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Currency Factors

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Abstract. We examine the ability of existing and new factor models to explain the comovements of G10 currency changes, measured using "currency baskets." A clustering technique reveals a clear two-block structure in currency comovements, with the first block containing mostly the dollar currencies and the other the European currencies. A factor model incorporating this "clustering" factor and two additional factors, a commodity currency factor and a "world" factor based on trading volumes, fits currency basket correlations much better than extant factors, such as value and carry, do. In particular, it explains on average about 60% of currency variation and generates a root mean squared error relative to sample correlations of only 0.11. The model also fits comovements in emerging market currencies well. Economically, the correlations between currency baskets underlying the factor structure are inversely related to the physical distances between countries.

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Introduction

According to recent Bank for International Settlement (BIS) surveys, more than half of the trading volume in foreign exchange arises from trades with "financial" customers, institutional investors, mutual funds, hedge funds, and other portfolio managers (Rime and Schrimpf 2013), a phenomenon that mostly reflects increasing globalization of financial markets over time. While foreign exchange (FX) transactions often arise merely as a by-product of buying or selling international securities, profit-seeking active currency management has become more commonplace, increasing the need for models to explain the risks and comovements of currencies. There is also renewed interest in the pricing of currency risk in international equities (see, e.g., Brusa et al. 2014), but standard theory (see the famous Adler and Dumas 1983 survey) suggests that all currency risks are priced for each equity market. A parsimonious currency factor model may therefore simplify the implementation of international equity pricing models. It may also help characterize currency commovements for determining optimal currency hedge ratios (see De Roon et al. 2009, Campbell et al. 2010).

The academic literature so far has focused almost exclusively on detecting currency factors that generate attractive return profiles. Important currency factors include the carry factor of Lustig et al. (2011), the currency-momentum factor (see, e.g., Burnside et al. 2011, Menkhoff et al. 2012b), the currency-volatility factor (see, e.g., Menkhoff et al. 2012a), and currency-value factors (see, e.g., Menkhoff et al. 2017). Currency, value, and momentum feature in practitioner indices created by Deutsche Bank. Practitioners also recognize a commodity factor in currencies, and the Australian and Canadian dollar are typically categorized as "commodity currencies" (see Chen and Rogoff 2003 and Ready et al. 2017). Just as the model of Fama and French (1996) for equities is also a good risk model to explain equity return comovements (see Bekaert et al. 2009 and Hou et al. 2011), it is possible that these factors are effective in explaining currency comovements.

Developing an adequate factor model for currency movements raises special issues, however. Taking the dollar as the numéraire currency, a factor model that explains bilateral dollar movements perfectly, also fits, by triangular arbitrage, other bilateral exchange rates perfectly, whatever the perspective. However, if the fit is imperfect, then a good dollar model may be a poor yen model and vice versa. In this paper, we examine various factor models to explain currency comovements from a *global* perspective. That is, we attempt to identify a factor model that fits the data well, whatever the currency perspective is.[1] To facilitate a global perspective on currency comovements, we use

Table 1. Bilateral Currency Correlations

Panel A	$\Delta s_{AUD,USD}$	$\Delta s_{CAD,USD}$	$\Delta s_{EUR,USD}$	$\Delta s_{JPY,USD}$	$\Delta s_{NZD,USD}$	$\Delta s_{NOK,USD}$	$\Delta s_{SEK,USD}$	$\Delta s_{CHF,USD}$
$\Delta s_{CAD,USD}$	0.55							
$\Delta s_{EUR,USD}$	0.40	0.33						
$\Delta s_{JPY,USD}$	0.17	0.03	0.45					
$\Delta s_{NZD,USD}$	0.69	0.43	0.47	0.25				
$\Delta s_{NOK,USD}$	0.44	0.41	0.83	0.38	0.46			
$\Delta s_{SEK,USD}$	0.43	0.40	0.81	0.37	0.47	0.82		
$\Delta s_{CHF,USD}$	0.31	0.20	0.84	0.53	0.41	0.74	0.71	
$\Delta s_{GBP,USD}$	0.34	0.31	0.69	0.36	0.42	0.66	0.63	0.60
Panel B	$\Delta s_{AUD,JPY}$	$\Delta s_{CAD,JPY}$	$\Delta s_{EUR,JPY}$	$\Delta s_{USD,JPY}$	$\Delta s_{NZD,JPY}$	$\Delta s_{NOK,JPY}$	$\Delta s_{SEK,JPY}$	$\Delta s_{CHF,JPY}$
$\Delta s_{CAD,JPY}$	0.77							
$\Delta s_{EUR,JPY}$	0.60	0.65						
$\Delta s_{USD,JPY}$	0.64	0.85	0.58					
$\Delta s_{NZD,JPY}$	0.80	0.68	0.61	0.57				
$\Delta s_{NOK,JPY}$	0.63	0.68	0.87	0.59	0.61			
$\Delta s_{SEK,JPY}$	0.62	0.67	0.85	0.57	0.61	0.86		
$\Delta s_{CHF,JPY}$	0.46	0.46	0.83	0.42	0.50	0.76	0.72	
$\Delta s_{GBP,JPY}$	0.58	0.67	0.76	0.63	0.59	0.75	0.73	0.63
Panel C	Δs_{AUD}	Δs_{CAD}	Δs_{EUR}	$\Delta s_{JPY/USD}$	Δs_{NZD}	Δs_{NOK}	Δs_{SEK}	Δs_{CHF}
Δs_{CAD}	−0.22							
Δs_{EUR}	−0.20	−0.32						
$\Delta s_{JPY/USD}$	−0.46	−0.82	−0.13					
Δs_{NZD}	−0.10	−0.24	−0.14	−0.32				
Δs_{NOK}	−0.19	−0.27	−0.04	−0.21	−0.15			
Δs_{SEK}	−0.19	−0.27	−0.04	−0.20	−0.14	−0.04		
Δs_{CHF}	−0.14	−0.27	0.01	0.11	−0.08	−0.01	−0.02	
Δs_{GBP}	−0.24	−0.36	−0.08	−0.27	−0.17	−0.09	−0.09	−0.03

Notes. The table presents correlation matrices for all currency pairs relative to the U.S. dollar and Japanese yen in panels A and B, respectively. Panel C reports correlations in panel A minus those in panel B. The sample extends from January 1973 to December 2015.

the concept of a "currency basket." A currency basket averages all bilateral currency changes relative to one particular currency. As we show formally, the 10 currency baskets for the G10 currencies span all possible bilateral currency changes. We then examine the relative explanatory power of the extant and various new factors for currency basket comovements.

We use a clustering technique to introduce several new currency factors. When selecting two clusters, a very clear factor structure emerges, with the dollar currencies (Australian, Canadian, New Zealand, and U.S.) and the Japanese yen in one block and the European currencies in the other. When using three clusters, a commodity-type currency factor also emerges. Combining these statistical factors with a "market" factor, based on currency trading volumes, and a commodity currency factor, we propose several parsimonious factor models and run a horse race versus models incorporating the existing factors.

Among the extant currency factors, the carry and value factors exhibit the highest explanatory power for currency basket variation. This is not surprising, because both factors are relatively highly correlated with the first principal component in bilateral currency rates. However, a new parsimonious factor model incorporating the two-block clustering factor, a commodity factor, and the market factor easily beats factor models created from extant risk factors, even models that feature double as many factors. The new factor model explains on average about 60% of the variation in currency baskets. Moreover, the root mean squared error (RMSE) relative to sample correlations is only about 0.11, which is statistically significantly better than any model based on extant risk factors.

Our proposed factor structure has economic content. We demonstrate that currency basket correlations intuitively decrease with the physical distance between the corresponding countries, which in turn affects our factor identification. This result is reminiscent of the gravity result of Lustig and Richmond (2020), who link the betas in regressions of bilateral exchange rate changes on currency baskets to gravity variables, showing currencies of peripheral countries to have higher factor loadings. Using specific international kernel models, they show that these factor loadings are inversely related to exposures to global shocks. The currency basket framework shows that the betas first and foremost reflect a common component (the variability of the home currency basket), in addition to the comovement between home and foreign currency baskets. Using the link between currency changes and pricing kernels, we show that countries with pricing kernels exhibiting similar (dissimilar) exposure to the global pricing kernel have currency baskets that are positively (negatively) correlated.

The remainder of the article is organized as follows. In Section 1, we describe our methodology and introduce the currency basket concept. Section 2 explains our clustering technique and introduces a new factor model for currency returns. In a contemporaneous paper and using a very different methodology, Greenaway-McGrevy et al. (2018) also find a two-factor structure in bilateral exchange rates. Our results are also consistent with the findings in Maurer et al. (2019) who compute principal components from 55 bilateral exchange rates, identifying two major global risk sources in foreign exchange markets. Section 3 examines the explanatory power of the standard currency factors for currency comovements. Section 4 runs a horse race of a variety of different factor models, using primarily the RMSE for correlations as the metric. In Section 5, we investigate the recent factor model of Verdelhan (2018) and reinterpret the results in Lustig and Richmond (2020). We show that Verdelhan's (2018) two dollar factors,[2] which have, mostly by design, very strong explanatory power for contemporaneous bilateral exchange rate changes with respect to (w.r.t.) the U.S. dollar, nonetheless have poor "global" explanatory power. In Section 6, we examine the explanatory power of our new factor model for emerging market currencies, showing it to explain a smaller portion of their variation but to fit comovements only slightly worse as for developed currencies. Section 7 concludes.

1. Explaining Currency Comovements

We study the G10 currencies—Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Eurozone euro (EUR), Japanese yen (JPY), Norwegian krone (NOK), Swedish krona (SEK), New Zealand dollar (NZD), British pound (GBP), and U.S. dollar (USD). The G10 currencies are the most traded developed currencies in the world. According to the latest BIS survey (BIS 2019), the G10 currencies account for 89% of the total trading volume over the 2001–2019 period. They are also generally considered the most liquid currencies in the world[3] and feature prominently in various currency products in the practitioner world (see Bekaert and Panayotov 2020, for concrete references). We use end-of-month mid-spot rates from Barclays Bank International (BBI) and WM/Reuters (WMR) that are available via Datastream to calculate (logarithmic) currency changes. The time period is from January 1973 to December 2015. For the euro before 1999, we use Deutsche mark rates.

1.1. Bilateral Correlations

To set the stage, Table 1, panels A and B, show bilateral correlations from two perspectives. Panel A takes the perspective of a U.S. dollar investor and reproduces the full sample correlation matrix for all currency pairs relative to the dollar expressed in foreign currencies per dollar (e.g., CAD/USD is the amount of Canadian dollar equivalent to one U.S. dollar). Panel B takes the perspective of a yen investor. Panel C provides the differences between correlations in panels A and B.

Most of the pairwise currency correlations are higher in panel B compared with those in panel A, presumably reflecting a volatile Japan specific factor. Thus, the correlation differences in panel C are mostly negative and often quite substantially so. For example, the correlation between the CAD and the GBP is 0.36 higher from the yen than from the dollar perspective, but for the CHF and GBP, the difference is tiny.

Clearly, the correlation structure among currencies is numéraire dependent. A factor model that fits "dollar-based" correlations well may not fit "yen-based" correlations well. However, Table 1 hints at an overall factor structure in currencies in that the "dollar currencies" (USD, AUD, CAD, and NZD) are more correlated with each other and less correlated with the other currencies in both panels. We seek to find factor models for currencies that maximize overall fit, across all base currencies.

1.2. Currency Baskets

Explaining currency comovements globally is nontrivial. The 10 currencies imply 45 different currency pairs, which are linearly dependent through the triangular arbitrage relation. To resolve this problem, we introduce the concept of a *currency basket*. A currency basket is an equally weighted average appreciation of one currency relative to a basket of all currencies in our sample. In other words, the currency i-basket is calculated as

$$CB_i = \frac{1}{9}\sum_{j=1}^{10} \Delta s_{j,i}, \tag{1}$$

where $\Delta s_{j,i}$ is the log spot-rate change of currency i w.r.t. currency j that is, the (logarithm) change in the value of currency i relative to currency j. For example, the U.S. dollar basket denoted by CB_{USD} is an equally weighted average of log changes in the value of the U.S. dollar w.r.t. AUD, CAD, CHF, EUR, JPY, NOK, SEK, NZD, and GBP. Note that $\Delta s_{i,i} = 0$.

Under the absence of triangular arbitrage, we can replicate all bilateral rates by having only $N-1$ nonrepeated exchange rates. It should therefore not be surprising that our 10 currency baskets span all bilateral rates. Because the concept of the currency basket is essential to this article, we show this spanning property in some detail. Triangular arbitrage implies that

$$\Delta s_{k,i} = \Delta s_{j,i} + \Delta s_{k,j} \quad \forall j. \tag{2}$$

Table 2. Descriptive Statistics of Currency Baskets

	CB_{USD}	CB_{AUD}	CB_{CAD}	CB_{CHF}	CB_{EUR}	CB_{JPY}	CB_{NOK}	CB_{SEK}	CB_{NZD}	CB_{GBP}
Mean	0.24%	−1.16%	−0.62%	3.67%	−0.83%	2.61%	−0.51%	−1.25%	−1.20%	−0.96%
S.D.	7.92%	9.63%	7.63%	8.34%	5.83%	10.48%	6.42%	6.89%	9.78%	7.40%
AC(1)	0.053	0.017	−0.023	−0.005	0.014	0.050	−0.054	0.100	−0.044	0.033

Notes. The table presents annualized means, standard deviations (S.D.), and the first-order autocorrelation coefficient (AC(1)) of currency baskets (CB). The mean is annualized by multiplying by 12. The standard deviation is annualized by multiplying by $\sqrt{12}$. The CB factors are equally weighted average log changes of the indicated G10 currencies relative to the other currencies. The sample extends from January 1973 to December 2015.

In the absence of arbitrage, this equation holds for any third currency. Therefore, we can add up "n" of those triangular equations for a basket of third currencies to find the relation between the log change of a bilateral exchange rate and the currency baskets. From Equation (2), we have

$$\Delta s_{k,i} = \frac{1}{n} n \; (\Delta s_{k,i}) = \frac{1}{n} \sum_{j=1}^{n} \left[\Delta s_{j,i} + \Delta s_{k,j} \right] , \tag{3}$$

$$\Delta s_{k,i} = \frac{1}{n} \left[\sum_{j=1}^{n} \Delta s_{j,i} \right] - \frac{1}{n} \left[\sum_{j=1}^{n} \Delta s_{j,k} \right]. \tag{4}$$

If there are "n" currencies, then there will be "$n-1$" exchange rates ($\Delta s_{i,i} = \Delta s_{k,k} = 0$). Therefore,

$$\Delta s_{k,i} = \frac{n-1}{n} \; \frac{1}{n-1} \left[\sum_{j=1}^{n} \Delta s_{j,i} \right] - \frac{n-1}{n} \; \frac{1}{n-1} \left[\sum_{j=1}^{n} \Delta s_{j,k} \right], \tag{5}$$

$$\Delta s_{k,i} = \frac{n-1}{n} CB_i - \frac{n-1}{n} CB_k. \tag{6}$$

Equation (6) shows that the appreciation of currency k w.r.t. currency i ($\Delta s_{k,i}$) is spanned by the average appreciation of a basket of currencies w.r.t. currency i (CB_i) minus the average appreciation of a basket of currencies w.r.t. currency k (CB_k). Empirically, with nine bilateral exchange rates among G10 currencies, combining Equations (1) and (6) implies that

$$\Delta s_{k,i} = \frac{9}{10} CB_i - \frac{9}{10} CB_k. \tag{7}$$

Table 2 reports summary statistics on currency baskets. Over the sample period, CB_{SEK} (CB_{CHF}) has the highest annualized depreciation (appreciation) rate of 1.2% (3.7%). Annualized volatilities range between 5.8% for CB_{EUR} and 10.5% for CB_{JPY}. Currency baskets show little serial correlation with the first-order autocorrelations never higher than 0.10 in absolute value.

Note that the U.S. dollar basket (CB_{USD}) approximately corresponds to the dollar factor introduced by Lustig et al. (2011). Whereas it is likely that this factor explains bilateral exchange rate changes from the dollar perspective well, Table 3 compares the contemporaneous explanatory power of currency baskets for bilateral exchange rates from different currency perspectives. That is, we run regressions of the form

$$\Delta s_{j,i} = a_j + b_j \; CB_k + e_{j,i}, \;\; \text{for all currency perspectives } i \text{ and currency baskets } k, \tag{8}$$

and we report the average adjusted R^2s.

Table 3. Explanatory Power of Currency Baskets

	Average adjusted R^2											
	USD	AUD	CAD	CHF	EUR	JPY	NOK	SEK	NZD	GBP	Top 3	Top 5
USD rates	**55.4**%	11.5%	23.6%	16.0%	16.1%	10.7%	14.2%	13.7%	10.1%	6.2%	71.8%	81.5%
AUD rates	8.0%	**65.5**%	11.7%	17.8%	17.8%	9.7%	10.9%	10.2%	20.4%	9.2%	79.0%	87.4%
CAD rates	23.4%	16.4%	**53.4**%	21.9%	18.0%	10.1%	11.4%	11.1%	10.9%	7.1%	69.6%	81.5%
CHF rates	13.0%	15.9%	21.1%	**58.9**%	17.3%	10.2%	9.2%	8.1%	10.6%	5.7%	70.0%	79.3%
EUR rates	13.6%	15.2%	16.3%	21.8%	**38.7**%	10.3%	13.8%	13.4%	11.9%	8.2%	56.5%	74.3%
JPY rates	7.7%	11.3%	10.6%	8.6%	8.5%	**69.9**%	10.6%	9.7%	9.1%	4.9%	83.2%	88.2%
NOK rates	13.5%	13.2%	13.1%	15.0%	14.9%	10.4%	**43.5**%	14.6%	11.7%	7.4%	63.4%	78.0%
SEK rates	13.4%	12.7%	12.6%	13.3%	13.8%	10.0%	14.1%	**47.6**%	10.9%	6.9%	65.7%	79.1%
NZD rates	8.6%	20.8%	8.1%	12.7%	13.9%	8.7%	11.9%	9.8%	**66.3**%	7.0%	80.2%	87.2%
GBP rates	10.1%	12.8%	11.4%	11.8%	10.7%	8.8%	9.1%	8.9%	10.2%	**51.7**%	69.7%	79.9%
All rates	16.7%	19.5%	18.2%	19.8%	17.0%	15.9%	14.9%	14.7%	17.2%	11.4%	70.9%	81.7%
Off-diagonal	12.4%	14.4%	14.3%	15.4%	14.6%	9.9%	11.7%	11.1%	11.8%	6.9%		

Notes. The table presents average adjusted R^2s of regressing bilateral exchange rates on currency baskets from different currency perspectives. For example, the fifth row of the second column (23.4%) is the average adjusted R^2 of the following regression: $\Delta s_{j,\mathrm{CAD}} = a_j + b_j \, CB_{\mathrm{USD}} + e_{j,\mathrm{CAD}}$, $j \in$ {G10 currencies}. We report the explanatory power of the best three and five currency baskets of each, in the columns indicated by top 3 and top 5, respectively. The all rates (off-diagonal) rows report the average of the columns (excluding the diagonal entry). The sample extends from January 1973 to December 2015.

Table 4. Correlations Between the G10 Currency Baskets

	CB_{USD}	CB_{AUD}	CB_{CAD}	CB_{CHF}	CB_{EUR}	CB_{JPY}	CB_{NOK}	CB_{SEK}	CB_{NZD}	CB_{GBP}
CB_{USD}	1.00									
CB_{AUD}	−0.03	1.00								
CB_{CAD}	0.54	0.27	1.00							
CB_{CHF}	−0.39	−0.46	−0.56	1.00						
CB_{EUR}	−0.39	−0.45	−0.45	0.51	1.00					
CB_{JPY}	0.09	−0.30	−0.25	0.11	−0.13	1.00				
CB_{NOK}	−0.38	−0.30	−0.26	0.24	0.39	−0.26	1.00			
CB_{SEK}	−0.38	−0.28	−0.27	0.16	0.35	−0.25	0.42	1.00		
CB_{NZD}	−0.17	0.44	0.01	−0.31	−0.35	−0.25	−0.32	−0.26	1.00	
CB_{GBP}	−0.07	−0.30	−0.15	−0.03	0.08	−0.10	0.04	0.00	−0.22	1.00
CF_{abs}	0.53	0.60	0.57	−0.65	−0.74	0.19	−0.66	−0.62	0.48	−0.38
45FPC	−0.30	−0.81	−0.59	0.72	0.64	0.35	0.46	0.41	−0.66	0.26

Notes. The top panel presents monthly correlations between the currency baskets. The bottom panel presents monthly correlations between our simple currency factor (CF_{abs}) and the FPC of the 45 nonoverlapping bilateral exchange rates (45FPC) on the one hand and the currency baskets on the other. The sample extends from January 1973 to December 2015.

Not surprisingly, each currency basket has the highest explanatory power for its own bilateral rates. For example, the second column of Table 3 shows that the U.S. dollar basket (CB_{USD}) explains 55.4% of the variation in the bilateral exchange rates against the U.S. dollar (USD rates), which is analogous to the explanatory power documented in Verdelhan (2018). However, the explanatory power of CB_{USD} is low for the other bilateral rates, varying from 7.7% for JPY rates to 23.4% for CAD rates. Moreover, it is the Swiss franc basket (CB_{CHF}), not CB_{USD}, which has the highest explanatory power among the G10 currency baskets, explaining on average 19.8% of all exchange rate variation. The last row represents the average off-diagonal adjusted R^2s. The currency baskets CB_{AUD}, CB_{EUR}, and CB_{CHF} deliver the highest explanatory power among the G10. Clearly, by triangular arbitrage, there is dependence among these bilateral rates. Yet, Table 3 shows that it is not obvious which combination of currencies would capture correlations well for all currency perspectives. By focusing on currency baskets, we collapse a total of 45 different bilateral rates that are codependent into 10 manageable baskets.

In the last two columns on the right, we use the top-three or top-five currency baskets in each row to explain bilateral currency movements and report the adjusted R^2s. These always include the own basket. By the spanning argument that we discussed earlier, the R^2 rapidly increases and reaches on average 82% with five baskets. For the remainder of our paper, we examine which factor models best describe the correlation structure of the currency baskets. These models then automatically also capture comovements between any bilateral rates.

1.3. Currency Baskets and Currency Comovements

In Table 4, we report the currency basket correlations. They range between −0.56 for the CHF/CAD pair and 0.54 for the CAD/USD pair. It is already apparent that the correlations are linked to geography, with the European currency baskets mostly positively correlated. That correlations can go negative is not surprising, given that bilateral rates may appear with different signs in two different currency baskets, but has also an intuitive economic interpretation.

Consider a complete markets economy, so that currency changes reflect the difference in the log pricing kernels in the two countries:

$$\Delta s_{j,i} = m_i - m_j. \tag{9}$$

To derive the equivalent expression for currency baskets, it is useful to define a "global" pricing kernel as the equally weighed average of individual pricing kernels: $m_g = \frac{1}{n}\sum_{j=1}^{n} m_j$. Technically, the global pricing kernel thus adds variables expressed in different units, but they can be made "dimensionless" by subtracting the logarithm of the expected value of the pricing kernel (see Bakshi et al. 2020). This would be equivalent to redefining currency baskets with currency returns rather than just exchange rate changes, which we explore in Section 1.4. Combining the currency basket and global kernel definitions with Equation (9), it follows that

$$CB_i = \frac{n}{n-1}[m_i - m_g]. \tag{10}$$

The currency basket for country i is proportional to the difference between the pricing kernel for country i and the global pricing kernel, that is, to the "idiosyncratic" component of the pricing kernel.

To obtain intuition on what derives comovements between currency baskets, assume that each pricing kernel has a "systematic" component and a country-specific component, which we assume are uncorrelated across countries:

$$m_i = \mathrm{a}_i m_g + \bar{m}_i, \tag{11}$$

where $\bar{m}_i$ is orthogonal to m_g. Hence, the currency basket's variation now depends on a systematic and idiosyncratic component, and the covariance between currency baskets has a simple, intuitive expression:

$$CB_i = \frac{n}{n-1}((a_i - 1)m_g + \bar{m}_i), \tag{12}$$

$$cov[CB_i, CB_j] = \left(\frac{n}{n-1}\right)^2 (a_i - 1)(a_j - 1)\sigma_g^2, \tag{13}$$

where σ_g is the volatility of the global kernel.

Consequently, if two pricing kernels have jointly high ($a_i,\ a_j > 1$) or jointly low exposure ($a_i,\ a_j < 1$) to the global pricing kernel, then the currency baskets of the two corresponding countries are positively correlated; if not, then they show negative correlation. This strong separation in currency basket correlations is apparent in Table 4, which nicely circumvents the common factor issue in bilateral exchange rate correlations, as seen in Table 1.

1.4. Numéraire Issues

All of our computations use logarithmic exchange rate changes. This generates two unit issues. First, investors care about returns and not just currency changes. Second, by considering various nominal currency baskets simultaneously, we aggregate economic variables expressed in different currencies. Both issues are, in fact, immaterial, given our objective of creating a factor model that works from all currency perspectives. The main reason for this is that the variability of currency changes is almost an order of magnitude larger than the variation of interest and inflation differentials, and thus nominal currency changes are the main driver of currency return comovements.

To verify this, we compute excess bilateral exchange rate returns, as well as two real concepts: bilateral real exchange rate changes and real foreign exchange returns. The interest rate and inflation data are non-seasonally adjusted and available on Datastream. The excess returns are calculated as the one-month exchange rate changes plus the monthly interest rate differentials. Correlating the equivalent currency baskets in excess return space with the currency baskets using currency changes, the lowest correlation is observed for the GBP currency basket, equaling 99.7%.

In an integrated economy, a world pricing kernel should price real returns in various countries; alternatively, if purchasing power parity holds, then the real return from investing in any country would be equalized, whatever the numéraire perspective. We therefore also formulate the currency baskets in real return space. Real exchange rate changes are calculated as one-month exchange rate changes plus monthly inflation rate differentials. Real returns are computed as nominal exchange rate changes plus the foreign interest rate deflated by domestic inflation. Here, the correlations between "real" currency baskets (real currency changes) and our nominal exchange rate ones vary between 95.6% for the EUR and 99.1% for the AUD. For actual real returns, the correlations vary between 98.68% for the USD and 99.92% for the CAD perspective.

1.5. Factor Models

With F denoting a set of factors and β_j the vector of factor exposures, we estimate a variety of linear factor models:

$$CB_j = a_j + \beta_j' \times F + e_j, \tag{14}$$

To compare the performance of different factor models, we use two sets of statistics.

First, we examine the significance of the betas in Equation (14) and calculate a global R^2 as the equally weighted average of the R^2s for each CB_j. Our conclusions are robust to averaging R^2s based on trading volumes (results are available upon request).

Second, we examine how well the various factor models explain the comovement structure present in exchange rates, focusing on the correlation fit of various currency factor models. The covariance matrix produced by a particular factor model with factor covariance matrix $\boldsymbol{V}_F$ is, as usual,

$$Cov_F = \boldsymbol{\beta}_F{}'\ \boldsymbol{V}_F\ \boldsymbol{\beta}_F, \tag{15}$$

where $\boldsymbol{\beta}_F$ is the $10 \times K$ matrix of factor loadings, K is the number of factors, and Cov_F is the model-implied covariance matrix for the currency basket factors. We then compute the correlation RMSE (root mean square error) for model F as

$$RMSE_F = \sqrt{\frac{1}{45}\sum_j \sum_i (\hat{\rho}_{i,j} - \rho_{i,j})^2}, \tag{16}$$

where $\hat{\rho}_{i,j}$ is the sample correlation between CB_i and CB_j, $\rho_{i,j}$ is the model implied correlation between currency i and j, $\rho_{i,j} = \frac{\beta_{F,i} V_F \beta_{F,j}'}{\hat{\sigma}_i \hat{\sigma}_j}$, and $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are the sample volatilities. There are 45 (= $(10 \times 9)/2$) sample correlations to be fit.

To account for sampling error in those computations, we conduct a bootstrap exercise, in which we bootstrap the 10 currency baskets with replacement. The bootstrap creates artificial samples of equal length to our sample by randomly selecting and concatenating blocks of six months of currency basket changes. The contemporaneous correlation structure between currencies is therefore preserved. For each random sample, we estimate the correlation matrix, as well as the factor model. Then, we use the factors exposures to compute model-implied correlations and finally the RMSEs. We use 1,000 replications.

2. A New Factor Model for Currency Returns

Here we propose a new currency factor model that incorporates a statistical factor, a factor based on trading volumes (akin to the market model often used in equity trading) and a commodity currency factor. Importantly, an intuitive clustering technique uncovers a very prevalent two-block factor structure in currencies, which is the main focus of this section.

2.1. Cluster Analysis

The correlations in Tables 1 and 4 suggest a two- or three-factor structure in currencies. Dollar rates seem highly correlated, as are rates within continental and Scandinavian Europe. To investigate this formally, we rely on a clustering technique introduced by Ormerod and Mounfield (2000), who examine the clustering of currencies just before the euro was introduced. Ahn et al. (2009) apply the algorithm in a stock portfolio formation context to create "basis assets." The resulting basis assets are correlated with the standard firm characteristics, display significant dispersion in returns, and generate a relatively well-conditioned return covariance matrix.

The algorithm starts by defining a distance measure, which is a negative function of correlation:

$$d_{ij} = \sqrt{2 \times \left(1 - \hat{\rho}_{i,j}\right)}, \tag{17}$$

where $\hat{\rho}_{ij}$ denotes the sample correlation between currency baskets i and j, CB_i and CB_j, respectively. Perfectly positively correlated currency baskets have the minimum distance of 0, whereas perfectly negatively correlated currency baskets have the maximum distance of 2. Note that $d_{ii} = 0$. The clustering algorithm then creates clusters aiming to maximize within-group correlation and minimize across-group correlations.

An obvious way to use the distance concept to cluster currencies into N clusters is to find the combination of currencies that minimizes the total distance between currency baskets within a cluster. This "absolute" clustering algorithm therefore minimizes,

$$SD(N) = \sum_{k=1}^{N} \sum_{i,j} d_{ij}^{(k)}, \ \ (i,j \in k\text{th cluster}), \tag{18}$$

where k indexes a cluster of currencies, N is the number of clusters, and SD stands for the sum of distances between all members of the cluster.

In other words, to cluster currencies, we first consider all possible allocations of G10 currency baskets in N different clusters ($1 \leq N \leq 10$) and calculate their in-cluster distance as the sum of distances among all members of each cluster. Then, we calculate the total distance as the sum of all in-cluster distances for each possible allocation. The currency allocation that minimizes the total distance for each N constitutes the optimal clustering of the G10 currency baskets in N clusters.

Given our limited set of currencies, we can easily consider all possible combinations of currencies for a given number of N clusters. However, in the aforementioned papers, the authors apply a sequential clustering procedure. We relegate a discussion of this alternative, but suboptimal procedure to the online appendix, focusing the discussion here on the results using the absolute algorithm.

2.2. Optimal Currency Clusters

To gain some intuition regarding the methodology, Table 5 reports the results of clustering G10 currency baskets in N clusters, with N varying from 0 to 10. The end points are trivial: for 10 clusters, each currency basket is in its own cluster, and the total distance is zero; for the case of one cluster, the distance reflects the average correlation of all currency baskets. For nine to five clusters, optimal clustering is achieved by pairing currencies along regional lines, starting with CB_{CAD} and CB_{USD}, then CB_{CHF} and CB_{EUR}, CB_{NZD} and CB_{AUD}, CB_{NOK} and CB_{SEK}, and finally CB_{JPY} and CB_{GBP}.

To cluster the G10 currency baskets in three clusters, the algorithm produces (CB_{AUD}, CB_{CAD}, and CB_{NZD}), (CB_{CHF}, CB_{EUR}, CB_{NOK}, and CB_{SEK}), and (CB_{USD}, CB_{JPY}, and CB_{GBP}), with a total distance of 14.830 (average within-cluster correlation of 0.577). Whereas the second factor comprises European currencies, the first factor contains three well-known "commodity" currencies.

Figure 1 shows how the clustering algorithm lowers the total distance for all clusters, $N = 1$ through 10. Because we seek to construct a parsimonious factor model, we focus on $N = 2$. For two clusters, the worst grouping generates a total distance of 53.4, which corresponds to an average within-cluster correlation of 0.125. When we use the algorithm to minimize the distance, it more than halves to 26.2, and the average within-cluster correlation is much higher at 0.416.

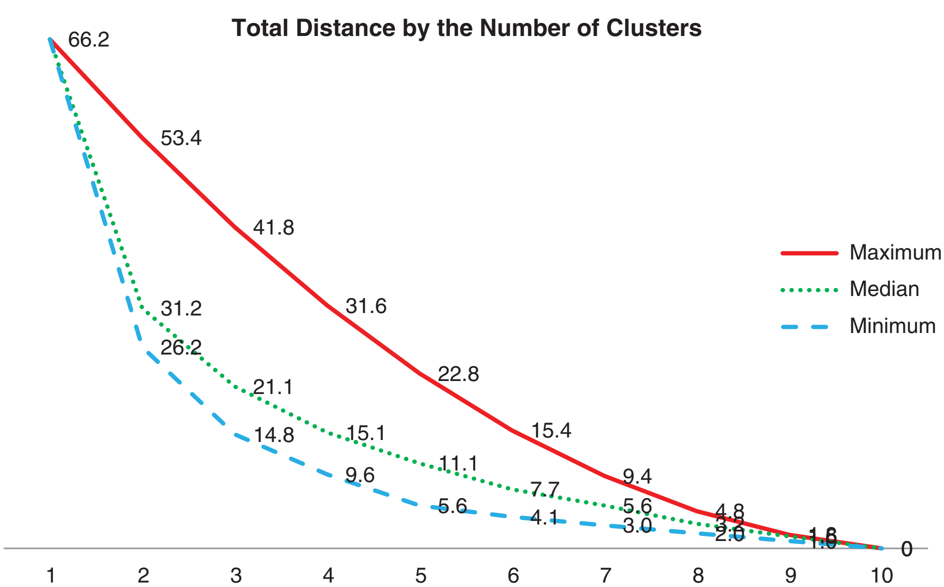
Optimal clustering for two clusters puts the dollar currencies plus the Japanese yen (CB_{USD}, CB_{AUD}, CB_{CAD}, CB_{NZD}, and CB_{JPY}) in the same block, and the European currencies (CB_{CHF}, CB_{EUR}, CB_{NOK}, CB_{SEK}, and CB_{GBP}) in the other block. The currency basket correlations reported in Table 4 confirm that currency baskets are more positively correlated within these blocks and more negatively correlated across the blocks. Note that the first block involves all "dollar" currencies plus the Japanese yen, whereas the other block involves all European currencies. Therefore, the

Table 5. The Minimum-Distance Currency Clusters

Number of clusters	Optimal clusters	Total distance	Average correlation	
			Within	Across
10	(CAD) (USD) (AUD) (CHF) (EUR) (JPY) (NOK) (SEK) (NZD) (GBP)	0	— 1.000	−0.104
9	(CAD, USD) (AUD) (CHF) (EUR) (JPY) (NOK) (SEK) (NZD) (GBP)	0.961	0.539 0.958	−0.119
8	(CHF, EUR) (CAD, USD) (AUD) (JPY) (NOK) (SEK) (NZD) (GBP)	1.954	0.523 0.920	−0.133
7	(NZD, AUD) (CHF, EUR) (CAD,USD) (JPY) (NOK) (SEK) (GBP)	3.015	0.494 0.883	−0.147
6	(NOK, SEK) (NZD, AUD) (CHF, EUR) (CAD, USD) (JPY) (GBP)	4.096	0.475 0.850	−0.161
5	(JPY, GBP) (NOK, SEK) (NZD, AUD) (CHF, EUR) (CAD, USD)	5.582	0.359 0.786	−0.162
4	(CHF, JPY, GBP) (EUR, NOK, SEK) (NZD, AUD) (CAD, USD)	9.608	0.263 0.672	−0.184
3	(CAD, NZD, AUD) (CHF, EUR, NOK, SEK) (USD, JPY, GBP)	14.830	0.224 0.577	−0.224
2	(CHF, EUR, NOK, SEK, GBP) (CAD, USD, NZD, AUD, JPY)	26.170	0.125 0.416	−0.287
1	(CHF, EUR, NOK, SEK, JPY, GBP, CAD, USD, NZD, AUD)	66.224	−0.104 0.096	—

Notes. The table presents the optimal clusters of G10 currency baskets. In this absolute clustering technique, we consider all possible allocations of G10 currency baskets in N clusters ($1 \leq N \leq 10$) and calculate their in-cluster distance as the sum of distances among all members of the cluster. The average correlation "within" simply is the equally weighted average of all "within" the cluster correlations, where the top number excludes the correlation with the own currency and the bottom number includes it (i.e., assigns a 1 to the correlation with oneself). The "across" averages all correlations between currencies not in the same cluster.

Figure 1. (Color online) How Optimal Clustering Lowers Distance



Notes. The figure presents the minimum (dashed line), median (dotted line), and maximum (solid line) distance for various numbers of absolute clusters of G10 currency baskets. The distances are computed as in Equation (17).

countries in each currency block share commonality in language, borders, legal origin, culture, and resources, or have colonial linkages, which are features stressed in recent work by Lustig and Richmond (2020). We explicitly link our work to theirs in Section 5. Recall from Section 1.3 that high currency basket correlations may reflect pricing kernels with similar exposures to the world pricing kernel. Such an interpretation is plausible for the "dollar" block countries on the one hand and the European countries on the other. The latter is especially not surprising, given the efforts at bringing about economic and financial integration within Europe in the context of the European Union and the European Free Trade Zone. The result is also reminiscent of the results of Greenaway-McGrevy et al. (2018), who identify a "dollar" and "euro" factor in bilateral exchange rates.

Based on these currency blocs, we introduce a currency factor (CF_{abs}) as the sum of the dollar currency baskets plus the Japanese yen basket, as follows:

$$CF_{abs} = CB_{USD} + CB_{AUD} + CB_{CAD} + CB_{NZD} + CB_{JPY}. \quad (19)$$

Because each currency pair appears in two currency baskets with opposite signs, the sum of all currency baskets equals zero; that is,

$$CB_{USD} + CB_{AUD} + CB_{CAD} + CB_{CHF} + CB_{EUR} + CB_{JPY} + CB_{NOK} + CB_{SEK} + CB_{NZD} + CB_{GBP} = 0. \quad (20)$$

Therefore, the sums of currency baskets in the two blocks are perfectly negatively correlated and can be collapsed into one factor. In addition, from the definition of a currency basket in Equation (1), it follows that

$$CB_{USD} + CB_{AUD} + CB_{CAD} + CB_{NZD} + CB_{JPY} = (-\Delta s_{AUD,USD} - \Delta s_{CAD,USD} - \Delta s_{JPY,USD} - \Delta s_{NZD,USD} + \Delta s_{EUR,USD} + \Delta s_{NOK,USD} + \Delta s_{SEK,USD} + \Delta s_{CHF,USD} + \Delta s_{GBP,USD}) \times 5/9. \quad (21)$$

Thus, the CF_{abs} factor represents an investment strategy of longing dollar currencies as well as Japanese yen and shorting European currencies.

2.3. A New Factor Model

The bottom row of Table 4 presents the correlations between our CF_{abs} factor and the currency baskets. Not surprisingly, CF_{abs} is positively correlated with CB_{USD}, CB_{AUD}, CB_{CAD}, CB_{NZD}, and CB_{JPY}, and negatively correlated with CB_{CHF}, CB_{EUR}, CB_{NOK}, CB_{SEK}, and CB_{GBP}. Its absolute correlation with currency baskets varies from 19% (CB_{JPY}) to 75% (CB_{EUR}), averaging 54.2%, making it an excellent currency factor candidate.

In Table 6, the first column shows regression coefficients from regressing the currency baskets onto the CF_{abs} factor. The coefficients are highly statistically significant for all currency baskets, with R^2s ranging between 14% and 43%, averaging 32%. As expected, the coefficients for the dollar rates (and the JPY) are positive, and those for the European rates are negative. The online appendix shows that the CF_{abs} factor is also highly correlated with the bilateral rates directly, with its explanatory power better than any "off-diagonal" currency basket. In addition, we repeat the analysis in Table 6 for all possible cluster factors to assess the contribution of optimal clustering. The distribution of average R^2s across all possible cluster factors ranges between 6% and 32% with a median of 15%.

We consider two avenues to come up with a parsimonious model for currency comovements. First, we create three clusters instead of two clusters. Going back to Table 5, this yields two currency factors (as the third one is colinear with the other two):

$$CF_{31} = CB_{USD} + CB_{GBP} + CB_{JPY}, \quad (22)$$

$$CF_{32} = CB_{AUD} + CB_{CAD} + CB_{NZD}. \quad (23)$$

The first factor combines the USD with the GBP and JPY; whereas the second factor combines all of the other dollar rates and hence is close to what practitioners would dub a commodity currency basket (which would also involve the NOK). The third cluster contains the remaining non-UK European currencies. In Table 6 (panel B), we regress the currency baskets on both factors, showing that both are highly statistically significant for all currency baskets. The R^2s now range between 35% and 59%, averaging 48%.

Second, we continue to use the CF_{abs} factor but add two "economic" factors. The first is the commodity factor (CF_{com}), computed as the sum of commodity-driven currency baskets, including CB_{AUD}, CB_{CAD}, CB_{NZD}, and CB_{NOK}:

$$CF_{com} = CB_{AUD} + CB_{CAD} + CB_{NZD} + CB_{NOK}. \quad (24)$$

Because of their link with commodity prices, we expect commodity currencies to be naturally correlated, and the clustering algorithm endogenously creates a commodity factor. The second is the market factor (CF_{TW}), computed as the trading-volume-weighted average of all G10 currency basket returns:

$$CF_{TW} = \sum_{i}^{10} w_i CB_i, \ i \in \{\text{G10 currencies}\}. \quad (25)$$

where i indexes the G10 currencies and w_i represents the trading-volume weights reported by the Bank for International Settlements (BIS) every three years from 1998 to 2013. We fix the weights before 1998 at the 1998 weights. In addition, the BIS weights include non-G10 currencies and add up to 200%, because each currency trade is counted twice, given two trading parties. Thus, we calculate a new weight for each G10 currency as its BIS weight divided by the sum of all G10

Table 6. Explanatory Power of Various Currency Factors

	Panel A	Panel B		Panel C			Panel D		
	CF_{abs}	CF_{31}	CF_{32}	CF_{abs}	CF_{com}	CF_{TW}	CF_{31}	CF_{32}	CF_{TW}
CB_{USD}	0.20	0.38	0.15	−0.03	0.19	2.34	−0.05	0.12	2.25
	(12.56)	(17.96)	(9.15)	(−3.09)	(12.98)	(43.77)	(−3.41)	(18.84)	(44.99)
CB_{AUD}	0.27	−0.06	0.40	0.17	0.27	−0.59	0.00	0.40	−0.30
	(12.17)	(−3.34)	(19.06)	(6.93)	(10.66)	(−4.12)	(0.04)	(18.63)	(−2.32)
CB_{CAD}	0.21	0.14	0.25	−0.05	0.35	1.37	−0.09	0.24	1.22
	(15.14)	(5.88)	(13.53)	(−2.01)	(11.59)	(10.81)	(−2.92)	(15.31)	(10.39)
CB_{CHF}	−0.26	−0.24	−0.33	−0.08	−0.27	−0.72	−0.22	−0.33	−0.10
	(−18.55)	(−8.86)	(−19.84)	(−2.99)	(−7.52)	(−4.75)	(−5.60)	(−19.83)	(−0.74)
CB_{EUR}	−0.21	−0.22	−0.24	−0.24	0.04	0.34	−0.30	−0.24	0.45
	(−24.48)	(−13.10)	(−23.14)	(−14.97)	(1.78)	(3.75)	(−14.72)	(−24.45)	(6.18)
CB_{JPY}	0.10	0.47	−0.07	0.66	−0.92	−1.79	0.67	−0.06	−1.08
	(3.80)	(16.15)	(−3.76)	(29.27)	(−34.02)	(−16.27)	(13.65)	(−3.03)	(−6.41)
CB_{NOK}	−0.20	−0.26	−0.21	−0.37	0.28	0.55	−0.24	−0.21	−0.13
	(−17.83)	(−11.27)	(−17.14)	(−19.55)	(12.56)	(6.31)	(−7.26)	(−17.06)	(−1.40)
CB_{SEK}	−0.20	−0.28	−0.22	−0.18	0.00	−0.34	−0.24	−0.21	−0.21
	(−13.13)	(−11.08)	(−12.75)	(−7.22)	(0.05)	(−1.85)	(−7.89)	(−12.97)	(−1.58)
CB_{NZD}	0.22	−0.08	0.35	0.25	0.10	−1.32	0.09	0.36	−0.93
	(10.94)	(−3.48)	(14.44)	(8.36)	(3.74)	(−7.01)	(2.78)	(15.01)	(−5.65)
CB_{GBP}	−0.13	0.16	−0.08	−0.12	−0.04	0.18	0.38	−0.07	−1.16
	(−7.61)	(5.09)	(−4.08)	(−3.91)	(−1.26)	(1.23)	(9.34)	(−3.87)	(−8.08)
Adjusted R^2	0.32	0.48			0.58			0.58	
	[0.14, 0.43]	[0.35, 0.59]			[0.41, 0.81]			[0.46, 0.70]	

Notes. The table presents results of regressing G10 currency baskets on our suggested currency factors. Our suggested currency factors include a simple factor based on two absolute clusters, as in Equation (19), CF_{abs}, a commodity currency factor as in Equation (24), CF_{com}, a currency trading-volume weighted factor as in Equation (25), CF_{TW}, and two currency factors based on three absolute clusters, CF_{31} and CF_{32} as in Equations (22) and (23), respectively. We combine these factors in one univariate (panel A) and three multivariate models (panels B–D). The *t*-statistics use White standard errors and are reported in parentheses. The table also reports the average R^2s, as well as their ranges in brackets. The sample extends from January 1973 to December 2015.

currencies' BIS weights (see Table A.1 in the appendix). The weights are highest for the dollar (around 50%), followed by the euro (around 20%) and the yen (around 10%). Therefore, the factor may have significant correlation with the "dollar" factor, examined in Verdelhan (2018), but economically makes more sense when the goal is to explain currency correlations globally.

The explanatory power of these two factors in isolation is quite substantial but somewhat lower than that of the CF_{abs} factor (full results are relegated to the online appendix). In Table 6, panel C, we report the results from a regression of the currency baskets onto this first candidate factor model with three factors, including CF_{abs}, CF_{com}, and CF_{TW}. The bulk of the individual coefficients is highly statistically significant with only 3 out of 30 not significant at the 10% level. The R^2s now range from 41% to 81%, and average 58%. Although it is always hard to interpret partial regression coefficients, the dominance of the USD and the EUR in the TW factor implies that their currency baskets and the currency baskets highly positively correlated with them (the CAD and NOK, respectively) load positively on this factor with very high *t*-statistics.

As a second candidate model, we supplement the CF_{31} and CF_{32} factors with the CF_{TW} factor. Recall that the CF_{32} factor is almost a commodity factor, so adding the trade-weighted market factor makes the most sense. Panel B in Table 6 shows the explanatory power of this candidate factor model. The model's explanatory power is equally impressive, with the coefficients mostly highly statistically significant and only four coefficients not significant at the 10% level. The R^2s range from 46% to 70%, but also average 58%.

3. Standard Currency Factors

The extant currency literature has spawned a number of factors inspired by risk considerations (e.g., carry), economic value (factors based on purchasing power parity), or trading models (momentum). Here, we provide a new perspective by examining the ability of these currency factors to explain the correlation structure among currency changes. The portfolios are, consistent with the literature, computed from a USD perspective. Given that they are spread portfolios, expressing them in a different currency would generate highly correlated return profiles (see, e.g., Bekaert and Panayotov 2020).

3.1. The Factors

Carry, Volatility, Value, and Momentum Factors. To construct the carry, volatility, value, and momentum factors, we use currencies from the 28 Organisation for Economic Co-operation and Development (OECD) countries.[4] We create equally weighted portfolio returns of roughly the top and bottom third of the currencies, ranked according to the characteristic.[5] Our factor is the difference in returns on these portfolios, where returns include exchange rate changes and interest rate differentials. Rebalancing is monthly, unless otherwise mentioned. As a robustness check, we construct currency carry, volatility, value, and momentum factors for 15 developed currencies used in several studies and report the results in the online appendix. The results are similar to the ones reported here.

The carry factor results from sorting available currencies on their one-month interest rate. This factor is similar to the one-month carry factor of Lustig et al. (2011). There is no consistent measurement of currency volatility factors in the literature. Pojarliev and Levich (2008) use a nontraded level factor, but Menkhoff et al. (2012a) use volatility changes as a nontraded risk factor. To be consistent with the other factors, our currency volatility factor represents the return of going long a portfolio of high volatility currencies and going short a portfolio of low volatility currencies, where volatility is calculated as the cross-currency and time-series average of absolute log daily exchange rate changes over the last 22 days.[6]

We use purchasing power parity (PPP) to reflect currency value. If exchange rates revert back to their long-term PPP values (see, e.g., Mark 1995 for empirical evidence), then similar deviations from PPP can be a source of currency comovements. We create a PPP factor return in three steps. First, we obtain PPPs for our 28 OECD countries. These PPPs reflect annual averages of monthly values and vary over the year. The OECD constructs PPPs for detailed items that are part of gross domestic product and aggregates them using relative expenditures. Second, for each month and each currency, we create a currency value index as a currency's nominal exchange rate divided by its PPP last year. For example, the value ratio for GBP/USD is $\frac{S_t^{\text{GBP/USD}}}{PPP_{t-12}^{\text{GBP/USD}}}$, where $S_t^{\text{GBP/USD}}$ is the average daily GBP/USD spot rate over the last three months and $PPP_{t-12}^{\text{GBP/USD}}$ is the average annual PPP for GBP/USD over the last year. The sorting exercise uses these valuation ratios relative to the USD; the value factor goes long (short) undervalued (overvalued) currencies. Here, rebalancing is every three months.

The currency momentum factor results from sorting on past one-month returns and is thus similar to the one-month momentum factor of Menkhoff et al. (2012b).

Commodity Factor. The values of the commodity currencies (AUD, CAD, NOK, and NZD) are correlated with commodity prices. In addition, changes in commodity prices have predictive power for currency carry returns (Bakshi and Panayotov 2013), as well as for bilateral forex returns (Aloosh 2012). Our commodity price factor uses monthly changes in the Raw Industrials Sub-Index of the CRB Spot Commodity Index, which is available from Datastream for the period from January 1951 to December 2015. Because the cluster algorithm already identifies a commodity currency factor, we do not consider such a factor. However, we do verify the performance of the so-called IMX factor (Ready et al. 2017), which represents a long position in importers and a short position in exporters of finished goods. The importing countries include the usual commodity countries, and Japan is a salient example of an exporter country, implying that the factor is correlated with the commodity currency factor and the standard carry factor.

World Equity Factor. Finally, we include a global equity factor. Whereas the correlation between equity returns and currency returns is low for developed markets (see Bekaert and Hodrick 2017), some standard currency factors (such as carry) show nonnegligible equity exposure (see Lustig et al. 2011). To proxy for equity risk in the markets of the G10 currencies, we construct an equally weighted world equity market return (in domestic currencies) based on MSCI equity price indices in Australia, Canada, Europe (an index of equity markets in the Eurozone), Japan, Norway, Singapore, Sweden, Switzerland, the United Kingdom, and the United States.[7]

3.2. Factor Regressions

In Table 7, we examine the explanatory power of the aforementioned existing currency factors for the variation in our 10 currency baskets. The top panel shows results for univariate regressions, and the bottom panel shows the multivariate regression.

In the top panel, for the carry, value, and equity factors, the overwhelming majority of the factor loadings are statistically significantly different from zero. The carry and value factors explain on average 10% and 9% of the variation in the currency basket factors respectively, but the R^2 is only 4% on average for the equity factor. The average R^2s for the other factors are even lower. Most of the factor exposures make economic sense. For example, the typical funding currencies (JPY and CHF) load significantly negatively on the carry factor, whereas the typical investment currencies (AUD and NZD) have significantly positive betas. The AUD, NOK, NZD, and SEK baskets are the most exposed to commodity price changes.

In the bottom panel, we see that the number of significant factors varies from currency to currency, being as low as 1 for the GBP and NZD and as high as 5 for the USD. The commodity factor is surprisingly not significant for CAD. Interestingly, every factor is significant at least once, but carry is significant for 6 and value for 5 out of 10 currencies. The R^2 for the multivariate models varies between 10% and 32% and is 18% on average.

The table reveals that the carry and value factors are the most promising candidates to feature in a factor model aimed at explaining currency comovements. However, the explanatory power is distinctly lower than the explanatory power of the new factors that we proposed in Section 2. Of course, the models here were not developed to maximize explained variation in currency changes or fit their comovements.

4. The Fit of Various Factor-Based Models

We have now introduced a total of 11 factors, 5 new ones and 6 factors that have been considered before, mostly in pricing exercises. We now determine which model best fits the comovements across currency changes. Before we run various horse races, we examine the correlations between the factors and their relationship to the standard principal components.

4.1. Factor Correlations

To obtain further intuition on these factors, Table 8 produces their correlation matrix and their correlations with the first three principal components of the bilateral currency changes. Note that, in an N-currency world, we have $N(N-1)/2$ different pairs. Thus, there are 45 nonrepeated bilateral rates among the G10 currencies. We denote the first three principal components by 45FPC, 45SPC, and 45TPC, respectively.

It is not surprising that our clustering technique yields a factor that is highly correlated with the first principal component (the correlation with 45FPC is −83%). However, CF_{abs} is also highly correlated with the second principal component (53%). Going back to Table 4, we note that CF_{abs} is highly correlated (above 0.50 in absolute magnitude) with all individual currency baskets, with the exception of the JPY and GBP baskets. Moreover, the currency commodity factor (CF_{com}) is more highly correlated with 45FPC (at −90%) than is our clustering factor. In Table 4, we added a line with correlations between the first principal component and the various currency baskets: the highest correlation (in absolute magnitude) is observed for the AUD. The CF_{TW} factor is 77% (41%) correlated with the second (third) principal component, but barely at all with the 45FPC. Recall that the trading-volume-weighted factor is dominated by the dollar currency basket, which implies that the first principal component in bilateral currency changes is not dominated by dollar variation. The two factors resulting from selecting three clusters, CF_{31}, and CF_{32}, are −37% correlated. The factor CF_{31} includes two important currency baskets (CB_{USD} and CB_{JPY}); it is not highly correlated with the first principal component, 45FPC, but it is highly correlated with 45SPC (87%). It is CF_{32} that is very highly correlated with 45FPC (−97%)! Therefore, the cluster of AUD, CAD, and NZD is the set of currencies that best approximates the first principal component in the G10 currencies. Our clustering factors being most correlated with the first two principal components is also reminiscent of a result of Maurer et al. (2019), who find that the first two principal components summarize the common global variation in 55 bilateral exchange rates.

Among the extant currency factors, the currency carry trade factor (denoted by *Carry*) is 54% correlated with the first principal component, 4% correlated with the second principal component, and 25% correlated with the third principal component. That a conditional return strategy shows strong correlation with the unconditional correlation structure in currencies is at first glance surprising. However, Hassan and Mano (2019) show that a significant portion of carry returns is driven by static exposures to prototypical carry currencies. This finding is also consistent with results of both Greenaway-McGrevy et al. (2018) and Maurer et al. (2019). The currency value factor (denoted by *Value*) and the equally weighted world equity market return (denoted by *Equity*) are, respectively, 49% and 33% correlated with 45FPC. The changes in the CRB Spot Commodity Index (denoted by *Commodity*) and the currency-volatility factor (denoted by *Volatility*) are 33% and 35% correlated with 45SPC, respectively.

Table 7. Explanatory Power of Extant Currency Factors for Currency Baskets

Panel A

	Carry	*Commodity*	*Value*	*Volatility*	*Momentum*	*Equity*
CB_{USD}	0.25	−0.25	0.33	−0.41	−0.05	−0.08
	(4.11)	(−6.21)	(5.30)	(−5.99)	(−0.79)	(−2.51)
CB_{AUD}	0.45	0.20	0.38	0.10	−0.11	0.21
	(5.84)	(4.33)	(5.91)	(1.00)	(−1.57)	(6.49)
CB_{CAD}	0.39	0.02	0.41	−0.21	−0.10	0.11
	(5.71)	(0.40)	(6.02)	(−2.94)	(−1.73)	(4.41)
CB_{CHF}	−0.51	−0.04	−0.50	0.18	0.21	−0.16
	(−7.35)	(−1.12)	(−7.34)	(2.53)	(3.43)	(−6.62)
CB_{EUR}	−0.22	0.01	−0.17	0.16	0.02	−0.09
	(−5.10)	(0.62)	(−4.30)	(3.09)	(0.53)	(−4.46)
CB_{JPY}	−0.43	−0.25	−0.31	−0.30	0.14	−0.16
	(−6.14)	(−3.06)	(−5.02)	(−2.89)	(1.86)	(−3.60)
CB_{NOK}	−0.14	0.11	−0.20	0.21	0.04	0.01
	(−2.69)	(3.92)	(−3.55)	(4.00)	(0.79)	(0.28)
CB_{SEK}	−0.13	0.10	−0.21	0.20	0.01	0.03
	(−3.30)	(3.86)	(−4.01)	(3.75)	(0.16)	(0.97)
CB_{NZD}	0.31	0.10	0.28	0.16	−0.12	0.17
	(3.97)	(2.08)	(4.53)	(1.97)	(−1.94)	(4.68)
CB_{GBP}	0.02	−0.01	0.00	−0.11	−0.04	−0.04
	(0.42)	(−0.42)	(−0.10)	(−1.91)	(−0.94)	(−1.62)
Adjusted R^2	0.10	0.03	0.09	0.03	0.01	0.04
	[0.02, 0.17]	[0.00, 0.06]	[0.05, 0.17]	[0.01, 0.04]	[0.00, 0.01]	[0.00, 0.07]

Panel B

	Carry	*Commodity*	*Value*	*Volatility*	*Momentum*	*Equity*
CB_{USD}	0.20	−0.21	0.23	−0.36	0.05	−0.08
	(2.40)	(−6.17)	(2.89)	(−6.53)	(0.84)	(−2.94)
CB_{AUD}	0.41	0.13	0.01	−0.03	0.12	0.13
	(3.29)	(3.52)	(0.12)	(−0.37)	(1.97)	(3.60)
CB_{CAD}	0.26	0.00	0.19	−0.26	0.07	0.06
	(2.86)	(−0.10)	(2.21)	(−4.71)	(1.21)	(2.56)
CB_{CHF}	−0.38	0.00	−0.15	0.28	−0.01	−0.09
	(−4.54)	(−0.01)	(−1.79)	(4.56)	(−0.13)	(−3.51)
CB_{EUR}	−0.30	0.03	0.09	0.22	−0.07	−0.07
	(−4.25)	(1.36)	(1.24)	(4.38)	(−1.44)	(−3.39)
CB_{JPY}	−0.49	−0.17	0.12	−0.16	−0.09	−0.05
	(−4.47)	(−2.66)	(1.07)	(−1.91)	(−1.31)	(−1.40)
CB_{NOK}	−0.02	0.10	−0.20	0.19	−0.01	0.00
	(−0.25)	(3.84)	(−2.94)	(4.04)	(−0.26)	(0.11)
CB_{SEK}	0.01	0.08	−0.26	0.17	−0.06	0.03
	(0.09)	(3.01)	(−2.40)	(2.95)	(−0.99)	(0.87)
CB_{NZD}	0.18	0.03	0.09	0.09	0.03	0.12
	(1.04)	(0.65)	(0.64)	(1.25)	(0.46)	(2.77)
CB_{GBP}	0.13	0.01	−0.12	−0.12	−0.05	−0.04
	(1.16)	(0.19)	(−1.19)	(−2.16)	(−0.95)	(−1.69)
Adjusted R^2				0.18		
				[0.10, 0.32]		

Notes. The table presents results of regressing currency baskets on the extant currency factors. Panel A reports results of univariate regressions, and panel B reports the results of multivariate regressions. The *t*-statistics are based on White standard errors and reported in parentheses. The table also reports the average R^2s, as well as their ranges in brackets. The sample extends from January 1973 to December 2015.

The currency-momentum factor (denoted by *Momentum*) is not highly correlated with any of the top-three principal components. Thus, the carry and value factors are most highly correlated with the first principal component of exchange rate changes, consistent with their relatively high explanatory power for currency variation, reported in Table 7.

To create factor models using the factors that we introduced, it is important that the factors are not multicollinear. The correlation table shows that this is

Table 8. Correlations Among Currency Factors and with the Top Three Principal Components

	Panel A			Panel B									
	45FPC	45SPC	45TPC	CF_{abs}	CF_{com}	CF_{TW}	CF_{31}	CF_{32}	*Carry*	*Commodity*	*Value*	*Volatility*	*Momentum*
CF_{abs}	−0.83	0.53	−0.16										
CF_{com}	−0.90	−0.31	−0.05	0.60									
CF_{TW}	0.03	0.77	0.41	0.32	−0.36								
CF_{31}	0.22	0.87	0.09	0.23	−0.54	0.74							
CF_{32}	−0.97	−0.12	−0.13	0.77	0.94	−0.21	−0.37						
Carry	−0.54	−0.04	0.25	0.38	0.47	0.07	−0.09	0.49					
Commodity	−0.11	−0.33	−0.02	−0.08	0.25	−0.37	−0.35	0.17	0.13				
Value	−0.49	0.07	0.22	0.40	0.38	0.19	0.01	0.43	0.83	0.06			
Volatility	0.05	−0.35	−0.13	−0.20	0.10	−0.33	−0.35	0.02	0.13	0.19	0.03		
Momentum	0.15	0.03	−0.07	−0.09	−0.12	−0.01	0.03	−0.13	−0.43	−0.11	−0.39	−0.14	
Equity	−0.33	−0.19	0.00	0.17	0.40	−0.28	−0.26	0.36	0.30	0.24	0.20	0.14	−0.09

Notes. The table presents monthly correlations between the various currency factors considered in this article and between the factors and the first, second, and third principal components (FPC, SPC, and TPC, respectively) of the 45 nonoverlapping bilateral exchange rates. The sample extends from January 1973 to December 2015.

clearly not the case. The highest correlations observed are those between the CF_{31} and the CF_{TW} factors (at 74%), and between the carry and value factors at 0.83. The factors CF_{com} and CF_{32} are naturally highly correlated, but they are never considered together.

4.2. Horse Race Between Factor Models

We now focus on the RMSE in correlation space to determine the factor model that best fits the currency basket comovements. Results using covariances rather than correlations are very similar and available in the online appendix. The RMSE can be viewed as the average correlation distance between the model and the data. Statistically, it is estimated rather precisely with the bootstrap procedure described before revealing a standard error of about 0.01 to 0.02. This is quite low, given the economic magnitude of currency basket correlations, which vary between −56% and +54% (see Table 4).

To set the stage, Table 9 reports the RMSE for univariate factor models using all 11 factors that we consider in this article. This exercise immediately reveals the value of the new CF_{abs} factor, which has an RMSE of only 0.176, with the 95% confidence interval being [0.153, 0.201]. Most of the other factors have RMSEs that are far above this interval. The second-best individual factor among the new factors is CF_{32} with an RMSE of 0.201. Among the extant factors, the best factor is the carry factor with an RMSE of 0.244, but its confidence interval does not overlap with that for CF_{abs}.

In Table 10 (panel A), we compare the fit of various multivariate models. We start with the two three-factor models that we proposed in Section 2. The three-factor models significantly reduce the RMSE, bringing it down to 0.112 for the model incorporating CF_{abs}, and to 0.131 for the model with the two clustering factors. In an absolute sense, a correlation error of about 10% seems small, and these models thus match the data correlations rather well. The RMSE difference between the two models is small economically and is also not statistically significant in that the RMSE generated by the second model is within the 95% convince interval of the first one.

The rest of panel A investigates the fit of various combinations of the extant currency factors. When we use all six factors, the RMSE is 0.206, almost twice the RMSE of our parsimonious model. When we drop the two worst-performing factors (volatility and momentum), the fit does not improve. We also report the RMSE for two three-factor models adding to carry and value, either the equity factor or the commodity price factor. Both models perform similarly with an

Table 9. Horse Race Part 1: Univariate Factors

	RMSE	BS-RMSE	SE	95% confidence interval	
CF_{abs}	0.176	0.177	0.012	0.153	0.201
CF_{com}	0.231	0.241	0.013	0.216	0.265
CF_{TW}	0.281	0.280	0.021	0.238	0.322
CF_{31}	0.299	0.298	0.021	0.257	0.340
CF_{32}	0.201	0.210	0.011	0.190	0.231
Carry	0.244	0.253	0.019	0.215	0.291
Commodity	0.298	0.297	0.019	0.259	0.335
Value	0.245	0.256	0.019	0.218	0.293
Volatility	0.289	0.287	0.019	0.250	0.324
Momentum	0.300	0.299	0.018	0.263	0.334
Equity	0.288	0.289	0.019	0.253	0.326

Notes. This table presents the RMSEs and the bootstrap results for the RMSE of implied correlations for various univariate models with factors tested in Tables 6 and 7. We bootstrap the G10 currency baskets simultaneously with replacement. For each random sample, we estimate the correlation matrix, as well as the factor model. Then, we use the factor exposures to compute model-implied correlations and finally the RMSEs. We use a block bootstrap with six-month blocks to create samples of the same size as the actual sample. The number of replications is 1,000. The sample extends from January 1973 to December 2015. SE stands for standard error, that is, the standard deviation of the RMSE over the bootstrapped (BS) samples.

Table 10. Horse Race Part 2: Multivariate Models

	Panel A				
	RMSE	BS-RMSE	SE	95% confidence interval	
$CF_{abs} + CF_{com} + CF_{TW}$	0.112	0.121	0.010	0.102	0.140
$CF_{31} + CF_{32} + CF_{TW}$	0.131	0.139	0.009	0.120	0.157
Carry + *Volatility* + *Commodity* + *Momentum* + *Value* + *Equity*	0.206	0.223	0.019	0.186	0.261
Carry + *Commodity* + *Value* + *Equity*	0.225	0.241	0.021	0.200	0.281
Carry + *Commodity* + *Value*	0.229	0.243	0.021	0.202	0.285
Carry + *Value* + *Equity*	0.233	0.248	0.020	0.208	0.287

	Panel B				
	RMSE	BS-RMSE	SE	95% confidence interval	
$CF_{abs} + CF_{com} + CF_{TW}$ + *Carry*	0.108	0.119	0.009	0.101	0.137
$CF_{abs} + CF_{com} + CF_{TW}$ + *Value*	0.110	0.120	0.010	0.101	0.138
$CF_{abs} + CF_{com} + CF_{TW}$ + *Carry* + *Value*	0.107	0.118	0.009	0.100	0.136
$CF_{31} + CF_{32} + CF_{TW}$ + *Carry*	0.120	0.131	0.009	0.113	0.149
$CF_{31} + CF_{32} + CF_{TW}$ + *Value*	0.126	0.135	0.009	0.116	0.153
$CF_{31} + CF_{32} + CF_{TW}$ + *Carry* + *Value*	0.120	0.130	0.009	0.112	0.148

Notes. This table presents the RMSEs and the bootstrap (BS) results for the RMSE of implied correlations for various currency multivariate models with factors tested in previous tables. Panel A reports models that included either extant or new currency factors. Panel B reports some models that included both extant and new currency factors together. We bootstrap the G10 currency baskets simultaneously with replacement. For each random sample, we estimate the correlation matrix, as well as the factor model. Then, we use the factor exposures to compute model-implied correlations and finally the RMSEs. We use a block bootstrap using six-month blocks to create samples of the same size as the actual sample. The number of replications is 1,000. The sample extends from January 1973 to December 2015.

RMSE of 0.23. The RMSEs generated by these models are also outside the 95% confidence intervals generated by the bootstrap for our two three-factor models. We conclude that the proposed new models are far superior to models created from extant currency factors in fitting currency comovements.

We also verify whether using the IMX factor of Ready et al. (2017) instead of the extant commodity price factor improves the performance of the existing factor models. Because our IMX sample is somewhat shorter than the sample used here, we relegate the results to the online appendix. The IMX factor explains more currency basket factor variation than the commodity price factor; however, the overall performance of the factor models using existing factors actually worsens. The RMSE of the IMX factor is higher than that for carry and value; and multivariate models incorporating the IMX factor produce the same RMSE as the models reported in this section.

It is still conceivable that the extant currency factors can help the fit of our proposed model. We address this issue in panel B of Table 10. We focus on the carry and value factors, which are the best extant currency factors. Adding these factors does decrease the RMSE most of the time, but the decrease is both economically and statistically insignificant.

We conclude that a parsimonious factor model—which uses a factor obtained from a simple clustering method that groups mostly the dollar currencies, a commodity currency factor, and a trading-volume-weighted "market" factor—fits currency comovements very well and does so better than any other factor model extracted from the extant currency factors. Note that Greenaway-McGrevy et al. (2018) also find that "carry" does not survive their factor identification procedure, but they do not examine other extant currency factors.

5. Comparison with Recent Factor Models

In this section, we link our results to some recent academic studies regarding currency factors.

5.1. A Dollar Factor

Lustig et al. (2011) introduce the U.S. dollar factor as a common currency factor. It is essentially the average excess return for a U.S. investor to investing in all the foreign currencies and is thus closely related to our dollar basket (CB_{USD}). Verdelhan (2018)'s version represents the average changes in exchange rates across six interest-rate-sorted portfolios.[8] He also constructs an alternative dollar factor as the spread of average exchange rate changes between high and low "dollar beta" portfolios. Then, he shows that both dollar factors account for a large share of bilateral exchange rate variation against the U.S. dollar. He identifies the dollar beta factor as a key "global" risk factor and links its explanatory power for currency movements to its comovements with different macroeconomic variables (in particular capital flows). Importantly, he shows that both dollar factors explain much more of

Table 11. Revisiting Verdelhan (2018): Regression of Bilateral Exchange Rate Changes vs. Currency Baskets

	Panel A				Panel B		
	Carry	CB_{USD}	R^2		*Carry*	CB_{USD}	R^2
$\Delta s_{AUD,USD}$	−0.45	1.05	0.52	CB_{AUD}	0.49	−0.17	0.16
	(−6.16)	(19.90)			(6.16)	(−2.90)	
$\Delta s_{CAD,USD}$	−0.25	0.50	0.34	CB_{CAD}	0.27	0.45	0.37
	(−4.44)	(9.87)			(4.44)	(7.96)	
$\Delta s_{CHF,USD}$	0.39	1.16	0.74	CB_{CHF}	−0.43	−0.29	0.31
	(6.59)	(23.14)			(−6.59)	(−5.24)	
$\Delta s_{EUR,USD}$	0.14	1.12	0.79	CB_{EUR}	−0.15	−0.25	0.19
	(4.18)	(36.61)			(−4.18)	(−7.29)	
$\Delta s_{JPY,USD}$	0.44	0.67	0.40	CB_{JPY}	−0.49	0.25	0.14
	(6.23)	(8.85)			(−6.23)	(3.01)	
$\Delta s_{NOK,USD}$	0.06	1.16	0.75	CB_{NOK}	−0.07	−0.29	0.15
	(1.33)	(28.74)			(−1.33)	(−6.42)	
$\Delta s_{SEK,USD}$	0.05	1.18	0.73	CB_{SEK}	−0.05	−0.31	0.14
	(1.40)	(27.67)			(−1.40)	(−6.63)	
$\Delta s_{NZD,USD}$	−0.34	1.18	0.54	CB_{NZD}	0.38	−0.31	0.12
	(−5.19)	(18.99)			(5.19)	(−4.49)	
$\Delta s_{GBP,USD}$	−0.04	0.97	0.57	CB_{GBP}	0.04	−0.08	0.00
	(−0.79)	(19.69)			(0.79)	(−1.39)	

Notes. The table reports coefficients and adjusted R^2s from regressing bilateral exchange rates against the U.S. dollar on the carry and the U.S. basket factor (Panel A) and from regressing currency basket factors on the carry and the U.S. basket factor (Panel B). The *t*-statistics are based on White standard errors and reported in parentheses. The sample extends from January 1973 to December 2015.

bilateral currency variation than does the carry factor, which we have shown to be one of the better extant currency factors.

In this section, we show that the dollar factor's explanatory power measured in Verdelhan (2018) is not surprising and reinterpret it in terms of currency basket correlations. In addition, being numéraire dependent, it fits currency variation in other countries poorly.

Consider the main regression in Verdelhan (2018):

$$\Delta s_{k,USD,t+1} = \alpha + \beta \; Carry_{t+1} + \gamma \; CB_{USD,t+1} + \epsilon_{t+1}, \qquad (26)$$

where, $Carry_{t+1}$ is the difference in returns between portfolios of high- and low-interest-rate currencies. Now, using Equation (7), $\Delta s_{k,USD,t+1} = \frac{9}{10} CB_{USD,t+1} - \frac{9}{10} CB_{k,t+1}$, and Equation (26), it follows that

$$CB_{k,t+1} = -\frac{10}{9}\alpha - \frac{10}{9}\beta \; Carry_{t+1} + \left(1 - \frac{10}{9}\gamma\right) CB_{USD,t+1} - \frac{10}{9} \; \epsilon_{t+1}. \qquad (27)$$

The results of regressions (26) and (27) are reported in panels A and B, respectively, of Table 11. As can be seen, the coefficients of carry in panel B are equal to $-\frac{10}{9} \times$ the coefficients of *Carry* in panel A. In addition, the coefficients of CB_{USD} in panel B are equal to $1 - \frac{10}{9} \times$ the coefficients of CB_{USD} in panel A. Thus, high coefficients in Verdelhan's regression in fact reflect a *low* currency basket beta with respect to the U.S. dollar basket, where the beta is conditional on the covariance with the carry factor. Finally, the adjusted R^2s in panel A are much higher than those in panel B.

The presence of a common component on the left- and right-hand sides in Equation (26) leads to a somewhat different interpretation of Verdelhan's results. First, the coefficients in Equation (26) are difficult to interpret. For example, the "dollar factor" has virtually no independent effect on CB_{GBP}, yielding an insignificant −0.08 coefficient; yet, regression (26) produces a coefficient of 0.97 (which is, of course, nothing but $9/10 - 9/10 \times -0.08$), with a huge *t*-statistic. Second, the explanatory power of *Carry* and CB_{USD} for bilateral exchange rates, using Equation (26) (in panel A) is artificially high, because we use a component in the left-hand-side variable as a right-hand-side explanatory variable. For example, in the last row of panels A and B, the R^2 of *Carry* and CB_{USD} is 57% for changes in the GBP/USD, while in fact the adjusted R^2 of *Carry* and CB_{USD} is virtually 0% for CB_{GBP}. In contrast, in the second row, the R^2 of *Carry* and CB_{USD} is the lowest at 34% for changes in the CAD/USD, while in fact the R^2 of *Carry* and CB_{USD} is the highest at 37% for CB_{CAD}, which is expected, given the close economic ties between the United States and Canada.

Verdelhan (2018) excludes the left-hand-side exchange rate in the composition of his dollar factor, but it is easy to see that this does not resolve the "common variation" problem.[9] Moreover, this now aggravates the problem that the factor is not common across even bilateral rates relative to the dollar. Furthermore, as we have shown before, the original dollar factor, CB_{USD}, is not a suitable common factor for all bilateral rates.

To resolve the numéraire currency problem, Verdelhan (2018) proposes the difference in exchange rate changes between high and low dollar beta portfolios, hereafter denoted by *HML$*.[10] To create such a portfolio, he regresses currency changes in a rolling fashion on the carry and dollar baskets and sorts currencies in six groups according to their dollar basket exposures, taking the difference between the first and sixth portfolio. From our earlier analysis, this exercise essentially sorts on the dollar basket exposure of other currency baskets and is therefore potentially a valid global risk factor.

Whereas Boudoukh et al. (2018) and Maurer et al. (2018) criticize the risk-pricing implications of this dollar factor, we focus on its ability to explain global currency variation. We now show that our simple currency factor (CF_{abs}) has more explanatory power for currency variation than the *HML$* factor. We run the following horse race regressions:

$$CB_{k,t+1} = \alpha + \beta\, HML_{t+1} + \gamma\ \mathrm{CF}_{\mathrm{abs},t+1} + \epsilon_{t+1}. \tag{28}$$

The results are reported in Table 12. Panels A and B show the results for univariate regressions, and panel C shows the explanatory power of *HML$* and CF_{abs} jointly for the bivariate regression. The adjusted R^2s reported in panel B (CF_{abs}) are mostly higher than in panel A (*HML$*). The *HML$* has much higher explanatory power (an R^2 of 71%) for the U.S. dollar basket factor, whereas CF_{abs} has more balanced explanatory power for all other baskets compared with the HML$ factor (an average R^2 of 34% vs. only 17% for *HML$*). Furthermore, the *HML$* coefficient is significant for only 7 out of 10 currency baskets, while the CF_{abs} coefficient is significant for all G10 currency baskets.

It is puzzling that Verdelhan's global factor has such high correlation with the dollar basket. After all, being created by differencing a basket of currencies with high dollar betas versus one with low dollar betas, the factor should be dollar neutral. However, Verdelhan includes pegged currencies such as the United Arab dirham, the Saudi riyal, the Kuwaiti dinar, and the Hong Kong dollar in his post-1999 sample. These currencies have naturally very low dollar betas by construction, given that they are pegged to the dollar.[11]

When we put both factors together in panel C, the adjusted R^2s increase, which shows that the two factors contain different information. The coefficient of CF_{abs} remains statistically significant for all currency baskets, except for the USD basket factor, whereas the coefficient of *HML$* is not significant for the CAD, CHF, JPY, and NOK basket factors.

The bottom panel of the table reports the results of the comovement fitting horse race. The *HML$* factor has an RMSE of 0.214 relative to the data correlations, which is higher than the 0.192 RMSE generated by our CF_{abs} factor. Moreover, the bivariate model has a better RMSE (of 0.161) than the univariate models. We conclude that the explanatory power of our simple currency factor (CF_{abs}) is higher than that of the global dollar factor of Verdelhan (2018). Nonetheless, given the nature of Verdelhan's regression, his global factor is related to the factor structure that we uncover. If the regressions were done unconditionally (instead of using rolling samples); if they were not conditioned on the carry factor and if they used only the G10 currency set (Verdelhan uses more than 20 currencies), then the procedure would effectively sort on the beta with respect to the USD currency basket and likely yield a factor closely related to CF_{abs}. In fact, when verifying the identity of the currencies in the high and low beta baskets, we find that the CAD and the AUD feature frequently in the low beta buckets and the European currencies feature frequently in the high beta buckets. Pegged currencies are always present in the low beta category, biasing the dollar factor to be non-dollar neutral.

We also examine the explanatory power of the dollar-carry factor introduced in Lustig et al. (2014), which goes long in a basket of foreign currencies and short in the dollar whenever the average foreign short-term interest rate is above the U.S. interest rate and vice versa. However, the explanatory power of the dollar-carry factor is much lower than even the *HML$*. The results are reported in the online appendix.

5.2. Reinterpreting the Currency Factor Structure in Lustig and Richmond (2020)

Lustig and Richmond (2020), LR henceforth, detect an interesting pattern in cross-currency correlations. They regress bilateral exchange rate changes on "base factors," which are closely related to our currency baskets. They then show that the betas in these regressions and the R^2s can be interpreted using a gravity model: they are lower the closer the countries are in terms of distance and other variables measuring economic closeness.

There is a nice analogy between our currency basket results and the LR results. Our clustering model measures distance as a negative function of correlation and finds a factor structure that puts countries that are geographically close (the European currencies) within one block (at least when we use two clusters). In addition, when we run a regression of the bilateral distances (*BDistance*) between the currency baskets as used in the clustering algorithm on the population-weighted physical distance (*PDistance*) between the involved countries (as used in LR),[12] we obtain

$$BDistance_{i,j} = 1.33 + 1.86 \times 10^{-5}\ PDistance_{i,j} + \ \mathrm{e}$$
$$t-statistics: (95.56)\ (7.77). \quad (\mathrm{R}^2 = 0.09) \tag{29}$$

This means that, for every physical mile, the "correlation" distance increases by 1.86×10^{-5}. The coefficient is highly statistically significant. We also run the same regression using actual correlations between the currency baskets, finding statistically significant negative coefficients which imply that correlations decrease roughly 2% per 1,000 miles. Hence, currency basket correlations also follow a gravity model.

To illustrate our connection with LR further, let us consider the U.S. dollar as the base currency. In that case, the base factor in LR is in fact equivalent to our

Table 12. Explanatory Power of the Global Dollar Factor (*HML$*) of Verdelhan (2018) vs. Our Currency Factor (CF_{abs})

	Panel A		Panel B		Panel C		
	HML$	R^2	CF_{abs}	R^2	*HML$*	CF_{abs}	R^2
CB_{USD}	0.006	0.71	0.20	0.29	0.007	−0.03	0.71
	(21.47)		(10.38)		(15.05)	(−1.61)	
CB_{AUD}	0.001	0.03	0.25	0.32	−0.004	0.39	0.42
	(1.62)		(10.82)		(−4.47)	(11.67)	
CB_{CAD}	0.004	0.27	0.22	0.36	0.001	0.17	0.38
	(6.00)		(12.18)		(1.58)	(4.70)	
CB_{CHF}	−0.003	0.23	−0.23	0.41	0.000	−0.21	0.41
	(−5.51)		(−14.73)		(−0.50)	(−6.51)	
CB_{EUR}	−0.004	0.47	−0.21	0.62	−0.001	−0.16	0.66
	(−12.56)		(−20.31)		(−3.29)	(−8.89)	
CB_{JPY}	0.002	0.04	0.14	0.07	0.000	0.12	0.07
	(2.05)		(4.00)		(0.20)	(1.85)	
CB_{NOK}	−0.003	0.23	−0.21	0.47	0.000	−0.21	0.47
	(−5.76)		(−14.72)		(−0.11)	(−7.19)	
CB_{SEK}	−0.004	0.28	−0.22	0.41	−0.001	−0.18	0.43
	(−8.82)		(−10.70)		(−2.06)	(−5.98)	
CB_{NZD}	0.000	0.00	0.19	0.23	−0.004	0.35	0.39
	(0.58)		(9.65)		(−5.35)	(9.31)	
CB_{GBP}	0.000	0.00	−0.14	0.16	0.003	−0.24	0.26
	(−0.77)		(−6.71)		(4.36)	(−7.40)	
Adjusted R^2 (all)		0.23		0.33			0.42
Adjusted R^2 (non-dollar)		0.17		0.34			0.39
RMSE correlation	0.214		0.192			0.161	
Block bootstrap 95% confidence interval	[0.163, 0.234]		[0.118, 0.215]			[0.127, 0.182]	

Notes. The table compares the explanatory power of the global dollar factor (*HML$*) of Verdelhan (2018) and our cluster currency factor. Panel A reports the results of regressing currency baskets on *HML$*. Panel B reports the results of regressing currency baskets on our currency factor. Panel C reports the results of regressing currency baskets on *HML$* and our currency factor. The *t*-statistics are based on White standard errors and reported in parentheses. The RMSE is the root mean squared error of the implied correlations. The global dollar factor (*HML$*) of Verdelhan (2018) is available from November 1988 to December 2010.

USD currency basket, and they essentially regress bilateral currency changes relative to the dollar onto CB_{USD}. Thus, it is a simpler version of the Verdelhan regression (without the carry factor) in Equation (26).[13] We replicate this regression for bilateral dollar rates and also estimate its currency basket analogue. To conserve space, we relegate the estimation results to the online appendix. In fact, the coefficients are very similar to those of the Verdelhan regressions reported in Table 11, to which we will refer in our discussion.

Recalling that a bilateral currency change is approximately the difference between two currency baskets, it is straightforward to derive that $FL_{LR} = \frac{9}{10}[1 - FL_{CB}]$, where FL_{LR} stands for the factor loading in LR, and FL_{CB} is the factor loading of regressing currency baskets on the USD basket. Therefore, the regression beta in LR, everything else equal, is decreasing in the comovement between the currency baskets. The empirical results confirm this intuition. Within Europe, the LR beta is smallest for the economically close United Kingdom and higher for the farther-away Scandinavian countries. In this article, we use currency baskets to represent currency comovements. From that perspective, the CAD and JPY currency baskets are the only ones positively correlated with the USD basket (they show a short "distance"), as shown in Table 4. The JPY and CAD have the lowest betas and R^2s in the LR regressions. Panel B of Table 11 shows that if we recast the LR regressions in our currency basket framework, then the CAD and JPY baskets are the only baskets with positive betas.

In the final part of their paper, LR use kernel models under complete markets to show that the base

Table 13. Explaining the Variation in 21 Emerging Currency Baskets

	R^2	RMSE
CF_{abs}	0.05	0.189
	[0.01, 0.21]	[0.154, 0.218]
$CF_{31} + CF_{32}$	0.12	0.158
	[0.00, 0.34]	[0.129, 0.189]
$CF_{abs} + CF_{com} + CF_{TW}$	0.15	0.151
	[0.02, 0.40]	[0.123, 0.187]
$CF_{31} + CF_{32} + CF_{TW}$	0.15	0.151
	[0.02, 0.40]	[0.123, 0.188]
$CF_{abs} + CF_{com} + CF_{EM}$	0.15	0.161
	[0.03, 0.37]	[0.135, 0.186]
$CF_{abs} + CF_{com} + CF_{TW} + CF_{EM}$	0.19	0.151
	[0.05, 0.42]	[0.127, 0.184]
$CF_{31} + CF_{32} + CF_{EM}$	0.16	0.158
	[0.03, 0.37]	[0.134, 0.185]
$CF_{31} + CF_{32} + CF_{TW} + CF_{EM}$	0.19	0.152
	[0.05, 0.42]	[0.128, 0.184]
All extant	0.13	0.172
	[0.02, 0.27]	[0.151, 0.196]

Notes. The table presents results of regressing 21 emerging currency baskets on our suggested currency factors. The emerging currency baskets are the average appreciation rate of the emerging currency w.r.t. 30 currencies (20 other emerging currencies and the G10 currencies). Our suggested currency factors include a simple factor based on two absolute clusters as in Equation (19), CF_{abs}, two currency factors based on three absolute clusters as in Equations (22) and (23), CF_{31} and CF_{32}, a commodity currency factor as in Equation (24), CF_{com}, a G10 currency trading-volume-weighted factor as in Equation (25), CF_{TW}, and an emerging currency trading-volume-weighted factor as in Equation (25), CF_{EM}, where the weights are from the BIS and reported in the appendix. The *t*-statistics are based on White standard errors and reported in parentheses. The table also reports the average R^2s, as well as their ranges in brackets. The RMSE is the root mean squared error of implied correlations. The sample extends from July 1993 to December 2015.

factor loading in their regressions is inversely related to exposure to a common "global" shock. We can recast the LR findings and their link with pricing kernels in terms of currency basket regressions with minimal assumptions. Section 1.3 shows that currency basket covariances economically reflect exposures to the global pricing kernel. Presumably, currency baskets of nearby countries should be highly correlated (and they are; see Table 5) and show positive currency basket betas, as they have similar exposures to the global kernel. In the LR world, the currencies of these countries show low bilateral currency betas and low "systematic" risk. Peripheral countries relative to the base country, in contrast, have high LR betas, and thus low currency basket betas, reflecting dissimilar exposures to the global pricing kernel, relative to the base currency.

6. Explaining Emerging Market Currencies

As an out-of-sample exercise, we verify how well the various factor models fit currency variation and correlations in 21 emerging markets. Our sample period here extends from July 1993 to December 2015.[14] We consider our two new models, one involving CF_{abs}, the currency commodity factor (CF_{com}), and the market factor (CF_{tw}), and the other involving the two cluster factors (CF_{31} and CF_{32}) and the market factor. We also consider the performance of our cluster factors separately.

Our set of emerging countries includes Brazil (BRL), Chile (CLP), China (CNY), Columbia (COP), Czech Republic (CZK), Hungary (HUF), Israel (ILS), Indonesia (IDR), India (INR), Mexico (MXN), Malaysia (MYR), Peru (PEN), Philippines (PHP), Poland (PLN), Romania (RON), Russia (RUB), South Africa (ZAR), South Korea (KRW), Taiwan (TWD), Thailand (THB), and Turkey (TRY). Note that our currency baskets in this case include both emerging and developed currencies (i.e., 31 currencies). For example, the China yuan basket factor is the average appreciation of Chinese yuan with respect to 20 other emerging market currencies and the 10 G10 currencies. However, we only consider correlations between the emerging market baskets.

Table 13 presents the results in terms of average R^2s and RMSEs with confidence bands based on a block bootstrap. The parameter estimates are reported in the online appendix. A single CF_{abs} factor model explains on average 5% of the variation in emerging currency baskets, and the coefficient on CF_{abs} is statistically significant for 18 out of 21 emerging currency baskets (see the online appendix). The RMSE of its implied currency correlations is 0.189. Therefore, the CF_{abs} factor explains less variation in emerging currency baskets than in G10 currency baskets and also fits the correlations among the 21 emerging currency baskets slightly worse than those among the G10 currency baskets. As the online appendix shows, the large idiosyncratic risk displayed by emerging market currencies both explains the low R^2s and reduces correlations among emerging market baskets, making the correlation error larger in relative terms.

Bivariate regressions of the 21 emerging currency baskets on the two cluster factors, CF_{31} and CF_{32}, reveal that this model explains on average 12% of currency variation. The coefficients for CF_{31} and CF_{32} are statistically significant for 17 and 9 out of 21 emerging currency baskets, respectively. The RMSE of its implied currency correlations is 0.158.

We now consider the performance of the two new three-factor models. In multivariate regressions of the 21 emerging currency baskets on our suggested three factors CF_{abs}, CF_{com}, and CF_{TW} jointly, the commodity factor, CF_{com}, is statistically significant for five currencies and CF_{TW} is significant for 15 currencies. Our new three-factor model explains on average 15% of the emerging currency basket variation, and the

RMSE of its implied currency correlations is 0.151. The alternative three-factor model, combining our currency trading-volume-weighted factor, CF_{TW}, with CF_{31} and CF_{32}, generates an average R^2 of 15%, and the coefficient on CF_{TW} is statistically significant for 13 out of 21 emerging currency baskets. The RMSE of the model's implied currency correlations is 0.151, which is only slightly lower than that of the two-factor model. In fact, the improvements produced by the two three-factor models relative to the model with the two cluster factors are economically and statistically insignificant, but they are significant relative to CF_{abs}.

Whereas emerging market currencies have more country-specific risk than developed currencies, it is conceivable that there is an emerging market factor. Emerging markets is still a popular asset class among institutional investors, and currency hedging for emerging market investments remains uncommon, potentially inducing comovements between the asset class and emerging market currencies overall. Moreover, emerging market currency exposure may reflect "carry trade" exposure, with many emerging market currencies featuring relatively high interest rates.

We therefore create an emerging market currency factor (CF_{EM}) using relative trade weights, as we did for the G10 currencies. The trade weights are reported in the appendix.[15] We either replace the CF_{TW} factor in our preferred three-factor model by the corresponding emerging market factor or we add it to the basic three-factor model. However, replacing the CF_{TW} factor by the emerging market factor worsens the performance of the model, with the R^2s failing to improve and the RMSEs becoming slightly worse. When we use the four-factor model, the R^2 modestly increases to 19%, but the correlation fit is similar to that of the parsimonious three-factor model. Yet, 12 of the emerging market currencies have significant exposure to the emerging market factor. Performing the same analysis for the alternative three-factor model with CF_{31} and CF_{32}, we find analogous results. We conclude that our new factor model also provides the best fit for emerging market currencies. Finally, we also verify the performance of the extant factors, finding that they explain less of the variation of emerging market currency baskets and generate higher RMSEs (see the last lines in Table 13).

7. Conclusions

We examine various factor models to explain currency (co) movements and document their fit with the data. Rather than studying bilateral rates with a specific numéraire currency, as is customary in the literature, we focus on "currency baskets," representing the average of each currency's changes relative to all other currencies. For the G10 currencies, studying 10 currency baskets is equivalent to studying all 45 unique currency pairs. This methodology, together with a clustering technique, helps us detect a clear factor structure in currency comovements suggesting two currency blocks. One block includes the dollar rates and the yen, and the other block includes the European currencies.

The new factor is a very significant determinant of variation in the 10 currency baskets. When combined with a currency commodity factor (including the AUD, NZD, CAD, and NOK) and a market factor, which we construct from the currency baskets using trade volumes from the BIS, a parsimonious factor model results that explains on average 58% of the changes in the various currency baskets. It also fits the currency basket correlations quite well, generating an RMSE of only 0.11. In addition, this parsimonious model also has significant explanatory power for emerging currency baskets and fits their comovements well, with an RMSE of only 0.15.

We also compare the performance of the new model with that of extant currency factors, including the carry, volatility, value, and momentum currency factors, a commodity prices factor, and a world-equity factor. The carry and value factors fit currency variances and correlation much better than the other extant currency factors. However, any factor model created from the extant currency factors performs much less well than the new factor model. This result also extends to emerging market currency baskets. In addition, we find that our new currency factor—the sum of "dollar" basket factors and the yen— has more explanatory power for global currency variation than the global dollar factor of Verdelhan (2018). Economically, the correlations between currency baskets underlying the factors are inversely related to the physical distances between countries. Moreover, the factor structure in currency baskets is related to the exposure of the corresponding pricing kernels with respect to the global pricing kernel.

With active currency management becoming more commonplace, our findings should help currency managers and international investors to better explain the risks and comovements of currencies worldwide. In the "active" currency management space, the factors used to measure risk exposures and performance typically include a carry, momentum, and value factor (see, e.g., the often-used Deutsche Bank factors in Pojarliev and Levich 2008). Tradeable baskets corresponding to our three factors do not exist, but it would be straightforward to create them (see the online appendix). The new factor model can also be of help in implementing international asset pricing models, where, unless very restrictive assumptions are made (see especially Sercu 1980), the risk premium on stocks involves a

Table A.1. The G10 Currency Trade Weights

Date	USD	AUD	CAD	CHF	EUR	JPY	NOK	SEK	NZD	GBP
April 30, 1998	50.6%	1.7%	2.0%	4.1%	22.1%	12.6%	0.1%	0.2%	0.1%	6.4%
April 30, 2001	48.9%	2.3%	2.4%	3.3%	20.6%	12.8%	0.8%	1.4%	0.3%	7.1%
April 30, 2004	47.9%	3.3%	2.3%	3.3%	20.4%	11.3%	0.8%	1.2%	0.6%	9.0%
April 30, 2007	47.8%	3.7%	2.4%	3.8%	20.7%	9.6%	1.2%	1.5%	1.1%	8.3%
April 30, 2010	47.1%	4.2%	2.9%	3.5%	21.7%	10.5%	0.7%	1.2%	0.9%	7.2%
April 30, 2013	48.7%	4.8%	2.6%	2.9%	18.7%	12.9%	0.8%	1.0%	1.1%	6.6%

Notes. This table reports currency trade weights for the G10 currencies. The weights are based on trade volumes reported by BIS. The trade weights are renormalized to add to 1.

currency risk premium for each currency. This renders real-world applications of such models rather impractical. Our factor model can be used to reduce the dimensionality of these models, requiring the specification of at most three currency risk premiums overall and three currency factor exposures for each country. Another application involves models of optimal currency hedging. To determine optimal hedge ratios, a full covariance matrix of all currencies and their covariances with the underlying asset returns (e.g., equity returns) is required. Because equity returns and currencies tend to show small long-term correlations, the covariances among currencies are particularly important, and poor estimates of these covariances can lead to hedge coefficients that are hard to interpret. With the factor model, the problem is again reduced to estimating a parsimonious covariance matrix for the factors, and the exposures of currencies and equities to these currency factors. In addition, the currency basket concept can be useful if an investor wants to hedge an (equally weighted) international equity portfolio, as a regression of portfolio returns on its domestic currency basket suffices to measure currency exposure.

Of course, much additional research is needed. We have only studied our factors in terms of their ability to fit comovements and have not considered the returns associated with them or their ability to price the cross-section of currency portfolios. We have also focused on unconditional correlations, and it is well known that currency correlations vary through time (see, e.g., Hau and Rey 2006). They may also depend on economic conditions (see Christiansen et al. 2011) or be affected by structural shifts such as the introduction of the euro in 1999. It should be straightforward to use high-frequency data to extend our methodology to a conditional framework. Finally, the currency basket concept that we introduced has additional applications; it is potentially a useful tool to create numéraire independent global returns (see Aloosh 2017).

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Appendix

Tables A.1 and A.2 present the currency trading weights reported by the Bank for International Settlements (BIS). In the online appendix, Table A4 reports the explanatory power of the trade-weighted currency factor (CF_{TW}). As the USD is the most traded currency (more than 47% of all currency trading volumes), it has a high weight in the construction of CF_{TW}, and thus the CF_{TW} has a high explanatory power for CB_{USD}, with an R^2 of 83%. However, CF_{TW} has lower average explanatory power than CF_{abs} (see Table 6).

Table A.2. Emerging Currency Trade Weights

Date	BRL	CLP	CNY	COP	CZK	HUF	ILS	IDR	INR	MXN	MYR	PEN	PHP	PLN	RON	RUB	ZAR	KRW	TWD	THB	TRY
April 30, 1998	8.0%	4.0%	0.0%	0.0%	12.0%	0.0%	0.0%	4.0%	4.0%	20.0%	0.0%	0.0%	0.0%	4.0%	0.0%	12.0%	16.0%	8.0%	4.0%	4.0%	0.0%
April 30, 2001	9.8%	3.9%	0.0%	0.0%	3.9%	0.0%	2.0%	0.0%	3.9%	15.7%	2.0%	0.0%	0.0%	9.8%	0.0%	5.9%	17.6%	15.7%	5.9%	3.9%	0.0%
April 30, 2004	4.9%	1.6%	1.6%	0.0%	3.3%	3.3%	1.6%	1.6%	4.9%	18.0%	1.6%	0.0%	0.0%	6.6%	0.0%	9.8%	11.5%	18.0%	6.6%	3.3%	1.6%
April 30, 2007	4.7%	1.2%	5.9%	1.2%	2.4%	3.5%	2.4%	1.2%	8.2%	15.3%	1.2%	0.0%	1.2%	9.4%	0.0%	8.2%	10.6%	14.1%	4.7%	2.4%	2.4%

Table A.2. (Continued)

Date	BRL	CLP	CNY	COP	CZK	HUF	ILS	IDR	INR	MXN	MYR	PEN	PHP	PLN	RON	RUB	ZAR	KRW	TWD	THB	TRY
April 30, 2010	6.3%	1.8%	8.1%	0.9%	1.8%	3.6%	1.8%	1.8%	9.0%	11.7%	2.7%	0.0%	1.8%	7.2%	0.9%	8.1%	6.3%	13.5%	4.5%	1.8%	6.3%
April 30, 2013	7.0%	1.9%	13.9%	0.6%	2.5%	2.5%	1.3%	1.3%	6.3%	15.8%	2.5%	0.6%	0.6%	4.4%	0.6%	10.1%	7.0%	7.6%	3.2%	1.9%	8.2%

Notes. This table report currency trade-weights for emerging currencies. The weights are based on trade volumes reported by BIS (Bank for International Settlements). The trade weights are renormalized to add to 1.

Endnotes

[1] In an analogous but different effort, Panayotov (2020) attempts to identify global risk factors using currency perspective-invariant carry trades.

[2] Verdelhan (2018) sorts currencies in six portfolios each month according to their interest rates, recording their equally weighted exchange rate changes. His first factor is the average across all six portfolios, and the second factor is the spread portfolio of currencies with high versus low exposure to the level factor.

[3] Prominent studies of FX liquidity mostly focus on these currencies, but Karnaukh et al. (2015) do show that bid-ask spreads are lower for the SGD than for the SEK, NZD, and NOK.

[4] These countries include Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Europe (Euro), Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, South Korea, Spain, Sweden, Switzerland, Turkey, and the United Kingdom.

[5] The number of currencies available varies over time; and when not a multiple of 3, the ranking puts more currencies in the middle group. Further details are given in the online appendix.

[6] To check robustness to alternative measurements, we also check the performance of nontraded volatility risk factors, both in levels and in changes. Inspired by further analysis in Menkhoff et al. (2012a), we also construct a high-low spread portfolio based on ranking currencies with respect to their exposure to volatility changes (using 36 months of rolling data), and one based on a mimicking volatility factor, regressing volatility shocks on the bilateral currency excess returns versus the dollar. We report the results in the online appendix. The results are mostly similar to the ones reported here, with the first three factors actually performing slightly worse in terms of R^2 and RMSE, compared with the spread factor that we employ. However, the volatility-mimicking factor performs better, without altering our main conclusions.

[7] Because equity market data for New Zealand are not available for the full sample, we use equity market data for Singapore instead. We also construct a value-weighted world equity market return as the market-capitalization-weighted average of these equity market returns, based on market values available on Datastream, which produces very similar but slightly weaker results.

[8] Verdelhan's (2018) U.S. dollar factor is more than 97% correlated with the U.S. dollar basket factor.

[9] Assume that EUR/USD is the left-hand-side variable in the regression. If we exclude it in the composition of the dollar basket factor, as well as in the euro basket factor, then we have $CB_{\$} = \frac{1}{9}\left[\Sigma_j^9 \Delta s_{j,\$}\right]$ and $CB_{€} = \frac{1}{9}\left[\Sigma_j^9 \Delta s_{€,j}\right]$, and, as a result, $\Delta s_{€,\$} = \frac{8}{9}CB_{\$} - \frac{8}{9}CB_{€}$. As can be seen, the dollar basket factor ($CB_{\$}$) is still a part of the left-hand-side variable. Thus, our concern is valid, even after excluding the EUR/USD exchange rate changes ($\Delta s_{€,\$}$ *and* $\Delta s_{\$,€}$) in the composition of the basket factors, $CB_{\$}$ and $CB_{€}$, respectively. The supportive empirical evidence is available on request.

[10] In the working paper version, Verdelhan also proposes to use the numéraire currency basket factor as the explanatory variable (e.g., a pound basket factor for the bilateral rates w.r.t. the British pound). Obviously, such factors are not truly global, and all perform poorly in terms of global fit (see Table 3).

[11] That their tight link with the dollar results in a *low*, and not a high, beta is again due to the nature of the Verdelhan regression: with a pegged currency, the dependent variable has little variation and is regressed onto the dollar basket, which has plenty of variation. This results in a low beta. In our rewritten currency basket regression, the pegged currencies would naturally show high dollar basket betas.

[12] Head and Mayer (2002) introduce physical distance between countries i and j as a population-weighted average distance between their 25 more populated cities; $PDistance_{i,j} = \Sigma_{k\in i}\left(\frac{p_k}{p_i}\right)\Sigma_{l\in j}\left(\frac{p_l}{p_j}\right)d_{k,l}$, where p_k and p_l are populations of cities k and l, respectively, p_i and p_j are populations of countries i and j, respectively, and $d_{k,l}$ is the distance between two cities k and l.

[13] LR exclude the bilateral exchange rate on the left-hand side from the base factor on the right-hand side. As shown in note 9, this does not resolve the common variation issue in the dependent and independent variables.

[14] Because the sample is shorter than for the G10 currencies, the results are not exactly comparable with our previous results. However, in results available on request, we find that the RMSEs for the shorter sample are similar (albeit slightly higher) than for the full sample (0.12–0.13 for the two new three-factor models).

[15] In the latest BIS survey, the Mexican peso was the most traded emerging market currency (almost 15.8% of trading volume), followed by the Chinese renminbi (13.9%) and the Russian ruble (10.1%). Earlier in the sample, the South African rand (ZAR) was the second most traded currency, but the Mexican peso always has comprised an important part of trading volumes. In contrast, the CNY represented a rather negligible part of trading volumes until 2007; since then, its weight has increased to over 13%.

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