Can global uncertainty promote international trade?

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Article history:
Received 16 September 2019
Received in revised form 19 May 2020
Accepted 20 May 2020
Available online 30 May 2020

Research data related to this submission:
https://github.com/mwaugh0328/can-uncertainty-promote-trade

Common wisdom holds that uncertainty impedes trade—yet we show that uncertainty can fuel more trade in a simple general equilibrium trade model with information frictions. In equilibrium, increases in uncertainty increase both the mean and variance in returns to exporting. This implies that trade can increase or decrease with uncertainty, depending on preferences. Under general conditions on preferences, we characterize the importance of these forces using a sufficient statistics approach. Higher uncertainty leads to increases in trade because agents receive improved terms of trade, particularly in states of nature in which consumption is most valuable. Trade creates value, in part, by offering a mechanism for risk sharing, and risk sharing is most effective when both parties are uninformed.

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1. Introduction

In any discussion of frictions in cross-border trade, inevitably one that arises concerns information and uncertainty. Portes and Rey (2005) show that the volume of phone calls between two countries predicts how much they trade. Gould (1994) and Rauch and Trindade (2002) argue that immigrants trade more with their home countries. The argument is so simple that it needs no formalization: information frictions create uncertainty, and this uncertainty deters risk-averse exporters. In this paper, we show that uncertainty can fuel more trade in a simple general equilibrium trade model.

Anecdotal evidence suggests that the effects of uncertainty on trade are far from clear. There has been increased uncertainty about the future international trading environment, with the United States government adopting a hostile stance to existing trade agreements and others threatening retaliation. In particular, measures of policy uncertainty have increased dramatically since late 2016; see, e.g., Fig. 1. Despite this uncertain environment, U.S. exports have grown by 17% since early 2016.
In a simple general equilibrium trade model with information frictions, we show how outcomes of this nature—uncertainty-fueled booms in trade—are possible. We deliver two insights about the relationship between uncertainty and international trade. The first concerns mechanics: Uncertainty increases both the mean and the variance in the returns to exporting. The implication is that trade can increase or decrease with uncertainty, depending on preferences over these different forces. Under general conditions on preferences, we characterize the importance of these forces using a sufficient statistics approach. Once one understands certain risk, prudence, and temperance properties of preferences, changes in mean and variance are sufficient to characterize the change in trade flows to aggregate uncertainty. In the commonly used CES case, these sufficient statistics simply boil down to functions of the elasticity of substitution across home and foreign varieties, or “trade elasticity.”

The second insight regards interpretation: Uncertainty facilitates cross-country risk sharing and, hence, more trade. When uncertainty is high, other countries do not realize that bad states of nature are prevailing domestically. Their exports provide the home country with lots of goods in exactly the states in which consumption is most needed. In contrast, when uncertainty is low, this risk-sharing mechanism is muted; informed countries substitute away from trade in states in which they would prefer to not insure their trading partner. Thus, one interpretation of our results is that uncertainty-fueled increases in trade occur because risk sharing is most effective when both parties are uninformed.

The role of theory is to clarify thinking. Thinking about uncertainty and trade has been focused on one channel. The model uncovers multiple ways in which trade and uncertainty are related. The results guide our thinking about what constitutes aggregate uncertainty and what its cross-border effect might be. Our results do not prove that uncertainty increases trade. But if uncertainty is a trade barrier, the results teach us that many standard parameterized trade models are logically inconsistent with that finding. Either our thinking about uncertainty and trade, or our models, should change.

We demonstrate these results in a standard, simple general equilibrium trade model—a two-good, two-country Armington model. We introduce cross-country uncertainty in the most obvious way: Each country experiences a random shock that affects its export choice. Home firms observe home shocks perfectly. Foreigners observe foreign shocks perfectly. But each group observes the other’s shocks imperfectly, with a noisy signal. Then every firm chooses how much to export to an international market. The international relative price clears that market, goods are immediately shipped to their destination country, and agents consume.

In our model, uncertainty is about another country’s endowment. But endowment risk is a simple economic primitive that is a stand-in for many other sorts of uncertainty. For example, uncertainty about the quality of foreign exports or preference shocks is isomorphic to the model we wrote down. Similarly, the endowment could represent production, net of deadweight “iceberg” losses, created by trade policy. Trade policy uncertainty creates randomness in the residual supply. What matters to an exporting firm is not the source of uncertainty or the quantity or quality of foreign exports; it cares about the distribution of the relative price of their good. Section 3 establishes that the key to our results is that this relative price has a distribution that is right-skewed. Such a distribution arises naturally in this context because terms of trade are never negative, but can be arbitrarily large. If some uncertain outcome makes the right-skewed terms of trade more variable, it raises risk—but, under some plausible conditions, also raises the average relative price. These competing mean and variance forces are central to our analysis, and arise in settings in which aggregate uncertainty affects trade.

Our analysis proceeds in several steps. First, we consider the effects of uncertainty on the terms of trade. The key insight is that—in general equilibrium—uncertainty affects not only the volatility, but also the expected terms of trade. Mathematically, the mechanism is that uncertainty impairs home agents’ ability to condition their exporting behavior on the foreign country’s state (and vice versa). As a result, home and foreign exports covary less. In equilibrium, the terms of trade depend on the ratio of
home and foreign exports. If home and foreign exports are always proportional, the terms of trade are constant. Less coordination creates more volatile terms of trade. The mechanism encodes the conventional wisdom that uncertainty deters risk-averse exporters from exporting.

This conventional wisdom, however, is incomplete. A fall in export covariance causes the numerator and denominator of the terms of trade to covary less, while always remaining positive. This results in terms of trade that occasionally reach a very high level, but never fall below zero. When such a positive ratio varies more, its mean increases. Thus, high uncertainty, which results in more volatile terms of trade, also increases the expected level of the terms of trade, making exporting more lucrative on average.

The effect of information on the risk and the expected return from exporting permeates a broad class of general equilibrium trade models. However, how risk and return affect the incentives—to export and which effect dominates—depends on preferences and their parameters. While our analysis ultimately identifies fundamental features of preferences that cause information to affect trade volumes one way or another we begin with a specific but commonly used form of preferences to identify these forces in a well-understood setting and build intuition for how and why they arise. With constant elasticity of substitution (CES) preferences, these comparative statics boil down to functions of the elasticity of substitution across home and foreign varieties. If goods are highly substitutable, the risk effect dominates and information frictions decrease trade. When goods are less substitutable, the rise in the expected terms of trade more than offsets the increase in risk, and firms choose to export more when information is less precise. In other words, uncertainty facilitates trade.

With other preferences outside the CES class, the same forces are at play, but may result in different net effects on trade volume. One possibility is that the increase in the terms of trade can reduce exports. The logic is that if I am expecting to get lots of the foreign good back in return for my exports, and I like a balanced consumption bundle, then I should export less when the relative price of my good rises. Otherwise, I will have too much of the foreign good to consume. Another possibility is that when the terms of trade become more uncertain, an agent chooses to export more for purely precautionary reasons. Our general results characterize preferences where substitution or precautionary effects dominate. We can distinguish these well-understood substitution and precaution effects from our equilibrium terms of trade effect.

These arguments have a tight link to risk-sharing and insurance motives. We point out how the change in covariance, which governs risk sharing, affects the mean return to trade. How uncertainty actually affects trade volume depends on which of these forces—the increase in risk or increase in return—dominate. One obvious reason that uncertainty might encourage more trade is that agents have precautionary motives to trade. Agents who export, not knowing how much of the foreign good they will get in return, might export more to make sure they get enough of the foreign good back. For preferences with the right type of curvature, precautionary exporting emerges. But even when preferences do not normally induce precautionary behavior, we show that equilibrium movements in the terms of trade can induce countries to export more in the face of more mutual uncertainty. Just as borrowing constraints can change interest rate dynamics to induce precautionary behavior in a savings problem, equilibrium movements in the terms of trade can induce precautionary exporting in trade models with a wide range of non-precautionary preferences.

In other words, the terms of trade vary, in such a way as to share risk between countries (Cole and Obstfeld, 1991). When uncertainty is low, the terms of trade vary less and pose less risk to the exporter. But terms of trade that are not variable cannot hedge risk effectively. As uncertainty rises, and the terms of trade are less predictable, they also covary more negatively with endowments, so as to hedge each country’s risk. This is what makes trade more attractive.

Our results rely on the assumption that there are no financial instruments or contracts that formally share risk. Just as in Newbery and Stiglitz (1984), we eliminate such instruments because we cannot logically study uncertainty if all uncertainty can be hedged and thereby effectively eliminated. We relax this restriction and describe the average amount of trade in settings in which some agents can write fully state-contingent contracts and others cannot. We find that allowing more risk-sharing works just like reducing uncertainty. If you can condition exports on the realized price, then it is just like knowing the price. Both reduce the average amount of trade.

Our results are most applicable to existing trading relationships. Our argument does not apply when two countries are new trading partners and many new trading relationships are potentially being formed. The reason is that new trading relationships surely involve fixed costs to set up. Uncertainty affects the willingness to bear those fixed costs in a way that is not captured by this model. However, much of the world’s trade takes place between trading partners that are already established, like the U.S. and Mexican car manufacturers. The question there is not whether to start exporting, but how much to trade within an existing relationship. This question is a natural starting point because the setting is simpler, but also because the answer is more surprising.

1.1.1. Related literature

A handful of classic and more recent papers explore how openness to trade affects utility and economic volatility, which is close to the reverse of our question. Newbery and Stiglitz (1984) and Caselli et al. (2020) debate whether, in incomplete markets, trade can reduce welfare or increase economic volatility. While our focus is on trade volume rather than volatility, some of the mechanisms are similar. Specifically, their focus on the relative price of goods as a risk-sharing mechanism is present in our results. In their setting, opening to trade affects terms of trade volatility; in our setting, information about trading partners increases the coordination of exports, which reduces terms of trade volatility.
Closer to our main point are papers that measure the negative effect of uncertainty on trade. Some focus on firm-specific or product-specific uncertainty. Our model complements these findings by focusing on an element these theories abstract from, the role of uncertainty about a foreign economy. Other authors use the term “uncertainty” to mean volatility and show that volatility reduces trade (De Sousa et al., 2020). Our work does not dispute or contradict these facts. We also find that volatility reduces trade, but uncertainty does not. The difference between the two is information. A process can be volatile but predictable, like brightness over night and day. Our analysis leaves the volatility of shocks unchanged and simply varies the quality of the information about those shocks. This clarifies the distinction between aggregate and idiosyncratic risk and between volatility and uncertainty. In doing so, it informs measurement.

Several recent papers have measured the impact of the alleviation of information frictions on international trade. Steinwender (2018) showed that transatlantic connectivity through the introduction of the telegraph lowered average prices, lowered volatility, and increased imports of U.S. cotton in the mid 19th century. The fact that decreases in price uncertainty led to a reduction in prices and a reduction in price volatility supports our main mechanism. But our model changes the interpretation of their increase in trade volume. One interpretation, as suggested by Juhasz and Steinwender (2019) is that the uncertainty concerns product characteristics, not aggregate conditions. However, if the reduction in uncertainty were about aggregate shocks and did cause a surge in trade, then it would imply that agents’ preferences or market mechanics should be modeled differently.

In financial markets, lower uncertainty also frequently inhibits risk-sharing. The Hirshleifer (1971) effect arises when information precludes trade in assets whose payoffs are contingent on an outcome revealed by the information. Our effect is distinct because (1) our signals are not public, (2) the existence of two distinct consumption goods matters, and (3) our mechanism works through changes in the international relative price. We discuss the importance of each of these differences when we explore risk sharing in Section 3.3.

2. A benchmark equilibrium model of trade under uncertainty

This section develops a simple model with two countries, stochastic nationally differentiated endowments, and a cross-border information friction. The first two ingredients are standard ingredients of trade and international business cycle models, as in Armington (1969) and Backus et al. (1995). The cross-border information friction is that agents in each country know their own country’s aggregate endowment, but have imperfect information about the other country’s endowment. This information friction gives rise to aggregate uncertainty about the terms of trade. Below we discuss the economic environment and then discuss our modeling choices at the end of the section.

2.1. The economic environment

The economic environment is a repeated static model with the following features.

2.1.1. Preferences

There are two countries (x and y) and a continuum of agents within each country. We denote individual variables with lower case and aggregates with upper case. Agents like to consume two goods, x and y (which are nationally differentiated), and their utility flow each period is

\[ U(c_x, c_y) \]  (1)

where for now we only restrict \( U \) to be increasing and concave in both goods. Section 3 solves the model for the constant elasticity of substitution case; Section 4 characterizes the general case.

2.1.2. Endowments

Each agent in the domestic country has an idiosyncratic endowment of \( z_x \) units of good x, where \( \ln z_x \sim \mathcal{N}(\mu_x, \sigma_x^2) \). Agents in the foreign country have an idiosyncratic endowment of \( z_y \) units of good y, where \( \ln z_y \sim \mathcal{N}(\mu_y, \sigma_y^2) \). Thus, production of each good is nationally differentiated, as in Armington (1969). Most important is that we let the mean of these distributions be independent random variables; \( \mu_x \sim \mathcal{N}(\tilde{m}_x, s_x^2) \) and \( \mu_y \sim \mathcal{N}(\tilde{m}_y, s_y^2) \). Because they represent the average endowment realization for each country, \( \tilde{m}_x \) and \( \tilde{m}_y \) are aggregate shocks.

Endowment shocks are the source of uncertainty in the model. They could equivalently be quality or preference shocks. See Appendix B for an isomorphic model. Since trade policy is often modeled as a preference change, this could represent policy as well. What is important is that the shocks are aggregate, not firm-specific.

\[ \text{E.g., Allen (2014), Petropoulou (2011), Rauch and Watson (2004), and Eaton et al. (2011).} \]
2.1.3. Information

At the beginning of the period, agents in country $x$ observe their own endowment $z_x$ and the mean of their country's endowment $\mu_x$. Likewise, agents in country $y$ observe $z_y$ and $\mu_y$. Furthermore, agents know the distribution from which mean productivity is drawn and the cross-sectional distribution of firm outcomes. In other words, $m_x$, $m_y$, $s_x$, $s_y$, $\alpha_x$ and $\alpha_y$ are common knowledge.

Agents in each country receive signals about the other countries' aggregate endowment realization. Specifically, agents in country $x$ observe a signal about the $y$-endowment

$$\bar{m}_y = \mu_y + \eta_y$$ (2)

where $\eta_y \sim N(0, \xi_y^2)$. Similarly, agents in country $y$ observe a signal about the $x$-endowment

$$\bar{m}_x = \mu_x + \eta_x$$ (3)

where $\eta_x \sim N(0, \xi_x^2)$. Thus agents in each country receive an imprecise but unbiased signal about fundamentals in the foreign country. How precise or imprecise the signal is will depend on the variance of the noise, $\xi_x^2$ and $\xi_y^2$. Changing these variances allows us to vary fundamental uncertainty, in a continuous way, and study the response of the economy.

Let $\mathcal{S}_x$ denote the information set of an agent in the home country and $\mathcal{S}_y$ denote the information set of a foreign agent. All country $x$ choices will be a function of the three random variables in the home agents' information set: $\mathcal{S}_x = \{z_x, \mu_x, \bar{m}_y\}$. Likewise, country $y$ choices depend on $\mathcal{S}_y = \{z_y, \mu_y, \bar{m}_x\}$.

2.1.4. Bayesian updating

Agents in each country combine their signal (i.e., Eqs. (2) and (3)) with their prior knowledge of the endowment distribution to form posterior beliefs. Agents must form posterior beliefs over two outcomes: First, they must form a belief about the endowment realization in the foreign country; we will call these first-order beliefs. Second, those in the home country must form beliefs about the foreign country's belief about themselves; we will call these second-order beliefs. Characterizing first- and second-order beliefs are sufficient to characterize optimal actions.2

To compute country $x$'s first-order beliefs about country $y$'s endowment distribution, note that by Bayes' law, the posterior probability distribution is normal with mean $\bar{m}_y$ and variance $\xi_y^2$ given by

$$F(\mu_y | \mathcal{S}_x) = \Phi\left(\frac{\mu_y - \bar{m}_y}{\xi_y}\right) \quad \text{where} \quad \bar{m}_y = \frac{s_y^2 m_y + \bar{s}_y^2 \bar{m}_y}{s_y^2 + \bar{s}_y^2}, \quad \xi_y^2 = \frac{1}{s_y^2 + \bar{s}_y^2}$$ (4)

where the posterior mean is a precision weighted average of the signal and unconditional mean; $\Phi$ is the standard normal distribution. Similarly country $y$'s first-order belief about country $x$'s endowment distribution is:

$$F(\mu_x | \mathcal{S}_y) = \Phi\left(\frac{\mu_x - \bar{m}_x}{\xi_x}\right) \quad \text{where} \quad \bar{m}_x = \frac{s_x^2 m_x + \bar{s}_x^2 \bar{m}_x}{s_x^2 + \bar{s}_x^2}, \quad \xi_x^2 = \frac{1}{s_x^2 + \bar{s}_x^2}$$ (5)

Then to compute country $x$'s second-order belief—its belief about country $y$'s belief about itself—these second-order beliefs are

$$F(\bar{m}_x | \mathcal{S}_x) = \Phi\left(\frac{\bar{m}_x - \bar{\bar{m}}_x}{\bar{s}_x}\right) \quad \text{where} \quad \bar{\bar{m}}_x = \frac{s_x^2 \bar{m}_x + \bar{s}_x^2 \bar{\bar{m}}_x}{s_x^2 + \bar{s}_x^2}, \quad \bar{s}_x^2 = \frac{\bar{s}_x^2}{(s_x^2 + \bar{s}_x^2)^2}$$ (6)

$$F(\bar{m}_y | \mathcal{S}_y) = \Phi\left(\frac{\bar{m}_y - \bar{\bar{m}}_y}{\bar{s}_y}\right) \quad \text{where} \quad \bar{\bar{m}}_y = \frac{s_y^2 \bar{m}_y + \bar{s}_y^2 \bar{\bar{m}}_y}{s_y^2 + \bar{s}_y^2}, \quad \bar{s}_y^2 = \frac{\bar{s}_y^2}{(s_y^2 + \bar{s}_y^2)^2}$$ (7)

Here the second-order beliefs posterior mean ($\bar{\bar{m}}_x$, $\bar{\bar{m}}_y$) is a precision weighted average of a country's own realization and the unconditional mean.

A final note regarding notation. Since there is a one-to-one mapping between signals $\bar{m}$ and posterior beliefs $\bar{m}$, we will use posterior beliefs as a state variable rather than using signals. This simplifies the notational burden. Thus, we write $\mathcal{S}_x = \{z_x, \mu_x, \bar{m}_y\}$ and $\mathcal{S}_y = \{z_y, \mu_y, \bar{m}_x\}$.

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2 In fact, all higher orders of beliefs can matter for export choices. But because there are only two shocks observed by each country, the first two orders of beliefs are sufficient to characterize the entire hierarchy.
2.1.5. Price and budget set

Given their information sets, agents chose how much to export, \( t_x \) or \( t_y \). In return, they receive the other country’s goods at relative price \( p \), which is denominated in units of \( y \) good. For example, an agent who exports \( t_x \) units of the \( x \) goods receives \( p t_x \) units of \( y \) for immediate consumption. Finally, we assume that there is no secondary resale market or storage and we restrict exports and consumption to be nonnegative. This implies that country \( x \)'s budget set is:

\[
\begin{align*}
    c_x & \in [0, z_x - t_x], \\
    c_y & \in [0, p t_x], \\
  \end{align*}
\]

and country \( y \)'s budget set is:

\[
\begin{align*}
    c_x & \in \left[ 0, \frac{t_y}{p} \right], \\
    c_y & \in [0, z_y - t_y].
\end{align*}
\]

2.1.6. Timing

The timing protocol is as follows: First, agents see their endowments and receive signals about the foreign county’s endowment. Agents then make export decisions. Thus, they are exporting prior to knowing the actual price \( p \). This timing protocol allows information frictions to matter: Uncertainty about the foreign country’s endowment gives rise to aggregate uncertainty about the terms of trade and this uncertainty, in turn, feeds back into the decision to export.

2.1.7. Equilibrium

An equilibrium is given by export policy functions for domestic \( t_x(z_x, \mu_x, \bar{m}_y) \) and foreign \( t_y(z_y, \mu_y, \bar{m}_x) \) countries; aggregate exports \( T_x(\mu_x, \bar{m}_y), T_y(\mu_y, \bar{m}_x) \); a perceived price function \( \bar{p}(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y) \) for each country; and an actual price function \( p(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y) \) such that:

1. Given perceived price functions \( \bar{p}(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y) \), export policies maximize expected consumption of every firm in each country. Substituting budget sets (8) to (11) into utility \( E[U(c_x, c_y)] \), we can write this problem as

\[
\begin{align*}
    & t_x(z_x, \mu_x, \bar{m}_y) = \arg \max \mathbb{E}\left[ U\left(z_x - t_x, \bar{p}\left(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y\right)t_x\right) \bigg| \mathcal{F}_x \right] \\
    & t_y(z_y, \mu_y, \bar{m}_x) = \arg \max \mathbb{E}\left[ U\left(\frac{t_y}{\bar{p}(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y)} z_y - t_y \right) \bigg| \mathcal{F}_y \right]
\end{align*}
\]

Using the conditional densities (4), (5), (7), and (6), we can compute expectations as

\[
\begin{align*}
    & t_x(z_x, \mu_x, \bar{m}_y) = \arg \max \int \int U\left(z_x - t_x, \bar{p}\left(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y\right)t_x\right) dF(\mu_x, \mathcal{F}_x) dF(\bar{m}_x, \mathcal{F}_x) \\
    & t_y(z_y, \mu_y, \bar{m}_x) = \arg \max \int \int U\left(\frac{t_y}{\bar{p}(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y)} z_y - t_y \right) dF(\mu_y, \mathcal{F}_y) dF(\bar{m}_y, \mathcal{F}_y)
\end{align*}
\]

2. The relative price \( p \) clears the international market. Since every unit of \( x \)-good exported must be sold and paid for with \( y \) exports, and conversely, every unit of \( y \) exports must be sold and paid for with \( x \) exports, the only price that clears the international market is the ratio of aggregate exports:
where aggregate exports in each country are
\[ T_x(\mu_x, \bar{m}_x) = \int t_x(z_x, \mu_x, \bar{m}_x) dF(z_x|\mu_x) \] (17)
\[ T_y(\mu_y, \bar{m}_x) = \int t_y(z_y, \mu_y, \bar{m}_x) dF(z_y|\mu_y). \] (18)
which simply integrate over the individual heterogeneity within each country.

3. The perceived and actual price functions coincide:

\[ \tilde{p}(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y) = p(\mu_x, \mu_y, \bar{m}_x, \bar{m}_y) \forall (\mu_x, \mu_y, \bar{m}_x, \bar{m}_y). \] (19)
This definition is relatively straightforward. Agents maximize utility, markets clear, and then the mapping from endowments and signals to prices is consistent with agents’ expectations about prices.

2.1.8. Aggregation
Given the model assumptions, the individual trade and consumption policies are multiplicative in the idiosyncratic shock
\[ t_x(z_x, \mu_x, \bar{m}_x) = z_x \Psi(\mu_x, \bar{m}_x), \quad c_x(z_x, \mu_x, \bar{m}_x) = z_x(1 - \Psi(\mu_x, \bar{m}_x)), \] (20)
where \( \Psi(\cdot) \) is a function that depends only on the aggregate domestic endowment and the beliefs about aggregate foreign endowment. Appendix A contains all the proofs. With this decomposition, aggregate exports become \( T_x(\mu_x, \bar{m}_x) = f_x \Psi(\mu_x, \bar{m}_x), \) where \( f_x = \int z_x dF(z_x|\mu_x) = \exp(\mu_x - \sigma_x^2/2) \) represents the aggregate fundamental. This allows us to aggregate each economy and consider two representative agents, with utility \( \mathbb{E}[U(C_x, C_y)|\mathcal{F}] \) over aggregate consumption, computed as
\[ C_x(\mu_x, \bar{m}_y) = f_x - T_x(\mu_x, \bar{m}_y), \] (21)
\[ C_y(\mu_y, \bar{m}_x, \bar{m}_y) = p(\mu_y, \bar{m}_x, \bar{m}_y) T_x(\mu_x, \bar{m}_y) \] (22)
for the domestic country (and analogously for the foreign country, i.e., \( T_y(\mu_y, \bar{m}_x) = f_y \Psi(\mu_y, \bar{m}_x), \) where \( f_y = \int z_y dF(z_y|\mu_y) = \exp(\mu_y - \sigma_y^2/2) \).

The rest of the analysis focuses on how the trade policies of the representative agents change with uncertainty.

2.2. Remarks on the economic environment
Several comments regarding our modeling choices are in order. While our model makes a clear connection between fundamental frictions and aggregate uncertainty, we abstract from two aspects of uncertainty discussed in the literature. First, uncertainty could arise from firm-specific conditions and second, it could arise from product characteristics. While these issues are interesting, in this case uncertainty would be idiosyncratic rather than aggregate, and hence its effect on aggregate trade would only concern aggregation properties (e.g., distortions to the extensive margin of trade) rather than the uncertainty itself. Moreover, uncertainty does not mean volatility. The shocks in both countries have a fixed variance. Only the uncertainty—what is not known about those shocks—is changing.

Timing is a form of friction here, which allows the information friction to generate aggregate uncertainty. Forcing firms to export before knowing shocks or prices causes uncertainty about the foreign country’s endowment to matter; it is the aggregate uncertainty about the terms of trade that feeds back into the decision to export. Absent any timing friction, information frictions and uncertainty would play no role. Realistically, this modeling choice captures the idea that shipping lags and certain payment arrangements make exporting risky, because the terms of trade might not be known with certainty. For example, Hummel and Schaur (2010) and Hummel and Schaur (2013) demonstrate the time-intensive nature of trade and show how it shapes the...
cross-sectional pattern of trade. Similarly, Antras and Foley (2015), International Monetary Fund (2009, 2011), and Asmundson et al. (2011) present evidence on the cash-in-advance-like arrangements under which import transactions are often carried out.

The financial contracts agents have access to also matters. If a complete set of risk-sharing instruments exists, then agents can contract on outcomes of all unknown variables and the effects of information asymmetry would disappear. In other words, some market incompleteness is necessary for informational frictions to matter. If we want to explore the effects of information frictions, complete markets render that investigation impossible. When markets are partially incomplete, end results show that information frictions facilitate more risk sharing through movements in international prices than would otherwise be insured with financial instruments.

3. Trade and uncertainty with constant elasticity (CES) preferences

This section illustrates the main argument of the paper in the context of a specific utility function. We start with a form of CES preferences that is standard in the trade and international macro literature. The first two results, establishing how uncertainty changes the distribution of the terms of trade, are statistical arguments. They are preference-independent. They are included in this section because they are necessary for a complete explanation of the effect. Proposition 1 explores how changes in the terms of trade distribution induce changes in firms’ chosen export volume. That choice is preference-dependent. Section 4 generalizes Proposition 1 and develops sufficient statistics to determine if uncertainty promotes or inhibits trade.

We argue the following. First, we show how uncertainty—in general equilibrium—affects both the mean and variance of the terms of trade. Second, we characterize how changes in the volume of trade depend on both changes in the mean and variance of the terms of trade and trade elasticity.

As discussed above, we work with the following CES utility function:

$$E\left[ C_x^\theta + C_y^\theta \right] \quad \theta < 1.$$  

The restriction $\theta < 1$ is required for the function to be concave in both goods. How to interpret this utility function? Consider a consumer with CRRA utility $E\left((C^{1-\sigma} - 1)/(1-\sigma)\right)$ and a consumption bundle given by an aggregator $C = (C_x + C_y)^{1/\theta}$. Then this CES case is a special case in which $\sigma + \theta = 1$. If $0 < \theta < 1$, then $\sigma < 1$ and it is a risk-averse agent. These preferences are also considered in Dhingra and Morrow (2019). Lastly, expectations are conditional on the information set of the home country, which contains its own realization of the aggregate shock and the signal about the realization abroad.

3.1. Information and the terms of trade

How does uncertainty affect the stochastic properties of the terms of trade? We proceed in three steps: how uncertainty affects the covariance of exports, how the covariance affects the average terms of trade, and how the covariance affects volatility in the terms of trade. This set of results holds for all elasticity parameters $\theta$ and, as we see later, holds for a much broader class of preferences as well.

Result 1 states that more uncertainty (less precise information about the other country’s endowment) decreases the covariance between aggregate exports.

**Result 1.** Uncertainty Reduces the Covariance of Aggregate Exports. In a neighborhood around complete certainty ($\tilde{s}_x^2$ and $\tilde{s}_y^2$ equal zero), more uncertainty moves the covariance between aggregate exports toward zero.

The intuition behind this result is easy to understand: Agents cannot condition their action on a variable that is not known to them. In our context, this implies that home agents cannot export conditional on foreign outcomes if the foreign state is unknown. Thus, when signal precision approaches zero, the covariance of exports must be zero.

In contrast, as each country becomes less uncertain about the other, they are able to trade in a more sophisticated way. By “sophisticated”, we mean that the home country’s export decision is better informed about the foreign country’s endowment and, in turn, the resulting terms of trade. This leads to more coordinated actions and a stronger covariance in export behavior.

As an example, in the substitute case (i.e., $\theta > 0$), accurate information about a high endowment realization in the foreign country suggests that foreigners will export a lot. Foreign goods will be abundant and cheap; home goods will be relatively expensive. The expectation of high returns to exports incentivizes agents at home to export more. Both countries export more together, i.e., actions are positively covarying.

Fig. 2 illustrates this point. The top panel illustrates the case in which there is perfect information; one sees that exports are highly correlated across countries. When one country exports more, the other country has a strong desire to export more as well. The bottom panel illustrates the opposite extreme. Here neither country has any information about the other country, and thus their exports are independent.

The fact that information allows exports to covary underpins the following result: In equilibrium, higher uncertainty increases both the mean and the variance of the terms of trade.

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3 Appendix D conducts comparative statics for various levels of risk aversion $\sigma$. The restriction $\sigma = 1 - \theta$ adds tractability to the analysis but does not change any of the results qualitatively.
Result 2. Uncertainty Increases Mean and Variance of Terms of Trade. If the unconditional expectations and variances of aggregate exports are kept constant, an increase in uncertainty:

1. Increases the expected terms of trade for both countries. Furthermore, if countries are symmetric, the average terms of trade can be expressed as

\[ E[p] = 1 + \mathbb{C}^{V^2}[T_x] \left(1 - \text{corr}[T_x, T_y]\right), \]  

where \( \mathbb{C}^{V^2} \) is the squared coefficient of variation and \( \text{corr} \) is the correlation.

2. Increases the volatility of the terms of trade for both countries.

The terms of trade are the price that clears the international export market. The only price that clears that market is the ratio of exports. When home and foreign exports covary, the numerator and denominator of the terms of trade covary. Imagine an extreme case in which home and foreign exports had perfect correlation. Home exports were exactly proportional to foreign exports, in every state. Then the ratio of home and foreign exports would be constant. That would imply constant terms of trade, with zero variance. When home and foreign exports covary less, the terms of trade become more volatile. This is the logic formalized in the previous result.

Fig. 3 illustrates the link between the reduction in export covariance and the volatility of the terms of trade. As the figure shows, more uncertainty means less covariance in exports, and thus the terms of trade are much more volatile (compare the black to the gray line).

Fig. 3 also shows the link between uncertainty and the average terms of trade. Greater uncertainty reduces export covariance, which causes the numerator and denominator of the terms of trade to covary less, while always remaining positive. This high-uncertainty case is depicted by the gray line. Notice that the high-uncertainty terms of trade occasionally spike. These are states in which home exports are quite low, and therefore scarce and valuable. With a sufficiently low productivity state, exports can become arbitrarily low, which makes the terms of trade arbitrarily high. Yet the terms of trade never fall below zero. By its nature, the process for the terms of trade is asymmetric.

This is not to say that this is an asymmetry that systematically favors one country over the other. When information is scarce, both countries simultaneously have high expected terms of trade. Indeed, high terms of trade for one country imply low terms of
trade for the other. But high expected terms of trade do not imply that expectation of inverse terms of trade is low. In short, information frictions increase the expected terms of trade for both countries.

Importantly, these results are proven, without reference to the preference specification. The relationship between export covariance and the properties of the terms of trade is a statement about the statistical properties of the ratio of two lognormal random variables. These statistical properties are independent of preferences. Therefore, these two results will carry over when we discuss the model with general preferences at the end. The first result about information that enables correlation is not general because of its sign. In some cases, preferences will make agents want to coordinate their exports negatively. It is always true that the only feasible level of coordination with no information is zero covariance. But less uncertainty might enable either positive or negative export covariance strategies, depending on preferences.

The previous results showed how uncertainty affected mean and variance in the terms of trade through a coordination motive. The next section connects these forces to firms’ decisions on how much to export.

3.2. How the terms of trade distribution affects the volume of trade

The second part of our argument links the expected terms of trade and its variance to the volume of trade. There are no surprises here. Our main point is not that agents react to changes in the mean and variance of the terms of trade in some strange way. With CES preferences and sufficient substitutability, firms export more when the return to exporting is higher and export less when exporting is risky. So conventional wisdom about uncertainty deterring trade is correct in the substitutable-good, CES model. The unexpected part of the relationship between uncertainty and trade comes from the previous section, in which uncertainty about others’ exports raises the expected returns to exporting one’s own good.

The next result shows that, as one would expect, an increase in the expected terms of trade—the return to exporting—increases the average level of exports. It also shows that higher variance in the terms of trade deters exports, because agents are averse to the risky return of exporting.

Proposition 1. Trade’s Response to Terms of Trade Mean and Variance. Suppose the terms of trade mean and variance change in $d\bar{E}_x[p]/\bar{E}_x[p]$ and $d\text{Var}_x[p]/\text{Var}_x[p]$, respectively. Then the sign of the change in exports is equal to the sign of the following expression:

$$\theta \frac{d\bar{E}_x[p]}{\bar{E}_x[p]} + \frac{C\text{Var}_x[p]}{2} \left( \theta(1-\theta)(2-\theta) \frac{d\bar{E}_x[p]}{\bar{E}_x[p]} - \theta(1-\theta) \frac{d\text{Var}_x[p]}{\text{Var}_x[p]} \right).$$

Proposition 1 tells us how much exports will rise or fall from a given percentage change in the expected mean or variance of the terms of trade. It reveals many facets of the relationship between the terms of trade $p$ and trade volume. First, it tells us that an improvement in the expected terms of trade, holding other moments equal, causes firms to export more. This is true for elasticity of substitution $0 < \theta \leq 1$ or for $\theta \geq 2$. The elasticity $\theta$ governs the size of the trade volume effect.
Variance in the terms of trade also changes with uncertainty. More variance—or risk—in the terms of trade deters trade. And this is true for any level of elasticity of substitution. Combined with (25), this result means that uncertainty deters trade through risk. This is the conventional wisdom—that is, noisier signals increase uncertainty, and this force deters exports.

We have identified two competing forces. Consistent with conventional wisdom, uncertainty creates risk and this deters trade. However, uncertainty also raises the return on trade, encouraging more trade. Which force wins?

The relative strengths of the mean and variance forces depend on the degree of uncertainty, as well as the elasticity of substitution. Fig. 4 illustrates this by plotting average exports of the home country as uncertainty increases in two alternative economies: The left panel considers a high substitution economy with $\theta = 0.8$, which features a nonmonotonic effect of uncertainty on trade with a large decreasing segment. The right panel considers a low substitution economy with $\theta = 0.3$ that generates an increasing relationship between uncertainty and trade.

The mean effect is always positive, meaning that increases in the mean terms of trade always increase average exports. The variance effect is always negative. More volatile terms of trade alone always deter trade. When there is a low degree of substitutability between home and foreign goods, the trade-increasing effect is stronger. This reflects the idea that as goods become more complementary, uncertainty matters more because agents want to ensure balanced consumption bundles across goods. As a consequence, agents export more to reach that balance in expectation. Since the trade-increasing mean effect is stronger and the trade-reducing variance effect is weaker for low substitutable goods, these low-$\theta$ economies are ones in which greater uncertainty promotes trade.

3.3. A risk-sharing interpretation

One way of understanding why uncertainty can facilitate trade is to explore why uncertainty enables better risk-sharing. In our trade model, countries would achieve full risk sharing if each country consistently exported half its endowment. In such a world, both countries would consume the same bundle: half of the home goods produced and half of the foreign goods produced in that period. Consumption in both countries would be perfectly correlated. This full risk-sharing world also achieves the maximum level of average trade. Exporting more than half of one’s endowment, on average, never makes sense. So if full risk sharing implies maximum trade, the question of why uncertainty promotes trade amounts to asking why uncertainty brings the world economy closer to full risk sharing.

In finance, the argument for why uncertainty facilitates risk sharing is well understood and is often called the “Hirschleifer effect.” Hirschleifer considers the example of two bettors, each with a ticket on an identical but independent lottery. The bettors can diversify their risk by splitting the two claims, so that if either lottery pays off, both get half the winnings. Now, suppose that both bettors observe noisy signals about the outcome of each lottery. The bettor whose claim is on the lottery with the more favorable signal would want to keep a larger claim on his own lottery. The signal reduces uncertainty about both lottery outcomes, but at the same time undermines risk-sharing. The only way both bettors will consistently share all their risk is if they know nothing about the lottery outcomes.

The analogy between financial markets and trade is not perfect. The fact that trade involves two or more goods, rather than two lottery tickets that pay identical currency units, matters. Risk-sharing in trade takes a different form. There are no ex ante agreements to share output. Instead, the mechanism for international risk-sharing is movement in the terms of trade. When agents in one country get a low endowment, their good is scarce; therefore their good is valuable, and they get lots of foreign goods in return for their exports. The abundance of foreign goods helps to insure the risk of a low endowment.

This insurance mechanism through terms of trade is already present in worlds with perfect information, as in Cole and Obstfeld (1991). What is new here is that uncertainty strengthens the terms of trade as a risk-sharing mechanism, because it
prevents countries from backing away from trade in states when they would prefer not insure their trading partner. To understand the logic of this argument, let us focus on how foreign beliefs are affected by changes in uncertainty, as illustrated by Fig. 5.

Suppose there is a low realization of domestic endowment $\mu_{x_{\text{low}}}$ (solid dark gray line) and uncertainty is very low (toward the left side of the figure). Then foreign firms expect domestic firms to export little, and those are states in which they would prefer to walk away from full insurance. The full insurance action would be for foreign to export lots, home to export little, and both to consume the same amount. But foreign agents do not want to export much at those terms. Just like the bettor who no longer wants to share his half ticket in return for one with lower odds, the foreign country who knows that the home endowment is low no longer wants to send lots away for little in return.

The previous logic breaks when uncertainty is high (toward the right side of the figure), as the foreigners then do not realize that the home endowment is low as their belief moves closer to the prior and away from the true realization. This is pure Bayesian updating. In this case, foreign will export more in a low-endowment ($\mu_{x_{\text{low}}}$) state, providing the home country with better insurance in exactly the states in which it is more needed.

Clearly, the arguments above about uncertainty increasing insurance in bad states applies in reverse for good states. In other words, if the foreigners know that the domestic endowment is high, they would exports lots. But as uncertainty increases, foreigners’ beliefs again move toward the prior and away from the true realization, decreasing exports. However, the lack of foreign exports under this scenario is not very costly for the home country, as it is enjoying a high endowment anyway (this is especially true with high substitutability across foreign and domestic goods). This asymmetry in the demands for insurance is at the core of our results, as the average response of trade to uncertainty is mainly driven by its response at low states, when the insurance premium is larger.

To further investigate how the relationship between uncertainty and trade varies across states, Fig. 6 plots moments of the terms of trade and the volume of exports conditional on the realization of the domestic state $\mu_{x}$, which is either high or low. Without loss of generality, we fix the belief about foreign endowment to its prior value $m_{p} = m_{y}$. We show results for a parametrization with the preferences in \((23)\) with a low level of substitutability $\theta = 0.3$, for which the average response of trade is increasing in uncertainty, as shown above. As a normalization, we express results for expected terms of trade and trade volume as multiples of the value under perfect information (zero uncertainty).

We observe that when the domestic country has a low endowment (solid line), the expectations and volatility of the terms of trade and the volume of exports conditional on the realization of the domestic state $\mu_{x}$, which is either high or low. Without loss of generality, we fix the belief about foreign endowment to its prior value $m_{p} = m_{y}$. We show results for a parametrization with the preferences in \((23)\) with a low level of substitutability $\theta = 0.3$, for which the average response of trade is increasing in uncertainty, as shown above. As a normalization, we express results for expected terms of trade and trade volume as multiples of the value under perfect information (zero uncertainty).

Now let us consider the opposite case with a high domestic endowment (dashed line). Moments of the terms of trade are still monotonically increasing (the increase in the volatility is not observable in the figure due to the scale), but in this case, the volatility effect dominates the average effect and exports decrease with uncertainty. When we average across all states, the strong positive response of exports to uncertainty in bad states dominates the weak negative response in good states, and we recover the result that for low substitutability, uncertainty increases trade on average.

By increasing returns to trade $p$ when the endowment is low and reducing returns when the endowment is high, uncertainty smooths out the utility of each country’s residents. That is, uncertainty improves risk-sharing. The left panel in Fig. 7 shows that cross-country correlation in exports decreases toward zero with higher mutual uncertainty, which illustrates the coordination
failure stated in Result 1 above. The right panel shows how cross-country correlation in utility—a commonly used measure of risk-sharing—increases with uncertainty.

4. The general case

Information reduces uncertainty. Conditioning on that information makes random variables more predictable, and thus less risky. That is the nature of information. Depending on agents’ desire to undertake precautionary savings, the reduction in risk could prompt them to export less or export more. In an equilibrium trade model, the risk effect is not the only effect of information. The other effect is to reduce the expected terms of trade. This shift in the terms of trade distribution can also move the desire to export in either direction. However, the combination of the mean effect and the variance effect reveals the total impact of information on trade. We now examine these forces more generally.

While CES preferences are commonly used and useful for illustrating our results and the mechanisms behind them, focusing on only one type of preferences raises questions: Is CES a special, knife-edge case that generates unusual results? It turns out that CES is not special or knife-edge. A broad class of preferences also has the property by which uncertainty promotes trade. But not all preferences have this property. This section delineates which preferences are like CES, in the sense that uncertainty promotes trade. We also offer practical guidance for those who wish to pursue aggregate uncertainty as a trade barrier. Our results reveal what sorts of preferences are required for mutual uncertainty to deter trade. They also clarify to what extent the CES results reflect risk aversion or good substitutability. With CES preferences, both are tied to one parameter. When we work with a general utility function, we can expose what effects come from each force. The general characterization of the forces at
work also uncovers a mathematical foundation for our risk-sharing interpretation: Uncertainty about trade raises the returns to trade because it facilitates better international risk-sharing.

4.1. How export uncertainty affects terms of trade

The results from Section 2.1, which describe the relationship between trade uncertainty and the mean and variance of the terms of trade, do not depend on any preference specification, i.e., CES preferences. They use the fact that the terms of trade are the ratio of the two countries’ exports. The result by which countries whose exports have lower covariance have higher expected terms of trade does use the fact that countries’ endowments are lognormally distributed. Is this result specific to lognormal variables?

The key to the relationship between trade uncertainty and the expected terms of trade lies in the distributional assumptions. What is essential is that exports can be arbitrarily close to zero but can never be negative. If neither country can ever exports a negative amount, then the ratio of exports, which is the terms of trade, are bounded below by zero. But if there exist states of the world in which either country would choose an export amount arbitrarily close to zero, then the ratio of exports can be arbitrarily large. If the terms of trade are $T_y/T_x$ and $T_x$ can be arbitrarily close to zero, then $p = T_y/T_x$ will occasionally be huge. The point is that the economics of exporting skew the distribution of the terms of trade. There is no way this distribution can be symmetric if it is bounded below and unbounded above. Exactly how skewed and what form the skewness takes depend on the distributions and preferences. But the histogram of the terms of trade, for either country, will always have a bigger right tail.

Once we understand that the terms of trade are a skewed distribution, we can see why signals about exports reduce the mean. Think of a skewed distribution as a function of a normal distribution. Left-skewed distributions would be a concave function of a normal; the concavity accentuates the left tail (bad events). In this case, we have a right-skewed distribution. This can be constructed as a convex function of a normal. Lemma 1 in the Appendix proves that the distribution of the terms of trade must be a somewhere-convex function of a normal probability density. Now, recall that by the definition of convexity, lotteries of convex functions have higher expected values than the median lottery realization. The more uncertain the lottery, the higher the expected value. Firms face terms of trade that are like this convex lottery. The more uncertain the lottery, the higher the expected terms of trade. The higher expected terms of trade are what make exporting more attractive.

For the CES case, Fig. 3 illustrates the same effect in a time-series plot. Recall that when information was scarce, the terms of trade were very volatile; this resulted in occasional spikes in the terms of trade that raised the average. The average terms of trade effect originates in the asymmetry of the terms of trade distribution. This asymmetry arises naturally, whenever exports are required to be nonnegative.

What does this convex lottery look like economically? Consider trade policy uncertainty of the following form: When you export goods, you may get very little in return; the least you can get is $\epsilon > 0$. But there is also a possibility that your good gets through, is relatively scarce, and earns an enormous rate of return. A firm that exports more in the face of trade uncertainty is gambling on the possibility that it is one of the few units that get into the foreign country. If it is, it earns enormous rents on its scarce good. This is a risky lottery, and firms dislike risk. But they also understand that the odds are stacked in their favor: The more uncertain the trade policy, the greater the possibility of winning an enormous rate of return.

4.2. How terms of trade moments affect exports: Sufficient statistics for preferences

With more general preferences, the uncertainty and trade relationship can work either way: Firms may export more or less when the expected terms of trade rise; they may export more or less when the variance increases, or mean and variance effects can trade off differently. We classify preferences, according to a few sufficient statistics, that allow us to say how firms with these preferences will react to trade uncertainty and why.

In a multi-good setting, risk aversion and its related higher-order risk preferences must reflect the interaction of preferences for the two goods. We encode risk attitudes with the following coefficients, which turn out to be the sufficient statistics for determining whether our export paradox holds:

- $\hat{\rho}_y^{(1)} = \rho_y^{(1)} \left(1 - \frac{U_y}{pU_{yy}}\right)$ where $\rho_y^{(1)} = -\frac{C_yU_{yy}}{U_y}$ relative risk aversion;
- $\hat{\rho}_y^{(2)} = \rho_y^{(2)} \left(1 - \frac{U_{yy}}{pU_{yyy}}\right)$ where $\rho_y^{(2)} = -\frac{C_yU_{yy}}{U_{yy}}$ relative prudence; and
- $\hat{\rho}_y^{(3)} = \rho_y^{(3)} \left(1 - \frac{U_{yyy}}{pU_{yyyy}}\right)$ where $\rho_y^{(3)} = -\frac{C_yU_{yyy}}{U_{yyyy}}$ relative temperance.

Coefficients without tildes are the standard single-good expressions for risk aversion, prudence, and temperance. Coefficients with tildes are adjusted by cross-good derivatives and the terms of trade to reflect the fact that there are two goods.

Before we proceed it is useful to interpret each of these statistics, so that we understand what economic forces we are discussing. Start with risk aversion. Note that $\rho_y = -C_yU_{yy}/U_y$ is the standard coefficient of relative risk aversion (RRA) for risky good $y$. Why adjust relative risk aversion for the two goods? If agents do not have any preference for correlated consumption of the two goods ($U_{xy} \leq 0$), then consumption of $y$ is a hedge for the risk of low consumption of $x$. If agents prefer correlated
consumption of both goods, then utility is very high when both goods are abundant and very low when both are scarce. This increases utility risk. Following Kihlstrom and Mirman (1974), when $U_{xy} > 0 \hat{\rho}_y > \rho_y$ the adjusting factor in the RRA amplifies risk aversion compared to a one-good case.

Prudence is a third derivative of preferences and is related to the desire for precautionary savings. It governs whether agents want to export more to insure a modicum of foreign consumption or export less when the return to exporting is riskier. Following Eeckhoudt et al. (2007), we adjust prudence for cross-prudence in $x$. Given a zero-mean $\delta$ random variable, an individual is cross-prudent in $x$ if the lottery $[(x, y + \delta); (x - k, y)]$ is preferred to the lottery $[(x, y); (x - k, y + \delta)]$; that is, higher $x$ consumption dampens the detrimental effects of risk in $y$. Eeckhoudt et al. (2007) show that cross-prudent preference for $x$ implies that $U_{xy} > 0$.

Temperance is a negative fourth derivative of utility, which can be interpreted as a preference for risk disaggregation (see Eeckhoudt and Schlesinger, 2006). Consider two zero mean random variables $\varepsilon_1$, $\varepsilon_2$. An individual is said to be temperate if the lottery $[\varepsilon_1; \varepsilon_2]$ is preferred to the lottery $[0; \varepsilon_1 + \varepsilon_2]$, where all outcomes of the lotteries have equal probability. A temperate individual prefers that risks to be spread across states. With multiple goods, relative temperance also implies that some risk in each good is preferred to concentrating all risk on one good.

Now, we use these sufficient statistics to characterize the relationship between the terms of trade moments and export volume. The first general proposition comes from a second-order Taylor approximation of the firm’s first-order condition. It says that the marginal rate of substitution of a unit of home good for a unit of foreign good should be equal to the risk-adjusted rate of exchange of the two goods. If this were not true, a firm could improve its utility by exporting less or more.

**Proposition 2.** Optimal Exports as a Function of Terms of Trade Conditional Moments. Optimal exports can be approximated, up to second-order, as a function of the conditional mean and variance of the terms of trade distribution, $T[f_x, E_x[p], Var_x[p]]$, and are determined by equating the marginal rate of substitution of $x$ at the expected terms of trade, with the risk-adjusted expected terms of trade.

$$
\frac{U_x(f_x - T, E_x[p])}{U_y(f_x - T, E_x[p])} = \left[1 - \frac{\rho^{(1)}_x}{\text{relative risk aversion}} \left(2 - \frac{\rho^{(2)}_y}{\text{relative prudence}}\right) \frac{C \nabla^2_x(p)}{2} \right].
$$

What is useful about this way of expressing the first-order condition is that it expresses the risk-adjusted terms of trade in terms of our first two sufficient statistics and the mean and variance of the terms of trade. Once we know the mean and variance of the terms of trade, and we know these two features of preferences, we can describe the firms’ optimal export condition.

Higher expected terms of trade make exporting more desirable, unless the term in square brackets is negative. If risk aversion and prudence are sufficiently high, then when a firm believes that it will get more foreign goods back in return for each unit of home exports, it reasons that it can send fewer exports and still have plenty of foreign goods to eat. So it exports less.

More variable terms of trade raise the coefficient of variation, $C \nabla^2_x(p)$. This deters exporting, unless the adjusted relative prudence term $2 - \frac{\rho^{(2)}_y}{\text{relative prudence}}E_x[p]$ is negative. When this prudence term is negative, the firm that faces more uncertain terms of trade exports more to ensure that they will have enough foreign good to consume, even if the rate of exchange turns out to be low. This effect is similar to the increase in precautionary savings observed when earnings are more volatile in consumption/savings problems.

The next result simply differentiates (26) with respect to the mean and variance of the terms of trade. The resulting expression clarifies how changes in the terms of trade distribution change the volume of exports.

**Proposition 3.** Trade’s response to terms of trade mean and variance. Suppose the terms of trade mean and variance change in $dE_x[p]/E_x[p]$ and $d\nabla_x[p]/\nabla_x[p]$, respectively. Then the sign of the change in exports is equal to the sign of the following expression:

$$
\left(1 - \frac{\rho^{(1)}_y}{E_x[p]} \right) \frac{dE_x[p]}{E_x[p]} + \frac{C \nabla^2_x(p)}{2} \left(\frac{\rho^{(1)}_y}{\text{relative risk aversion}}\right) \left(2 - \frac{\rho^{(2)}_y}{\text{relative prudence}}\right) \frac{d\nabla_x[p]}{\nabla_x[p]}. 
$$

What we learn from this is that it clarifies many reasons why exports might rise or fall in response to clearer mutual information. Information can change risk, it can change whether risk is highest when consumption is low or high, and it can change whether risk in one consumption good is high when the other is high or low. How a country responds to each of these changes depends on their preferences. Specifically, it depends on their risk aversion, temperance, and prudence, as described above.
4.2.1. How can information frictions boost trade?

The main question we aim to answer is how cross-border information frictions affect trade volume. We have described some competing effects: Endowment uncertainty raises the mean terms of trade, but also raises variance. This leaves the question of why these competing effects facilitate trade on average.

The mean effect of the terms of trade dominates the variance effect because of preferences. The combination of risk aversion, prudence, and temperance is not strong enough to overcome the higher mean returns to exporting. Under some preferences, higher risk and higher return correspond to less trade. But under commonly used preferences—like CES, with elasticities of substitution consistent with other trade facts ($\theta \approx 0.8$)—the net effect is more trade when uncertainty rises from a low level. This is not an oddity of CES preferences; a broad class of preferences produces the same effect. So, might President Trump’s, or anyone else’s, trade threats promote trade? Yes—if these threats create mutual uncertainty about the quantity of foreigners’ exports and if preferences are not too risk-averse, too prudent, or too temperate, this surge in trade is a logical equilibrium outcome.

4.2.2. When is uncertainty a barrier to trade?

So far, we have focused on cases in which uncertainty increases trade because these are the most surprising. In many cases, however, a researcher might want to build a model in which uncertainty is a barrier to trade. What preferences make that possible?

We can use the previous proposition to precisely define this set of preferences. Suppose information increases both the mean and the variance in equal proportions. Then applying Proposition 3 tells us that average exports will fall if

$$\frac{1-\bar{\rho}_y}{\bar{\rho}_y} + \frac{\mathbb{C}(p)\mathbb{V}(p)}{2} \begin{bmatrix} \frac{\bar{\rho}_y(2)-2}{\bar{\rho}_y(2)} & \frac{3-\bar{\rho}_y(3)}{\bar{\rho}_y(3)} \\ \rho_y & \rho_y \end{bmatrix} < 0. \quad (28)$$

The inequality requires that preferences exhibit high risk aversion ($\bar{\rho}_y(1) > 1$), low prudence ($\bar{\rho}_y(2) < 2$), high temperance ($\bar{\rho}_y(3) > 3$), or a combination of these. This condition describes a test that can be applied to any preferences to determine whether the mean and variance effect combine to deliver a decrease in trade from a rise in uncertainty.

Conceptually, the test is this: Preferences must have the feature that uncertain terms of trade deter, rather than promote, trade, and that this force is strong enough to overcome the increase in the expected terms of trade. High risk aversion helps; it tempers the reaction to changes in expected terms of trade and amplifies the effect of risk. For positive adjusted risk aversion ($\bar{\rho}_y(1) > 0$), low prudence implies a lower precautionary motive. Agents do not want to export more in the face of risk to ensure they have some foreign goods to consume. Instead, they export less to expose themselves less to the unknown rate of return. If the urge to step away from risk is strong, then resolving uncertainty about trade will reduce the terms of trade risk and promote more trade. Low prudence (low $\bar{\rho}_y(2)$) also helps to render trade volumes more sensitive to changes in terms of trade variance. Finally, high temperance helps because temperate agents who face more risk in their consumption of one good want to shift some of that risk to another good. In this case, exporting less is a way of shifting the composition of consumption risk.

4.2.3. Relating the general case and the CES case

The CES results we presented earlier for change in trade volume were a special case of this more general result. Differentiating our CES preferences (23) reveals that the risk aversion term is $1-\bar{\rho}_y(1) = \theta$; the prudence term is $2-\bar{\rho}_y(2) = \theta$; and the temperance term is $3-\bar{\rho}_y(3) = \theta$ (because cross-derivatives are equal to zero, and thus the adjusted preference and standard preference parameters are equal). Substituting these $\theta$ terms into (28), we find the following.

**Corollary.** For CES preferences with $\theta < 1$, an equal percent increase in the average and the volatility of terms of trade inhibits exports if

$$\theta \left[1 + \frac{\mathbb{C}(p)\mathbb{V}(p)}{2}(1-\theta)^2\right] < 0. \quad (29)$$

Since the second term is squared and thus always positive, the only way this expression can be negative is if $\theta < 0$ (goods are complementary). The general preference results now shed light on why the numerical CES results reverse at $\theta = 0$. This is the threshold at which risk aversion, prudence, and temperance combine to make the terms of trade mean effect smaller than the risk affect.

4.2.4. Sufﬁcient statistics in broader classes of models

These results give us conceptual guidance about how to assess the effect of information in models outside this class. They point to two sets of statistics that could be computed for any model. The first pair of statistics maps uncertainty into a mean and variance of the terms of trade. In our model, where the terms of trade is the ratio of exports, uncertainty raises both. It other models with frictions or different market clearing mechanisms, these are the two statistics we need to know from the equilibrium side of...
the model. The second pair of statistics we need governs how a firm’s export decision reacts to raising the expected return and the variance of the return to exporting. We derive such conditions in a frictionless model. In a richer model, similar conditions, depending on the derivatives of the frictions-adjusted marginal utility of exports, would emerge.

4.3. Completing the contracting space

So far, we have assumed away all instruments that agents might use to share international risk. Exchange rate futures, international equity holdings, profit-sharing contracts, and secondary markets could all help to share international risk. If we included a complete set of risk-sharing instruments, then agents could hedge the outcomes of all unknown variables and the effects of information asymmetry would disappear. In fact, just allowing a complete set of risk-sharing instruments, then agents could hedge the outcomes of all unknown variables and the effects of international equity holdings, pro-

Fig. 8. Completing the Market Reduces Exports. Notes: Equilibrium exports in the domestic country for different fractions of agents with price-contingent contracts $\alpha \in [0,1]$. Figure assumes a low elasticity of substitution $\theta = 0.3$ and perfect information (signal noise $\sigma^2 = 0$). Other parameters are $m_x = m_y = 0$, $s_x = s_y = 1$, and $\alpha_x = \alpha_y = \sqrt{2}$. The left panel plots aggregate export volume and the right panel plots exports by type of contract.

<table>
<thead>
<tr>
<th>Aggregate exports</th>
<th>Exports by type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass with contingent exports $\alpha$</td>
<td>Mass with contingent exports $\alpha$</td>
</tr>
<tr>
<td>$0.2$</td>
<td>$0.2$</td>
</tr>
<tr>
<td>$0.4$</td>
<td>$0.4$</td>
</tr>
<tr>
<td>$0.6$</td>
<td>$0.6$</td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.8$</td>
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<tr>
<td>$1.96$</td>
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<tr>
<td>$1.91$</td>
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</table>

We eliminate any signals about the other country’s productivity (the complete uncertainty environment) and study what happens as we change the fraction $\alpha$ of price-contingent exporters. The equilibrium relative price is still the ratio of foreign exports to home exports:

$$p = e^{\mu_x - \mu_y} \frac{1}{2(\sigma_x^2 - \sigma_y^2)} \times \left( \frac{\alpha \Psi_C(p) + (1 - \alpha)\Psi_N(\mu_y)}{\alpha \Psi_C(p) + (1 - \alpha)\Psi_N(\mu_y)} \right).$$

(30)

However, the total export of each country is now $\alpha$ times the export amount of price-contingent firms plus $1 - \alpha$ times the export amount chosen by noncontingent firms.

Having a more complete contracting space and having more information are similar: Both cause the average export volume to fall. When we solve the model with contracts numerically (for a low elasticity of substitution of $\theta = 0.3$), we see that as the number of price-contingent exporters rises, non-price-contingent firms export less and price-contingent firms export more. However, the responses are not very large quantitatively. Still, as the number of price-contingent exporters rises, each noncontingent firm is trading against an average foreign firm that is more likely to have chosen a price-contingent quantity. This is like trading against a foreign country that is better informed. The price-contingent export share rises, but is relatively flat on a per-firm basis. The non-contingent export share falls in aggregate and each firm exports less. The net effect is a decline in exports (Fig. 8).

This model extension demonstrates that introducing complete contingent contracts undermines the effect of asymmetric information. At the same time, however, completing the market and reducing information asymmetry work almost identically to reduce trade, for the same reasons. Conditioning exports on the outcome of a random variable and knowing that random variable before exports are chosen are functionally equivalent.
5. Discussion and conclusions

5.1. What mechanisms are we missing?

The model that we focused on is deliberately stark—two good, two-country, general-equilibrium Armington model. While the Armington model is a core model used in the study of cross-country trade flows and international business cycles, there are important mechanisms that it abstracts from.

One important mechanism is Ricardian specialization according to comparative advantage (see, e.g., Eaton and Kortum, 2002). With Ricardian specialization, uncertainty would presumably affect both the pattern of specialization and the quantities that countries export. In contrast, the Armington structure predetermines the pattern of specialization and our analysis of uncertainty only focuses on the quantity choice or the intensive margin of trade. In a similar context, where resources are allocated to multiple activities with uncertain returns, Van Nieuwerburgh and Veldkamp (2010) show that more uncertainty can either accentuate or reduce specialization, depending on the preference for early or late resolution of uncertainty. Thus, endogenizing the pattern of specialization and studying the effects of uncertainty on production choices is possible. It would require a different model or another layer of model. Therefore, we leave it as an interesting open topic for future work.

A second important mechanism not present here is the extensive margin of trade with firm entry and exit into exporting markets. These mechanisms have been studied in the context of trade policy uncertainty in real-option-value frameworks; see, e.g., Handley and Limão (2017) and Greenland et al. (2019) as an empirical investigation. A related mechanism is where firms must make exporting decisions, while learning about demand conditions, as in Sager and Timoshenko (2019). Uncertainty has three key effects on the extensive margin. First, it deters risk-averse firms from incurring a fixed cost. Second, uncertainty changes the mean terms of trade by altering foreign firms’ entry decisions. Those are similar to the two forces we describe. However, there is a new force at work, not present in our intensive margin analysis. That new force is the real option to not enter or shut down. Uncertainty typically makes such options more valuable. This could encourage more entrants because their perceived upside risk is larger and their downside is always limited by their ability to exit, shut down production, or refuse to trade.

5.2. Connecting theory and data

The theory predicts that goods with higher elasticity should exhibit a more positive relationship between uncertainty and trade. That prediction suggests testing for an interaction term between aggregate trade uncertainty and demand elasticity that should be significant when predicting trade volume. The prediction does not hold for all preferences, but should hold when agents have preferences that are similar to CES with $\theta < 1$. Similar preferences means that the sufficient statistics described in Section 4 are numerically close.

While estimating demand elasticities is a well-established practice, estimating aggregate trade uncertainty is not. What is aggregate trade uncertainty? Taking the model literally, it is uncertainty about a foreign endowment. Endowment uncertainty is challenging to measure directly. But what firms really care about is not how much anyone else is endowed with; they care about these endowments because they forecast their terms of trade. Thus endowment uncertainty is a theoretical stand-in for anything that makes the terms of trade harder to forecast, at the time when the export decision is made. That could be policy changes, changes in taste, shipping delays, or foreign productivity.

Finally, uncertainty is often interpreted to mean any second moment. Here, it has a more specific meaning. Uncertainty means harder to predict. One reason terms of trade might be harder to predict is that it is more volatile. However, a series could have a higher standard deviation simply because its magnitude is larger, in which case the mean should rise as well. Such a scaling effect is not what the model is about. Similarly, terms of trade may change when a policy provision expires, even if that change was expected long in advance. Expected changes create volatility, but are not uncertain. Uncertainty is the volatility of the unpredictable component of the terms of trade. When measuring uncertainty to test the model, it is worth considering what predictable component of the terms of trade might be removed.

5.3. Conclusions

Information frictions are often invoked as reasons for low levels of international trade. But in an equilibrium model, the link between information friction and trade volume is not simple. Our model shows how information also changes the expected terms of trade. It also highlights that in the face of risk, some types of agents may prefer to export more to ensure that they have a sufficient amount of the foreign good to consume. This depends on agents’ preferences.

With constant elasticity of substitution (CES) preferences, information frictions impede trade when goods are very substitutable. The decline in trade occurs because the increase in risk from lower-precision information deters trade, and that risk effect is stronger than the effect on the mean terms of trade, which encourages exporting. But with empirically plausible elasticity parameters, the opposite is true: Information frictions encourage trade. CES preference is not a special or anomalous case. We derive a broad class of preferences for which similar effects arise.

Our results demonstrate that, if we believe that information frictions are truly an important barriers to international trade, we need to amend standard trade models to be consistent with this belief. The could mean changing the elasticities or types of preferences used, adding new frictions that interact with information, or finding some way to change the relationship between uncertainty and the expected terms of trade.
Acknowledgements

We thank our editor, Guido Lorenzoni, our 3 anonymous referees, as well as David Backus, Xavier Gabaix, Réka Juhász, Matteo Maggiori, Natalia Ramondo, Thomas Sargent, and Stanley Zin; our discussants, Kunal Dasgupta, Kyle Handley, Alexander Monge-Naranjo, Jaromir Nosal, and Claudia Steinwender; and seminar participants at NYU, Princeton, Stanford, UPF, CREI, Maryland, Minnesota, Toulouse, NYU Stern, Philadelphia FED, Atlanta FED, Michigan, Banco de México, ITAM, SED 2014, EconCon 2014, ASSA 2015, Econometric Society 2015, XXI Vigo Macro Workshop, and NBER Summer Institute 2019. Andrea Chiavari, Callum Jones, and Pau Roldán provided excellent research assistance. Isaac Baley acknowledges funding from the European Union’s Horizon 2020 Research and Innovation Programme under the Marie Skłodowska-Curie grant agreement No. 705686—Global Policy Uncertainty.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jinteco.2020.103347.

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