Uncertainty and the Economy: The Evolving Distributions of Aggregate Supply and Demand Shocks*

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Abstract

We estimate the time-varying distribution of aggregate supply (AS) and aggregate demand (AD) shocks. We distinguish between traditional Gaussian uncertainty and "bad" uncertainty, associated with negative skewness. The Great Moderation is driven by a reduction in the volatility of AS shocks and the Gaussian component of AD shocks. The increased role of "bad" demand uncertainty implies that the conditional skewness of GDP growth and inflation has decreased over time. The correlation between AS/AD shocks and shocks to their conditional volatilities is generally strongly negative. The correlation between inflation and growth shocks has increased due to a decrease in AS volatility.

Keywords: uncertainty shocks, business cycles, Great Moderation, AS/AD shocks, skewness, deflation risk

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1 Introduction

A growing literature offers equilibrium models in which uncertainty shocks, that is, unexpected changes in the standard deviation of economic shocks, are important drivers of the business cycle (e.g., Justiniano and Primiceri, 2008, Bloom, 2009, Fernández-Villaverde and Rubio-Ramírez, 2013, or Fernández-Villaverde et al., 2015). Researchers generally use either econometric models of time-varying volatility for macro variables (with stochastic volatility and GARCH models perhaps most popular), or direct proxies for uncertainty, such as the VIX index (see Bloom, 2009).[1] In a recent survey of the literature on uncertainty shocks and business cycles, Fernández-Villaverde and Guerron-Quintana (2020) highlight the lack of research on skewness shocks, citing the prevalence of negative one sided shocks, which can help create deep recessions. Building on new research in finance,[2] in this paper we decompose macro-uncertainty into "bad" uncertainty, which is accompanied by negative skewness, and standard Gaussian uncertainty.

We start by decomposing macroeconomic shocks into aggregate demand (AD) and aggregate supply (AS) shocks, defined in the Keynesian tradition, see also Blanchard (1989). That is, AS (AD) shocks move inflation and real activity in the opposite (same) direction. This distinction is important, for example, because the appropriate monetary and fiscal policy responses may be quite different for adverse demand versus supply shocks. We embed this shock structure in a dynamic model with "macro risk" factors, which are state variables that govern the time-varying volatility, skewness and higher-order moments of supply and demand shocks. Technically, we use the Bad Environment-Good Environment model of Bekaert and Engstrom (2017, "BEGE" henceforth) where each shock consists of a "good environment" and a "bad environment" component shock. In the model, a total of four separate factors drive "good" (positively skewed) and "bad" (negatively skewed)

[1] Kozeniauskas, Orlik, and Veldkamp (2018) distinguish between uncertainty shocks measured from micro dispersion, belief heterogeneity or macro uncertainty, but show that volatile macro outcomes can create all three types of uncertainty consistent with the data correlations.

[2] Patton and Sheppard (2015) advocate the use of semi variances (which separately uses positive and negative returns) to create "bad" and "good" volatility, and their methodology has been widely applied (e.g., Kilic and Shaliastovich, 2019). Bekaert and Engstrom (2017) introduce a component model with positively and negatively skewed shocks, which is the inspiration for our model.

uncertainties of AS and AD shocks. As good uncertainty increases, the distribution for the shock becomes more positively skewed.[3] Increases in the bad-type of uncertainty may pull skewness into negative territory. Thus, the model can easily accommodate asymmetric business cycles (Sichel, 1993; Morley and Piger, 2012). The BEGE model accommodates a wide set of distributions, such as a simple Gaussian or extreme rare disaster distributions. In addition, our model allows for a flexible time-varying correlation structure between shocks that drive the level of macroeconomic variables versus shocks that affect uncertainty. Identification of the AS/AD structure is achieved through the non-Gaussianities of the model which is estimated using approximate maximum likelihood (Bates, 2006).

The main model assumes that the exposures of inflation and GDP growth to the structural shocks are time-invariant. In contrast, monetary and fiscal policy regimes (see e.g., Bianchi, 2013, or Baele et al., 2015) can potentially induce time-varying exposures. We therefore also estimate alternative models where the shock exposures are regime dependent, whereas the structural shocks still follow BEGE processes but with constant volatility.

We use the best fitting estimated model to derive three sets of results regarding: 1) the conditional distribution of macro variables, 2) the correlation of level and volatility shocks, and 3) the Great Moderation. First, the data suggest that the "good" component for both demand and supply shocks is Gaussian. However, the data also strongly support a "bad environment" demand component that is highly negatively skewed, which spikes in recessions and features a volatility process that is more transient than that of the Gaussian demand shock. The supply "bad environment" component is similar but less skewed and its volatility process is more persistent. Our macro risk measures generate different non-Gaussian behavior for real activity and inflation depending on whether Gaussian or "bad" risks dominate. In recent years, the conditional distributions for GDP growth and inflation show substantial negative skewness, suggesting increased macro vul-

[3]This distinction opens the possibility of expansionary uncertainty shocks, such as observed during the adoption of the internet in the late 90s.

nerabilities as well as deflation risk. Our work here generally contributes to the literature proposing and estimating models for GDP growth and inflation that admit conditional non-Gaussianities, starting with the regime switching models of Hamilton (1990) for GDP growth and Evans and Wachtel (1993) for inflation. Our results are consistent with the quantile regression results in Adrian, Boyarchenko, and Giannone (2019), showing an important and time-varying left tail in US GDP growth,[4] and Jensen et al. (2020) showing that GDP growth skewness has become more negative over the past three decades, ascribing it to increased leverage of households and firms. However, our focus is broader as we consider the joint distribution of GDP growth and inflation and show how it varies across AD and AS environments.

Second, our econometric model does not impose unrealistic restrictions on the correlation between volatility shocks and shocks to the levels of macroeconomic data. Carriero, Clark and Marcellino (CCM, 2018) point out that more often than not the estimation of uncertainty measures is not embedded in the econometric model used to identify shocks and the uncertainty measures are therefore inefficiently and/or inconsistently estimated (e.g., using a homoskedastic vector autoregression - VAR - to identify shocks). In illustrating the importance of this shortcoming within the context of a Bayesian VAR, CCM (2018) demonstrate that uncertainty indices produce significantly negative output effects, but ultimately uncertainty shocks are not as important as the shocks to the levels of the variables in the VARs themselves. In doing so, CCM (2018) make the important assumption that volatility and level shocks are independent. Alessandri and Mumtaz (2019) create an uncertainty index from 4 macro series, making the same independence assumption. However, level shocks may be naturally correlated with volatility shocks, with negative economic activity shocks being associated with higher volatility, mimicking the asymmetric volatility effect in equities (see, e.g., Engle and Ng, 1993). Instead, our model admits a very flexible time-varying level-volatility correlation.[5] We find positive

[4]Salgado, Guvenen, and Bloom (2019) show related results for micro-dynamics, that is, the skewness of the growth rates of employment, sales, and productivity at the firm level is time-varying and procyclical.

[5]By focusing on two structural shocks and being estimatable from just a few macro series, it is complementary to the reduced form methodology of Gorodnichenko and Ng (2017), who infer volatility shocks from a large panel of data, without imposing correlation restrictions, using a factor model approach.

(negative) correlation between demand shocks and shocks to Gaussian (bad) uncertainty for demand. Supply shocks are negatively correlated with shocks to bad supply uncertainty. Thus, the data support the notion that overall volatility shocks are negatively correlated with level shocks, although these correlations are time-varying and can even switch signs. Bloom et al. (2018) show that empirical impulse responses in a macro VAR can only be fit if they allow negative level shocks to be correlated with uncertainty shocks. This is consistent with our finding that volatility and level shocks are not independent and thus often occur simultaneously.

Third, we use the estimated conditional volatilities and their Gaussian and negatively skewed components to revisit the Great Moderation - a reduction in the volatility of many macroeconomic variables since the mid-1980s. We find it is attributed largely to strong decreases in the volatility of AS shocks and the Gaussian component of AD shocks. Meanwhile, the volatility of bad demand shocks has not experienced a significant decline. As a result, the frequency and severity of recessions, which are mostly associated with elevated bad volatility over the last 40 years, have not changed much over our sample. These results offer a refinement to the work of Jurado, Ludvigson and Ng (JLN, 2015), who find a strong counter-cyclical component to aggregate volatility. Our formal break tests confirm the observation in Jensen et al. (2021) that the recessions since the Great Moderation are in fact deeper that the pre-1984 ones, that is, the skewness of real GDP growth has significantly decreased over time. The same is true, but to a lesser extent, for inflation, since 1990, confirming the recent deflationary bias documented in Bianchi, Melosi and Rottner (2021). The decreased importance of supply shocks also drove up the correlation between inflation and GDP growth shocks.

The final section of the article shows that our supply shocks are significantly correlated to oil and factor productivity shocks, whereas both demand and supply shocks are significantly correlated with consumer confidence shocks. We further show that using financial variables would not alter the identification of our macro model and that our macro risks help capture bond return volatility but have little explanatory power for stock volatility.

2 A Dynamic Model with AD/AS Shocks

Section 2.1 outlines our definition of AD/AS shocks whereas Section 2.2 describes the modelling of the shock distribution and macro risk factors. In Section 2.2, we also discuss the modelling of level versus uncertainty shocks. In Section 2.3, we consider an alternative model with time-varying exposures to AD/AS shocks.

2.1 Defining aggregate supply and demand shocks

Consider a bivariate system in real GDP Growth (g_t) and inflation (π_t):

$$\begin{aligned} g_t &= E_{t-1}[g_t] + u_t^g, \\ \pi_t &= E_{t-1}[\pi_t] + u_t^\pi, \end{aligned} \tag{1}$$

where E_{t-1} denotes the expectation operator conditional on information available at time $t-1$. The variables u_t^g and u_t^π are reduced-form shocks. We model the reduced-form shocks as linear combinations of two structural shocks, labeled supply and demand, and denoted u_t^s and u_t^d, respectively:

$$\begin{aligned} u_t^\pi &= -\sigma_{\pi s} u_t^s + \sigma_{\pi d} u_t^d, \\ u_t^g &= \sigma_{gs} u_t^s + \sigma_{gd} u_t^d. \end{aligned} \tag{2}$$

The σ parameters are the loadings of the reduced-form shocks onto the supply and demand shocks. We assume the σ parameters are all positive to make clear the sign restrictions that we are imposing. In this sense, our use of sign restrictions is different from the common methodology in macroeconomics, pioneered by Faust (1998), Canova and De Nicolo (2002) and Uhlig (2005), to impose sign restrictions on impulse responses to aid identification. The first fundamental economic shock, u_t^s, is an aggregate supply shock, defined so that it moves GDP growth and inflation in opposite directions, as happens, for instance, in episodes of stagflation. The second fundamental shock, u_t^d, is an aggregate demand shock, defined so that it moves GDP growth and inflation in the same direction as would be the case in a typical economic boom or recession from the past few decades. Supply and demand shocks are assumed to be uncorrelated and, without loss of generality, to have unit unconditional variance.

While more complex shock structures can be entertained, this minimal structure encompasses many important economic shocks. For example, standard monetary policy shocks can be viewed as demand shocks (see, e.g., Ireland, 2011); factor productivity and commodity price shocks are supply shocks. In Section 6, we link our estimated shocks to various alternative economic shocks.

Note that the sample covariance matrix of the reduced-form shocks from the bivariate system in equation (1) only yields three unique moments, but we need to identify four σ coefficients in equation (2) to extract the supply and demand shocks. In particular, the unconditional covariance matrix for inflation and growth shocks is:

$$\begin{bmatrix} \sigma_{\pi s}^2 + \sigma_{\pi d}^2 & -\sigma_{\pi s}\sigma_{gs} + \sigma_{\pi d}\sigma_{gd} \\ -\sigma_{\pi s}\sigma_{gs} + \sigma_{\pi d}\sigma_{gd} & \sigma_{gs}^2 + \sigma_{gd}^2 \end{bmatrix}. \tag{3}$$

Hence, absent additional assumptions, a system with Gaussian shocks would be unidentified. Our model achieves identification because we assume that the structural shocks follow a non-Gaussian distribution with time-varying higher order moments.[6]

The main advantage of the definition for supply and demand shocks above is that it carries minimal theoretical restrictions (only a sign restriction). Moreover, once we have estimated the σ parameters in equation (2), we can simply invert the supply and demand shocks without further assumptions:

$$\begin{aligned} u_t^s &= \frac{\sigma_{\pi d} u_t^g - \sigma_{gd} u_t^\pi}{\sigma_{\pi d}\sigma_{gs} + \sigma_{\pi s}\sigma_{gd}}, \\ u_t^d &= \frac{\sigma_{\pi s} u_t^g + \sigma_{gs} u_t^\pi}{\sigma_{\pi d}\sigma_{gs} + \sigma_{\pi s}\sigma_{gd}}. \end{aligned} \tag{4}$$

[6]See Lanne and Luoto (2021) and Bekaert, Engstrom, and Ermolov (2022) for alternative identification methods through higher-order moments. Lanne, Meitz, and Saikkonen (2017) prove the feasibility of our identification approach.

2.2 Modeling macro risk factors

2.2.1 Defining macro risk factors

We define macro risk factors as the variables that capture the time-variation in the second and higher-order moments of supply and demand shocks. Statistically, we generalize the "bad environment-good environment" (BEGE) framework of Bekaert and Engstrom (2017) to accommodate potentially independent innovations to the level and volatility of supply and demand shocks.

Consider a generic shock, u_{t+1} (e.g., a supply or demand shock) to occur at time $(t+1)$. We model u_{t+1} as having two components:

$$u_{t+1} = \sigma_{up}\omega_{p,t+1} - \sigma_{un}\omega_{n,t+1}, \tag{5}$$

where $\omega_{p,t+1}$ and $\omega_{n,t+1}$ are individual component shocks. The volatility parameters σ_{up} and σ_{un} are restricted to be positive. The component shocks are independent and distributed as centered-gamma:

$$\begin{aligned} \omega_{p,t+1} &\sim \tilde{\Gamma}(p_t, 1), \\ \omega_{n,t+1} &\sim \tilde{\Gamma}(n_t, 1), \end{aligned} \tag{6}$$

where the expression $\omega_{p,t+1} \sim \tilde{\Gamma}(p_t, 1)$ denotes that the random variable $\omega_{p,t+1}$ follows a centered gamma distribution with shape parameter p_t and a unit scale parameter.[7] Consider the first term on the right hand side of equation (5), $\sigma_{up}\omega_{p,t+1}$. Because the $\omega_{p,t+1}$ shock is right skewed, we refer to it as a "good" shock (though it has zero mean and it may, of course, have negative realizations). The variance of this component of u_{t+1} is $\sigma_{up}^2 p_t$, which is a well-known feature of the gamma distribution, and its (unscaled) third moment is $2\sigma_{up}^3 p_t$. When p_t is time-varying, we refer to p_t as the "good variance" state variable. Similarly, the second term in u_{t+1}, $-\sigma_{un}\omega_{n,t+1}$, is negatively skewed, with

[7] The probability density function is $\phi(\omega_{p,t+1}) = \frac{1}{\Gamma(p_t)}(\omega_{p,t+1} + p_t)^{p_t - 1} e^{-\omega_{p,t+1} - p_t}$ with $\omega_{p,t+1} > -p_t$ and $\Gamma(p_t)$ representing the gamma function. This distribution has zero mean, unlike the standard gamma distribution.

variance $\sigma_{un}^2 n_t$ and third moment of $-2\sigma_{un}^3 n_t$. We thus refer to n_t as the "bad variance" state variable. Unscaled skewness decreases (increases) in n_t (p_t) and the demeaned gamma distribution converges to a Gaussian distribution for large p_t and n_t.

For an illustration of the density implied by equation (5), the upper part of Panel A in Figure 1 illustrates that the probability density function of $\sigma_{up}\omega_{p,t+1}$ (the "good" component) is bounded from the left and has an unbounded right tail. Similarly, the middle part of Panel A in Figure 1 shows that the probability density function of $-\sigma_{un}\omega_{n,t+1}$ (the "bad" component) is bounded from the right and has an unbounded left tail. Panel B of Figure 1 illustrates possible conditional distributions of u_t which could arise as a result of time variation in the shape parameters p_t and n_t. In particular, the probability density function at the top of Panel B in Figure 1 characterizes the situation where good volatility (as governed by p_t) is relatively large and the distribution has a pronounced right tail, while the probability density function in the bottom corresponds to the case where bad volatility is relatively large (i.e., a large value for n_t) with the distribution exhibiting a pronounced left tail.

To model the dynamics of macroeconomic uncertainty, we assume that the risk factors, p_t and n_t, follow simple autoregressive processes:

$$\begin{aligned} p_{t+1} &= \bar{p}(1-\rho_p) + \rho_p p_t + \sigma_{pp}\nu_{p,t+1}, \\ n_{t+1} &= \bar{n}(1-\rho_n) + \rho_n n_t + \sigma_{nn}\nu_{n,t+1}, \end{aligned} \tag{7}$$

where $\bar{p}$ and $\bar{n}$ are the unconditional means of the variables, and ρ_p and ρ_n govern their autocorrelation. The volatility parameters, σ_{pp} and σ_{nn} are restricted to be positive. The shocks to good and bad variance, $\nu_{p,t+1}$ and $\nu_{n,t+1}$ in equation (7), are gamma-distributed shocks and they also use p_t and n_t as their shape parameters:

$$\begin{aligned} \nu_{p,t+1} &\sim \tilde{\Gamma}(p_t, 1), \\ \nu_{n,t+1} &\sim \tilde{\Gamma}(n_t, 1). \end{aligned} \tag{8}$$

This is similar to other common "square root volatility" specifications in which the conditional volatility of the shock is proportional to the square root of the level of the series. For example, the conditional volatility of p_{t+1} is $\sigma_{pp}\sqrt{p_t}$. In the general model presented above, the following conditional moments for u_t follow from the properties of the gamma distribution:

$$\begin{aligned}
&E_t[u_{t+1}] = 0, \\
&E_t[u_{t+1}^2] = Var_t[u_{t+1}] = \sigma_{up}^2 p_t + \sigma_{un}^2 n_t, \\
&E_t[u_{t+1}^3] = 2\sigma_{up}^2 p_t - 2\sigma_{un}^2 n_t, \\
&E_t[u_{t+1}^4] - 3(E_t[u_{t+1}^2])^2 = 6\sigma_{up}^4 p_t + 6\sigma_{un}^4 n_t.
\end{aligned} \tag{9}$$

Thus, the BEGE structure implies that the conditional variance of macro shocks varies through time, and that the shocks may be conditionally non-Gaussian, with time variation in the higher order moments all driven by p_t and n_t. Moreover, the conditional variances of GDP and inflation vary through time as functions of these structural macroeconomic risk factors, $[p_t^s, n_t^s, p_t^d, n_t^d]'$, with the "$s$" superscript denoting supply variables and "d" denoting demand. In addition, the model also implies that the conditional covariance between inflation and GDP growth shocks is time-varying and can switch signs:

$$Cov_{t-1}[u_t^g, u_t^\pi] = -\sigma_{\pi s}\sigma_{gs} Var_{t-1}[u_t^s] + \sigma_{\pi d}\sigma_{gd} Var_{t-1}[u_t^d], \tag{10}$$

where the subscripts on the Cov and Var operators denote that they may vary over time. As can be seen in equation (10), when demand variance dominates the covariance is positive but when supply variance dominates it is negative.

2.2.2 Level and uncertainty shocks

To close the model, we must make assumptions regarding the correlation between the "level" shocks, $\omega_{n,t}$ and $\omega_{p,t}$, and the "volatility" or "uncertainty" shocks, $\nu_{n,t}$ and $\nu_{p,t}$. Without loss of generality, we assume that the two types of shocks (ω's and ν's) are independent, but we allow a flexible correlation between the level and volatility shocks by replacing equation (5) with:

$$u_{t+1} = \sigma_{up}((1-\lambda_p^2)^{\frac{1}{2}}\omega_{p,t+1} + \lambda_p \nu_{p,t+1}) - \sigma_{un}((1-\lambda_n^2)^{\frac{1}{2}}\omega_{n,t+1} + \lambda_n \nu_{n,t+1}), \tag{11}$$

where λ_p and λ_n are between 0 and 1. Although the formulation looks complex, it is simply structured to imply that the conditional correlation between the good component of u_{t+1} and p_{t+1} is equal to λ_p. Analogously, the conditional correlation between the bad component of u_{t+1} and n_{t+1} is $-\lambda_n$. Note that despite the complexity of the model in equation (11), the conditional variance, Var_t, of u_t is still $\sigma_{up}^2 p_t + \sigma_{un}^2 n_t$. Moreover, we have:

$$Cov_t[u_{t+1}, Var_{t+1}] = \lambda_p \sigma_{up}^3 \sigma_{pp} p_t - \lambda_n \sigma_{un}^3 \sigma_{nn} n_t. \tag{12}$$

Intuitively, positive bad (good) variance shocks lower (increase) the conditional covariance between level shocks and uncertainty shocks. When the bad variance state variable dominates, the model generates the macroeconomic counterpart of asymmetric volatility in finance (e.g., Heston, 1993): level shocks are negatively correlated with conditional volatility. This property then also potentially captures the positive association between increases in demand/supply uncertainty and real contractions featured in some New Keynesian models (see e.g. Bianchi, Kung and Tirskikh, 2023). In addition, it can potentially capture the intriguing links between realized volatility and contractions described in Berger, Dew-Becker, and Giglio (2021). They find that innovations in realized stock market volatility are robustly followed by contractions, while shocks to forward-looking uncertainty have no significant effect on the economy (once realized volatility is controlled for). In our model with $\lambda = 1$, the same shocks drive both the level and uncertainty variables. More precisely, a large "bad" component shock is associated with low output (inducing high realized volatility), implies skewed recessions, and is associated with high future uncertainty. For these links to appear in our model, there must be a strong link between macro and financial uncertainty, a link we test below in Section 6.

Because the specification in equation (11) requires two additional parameters and involves 4 latent shock variables, we also consider more parsimonious special cases. In one case we set the λ parameters equal to zero. Under this specification, $\nu_{p,t+1}$ and $\nu_{n,t+1}$ are "pure" volatility shocks, with no effect on the level of the overall macro shock. Thus, the covariance between the level shocks and the conditional variance is zero, as is the case in a standard Gaussian GARCH model. At the other extreme, the λ's equal 1. In this case, the level and risk factor shocks coincide. For example, when $\lambda_p = 1$, the good component of u_t is perfectly correlated with the shock to p_t. It is worth noting that even in this seemingly restrictive case, there is still independent variation between the observed macro shock, u_{t+1}, and the risk factors. To see this, note that when $\omega_{p,t+1} = \nu_{p,t+1}$, the conditional correlation between u_{t+1} and p_{t+1} is $Corr_t(u_{t+1}, p_{t+1}) = \frac{\sigma_{up}\sigma_{pp}p_t}{(\sigma_{up}^2 p_t + \sigma_{un}^2 n_t)^{\frac{1}{2}}(\sigma_{pp}^2 p_t)^{\frac{1}{2}}}$ which in general varies from 0 ($n_t \gg p_t$) to 1 ($p_t \gg n_t$).[8]

The model is also rich enough to approximate the popular "disaster risk" model (e.g., Gabaix, 2012, or Wachter, 2013), where Gaussian shocks are combined with a jump process delivering occasional severe negative shocks (e.g., following a Poisson distribution). Such a model emerges when the $\omega_{p,t}$ shock is (nearly) Gaussian, and the $\omega_{n,t}$ shock is very skewed (with n_t having a very low mean).

2.3 Accommodating time-varying exposures

In our model, the conditional distribution of the structural shocks is time-varying, inducing time variation in conditional variances and conditional skewness. Unless the uncertainty shocks are independent of the level shocks (for which we do not find empirical evidence), the BEGE framework can capture the phenomenon that negative shocks increase volatility more than positive shocks. A large literature (see, e.g., Bachmann and Moscarini, 2012, Decker, D'Erasmo and Boedo, 2016, or Ilut, Kehrig, and Schneider, 2018) provides micro-foundations of firm behavior suggesting that negative firm specific level shocks can lead to increased macro volatility. However, increased volatility can also

[8]There is also a large body of work in finance on the importance of "volatility of volatility" shocks, see, e.g., Bollerslev, Tauchen and Zhou (2009). The factor structure that we build in for the higher order moments of the structural shocks implies that the variance of the variance of u_t is also an affine function of p_t and n_t. Denoting the variance of the variance of u_t by q_t, we have: $q_t = \sigma_{up}^4\sigma_{pp}^2 p_t + \sigma_{un}^4\sigma_{nn}^2 n_t$. Note that q_t is not perfectly correlated with the conditional variance of u_t, but the model does imply an intuitive positive correlation between the variance of u_t and its variance of variance, q_t.

arise through stronger responses to shocks of constant size. Berger and Vavra (2019) provide evidence of the latter using micro-level data on the dispersion of price changes reacting to exchange rate pass-through. At the macro level, models featuring regime changes in the conduct of monetary policy (see, e.g., Bianchi, 2013, or Baele, Bekaert, Cho and Moreno, 2015), or changes in the monetary/fiscal policy mix (Bianchi and Ilut, 2017) might imply time-varying responses to the structural shocks.

We therefore also formulate a model where the exposures are regime dependent, whereas the structural shocks still follow BEGE processes. In particular, define the matrix of exposures:

$$M = \begin{bmatrix} -\sigma_{\pi s} & \sigma_{\pi d} \\ \sigma_{gs} & \sigma_{gd} \end{bmatrix}. \tag{13}$$

Whereas in our previous model, the coefficients in M are assumed constant over time, we now assume that they can take on two different values depending on the realization of a regime variable. In the first specification, we introduce a different regime variable, $s_{t,ij}$ ($i = \pi/g$, $j = s/d$) for each loading, so that they need not switch at the same time. Each $s_{t,ij}$ follows the usual Hamilton (1989) Markov chain with constant transition probabilities. Assuming independent regime variables, there are a total of $2^4 = 16$ different regime combinations. The estimation of M involves $2 \times 4 = 8$ parameters and there are 8 transition probabilities (2 per regime variable). We therefore assume that the structural shocks follow BEGE processes but with constant volatility:

$$\begin{aligned} &[u_t^\pi, u_t^g]' = M(s_t)[u_t^s, u_t^d]', \\ &u_t^s \sim \sigma_{ps}\tilde{\Gamma}(\bar{p}^s, 1) - \sigma_{ns}\tilde{\Gamma}(\bar{n}^s, 1), \\ &u_t^d \sim \sigma_{pd}\tilde{\Gamma}(\bar{p}^d, 1) - \sigma_{nd}\tilde{\Gamma}(\bar{n}^d, 1), \end{aligned} \tag{14}$$

where $s_t = \begin{bmatrix} s_{t,\pi s} & s_{t,\pi d} \\ s_{t,gs} & s_{t,gd} \end{bmatrix}$. Thus, the model has 24 parameters and with the imposition of unit variances for the structural shocks, effectively 22 parameters. In the second, more parsimonious, specification, the elements of M switch at the same time. That is, there is just one regime variable, s_t, taking on two values, and two different M matrices. Because there are now only two transition probabilities in total, this model saves 6 parameters relative to our first specification.

3 Estimating the Dynamic Model

In this section, we first discuss the data used to estimate the model outlined in Section 2. We then describe the estimation methodology and the model selection procedure with its associated results.

3.1 Measuring macro shocks

We use forecast revisions from survey data to operationalize equation (1), obviating the need for model selection in estimating reduced-form macro shocks. Not having to model conditional means helps mitigate the criticisms of CCM (2018) on the modeling of volatility shocks within an inconsistent econometric framework. The survey data are from the Survey of Professional Forecasters (SPF). Our sample is quarterly from 1968:Q4 to 2019:Q4. The number of respondents in the SPF varies over time and across macro variables being forecasted but a typical number of respondents is about 40.

To identify inflation shocks using the survey data, we use:

$$u_t^{\pi} = \hat{\pi}_t - \hat{\pi}_{t,t-1}, \tag{15}$$

where $\hat{\pi}_t$ is the forecast in quarter t for the percentage change in the GDP deflator in quarter t (π_t), and $\hat{\pi}_{t,t-1}$ the forecast for π_t in the previous quarter. Therefore, u_t^{π} represents the revision to the expectation for π_t between periods $t-1$ and t. Note that published data for π_t is generally not fully available until many quarters after (at least until $t+1$ for an advance release), so $\hat{\pi}_t$ need not equal the eventually published official value for π_t. The SPF survey is typically published around the 10^{th} day of the month in February, May, August and November of each year. As a concrete example, our measured revision to inflation for the period 2018:Q1 is equal to the average SPF forecast (as of early February 2018) for inflation for Q1 inflation minus the expectation for Q1 inflation that was measured in the previous SPF survey, published in early November of 2017. The inflation data forecasted in the survey corresponds to the percentage change in the GDP price deflator over the first (calendar) quarter of 2018; this data is first published (through an advance release) by the U.S. Bureau of Economic Analysis (BEA) in April.

Our use of survey revisions to measure economic shocks is perhaps uncommon, but we believe it is well justified. First, the true pace of economic activity is never directly observed, only estimated. One estimate of economic activity is the BEA advance release that is published one month after the quarter end. Another estimate is the BEA final (revised) release, which is published many quarters (and often years) after the fact. The latter measure is the one which is perhaps most often used in academic papers, but it is the least plausible candidate for being in the minds of economic agents due to the lag in publishing. For example, Ghysels, Horan, and Moench (2018) show that the use of real time data substantially reduces the predictive power of macro variables for bond returns, suggesting that investors do not anticipate future data revisions. In addition, GDP and inflation data most certainly are plagued by measurement error (see, e.g., Aruoba et al., 2016, for GDP and Lebow and Rudd, 2006, for inflation), which renders the structural modeling of shocks more difficult. In contrast, current-quarter nowcasts from survey data also offer viable estimates of economic activity - those of the survey respondents, and they have the advantage of being available in real time, and are therefore plausibly reflected in the beliefs of economic agents. Moreover, they should be less subject to measurement error noise.

Second and importantly, these revisions do correspond to a difference in realized value from its conditional expectation as in equation (1). Because of the law of iterated expectations, $\hat{\pi}_{t,t-1}$ is the conditional expectation for $\hat{\pi}_t$ in the previous quarter.

Third, Ang, Bekaert and Wei (2007) show that inflation expectations from the SPF provide more accurate forecasts of future inflation than statistical, Phillips curve and term structure models. Coibion and Gorodnichenko (2012, 2015) show that the predictability of forecast errors from SPF inflation forecasts (including predictability coming from forecast revisions) is consistent with models of information rigidities and cast doubt on full rational expectations models. Our estimates are therefore more consistent with actual expectation formation than econometric models estimated on revised data would be. Similarly, we measure shocks to the outlook for real activity as forecast revisions for the percentage change in real GDP growth.[9]

Finally, the use of forecast revisions implies that structural changes, including the policy shifts alluded to before, may well be accounted for in the forecasts of the survey respondents. Still, we must ensure that the shocks we use are serially uncorrelated, and we therefore pre-whiten the forecast revisions using past revisions and past forecasts. We relegate the results from the projections to Appendix I.A. We find some predictive power of past revisions for current inflation forecast revisions, but not for GDP forecast revisions. Fortunately, the resulting shocks are highly correlated with the actual forecast revisions (at 0.91 and 0.98 for inflation and GDP growth shocks, respectively). We use the pre-whitened series in all of our analysis.

Figure **??** depicts the resulting real GDP and inflation shocks, expressed as a percentage change at an annual rate. Shocks to real GDP shocks are generally larger earlier in the sample, and deeply negative spikes occur during recessions throughout the sample. Similarly, inflation variability is higher earlier in the sample and large positive and negative spikes are evident during recessions that occur early in the sample period. Later in the sample period, the overall variability of inflation decreases, and the shocks during recessions are notably negative.

In Appendix I.B we verify that these patterns appear largely consistent with patterns in macro shocks defined in a more standard VAR fashion. In particular, we extract GDP

[9]Mechanically, survey respondents fill out forecasts for nominal GDP and the GDP deflator separately, with their forecasts for real GDP being calculated as the ratio between the two.

growth and inflation shocks from a bivariate VAR, appended with SPF data as predictors. Because the VAR uses "final release" data (with revisions sometimes happening years later), and our forecast revisions use information available in the first month of the quarter to make an end-of-quarter prediction, high correlations would be surprising. Still, the correlation between our pre-whitened revisions and VAR residuals is 0.47 for inflation and 0.61 for GDP growth.

3.2 Estimating the macro risk model

The two types of models we estimate feature non-Gaussian shocks. For both models, the structural shocks are unobserved, but can be recovered from inflation and GDP growth shocks, conditional on estimates for M (respectively for $\mathrm{M}(s_t)$, where s_t collects the 4 $s_{t,i,j}$ regime variables, or represents the one regime variable). Thus, the transition from structural shocks to data simply reflects a change in variables.

For the first BEGE model, featuring time-varying shape parameters and Gamma distributed shocks (see also Bekaert, Engstrom and Ermolov, 2015; Bekaert and Engstrom, 2017), the conditional distribution of the shocks depends on the latent variables $p_{t,k}$ and $n_{t,k}$ $(k = d, s)$. Therefore, the likelihood function is not available and we use an estimation and filtering apparatus due to Bates (2006). The methodology is similar in spirit to that of the Kalman filter, but the Bates routine accommodates non-Gaussian shocks. We relegate a technical discussion to Appendix III.

For the second set of models, with regime switching loadings, conditional on the regime, the BEGE likelihood is actually available in closed form but must be computed numerically. For each observation, depending on the specification, there are either 16 or 2 different regimes and thus 16 or 2 different M matrices. The regime probabilities are filled in recursively as in Hamilton (1989). We also estimate several simpler variants of the main models (see Section 3.3), including models with Gaussian shocks. When both components in the BEGE distribution are Gaussian, the overall distribution is replaced by one Gaussian distribution. The estimation of the various regime-switching models is described in detail in Appendix III.

3.3 Model selection

The general model in equations (5)-(8) and (11) is quite highly parameterized. Therefore, we estimate a number of variations on the basic BEGE model for both supply and demand shocks to identify the most parsimonious specifications that are supported by the data. When we estimate the full model, the λ parameter, which governs the correlation between level and uncertainty shocks, is not precisely identified for either demand or supply shocks. Moreover, the full models do not rank very high on the AIC criterion relative to the extreme models involving λ=1 or λ=0, and are therefore not considered as part of the general parameter search. The model search focuses on Gaussian versus BEGE shocks, time-varying or constant shape parameters, and independent or coinciding level and uncertainty shocks. Specifically, the various specifications that we investigate include (leaving out supply/demand indicators):

1. $\omega_{p,t+1}$ and $\nu_{p,t+1}$ are (i) independent ($\lambda_p = 0$), (ii) coincide ($\lambda_p = 1$)
2. $\omega_{n,t+1}$ and $\nu_{n,t+1}$ are (i) independent ($\lambda_n = 0$), (ii) coincide ($\lambda_n = 1$)
3. p_t is time-varying or constant
4. n_t is time-varying or constant
5. $\omega_{p,t+1}$ is demeaned gamma or Gaussian
6. $\omega_{n,t+1}$ is demeaned gamma or Gaussian

Variations 1 and 2 impose different degrees of dependence between the good and bad components of u_{t+1}, and the shocks to the risk factors, $\nu_{p,t+1}$ and $\nu_{n,t+1}$. One variation in 3 and 4 restricts the good and/or bad variance risk factors to be constant. For instance, p_t being constant imposes $\rho_p = \sigma_{pp} = 0$, reducing the number of parameters, but also reducing the flexibility of the model, potentially to the detriment of the model fit of the data.

Variations 5 and 6 potentially replace the gamma distribution with the Gaussian distribution for $\omega_{p,t+1}$ and/or $\omega_{n,t+1}$. The Gaussian distribution requires one fewer parameter relative to the gamma distribution, but the Gaussian distribution cannot accommodate conditional skewness or other higher-order moments.

We guarantee the positive nature of volatility processes. In models featuring Gaussian shocks, we follow Bates (2006) and posit that the filtered volatility state variables conform to a gamma distribution with a lower bound at zero. In BEGE models, the $\nu_{p,t+1}$ and $\nu_{n,t+1}$ shocks are assumed to follow a demeaned gamma distribution. With mild parameter restrictions, this specification ensures that volatility remains positive (see, e.g., Gourieroux and Jasiak, 2006).

Note that for each shock, there are 6 possible models (either Gaussian or BEGE models, with static moments, or time-varying moments with either λ=0 or 1), yielding a total of $6^4 = 1296$ different models. However, because the sum of Gaussian variables is Gaussian, a number of specifications are degenerate, so we end up estimating 1226 different specifications (in addition, static Gaussian models are not identifiable because there are only 3 covariance parameters to identify 4 response parameters). For the regime switching model, we also estimate the Gaussian counterpart to the main model involving BEGE shocks.

Regarding inference for the parameters of the model, we use a bootstrapping procedure. First, we block-resample our raw (pre-whitened) GDP and inflation shocks, using blocks of length 28 quarters. We then estimate the model on the bootstrapped data. This is repeated for 1000 bootstrapped samples. The reported standard errors are the standard deviations of the estimated parameters across bootstrapped samples.

We use the small sample corrected Akaike information criterion (AICc) as the basis for model selection. Table 1 shows the selection criteria for the "pure" models. Gaussian models are shown in the upper panel with either static or stochastic volatilities, or with constant volatilities, but regime-switching loadings on the structural shocks. The next panel reports the BEGE counterparts. The full BEGE model has 18 parameters whereas the two regime-switching BEGE models have 22 and 16 parameters, respectively. We report both the full BEGE specifications for specifications with either all λ parameters equal to 1, or equal to 0. We report the log-likelihood (but recall that these models are not nested) and the AIC criteria.

For the constant M models, the BEGE models are substantially better than the Gaussian models. However, for the first regime-switching model, assuming BEGE shocks delivers much less of an advantage, perhaps because the regime-switching model in itself generates conditional non-Gaussianities. The more parsimonious regime-switching model delivers a similar likelihood and thus, is substantially better in terms of the AIC criterion. Yet, the "stochastic" BEGE models (where the component shocks have time-varying distributions) still deliver considerably lower AICs than the regime switching models. Among the BEGE models, the static BEGE model is much worse than the time-varying shape parameter models. Contrasting the model with independent level and uncertainty shocks (λ=0) to the one with correlated level and uncertainty shocks (λ =1), the latter is substantially better.

Not surprisingly, the overall best model in terms of the AIC criterion features the same shocks affecting both the level and volatility of demand and supply. This overall best model is slightly more parsimonious than the general model reported before, featuring some Gaussian and constant shape parameter components. Specifically, for the supply shock, the optimal AICc model is:

$$\begin{aligned}
u_t^s &= \sigma_{sp}\omega_{p,t}^s - \sigma_{sn}\omega_{n,t}^s, \\
n_t^s &= \bar{n}^s(1-\rho_n^s) + \rho_n^s n_{t-1}^s + \sigma_{nn}^s\omega_{n,t}^s, \\
\omega_{p,t+1}^s &\sim \mathcal{N}(0,\bar{p}^s), \\
\omega_{n,t+1}^s &\sim \tilde{\Gamma}(n_t^s,1).
\end{aligned} \tag{16}$$

One important finding is that the "good" environment component of supply shocks, $\omega_{p,t}^s$, is well-modeled using a Gaussian distribution as opposed to a gamma distribution (the latter requires an additional parameter) with a constant variance. Second, the bad component of supply shocks is gamma-distributed with time-varying volatility. A single shock, $\omega_{n,t}^s$, affects both the level of the supply shock and the shock to bad variance. This generates negative correlation (but not perfect negative correlation) between the overall supply shock and the bad variance shock.

For demand shocks, the optimal specification under AICc is:

$$
\begin{aligned}
& u_t^d = \sigma_{dp}\omega_{p,t}^d - \sigma_{dn}\omega_{n,t}^d, \\
& p_t^d = \bar{p}^d(1-\rho_p^d) + \rho_p^d p_{t-1}^d + \sigma_{pp}^d \omega_{p,t}^d, \\
& n_t^d = \bar{n}^d(1-\rho_n^d) + \rho_n^d n_{t-1}^d + \sigma_{nn}^d \omega_{n,t}^d, \\
& \omega_{p,t+1}^d \sim \mathcal{N}(0, p_t^d), \\
& \omega_{n,t+1}^d \sim \tilde{\Gamma}(n_t^d, 1).
\end{aligned}
\tag{17}
$$

We see that for demand shocks, the AICc selects a specification with two volatility factors. As was the case for supply shocks, the good component of demand shocks is distributed as Gaussian and the bad component is gamma-distributed. Moreover, for demand, as for supply, the same shock, $\omega_{n,t}^d$, affects both the level of u_t^d as well as the bad variance, n_t^d. The only difference between the specification chosen for demand shocks versus supply shocks is that AICc selects the good component of the shock to demand to feature a time-varying shape parameter.

Because for both supply and demand shocks AICc selects a Gaussian "good" component, which of course has zero skewness, we refer to these components below as the Gaussian volatility or Gaussian component rather than "good." Of course, both supply and demand shocks also feature a negatively skewed bad component of volatility that varies over time, which we continue to refer to as bad volatility.

4 Parameter estimates and implications

We report the parameter estimates in two parts. Section 4.1 focuses on the M-parameters which govern the loadings of inflation and GDP growth shocks on the structural shocks. They suffice to extract demand and supply shocks from the data. Section 4.2 focuses on the macro risk parameters.

4.1 Loadings of GDP growth and inflation shocks on AD/AS shocks

In Table 2, Panel A, we report the supply and demand loadings for GDP growth and inflation (the M-parameters). These are generally quite precisely estimated. Our estimates suggest that supply shocks contribute more to the unconditional variance of inflation shocks over this sample period than do demand shocks; the inflation supply and demand loadings are -0.48 and 0.23, respectively.[10] This implies that supply (demand) shocks account for about 81% (19%) of the inflation variance. Unconditionally, demand shocks, contribute more than supply shocks to the overall variance of real growth shocks: the real GDP growth demand and supply loadings are 0.93 and 0.56, respectively (in variance terms the relative contribution is approximately 73% versus 27%). This is not entirely surprising, as much of the recent recessions were mostly associated with demand shocks, but our sample includes the stagflation of the 70's and the Great Recession, which many argue had a significant supply component (see, e.g., Ireland, 2011, or Mulligan, 2012).

The top panels of Figures 3 and 4 depict the supply and demand shocks that we recover from this exercise. Both sets of shocks exhibit greater overall variability early in the sample period, followed by a secular decline in variability that perhaps reflects the so-called "Great Moderation", although deeply negative shocks occur during recessions throughout the entire sample. We also verifiy that the AS and AD shocks are indeed non-Gaussian. The aggregate demand shock has a skewness coefficient of -0.40 and its excess kurtosis is 2.21, while the aggregate supply shock features a skewness coefficient of -0.55 and excess kurtosis of 3.01. Not surprisingly, a Jarque-Bera (1980) test of the null of Gaussianity rejects with a p-value of less than 0.1%.

Our supply and demand shocks definitions do not necessarily comport with demand and supply shocks in, say, a New Keynesian framework (see, e.g., Woodford, 2003) or structural VARs in the Sims (1980) tradition. However, Appendix II shows that the

[10] To the extent that we find that some of the volatility factors are very persistent, the concept of unconditional moments should be interpreted cautiously.

long-term effects of the AS/AD shocks thus identified are consistent with the standard Keynesian interpretation (e.g., Blanchard, 1989, or Blanchard and Quah, 1989). That is, demand shocks have no long-run effect on the real GDP level, but do affect price levels in a statistically significant fashion. Supply shocks positively and significantly affect long term GDP levels but their negative long-term effect on prices is statistically insignificant.

4.2 Macro risk parameter estimates

The parameter estimates for the BEGE model are reported in Table 2, Panel B. The parameter σ_p represents the unconditional volatility of the supply and demand shocks due to the Gaussian component. This parameter is similar across supply and demand shocks (0.50 for supply; 0.40 for demand). As discussed, for supply shocks p_t^s is constant, but it follows a Gaussian stochastic volatility model for demand shocks.[11] For demand, this variable is very persistent with an autocorrelation parameter of about 0.97 and a relatively low innovation standard deviation (σ_{pp} is 0.37), but this parameter is imprecisely estimated. The bad environment components of both supply and demand shocks follow gamma processes. The scale parameter σ_n is much larger for demand shocks than for supply shocks, but recall that the total variance of this component is the squared scale parameter times the unconditional mean of n_t. This value is 0.75 for supply shocks, and 0.84 for demand shocks. Comparing these values to the squared σ_p parameters, the contribution of bad variances to the overall variances dominates that of the Gaussian variances.

The properties of the gamma-distributed bad environment state variable for demand shocks, n_t^d, contrast sharply with those of p_t^d. Its mean is 1.67, implying an unconditional skewness of -1.55. This generates substantial negative skewness for demand shocks. The bad environment shape parameter is also less persistent than the good environment variable, with an autocorrelation of only 0.67. Therefore, n_t captures short-lived periods of risk characterized by potentially deeply negative shocks.

[11] Note that because the selected processes for p_t are Gaussian for both supply and demand shocks, the parameter $\bar{p}$, the unconditional value of the shape parameter, is not identified. Without loss of generality, we set $\bar{p}$=1. This implies that, for example, σ_{dp}, represents the unconditional standard deviation of the Gaussian shock component for demand shocks.

In contrast, the supply bad-environment distribution is virtually Gaussian with the unconditional mean of the shape parameter equal to 30.48. This implies unconditional skewness of only -0.36. The bad environment risk factor for supply is much more persistent than for demand shocks with $\rho_{ns} = 0.985$, making supply driven recessions less spiky than demand driven recessions. The volatility shock parameter (σ_{nn}) is 1.29, similar to the corresponding coefficient for demand shocks (1.15), implying that bad environment supply and demand variances are quite variable.

The Bates estimation also yields filtered estimates for the shocks to the Gaussian and bad components for supply and demand. The components of the demand shocks are shown in the middle and bottom panels of Figure 3. The Gaussian shock is relatively variable early in the sample, but less so later on. The bad demand shock is mostly near zero, but spikes down during some recessions. Reminiscent of a "jump" shock, the extreme non-Gaussianity of this shock is clearly evident. An analogous decomposition for supply shocks is shown in the three panels of Figure 4. The constant variance Gaussian shock shows modest variation throughout the sample. The bad environment supply shock is only variable in the early part of the sample, showing downward sharp spikes during most recessions in the 70s. Given these figures, it is not surprising that level and uncertainty shocks are very negatively correlated with unconditional correlations of -0.87 and -0.88 for demand and supply, respectively, contradicting the independence assumption often maintained in the literature (e.g., Alessandri and Mumtaz, 2019).

5 Characterizing the History of Macroeconomic Volatility in the US

Having used the AIC to select optimal models of volatility for demand and supply shocks, and having estimated the optimal BEGE model parameters, we can now use the BEGE model as a lens to interpret the history of U.S. macroeconomic uncertainty over the sample period.

5.1 Time series estimates of uncertainty

The Bates estimation procedure allows us to filter time series estimates of the risk factors governing Gaussian and bad variances for supply and demand shocks. These are plotted in Figure 5. Starting with the demand variances, in Panels A and B, the Gaussian component of demand variance was relatively high in the 70s and especially in the early 80s, and subsequently decreased to lower levels. The bad demand variance shows much less pronounced low frequency variation but increases in most recessions, especially after 1980. Figure 5, Panels C and D, perform the same exercise for supply variances. The good variance is constant over time and low. The bad supply variance increases in most recessions throughout the sample period, but its level is very high until the mid eighties and low thereafter.

Figure 5, Panel E, graphs both demand and supply variances. The conditional variance of supply shocks is largest in the early part of the sample, dominating the conditional variance of demand shocks, consistent with stagflation incidences during that period. In the second half of the sample, the supply and demand conditional variances seem often indistinguishable, but the conditional demand variance peaks more sharply in the last 2 recessions.

The Gaussian and bad components of supply and demand variances map linearly into the conditional variances of inflation and real GDP growth, which are graphed in panels A and C of Figure 6. Both GDP growth and inflation variances were relatively high in the early part of the sample, and both trended down dramatically through the 1980s and 1990s. Nevertheless, their variances continued to spike up during recessions through the end of the sample. As shown in the top two panels to the right (Panels B and D), the secular decline in volatility for both real GDP and inflation owes largely to a decline in the Gaussian component of demand variance and the bad supply variance. The latter played a dominant role in the decline in inflation variability in the 1980s. Spikes in volatility during recessions owe to the bad variance components of both supply and demand.

From equation (10), it is evident that in an environment where demand (supply) variances dominate, the conditional covariance between inflation and real activity is positive (negative). The bottom panels of Figure 6 graph the conditional covariance between inflation and real GDP growth shocks, and its components. The covariance is mostly very negative up until around 1990 due to the high variance of the bad supply shock, but afterwards, this source of variation dramatically declines, resulting in a covariance closer to zero with short but sharp positive spikes during demand-driven recessions. We also show the good and bad supply and demand covariance components of the total covariance. For example, the slightly positive correlation between real GDP and inflation during the 1981-1982 recession reflects both elevated bad supply and bad demand variances, with offsetting effects on the covariance.

Figure 7 plots the conditional correlations between shocks to the level and uncertainty for supply and demand. That is, for supply shocks, we graph the correlation between u^s_{t+1} and $Var_{t+1}[u^s_{t+2}]$ (see equation (12)), and analogously for demand shocks. The top panel illustrates that for supply, the correlation between shocks to the level and total uncertainty is always negative. This is because the optimal specification has a constant good variance shock, while bad variance shocks are perfectly negatively correlated with the level shocks. When bad volatility rises (relative to good volatility), the correlation becomes more negative. Because of the elevated level of the bad supply variance in the seventies and eighties, the correlation is near -1 until the mid-eighties. As shown in the bottom panel, for demand, shocks to good volatility are positively correlated with level shocks whereas shocks to bad volatility are negatively correlated with the level. When good volatility is large (relative to bad volatility) the correlation between level shocks and total uncertainty can become positive, and vice versa. There is substantial time-variation but little sign-switching in this correlation, and it is predominantly negative. This finding casts doubt on models that, ex ante, impose independence between level and uncertainty shocks.

5.2 The Great Moderation

Our estimated time series for volatilities can contribute to the debate on the Great Moderation. The literature has mostly focused on overall output volatility and puts a "break point" for output volatility in the first quarter of 1984 (see McConnell and Perez-Quiros, 2000; Stock et al., 2002). For inflation, Baele et al. (2015) suggest a later date, the first quarter of 1990. Whereas most of the literature attributes the decreased volatility to either good luck, improvements in monetary policy (see, e.g., Cogley and Sargent, 2005, Benati and Surico, 2009, Sims and Zha, 2006, and Baele et al., 2015, and the references therein) or a combination of the two (e.g., Fernandez-Villaverde et al., 2015), we decompose the overall changes in volatility into changes in demand versus supply variances and bad versus Gaussian variability. We also address a more recent question asking whether the Great Moderation is over. Baele et. al. (2015) suggest that the Great Moderation for both inflation and output has ended, even before (for inflation) or just with the onset (for output) of the Great Recession. In contrast, Gadea, Gomez-Loscos and Perez-Quiros (2015) argue that the Great Moderation is alive and well, despite the Great Recession experience.

To test these various hypotheses, Table 3 reports simple dummy variable regressions where the dependent variables are the estimated conditional variances for inflation and GDP growth, as well as their AS versus AD and Gaussian versus bad components. The columns report the constant and the coefficients for two dummy variables. The first dummy variable is equal to 1 in the post-1985 (for GDP growth) or post-1990 (for inflation) part of the sample and is designed to identify changes in volatility associated with the onset of the Great Moderation. The second dummy is equal to 1 after 2006 and is designed to capture any possible reversal of the Great Moderation in the period beginning with the 2007-2008 Great Recession.

The results for the inflation variance are shown in the top panel. The overall level of the variance declined by about 80% of its prior level in the post-1990 period, consistent with the Great Moderation. Looking at the components of inflation variance, the main reason for the much lower volatility is the disappearance of bad supply variances as reflected in the stagflations in the early part of the sample. However, there is also a statistically significant decrease in demand-induced inflation variance, but only due to the Gaussian demand component; the more pernicious bad volatility, which is strongly associated with demand recessions, remains. Turning to the third column, there is no statistical evidence of any volatility changes in the post-2006 period. Thus, recessionary inflation risk for AS driven recessions has waned, but deflation risk for AD recessions has remained. Under the assumption that monetary policy can affect the distribution of demand shocks but not supply shocks, these results suggest that monetary policy changes played only a modest role in the Great Moderation for inflation, e.g., monetary policy had little effect on “bad” component risk.

The results for real GDP growth, shown in the bottom panel, tell a similar story. There is a dramatic decline in the overall variance level for real GDP growth which falls by over 60%, consistent with the Great Moderation. Over half of this is due to a significant decrease in the bad supply variance post 1985. The remaining decrease is due to a drop in Gaussian demand volatility. However, there is no evidence of a decline due to changes in bad demand volatility during the Great Moderation, nor is there any evidence that volatility changed in the post-2006 period, except for a further small drop in Gaussian demand volatility. While the bad supply variance, shown in Table 3, decreased largely because of a lack of large negative supply shocks between 1985 and 2007, bad demand uncertainty remains elevated. This is consistent with the results in Bianchi, Kung and Tirskikh (2023), who also find demand uncertainty to be relatively more important after the Great Recession. Overall, we should not expect recessions to be less variable in the future than they were in the past, even though the Great Moderation appears to still apply for overall volatility.

Table 4 repeats the same exercise for skewness. To complement these tests, Figure 8 graphs the standard (scaled) conditional skewness over time. The standard (scaled) conditional skewness for both inflation (Panel A) and real GDP growth (Panel B) decreased substantially and significantly after the Great Moderation dates. For inflation, this decrease is partly due to a decrease in the unscaled (by the standard deviation to the third power) centered third moment, which is fully attributable to the fall in skewness of the bad supply component. As Figure 8, Panel A, shows, skewness was positive for the early portion of the sample when the supply variance components dominated. In the latter portion of the sample, inflation skewness declines and occasionally moves down sharply, particularly during recessions when the bad variance spikes in the absence of a large Gaussian component. An important conclusion is that deflation risk is more pronounced than the risk of high positive inflation in recent recessions in the U.S.

For GDP growth, unscaled skewness, however, has not changed and even slightly increased. Thus, the decline in conditional skewness for GDP growth is due to the decline in the conditional variance, which, as documented in Table 3, is due to the decline of Gaussian demand and bad supply variances. This leaves the more pernicious and negatively skewed bad demand variance to dominate recessions. An important implication is that risk is in fact higher than before, notwithstanding the low volatility observed in normal periods. Figure 8, Panel B, also shows how the skewness of real GDP growth has become more negative over time. This confirms the intuition of the "deepening" of recessions explored in Jensen et al. (2020), who suggest to normalize the severity of the recession (the fall in GDP growth per unit of time) by the standard deviation. They show, focusing simply on GDP growth itself, that the skewness is lower over the 1984-2016 period than over the 1947-1984 period; and the ratio of downside over upside volatility higher. Despite being produced in a framework with only AS and AD shocks, these results are reminiscent of the "volatility paradox" generated in models with credit frictions (Adrian and Boyarchenko, 2012; Brunnermeier and Sannikov, 2014), where periods of low volatility of output growth may foreshadow future crises. Adrian, Boyarchenko and Giannone (2019) also show distinct negative conditional skewness of GDP growth, using quantile regressions. They show, consistent with our results, more variation in the left tail than in the right tail and explore how financial conditions affect "growth vulnerability."

5.3 The joint conditional distribution of growth and inflation

Our dynamic model implies a non-Gaussian bivariate conditional distribution for real GDP and inflation shocks that varies notably over time. To illustrate this, Figure 9 plots the conditional bivariate distribution for inflation and real GDP growth from four periods in our sample. Each panel shows the bivariate density, illustrated using iso-density contours. The total probability mass inside each contour is labeled in blue. In the upper left panel, we plot the distribution as of 1972Q4. This is an expansionary period according to the NBER. As discussed above, during expansionary periods in the 1970s the Gaussian component of the demand shocks together with the bad supply shock (which is not very skewed) dominates the distributions. As a result, an ellipsoid distribution emerges, consistent with a nearly Gaussian bivariate relationship. In 1974Q4, a recession was underway, and as shown by the upper right panel, the distribution expands notably as both supply and demand volatility increased. Moreover, due to a very high bad supply variance, a more negative correlation between real GDP and inflation is evident. Still, the overall distribution appears to be mostly Gaussian.

The left panel of the next set of graphs shows the distribution from the expansionary period 2004Q4. The distribution is narrower in all directions compared to the previous panels, consistent with the Great Moderation. However, it is also evidently less Gaussian. Because the bad supply shock is now much less variable, the distinct "bad" tail over low GDP growth also features low inflation. This is a manifestation of the "bad" AD risks being more prevalent even in an expansionary period. Finally, the right panel shows the distribution after the onset of the Great Recession in 2009Q1. Due to a surge in bad demand volatility, the distribution appears wide and highly non-Gaussian with a vastly expanded heavy tail towards outcomes characterized by low inflation and low growth, suggesting dominant AD risks.

6 Further Interpretation of the Results

In this section, we provide three additional empirical exercises. In Section 6.1, we examine the link between our demand and supply shocks and several economic variables that have been associated with supply or demand shocks in various New Keynesian models. In Section 6.2, we explore whether the identification of our dynamic model would change when financial variables are used. In Section 6.3, we compare our estimates of macro uncertainty with the measures in JLN (2015). We also analyze how much macro uncertainty contributes to stock and bond return volatility, adding to the analysis in Engle, Ghysels and Sohn (2013) for stocks and Bekaert, Engstrom and Ermolov (2021) for bonds.

6.1 Further interpretation of Supply and Demand Shocks

New Keynesian models use a variety of variables which are interpreted as "demand" or "supply" variables. For example, Bianchi, Kung, and Tirskikh (2023) entertain factor productivity as a supply variable and a discount rate factor as a demand variable. We employ three clear "supply" variables: a) the real return on a Thomson Reuters/Jefferies Commodity Research Bureau commodity futures price index (from Thomson Reuters) (b) the West Texas Intermediate (WTI) spot oil price real return (from St. Louis's Fed FRED database). c) Total productivity growth adjusted for utilization (Fernald, 2014). Our economic proxies for demand shocks are less clear cut. Many articles use preference shocks as a prototypical demand shock (see, e.g., Ireland, 2011), but such consumer preferences are difficult to measure. We use the consumer confidence index from the University of Michigan Surveys of Consumers. However, this well known that changes in consumer confidence may not simply reflect sentiment changes ("animal spirits"), but may also reflect changes in expected productivity growth over a relatively long horizon (see, e.g., Barsky and Sims, 2012) and thus constitute a supply shock. Another potential demand shock is a change in financial conditions, which we proxy by the National Financial Conditions Index (NFCI) from the Federal Reserve Bank of Chicago (see, e.g., Alessandri and Mumtaz, 2019). Finally, we use the government spending to GDP ratio from St.

Louis's Fed FRED database, which has often been proposed as a demand shock (see Ravn, Schmitt-Grohé, and Uribe, 2012).

We extract shocks to these variables using the residuals from a first-order VAR with a constant and all 6 variables. Both AIC and BIC criteria pick 1 lag. We next regress these macro shocks on our AD and AS shocks and a constant. Given the data availability, our sample for this exercise is 1971:Q1-2019:Q4 (versus 1969:Q2- 2019:Q4 for the rest of the paper). To facilitate economic interpretation, we rescale both demand and supply shocks to have unit variance in the 1971:Q1-2019:Q4 sample.

Table 5 presents the results. For the variables capturing supply shocks, two out of three variables show the expected effect: WTI oil return shocks load negatively and significantly on the supply shock: a one standard deviation negative supply shock is associated with 1.82% increase in the oil spot price. Total productivity growth loads positively on the supply shock, although the coefficient is only significant at the 10% level. Consistent with consumer confidence capturing both preference shocks and expected factor productivity shocks, its shocks load significantly on both AD and AS shocks. Financial conditions do not show any relationship to our identified AS and AD shocks. In Section 6.2, we further show that our economic shocks show little interaction with financial shocks. Finally, federal government spending to GDP ratio shocks load negatively on the demand shock. While at first glance surprising, a very active literature debates whether government spending may crowd out consumption and investment spending, with, for instance, Furceri and Sousa (2011) concluding that government spending crowds out both private consumption and investment. Alternatively, the result may reflect that government spending is increased when private demand is weak.

The final two rows of Table 5 show how financial returns load on supply and demand shocks. Their effect should depend on how these shocks affect expectations. Positive economic activity shocks should induce a positive cash flow effect, important for equities, and may also decrease risk aversion (as would be the case in a habit model). The latter effect should also induce a positive effect on equities, but the effect on bonds is not clear as a decrease in risk aversion may increase or decrease interest rates (Wachter, 2006) even though it may lower the term premium component of yields. The economic activity shock may also be accompanied with an increase in the real interest rate. While the overall effect on equities should be positive, the economic activity shock may well lower bond prices (through interest rate increases). For a demand shock, such an effect is then exacerbated by an increase in inflation (expectations), whereas for supply shocks the negative effect may be counteracted by lower inflation. The empirical effects are in line with expectations: both supply and demand shocks generate strong positive effects on stock returns. However, for bond returns, demand shocks generate a statistically significant negative response, and supply shocks generate an insignificantly positive response. Overall, our macro shocks are associated with meaningful economic and financial effects.

6.2 The interaction of financial and real shocks

Several articles suggest that there are spillovers from the financial to the real economy (e.g., Cheng, Liao and Schorfheide, 2016, or Berger, Dew-Becker and Giglio, 2020). However, our model does not use any financial variables. Here, we conduct three explicit tests for the effects of financial factors on our inferences, assessing whether financial variables a) predict our fundamental shocks (which would violate the white noise properties of the shocks) b) predict our macroeconomic squared shocks (which simply establishes a link between macro and financial uncertainty) c) predict squared macro residuals scaled by the estimated conditional variance, that is, we evaluate the correct specification of our estimated conditional macro volatilities. The latter test is the most important, as it tests whether critical financial variables were missed in creating our conditional volatilities. As financial variables, we use quarterly realized volatilities of stock returns and yield changes; the credit spread and the term spread. We report the results in Table 6.

The results are quite encouraging. For test (a) (first and fourth column for inflation and GDP growth, respectively), which is a model specification test, we find that the term spread (at the 1% level) and the realized Treasury bond return volatility (at the 10% level) are significant predictors of inflation residuals. The term spread is also significant at the 5% level for GDP growth residuals, although overall there is no predictability of GDP growth residuals: the R^2 of 3% is statistically and economically insignificant. Test (b) (second and fifth column for inflation and GDP growth, respectively) simply verifies whether any of the financial variables are correlated with our macro risks (the conditional variance of inflation and GDP growth), and does not constitute a model specification test. We find surprisingly little correlation with only the credit spread positively and significantly associated with future squared inflation residuals (at the 5% level). Most important is test (c) (third and sixth column for inflation and GDP growth, respectively), which is a direct test of the correct specification of our macro risk variables. We observe no rejections of the null of no predictability for test (c) for either inflation or GDP growth, for any of the 4 financial variables.

To make sure financial variables do not affect our inference, we therefore conduct a robustness check, where we also use the term spread to pre-whiten our macro shocks. The resulting pre-whitened shocks have a correlation of greater than 0.99 with our standard set of pre-whitened shocks for both GDP and inflation, so we strongly suspect that the addition of these variables would not make a material difference to our results.

6.3 Comparing Uncertainty Measures

In this section, we examine the correlation between our macro risk measures and a number of available uncertainty measures in the literature. We start with the uncertainty indices of Jurado, Ludvigson, and Ng (2015) (hereafter JLN) and Ludvigson, Ma, and Ng (2021) (hereafter LMN). They propose three different indices. First, their main measure is "macroeconomic uncertainty" which is computed from the forecast errors of real variables (e.g., real GDP growth or unemployment), nominal variables (e.g. various price indices), and financial variables (e.g., bond yields or equity returns). Second, they also provide a "financial uncertainty" measure which is computed using the forecast errors of financial variables alone and "real uncertainty", which is only based on forecast errors of real activity. We regress these uncertainty measures based on quarterly forecast errors (the h=3 specification in the JLN/LMN articles) contemporaneously on our state variables with the results reported in Table 7, Panel A.[12] Our state variables have the most explanatory power for macroeconomic uncertainty and real uncertainty with R^2:s of around 50%; the R^2 for financial uncertainty is only 13.49%. In line with economic intuition, the regression coefficients are also predominantly positive (with the exception of the financial uncertainty loading on p_t^s, which is statistically and economically insignificantly negative). Bad demand variance (n_t^d) is statistically significant for all three uncertainty measures but only at the 10% level for financial uncertainty. For LMN macro uncertainty, Gaussian demand uncertainty (p_t^d) is also statistically significant (at the 5% level), whereas for real uncertainty, bad supply uncertainty is statistically significant.

Table 7, Panel B, shows the explanatory power of our state variables for a number of other uncertainty indices for which the sample size varies due to the data availability. We consider four different series:

1) The uncertainty index of Bekaert, Engstrom, and Xu (2022), a proxy to macroeconomic uncertainty created from financial variables.

2) The economic policy uncertainty index of Baker, Bloom, and Davis (2016) (the headline three component index and the news-based component)

3) The VXO from the Chicago Board Options Exchange, representing stock implied volatility. This index is very highly correlated to the VIX, but is available for a longer period of time.

The explanatory power of our macro state variables varies and is strongest for VXO. Again, the coefficients are predominantly positive. However, the news-based economic policy uncertainty loads negatively and significantly on the bad supply variance, n_t^s, which may simply indicate that news uncertainty was low during the supply shocks of the 70s (for example, economic policy uncertainty loads positively on n_t^s, but the former series only starts in 1985). The coefficient on bad demand variance (n_t^d) is consistently and significantly positive and economically large.

We next contribute to a large literature linking macro uncertainty to financial uncer-

[12]The results based on monthly forecast errors (the h=1 specification in JLN/LMN) are very similar.

tainty, by studying the predictive power of our macroeconomic state variables for future realized return variances. We construct realized variances for the aggregate stock market portfolio and 5-year nominal zero-coupon Treasury bonds as the sum of daily squared returns during the quarter. As above, our measure of the aggregate stock market return is the NYSE/AMEX/NASDAQ stock market return index from WRDS. We use nominal zero-coupon yields from Gürkaynak, Sack, and Wright (2007) to construct 5-year nominal bond returns. The results are reported in Table 8. In Specification 1 the only predictors are our macro state variables, in Specification 2 the only predictor is the lagged realized variance, and Specification 3 combines all predictors.

Macro state variables have substantial predictive power for realized bond return variances before and after controlling for the lagged realized variance. This confirms the evidence in Bekaert, Engstrom and Ermolov (2021) showing high R^2:s in similar quarterly regressions. We additionally show that the signs are positive for the demand variance, but the n_t^s coefficient is statistically and economically significantly negative. Note that both supply and demand state variables increase the variance of inflation but demand variables increase deflation risk, whereas supply variables increase the risk of high inflation. The results suggest that deflation risk increases bond variances more than does inflation risk.

The predictive power of macro state variables for stock return variances is weak and disappears completely after controlling for the lagged realized variance. While this may be surprising at first, several articles suggest that risk aversion variation may be a very important driver of stock market uncertainty, see e.g. Bekaert, Engstrom, and Xu (2022) and Asgharian, Christiansen and Hou (2023). In addition, early work on the macroeconomic sources of stock market volatility (Schwert, 1989; Engle, Ghysels and Sohn, 2013) found insignificant results for inflation volatility and weak to somewhat stronger results for industrial production volatility. Overall, while using a very different framework, our results also confirm the findings in Baele, Bekaert and Inghelbrecht (2010), showing macro factors to be important in fitting bond return volatility, but not in explaining stock market volatility, where the "variance premium," often used as an indicator of risk aversion, is much more critical. However, when we look at the contemporaneous link between realized variances and our state variables in the fourth column in Table 8, the results for bond return variances do not change much, but the R^2 for stock return variances shoots up to 23.91%. The main reason is that the coefficient on n_t^d is now much larger and significant at the 10% level. Because the n_t^d variable has a strong association with recessions, this indirectly corroborates the findings in Berger, Dew-Becker and Giglio (2020), who find that innovations in realized stock volatility are followed by contractions, while shocks to forward-looking uncertainty are not.

7 Conclusion

In this article, we develop a new dynamic model for real GDP growth and inflation using forecast revisions from the SPF obviating any complex modeling of conditional mean macro-dynamics. The shocks are driven by aggregate demand and aggregate supply shocks, featuring BEGE dynamics, which accommodate time-varying non-Gaussian distributions with good and bad volatility. We find both aggregate demand and supply shocks to be negatively skewed and leptokurtic, but their "good" components are Gaussian. Our model delivers several empirical findings regarding macroeconomic dynamics.

First, we differentiate models with various degrees of correlation between level and volatility shocks, finding that in the best model volatility and level shocks are on average negatively correlated. Second, we characterize the time-variation in these supply and demand macro risks and their resulting effect on the conditional variances of inflation and GDP growth. We show that the Great Moderation largely reflects secular and large declines in supply variances and Gaussian demand variances. However, there is little evidence that the bad demand variance has decreased over time, and this variance almost invariably peaks in recent recessions. Third, the conditional skewness of both GDP growth and inflation has decreased over time, heightening macro vulnerabilities. With supply shocks subdued over the last 20 years, the prevailing macro risk with regard to price movements was one of deflation, not inflation.

Our work provides alternative measurement of macro uncertainty. In a leading paper in this literature, JLN, the uncertainty index is based on 114 different time series, comprising price and output indices, and some financial time series as well. While our uncertainty measures are based on only two macro series, they admit an easy economic interpretation of various types of uncertainty and are not contaminated by financial data. We find that our macro risk factors are significantly correlated with their macro and real uncertainty indices (but not with their financial uncertainty index).

Our model and findings contribute to both the empirical and theoretical literature on uncertainty and business cycles in various ways. For example, a number of articles (e.g., Caggiano, Castelnuovo and Groshenny, 2014, or Alessandri and Mumtaz, 2019) show that the effects of uncertainty shocks are much larger in recessions. This prompts researchers to favor models with regime dependent exposures to economic shocks, but our model incorporates such effects endogenously, as the relative importance of bad volatility varies over time. Moreover, a model which explicitly accommodates time-varying exposures to the structural shocks (while shutting down time-varying volatility) fits the data much worse than our selected model.

Our parametric model may also prove useful in theoretical real business cycle models. As Fernández-Villaverde and Guerron-Quintana (2020) discuss, equilibrium models often generate overly small effects of uncertainty shocks. Accounting for the strong conditional non-Gaussianities in the macro data in a tractable fashion as our model does, can be helpful. While there are alternative models that may fit non-Gaussianities and time-varying volatilities in macro data (e.g., the rare disaster models in Gabaix, 2012, and Wachter, 2013), the case for the BEGE model was recently bolstered by Bakshi and Chabi-Yo (2012), Chabi-Yo and Liu (2020), and Chabi-Yo and Loudis (2020), who show that a tractable representative agent model with BEGE dynamics for the macro fundamentals outperforms alternative non-Gaussian models in fitting asset prices.

Our research comes at time when events like the COVID crisis and Ukraine-Russia war may engulf the economy with dramatic "bad" uncertainty of both demand and supply shocks.[13] Obviously, the efficacy of various policy responses may well depend on whether AS or AD shocks dominate and even which type of uncertainty dominates, if uncertainty has indeed a causal effect on business cycles, as asserted in the new business cycle literature. A framework with time-varying non-Gaussian distributions of AS and AD shocks may prove particularly useful for future macro modeling.

[13]Bekaert, Engstrom and Ermolov (2020) find that an AS shock accounts for 57% of the hugely negative COVID shock in the second quarter of 2020.

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Figure 1: Components of Bad Environment - Good Environment Distribution.

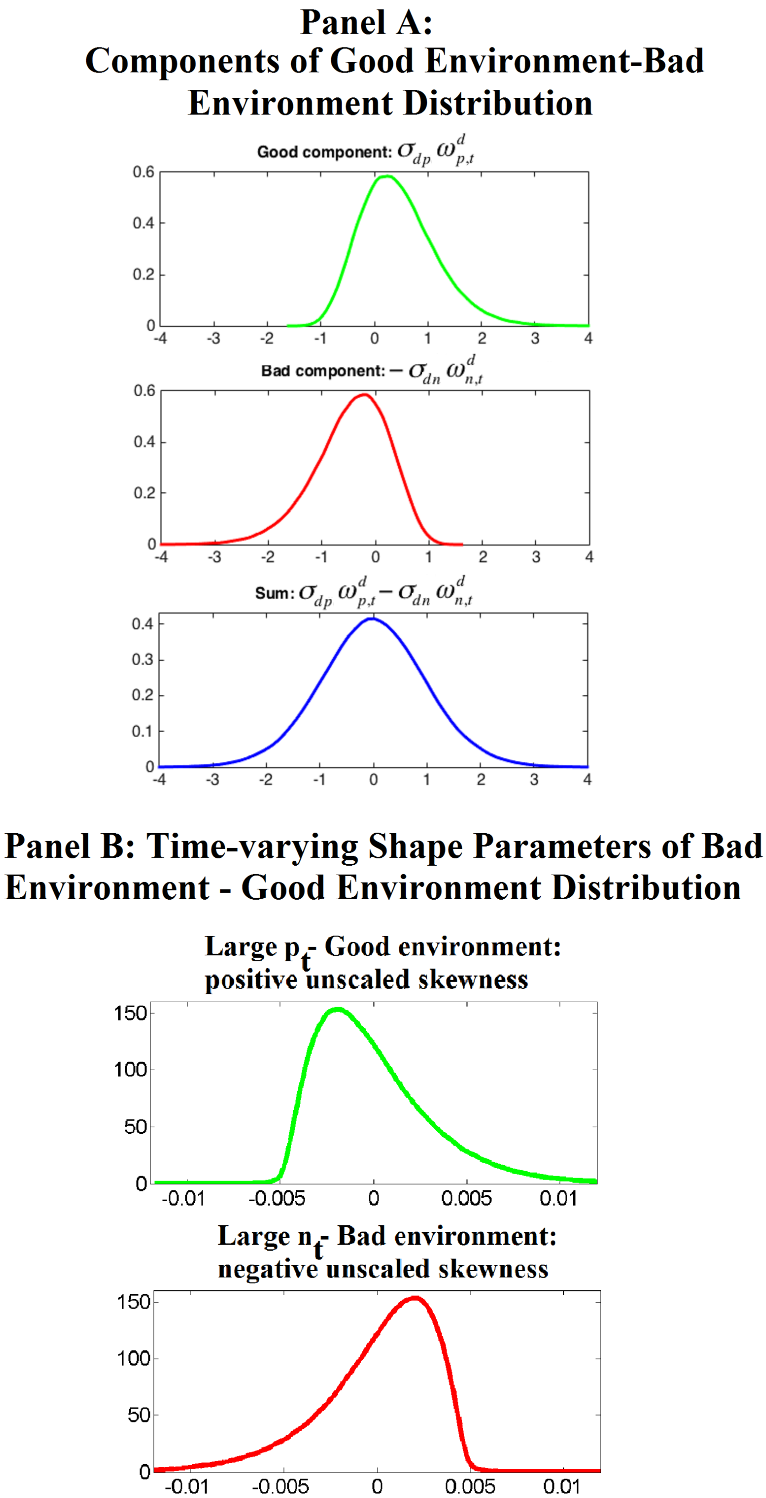


Figure 2: Real GDP Growth and Inflation Shocks. Shocks are expressed in percentages at an annual rate. The sample is quarterly 1968:Q4-2019:Q4. Shading corresponds to NBER Recessions.

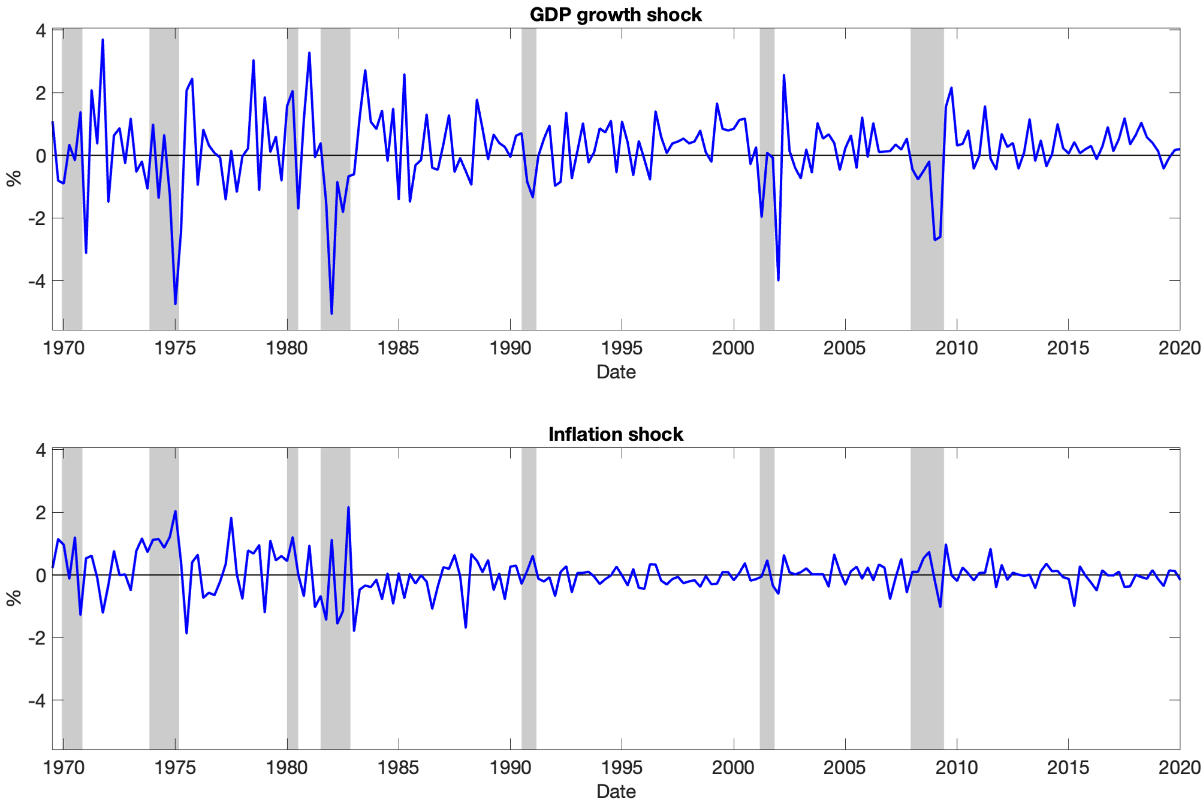


Figure 3: Demand Shock Decomposition. The sample is quarterly 1968:Q4-2019:Q4. Shocks are expressed in percentages at an annual rate. Shading corresponds to NBER Recessions. Demand shock dynamics is $u_t^d = \sigma_{dp}\omega_{p,t}^d - \sigma_{dn}\omega_{n,t}^d$ with $\omega_{p,t}^d \sim \mathcal{N}(0, p_{t-1}^d)$ and $\omega_{n,t}^d \sim \tilde{\Gamma}(n_{t-1}^d, 1)$. Furthermore, $p_t^d = \bar{p}^d + \rho_p^d(p_{t-1}^d - \bar{p}^d) + \sigma_{pp}^d\omega_{p,t}^d$ and $n_t^d = \bar{n}^d + \rho_n^d(n_{t-1}^d - \bar{n}^d) + \sigma_{nn}^d\omega_{n,t}^d$. $\mathcal{N}(0, p_t)$ denotes a zero-mean Gaussian distribution with variance p_t. $\tilde{\Gamma}(n_t, 1)$ denotes a centered gamma distribution with shape parameter n_t and a unit scale parameter.

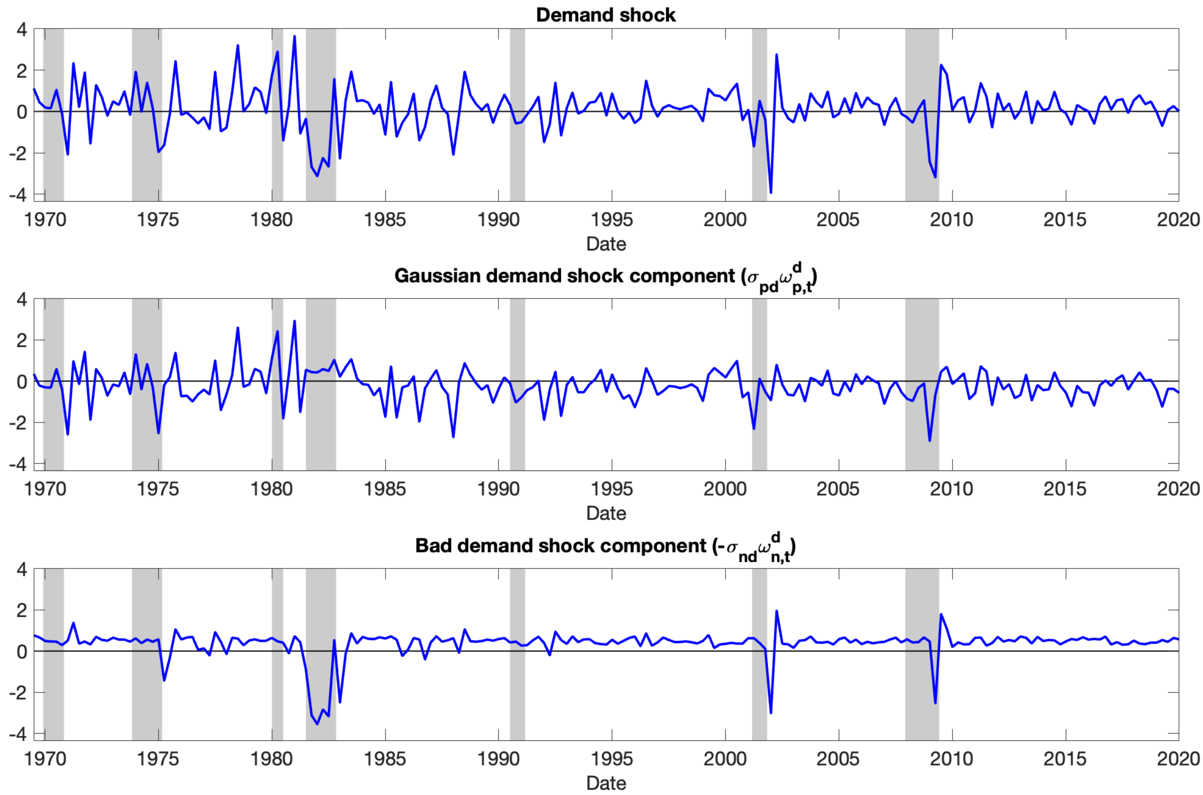


Figure 4: Supply Shock Decomposition. The sample is quarterly 1968:Q4-2019:Q4. Shocks are expressed in percentages at an annual rate. Shading corresponds to NBER Recessions. Supply shock dynamics is $u_t^s = \sigma_{sp}\omega_{p,t}^s - \sigma_{sn}\omega_{n,t}^s$ with $\omega_{p,t}^s \sim \mathcal{N}(0, \bar{p}^s)$ and $\omega_{n,t}^s \sim \Gamma(n_{t-1}^s, 1)$. Furthermore, $n_t^s = \bar{n}^s + \rho_n^s(n_{t-1}^s - \bar{n}^s) + \sigma_{nn}^s\omega_{n,t}^s$. $\mathcal{N}(0,p)$ denotes a zero-mean Gaussian distribution with variance p. $\tilde{\Gamma}(n_t, 1)$ denotes a centered gamma distribution with shape parameter n_t and a unit scale parameter.

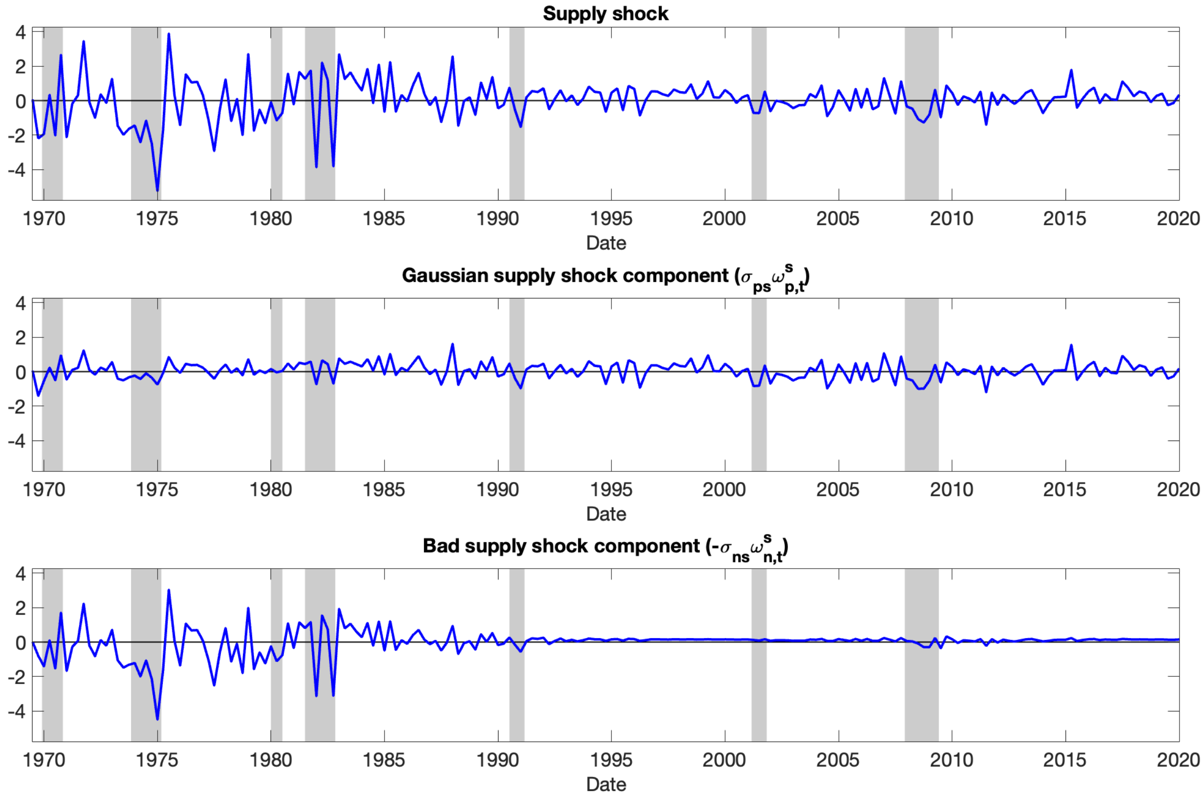


Figure 5: Conditional Demand and Supply Variances. The sample is quarterly 1968:Q4-2019:Q4. Shading corresponds to NBER Recessions.

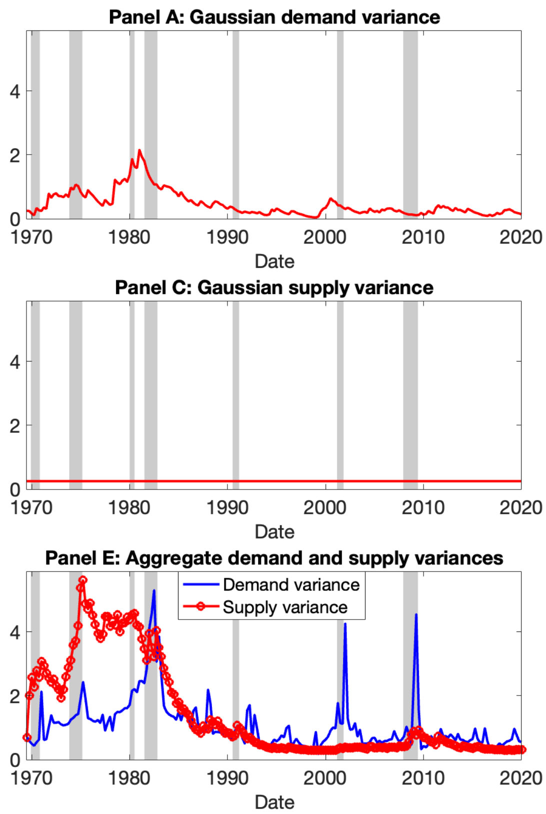


Figure 6: Conditional Second Moments of Real GDP Growth and Inflation. The sample is quarterly 1968:Q4-2019:Q4. Real GDP growth and inflation rates are annualized. Shading corresponds to NBER Recessions.

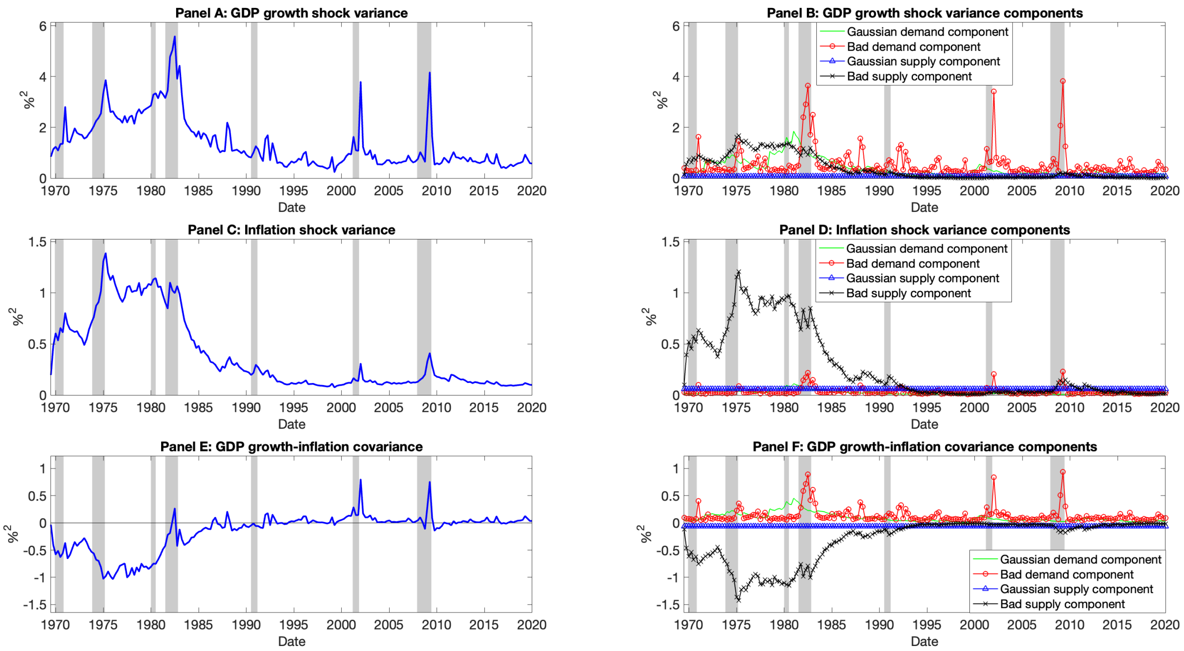


Figure 7: Conditional Correlations between Level and Variance Shocks. The sample is quarterly 1968:Q4-2019:Q4. Shading corresponds to NBER Recessions.

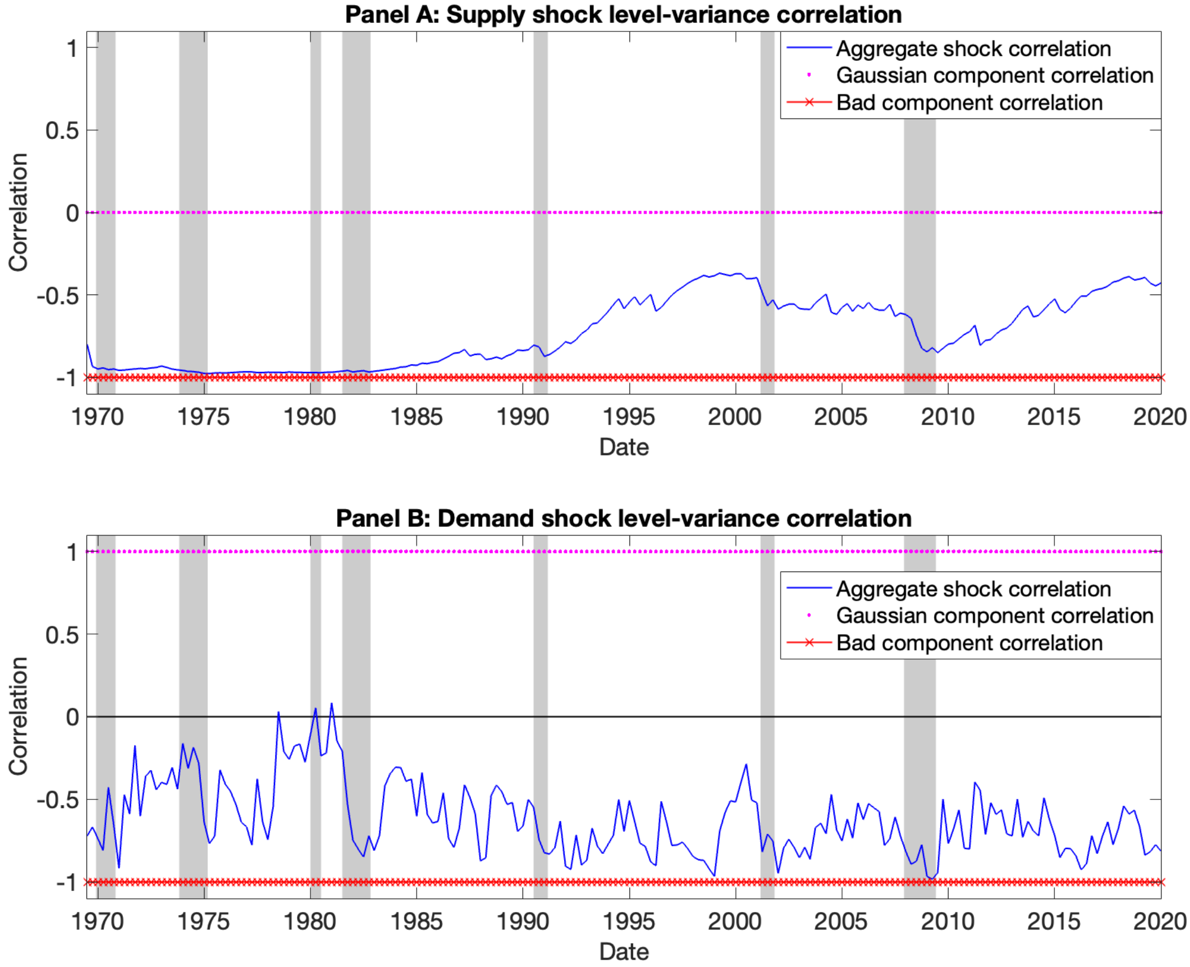


Figure 8: Conditional Third Moments of Real GDP Growth and Inflation. The sample is quarterly 1968:Q4-2019:Q4. Shading corresponds to NBER Recessions.

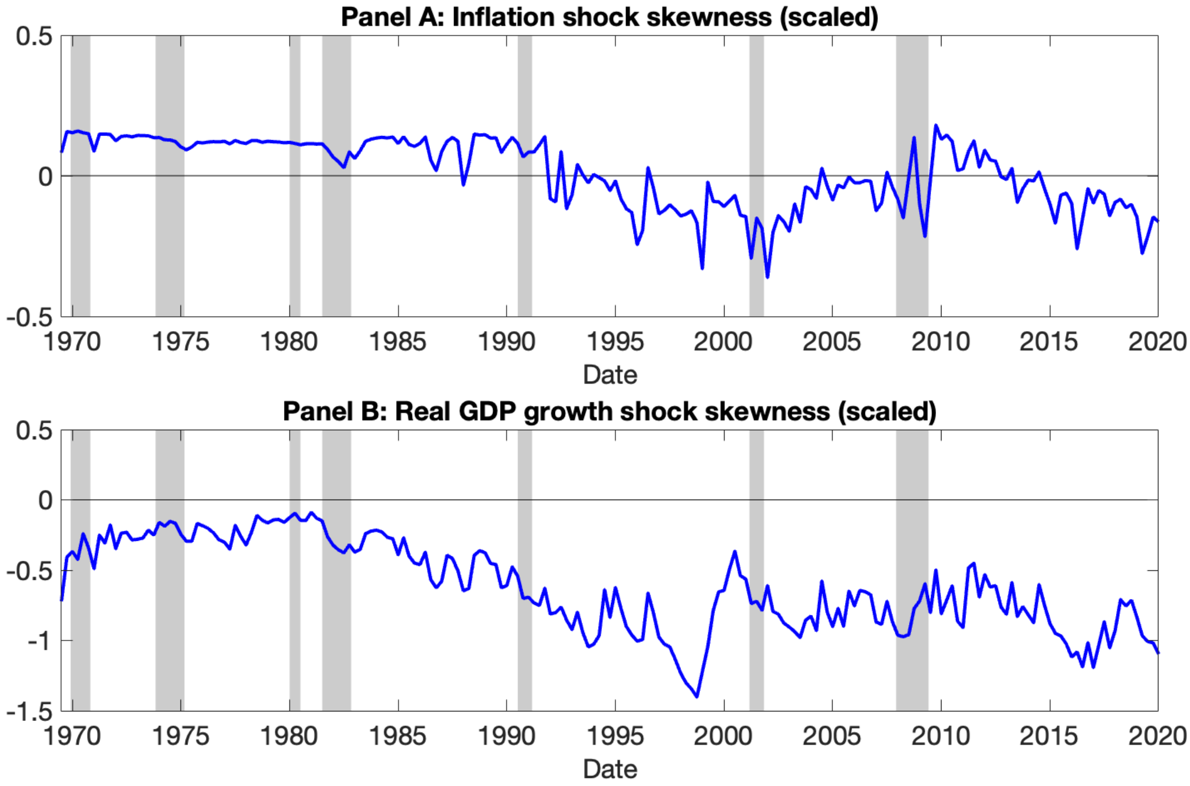


Figure 9: Conditional Contour Plots of Joint Real GDP Growth - Inflation Distributions. Numbers correspond to percentiles. Values are annualized. Plots are constructed by simulation.

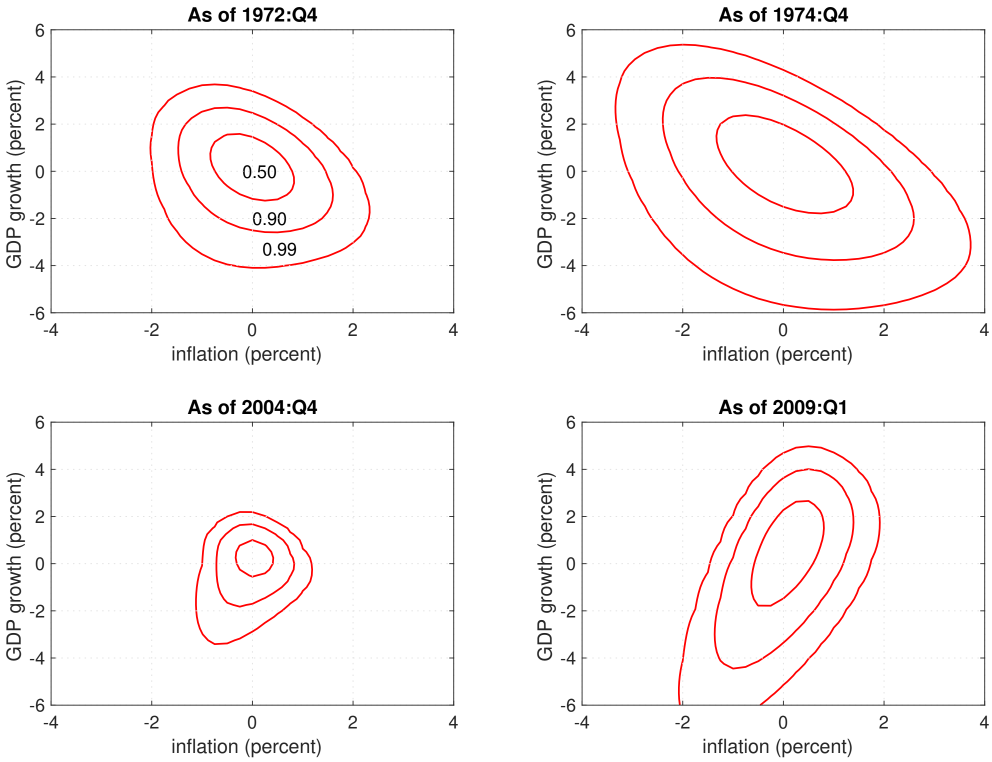


Table 1: Model Selection. We estimate 1,226 models combining Gaussian and BEGE shocks, time-varying and constant shape parameter processes, independent or perfectly correlated level and uncertainty shocks. We report the log-likelihood and AIC for 7 "pure" models in each class. The last line reports the model with the lowest AIC.

Model	log-likelihood	AICc
Gaussian static	-501.62	1009.30
Gaussian stochastic volatility	-435.80	889.30
Gaussian regime-switching loadings (4 elements of M switching separately)	-433.98	902.40
BEGE static	-458.57	937.07
BEGE stochastic volatility ($p_t^{d/s}$ and $n_t^{d/s}$ time-varying, λ=1)	-407.35	852.37
BEGE stochastic volatility ($p_t^{d/s}$ and $n_t^{d/s}$ time-varying, λ=0)	-418.31	874.34
BEGE regime-switching loadings (4 elements of M switching separately)	-425.93	900.23
BEGE regime-switching loadings (4 elements of M switching together)	-426.68	886.66
Optimal stochastic volatility (p_t^d time-varying Gaussian λ_p^d=1, n_t^d time-varying BEGE $\lambda_n^d = 1$, p_t^s static Gaussian, n_t^s time-varying BEGE λ_n^s=1)	-407.03	843.07

Table 2: Parameter Estimates. Standard errors, computed from 1,000 bootstrap runs, are in parentheses. The i.i.d. bootstrap uses the pre-whitened shocks and the model is re-estimated for each replication. Note that $\bar{p}$ is missing, because the "good" components of both demand and supply shocks are Gaussian.

Panel A: Loadings of Reduced-form Shocks onto Supply and Demand Shocks		
	u_t^π	u_t^g
u_t^s	-0.4738	0.5610
	(0.0606)	(0.2001)
u_t^d	0.2273	0.9286
	(0.0990)	(0.1495)
Panel B: Bad Environment-Good Environment Parameter Estimates		
	u_t^s	u_t^d
σ_p	0.5036	0.3998
	(0.0929)	(0.0869)
ρ_p	-	0.9707
		(0.1378)
σ_{pp}	-	0.3678
		(0.4305)
σ_n	0.1565	0.7103
	(0.4142)	(0.2376)
$\bar{n}$	30.4801	1.6654
	(9.3056)	(2.1696)
ρ_n	0.9851	0.6660
	(0.1007)	(0.0863)
σ_{nn}	1.2904	1.1463
	(0.4883)	(0.2442)

Table 3: The Great Moderation Variance Decomposition. The sample is quarterly 1968Q4-2019Q4. Coefficients are OLS regression coefficients from regressing the dependent variable on a constant and two dummies (as described in the headings). Newey-West (1987) standard errors computed with 20 lags are in parentheses. The asterisks, *, ** and ***, correspond to statistical significance at the 10, 5 and 1 percent levels, respectively.

Panel A: Aggregate Inflation			
Dependent variable	Constant	Dummy 1991Q1-	Dummy 2007Q1-
Aggregate variance	0.7016***	-0.5714***	0.0127
	(0.1324)	(0.1358)	(0.0247)
Supply variance	0.6248***	-0.5351***	0.0141
	(0.1264)	(0.1294)	(0.0191)
Gaussian supply variance	0.0569	0.0000	0.0000
	(constant)	(constant)	(constant)
Bad supply variance	0.5679***	-0.5351***	0.0141
	(0.1264)	(0.1294)	(0.0191)
Demand variance	0.0768***	-0.0363**	-0.0014
	(0.0129)	(0.0140)	(0.0078)
Gaussian demand variance	0.0395***	-0.0272***	-0.0014
	(0.0075)	(0.0075)	(0.0019)
Bad demand variance	0.0372***	-0.0091	-0.0001
	(0.0066)	(0.0078)	(0.0074)

Panel B: Real GDP Growth			
Dependent variable	Constant	Dummy 1985Q1-	Dummy 2007Q1-
Aggregate variance	2.4898***	-1.5565***	-0.1360
	(0.3202)	(0.3375)	(0.1708)
Supply variance	1.0810***	-0.8974***	-0.0380
	(0.1184)	(0.1319)	(0.0497)
Gaussian supply variance	0.0798	0.0000	0.0000
	(constant)	(constant)	(constant)
Bad supply variance	1.0012***	-0.8974***	-0.0380
	(0.1184)	(0.1319)	(0.0497)
Demand variance	1.4088***	-0.6591**	-0.0980
	(0.2722)	(0.2777)	(0.1320)
Gaussian demand variance	0.7579***	-0.4983***	-0.0764*
	(0.1403)	(0.1439)	(0.0452)
Bad demand variance	0.6508	-0.1608	-0.0216
	(0.1509)	(0.1567)	(0.1160)

Table 4: The Great Moderation Skewness Decomposition. The sample is quarterly 1968Q4-2019Q4. Reported coefficients are OLS regression coefficients from regressing the dependent variable on a constant and both dummies. Standard errors in parentheses are Newey-West (1987) standard errors computed with 20 lags. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively. Note that good demand and supply components are Gaussian and, thus, have 0 skewness.

Panel A: Aggregate Inflation			
Dependent variable	Constant	Dummy 1991Q1-	Dummy 2007Q1-
Skewness	0.1147***	-0.2001***	0.0400
	(0.0060)	(0.0277)	(0.0456)
Unscaled centered 3$^{\text{rd}}$ moment	0.0722***	-0.0764***	0.0021
	(0.0187)	(0.0190)	(0.0022)
Supply component (bad)	0.0842***	-0.0794***	0.0021
	(0.0187)	(0.0192)	(0.0028)
Demand component (bad)	-0.0120***	0.0029	0.0001
	(0.0021)	(0.0025)	(0.0024)
Panel B: Real GDP Growth			
Dependent variable	Constant	Dummy 1985Q1-	Dummy 2007Q1-
Skewness	-0.2543***	-0.4957***	-0.0817
	(0.0264)	(0.0793)	(0.0948)
Unscaled centered 3$^{\text{rd}}$ moment	-1.0344***	0.3697*	0.0352
	(0.1999)	(0.2082)	(0.1578)
Supply component (bad)	-0.1758***	0.1576***	0.0067
	(0.0208)	(0.0232)	(0.0087)
Demand component (bad)	-0.8586***	0.2121	0.0286
	(0.1990)	(0.2067)	(0.1531)

Table 5: Economic Interpretation of Demand and Supply Shocks. The sample is quarterly 1971:Q1-2019:Q4. Values in the second and third columns are coefficients from OLS regression of shocks in the first column on demand and supply shocks (and a constant, with a constant coefficient not being reported). Newey and West (1987) standard errors computed with 40 lags are in parentheses. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Economic shock	Demand shock loading	Supply shock loading	R^2
Thomson Reuters/Jefferies CRB index real return (percentages)	0.94	0.16	1.91%
	(0.59)	(0.38)	
WTI real return (percentages)	2.37	-1.82**	2.99%
	(1.76)	(0.78)	
TFP change (percentages)	0.01	0.07*	0.75%
	(0.06)	(0.04)	
Consumer confidence index	1.44***	1.55***	18.58%
	(0.36)	(0.28)	
NFCI	0.09	0.02	2.74%
	(0.09)	(0.03)	
Federal government spending/GDP (percentages)	-0.06***	0.01	2.79%
	(0.02)	(0.03)	
Excess aggregate stock market return (percentages)	2.46***	3.11***	20.37%
	(0.77)	(0.87)	
Excess 5 year nominal Treasury bond return (percentages)	-0.92***	0.36	10.26%
	(0.22)	(0.32)	

Table 6: Predictability of Squared Residuals with Financial Variables. The sample is quarterly 1968:Q4-2019:Q4. Values are OLS regression coefficients from regressing the column variables on row variables and a constant with the constant coefficient not being reported. Values in parentheses are the proportion of times the absolute value of the t-statistic (for coefficients)/F-statistic (for R^2:s) exceeded the sample counterpart in 1,000 bootstrap runs under the null of no predictability. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	Inflation			Real GDP growth		
	u^{π}_{t+1}	$(u^{\pi}_{t+1})^2$	$\frac{(u^{\pi}_{t+1})^2}{\hat{\sigma}^2_t(u^{\pi}_{t+1})}$	u^{g}_{t+1}	$(u^{g}_{t+1})^2$	$\frac{(u^{g}_{t+1})^2}{\hat{\sigma}^2_t(u^{g}_{t+1})}$
10 year Treasury bond realized return volatility	-0.23*	0.07	0.12	-0.23	0.29	-0.01
	(0.06)	(0.92)	(0.60)	(0.25)	(0.65)	(0.98)
Aggregate equity realized return volatility	0.01	-0.01	0.00	-0.01	0.03	0.02
	(0.10)	(0.26)	(0.98)	(0.31)	(0.53)	(0.32)
Aaa-Baa credit spread	-0.11	0.56**	0.13	0.27	1.14	0.02
	(0.41)	(0.04)	(0.65)	(0.32)	(0.21)	(0.97)
10 year-3 month Treasury spread	0.10***	-0.11	-0.08	0.13**	-0.32	-0.06
	(0.01)	(0.11)	(0.29)	(0.04)	(0.21)	(0.63)
R^2	0.10***	0.13**	0.01	0.03	0.05*	0.01
	(<0.01)	(0.03)	(0.74)	(0.15)	(0.10)	(0.75)

Table 7: Loadings of Various Uncertainty Variables on Macro State Variables. The sample is quarterly 1969:Q2-2019:Q4. Values in columns 2-4 are coefficients from a contemporaneous OLS regression of variables in the first column on macroeconomic state variables (and a constant, with the constant coefficient not being reported). All independent and dependent variables are scaled to unit variance. Newey and West (1987) standard errors computed with 20 lags are in parentheses. The asterisks, ** and ***, correspond to statistical significance at the 5 and 1 percent levels, respectively.

Panel A: Ludvigson, Ma, and Ng (2021) measures (1968:Q4-2019:Q4)				
Variable	p_t^d	n_t^d	n_t^s	R^2
Macro uncertainty	0.45**	0.42***	0.07	49.14%
	(0.20)	(0.12)	(0.16)	
Financial uncertainty	-0.08	0.27*	0.25	13.49%
	(0.21)	(0.16)	(0.24)	
Real uncertainty	0.03	0.29***	0.58***	50.57%
	(0.22)	(0.11)	(0.22)	

Panel B: Other uncertainty measures				
Variable	p_t^d	n_t^d	n_t^s	R^2
Bekaert, Engstrom, and Xu (2022) (1986:Q2-2019:Q4)	-0.08	0.43***	0.13	23.27%
	(0.16)	(0.08)	(0.15)	
Economic policy uncertainty (1985:Q1-2019:Q4)	-0.10	0.32***	0.30**	20.67%
	(0.08)	(0.07)	(0.12)	
News-based economic policy uncertainty (1969:Q2-2014:Q3)	0.19	0.42***	-0.45***	24.10%
	(0.12)	(0.14)	(0.16)	
VXO	0.02	0.50***	0.23	35.74%
	(0.14)	(0.08)	(0.16)	

Table 8: Realized Return Variances and Macro State Variables. The sample is quarterly 1969:Q2-2019:Q4. Values are coefficients from OLS regressions of dependent variables on macroeconomic state variables (and a constant, with the constant coefficient not being reported). Realized variances are computed as the sum of daily returns inside the quarter. All independent and dependent variables are scaled to unit variance. Newey and West (1987) standard errors computed with 20 lags are in parentheses. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

Panel A: 5 Year Nominal Bond Realized Variance				
	Specification 1	Specification 2	Specification 3	Specification 4
Dependent variable	RV_{t+1}	RV_{t+1}	RV_{t+1}	RV_t
RV_t		0.57***	0.32**	
		(0.0393)	(0.15)	
p_t^d	0.81***		0.57**	0.74***
	(0.25)		(0.29)	(0.22)
n_t^d	0.22***		0.12***	0.32***
	(0.07)		(0.03)	(0.06)
n_t^s	-0.41***		-0.28*	-0.40***
	(0.12)		(0.15)	(0.06)
R^2	38.08%	32.90%	44.47%	37.92%
Panel B: Aggregate Stock Market Realized Variance				
	Specification 1	Specification 2	Specification 3	Specification 4
Dependent variable	RV_{t+1}	RV_{t+1}	RV_{t+1}	RV_t
RV_t		0.53***	0.54***	
		(0.10)	(0.12)	
p_t^d	-0.03		0.02	-0.09
	(0.11)		(0.05)	(0.13)
n_t^d	0.23		-0.04	0.49*
	(0.16)		(0.04)	(0.28)
n_t^s	-0.11**		-0.06	-0.10
	(0.05)		(0.04)	(0.07)
R^2	5.80%	27.80%	28.17%	23.91%

Appendix I: GDP Growth and Inflation Shocks

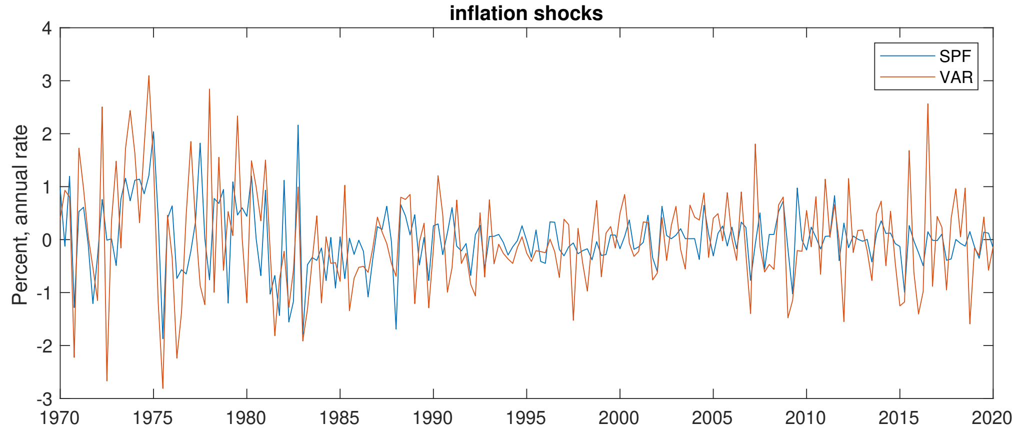
Appendix I.A: Pre-whitening Forecast Revisions

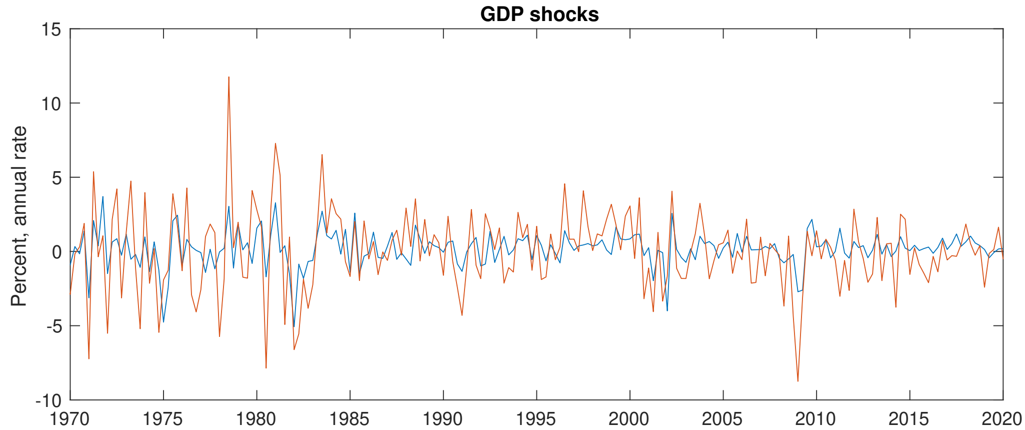
Pre-whitening Forecast Revisions Using Past Revisions and Forecasts. The sample is quarterly 1968:Q4-2019:Q4. Values are OLS coefficients from regressing inflation and real GDP growth shocks, u_t^π and u_t^g, respectively, on their lagged values and Survey of Professional Forecasters forecasts from the previous period π_{t-1}^{SPF} and g_{t-1}^{SPF}, respectively. OLS standard errors are in parentheses. The asterisks, ***, correspond to statistical significance at the 1 percent level.

	constant	u_{t-1}^π	u_{t-1}^g	π_{t-1}^{SPF}	g_{t-1}^{SPF}
u_t^π	-0.03	0.31***	0.12***	-0.01	0.02
	(0.13)	(0.07)	(0.04)	(0.03)	(0.02)
u_t^g	-0.47	0.03	0.20	0.03	0.00
	(0.56)	(0.30)	(0.17)	(0.13)	(0.10)

Appendix I.B: VAR Shocks with Revised Data

Real GDP Growth and Inflation Shocks. Shocks are expressed in percentages at an annual rate. VAR shocks are from a bivariate VAR with final (revised) GDP growth and inflation data. SPF shocks are constructed as discussed in section 3.1. The sample is quarterly 1968:Q4-2019:Q4.





Appendix II: GDP Growth and Inflation Impulse Responses to Aggregate Demand and Aggregate Supply Shocks

Long-term Growth Implications of Aggregate Demand and Supply Shocks: A Local Projections Approach. Values are OLS regression coefficients from regressing cumulative real GDP growth between time t and $t+n$ on time t aggregate demand and aggregate supply shocks and time $t-1$ inflation and GDP growth, and a constant (with only aggregate demand and aggregate supply coefficients being reported). Newey and West (1987) standard errors computed with 80 lags are in parentheses. The data are final revised 1969:Q2-2019:Q4 data. The asterisks, *, **, and ***, correspond to statistical significance at the 10, 5, and 1 percent levels, respectively.

	Demand shock	Supply shock
Contemporaneous real GDP growth (n=0 quarters)	0.33%***	0.22%***
	(0.05%)	(0.03%)
Cumulative 5 year real GDP growth (n=19 quarters)	-0.19%	0.83%***
	(0.19%)	(0.26%)
Contemporaneous inflation (n=0 quarters)	0.06%**	-0.05%*
	(0.01%)	(0.03%)
Cumulative 5 year inflation (n=19 quarters)	1.07%**	-0.39%
	(0.42%)	(0.25%)

Appendix III: Maximum Likelihood Estimation of Demand and Supply Shocks Parameters

Appendix III.A: Change of Variables Technique

To evaluate the joint log likelihood of shocks to GDP and inflation, $\begin{bmatrix} u_t^g & u_t^\pi \end{bmatrix}$, we use a change of variables technique. Specifically, we exploit the linearity of the system,

$$\begin{bmatrix} u_t^g \\ u_t^\pi \end{bmatrix} = M \begin{bmatrix} u_t^s \\ u_t^d \end{bmatrix}$$

where M contains the loading parameters as in Equation (13). We write the joint log likelihood of GDP growth and inflation observations, $loglike\left(u_t^g, u_t^\pi\right)$, as follows

$$loglike\left(u_t^g, u_t^\pi\right) \quad = \quad \ln\left(|M|\right) + loglike\left(u_t^d, u_t^s\right)$$

$$= \quad \ln\left(|M|\right) + loglike\left(u_t^d\right) + loglike\left(u_t^s\right)$$

where in the second line we have additionally exploited the assumption of independence between supply and demand shocks. Operationally, conditional on the model parameters,.we first invert the supply and demand shocks using:

$$\begin{bmatrix} u_t^d \\ u_t^s \end{bmatrix} = M^{-1} \begin{bmatrix} u_t^g \\ u_t^\pi \end{bmatrix}$$

We then evaluate the (approximate) maximum likelihood of supply and demand shocks using the methodologies described below for stochastic volatility or regime switching models, depending on the model begin estimated.

Appendix III.B: Penalized Maximum Likelihood

To help stabilize estimation, we include a penalty term for all likelihood methods, which facilitates convergence of parameter estimation and at the same time assures that the estimated model matches the unconditional second-order moments of GDP growth and inflation reasonably well. In particular, we add a penalty term to the likelihood of all models equal to

$$\left(umom_{mod} - \widehat{umom}\right)' \cdot \widehat{V}_{umom}^{-1} \cdot \left(umom_{mod} - \widehat{umom}\right)$$

where $\widehat{umom}$ is a 3-vector containing the unconditional sample standard deviations of real GDP growth, inflation, and their unconditional correlation. The 3x3 matrix, $\widehat{V}$, is the estimated covariance matrix of $\widehat{umom}$, which we calculate using a block-boostrap of the sample data $\begin{bmatrix} u_t^g & u_t^\pi \end{bmatrix}$ with block lengths of 28 quarters. The vector $umom_{mod}$ are the unconditional moments implied by the model for a given set of model parameters.

III.C: Estimation of Stochastic BEGE Models

The estimation procedure is a version of Bates (2006) algorithm for the component model of two gamma distributed variables. The step-by-step estimation strategy for the demand shock is described below. The estimation for the supply shock is identical.

The methodology below is an approximation, because, in order to facilitate the computation, at each time point the conditional distribution of state variables p_t^d and n_t^d is assumed to be gamma, although the distribution does not have a closed form solution. The choice of the approximating distributions is discussed in details in section 1.3 of Bates (2006). Here the gamma distributions are used, because they are bounded from the left at 0, which ensures that the shape parameters of the gamma distribution in the model (p_t^d and n_t^d) will always stay positive, like they should.

The system to estimate is:

$$
\begin{aligned}
&u_{t+1}^d = \sigma_{dp}\omega_{p,t+1}^d - \sigma_{dn}\omega_{n,t+1}^d,\\
&\omega_{p,t+1}^d \sim \Gamma(p_t^d, 1) - p_t^d,\\
&\omega_{n,t+1}^d \sim \Gamma(n_t^d, 1) - n_t^d,\\
&p_{t+1}^d = \bar{p}^d(1-\rho_p^d) + \rho_p^d p_t^d + \sigma_{pp}^d \omega_{p,t+1}^d,\\
&n_{t+1}^d = \bar{n}^d(1-\rho_n^d) + \rho_n^d n_t^d + \sigma_{nn}^d \omega_{n,t+1}^d.
\end{aligned}
$$

The following notation is defined:

$U_t^d \equiv \{u_1^d, ..., u_t^d\}$ is the sequence of observations up to time t.

$F(i\phi, i\psi^1, i\psi^2|U_t^d) \equiv E(e^{i\phi u_{t+1}^d + i\psi^1 p_{t+1}^d + i\psi^2 n_{t+1}^d}|U_t^d)$ is the next period's joint conditional characteristic function of the observation and the state variables.

$G_{t|s}(i\psi^1, i\psi^2) \equiv E(e^{i\psi^1 p_t^d + i\psi^2 n_t^d}|U_s^d)$ is the characteristic function of the time t state variables conditioned on observing data up to time s.

At time 0, the characteristic function of the state variables $G_{0|0}(i\psi^1, i\psi^2)$ is initialized. As mentioned above, the distribution of p_0^d and n_0^d is approximated with gamma distributions. Note that the unconditional mean and variance of p_t^d are $E(p_t^d) = \bar{p}^d$ and $Var(p_t^d) = \frac{\sigma_{pp}^2}{1-\rho_p^{d2}}\bar{p}^d$, respectively. The approximation by the gamma distribution with the shape parameter k_0 and the scale parameter σ_0^p is done by matching the first two unconditional moments. Using the properties of the gamma distribution, $k_0^p = \frac{E^2 p_t^d}{Var(p_t^d)}$ and $\theta_0^p = \frac{Var(p_t^d)}{E(p_t^d)}$. Thus, p_0^d is assumed to follow $\Gamma(k_0^p, \theta_0^p)$ and n_0^d is assumed to follow $\Gamma(k_0^n, \theta_0^n)$, where k_0^n and θ_0^n are computed in the same way. Using the properties of the expectations of the gamma variables, $G_{0|0}(i\psi^1, i\psi^2) = e^{-k_0^p \ln(1-\theta_0^p i\psi^1) - k_0^n \ln(1-\theta_0^n i\psi^2)}$. Given $G_{0|0}(i\psi^1, i\psi^2)$, computing the likelihood of U_T^d is performed by repeating the steps 1-3 below for all subsequent values of t.

Step 1. Computing the next period's joint conditional characteristic function of the observation and the state variables:

$$
\begin{aligned}
&F(i\Phi, i\psi^1, i\psi^2|U_t^d) = E(E(e^{i\Phi(\sigma_{dp}\omega_{p,t+1}^d - \sigma_{dn}\omega_{n,t+1}^d) + i\psi^1(\bar{p}^d + \rho_p^d p_t^d + \sigma_{pp}^d \omega_{p,t+1}^d) + i\psi^2(\bar{n}^d(1-\rho_n^d) + \rho_n^d n_t^d + \sigma_{nn}^d \omega_{n,t+1}^d)}|U_t^d)\\
&= E(e^{i\psi^1 \bar{p}^d(1-\rho_p^d) + i\psi^2 \bar{n}^d(1-\rho_n^d) + (i\psi^1 \rho_p^d - \ln(1 - i\Phi\sigma_{dp} - i\psi^1 \sigma_{pp}^d) - i\Phi\sigma_{dp} - i\psi^1\sigma_{pp}^d)p_t^d + (i\psi^2 \rho_n^d - \ln(1 + i\Phi\sigma_{dn} - i\psi^2\sigma_{nn}^d) + i\Phi\sigma_{dn} - i\psi^2 \sigma_{nn}^d)n_t^d}|U_t^d)\\
&= e^{i\psi^1 \bar{p}^d(1-\rho_p^d) + i\psi^2 \bar{n}^d(1-\rho_n^d)} G_{t|t}(i\psi^1\rho_p^d - \ln(1 - i\Phi\sigma_{dp} - i\psi^1\sigma_{pp}^d) - i\Phi\sigma_{dp} - i\psi^1\sigma_{pp}^d, i\psi^2\rho_n^d - \ln(1 + i\Phi\sigma_{dn} - i\psi^2\sigma_{nn}^d) + i\Phi\sigma_{dn} - i\psi^2\sigma_{nn}^d).
\end{aligned}
$$

Step 2. Evaluating the conditional likelihood of the time $t+1$ observation:

$$p(u_{t+1}^d|U_t^d) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d})d\Phi,$$

where the function F is defined in step 1 and the integral is evaluated numerically.

Step 3. Computing the conditional characteristic function for the next period, $G_{t+1|t+1}(i\psi^1, i\psi^2)$:

$$G_{t+1|t+1}(i\psi^1, i\psi^2) = \frac{\frac{1}{2\pi}\int_{-\infty}^{\infty} F(i\Phi, i\psi^1, i\psi^2|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi}{p(u_{t+1}^d|U_t^d)}.$$

As above, the function $G_{t+1|t+1}(i\psi^1, i\psi^2)$ is also approximated with the gamma distribution via matching the first two moments of the distribution. The moments are obtained by taking the first and second partial derivatives of the joint characteristic function:

$$
\begin{aligned}
E_{t+1}p_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^1}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi, \\
Var_{t+1}p_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^1\psi^1}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi - E_{t+1}^2 p_{t+1}^d, \\
E_{t+1}n_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^2}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi, \\
Var_{t+1}n_{t+1}^d &= \frac{1}{2\pi p(u_{t+1}^d|U_t^d)}\int_{-\infty}^{\infty} F_{\psi^2\psi^2}(i\Phi, 0, 0|U_t^d)e^{-i\Phi u_{t+1}^d}d\Phi - E_{t+1}^2 n_{t+1}^d,
\end{aligned}
$$

where F_{ψ^i} denotes the derivative of F with respect to ψ^i. The expressions inside the integral are obtained in closed form by taking the derivative of the function $F(i\Phi, i\psi^1, i\psi^2|U_t^d)$ in step 1, and integrals are evaluated numerically. Using the properties of the gamma distribution, the values of the shape and the scale parameters are $k_{t+1}^p = \frac{E_{t+1}^2 p_{t+1}^d}{Var_{t+1}p_{t+1}^d}$ and $\theta_{t+1}^p = \frac{Var_{t+1}p_{t+1}^d}{E_{t+1}p_{t+1}^d}$, respectively. The expressions for k_{t+1}^n and θ_{t+1}^n are similar.

The total likelihood of the time series is the sum of individual likelihoods from step 2: $L(Y_T) = \ln p(u_1^d|k_0^p, \theta_0^p) + \sum_{t=2}^T \ln p(u_{t+1}^d|U_t^d)$.

Appendix III.D: Estimation of Regime-switching Models

These models are of the general form

$$\begin{bmatrix} u_t^g \\ u_t^\pi \end{bmatrix} = M(s_t) \begin{bmatrix} u_t^s \\ u_t^d \end{bmatrix}$$

where $M(s_t)$ is a state-dependent loading matrix depending on the latent variable s_t. For these models, we assume that the distributions of $\begin{bmatrix} u_t^s & u_t^d \end{bmatrix}$ are independent, static, and follow either a Gaussian or BEGE distribution. We first use a parsimonious model in which all four elements of M depend on the same binary state variable, so that

$$M(s_t) = \left\{ \begin{matrix} \begin{bmatrix} -\sigma_{\pi,s}^1 & \sigma_{\pi,d}^1 \\ \sigma_{g,s}^1 & \sigma_{g,d}^1 \end{bmatrix} & s_t = 1 \\ \begin{bmatrix} -\sigma_{\pi,s}^2 & \sigma_{\pi,d}^2 \\ \sigma_{g,s}^2 & \sigma_{g,d}^2 \end{bmatrix} & s_t = 2 \end{matrix} \right\}.$$

We also consider a more flexible model in which each of the four parameters in M may switch independently.

$$M(s_t) = \left\{ \begin{matrix} \begin{bmatrix} -\sigma_{\pi,s}^1 & \sigma_{\pi,d}^1 \\ \sigma_{g,s}^1 & \sigma_{g,d}^1 \end{bmatrix} & s_t = 1 \\ \begin{bmatrix} -\sigma_{\pi,s}^2 & \sigma_{\pi,d}^1 \\ \sigma_{g,s}^1 & \sigma_{g,d}^1 \end{bmatrix} & s_t = 2 \\ \begin{bmatrix} -\sigma_{\pi,s}^1 & \sigma_{\pi,d}^2 \\ \sigma_{g,s}^1 & \sigma_{g,d}^1 \end{bmatrix} & s_t = 3 \\ \ldots & \\ \begin{bmatrix} -\sigma_{\pi,s}^2 & \sigma_{\pi,d}^2 \\ \sigma_{g,s}^2 & \sigma_{g,d}^2 \end{bmatrix} & s_t = 16 \end{matrix} \right\}.$$

In all cases, the state variable is assumed to evolve according to a constant Markov transition matrix. For the 2-state version, this involves estimating two transition probabilities, p_{11} and p_{22}. For the 16-state version, we assume that each of the 4 switching parameters evolve independently, which requires estimation of 8 transition probabilities (two per transitioning parameter).

In the context of the change- of variables technique described above, we follow the classic methodology of Hamilton (1989) for estimating Markov regime-switching models. Pseudocode for that algorithm is as follows. Let $s_{10,t}$ be the vector of ex-ante state probabilities for each time period. Let $s_{11,t}$ be the ex-post state probabilities for each observation. P is the transition matrix.

- initialize state probabilities for period 1, $s_{11,1}$
- for $t = 2 : T$
  - calculate ex-ante state probabilities, $s_{10,t} = P \cdot s_{11,t-1}$
  - evaluate log likelihoods for each state
    - for $i = 1 : 2$
      - $\left[u^s_{t,i} \quad u^d_{t,i}\right] = M_i^{-1}\left[u^\pi_t \quad u^g_t\right]$
      - calculate $loglike\left(u^\pi_{t,i}, u^g_{t,i}\right) = \ln\left(|M|\right) + loglike\left(u^s_{t,i}\right) + loglike\left(u^d_{t,i}\right)$
    - end
  - total observation likelihood is $s'_{10,t} \cdot \exp\left[loglike\left(u^\pi_{t,1}, u^g_{t,1}\right) \quad loglike\left(u^\pi_{t,2}, u^g_{t,2}\right)\right]$
  - update state probabilities, for $i = 1, 2$, $s^i_{11,t} = s^i_{10,t} \cdot \exp\left(loglike\left(u^\pi_{t,i}, u^g_{t,i}\right)\right)$ / total observation likelihood
- end