Explaining Return Anomalies with a Two-factor ICAPM Model

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Abstract. A two-factor Intertemporal Capital Asset Pricing Model (ICAPM) explains returns to risk associated with fundamentals with zero alpha. In contrast, the model identifies non-zero alphas for returns involving price movements rather than fundamentals, such as price momentum and price drifts. Further, these alphas are identified as mispricing under the ICAPM: In earning alpha, investors adopt factor exposures that differ from those a multiperiod investor rationally desires. Indeed, momentum crashes are explained as departure from optimal exposures.

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1. Introduction

Investment returns unexplained by a specified risk factor model have been nominated as "anomalies." There are two reactions to these discoveries. In some cases, the anomaly is nominated as a "mispricing," with behavioral explanations supporting the attribution. In other cases, the discovery leads to the revision of the factor model to include the anomaly, such as in the four, five, and six factor extensions of the Fama and French (1993) three-factor model in Carhart (1997) and Fama and French (2015, 2018). Thus the anomaly disappears.

The joint hypotheses problem of Fama (1991) haunts the endeavor: Without an agreed-upon asset pricing model, the differentiation of "normal" returns for risk ("beta") from "abnormal" returns ("alpha") remains elusive. Indeed, Kozak, Nagel, and Santosh (2018) explain that, without a model of preferences and beliefs, distinguishing the two and attributing returns to rational versus behavioral factors is a fool's errand. Perhaps we will never arrive at the definitive model but, absent a "best model," conviction is increased with a "good model" based on sound theory. This paper takes Intemporal Capital Asset Pricing Model (ICAPM) of Merton (1973), as implemented in Penman and Zhu (2022) and Penman, Zhu, and Wang (2022), to the task.

The model has features that discriminate on risk versus mispricing. For rational pricing under the ICAPM, it requires specified betas on the factors with zero alpha. For a wide range of portfolios involving risk in fundamentals, those requirements are satisfied. No anomalies. However, for portfolios formed on price movements, including price momentum and price drift, they are not. Not only are non-zero alphas observed, but sensitivity to factors differ from that which a rational intertemporal investor desires under the ICAPM. That invokes a mispricing interpretation: In pursuing alpha, the investor takes positions inconsistent with rational risk pricing, with the alpha earned then at risk. Indeed, momentum crashes are explained as departure from optimal exposures.

1.1 The Benchmark Two-Factor Model

The two factors in the model are the market factor and a long-short factor based on firms' fundamental information that has the features of a hedge portfolio in the Merton (1973) intertemporal capital asset pricing model. The model is thus labeled as an ICAPM. The following features lend credence to this model as a "good model," one that goes some way in satisfying the requirements in Kozak, Nagel, and Santosh (2018).

(a) The model is based on standard consumption-based asset pricing theory under which investments yield future consumption, with that consumption at risk. This theory underlies the ICAPM of Merton (1973) and Breeden (1979). In Penman and Zhu (2022), the consumption asset pricing model is restated in terms of accounting numbers. Dividends that buy consumption are paid out of book value, so the risk to future book value and the earnings that generate them substitute for consumption at risk. The fundamental factor is formed from observed accounting numbers that project those earnings and book values at risk.

(b) In Penman and Zhang (2020) the accounting principles that determine these numbers convey information about the discount factor in the general, no-arbitrage pricing model, as a matter of theory.

(c) In Penman and Zhu (2022) and in accordance with the theory, those same accounting numbers indicate the risk to future earnings empirically, forecasting forward returns, forward return betas, forward fundamental betas, and ex post risk around expected returns, both return variance and tail risk. This is risk that investors experience in real time.

(d) The fundamental factor formed from these accounting numbers dominates most extant factor models on Sharpe Ratios and in standard spanning tests in Penman and Zhu (2022), as do the underlying accounting features in Penman and Zhang (2022).

(e) The fundamental factor exhibits the features of a hedging factor against loss to consumption in the ICAPM of Merton (1973). In terms of that model, it appears to be a proxy that mimics state variables that convey risk to future investment-consumption opportunities. While the long side of the portfolio exhibits relative high beta with respect to the market portfolio in both up markets and down markets, the short side has relatively low beta in down markets while preserving some upside. Correspondingly, in Penman, Zhu, and Wang (2022) the factor protect returns in down markets when consumption and wealth are lower but adds to returns in up markets. In short, the short side provides insurance in "bad times" and the long side a reverse hedge for speculators.

(f) In Penman, Zhu, and Wang (2022), this two-factor model explains returns for a variety of risky test portfolios with zero alphas, including those underlying extant factor models such as CAPM beta, book-to-price ("value versus growth"), firm size, earnings-to-price, various profitability measures, investment, and betting against beta (BAB). For all portfolios, increasing returns to these characteristics are explained by exposure to the market portfolio with a beta of approximately 1.0 and increasing exposure to the fundamental factor (with zero alphas). That accords with the ICAPM: Stocks have the same unit beta exposure to the market, with differences in their returns explained by exposure to the hedge portfolio. It is also consistent with the Merton-consistent conditional CAPM where a conditional beta applied to the market risk premium is 1.0. Conditional betas are hard to observe, but a portfolio added to the market portfolio captures the varying conditional beta if it yields higher returns in up-markets when the market is up and lower returns when it is down. That is a property of the fundamental hedging factor so, when added to the market in a regression, the beta on the market is 1.0 with exposure to the factor explaining additional return. Correspondingly, the increasing returns are explained by decreasing exposure to the short side of the factor providing hedging insurance.

When an alpha return is identified by a given model, the issue becomes one of attributing the anomaly to risk (not captured by the model) or to mispricing, the latter a keen focus of active investors. The findings in the last point (f)—zero alphas for risky portfolios—raise the specter that the model might serve to detect alphas if mispricing is operating. Indeed, there was an exception observed in the predecessor paper. Momentum was investigated because it appears as a factor (sometimes called UMD) in some factor models, and significant non-zero alphas were observed. However, momentum is a pricing feature, not a fundamental feature and many papers claim that momentum returns are due to mispricing—"the biggest challenge to market efficiency" in Fama (2013). That begs the question of whether the two-factor model detects mispricing from price movements more generally, including those from price drifts associated with earnings announcements, revenue surprises, analysts' forecasts, and price volatility. That is the subject of this paper.

The ICAPM has features that aid the endeavor. First, under the ICAPM, higher mean returns are explained by sensitivity to the market portfolio and increasing sensitivity to the hedge factor. Thus, if non-zero alpha is observed without those features, mispricing is suspected; the pricing is inconsistent with rational intertemporal investing. Second, the observed alpha can only be attributed to risk under the ICAPM if it can be rationalized as proxying for state variables that indicate changes in future investment opportunities that require a risk premium. If not, a mispricing conclusion is supported.

To the first point, this paper reports that the alpha returns are associated price movements are also associated with exposures to the ICAPM factors that are inconsistent with intertemporal investing; investors exploiting this apparent mispricing are at risk by departing from rational investing. Indeed, the paper shows that momentum crashes are so explained. In effect the alphas are at risk. To the second point, one strains to interpret momentum under the ICAPM. While the joint hypothesis problem still haunts, that lends support to the mispricing interpretation.

A further feature enhances the identification. Behavioral theory has been offered to explain perceived departure from "rational" pricing. So the paper also tests whether mispricing indicated by the model coincides with features in behavioral theory like investor sentiment, limited attention, and overconfidence. Given the validity of the behavioral theories, this test provides further validation. However, this is a joint test: If the ICAPM identifies mispricing, this is a test of whether these behavioral theories explain identified mispricing as they claim. Some do, some don't.

Data

The sample covers all U.S. firms on Compustat files for any of the years, 1982-2020, which also have required stock prices and monthly returns on CRSP files. Financial firms (SIC codes 6000-6999) are excluded as their (fair value) accounting is, in part, under different accounting principles to those evoking the fundamental factor. So are utility firms (SIC codes 4900-4949) where accounting numbers are partially due to regulation. Firms were deleted for any year in which Compustat reports a missing number for annual book value of common equity, income before extraordinary items, common shares outstanding, or total assets. Firms with negative book value for common equity or a per-share price of less than 50 cents were also eliminated.

The return on the market portfolio in the two-factor model is that for the CRSP value-weighted index and monthly returns for the fundamental factor are also from CRSP. Returns associated with many of anomalies are from the Hou-Xue-Zhang (HXZ) website at http://global-q.org/factors.html where a technical document explains how anomaly return portfolios are constructed. Returns to volatility are from the Kenneth French Data Library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research. “Mispricing factor” returns are from the Stambaugh website at http://finance.wharton.upenn.edu/~stambaug/ and some behavioral factor returns were supplied by Kent Daniel at Columbia. (We are enormously grateful to have these data.) other portfolios in the tests were constructed by us.

The construction of the fundamental hedging factor is described in the predecessor papers. It is a “factor mimicking” long minus short portfolio, with the long side based on fundamentals yielding higher returns in up markets and the short side hedging returns in down markets. We refer to the long-short factor as FH and the short side FH-S. The factor is reformed each March 31, 1982-2020, from firms’ most recent annual financial statement at those dates, with monthly returns for the factor running from April 1982 to December 2020. That is the test period. Both equal-weighted and value-weighted portfolios are formed. However, only results for equal-weighted portfolios are reported, but there is little difference with value-weighted portfolios unless noted. The hedge feature is associated with firm characteristics, and equal-weighted portfolios capture the average hedging exposures for all firms in the portfolio. Value-weighting weights towards large market-capitalization firms whose accounting characteristics indicate they may provide less of a hedge or reverse-hedge. Tests are also run with the hedge portfolio excluding microcaps, that is, firms with a market capitalization below the 20th percentile of NYSE firms with little difference in results unless noted. Excluding microcaps is common in factor tests but possibly removes firms providing the desired hedging opportunities where the sensitivity to the hedge may be particularly important. However, a portfolio excluding microcaps may be more tradable.

2. Exploring Anomalies

Our empirical analysis classifies anomalies into four groups:

(1) Anomalies observed with investment based on fundamentals for which there are *a priori* reasons that they price risk and return. If the two-factor model explains these returns with zero alpha, that gives credence to the model for identifying alphas that indicate mispricing. Further, observed betas on factors for the pricing of risk serve as benchmarks for assessing departure from rational pricing

(2) Anomalies associated with price movements rather than fundamentals. These include price momentum where mispricing is a common attribution, but also price movements following news announcements like the post earnings announcement drift: Does the two-factor model that explains returns in group (1) with zero alpha detect alpha in these returns?

(3) This group consists of anomalies that have been nominated as mispricing in the literature based on behavioral theory, like the accrual anomaly and the 11 "mispricing" factors in Stambaugh, Yu, and Yuan (2012). Are these returns explained by the model?

(4) This group involves long minus short factors based on behavioral theory, the returns to which have been interpreted as evidence of pricing that differs from "rational" risk

pricing. We ask whether these portfolio returns are explained by the two-factor model or whether the model indicates alphas consistent with the behavioral theories.

The tests involve time-series regressions of monthly returns for portfolios formed on conjectured anomaly variables on the two factors in the model. They involve asking whether increasing returns to portfolios formed from ranking on the anomaly variable ard explained by beta on the market portfolio and an increasing sensitivity to the fundamental factor with zero alpha. If so, the model explains the anomaly returns. With the fundamental factor a long minus short return (with zero net investment), an increasing coefficient indicates sensitivity to risk in the revere-hedge feature, the risk in the long side. That involves less exposure to the insurance side of the factor, the short side which has low down-side beta. Complementary tests assess whether increasing returns display decreasing exposure to the hedge (insurance) side of the factor. Conditional upon the integrity of the model, tests that yield non-zero alpha and/or increasing returns not explained by the factors indicate mispricing.

There are many tests here, so we remind readers that significant alphas can be observed by chance even though they are zero. And the returns are for particular period of time when some trading strategies may have delivered non-zero alpha ex post but not ex ante. The long sample period mitigates.

Group 1: Anomalies Associated with Risk

For a model to be credible as a benchmark, it is important to demonstrate its integrity when *a priori* there is risk and return to explain. Much of this has been done in the predecessor paper, Penman, Zhu, and Wang (2022), where the two-factor model is shown to explain returns to risk in factor models, including returns to CAPM beta, betting against beta, B/P (value versus growth), firm size, E/P, return on equity (ROE), operating profitability, cash flow-to-price, and investment. Table 1 adds additional tests. These are chosen with an *a priori* attribution to risk and return based on the accounting underlying the measure. A short explanation is provided in the captions to panels in the table, but a more detailed justification is in the appendix to Penman and Zhu (2022).

Table 1 reports tests for decile portfolios formed on the change in net operating assets, change in gross property, plant, and equipment plus inventory, enterprise P/E, and sales-to-price, all of which have been said to be associated with anomalous returns. For brevity, results are presented only for the top and bottom three portfolios, the portfolios typically involved in the formation of "factor mimicking" portfolios. On the left-hand side of each panel, monthly returns for these portfolios are regressed on the excess return on market portfolio and the fundamental hedge factor (FH), the two factors in the model. The right-hand-side tests are for the same portfolios but with the excess return on market portfolio and the short (hedging) side of FH, FH-S.

Though the ranking variables in each panel yield a spread of returns, the alphas are zero in nearly all portfolios in the left-hand side of the panels, and with the high R^2 indicating a significant part of the return explained by the two factors. In the few cases where t-statistics imply a statistically significant alpha, the alphas returns are small relative to the mean excess returns for the portfolio and are typically out of pattern. (The significant positive alpha for ΔNOA portfolio 2 is an exception, but that is not the portfolio with the lowest ΔNOA.)

Further, for all portfolios, returns are explained by a beta of approximately 1.0 on the market factor, β_1, with increasing returns associated with increasing sensitivity to the FH factor, β_2. The latter is an increasing exposure to the risky, reverse-hedge side of the factor and foregoing the short-side insurance feature.

These features are mirrored in the right-hand-side regressions with the insurance feature. High (low) returns explained by high (low) exposure to FH on the left-hand side typically correspond to a low (high) exposure to the short-side hedging portfolio, FH-S (though less so for sales-to-price). The coefficients on the market portfolio and this hedge approximately sum to 1.0 for all portfolios: The investor allocates a dollar of investment to the market portfolios and the hedge, with returns and the alphas increasing with lower sensitivity to the short-side hedge. The increasing alphas are the reward to exposure to the long side.

The results here are the same as for the risk-based portfolios in Penman, Zhu, and Wang (2022) and with similar sensitivity coefficients on the factors that benchmark rational pricing. These accumulated findings provide some confidence that, when return are due to mispricing rather than risk, the model might detect it, the issue to which we now proceed.

Group 2: Anomalies Based on Price Movements

The variables investigated in Group 1 involve fundamentals. Group 2 deals with anomalies involving price movements without a direct connection to fundamentals. These potentially involve mispricing and many papers have nominated them as such. Tables 2 and 3 report the findings.

Returns to price momentum were documented in U.S, stocks in Jegadeesh and Titman (1993), with many follow-up papers confirming the phenomenon. See Asness, Moskowitz, and Pedersen (2013). Although momentum has been a basis for a risk factor in some factor models, there is difficulty rationalizing why rising (falling) prices indicate higher (lower) risk. Rather, many papers claim that momentum involves prices moving away from fundamentals, and momentum has been associated with price crashes in Daniel and Moskowitz (2016).

Penman, Zhu, and Wang (2022) reported significant alphas for the two-factor model for portfolios formed on price momentum, prompting the further investigation in this paper. Momentum was defined as the price change over the 12 months prior to one month before portfolio formation, a common measure. Panel A reports the same tests with momentum defined as the per-share price at the date of the portfolio formation relative to the highest (split-adjusted) price over the prior 12 months (52w12 on the HXZ website). Results are similar, though the spread of returns over portfolios is higher with the earlier momentum measure. Positive alphas are reported for high momentum and negative alphas for low momentum: Momentum returns are not fully explained by the model. Given the ability of the model to convey returns for risk, as it does with Group 1, this suggests the model is detecting mispricing.

Further observations reinforce this interpretation.

First, while betas on the market portfolio are close to 1.0 for all portfolios in Group 1, they are declining with higher momentum here: Higher returns with lower beta, and consistent with Daniel and Moskowitz (2016) where high (low) momentum is associated with low (high) market beta. That means lower exposure to the wealth and consumption that the market generates.

Second, the higher returns and alphas for high momentum portfolios involve lower β_2 sensitivity to the FH factor while that is not so for with low momentum returns. Higher returns are explained by exposure to reverse hedges under the ICAPM, as in Group 1, but high momentum takes the investor away from that exposure. Yet higher returns are realized.

Third, lower sensitivity to the reverse hedge means higher exposure to the hedge (in Group 1) but that is not the case in the right-hand-side regressions with FH-S here: The β_2^* coefficient on FH-S is decreasing with higher momentum. That, too, is consistent with the momentum crashes findings in Daniel and Moskowitz (2016): With high (low) momentum stocks being low (high) beta, returns in subsequent up market are lower for high momentum stocks and for the long-short position. The beta in that paper is the conditional beta (with respect to the market return premium). In terms of the ICAPM, high momentum investing takes the investor away from the return-enhancing reverse hedge and thus lower sensitivity to the market in up markets.

Fourth, if the observed alphas are attributable to risk under the ICAPM, momentum must exposure investors to changes in information about future investment-consumption opportunities. We cannot conceive of such a scenario.

Similar results are observed with momentum calculated as the stock return over the prior six months and with industry momentum. The mispricing conclusion runs into the argument that some investors would arbitrage away the mispricing. The results here would then indicate a limit to arbitrage: With momentum taking away the upside in up markets, the arbitrageur takes on risk of losing that return. Kozak, Nagel, and Santosh (2018) argue that arbitrageurs are inevitably exposed to risk. The model here articulates that risk.

Results for the other price movements in the remaining panels in Table 2 are similar, though there is some variation. The price movements are those following the reporting of the fundamental information indicated in each panel. In an efficient market, prices react quickly to new information, taking away any opportunity to earn abnormal returns from trading on the news. Thus any post-announcement return indicates slow price adjustment to the information, a trading opportunity (subject to arbitrage limits) unless the post-announcement return can be attributed to risk.

Returns following earnings announcements in Panel B, often referred to as PEAD, post earnings announcement drift, are for portfolios formed on standardized unexpected earnings (Sue) reported each fiscal quarter. This an anomaly that has been observed for very long time, with numerous attempts to explain it. Returns are over the month following the report. Unlike momentum, the exposure to the market portfolio is fairly constant across portfolios. Like momentum, significant alphas are observed. And, as for momentum portfolios, higher alphas are associated with negative sensitivity to FH, taking the PEAD investor from returns to the factor: the PEAD investor loses this potential upside. And, as with momentum strategies, the lower exposure to FH returns is not compensated with higher exposure to the security in the FH-S hedge. This, again, suggests deviation form rational investing under the ICAPM. PEAD returns are typically observed for periods greater than a month, and results are similar with returns over six months following the information arrival. And results are similar with the partitioning variable being the market reaction to the earnings report, a measure of under- and -over- reaction to the news that potentially results in the return drift.

Panels C and D report similar results for revenue surprises and analysts' forecast revisions, again with significant alphas that differ over the high and low portfolios, and with the β_2 coefficients indicating a negative sensitivity to FH with positive alphas. The results for revenue surprises contrast with those for Sales/Price in Table 1: While Sales/Price yields zero alpha and pricing consistent with an ICAPM model, the arrival of sales (revenue) news is followed by price movements with non-zero alphas.

The final panel in Table 2 forms portfolios on quarterly R&D-to-sales. The conservative accounting principles underlying the fundamental hedge factor deem R&D expenditures as particularly risky and thus are expensed against sales revenue: The R&D into a new product might not be successful in generating sales. Thus, if R&D to gain future sales is high relative to the sales that a firm can currently generate, R&D/Sales conveys risk. Penman and Zhang (2020) connect that to priced risk theoretically, so the measure should be positively associated with returns. That is the case in Panel E, though the differences are largely in the extremes. Significant non-zero alphas are observed, suggesting the two-factor model does not capture this risk. However, casual observation suggests investors get over-excited about tech firms, and that can explain the post announcement alpha returns here. In evidence, β_2 is negative for high return, positive alpha portfolios and positive for low return, negative alpha portfolios. The β_2^* coefficients corroborate. As with momentum and the return drifts in this table, that is inconsistent with the exposures the multiperiod investor desires under the ICAPM. That indicates mispricing: Even though the investor earns higher returns, she does so by losing wealth when the markets is up; correspondingly, the lower return portfolios maintain that exposure.

Table 3 examines the low volatility (low-vol) anomaly documented in Ang, Hodrick, Xing, and Zhang (2006) among others. This variable is not a price change between two points of time, but the volatility of stock returns involves price changes. Risk is typically associated with high volatility, so the higher returns observed with low volatility are puzzling. Further, higher price swings unrelated to fundamentals, if exploited, would yield returns from mispricing, low volatility less so. The phenomenon has been attributed to behavioral factors as well as limits to arbitrage, for example in Baker, Bradley, and Wurgler (2011). Table 3 reports little difference in returns across portfolios, except in the high variance portfolio 10. There the lower return is due to negative beta on FH and, correspondingly, this portfolio has the highest β_2^* sensitivity to the insurance hedge FH-S: these returns are explained by the two-factor model with zero alpha. That makes sense: Seeing high volatility, investors by insurance. In contrast, low-vol portfolios have positive beta on FH and lower sensitivity to the FH-S hedge, explaining the higher returns.

However, low-vol portfolios also return significantly positive alphas. Notably, while nearly all portfolios in other tests in this and the predecessor paper have a beta on the market return of approximately 1.0, the low-vol portfolios here have low market beta; investors appear to earn higher returns but forego exposure to the wealth generated by the market portfolio. That is inconsistent with the preferences of the intertemporal investor. That is also the feature with the positive alphas with high momentum in Table 2. The higher returns with higher alphas here could be due to some added risk with low vol that is not captured by the model, but that is difficult to rationalize.[1]

Group 3: Anomalies Nominated as Mispricing Due to Behavioral Features

Stambaugh, Yu, and Yuan (2012) observe returns to 11 anomalies that are predicted by investor sentiment. With investor sentiment deemed to drive prices away from fundamentals, the returns are attributed to mispricing. In Stambaugh and Yuan (2017), two "mispricing factors" are formed by averaging over these anomalies. Added to market and size factors, the resulting factor model not only explains a large set of anomalies but also outperforms the Hou, Xue, and Zhang (2015) investment CAPM and the Fama and French (2015) five-factor model. This section tests whether the returns to the 11 so-called mispricing anomalies are explained by the two-factor ICAPM model. We report findings for all 11 anomalies, even though some are involved in the earlier

[1] When tests are run with value-weighted portfolios, the alphas (and *t*-statistics) for portfolios 1 to 3 are 0.003 (2.88), 0.002 (2.87), and 0.001 (1.61), respectively, diminished from those in the table. So, low-vol alphas appear to be more associated with small firms. Results are similar with the total volatility measure on the HXZ website.

tests. As with Group 2, tests for mispricing are based on both observed alphas and sensitivity betas on the hedge factor.

The findings are in Table 4 in the same format as the earlier tables but with monthly returns running only through December 2016, the availability on the Stambaugh website. Panel A deals with the anomalies grouped into the MGMT factor in Stambaugh and Yuan (2017) (involving features that management can influence) and Panel B those in the PERF factor (involving performance features). Note that the long and short positions are in the top and bottom decile of rankings, more extreme portfolios relative to the top and bottom 30% for the long and short sides on the FH factor.

For the anomalies involved in the MGMT factor, the two-factor model explains returns for Accruals, Asset Growth, and Investment-to-Assets, with alphas not significantly different from zero and with differences between returns for long and short positions explained by the differences in hedge exposures, β_2 and β_2^*. Investment-to-Assets was investigated in Panel B of Table 1, with similar results, and Asset Growth covers a wider set of assets. Accruals are realizations of earnings that indicate lower risk, so the findings are those predicted by accounting principles conveying risk and return, the principles underlying the fundamental factor. Accordingly, the accrual anomaly is not explained as mispricing.[2]

For the remaining three MGMT anomalies—Net Share Issues, Composite Equity Issues, and Net Operating Assets—returns are explained by the model with high R^2 and β_2 and β_2^* coefficients indicating returns from exposure to the hedge factors. However, non-zero alphas are

[2] Results are similar with operating accruals and total accruals, though there is not much of a spread of returns to be explained with the later. We found similar results for accrual decile portfolios on the Kenneth French website. However, we observed alphas significantly different from zero for portfolios formed on discretionary accruals (pda on the HXZ website) though only in the extreme decile portfolios.

observed, consistent with the mispricing explanation. Those negative alphas are only on the short side, as observed in Stambaugh and Yuan (2017) who then observe that that the short-side returns are higher in times of higher investor sentiment. They then attribute them to constraints on short selling; sentiment induces mispricing in both long side and short side stocks, but only the former can be exploited. If so, the two-factor model identifies returns to mispricing that cannot be exploited; limits to arbitrage.

For PERF anomalies is Panel B of Table 4, non-zero alphas are observed for all anomalies, though those for O-score are only marginally significant. The alphas are higher (in absolute terms) for the short positions. The momentum results are similar to those in Table 2 (and in the predecessor companion paper where the momentum measure is the same as that here.) As with those findings, the returns and alphas for the long and short portfolios are suggestive of mispricing, reinforced by the observation that they are associated with relative β_2 and β_2^* exposures to the reverse hedge and the hedge and in the opposite direction to that for the pricing of risk in Table 1. If sentiment is the driver, it takes prices away from the rational pricing of risk.

Returns to the Piotroski (2000) F-score are claimed to be mispricing, largely because the score is based on measures indicating safe, low-risk firms, yet a higher score yields higher returns on average. Panel C of Table 4 reports returns for the seven F-score portfolios on the HXZ website with the score differentiating returns only in the extremes. However, the two-factor model explains returns for these portfolios with high R^2. The lowest F-score portfolio has a marginally significant negative alpha. Some higher scores return positive alphas, though not for the highest

F-score. Though the results are mixed, there is little return here that is not explained by the model.[3]

Group 4: Behavioral Factors

In a challenge to risk factor models, some papers have formed long-short portfolios based on behavioral theories involving psychological biases. They are motivated by the point in Daniel and Titman (1997) that factors in those models may capture common movement in returns but that covariance is determined by behavioral factors rather than common risk exposure. The papers report that behavioral factors dominate those in extant risk models in explaining returns, with the inference that behavioral features lead to mispricing of risk as expressed in those models

Table 5 reports tests of whether long minus short portfolios formed on behavioral theory are explained by the two-factor model. In Panel A, two behavioral factors based on investor sentiment are MGMT and PERF constructed from the anomalies in Group 3. In Panel B it is the PEAD and FIN factors of Daniel, Hirshleifer, and Sun (2020). PEAD is said to capture limited attention of investors over the short term and FIN their overconfidence that leads to overreaction to private information and under reaction to public information and that affects longer-run pricing.

With MGMT and PERF constructed from the anomalies in Table 4, the findings are expected. The long-short factors yield negative beta to the market portfolio. With the long side of the MGMT (PERF) anomalies in Table 4 typically having positive (negative) beta to FH, returns are explained partially by significant positive (negative) exposure to FH here. However, significant

[3] The HWZ construction of portfolios does not quite matched that in the original Piotroski paper where F-score portfolios were formed within book-to-price portfolios. Penman, Zhu, and Wang (2022) report that the two-factor model explains for book-to-price portfolios.

positive alphas are reported for both, returns not fully explained by the model. If those returns are indeed due to mispricing, the two-factor model detects them. Though not a behavioral factor, Panel A adds the liquidity factor of Pástor and Stambaugh (2003) where higher returns are said to compensate for lower liquidity. The returns are explained with zero alpha.

As with post earnings announcement drift in Table 2, the two-factor model also identifies alpha for PEAD in Panel B. Again, the positive alpha is observed with a negative coefficient on FH, suggesting mispricing, as before. However, the returns to FIN are explained by the model with zero alpha. The β_2 coefficient on FH is quite high, one to one, and the corresponding β_2^* exposure to the FH-S hedge is negative. FIN is based on net share issues over one year and composite share issue over five years, both investigated in Table 4, so the findings there would suggest similar results here. Perhaps the difference is explained by value-weighted portfolios in FIN but equal-weighted in Table 4. If the behavioral theory holds, that says the behavior is more pronounced with smaller firms.

3. Conclusion

The ICAPM model has features that not only identify anomalous returns but also can explain them. Those features are sensitivity to the market portfolio and to hedge factors that capture exposure to changes in information about future investment-consumption opportunities. If asset returns are compensation for risk, the model explains the risk with zero alpha, as with the portfolios in Group 1 in this paper. However, if an alpha is identified, an attribution to mispricing is strengthened if returns exhibit betas on the market portfolio and the hedge factor that are inconsistent with those a rational intertemporal requires; the perceived mispricing takes the investor away from exposures that bears on his or her utility from consumption. That is the case with alphas observed with anomalies in Group 2, 3, and 4 in the paper, provoking the mispricing interpretation. Momentum returns are the primary exhibit, for those are difficult to rationalize as compensation for risk: Pursuing momentum alpha returns takes the investor away from the investment allocations for a rational investor under the ICAPM. That coincides with the "momentum crashes" observations in previous papers where momentum takes the investor away from the upside on the market portfolio that is desired to deliver wealth and consumption. However, the same features are observed in with other price movements such as post earnings announcement drift, returns following revenue surprises, analysts forecast revisions, R&D reporting, and many other anomalies attributed to mispricing in the literature.

The ICAPM requires the identification of state variables that bear on risk to future consumption. That is satisfied in the paper by a single factor based of firms' fundamental information. The inferences in the paper thus depend on how well this factor "does the job." Presumably it is not a perfect proxy, but it does have the properties that suggest it serves the purpose well. It emanates from a consumption-based asset pricing model where fundamental accounting numbers enter as indicating risk to consumption under both accounting theory and asset pricing theory. Empirically, it exhibits the properties of a hedge portfolio, adding to returns in up markets and providing a hedge in down markets. And, when applied to evaluate returns in portfolios deemed to be risky, it prices those returns with betas on the market portfolio and both the long and short side of the factor that are those predicted under the ICAPM, and with zero alphas. Thus, when alphas are observed, one is more confident with attribution to mispricing. That confidence is enhanced if the alphas are associated with a departure from rational pricing, as here.

The joint hypothesis problem still lurks in the background, or course. The attribution to mispricing is enhanced in the paper by showing that, when mispricing is indicated by the two-factor model, it is also explained by behavioral theory with factors such as sentiment, attention bias, and overconfidence determining the mispricing. That attribution would be more secure if we had a behavioral model that showed how preferences, tastes, or beliefs formally determine asset prices, but that we do not have.

Hou, Xue, and Zhang (2015) report that many so-called anomalies disappear when benchmarked against their *q*-investment model, as with Group 1 anomalies here. Many anomalies not explained by the investment model are the mispricing variables identified here. So the two papers come to similar conclusions. However, their paper leaves open the question of whether the documented anomalies are returns to risk not captured by their model or mispricing detected by the model. Their production-investment model does not associate these anomalies with investor preferences, tastes, or beliefs, a requirement in Kozak, Nagel, and Santosh (2018) for attribution. While not incorporating the preferences and beliefs of "non-rational" behavior to explain pricing, the ICAPM does explain pricing of the multiperiod "rational" consumer-investor, and it is departure from this benchmark pricing that reinforces the mispricing interpretation.

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Table 1

Explaining Group 1 Anomalies with the Two-Factor Model

Group 1 anomalies are those where *a priori* the anomaly indicates risk and return. The table reports tests of whether returns for portfolios formed on these variables are explained by factors capturing risk exposures in the two-factor model. The calculation of the variables and the portfolio formation each June 30 are described in the technical document on the HXZ website at http://global-q.org/factors.html. Portfolio returns are value weighted. The fundamental hedging factor (FH) is reformed each March 31. On the left-hand side of each panel, monthly portfolio excess returns are regressed on excess return on the CRSP market index, MKT - R_f, and on the return for the fundamental hedging factor, FH_t. On the right-hand side of each panel, they are regressed on the CRSP market index and the excess return on the short side of that factor, FH-S_t. The return period runs from April 1982 to December 2020. *t*-statistics are in parentheses.

Regression on left-hand side:: ΔNOA Portfolio Excess $\text{Return}_t = \alpha + \beta_1(\text{MKT}_t - R_{ft}) + \beta_2 \text{FH}_t + e_t$

Regression on right-hand side: ΔNOA Portfolio Excess $\text{Return}_t = \alpha + \beta_1^* (\text{MKT}_t - R_{ft}) + \beta_2^*(\text{FH-S}_t - R_{ft}) + \varepsilon_t$

Panel A: Portfolios Formed on Change in Net Operating Assets (ΔNOA_t/Total $Assets_{t-1}$)

This variable measures the addition to the balance sheet from operating earnings and new investment in the operations. Both are realizations that indicate lower returns in theory.

		Hedge Portfolio = FH Factor				Hedge Portfolio = FH-S			
ΔNOA Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.785	-0.000	1.066	0.071	0.83	0.000	0.920	0.113	0.84
		(-0.47)	(46.91)	(1.62)		(0.29)	(22.32)	(3.87)	
2	1.055	0.003	0.932	0.028	0.85	0.004	0.851	0.064	0.85
		(3.96)	(48.63)	(0.75)		(4.58)	(24.32)	(2.58)	
3	0.869	0.001	0.902	0.103	0.85	0.002	0.901	-0.011	0.85
		(1.68)	(50.31)	(2.99)		(2.63)	(27.08)	(-0.46)	
7	0.848	0.001	0.923	0.059	0.86	0.00	1.011	-0.080	0.87
		(1.59)	(52.49)	(1.75)		(1.98)	(31.61)	(-3.54)	
8	0.776	0.001	0.993	-0.113	0.86	0.000	0.944	0.054	0.86
		(1.22)	(50.92)	(-3.03)		(0.47)	(26.22)	(2.11)	
9	0.709	-0.000	1.069	-0.135	0.89	-0.001	0.958	0.109	0.90
		(-0.01)	(59.16)	(-3.90)		(-0.92)	(29.02)	(4.65)	
10(High)	0.422	-0.002	1.152	-0.330	0.86	-0.004	0.826	0.311	0.87
		(-2.13)	(48.59)	(-7.26)		(-4.01)	(20.01)	(10.63)	

Panel B: Portfolios Formed on Change in Gross Property, Plant, and Equipment and Inventory (ΔPIA = (ΔGrossPPE + ΔInventory)$_t$/Total Assets$_{t-1}$

The is the component of ΔNOA due to fresh investment in PPE and inventory. This investment is the realization of uncertain investment opportunities that indicate lower return in theory.

	Hedge Portfolio = FH Factor					Hedge Portfolio = FH-S			
ΔPIA Decile	**% Mean Excess Return**	α	β_1	β_2	**R^2**	α	β_1^*	β_2^*	**R^2**
1(Low)	0.882	0.001	1.010	0.091	0.85	0.002	0.804	0.161	0.86
		(0.82)	(49.04)	(2.32)		(2.08)	(21.97)	(6.19)	
2	0.713	-0.001	0.987	0.134	0.87	-0.000	0.987	-0.016	0.86
		(-1.21)	(53.94)	(3.81)		(-0.11)	(28.91)	(-0.64)	
3	0.835	0.001	0.935	0.066	0.86	0.001	0.990	-0.053	0.86
		(1.26)	(52.71)	(1.93)		(1.77)	(30.42)	(-2.32)	
7	0.809	0.001	0.974	-0.044	0.86	0.001	0.947	0.028	0.86
		(1.32)	(51.78)	(-1.23)		(1.08)	(27.39)	(1.12)	
8	0.948	0.004	0.947	-0.241	0.86	0.002	0.997	-0.013	0.84
		(4.37)	(48.21)	(-6.40)		(2.42)	(26.47)	(-0.49)	
9	0.562	-0.001	1.065	-0.170	0.86	-0.002	0.938	0.126	0.87
		(-1.30)	(51.07)	(-4.27)		(-2.38)	(24.56)	(4.65)	
10(High)	0.571	-0.002	1.133	-0.153	0.81	-0.002	0.882	0.227	0.83
		(-1.44)	(42.55)	(-2.99)		(-2.06)	(18.72)	(6.79)	

Panel C: Portfolios Formed on an Enterprise P/E (EntP/E = Enterprise Price$_t$/ Operating Income before Depreciation$_t$)

This variable is the inverse of an E/P ratio for operations. Like E/P it is a yield related to risk and return in theory, with price incorporating the discount for risk.

		Hedge Portfolio = FH Factor				Hedge Portfolio = FH-S			
EntP/E Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	1.012	0.002	1.015	0.311	0.71	0.002	1.235	-0.213	0.69
		(1.11)	(33.38)	(6.31)		(1.65)	(11.97)	(-2.36)	
2	0.886	0.001	0.946	0.203	0.76	0.002	1.217	-0.256	0.75
		(1.02)	(38.10)	(5.04)		(1.33)	(14.76)	(-3.55)	
3	0.853	0.002	0.842	0.163	0.76	0.002	1.146	-0.285	0.76
		(1.73)	(38.31)	(4.58)		(1.90)	(15.90)	(-4.52)	
7	0.724	0.001	0.878	0.051	0.82	0.001	0.879	-0.002	0.82
		(0.65)	(46.65)	(1.68)		(0.86)	(14.20)	(-0.05)	
8	0.741	0.001	0.951	-0.066	0.87	0.001	0.719	0.215	0.87
		(0.74)	(55.34)	(-2.36)		(0.87)	(12.97)	(4.43)	
9	0.777	0.001	0.945	-0.105	0.86	0.001	0.664	0.261	0.86
		(1.40)	(53.96)	(-3.69)		(1.45)	(11.75)	(5.29)	
10(High)	0.720	-0.000	1.167	-0.376	0.87	-0.000	0.209	0.894	0.90
		(-0.18)	(54.73)	(-10.88)		(-0.23)	(3.35)	(16.42)	

Panel D: Portfolios Formed on Sales-to-Price ($Sales_t/Price_t$)

Sales-to-price isolates the revenue component of the enterprise P/E multiple in Panel C and in the E/P ratio. Price in the denominator incorporates the discount for risk.

	Hedge Portfolio = FH Factor						Hedge Portfolio = FH-S			
Sales/Price Decile	**%Mean Excess Return**	α	β_1	β_2	**R^2**		α	β_1^*	β_2^*	**R^2**
1(Low)	0.690	0.001	1.028	-0.299	0.90		-0.001	1.017	0.044	0.88
		(1.37)	(57.97)	(-8.81)			(-1.11)	(29.01)	(1.75)	
2	0.753	0.001	0.898	-0.029	0.86		0.001	0.999	-0.081	0.87
		(1.33)	(52.13)	(-0.89)			(0.87)	(31.98)	(-3.64)	
3	0.795	0.001	0.869	0.019	0.86		0.001	0.985	-0.099	0.87
		(1.81)	(51.72)	(0.60)			(1.80)	(32.63)	(-4.63)	
7	0.891	0.000	0.993	0.203	0.80		0.002	0.829	0.112	0.80
		(0.28)	(41.98)	(4.49)			(1.94)	(18.93)	(3.62)	
8	0.855	0.000	0.960	0.160	0.77		0.002	0.868	0.058	0.76
		(0.40)	(38.29)	(3.34)			(1.57)	(18.68)	(1.75)	
9	0.891	-0.001	1.118	0.284	0.74		0.002	0.749	0.274	0.76
		(-0.80)	(36.33)	(4.82)			(1.19)	(13.58)	(7.02)	
10(High)	1.001	-0.001	1.142	0.361	0.71		0.003	0.716	0.312	0.76
		(-0.42)	(33.79)	(5.58)			(1.84)	(11.78)	(7.23)	

Table 2

Detecting Mispricing with the Two-Factor Model for Group 2 Anomalies Based on Price Movements

Group 2 anomalies are those where the anomaly is based on price movements. The table reports tests of whether returns for portfolios formed on these anomaly variables are explained by the two-factor model. Returns in Panels A-D are value-weighted returns from the HXZ website at http://global-q.org/factors.html where a technical document describes the calculation of the anomaly variables. R&D-to-Sales (R&D/S) in Panel E is constructed by the authors and the returns are equal-weighted. The fundamental hedging factor (FH) is reformed each March 31. On the left-hand side of each panel, monthly excess portfolio returns are regressed on excess return on the CRSP market index, MKT - R_f, and on the return for the fundamental hedging factor, FH_t. On the right-hand side of each panel, they are regressed on the CRSP market index and the excess return on the short side of that factor, $FH\text{-}S_t$, as in Table 1. The return period runs from April 1982 to December 2020. *t*-statistics are in parentheses.

Panel A: Portfolios Formed on Price Momentum

Momentum (Mom) is the per-share price at the date of portfolio formation relative to the price high during the previous 12 months (52w12 on the HXZ website)

		Hedge Portfolio = FH Factor				Hedge Portfolio = FH-S			
Mom Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.345	-0.006	1.487	-0.092	0.71	-0.004	0.487	0.865	0.84
		(-2.71)	(31.35)	(-1.03)		(-2.96)	(7.77)	(19.22)	
2	0.500	-0.003	1.269	0.003	0.82	-0.002	0.718	0.470	0.89
		(-2.67)	(43.90)	(0.05)		(-2.35)	(17.42)	(15.89)	
3	0.652	-0.001	1.132	0.035	0.87	-0.000	0.792	0.287	0.91
		(-1.34)	(53.21)	(0.88)		(-0.40)	(24.00)	(12.10)	
7	0.727	0.001	0.921	-0.007	0.97	0.001	0.963	-0.036	0.97
		(3.05)	(108.31)	(-0.47)		(2.86)	(64.08)	(-3.29)	
8	0.757	0.002	0.872	-0.037	0.95	0.002	0.955	-0.067	0.95
		(4.46)	(88.02)	(-1.96)		(3.75)	(55.35)	(-5.38)	
9	0.776	0.002	0.853	-0.041	0.93	0.002	0.970	-0.095	0.93
		(4.33)	(71.49)	(-1.83)		(3.64)	(47.33)	(-6.47)	
10(High)	0.753	0.003	0.847	-0.102	0.90	0.002	0.937	-0.066	0.90
		(4.09)	(60.62)	(-3.85)		(2.72)	(37.43)	(-3.69)	

Panel B: Portfolios Formed on Post Earnings Announcement Drift

Portfolios are formed on quarterly standardized unexpected earnings reported (Sue on the HXZ website). Returns are over the month following the earnings announcement.

		Hedge Portfolio=FH				Hedge Portfolio=FH-S			
Sue Decile	**% Mean Excess Return**	α	β_1	β_2	**R^2**	α	β_1^*	β_2^*	**R^2**
1(Low)	0.564	-0.003	1.106	0.048	0.83	-0.002	0.973	0.105	0.84
		(-2.64)	(46.77)	(1.06)		(-2.22)	(22.65)	(3.44)	
2	0.457	-0.003	0.985	0.054	0.84	-0.002	0.848	0.107	0.85
		(-3.25)	(47.87)	(1.38)		(-2.73)	(22.79)	(4.05)	
3	0.659	-0.001	0.988	0.108	0.83	-0.000	0.790	0.152	0.84
		(-1.43)	(46.53)	(2.65)		(-0.29)	(20.77)	(5.64)	
7	0.949	0.003	0.930	-0.082	0.86	0.003	0.912	0.025	0.85
		(3.63)	(49.88)	(-2.30)		(3.13)	(26.48)	(1.02)	
8	0.816	0.002	0.934	-0.075	0.86	0.001	0.955	-0.008	0.86
		(2.00)	(50.44)	(-2.12)		(1.39)	(27.91)	(-0.33)	
9	0.863	0.002	0.984	-0.111	0.86	0.001	0.965	0.029	0.86
		(2.24)	(49.94)	(-2.95)		(1.49)	(26.45)	(1.10)	
10(High)	0.923	0.003	0.945	-0.078	0.84	0.002	1.147	-0.159	0.86
		(2.98)	(47.57)	(-2.06)		(2.11)	(32.62)	(-6.37)	

Panel C: Portfolios Formed on Revenue Surprises

Revenue surprises are the reported change in quarterly revenue from four quarters ago (rs on the HXZ website). Returns are those over the subsequent month.

		Hedge Portfolio = FH Factor				Hedge Portfolio = FH-S			
rs Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.540	-0.003	1.020	0.201	0.80	-0.002	0.826	0.138	0.80
		(-3.04)	(41.80)	(4.30)		(-1.53)	(18.43)	(4.33)	
2	0.604	-0.003	1.064	0.237	0.81	-0.001	0.849	0.151	0.81
		(-2.98)	(44.05)	(5.13)		(-1.17)	(19.06)	(4.78)	
3	0.649	-0.001	0.970	0.096	0.84	-0.000	0.879	0.065	0.84
		(-1.41)	(48.45)	(2.50)		(-0.52)	(23.87)	(2.49)	
7	0.689	0.000	0.964	-0.080	0.87	-0.000	0.942	0.028	0.87
		(0.28)	(52.27)	(-2.28)		(-0.34)	(27.69)	(1.14)	
8	0.735	0.001	0.948	-0.058	0.86	0.000	0.967	-0.009	0.86
		(0.80)	(51.55)	(-1.63)		(0.29)	(28.53)	(-0.37)	
9	0.735	0.001	0.972	-0.112	0.85	-0.000	1.086	-0.082	0.85
		(0.90)	(48.46)	(-2.91)		(-0.19)	(29.50)	(-3.13)	
10(High)	0.939	0.003	0.963	-0.149	0.86	0.002	1.072	-0.073	0.86
		(3.62)	(50.57)	(-4.08)		(2.29)	(30.36)	(-2.91)	

Panel D: Portfolios Formed on Revisions in Analysts' Earnings Forecasts

Forecast revisions are updates in sell-side analysts' consensus earnings forecasts (re on the HXZ website). Returns are over the six months following the revision.

		Hedge Portfolio = FH Factor					Hedge Portfolio = FH-S			
Re Decile	**% Mean Excess Return**	α	β_1	β_2	R^2		α	β_1^*	β_2^*	R^2
1(Low)	0.570	-0.004	1.361	0.192	0.72		-0.002	0.676	0.573	0.74
		(-2.22)	(33.10)	(2.59)			(-1.46)	(6.19)	(6.46)	
2	0.626	-0.002	1.184	0.151	0.82		-0.001	0.715	0.390	0.83
		(-1.79)	(43.83)	(3.09)			(-0.91)	(9.97)	(6.68)	
3	0.627	-0.001	1.076	0.131	0.84		-0.001	0.796	0.228	0.85
		(-1.34)	(47.85)	(3.23)			(-0.53)	(12.98)	(4.58)	
7	0.738	0.002	0.922	-0.159	0.91		0.001	0.918	0.022	0.90
		(2.69)	(63.09)	(-6.05)			(1.71)	(21.92)	(0.64)	
8	0.707	0.001	1.004	-0.267	0.92		0.000	0.846	0.168	0.90
		(1.72)	(64.42)	(-9.51)			(0.47)	(18.36)	(4.48)	
9	0.843	0.002	1.018	-0.181	0.90		0.002	0.645	0.346	0.90
		(2.82)	(57.29)	(-5.66)			(2.63)	(13.86)	(9.13)	
10(High)	1.026	0.004	1.070	-0.199	0.77		0.004	0.389	0.614	0.80
		(2.86)	(35.67)	(-3.69)			(3.16)	(5.13)	(9.97)	

Panel E: Portfolios Formed on Quarterly R&D-to-Sales

Returns are over the six months following the quarterly report.

		Hedge Portfolio=FH				Hedge Portfolio=FH-S			
R&D/S Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.480	-0.004	1.140	0.133	0.82	-0.003	1.002	0.099	0.82
		(-3.09)	(39.95)	(2.55)		(-2.57)	(19.13)	(2.74)	
2	0.818	-0.000	1.043	0.234	0.71	0.001	0.950	0.053	0.70
		(-0.19)	(29.65)	(3.66)		(0.62)	(14.46)	(1.16)	
3	0.718	-0.001	0.938	0.264	0.67	0.001	0.876	0.025	0.65
		(-0.44)	(27.25)	(4.20)		(0.47)	(13.52)	(0.57)	
7	1.028	0.004	0.940	-0.206	0.77	0.003	0.835	0.105	0.76
		(3.55)	(33.15)	(-3.99)		(2.78)	(15.87)	(2.89)	
8	0.893	0.003	1.011	-0.298	0.78	0.001	1.066	-0.016	0.77
		(2.28)	(34.12)	(-5.52)		(1.08)	(18.80)	(-0.40)	
9	0.906	0.003	1.093	-0.341	0.78	0.001	1.060	0.060	0.77
		(1.91)	(33.95)	(-5.81)		(0.68)	(17.19)	(1.40)	
10(High)	1.056	0.005	1.134	-0.662	0.75	0.002	0.807	0.331	0.72
		(3.07)	(28.99)	(-9.27)		(1.15)	(10.63)	(6.31)	

Table 3

Explaining the Low Volatility Anomaly with the Two-factor Model

Volatility decile portfolios are those from the Kenneth French Data Library formed each month based on the variance of returns (VAR) over the prior 60 days. Returns for these portfolios are excess equal-weighted monthly returns. Mean excess returns are monthly percentage returns. On the left-hand side of each panel, excess portfolio returns are regressed on excess return on the CRSP market index, MKT - R_f, and on the return for the fundamental hedging factor, FH_t. On the right-hand side of each panel, they are regressed on the CRSP market index and the excess return on the short side of that factor, $FH\text{-}S_t$. The return period runs from April 1982 to December 2020 with the hedging portfolios reformed each March 31. *t*-statistics are in parentheses.

		Hedge Portfolio=FH				Hedge Portfolio=FH-S			
VAR Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.859	0.004	0.501	0.110	0.58	0.005	0.342	0.120	0.59
		(4.56)	(24.88)	(2.85)		(6.06)	(9.37)	(4.62)	
2	0.943	0.003	0.723	0.135	0.75	0.004	0.559	0.121	0.76
		(3.62)	(36.93)	(3.60)		(5.30)	(15.69)	(4.79)	
3	0.978	0.003	0.824	0.166	0.79	0.004	0.595	0.171	0.80
		(2.88)	(40.70)	(4.28)		(4.93)	(16.41)	(6.66)	
7	1.095	0.002	1.074	0.116	0.77	0.004	0.436	0.517	0.87
		(1.80)	(38.71)	(2.18)		(4.72)	(11.33)	(18.96)	
8	1.085	0.002	1.165	0.052	0.76	0.004	0.351	0.672	0.89
		(1.30)	(36.68)	(0.85)		(4.19)	(9.11)	(24.60)	
9	0.949	0.000	1.269	-0.006	0.73	0.002	0.225	0.870	0.92
		(0.07)	(34.28)	(-0.08)		(2.39)	(6.02)	(32.91)	
10(High)	0.540	-0.003	1.388	-0.308	0.55	-0.002	-0.368	1.498	0.88
		(-1.05)	(22.32)	(-2.59)		(-1.17)	(-6.10)	(34.98)	

Table 4

Explaining Group 3 Mispricing Anomalies with the Two-Factor Model

Group 3 anomalies are those attributed to mispricing with behavioral explanations. In Panels A and B they are the "mispricing anomalies" nominated as such in Stambaugh, Yu, and Yuan (2012) because they are predicted by investor sentiment. In Panel C it is the alleged mispricing of the quality F-score. The table reports tests of whether returns for portfolios formed on these anomaly variables are explained by the two-factor model. Anomaly variables In Panels A and B are defined in the appendix to Stambaugh and Yu (2017). Long and short portfolios are for the top and bottom deciles from ranking on the variables each month, respectively. Returns are equal-weighted returns from the Stambaugh website at http://finance.wharton.upenn.edu/~stambaug/. The Panel C value-weighted returns are from the HXZ website at http://global-q.org/factors.html where a technical document describes the calculation of the score. The fundamental hedging factor (FH) is reformed each March 31. On the left-hand side of each panel, monthly excess portfolio returns are regressed on excess return on the CRSP market index, MKT - R_f, and on the return for the fundamental hedging factor, FH_t. On the right-hand side of each panel, they are regressed on the CRSP market index and the excess return on the short side of that factor, $FH\text{-}S_t$. The return period runs from April 1982 to December 2016. *t*-statistics are in parentheses.

Panel A: Anomalies Underlying the MGMT Factor

These are anomalies deemed to be under management control. The rational for treating these anomalies as mispricing is in the appendix to Stambaugh and Yu (2017).

Net Share Issues

Annual log change in shares outstanding adjusted for stock splits.

	% Mean Excess Return	Hedge Portfolio = FH				Hedge Portfolio = FH-S			
		α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
Short	0.262	-0.005	1.174	-0.003	0.90	-0.005	1.036	0.119	0.90
		(-5.48)	(56.50)	(-0.08)		(-5.68)	(28.59)	(4.58)	
Long	0.829	0.001	0.898	0.140	0.92	0.002	1.063	-0.156	0.93
		(1.97)	(68.85)	(5.69)		(3.78)	(48.97)	(-10.01)	

Composite Equity Issues

Growth in total market equity not attributable to the stock return.

	% Mean Excess Return	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
Short	0.368	-0.004	1.187	-0.045	0.90	-0.004	1.011	0.155	0.91
		(-4.26)	(58.73)	(-1.17)		(-4.74)	(29.24)	(6.26)	
Long	0.900	0.001	0.875	0.239	0.79	0.003	1.016	-0.146	0.79
		(1.28)	(39.58)	(5.71)		(3.06)	(25.56)	(-5.11)	

Accruals

Annual change in non-cash working capital minus depreciation and amortization expense all divided by average total assets over the prior two years, as in Sloan (1996).

Short	0.417	-0.001	1.183	-0.402	0.85	-0.003	0.831	0.343	0.86
		(-0.63)	(44.92)	(-8.08)		(-3.05)	(18.53)	(10.66)	
Long	0.745	0.000	1.157	-0.085	0.83	0.000	0.897	0.231	0.85
		(0.24)	(43.35)	(-1.68)		(0.11)	(19.87)	(7.14)	

Net Operating Assets

Operating assets minus operating liabilities scaled by total assets.

Short	0.153	-0.005	1.109	-0.122	0.89	-0.006	0.956	0.144	0.89
		(-5.31)	(54.19)	(-3.14)		(-6.56)	(26.80)	(5.63)	
Long	0.907	0.001	1.131	0.025	0.88	0.001	1.133***	-0.004	0.88
		(1.28)	(52.96)	(0.63)		(1.58)	(29.68)	(-0.15)	

Asset Growth

Annual growth rate in total assets.

Short	0.400	-0.002	1.205	-0.290	0.89	-0.004	1.070	0.146	0.89
		(-1.92)	(55.19)	(-7.03)		(-4.12)	(26.72)	(5.10)	
Long	0.848	0.001	1.026	0.073	0.83	0.002	0.860	0.135	0.84
		(0.96)	(44.61)	(1.68)		(1.98)	(21.40)	(4.67)	

Investment-to-Assets

Annual change in gross property plant and equipment and inventories scaled by lagged total assets.

Short	0.399	-0.002	1.117	-0.204	0.82	-0.003	0.834	0.264	0.84
		(-1.59)	(40.91)	(-3.94)		(-2.74)	(18.00)	(7.94)	
Long	0.844	0.001	1.017	0.076	0.86	0.002	0.808	0.171	0.87
		(1.07)	(48.99)	(1.94)		(2.38)	(22.84)	(6.73)	

Panel B: Anomalies Underlying the PERF Factor

These are anomalies said to be related to performance. The rational for treating these anomalies as mispricing is in the appendix to Stambaugh and Yu (2017).

	Hedge Portfolio = FH				Hedge Portfolio = FH-S			
% Mean Excess Return	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2

Distress

Failure probability estimated with a logit model with accounting and price variables.

Short	0.093	-0.011	1.698	0.113	0.77	-0.009	1.092	0.506	0.81
		(-5.32)	(36.75)	(1.29)		(-5.05)	(14.54)	(9.40)	
Long	0.813	0.004	0.847	-0.244	0.74	0.002	0.882	-0.005	0.73
		(3.51)	(32.22)	(-4.92)		(1.88)	(18.27)	(-0.14)	

O-Score

Probability of bankruptcy predicted from accounting data.

Short	0.478	-0.001	1.216	-0.309	0.81	-0.002	0.619	0.542	0.89
		(-0.76)	(39.65)	(-5.33)		(-2.31)	(14.47)	(17.66)	
Long	0.706	0.002	1.036	-0.298	0.85	-0.000	1.008	0.055	0.83
		(2.19)	(45.26)	(-6.88)		(-0.04)	(23.45)	(1.79)	

Momentum

Stock return from 12 months to 2 months before portfolio formation.

Short	-0.013	-0.011	1.429	0.231	0.67	-0.008	0.823	0.494	0.71
		(-5.00)	(28.75)	(2.46)		(-4.17)	(9.93)	(8.33)	
Long	1.035	0.006	1.099	-0.409	0.73	0.003	0.889	0.221	0.72
		(3.61)	(30.44)	(-5.99)		(1.96)	(13.59)	(4.72)	

Gross Profitability Premium

Annual gross profit (revenue minus cost of goods sold) to total assets

Short	0.615	-0.005	1.211	0.368	0.79	-0.002	1.277	-0.095	0.78
		(-3.39)	(39.77)	(6.38)		(-1.39)	(22.56)	(-2.34)	
Long	0.920	0.004	0.894	-0.072	0.81	0.003	1.001	-0.085	0.81
		(3.63)	(40.27)	(-1.72)		(3.05)	(25.44)	(-3.00)	

Return on Assets

Quarterly income before extraordinary items to lagged quarter total assets.

Short	0.253	-0.005	1.356	-0.186	0.80	-0.006	0.820	0.478	0.85
		(-3.20)	(38.61)	(-2.80)		(-4.27)	(15.10)	(12.30)	
Long	0.842	0.004	0.981	-0.250	0.86	0.001	1.073	-0.053	0.85
		(3.83)	(46.92)	(-6.31)		(1.57)	(27.57)	(-1.89)	

Panel C: Portfolios Formed on F-score

The Piotroski (2000) F-score is calculated from 9 accounting measures deemed to indicate firm quality and safety. The portfolios are based on quarterly data and returns are for the following 12 months.

		Hedge Portfolio=FH				Hedge Portfolio=FH-S			
F Decile	**% Mean Excess Return**	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
1(Low)	0.516	-0.003	1.196	-0.231	0.84	-0.003	0.652	0.477	0.90
		(-2.18)	(44.34)	(-4.46)		(-3.38)	(16.62)	(17.16)	
2	0.715	-0.000	1.069	-0.120	0.90	-0.001	0.774	0.259	0.92
		(-0.39)	(58.32)	(-3.42)		(-0.79)	(26.28)	(12.40)	
3	0.798	0.001	1.012	-0.092	0.94	0.000	0.935	0.074	0.94
		(1.36)	(80.01)	(-3.80)		(0.64)	(40.28)	(4.52)	
4	0.793	0.001	0.983	-0.068	0.96	0.000	0.977	0.013	0.96
		(1.76)	(95.99)	(-3.47)		(0.92)	(50.95)	(0.96)	
5	0.791	0.001	0.931	-0.055	0.96	0.001	0.969	-0.026	0.96
		(2.45)	(92.46)	(-2.83)		(1.63)	(51.86)	(-1.95)	
6	0.898	0.002	0.967	-0.028	0.93	0.002	0.980	-0.007	0.93
		(2.95)	(73.29)	(-1.12)		(2.72)	(40.14)	(-0.42)	
7(High)	0.792	0.001	0.933	-0.024	0.84	0.001	0.984	-0.040	0.84
		(1.01)	(44.96)	(-0.60)		(0.80)	(25.72)	(-1.46)	

Table 5

Explaining Group 4 Behavioral Factors with the Two-Factor Model

The table reports tests of whether returns for long-short factors formed on the basis of behavioral theory are explained by the two-factor model. MGMT and PERF returns are equal-weighted returns from the Stambaugh website at http://finance.wharton.upenn.edu/~stambaug/. LIGV returns are value-weighed returns from the same website. PEAD and FIN returns are value-weighted returns for portfolios constructed in Daniel, Hirshleifer, and Sun (2020) supplied by Kent Daniel. They are reformed at the end of each June. The fundamental hedging factor (FH) is reformed each March 31. On the left-hand side of each panel, monthly portfolio excess returns for the factors are regressed on excess return on the CRSP market index, MKT - R_f, and on the return for the fundamental hedging factor, FH_t. On the right-hand side of each panel, they are regressed on the CRSP market index and the excess return on the short side of that factor, $FH\text{-}S_t$. The return period runs from April 1982 to December 2016 for MGMT and PERF and until December 2020 for PEAD and FIN. *t*-statistics are in parentheses.

Panel A: Investor Sentiment Factors in Stambaugh and Yuan (2017) and Liquidity Factor

The MGMT and PERF are formed from averaging long-short returns in Panels A and B in Table 4, respectively. The liquidity factor, LIQV, is formed from long-short portfolios from ranking firms on liquidity betas.

	Hedge Portfolio = FH				Hedge Portfolio = FH-S			
Factor	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
MGMT	0.005	-0.265	0.471	0.39	0.008	0.029	-0.300	0.371
	(3.90)	(-10.37)	(9.76)		(6.97)	(0.62)	(-9.06)	
PERF	0.013	-0.387	-0.456	0.19	0.009	-0.123	-0.178	0.149
	(6.73)	(-9.23)	(-5.76)		(4.71)	(-1.60)	(-3.24)	
LIQV	0.003	0.062	0.148	0.01	0.004	0.051	-0.006	0.003
	(1.57)	(1.57)	(1.96)		(2.36)	(0.71)	(-0.11)	

Panel B: Limited Attention and Confidence Factors in Daniel, Hirshleifer, and Sun (2020)

The PEAD factor is called limited attention and is measured with post earnings announcement drift following quarterly earnings announcements. FIN is called confidence and is constructed from returns following share issues and repurchases that are nominated as overpricing and underpricing due to investors' overconfidence.

	Hedge Portfolio = FH				Hedge Portfolio = FH-S			
Factor	α	β_1	β_2	R^2	α	β_1^*	β_2^*	R^2
PEAD	0.008	-0.004	-0.108	0.02	0.007	0.010	0.010	0.00
	(6.93)	(-0.13)	(-1.96)		(6.65)	(0.24)	(0.32)	
FIN	0.002	-0.209	1.020	0.47	0.008	0.150	-0.530	0.44
	(0.91)	(-4.68)	(11.66)		(4.51)	(2.20)	(-10.90)	