Test-Optional Admissions*

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Abstract

The Covid-19 pandemic has accelerated the trend of many colleges moving to test-optional, and in some cases test-blind, admissions policies. A frequent claim is that by not seeing standardized test scores, a college is able to admit a student body that it prefers, such as one with more diversity. But how can observing less information allow a college to improve its decisions? We argue that test-optional policies may be driven by social pressure on colleges' admission decisions. We propose a model of college admissions in which a college disagrees with society on which students should be admitted. We show how the college can use a test-optional policy to reduce its "disagreement cost" with society, regardless of whether this results in a preferred student pool. We discuss which students either benefit from or are harmed by a test-optional policy. In an application, we study how a ban on using race in admissions may result in more colleges going test optional or test blind.

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1. Introduction

With college admissions in the United States under increasing scrutiny, there is a vibrant debate about the role of standardized test scores. The last decade has seen an increase in colleges going *test optional*, i.e., not requiring applicants to submit standardized test scores. The University of Chicago made waves when it adopted this policy in 2018, and by 2019, one third of the 900+ colleges that accepted the Common Application did not require test scores.

For obvious reasons, the Covid-19 pandemic dramatically increased the adoption of test-optional policies: in the 2021–22 application season, 95% of Common-Application colleges did not require test scores. But even after the pandemic's physical disruptions receded in the U.S., most colleges have decided to stay test optional, at least for the near term. None of the Ivy league schools currently require tests; Harvard University has extended its test-optional policy until at least 2026, and Columbia University recently announced that it is permanently test optional. Furthermore, although our paper emphasizes college admissions, the shift away from requiring standardized tests is also pervasive in other segments of education.[1]

Proponents of test-optional admissions often cite concerns that standardized testing may disadvantage low-income students and students of color. Indeed, many schools that go test optional claim to do so in order to increase the racial and income diversity on campus.[2] But private schools, at least, always had a choice of how to use test scores in admissions. A test-mandatory college is free to admit students with low test scores if they are strong on other dimensions. Moreover, test scores are unlikely to be completely uninformative, and other components of applications, including letters of recommendation and college essays, may also be subject to racial and income disparities.[3] Indeed, MIT reinstated its testing re-

[1] According to Forbes magazine in January 2022, "The most public break-up [with standardized tests] has been in undergraduate admissions and the SAT/ACT, but kindergarten, high school, and graduate school admission offices have also been rejecting standardized tests . . . [there is a] near-universal shift away from standardized tests that started before the pandemic but has accelerated in the last eighteen months."

[2] For example, when George Washington University went test optional in 2015, a school official explained that "The test-optional policy should strengthen and diversify an already outstanding applicant pool and will broaden access for those high-achieving students who have historically been underrepresented at selective colleges and universities, including students of color, first-generation students and students from low-income households".

[3] In a 2016 Washington Post opinion titled 'Letters of recommendation: An unfair part of college admissions,' John Boeckenstedt from DePaul University argues that: "If you wanted to ensure that kids from more privileged backgrounds have a better chance to get into the schools with the most resources, letters of recommendation would be one of the things you'd start with."

quirement for the 2022-23 admissions cycle, arguing that "standardized tests help us identify socioeconomically disadvantaged students who lack access to advanced coursework or other enrichment opportunities that would otherwise demonstrate their readiness for MIT." Similarly, a 2020 report by the University of California found that standardized test scores help predict student success, across demographic groups and disciplines, even after controlling for high school GPA (UC Academic Senate, 2020).

Hence a puzzle: if a college can use test scores as it would like, why would it choose not to have access to a student's score? Why throw away potentially valuable information? We make this point formally in Section 2. We show that under a broad set of conditions—including differential costs of test preparation and different distributions of test scores for reasons unrelated to ability—a college that can freely use information cannot benefit from going test optional.[4] The reason is straightforward: a college has the option of replicating test-optional outcomes in a test-mandatory environment.[5]

So why, then, would a college choose to go test optional? After discussing a few alternative theories in Section 2, we propose that social pressure may be a driving force. When, say, Harvard admits a low-scoring student while rejecting a high-scoring student with an otherwise similar GPA, it may be subject to social pressure from a community that disagrees with the weight that Harvard puts on tests versus legacy status or racial diversity. Indeed, in a 2022 PEW research survey, only 26% of respondents thought that race or ethnicity should be even a minor factor in college admissions, with 25% for legacy status. By contrast, 39% thought that test scores should be a major factor, and an additional 46% thought they should be a minor factor. Such social pressure is exemplified by two lawsuits challenging the admissions policies of Harvard and the University of North Carolina, which are currently under review by the U.S. Supreme Court.

We develop the argument that a college can combat social pressure by going test optional. Broadly, by hiding score disparities among students who do not submit their test scores, the college can lower the cost of disagreement with society. The lower disagreement cost may also allow the college to admit students it likes more, based on diversity, extracurriculars,

[4] Our conditions do preclude prohibitive costs of sitting, as opposed to studying, for the test. As elaborated in Section 2, we do not find the cost of sitting for a test to be a compelling rationale for going test optional. In fact, prior to the Covid-19 pandemic, 25 U.S. states required either the SAT or ACT for high school graduation.

[5] This presumes that colleges can commit to their admissions policy, which is our focus in this paper. But Section 2 notes that if a school lacks commitment power, then under natural conditions, a test-optional policy would unravel to all students submitting scores.

or legacy preferences. Importantly, our argument does not rely on any naivety: we assume that society is Bayesian and understands that students who don't submit scores tend to have lower scores. Also important, we show that being test optional can help a college regardless of whether, for any given group of students, it wishes to be less selective than society (i.e., to use a lower test-score threshold) or more selective (a higher threshold). In one application of this framework, we study when the inability to use race in admission decisions may result in more schools becoming test optional or even test blind.

In more detail, our model in Section 4 has a college with preferences over which students to admit, based on both their non-test observable characteristics (e.g., GPA, race, SES, extracurriculars, and legacy status) and test scores. Society has its own preferences. Society does not make any strategic decisions, but the college places some value on minimizing disagreement between its admission decisions and those that society would make. The college commits to an admissions policy: an acceptance rule mapping observables and test scores into an admission decision, and also—in a test-optional regime—an *imputed* test score that it assigns to students who don't submit scores (as a function of non-test observables). A student submits their test score if and only if it is higher than the score the college would impute. Society passively assesses test scores in a Bayesian manner: non-submitters are evaluated based on their expected test score, given non-test observables and submission behavior.

Whenever society disagrees with the college's admission decision, the college incurs a *disagreement cost*. If the college accepts an applicant that society wants to reject, this cost is proportional to society's disutility from acceptance. If the college rejects an applicant society wants to accept, this cost is proportional to society's disutility from rejection. The college chooses its admissions policy—both the imputation and acceptance rules—to maximize its ex-ante expected utility from admissions decisions less disagreement costs.

When a college can freely choose its imputation rule, the college can't be worse off under test optional than test mandatory. It could simply replicate the test-mandatory outcome by imputing a low enough test score that all students submit. Our key insight, though, is that the college can benefit—strictly—from going test optional.

To see how, consider the case of a student with non-test observables such that the college is less selective than society: the college has a lower test-score bar than society to admit this type of applicant. For instance, take students who excel in fencing and suppose the college values able fencers more than society.[6] One option for the college is to impute a very high test score for fencers, with the policy of admitting all those with the imputed score (or higher). Then none of the fencers submit their scores, and all of them are admitted. The cost for the college is that it may be admitting some very low-scoring fencers. The benefit, though, is that bringing high-scoring fencers into the non-submission pool reduces disagreement costs from admitting some fencers that the college wanted but society did not. Indeed, if society is willing to accept fencers with average test scores, then imputing a very high score allows the college to accept all of these now-undifferentiated fencers at *zero* disagreement cost. At the extreme, if the college prefers to admit every fencer regardless of test score, it obtains its first best for this group—they are all admitted, with no disagreement cost.

Now consider students with observable characteristics at which the college is *more* selective than society. Suppose the college prefers to admit applicants from New Jersey only if they score above 55, whereas society loves the Garden State and would like to admit any of its students with a score above 25. If test scores are submitted, the college incurs a disagreement cost for any rejected applicant with a score above 25. Consequently, under test mandatory, the college uses a score threshold between 25 and 55, say 40. Under test optional, however, the college can do strictly better among New Jerseyans by imputing a score between 40 and 55 and then rejecting non-submitters. Imputing the score of 40 would replicate the test-mandatory admissions outcome but lower the disagreement cost because all New Jerseyans with scores below 40 don't submit; now there is no differentiation between those below 25, where there is no disagreement, and those in the 25–40 range, where there is disagreement. The college may do even better by imputing a score strictly above 40, which would improve, from its perspective, its New Jerseyan student body.

We show in Subsection 6.2 that the above examples encapsulate the general logic for how a college can benefit from going test optional. Notice that in these examples, fencers benefit—some weakly and some strictly—from a school going test optional, whereas New Jerseyans are hurt. Subsection 6.2 establishes that these consequences for student welfare hold generally: student groups for whom the college is less selective than society benefit from test optional, while student groups for whom the college is more selective are hurt.

For test optional to never harm a college, the college must judiciously choose its imputation rule. In practice, we see many schools promising that non-submitters will be treated "fairly". The University of Southern California's statement is representative: "applicants will not

[6] The New York Times reports in October 2022 that "a way with the sword can help students stand out in the college admissions game... because each good school, especially Ivy League schools, have fencing."

be penalized or put at a disadvantage if they choose not to submit SAT or ACT scores." Although it is ambiguous what such policies really mean, we propose that they correspond to a *no adverse inference* imputation rule: a student who does not submit a test score is imputed their expected test score given other observables, but crucially, not conditioning on non-submission. Subsection 6.3 studies test-optional outcomes under this or some other given imputation rule. We establish a sense in which students with good non-test observables (and low test scores) benefit when a college goes test optional because it increases their admission rate. Students with intermediate observables (and intermediate scores) are harmed. Other students are unaffected.

When constrained to use an imputation rule like no adverse inference, colleges may be worse off under test optional than test mandatory (by contrast with flexible imputation). Determining whether test optional is attractive to the college requires more structure on the environment. We turn to an extended example in Section 7, where we study how affirmative-action regulations affect a college's preference over test-score regimes. Our interest stems from the ongoing U.S. Supreme Court cases on college admissions, which are widely expected to result in the Supreme Court severely limiting race-conscious admissions.

Our extended example considers a college with affirmative-action preferences: conditional on all other characteristics (test scores and some non-test observables), it also has preferences over a student's group membership, e.g., their race. Society has the same preferences as the college over other characteristics, but its preferences are group-neutral. Our specification is such that when affirmative action is allowed—the college can condition its admissions rule on group membership—the college will choose the test-mandatory regime. The college can use different score thresholds for admitting students of different groups, and it values test scores enough to outweigh the disagreement cost. If affirmative action is banned, however, then the college may switch to *test blind*.[7] The intuition is that if students in the college's favored group have lower test scores, then the college values tests less when it cannot condition on group membership, and so it now prefers to go test blind to reduce disagreement costs. We discuss how banning affirmative action may thus backfire: society prefers the college use tests but not use group membership in admissions, but society may be better off when the college uses both rather than neither. Interestingly, commentators have suggested that if the Supreme Court rules affirmative action illegal, colleges like Harvard may make their current test-optional policies permanent (New Yorker, January 2022).

[7] Test blind is when students simply cannot submit tests scores, or the college ignores test scores entirely. In our model, this is equivalent to test optional in which non-submission is imputed as the highest test score.

Related literature. There are empirical papers studying test-optional (or test-blind) college admissions, using data from prior to the Covid-19 pandemic (e.g., Belasco, Rosinger, and Hearn, 2015; Saboe and Terrizzi, 2019; Bennett, 2022). In a recent review, Dynarski, Nurshatayeva, Page, and Scott-Clayton (2022, pp. 53–54) conclude from these studies that test-optional policies have had limited effect on increasing diversity and applications, but may have helped colleges boost their public rankings by raising the average (submitted) standardized test score of enrolled students.

Economic theory has also studied how standardized test scores should be used in college admissions. Krishna, Lychagin, Olszewski, Siegel, and Tergiman (2022) propose pooling test scores into coarse categories to reduce the wasteful costs of test preparation. Lee and Suen (2023) study how low-powered selection—such as putting less weight on test scores—may help a college by reducing students' incentives to improve their test scores.[8] Garg, Li, and Monachou (2021) assume that some students have no access to standardized tests, which means that a test-optional/blind policy broadens the applicant pool even though it provides less information about those who do apply. Borghesan's (2022) structural analysis of college admissions also emphasizes students' costs of taking standardized tests: going test blind reduces a college's information but allows students with high test-taking costs to apply. He predicts that this policy would reduce student quality at top schools without increasing diversity. Related to costly test-taking is Ottaviani's (2020) model of prize allocation with costly application. He shows that using more noisy measures of applicant quality can enlarge the applicant pool; while he is motivated by the allocation of grants, the logic can also be applied to selective colleges' admissions.

In contrast to the papers in the preceding paragraph, our argument for why colleges benefit from going test optional does not rely on the cost of obtaining or improving test scores, nor on the cost of applying to a college. While we discuss these factors in Section 2, our model of social pressure assumes that students are simply endowed with a test score and application is costless. Indeed, at least prior to Covid-19, 25 U.S. states required students to take the SAT or ACT in order to graduate high school.[9]

[8] More broadly, in a "muddled information" framework (Frankel and Kartik, 2019), Ball (2022) and Frankel and Kartik (2022) explore how a decisionmaker should commit to underutilize manipulable information to improve decision accuracy.

[9] In their empirical studies, Goodman (2016) and Hyman (2017)) find that such policies increase college enrollment rates of low-income students, either because the students discover they are higher-achieving than they thought or because colleges discover and then recruit students through such testing. More generally, scholars have suggested that eliminating application barriers for low-income students can increase the number of students that apply to and enroll in selective colleges (Hoxby and Avery, 2012; Hoxby and Turner, 2013;

Some theoretical papers on college admissions have also studied the specific issue of affirmative action (e.g., Abdulkadiroglu, 2005; Chade, Lewis, and Smith, 2014; Fershtman and Pavan, 2021; Brotherhood, Herskovic, and Ramos, 2022), which we take up in Section 7.[10] Most related to our work is Chan and Eyster (2003), who model a college that values both student quality and diversity. When affirmative action is banned, the college may adopt an admission rule that puts less weight on academic qualifications, such as standardized test scores, in order to promote diversity. The logic is related to that of statistical discrimination (Phelps, 1972; Arrow, 1973), except that instead of race serving as a signal of qualification, qualification serves as a signal of race. Notably, Chan and Eyster (2003) do not provide a rationale for why a college strictly benefits from not observing test scores; in their model, being test blind is equivalent to being test mandatory and putting zero weight on tests. By contrast, in our model, social pressure can lead a college to strictly prefer test-blind (or test-optional) admissions to test mandatory.

Our paper also contributes to the large literature on voluntary disclosure of verifiable information. The canonical result here is that of "unravelling" (Grossman, 1981; Milgrom, 1981), which corresponds to all students submitting their scores even when it is optional. It is reported, however, that fewer than half of U.S. college applicants who applied early decision in Fall 2022 submitted test scores. Unraveling does not arise in our model because we assume the college can commit to how it will treat students who do and do not submit their score.

Finally, in our model, the college's and society's information depends on which students submit test scores. This is determined by the testing regime and, under test optional, the college's imputation rule. Our work is thus related to the large and growing literature on Bayesian persuasion and information design (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019). For example, Liang, Lu, and Mu (2022) explore how a designer may ban the use of certain inputs, such as test scores, because of a disagreement with how a decisionmaker would use those inputs. As is standard with Bayesian persuasion, their designer only cares about information insofar as it affects the decisionmaker's actions. In our model, by contrast, the college both controls information and makes admission decisions; but society observes the same information, which affects the college's social pressure costs. To put it differently,

Goodman, Gurantz, and Smith, 2020).

[10] Various other papers model aspects of college admissions that we do not address, such as early admissions (e.g., Avery and Levin, 2010), managing enrollment uncertainty (e.g., Che and Koh, 2016), college tuition determination (e.g., Fu, 2014), and which colleges a student should apply to (e.g., Chade and Smith, 2006; Ali and Shorrer, 2021).

Liang et al. (2022) explain why society may choose to prevent a college from using test scores; we show why a college may itself choose to not see test scores. Like us, Liang et al. (2022) also discuss why a college may choose to not see test scores if society bans affirmative action; rather than social pressure, their mechanism involves a conflict of preferences between the college and it admissions officers.

2. A Puzzle

This section formalizes our motivating puzzle: under a broad set of conditions, a college can always do at least well under test mandatory as under test optional. The intuition is simply that more information cannot hurt the college, if it is free to use information as it would like—even though how the college uses information can affect choices students make. By making explicit a set of assumptions behind the result, our formalization also allows us to discuss various reasons why the result may fail. Readers primarily interested in seeing our analysis of social pressure can skip forward to Section 3.

2.1. An Impossibility Result

A student is considering applying for admission to a college. If the student applies, the college will choose whether to admit this student based on their non-test observables and, if submitted, their test standardized scores.

Prior to taking a standardized test, the student is endowed with publicly observable characteristics x; privately observed characteristics z; a "holistic" component h that is observed by the college (if the student applies) but not the student; and an underlying ability level a that isn't directly observed by the student or the college. The exogenous variables x, z, h, and a follow some commonly known joint distribution, which may have arbitrary correlation across variables. The college has some net utility $u^c(x, z, h, a)$ for admitting the student, with its preferred admission threshold normalized to $u^c = 0$.

We think of the public observables x as representing features that the college can see in the student's application: GPA and other measures of classroom performance, extra-curricular achievements, legacy status, etc. The private characteristics z represent features that the college cannot directly observe: aspects of the student's interests and upbringing, say. Some features such as race or socioeconomic status might lie in x or in z, depending on what information the college collects on its application. The holistic variable h can be anything the college assesses that the student does not know when applying, e.g., match quality, or some unpredictability in how the college evaluates the student's personal characteristics. Finally, we interpret ability a as representing some aspect of how well a student will perform in college, if admitted.

The college can make inferences about ability a (and the private characteristics z) through any correlation it has with x and h. The college can also potentially learn additional information from a standardized test score t, and from a supplementary cheap-talk message m that the student submits with their application. This supplementary message can represent a component of the student's personal statement. The game is as follows.

1. One of the *test-mandatory*, *test-optional*, or *test-blind* testing regimes is determined.
2. The college publicly commits to a mapping from x, m, h, and t (if observed), to a probability of admission.
3. The student's features x, z, h, and a are realized, and the student learns x and z.
4. The student chooses *test-preparation* effort e at a cost $\varphi_{\text{effort}}(e|x, z)$ that depends arbitrarily on e, x, and z. This effort choice will not be observable to the college.
5. The student realizes a test score t drawn from a distribution that depends arbitrarily on x, z, h, a, and e.
6. The student chooses whether to apply to the college at a cost $\varphi_{\text{apply}}(x, z, e)$ that depends arbitrarily on x, z, and e.[11] If the student applies:
   (a) In the test-optional regime, the student chooses whether to disclose the test score t. The test score is automatically disclosed in the test-mandatory regime, and is not disclosed in the test-blind regime.
   (b) The college observes x and h. The student may also send an arbitrary supplementary message m at no cost.
7. If the student applies, admission is determined by the college's admission rule. The student gets a gross payoff $v_{\text{admit}}(x, z, e)$ if admitted, and a gross payoff 0 otherwise. They also incur the costs φ_{effort} and, if they applied, φ_{apply}. So, for example, an admitted student's net payoff is $v_{\text{admit}}(x, z, e) - \varphi_{\text{effort}}(e|x, z) - \varphi_{\text{apply}}(x, z, e)$.

In this game, we show below that it is impossible for the college to strictly prefer test-

[11] We could also allow for the cost φ_{apply} to depend on an endogenous "application effort" that generates an additional signal for the college; we omit that for simplicity.

optional or test-blind admissions to test-mandatory.[12] To be clear, we do *not* interpret this impossibility result as implying that colleges in the real world cannot benefit from going test optional or test blind. Rather, the model and result point us to the assumptions that must be violated when a college might in fact benefit from going test-optional.

Proposition 1. *The college prefers test mandatory to test optional, and prefers test optional to test blind. In particular, for any test-optional equilibrium, there is a test-mandatory equilibrium in which the college's expected payoff is (weakly) higher. For any test-blind equilibrium, there is a test-optional equilibrium in which the college's expected payoff is (weakly) higher.*

The proof of the proposition is almost trivial, given the assumptions. Here is how test mandatory can replicate the test-optional outcome. (An analogous argument shows that test optional can replicate test blind.) Take any test-optional equilibrium, which consists of the college's admission rule and the student's strategy. Now suppose the college chooses (perhaps sub-optimally) a test-mandatory admissions rule that sets an acceptance probability equal to what a student with the same x, h, and t *would have gotten* in the test-optional equilibrium. More precisely, the college asks the student to report in their supplementary message m whether they would have disclosed under test optional, along with any other supplementary information they were previously submitting.[13] If the student would have submitted, the college assigns them the test-optional acceptance probability for a student with that test score (and given everything else the college observes); if not, the college assigns them the acceptance probability for a student who did not submit a score. This college strategy ensures that the incentives for the student to choose test-preparation effort, to apply, and to (report whether they would) submit a test score are identical to those under the test-optional equilibrium. It is thus a best response for the student to act the same as under test optional, and we have replicated that outcome.

The above argument only establishes that the college cannot do worse with test mandatory

[12] For simplicity, we adopt Bayes Nash equilibrium as our solution concept, but the argument applies with any standard concept.

[13] We assume that doing so is feasible under the test-mandatory message space: this is assured, for example, if the college chooses the message space in each regime. Note that we only need the student to indicate whether they would have submitted under test optional if the college cannot predict that based on its observation of x and t. But in those cases, allowing the supplementary cheap-talk message could play an indispensable role. Albeit in a different model, Hancart (2023, Section 2.1) shows that a form of test optional can do strictly better than test mandatory in the absence of cheap talk; when cheap talk is permitted, however, even in his model there would be no benefit from going test optional.

than test optional, but one would generally expect that observing more information about the test score would allow the college to do strictly better.

2.2. Ways Out of the Puzzle

In light of Proposition 1, we now discuss how breaking some of its underlying assumptions might—or might not—lead the college to prefer test-optional admissions over test mandatory.

College lacks commitment power. We assumed the college can commit to its admission rule. Suppose the college lacks such commitment power: no matter what the college announces about its admission rule, these claims are unenforceable and students know not to believe them. The college ends up admitting students according to what it finds ex-post optimal given the information provided. We expect that under reasonable monotonicity assumptions, a college without commitment power still cannot do better under test optional than test mandatory. The logic is now quite different from that of Proposition 1, however. The issue now is that the test-optional equilibrium unravels, as in classic voluntary-disclosure models (e.g., Milgrom, 1981). Despite a nominally test-optional policy, all students end up submitting their scores because not submitting will, in equilibrium, be met with the skepticism of a low score and hurt admission. Such unraveling suggests that a lack of college commitment power is unlikely, on its own, to explain why colleges might go test optional.

Additional costs. We allowed for the students to have arbitrary type-dependent costs of studying for the test and of applying to the college. We did not allow for a direct cost of sitting for the test, however, nor of submitting a test score.

It is easy to see how adding these costs breaks our replication argument and can in fact flip the result. A student who pays a cost of sitting for the test and/or submitting the test score can avoid these costs only if the college is test optional or test blind. In the presence of such costs, colleges potentially face a genuine tradeoff: requiring test scores deters applications while yielding more information about students who do apply (cf. Garg et al., 2021).

There is, of course, a large cost of sitting for the test during a pandemic—even infinite, when test centers are shut down. But both prior to the Covid-19 pandemic, and after its effects have receded, our view is that this cost is not particularly large.[14] (Let us reiterate

[14] For instance, the SAT takes about 3 hours to sit—about half a day of school, while a typical U.S. student is expected to go to school for about 180 days a year for 12 years prior to college. The SAT currently

that this test-sitting cost would be separate from any costs of studying and preparing for test, which we view as significant but are already part of the model.) We also note that, if the cost of sitting—as opposed to preparing for—the test or the cost of submitting the score were the main benefits for a college being test optional, then subsidizing test-taking or score-submission for the relevant groups of applicants, as is already done to some extent, would likely be a more efficient way to increase participation. All that said, we acknowledge that students may still perceive these costs as significant.

Non-equilibrium behavior. A related way that the impossibility result can fail is if students don't follow our predictions of equilibrium behavior. Students may make different application or test-preparation decisions when facing a test-optional college rather than a test-mandatory college with the same acceptance probabilities. For instance, colleges would certainly want to switch from test mandatory to test optional if many students happened to follow the behavioral rule that they will not apply to test-mandatory colleges.[15] Even a student who plans on taking the test and submitting their score to test-optional colleges might, for reasons of principle, be unwilling to apply to a test-mandatory college.

Constraints on the college's admissions rule. Our replication argument assumed that the college was able to choose any admission rule it wanted. If the college is constrained in setting this rule, the impossibility result could fail. As an extreme case, imagine that the college has no flexibility at all: it is required to evaluate students with a test score by one rule, and students without a test score by another rule. If the admission rule for students with test scores were to put too much weight on tests, the college might very well prefer not to see tests at all. Such an exogenous admission rule might be prescribed by the government, or it might be that admissions officers make decisions according to their own views and cannot be incentivized to act differently. Our view is that government policies are not a major constraint on colleges, at least as long as a college is private and is not violating civil rights laws. And on the organizational side, colleges invest a lot of money

has a monetary cost of $60, but low income students in the US can get this fee waived; fee waivers are automatic for students eligible for federally subsidized school lunches. Students can then submit their SAT scores to four colleges at no cost and they pay $12 per submission after that, but again these fees are waived for low income students. (Fees link.)

[15] In the context of applications to graduate schools, Dr. Kim Yi Dionne, a professor at UC Riverside, writes on Twitter: "Students at the minority-serving institution where I work are ABSOLUTELY taking schools off their list if they require the GRE."

into their admission process and presumably can direct their admissions offices to at least approximately follow the rules they want.[16]

Social pressure. The story we seek to explore is related to the previous one, but milder. We posit that colleges do have the flexibility to choose arbitrary admission rules, but that their choices may be costly. In particular, we suppose that colleges face pressure from some third party—we call this "society"—over their admissions decisions. "Society" here represents any external group that might scrutinize admission decisions, and have preferences over who ought to be admitted: alumni, parents, local governments, the popular press, and even the judicial branch. If the college's admissions decisions disagree with society—the college rejects students that the society thinks should be admitted, or accepts students that society thinks should not be accepted—then the college has to pay some cost of social pressure.

In the limit as these disagreement costs go to infinity, this model effectively converges to one where the college simply follows an exogenous rule (in this case, society's admission rule). When disagreement costs are less extreme, though, the college faces a choice over how to try and maximize its "underlying utility" over the admitted students while minimizing disagreement costs.

Before turning to our formal model of how social pressure affects admissions decisions under different testing regimes, the next section provides an illustrative example showing how a college subject to social pressure can be strictly better off by *not* seeing information.

3. An Illustrative Example

Consider a single student who has applied to a college. (An alternative interpretation is that of a mass of students who share common observable characteristics.) The student's test score t is drawn from a uniform distribution between 0 and 100. Society's utility from admitting the student is $u^s(t) = t - 40$, and its utility from not admitting the student is normalized to 0. So, ignoring indifference, society wants to admit the student if and only if their test score is above 40. The college receives some information about the student's test score—we will consider different possibilities below—and then chooses whether to accept or

[16] Admittedly, admission officers might put more weight on test scores than the college seeks, owing to their intrinsic preferences or beliefs. Or, they might do so due to incentives and moral hazard: evaluating test scores may be easier than evaluating more subjective features, and admission officers may only find it worthwhile to conduct the costly holistic assessment when they lose access to test scores.

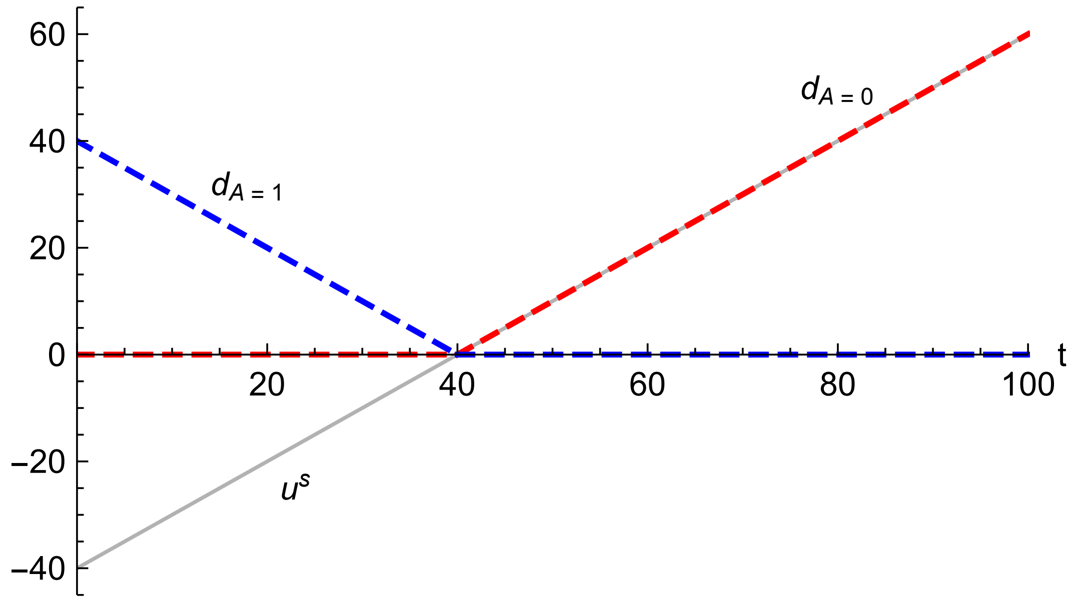


Figure 1 – Disagreement cost from accepting ($A = 1$) and rejecting ($A = 0$) an student.

reject the student. Society then judges the college's decisions given the available information. Importantly, the college and society have the same information; information asymmetry between them is not our driving mechanism. Rather, what is crucial is that the college faces disagreement costs from social pressure for making decisions that society disagrees with.

Disagreement cost. The disagreement cost is proportional to the extent of society's disagreement with the college's decision, given the available information. If the college accepts the student and society would also prefer to accept them (i.e., $\mathbb{E}[u^s(t)] > 0$), or the college rejects the student and society would also prefer to reject ($\mathbb{E}[u^s(t)] < 0$), then the college bears no disagreement cost. That is, in each of those cases, the respective disagreement costs $d_{A=1}$ and $d_{A=0}$ are both 0, where $A = 1$ denotes acceptance and $A = 0$ denotes rejection. However, if the college rejects the student when society prefers to accept, the college bears a disagreement cost of $d_{A=0} = \mathbb{E}[u^s(t)] > 0$. Likewise, if the college accepts an student that society prefers to reject, the disagreement cost is $d_{A=1} = -\mathbb{E}[u^s(t)] > 0$. See Figure 1.

Why not observe test scores? We now illustrate how the college can reduce disagreement costs by not observing test scores.

First consider *test mandatory*: the student's test score is observed. If the college chooses to accept regardless of the test score, it bears a disagreement cost of $40-t$ whenever the score is below 40 (and 0 otherwise), and so the the expected disagreement cost is $\int_0^{40} \frac{1}{100}(40-t)\mathrm{d}t = 8$. Analogously, if the college instead chooses to reject regardless of test score, it bears an expected disagreement cost of $\int_{40}^{100} \frac{1}{100}(t-40)\mathrm{d}t = 18$.

Now consider *test blind*: the student's test score is not observed. Here, having no information beyond the uniform prior over the test score, society evaluates the student as if their test score were equal to the expected value $\mathbb{E}[t] = 50$. If the college chooses to accept the student, it now faces a disagreement cost of 0: absent test score information, society agrees that the student should be accepted. So if the college were going to accept the student regardless of their test score, then hiding the test score reduces its expected disagreement cost from 8 to 0.

If the test-blind college rejects the student, it does face a disagreement cost: society's expected utility from admitting the student is $\mathbb{E}[t] - 40 = 10$, and so the college's disagreement cost from rejection is 10. Nonetheless, hiding the test score reduces the expected disagreement cost of rejecting all applicants from 18 to 10.

The upshot is that, for either decision the college makes—so long as it is independent of the test score when that is observed—the college can reduce expected disagreement cost by hiding the test score, i.e., going test blind. The fundamental reason is that both disagreement cost curves $d_{A=1}(t) = \max\{40-t, 0\}$ and $d_{A=0}(t) = \max\{t-40, 0\}$ are convex, as seen in Figure 1. Mathematically, the reduction of expected disagreement cost by going test blind is simply a consequence of Jensen's inequality.

A tradeoff. The downside of going test blind is that the college may want to condition its decisions on test scores. We have not specified the college's preferences over admission decisions here, either gross or net of the disagreement cost it bears. But perhaps the college seeks to admit only those students with test scores above, say, 20 (which is a lower threshold than society) or 60 (a higher threshold). Either way, not observing the test score harms the quality of decisions the college can make.

In the rest of the paper, we explore the tradeoff a college faces between reducing disagreement cost and using information to make better decisions in the context of test-optional (or test-blind) policies. We study how test-optional colleges decide which applicants to admit, how students choose whether to submit test scores, and how the resulting outcomes differ from a test-mandatory benchmark.

4. A Model of Admissions under Social Pressure

We model a student applying to a college, with a broader "society" playing a passive role. (The student can be viewed as being a representative applicant from a mass of students, and we will sometimes use the plural students for exposition.) The student is endowed with some publicly-observable characteristics and a test score, which is their private information. In a test-mandatory regime, the student mechanically submits their test score, making it public to the college and society. In a test-optional regime, the student chooses whether to submit their score. In either regime, the college chooses whether to admit the student based on their observable characteristics and, if submitted, their test score. Both the college and society have preferences over whether the student should be admitted as a function of their observables and their true test score. The college also places some weight on reducing disagreement between its admission decision and the decision society would want it to make, given all available information.

4.1. Model Primitives

Observables and test scores. Formally, the student/applicant has a *type* $(x, t) \in \mathcal{X} \times \mathbb{R}$, where x is an *observable* (or vector of *observables*) and t is the *test score*. The distribution of observables is given by F_x and the test score has conditional distribution $F_{t|x}$.[17]

The observable x is public information to all players. The test score t is private information to the student, which may be submitted ($S = 1$) or not ($S = 0$). Submitting the score makes it observable to all other players. Our primary interest is in two college-admission regimes: *test mandatory*, in which test scores must be submitted, and *test optional*, in which scores may be submitted. We will also talk about *test blind*, wherein the score cannot be submitted.

Preferences. The college decides whether to admit the student (denoted $A = 1$) or not ($A = 0$), based on observables x and, if submitted, the test score t. The student strictly prefers a higher probability of being admitted. Society's utility and the college's material or

[17] More precisely, $\mathcal{X}$ is a measurable space and F_x is a probability measure on that space. To simplify some technicalities, we assume that for each x, $F_{t|x}$ is either continuous or is discrete with no accumulation points, and that all relevant expectations exist.

"underlying" utility if the student is accepted are given, respectively, by

$$u^s(x,t) := v^s(x) + w^s(x)t,$$
$$u^c(x,t) := v^c(x) + w^c(x)t,$$

where the superscripts have the obvious mnemomic (*s*ociety and *c*ollege), and each $w^i(\cdot) > 0$ for $i = s, c$. We view monotonicity of these preferences in the test score as natural; the affine specifications aid subsequent interpretation and tractability. Both society's and the college's underlying utility are normalized to 0 if the student is not admitted.

In addition to its underlying utility, the college suffers disutility from social pressure on its admission decision. To formalize that disutility, let t^s denote the test score society treats the student as having; this will be determined endogenously. Anticipating equilibrium, think of $t^s = t$ if the score is submitted, and $t^s = \mathbb{E}[t|x, S = 0]$ under non-submission. For any t^s, society's *disagreement* with the college's decision is given by

$$d(x, t^s, A) := \begin{cases} \max\{u^s(x, t^s), 0\} & \text{if } A = 0, \\ \max\{-u^s(x, t^s), 0\} & \text{if } A = 1. \end{cases} \tag{1}$$

The assumed linearity of $u^s(x,t)$ in the test score t means that we can interpret $u^s(x, t^s)$ as society's expected benefit from admitting the student when t^s is the expected test score given all available information. Hence, society's disagreement can be understood as follows: there is no disagreement if, given the available information, society's preferred decision is the same as the college's decision; but when there is a conflict in preferred decisions, then disagreement is linear in the magnitude of society's expected benefit from its preferred decision. As before, the monotonicity here is natural; linearity is for tractability.

The college's overall payoff U^c is its underlying utility less the (scaled) disagreement:

$$U^c(x, t, t^s, A) := Au^c(x,t) - \delta d(x, t^s, A), \tag{2}$$

where $\delta > 0$ is a parameter capturing the extent of social pressure on the college. We refer to $\delta d(\cdot)$ as the *disagreement cost* to the college.

Imputation and admission policies. The college's admissions policy has two components, one of which—how to treat students who don't submit test scores—is irrelevant under test mandatory.

First, given the student's observable x, we assume that the college treats non-submission of a test score as equivalent to some specific test score, which we call the *imputation.* More precisely, there is an *imputation rule* $\tau : \mathcal{X} \to [-\infty, +\infty]$,[18] with $\tau(x)$ the imputation for observable x. We will be interested in two settings: either the college can choose the imputation rule arbitrarily, which we call *flexible imputation*, or the imputation rule is exogenously given, which we call *restricted imputation.*

Second, the college chooses an *acceptance rule* $\alpha : \mathcal{X} \times [-\infty, +\infty] \to [0, 1]$, where $\alpha(x, \hat{t})$ is the probability of admitting a student with observable x and imputed/submitted test score $\hat{t}$. We stress that the acceptance rule cannot (directly) condition on the student's true test score, and it does not distinguish between imputed and submitted scores—this captures our notion that imputing a score means treating a non-submitting student as if they have submitted that imputed score. As in Chan and Eyster (2003), we assume that α must be *monotonic* in the sense that for any x, $\alpha(x, \cdot)$ is weakly increasing.

College's problem. Since the college's acceptance rule is monotonic, there is a simple best response for the student: submit their score if $t > \tau(x)$ and don't submit if $t \leq \tau(x)$. We restrict attention to the student playing this strategy. Given this student strategy, we assume society is Bayesian in evaluating the student. In particular, if the student submits their test score, then $t^s = t$; if the student does not submit, then $t^s = L(\tau(x)|x)$, where L (mnemonic for "lower expectation") is defined by

$$L(\tilde{t}|x) := \mathbb{E}[t|t \leq \tilde{t}, x].^{19}$$

The college's problem is to choose—commit to—its imputation rule τ (under test optional with flexible imputation) and its acceptance rule α, to maximize its expected payoff U^c, anticipating the student's best response and society's Bayesian inferences.

[18] The co-domain is the extended reals for technical convenience when test scores can be arbitrarily small or large; if test scores lie in a compact set, then we could take the co-domain of τ to be that compact set.

[19] For $\tilde{t} \leq \inf \operatorname{Supp}[F_{t|x}]$, we set $L(\tilde{t}|x) = \inf \operatorname{Supp}[F_{t|x}]$.

4.2. Ex-Post Utility

Observe that when $t^s = t$, as will be the case if the student submits their score, Equation 1 and Equation 2 imply that the college's net benefit from admitting the student is given by

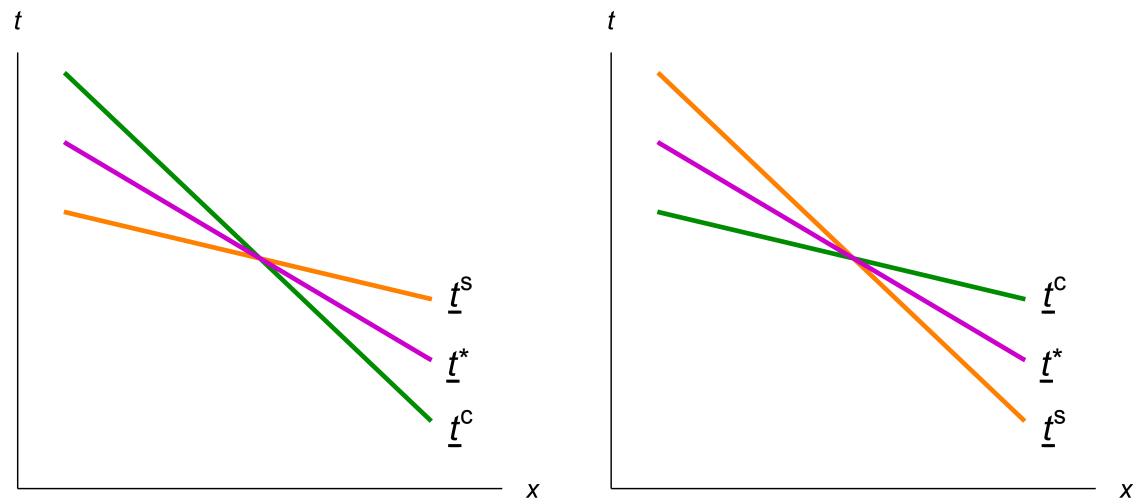
$$\begin{aligned} U^c(x,t,t,1) - U^c(x,t,t,0) &= u^c(x,t) - \delta\left[d(x,t,1) - d(x,t,0)\right] \\ &= u^c(x,t) + \delta u^s(x,t) \\ &\propto \frac{1}{1+\delta} u^c(x,t) + \frac{\delta}{1+\delta} u^s(x,t) \\ &=: u^*(x,t). \end{aligned} \tag{3}$$

We refer to $u^*(x,t)$ as the *college's ex-post utility.* For a score-submitting student, our disagreement cost formulation implies that the college's net benefit from admission is equivalent (i.e., proportional to) to a convex combination of the college's underlying utility and society's utility. If the student submits their score, the college's payoff is maximized by admitting the student if and only if (modulo indifference) $u^*(x,t) > 0$.

For $i \in \{c, s\}$, we refer to $\underline{t}^i(x)$ such that $u^i(x, \underline{t}^i(x)) = 0$ as the college/society's test-score *bar* for admission: it is the score threshold such that each would—if unencumbered by social pressure—prefer to admit the student with observable x if and only their score is above that threshold. We denote the ex-post utility bar by $\underline{t}^*(x)$; it is defined by $u^*(x, \underline{t}(x)) = 0$ and is the threshold above which, accounting for social pressure, the college wants to admit the student.[20] We say that the college is *less selective* than society at observable x if $\underline{t}^c(x) < \underline{t}^s(x)$, while it is *more selective* if $\underline{t}^c(x) > \underline{t}^s(x)$. In either case, the ex-post utility bar $\underline{t}^*(x)$ is in between the two parties' bars, and it monotonically shifts from $\underline{t}^c(x)$ to $\underline{t}^s(x)$ as the social-pressure intensity parameter δ increases.

Figure 2 illustrates with a leading specification in which $x \in \mathbb{R}$, and for each $i \in \{c, s\}$, $u^i(x,t) = a^i + x + w^i \times t$. In this specification, the college weights test scores more than society when $w^c > w^s$, and weights test scores less than society when $w^c < w^s$. The three lines indicate the respective test-score bars at each x. When the college weights test scores less, as in the figure's left panel, at low x it is more selective (has a higher bar) than society, but at high x it is less selective (has a lower bar); and the reverse when the college weights test scores more than society, as in the right panel.

[20] More explicitly, since $u^i(x,t) = v^i(x) + w^i(x)t$ and $u^*(x,t) = \left(u^c(x,t) + \delta u^s(x,t)\right)/(1+\delta)$, we compute $\underline{t}^i(x) = -w^i(x)/v^i(x)$ and $\underline{t}^*(x) = -\left(w^c(x) + \delta w^s(x)\right) / \left(v^c(x) + \delta v^s(x)\right)$.



(a) College weights tests less than society: $w^c < w^s$.

(b) College weights tests more than society: $w^c > w^s$.

Figure 2 – Test score admission bars for society ($\underline{t}^s$), the college's underlying utility ($\underline{t}^c$), and ex-post utility ($\underline{t}^*$). For this figure, $x \in \mathbb{R}$ and $u^i = a^i + x + w^i \times t$.

4.3. Discussion of the Model

4.3.1. Restricted imputation rules

With flexible imputation, the college can arbitrarily choose how to impute missing test scores. With restricted imputation, we consider the other extreme, in which an imputation rule is exogenously specified. Although our analysis will not have any results tied to particular restricted imputation rules, we allow for them to cover some colleges' practice of publicly promising not to "penalize" or "disadvantage" students who don't submit scores. We interpret such promises as mapping to some version of what we call the *no adverse inference* imputation rule, $\tau(x) = \mathbb{E}[t|x]$. Contrast this expression to the Bayesian imputation rule used by society, in which $t^s = \mathbb{E}[t|x, S = 0]$: no adverse inference updates based on observables but not on the choice not to submit. That is, the college imputes test scores as if students who did not submit chose to do so non-strategically.[21]

Even when ignoring the submission decision, the college might condition its expectation not on the full vector of observables but on some subset of relevant components. For instance, if the observable vector $x = (x_0, x_1)$ has component x_1 corresponding to "grades" and

[21] After switching to test optional in 2020, Dartmouth announced "Our admission committee will review each candidacy without second-guessing the omission or presence of a testing element."

component x_0 corresponding to "demographics" (race, gender, family income, neighborhood of residence), the college might impute $\tau(x) = \mathbb{E}[t|x_1]$. This gives the expectation of t conditional on grades but not on demographics (and not on the decision to submit). Indeed, certain features such as race or gender may be legally protected categories, in which case it might be forbidden to impute scores differently based on these factors—even if they are in fact predictive of test scores.[22] In the limiting case, a college might deem *all* observables irrelevant, in which case it would impute $\tau(x) = \mathbb{E}[t]$ identically for all applicants. This constant imputation rule can be thought of as "scaling up" the importance of all the non-test factors for those who do not submit. Imputing $\tau(x) = \mathbb{E}[t|x]$ corresponds instead to scaling up observables that predict test scores, which might end up putting extra weight on academic versus nonacademic factors.

4.3.2. Key assumptions

Simplifications. Relative to the framework discussed in Section 2, the current model makes a number of simplifying assumptions in order to focus on the channel of social pressure as an explanation for going test optional. For instance, we abstract away from a student's decision of how much to study for, or whether to even take, the test. Instead, we endow students with a test score. We then give the college and society a reduced form preference over these test scores rather than microfounding any inference over underlying ability. We also don't model the student's application decision.

One other simplifying assumption to flag is that we model the college as having a fixed underlying utility threshold for admission. In particular, even if a switch from test mandatory to test optional leads to a different number of admitted students, the college does not raise or lower its threshold for admission in order to keep its class size constant. We return to this point in the Conclusion.

Student submission behavior. We assume that students submit a test score if their true score t is strictly above the college's imputed value $\tau(x)$, and they withhold the score if t is weakly below $\tau(x)$. Higher test scores can only help admission chances.[23] So, as

[22] Society, too, might only factor in certain components of observables: for instance, setting $t^s = \mathbb{E}[t|x_1, S = 0]$. We discuss this sort of non-Bayesian updating rule for society in the Conclusion.

[23] Recall the assumption that, at each observable x, the college's acceptance probability must be weakly increasing in the test score. In fact, we can show that dropping this monotonicity assumption would not change our results. But it is both plausible for the setting and simplifies the student submission behavior. The monotonicity assumption can also be microfounded by allowing "free disposal" of test scores, i.e., a student with test score t can costlessly reduce it to any value less than t.

discussed, this strategy guarantees a student the highest chance of admission. While there may be other optimal student strategies (when submitting a test score t would lead to the same acceptance probability as not submitting),[24] a student can safely follow the strategy we focus on even if they do not know which (monotonic) acceptance rule the college is using.

Of course, while the strategy is robust to a student's uncertainty over the college's acceptance rule, it is sensitive to the student's knowledge about their imputed test score. In the real world, students face uncertainty about about how colleges treat missing test scores. Our model makes an admittedly strong assumption of "equilibrium knowledge" here. One interpretation of our model is that it captures a hypothetical world in which test-optional colleges ask for test scores on the last line of an electronic college application, and that same line states the score that would be imputed if the student chose not to submit.

5. Test-Mandatory Admissions

In a test-mandatory regime, the college faces a simple problem, as both the college and society always know the student's score. In light of social pressure, the college simply maximizes its ex-post utility for each (x, t); its admission decision is determined by the ex-post bar.

Proposition 2. *In a test-mandatory regime, the college admits a student with observable x if $u^*(x, t) > 0$ (equivalently, $t > \underline{t}^*(x)$) and rejects the student if $u^*(x, t) < 0$ (equivalently, $t < \underline{t}^*(x)$).*

As the social-pressure intensity parameter δ increases, the college becomes less selective at observable x if, based on its underlying utility, it is more selective than society ($\underline{t}^c(x) > \underline{t}^s(x)$), and conversely if it is less selective than society. Plainly, the student with observable x benefits in the former case and is harmed in the latter case.[25]

[24] In particular, when $t = \tau(x)$, the student is necessarily treated identically regardless of whether they submit; the behavior of these student types is immaterial if there are no mass points in the score distribution at $t = \tau(x)$.

[25] Benefit/harm here is in the sense of set inclusion. For example, suppose the college is more selective than society at x. Then a student with that observable may be rejected when social pressure intensity is low, and admitted when intensity is high; or they may receive the same outcome at both intensities.

6. Test-Optional Admissions

6.1. Optimal Acceptance Rule

In a test-optional regime, our college has two instruments: the imputation rule and the acceptance rule. Only the imputation rule affects students' score submission, and in turn the college's and society's information. Moreover, the only decision that students make is whether to submit their score. So, no matter the imputation rule, the college's optimal acceptance rule simply maximizes its ex-post utility given students' submission behavior. Formally, recalling that $L(\tau(x)|x)$ is the average test score of non-submitters with observable x given the imputation $\tau(x)$:

Lemma 1. *Consider a test-optional regime with any imputation rule τ. The college has an optimal acceptance rule in which a student with observable x and imputed/submitted score $\hat{t}$ is accepted if (i) $\hat{t} > \tau(x)$ and $u^*(x, \hat{t}) > 0$ or if (ii) $\hat{t} = \tau(x)$ and $u^*(x, L(\tau(x)|x)) > 0$, and is rejected otherwise.*

Any optimal admission rule must have the college making ex-post optimal decisions on path. The lemma's acceptance rule also specifies rejecting any student who has a test score below the imputed level but who chooses, off path, to submit. When the non-submitters are accepted, we could replace this behavior with any other monotonic rule and the outcome would be the same. When the non-submitters are rejected, though, monotonicity of the admission rule requires the college to also reject any score submission below the imputed score. In this latter case, commitment to the policy may be necessary: off path, the college may be rejecting students that it ex-post prefers to accept. For example, suppose test scores at some observable x are distributed uniformly between 0 and 100, and the imputation is $\tau(x) = 50$. Students with scores between 0 and 50 don't submit, leading to an average score of 25 for non-submitters. If the college's ex-post bar for acceptance is in between 25 and 50, say $\underline{t}^*(x) = 40$, then the college will reject the non-submitters. The college must then reject all off-path submissions of scores below 50, including—ex-post suboptimally—those above its ex-post bar of 40.

6.2. Flexible Imputation

We now turn to studying optimal admission policies under flexible imputation. Clearly, the college can ensure that it is no worse off than under test mandatory: after all, the imputation rule $\tau(\cdot) = -\infty$ ensures that all students submit their scores. But when and how can the college do better?[26]

In choosing its imputation $\tau(x)$ for some observable x, the college trades off making better admission decisions with reducing disagreement cost. Raising $\tau(x)$ leads fewer students to submit their test scores. The cost is that the college now has less information with which to make admissions decisions. The benefit is that by pooling together a larger set of test scores (those of the non-submitters), the college can reduce the disagreement cost it bears with society, as we saw in Section 3. In particular, consider two students who are both rejected or both accepted. If their test scores are either both below society's bar $\underline{t}^s(x)$ or both above, the disagreement cost is the same regardless of whether these students submit their scores or are pooled together. But if these students are on opposite sides of society's bar, then the disagreement cost is lower when the students are pooled together.

When solving for the optimal admissions policy, the college's problem is separable across observables. That is, we can optimize at each observable x and then "stitch" together the solutions across x's to get the globally optimal admission policy.

Given some fixed x, it is useful to consider separately the case in which the college is less selective than society ($\underline{t}^c(x) < \underline{t}^*(x) < \underline{t}^s(x)$) and the case in which it is more selective ($\underline{t}^s(x) < \underline{t}^*(x) < \underline{t}^c(x)$).[27] For both cases, we will assume that the imputation level $\tau(x)$ is set such that any submitted score $t > \tau(x)$ is accepted. This is without loss: if the college were to reject imputed/submitted scores up to some threshold $t' > \tau(x)$,[28] then it could instead raise the imputation level to t', still reject non-submitters, and now accept all submitted scores. This alternative policy leads to the same admission decisions but weakly lowers disagreement costs by pooling a superset of scores. Note that, by Lemma 1, optimally accepting any $t > \tau(x)$ implies $\tau(x) \geq \underline{t}^*(x)$.

[26] Lemma 1 says that given an imputation rule, admission decisions are made to maximize the ex-post utility (on path). We caution, however, that the lemma does *not* imply that solving for the optimal imputation rule is a problem of "Bayesian Persuasion" (Kamenica and Gentzkow, 2011)—even with the constraint of information being generated by an imputation rule—in which the receiver's decisions are determined by the ex-post utility and the sender has some utility function over the "unknown state" t and the receiver's decision. The reason is that, as illustrated in Section 3, different information structures can lead to different disagreement costs even when the same set of students is admitted.

[27] The remaining case, $\underline{t}^c(x) = \underline{t}^*(x) = \underline{t}^s(x)$, is trivial, as there is no disagreement at the observable x. The first-best is achieved when the college uses imputation $\tau(x) = \underline{t}^*(x)$ and accepts a student if and only they submit a score $t > \tau(x)$.

[28] Lemma 1 implies that we can restrict attention to deterministic (and monotonic) acceptance rules; each such rule is described by some threshold such that a student is admitted if and only if they submit a score above that threshold.

College is less selective than society. When the college is less selective, setting $\tau(x) = \underline{t}^*(x)$ and rejecting non-submitters replicates not only the test-mandatory admission decisions, but also the college's test-mandatory payoff. This is because all of the scores being pooled together are below society's acceptance threshold $\underline{t}^s(x)$. Furthermore, the college does worse if it sets $\tau(x) > \underline{t}^*(x)$ and then rejects non-submitters: it is now rejecting students that it preferred to accept even if it had to pay a disagreement cost to do so. Altogether, if the college rejects non-submitters, then it cannot improve on setting $\tau(x) = \underline{t}^*(x)$ and replicating the test-mandatory outcome.

The college might improve on test mandatory, however, by accepting non-submitters at some observable. Monotonicity of the acceptance rule means that the college would then accept all students with this observable. With all of these students being accepted, the college would minimize disagreement costs by setting the imputation level to infinity, so that none of these students submit scores.[29] Of course, relative to test mandatory, the college would then be admitting too many low-scoring students. Hence:

Proposition 3. *Consider flexible imputation and some observable x. When the college is less selective than society ($\underline{t}^c(x) < \underline{t}^*(x) < \underline{t}^s(x)$), it is optimal for the college to either:*

1. *Impute $\tau(x) = \infty$ and accept students regardless of imputed/submitted score $\hat{t}$; or*
2. *Replicate the test-mandatory outcome by imputing $\tau(x) = \underline{t}^*(x)$, rejecting students with imputed/submitted score $\hat{t} \leq \underline{t}^*(x)$, and accepting students with $\hat{t} > \underline{t}^*(x)$.*

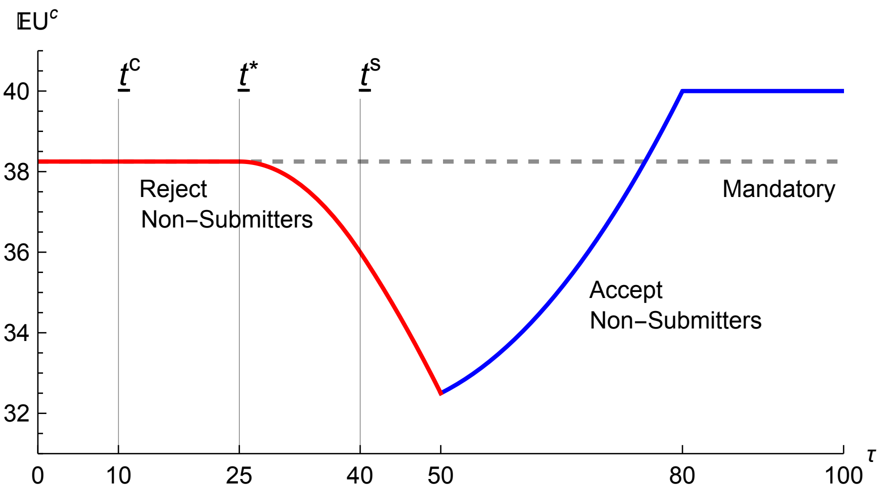
Figure 3a and Figure 3b illustrate the two possibilities.

College is more selective than society. Let us turn to observables at which the college is more selective than society. Unlike when the college is less selective, the college can improve on test mandatory by imputing the ex-post optimal bar, rejecting non-submitters, and accepting submitters. Pooling together the scores of all the rejected students now reduces disagreement cost because society prefers to reject some of those students (those with $t < \underline{t}^s(x)$) and accept others ($t \in (\underline{t}^s(x), \underline{t}^*(x))$. In general, the college might do even better by choosing a higher imputation, altering the set of admitted students.

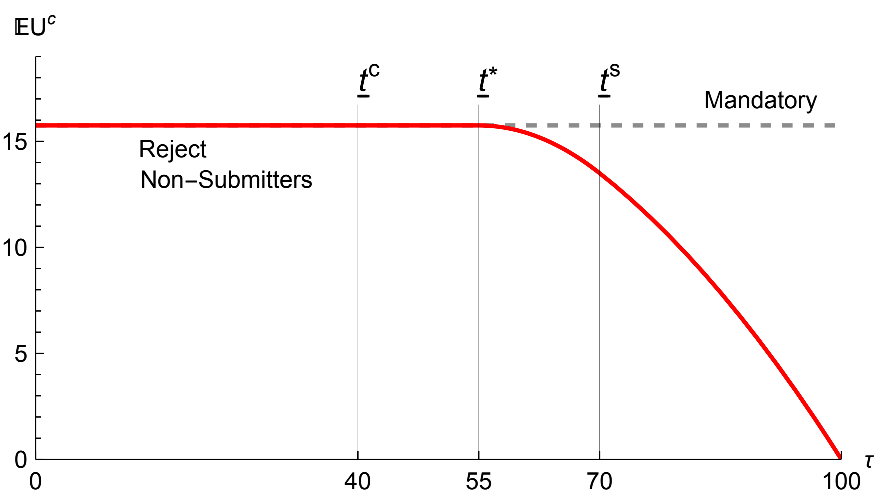
Proposition 4. *Consider flexible imputation and some observable x. When the college is more selective than society ($\underline{t}^s(x) < \underline{t}^*(x) < \underline{t}^c(x)$), the college optimally chooses imputation*

[29] If $\mathbb{E}[t|x] > \underline{t}^s(x)$, then any large enough $\tau(x)$ would also be optimal as that would ensure that society prefers to accept the pool of non-submitters, resulting in zero disagreement cost.

(a) The college's payoff is maximized by setting $\tau = \infty$ and accepting non-submitters.



(b) The college's payoff is maximized by setting $\tau = \underline{t}^*$ and rejecting non-submitters.



Fix some observable x. The distribution of test scores given x is $t \sim U[0, 100]$. Utilities are $u^c(x, t) = t - \underline{t}^c$, $u^s(x, t) = t - \underline{t}^s$, and $\delta = 1$, implying $\underline{t}^* = \frac{1}{2}(\underline{t}^c + \underline{t}^s)$.

Figure 3 – College's test-optional payoff as a function of the imputed test score, when the college is less selective than society.

$\tau(x) \in [\underline{t}^*(x), \underline{t}^c(x)]$; *it rejects students with imputed/submitted score* $\hat{t} \leq \tau(x)$ *and it accepts students with* $\hat{t} > \tau(x)$.

The proposition's proof establishes that the optimal $\tau(x)$ is determined by comparing the function $L(\cdot|x)$, which gives the average test score of non-submitters, with society's bar $\underline{t}^s(x)$. Specifically, letting t° be the score at which $L(t^\circ|x) = \underline{t}^s(x)$,[30] the college sets

$$\tau(x) = \begin{cases} \underline{t}^*(x) & \text{if } t^\circ \leq \underline{t}^*(x) \\ t^\circ & \text{if } t^\circ \in (\underline{t}^*(x), \underline{t}^c(x)) \\ \underline{t}^c(x) & \text{if } t^\circ \geq \underline{t}^c(x). \end{cases}$$

For the intuition behind Proposition 4, let us consider the case in which $t^\circ \in (\underline{t}^*(x), \underline{t}^c(x))$. The optimal admissions policy then involves setting $\tau(x) = t^\circ$, rejecting non-submitters, and accepting submitters.[31] This imputation makes society indifferent over whether to accept the pool of non-submitters, as their expected test score is $L(\tau(x)|x) = \underline{t}^s(x)$. Moreover, society wants to accept any submitter, since their score is $t \geq \tau(x) > \underline{t}^s(x)$. So the disagreement cost is zero. Now consider a marginal change of the imputation level $\tau(x)$ from t° to t'. On the one hand, raising the imputation level $\tau(x)$ to $t' > t^\circ$ cannot help. Doing so and then rejecting the larger pool[32] yields the same set of admitted students and the same disagreement cost as setting $\tau(x) = t^\circ$ and then rejecting students with scores $t \in (t^\circ, t']$; there is no benefit from pooling the scores of these marginal students with those below t° since society does not strictly prefer to reject the pool of non-submitters. But the latter policy is dominated by the originally proposed policy of setting $\tau(x) = t^\circ$ and accepting students with scores $t \in (t^\circ, t']$, as they provide positive ex-post utility. On the other hand, lowering the imputation level to $t' < t^\circ$ also cannot help. Doing so and then rejecting students with $t \in (t', t^\circ]$ yields the same set of admitted students but higher disagreement cost, since society strictly prefers to reject the pool of non-submitters when $\tau(x) = t'$; doing so and then accepting applicants

[30] If $L(\cdot|x)$ is everywhere below $\underline{t}^s(x)$, let $t^\circ = -\infty$, and if $L(\cdot|x)$ is everywhere above $\underline{t}^s(x)$, let $t^\circ = \infty$. Otherwise, for simplicity of discussion here, we assume that there is a unique solution to $L(t^\circ|x) = \underline{t}^s(x)$, as is guaranteed when the distribution of $t|x$ is atomless and has interval support.

[31] To see why this acceptance policy is optimal given the imputation $\tau(x) = t^\circ$, notice that disagreement cost is zero regardless of whether non-submitters are accepted or rejected, because $L(\tau(x)|x) = \underline{t}^s(x)$. Since $t^\circ < \underline{t}^c(x)$, it is better for the college to reject non-submitters at this imputation level. It is better to accept submitters, on the other hand, because $t^\circ > \underline{t}^*(x)$.

[32] For any marginal change, the college will still prefer to reject the pool, since the expected test score of non-submitters is strictly below $\underline{t}^*(x)$.

with $t \in (t', t^\circ]$ yields a worse set of admitted students from the college's perspective, as $t < \underline{t}^c(x)$, but identical (zero) disagreement cost.

Figure 4 illustrates two examples of Proposition 4. Panel 4a shows a case in which the optimal $\tau(x)$ is in $(\underline{t}^*(x), \underline{t}^c(x))$. Panel 4b shows a case in which the optimal $\tau(x)$ is equal to $\underline{t}^c(x)$, and the college achieves its first best: it accepts students if and only if $t > \underline{t}^c(x)$, and it incurs no disagreement cost. Although not illustrated in the figure, it is also possible that the optimal $\tau(x) = \underline{t}^*(x)$.

How are students affected? The outcomes of a college-optimal admissions policy under test-optional admissions have clear-cut and intuitive implications for student welfare relative to the outcomes of test-mandatory admissions.

Students benefit from test optional at observables where the college is less selective than society. Specifically, at these observables, Proposition 3 implies that either the college replicates the test-mandatory admissions, or it admits all students. In the latter case, high-scoring students (with $t > \underline{t}^*(x)$)) are indifferent between test optional and test mandatory, but low-scoring students ($t < \underline{t}^*(x)$) strictly benefit.

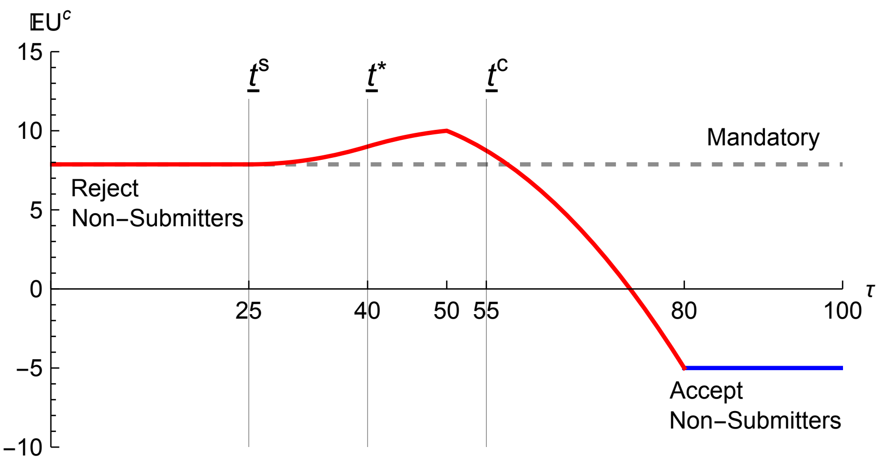
By contrast, students are harmed by test optional at observables where the college is more selective than society. Specifically, Proposition 4 implies that when the optimal imputation is $\tau(x) = \underline{t}^*(x)$, the test-mandatory outcome is replicated for all students. But when the optimal imputation is $\tau(x) > \underline{t}^*(x)$, intermediate-scoring students (with $t \in (\underline{t}^*(x), \tau(x)]$) are rejected under test optional while they would have been accepted under test mandatory, whereas the outcomes for low- and high-scoring students ($t < \underline{t}^*(x)$ and $t > \tau(x)$, respectively) are unchanged.

6.3. Restricted Imputation

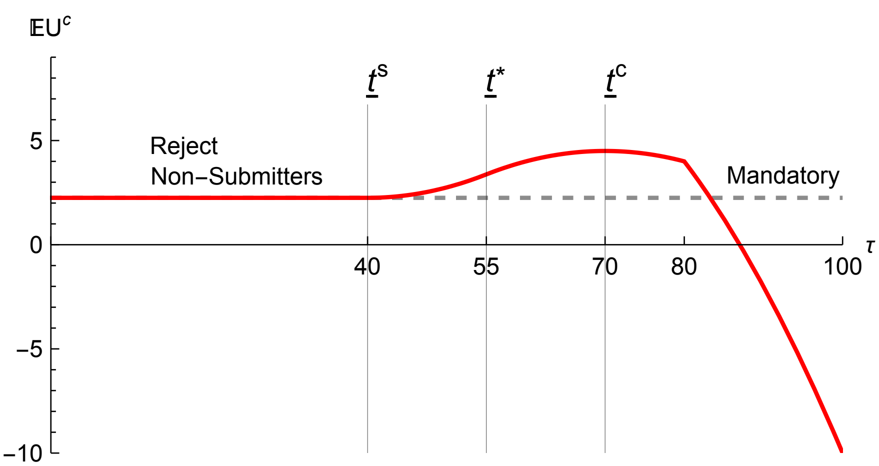
We now turn to test-optional admissions when the imputation rule $\tau(\cdot)$ is exogenously given. The college only optimizes over its acceptance rule. As discussed in Subsection 4.3.1, many colleges announce publicly a policy that we interpret as no adverse inference imputation. Restricted imputation also subsumes test-blind admissions, as that is equivalent to the imputation rule $\tau(\cdot) = \infty$.

The optimal acceptance rule. As with flexible imputation, we can solve for the optimal acceptance rule under restricted imputation separately for each observable x. We can readily

(a) The college's payoff is maximized by setting $\tau \in (\underline{t}^*, \underline{t}^c)$ and rejecting non-submitters.



(b) The college achieves its first best by setting $\tau = \underline{t}^c$ and rejecting non-submitters.



Fix some observable x. The distribution of test scores given x is $t \sim U[0, 100]$. Utilities are $u^c(x,t) = t - \underline{t}^c$, $u^s(x,t) = t - \underline{t}^s$, and $\delta = 1$, implying $\underline{t}^* = \frac{1}{2}(\underline{t}^c + \underline{t}^s)$.

Figure 4 – College's test-optional payoff as a function of the imputed test score, when the college is more selective than society.

deduce an optimal acceptance rule using Lemma 1:

Proposition 5. *Consider some observable x and imputation level $\tau(x)$. An optimal acceptance rule for the college is as follows. A student with submitted score $t > \tau(x)$ is accepted if and only if $t > \underline{t}^*(x)$; a student with submitted score $t < \tau(x)$ is rejected; and a student with imputed/submitted score $\tau(x)$ is accepted if and only if $L(\tau(x)|x) > \underline{t}^*(x)$.*

The proposition says that the college's acceptance rule on path is determined by comparing a student's expected score—the score itself if submitted, or $L(\tau(x)|x)$ if not submitted—with the ex-post bar. (Submission of $t \leq \tau(x)$ only occurs off path.) Whether the college is more or less selective than society does not affect the college's optimal acceptance rule; the distinction matters under flexible imputation (Subsection 6.2) only because it affects the optimal imputation.

To better understand the admissions policy under restricted imputation, we can consider exogenously varying the imputation $\tau(x)$ at a given x. In that case, there is a threshold $T(x)$ such that if $\tau(x) < T(x)$, then it is optimal to reject non-submitters, whereas if $\tau(x) > T(x)$, then it is optimal to accept non-submitters.[33] Figure 3 and Figure 4 illustrate, at some fixed observable x, how the college's payoff and its decision of whether to accept non-submitters may depend on the imputation level.

How are students affected? Whether students at an observable x benefit from test optional under restricted imputation (relative to test mandatory) depends on how the imputation level $\tau(x)$ and the lower expectation $L(\tau(x)|x)$ compare with the ex-post bar $\underline{t}^*(x)$. To understand how these vary with observables, we must make further assumptions.

Accordingly, define a *path of increasing observables* as a parameter $q \in [0, 1]$ determining the observable $x(q)$, with the following properties: (i) $u^c(x, t) = v^c(x) + t$ and $u^s(x, t) = v^s(x) + t$, with $v^c(x(q))$ and $v^s(x(q))$ both increasing in q; (ii) the distribution of $t|x(q)$ is MLRP-increasing in q,[34] and (iii) $\tau(x(q))$ is increasing in q. Property (i) guarantees that the ex-post bar $\underline{t}^*(x(q))$ is decreasing in q, while properties (ii) and (iii) guarantee that the

[33] $T(x) \geq \min\{\underline{t}^s(x), \underline{t}^c(x)\}$, implying that if the imputation is below both the college's and society's bars, then it is optimal to reject non-submitters. In fact, $T(x) = \infty$ if $\mathbb{E}[t|x] \leq \underline{t}^*(x)$. If $\mathbb{E}[t|x] > \underline{t}^*(x)$, then so long as the distribution of test scores conditional on x has full support and is atomless, $T(x)$ is the unique solution to $L(T(x)|x) = \underline{t}^*(x)$.

[34] I.e., for each x, there is a test-score density/probability mass function $f(t|x)$ such that the monotone likelihood ratio property (MLRP) holds: $q > q'$ and $t > t'$ imply $f(t|x(q))f(t'|x(q')) \geq f(t'|x(q))f(t|x(q'))$.

expected test score conditional on not submitting, $L(\tau(x(q))|x(q))$, is increasing in q.[35] Note that property (iii) is implied by property (ii) when τ is the no adverse inference rule defined by $\tau(x) = \mathbb{E}[t|x]$.

A path of increasing observables yields straightforward implications for which students benefit or are harmed by test-optional admissions with restricted imputation, as can be seen using Figure 5. Students with "low" observables (those with q such that $\tau(x(q)) < \underline{t}^*(x(q))$) are unaffected. Under both test-optional and test-mandatory admissions, these students are accepted if and only if their test score is above the ex-post bar. Students with "medium" observables (q such that $\tau(x(q)) > \underline{t}^*(x(q))$ but $L(\tau(x(q))|x(q)) < \underline{t}^*(x))$ are harmed. If their test score is low ($t \leq \underline{t}^*(x)$) then they are rejected under both regimes, and if their test score is high ($t > \tau(x)$) then they are admitted either way. But if their score is in between, they are accepted under test mandatory and rejected under test optional. Finally, students with "high" observables (q such that $L(\tau(x(q))|x(q)) > \underline{t}^*(x)$) benefit. If their test score is high ($t > \underline{t}^*(x(q))$) then they are admitted under both regimes. If their score is low ($t \leq \underline{t}^*(x(q))$), they are rejected under test mandatory and are accepted without submitting their score under test optional.

Restricted vs. flexible imputation. Under restricted imputation, given a path of increasing observables, students with good observables benefit under test optional while students with medium observables are harmed. By contrast, under flexible imputation, it is students with observables at which the college is less selective than society that benefit and those with observables at which the college is more selective that are harmed. Looking back at Figure 2, we see that these predictions may go in the same qualitative direction, or may go in opposite directions.[36] In the figure's left panel, where the college weights tests less than society, the college is less selective than society at higher observables. Hence, students with higher observables benefit from test optional under both flexible and restricted imputation. In Figure 2's right panel, where the college weights tests more than society, we have the re-

[35] $\underline{t}^*(x(q))$ is decreasing in q from the definition of $\underline{t}^*$ and that property (i) immediately implies that $\underline{t}^c(x(q)$ and $\underline{t}^s(x(q))$ are both decreasing in q. $L(\tau(x(q))|x(q))$ is increasing in q given properties (ii) and (ii) because of the well-known fact that if $t \sim G$, $t' \sim G'$, and G' MLRP-dominates G, then for any two thresholds $\hat{t} \leq \hat{t}'$, it holds that $\mathbb{E}[t|t < \hat{t}] \leq \mathbb{E}[t'|t' < \hat{t}']$.

[36] For the example in Figure 2, utilities were defined as $u^i(x,t) = a^i + x + w^i t$, with $x \in \mathbb{R}$ and $w^i > 0$; we can rescale these utilities as $u^i(x,t) = \frac{a^i}{w^i} + \frac{x}{w^i} + t$. Then take $x(q)$ to be any increasing function. We have a path of increasing observables as long as test scores are MLRP-increasing in q and $\tau(x(q))$ is increasing as well. The simplest case satisfying both requirements is when the distribution of test scores is independent of observables and $\tau(x) = \mathbb{E}[x|t] = \mathbb{E}[t]$ is the no adverse inference imputation rule.

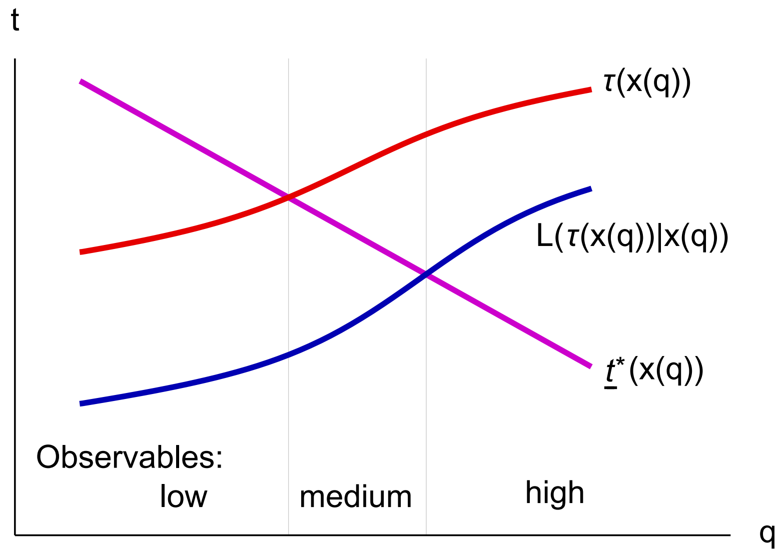


Figure 5 – Restricted imputation along a path of increasing observables.

verse: the college is more selective at higher observables. In this case, the predictions about which students benefit from test optional flip depending on whether imputation is flexible or restricted.

Under flexible imputation, the college always benefits (at least weakly) from going test optional. Notably, this benefit accrues at every observable x. By contrast, under restricted imputation, the college may or may not benefit from going test optional at any specific x—it depends on the imputation level $\tau(x)$. Consequently, aggregating across observables, the college may or may not benefit from going test optional.

We now turn to an extended example in which we make specific assumptions that allow us to say more about when a college benefits from not seeing test scores absent flexible imputation. The example is in the context of a college's response to a ban on affirmative action.

7. Effects of a Ban on Affirmative Action

This section illustrates how, within our framework, banning affirmative action can push a college from test-mandatory admissions to test-blind admissions.[37] As discussed in the

[37] We study test blind rather than test optional for simplicity; as noted previously, test blind is equivalent to test optional when non-submitters are imputed sufficiently high test scores.

Introduction, this is consistent with the view espoused by commentators.

In a nutshell, our idea is as follows. There are two groups of students. Relative to society, the college has a preference for admitting students from the group that has lower test scores on average. When affirmative action is allowed, the college can treat applicants from different groups differently. In that case, the college always prefers test mandatory, since observing test scores lets it better determine which applicants to accept from each group. But if affirmative action is banned, the college must use a single admissions rule for both groups. Now the college wants to put less weight on tests than does society, since a low test score is associated with being from the college's favored group. This disagreement can push the college to want to switch to test blind.

7.1. A Model of Affirmative Action

There are two potentially observable non-test dimensions, $x = (x_0, x_1)$. Dimension x_0 is binary, with realizations in $\{r, g\}$ (red and green). Dimension x_1, which may represent some aggregate of GPA and/or extra-curricular achievement, takes continuous values in $\mathbb{R}$. For simplicity, test scores are binary, with values normalized to 0 and 1.

The college and society have identical preferences over all factors except for the type dimension x_0. Society does not care about this dimension, but all else equal, the college wants to admit green types over red types.[38] Specifically, extending our leading linear specification discussed at the end of Subsection 4.2, we assume that

$$\begin{aligned} u^s(x,t) &= x_1 + t, \\ u^c(x,t) &= x_1 + t + \beta \mathbb{1}_{x_0=g} - c, \end{aligned}$$

with $\beta > c > 0$, and $\mathbb{1}_{x_0=g}$ an indicator for green types. The parameter β scales how much of a bonus the college gives to green types over red types. The parameter c is not essential to our analysis, but it allows for the college and society to have different test-score bars for both red and green students. It can be interpreted as the (opportunity) cost for a college of admitting any student. We have normalized the analogous constant in society's utility to zero. The assumption $\beta > c > 0$ implies that the college has a lower test-score bar than society for green types and a higher one for red types. Note that the the college's ex-post

[38] We could allow for society to have preferences over a student's x_0 dimension as well; what is important is that the college favors green types more than society does.

utility is

$$u^*(x,t) = x_1 + t + \frac{\beta}{1+\delta}\mathbb{1}_{x_0=g} - \frac{c}{1+\delta}.$$

Let $x_0 = g$ with probability $q \in (0,1)$ and $x_0 = r$ with probability $1-q$. We assume that the distribution of test scores depends on x only through x_0: $\Pr(t=1|x=(x_0,x_1)) = p_{x_0} \in (0,1)$. Our primary interest is in the case of $p_r > p_g$, meaning that green types, which are favored by the college, have a worse distribution of test scores. This may correspond to green students being an underrepresented demographic group, for instance. But we also allow for the opposite case of $p_r < p_g$, in which the college's favored group has a better test score distribution. Here, green students may correspond to those from rich families, who have better access to test preparation, and are favored by the college because of donor considerations. If the green students correspond to legacy applicants, it may be that either $p_r < p_g$ or $p_r > p_g$.

We take x_1 to be independent of both x_0 and t. For tractability, we also assume that x_1 is uniformly distributed over a large-enough interval. Specifically, $x_1 \sim U[\underline{x}_1, \overline{x}_1]$, with $\underline{x}_1 < c - \beta - 1$ and $\overline{x}_1 > c$. The inequality on $\underline{x}_1$ guarantees that there are students with x_1 low enough that neither the college nor society wants to admit them, even if they are otherwise as desirable as possible ($x_0 = g$ and $t = 1$). The inequality on $\overline{x}_1$ guarantees that there are students with x_1 high enough that the college and society want to admit them even if they are otherwise as undesirable as possible ($x_0 = r$ and $t = 0$).

We will consider the college's choice over whether to be test mandatory or test blind in two observability regimes. First, we allow both dimensions of x to be observable, which we call *affirmative action allowed*. Then we consider only x_1 to be observable, with the dimension x_0 unobservable; we call this regime *affirmative action banned*. We interpret the switch from the first to the second regime as a policy change where society—which does not intrinsically care about x_0—bans the use of that dimension in admissions. This may represent a law or court decision forbidding the use of race or legacy status in admissions.[39]

[39] Note that we assume that when x_0 is unobservable to the college, it is also unobservable to society. While society does not value x_0 directly, the observability of x_0 to society could still matter for the calculation of the college's social costs. This is because, if society can observe x_0 but cannot observe test scores, then it would expect a different test score for green students (p_g) than red students (p_r). We assume that a law preventing the college from making inferences of this form also stop society from making/penalizing the college based on such inferences.

7.2. Results

Affirmative action allowed. Consider first the case when affirmative action is allowed.

Under test mandatory, the college can choose a distinct threshold of x_1 above which to admit students at each (x_0, t) pair.[40] This threshold is determined by setting the ex-post utility to 0. Since the college favors green students, its x_1 threshold will be lower by $\beta/(1+\delta)$ for green students than for red students at each score level t. From society's perspective, the college uses an x_1 threshold that is too low for green students and too high for red students—but crucially, the gap between society's preferred threshold and what the college uses does not vary with t.[41]

Under test blind, the college chooses an admissions threshold on dimension x_1 that depends on the student's type x_0 but not the test score t. However, x_0 is informative about t: the college and society evaluate students of type x_0 as if they have the expected test score $\mathbb{E}[t|x_0] = p_{x_0}$. If $p_r > p_g$, the college's preference for green students is countered by the fact that green students have lower test scores on average than red students. So the college will now use a lower x_1 threshold for green students than red students only if the preference parameter β is sufficiently large: specifically, if and only if $\beta/(1+\delta) > p_r - p_b$. Regardless, the gap between the college's chosen x_1 threshold and society's preferred threshold is the same as under test mandatory, for any test score t—that gap did not depend on the test score, and utilities are linear in the test score.

We can now establish:

Proposition 6. *If affirmative action is allowed, then the college prefers test mandatory to test blind.*

The reason is that going test blind leads to a set of students that the college prefers less, but in the current specification there is never a countervailing benefit of reducing disagreement cost. The latter point stems from two sources. First, as noted above, for any given x_0 type (and test score, under test mandatory), the gap between society's preferred x_1 threshold and what the college uses is independent of the regime, even though these thresholds do shift across regimes. Second, our assumption of a uniform distribution of x_1 means that the total disagreement cost for students of a given x_0 type (at a given test score, or averaging over test scores) only depends on the size of the gap.

[40] Since we will be comparing test mandatory with test blind, it turns out to be convenient for our analysis to take the perspective of x_1 admissions thresholds rather than test score thresholds.

[41] The gap is $(\beta - c)/(1+\delta)$ for green students and $c/(1+\delta)$ for red students.

Affirmative action banned. Now consider the case when affirmative action is banned.

Under test mandatory, the observed test score is informative about a student's type x_0. Specifically, since there are a fraction q of green types in the population and the probability of test score $t = 0$ for a student of type x_0 is $1 - p_{x_0}$, we compute the probability of a student being green conditional on $t = 0$ as

$$P_g^0 := \Pr(x_0 = g | t = 0) = \frac{q}{q + (1-q)\frac{1-p_r}{1-p_g}}.$$

Analogously, conditional on $t = 1$, the probability of a green type is

$$P_g^1 := \Pr(x_0 = g | t = 1) = \frac{q}{q + (1-q)\frac{p_r}{p_g}}.$$

Let $\Delta := P_g^0 - P_g^1$ be the difference between these two quantities, i.e., a low test score implies a Δ higher probability of $x_0 = g$ than a high test score. Note that $\Delta > 0$ if $p_r > p_g$, whereas $\Delta < 0$ if $p_r < p_g$. Based on the inference of x_0 from t, the college's underlying utility gives a bonus of $\beta\Delta$ to students with low test scores relative to those with high scores. As a result, the college now values a high test score $1 - \beta\Delta$ units higher than a low score, whereas society still values it 1 unit higher. That is, unlike when affirmative action is allowed, the gap between society's preferred x_1 admissions threshold and what the college chooses now varies with the test score.[42] We impose the assumption that $\beta\Delta < 1$, so the college still prefers students with higher test scores.

There is now an avenue for test blind to help the college. Under test blind, since the college evaluates all students as having $\Pr(x_0 = g) = q$ and $\mathbb{E}[t] = qp_g + (1-q)p_r$, it is as if the college's utility from any student is $x_1 + \mathbb{E}[t] + q\beta - c$. Analogously, it is as if society's utility from any student is $x_1 + \mathbb{E}[t]$. If $c = q\beta$, which means the college and the society seek to admit the same number of students overall, then it is as if their utilities agree, and the college implements its preferred admissions policy—subject to being test blind and no affirmative action—at zero disagreement cost. More generally, the disagreement cost is always lower under test blind than test mandatory. Whether the reduced disagreement cost

[42] Absent affirmative action, it is as if the college's underlying utility from a student is $x_1 + t + \beta P_g^t - c$, and so the college's gain from a student with test score $t = 1$ over $t = 0$ is $1 + \beta P_g^1 - \beta P_g^0 = 1 - \beta\Delta$. Given its underlying utility, the college's ex-post utility from a student is $x_1 + t + \frac{\beta P_g^t - c}{1+\delta}$. The gap between the college's chosen x_1 admissions threshold with society's preference is the term $\frac{\beta P_g^t - c}{1+\delta}$, which varies with t so long as $P_g^0 \neq P_g^1$, or equivalently $\Delta \neq 0$.

outweighs the allocative loss from being test blind depends on parameters, specifically the intensity of social pressure δ and the college's bonus to low-scoring students $\beta\Delta$.

Proposition 7. *Suppose affirmative action is banned. If* $(1+\delta)(2\beta\Delta - 1) \geq (\beta\Delta)^2$*, then the college prefers test blind, and otherwise the college prefers test mandatory.*

Recall we assume $\beta\Delta < 1$. Proposition 7 implies that if $\beta\Delta \leq 1/2$, the college always prefers test mandatory: the allocative losses ("admission mistakes") from not observing test scores are larger than those from simply implementing society's preferred decision rule and incurring no disagreement. When $\beta\Delta \in (1/2, 1)$, there is a trade-off, and test blind will be preferred if the intensity of social pressure, δ, is sufficiently large. The following corollary develops this and other comparative statics.

Corollary 1. *Suppose that affirmative action is banned (x_0 is unobservable) and that a low test score is associated with $x_0 = g$ ($\Delta > 0$).*

1. *There is some $\beta^* \in \left(\frac{1}{2\Delta}, \frac{1}{\Delta}\right)$ such that the college prefers test mandatory when $\beta < \beta^*$ and prefers test blind when $\beta > \beta^*$.*
2. *There is some $\Delta^* \in \left(\frac{1}{2\beta}, \frac{1}{\beta}\right)$ such that the college prefers test mandatory when $\Delta < \Delta^*$ and prefers test blind when $\Delta > \Delta^*$.*
3. *There is some δ^* such that the college prefers test mandatory when $\delta < \delta^*$ and prefers test blind when $\delta > \delta^*$. If $\beta\Delta \leq 1/2$, then $\delta^* = 0$ (so the college prefers test mandatory for all δ); if $\beta\Delta \in (1/2, 1)$, then $\delta^* > 0$ (so the college prefers test blind if and only if δ is large enough).*

7.3. Society's Preferences

We now consider society's payoff under different affirmative action and testing regimes. Society's realized utility for an individual student is $Au^s(x,t)$, where the dummy variable A indicates whether the student is admitted. We assume that society's objective is to maximize its expected utility across the pool of applicants.

Proposition 8. *Society's preferences over affirmative action and testing regimes are as follows:*

1. *Fixing the testing regime as mandatory or blind, society prefers banning affirmative action to allowing affirmative action.*

2. *Fixing affirmative action as banned or allowed, society prefers test mandatory to test blind.*

3. *Suppose society chooses the affirmative action regime and then the college chooses the testing regime. Then banning affirmative action can harm society. In particular, if* $\beta\Delta \in (1/2, 1)$*, there exist thresholds* $0 < \underline{\delta} \leq \overline{\delta} < \infty$ *such that (i) if affirmative action is banned, the college chooses test blind if* $\delta > \underline{\delta}$*, and (ii) society is harmed by banning affirmative action if* $\delta > \overline{\delta}$*, while it benefits if* $\delta < \overline{\delta}$*.*[43]

The first two parts of the proposition are intuitive, since society does not want the admission decision to depend on whether a student is red or green (which suggests part 1) but does want the decision to depend on the test score (which suggests part 2). If society could choose both the testing and affirmative action regimes, it would ban affirmative action and choose test mandatory. However, part 3 of the proposition cautions that if society chooses the affirmative action regime and the college subsequently chooses the testing regime, society can be worse off by banning affirmative action. Specifically, when δ is large enough, banning affirmative action backfires because the college's response of going test optional results in a student pool that society likes less than under test mandatory and affirmative action allowed. Indeed, as δ gets arbitrarily large, society's payoff is arbitrarily close to society's first best when affirmative action is allowed and there is mandatory testing, while it is bounded away when affirmative is banned and the college goes test blind. But when δ is intermediate (between the thresholds $\underline{\delta}$ and $\overline{\delta}$ in Proposition 8 part 3), society is better off by banning affirmative even though it results in the college going test blind.[44]

8. Conclusion

Our paper begins by asking why a college would choose to obtain less information about students by using a test-optional (or test-blind) admissions policy. We formalize an impossibility result in Section 2: under a broad set of conditions, a college that can use test scores as it likes does at least as well with test-mandatory admissions. Our main contribution is to offer a resolution to this "puzzle": going test optional helps a college alleviate social pressure regarding the students it admits. Specifically, we introduce and solve a model of college

[43] If $\beta\Delta \leq 1/2$, the college never goes test blind, and so, by part 1 of the proposition, society always benefits from banning affirmative action.

[44] It is possible that $\underline{\delta} = \overline{\delta}$, in which case whenever a ban on affirmative action leads to test optional, society is harmed by the affirmative-action ban.

admissions in which a college faces costs from making admission decisions that an external observer, society, disagrees with. Society has the same information as the college, and society is Bayesian in how it assesses students who don't submit scores. The college commits to an imputation rule—stipulating, as a function of a student's observable characteristics, the test score assigned to non-submitters—and an acceptance rule specifying whether a student with any given observables and test score is admitted.

Our results in Subsection 6.2 establish that when a college can flexibly choose its imputation rule, a test-optional regime is always weakly better for the college than a test-mandatory one. Test optional is often strictly better, reducing the college's cost from social pressure and/or delivering a student body it likes more. In Subsection 6.3, we study restricted imputation rules. Here, we find that going test optional may or may not benefit a college. For both flexible and restricted imputation, we identify which students benefit and which students are hurt by test optional. In Section 7 we explore an extended example of restricted imputation, illustrating that our framework can explain how a ban on affirmative action can result in a college choosing to go test blind. We now close by discussing how our conclusions would be affected by some alternative modeling assumptions.

8.1. Capacity Constraints

Our model assumes that a college admits any student that provides it a utility above some fixed threshold, normalized to zero. This abstracts away from "capacity constraints": if a college accepts more applicants of one type, it may mechanically have to accept less applicants of other types.

In general, introducing a capacity constraint would affect our analysis, because in our model the number of students the college admits need not be the same under test optional as test mandatory. It could be that in our model going test optional benefits students of one group without affecting students in another group (e.g., this happens under flexible imputation if the college is less selective than society for the former group but equally selective for the latter group). But with a capacity constraint, if students from one group benefit from test optional, then students from some other group will necessarily be harmed. This externality could raise important equity concerns in practice.

8.2. Ex-post Optimal Acceptance

Next, suppose the college commits to evaluating non-submitters by their imputed score, but cannot commit to its acceptance rule. That is, the college admits students ex-post optimally given their imputed/submitted scores.

In this case, a college's acceptance decision is simply determined by whether a student's imputed/submitted score is above or below the ex-post optimal bar, $\underline{t}^*(x)$. Under flexible imputation, it follows from Proposition 3 that the outcome is unchanged at observables at which the college is less restrictive than society. But when the college is more restrictive, the college can no longer set $\tau(x) > \underline{t}^*(x)$ and reject non-submitters, which could have been optimal (Proposition 4); the problem now is that the college must accept students who submit scores above $\underline{t}^*(x)$. Consequently, the college now optimally sets $\tau(x) = \underline{t}^*(x)$ and rejects non-submitters.

This means that a test-optional college now accepts all the students it would under test mandatory, and possibly additional ones (if, for some observable x at which it is less selective than society, it chooses $\tau(x) = \infty$ and accepts all students with observable x). Hence, all students benefit, at least weakly, from test optional. Of course, this conclusion relies crucially on the college not having a capacity constraint.

8.3. Alternative Restricted Imputation Rules

The restricted imputation rule we have highlighted is that of no adverse inference: $\tau(x) = \mathbb{E}[t|x]$. There are at least two other rules that appear salient.

First, colleges' claims to not punish non-submitters can be interpreted as a promise to impute missing test scores as equal to those of an average submitted score: $\tau(x) = \mathbb{E}[t|x, S = 1]$. Notice that for any observable x, we cannot have a range of scores being submitted: a student with the lowest such score would, instead, not submit their score. Hence, this form of "equal treatment" effectively unravels to no student submitting their score.

Second, the reason that a college may not be able to flexibly impute missing scores is that it lacks commitment power, and instead it can only impute via Bayes rule: $\tau(x) = \mathbb{E}[t|x, S = 0]$. Now, for any x, if there is a range of scores not being submitted, a student with the highest such score would instead submit. Hence, this imputation rule effectively unravels to every student submitting their score.

The upshot, then, is that under either of these alternative forms of restricted imputation, test optional would collapse to either test blind or mandatory.

8.4. Non-Bayesian Society

We have assumed that society is Bayesian: it evaluates non-submitters ($S = 0$) with observable x as if they have the test score $t^s = \mathbb{E}[t|x, S = 0]$. This means that if students from certain groups have lower scores on average than others, then society accounts for that. We view Bayesian updating as a way of tying our hands, showing that our mechanism still goes through even when society can't be systematically misled. In practice, one source of non-Bayesianism could be that society is unwilling to judge students based on the test score distribution of their demographic group. For instance, if the observable vector $x = (x_0, x_1)$ has component x_1 corresponding to grades and component x_0 corresponding to demographics, society might evaluate non-submitters as having test score $t^s = \mathbb{E}[t|x_1, S = 0]$.

A college that faces such a non-Bayesian society may get an additional benefit from going test optional.

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Appendix

A. Proofs for Section 2 and Section 5

Proof of Proposition 1. Omitted, given the argument in the text after the proposition. □

Proof of Proposition 2. Under test mandatory, the student's score is always observed by both the college and society. So the college's problem for any observed (x, t) is to choose $A \in \{0, 1\}$ to maximize

$$AU^c(x, t, t, 1) + (1 - A)U^c(x, t, t, 0).$$

From the definition of the ex-post utility function $u^*(x, t)$ in (3), it is equivalent for the college to maximize $Au^*(x, t)$, which implies the result. □

B. Proofs for Section 6

Proof of Lemma 1. Fix test optional with some imputation rule τ. Consider a student with observable x and imputed/submitted score $\hat{t}$. Given our assumption that the student submits if they have score $t > \tau(x)$ and does not submit if $t \leq \tau(x)$, whether the imputed/submitted score $\hat{t}$ is on path or off path depends only on the support of the score distribution $F_{t|x}$. If $\hat{t}$ is off path, then any college acceptance decision is optimal. Since any $\hat{t} < \tau(x)$ is necessarily off path, it is optimal to reject such $\hat{t}$. There are two remaining cases:

1. $\hat{t} > \tau(x)$, and it is on path. Then the student must have submitted $\hat{t}$, and so by the logic of Proposition 2, it is optimal for the college to accept the student if $u^*(x, \hat{t}) > 0$ and reject the student otherwise.
2. $\hat{t} = \tau(x)$, and it is on path. Then $\hat{t}$ is an imputed score. By similar reasoning to that in Proposition 2, the expected utility gain from accepting these students types is proportional to the ex-post utility $u^*(x, L(\tau(x)|x))$, and so it is optimal to accept if that ex-post utility is positive and reject otherwise. □

We note that the resulting acceptance rule is monotonic, as $L(\tau(x)|x) \leq \tau(x)$ and $u^*(x, \cdot)$ is increasing.

We next state and prove a lemma that will be used in the proof of Proposition 3 and Proposition 4. We write $\mathbb{1}_Y$ to denote the indicator function for the event Y.

Lemma 2. *Fix observables x, and consider two possible imputation levels $\tau^l < \tau^h$.*[45]

1. *It holds that*

$$\begin{aligned}
&\mathbb{E}[d(x, L(\tau^h|x), A) \cdot \mathbb{1}_{t \le \tau^h}] \\
&\quad \le \mathbb{E}[d(x, L(\tau^l|x), A) \cdot \mathbb{1}_{t \le \tau^l}] + \mathbb{E}[d(x, \mathbb{E}[t|t \in (\tau^l, \tau^h]], A) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}] \\
&\quad \le \mathbb{E}[d(x, L(\tau^l|x), A) \cdot \mathbb{1}_{t \le \tau^l}] + \mathbb{E}[d(x, t, A) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}].
\end{aligned}$$

2. *If $\mathbb{E}[t|x, \tau^l < t \le \tau^h] \le \underline{t}^s(x)$ or $\underline{t}^s(x) \le L(\tau^l|x)$, then*

$$\begin{aligned}
&\mathbb{E}[d(x, L(\tau^h|x), A) \cdot \mathbb{1}_{t \le \tau^h}] \\
&\quad = \mathbb{E}[d(x, L(\tau^l|x), A) \cdot \mathbb{1}_{t \le \tau^l}] + \mathbb{E}[d(x, \mathbb{E}[t|t \in (\tau^l, \tau^h]], A) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}].
\end{aligned}$$

3. *If $\tau^h \le \underline{t}^s(x)$ or $\underline{t}^s(x) \le \tau^l$, then*

$$\mathbb{E}[d(x, \mathbb{E}[t|t \in (\tau^l, \tau^h]], A) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}] = \mathbb{E}[d(x, t, A) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}].$$

In words, Lemma 2 compares the disagreement incurred on the pool of non-submitters with $t \le \tau^h$ (having expected test score $L(\tau^h|x)$) to an artificial scenario in which this pool is "broken up" into two distinct pools, one with $t \le \tau^l$ (expected test score $L(\tau^l|x)$) and another with $t \in (\tau^l, \tau^h]$ (expected test score $\mathbb{E}[t|x, t \in (\tau^l, \tau^h]]$). The same admission decision is made in the broken up pools as in the original pool. Breaking up the pools weakly increases disagreement. But when it happens that society would make the same decision on the low pool as the high pool—both of $L(\tau^l|x) \le \mathbb{E}[t|x, t \in (\tau^l, \tau^h]]$ are on the same side of $\underline{t}^s$—the disagreement is in fact unchanged. The lemma then shows that turning the second pool on $t \in (\tau^l, \tau^h]$ into a separating region weakly increases disagreement further, but leaves disagreement unchanged if both of $\tau^l < \tau^h$ are on the same side of $\underline{t}^s$.

Proof of Lemma 2. Part 1 follows from convexity of the disagreement function $d(x, t^s, A)$ in t^s. Part 2 and part 3 follow from the linearity of $d(x, t^s, A)$ on the domain $t^s \le \underline{t}^s$ and on the domain $t^s \ge \underline{t}^s$. For part 2, we also apply the fact that $L(\tau^l|x) \le \mathbb{E}[t|x, \tau^l < t \le \tau^h]$, and hence the assumptions guarantee that $L(\tau^l|x)$ and $\mathbb{E}[t|x, \tau^l < t \le \tau^h]$ are both on the

[45] Recall that $L(t'|x) = \mathbb{E}[t|x, t \le t']$, and that disagreement is given by $d(x, t^s, A)$ for t^s equal to the test score that society treats a student as having and $A \in \{0, 1\}$ the admission decision.

same side of $\underline{t}^s(x)$. For part 3, we similarly recall that $\tau^l < \tau^h$, and hence the assumptions guarantee that τ^l and τ^h are both on the same side of $\underline{t}^s$. $\square$

Proof of Proposition 3. Fix some observable x at which the college is less selective than society. To reduce notation, the rest of this proof omits the x argument in τ, $\underline{t}^c$, $\underline{t}^*$, $\underline{t}^s$, and $L(t') = \mathbb{E}[t|x, t \leq t'])$.

The college's payoff of imputing τ is constant over $\tau \in [-\infty, \underline{t}^*]$: for any of these imputations, Lemma 1 implies that the college rejects students with scores $t \leq \underline{t}^*$ and accepts those with scores $t > \underline{t}^*$; since $\underline{t}^* < \underline{t}^s$, Lemma 2 part 3 implies that the disagreement cost does not change. To prove the result, then, it is sufficient to establish that the payoff from imputing $\tau \in [\underline{t}^*, \infty]$ is decreasing (weakly) and then increasing (weakly).

Let $t^\dagger := \sup\{t | L(t) \leq \underline{t}^*\}$. First, we will show that the college's expected payoff is decreasing in τ over the domain $\tau \in [\underline{t}^*, t^\dagger)$. Note that when $\tau \in [\underline{t}^*, t^\dagger)$, Lemma 1 implies that it optimal for the college to reject students with imputed/submitted score $\hat{t} \leq \tau$. That is, non-submitters are rejected. Submitters with $t > \tau$ are admitted, since $\tau \geq \underline{t}^*$.

Consider any $\tau^l < \tau^h$ in the interval $[\underline{t}^*, t^\dagger)$. We seek to show that the college's expected payoff (among students with the given observable x) is higher from choosing $\tau = \tau^l$ than $\tau(x) = \tau^h$. We consider three cases:

- Suppose $\tau^l < \tau^h \leq \underline{t}^s$. Then the college's expected payoff $\mathbb{E}[Au^c(x,t) - \delta d(x, t^s, A)|x]$ can be written as

  $$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau, \infty)}|x] - \delta \mathbb{E}[-u^s(x,t)\mathbb{1}_{t \in (\tau, \underline{t}^s)}|x],$$

  because the college only accepts students with $t > \tau$ and only incurs a disagreement cost for the students it accepts with $\tau < t \leq \underline{t}^s$. (There is no disagreement cost for rejecting the non-submission pool, which has $\mathbb{E}[t|t \leq \tau] < \underline{t}^s$, since $\mathbb{E}[t|t \leq \tau] < \underline{t}^*$ for all $\tau < t^\dagger$; and there is no disagreement cost for accepting the applicants with $t \geq \underline{t}^s$.) Hence, the college's expected payoff at $\tau = \tau^l$ minus that at $\tau = \tau^h$ simplifies to

  $$\begin{aligned} &\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}|x] - \delta \mathbb{E}[-u^s(x,t)\mathbb{1}_{t \in (\tau^l, \tau^h]}|x] \\ &\quad = \mathbb{E}[(u^c(x,t) + \delta u^s(x,t)) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}|x]. \end{aligned} \tag{4}$$

  Moreover, it holds that $u^c(x,t) + \delta u^s(x,t) > 0$ for all $t \in (\tau^l, \tau^h]$, because $u^c(x,t) + \delta u^s(x,t)$ has the sign of $u^*(x,t)$, and $u^*(x,t) > 0$ at all $t \geq \tau^l \geq \underline{t}^*$. Hence, expression

(4) is non-negative.

- Suppose $\underline{t}^s \leq \tau^l < \tau^h$. Then the college's expected payoff $\mathbb{E}[Au^c(x,t) - \delta d(x, t^s, A)|x]$ can be written as
$$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau, \infty)}|x],$$
since the college only accepts students with $t > \tau$ and does not incur a disagreement cost for any student. (There is no disagreement cost for rejecting the non-submission pool, which has $\mathbb{E}[t|t \leq \tau] < \underline{t}^s$, since $\mathbb{E}[t|t \leq \tau] < \underline{t}^*$ for all $\tau < t^\dagger$; and there is no disagreement cost for accepting the students with $t > \tau \geq \underline{t}^s$.)

  Hence, the college's expected payoff at $\tau = \tau^l$ minus that at $\tau = \tau^h$ simplifies to
$$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau^l, \tau^h]}|x],$$
which is non-negative because $u^c(x,t) > 0$ for all $t \in (\tau^l, \tau^h]$ given that $\tau^l \geq \underline{t}^s > \underline{t}^c$.

- Suppose $\tau^l < \underline{t}^s < \tau^h$. This case is implied by the two above: Moving from τ^l to τ^h is a sum of a move from τ^l to $\underline{t}^s(x)$ (reducing payoffs, as in the first case), plus a move from $\underline{t}^s(x)$ to τ^h (reducing payoffs, as in the second case).

Next, we show that the college's expected payoff is increasing in τ over the domain $\tau \in [t^\dagger, \infty]$. Note that when $\tau \in [t^\dagger, \infty]$, Lemma 1 implies that it is optimal for the college to accept non-submitters (who have expected test score of $L(\tau) > \underline{t}^*$) as well as submitters with $t > \tau$. Hence, all students are accepted, no matter $\tau \in [t^\dagger, \infty]$. Moreover, pooling more students by raising τ always weakly reduces disagreement costs, if raising τ does not change acceptance decisions (Lemma 2 part 1). Hence, raising τ in this range benefits the college, at least weakly.[46] $\square$

Proof of Proposition 4. Fix some observable x at which the college is more selective than society. For notational simplicity, the rest of this proof omits the x argument in τ, $\underline{t}^c$, $\underline{t}^*$, $\underline{t}^s$, and $L(t') = \mathbb{E}[t|x, t \leq t']$.

The college's payoff of imputing τ is increasing over $\tau \in [-\infty, \underline{t}^*]$: for any of these imputations, Lemma 1 implies that the college rejects students with scores $t \leq \underline{t}^*$ and

[46] Raising τ strictly reduces disagreement costs when $\tau > \underline{t}^s$ and $L(\tau) < \underline{t}^s$, at least as long as τ is in the support of t; this is where society wants to reject the non-submission pool but wants to accept students with $t \simeq \tau$. Raising τ does not change disagreement costs when $L(\tau) > \underline{t}^s$ or when $\tau < \underline{t}^s$; in either case, society wants the same decision for students with $t \simeq \tau$ as it does for the pool.

accepts those with scores $t > \underline{t}^*$; since $\underline{t}^s < \underline{t}^*$, Lemma 2 part 1 implies that the disagreement cost increases in the imputation. Without loss, then, we can restrict attention to $\tau \geq \underline{t}^*$.

- First suppose $L(\underline{t}^*) \geq \underline{t}^s$. We claim that it is optimal to set $\tau = \underline{t}^*$. (If there is a solution t° to $L(t^\circ) = \underline{t}^s$, then this corresponds to the case of $t^\circ \leq \underline{t}^s$.)

  Under $\tau = \underline{t}^*$, Lemma 1 implies that the college rejects the pool of non-submitters with $t \leq \underline{t}^*$ (since $L(\underline{t}^*) \leq t^*$) and accepts submitters with $t > \underline{t}^*$. The college's expected payoff $\mathbb{E}[Au^c(x,t) - \delta d(x, t^s, A)|x]$ at $\tau = \underline{t}^*$ can be written as

$$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\underline{t}^*, \infty)}|x] - \delta \mathbb{E}[u^s(x, L(\underline{t}^*)) \cdot \mathbb{1}_{t \in (-\infty, \underline{t}^*]}|x]. \tag{5}$$

  Consider, instead, $\tau = \tau^h > \underline{t}^*$. There are two possibilities:

  - If $L(\tau^h) \leq \underline{t}^*$, then the college rejects non-submitters (by Lemma 1), and so (applying Lemma 2 parts 2 and 3) the college's expected payoff can be written as

$$\begin{aligned}&\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau^h, \infty)}|x] - \delta \mathbb{E}[u^s(x, L(\tau^h)) \cdot \mathbb{1}_{t \in (-\infty, \tau^h]}|x] \\ &\quad = \mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau^h, \infty)}|x] - \delta \mathbb{E}[u^s(x, L(\underline{t}^*)) \cdot \mathbb{1}_{t \in (-\infty, \underline{t}^*]}|x] - \delta \mathbb{E}[u^s(x,t) \cdot \mathbb{1}_{t \in (\underline{t}^*, \tau^h]}|x].\end{aligned} \tag{6}$$

    The expected payoff of setting $\tau = \underline{t}^*$ minus that of setting $\tau = \tau^h$ is given by the difference of expressions (5) and (6), which simplifies to

$$\mathbb{E}[(u^c(x,t) + \delta u^s(x,t)) \cdot \mathbb{1}_{t \in (\underline{t}^*, \tau^h]}|x].$$

    We now observe that $u^c(x,t) + \delta u^s(x,t)$ has the sign of $u^*(x,t)$, which is positive on $t > \underline{t}^*$, implying that the college prefers setting $\tau = \underline{t}^*$ to $\tau = \tau^h$.

  - If $L(\tau^h) > \underline{t}^*$, then by Lemma 1, the college accepts non-submitters as well as the submitters with $t > \tau^h$. That is, it accepts all students. Since $L(\tau^h) \geq L(\underline{t}^*) \geq \underline{t}^s$, it faces no disagreement costs, and its expected payoff is simply $\mathbb{E}[u^c(x,t)|x]$. Subtracting this from (5) yields the expected payoff difference of setting $\tau = \underline{t}^*$ and $\tau = \tau^h$:

$$-\mathbb{E}[(u^c(x,t) + \delta u^s(x, L(\underline{t}^*))) \cdot \mathbb{1}_{t \in (-\infty, t^*]}|x].$$

    We now observe that this expression is positive by definition of the ex-post bar $\underline{t}^*$ (i.e., $u^c(x,t) + \delta u^s(x,t) \leq 0$ for $t \leq t^*$), the linearity of $u^s(x, \cdot)$, and $L(\underline{t}^*) \leq t^*$. Hence, the college prefers setting $\tau = \underline{t}^*$ to $\tau = \tau^h$.

- Second, suppose $L(\underline{t}^c) \leq \underline{t}^s$. Then we claim that it is optimal to set $\tau = \underline{t}^c$. (If there is a solution t° to $L(t^\circ) = \underline{t}^s$, then this corresponds to the case of $t^\circ \geq \underline{t}^c$.)

  The argument is straightforward: setting $\tau = \underline{t}^c$ gives the college its first-best payoff. It admits students with $t > \underline{t}^c$, and it rejects students with $t \leq \underline{t}^c$ at zero disagreement cost, since students with $t \leq \underline{t}^c$ don't submit their test score and the pool has average score $L(\underline{t}^c) \leq \underline{t}^s$.

- Finally, suppose $L(\underline{t}^*) < \underline{t}^s$ and $L(\underline{t}^c) > \underline{t}^s$. In this case, we separately consider the cases that the conditional distribution of test scores is continuous or is discrete.

  – With a continuous distribution of test scores, $L(t)$ is continuous, and so we can find $t^\circ \in (\underline{t}^*, \underline{t}^c)$ such that $L(t^\circ) = \underline{t}^s$. In this case, we will show that it is optimal to set $\tau = t^\circ$.

    At $\tau = t^\circ$, the college rejects non-submitters and faces no disagreement cost, so the college's expected payoff $\mathbb{E}[Au^c(x,t) - \delta d(x, t^s, A)|x]$ at $\tau = t^\circ$ is

    $$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (t^\circ, \infty)}|x]. \tag{7}$$

    At any $\tau \in [\underline{t}^*, t^\circ)$, the college also rejects non-submitters and faces no disagreement cost, so its expected payoff is

    $$\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau, \infty)}|x], \tag{8}$$

    which is clearly less than (7) because $u^c(x,t) < 0$ on $(\tau, t^\circ]$.

    At $\tau > t^\circ$, we can consider two possibilities: $L(\tau) \leq \underline{t}^*$ or $L(\tau) > \underline{t}^*$. If $\tau > t^\circ$ and $L(\tau) \leq \underline{t}^*$, then the college rejects the pool of non-submitters, and its expected payoff is

    $$\begin{aligned}
    &\mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (\tau, \infty)}|x] - \delta \mathbb{E}[u^s(x, L(\tau))\mathbb{1}_{t \in (-\infty, \tau]}] \\
    &\quad = \mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (t^\circ, \infty)}|x] - \mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (t^\circ, \tau]}|x] - \delta \mathbb{E}[u^s(x, \mathbb{E}[t|t \in (t^\circ, \tau]])\mathbb{1}_{t \in (t^\circ, \tau]}] \\
    &\quad = \mathbb{E}[u^c(x,t) \cdot \mathbb{1}_{t \in (t^\circ, \infty)}|x] - \mathbb{E}[(u^c(x,t) + \delta u^s(x,t))\,\mathbb{1}_{t \in (t^\circ, \tau]}] \qquad (9)
    \end{aligned}$$

    where the second equality applies Lemma 2 part 2 (breaking up the pool from $(-\infty, \tau]$ into $(\infty, t^\circ] \cup (t^\circ, \tau]$) and the third applies part 3. Observing that $u^c(x,t) + \delta u^s(x,t) > 0$ on all $t > \underline{t}^*$ implies that (9) is less than (7). Finally, if $\tau > t^\circ$ and $L(\tau) > \underline{t}^*$, then the college accepts the pool of non-submitters and pays no disagreement costs, and its expected payoff is

$$\mathbb{E}[u^c(x,t)|x] = \mathbb{E}[u^c(x,t)\cdot \mathbb{1}_{t\in(-\infty,t^\circ]}|x] + \mathbb{E}[u^c(x,t)\cdot \mathbb{1}_{t\in(t^\circ,\infty)}|x], \tag{10}$$

which is less than (7) since the first term is weakly negative.

– Now consider a discrete distribution of test scores. If there exists t° such that $L(t^\circ) = \underline{t}^s$, then the argument follows exactly as in the continuous case. Otherwise, let $t^- := \max\{t|L(t) < \underline{t}^s\} \geq \underline{t}^*$ and let $t^+ := \min\{t|L(t) > \underline{t}^s\} \leq \underline{t}^c$. (Our assumption that a discrete test score distribution does not have an accumulation point ensures that t^- and t^+ are well defined; note also that there is no score $t \in (t^-, t^+)$ that is in the support of the distribution, since such t would solve $L(t) = \underline{t}^s$.) An analogous argument as above shows that the college's expected payoff is increasing in τ for $\tau \leq t^-$, and is decreasing in τ for $\tau \geq t^+$. Hence, the college's payoff is maximized by setting τ equal to either t^- or t^+. □

Proof of Proposition 5. Follows from Lemma 1. □

C. Proofs for Section 7

As a preliminary observation, we can write the college's loss relative to first best as its allocative loss plus the cost of social pressure. At a given (x_0, t) pair of test scores and group memberships, the assumption of a uniform distribution over x_1 implies that the college's allocative loss depends only on the difference between the college's chosen x_1-cutoff for admission and the college's ideal x_1 cutoff. Specifically, let $f := \frac{1}{\overline{x}_1 - \underline{x}_1}$ be the (constant) density of the x_1 distribution on its support. If the college's chosen cutoff is r above its ideal cutoff, then its allocative loss on this (x_0, t) pair is

$$\int_0^r f x \mathrm{d}x = \frac{f}{2} r^2. \tag{11}$$

Society's (allocative) loss is given by the same formula, when the chosen cutoff is r above society's preferred cutoff.

Proof of Proposition 6. Suppose that affirmative action is allowed. Here, there is no interaction between the college's decisions at different realizations of x_0. So, it suffices to show that test mandatory would be preferred to test blind for any fixed $x_0 = x_0'$ in $\{r, b\}$.

Fixing $x_0 = x_0'$, let $h := u^c(x_0', x_1, t) - u^s(x_0', x_1, t) = \beta \mathbb{1}_{x_0'=g} - c$ be the difference between the college's and society's utility for admitting an applicant of type $x_0 = x_0'$, which does not depend on x_1 or t. It then holds that $u^c(x_0', x_1, t) - u^*(x_0', x_1, t) = \frac{\delta}{1+\delta} h$, and that $u^*(x_0', x_1, t) - u^s(x_0', x_1, t) = \frac{1}{1+\delta} h$. Given its information, the college sets x_1 admissions cutoffs at the value of x_1 setting the expectation of $u^*(x_0', x_1, t)$ to 0. Note that the college's ideal x_1-cutoff for applicants in group $x_0 = x_0'$ with test score t is $-t - h$, whereas society's ideal x_1-cutoff is $-t$.

The college's loss under test mandatory. At (x_0', t), the college's chosen x_1-cutoff for admission is $\frac{\delta}{1+\delta} h$ above its ideal point, yielding allocative loss (from (11)) of

$$\frac{f}{2} \frac{h^2 \delta^2}{(1+\delta)^2}. \tag{12}$$

Similarly, the college's chosen x_1-cutoff for admission is $-\frac{1}{1+\delta} h$ above society's ideal point, leading to an allocative loss for society of $\frac{f}{2} \frac{h^2}{(1+\delta)^2}$. The college then pays a social pressure cost equal to δ times that, or

$$\frac{f}{2} \frac{\delta h^2}{(1+\delta)^2}. \tag{13}$$

Both of these expressions are independent of t, meaning that these expressions also represent the college's losses averaged over test scores.

The college's loss under test mandatory, for applicants with $x_0 = x_0'$, is the sum of (12) and (13).

The college's loss under test blind. With unobservable test scores, the players evaluate applicants of type $x_0 = x_0'$ as if they have the expected test score of $p_{x_0'}$. The college's chosen x_1 cutoff for applicants of type $x_0 = x_0'$ sets $u^*(x_0', x_1, p_{x_0'})$ to 0, i.e., a cutoff of $x_1 = -p_{x_0'} - \frac{h}{1+\delta}$.

To calculate the college's allocative losses, we compare the college's chosen (test-independent) x_1 admissions cutoffs to its (test-dependent) ideal cutoffs. Recall that the college's ideal cut-off at test score t is $x_1 = -t - h$. So at $t = 1$, the college's chosen cutoff is $1 - p_{x_0'} + \frac{\delta}{1+\delta} h$ above its ideal point; at $t = 0$, the college's chosen cutoff is $-p_{x_0'} + \frac{\delta}{1+\delta} h$ above its ideal point. The college's expected allocative loss over test scores, once again plugging into (11), is therefore given by

$$p_{x_0'}\frac{f}{2}\left(1-p_{x_0'}+\frac{\delta}{1+\delta}h\right)^2+(1-p_{x_0'})\frac{f}{2}\left(-p_{x_0'}+\frac{\delta}{1+\delta}h\right)^2=\frac{f}{2}\frac{h^2\delta^2}{(1+\delta)^2}+\frac{f}{2}p_{x_0'}(1-p_{x_0'}). \tag{14}$$

To calculate social costs, we compare the college's chosen x_1 admissions cutoff not to society's ideal cutoff, but to society's preferred cutoff given that test scores are not observed. Society's preferred x_1-cutoff is given by $-p_{x_0'}$. The chosen cutoff is $-\frac{h}{1+\delta}$ above society's preferred cutoff. We can now plug into (11) to calculate society's loss relative to its preferred cutoff (given its information) as $\frac{f}{2}\frac{h^2}{(1+\delta)^2}$. The college's social pressure cost is δ times that, or

$$\frac{f}{2}\frac{\delta h^2}{(1+\delta)^2}. \tag{15}$$

The college's loss under test blind, for applicants with $x_0 = x_0'$, is the sum of (14) and (15).

Comparison. Comparing equations (13) and (15), the social pressure cost under test blind is identical to that under test mandatory. And comparing equations (12) and (14), the allocative loss is higher under test blind. Hence, the college prefers test mandatory. □

Proof of Proposition 7. Suppose that affirmative action is banned. Let $ET := \mathbb{E}[t] = qp_r + (1-q)p_g$ be the average test score in the population, i.e., the share with test score $t = 1$. Recall that $P_g^t = Pr(x_0 = g|t)$. We will now calculate the college's loss in each testing regime.

In each case, we will evaluate the college's allocative loss relative to a benchmark where the college must make decisions independently of the unobservable x_0 type. The college's ideal cutoff at test score t, given that it must pool together applicants across the two x_0 types, is $-t - \beta P_g^t + c$.

The college's loss under test mandatory. Society's ideal x_1-cutoff for admitting a student of with test score t is $-t$. The college's chosen cutoff, setting the expected ex post utility to 0, is $-t - \frac{1}{1+\delta}(\beta P_g^t - c)$.

To calculate the allocative loss, observe that the college's chosen cutoff is $\frac{\delta}{1+\delta}(\beta P_g^t - c)$ above its ideal cutoff at test score t. Plugging into (11), its allocative loss across the two test scores is given by

$$(1-ET)\frac{f}{2}\left(\frac{\delta}{1+\delta}(\beta P_g^0 - c)\right)^2 + ET\frac{f}{2}\left(\frac{\delta}{1+\delta}(\beta P_g^1 - c)\right)^2. \tag{16}$$

To calculate the loss due to social pressure, observe that the chosen x_1-cutoff is $-\frac{1}{1+\delta}(\beta P_g^t - c)$ above society's preferred cutoff. The college's expected loss due to social pressure (plugging this difference into (11) for each test score, taking expectation over test scores to find society's loss, and then multiplying by δ) is therefore

$$\delta(1-ET)\frac{f}{2}\left(\frac{1}{1+\delta}(\beta P_g^0 - c)\right)^2 + \delta ET\frac{f}{2}\left(\frac{1}{1+\delta}(\beta P_g^1 - c)\right)^2. \tag{17}$$

The college's total loss is (16) plus (17).

The college's loss under test blind. The average test score is ET, and so society's preferred x_1-cutoff is $-ET$. The college's chosen cutoff, setting the expected ex post utility to 0, is $-ET - \frac{1}{1+\delta}(\beta q - c)$, where q is the probability of $x_0 = g$.

Once again, we calculate the college's allocative loss relative to its ideal point with observable t but unobservable x_0. At test score t, the chosen cutoff minus the ideal cutoff is

$$t - ET + \beta P_g^t - \frac{q\beta}{1+\delta} - \frac{c\delta}{1+\delta}$$

Plugging into (11) and taking the expectation across test scores, the college's allocative loss is given by

$$(1-ET)\frac{f}{2}\left(-ET + \beta P_g^0 - \frac{q\beta}{1+\delta} - \frac{c\delta}{1+\delta}\right)^2 + ET\frac{f}{2}\left(1 - ET + \beta P_g^1 - \frac{q\beta}{1+\delta} - \frac{c\delta}{1+\delta}\right)^2. \tag{18}$$

The difference between the college's chosen cutoff and society's preferred cutoff is $-\frac{1}{1+\delta}(\beta q - c)$.

Plugging into (11) and multiplying by δ, the college's loss from social pressure is

$$\frac{f}{2}\frac{\delta(\beta q - c)^2}{(1+\delta)^2}. \tag{19}$$

The college's total loss is (18) plus (19).

Comparison. The net benefit of choosing test blind rather than test mandatory is given by the loss from test mandatory minus the loss from test blind, i.e.,

$$(16) + (17) - (18) - (19).$$

Substituting in $q = (ET)P_g^1 + (1-ET)P_g^0$ and $\Delta = P_g^0 - P_g^1$ and then simplifying with routine algebra, we can rewrite this net benefit as

$$\frac{f}{2}\frac{ET(1-ET)}{1+\delta}\left((1+\delta)(2\beta\Delta - 1) - (\beta\Delta)^2\right).$$

The above expression is weakly positive if and only if $(1+\delta)(2\beta\Delta - 1) \geq (\beta\Delta)^2$. □

Proof of Corollary 1. Suppose that affirmative action is banned. Proposition 7 establishes that the college prefers test blind to test mandatory if and only if

$$(1+\delta)(2\beta\Delta - 1) \geq (\beta\Delta)^2. \tag{20}$$

Recall we maintain the assumptions that $\beta > 0$, $\beta\Delta < 1$, and for this corollary, $\Delta > 0$. We prove each part of the corollary in turn:

1. Rewriting (20), the college prefers test blind if and only if

$$-\Delta^2\beta^2 + 2\Delta(1+\delta)\beta - (1+\delta) \geq 0.$$

   The LHS is a concave quadratic that is negative at $\beta = \frac{1}{2\Delta}$ (equal to $-1/4$) and positive at $\beta = \frac{1}{\Delta}$ (equal to δ). Hence, there exists $\beta^* \in (\frac{1}{2\Delta}, \frac{1}{\Delta})$ such that the college prefers test blind when $\beta > \beta^*$ and test mandatory when $\beta < \beta^*$. Using the quadratic formula, $\beta^* = \frac{1+\delta-\sqrt{\delta(1+\delta)}}{\Delta}$.

2. Since (20) is symmetric with respect to β and Δ, the argument of the previous part goes through unchanged after swapping β and Δ. We get $\Delta^* = \frac{1+\delta-\sqrt{\delta(1+\delta)}}{\beta}$.

3. If $\beta\Delta \in (0, 1/2)$, then the LHS of (20) is nonpositive (since $2\beta\Delta - 1 \le 0$) and the RHS is strictly positive, implying that test mandatory is optimal.

   If $\beta\Delta > 1/2$, then we can rewrite (20) as $\delta \ge \frac{(1-\beta\Delta)^2}{2\beta\Delta-1}$, and hence the result holds for $\delta^* = \frac{(1-\beta\Delta)^2}{2\beta\Delta-1} > 0$. □

Proof of Proposition 8. As in equation (11), at a given (x_0, t), society's loss relative to its first best when the college's chosen x_1-cutoff for admission is r above society's ideal cutoff is $\int_0^r f x \mathrm{d}x = \frac{f}{2}r^2$. Society's expected loss across all values of x_0 and t is equal to the expectation of $\frac{f}{2}r^2$ over the distribution of r, with r the difference between the chosen cutoff (which may depend on x_0 and t) and society's ideal cutoff (which depends only on t). Since the loss $\frac{f}{2}r^2$ is convex in r, mean-preserving spreads in the distribution of these cutoff differences make society worse off.

Part 1. Fix any testing regime. The distribution of cutoffs at each test score when affirmative action is allowed is a mean-preserving spread of the distribution when affirmative action is banned. Hence, society prefers banning affirmative action.

Part 2. First, suppose that affirmative action is allowed. Fix some type $x_0 = x_0'$, at which the college has a utility bonus of $h := u^c(x_0', x_1, t) - u^s(x_0', x_1, t) = \beta \mathbb{1}_{x_0'=g} - c$ relative to society. Under test mandatory, at each test score, the chosen x_1-cutoff is $\frac{h}{1+\delta}$ above society's ideal cutoff. Under test blind, at $t = 1$, the chosen cutoff is $1 - p_{x_0'} + \frac{h}{1+\delta}$ above society's cutoff; and at $t = 0$, the chosen cutoff is $-p_{x_0'} + \frac{h}{1+\delta}$ above society's cutoff. Hence, under test blind, at each type x_0', the distribution of the chosen cutoff minus society's cutoff is given by

$$\begin{cases} 1 - p_{x_0'} + \frac{h}{1+\delta} & \text{w/ prob } p_{x_0'} \\ -p_{x_0'} + \frac{h}{1+\delta} & \text{w/ prob } 1 - p_{x_0'}. \end{cases}$$

This distribution is a mean-preserving spread of the constant $\frac{h}{1+\delta}$. Hence, society is worse off under test blind for each realization x_0' of x_0, and so is worse off in expectation.

Next, suppose that affirmative action is banned. As also defined in the proof of Proposition 7, we let $ET := \mathbb{E}[t] = qp_r + (1-q)p_g$ denote the average test score in the population, i.e., the share of students with test score $t = 1$. At test score t, the college's ideal x_1-cutoff is $-t - \beta P_g^t + c$ (recall $P_g^t = \Pr(x_0 = g|t)$), and society's ideal x_1-cutoff is $-t$.

Under test mandatory with affirmative action banned, the college's chosen x_1-cutoff is $-\frac{1}{1+\delta}(\beta P_g^t - c)$ above society's ideal point at test score t. That is, a share ET of students have cutoffs $-\frac{1}{1+\delta}(\beta P_g^1 - c)$ above society's ideal point, and a share $1 - ET$ have cutoffs $-\frac{1}{1+\delta}(\beta P_g^0 - c)$ above. Plugging in $q = (ET)P_g^1 + (1 - ET)P_g^0$ and $\Delta = P_g^0 - P_g^1$, some algebra yields that the distribution of chosen cutoffs minus society's ideal cutoffs is

$$\begin{cases} -\frac{1}{1+\delta}(\beta q - c) + (1 - ET)\frac{\beta\Delta}{1+\delta} & \text{w/ prob } ET \\ -\frac{1}{1+\delta}(\beta q - c) - ET\frac{\beta\Delta}{1+\delta} & \text{w/ prob } 1 - ET. \end{cases} \tag{21}$$

Under test blind with affirmative action banned, the college's chosen x_1 cutoff is $-ET - \frac{1}{1+\delta}(\beta q - c)$. This means that for the ET share of applicants with $t = 1$, the chosen x_1-cutoff is $-\frac{1}{1+\delta}(\beta q - c) + (1 - ET)$ above society's ideal cutoff of -1; for the $1 - ET$ share with $t = 0$, the chosen cutoff is $-\frac{1}{1+\delta}(\beta q - c) - ET$ above society's ideal cutoff of 0. That is, the distribution of chosen cutoffs minus society's ideal cutoffs is

$$\begin{cases} -\frac{1}{1+\delta}(\beta q - c) + (1 - ET) & \text{w/ prob } ET \\ \frac{1}{1+\delta}(\beta q - c) - ET & \text{w/ prob } 1 - ET. \end{cases} \tag{22}$$

Since $\beta\Delta < 1$ (by assumption) and $1 + \delta > 1$, the distribution in (22) is a mean-preserving spread of that in (21). Hence, when affirmative action is banned, society prefers test mandatory to test blind.

Part 3. From Proposition 7, if $(1 + \delta)(2\beta\Delta - 1) < (\beta\Delta)^2$, then the college chooses test mandatory under an affirmative action ban. If $(1 + \delta)(2\beta\Delta - 1) > (\beta\Delta)^2$, which implies $\beta\Delta > 1/2$, the college chooses test blind under an affirmative action ban.

So, when $\beta\Delta \in (0, 1/2]$, society prefers to ban affirmative action: it prefers test mandatory and no affirmative action to test mandatory with affirmative action (by part 1).

Now suppose that $\beta\Delta > 1/2$. Let $\underline{\delta} := \frac{(\beta\Delta)^2}{2\beta\Delta - 1} - 1$ be the solution to $(1 + \delta)(2\beta\Delta - 1) = (\beta\Delta)^2$. For $\delta < \underline{\delta}$, the college chooses test mandatory, in which case society prefers to ban affirmative action. For $\delta > \underline{\delta}$, the college chooses test blind. In this case, we need to compare society's payoff of test mandatory plus affirmative action versus test blind plus no affirmative action.

The distribution of chosen x_1-cutoffs minus society ideal cutoffs under test mandatory plus affirmative action is

$$\begin{cases} \frac{\beta - c}{1+\delta} & \text{w/ prob } q \\ \frac{-c}{1+\delta} & \text{w/ prob } 1-q. \end{cases}$$

Society's corresponding payoff loss is

$$\frac{f}{2(1+\delta)^2}\left(c^2 - 2cq\beta + q\beta^2\right). \tag{23}$$

The distribution of cutoffs minus society ideal points under test blind plus no affirmative action is given by (22). Society's payoff loss is correspondingly

$$\frac{f}{2(1+\delta)^2}\left((\beta q - c)^2 + (1-ET)ET(1+\delta)^2\right), \tag{24}$$

with $ET = qp_g + (1-q)p_r$.

The difference of (24) minus (23), which is positive when test mandatory plus affirmative action is better than test blind with no affirmative action and negative when worse, is given by

$$\frac{f}{2(1+\delta)^2}\left(ET(1-ET)(1+\delta)^2 - q(1-q)\beta^2\right). \tag{25}$$

This expression has the same sign as $ET(1-ET)(1+\delta)^2 - q(1-q)\beta^2$. So, let $\delta' := \beta\sqrt{\frac{q(1-q)}{ET(1-ET)}} - 1$ be the solution to $ET(1-ET)(1+\delta)^2 - q(1-q)\beta^2 = 0$. When $\delta > \delta'$, society prefers test mandatory plus affirmative action to test blind plus no affirmative action; when $\delta < \delta'$, society prefers test blind plus no affirmative action to test mandatory plus affirmative action.

Finally, let $\overline{\delta} = \max\{\underline{\delta}, \delta'\}$. We now see that (i) when $\delta > \underline{\delta}$, the college chooses test blind if affirmative action is banned; and (ii) taking into account the college's response in choosing its testing regime, society prefers to ban affirmative action if $\delta < \overline{\delta}$, and prefers to allow affirmative action if $\delta > \overline{\delta}$. $\square$