Central Bank Credibility and Fiscal Responsibility*

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Abstract

We consider a New Keynesian model with strategic monetary and fiscal interactions. The fiscal authority maximizes social welfare. Monetary policy is delegated to a central bank with an anti-inflation bias that suffers from a lack of commitment. The impact of central bank hawkishness on debt issuance is non-monotonic because increased hawkishness reduces the benefit from fiscal stimulus while simultaneously increasing real debt capacity. Starting from high levels of hawkishness (dovishness), a marginal increase in the central bank's anti-inflation bias decreases (increases) debt issuance.

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1 Introduction

Conventional wisdom holds that a more hawkish central bank will promote fiscal responsibility. Policymakers will refrain from pursuing a debt-fueled government expansion if they expect the central bank to counteract it with high interest rates.[1] Indeed, the notion that a conservative central bank can serve to discipline fiscal policy is often used as a justification for moving central banks towards inflation targeting.

This mechanism is supported by a number of historical episodes where fiscal discipline has improved in countries pursuing monetary stabilization.[2] Moreover, it can also explain cases where a relaxation of monetary stance can lead to fiscal expansions. For instance, in August 2020 the U.S. Federal Reserve changed its monetary framework towards average inflation targeting. Because this new policy framework allowed for inflation to temporarily overshoot the 2 percent target, this change was interpreted by many as a shift towards a more dovish monetary policy.[3] In the ensuing months, the U.S. Congress took public debt to unprecedented levels by passing two massive fiscal stimulus bills, and this was followed by the highest level of inflation since the early 1980s.[4]

Despite numerous examples consistent with the conventional wisdom, there are also cases of the reverse, where the introduction of monetary stabilization was followed by a deterioration of fiscal discipline.[5] For example, consider the adoption of the Euro in 1999. Greece—which had experienced inflation levels averaging above 10 percent in the previous two decades—saw inflation drop precipitously. However, rather than responding with a more restrained fiscal policy, the Greek government facing historically low interest rates undertook a large debt buildup.[6] This contrasted with Germany—which also saw its inflation level and volatility decline following Euro adoption—but did not pursue a fiscal expansion and instead pursued fiscal reforms.[7]

Why would a more conservative central bank improve fiscal discipline in some cases but worsen it in others? In this paper, we explore this question using a simple two-period New Keynesian model with monetary and fiscal interactions. A key feature of

[1] This idea goes back at least as far as Ricardo (1824). See Sims (2016) and Orphanides (2018) for more recent discussions.

[2] Fatas and Rose (2001), for example, find that belonging to a currency board results in an improvement in fiscal outcomes.

[3] See Board of Governors of the Federal Reserve System (2020) and Politi and Smith (2020)

[4] Similar developments occurred in the Euro Area, where the European Central Bank also announced a change to its strategy in 2021, and what ensued was a record increase in public debt across the Euro area, even in countries with historically large surpluses like Germany.

[5] See for example the analysis in Frieden (2018) and Tornell and Velasco (1995, 1998)

[6] See for example the discussion in Jalles et al. (2021).

[7] See Halac and Yared (2018) and the references therein for a discussion of those reforms.

our framework is that monetary policy is delegated to a central bank that does not have the same welfare as the fiscal authority and puts more weight on fighting inflation, as in Rogoff (1985). Our analysis considers how public debt issuance responds to changes in the central bank's hawkishness.

We show that there are two forces at play. On the one hand, a more hawkish central bank is more inclined to keep real interest rates high to offset fiscal stimulus to prevent the accompanying inflation. On the other hand, a more hawkish central bank is less likely to inflate away debt ex post, and this increases real debt capacity by inducing lower future debt devaluation in response to additional borrowing. In the face of a more hawkish central bank, the first force induces *more* fiscal discipline, since a fiscal authority sees a lower benefit of stimulus. However, the second force induces *less* fiscal discipline, since a fiscal authority sees a greater opportunity to borrow.

Our analysis evaluates the relative strength of these two forces, and our main result is that the impact of central bank hawkishness on debt issuance is non-monotonic. Starting from high levels of hawkishness, a marginal increase in the central bank's anti-inflation bias decreases debt issuance, whereas the opposite happens starting from low levels of hawkishness. This non-monotonicity emerges because the starting level of real debt capacity is higher for countries with more hawkish central banks. Thus, they do not respond on the margin to an increase in real debt capacity induced by an increase in central bank hawkishness. They instead become more fiscally disciplined in anticipation of more counteraction of stimulus by a more inflation-averse central bank. In contrast, countries with very dovish central banks are debt constrained, and they increase their debt in response to an increase in real debt capacity. In those cases, an increase in central bank hawkishness leads to less fiscal discipline.[8]

Our results provide us with a framework to interpret our motivating case studies. The increase in public debt in the U.S. following the adoption of the New Monetary Policy framework in 2020 can be interpreted as the optimal response of a fiscal authority that is not debt constrained and that is anticipating a more accommodative monetary policy. The increase in public debt in Greece following entry into the Euro in 1999 can in contrast be interpreted as the optimal response of a fiscal authority that is debt constrained and taking advantage of its enhanced debt capacity following monetary stabilization. Moreover, Germany's fiscal belt tightening following Euro adoption can also be rationalized by our model once we consider Germany's different starting point relative to Greece. Germany's pre-Euro central bank was much more hawkish than Greece's, implying that Germany

[8] Those governments issue more debt even though the degree of central bank accommodation conditional on the level of real debt issuance is lower.

was further away from its debt capacity. Our model predicts that a country not bound by its debt capacity would become more fiscally disciplined in the face of an increase in central bank hawkishness.

Related Literature This paper contributes to the literature on monetary and fiscal interactions.[9] Relative to this literature, we do not assume that policies are exogenous, but we consider policies chosen by a monetary and a fiscal authority that interact strategically with one another in the absence of commitment.[10]

The work of Dixit and Lambertini (2003) and Adam and Billi (2008) also study strategic monetary and fiscal interactions. In contrast to their analysis, our model explicitly considers the role of government debt and the possibility of debt devaluation via inflation, which they abstract from. In this regard, we build on Alvarez et al. (2004), Chari and Kehoe (2007), and Aguiar et al. (2015), among others, who consider the monetary authority's commitment to preserving the value of public debt. In contrast to this work, we introduce sticky prices, which endows the central bank with an additional role in supporting a fiscal expansion with a monetary one. The combination of this feature with the possibility of a debt devaluation is what generates the non-monotonic results in our model.

This paper more broadly builds on the literature that studies the time consistency of monetary policy (e.g., Kydland and Prescott (1977), Calvo (1978), and Barro and Gordon (1983)). As in the seminal contribution of Rogoff (1985), we consider an environment in which monetary policy is delegated to a central bank with an anti-inflation bias. Using this framework, we show that a higher anti-inflation bias not only increases a central bank's incentives to counteract an inflationary fiscal stimulus, but it also increases a government's debt capacity.

Finally, this paper also relates to the literature on fiscal responsibility and fiscal rules.[11] Relative to this literature, we consider the extent to which delegation of monetary policy serves as an indirect fiscal rule, which changes a government's debt capacity.

[9] See Leeper and Leith (2016) for a survey.

[10] As such, our discussion does not touch on questions of determinacy or monetary versus fiscal dominance. In our framework, inflation is jointly determined by the sequential decisions of the monetary and fiscal authorities.

[11] This literature includes, among others, Halac and Yared (2014, 2018, 2022), Azzimonti et al. (2016), Dovis and Kirpalani (2020), and Bouton et al. (2020).

2 Model

We consider a simple two-period New Keynesian model with $t = 0, 1$. At each date, households choose consumption across varieties, labor, and savings. Monopolistically competitive firms sell consumption varieties. Prices are sticky at date 0 and flexible at date 1. At each date, the government chooses proportional consumption taxes, lump sum taxes, government spending, government debt, and the price level.[12] The government also chooses the nominal interest rate at date 0.

2.1 Households

There is a continuum of mass 1 of households that have the following preferences over a consumption bundle $C_t \geq 0$, labor $N_t \geq 0$, and government spending $G_t \geq 0$:

$$\sum_{t=0,1} \left((1-\mu) \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) + \mu \log G_t \right), \tag{1}$$

where $\mu \in (0, 1)$ and $\varphi \geq 0$.[13] Moreover, C_t satisfies $C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}} \right)^{\frac{1}{1-\sigma^{-1}}} dj$, where $C_{j,t} \geq 0$ is the household consumption of variety j at date t and $\sigma > 1$. We define G_t analogously as composed of government consumption of varieties $G_{j,t} \geq 0$, where $G_t = \left(\int_0^1 G_{j,t}^{1-\sigma^{-1}} \right)^{\frac{1}{1-\sigma^{-1}}} dj$.

The household budget constraint at date 0 is

$$(1+\tau_0) \int_0^1 P_{j,0} C_{j,0} dj + P_0 T_0 + \frac{1}{1+i} B = W_0 N_0 + \phi_0,$$

and at date 1 is:

$$(1+\tau_1) \int_0^1 P_{j,1} C_{j,1} dj + P_1 T_1 = W_1 N_1 + \phi_1 + B.$$

$P_{j,t}$ is the price of consumption variety j at date t, and $P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} dj$ is the price

[12] We assume that the government can coordinate firm behavior by setting the price level via monetary policy. This is necessary since the model has a finite horizon with limited commitment, and the price level needs to be pinned down in the final period.

[13] We consider balanced growth path preferences, as they imply a globally concave policy problem under flexible prices (i.e., a globally concave implementability condition). Our results extend to other preferences for which this is the case. The analog of Assumption 1 in Debortoli et al. (2017) would be required in that case.

index. The variable i is the nominal interest rate at date 0, B are nominal government bonds purchased by households at date 0, W_t is the nominal wage at date t, τ_t is a proportional consumption tax at date t, T_t is a lump sum tax at date t, and ϕ_t is the before-price adjustment costs profit from firms owned by the households at date t.

2.2 Firms

There is a continuum of mass 1 of firms, each indexed by j, corresponding to the variety produced by the firm. The production function is

$$N_{j,t} = C_{j,t} + G_{j,t}, \tag{2}$$

where $N_{j,t}$ represents the labor employed by firm j at date t. Firm profits before price adjustment costs at t equal

$$P_{j,t}\left(C_{j,t} + G_{j,t}\right) - W_t N_{j,t}.$$

At date 0, firms face quadratic price adjustment costs (as in Rotemberg (1982)) equal to

$$\frac{\alpha}{2}\left(P_{j,0} - 1\right)^2 P_0\left(C_0 + G_0\right), \tag{3}$$

so that deviations from a normalized price of 1 are costly for firms. This cost is proportional to aggregate nominal output $P_0\left(C_0 + G_0\right)$ and indexed by $\alpha > 0$ which parameterizes the degree of price stickiness. To faciliate exposition and with no bearing on our results, we assume that this cost corresponds to a transfer payment to workers.[14]

There are no price adjustment costs at date 1. Observe that the resource constraint of the economy requires that $\int_0^1 N_{j,t} dj = N_t$.

2.3 Government

The government budget constraint at date 0 is

$$\int_0^1 P_{j,0} G_{j,0} dj = \tau_0 \int_0^1 P_{j,0} C_{j,0} dj + P_0 T_0 + \frac{B}{1+i}, \tag{4}$$

[14] As such, the term ϕ_t in the household budget constraint corresponds to $P_{j,t}\left(C_{j,t} + G_{j,t}\right) - W_t N_{j,t}$. Our main results hold if the price adjustment cost is a resource cost or under Calvo pricing where only an exogenous fraction of firms are able to change their price. We consider this setting for expositional simplicity.

and at date 1 is

$$\int_0^1 P_{j,1} G_{j,1} dj = \tau_1 \int_0^1 P_{j,1} C_{j,1} dj + P_1 T_1 - B. \tag{5}$$

Government policy corresponds to the set $\left\{\left\{\{G_{j,t}\}_{j\in[0,1]}, \tau_t, T_t\right\}_{t=0,1}, B, i, P_0, P_1\right\}$. The government chooses varieties $G_{j,t}$ that are optimal conditional on G_t and prices $P_{j,t}$:

$$G_{j,t} = G_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma}. \tag{6}$$

The value $B \in [0, \overline{B}]$ for some finite, but arbitrarily large $\overline{B} > 0$.

We assume from hereon that taxes are exogenously set with:

$$\tau_t = -\frac{1}{\sigma} \tag{7}$$

and

$$T_t = T + \frac{1}{\sigma} C_t \text{ for } T < \mu (1-\mu)^{-\frac{1}{1+\varphi}}. \tag{8}$$

The assumption in (7) is standard in New Keynesian models as implies the absence of monopoly distortions in equilibrium. Note that the upper bound on T in (8) is satisfied under high enough μ. We discuss the implications of these assumptions on fiscal policy in Section 3.4.

3 Competitive Equilibrium

In this section, we define the necessary and sufficient conditions for a competitive equililibrium in which households and firms maximize their payoffs subject to their budget constraint given government policy. Using this characterization, we explain the implications of the assumptions on fiscal policy in (7) and (8).

3.1 Household Optimality

Optimal consumption across varieties implies that

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t}\right)^{-\sigma}. \tag{9}$$

Moreover, the household's intratemporal condition taking into account (7) is

$$C_t N_t^{\varphi} = \frac{\sigma}{\sigma - 1} \frac{W_t}{P_t}, \tag{10}$$

and the household's intertemporal condition is

$$1 = (1 + i) \frac{P_0}{P_1} \frac{C_0}{C_1}. \tag{11}$$

3.2 Firm Optimality

At date 0, firms maximize profits given the production function (2), price adjustment cost (3), and demand (9). The date 0 firm problem can be written as

$$\max_{P_{j,0}} \left((P_{j,0} - W_0) \left(\frac{P_{j,0}}{P_0} \right)^{-\sigma} - \frac{\alpha}{2} (P_{j,0} - 1)^2 P_0 \right) (C_0 + G_0) .$$[15]

The first order conditions yield

$$\frac{W_0}{P_0} = \frac{\sigma - 1}{\sigma} \frac{P_{j,0}}{P_0} + \frac{\alpha}{\sigma} \left(\frac{P_{j,0}}{P_0} \right)^{\sigma+1} (P_{j,0} - 1) P_0. \tag{12}$$

Since all firms are identical, $P_{j,0} = P_0$, and this condition becomes

$$\frac{W_0}{P_0} = \frac{\sigma - 1}{\sigma} + \frac{\alpha}{\sigma} (P_0 - 1) P_0. \tag{13}$$

By analogous reasoning, first order conditions at date 1, taking into account the absence of price adjustment costs, yield

$$\frac{W_1}{P_1} = \frac{\sigma - 1}{\sigma}. \tag{14}$$

3.3 Aggregation

We now characterize the allocations at date 0 and 1 as a function of policy. Since $C_{j,t} = C_t$, $G_{j,t} = G_t$, and $N_{j,t} = N_t$ for all j and t, equation (2) implies an aggregate resource constraint at $t = 0, 1$:

$$N_t = C_t + G_t. \tag{15}$$

[15] At date 0, dynamic considerations do not have to be made since all firms can change their prices flexibly at date 1.

Combining (13) and (14) with the intratemporal condition (10) taking into account (7), we achieve

$$C_0 N_0^{\varphi} = 1 + \frac{\alpha}{\sigma - 1}\left(P_0 - 1\right) P_0 \text{ and} \tag{16}$$

$$C_1 N_1^{\varphi} = 1. \tag{17}$$

By analogous reasoning, government budget constraints (4) and (5) taking into account (6), (7) and (8), can be rewritten as

$$G_0 = T + \frac{C_0}{C_1}\frac{B}{P_1} \text{ and} \tag{18}$$

$$G_1 = T - \frac{B}{P_1}. \tag{19}$$

We can use this aggregation to characterize necessary and sufficient conditions for a competitive equilibrium.

Lemma 1 *Given (7) and (8), the set* $\left\{\left\{P_t, C_t, G_t, N_t\right\}_{t=0,1}, B, i\right\}$ *is a competitive equilibrium if and only if it satisfies (11) and (15) − (19).*

3.4 Discussion of Assumptions

The assumptions on taxes in (7) and (8) allow us to focus on monetary and fiscal interactions. To see why, it is useful to consider the first best benchmark in the absence of price adjustment costs. Maximization of welfare (1) subject to the resource constraint (15) yields the first best allocation, which admits $C_{j,t} = (1-\mu)^{\frac{\varphi}{1+\varphi}}$, $G_{j,t} = \mu\left(1-\mu\right)^{-\frac{1}{1+\varphi}}$, and $N_{j,t} = (1-\mu)^{-\frac{1}{1+\varphi}}$ for all j and t.

Observe that the first best allocation can be implemented as a competitive equilibrium with $\tau_t = -1/\sigma$, $P_t = 1$, and $T_t = \mu\left(1-\mu\right)^{-\frac{1}{1+\varphi}} + \frac{1}{\sigma}\left(1-\mu\right)^{\frac{\varphi}{1+\varphi}}$ for $t = 0, 1$, with $B = 0$. Intuitively, the monopolistic power of firms results in a labor wedge, which can be undone with a consumption subsidy of $1/\sigma$. Moreover, lump sum taxes can be chosen so that total tax revenue net of the consumption subsidy equals the first best level of government spending.[16]

Thus, the assumption on the value of τ_t in (7) implies that there is no role for monetary policy to undo monopoly distortions, since tax rates have already been set to do so. Importantly though, the assumption on T_t in (8) implies that tax revenue is not large

[16]By Ricardian Equivalence, multiple combinations of T_t and B could potentially satisfy the government budget constraints in this case.

enough to support the first-best value of government spending, and this provides a role for monetary policy in supporting the fiscal policy goal of increasing spending.

More specifically, consider the following observations. First, note from (19) that if $B > 0$, then an increase in P_1 increases G_1. By devaluing the debt via inflation, the government can increase public spending. Observe further that because of the assumption in (8), this will increase G_1 towards the first best level from below.

Second, observe that if $B = 0$, then $G_0 = G_1 = T$, where this follows from (18) and (19). This means that a spending increase at date 0 is infeasible without debt issuance. Note further that the solution that maximizes social welfare subject to the additional constraint that $G_0 = G_1 = T$ can be implemented with $P_0 = 1$. This means that in the absence of any public debt issuance, there is no social benefit from a monetary expansion that increases P_0 above 1. This follows from the assumption in (7), since absent this assumption, distortions due to monopoly power could be reduced with an increase in P_0.

Finally, observe that conditional on $B/P_1 > 0$, an increase in P_0 starting from $P_0 = 1$, holding C_1 and G_1 constant increases G_0, which is beneficial if it is approaching the first best level from below. More specifically, for a given $B/P_1 > 0$ and C_1, an increase in C_0 increases G_0 by increasing the right hand side of (18). In other words, a stimulus to consumption C_0 reduces the gross real interest rate (which equals C_1/C_0), thus allowing for more government spending for a given value of real debt issuance. Moreover, note that an increase in the price level P_0 is an indirect consumption subsidy which increases the right hand side of (16), thus increasing C_0 and G_0. This is a useful observation for establishing our later results.

4 Strategic Monetary and Fiscal Interactions

We consider the following game between a fiscal authority and a monetary authority. The fiscal authority shares the same preferences as society (1). The monetary authority's welfare is

$$\sum_{t=0,1} \left((1-\lambda) \left((1-\mu) \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) + \mu \log G_t \right) - \lambda H\left(P_t\right) \right) \tag{20}$$

for $\lambda \in (0,1)$ and $H(\cdot) \geq 0$ which is strictly convex and satisfies $H(1) = H'(1) = 0$. The value of λ captures how committed the monetary authority is to price stability versus maximizing social welfare. If $\lambda = 0$, the monetary and fiscal authorities share the same preferences, and if $\lambda = 1$, the monetary authority only cares about minimizing inflation.[17]

Observe that given that the values of B, P_0, and P_1, conditions (15) − (19) determine the allocations $\{C_t, G_t, N_t\}_{t=0,1}$. This means that the fiscal authority's and monetary authority's welfare (1) and (20) are determined by B, P_0, P_1.

Taking this into account, we consider a game with the following sequence of events.

1. The fiscal authority chooses B.
2. The monetary authority chooses P_0.
3. The date 0 market opens and clears.
4. The monetary authority chooses P_1.
5. The date 1 market opens and clears.

An important feature of this environment is that the monetary authority lacks commitment. It is unable to pre-commit to monetary policy before fiscal policy is chosen at date 0. Moreover it is unable to pre-commit to the price level at date 1 until after date 0 policies are chosen and the date 0 market has opened and cleared.[18] This is an important feature for generating the main insights of this model.[19,20]

5 Main Results

We use backward induction to characterize optimal policies chosen by the monetary and fiscal authorities. We then use this analysis to describe the equilibrium under extreme monetary bias. Finally, we describe policies under intermediate monetary bias for an analytical example.

[17] Without loss of generality, we can replace the monetary authority's inflation cost with $H(P_t - P_{t-1})$, so that its disutility is a function of price changes versus price levels. This is equivalent to our formulation at date 0 since $P_0 - P_{-1} = P_0 - 1$. But it may not be equivalent at date 1 since $P_1 - 1$ may not equal $P_1 - P_0$. However, because prices are flexible at date 1, this normalization has no bearing on our results.

[18] Observe that we have ignored the fiscal authority's decision at date 1 since it is implied by the government budget constraint at date 1.

[19] If instead the monetary authority chooses the price level before the fiscal authority chooses debt, the equilibrium policies change, but our main results regarding comparative statics on λ around the extremes do not change. If instead the monetary and fiscal authorities move simultaneously, then multiple equilibria emerge.

[20] We let the central bank choose the price level at date 0 in order to have a symmetric set of tools across dates. An equivalent formulation has the central bank choose the nominal interest rate i at date 0.

5.1 Characterization by Backward Induction

5.1.1 Monetary Policy at Date 1

At date 1, the monetary authority takes B as given as it solves the following program:

$$\max_{C_1,G_1,N_1,P_1} \left\{(1-\lambda)\left((1-\mu)\left(\log C_1 - \frac{N_1^{1+\varphi}}{1+\varphi}\right) + \mu \log G_1\right) - \lambda H(P_1)\right\} \quad (21)$$
$$\text{s.t. } (15), \ (17), \text{ and } (19).$$

The solution to the relaxed problem that ignores (17), achieves the following first order condition with respect to G_1

$$-(1-\mu) N_1^{\varphi} + \mu \frac{1}{G_1} = \frac{\lambda}{1-\lambda} H'\left(\frac{B}{T-G_1}\right) \frac{B}{(T-G_1)^2}. \quad (22)$$

and also satisfies (17). Therefore policy at date 1 is characterized by (15), (17), (19), and (22).

Lemma 2 *The solution to* (21) *has the following properties:*

1. *An increase in B increases P_1 and decreases G_1, and*

2. *An increase in λ decreases P_1 and decreases G_1.*

If the inherited debt is higher, this tightens the government budget constraint and results in lower levels of government spending for a given price level. As such, the central bank is more motivated to devalue the debt via inflation if the debt is higher.

Since B is bounded from above by $\overline{B} > 0$, G_1 is bounded from below by $\underline{G}_1(\lambda)$ which is the value of G_1 that solves (21) for $B = \overline{B}$. Note that $\lim_{\lambda \to 1} \underline{G}_1(\lambda) = 0$ and $\lim_{\lambda \to 0} \underline{G}_1(\lambda) = T$. Intuitively, if $\lambda = 1$, the central bank is very hawkish and does not tolerate any inflation. It will choose $P_1 = 1$ for any value of B and therefore, the level of spending is bounded from below by the non-negativity limit.[21] In contrast if $\lambda = 0$, then the central bank is very dovish and will choose $P_1 = \infty$ if $B > 0$, and therefore, debt is fully inflated away and the budget is always balanced.

There are two important observations. First, note that for the fiscal authority, choosing B at date 0 is equivalent to choosing G_1 directly, subject to the constraint that $G_1 \geq \underline{G}_1(\lambda)$, and taking into account that C_1 and N_1 will be determined according to (15) and

[21] Note in that case that if $\overline{B} > T$, then the choice of $B = \overline{B}$ at date 0 is equivalent to $B = T$, so that the date 0 government does not need to consider values of B that exceed T.

(17) given G_1.[22] For the remainder of our analysis, it will be useful for us to consider the fiscal authority as choosing G_1 directly, recognizing that higher values of G_1 are associated with lower choices of B. With that in mind, let us define $C_1^*(G_1)$ and $N_1^*(G_1)$ as the values of C_1 and N_1 that satisfy (15) and (17) given G_1.

Second, note that the monetary authority's policy at date 0 will have no effect on the monetary authority's policy at date 1, since the latter will be fully determined by the level of debt B chosen by the fiscal authority at date 0. Thus, the monetary authority at date 0 focuses on maximizing date 0 welfare.

5.1.2 Monetary Policy at Date 0

At date 0, the monetary authority takes the choice of B, and therefore G_1 and the implied P_1 from (22) as given. To facilitate the analysis, note that (18) and (19) can be combined to yield

$$\frac{T-G_0}{C_0}+\frac{T-G_1}{C_1}=0. \tag{23}$$

The monetary authority thus solves the following problem:

$$\max_{C_0,G_0,N_0,P_0}\left\{(1-\lambda)\left((1-\mu)\left(\log C_0-\frac{N_0^{1+\varphi}}{1+\varphi}\right)+\mu\log G_0\right)-\lambda H(P_0)\right\} \tag{24}$$

$$\text{s.t.}$$

$$(15),\ (16),\ (23),\ C_1=C_1^*(G_1),\ \text{and}\ N_1=N_1^*(G_1).$$

Observe that the value of G_1 that constrains this program maps directly into the value of real debt B/P_1, which the central bank at date 0 takes as given. We now characterize the optimal monetary policy at date 0.

Lemma 3 *The solution to (24) has the following properties:*

1. *If $B/P_1=0$, then $P_0=1$,*

2. *There exists $\upsilon>0$ such that $P_0>1\ \forall B/P_1\in(0,\upsilon)$, and*

3. *P_0 is locally decreasing in $\lambda\ \forall B/P_1\in(0,\upsilon)$.*

If $B/P_1=0$, there is no value from monetary stimulus for the central bank, since this would increase labor and consumption with no impact on government spending given the

[22] The implied value of P_1 determined by (22) is payoff irrelevant for the date 0 fiscal authority which places no weight on date 1 inflation.

budget is balanced. In contrast, starting from $B/P_1 > 0$, the monetary authority may wish to expand monetary policy, because increasing P_0 is an indirect labor subsidy. It results in higher aggregate demand for goods, which stimulates firm demand for workers, and which boosts wages, resulting in higher consumption and labor. Since households now face higher consumption at date 0 versus date 1, real interest rates faced by the fiscal authority decline, allowing for an increase in government spending toward the efficient level. Observe that the extent to which the central bank will accommodate the fiscal stimulus depends on its level of hawkishness, with more hawkish central banks accommodating the stimulus by less. Define by $\{C_0^*(G_1), G_0^*(G_1), N_0^*(G_1), P_0^*(G_1)\}$ the central bank's strategy at date 0.

5.1.3 Fiscal Policy at Date 0

The fiscal authority at date 0 takes as given the strategy of the date 0 monetary authority $\{C_0^*(G_1), G_0^*(G_1), N_0^*(G_1), P_0^*(G_1)\}$ and the strategy of the date 1 monetary authority $\{C_1^*(G_1), G_1^*(G_1)\}$, and it solves the following program:

$$\max_{G_1} \left\{ \begin{array}{c} (1-\mu)\left(\log\left(C_0^*(G_1)\right) - \frac{N_0^*(G_1)^{1+\varphi}}{1+\varphi}\right) + \mu \log\left(G_0^*(G_1)\right) \\ (1-\mu)\left(\log C_1^*(G_1) - \frac{N_1^*(G_1)^{1+\varphi}}{1+\varphi}\right) + \mu \log G_1 \end{array} \right\} \tag{25}$$

s.t.

$$G_1 \in \left[\underline{G}_1(\lambda), T\right]. \tag{26}$$

Given the reaction functions of the date 0 and date 1 central banks, the fiscal authority decides on how to allocate government spending between dates 0 and 1.

5.2 Extreme Bias

We now present the main result of the paper.

Proposition 1 *There exists $\overline{\lambda}, \underline{\lambda} \in (0,1)$ with $\overline{\lambda} > \underline{\lambda}$ such that in equilibrium*

1. *If $\lambda > \overline{\lambda}$ then B/P_1 is weakly decreasing in λ, and $B/P_1 \to 0$ as $\lambda \to 1$, and*
2. *If $\lambda < \underline{\lambda}$, then B/P_1 is weakly increasing in λ, and $B/P_1 \to 0$ as $\lambda \to 0$.*

To understand this result, suppose that $\lambda = 1$, so that the central bank is extremely hawkish and $P_0 = P_1 = 1$. Then there is no value for the fiscal authority from debt issuance, since issuing debt would tilt government spending towards date 0 while taking away from government spending at date 1 on a one for one basis, where this follows from the fact that the economy is identical in the two periods and households value consumption identically across periods. Given that government spending enters symmetrically across dates in the fiscal authority's welfare, it is optimal to not borrow and to smooth government spending across periods.

In contrast, suppose that $\lambda = 0$. In the face of an extremely dovish central bank at date 0, the fiscal authority values fiscal stimulus, because it knows that a dovish central bank will accommodate the stimulus and maximize social welfare. However, the central bank is also dovish at date 1 and suffers from lack of commitment, and the private sector anticipates that the central bank will devalue the debt at date 1. Therefore, nominal interest rates for any debt issues are infinity, making it impossible for the central bank to issue any debt and engage in fiscal stimulus. Therefore, even though it would be optimal for the fiscal authority to borrow, it is unable to.

To see what this means for comparative statics, consider a situation in which λ is close to 1 and the government is borrowing with $B/P_1 > 0$. Then a marginal increase in λ reduces B/P_1. A higher anti-inflation bias makes the benefit to the fiscal authority from stimulus lower, since the stimulus is less accommodated by the central bank. This reduces the incentive to issue debt, which results in lower stimulus and lower inflation. Intuitively, the value of $\underline{G}_1(\lambda)$ in the fiscal authority's program in (25) − (26) is not binding, which means that the main consideration for the fiscal authority is the extent to which the monetary authority will accommodate the stimulus at date 0.

In contrast, consider a situation in which λ is close to 0 with $B/P_1 > 0$. A marginal increase in λ increases B/P_1. This is because a higher anti-inflation bias increases debt capacity, since the central bank is more committed to not devaluing the debt in the future. This facilitates the issuance of debt for the fiscal authority, which results in greater stimulus. This comparative static stems from the lack of commitment of the central bank. The marginally more hawkish central bank would like to commit to either accommodating the stimulus by less at date 0 or to devaluing the debt by more at date 1 in order to dissuade fiscal stimulus. However, it is unable to do so.

5.3 Analytical Example

To facilitate analysis away from the extremes, we can consider a special case, where

$$\mu \to 1,\ \varphi = 0, \text{ and } H(P) = \frac{\kappa}{2}\left(P\left(P-1\right)\right)^2, \text{ for some } \kappa > 0. \tag{27}$$

Under this formulation a desire to increase government spending dominates social welfare considerations in (1) since $\mu \to 1$.[23] Moreover, labor is perfectly elastic, which means that (16) and (17) become:

$$\begin{aligned} C_0 &= 1 + \frac{\alpha}{\sigma - 1} (P_0 - 1) P_0 \text{ and} \\ C_1 &= 1. \end{aligned} \tag{28}$$

Date 0 consumption C_0 is proportional to monetary policy expansion P_0, whereas C_1 is constant. As such, the gross real interest rate equals C_0^{-1} and is decreasing in C_0 and P_0. Observe that further that (23) taking into account (28) yields:

$$G_0 = T + C_0 (T - G_1).$$

Finally, observe that $H(P_0)$ is equal to $\kappa \left(\frac{\sigma-1}{\alpha}\right)^2 (C_0 - 1)^2 / 2$. These observations imply that the central bank's problem at date 0 in (24) can be represented as

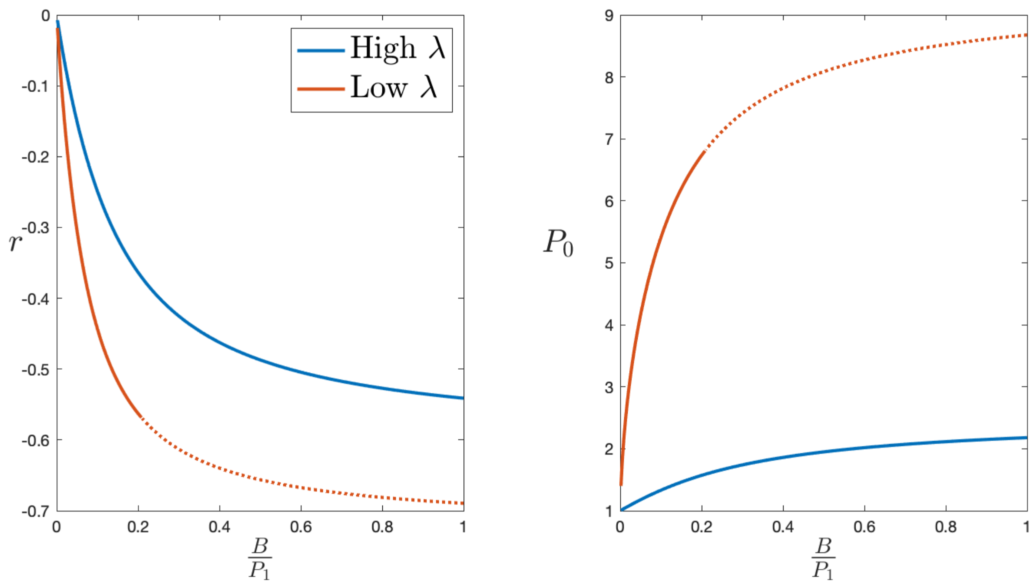
$$\max_{C_0} \left\{ (1 - \lambda) \log (T + C_0 (T - G_1)) - \lambda \kappa \left(\frac{\sigma - 1}{\alpha} \right)^2 (C_0 - 1)^2 / 2 \right\}, \tag{29}$$

Figure 1 displays the central bank's reaction function in the solution to (29) and what it implies for how the real interest rate $r = C_0^{-1}$ and inflation P_0 depend on a hypothetical value of issued real debt B/P_1 (which is inversely related with the level of spending G_1). Observe that as the level of real debt increases, so does the degree of central bank accommodation, with lower real interest rates C_0^{-1} and higher levels of inflation P_0.[24] Importantly, the extent of accommodation depends on the anti-inflation bias λ. For a given level of real debt issuance, real interest rates are higher and inflation is lower the more hawkish the central bank. Moreover, a more hawkish central bank is less accommodative on the margin as debt increases relative to a less hawkish one. However, note that a more accommodative central bank also makes higher level of real debt issuance infeasible by limiting debt capacity. These observations lead to the following proposition.

[23] This assumption means that the fiscal authority under $\mu = 1$ prefers maximal inflation at date 0 if $G_1 < T$, which is not the case in our benchmark model where $\mu < 1$, since inflation has direct costs on social welfare through subsidization of labor.

[24] The negative response of real interest rates to debt issuance is driven by the quasilinearity in this setting. Without this assumption, real interest rates can increase in response to debt issuance, with a smaller increase under a more dovish central bank.

Figure 1: Impact of Debt Issuance on Real Interest Rates and Inflation



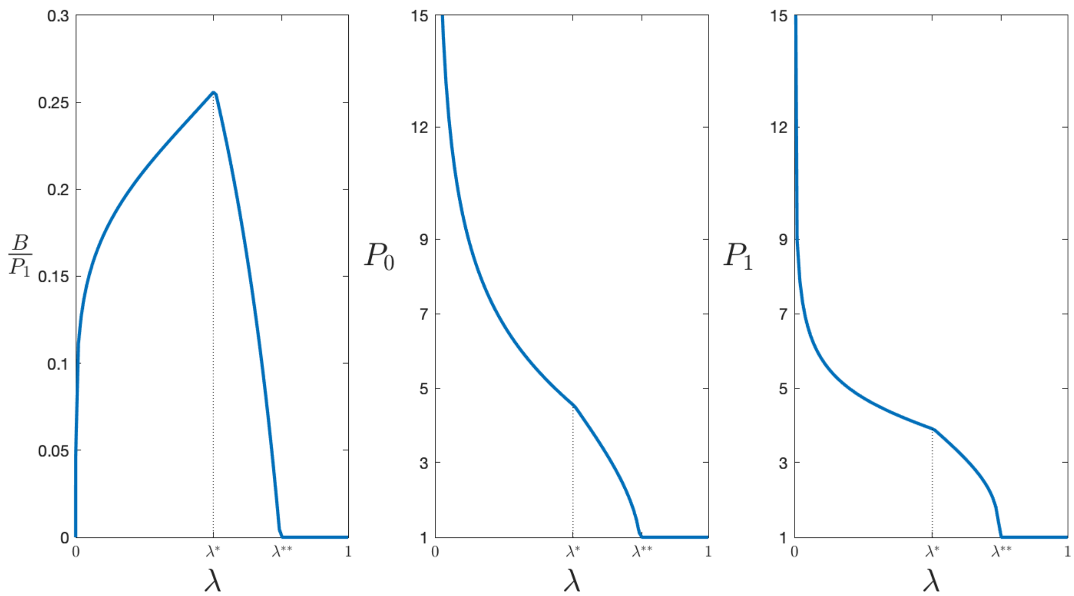
Note: This figure represents the date 0 central bank's reaction function conditional on the fiscal authority's real debt issuance B/P_1 in the analytical example. The left panel represents the real interest rate r. The right panel represents the price level P_0. These are drawn for high and low levels of central bank hawkishness λ. The dotted line corresponds to debt levels that exceed the economy's endogenous debt capacity conditional on λ.

Proposition 2 *Consider an economy under condition (27). In equilibrium, $\exists \lambda^*, \lambda^{**} \in (0,1)$ with $\lambda^{**} > \lambda^*$ such that (i) if $\lambda < \lambda^*$, then B/P_1 is strictly increasing in λ, if (ii) $\lambda \in (\lambda^*, \lambda^{**})$, then B/P_1 is strictly decreasing in λ, and if (iii) $\lambda > \lambda^{**}$, then $B/P_1 = 0$.*

Figure 2 displays the result of Proposition 1 graphically. If $\lambda > \lambda^*$, the constraint that $G_1 \geq \underline{G}_1(\lambda)$ is not binding for the fiscal authority, so it is unconstrained in its borrowing. An increase in the anti-inflation bias of the central bank reduces the benefit from debt issuance, since the central bank accommodates stimulus by less. Once the bias becomes high enough with $\lambda > \lambda^{**}$, there is no further benefit from debt issuance. By contrast, if $\lambda < \lambda^*$, the constraint that $G_1 \geq \underline{G}_1(\lambda)$ is binding, and a marginal increase in the anti-inflation bias increases real debt capacity and relaxes the borrowing limit of the fiscal authority, resulting in more real debt issuance. Figure 2 also displays the effect of central bank hawkishness on inflation through P_0 and P_1. Higher values of λ cause both P_0 and P_1 to decline; a more hawkish central bank results in more price stability.[25]

[25] We can also show that an increase in price flexibility (reduction in α) or increase in competition (increase in σ) both increase λ^*. Both of these factors increase the central bank's incentives to increase

Figure 2: Equilibrium Debt Issuance and Inflation



Note: This figure represents the equilibrium as a function of central bank hawkishness λ in the analytical example. The left panel represents real debt issuance B/P_1 The middle panel represents the date 0 price level P_0. The right panel represents the date 1 price level P_1.

6 Concluding Remarks

We have presented a model of monetary and fiscal interactions in which the effect of central bank hawkishness on fiscal outcomes is non-linear. The model allows for an interpretation of different historical episodes, and a natural next step for future research is a systematic empirical analysis combined with a quantification of the model in a dynamic environment. Such an analysis is challenging, as it would require solving a dynamic game—with monetary and fiscal state variables—between the monetary and fiscal authority.

Our model has three important implications for the implementation of monetary reform, where monetary reform can be interpreted as an increase in the central bank's inflation aversion. First, if the government is constrained in its ability to borrow by the market's expectation of debt devaluation, monetary reform should not be pursued in a vacuum. It should be paired with fiscal reform such as the adoption of credible fiscal rules

inflation conditional on the level of hawkishness λ, as they imply a larger decline in real interest rates C_0^{-1} for any given level of inflation P_0. Thus, the central bank has more scope for stimulating the economy at little cost to inflation. Because a fiscal authority expecting a more expansionary central bank is more likely to be debt constrained, the value of λ^* increases.

in order to prevent the deterioration of fiscal discipline. Such reforms would lead to a simultaneous improvements in monetary credibility and fiscal discipline, which reinforce each other.

A second implication of our model is that the degree of political support for monetary reform will depend both on the government's current monetary framework and its fiscal goals.[26] Support for monetary reform by the fiscal authority can be viewed as support for the ensuing expanded debt capacity. In contrast, backlash against monetary reform can be viewed as disapproval of the anticipated undoing of fiscal stimulus by the central bank.

A final consequence of our model is a conundrum that results from the first and second implications: support from policymakers for monetary reform is greatest in environments where it is least effective. The government, and in particular the fiscal authority, is inclined towards appointing a more conservative central banker when the direct effect on inflation reduction is partly offset by the indirect effect due to a loosening of the government's borrowing constraint, which in turn raises the central bank's incentive to generate inflation.

[26] For interior values of μ, the fiscal authority in our model prefers a central bank with an intermediate degree of hawkishness because such a central bank allows for some government borrowing and some accommodation of fiscal stimulus.

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Online Appendix

Proof of Lemma 1

Necessity follows from our discussion in the text. Sufficiency follows by using $\Big\{\{\tau_t, T_t, P_t, C_t, G_t, N_t\}_{t=0,1},$ $B, i\Big\}$ given (7) and (8) to construct the values of $\{C_{j,t}, N_{j,t}, W_t, P_{j,t}\}_{t=0,1}$ that satisfy all optimality conditions and budget constraints.■

Proof of Lemma 2

Step 1. Let us consider how G_1 is determined. The relaxed problem is strictly concave which means that the first order condition defines the unique global optimum. Equation (17) implies that C_1 and G_1 are negatively related, which means that $N_1 = C_1 + G_1$ is strictly increasing in G_1. Therefore, the left hand side of (22) is decreasing in G_1 and the right hand side of (22) increasing in G_1. Since the right hand side of (22) is increasing in B, this implies that G_1 is decreasing in B.

Step 2. Analogous argument to step 1 imply that G_1 is decreasing in λ.

Step 3. Let us consider how P_1 is determined. Substitute (19) into (22) to achieve

$$\left[-\left(1-\mu\right)\left(C_1+G_1\right)^{\varphi}G_1+\mu\frac{1}{G_1}\right]B=\frac{\lambda}{1-\lambda}H'\left(P_1\right)P_1^2. \tag{A.1}$$

From step 1, higher B is associated with lower G_1, which means that the left hand side of (*A*.1) is increasing in B. Therefore, since the right hand side of (*A*.1) is increasing in P_1, this means that P_1 is increasing in B.

Step 4. To consider how P_1 changes with respect to λ, we first establish that $P_1 > 1$. Suppose by contradiction that $P_1 \leq 1$. Consider a perturbation that increases P_1 in order to increase G_1 by some $\varepsilon > 0$ arbitrarily small. The change in welfare taking into account (17) is

$$-\left(1-\mu\right)\frac{1}{C_1}+\mu\frac{1}{G_1}.$$

We can establish that $G_1/C_1 < \mu/\left(1-\mu\right)$, implying that this term is positive and that the perturbation raises welfare. To see why, note that (17) implies that

$$G_1^{1+\varphi}\frac{C_1}{G_1}\left(\frac{C_1}{G_1}+1\right)^{\varphi}=1. \tag{A.2}$$

Suppose by contradiction that $C_1/G_1 \leq \left(1-\mu\right)/\mu$. Taking into account that (8) and (19) implies that $G_1 < \mu\left(1-\mu\right)^{-\frac{1}{1+\varphi}}$, it follows that

$$G_1^{1+\varphi}\frac{C_1}{G_1}\left(\frac{C_1}{G_1}+1\right)^{\varphi}<\mu^{1+\varphi}\left(1-\mu\right)^{-1}\left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{\mu}\right)^{\varphi}=1,$$

which violates (*A*.2). Therefore, $G_1/C_1 < \mu/\left(1-\mu\right)$ and the perturbation strictly increases welfare. Therefore, $P_1 > 1$ for all $\lambda \in (0, 1)$.

Consider a central bank with hawkishness λ' choosing $P_1\left(\lambda'\right)$ and another central bank with hawkishness $\lambda'' > \lambda'$ choosing $P_1\left(\lambda''\right)$. For both central banks to be weakly prefering their policy choice, it is necessary that they weakly prefer to not mimic each other, which means that

$$\left(\frac{\lambda''}{1-\lambda''}-\frac{\lambda'}{1-\lambda'}\right)\Big(H(P_1(\lambda')-H(P_1(\lambda''))\Big)\geq 0$$

Since $\lambda'' > \lambda'$ and $P_1(\lambda')$ and $P_1(\lambda'')$ both exceed 1, with $H(P)$ increasing for $P > 1$, it follows that this condition can only hold if $P_1(\lambda') \geq P_1(\lambda'')$. Therefore, P_1 decreases in λ. ■

Proof of Lemma 3

Proof of part (i). If $B = 0$ then $G_1 = T$ and $G_0 = T$, and the first best allocation conditional on $G_0 = T$ can be implemented with $P_0 = 1$.

Proof of part (ii) Suppose that $\forall B/P_1 \in (0, \upsilon)$ for some $\upsilon > 0$ arbitrarily small. We establish this result in two steps.

Step 1. We first establish that $P_0 \neq 1$. Consider a perturbation that increases P_0 by some $\varepsilon > 0$ arbitrarily small. Using implicit differentiation taking into account (15), (16), and (23), we can derive the ensuing change in C_0, G_0, and N_0. The change in central bank welfare from the perturbation taking into account (16) is

$$(1-\lambda)\left(-(1-\mu)N_0^{\varphi} + \mu\frac{1}{G_0}\right)\frac{\partial G_0}{\partial P_0} - \lambda H'(P_0). \tag{A.3}$$

Equations (15), (16), and (23) imply that $\frac{\partial G_0}{\partial P_0} > 0$. Moreover, analogous reasoning to Step 4 in the proof of Lemma 2 taking into account that $P_0 = 1$ implies that $-(1-\mu)N_0^{\varphi} + \mu\frac{1}{G_0} > 0$. Taking into account that $H'(1) = 0$, it follows that the sign of ($A.3$) is strictly positive.

Step 2. We next establish that $P_0 > 1$. We first show that constraint (16) is equivalent to

$$C_0(C_0 + G_0)^{\varphi} \leq 1 + \frac{\alpha}{\sigma - 1}(P_0 - 1)P_0 \;. \tag{A.4}$$

Suppose that the solution to the relaxed problem admits ($A.4$) as a strict inequality. Then necessarily, the solution admits $P_0 = 1$. Consider a perturbation which increases G_0 by some ε arbitrarily small and which also increases C_0 so as to satisfy (23). The change in welfare is

$$(1-\lambda)\left(\left(\frac{1}{C_0} - (1-\mu)N_0^{\varphi}\right)\frac{\partial C_0}{\partial G_0} - (1-\mu)N_0^{\varphi} + \mu\frac{1}{G_0}\right) - \lambda H'(P_0). \tag{A.5}$$

Given $P_0 = 1$, ($A.4$) which holds as a strict inequality, and the fact that $\frac{\partial C_0}{\partial G_0} > 0$, it follows that ($A.5$) is strictly larger than

$$(1-\lambda)\left(-(1-\mu)\frac{1}{C_0} + \mu\frac{1}{G_0}\right). \tag{A.6}$$

Observe that as $B/P_1 \to 0$, satisfaction of (23) requires $C_0 \to C_1$ and $G_0 \to T$. Using this observation, it follows that satisfaction of (23) requires $C_0/G_0 > (1-\mu)/\mu \; \forall B/P_1 \in (0, \upsilon)$ for some $\upsilon > 0$ arbitrarily small. Thus, analogous reasoning to Step 4 in the proof of Lemma 2 implies that ($A.6$) is strictly positive. Therefore, the solution to the relaxed problem is equal to the solution to constrained problem.

Now suppose by contradiction that the solution admits $P_0 < 1$. Consider a perturbation that increases P_0 to 1, holding C_0 and G_0 constant. This perturbation satisfies all constraints of the relaxed problem and strictly increases welfare. Therefore, $P_0 \geq 0$ and by Step 1, $P_0 > 0$.

Proof of part (iii). This follows from analogous reasoning to Step 4 in the proof of Lemma 2.■

Proof of Proposition 1

Proof of part (i). Take $\lambda \to 1$, where $\underline{G}_1(\lambda) \to 0$, $P_0 \to 1$, $P_1 \to 1$. Consider the program of the fiscal authority which can be rewritten as

$$\max_{C_0,G_0,N_0,C_1,G_1,N_1} \left\{ \begin{array}{c} (1-\mu)\left(\log C_0 - \frac{N_0^{1+\varphi}}{1+\varphi}\right) + \mu \log G_0 \\ (1-\mu)\left(\log C_1 - \frac{N_1^{1+\varphi}}{1+\varphi}\right) + \mu \log G_1 \end{array} \right\}$$

$$\text{s.t.}$$

$$C_t + G_t = N_t \text{ for } t = 0, 1,$$

$$C_t N_t^{\varphi} = 1 \text{ for } t = 0, 1, \text{ and} \tag{A.7}$$

$$\frac{T - G_0}{C_0} + \frac{T - G_1}{C_1} = 0. \tag{A.8}$$

Observe that ($A.8$) is equivalent to a weak inequality constraint

$$\frac{T - G_0}{C_0} + \frac{T - G_1}{C_1} \geq 0. \tag{A.9}$$

This is because the solution in the absence of this constraint admits

$$C_t N_t^{\varphi} = \frac{1-\mu}{\mu} G_t N_t^{\varphi} = 1,$$

which is the first best allocation, which violates ($A.9$). Therefore, the solution to the relaxed problem with ($A.9$) is equivalent to the solution to the constrained problem. Observe that ($A.9$) can be rewritten as

$$C_1 (T - G_0) + C_0 (T - G_1) \geq 0, \tag{A.10}$$

which is a globally convex constraint. Let ψ correspond to the Lagrange multiplier on ($A.10$), and consider the relaxed problem that ignores ($A.7$). First order conditions yield

$$\begin{aligned} \frac{1}{C_0} - (C_0 + G_0)^{\varphi} + \psi (T - G_1) &= 0 \\ \frac{1}{C_1} - (C_1 + G_1)^{\varphi} + \psi (T - G_0) &= 0 \\ \frac{\mu}{1-\mu}\frac{1}{G_0} - (C_0 + G_0)^{\varphi} - \psi C_1 &= 0 \\ \frac{\mu}{1-\mu}\frac{1}{G_1} - (C_1 + G_1)^{\varphi} - \psi C_0 &= 0 \end{aligned}$$

Since the program is concave and the constraint set convex, the solution is unique. Observe that $G_0 = G_1 = T$ satisfies the first order conditions so that it constitutes the solution. Moreover, condition ($A.7$) is satisfied, so that the solution to the relaxed problem is the solution to the constrained problem. Therefore, $B/P_1 = 0$. The statement of the proposition follows by continuity given that $B/P_1 \geq 0$.

Proof of part (ii). As $\lambda \to 0$, $\underline{G}_1(\lambda) \to T$, which means that $B/P_1 \to 0$. The statement of the proposition follows by continuity given that $B/P_1 \geq 0$.■

Proof of Proposition 2

The equilibrium value of B/P_1 is inversely proportional to the value of G_1. Therefore, we establish this result by focusing on the value of G_1. Define $G_1^*(\lambda)$ as the solution to the unconstrained problem of the fiscal authority. Observe that this value represents the solution to the below unconstrained problem:

$$\max_{G_1} \{\log G_0^*(G_1, \lambda) + \log G_1\}, \tag{A.11}$$

where $G_0^*(G_1, \lambda)$ denotes the best response of the date 0 monetary authority with hawkishness λ. First order conditions yield

$$\frac{1}{G_0} \frac{\partial G_0^*(G_1, \lambda)}{\partial G_1} + \frac{1}{G_1} = 0. \tag{A.12}$$

To determine $G_0^*(G_1, \lambda)$, note that the date 0 central banks' problem (24) can be represented as

$$\max_{G_0} \left\{ \eta(\lambda) \log G_0 - \frac{\left(\frac{1}{T - G_1}(G_0 - T) - 1\right)^2}{2} \right\}$$

for

$$\eta(\lambda) = \frac{1}{\kappa} \frac{1-\lambda}{\lambda} \left(\frac{\alpha}{\sigma - 1}\right)^2,$$

Observe that the function $\eta(\lambda)$ is a strictly decreasing function of λ. Define

$$\lambda^{**} = \left(1 + \kappa \left(\frac{\sigma - 1}{\alpha}\right)^2\right)^{-1}, \tag{A.13}$$

and observe that $\eta(\lambda^{**}) = 1$. The first order condition implies that

$$0 = G_0^2 - G_0(2T - G_1) - \eta(\lambda)(T - G_1)^2. \tag{A.14}$$

Implicit differentiation of ($A.14$) yields

$$\frac{\partial G_0^*(G_1, \lambda)}{\partial G_1} = -\frac{G_0 + \eta(\lambda) 2 (T - G_1)}{G_1 + 2(G_0 - T)} < 0. \tag{A.15}$$

After substitution, ($A.12$) can be rewritten as

$$\frac{1}{G_1}\left(-\frac{1 + \eta(\lambda) 2(T - G_1) G_0^{-1}}{1 + 2(G_0 - T) G_1^{-1}} + 1\right) = 0. \tag{A.16}$$

Observe that ($A.16$) is satisfied for $G_1 = T$. Thus, $G_1 = T$ is a local maximum or a local minimum in the date 0 fiscal authority's problem.

Using these observations, we prove the proposition in three steps. First, we establish that if $\lambda < \lambda^{**}$, then $G_1^*(\lambda) < T$ and is strictly increasing in λ. Second, we establish that if $\lambda \geq \lambda^{**}$, then $G_1^*(\lambda) = T$. Finally, we combine these results with the observation that $\underline{G}_1(\lambda)$ is strictly decreasing in λ to complete the proof.

Step 1. We establish that if $\lambda < \lambda^{**}$, then $G_1^*(\lambda) < T$ and is strictly increasing in λ.

Step 1a. We establish that $G_1^*(\lambda) < T$. Suppose by contradiction that $G_1^*(\lambda) = T$. Consider the necessary second order condition to the date 0 fiscal authority's problem by differentiating ($A.16$) with respect to G_1, taking into account that the term in parentheses in ($A.16$) evaluated at $G_1 = T$ is zero and that $\frac{\partial G_0^*(T,\lambda)}{\partial G_1} = -1$:

$$\frac{1}{G_1}\left(-\partial\frac{\left(\frac{1+\eta(\lambda)2(T-G_1)G_0^{-1}}{1+2(G_0-T)G_1^{-1}}\right)}{\partial G_1}+\partial\frac{\left(\frac{1+\eta(\lambda)2(T-G_1)G_0^{-1}}{1+2(G_0-T)G_1^{-1}}\right)}{\partial G_0}\right)<0. \tag{A.17}$$

Inequality ($A.17$) evaluated at $G_0 = G_1 = T$ yields

$$\frac{2}{T^2}\left(\eta\left(\lambda\right)-1\right)<0. \tag{A.18}$$

However, ($A.18$) cannot hold if $\lambda < \lambda^{**}$ since $\eta(\lambda) > 1$. Therefore, $G_1 = T$ is a local minimum if $\lambda < \lambda^{**}$, which means that $G_1^*(\lambda) < T$.

Step 1b. We establish that $G_1^*(\lambda) < T$ is uniquely determined. Note that ($A.16$) taking into account that $G_1 < T$ can be rewritten as

$$\eta\left(\lambda\right)=\frac{G_0^2-TG_0}{TG_1-G_1^2}. \tag{A.19}$$

Combining ($A.14$) and ($A.19$), we achieve:

$$G_0=\eta\left(\lambda\right)\left(2G_1-T\right), \tag{A.20}$$

which implies that since $G_0 > 0$, it follows that $G_1 > T/2$. Substitution of ($A.20$) into ($A.19$) yields an equation defining G_1:

$$\left(4\eta\left(\lambda\right)+1\right)G_1^2-\left(4\eta\left(\lambda\right)+3\right)TG_1+\left(\eta\left(\lambda\right)+1\right)T^2=0. \tag{A.21}$$

Observe that the left hand side of ($A.21$) is convex in G_1, exceeds 0 if $G_1 = 0$ and $G_1 = T$ (since $\lambda < \lambda^{**}$), and is below 0 for $G_1 = T/2$. It thus follows that there is a unique value of $G_1 > T/2$ that satisfies ($A.21$).

Step 1c. Equation ($A.21$) defines $G_1^*(\lambda)$. Given Step 1b, observe that from the convexity of the left hand side of ($A.21$), it follows that the the left hand side of ($A.21$) is strictly increasing in G_1 at $G_1 = G_1^*(\lambda)$, so that

$$\left(4\eta\left(\lambda\right)+1\right)2G_1-\left(4\eta\left(\lambda\right)+3\right)T>0. \tag{A.22}$$

Implicit differentiation of ($A.21$) with respect to λ yields

$$\frac{\partial G_1^*\left(\lambda\right)}{\partial\lambda}=-\eta'\left(\lambda\right)\frac{\left(2G_1-T\right)^2}{\left(4\eta\left(\lambda\right)+1\right)2G_1-\left(4\eta\left(\lambda\right)+3\right)T}>0, \tag{A.23}$$

where we have applied ($A.22$) and the fact that $G_1 > T/2$ to sign ($A.23$). This establishes $G_1^*(\lambda)$ is strictly increasing in λ for $\lambda < \lambda^{**}$.

Step 2. We now establish that if $\lambda \geq \lambda^{**}$, then $G_1^*(\lambda) = T$.

Step 2a. We first establish that if $\lambda = \lambda^{**}$, then $G_1^*(\lambda) = T$. Suppose that this were not the case and that the solution admits $G_1^*(\lambda) < T$. Equation ($A.21$) then defines $G_1^*(\lambda)$ and the same arguments as in Step 2b imply that $G_1^*(\lambda)$ is uniquely determined. Observe that if $\lambda = \lambda^{**}$, then $G_1 = T$ solves ($A.21$), contradicting the fact that the solution admits $G_1^*(\lambda) < T$. Therefore, $G_1^*(\lambda) = T$.

Step 2b. We now establish that $G_1^*(\lambda) = T$ for all $\lambda > \lambda^{**}$. Consider the contradiction assumption that $G_1^*(\lambda') = \hat{G}_1 < T$ for some $\lambda' > \lambda^{**}$. Weak optimality for the fiscal authority at date 0 conditional on $\lambda = \lambda'$ requires

$$\log\left(G_0^*\left(\hat{G}_1\right), \lambda'\right) + \log \hat{G}_1 \geq 2\log T. \tag{A.24}$$

Strict optimality for the fiscal authority at date 0 conditional on $\lambda = \lambda^{**}$ requires

$$2\log T > \log\left(G_0^*\left(\hat{G}_1\right), \lambda^{**}\right) + \log \hat{G}_1. \tag{A.25}$$

Combining ($A.24$) and ($A.25$) we achieve

$$\log\left(G_0^*\left(\hat{G}_1\right), \lambda'\right) > \log\left(G_0^*\left(\hat{G}_1\right), \lambda^{**}\right). \tag{A.26}$$

Implicit differentiation of ($A.14$) yields

$$\frac{\partial G_0^*\left(G_1, \lambda\right)}{\partial \lambda} = -\frac{1}{\lambda^2}\frac{\left(T - G_1\right)^2}{G_1 + 2\left(G_0 - T\right)} < 0,$$

which contradicts ($A.26$). Therefore, $G_1^*(\lambda) = T$ for all $\lambda > \lambda^{**}$.

Step 3. Observe that the constrained problem of the first authority at date 1 implies that the equilibrium value of G_1 must satisfy

$$G_1 = \max\left\{G_1^*\left(\lambda\right), \underline{G}_1(\lambda)\right\}.$$

Observe that $\lim_{\lambda\to 0} \underline{G}_1\left(\lambda\right) = T > \lim_{\lambda\to 0} G_1^*\left(\lambda\right)$ (from step 1a). Moreover, $\lim_{\lambda\to 1} \underline{G}_1\left(\lambda\right) < T < \lim_{\lambda\to 1} G_1^*\left(\lambda\right) = T$ (from step 2b). Therefore, $G_1^*\left(\lambda\right) = \underline{G}_1(\lambda)$ for some interior value of λ. Moreover, since $G_1^*\left(\lambda\right)$ and $\underline{G}_1(\lambda)$ are both monotonic, this interior point is unique, and can be labeled by λ^*. It follows that $G_1 = \underline{G}_1(\lambda)$ if $\lambda < \lambda^*$, with G_1 decreasing in λ if $\lambda < \lambda^*$. Moreover $G_1 = G_1^*\left(\lambda\right)$ if $\lambda > \lambda^*$, with G_1 strictly increasing in λ for $\lambda \in (\lambda^*, \lambda^{**})$ and $G_1 = T$ for $\lambda > \lambda^{**}$. ■