The Macroeconomics of Stakeholder Equilibria*

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Abstract

We propose one route to a more inclusive society. Our context is the prevailing one of high wealth inequality where stockholders alone supply the stochastic discount factor governing the allocation of capital. A large and pervasive pecuniary externality is thus imposed on non-stockholder workers, something we view as antithetical to the notion of an inclusive society. Accordingly, the paper explores the extent to which the externality can be purely privately internalized solely (without wealth redistribution) through a combination of bond trading between workers and stockholders and egalitarian present value wage bargaining in the labor market. In this incomplete financial market setting, endogenous countercyclical Coasian worker property rights arise as a natural consequence of egalitarian bargaining. This shifting distribution of property rights manifests itself in the form of wage payments representable as endogenous low-risk present value wage assets characterized by efficient wage markups, allowing roughly 60% of the pecuniary externality to be internalized in the benchmark case. As wealth inequality grows, increasing firm desire to retain bonds for precautionary purposes leads to massive declines in default free interest rates with worker income insurance increasingly provided by the wage asset whose value becomes progressively disassociated from labor productivity. Under extreme inequality, this feedback loop, what we refer to as the "polarization trap," restricts the stakeholder economy to internalize only 50% of the externality. Macroeconomically, the economy gravitates towards *cyclical stagnation* rather than *secular stagnation*. For all degrees of wealth inequality, however, the long-run decentralized equilibrium is constrained efficient (exclusive equity ownership is retained), because this stakeholder economy's increasing *centralization* of capital ownership serves as the engine of economic growth.

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1 Introduction

Under current economic conditions of highly concentrated equity ownership, investment decisions are taken at the direction of a small measure of the population. This creates a pervasive pecuniary externality: non-stockholders, who represent the overwhelming fraction of the population, have no voice in the firm's investment decisions though they may be substantially affected by them. In the spirit of the Coase Theorem, we propose a simple decentralized mechanism by which this externality can be largely privately internalized through non-competitive labor market arrangements, and suggest measures by which the degree of internalization can be assessed. As such, the model suggests one possible route to a more inclusive, stakeholder economy at least as regards the relationship between firm owners and their workers.[1,2]

Critical to our analysis are three main ingredients. First, we assume restricted financial market participation: all equity ownership and trading is exclusively concentrated in a small stockholder measure of the population. Accordingly, the economy's investment plan is determined exclusively by the stockholder's stochastic discount factor, thereby creating the basis of the pecuniary externality.[3]

The second major ingredient is to allow default-free bond trading between stockholders (the firm) and its non-stockholder-workers. Consistent with our emphasis on a purely private internalization of the externality, these bonds are assumed to be issued exclusively by the firm. Accordingly, workers end up financing a portion of the firm's capital structure. We thus eschew the availability of government

[1]The present model is not the first attempt to consider the notion of a stakeholder economy in a formal setting. As far as we understand that honor goes to Allen et al. (2015) and Magill et al. (2015). The latter paper, in particular, has a comprehensive discussion of the stakeholder concept and its historical evolution. A stakeholder economy is one in which the the firms' objective is not only firm value maximization but also takes account of the effects of its investment decisions on its workers, customers and the communities in which it operates facilities. An exclusive focus on firm value maximization otherwise imposes adverse pecuniary externalities on the other groups affected by its investment (or, more recently, disinvestment) decisions. The present paper explores the decentralized Coasian internalization of one such externality through a more expansive notion of wage determination in a dynamic macroeconomic setting. In Allen et al. (2015), there are two firm types, a shareholder firm where profit maximization rules and a stakeholder firm that explicitly internalizes (in reduced form) the effects of its decisions on its other stakeholders. In particular, the stakeholder firm chooses pricing decisions jointly to maximize profits and to avoid bankruptcy in which case the firm's other stakeholder benefits are lost. It is in this sense that stakeholder interests are brought into the firm's objective function. There is marginal cost uncertainty and demand uncertainty with product market competition between the two types of firms. The paper is concerned how the two firms pricing decisions differ in this setting. Under certain circumstances, the stakeholder firm is the more valuable of the two; the reverse is also possible. There is, however, no investment decision or explicit labor market, and thus no consideration of the pecuniary externality we investigate.

[2]Jacob Fugger (1459-1525), whose overall wealth is estimated to be comparable to 2% of Europe's GDP at that time, was perhaps a first provocateur to inspire the idea of a *stakeholder equilibrium*. He initiated a housing project for Ausburg's working poor, referred to as the *Fuggerrei*, and its resulting settlement remains in service even 500 years later, housing the poor as usual (see: *the Richest Man Who Ever Lived: The Life and Times of Jacob Fugger* by G. Steinmetz, 2015). A plaque in Ausburg describes the Fuggerei as follows: *The brothers Uhlig, George and Jacob Fugger of Augsburg, who are convinced they were born to serve the city and feel obligated to return property received from all mighty and just God, have out of piety and as a model of openhearted generosity, given, granted, and dedicated 106 homes with all fixtures to the diligent and hardworking but poor fellow citizens.*

[3]While this assumption may appear extreme, it is widely made in the literature, and reflects, to a reasonable approximation, the reality that 10% of the US population owns approximately 90% of all firm equity. Since Siconolfi's (1988) seminal work, this type of market incompleteness has been a popular modeling choice in the macro-finance literature. Notable references are Basak and Cuoco (1998), Guvenen (2009), and Saito (1996) to mention but a few. None of them, however, identify the "hidden" pecuniary externality restricted asset market participation implies and its resulting implications for unemployment fluctuations and workers' social insurance.

bonds; equivalently, the government's budget is in perpetual balance.[4] Private safe bond trading is critical to the analysis to follow. Most importantly, without bond trading it is unreasonable to presume that non-stockholder-workers would commit to any form of present value wage bargaining, something we will assume, in an otherwise profoundly incomplete financial market setting. In fact, to keep the bond market 'usefully liquid for risk sharing purposes,' stockholders (firms) must be encouraged to trade ample quantities of bonds rather than retaining them for their own precautionary savings needs. How the labor market both responds to and encourages the reluctance of firms to supply safe assets will form much of the discussion in the present paper.[5] As a consequence of any reduction in bond trading, achievable risk sharing between stockholders and workers is reduced. The labor market arrangement we propose endogenously create a partial substitute.

Our third major ingredient is to assume Kalai (1977)-Rawls (1971)-Nash (1950) egalitarian present value wage bargaining between firms and their workers. It operates in the following way: *stockholders and workers apportion to one another the joint surplus created by an employment match in accordance with equating the resulting lifetime welfare gains to each party rather than the respective present value wealth gains accruing to each as in standard Nash bargaining. Relative stockholder/worker risk aversion, disparate human capital levels, and any other distinctive feature of agent preferences or circumstances can in principle be considered, befitting the spirit of a more inclusive stakeholder society.* What we find both surprising and hopeful is the fact that a change only in the nature of wage bargaining is sufficient to eliminate 60% of the pecuniary externality without the necessity of equity wealth redistribution.

Bond trading and egalitarian wage bargaining interact in the following way. Under this incomplete financial market arrangement, the ratio of worker to stockholder marginal utility of consumption is not constant but will be seen to follow a countercyclical stochastic process, a quantity we refer to as distribution risk. The Coasian distribution of the property right to wage payments noted above turns out to be directly proportional to distribution risk and thus is itself countercyclical, but this "Coasian" solution equally balances the benefits to employment of firms and their workers. As a result, a semi-safe present value wage contact is endogenously created that provides workers with supplementary (semi-financial) collateral to facilitate their consumption smoothing by ensuring sufficient bond market liquidity.[6] Egalitarian wage bargaining may thus be viewed as a generalization of the wage bargaining process that attempts to compensate workers for their diminished ability to smooth out their consump-

[4]Although we do not deal explicitly with this feature, workers thus have an incentive to prevent the firm from becoming leveraged to the point of bankruptcy. In this sense, the present structural assumption is in the spirit of Allen et al. (2015).

[5]The present perspective is thus consistent with a Phelps-Phillips relation, reflecting a tradeoff between real rates of interest and agggregate employment volatility (Phelps, 1994).

[6]The notion that employers and employees regard their salary payments as an asset is coming to the fore particularly in countries where acute labor shortages exist. In South Korea the Suprema Company offers 500 MM won loans, the equivalent of $420K, to employees for the purchase of homes close to their place of employment with the idea of improving their work-life balance. There are two implications of this initiative relative to the present modeling perspective. First, the firm is specifically concerned with employee welfare, as related to working conditions and salary in the condition of lifetime employment, something our bargaining scheme presumes. Second, and more directly relevant, it implies the creation of an employee's implicit wage asset against which it may borrow to purchase a home.

tion flow via the limited selection of securities available to them. In equilibrium, the remaining residual distribution risk measures the degree to which the stakeholder externality is not internalized, leading to socially desirable wage markups.[7]

Under these arrangements, the long run mean levels of the capital stock and the consumption of both agents are (constrained) first best for all levels of wealth inequality; that is, no wealth redistribution is necessary and thus the classical Coase Theorem applies. In conjunction with perfect risk sharing within the family (an assumption to be adopted in Section 2) wage markups lead effectively to a privately arranged Universal Basic Income (UBI) for all workers.[8] Business cycle deviations from the long-run growth path are not first best, however, and it is in this latter sense that the internalization of the pecuniary externality is partial.

The model's parameterization procedure allows us to establish a correspondence between the Baseline model's theoretical predictions and measured data for the US economy. We choose the data period 1970-2015, which Eggertsson et al. (2019b) identify as a period of "secular stagnation," as characterized by high labor market volatility, high unemployment, and near zero real interest rates. In undertaking this exercise, we imagine that egalitarian bargaining was in effect during this period. Magill et al. (2015) indeed provide some qualitative evidence. While the model provides a good fit to the data, our stakeholder motivation requires a radically different interpretation. In the stakeholder economy, these phenomena are purely cyclical (business cycle frequency) rather than secular, and thus policies designed to remove undesirable secular phenomena would be inapproprate to the present setting. Rather than secular phenomena, model results reflect the interaction of the bond market and wage asset creation in ways that differentially affects the contracting parties but in a manner that each is benefitted. To illustrate, whereas Fahri and Werning (2016) advocate macroprudential policies that restrict debt issuance, in an environment of high wealth inequality the present model will call for increased private debt issuance to moderate high labor market volatility.

The model framework is sufficiently flexible to allow us progressively to increase wealth inequality either by allowing capital ownership gradually to concentrate in a smaller and smaller measure of stockholders, while keeping the measure of non-stockholder-workers constant, or by hypothesizing technological change that increases the share of income to capital. Whatever the origin of increasing inequality, the economy progresses through a series of corresponding phases as the interaction of bond trading and wage asset creation changes. To this end, we present a series of "policy experiments" which describe how the stakeholder equilibrium would be affected by increasing wealth inequality.[9]

[7] As demonstrated in Section 5, the *efficient wage mark-up* can also be interpreted as an *increasing* labor income share, standing 180 degrees to the *constant* labor income share on the textbook competitive balanced growth path.

[8] There is a long tradition in economics of considering the family unit as the building block of economic decision making, something our "perfect insurance within the family" assumption implicitly adopts. Furthermore, there is no endorsement for driving wages as low as possible, an equilibrium outcome also ignored in the present model where period wages are above the competitive level, especially in low productivity times. See Liberatore (*Principles of Political Economy*, 1891), Marshall (*Principles of Economics, Volume I*, 1898) and Bagehot (*Economic Studies*,1888).

[9] A "policy experiment" in the present context constitutes two different steps. The first is one that seeks an answer to the

Near Complete Case Firm (stockholder) income risk is modest, allowing firms willingly to supply bonds as indicated by low equilibrium bond prices and high positive real interest rates. We interpret these positive rates as suggesting that it is stockholders who are effectively paying the cost of commitment to honor the firm's long-term wage contract. Sufficient bond trading thus creates a Coasian assignment of property rights unambiguously in favor of workers in this case. Indeed, in a moderate-wealth-inequality decentralized economy characterized by highly liquid bond trading and positive real interest rates, egalitarian wage bargaining leads to an equilibrium outcome virtually identical to the analogous standard DMP complete markets Nash bargaining setup of, e.g., Andolfatto (1996).[10] As a consequence, residual equilibrium distribution risk is relatively low.

Baseline Case The Baseline case, identified by a higher degree of wealth inequality, is one where bond trading becomes less liquid leading to near-zero interest rates and high-price, low-discount-volatility wage assets that serve both agents and maintain a "sufficient" internalization of the externality. These wage assets possess both low financial and business cycle risk, the former feature presenting itself as a discount-rate-protected safe wage asset for the firm to "hire," while the latter manifests itself as a nearly fixed income wage asset for workers. Under the Baseline stakeholder equilibrium, the labor market volatility is largely indexed to (small) fluctuations in the discount rate applicable to the discount-protected safe wage asset, thereby exchanging real productivity risk for small financial risk.[11]

High Wealth Inequality By scaling up the aforementioned effects, extreme wealth inequality creates a "polarization trap" in which firm-owners' extreme precautionary demand for bonds provokes a safe asset shortage characterized by dramatically higher distribution risk and negative safe asset rates. Egalitarian bargaining responds by effecting equilibrium "sticky wages," measured in present value terms and insensitive to productivity changes, which we refer to as a "semi-fixed wage asset." The latter phenomenon drives up labor market unemployment and vacancy volatility, thus contributing further to driving down the safe asset rate to negative territory. Key to this "polarization trap" is the failure of stockholders fully to internalize the elevated pecuniary externality (distribution risk) as a market equilibrium outcome. This "safe-asset shortage" further creates the twin consumption-smoothing dis-

following question: presuming that the US society has been making a effort to address the stakeholder externality we identify here, to what extent had the US economy achieved the internalization of that externality, albeit partially, for the period 1970-2015? Accordingly, our empirical finding on the first inquiry suggests that between 1970 and 2015, the US appears to have achieved 60% partial internalization of the externality. Taking the latter case as a useful benchmark, we next consider a second-type policy experiment to analyse how our stakeholder equilibrium would evolve with rising wealth inequality.

[10] Accordingly, egalitarian wage bargaining, which we argue is particularly appropriate to an incomplete markets setting, is fully consistent with its complete markets analogue.

[11] In the latter sense, the period 1970-2015 could have been regarded as the business-cycle episode of "Minsky cycles," with a more positive connotation, under our stakeholder equilibrium hypothesis. As comprehensively discussed in Eggertsson et al. (2019b), however, the 1970-2015 period is often regarded as the era of "Secular Stagnation" characterized by a persistent decline in interest rates resulting from a variety of slow-moving secular forces.

tortions of the stockholder's non-homothetic and the worker's buffer-stock-like savings behavior.[12,13] But even in this case, egalitarian bargaining retains the capacity to internalize the pecuniary externality through "socially efficient" mean level wage markups.[14]

Underlying these three results is a socially efficient long-term growth mechanism which we term the "Musk-Tesla effect": growth arises endogenously from the increasing centralization of capital ownership, isomorphic to an increasingly concentrated measure of hyper-productive "superstar-stockholder-managers," that allows the economy to achieve sustainable output growth without resorting to exogenous population or exogenous labor productivity growth, as is more typically assumed.[15,16] Accordingly, the economy exhibits a "pseudo-balanced growth path" which coincides with the (constrained) socially efficient allocation: as wealth inequality grows, output, investment, and capital stock all grow at the same rate while the equilibrium real rate of interest, often termed Wicksell's *natural rate of interest,* remains constant and the share of income to labor increases. To our knowledge, this "Musk-Tesla" effect is the only growth mechanism that reconciles the constant positive long-term rate of interest with the increasing concentration of capital, lending support to Piketty's (2014) perspective.

An outline of the paper can now be presented. Section 2 describes the model while Section 3 details our theoretical claim that the steady state time path of the decentralized economy under egalitarian wage bargaining coincides with its constrained efficient counterpart for all levels of wealth inequality. In Section 4, we explore the business cycle properties of the Baseline economy. Section 5 undertakes a

[12]Evidence for this buffer stock-savings-story, at least for workers, comes from the elasticity of worker flow consumption with respect to the present value of the egalitarian contract wage, a statistic often referred to as the worker's Marginal Propensity to Consume out of Permanent Income (MPCPI, hereafter). In this model the behavior of the worker's MPCPI confirms the countercyclical precautionary savings found in the data as an equilibrium outcome. What strikes us as surprising, however, is an MPCPI close to one observed in the "polarization trap" case even though workers' liquid wealth is significantly positive, in contrast to Deaton's (1991) hypothesis of an asymptotic MPCPI of one only as wealth declines to zero. What differentiates the model's result from Deaton's (1991) conjecture, is the general equilibrium effect of a "safe-asset shortage" on elevated unemployment fluctuations. The latter effect will become the focal point of Section 5.5.

[13]Stockholders, in contrast, behave as "non-homothetic savers" (Mian et al. (2021)), desiring to increase their bond holdings even as they own the larger fraction of them in equilibrium, despite the present model assuming that both agents have 'conventional' homothetic preferences. In a polarization trap, however, stockholders display their equilibrium "non-homothetic" saving pattern vis-à-vis safe bonds with negative returns. In response, workers retain their "homothetic" saving behavior, suggesting they will save less due to the negative rates. The latter observation can be viewed as a restatement of Deaton's high MPCPI result. See Section 5.5.

[14]It is somewhat ironic that growing wealth inequality, in conjunction with egalitarian wage bargaining and the partial internalization of the pecuniary externality it creates, leads to the private creation of a UBI, long a goal of social progressives. More striking is the fact that the primitive idea of 'wage markups' as an effective UBI in an economy with the increasing concentration of wealth in progressively fewer hands dates back to Jean Charles Léonard de Sismondi (1773-1842), a Swiss economist and historian, often depicted as a "neglected advocate" for worker property rights.

[15]This Musk-Tesla effect, at least symbolically, reflects Lenin's (1916) dystopian prediction and the recent empirical finding of the intensified concentration of productive capital for the past one hundred years. Lenin (1916) gathered census statistics in the early 1900s to reach his self-constituted indictment of the future of capitalism, summarized by his dicta: "*the enormous growth of industry and the remarkably rapid concentration of production. . . are one of the most characteristic features of capitalism...Concentration of production, however, is much more intense than the concentration of workers, since labor in the large enterprises is much more productive.*" Kwon et al. (2022) document that the concentration of corporate businesses in terms of the asset and sales share of the top enterprises in the US has been accelerating for the past 100 years. Related evidence also suggests that top firms may produce more with fewer workers, exhibiting "scale without mass."

[16]This effect also corresponds to Marx's (1867) "Marxian-centralization-driven growth," a rationale for his own dystopian prediction of the capitalist society's demise. Under the present stakeholder equilibrium, however, the Marxian centralization is seen to "metamorphose" into a constrained-efficient Musk-Tesla superstar growth regime.

series of hypothetical policy experiments to explore the effects of increasing wealth inequality on stakeholder economy dynamics. The interaction of the labor and financial markets as wealth inequality rises receives the bulk of the attention as they are intimately related in a stakeholder economy. Section 5.6 also undertakes the analogous discussion when the growth in wealth inequality is driven by technology innovations rather than a declining measure of stockholders, while in Section 5.7 we revisit the data in light of Section 5's conclusions. Section 6 concludes.

2 A Stakeholder Model

We consider a discrete-time infinite horizon economy with two distinct infinitely-lived agent types, "stockholders" and "non-stockholder-workers," with the latter type simply referred to as "workers" for economy of presentation.[17] These groups are uniformly distributed, respectively, on sets of positive Lebesgue measure, μ_s and μ_n, normalized to $\mu_n = 1$. Both supply labor in the model, and both possess GHH (Greenwood et al. (1988)) period utility, on which is imposed "catching up with the Joneses" (Abel, 1990) habit formation. The agent groups differ only in the magnitude of their habit parameters and their disutility of supplying hours. In the present setting, a larger habit parameter leads to the accumulation of greater wealth. Accordingly, we will refer to the habit parameters as precautionary savings parameters going forward. Differences in the precautionary savings and work disutility parameters are necessary for the model to display a reasonable wealth Gini coefficient.

2.1 Stockholders

A stockholder, endowed with one unit of time per period, supplies labor services to the (representative) firm and trades securities – both equity claims to the firm's net income stream, and a one-period default-free real bond (henceforth referred to simply as the "bond") issued by the firm. Being an owner of the firm, a stockholder is assumed to have a permanent relationship with it and to trade her labor services in an exclusive stockholder labor market. This market is characterized by employment adjusting only along the intensive margin; i.e., the labor income risk of a stockholder originates entirely from fluctuations in wages and hours worked. Given her information set Ω_0^s, the representative stockholder maximizes her lifetime expected utility as given by:

$$V^s(\Omega_0^s) = \max_{\{h_t^s, c_t^s, e_{t+1}^s, b_{t+1}^s\}} \mathbb{E}_0^s \sum_{t=0}^{\infty} \beta^t [u^s(c_t^s - \chi^s \mathbf{c}_{t-1}^s - H(h_t^s))] \tag{1}$$

$$\text{s.t. } c_t^s + p_t^e e_{t+1}^s + p_t^b b_{t+1}^s \leq w_t^s h_t^s + (p_t^e + d_t) e_t^s + p_t^b b_t^s \tag{2}$$

[17] Many of the model's features are entirely standard. The informed reader may wish to skim Sections 2.1-2.4 for notation only. The influence of financial market incompleteness on the outcome of wage bargaining is discussed beginning in Section 2.5.

where $u^s(\cdot)$ denotes her period utility function, c_t^s her period t consumption, h_t^s her period t labor hours, χ^s her habit parameter and $H(\cdot)$ her disutility of labor hours measured in units of consumption. The expression $\mathbf{c}_{t-1}^s$ represents the average consumption level across the entire stockholder group in the previous period:

$$\mathbf{c}_{t-1}^s \equiv \frac{1}{\mu_s} \int c_{t-1}^s d\varkappa,$$

with $\varkappa$ standing for the measure of stockholders. With d_t representing the period t per share dividend payment, e_t^s and b_t^s denote, respectively, the stockholder's period t stock and bond holdings with corresponding period t equilibrium prices p_t^e and p_t^b. Lastly, w_t^s is the stockholder's period t wage, while $\mathbb{E}_t^s \equiv \mathbb{E}(\cdot|\Omega_t^s)$ denotes her expectations operator conditional on her information set Ω_t^s. The parameter β is the economy-wide subjective discount factor, identical for all agents. Stockholders regard all prices, p_t^e, p_t^b and w_t^s, as exogenous. In all cases a $\tilde{}$ above a quantity identifies it as a random variable.[18]

Stockholders' GHH period utility is assumed to be of the form:

$$u^s(c_t^s - \chi^s \mathbf{c}_{t-1}^s - H(h_t^s)) = \frac{(c_t^s - \chi^s c_{t-1}^s - B_s (h_t^s)^{\psi})^{1-\gamma} - 1}{1-\gamma}. \tag{3}$$

While the parameters χ^s and B_s are unique to the stockholders, ψ, the labor supply parameter and γ, the coefficient of relative risk aversion, will be common to both agent types.[19]

2.2 Workers

Workers differ from stockholders in their investment opportunity sets and job opportunity sets. First, workers do not participate in the equity market, although they can freely trade the firm's default-free bonds. Second, workers trade their services exclusively in a separate labor market with two distinct characteristics: (1) variation in employment at both the extensive and intensive margins, and (2) firms and workers egalitarian wage bargain in a context of DMP (Diamond (1982), Mortensen (1992), and Pissarides (1990)) search and matching frictions. As we show, the outcome of this wage bargaining process is endogenously influenced by the asymmetric security trading opportunities and the consequent imperfect income insurance implicitly provided by stockholders.

Following Merz (1995), each worker is viewed as a large extended family containing a continuum of family members uniformly distributed on a set of Lebesgue measure one. Each worker-family consists of employed and unemployed workers, who pool their financial and labor incomes (perfect risk sharing within the family) before choosing per-capita consumption and asset holdings.[20] The latter ob-

[18] The same quantity without the $\tilde{}$ is to be interpreted as some fixed realization thereof.

[19] First order conditions for problem (1)-(2) and all other problem formulations in the present paper can be found in Part B of the Appendix.

[20] In the Appendix, we demonstrate a construction of the representative family that guarantees perfect within-family risk sharing, thereby implying formulation (4) to follow.

servation implies that the representative non-stockholder family, given its information set Ω_0^n, solves[21]:

$$V^n\left(\Omega_0^n\right) = \max_{\left\{c_t^n, h_t^n, c_t^{n,e}, c_t^{n,u}, b_{t+1}^n\right\}} \mathbb{E}_0^n \sum_{t=0}^{\infty} \beta^t \left(u^n\left(\widetilde{c}_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L\left(\widetilde{h}_t^n\right)\right)\right) \quad (4)$$

$$\text{s.t. } c_t^n = n_t c_t^{n,e} + (1-n_t)\, c_t^{n,u}, \quad (5)$$

$$n_t c_t^{n,e} + (1-n_t)\, c_t^{n,u} + p_t^b b_{t+1}^n \leq w_t^n h_t^n n_t + b\,(1-n_t) + b_t^n - T_t, \text{ and} \quad (6)$$

$$n_{t+1} \leq (1-\rho)\, n_t + s_t\,(1-n_t)\,. \quad (7)$$

In the above problem, $u^n(\cdot)$ denotes a representative worker's period utility function, $c_t^{n,e}$ and $c_t^{n,u}$, respectively, his period t consumption when employed and when unemployed, c_t^n, the total consumption of his family, $L(\cdot)$ his disutility of labor function measured in consumption units, χ^n his precautionary parameter, and h_t^n his period t labor hours supplied when employed. The expression b_t^n denotes the family's period t bond holdings; w_t^n is the worker's wage determined through the bargaining process, while b represents unemployment benefits and T_t is the lump sum tax levied on workers by the government to finance these benefits. Accordingly, equation (6) is the representative worker-family's budget constraint. The n_t term represents the measure of workers actually at work in period t while $\mathbb{E}_t^n \equiv \mathbb{E}\left(\cdot \mid \Omega_t^n\right)$ is the expectations operator conditional on their information set Ω_t^n. Equation (7) describes the evolution of the fraction of workers who are employed as a function of the exogenous separation rate ρ and s_t, the period-t fraction of unemployed workers matched to the firm. Both quantities are assumed exogenous to the worker family. As with stockholders, workers also take all prices as exogenous to their decision problem.

Note that workers' hours are supplied under the condition that the (hourly) wage equals the marginal rate of substitution of consumption for leisure, and that $\mathbf{c}_{t-1}^n = \int c_{t-1}^n d\omega$, analogous to the stockholder's counterpart, is average worker consumption in the previous period with ω the measure of workers. The representative worker's period utility is specialized to the form

$$u^n(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n)) = \frac{(c_t^n - \chi^n c_{t-1}^n - n_t B_n (h_t^n)^{\psi})^{1-\gamma} - 1}{1-\gamma}. \quad (8)$$

While the parameters χ^n and B_n are unique to the workers, ψ and γ are the same as for stockholders. We next describe the functioning of the labor market and its wage determination process.

[21] A more "structural" form of the contemporaneous utility is to introduce search effort per worker seeking employment: $u^n(c_t^n - n_t L(h_t^n) - (1-n_t)L(e_t))$, where e_t is period t search effort. However, empirical studies show that time devoted to search effort is modest. Krueger and Mueller (2011) estimate, for example, that formal search activities typically consume less than 10% of an unemployed person's week days (8 hours). Therefore, without loss of too much generality, we simplify and assume that $L(e_t) = L(0) = 0$.

2.3 Search in the labor market for non-stockholder-workers

There is one infinitely lived representative firm that behaves competitively. The firm hires n_t workers, and posts v_t vacancies in order to attract new workers for its period $t+1$ production. The total number of unemployed workers who search for a job in period t is $u_t \equiv 1 - n_t$. Following basic DMP search theory, we postulate a standard matching technology in the labor market for non-stockholder-workers,

$$m(\nu_t, 1-n_t) = \sigma_m \nu_t^{\sigma}(1-n_t)^{1-\sigma},$$

with $m_t \equiv m(\nu_t, 1-n_t)$ representing "matches," the number of newly hired workers. The exponents σ and $(1-\sigma)$ describe, respectively, the elasticity of matches with respect to vacancies and unemployment, while σ_m is a scale parameter. Accordingly, the probability that the firm fills a vacancy in period t, q_t, is given by $q_t = {}^{m(\nu_t, 1-n_t)}/_{v_t} = {}^{m_t}/_{v_t}$, while the probability that an unemployed worker finds a job in period t, s_t, is given by $s_t = {}^{m(\nu_t,1-n_t)}/_{1-n_t} = {}^{m_t}/_{u_t}$. Labor market tightness, θ_t, is measured by $\theta_t = v_t/u_t$. Both quantities, q_t and s_t, are assumed exogenous from the perspectives of both the firm and an individual worker. Equation (7) can thus also be written as $n_{t+1} = (1-\rho)\, n_t + m_t$.

2.4 The Firm

Each period, the firm produces output, y_t, according to the following aggregate production function:

$$y_t = f(k_t, \mu_s h_t^s, n_t h_t^n) z_t = z_t A k_t^{\alpha}((\mu_s h_t^s)^{\mu}(h_t^n n_t)^{1-\mu})^{1-\alpha} \tag{9}$$

where z_t, k_t, $\mu_s h_t^s$, and $n_t h_t^n$ denote, in sequence, the period t aggregate productivity shock, the capital stock, aggregate labor (hours) supplied by the stockholders, and aggregate labor hours supplied by those workers actually working. The exponents $\mu = {}^{\mu_s}/_{1+\mu_s}$ and $(1-\mu)$ represent, respectively, the normalized measures of stockholders and workers, while A is a scale parameter. The technology shock process, $\{\tilde{z}_t\}$, follows the standard autoregressive form:

$$\log \tilde{z}_{t+1} = 0.95 \log \tilde{z}_t + \tilde{\varepsilon}_{t+1}, \text{ with } \{\tilde{\varepsilon}_t\} \text{ i.i.d. distributed } N(0, \sigma_{\varepsilon}^2).$$

The firm owns the (physical) capital stock, k_t, and employs n_t workers in period t. A fraction δ of the capital depreciates during production each period, while being supplemented by new investment i_t. Similarly, a fraction ρ of the existing workforce leaves the firm, which then recruits m_t new hires through the frictional labor market. The latter is characterized by the hiring rate, x_t, defined as the ratio of new hires $q_t v_t$ to the existing pool of employed workers: i.e., $x_t \equiv \frac{q_t v_t}{n_t} = \frac{\text{new hires}}{\text{existing work force}}$.

One perfectly divisible equity share, price p_t^e, and one-period default-free bonds which it issues at the price p_t^b represent the firm's capital structure. The total supply of corporate bonds is assumed constant over time and equals a fraction φ of the steady state capital stock owned by the firm. In each period, the firm makes net interest payments $\left(\varphi\bar{k} - p_t^b\varphi\bar{k}\right)$ to bondholders where the $\bar{}$ above k denotes its certainty steady state value.

Given our emphasis on multi-period egalitarian bargaining wage contracts and debt contracts in the firm's capital structure, we presume this single firm lives forever. The firm's decision problem is to maximize its pre-dividend stock market value $d_t + p_t^e$ on a period-by-period basis given its information set $\Omega_t^f = \Omega^f\left(k_t, z_t, q_t, n_t\right)$, while simultaneously honoring its commitment to the employed workers in accordance with the terms of its multi-period bargaining contract:

$$\max_{\{i_t, h_t^s, x_t\}} V_t^f \equiv d_t + p_t^e \equiv d_t + \mathbb{E}_t(\tilde{M}_{t,t+1}^s(\tilde{p}_{t+1}^e + \tilde{d}_{t+1})|\Omega_t^f) \tag{10}$$

$$\text{s.t.}\quad d_t \equiv f(k_t, \mu_s h_t^s, n_t h_t^n)z_t - i_t - \mu_s w_t^s h_t^s - w_t^n h_t^n n_t - \frac{\kappa}{2}x_t^2 n_t - \varphi\bar{k} + p_t^b\varphi\bar{k} \tag{11}$$

$$k_{t+1} = (1-\delta)k_t + G(\frac{i_t}{k_t})k_t \tag{12}$$

$$n_{t+1} = (1-\rho)\,n_t + x_t n_t, \tag{13}$$

[22]

where $\{\widetilde{M}_{t,t+1}^s\}$ denotes the stockholder's intertemporal marginal rate of substitution (IMRS), and represents the economy-wide stochastic discount factor (SDF) for all valuation purposes.

Following Christiano et al. (2008) and Gertler and Trigari (2009), the term $\frac{\kappa}{2}x_t^2 n_t$ represents a (quadratic) adjustment cost to altering the employment level of workers within the firm, where κ is a vacancy cost parameter.[23] In constraint (12), the term $G(\frac{i_t}{k_t})k_t$ is a postulated cost of adjusting the firm's capital stock from its current level.[24] We adopt the standard specification of Jermann (1998) and Kaltenbrunner and Lochstoer (2010):

$$G(\frac{i_t}{k_t}) = \frac{a_1}{1-\frac{1}{\xi}}(\frac{i_t}{k_t})^{1-\frac{1}{\xi}} + a_2,$$

where parameters a_1 and a_2 are chosen so that $G(\delta) = \delta$, and $G_1(\delta) = 1$.

In the next section we describe what egalitarian bargaining entails.

[22]Note that in problem (10) to choose the hiring rate x_t is to choose the number of vacancies v_t.

[23]The quadratic labor adjustment cost specification, with the parameter κ having the interpretation of a vacancy cost, is employed to guarantee that the wage bargaining is independent of the employment size of the firm (the firm's scale). See Gertler and Trigari (2009).

[24] With these identifications, the elasticity parameter $\xi \equiv -\frac{1}{G_{11}(\delta)\delta} > 0$ is independent of the determination of the model's steady-state equilibrium; i.e. the steady state is not affected by the positive value ξ; $\xi = \infty$ corresponds to the case of no adjustment costs. Without this feature, it is impossible for models of this type to replicate observed non-trivial equity risk premia. Increasing endogenous risk premia will play a key role in creating "shortages" of safe assets (default-free bonds) in scenarios of high wealth inequality. This interaction will become the focal point of Section 5.

3 Characterizing the egalitarian wage bargaining problem

Under egalitarian wage bargaining, the aggregate match surplus when a worker is hired is allocated so that the additional lifetime welfare benefits to each party are equated. To operationalize this concept, the firm's matching surplus and the non-stockholder-worker employment and unemployment values must first be defined in terms of current consumption and then transformed into the corresponding marginal welfare benefits to stockholders and worker-families using their respective marginal utility expressions. We first measure the matching surplus in consumption terms to each agent.

3.1 Firm's shadow value of hiring one additional worker

The "job value" to the firm of hiring one additional worker, J_t, is first specified. As is customary in the DMP literature, J_t is the present value, using an appropriate discount rate, of the flow benefit that the firm gains from an added worker, measured as of the time t when the worker begins his job. It is directly derived from the envelope condition, ${}^{\partial V_t^f}/{}_{\partial n_t}$, of the firm value V_t^f in formulation (10): that is,

$$J_t = \frac{\partial V_t^f}{\partial n_t} \tag{14}$$
$$\text{s.t. } n_{t+1} = (1-\rho)n_t + x_t n_t = (1-\rho+x_t)n_t.$$

A restatement of envelope condition (14) leads to the firm's optimal hiring decision problem in a DMP environment as satisfying:

$$J_t = \max_{\{x_t\}} \left[h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t - w_t^n h_t^n - \frac{\kappa}{2} x_t^2 + (1-\rho+x_t)\,\mathbb{E}_t\left[\tilde{M}_{t+1}^s \widetilde{J}_{t+1}\right]\right] \tag{15}$$

where $h_t^n f_3(k_t, \mu_s h_t^s, n_t h_t^n) z_t$ represents the worker's flow labor productivity. In a nutshell, the firm faces the Bellman equation vis-à -vis the *job value*, J_t, to determine its *optimal* hiring rate (equivalently, its job vacancy postings). Moreover, the solution to formulation (15) is identical to that of the equation defining the firm's optimal hiring decision for workers, equation (49) in the Appendix, suggesting that the firm's job value (matching surplus to the firm) in a DMP environment is consistent with the investment plan of the firm as an institution participating in the capital market.

3.2 Non-stockholder-worker's shadow value of employment

The period t present discounted value to a non-stockholder-worker of employment in terms of period t consumption, denoted by EP_t, is defined recursively by:

$$EP_t = w_t^n h_t^n + (1-\rho)\beta\mathbb{E}_t\widetilde{\Lambda}_{t,t+1}^n\widetilde{EP}_{t+1} + \rho\beta\mathbb{E}_t\widetilde{\Lambda}_{t,t+1}^n\widetilde{U}_{t+1}$$

where $\beta\Lambda^n_{t,t+1} \equiv \beta\lambda^n_{t+1}/\lambda^n_t$ is his IMRS, and U_t denotes his present discounted value of unemployment in terms of current consumption in period t. It is recursively defined by the corresponding relationship:

$$U_t = L(h^n_t) + b + s_t\beta\mathbb{E}_t\widetilde{\Lambda}^n_{t,t+1}\widetilde{EP}_{t+1} + (1-s_t)\beta\mathbb{E}_t\widetilde{\Lambda}^n_{t,t+1}\widetilde{U}_{t+1}.$$

Here, the value of being unemployed is the sum of the worker's current disutility of supplying hours, $L(h^n_t)$, his unemployment benefit b, and the discounted values of being employed or unemployed next period, weighted by their relative likelihoods where an unemployed workers has a probability s_t of finding a new job. Each of these quantities is foregone when the unemployed worker accepts employment and each is measured in terms of final goods consumption. Accordingly, the workers' matching shadow value of being hired is then defined as the difference $EP_t - U_t$.

3.3 Distribution risk

In equilibrium, the extent of (partial) risk sharing that results from stockholders and workers interacting in the bond market will influence the outcome of the egalitarian wage bargaining process and will in turn be affected by it. To measure the aggregate effect, the ratio between the stockholder's and worker's marginal utilities, the expression

$$\phi_t \equiv \frac{u^s_1(c^s_t - \chi^s\mathbf{c}^s_{t-1} - H(h^s_t))}{u^n_1(c^n_t - \chi^n\mathbf{c}^n_{t-1} - n_tL(h^n_t))} = \frac{\lambda^s_t}{\lambda^n_t}, \tag{16}$$

is introduced to characterize the extent of risk-sharing between these two groups. Going forward, $\{\widetilde{\phi}_t\}$ will be our "distribution risk" measure. If $\{\widetilde{\phi}_t\}$ is constant across time and across all states, relation (16) coincides with the efficient risk-sharing condition.[25] Alternatively, suppose that $\{\widetilde{\phi}_t\}$ is constant across period t states for each t, but is time-varying. In this event, a larger ϕ_t is evidence of a greater share of aggregate income to workers, while a smaller ϕ_t suggests a greater share to stockholders. Suppose, in addition, that ϕ_t were to be time-varying *and* countercyclical over the business cycle. This countercyclical behavior means that when a high-productivity state is realized, a smaller ϕ_t ensues and stockholders reap most of the benefits from that high productivity state; alternatively, when a low-productivity state is realized, a greater share of aggregate income goes to the workers; i.e., the normally low payment to stockholders is further reduced by labor's priority claim on output. Accordingly, the countercyclical nature of $\{\widetilde{\phi}_t\}$, if verified, captures the idea that the shares of income going to labor and capital are not equally risky and that stockholders, via the institution of the firm, partially insure their workers. "Distribution Risk" (variation in $\{\widetilde{\phi}_t\}$) is thus largely borne by owners of the firm and, as such, may be viewed as a gesture by stockholders to stakeholder capitalism. It is assumed to be

[25]The "Efficicient Fluctuations" scenario of Section 6 illustrates this feature.

uninsurable. In what follows it will be convenient to use $\sigma_{\widetilde{\phi}}$ as a measure of the degree of effective market incompleteness. It is also a measure of the extent of the residual pecuniary externality after bond trading and wage asset creation have occured.

No *a priori* assumption concerning either the source or cyclicality of distribution risk is made; rather, distribution risk (defined as per (16)) is generated entirely endogenously. In the present model , however, it will prove to be *countercyclical* over the business cycle, a feature not totally unexpected in a context where stockholders partially insure workers: stockholders' consumption is thus more risky, falling more than proportionately in recessions, causing $u_1(c_t^s - \chi^s \mathbf{c}_{t-1}^s - H(h_t^s))$ to increase relative to $u_1(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t L(h_t^n))$ and thus ϕ_t to increase, with the opposite for expansions. Proposition 3.2, to follow, will identify countercyclical distribution risk, $\{\widetilde{\phi}_t\}$, as influencing the assignment of property rights between workers and stockholders, namely, by strengthening worker propery rights in recessions and diminishing them in expansions, leading to diminished worker income variation and thus greater stockholder consumption volatility.[26] Accordingly, countercyclical variation in $\{\widetilde{\phi}_t\}$ can also be thought of as measuring the "cost of inequality" to the firm since it increases the firm's capital income risk and thus shareholder consumption risk.

3.4 Egalitarian wage bargaining

With no agency problem between firm owners and managers, we may identify the firm's matching surplus with the marginal benefit to the representative stockholder of adding one additional worker. Accordingly, the firm's matching surplus accruing to stockholders in welfare terms, denoted by $V_{n_t}^s$, can be formulated as:

$$V_{n_t}^s \equiv \frac{\partial V_t^s}{\partial n_t} = \lambda_t^s J_t \tag{17}$$

where $V_t^s \equiv V^s(\Omega_t^s)$ denotes the value function of a representative stockholder.

Similarly, a worker's matching surplus in welfare terms, $V_{n_t}^n$, can be readily identified with the marginal welfare benefit (to the worker-family) of one additional worker being hired:

$$V_{n_t}^n \equiv \frac{\partial V_t^n}{\partial n_t} = \lambda_t^n \left(EP_t - U_t \right). \tag{18}$$

Identifying each matching surplus with its marginal welfare benefit to the corresponding agent is reasonable in a situation where two heterogeneous agents with potentially different attitudes toward risk bargain over the wage. Indeed, the existing game theory literature holds that the division of the joint matching surplus can be significantly affected by heterogeneity in the agents' risk aversion coefficients. With these observations in mind, KRN egalitarian wage bargaining postulates that the division of the

[26]Empirically, the labor income share is much less risky than the share going to capital; labor's claim on output is largely fixed and negotiated prior to the actual realization of the output.

surplus (in welfare terms) is renegotiated every period to apply to all workers, just hired or not, so as to satisfy:

$$\eta \left(V_t^s\left(\Omega_t^s\right) - \bar{V}_t^s\right) = (1-\eta)\left(V_t^n\left(\Omega_t^n\right) - \bar{V}_t^n\right), \tag{19}$$

where $\bar{V}_t^n$ and $\bar{V}_t^s$, respectively, denote time-varying but non-stochastic disagreement points and η an exogenously given bargaining parameter.[27] Equation (19) illustrates the "Equal Gains Principle" in welfare terms and takes into account that in each period, workers' working hours are determined competitively according to the following condition:

$$MRS_{c,l}^n = w_t^n \tag{20}$$

where $MRS_{c,l}^n$ represents the worker's marginal rate of substitution for leisure vs. consumption.

To further clarify the KRN egalitarian wage bargaining concept, we note the distinction between the bargained flow wage, $\{\widetilde{w}_t^n\}$, and its present value counterpart $\{\widetilde{W}_t^{\mathrm{pv}}\}$. Andolfatto (1996), Shimer (2004, 2005), Hall (2017) and the literature following directly from these studies model bargaining as occurring over the present value of the wage flow to workers, assuming a long-term contract, and egalitarian bargaining follows suit. Accordingly, W_t^{pv}, the present value of the flow of KRN bargained wages, $\{\widetilde{w}_t^n\}$, is defined by:

$$\begin{aligned} W_t^{\mathrm{pv}} &= w_t^n + \mathbb{E}_t\left[\sum_{j=1}^{\infty}(1-\rho)^j\left(\prod_{k=1}^{j}\cdot\widetilde{M}_{t,t+k}^s\right)\widetilde{w}_{t+j}^n\right] \\ &= w_t^n + (1-\rho)\cdot\mathbb{E}_t\left[\widetilde{M}_{t,t+1}^s\widetilde{W}_{t+1}^{\mathrm{pv}}\right]. \end{aligned}$$

In all studies of wage bargaining under complete financial markets, $\{\widetilde{W}_t^{\mathrm{pv}}\}$ is the relevant quantity for employment allocations (the pattern of $\{\widetilde{n}_t\}$), while the associated flow wage stream ($\{\widetilde{w}_t^n\}$) is largely irrelevant: with complete markets, workers can transform W_t^{pv} into any preferred flow wage stream by trading in the financial market.[28] In the present paper, where financial markets are not complete, although W_t^{pv} fully maintains its allocative role, the associated flow wage stream becomes a powerful determinant of the worker's ultimate consumption pattern, $\{\widetilde{c}_t^n\}$, and welfare.

From the firm's perspective, the present value of the present and future output per worker, X_t, as defined by

[27] Egalitarian wage bargaining modifies the axiom of scale invariance which fails to apply when agent utilities differ, in the following way. Under Nash bargaining, if the surplus doubles, the allocation to both agents doubles (scale invariance). Under egalitarian bargaining the added surplus is allocated so that the additional welfare benefits to each agent are equated.

[28] The same comment applies to the dividend stream: stockholders can transform $\{\widetilde{d}_t\}$ into any desired capital income stream in a complete finanical market setting. Thus, its precise form is arbitrary except its present value.

$$X_t = \left(\frac{y_t}{n_t}\right) + \mathbb{E}_t\left[\sum_{j=1}^{\infty}(1-\rho)^j \cdot \left(\prod_{k=1}^{j}\widetilde{M}^s_{t,t+k}\right)\left(\frac{\widetilde{y}_{t+j}}{\widetilde{n}_{t+j}}\right)\right]$$
$$= \left(\frac{y_t}{n_t}\right) + (1-\rho)\cdot\mathbb{E}_t\left[\widetilde{M}^s_{t,t+1}\widetilde{X}_{t+1}\right],$$

is the relevant quantity where the $(1-\rho)$ factor accounts for separations. The present value X_t essentially measures the value to the firm of a new hire's human capital.[29]

We conclude this section by detailing how the flow wage pattern embodies the variable 'property right' $\{\widetilde{\phi}_t\}$ arising as an outcome of egalitarian bargaining. This is accomplished through a series of propositions. All proofs are found in the Technical Appendix.

Proposition 3.1. *KRN egalitarian bargaining generalizes the standard DMP present-value Nash bargaining to accommodate a time varying sharing parameter: i.e.*

$$\frac{EP_t - U_t}{\eta_t} = \frac{J_t}{(1-\eta_t)}, \tag{21}$$

where the joint match value, S_t, is given by

$$S_t = EP_t - U_t + J_t.$$

Corollary 3.1. *KRN egalitarian wage bargaining is a form of present value bargaining.*

Corollary 3.2. *When financial markets are complete, egalitarian and present value Nash bargaining coincide since $\phi_t \equiv 1$ for all t.*

We next explore the implications of Proposition 3.1 for the period-by-period flow wage $\{\widetilde{w}^n_t\}$. It will be useful to define the distribution of worker property rights as the (time-varying) effective sharing rule, $\{\widetilde{\eta}_t\}$, where

$$\widetilde{\eta}_t = \frac{\eta}{(1-\eta)\frac{1}{\widetilde{\phi}_t} + \eta}. \tag{22}$$

In (22), η is the egalitarian bargaining parameter in (19) and $\{\widetilde{\phi}_t\}$ represents endogenous distribution risk as per (16).[30] Proposition 3.2 follows.

[29] By production function (9) the present value of output per worker is a constant fraction of the marginal output contribution of a newly hired worker. We use the former measurement rather than the perhaps-more-consistent latter one to conform better to the literature and to potential data sources.

[30] The variable sharing rule guarantees endogenous factor share variation, something that is generally believed to be important for successful asset pricing. See Lansing (2015) and Favilukis and Lin (2015). Drautzburg et al. (2017) generate factor share variation by placing an exogenous process on the Nash bargaining parameter calibrated to reflect policy changes.

Proposition 3.2. *Under the assumptions of Section 2.1-2.4, and the requirement that wages in all matches, new or existing, are renegotiated every period following the aggregate shock, egalitarian wage bargaining implies that the distribution of Coasian bilateral property rights is time varying as described by (22). Furthermore, the worker's per period wage bill is given by*

$$w_t^n h_t^n = (1-\eta_t)\zeta_t + \eta_t \pi_{n_t} \tag{23}$$

where ζ_t and π_{n_t}, respectively, represent an employed worker's dynamic reservation value and the match benefit to the firm of one marginally added worker. These quantities are made explicit below:

$$\zeta_t = b + (c_t^{n,e} - c_t^{n,u}) - \chi^n(\mathbf{c}_{t-1}^{n,e} - \mathbf{c}_{t-1}^{n,u}) = b + (L(h_t^n) - L(0))$$

and

$$\pi_{n_t} = \left[h_t^n f_3(k_t, \mu_s h_t^s, h_t^n n_t) z_t + \frac{\kappa}{2} x_t^2 + \kappa x_t s_t\right]$$

.

Representation (22) makes apparent the role of financial market incompleteness in generating variable distribution risk and thus a variable property right manifest as the time-varying sharing rule, $\{\widetilde{\eta}_t\}$. Up to a first-order approximation, relation (22) implies

$$\widehat{\widetilde{\eta}}_t = (\text{positive constant}) \cdot \widehat{\widetilde{\phi}}_t.\text{[31]} \tag{24}$$

Since distribution risk will prove to be countercyclical in this model, the strength of the property right will be as well. It is by this mechanism that the pecuniary externality is partially internalized. It also causes the firm's SDF to be countercyclical, a feature that substantially affects the volatility of investment and vacancy postings, since both decisions (investment in physical vs. human capital) are evaluated on the same discounted present value basis.[32] In this way the "pecuniary externality" manifests itself throughout the entire economy. We are now in a position to define equilibrium.

3.5 Stakeholder Equilibrium

Market clearing in the stakeholder economy requires that for all t,

$$e_t = \int e_t^s d\varkappa = 1,$$

[31] A $\wedge$ on a variable denotes log deviations from the corresponding steady-state value. The latter values are distinguished by a — above them.

[32] Low relative stockholder consumption in low-output states signals a high future expected growth rate in stockholder consumption and thus a high default-free rate with the reverse (low rates) being true in high-output states.

$$\phi\bar{k} = \int b_t^s d\varkappa + \int b_t^n d\omega,$$

$$c_t = \int c_t^s d\varkappa + \int c_t^n d\omega,$$

$$y_t = c_t + i_t + \frac{\kappa}{2} {x_t}^2 n_t,$$

Lump sum employment taxes are levied on workers to balance the government's budget constraint:

$$T_t + (1 - n_t)b = 0.$$

Internal consistency also requires that $\mathbf{c}_t^n = c_t^n$ and $\mathbf{c}_t^s = c_t^s$ for all t. Equilibrium is then defined as follows:

Definition 3.1 (Stakeholder Equilibrium). *Under the above market-clearing conditions, a decentralized stationary recursive stakeholder equilibrium is defined as a set of decision rules* $\{c_t^s(\cdot), c_t^n(\cdot), h_t^s(\cdot), h_t^n(\cdot), e_{t+1}^s(\cdot), i_t(\cdot), b_{t+1}^s(\cdot), b_{t+1}^n(\cdot), v_t(\cdot)\}$*, and a set of wage and price functions* $\{w_t^s(\cdot), w_t^n(\cdot), p_t^e(\cdot), p_t^b(\cdot)\}$ *given the information set of aggregate states* $\Omega_t = \{k_t,\ n_t,\ b_t^n,\ z_t\}$ *such that (i)* $\{c_t^s(\cdot), h_t^s(\cdot), e_{t+1}(\cdot), b_{t+1}^s(\cdot)\}$ *solve the stockholder's intertemporal problem (1) given the information set* $\Omega_t^s = \{e_t,\ b_t^s,\ w_t^s,\ p_t^e,\ p_t^b,\ c_{t-1}^s\}$*, (ii)* $\{c_t^n(\cdot), h_t^n(\cdot), b_{t+1}^n\}$ *solve the non-stockholder-worker family's intertemporal problem (4) given its information set* $\Omega_t^n = \{b_t^n,\ w_t^n,\ p_t^b,\ s_t,\ c_{t-1}^n\}$*, (iii)* $\{w_t^n(\cdot)\}$ *satisfies the wage bargaining condition (21), (iv)* $\{i_t(\cdot), x_t(\cdot)\}$ *solve the firm's intertemporal problem (10) given the information set* Ω_t^f $\{k_t,\ z_t,\ q_t,\ n_t\}$*, (v)* $\{p_t^e(\cdot), d_t(\cdot)\}$ *satisfy the Lucas (1978a) asset pricing equation, i.e.,* $p_t^e = \mathbb{E}_t[\widetilde{M}_{t,t+1}^s(p_{t+1}^e + d_{t+1})]$*, while* $\{p_t^b(\cdot)\}$ *satisfies equations* $p_t^b = \mathbb{E}_t[\beta\widetilde{\Lambda}_{t,t+1}^i]$ *(*$i = s$ *or* n*), and (vi) the economy follows the two laws of motion, (12) and (13). Rational expectations are assumed for all agents.*

Despite the assumed financial market incompleteness, a restricted sense of optimality holds in the long run.

Definition 3.2. *A Constrained Efficient Equilibrium for the present model context is defined as the solution to the following central planning formulation:*

$$\max_{\{c_t^s, c_t^n, h_t^s, h_t^n, k_{t+1}, n_{t+1}, i_t, v_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mu_s \cdot \tau^s \cdot u(c_t^s - \chi^s \mathbf{c}_{t-1}^s - H(h_t^s)) + \tau^n \cdot u(c_t^n - \chi^n \mathbf{c}_{t-1}^n - n_t H(h_t^n))\right] \quad (25)$$

subject to

$$\mu_s c_t^s + c_t^n + \frac{\kappa}{2}\left(\frac{q_t v_t}{n_t}\right)^2 n_t + i_t = f(k_t, \mu_s h_t^s, n_t h_t^n) z_t$$

$$k_{t+1} = (1-\delta)k_t + G(\frac{i_t}{k_t})k_t$$

$$m_t = m(v_t, 1 - n_t) = \sigma_m (v_t)^{1-\sigma} (1 - n_t)^{\sigma}$$
$$n_{t+1} = (1 - \sigma) n_t + m(v_t, 1 - n_t)$$
$$x_t = \frac{q_t v_t}{n_t} = \frac{m_t}{n_t}$$

and $\overline{W}_0^s$ and $\overline{W}_0^n$, respectively, coincide with their decentralized counterparts; i.e., stockholders receive all equity income.

The following propositions make clear how the stakeholder and constrained efficient economies differ.

Proposition 3.3. *(a "Coasian result") Under the choice of functional forms (3), (8) and (9) and for the parameterizations to be detailed in Section 4, the decentralized model economy achieves the constrained-efficient steady-state capital stock, investment, individual agent consumption, employment and hiring rate provided*

a.
$$\overline{\eta} = \frac{\eta}{(1-\eta)(1/\overline{\phi}) + \eta}, \quad \text{where} \quad \overline{\phi} = \frac{u_1^s(\overline{c}^s - \chi^s \overline{\mathbf{c}}^s - H(\overline{h}^s))}{u_1^n(\overline{c}^n - \chi^n \overline{\mathbf{c}}^n - \overline{n} L(\overline{h}^n))}, \quad \text{and}$$

b. the Negishi weight τ^s, τ^n are chosen to satisfy $\frac{\tau^n}{\tau^s} = \overline{\phi}$.

c. $(1 - \overline{\eta})$ equals the elasticity of the matching technology with respect to vacancies, $(1 - \sigma)$; i.e.,

$$\sigma = \overline{\eta}.$$

We refer to requirement (c) of Proposition 3.3 as a "revisionist Hosios (1990) condition" appropriate to egalitarian wage bargaining. It is of note that despite the presence of the pecuniary externality, there are no steady state distortions in any macro variables and thus no distortion in steady state welfare in the present model, in contrast to the wide class of models detailed in Dávila et al. (2012).[33,34] Corollary 3.3 ties Proposition 3.3 back to egalitarian wage bargaining.

Corollary 3.3. *The steady-state egalitarian bargained wage is efficient; that is, it is consistent with a constrained efficient allocation of resources in the steady state.*

[33] This fact in turn suggests that the welfare losses of introducing uncertainty into the model relative to steady state welfare will be modest since the technology shock standard deviation we impose, σ_ε, is small relative to other heterogeneous agent models.

[34] Dávila et al. (2012) explore the efficiency of competitive equilibrium in a related framework, but with no security trading. Capital accumulation via precautionary savings is thus the only mechanism for consumption smoothing. Heterogeneous wealth levels are provoked via idiosyncratic human capital endowment shocks while wages are competitively determined. Inefficient under- or over-accumulation of capital can result. In the present context of aggregate shocks, egalitarian present value wage bargaining under commitment overcomes the pecuniary externality arising from limited financial participation to achieve the long-run first-best steady-state capital stock. Andolfatto (1996) shows that any decentralized economy with DMP labor market frictions is first best conditional on its matching technology. Financial markets are complete in Andolfatto's (1996) formulation, however.

Proposition 3.3 and its corollary illustrate the Coase Theorem: egalitarian wage bargaining goes a long way to resolving the lack of insurance opportunities that would otherwise be available to workers in a complete markets setting. As Proposition 3.4 suggests, resolution is not perfect, however, off the steady state:

Proposition 3.4. *Let $H_t = (\mu_s h_t^s)^\mu \left(h_t^n n_t\right)^{1-\mu}$ represent the aggregate labor input for the economy of Section 2. Then, in response to business cycle variation, the aggregate marginal products of capital and labor, respectively,*

$$\alpha \frac{y_t}{k_t} \text{ and } (1-\alpha)\frac{y_t}{H_t}$$

do not support an efficient allocation away from the steady state in the stakeholder economy.

Equivalently, business cycle deviation from the steady state are not constrained-efficient, as noted in the introduction. In the remainder of this paper we explore both the nature of stockholder-worker risk sharing off the steady state, where the externality is not fully internalized, and its changing nature as wealth inequality grows.

4 Stakeholder Model Performance

4.1 The Baseline Scenario

In the remaining sections of the paper, we solve for the dynamic stakeholder equilibrium and study its properties under a variety of scenarios with the present section devoted to the Baseline case. We do this assuming the entirely standard production and utility functions introduced earlier, while believing that changes in labor market arrangements of the type we propose should not alter fundamental underlying technology or preferences. Model generated data are compared with data characterizing the period 1970-2015 for the US economy. We do not claim that this data period is one where egalitarian bargaining was in effect but undertake the comparison to see if model performance is reasonable. As such we model a "parallel economy" and ask what it would have "looked like" if stakeholder considerations of the type we consider had been in effect during the 1970-2015 period. Our goal is thus not to match the data but to uncover the potential degree of stakeholder externalities from it.

4.2 Baseline Calibration

The assumed utility and production function forms were detailed earlier ((3), (8), and (9)), while all exogenous parameter values and their empirical justifications are found in Table 1. The remaining parameters (χ^n, χ^s, B^n, and B^s) are determined endogenously within the model in order to generate realistic stationary wealth distributions and labor market quantities.

Table 1: **Exogenous Parameters**

	Parameter	Value	Attribution
α	(production parameter)	$\alpha = .36$	Cooley and Prescott (1992); commonplace[(i)]
μ_s	(stock market participation rate)	$\mu_s = .11$	Lansing (2015)
φ	(leverage parameter)	$\varphi = .4$	proposed in Kandel and Stambaugh (1992); Rouwenhorst (1995) [(ii)]
ξ	(capital cost-of-adjustment parameter)	$\xi = .45$	Guvenen (2009)
δ	(quarterly capital depreciation rate)	$\delta = .025$	Kydland and Prescott (1982); Kaltenbrunner and Lochstoer (2010)
γ	(coefficient of relative risk aversion)	$\gamma = 2$	commonplace [(iii)]
η	(Nash bargaining parameter)	$\eta = .5$	commonplace [(iv)]
ψ	(Frisch labor supply parameter)	$\psi = 1.4$	Jaimovich and Rebelo (2009) [(v)]
A	(production function coefficient)	$A = 1.25$	arbitrary; pure scale parameter
ρ	(quarterly separation rate)	$\rho = .10$	Davis, Haltiwanger and Schuh (1996)[(vi)]
σ	(match elasticity)	$\sigma = \bar{\eta}$	The Hosio condition (See Table 2)
b	(unemployment benefit)	$b = 0$	see extensive footnote [(vii)]
β	(economy-wide quarterly subjective discount factor)	$\beta = .99$	commonplace; yields an annualized average capital return of 4%
σ_ϵ	(tehnology shock SD)	$\sigma_\epsilon = .000712$	Cooley and Prescott (1992)

(i) When factor markets are competitive the parameter α is typically calibrated to reproduce the observed share of US capital income in total value added. While the labor market of the present model is not competitively determined, we nevertheless retain its most commonly assumed value.

(ii) Besides reflecting actual market leverage, $\varphi = .4$ is sufficiently large to allow the levels of wealth inequality that are explored in the model.

(iii) Mehra and Prescott (1985) argue that $(0, 10)$ is a reasonable range of possible values. By choosing a low value of γ, we argue that our results are generic and not dependent on high levels of risk aversion.

(iv) The existing literature is neutral on this value; a choice of $\eta = .5$ does not bias our results in favor of either agent.

(v) Choosing $\psi = 1.4$ implies a Frisch elasticity of labor supply of $\left[\frac{1}{(1.4-1)}\right] = 2.5$.

(vi) With $\rho = .10$, the expected duration of employment of a worker before separation is approximately 2.5 years.

(vii) By choosing $b = 0$, we obtain the model's steady state pure value of leisure normalized by output equal to .5, a reasonable value to match unemployment volatility as an important cross-check for the model.

The free parameters B_s and B_n are selected to match the following wage and hours ratios:

$$\bar{w}^s/\bar{w}^n = 1.57, \text{ and } \bar{h}^s/\bar{h}^n = .59.$$

In particular, without an hours ratio less than one, the stockholder's equilibrium wage rate may be less than that of the workers. Heathcote et al. (2010) report a male college wage premium of 1.4 in 1980, and college-educated persons are more likely to be stockholders; the chosen wage ratio thus roughly approximates the Heathcote et al. (2010) estimate. The hours ratio follows accordingly (in the Baseline case, $\bar{h}^s = .2$ and $\bar{h}^n = .34$).[35]

Let W^n and W^s denote the wealth states, respectively, in terms of consumption of a representative non-stockholder-worker and a representative stockholder. Of special relevance is the steady-state Gini coefficient for wealth, $\bar{G}^w$, where a $^-$ above a variable indicates its steady-state value. In the present

[35] Lansing (2015), in contrast, exhibits an hours supply ratio of .225 in order to achieve a wage ratio of 2 between capitalists and workers. In Lansing (2015), workers hold no bonds, however (zero wealth).

two-agent economy,

$$\bar{G}^{\mathrm{w}} = 1 - \left(\frac{\mu_s}{1+\mu_s} + \frac{\bar{W}^n}{\bar{W}^s + \bar{W}^n} \right) \tag{26}$$

where $\bar{W}^s = \mu_s \bar{b}^s + \bar{p}^e$, $\bar{W}^n = \bar{b}^n$, and $\mu_s \bar{b}^s + \bar{b}^n = \beta\varphi\bar{k}$.[36] The magnitude of χ^s, relative to χ^n, will be a critical determinant of the model's equilibrium steady state $\bar{G}^{\mathrm{w}}$.

The Baseline version of the model corresponds to the case where the top 10% of the population owns 90% of all financial assets (Poterba and Samwick (1995), Guvenen (2009), Lansing (2015)); i.e., $\bar{\Omega}^{\mathrm{w}} = .9$, which corresponds to a Gini coefficient of $\bar{G}^{\mathrm{w}} = .80$ via (26). With an agent's habit parameter χ influencing the extent of his precautionary savings, greater habit parameter values lead to greater wealth accumulation: χ^s must exceed χ^n for stockholders to possess the majority of the equilibrium wealth. In fact, there is a one-to-one mapping between the choice of habit parameters χ^n and χ^s according to the steady state relationship:

$$\chi^s = 1 + \frac{1}{\bar{\phi}^{\frac{1}{\gamma}} \left(\frac{\bar{c}^s}{\bar{c}^n} \right)} (\chi^n - 1) + \frac{1}{\bar{c}^s} \left[\frac{\bar{n}\,\bar{\Gamma}^n}{\bar{\phi}^{\frac{1}{\gamma}}} - \bar{\Gamma}^s \right] \tag{27}$$

where $\bar{\Gamma}^s = B_s \left(\bar{h}^s\right)^{\psi}$, $\bar{\Gamma}^n = B_n \left(\bar{h}^n\right)^{\psi}$ and $0 \leq \chi^n \leq .299$.[37], [38] This is of the form $\chi^s = a + b\chi^n$ for constants a and b. In order for the wealth distribution to be stationary, however, χ^s cannot be too large; elementary calculations suggest a value in the range [.5, .7]. Consistent with $\bar{\Omega}^W$, relationship (27) and these latter considerations we choose $\chi^s = .61$ and $\chi^n = .08$ for the Baseline calibration.[39],[40],[41]

[36] More generally, $\mu_s b^s_{t+1} + b^n_{t+1} = q^b_t \varphi k_t$. At any time t, the $G^{\mathrm{w}}_t = 1 - \left(\frac{\mu_s}{1+\mu_S} + \frac{W^n_t}{W^s_t + W^n_t} \right)$. See the Appendix for details.

[37] Note that the second term in (33) is negative.

[38] If χ^n exceeds this range, the argument of the worker's utility function becomes negative due to the presence of negative work disutility.

[39] Qualitatively, this particular choice of habit parameters enjoys substantial theoretical support. Hornstein and Uhlig (2000) emphasize the self-selection of agents: agents who become accustomed to a high consumption level, i.e., have habit formation preferences, are more likely to build up large precautionary capital stocks (i.e., become stockholders) than agents who do not. In a classic study, Becker (1980) shows that if heterogeneity across households takes the form of differential subjective discount factors, then the household with the lowest rate of discount (i.e., the most patient household, the one with the highest β) owns all the capital and earns wage income in the long-run steady state, while all other households receive only wage income. This study suggests that an unequal wealth distribution has its origin in preference heterogeneity. More recently, Diaz et al. (2003) show that in a heterogeneous agent economy, identical habit formation preferences encourage a more uniform wealth distribution relative to standard preferences, suggesting that a skewed wealth distribution will obtain only if heterogeneity in habit parameters (differential habits) is allowed. Lastly, Fuhrer (2000) shows that to be consistent with the VAR finding of a hump-shaped response of consumption to income, the aggregate consumption function should derive from two distinct groups of agents, i.e., a group of agents with habit-formation preferences and a group who live for the moment (low habit). This is the Baseline calibration. We also consider cases of both higher and lower consumption and wealth inequality, and compare these results with those of our Baseline calibration. Higher extremes of wealth inequality can be achieved by varying μ_s and especially the χ^s and χ^n parameters, as noted above.

[40] The $\bar{G}^W = .80$ is the wealth Gini coefficient averaged between 1971 and 2013 from Kuhn et al. (2020). Quadrini (2000), Krusell and Smith (1998), and Favilukis (2013) also report that the $\bar{G}^W$ range is between .75 and .81.

[41] The model is solved using techniques comprehensively discussed in Jermann (1998) and Kliem and Uhlig (2016) of log-normal-log-linear approximation about the model's certainty steady state, a procedure similar to the second-order updating of the steady-state in Schmitt-Grohe and Uribe (2004). Reported statistics are averages based on 1,000 independent runs, each 1,000 periods in length. Log-normal formulae are applied to compute the relevant asset prices and returns (see Jermann (1998) and the Technical Appendix). Also see the Technical Appendix for details on the steady state characterization.

4.3 Baseline Parametrization: Macroeconomic Properties

In this section, we explore the aggregate behavior of the Baseline economy together with an Investment Wedge scenario where an additional source of uncertainty, often interpreted as a "demand shock," is introduced. The relevant statistics are found in Table 2, Panels A and B including comprative data for the *generic* historical period 1970-2015, generic in the sense that its business cycle relative volatilities and correlation patterns are typical of post-war business cycles (Cooley and Prescott (1995)).

4.3.1 Macro-aggregate and Quantities

Our initial discussion is largely based on Table (2.A).[42] We see that the standard deviation of the growth rates of output, $\text{SD}(\Delta\tilde{y}) = .65$, and aggregate consumption, $\text{SD}(\Delta\tilde{c}) = .56$, largely illustrate the corresponding data for the 1970-2015 period, the former value falling somewhat short with the latter somewhat in excess. Accordingly, the models $\text{SD}(\Delta\tilde{c})/\text{SD}(\Delta\tilde{y})$ is large but not dramatically so. As in the data, $\text{SD}(\Delta\tilde{c})$ is less than $\text{SD}(\Delta\tilde{y})$, befitting any legitimate business cycle model.[43] Finally, the correlations of investment growth and consumption growth with output are excessive but within an acceptable range of the data. Generally, the model reasonably replicates most macro aggregate statistics for this generic historical period. Labor market and investment volatility, however, merit further discussion.

As for the key labor market quantities, including unemployment, u, vacancies, v, and labor market tightness, v/u, the Baseline economy displays atypically (but empirically valid) high volatility for this class of models.[44] Correlations of each of these variables, individually, with output, their standard deviations and their standard deviations relative to output well match the data from the historical period, perfectly on sign and close in magnitude, the latter measure nearly perfectly so for unemployment.[45]

The cost to the society of implementing stakeholder equilibrium is represented by a *risk tolearance* measure, $\text{SD}\widetilde{W}^{\text{pv}}/\mathbb{E}\widetilde{W}^{\text{pv}}$, the inverse of the financial Sharpe ratio: it measures how much volatility workers are willing to tolerate per unit of their present-value wage compensation. The Baseline model reports $\text{SD}\widetilde{W}^{\text{pv}}/\mathbb{E}\widetilde{W}^{\text{pv}} = .12$, compared with its efficient-fluctuation counterpart, .20, where the pecuniary externality is fully internalized. Under the egalitarian regime of $\overline{G}^{\text{w}} = .80$, workers are thus only willing to tolerate 60% of volatility relative to the fully internalized efficient economy. As confirmed in Table 6 of

[42] For ease of reference, the notation "Table (X.Y)" is adopted going forward where X denotes the Table number and "Y" the specific panel within it.

[43] While we have no BEA data on stockholder vs. non-stockholder consumption growth, the values generated by the model ($\text{SD}(\Delta\tilde{c}^s) = 2.01$, and $\text{SD}(\Delta\tilde{c}^n) = 1.29$), with a ratio $\text{SD}(\Delta\tilde{c}^s)/\text{SD}(\Delta\tilde{c}^n) = 1.56$ (see Table 7.C), accord well with independent estimates: Malloy et al. (2009b) report an empirical $\text{SD}(\Delta\tilde{c}^s)/\text{SD}(\Delta\tilde{c}^n) = 1.63$, while Mankiw and Zeldes (1989) report a figure of 1.60. As both figures from the literature are annualized quantities, we follow suit: the model's consumption data has been annualized accordingly.

[44] The general inability of the class of models to replicate labor market volatilities is often referred to as the Shimer (2004, 2005) puzzle.

[45] As is more evident in Section 5.4.2, however, the present high labor market volatility is not necessarily particularly harmful to workers, because it is part of the mutually arranged *discount-protected* wage asset created through egalitarian bargaining between the firm and worker.

Table 2: **Baseline Case**

	Data	Baseline Model	
	1970 – 2015	TFP only	Investment Wedge
Panel A: Macro and Labor Market Statistics			
Macro variable[i]			
Standard deviation			
$\text{SD}(\Delta y)$	.82	.65	.67
$\text{SD}(\Delta c)$	.49	.56	.59
$\text{SD}(\Delta i)$	4.00	1.27	3.60
Correlation with output growth			
$\text{corr}(\Delta c, \Delta y)$	.63	.79	.72
$\text{corr}(\Delta i, \Delta y)$	.75	.85	.15
Labor market variable[ii]			
Standard deviation			
$\text{SD}(\tilde{y})$	1.51	1.55	1.58
$\text{SD}(\tilde{v})$	13.16	18.65	20.93
$\text{SD}(\tilde{u})$	11.79	10.91	12.17
$\text{SD}(\tilde{\theta})$	25.38	28.17	31.42
Relative standard deviation			
$\text{SD}(\tilde{v})/\text{SD}(\tilde{y})$	8.72	12.02	13.27
$\text{SD}(\tilde{u})/\text{SD}(\tilde{y})$	7.81	7.03	7.71
$\text{SD}(\tilde{\theta})/\text{SD}(\tilde{y})$	16.81	18.16	19.92
Correlation with filtered output			
$\text{corr}(\tilde{v}, \tilde{y})$	.89	.99	.92
$\text{corr}(\tilde{u}, \tilde{y})$	$-.87$	$-.88$	$-.86$
$\text{corr}(\tilde{\theta}, \tilde{y})$	.905	.99	.94
Panel B: Secular-Stagnation-related Statistics			
Labor income share ℓ^{s}[iii]			
$\text{SD}(\tilde{\ell}^{s})/\text{SD}(\tilde{y})$	.73	.67	.66
$\text{corr}(\tilde{\ell}^{s}, \tilde{y})$	$-.10$	$-.20$	$-.20$
Mean safe-asset rate			
$\mathbb{E}\tilde{r}^{b}$	.92	$-.22$	-1.43
Panel C: Unobservable Externality-related Statistics			
Distribution risk ϕ[iv]			
$\text{SD}(\tilde{\phi})/\text{SD}(\tilde{y})$	–	9.28	10.14
$\text{corr}(\tilde{\phi}, \tilde{y})$	–	$-.86$	$-.81$
Price of stakeholder-societal risk[v]			
$\text{SD}\widetilde{W}^{\text{pv}}/\mathbb{E}\widetilde{W}^{\text{pv}}$	–	.12	.16

(i) All series are computed in growth rate.

(ii) Actual and model data H.P. filtered with smoothing parameter 1600.

(iii) ℓ^{s} denotes the labor share of income, computed as $\ell_t^s = \frac{w_t^n h_t^n n_t}{w_t^n h_t^n n_t + \mu_s w_t^s + r_t k_t}$.

(iv) *ex post* equilibrium distribution risk.

(v) This ratio, an analogue of the financial Sharpe ratio, measures how much volatility (in percentage points) workers would tolerate per unit of thier present-value wage compensation.

Section 5, the Baseline economy achieves the same wage compensation, $\mathbb{E}\widetilde{W}^{\mathrm{pv}}$, as its efficient counterpart. In this sense, the Baseline economy creates the *efficient wage mark-up*, irrespective of whether the pecuniary externality is fully internalized or only partially so.[46]

In contrast, the low measure of investment volatility is also a principal Baseline model feature, reflecting the close interaction of the bond and labor market under the stakeholder formulation.[47] As we will develop more fully in Section 5, the firm's precautionary demand for default-free bonds increases under the Baseline degree of wealth inequality, causing the share of risky investment in physical capital to decline with discount-protected, fixed-income-like wage assets to hire being created and acquired in response. Accordingly, the reduced provision of safe bonds, i.e., a fall in the firm's short-term *financial leverage*, is supplanted by the enhanced "supply" of long-term wage assets, which represents an increase in the firm's long-term *operating leverage* (see Section 5.4.2). The latter effect further reinforces a fall in the share of risky investment in the firm's financial portfolio, and thus suggests that under egalitarian bargaining, this *operating leverage* effect amounts to a privately arranged macro-prudential-type tax levied on the firm's investment in the attempt to internalize the externality.[48]

In the Baseline model, relative "distribution risk," $\sigma_{\widetilde{\phi}}/\sigma_{\widetilde{y}}$, is 9.28, suggesting a substantial residual pecuniary externality that market participants have not been able to internalize fully. Note that for the conditional first-best economy and the complete markets but otherwise equivalent DMP model of Andolfatto (1996) (Table 5), there is zero distribution risk. Confirming our earlier speculation that distribution risk must be countercyclical (and thus also the property right assignment), $\mathrm{corr}(\widetilde{\phi}_t, \widetilde{y}_t) = \mathrm{corr}(\widetilde{\eta}_t, \widetilde{y}_t) = -.86$ (Table 2.C); risk redistribution favoring worker insurance needs is active. The latter observation materializes as the labor share's countercyclicality in the stakeholder economy (Table 2.B).

4.3.2 Investment Shocks: An Extension

The Baseline model's low relative investment volatility can be "rectified" by adding a "demand shock" to the Baseline formulation in the form of an investment wedge, $\{\widetilde{\varpi}_t\}$, as per Greenwood et al. (1988) and Chari et al. (2007). The demand shock is assumed orthogonal to the productivity disturbance $\{\widetilde{z}_t\}$ and effectively makes variable the cost of investment in terms of consumption.[49] It

[46] As demonstrated in Section 5, the *efficient wage mark-up* can also be interpreted as the *increasing* labor income share on a "Musk-Tesla superstar" growth path, standing 180 degrees to the *constant* labor income share on the textbook competitive balanced growth path.

[47] Boldrin et al. (2001), however, argue for low relative investment volatility, $\sigma_{\widetilde{i}}/\sigma_{\widetilde{y}} = 2.39$, as the appropriate estimate.

[48] Donaldson and Kim (2018) show that a representative-agent version of the baseline model, featuring Nash bargaining without any additional "wage mark-up," exhibits investment volatility comparable to standard RBC formulation.

[49] Recent empirical studies appear to rationalize this "orthogonal" investment wedge: in the US manufacturing sector, investment-related tax incentives play a key role in the short-run investment volatility at the firm level, while employment is disassociated with investment. The former finding is underlined by Zwick and Mahon (2017), while the latter is confirmed by Pierce and Schott (2016).

manifests itself through a modified dividend definition in the firm's problem (10):

$$d_t = f(k_t, \mu_s h_t^s, n_t h_t^n) z_t - \left(\frac{1}{\varpi_t}\right) \cdot i_t - \mu_s w_t^s h_t^s - w_t^n h_t^n n_t - \frac{\kappa}{2} x_t^2 n_t - \varphi \bar{k} + p_t^b \varphi \bar{k}, \tag{28}$$

where $\log \widetilde{\varpi}_{t+1} = \rho_\varpi \log \varpi_t + \widetilde{\epsilon}_{t+1}^\varpi$, $\widetilde{\epsilon}_t^\varpi \sim N(0, \sigma_{\epsilon^\varpi}^2)$, and $\mathrm{cov}(\widetilde{\epsilon}_t^\varpi, \widetilde{\epsilon}_t) = 0$. As such ρ_ϖ and $\sigma_{\epsilon^\varpi}^2$ become free parameters for the case reported in Table 2.[50]

The results of this alternative simulation are presented in the right-most column of Table 2 under the heading "Investment Wedge." Two features stand out. First, investment growth volatility is roughly three times as great as in the Baseline and largely in line with its empirical counterpart. It may be further increased by increasing $\sigma_{\epsilon^\varpi}^2$. Second, relative to output, the volatility and correlation measurements for all the labor market variables are not materially affected by the presence of the "wedge."[51] Accordingly, the observed high investment volatility in the stakeholder economy must be created mostly through the "demand shock" channel, suggesting that the Prescott (1986) channel of "supply shocks" is the primary player in the economy's risk sharing mechanism. With the addition of the uncertain investment wedge only increasing the volatility of all macro quantities across the board without materially altering correlations or relative volatilities, we elect to omit it from further analysis and focus on understanding how the economy responds to the single aggregate shock $\{\widetilde{z}_t\}$ only.[52]

In conclusion, we argue that the proposed model is a reasonable business cycle formulation and move on to a more detailed study of the model's evolving risk sharing mechanisms and their joint effect on labor and bond markets.

5 Inspecting the Mechanism

5.1 Overview: Polarization Traps, Cyclical Stagnation and the Stakeholder Society

This section presents a series of policy experiments which describe how the stakeholder equilibrium is affected by increasing wealth inequality. Recall that for all cases the entire equity ownership of the capital stock remains with the stockholder class. Under this restriction, increasing inequality is affected either by reducing the measure of stockholders or increasing the capital's share in production, while the measure of stockholders remains unchanged. What is of interest is how the ownership pattern of

[50] They are chosen via a standard "hyperparameter search" to target the SD of the real default free rate of 2.31% (annualized), and a quarterly SD of investment growth of 4.00%, both figures reflecting US data for the period 1970.Q1-2015.Q4. At the overall minimum, $\sigma_{\widetilde{r}^b} = 2.59\%$ and $\sigma_{\Delta \log \widetilde{i}_t} = 3.60\%$.

[51] Not surprisingly, $\mathrm{corr}(\Delta \widetilde{y}_t, \Delta \widetilde{i}_t)$ declines substantially relative to its Baseline counterpart since an orthogonal direct source of investment uncertainty has been added.

[52] Further discussion of the present "investment-wedge" model, including its asset-pricing implications, can be found in Donaldson and Kim (2020).

bond holdings interacts with egalitarian wage bargaining in determining the pattern of equilibrium interest rates and risk sharing. This interaction will suggest how a stakeholder economy may evolve in the future and how its macroeconomic properties may change with increasing wealth inequality. By Proposition 3.3, we already know that the first best equilibrium steady state and the stakeholder equilibrium steady state will coincide irrespective of the level of wealth inequality may be. It is thus in the business cycle properties that the consequences of the stakeholder economy are manifest, representing the consequences of incomplete pecuniary externality resolution.

Tables 5-7 provide macroeconomic and financial statistics for three model configurations reflecting the increasing concentration of capital ownership. Each satisfies the assumption required for Propositions 3.1-3.3. For comparison purposes, additional scenarios are also presented under the broad heading of constrained "Efficiency." The DMP entry refers to the present model but with complete financial markets and a representative agent; it is essentially Andolfatto's (1996) model, where the resulting allocation is efficient conditional on its matching technology. In contrast, the RBC entries (Table 7), also efficient, derive from the classic model of Hansen (1985). The column headed by the phrase "Efficient Fluctuations" represents the equilibrium fluctuations associated with the solution to Problem (25), the central planner's version of the decentralized Coasian formulation. As stated in Proposition 3.3, this formulation shares the same steady state as its decentralized counterpart, but its equilibrium reaction to technology shocks differs. To clarify, as wealth inequality changes, so does the model's constrained efficient steady state. *What does not change, however, is the pattern of efficient deviations about the steady state.* Thus we provide one set of volatility values in Tables 5, 6, and 7 for all levels of wealth inequality under the "Efficient Fluctuations" designation. Of the cases we consider, the category "Polarization Traps" encompasses those cases where wealth inequality is the most extreme.

Behind these results, three interrelated mechanisms are at work. The first is endogenous growth in real output as the measure of stockholders declines, while the second is egalitarian wage bargaining. The increasing pattern of precautionary savings as wealth inequality increases represents the third. Each is detailed in the next sections.

5.2 Calibration

The transition to higher (lower) wealth inequality is first accomplished by progressively reducing the measure of stockholders, while adjusting the precautionary parameters accordingly. As μ declines from $\mu = .20$ (low inequality) to $\mu = .10$ (Baseline), for example, the per capita wealth of each stockholder roughly doubles, manifest as $\overline{G}^{w}$ increasing from $\overline{G}^{w} = .7$ to $\overline{G}^{w} = .8$. As μ further decreases from $\mu = .10$ to $\mu = .075$, $\overline{G}^{w}$ further increases to $\overline{G}^{w} = .83$, consistent with its recent empirical counterpart. Accordingly, the Baseline case and the $\overline{G}^{w} = .83$ case are both viewed as "high wealth inequality" ones.

Second, we increase the (competitive) share of income to capital, α, to $\alpha = .383$ while leaving the stockholders' measure unchanged from the Baseline $\mu = .10$ value. Table 3 presents the full set of critical wealth-inequality-related parameter choices for the cases we consider.

We note that the efficiency claims of Proposition 3.3 apply to all the model variations detailed in Table 3.

Table 3: **Capital Ownership Concentration Parameters**

α	μ	γ	χ^s	χ^n	$\overline{\phi}$	$\bar{\eta}$	$\bar{G}^W$	$\overline{b}^n/\overline{w}^n\overline{h}^n$ (i)
High Wealth Inequality								
.36	.075	2	.67	.087	.78	.44	.83 (ii)	4.89 months
Baseline								
.36	.10	2	.62	.087	.76	.43	.80 (iii)	5.01 months
Low Wealth Inequality								
.36	.20	2	.45	.087	.66	.40	.70	5.67 months
Technological Change								
.383 (iv)	.10	2	.64	.087	.73	.42	.80	5.55 months

(i) This ratio measures the precautionary savings of workers by expressing their bond holdings relative to their mean period-by-period labor income.
(ii) $\bar{G}^W = .83$ is the highest wealth Gini coefficient observed across all OECD countries in 2016.
(iii) The $\bar{G}^W = .80$ is the wealth Gini coefficient averaged between 1971 and 2013 from Kuhn et al. (2020). Quadrini (2000), Krusell and Smith (1998), and Favilukis (2013) also report that the $\bar{G}^W$ range is between .75 and .81.
(iv) This parameter change follows from Caballero et al.'s (2017) empirical finding for the recent 2008-2015 historical period.

5.3 Endogenous Growth

As evident in Table 4, increased wealth inequality in the present model is associated with a higher steady-state (mean) capital stock. It follows that steady-state wages are higher for both agents, leading to higher per capita consumption and higher welfare for each agent. As their measure is declining, the mean per capita consumption of stockholders grows more rapidly than per capita worker-family consumption. The wealth per capita of workers also grows more modestly, from 1.17 to 1.34 units of consumption, while that of stockholders grows much more dramatically, from $\overline{W}^s/\mu_s = 52.8$ when $\overline{G}^W = .70$ to $\overline{W}^s/\mu_s = 162$ when $\overline{G}^W = .83$ (phenomena characteristic of the U.S. economy over the past 20 years) largely due to a reduction in their measure. Nevertheless, a positive "trickle-down" effect is evident: the worker's share of total income increases from $\overline{\ell}^s = .50$ when $\overline{G}^W = .70$ to $\overline{\ell}^s = .58$ when

$\overline{G}^{\text{w}} = .83$. Growing mean unemployment is another direct implication, as observed.

Why do we observe these effects? With the equilibrium real rate of interest remaining positive and constant in the long run, the increase in the capital stock is only made possible by the endogenous increase in the effective labor input from stockholders (for more details, see part D of the Appendix). To see this, notice that the aggregate mean labor input across both agent groups, $\overline{H}$, is given by

$$\overline{H} = \left(\mu_s \overline{h}^s\right)^{\frac{\mu_s}{1+\mu_s}} \left(\overline{n}\overline{h}^n\right)^{1-\frac{\mu_s}{1+\mu_s}},$$

and recall from our calibration that $\overline{h}^s$ and $\overline{h}^n$ are fixed across all cases, being judiciously chosen to yield a reasonable approximation to the observed $\overline{w}^s/\overline{w}^n$ ratio.[53]

Table 4: **Mean Values**

	$\overline{G}^{\text{w}} = .70$ ($\mu = 0.20$)	$\overline{G}^{\text{w}} = .80$ ($\mu = 0.10$)	$\overline{G}^{\text{w}} = .83$ ($\mu = 0.075$)	Technological Change $1-\alpha = .617$; $\mu = .10$
$\bar{y}$	1.07	1.22	1.28	1.34
$\bar{k}$	11.01	12.55	13.14	14.63
$\bar{c}^s$	1.00	1.77	2.27	2.18
$\bar{c}^n$	.57	.71	.77	.83
$\bar{b}^s$	13.65	33.95	48.52	43.57
$\bar{b}^n$	1.17	1.30	1.35	1.66
$\mu_s \cdot \bar{b}^s$	3.41	3.77	3.93	4.84
$\bar{w}^s$	2.86	3.56	3.81	4.14
$\bar{w}^n$	1.83	2.28	2.44	2.65
$\bar{\ell}^{s\,\text{(i)}}$	.50	.56	.58	.54
$\bar{p}^e$	7.15	7.90	8.24	10.12
$\bar{V}^s$	-668.47	-575.07	-544.95	-482.15
$\bar{V}^n$	-1228.10	-1000.01	-935.65	-850.63
$\bar{u}/(1+\mu_s)$	8%	9%	9.25%	9%
$\bar{r}^{\text{(ii)}}$	.04	.04	.04	.04

(i) $\bar{\ell}^s = \bar{w}^n \bar{h}^n \bar{n}/\bar{y}$, as earlier.
(ii) returns annualized.

The engine of growth arises purely from the declining measure of increasingly effective "Musk-Tesla superstars."[54] As μ_s (already small) tends to zero, there is very little reduction in the $\left(\overline{n}\overline{h}^n\right)^{1-\frac{\mu_s}{1+\mu_s}}$ term, but a very large proportionate increase in $\left(\mu_s \overline{h}^s\right)^{\frac{\mu_s}{1+\mu_s}}$. Somewhat lightly, we describe this phenomenon as the "Musk-Tesla superstar" effect: a shrinking measure of very capable people become

[53] In the present context, $\overline{n}$ is independently chosen to target the steady state vacancy-unemployment ratio, $\overline{v}/\overline{u} = 1$, which represents the standard calibration in the related literature.

[54] More standard growth formulations argue that TFP innovation is roughly an increasing function of the total population, with a decline in the latter provoking a decline in the former, leading to an end to growth (Jones (2020)). The present model economy ceases to grow only in the limit as μ_s tends to zero, however.

increasingly productive. This "Musk-Tesla superstar" effect, where the ownership of capital becomes concentrated in fewer and fewer stockholders, indirectly reflects Lenin's (1916) dystopian prediction and Kwon et al.'s (2022) recent finding of the increasing concentration of productive capital.[55]

As noted above in Table 4, however, the present economy exhibits the presence of a "pseudo-balanced growth path": while output, capital stock and investment all grow at the same rate (with declining μ_s) and the equilibrium real rate of interest remains constant, as would be the case in an analogous competitive labor market growth model, *the share of income to labor under the present construct is increasing*. The latter phenomenon, in stark contrast to the *constant* labor income share on a competitive growth path, is the direct consequence of egalitarian wage bargaining: the worker's Coasian property right strengthens with increasing wealth inequality on the growth path. As its very name suggests, it is in this sense that egalitarian wage bargaining invokes the classical Coase Theorem: indeed, the present decentralized counterpart also achieves the socially efficient steady state growth path. As a corollary, the classical Chamley-Judd zero capital tax result applies, reflecting the true sense of the Coase Theorem: Coasian bargaining always achieves a superior outcome to standard Pigovian tax prescriptions. The fact that the long-term interest rate, interchangeable with the return on capital, is constant, independent of the concentration of capital, is supportive of Piketty's "Capital in the Twenty-First Century" (2014) perspective. With a standard neoclassical production function, more capital normally dictates a declining return on capital, something that, *per se*, constitutes a major criticism of Piketty (2014). This effect is not observed in the present model where the Musk-Tesla effect "props up" capitalism.

5.4 Present Value Wage Bargaining and Partial Internalization

5.4.1 Partial Internalization in the Coasian Sense: A Social Planner's View

We divide this section by first discussing the cyclical behavior of the planner's efficient allocation described earlier. Tables 5, 6, and 7 contain the relevant data. In doing so we uncover how the planner would internalize the externality in a long-run Coasian sense, all the while constrained to respect the existing level of wealth inequality.

Note first that the planner's short-run constrained-efficient equilibrium fluctuations are invariant across all the levels of wealth inequality: the "Musk-Tesla" effect and the concurrent gradual wealth polarization do not affect the planner economy's business cycle risks arising from RBC-type productivity shocks. This phenomenon stands in stark contrast to the market equilibrium counterparts we will discuss in the later sections: under intensifying wealth inequality, the presence of the pecuniary externality, manifested as $\mathrm{SD}(\widetilde{\phi})$ and its "partial internalization," fundamentally alter business-cycle-frequency fluctuations. Thus we consider the efficient scenario as the zenith of the Coasian principle

[55] Marx (1867) also makes a related point by saying that "*capital grows in one place to a huge mass in a single hand, because it has in another place been lost by many,*" which rationalizes *centralization* as distinct from *accumulation and concentration.*

applied in the present context. It also serves as a benchmark for the other cases of "partial internalization." We defer a precise definition of this latter term until the next section.

As a rough summary, the planner chooses to allow greater variation in prices (wages, broadly defined) in order to reduce employment-related quantity variation. Equivalently, the planner's allocation is one where wages largely absorb productivity-shock-induced business cycle variation rather than employment. In particular, we see (Table 6) that $\text{SD}(\widetilde{W}^{\text{pv}}_{\text{efficient}}) = 4.12$ in the efficient allocation, a value greater than its counterpart in each of the egalitarian scenarios while the elasticity of employment relative to changes in the allocation price $\widetilde{W}^{\text{pv}}$, $\partial \log \widetilde{n}_t / \partial \log \widetilde{W}^{\text{pv}}_t = .003$, is essentially zero: in this sense the present value wage is essentially absorbing the business cycle shocks in their entirety.

By contrast, the egalitarian wage, arising as a market equilibrium outcome, features a monotonic reduction in its volatility, $\text{SD}(\widetilde{W}^{\text{pv}})$, in response to widening wealth inequality (Table 6). In the social planner's eyes, the present value of egalitarian bargained wages in new jobs is too "sticky" even in the near-complete scenario ($\overline{G}^{\text{w}} = .70$), because private agents do not fully take into account the social benefits of efficient wage flexibility or, equivalently, the social costs of wage stickiness: the present value wage is only 85% as volatile as its (constrained) efficient counterpart, i.e., $\text{SD}(\widetilde{W}^{\text{pv}}_{\text{near complete}}) = 3.51 < 4.12 = \text{SD}(\widetilde{W}^{\text{pv}}_{\text{efficient}})$. As a counterpart to this declining wage volatility, employment (quantity variation) becomes more responsive to changes in W^{pv}_t (price variation): $\partial \log n_t / \partial \log W^{\text{pv}}_t = .10$ when $\overline{G}^{\text{w}} = .70$, increasing to $\partial \log n_t / \partial \log W^{\text{pv}}_t = .76$ when $\overline{G}^{\text{w}} = .83$, suggesting an unambiguous trade-off between the price and quantity variations in the decentralized market equilibrium. In the first-best constrained equilibrium this latter trade-off is essentially absent, a fact that suggests the central planner is most concerned with minimizing unemployment. The extremely low responsiveness of unemployment in the efficient case is also seen in the relative (to $\text{SD}(\widetilde{y})$) volatilities of vacancies, unemployment and tightness. In all the egalitarian cases, by contrast, these volatility values are substantial multiples of the corresponding efficient case equivalents (Table 5).[56]

Nevertheless, workers in all the decentralized scenarios benefit from egalitarian wage bargaining $\{\widetilde{W}^{\text{pv}}\}$ on the "risk and return dimensions," since the average wage compensation, $\mathbb{E}(\widetilde{W}^{\text{pv}})$, invariably achieves its constrained efficient level. We view this efficient yet decentralized equilibrium outcome, $\mathbb{E}(\widetilde{W}^{\text{pv}})$, as the "partial internalization" of the pecuniary externality in the decentralized economy (Table 6). As a corollary argument, the price of stakeholder-societal risk, $(\text{SD}\widetilde{W}^{\text{pv}} / \mathbb{E}\widetilde{W}^{\text{pv}})^{-1}$, is seen to become "higher" than its efficient counterpart, because $\text{SD}(\widetilde{W}^{\text{pv}})$ diminishes while $\mathbb{E}(\widetilde{W}^{\text{pv}})$ remains unchanged due to the "partial internalization." Alternatively, the magnitude, $\text{SD}\widetilde{W}^{\text{pv}} / \mathbb{E}\widetilde{W}^{\text{pv}}$, can also be viewed as quantifying the "partial internaliztion" of the stakeholder economy of interest relative to its efficient benchmark, $\text{SD}\widetilde{W}^{\text{pv}}_{\text{efficient}} / \mathbb{E}\widetilde{W}^{\text{pv}}_{\text{efficient}}$.

Lastly, we comment on the relative consumption growth variation of workers and capital owners

[56] These results remind us that the transition to a decentralized stakeholder economy will not necessarily be free of undesirable consequences.

under the planner's direction (Table 7): the firm owners experience very little consumption volatility with the opposite being true for the workers, an unexpected comparison that is partially reversed in the egalitarian cases. We also obtain $\text{corr}(\Delta\tilde{c}_t^s, \Delta\tilde{c}_t^n) \neq 1$ in the planner's case largely because the planner is constrained to respect that stockholders own all the capital and receive all the capital income. We attribute these relative magnitudes to the planner's desideratum that firm owners should accommodate hypothetical "long-run consumption risk" related to a growth path or tolerate investment-related shocks, if they exist, in return for reduced short-run consumption risk. The latter point, in particular, can be rationalized by the observation that investment-related shocks are concentrated exclusively on stockholders, while keeping labor market volatility virtually unchanged, as demonstrated in the Baseline model (Table 2). In her wisdom the planner apparently compensates workers in the form of higher $\mathbb{E}(\widetilde{W}^{\text{pv}})$, in an environment where workers must tolerate more short-run consumption risk (Tables 6 and 7). This latter observation can be interpreted as the planner's decision that workers, in return for greater short-run consumption risk, should enjoy more stable long-run consumption and reduced unemployment risk possibly allowing them more opportunity to accumulate (unspecified) "human capital."[57] In summary, the planner appears to agree in large part with Lucas's (1987) conclusive prescription: the welfare consequences of even "small" improvements in economic growth enormously outweigh any sophisticated stabilization policies attempting to eliminate short-run business cycle variation.

With these observations as background, we next study in detail how the decentralized economy partially internalizes the externality. It does so by creating an allocation very different from the central planner's. Our understanding is enhanced by first exploring the changing model outcomes as wealth becomes concentrated in a declining measure of stockholders.

5.4.2 A Financial Market View: Wage Assets

Preliminaries A discussion of wage behavior in a decentralized market environment inevitably initiates the topic of risk sharing since the form of the equilibrium wage contract influences the ability of workers to mitigate their income risk. We view workers as confronting two distinct but related risks: (1) the risk of episodes of prolonged unemploment, and (2) conventional productivity-shock-related high-frequency income variation. Accepting this dichotomy for the moment, it is natural to view the precautionary accumulation of bonds as the primary operative device in the first case and the pattern of the wage payments as influencing the second. In the standard DMP scenario where Nash and egalitarian wage bargaining coincide, the present-value wage contract is the main tool for consumption smoothing, because present value bargaining in a complete-market world makes it possible for workers to smooth consumption by distributing the present value of their flow wages in any desired pattern

[57] In this respect, we consider Kehoe et al.'s (2019) research program, incorporating explicit human capital accumulation into a DMP framework, to be an important step forward.

across dates and states over the desired time horizons.

Accordingly, the precautionary demand for bonds mainly provides insurance against unemployment risk, while playing a tertiary role in the worker's consumption smoothing. In a nutshell, the complete-market DMP scenario produces a separation result between *insurance* (against unemployment risk) and *liquidity* (for consumption smoothing). It also implies that the flow wage streams, smooth or flexible, will be irrelevant for labor market allocations, as long as financial market are complete. When financial markets are incomplete, however, egalitarian bargained wages do not necessarily create the separation and irrelevance results described above. Indeed, it is not immediately clear that present value wage bargaining even makes sense for stockholders and workers in an incomplete markets setting, a concern we address shortly.

Near-complete case To obtain better insight into the wage mechanism, we first consider the low-wealth-inequality case ($\bar{G}^{w} = .70$). In the present model, the degree of the residual externality due to market incompleteness is manifested as the volatility of distribution risk, $\mathrm{SD}(\widetilde{\phi})$, while the economy's decentralized response is captured in the behavior of the present-value wage, $\{\widetilde{W}_t^{\mathrm{pv}}\}$, and its flow wage constituent, $\{\widetilde{w}_t^n\}$. Note that the litmus test for efficient risk sharing is $\mathrm{SD}(\widetilde{\phi}) \approx 0$, a statistic characteristic of both the standard complete markets DMP model and the social planner's version of the present construct (Table 6).

Despite representing a moderate level of inequality, the $\bar{G}^{w} = .70$ case nevertheless displays a non-trivial amount of distribution risk, $\mathrm{SD}(\widetilde{\phi})/\mathrm{SD}(\widetilde{y}) = 2.92$, implying significant market incompleteness. Our first unanswered question, thus, is whether present-value wage bargaining in this setting is generally "implementable" or not. To illustrate this point, consider the simplest scenario in which the flow wage is set equal to a single one-time present-value payment at some time t, i.e., $w_t^n = W_t^{\mathrm{pv}}$, and zero afterward. Such a "one-shot" payment would not be equivalent to the worker's desired stream of payments over his lifetime unless he could allocate the single payment across future states and dates. Another potential problem with this scenario is future commitment: if bargaining were to reopen at a later date, it could disincentivize the worker to agree to accept the previously agreed-upon subsequent zero flow wage rate. In response, the firm could not commit to a present-value wage arrangement, knowing the worker would never fulfill his implicit promise, etc. To wit, present value bargaining invokes the issue of agents' ability to commit. Overcoming these difficulties in the present incomplete market setting would require, we propose, at the very least default-free bond trading, often referred to as Telmer's (1993) near-completeness mechanism: Telmer (1993) observes that in a dynamic stochastic setting bond trading alone is sufficient to achieve near-to-perfect risk sharing, if the option is available.

We next argue that egalitarian wage bargaining coupled with default-free bond trading achieves near completeness in the Telmer (1993) sense. *Ceteris paribus*, egalitarian wage bargaining yields the variable $\{\widetilde{\eta}_t\}$ having the interpretation of cyclical worker property rights and satisfying Proposition

3.1:

$$\frac{EP_t - U_t}{\eta_t} = \frac{J_t}{1-\eta_t}. \tag{29}$$

Unlike the standard DMP model, however, the equilibrium bargaining condition (29) must also be consistent with the general equilibrium bond market clearing condition:

$$\mathbb{E}_t \widetilde{M}^s_{t,t+1} = p^b_t = \mathbb{E}_t \widetilde{M}^n_{t,t+1}. \tag{30}$$

Due to these general equilibrium effects, we will interpret conditions (29) and (30) as jointly constituting the egalitarian wage contract in the present incomplete market environment.[58]

Accordingly, we propose to characterize the degree of market incompleteness in the present setting by three related quantities: (i) an "affordable" bond price, one that earns workers a significant positive return, (ii) a countercyclical discount rate $[\mathbb{E}_t \widetilde{M}^s_{t+1}]^{-1}$, where in equilibrium, $[\mathbb{E}_t \widetilde{M}^s_{t+1}]^{-1} = 1/p^b_t$, and (iii) the volatility level of *ex post* distribution risk (our measure of the severity of the pecuniary externality), where $\widetilde{\eta}_t$ is in fixed proportion to $\widetilde{\phi}_t$. How these elements are related to one another will be seen as follows. First, the volatility of distribution risk, SD$(\widetilde{\phi})$, moves directly with the bond price in a "reduced-form" sense: *pari passu*, they increase or decrease together. Second, a countercyclical discount rate, *ceteris paribus*, suggests the enhanced stockholder provision of safe assets (default-free bonds) in recessions, leading to a procyclical corr$(\widetilde{y}_t, \widetilde{b}^s_{t+1})$ and a positive default-free rate in recessions: stockholders effectively sell bonds to workers at low prices in bad times (high unemployment) while buying them back at high prices in the subsequent period, thereby allowing workers to earn high returns, a pattern especially helpful to workers since unemployment is highly persistent across all the reported cases.[59] Taken together, these observations, if confirmed, suggest that stockholders' pattern of bond trading is one device by which they encourage workers to agree to bargain over the present value wage in an incomplete market, the stockholders themselves paying the "cost of commitment" in the form of high interest payments.[60] To summarize, "affordable" bond pricing and stockholders' abundant provision of default-free bonds in low-output states constitute the principal ingredients of the near-to-complete equilibrium mechanism.

[58] It differs from its purely private equilibrium counterpart in which workers and the firm agree on a bargained wage satisfying only condition (29). In the standard DMP model $\widetilde{\eta}_t$ is constant either because the firm and its workers are risk-neutral or financial market completeness is assumed. As a result, condition (30) is irrelevant and condition (29) with constant $\widetilde{\eta}_t$ defines the uniquely agreed upon aggregate and private equilibrium Nash contract.

[59] corr$(\widetilde{u}_t, \widetilde{u}_{t+1}) > .9$ in all cases. See part H of the Appendix.

[60] Here we are asserting that the provision of safe assets by the firm can be regarded as a type of commitment to make present value wage bargaining implementable in an incomplete markets environment. Thus we propose abundant bond trading as part of wage bargaining, through the bond market rather than within the firm, an observation reminiscent of "efficiency wage" models of unemployment. In efficiency wage models of unemployment (e.g., Shapiro and Stiglitz (1984)), workers must pay "entrance fees" or "performance bonds" to the firm to dissuade them from shirking in equilibrium. The present model features the opposite outcome: the firm offers safe assets (default-free bonds) for sale to workers at attractive prices each period as a commitment device, indirectly allowing bond trading between capital owners and workers. This mechanism is costly to firm owners as bond sales to workers increase the firm's financial leverage, making the dividend stream and stockholder consumption more volatile.

As its name implies, the near-to-complete $\bar{G}^{w} = .70$ case illustrates well the scenario described above. The associated positive default-free rate of $\mathbb{E}\tilde{r}^{b} = 2.9\%$ (Table 5) not only identifies a relatively affordable bond price satisfying (30), but also confirms an ample provision of bonds by stockholders in low-output states, as reflected in $\text{corr}(\tilde{y}_t, \tilde{b}^{s}_{t+1}) = +.78$ (Table 7).[61,62] Direct confirmation of this latter assertion follows from $\text{corr}(\Delta\tilde{c}^{s}_t, \Delta\tilde{c}^{n}_t) = .95$ ($\overline{G}^{w} = .70$), a figure that nearly matches its DMP complete markets counterpart where, by construction, markets are complete (Table 7.C). Taken together these mechanisms have the overall effect of a comparatively low *ex post* distribution risk volatility $\text{SD}(\tilde{\phi}_t)$, (Table 5.B). Accordingly, the $\bar{G}^{w} = .70$ case can be identified with Telmer's (1993) near-to-complete mechanism, fully activated, allowing the egalitarian bargained wage to approximate closely its Nash counterpart in a fully complete financial market environment.

Two further implications merit discussion. The first concerns the effects of near-to-completeness ($\overline{G}^{w} = .70$) on labor market volatility. Shimer (2005) emphasizes the role of "sticky" wages, measured in present value terms, as critical to the amplification of productivity shocks on labor market volatility. In particular, Nash bargaining, as in the DMP formulation, exhibits a present value wage too flexible to generate the observed volatility of unemployment. The elasticity of the present value of the egalitarian bargained wage with respect to that of productivity, $\partial \log \widetilde{W}^{pv}_t / \partial \log \tilde{X}_t$, rationalizes Shimer's (2005) point (Table 7.A). In the DMP case this statistic exceeds one, suggesting an extremely flexible present value wage. As a result, labor market volatility across all the variables of interest, $\tilde{v}$, $\tilde{u}$, and $\tilde{\theta}$ is extremely counterfactually low, especially unemployment (Table 5).

The above observations suggest that the $\overline{G}^{w} = .70$ near-to-complete case should closely reflect the same properties as the DMP version and this is indeed the case. The elasticity $\partial \log \widetilde{W}^{pv}_t / \partial \log \tilde{X}_t = .94$, in particular, reflects continued high present value wage flexibility. As a result, the risks of the three principal labor market variables, $\text{SD}(\tilde{v})$, $\text{SD}(\tilde{u})$, and $\text{SD}(\tilde{\theta})$, are not significantly higher than in the DMP case, especially $\text{SD}(\tilde{u})$, while continuing to be well below the corresponding empirical values (Table 5.A). The relative volatilities produced in the $\overline{G}^{w} = .70$ case, in fact, closely resemble those reported in Andolfatto (1996), whose DMP complete markets model cum Nash bargaining otherwise closely approximates the one presented here.

While abundant bond trading is a precondition for committed present value wage bargaining, its very existence makes itself less critical for worker consumption smoothing, our second implication. It is as though bond trading attenuates its own raison d'être in the following sense: viewed as a wage asset the present value egalitarian wage provides the principal consumption smoothing device for

[61] Recall that stockholders initially own all the bonds and elect to sell some to workers. A correlation of $\text{corr}(\tilde{y}_t, \tilde{b}^{s}_{t+1}) = +.78$ suggests they wish to retain fewer of the bonds in low-output states, making them more plentifully available to workers at lower prices.

[62] As will be discussed more thoroughly in Section 5.5, this trading pattern suggests that workers are accumulating bonds for precautionary purposes in the anticipation of a possibly extended period of high unemployment ($\text{corr}(\tilde{y}_t, \tilde{b}^{n}_{t+1}) = -.78$), saving behavior consistent with the so-called "buffer-stock savings" characteristic of U.S. data for periods surrounding recessions (Figure 1b). The result of these interactions is a near-to-complete financial market in the $\overline{G}^{w} = .70$ case.

Table 5: **Polarization Traps: Comparison**

Panel A: Macro and Labor Market Statistics								
	DATA		**MODEL**					
			Polarization Traps		Baseline	Near-Completeness	Efficiency	
	1970 – 2015	2008– 2015[i]	Tech. Change	$\bar{G}^W = .83$	$\bar{G}^W = .80$	$\bar{G}^W = .70$	Efficient Fluctuations[ii]	DMP[iii]
Macro variable								
Standard deviation								
$\text{SD}(\Delta y)$	.82	.72	.81	.80	.65	1.14	1.22	.87
$\text{SD}(\Delta c)$	.49	.61	1.03	1.11	.56	.93	1.22	.23
Correlation with output growth								
$\text{corr}(\Delta c, \Delta y)$	.63	.73	.96	.93	.79	.99	1.00	.99
Labor market variable								
Standard deviation								
$\text{SD}(\tilde{y})$	1.51	1.04	1.67	1.77	1.55	1.68	1.64	1.18
Relative standard deviation								
$\text{SD}(\tilde{v})/\text{SD}(\tilde{y})$	8.72	10.15	19.15	16.82	12.02	5.46	.92	4.08
$\text{SD}(\tilde{u})/\text{SD}(\tilde{y})$	7.81	10.99	10.89	10.12	7.03	2.83	.78	2.44
$\text{SD}(\tilde{\theta})/\text{SD}(\tilde{y})$	16.81	20.50	28.76	25.71	18.16	7.87	1.52	5.43
Correlation with filtered output								
$\text{corr}(\tilde{v}, \tilde{y})$	.89	.91	.79	.81	.99	.98	1.00	.82
$\text{corr}(\tilde{u}, \tilde{y})$	−.87	−.87	−.99	−1.00	−.88	−.65	−.64	−.79
$\text{corr}(\tilde{\theta}, \tilde{y})$	.905	.90	.90	.92	.99	.91	.93	.97
Panel B: Secular-Stagnation-related Statistics								
Labor income share ℓ^s								
$\text{SD}(\tilde{\ell}^s)/\text{SD}(\tilde{y})$	.73	1.15	1.34	1.11	.67	.14	.05	.26
$\text{corr}(\tilde{\ell}^s, \tilde{y})$	−.10	.16	.32	.27	−.20	−.50	.98	.26
Mean safe-asset rate								
$\mathbb{E}\tilde{r}^b$	.92	−2.85	−5.16	−5.43	−.22	2.865	3.32	3.98
Panel C: Unobservable Externality-related Statistics								
Distribution risk ϕ								
$\text{SD}(\tilde{\phi})/\text{SD}(\tilde{y})$	NA	NA	15.68	14.30	9.28	2.92	.00	.00
$\text{corr}(\tilde{\phi}, \tilde{y})$	NA	NA	−.46	−.52	−.86	−.99	.00	.00

(i) Eggertsson et al. (2019b) identify the period 2008-2015 where the ZLB is biding during the overall Secular Stagnation era, accordingly emblematic of the financial crisis.
(ii) These statistics apply to the economy represented by Problem (25) and independent of any levels of wealth inequality and parameterization as per Table 2.
(iii) This represents the class of DMP-RBC models with Nash wage bargaining, essentially Shimer (2005).

Table 6: **Wage Propagation Mechanism: The Planner's Perspective**

	Polarization Traps		Baseline	Near-Completeness	Efficiency	
	Tech. Change	$\bar{G}^W = .83$	$\bar{G}^W = .80$	$\bar{G}^W = .70$	Efficient Fluctuations[(i)]	DMP
Cyclicality						
$\text{corr}(\widetilde{W}^{\text{pv}}, \tilde{y})$	.86	.87	.82	.91	.96	.96
Volatility Effects						
$\text{SD}(\widetilde{W}^{\text{pv}})$	2.50	2.03	2.57	3.51	4.12	.56
$\text{SD}(\tilde{\phi})/\text{SD}(\tilde{y})$	15.68	14.30	9.28	2.92	.00	.00
Mean Effects[(ii)]						
$\mathbb{E}(\widetilde{W}^{\text{pv}})$	24.27	22.41	20.90	16.76	The same as[(iii)] the decentralized counterparts	20.97
Price of Stakeholder-societal risk						
$\text{SD}\widetilde{W}^{\text{pv}}/\mathbb{E}\widetilde{W}^{\text{pv}}$	.10	.09	.12	.21	See the efficient counterpart of each case, i.e., $\text{SD}\widetilde{W}^{\text{pv}}_{\text{efficient}}/\mathbb{E}\widetilde{W}^{\text{pv}}_{\text{efficient}}$	.03
$\text{SD}\widetilde{W}^{\text{pv}}_{\text{efficient}}/\mathbb{E}\widetilde{W}^{\text{pv}}_{\text{efficient}}$	.17	.18	.20	.245		.03
Labor Supply						
$\partial \log \tilde{n}/\partial \log \widetilde{W}^{\text{pv}}$	.42	.76	.36	.10	.003	.27

(i) These labor market statistics apply to the economy represented by Problem (25) and are independent of any levels of wealth inequality and parameterization as per Table 2. Thus wealth inequality does not matter for business cycle fluctuations, as in standard RBC models.

(ii) These mean effects define the twin key notions of efficient wage markups and partial internalization in the present model.

(iii) As wealth inequality changes, so do the model's efficient steady state and corresponding mean values. What does not change, however, is the pattern of efficient deviations about the steady state, as noted in footnote (i).

Table 7: **Inspecting the Mechanism: Comparison**

	Polarization Traps		Baseline	Near-Completeness	Hall (2017)[ii]	Efficiency		
	Tech. Change	$\bar{G}^W = .83$	$\bar{G}^W = .80$	$\bar{G}^W = .70$		Efficient Fluctuations	DMP	RBC[iii]
A. Labor Market Elasticities[i]								
$\partial \log \widetilde{\theta}_t / \partial \log [\mathbb{E}_t \widetilde{M}_{t+1}]^{-1}$	-1.48	-2.47	-28.29	-2.34	-8.56	$-.69$	.24	n/a
$\partial \log \widetilde{\theta}_t / \partial \log \widetilde{X}_t$	-2.81	2.44	8.22	1.64	$-.16$	.45	11.88	n/a
$\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t$	.37	.55	.992	.94	.997	.92	1.20	1.00
B. Saving Behaviors								
Workers								
$\partial \log \widetilde{c}_t^n / \partial \log \widetilde{W}_t^{\text{pv}}$	.96	.91	.36	.41	n/a	.50	.58	1.16
$\text{corr}\left(\widetilde{y}_t, \widetilde{b}_{t+1}^n\right)$	.58	.36	$-.35$	$-.78$	n/a	.00	n/a	n/a
Capital Owners								
$\partial \log(1+\widetilde{r}_{t,t+1}) / \partial \log \widetilde{b}_{t+1}^s$	$-.22$	$-.33$	$-.22$	$-.06$	n/a	.00	.00	.00
$\text{corr}\left(\widetilde{y}_t, \widetilde{b}_{t+1}^s\right)$	$-.58$	$-.36$	.35	.78	n/a	.00	n/a	n/a
C. Consumption Heterogeneity[iv]								
$\text{SD}(\Delta\tilde{c}^s)$	2.45	2.23	2.01	2.07	n/a	.05	.45	.63
$\text{SD}(\Delta\tilde{c}^n)$	4.17	3.54	1.29	2.08	n/a	3.09	.45	.63
$\text{SD}(\Delta\tilde{c}^s)/\text{SD}(\Delta\tilde{c}^n)$	.59	.63	1.56	.996	n/a	.02	1.00	1.00
$\text{corr}(\Delta\tilde{c}^s, \Delta\tilde{c}^n)$	.06	.18	.70	.95	n/a	.77	1.00	1.00
D. Discounts and Financial Statistics								
$\partial \log \widetilde{y}_t / \partial \log [\mathbb{E}_t \widetilde{M}_{t+1}]^{-1}$	$-.30$	$-.31$	-1.36	$-.71$	n/a	$-.69$	.05	.10
$\mathbb{E}\tilde{r}^b$	-5.16	-5.43	$-.22$	2.865	4.96	3.32	3.98	4.00
$\text{SD}\tilde{r}^b$	5.40	3.05	2.01	2.59	6.11	3.16	.36	.23
$\mathbb{E}(\tilde{r}^e - \tilde{r}^b)$	5.19	5.625	3.37	2.12	n/a	1.56	0.02	0.00
E. Fixed-Income Wage Asset								
$\text{corr}(\Delta\tilde{w}^n, \Delta\tilde{c}^n)$	.93	.95	.67	.98	n/a	.99	.99	1.00
$\text{corr}(\Delta\tilde{w}^n, \Delta\tilde{y})$	.67	.60	-.20	.99	n/a	.99	.99	.97

(i) Elasticities are measured by regression coefficients with the first term as the dependent variable.

(ii) We replicate Hall's (2017) Table 5 in terms of elasticities.

(iii) RBC refers to Hansen's (1985) classic business cycle wage model. Elasticity is with respect to the competitive wage.

(iv) $\Delta\tilde{c}^s$ represents the annualized growth rate of shareholder consumption and analogously for $\Delta\tilde{c}^n$. Malloy et al. (2009) report $\text{SD}(\Delta\tilde{c}^s)/\text{SD}(\Delta\tilde{c}^n) = 1.63$ from the US data, while Mankiw and Zeldes (1989) report a figure of 1.60.

workers in this scenario. As a consequence, workers' equilibrium bond trading serves largely as insurance against unemployment risk. Such a separation result is confirmed by the statistic $\text{corr}(\Delta\widetilde{c}_t^n, \Delta\widetilde{w}_t^n) =$ .98. It signals that the worker's flow egalitarian wage path, directly drived from his present value wage arrangement, closely tracks his flow consumption, while also reinforcing the argument that the worker's countercyclical bond trading pattern ($\text{corr}(\widetilde{y}_t, \widetilde{b}_{t+1}^n) = -.78$) facilitates worker unemployment insurance rather than consumption risk sharing. From another perspective, $\text{corr}(\Delta\widetilde{c}_t^n, \Delta\widetilde{w}_t^n) = .98$ is almost identical to its DMP and efficient counterparts, confirming, once again, effective near market completeness derived from egalitarian wage bargaining in the $\bar{G}^{\text{w}} = .70$ case.

Having observed that workers' flow wage growth closely tracks their consumption growth when $\bar{G}^{\text{w}} = .70$, the observed simultaneous countercyclical pattern of the labor income share, $\text{corr}(\widetilde{y}_t, \widetilde{\ell}_t^s) =$ $-.50$, can be easily understood. In the present model, the stockholder's labor income share, being competitively determined in a CRS production setting, is constant, while the worker's share of output, being non-competitively determined, is not. With this observation in mind, the labor share of total income, including stockholder's share, can be decomposed as

$$\widetilde{\ell}_t^s = \text{const.} + \frac{\widetilde{w}_t^n \widetilde{h}_t^n \widetilde{n}_t}{\widetilde{y}_t} \approx \text{const.} + \frac{\widetilde{c}_t^n}{\widetilde{y}_t}, \tag{31}$$

since the interest income to workers is small (rates are low and the per capita bondholding of workers is small relative to firm owners). In other words, $\widetilde{c}_t^n$, total consumption aggregated across workers, being largely made up of their flow wage income, enables us to approximate the labor share $\widetilde{\ell}_t^s$ by the ratio of workers' consumption to total output in expression (31). If workers' consumption smoothing schedules are satisfactorily arranged over the business cycle, the ratio $\widetilde{c}_t^n/\widetilde{y}_t$ should be countercyclical, which will, in turn, lead to a countercyclical labor share statistic by relationship (31). The $\bar{G}^{\text{w}} = .70$ exactly makes the case: $\text{corr}(\widetilde{y}_t, \widetilde{\ell}_t^s) = -.50$.

The classical Coasian principle provides yet another perspective from which to rationalize the countercyclical labor income share. From the social planner's perspective, the cyclical pattern of *ex post* distribution risk, $\{\widetilde{\phi}_t\}$, is directly and positively associated with the worker's surplus sharing rule, $\{\widetilde{\eta}_t\}$, which we identify as reflecting an implicit "property right." The classic Coase theorem suggests that the distribution of "property rights" should not matter in internalizing the given externality, implying that the cyclicality of distribution risk could take any sign. In the $\bar{G}^{\text{w}} = .70$ scenario, nevertheless, it is unambiguously countercyclical: $\text{corr}(\widetilde{\phi}_t, \widetilde{y}_t) = -1$, with the natural interpretation that "worker property rights" increase in recessions.

In summary, stockholders are willing to supply bonds to workers at attractive prices leading to a near-to-complete effective financial market, thereby legitimizing present value (egalitarian) wage bargaining. Bond trading is then simultaneously freed up to allow workers to self-insure against prolonged periods of unemployment. Strong countercyclical worker property rights may thus be viewed as a bellwether for near-complete stockholder-worker consumption risk sharing.

The "Musk-Tesla" case This scenario reflects a different polar extreme: wealth inequality is greatest when $\bar{G}^{\mathrm{w}} = .83$. Distribution risk, $\widetilde{\phi}_t$, is now far more volatile than in the near-to-complete case, its volatility having increased monotonically from $\mathrm{SD}(\widetilde{\phi})/\mathrm{SD}(\widetilde{y}) = 2.90$ when $\overline{G}^{\mathrm{w}} = .70$ to $\mathrm{SD}(\widetilde{\phi})/\mathrm{SD}(\widetilde{y}) = 14.30$ when $\bar{G}^{\mathrm{w}} = .83$, suggesting a more pronounced pecuniary externality and an (expected) increase in worker property rights volatility, $\mathrm{SD}(\widetilde{\eta})$, by (29).

A natural concomitant of elevated distribution risk is a heightened demand for bonds by both agents. As a result, the equilibrium bond price skyrockets and default-free rates turn negative. In fact, as inequality grows, the default-free rate monotonically declines from $\mathbb{E}\widetilde{r}^b = 2.9\%$ ($\overline{G}^{\mathrm{w}} = .70$) to $\mathbb{E}\widetilde{r}^b = -5.43\%$ ($\overline{G}^{\mathrm{w}} = .83$), suggesting a steady increase in the precautionary demand for bonds. We note that high asset prices and low default-free returns have been characteristics of the Post-Financial-Crisis period of rising wealth inequality in the U.S.

As noted in the near-complete case, countercyclical discount rates, *ceteris paribus*, would normally allow workers to earn positive returns by buying bonds in highly persistent low-output, high-unemployment states. High prices notwithstanding, workers would be encouraged in this way to participate in "present value bargaining" assuaging worker fears of stockholder "limited commitment." Nevertheless, an even greater effect appears to supplant this countercyclical discount mechanism in the Musk-Tesla case: stockholders actually seek to reduce rather than enhance *their own* provision of safe assets, especially in low-output states as evidenced by the shift in $\mathrm{corr}(\widetilde{y}_t, \widetilde{b}^s_{t+1})$ from .78 ($\overline{G}^{\mathrm{w}} = .70$) to $-.36$ ($\overline{G}^{\mathrm{w}} = .83$) (Table 7.B). We diagnose the reason for this dramatic change through the lens of relationship (29), which allows us to relate J_t, the value to the firm of a new hire, to the firm's share of the joint match surplus S_t in the following way:

$$(1 - \eta_t)S_t = J_t = \frac{\partial V_t^f}{\partial n_t} \tag{32}$$

where $V_t^f = d_t + p_t^e$ is the *cum dividend* stock market value, the maximization of which is the firm's objective.

With the anticipation of more volatile yet still countercyclical distribution risk $\{\widetilde{\phi}_t\}$, by (22) the same is true for the worker's property rights $\{\widetilde{\eta}_t\}$. By (32), the volatility of the profit share to the firm of creating vacancies, J_t, increases, thereby reducing the firm's value at the employment margin, thus putting more downward pressure on the equity price p_t^e for the given safe interest rate. With the resulting increase in the equity risk premium, portfolio rebalancing considerations motivate stockholders to seek to elevate their precautionary demand for bonds, resulting in a higher equilibrium bond price. This increase in p_t^b, and the reduced availability of bonds to workers attendant upon it, further anticipating a high value of $\{\widetilde{\phi}_t\}$ particularly in low-output states, confirms stockholder expectations.[63] In equilibrium, this feedback loop from p_t^b to $\sigma_{\widetilde{\phi}}$ leads to less attractive capital ownership, with the equity premium increasing from 2.12% ($\overline{G}^{\mathrm{w}} = .70$) to 5.63% ($\overline{G}^{\mathrm{w}} = .83$) (Table 7.D). With the reduction in safe assets at the trough of the business cycle, $\mathrm{corr}(\Delta\widetilde{c}_t^s, \Delta\widetilde{c}_t^n)$ falls to the near-to-acyclical value of .18, signaling that Telmer's (1993) bond trading mechanism is playing only a tertiary role (Table 7.C).

Accordingly, a ripple effect due to the changing pattern of stockholder bondholding to $\mathrm{corr}(\widetilde{y}_t, \widetilde{b}_{t+1}^s) = -.36$, overshadows job search and matching in the labor market. Via negative default free rates, it is workers who appear to be paying stockholders a commitment "fee" to retain their participation in the bond market, a complete reversal of the near-complete pattern. In summary, deepening market incompleteness goes hand-in-hand with strong fears on the workers' side of "limited stockholder commitment."

In this environment of extreme financial market incompleteness, it is not obvious how present value bargaining should be bilaterally managed, especially from the worker's perspective. As noted in the discussion in Section 5.4.2, bargaining over the expected present value of egalitarian wages in new matches does not pin down the timing or magnitude of flow wage payments. It is also not the case that any pattern of lifetime wages with the same discounted present value can be created: in contrast to the near complete scenario, the wage rule of a one-time single large payment, for example, would be insufficient to replicate any desired stream of flow wages of equal present value over the lifetime via the financial market, especially in the midst of a safe asset shortage evidenced by negative rates. The choice of an equilibrium flow wage selection rule thus seems inevitable. Accordingly, we propose the *continuous-renegotiation wage rule* defined by formulation (23) in Proposition 3.2 where $\{\widetilde{\eta}_t\}$ follows a known stationary Markov process.[64] The absence of the Markov property would otherwise suggest history dependence on the Coasian distribution of bilateral property rights, something the scenario thus far is unable to preclude.[65]

The continuous-renegotiation wage rule thus bears a resemblance to Acemogolu's (2003) version of the Coase theorem, in the sense that Markovian, time-varying $\widetilde{\eta}_t$ in (23) not only represents the assignment of payoff-relevant "property rights" between two parties, which are not fully enforceable due to commitment concerns, but also a resulting failure to achieve (constrained) Pareto-efficient alloca-

[63]Indeed the one positive inducement to stockholders is, in fact, the average negative rate on bonds: workers are paying real consumption units to stockholders for selling bonds to them to the extent of 5.23% of their maturity value on average, which exceeds the steady-state return on capital.

[64]The entire economy is stationary Markovian. The same is thus true of $\{\widetilde{\phi}_t\}$ and the same then holds for $\{\widetilde{\eta}_t\}$.

[65]The latter observation would be a likely event only when wages across all matches, new or existing, are renegotiated every period, thereby reaching the maximum frequency of renegoitiation, otherwise bargained wages might rely on any history of distribution risk, $\{\eta_t\}$ $(t = 0, 1, 2, ...)$. In one extreme scenario, suppose, a firm-worker pair meets at time zero and bargains over $\widetilde{W}_0^{\mathrm{pv}}$, being equated to a sequence of state-contingent flow wages over lifetime, once and for all. It is clear, then, that any future flow wages, say, at time t should be a function of an initial distribution risk, η_0, requiring history dependence and thus mutual commitment, particularly when these wage arrangements will fall entirely within the realm of employer-employee relationships without resorting to any trades in the financial market. However, this scenario is unlikely to materialize in an environment where a shortage of safe assets might trigger concerns over limited commitment, as is evident in the Musk-Tesla scenario.

tions.[66] There is *prima facie* evidence of the latter observation: as wealth inequality rises, $\text{corr}(\widetilde{\phi}_t, \widetilde{y}_t)$, which directly gauges the worker's pay-off relevant property rights, falls to $-.52$ from an estimate of $-.99$. As a concomitant to intensifying wealth inequality, the declining pattern of $\text{corr}(\widetilde{\phi}_t, \widetilde{y}_t)$ can be interpreted as attenuated Coasian property rights on the worker side: the distribution of "property rights" in the Coasian sense thus appears to shift in favor of capital owners, an observation reflecting social commentary on the present US economy. From Table 6, the Musk-Tesla scenario is clearly one where market agents *cannot fully internalize ex ante* distribution risk,[67] but they have the ability to internalize it partially, through achieving the efficient level of $\mathbb{E}\widetilde{W}_t^{\text{pv}}$. To summarize, we argue that the flow wage rule (23) in Proposition 3.2 should be regarded as a dominant equilibrium selection rule in an environment where the firm's temptation to bargain in a discretionary manner cannot be ruled out, with workers' fears of limited commitment being correspondingly intensified.

Under the wage rule (23), the wage-productivity elasticity $\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t$, measured in present value terms, dramatically declines from .94 in the $\overline{G}^{\text{w}} = .70$ case to .55 when $\overline{G}^{\text{w}} = .83$.[68] That is, in the Musk-Tesla scenario, the phenomenon of what we refer to as the *semi-fixed wage asset* arises as an equilibrium outcome, implying that the expected present value of bargained wages in new jobs tends to be "sticky," amplifying the effect of productivity shocks and creating greater fluctuations in unemployment. All the key labor market variables thus experience significant volatility increases, both absolute and relative to output (Table 5.A).

Semi-fixed wage asset creation also means that the present value of bargained wages becomes increasingly disassociated from the present value of labor productivity, eventually creating more downward pressure on p_t^e through channel (32). Accordingly, the endogenous risk premia increase reduces the shareholder's provision of bonds so dramatically relative to the near-completeness case, that the sharp increase in bond prices produces an elasticity of $\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t$, strictly less than 1, another defining feature of the semi-fixed wage asset. This feedback loop, a vicious circle that defines what we refer to as a *polarization trap*, is novel: Equilibrium "sticky" wages, measured in present value terms and referred to as the semi-fixed wage asset, are a concomitant of the phenomenon of safe asset shortages. Key to this polarization trap is a failure to fully internalize the pecuniary externality $\{\widetilde{\phi}_t\}$, arising as a market outcome in a high-inequality economy. In a polarization trap under egalitarian bargaining, private agents do not fully internalize the efficient risk-sharing effects of sufficient safe asset provi-

[66]The exclusion of any histories of distribution risk in the *continuous-renegotiation* wage rule is comparable, at least in spirit, to the one that defines Markov perfect equilibrium in dynamic games. Markov perfect equilibrium in a dynamic game generally identifies itself as a situation in which agents cannot commit to each other, by writing "enforceable" contracts, thus seeking self-enforcing (equilibrium) outcomes. This concept is directly applicable to our Coasian context, where the inability to write "enforceable" contracts due to limited commitment may be viewed as "transaction costs," one of the key elements in the classic Coase theorem. In fact, overcoming a revisionist "transaction costs" would validate the classical Coase theorem in a bargaining game, resulting in what is often referred to as the *political* Coase theorem (Acemogolu (2003)).

[67]That is, the pecuniary externality $\{\widetilde{\phi}_t\}$, by transforming $\{\widetilde{\phi}_t\}$ into countercyclical and substantially volatile payoff-relevant property rights, $\{\widetilde{\eta}_t\}$.

[68]Incidentally, this fact mirrors a recent disconnect between wages and productivity in the US: the former has been static when the latter has risen.

sion, and thus the present value of bargained wages ends up being "sticky," thereby accelerating labor market volatility and driving down real interest rates into negative territory. Nevertheless, egalitarian bargaining arrangements have the capacity to substantially, though partially, internalize the pecuniary externality (*ex ante* distribution risk) by making it still possible to achieve the efficient level of $\mathbb{E}\widetilde{W}_t^{\text{pv}}$ (Table 6).

Remark 5.1. *In a heterogeneous agent set-up, Caballero and Farhi (2018) propose the safety trap hypothesis in which a shortage of safe assets triggers a deflationary equilibrium with an endogenous risk premium. While close in spirit to Caballero and Farhi's (2018) safety trap, the polarization trap developed here differs in some crucial aspects. First, the polarization trap is an episode of business-cycle fluctuations, representing deviations from a polarization-led efficient growth path. In contrast, the safety trap is a "news-driven" deflationary equilibrium characteristic of stochastic steady states, although it does not exclude the possibility of converging to a deterministic steady state with positive long-term rates after the realization of adverse "news shocks."*[69] *Second, in a polarization trap, the key to elevated aggregate volatility is the endogenous wage mechanism that generates increasingly "sticky" wages, measured in present value terms, in response to a shortage of safe assets. The safety trap, by contrast, arises only when the Zero Lower Bound (ZLB) is binding in a New-Keynesian-style economy with fixed prices. Lastly, negative safe asset rates arise as an equilibrium outcome in a polarization trap due to imperfect risk redistribution over the business cycle, while corresponding negative natural rates in the Caballero and Fahri model present themselves as a deus ex machina to replicate an equilibrium safety trap.*

To conclude the Musk-Tesla scenario, we explore the now-procyclical labor income share characteristic of both this case and the more recent historical period (Table 5.C). The intuition follows once again from the diminished risk sharing most evident in $\text{corr}(\Delta\widetilde{c}_t^s, \Delta\widetilde{c}_t^n) = .18$, a far cry from the corresponding value of .95 in the near-completeness case. Further evidence is provided by change in the relative shareholder/worker consumption growth volatility, $\text{SD}(\Delta\widetilde{c}^s)/\text{SD}(\Delta\widetilde{c}^n)$: while nearly 1 in the near-complete case, it is now .63.[70] Since effective consumption insurance must at a minimum raise worker consumption in low-output states, the evidence suggests this is no longer the case. A natural consequence is that workers' consumption share of output is becoming procyclical, and, by (31), the labor share of income as well.[71] The declining negativity of $\text{corr}(\widetilde{\phi}_t, \widetilde{y}_t)$ in response to an increasing wealth gap ($\overline{G}^{\text{w}} = .83$ vs. $\overline{G}^{\text{w}} = .70$) thus suggests that the distribution of "property rights" has shifted in favor of capital. Perhaps ironically, $\text{corr}(\Delta\widetilde{c}_t^n, \Delta\widetilde{w}_t^n) = .95$, which is similar to the counterpart statistic in the $\overline{G}^{\text{w}} = .70$ case, confirming that most worker consumption smoothing is being arranged through the wage contract within the firm and not by the ultra-thin bond market.

[69] More specifically, the "fear" of a bad Poisson event is a primary driving force for Caballero and Farhi's (2018) safety trap model. As long as such "fear" is not resolved, it is possible to have a *permanent* safety trap, comparable to the conventional Secular Stagnation hypothesis.

[70] $\text{SD}(\Delta\widetilde{c}^n) = 3.54$ in the present case versus $\text{SD}(\Delta\widetilde{c}^n) = 2.08$ under near-completeness, where $\text{SD}(\Delta\widetilde{c}^s)$ does not increase much at all.

[71] This suits stockholders well: their own consumption is stabilized if the workers receive their greatest share of the firm's output in high-output states where they can most afford it and vice-versa in low-output states.

The Baseline scenario lies midway between the prior cases with a relative distribution risk value of $\text{SD}(\widetilde{\phi})/\text{SD}(\widetilde{y}) = 9.28$. We are then led to suspect that the mean default-free rate $\mathbb{E}\widetilde{r}^b$ will also take an intermediate value, which turns out to be virtually a zero rate of interest: in equilibrium $\mathbb{E}\widetilde{r}^b = -.22\%$ (annualized). We next interpret this case.

Baseline Case As in the Musk-Tesla scenario and under the same logic, the capital owner's anticipation of a higher distribution risk $\text{SD}(\widetilde{\eta}) = \text{SD}(\widetilde{\phi})$ puts downward pressure on the equity price p_t^e through the firm's "capital risk" channel (32), which, in turn, drives up the risk premium to $\mathbb{E}(\widetilde{r}^e - \widetilde{r}^b) = 3.37\%$, while driving down the default-free return essentially to zero due to portfolio rebalancing by stockholders. Nevertheless, the stockholder's provision of default-free bonds, though diminished, is still operating as in the near-completeness scenario: $\text{corr}(\widetilde{b}^s_{t+1}, \widetilde{y}_t)$, while less than half its value in the $\overline{G}^{\text{W}} = .70$ case, is still positive. Further evidence for the operative Telmer (1993) consumption risk-sharing mechanism is $\text{corr}(\Delta\widetilde{c}^s_t, \Delta\widetilde{c}^n_t) = .70$, suggesting less effective bond trading than its near-complete counterpart, but nevertheless comparable to the efficient case: $(\text{corr}(\Delta\widetilde{c}^s_t, \Delta\widetilde{c}^n_t) = .77)$. To summarize, shareholder enhanced capital risk, the main driver for the shortage of safe assets, is no longer dominant in contrast to the Musk-Tesla case.

Indeed, all the evidence points to the conclusion that bond trading in the baseline scenario promotes consumption-risk sharing, to the extent that both workers' and stockholders' (annualized) consumption growth volatilities are the lowest among the three cases (Table 7.C). In this regard, the bond trading pattern in the Baseline scenario is close in spirit to Telmer's (1993) original scheme: bond trading directly affects the worker's consumption insurance plans through borrowing and lending. Yet, as is evident in Table (5.B), all the primary labor market variables in the baseline version indicate significant volatility hikes, coming close to their empirical counterparts in the US, unlike the near-complete case. These volatility hikes are puzzling since the elasticity $\partial \log \widetilde{W}_t^{\text{PV}} / \partial \log \widetilde{x}_t$, being close to 1, would obviate the need for large labor market fluctuations, insofar as productivity shocks are a single driving force. In the baseline version, however, this is not the case: both the aforementioned elasticity and labor market volatility are high.

To rationalize this seeming inconsistency, we look to the default-free rate, $\mathbb{E}\widetilde{r}^b$, for a reconciliation. Indeed it is nearly zero: an annualized rate of $\mathbb{E}\widetilde{r}^b = -.22\%$ becomes an $\mathbb{E}\widetilde{r}^b = -.06\%$ quarterly. Accordingly, the near zero real rate of interest implies a near-to-unitary financial discount factor, thereby creating a price "run-up" effect on all present value variables, other things equal. In particular, the wage assets, $\widetilde{W}_t^{\text{PV}}$, become "safe assets" in the sense that they are protected against moderate "discount rate shocks," with their common discounts displaying a virtually constant factor $(1 - \rho)$, other things constant.[72] This suggests that workers can use $\widetilde{W}_t^{\text{PV}}$ as "collateral" for "issuing" any flow wage stream with near-complete insurance against "discount rate shocks." The latter type of "safe" wage assets, if

[72] Due to the zero rate of interest, it follows that $\widetilde{M}_{t+1}(1 - \rho) \approx (1 - \rho)^{1+t}$.

suitably "securitized," may also present itself as an altenative asset to stockholders, creating a disincentive for them to accumulate more safe bonds in an environment where the capital risk channel (32) is triggered and equity risk premia are rising. More specifically, if "securitized" wage assets possess both low financial and business cycle income risk, then the former feature will present itself as a discount-protected safe wage asset for the firm to "hire," with the latter manifesting itself as a fixed-income safe wage asset to workers. In this sense, the reduced provision of safe short-term bonds (a decrease in the firm's short-term *financial leverage*) can be supplanted by the enhanced "supply" of long-term "safe" wage assets with near-to-complete insurance against financial shocks (an increase in the firm's long-term *operating leverage*). Provided it does not much alter their own consumption pattern, stockholders are otherwise willing to sell bonds paying such a low rate. Bonds have become like "money" for stockholders: the interest cost to either agent for risk sharing is essentially zero. Accordingly, workers can cheaply buy or sell bonds allowing themselves to 'securitize' their long term W_t^{pv} contract by creating any flow wage stream consistent with it. "Active" bond trading thus materializes, obviating the "commitment" issues associated with the equilibrium W_t^{pv} contract satisfying conditions (29) and (30), and facilitating present value bargaining arrangements by creating alternative "safe" wage assets at least along the dimension of financial risk.[73]

Along the prior line of reasoning, the baseline scenario, we conjecture, is one where this newly created *safe asset* identifies itself as an insurance policy against the twin perils of the worker's consumption risk and the stockholder's discount rate risk (financial risk). To understand this logic, imagine a fictitious *employment intermediary* that serves to design and circulate a financial contract, what we identify as the *fixed-income safe wage asset*, for the firm-worker pairs concerned. The contract is characterized by several risk-transferring requirements to be attractive to both agents. First, both parties agree that the productivity elasticity $\partial \log \widetilde{W}_t^{\mathrm{pv}} / \partial \log \widetilde{X}_t$ should be comparable to its efficient counterpart, which is close to 1, thus eliminating the need for large variations at the employment margin vis-à-vis any labor productivity shocks. Second, the associated flow wage streams $\{\widetilde{w}_t^n\}$ should be characterized by a low value of $\mathrm{corr}(\Delta\widetilde{w}_t^n, \Delta\widetilde{y}_t)$, suggesting the wage flow exhibits little systematic business cycle risk. Third, under the regime of zero interest rates, the wage asset $\widetilde{W}^{\mathrm{pv}}$ should display little discount rate risk, so that it can be accordingly "collateralized and securitized" as a safe asset against financial risk. More specifically, the wage-discount elasticity, $\partial \log \widetilde{W}_t^{\mathrm{pv}} / \partial \log[\mathbb{E}_t M_{t+1}]^{-1}$, should be "rigid": $|\partial \log \widetilde{W}_t^{\mathrm{pv}} / \partial \log[\mathbb{E}_t M_{t+1}]^{-1}| \ll 1$ (Table 8). Fourth, large unemployment fluctuations, if present, should be correlated with *realized* discount rate shocks (financial shocks), reflecting a high value of the elasticity of market tightness with respect to the discount rate, $\partial \log \theta_t / \partial \log[\mathbb{E}_t M_{t+1}]^{-1}$, often referred to as "Hall's (2017) discount channel."[74]

[73] The contractual relationship between the firm and worker now evolves from an "insider" one within the orgainzation of the firm (the $\overline{G}^{\mathrm{w}} = .70$) to the "outsider" one through a securitization process encouraged by bond trading.

[74] Hall's (2017) elasticity of market tightness with respect to the discount rate, $\partial \log \theta_t / \partial \log[\mathbb{E}_t M_{t+1}]^{-1} = -8.56$, represents his key labor market propagation mechanism. A one percent increase in the discount rate leads to a greater than eight percent decrease in market tightness for Hall's (2017) "credible bargaining model." Striking is the fact that in the present model, where the discount rate is endogenous while for Hall (2017) it is exogenously specified, the corresponding decrease in tightness exceeds

It turns out that the Baseline version exactly illustrates the latter equilibrium phenomena. As evidenced by $\text{corr}(\Delta\widetilde{w}_t^n, \Delta\widetilde{y}_t) = -.20$, the least over the three scenarios under consideration, the *fixed-income securitization* of wage assets arises as an equilibrium outcome in the present scenario. That is, the wage asset $\widetilde{W}_t^{\text{pv}}$ produces a "smooth" stream of flow wages, bearing a resemblance to a fixed-income security. The wage-productivity elasticity $\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t$, indeed, confirms our reasoning by taking a value of .992, close to the benchmark for wage flexibility (Table 7.A). The *ex post* labor market volatility in response to discount rates, $\partial \log \widetilde{\theta}_t / \partial \log [\mathbb{E}_t \widetilde{M}_{t+1}]^{-1}$, assumes a huge value of -28.16, implying the high volatility of key labor market variables (Tables 5 and 7). However, the quarterly standard deviation of equilibrium discount rates, $\sigma_{\widetilde{r}^b} = .50$, is the least over the three scenarios of interest (Table 8), confirming the wage asset as a discount-protected one. It appears that workers have no reason to refuse the latter contract, from their *ex ante* perspective. The wage-discount elasticity, $|\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log [\mathbb{E}_t M_{t+1}]^{-1}| = .39$, largely satisfies the "safety" requirement for further securitization described below (Table 8). Table 8 summarizes the discussion thus far.

Several additional comments are in order. Using this *fixed-income safe wage asset* as collateral, workers can insure themselves against business cycle consumption risk, producing a "smooth" stream of flow wages, while the firm is largely insured against variations in financial discount rates. The former insurance arrangement is reinforced by the *flexible* wage-productivity elasticity, which obviates the need for substantial variations in labor quantity. By contrast, the latter insurance scheme is viewed as a "discount-protected security," with any residual unemployment volatility being largely indexed to (small) fluctuations in discount rates, not in productivity, thereby exchanging *real* productivity risk for *small* financial risk. Insofar as the equilibrium volatility of discount rates, $\sigma_{\widetilde{r}^b}$, is expected to be low and stable under the regime of zero rates of interest, this alternative *safe* asset is thus bilaterally beneficial: *ex ante*, workers will expect to experience a small risk of unemployment, while being able to insure themselves against regular "high-frequency" business-cycle risk.[75] The firm's discount rate shocks are now fully "washed out" by the twin advantages of flexibility, i.e., promised wage flexibility as regards (present-value) productivity and (un)employment flexibility in response to discount rate shocks, thereby incentivizing the firm to "hire" this securitized safe wage asset, instead of reducing the provision of short-term safe bonds.

Hall's (2017) figure by a factor of more than three (-28.19 percent, Table (7.A)). The economics behind Hall's model is that some unspecified financial crisis creates high discount rate shocks with the annualized standard deviation of 6.11%, reducing investment in hiring and increasing unemployment. The present model, by contrast, accounts for the whole business cycle risk of the persistent pseudo "Liquidity Trap" era, the 1970-2015 period, in terms of the general equilibrium effect of zero real rates of interest with empirically reasonable small risk, implying the creation of alternative 'safe' wage assets. Thus, the former model might underestimate the tightness-discount elasticity, due to the excessive volatility of discounts presenting itself as an infrequent, one-time financial crisis. As regards the elasticity of tightness with respect to the present value of labor productivity, $\partial \log \widetilde{\theta}_t / \partial \log \widetilde{X}_t$, Hall (2017) records a somewhat counterintuitive $-.16$ percent, whereas the Baseline presents a more plausible positive 8.22 percent (Table (7.A)). Both baseline elasticities are consistent with high tightness volatility and its high correlation with output reported in Table 2.

[75] In the baseline version, the quarterly standard deviation of HP-filtered output, $\sigma_{\widetilde{y}}$, is 1.55 (Table 5). Thus, if the quarterly standard deviation of equilibrium discount rates is strictly smaller than the prior output counterpart, then workers can form the expectation that they might suffer from high unemployment risk with infrequent and small chances.

Table 8: **Discount Rate Channel**

	Polarization Trap $\bar{G}^W = .83$	Baseline $\bar{G}^W = .80$	Near Completeness $\bar{G}^W = .70$
$\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log [\mathbb{E}_t \widetilde{M}_{t,t+1}]^{-1}$	$-.395$	.39	-2.04
$\partial \log \widetilde{\theta}_t / \partial \log [\mathbb{E}_t \widetilde{M}_{t,t+1}]^{-1}$	-2.47	-28.29	-2.34
$\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t$	.55	.992	.94
$\mathbb{E}(\widetilde{r}^b)^{(\text{i})}$	-1.36	$-.055$	.72
$\sigma_{\widetilde{r}^b}{}^{(\text{ii})}$	.76	.50	.65

(i) Quarterly rates.
(ii) Quarterly standard deviations.

In summary, the Baseline case is one where asset price "run-ups" due to zero rates of interest offset the adverse effect of increasing equity risk, averting a polarization trap as in the Musk-Tesla case. First recall that the twin characteristics of a polarization trap are the "sticky" wage-productivity elasticity, $\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log \widetilde{X}_t \ll 1$ and unambiguous negative rates of interest, reflecting a shortage of safe assets (Table 8). In contrast, the creation of a fixed-income safe wage asset in the present scenario helps provide a disincentive for stockholders to accumulate more safe bonds, thereby avoiding the safe asset shortage characteristic of the Musk-Tesla case.[76]

Concluding our discussio of the Baseline case, we note that the countercyclical pattern of the labor income share, $\text{corr}(\widetilde{y}_t, \widetilde{\ell}_t^s) = -.20$, remains negative despite assuming a value less than one-half that of its near-complete counterpart. Since the worker's intertemporal substitution effect, $\text{SD}(\Delta \widetilde{c}^n)$, is smooth enough due to the fixed-income securitization, the labor share can still be approximated by the $\widetilde{c}_t^n / \widetilde{y}_t$ ratio using formula (31). What differs from the near-complete case, however, is that workers experience greater contemporaneous unemployment risk, which in turn renders flow wage income more volatile, thus reducing the countercyclicality of the labor share $\widetilde{\ell}_t^s$ and simultaneously amplifying its volatility (Table 5.C). Nevertheless, the Coasian principle also offers insight into the countercyclical labor share in the present case: the distribution of "property rights" still favors the worker side, resulting in $\text{corr}(\widetilde{\phi}_t, \widetilde{y}_t) = \text{corr}(\widetilde{\eta}_t, \widetilde{y}_t) = -.86$ with the interpretation that the "worker property rights" would shift up *pari passu* in recessions confirming an increased labor income share $\widetilde{\ell}_t^s$.

[76]Contrasted with the textbook Phillips curve, the tightness-discount elasticity $\partial \log \widetilde{\theta}_t / \partial \log [\mathbb{E}_t \widetilde{M}_{t,t+1}]^{-1}$ may be termed a Phelps-Phillips relation (curve), reflecting a trade-off between real rates of interest and aggregate employment (Phelps (1994)). The Baseline scenario underscores the Phelps-Phillips relation, particularly coupled with the "rigid" wage-discount elasticity $\partial \log \widetilde{W}_t^{\text{pv}} / \partial \log [\mathbb{E}_t \widetilde{M}_{t,t+1}]^{-1}$, by substituting short-term financial leverage (via safe bonds) for long-term operating leverage (via fixed-income wage assets). The other two cases, by contrast, exhibit "slack" Phelps-Phillips relations due, in large part, to the safe-asset shortage or Telmer's near-completeness mechanism, respectively (Table 8). Moreover, the Musk-Tesla case, reminiscient of the 2008 financial crisis, is one where the "securitization externality," a concept often emphasized in the literature (e.g. Caballero and Farhi (2018)), prevails in the sense that alternative 'safe' wage assets cannot be created by private agents, otherwise averting a polarization trap.

5.5 Bond Trading and Saving Patterns

We initiate our discussion by observing that as wealth inequality increases, $\text{corr}(\widetilde{b}^n_{t+1}, \widetilde{y}_t)$ switches signs from $-.78$ ($\overline{G}^W = .70$) to .36 ($\overline{G}^W = .83$), with the intermediate ($\overline{G}^W = .80$) value of $-.35$ while the reverse pattern holds for stockholders (Table 7.B). As noted earlier, we focus on interpreting the bond trading patterns in light of the precautionary savings motive for which there is ample evidence.[77] We explore the same three scenarios as in Section 5.4.2.

Near completeness With $\text{corr}(\widetilde{b}^n_{t+1}, \widetilde{y}_t) = -.78$, workers are increasing their bondholdings in low productivity states and reducing them in high productivity ones. Stockholders are doing the opposite, $\text{corr}(\widetilde{b}^s_{t+1}, \widetilde{y}_t) = +.78$, increasing their holdings in high productivity states and selling them to workers in low productivity states. Under the regime of countercyclical discount rates, stockholders are selling bonds to workers at low prices and buying them back from workers at high prices, in which event they pay workers high returns ($\mathbb{E}\widetilde{r}^b = 2.86\%$). This trading pattern suggests that workers are accumulating bonds for precautionary purposes in the face of possibly extended periods of high unemployment, which is their principle risk.

Related evidence for this conclusion comes from the elasticity of worker flow consumption with respect to the present value of the contract wage: $\partial \log c^n_t / \partial \log W^{\text{pv}}_t$ (Table 7.B). With net bond income small for workers relative to their contract wage, the latter's present value, W^{pv}_t, closely approximates a worker's permanent income (PI). Accordingly, we will refer to $\partial \log c^n_t / \partial \log W^{\text{pv}}_t$ as the workers marginal propensity to consume out of his permanent income (MPCPI). From the perspective of Friedman's (1957) Permanent Income Hypothesis (PIH), this elasticity should be approximately one, which is the case for a classic RBC model (Table (7.B), next to right-most column). In the present model (as in the DMP version) this elasticity is much less than one ($\partial \log c^n_t / \partial \log W^{\text{pv}}_t = .41$) suggesting that workers eagerly acquire bonds under low wealth inequality when their PI increases, a fact that suggests there are lots of bonds to be acquired at favorable prices, pointing to a well-functioning bond market with

[77] There are two related motivations for worker bond trading, direct contemporaneous consumption smoothing, and the accumulation of bonds for precautionary purposes. If the former motivation played a dominant role, then workers would seek to accumulate lots of bonds from firm owners at low prices in high productivity-high output-high wage states, while selling them back to the firm owners at high prices (or borrowing at low rates) when productivity, wages and output were low: what is described in this trading scenario is essentially the bond trading pattern in Guvenen (2009). The scenario just described is consistent with $\text{corr}(\widetilde{b}^n_{t+1}, \widetilde{y}_t) > 0$, countercyclical bond prices and procyclical discount rates. None of these effects is observed in the present model — discount rates are countercyclical, while worker bond holdings are countercyclical conditional on the degree of wealth inequality.

cross-agent consumption allocations close to Pareto-optimal.[78] As noted earlier, $\text{corr}(\Delta\widetilde{c}_t^s, \Delta\widetilde{c}_t^n) = .95$.[79]

We interpret workers' desire to accumulate bonds in low-productivity states as an illustration of Carroll's (1992) buffer stock savings story. Carroll (1992) proposes that workers will increase their financial asset holdings (bonds in the present case) in low-productivity states to protect their consumption against prolonged periods of unemployment.[80] Although the present model presumes perfect risk sharing within the family, unemployment is both countercyclical, highly persistent, and quite high when $\overline{G}^W = .70$, ($\text{corr}(\widetilde{u}_t, \widetilde{u}_{t-1}) = .92$ and $\overline{u}^r = 10\%$ for the worker group), suggesting significant (and persistent) average consumption reductions within the worker-family in low-productivity states. The countercyclical worker bondholding in the present model is consistent with Carroll's (1992) view. Furthermore, the steady-state ratio $\overline{b}^n/\varpi^n \overline{h}^n$ is small across all the wealth inequality cases, the equivalent of approximately 5 months of flow labor income (Table 3), suggesting that bondholding represents a relatively small fraction of a worker's average wealth and thus that an increase in bond holdings in low-output states has a small immediate impact on worker consumption and welfare.[81]

Remark 5.2. *Note that the stockholder-worker in the standard DMP model is also a "buffer-stock saver" in the face of employment fluctuations. Other things constant,* $\partial \log \widetilde{c}_t^n / \partial \log \widetilde{W}_t^{pv} = .58$ *in Table (7.B), which suggests that this elasticity is still a good benchmark for labor market studies, although its Nash-bargained wage is too responsive in the face of productivity shocks (Shimer's (2005) critique). In the DMP model, the worker is also a capital stock owner for whom investment plans can be devised to smooth out his consumption, which accounts*

[78] Friedman's (1957) original PIH implies that the marginal propensity to consume out of transitory income shocks is about one-third, not much different from the Baseline estimate (.36). Rigorously speaking, our MPC comes out of variations in the worker's permanent income *per se*, measured in present value terms, while the Friedman counterpart depends on transitory "windfall" shocks. The present formulation $\partial \log c_t^n / \partial \log W_t^{pv}$ does not, however, make a sharp distinction between variations in transitory and permanent income, as business cycle risk is a primary driving force in all decentralized cases. More in support of the present paper's perspective, Friedman (1957) repeatedly emphasizes the importance of precautionary savings induced by uncertainty over future levels of labor income. Carroll (2009) demonstrates that the MPCPI will be between 0 and 1 provided that the worker's precautionary saving effects are substantial and an impatience condition preventing explosive savings is satisfied. In the present model framework, the χ^n parameter not only controls the worker's precautionary savings motive but also represents his relative impatience ($\chi^s > \chi^n$).

[79] For these reasons, a MPCPI $\ll 1$ also represents the degree of the worker's precautionary savings motive for buffer stock purposes.

[80] In a simple partial equilibrium consumption-savings context, Deaton (2001) illustrates the same phenomenon as in the Baseline case. Using a two-state exogenous growth model of Hamilton (1989), he presents a generic solution where the agent increases his holding of bonds at the start of a low output growth state and dissaves whenever the good state returns. In Deaton's (2001) growth model, "when consumers follow their optimal consumption policy, savings is countercyclical." He interprets this as follows: when the bad state strikes, consumers start to save as part of 'riding it out.' When high growth boom times return, they seek to reduce their asset holdings. In Deaton's (2001) model "the savings ratio falls sharply at the end of a boom, rises at the start of a slump and is zero in a well-established boom." The present case illustrates this same phenomenon in a general equilibrium setting.

[81] The remaining question is whether the savings patterns described above, especially that of workers, are empirically plausible. Carroll (1992) provides some supporting evidence. Anecdotally, Carroll (1992) reports that an August 1991 Gallup Poll in Britain, while Britain was in recession, found that 60% of households thought it a good time to increase their savings when 73% expected unemployment to rise. One of the poll directors is quoted as saying, "consumers everywhere were inclined toward precautionary savings to provide a cushion against the threat of prolonged unemployment." Figure 8 of Carroll (1992) shows that, on average, the US personal savings rate does not decline as the economy enters a recession. Lastly, Figure 9 of the same paper indicates that US households typically reduce their debt burden during recessions when income is temporarily low, a phenomenon also observed during the Covid-19 pandemic, and consistent with Figure 1b. Taken together, this evidence is consistent with workers' countercyclical equilibrium bond holdings in both the $\overline{G}^W = .70$ and Baseline cases. In addition, the results just discussed represent equilibrium outcomes, whereas Carroll's (1992) and Deaton's (1991) analyses are partial equilibrium.

for the higher MPCPI (.58 (DMP) vs .41 in the $\overline{G}^{W} = .70$ *case).*

Since it is the stockholders who trade bonds with workers their desired trading pattern must be consistent with the scenario described above. In equilibrium, stockholders are willing to acquire more bonds in high-output states at high prices (discount rates are low) and then sell some of them to workers (reduce their holdings) in low output-low price-high discount states ($\text{corr}(\widetilde{b}^{s}_{t+1}, \widetilde{y}_t) = .78$) for two reasons. First, this trading pattern tends to stabilize stockholders' own consumption at relatively little cost: they are vastly wealthier than workers (Table 7) and, in any event, own most of the bonds. Second, the willingness of stockholders to trade bonds facilitates the mutual acceptance of present value wage bargaining in the present incomplete markets setting. Workers, in particular, are indifferent to the time pattern of their wage payments and thus are willing to bargain over their present value when markets are complete and transaction costs low.[82] Each of theses conditions is largely satisfied in the present case.

As a concluding observation, we note that bond trading under near completeness resembles the "unemployment savings account" proposed by Feldstein (2005). In low-output states of nature, the firm readily sells default-free bonds to workers at attractive prices, behaving as if it were "persuading" workers to save in their "unemployment savings accounts." Feldstein's (2005) policy proposal is thus implemented as a private market equilibrium outcome in the present model when $\overline{G}^{W} = .70$. These observations confirm bond trading as facilitating unemployment insurance while egalitarian wage bargaining cum asset creation facilitates consumption smoothing.

The Musk-Tesla Case As noted earlier, what distinguishes this case the most is the radical change in the pattern of stockholder bond trading leading to a "safe asset shortage," as discussed in Section 6.3.2 — $\text{corr}(\widetilde{b}^{n}_{t+1}, \widetilde{y}_t) = +.36$, and $\text{corr}(\widetilde{b}^{s}_{t+1}, \widetilde{y}_t) = -.36$ (Table 7.B). We confirm the intuition by considering the changing pattern of the elasticity of the bond's return to changes in stockholder bond holdings, $\partial \log(1+\widetilde{r}_{t,t+1}) / \partial \log \widetilde{b}^{s}_{t+1}$, as wealth inequality increases (Table 7.B). Adopted from Mian et al. (2021), this statistic measures agents' non-homothetic savings behavior in the sense that as the price of safe assets rises, their demand increases as well, mimicking the demand pattern for certain luxury goods. While Mian et al. (2021) introduces non-homothetic preferences to capture the aforementioned savings behavior, the present model replicates it as an equilibrium outcome under high wealth inequality. Based on the earlier discussion, we would anticipate this quantity as being negative: as stockholders wish to retain more bonds, prices rise and rates fall. In the present environment, this elasticity changes from $-.06$ ($\overline{G}^{W} = .70$) to $-.33$ ($\overline{G}^{W} = .83$) as wealth inequality grows, suggesting a greater and more consistent price reaction to increased stockholder demand when inequality is high. In contrast to the present model, the standard DMP and RBC models without the pecuniary externality should display

[82] As such, workers-households can distribute the present value of their real wages over their infinite lifetime in any pattern that suits them best.

homothetic savings behavior suggesting an aforementioned elasticity value of about zero, as our calculations reveal (Table 7.B). The low wealth inequality case $\overline{G}^{\mathrm{W}} = .70$ displays an elasticity value of $\partial \log(1+\widetilde{r}_{t,t+1})/\partial \log \widetilde{b}^{s}_{t+1} = -.06$, close to zero, and confirming the near absence of an effective pecuniary externality in that case. As expected, this elasticity is also zero in the DMP case with no pecuniary externality by construction.

Alongside the stockholder's non-homothetic saving pattern in the present case, the worker MPCPI, $\partial \log \widetilde{c}^{n}_{t}/\partial \log \widetilde{W}^{\mathrm{PV}}_{t}$, reaches .91. A high MPCPI close to one typically signals that workers have nearly zero liquid wealth and the inability to borrow further, a case reminiscent of Deaton's (1991) hypothesis about an asymptotic MPCPI of one: a "liquidity-constrained" consumer will eventually run down his (initial) positive "buffer" assets to zero, ending up being an MPCPI of one in a partial-equilibrium context.[83] What differentiates our result from Deaton's conjecture, however, is that it is the general equilibrium effect of a safe-asset shortage on elevated unemployment fluctuations that drives up $\partial \log \widetilde{c}^{n}_{t}/\partial \log \widetilde{W}^{\mathrm{PV}}_{t}$ to .91, despite the fact that the worker buffer assets, on average, are equal to 4.89 months of labor income (Table 3). The economic reasoning behind these theoretical findings is straightforward. In a polarization trap, where the twin perils of high distribution risk and large unemployment fluctuations are anticipated, the substantially negative rate of safe bonds discourages further safe asset accumulation by workers who have "homothetic" precautionary saving purposes, unlike stockholders who are now becoming non-homothetic savers.[84] To obtain a similar MPCPI of .91, by contrast, a benchmark buffer-stock model would require an equivalent of .64 month of labor income as the worker's average buffer assets in an environment where large idiosyncratic risks and low risk aversion are presumed. Thus, despite being a "stripped-down" version of the Carroll-Deaton class of buffer-stock models, the present model suggests that considerable buffer stock savings, approximately 5 months of labor income in the Musk-Tesla case (Table 3), in conjunction with substantially negative interest rates are consistent with a "Deaton-asymptotic MPC" close to one. The semi-fixed wage asset, the worker's alternative income stabilization device in a polarization trap, further 'crowds out' safe bond accumulation, creating another disincentive for workers to buy bonds at substantially disadvantageous prices.[85]

The high MPCPI result in the present scenario resembles an episode of "deleveraging shocks" in a liquidity trap environment as frequently emphasized in the maro-prudential policy literature. The root cause of "deleveraging shocks" in the present context is a shortage of safe assets, and thus the provision of conditional safe assets, if implementable, should be promoted or equivalently, more borrowing should be undertaken by high-MPCPI workers, standing 180 degrees to standard macro-prudential

[83]What it is meant by "liquidity-constrained" here is that workers have a borrowing constraint. Our model framework and Carroll's (2009) buffer-stock model, for instance, do not impose any explicit borrowing constraints.

[84]The present model assumes that both agents have 'conventional' homothetic preferences.

[85]In contrast, note that the worker's buffer assets in the near-complete case are equivalent to 5.67 months of labor income. Relative to the near-complete scenario, the worker's buffer-stock assets in the Musk-Tesla case decline by 14% while the worker's MPC increases by more than 200%. Such is the power of negative rates and the available semi-fixed wage asset to alter savings behavior in equilibrium.

doctrines.[86] Moreover, and equally importantly, the worker MPCPIs in the present model are structural relations: the MPCPIs are not policy-invariant variables, as they would change *pari passu* with polarization-driven economic growth arising from the Musk-Tesla growth factor. This observation is a reminder of Lucas's (1976) famous critique that the consumption function, and thus its associated MPC, are not invariant to changing macroeconomic scenarios.

The Baseline The baseline version is again a transition case between the prior two. As described in Table 7, there is evidence both of reduced stockholder willingness to supply bonds to workers (measured by the shareholder's non-homothetic saving behavior, $\partial \log(1+\widetilde{r}_{t,t+1})/\partial \log \widetilde{b}^s_{t+1} = -.22$) and the sufficient provision of default-free bonds to workers ($\text{corr}(\widetilde{b}^s_{t+1}, \widetilde{y}_t) = -.36$) relative to the Musk-Tesla case. In fact, we need to explain why $\text{SD}(\Delta\widetilde{c}^n)$, and $\partial \log \widetilde{c}^n_t / \partial \log \widetilde{W}^{\text{PV}}_t$ assume values the least over all the cases we consider. While these facts alone suggest bountiful bond trading and the enhanced ability of workers to construct consumption streams different from their flow wage streams, risk sharing is nevertheless not perfect ($\text{corr}(\Delta\widetilde{c}^s_t, \Delta\widetilde{c}^n_t) = .70$), and much less perfect than in the $\overline{G}^{\text{W}} = .70$ case, where $\text{corr}(\Delta\widetilde{c}^s_t, \Delta\widetilde{c}^n_t) = .95$. The worker's MPCPI, $\partial \log \widetilde{c}^n_t / \partial \log \widetilde{W}^{\text{PV}}_t$, also declines somewhat from its corresponding $\overline{G}^{\text{W}} = .70$ value, taking a value of .36, comparable to Friedman's (1957) original MPC of one-third.

Despite the fact that the relative distribution risk volatility increases by a factor of three relative to its near-complete counterpart ($\text{SD}(\widetilde{\phi})/\text{SD}(\widetilde{y}) = 9.28$, vs. $\text{SD}(\widetilde{\phi})/\text{SD}(\widetilde{y}) = 2.92$ under near completeness), the degree of precaution savings, as measured by MPSPI $= 1 -$ MPCPI, increases by a much more modest 8.47% (.59 vs. .64 in the Baseline where MPCPI $= .36$). As the near-to-zero interest rate environment of the present case suggests a "liquidity trap" where all types of default free assets are in short supply, it is surprising that workers' MPSPI is not closer to zero (MPCPI closer to one). This latter effect is not observed because of the presence of the wage asset in workers' portfolios priced as an infinitely-lived Lucas (1978) tree at $\widetilde{W}^{\text{PV}}_t$. In an environment of very low interest rates, all asset values substantially increase, implicitly increasing asset availability. Asset values do not explode, however, as the phenomenon under discussion is exclusively business cycle related: average default free rates in the very long run are around 4% (Table 4).

In summary, despite the presence of low interest rates, bond trading is not discouraged; bonds are modestly plentiful promoting a significant level of risk sharing. In this sense, it appears that a "policy" of effective Forward Guidance arises endogenously within the model purely as a market outcome

[86] Borrowers with a high MPCPI close to one cannot borrow anymore in the face of "deleveraging shocks," resulting in a sharp decrease in aggregate demand due to nominal rigidities and their resulting aggregate demand externalities. Accordingly, the corresponding economy enters a (Keynesian) liquidity trap. In response, available macro-prudential policy prescriptions for the above episode of "deleveraging shocks" (e.g., Farhi and Werning (2016) and Korinek and Simsek (2016)) are found to restrict "excessive" leverage mostly undertaken by high-MPCPI borrowers. To fulfill such a policy goal, a macro-prudential tax on borrowing is imposed, and thus the borrower's MPCPI becomes a key policy variable and sufficient statistic for the design of optimal macro-prudential taxation.

without a need for eloquent announcements from monetary authorities.

5.6 Technological Change: An Alternative Route to Increasing Inequality

As an alternative to reducing the measure of stockholders, we propose a zeitgeist-inspired thought experiment where "capital-biased" technological change increases the share of income to capital in all states of the Baseline economy. In particular, we increase the production parameter α from .36 to .383 while otherwise retaining the baseline parameterization (Table 1). Our motivation for this parametric change follows from Caballero et al.'s (2017) empirical finding of a corresponding increase in α for the recent 2008-2015 historical period. We propose to view it as a result of increased "automation." This means a greater share of income is accruing to an unchanging baseline measure of the population ($\mu = .10$).

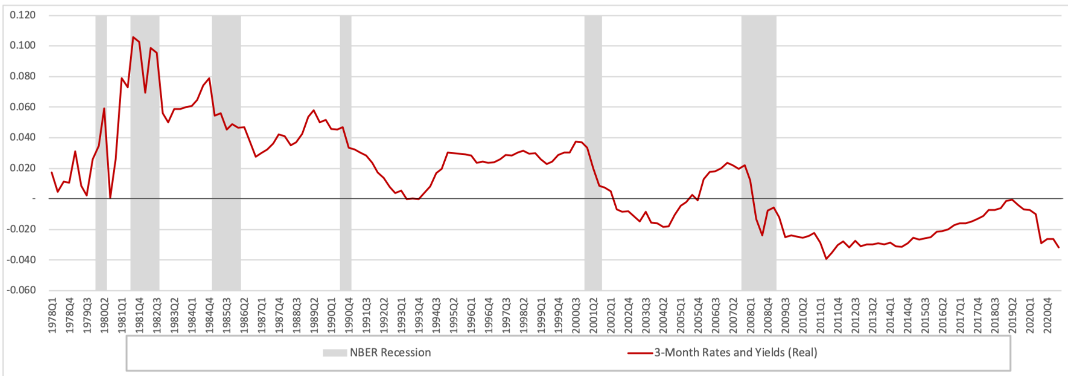
Relative to the Baseline, the increased share of income to capital stimulates investment along the growth path, leading to substantial capital deepening (Table 4). With more capital, labor becomes more productive and the mean wages of both agents increase (Table 4). As a result, mean consumption and welfare of both agents also improves, proportionately more for the stockholders who own the additional capital. Although stockholders are clearly the primary beneficiaries of the change, this "automation" effect is nevertheless an alternative engine of (socially efficient) economic growth to the Musk-Tesla mechanism. Our discussion of the present "automation" mechanism is thus relegated to part I of the Appendix. What differs from the Musk-Tesla case is the decline in the workers' overall income *share* relative to the baseline economy, a fact consistent with recent empirical findings of a declining labor share.

5.7 Empirics

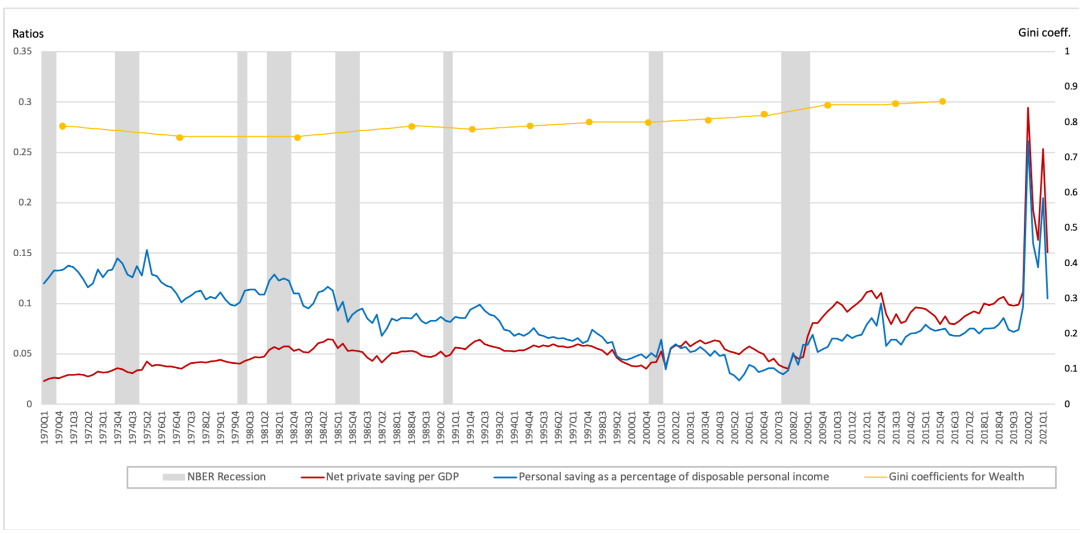
The prior analysis strongly suggests that increasing wealth inequality in a stakeholder society is likely to be accompanied by progressively declining real interest rates and greater labor market volatility, the former phenomenon reflecting the increasing shortage of safe assets and the latter reflecting the stakeholder economy's response by creating the risk-sharing-sensitive wage asset alternative. Are these implications reasonable? While we do not propose that egalitarian bargaining, the foundation of the stakeholder society, has been a feature of labor negotiation in the past, it is curious that recent patterns in the data illustrate these conclusions. This is evident in Table 5 and Figure 1 below where we briefly compare data from 2008-2015, a period of unquestioned high wealth inequality,[87] with its earlier, more comprehensive counterpart. Four significant data changes (facts) are evident:

1. The cyclical behavior of the labor share changes sign from a negative $\text{corr}(\widetilde{y}_t, \widetilde{\ell}^s_t) = -.10$ in the

[87] The period 2008-2015 is what Eggertsson et al. (2019a) refers to the Secular Stagnation era where the ZLB is binding.



(a) Time Series of U.S. Real Interest Rates
1978.1-2020.4



(b) Time Series of U.S. Net Private Savings and Net Personal Savings with the U.S. Gini Coefficient: 1970.1 2021.4

Figure 1: "Secular Stagnation"

earlier period to a positive $\text{corr}(\widetilde{y}_t, \widetilde{\ell}^s_t) = .16$ for the more recent one.

2. Increasing labor market volatility: the volatilities of vacancies, unemployment (and thus also employment) and labor market tightness all increase for the 2008-2015 period, both absolutely and relative to output.

3. Real interest rates turn negative from a positive $\mathbb{E}\widetilde{r}^b = .92$ (1970-2015) to a negative $\mathbb{E}\widetilde{r}^b = -2.85$ (2008-2015).

4. Referring to Figure 1b, savings as a share of GDP appears generally to increase prior to the start of an NBER-dated recession, while then declining in the immediate post-recessionary period. We interpret this phenomenon as evidence of substantial precautionary savings, a fact argued in

Carroll (1992, 2009).

Within the present model framework, these four phenomena will be shown as consequences of increasing wealth inequality. They are especially evident as the economy transitions from the Baseline case $\overline{G}^{\mathrm{w}} = .80$ to either "Polarization" scenario. See Table 5, where we restrict our attention to entries corresponding to $\overline{G}^{\mathrm{w}} = .70$, .80 (Baseline) and .83, where higher inequality arises from a shrinking measure of stockholders.

Considering the labor share (Fact 1), the corresponding model statistics change sign in a parallel fashion: when $\overline{G}^{\mathrm{w}} = .80$, $\mathrm{corr}(\widetilde{y}_t, \widetilde{\ell}_t^s) = -.20$ increasing to $\mathrm{corr}(\widetilde{y}_t, \widetilde{\ell}_t^s) = .27$ in the Polarization Trap case ($\overline{G}^{\mathrm{w}} = .83$), reflecting the shift in property rights more in favor of stockholders as wealth inequality expands. Labor share volatility together with labor market volatility along the standard dimensions of vacancies, tightness, and unemployment all increase (Fact 2) in both the later data period and its model counterpart replicated by the transition from $\overline{G}^{\mathrm{w}} = .80$ to $\overline{G}^{\mathrm{w}} = .83$. The real interest rate decline evident in Figure 1 (Fact 3) is also a key feature of the model data: $\mathbb{E}\widetilde{r}^b = -5.43$ when $\overline{G}^{\mathrm{w}} = .83$, a decline from $\mathbb{E}\widetilde{r}^b = 2.87$ when $\overline{G}^{\mathrm{w}} = .70$. As noted in the discussion earlier in Section 5, the interest rate decline in the model is closely related to the changing pattern of stockholder bondholdings: as wealth inequality becomes more extreme, $\mathrm{corr}(\widetilde{y}_t, \widetilde{b}_{t+1}^s)$ changes sign from $+.35$ (Baseline) to $-.36$ ($\overline{G}^{\mathrm{w}} = .83$) (see Table 7.B). Lastly, the increase in Savings/GDP prior to NBER-dated recessions we interpret as evidence of substantial precautionary savings, as argued by Carroll (1992, 2009), a phenomenon integral to the understanding of our high wealth inequality cases.

Despite the lack of any direct evidence that egalitarian bargaining was predominant during the period of interest, these identifications nevertheless suggest that a stakeholder economy is not likely to display aggregate behavior at business-cycle frequencies radically different from what has been recently observed. Building upon the latter correspondence, furthermore, we reason that the (Baseline) model counterpart of the 1970-2015 historical period can internalize 60% of the pecuniary externality (Table 6): by the same criterion, the hypothetical "near-complete" case eliminates 86% of the externality, while the Polarization Trap scenario, lying at the extreme opposite, only internalizes 50%. More importantly, the benefit of these alternative stakeholder-societal policies can be identified by directly contrasting the Polarization Trap case with more traditional models of secular stagnation. Benigno and Fornaro (2017), for example, provide a theory of *stagnation traps* in which widespread pessimism among market agents could depress a Schumpeterian-growth-type economy with nominal wage rigidity, so that the economy might fall into a high unemployment and low productivity growth trap by hitting the ZLB. Stagnation traps thus are secular changes in unemployment and productivity growth rates, suggesting the possibility of multiple steady state equilibria. The Polarization Trap, by contrast, is an episode of business-cycle-frequency deviations from the (constrained) Pareto-efficient steady state level, as determined by the Musk-Tesla endogenous growth factor while, on average, achieving long-run efficiency by classic Coase Theorem logic. Thus, the stakeholder equilibrium, at worst, features *cyclical stagnation* rather than *secular stagnation*.

6 Conclusion

In this paper we propose one route to a more inclusive society. Our context is the prevailing one where a small measure of the population, stockholders, alone supply the stochastic discount factor that governs the allocation of capital, thus leading to a large and pervasive pecuniary externality. We view this externality as antithetical to the notion of an inclusive society. Accordingly, the paper explores the extent to which it can be purely privately internalized, without wealth redistribution, through a combination of bond trading in the firm's own debt and egalitarian present value bargaining in the labor market. Egalitarian bargaining, a generalization of Nash bargaining, presumes workers and stockholders share the surplus of an employment match by equating their lifetime welfare gains rather than sharing the present value wealth gains as in standard Nash bargaining theory.

In the model's incomplete financial market setting, countercyclical Coasian distribution of worker property rights arise as an endogenous natural consequence of egalitarian bargaining. This shifting distribution of property rights manifests itself in the form of wage payments representable as low-risk present value wage assets characterized by efficient wage markups that contribute to the worker's ability to share risks. In the long run, the decentralized equilibrium is constrained efficient, although its deviations from the steady state are not. Using measures appropriate to the model we find that in our benchmark case roughly 60% of the pecuniary externality can be internalized, representing a significant step towards a more inclusive society.

"At the end of the day," what does our analysis have to say about allocations in a "more inclusive society," without equity ownership reallocation? First and foremost, wage markups will increase with increasing wealth inequality. In response, firms seek to retain more of their bonds for precautionary purposes as expected risk premia rise. With risk sharing via bond trading thereby attenuated, worker income insurance will become increasingly arranged through the labor contract with the value of the wage asset becoming progressively disassociated from labor productivity, thus leading to firms' increasing bond retention with consequent further declines in default free interest rates into negative territory. Despite this feedback loop, what we refer to as the polarization trap, the stakeholder society nevertheless internalizes about 50 % of the externality, resulting in the twin consumption-smoothing distortions of the capital owner's non-homothetic saving behavior and workers' buffer-stock-like savings with the worker MPCPI becoming close to one. The latter phenomenon surprisingly resembles key features of the 2008-2015 financial crisis where US wealth inequality was historically high.[88]

[88] Capital owners who, at first glance, seem to be receiving the "short end of the stick" under "more inclusive" egalitarian bargaining do receive some benefits from this arrangement, however: since the allocations remain constrained efficient, the

We conclude with a general comment concerning the prevailing aggregate demand (AD) perspective for avoiding "liquidity traps." The typical AD story goes as follows. There are two distinct household groups in the economy, the constrained and the unconstrained. When the constrained get hit by "deleveraging shocks," well-tailored macro-prudential policies to overcome the ZLB would incentivize the unconstrained households to accelerate their own spending thereby offsetting the original demand decline. One difficulty with this AD reasoning lies in the observation that the unconstrained are usually the rich who represent too small a population to pick up any substantial aggregate demand shortfall. Moreover, a fall in the real interest rate, a desirable adjunct to AD policies under a ZLB regime, might only incentivize the rich to accumulate more safe assets rather than increasing their household spending, as not only in the present model, but also in Mian et al.'s (2021) analysis of the recent situation in the United States. In summary, the cost of inequality and the notion of "stakeholder capitalism" should be given greater consideration. We suggest that the modest proposal for doing so suggested in the present paper could be a reasonable step in that direction.

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classical Chamley-Judd zero capital tax should survive in the long run. Curiously, the fact that growth in the present model is motivated by increasing productivity of a shrinking base of capitalists, in the Musk-Tesla tradition, our model lends support to Piketty's "Capital in the 21st Century Perspective": even as the capital stock grows, the real return on capital does not diminish. In contrast to Piketty's (2014) "French Theodicy" of advocating wealth redistribution in the 21st Century, however, the present social planner offers a more modest "Coasian Theodicy": egalitarian present value wage bargaining leading to a shift in the distribution of "property rights" in favor of labor manifested as constrained efficient "wage markups," an important aspect of wage asset creation.

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