consumers. Such data are unavailable. In an effort to determine the robustness of our results to alternative proxies for the endowment processes, we re-solve the model with endowment growths measured by the growth rate of the industrial production indices for consumer nondurables in the US and Japan.[17] That is, we calibrate the Markov chain for the exogenous processes to a VAR that is estimated using these alternative measures of endowment growths.

The results of this exercise are displayed in Table 8. (For convenience, we include the corresponding results from the model calibrated to consumption data, as reported in Tables 2–6.) In solving the nonlinear equation system, we encountered severe conditioning problems when we set $\rho = -9$. As a result, we only report results for $\rho = 0.5$ and -3. The results do not differ substantially from those in Tables 2–6. The alternative measure of output implies somewhat more variability in equity-market risk premiums, although the mean risk premiums are somewhat lower. There is slightly more variability in excess bond returns and in the foreign-exchange forward premium. In no case is the substantive inference changed. We still find that first-order risk aversion marginally increases excess return predictability in foreign exchange, bond, and equity markets, but this effect falls far short of what is needed to explain observed data.

6. On the success of Epstein and Zin (1991)

Epstein and Zin (1991) are unable to reject the overidentifying restrictions implied by their single-country model with preferences incorporating first-order

[17] The industrial production index for consumer nondurables for the US is compiled by the Board of Governors of the Federal Reserve System. The corresponding index for Japan is from the OECD Main Economic Indicators. As with the previous results, we use monthly data from 1974:4 to 1990:1.

risk aversion, which suggests considerable support for this approach to asset pricing. Our approach is less successful. How can we explain the differences in findings?

According to the Euler equation (18), the implications of these models for asset returns are summarized in the behavior of the asset pricing operator

$$\frac{I_A(Z_{t+1})}{\mathrm{E}_t\,[I_A(Z_{t+1})]}\,Z_{t+1}^{\alpha}R_{t+1}^{-1}.$$

This operator is a function of R_{t+1}, the return to the aggregate wealth portfolio. Euler equation estimation requires an observable analogue to this asset pricing operator, and Epstein and Zin use the return on a value-weighted portfolio of equities as their empirical measure of R_{t+1}. This procedure is clearly subject to Roll's (1977) critique, a point acknowledged by Epstein and Zin. Furthermore, with this approach, the empirical asset pricing operator is a function of the returns on the equity assets being priced. The operator partially inherits the statistical properties of observed equity returns, so it has less difficulty replicating the behavior of observed excess equity returns. In contrast, we derive R_{t+1} by explicitly solving the model's equilibrium as a function of the growth rates of output and money in the two countries. Nowhere do we use data on asset returns in deriving the asset pricing operator. To ask the pricing operator, derived in this way, to replicate the stochastic properties of equity returns is a much tougher test of the model than the Epstein–Zin procedure. It is not surprising that we find more evidence against the model.

7. Conclusions

In this paper, we ask whether high levels of risk aversion can explain the observed predictability of excess returns within the context of a frictionless, representative agent model. We assume that agents' preferences display first-order risk aversion. This preference specification implies that agents respond more strongly to consumption risk than would be the case under conventional Von Neumann-Morgenstern preferences. Yet, even this more extreme form of risk aversion can explain only a small fraction of the predictability of excess returns found in the data. Furthermore, we find that the slope coefficients in equations predicting excess returns do not increase monotonically with increased risk aversion. The level of risk aversion affects not only the variability of risk premiums, but also the second moments of other endogenous variables which affect predictability. The resulting implications for the signs and magnitudes of these slope coefficients are ambiguous.

Taken together, the results of this paper suggest that the predictability of a set of asset market excess returns cannot be fully explained simply by modifying preference assumptions. A more promising approach may be to abandon the assumption that the empirical distribution in the data set is a good proxy for agents' subjective distribution over future variables. Rational optimizing models that do not impose this assumption include learning models, models with peso-problems, and some models with regime switching. It is hoped that these alternative approaches will have more success in explaining excess-return predictability than approaches based solely on modelling agents' aversion to consumption risk.

Furthermore, there are several important issues in modelling multi-country economies that our approach does not address. Characterizing consumption goods as either 'US' or 'Japanese' is clearly simplistic, since there are many traded goods (food, automobiles) that are produced and consumed in both countries. We also assume that US and Japanese consumers face the same transactions cost function when purchasing Japanese goods. This simplification ignores potentially important frictions such as shipping costs, tariff and non-tariff trade barriers, and the costs and risks associated with international payments systems. Finally, the assumption that consumers in all countries have identical preferences is itself open to question. A direction for future research is to construct multi-country models that distinguish the preferences and institutional constraints associated with individual countries, as well as the frictions associated with international trade and capital flows. Such a model would lose the analytical tractability of the 'world-wide' representative agent, but may generate more realistic co-movements between quantity variables and asset returns in a multi-country economy.

Appendix A: Description of data

Monthly data on three-month and six-month Eurodollar and Euroyen interest rates are from the Harris Bank database at the University of Chicago. Monthly exchange rates are taken from Citicorp Database Services daily bid and ask rates and are described in detail in Bekaert and Hodrick (1993).

The US and Japanese money supplies are quarterly M1 series from International Financial Statistics (IFS Series 34). Growth rates are deseasonalized by regressing on four dummies. The consumption data are Nondurables and Services from the OECD Quarterly National Accounts. The Japanese data include the Semi-durables category, as this category is included in the US Nondurables series. Per capita data were derived by using linear interpolations from annual population series (IFS Series 99z).

The transaction cost technology parameters are considered part of the exogenous environment and are calibrated from the model's implications for money demand. Eqs. (22) imply linear relationships between the logarithms of current dollar and yen velocities of circulation and the logarithms of the respective interest rate divided by one plus the interest rate. The calibration is done by linear regression using quarterly Eurocurrency interest data and velocity series computed using nominal GDPs, which are taken from OECD Quarterly National Accounts. GDP velocity is used because it implies more reasonable parameters for the transaction cost function than consumption velocity. The use of GDP velocity can be justified because money in actual economies intermediates many more transactions than just consumption. See Marshall (1992) for a fuller discussion.

Appendix B: Solution procedure

We numerically solve the Euler equations (17) and (18) for the endogenous variables v_t^x, v_t^y, (defined in Eqs. (21)) and R_{t+1} (the return to the aggregate wealth portfolio). We use a finite-state Markov chain to approximate the exogenous driving process as in Tauchen and Hussey (1991), and we solve the model exactly for this approximate driving process. Here, we describe the solution procedure in some detail.

Let e_t^x denote the total output of good x at date t, let e_t^y denote the output of good y at time t, and let M_{t+1}^x and M_{t+1}^y denote the supplies of dollars and yen respectively, available for use in mediating transactions at time t. (These money stocks are dated $t+1$ because it is assumed that the loss in value from inflation accrues to the agent in $t+1$.) Let g_t denote the vector of growth rates of outputs and money supplies in the two countries:

$$g_t \equiv \left\{\frac{e_t^x}{e_{t-1}^x}, \frac{e_t^y}{e_{t-1}^y}, \frac{M_{t+1}^x}{M_t^x}, \frac{M_{t+1}^y}{M_t^y}\right\}.$$

It is assumed that $\{e_t^x, e_t^y, M_{t+1}^x, M_{t+1}^y\}$ is an exogenous process whose law of motion is known.

First, we show how Eqs. (17) and (18) can be written in terms of g_t and the three endogenous processes $\{v_t^x, v_t^y, R_{t+1}\}$. Using (12), (13), and the requirement that, in equilibrium, the output of each good must either be consumed or used as transaction costs,

$$e_t^j = c_t^j + \psi^j(c_t^j, (M_{t+1}^j/P_t^j)), \quad j = x, y,$$

we can write consumption growths, marginal transaction costs, and inflation rates as functions of $\{g_t, v_t^x, v_t^y, R_{t+1}\}$:

$$\frac{c_{t+1}^x}{c_t^x} = \frac{e_{t+1}^x}{e_t^x}\left[\frac{1+\lambda(v_t^x)^{v-1}}{1+\lambda(v_{t+1}^x)^{v-1}}\right], \tag{B.1}$$

$$\frac{c_{t+1}^y}{c_t^y} = \frac{e_{t+1}^y}{e_t^y}\left[\frac{1+\zeta(v_t^y)^{\xi-1}}{1+\zeta(v_{t+1}^y)^{\xi-1}}\right], \tag{B.2}$$

$$\psi^x_{1t} = \lambda \nu (v^x_t)^{\nu - 1}, \tag{B.3}$$

$$\psi^y_{1t} = \zeta \xi (v^y_t)^{\xi - 1}, \tag{B.4}$$

$$\psi^x_{2t} = \lambda (1 - \nu)(v^x_t)^{\nu}, \tag{B.5}$$

$$\psi^y_{2t} = \zeta (1 - \xi)(v^y_t)^{\xi}, \tag{B.6}$$

$$\frac{P^x_{t+1}}{P^x_t} = \frac{v^x_{t+1}}{v^x_t} \frac{c^x_t}{c^x_{t+1}} \frac{M^x_{t+2}}{M^x_{t+1}}, \tag{B.7}$$

$$\frac{P^y_{t+1}}{P^y_t} = \frac{v^y_{t+1}}{v^y_t} \frac{c^y_t}{c^y_{t+1}} \frac{M^y_{t+2}}{M^y_{t+1}}. \tag{B.8}$$

The next step is to formally characterize R_{t+1}, the return to the aggregate wealth portfolio. Since we define the return to money inclusive of marginal transaction costs, $\psi^x_{2,t}$ and $\psi^y_{2,t}$, we must incorporate these marginal transaction costs into the definition of the portfolio weights for the aggregate wealth portfolio. Formally, let

$$\hat{W}_t \equiv W_t - \left[c^x_t + \psi^x_t + S_t \frac{P^y_t}{P^x_t} (c^y_t + \psi^y_t) \right] + \frac{M^x_{t+1}}{P^x_t} \psi^x_{2t} + S_t \frac{M^y_{t+1}}{P^x_t} \psi^y_{2t}, \tag{B.9}$$

where $\hat{W}_t$ denotes wealth available for asset purchases at time t, adjusted for marginal transaction costs. The portfolio weights on the aggregate wealth portfolio are defined in terms of $\hat{W}_t$. Let $w_{x,t+1}$ and $w_{y,t+1}$ denote the portfolio weight on M^x_{t+1} and M^y_{t+1}, respectively:

$$w_{x,t+1} \equiv \left[\frac{M^x_{t+1}}{P^x_t} (1 + \psi^x_{2t}) \right] / \hat{W}_t, \tag{B.10}$$

$$w_{y,t+1} \equiv \left[S_t \frac{M^y_{t+1}}{P^x_t} (1 + \psi^y_{2t}) \right] / \hat{W}_t. \tag{B.11}$$

Let $w_{i,t+1}$ denote the portfolio weight on asset i:

$$w_{i,t+1} \equiv \frac{z_{i,t+1}}{\hat{W}_t}, \quad i = 1, \ldots, n. \tag{B.12}$$

Note that the weights sum to unity:

$$\sum_{i=1}^{n} w_{i,t+1} + w_{x,t+1} + w_{y,t+1} = 1. \tag{B.13}$$

The return to the aggregate wealth portfolio is defined as follows

$$R_{t+1} \equiv \sum_{i=1}^{n} w_{i,t+1}R_{i,t+1} + w_{x,t+1}R_{x,t+1} + w_{y,t+1}R_{y,t+1}, \tag{B.14}$$

where $R_{x,t+1}$ and $R_{y,t+1}$ are defined in Eq. (16). Aggregate wealth evolves according to

$$W_{t+1} = \hat{W}_t R_{t+1}. \tag{B.15}$$

In a single-good nonmonetary model, the market return can be expressed as a function of the wealth/consumption ratio and the growth rate of consumption. It is convenient to express R_{t+1} in a similar way. To do so, define $\bar{c}_t \equiv W_t - \hat{W}_t$, and let the 'wealth/consumption ratio' $W_t/\bar{c}_t$ be denoted wc_t. Eq. (B.15) then implie

$$R_{t+1} = \frac{wc_{t+1}(\bar{c}_{t+1}/\bar{c}_t)}{wc_t - 1}. \tag{B.16}$$

The transaction cost functions ψ^x and ψ^y are homogeneous of degree one, so we use Euler's theorem, along with Eq. (23), to write

$$\left(\frac{\bar{c}_{t+1}}{\bar{c}_t}\right) = \left(\frac{c^x_{t+1}}{c^x_t}\right)\left(\frac{1+\psi^x_{1,t+1}}{1+\psi^x_{1,t}}\right). \tag{B.17}$$

By using Eqs. (B.1)-(B.8) and Eq. (B.17) in Eqs. (16), (19), and (B.16), we can write the endogenous processes $R_{x,t+1}$, $R_{y,t+1}$, Z_{t+1}, and R_{t+1} as functions of $\{g_{t+1}, v^x_t, v^x_{t+1}, v^y_t, v^y_{t+1}, wc_t, wc_{t+1}\}$. It follows that the three-equation system consisting of Eqs. (17) and (18) with $i = x$ and $i = y$, can be expressed in terms of $\{g_{t+1}, v^x_t, v^x_{t+1}, v^y_t, v^y_{t+1}, wc_t, wc_{t+1}\}$. Let this three-equation system be denoted

$$\mathrm{E}_t\left[f(g_{t+1}, v^x_{t+1}, v^x_t, v^y_{t+1}, v^y_t, wc_{t+1}, wc_t)\right] = 0, \tag{B.18}$$

where f is a known function.

We must find a stochastic process $\{v^x_t, v^y_t, wc_t\}$ which satisfies Eq. (B.18) for the given g_t process. As in Tauchen and Hussey (1991), we approximate g_t by a finite-state Markov chain using Gaussian quadrature. In the results in Section 5, each of the four elements of g_t takes on 4 values, implying 256 states of the economy. The endogenous processes, v^x_t, v^y_t, wc_t, are vectors with 256 elements each, to be determined by solving system (B.18). The conditional expectation is evaluated exactly (given the discrete approximation) since the state transition probabilities are known. We reduce the computational burden of this solution algorithm by assuming that the growth rates of c^x_t and c^y_t are observed, rather than the growth rates of output in the two countries. This enables us to solve system (B.18) recursively: the elements of $\{wc_t, v^x_t\}$ do not depend on the third equation in (B.18). Therefore, the 512 elements of $\{wc_t, v_t^x\}$ are found by simultaneously solving the 512 equations represented by the first two equations of (B.18). Given these values for $\{wc_t, v_t^x\}$, the 256 elements of v_t^y are found by solving the 256 equations represented by the last equation of (B.18). Having solved for $\{wc_t, v_t^x, v_t^y\}$, the remaining endogenous variables can be computed using Eq. (B.16) and Eqs. (B.1)–(B.8).

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