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FISCAL RULES AND DISCRETION UNDER LIMITED ENFORCEMENT

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We study a fiscal policy model in which the government is present-biased towards public spending. Society chooses a fiscal rule to trade off the benefit of committing the government to not overspend against the benefit of granting it flexibility to react to privately observed shocks to the value of spending. Unlike prior work, we examine rules under limited enforcement: the government has full policy discretion and can only be incentivized to comply with a rule via the use of penalties which are joint and bounded. We show that optimal incentives must be bang-bang. Moreover, under a distributional condition, the optimal rule is a maximally enforced deficit limit, triggering the maximum feasible penalty whenever violated. Violation optimally occurs under high enough shocks if and only if available penalties are weak and such shocks are relatively unlikely. We derive comparative statics showing how rules should be calibrated to features of the environment.

KEYWORDS: Private information, fiscal policy, deficit bias, enforcement constraints.

1. INTRODUCTION

COUNTRIES IMPOSE RULES ON THEIR GOVERNMENTS to constrain their policy decisions. Increasingly prevalent are fiscal rules, in place in 92 countries in 2015, compared to only seven countries in 1990. Fiscal rules are commonly implemented in the form of mandated deficit limits, debt limits, and spending limits. Yet, according to the International Monetary Fund, governments comply with these limits only about 50 percent of the time. Violations are relatively more frequent in countries with weak legal and political institutions, in circumstances where fiscal limits are set at relatively stringent levels, and following negative budget shocks.[1]

Whenever a fiscal limit is breached, a formal or informal enforcement mechanism is triggered. In the European Union, countries are subject to the Excessive Deficit Procedure, a formal procedure specifying a sequence of costly fiscal adjustments and potential sanctions which applies in case of any fiscal limit violation. In emerging economies, formal enforcement mechanisms tend to include automatic correction measures and formal sanctions, the latter often being part of fiscal responsibility laws.[2] Penalties for non-compliance

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[1]See IMF Fiscal Rules Data Set, 1985–2015 on the prevalence of fiscal rules. According to this data set, 89 countries in 2015 had debt rules or budget balance rules (at the national or supranational level). See Caselli et al. (2018) on the frequency of non-compliance, and Cordes, Kinda, Muthoora, and Weber (2015), Reuter (2017), and Larch and Santacroce (2020) on the patterns of non-compliance.

[2]See Cordes et al. (2015) and Lledó, Yoon, Fang, Mbaye, and Kim (2017) for descriptions of the formal enforcement mechanisms in the European Union and emerging economies. In 2015, 55 countries with a debt

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can also be informal. For example, financial markets may serve as an informal enforcement mechanism, as breaches of fiscal limits often lead to an increase in the sovereign spread that is costly to the government.[3] Other forms of informal penalties may arise, for instance, if a breach by the current government sets a precedent for future governments to follow.[4]

A penalty mechanism, formal or informal, is necessary to incentivize the government to respect the fiscal constraints that society puts in place. In fact, while compliance is not perfect, fiscal rules do influence policy, deterring overborrowing and preventing excessively large deficits when fiscal limits are respected.[5] Penalties are therefore critical to enforce rules and promote fiscal discipline. However, imposing penalties like those described above also implies costs, both for the government and for society, and any disciplinary penalties available to society are naturally limited in scope.

In this paper, we study the optimal design of fiscal rules under limited enforcement. What is the optimal structure of fiscal constraints when available penalties for violation are joint and bounded, and how are these penalties used in an optimal rule? Should governments violate their fiscal limits when the economy is in distress? How does an optimal rule depend on the severity of available penalties and the distribution of economic shocks?

Our analysis of fiscal rules builds on the approach used in Amador, Werning, and Angeletos (2006) and Halac and Yared (2014).[6] We consider a government that is present-biased towards public spending and privately informed about shocks affecting the value of this spending. Society chooses a fiscal rule to trade off the benefit of committing the government to not overspend against the benefit of granting it flexibility to react to shocks. Motivated by real-world rules and unlike prior work, we posit that this fiscal rule can only be enforced using penalties that are socially costly and limited in scope.

Our environment is a small open economy in which the government makes a borrowing decision. Prior to the choice of policy, a shock to the social value of deficit-financed government spending is realized. The government is present-biased: for any given shock, the government overvalues current spending relative to future welfare compared to society. This preference structure results naturally from the aggregation of heterogeneous, time-consistent citizens' preferences (Jackson and Yariv (2014, 2015)),[7] or as a consequence of turnover in a political economy setting (e.g., Aguiar and Amador (2011)). We assume that the shock to the value of spending is privately observed by the government, capturing the fact that rules cannot explicitly condition on all contingencies in fiscal policy. Furthermore, the government has full discretion when choosing policy, and society can only commit to a schedule of penalties as a function of the government's choice. A fiscal rule in this setting can thus be studied as a direct mechanism that specifies a borrowing

rule or budget balance rule had a formal enforcement procedure in place (see IMF Fiscal Rules Data Set, 1985–2015).

[3]For the European Union, Caselli et al. (2018) found that sovereign spreads of non-complying countries are on average higher by 50 to 150 basis points compared to countries in compliance.

[4]This was the case in Chile: the government's breach of its fiscal rule in 2009 resulted in lax fiscal policy by the next administration, which continued to ignore the rule despite criticism of fiscal irresponsibility. See Velasco and Arenas (2010).

[5]See, for example, the evidence in Caselli and Wingender (2018).

[6]See also Athey, Atkeson, and Kehoe (2005) and Amador and Bagwell (2013).

[7]Our formulation corresponds to the quasi-hyperbolic model; see Laibson (1997). Jackson and Yariv (2015) showed that when citizens' preferences are heterogeneous, every non-dictatorial aggregation method that respects unanimity is time inconsistent, and any such method that is utilitarian leads to a present bias.

level and penalty for each shock, satisfying the government's truthtelling and enforcement constraints in addition to the feasibility of penalties.

To describe the forces underlying our model, suppose first that fiscal rules could be perfectly enforced. Society then chooses a rule to optimally resolve the tradeoff between commitment and flexibility. Fully committing the government to a contingent debt plan would allow to implement the first-best policy in the absence of private information, whereas granting the government full flexibility would yield the first-best policy in the absence of a present bias. Given both private information and a present bias, however, a tradeoff arises, and the first best is not implementable. Amador, Werning, and Angeletos (2006) showed that, under perfect enforcement and certain distributional assumptions, the solution to this tradeoff is a fiscal rule that takes the form of a deficit limit. The government borrows within the limit if the shock to the value of spending is relatively low and it borrows at the limit if the shock is higher, without triggering any penalties.

Our focus is on understanding the optimal fiscal rule under limited enforcement. As is also true under perfect enforcement, a rule under limited enforcement must satisfy the government's truthtelling constraints: given a realized shock, the government must prefer its assigned debt level and penalty to those prescribed for any other shock. In addition, the rule must satisfy the government's enforcement constraints: given a realized shock, the government must prefer its assigned debt level and penalty to any other level of debt not prescribed for any shock. Any observable (*off-path*) deviation—where the government chooses a debt level corresponding to no shock—is optimally punished with the maximum feasible penalty conditional on the choice of debt. In fact, if penalties are severe enough, enforcement constraints are non-binding: the government always prefers to abide to the perfect-enforcement deficit limit to avoid punishment.

Our main result is a characterization of the optimal fiscal rule when enforcement constraints bind. We show that this rule takes the form of a *maximally enforced deficit limit*, which, if violated, leads to the maximum feasible penalty for the government. The rule is thus similar to that under perfect enforcement, but it differs in two aspects. First, the deficit limit imposed on the government is more relaxed than the perfect-enforcement limit. Second, the possibility of *on-path* penalties emerges, with the government breaching the deficit limit and triggering punishment under sufficiently high shocks. We study the conditions under which on-path penalties are optimal and examine how the tightness of the deficit limit and the use of penalties depend on features of the environment.

To obtain our characterization, we begin by establishing general properties of optimal incentive provision. We show that in any optimal rule, penalties must be *bang-bang*, so the government receives either no penalty or the maximum feasible penalty given the level of debt, depending on its policy choice. Using intermediate penalties to provide local incentives—as is common in many adverse selection settings—is feasible, but it is suboptimal in our model in which penalties are socially costly. Society is better off prescribing either no penalty, or the harshest penalty to maximize the range of shocks under which fiscal discipline is imposed. Our bang-bang result relies only on generic properties of the distribution of shocks, and it applies under both limited and perfect enforcement. This result thus has implications for other models of delegation with money burning, including some of those studied in Amador and Bagwell (2013): we find that optimal delegation requires money burning to take a bang-bang form.

We complete our characterization of optimal incentives by introducing an assumption on the distribution of shocks.[8] We show that under this assumption, optimal bang-bang penalties must be monotonic, with the government either never receiving a penalty or receiving the maximum penalty only under high enough shocks. Moreover, building on monotonicity, we are able to characterize the optimal borrowing schedule. We establish that our distributional assumption is sufficient, as well as necessary, for maximally enforced deficit limits to be the unique optimal fiscal rule.

The optimal deficit limit is determined according to the following tradeoff. On the one hand, society can set a relatively lax deficit limit that satisfies the enforcement constraint under all shocks and thus entails no penalties on path. On the other hand, society can prescribe a tighter deficit limit that improves discipline but requires costly penalties on path: the government respects the limit and receives no punishment under low enough shocks, but it breaches the limit and is punished with the maximum penalty under higher shocks. We prove that the optimal deficit limit is unique and provide a necessary and sufficient condition for this limit to feature on-path penalties.

Our results show that optimal fiscal rules take the form of those typically observed in practice, namely, fiscal limits that trigger penalties upon breach.[9] This is, for example, the case under the Excessive Deficit Procedure described above; there are fiscal limits in place, and sanctions (which may depend on the level of debt as in our model) are applied whenever these limits are violated. Violations are not rare under the Excessive Deficit Procedure, and in fact they occur in periods of economic distress in many countries. An implication of our results is that on-path violation need not represent a failure but may rather be a feature of the optimal rule. Furthermore, our analysis elucidates how fiscal rules should be designed taking the possibility of breach into account.

We use our characterization of the optimal rule to study how fiscal constraints and the use of penalties vary with features of the environment. As mentioned, the optimal deficit limit features no on-path violation if available penalties are sufficiently severe. We derive comparative statics showing that the harsher is the maximum penalty available to society, the tighter is the optimal deficit limit and the smaller is the range of shocks that trigger violation. This result is consistent with evidence that breaches are more common under weaker enforcement regimes (e.g., Reuter (2017)). It also shows that improvements in enforcement mechanisms (e.g., by introducing a fiscal council as recommended in Eyraud, Debrun, Hodge, Lledó, and Pattillo (2018b)) should be done jointly with a tightening of fiscal constraints. In terms of welfare, we find that increasing the severity of penalties benefits society in two ways: a more stringent deficit limit that improves fiscal discipline, and increased enforceability that reduces the frequency with which penalties need to be imposed.

The tightness of the optimal rule also depends on the distribution of shocks. Recall that given a shock distribution, any violations of the optimal deficit limit occur under sufficiently high shocks—in line with evidence that breaches occur under unexpectedly high spending pressures (e.g., Larch and Santacroce (2020)). We derive comparative statics by studying changes in the shock distribution which may involve changes in the mean and variance of the shocks. For example, we find that an increase in expected fiscal needs

[8]See Assumption 1 in Section 3.3. This assumption holds for a broad range of distributions, including uniform, exponential, log-normal, gamma, and beta for a subset of its parameters. Assumption 1 is similar to, but stronger than, the assumption used in Amador, Werning, and Angeletos (2006).

[9]The optimal rule that we derive can also be viewed as a fiscal rule with an escape clause which applies under specific circumstances and entails future costly measures and penalties. Rules of this form are common in reality, the Swiss debt brake being an illustrative example (see Beljean and Geier (2013)).

(due to high shocks becoming more likely and low shocks less likely) leads to a more relaxed optimal deficit limit and a smaller range of shocks under which violation occurs. Intuitively, if society anticipates high borrowing needs, then it is suboptimal to punish the government for breaking a very stringent deficit limit when these needs realize, as this occurs relatively often. Our results support the International Monetary Fund's recommendation to calibrate fiscal rules to forecasts of the economic cycle (Eyraud et al. (2018a)); we highlight that calibrations should seek not only to regulate fiscal discipline but also to curb the use of penalties under limited enforcement.

Related Literature. Our paper fits into the aforementioned literature on mechanism design that studies the tradeoff between commitment and flexibility.[10] This literature is mainly concerned with environments with perfect enforcement, whereas we examine rules under limited enforcement. Closely related is Amador and Bagwell (forthcoming), which studies the problem of regulating a privately informed monopolist in the absence of transfers and given an ex post participation constraint. The paper shows that optimal regulation takes the form of a threshold that is imposed unless the monopolist chooses to shut down. In contrast to this work, our analysis does not rely on the perfect enforcement of thresholds, and it allows for money burning, which is sometimes optimally used on path.

Another departure from the commitment-versus-flexibility literature is that we prove our results by using perturbation arguments, whereas most of the literature (including Amador, Werning, and Angeletos (2006)) uses Lagrangian methods. Our approach follows Athey, Atkeson, and Kehoe (2005), who used perturbation arguments to characterize optimal monetary policy rules under perfect enforcement. Since our problem is not globally concave, a guess-and-verify Lagrangian approach would not establish uniqueness of our solution, and would make it difficult to identify necessary and sufficient conditions for the optimality of maximally enforced deficit limits. We are able to provide these results using perturbations. Furthermore, our analysis elucidates which properties of the optimal rule rely on assumptions on the distribution function and which properties are general. In particular, using perturbations and unlike prior work, we derive the optimality of bang-bang money burning as a general result, independent of specific distributional conditions.

The work of Riley and Zeckhauser (1983) and Fuchs and Skrzypacz (2015) is related in that they also found bang-bang incentives to be optimal.[11] Their settings, however, feature transfers, and bang-bang follows directly from the linearity of payoffs. Essentially, these papers show that their principal's objective function admits a linear representation after substitution of truthtelling constraints, and that these constraints can be ignored after substitution (subject to monotonicity) as they are satisfied by the corner solutions to the relaxed problem. While the former applies to our model, the latter does not: truthtelling constraints, which are not linear in our setting, cannot be ignored in our perturbation arguments.[12] Our bang-bang result stems not only from the linearity of the objective func-

[10]In addition to the work previously cited, see Sleet (2004), Ambrus and Egorov (2013), and Halac and Yared (2018, 2022a). More broadly, our paper relates to the literature on principal-agent delegation, including Holmström (1977, 1984), Alonso and Matouschek (2008), and Ambrus and Egorov (2017).

[11]Bang-bang incentives are also necessary for optimality in dynamic games with moral hazard when shocks are sufficiently informative, such as in Abreu, Pearce, and Stacchetti (1990) and Sannikov (2007). These papers consider finite actions (and thus finite incentive constraints) and a continuum of shocks, whereas our model features a continuum of actions and shocks.

[12]This issue also arises and was discussed in Athey, Atkeson, and Kehoe (2005, p. 1444).

tion but also from the jointness of punishments, which implies that local incentive provision via intermediate penalties is too costly for society.[13]

Our paper is also related to a large literature on the political economy of fiscal policy.[14] Dovis and Kirpalani (2020), in particular, examined deficit limits that are endogenously enforced by reputational concerns. We instead study the enforcement of optimal fiscal rules under privately observed shocks and without restricting their structure.

Finally, our paper contributes to the literature on hyperbolic discounting and the benefits of commitment devices.[15] We characterize the optimal structure of penalties to resolve the commitment-versus-flexibility tradeoff that arises in a setting with hyperbolic discounting.

2. MODEL

We study a simple model of fiscal policy in which the government makes a borrowing decision. Our setting is similar to that analyzed in Amador, Werning, and Angeletos (2006) and Halac and Yared (2014). Our departure is in examining the design of fiscal rules under only limited, as opposed to perfect, enforcement.

2.1. *Setup*

Consider a small open economy. A shock to the economy $\theta > 0$ is drawn from a bounded set $\Theta \equiv [\underline{\theta}, \overline{\theta}]$, with a continuously differentiable probability density function $f(\theta) > 0$ and associated cumulative density function $F(\theta)$. The realization of this shock is privately observed by the government, so we refer to θ as the government's *type*.

The government chooses borrowing $b \in [\underline{b}, \overline{b}]$. Public spending is then given by $\omega + b$, where $\omega > 0$ denotes the exogenous resources of the government, namely, collected tax revenue net of any debt repayment.[16] Society can influence policy by imposing penalties which are mutually costly to the government and society itself. Specifically, society commits ex ante to a penalty schedule as a function of the government's borrowing choice, where the maximum feasible penalty conditional on b is $\overline{P}(b) \in (0, \infty)$.

Given a borrowing level b and a penalty P, social welfare is given by

$$\mathbb{E}\big[\theta U(\omega + b) + \delta\big(V(b) - P\big)\big], \tag{1}$$

where $\theta U(\omega + b)$ is the social utility from public spending given a realized shock θ, $V(b)$ is the social continuation value, and $\delta \in (0, 1)$ is the discount factor. Government welfare for a government type θ is instead given by

$$\theta U(\omega + b) + \beta\delta\big(V(b) - P\big), \tag{2}$$

where $\beta \in (0, 1)$. We take $U(\cdot)$ to be continuously differentiable, strictly increasing, and strictly concave. We also assume that for all $b \in [\underline{b}, \overline{b}]$, $V(b)$ is continuously differentiable and strictly concave and $\overline{P}(b)$ is continuous.

[13] In fact, if penalties were not joint but only costly to the government, the optimal rule (assuming a broad enough range of feasible penalties) would prescribe interior penalties to achieve first best.

[14] See Yared (2019) for a survey. For quantitative analyses of fiscal rules, see Bassetto and Sargent (2006) and Azzimonti, Battaglini, and Coate (2016).

[15] See, for example, Laibson (1997), Lizzeri and Yariv (2014), Bernheim, Ray, and Yeltekin (2015), and Bisin, Lizzeri, and Yariv (2015).

[16] Note that since ω is independent of θ, cross-subsidization across types is not possible, unlike in other models such as Atkeson and Lucas (1992) and Thomas and Worrall (1990).

There are three main features of this environment. First, since $\beta < 1$, the government's objective (2) given its realized type does not coincide with the social objective (1). Compared to society, the government overweighs the importance of current spending relative to the future continuation value and would thus want to overborrow: its preferred policy absent penalties satisfies $\theta U'(\omega + b) = -\beta\delta V'(b)$, whereas the social optimum sets $\theta U'(\omega + b) = -\delta V'(b)$. As mentioned in the Introduction, this payoff structure arises naturally when the government's preferences aggregate heterogeneous citizens' preferences (see Jackson and Yariv (2014, 2015)). This formulation can also be motivated by political turnover; for instance, preferences such as these emerge in settings with political uncertainty where policymakers place a higher value on public spending when they hold power and can make spending decisions (see Aguiar and Amador (2011)).

The second feature of our environment is that the realization of θ—which affects the marginal social utility of public spending—is privately observed by the government. One interpretation is that penalties imposed on the government cannot explicitly condition on the value of θ, even if this shock were observable. An alternative interpretation is that the exact cost of public goods is only observable to the policymaker, who may be inclined to overspend on these goods. Another possibility is that citizens have heterogeneous preferences or heterogeneous information about the optimal level of public spending, and only the government sees the aggregate (Sleet (2004), Piguillem and Schneider (2016)).

The third and most critical feature of our environment is that the government has full discretion when choosing borrowing. Society can only influence policy by specifying contingent penalties which are limited and socially costly. This is a main distinction from prior work, which assumes that available actions can be restricted arbitrarily and at no cost. The extent of penalties that society can impose on the government is captured by the exogenous maximum-penalty function $\overline{P}(b)$. As we will show in Section 3, previous analyses in which available actions can be arbitrarily restricted correspond to a special case of our model in which $\overline{P}(b)$ is sufficiently large for all $b \in [\underline{b}, \overline{b}]$.

2.2. *Fiscal Rules*

We study society's problem as a mechanism design problem in which society is the principal and the government is the agent. By the Revelation Principle, we can focus on policies which are truthfully implementable in direct mechanisms. We thus define a *fiscal rule* as a direct mechanism $(b(\theta), P(\theta))$, which specifies a borrowing level and penalty for each government type θ satisfying truthtelling, enforcement, and feasibility constraints.

The truthtelling constraint captures the fact that the government can misrepresent its type. A fiscal rule $(b(\theta), P(\theta))$ is such that a government of type θ prefers to pursue its assigned policy rather than that of any other type $\theta' \neq \theta$:

$$\begin{aligned}&\theta U\big(\omega + b(\theta)\big) + \beta\delta\big(V\big(b(\theta)\big) - P(\theta)\big)\\&\quad \geq \theta U\big(\omega + b\big(\theta'\big)\big) + \beta\delta\big(V\big(b\big(\theta'\big)\big) - P\big(\theta'\big)\big) \quad \text{for all } \theta, \theta' \in \Theta.\end{aligned} \tag{3}$$

The enforcement constraint captures the fact that the government can freely choose any feasible level of debt, including levels not assigned to any other government type. A fiscal rule $(b(\theta), P(\theta))$ is such that a government of type θ prefers to pursue its assigned policy rather than any other policy $b' \in [\underline{b}, \overline{b}]$ such that $b' \neq b(\theta')$ for all $\theta' \in \Theta$. This requires that type θ have no incentive to deviate to such a b', and thus any $b' \in [\underline{b}, \overline{b}]$, when this choice is maximally punished:

$$\begin{aligned}&\theta U\big(\omega+b(\theta)\big)+\beta\delta\big(V\big(b(\theta)\big)-P(\theta)\big)\\&\quad\geq\theta U\big(\omega+b'\big)+\beta\delta\big(V\big(b'\big)-\overline{P}\big(b'\big)\big)\quad\text{for all }\theta\in\Theta\text{ and all }b'\in[\underline{b},\overline{b}].\end{aligned}$$

This inequality holds for all $b'\in[\underline{b},\overline{b}]$ if and only if it holds when b' corresponds to type θ's flexible level of debt conditional on the maximum penalty. Specifically, let $b^p(\theta)$ be defined by

$$b^p(\theta)\in\arg\max_{b\in[\underline{b},\overline{b}]}\big\{\theta U(\omega+b)+\beta\delta\big(V(b)-\overline{P}(b)\big)\big\}.$$

Then a necessary condition for the enforcement constraint to be satisfied is

$$\begin{aligned}&\theta U\big(\omega+b(\theta)\big)+\beta\delta\big(V\big(b(\theta)\big)-P(\theta)\big)\\&\quad\geq\theta U\big(\omega+b^p(\theta)\big)+\beta\delta\big(V\big(b^p(\theta)\big)-\overline{P}\big(b^p(\theta)\big)\big)\quad\text{for all }\theta\in\Theta.\end{aligned}\tag{4}$$

Constraints (3) and (4) are clearly necessary for $(b(\theta),P(\theta))$ to be incentive compatible. Furthermore, if $(b(\theta),P(\theta))$ satisfies these constraints, then it can be supported by specifying the maximum penalty following any choice $b'\in[\underline{b},\overline{b}]$ by the government such that $b'\neq b(\theta')$ for all $\theta'\in\Theta$. Since such a choice is off path, it is without loss to assume that it is maximally punished.

Last, the feasibility constraint ensures that the penalty $P(\theta)$ is feasible given the level of debt:

$$P(\theta)\in\big[0,\overline{P}\big(b(\theta)\big)\big]\quad\text{for all }\theta\in\Theta.\tag{5}$$

A fiscal rule is incentive compatible if it satisfies (3)–(4), and it is incentive compatible and feasible, or incentive feasible for short, if it satisfies (3)–(5). The fiscal rule is optimal if it additionally maximizes social welfare, namely, if it solves

$$\begin{aligned}&\max_{(b(\theta)\in[\underline{b},\overline{b}],P(\theta))}\mathbb{E}\big[\theta U\big(\omega+b(\theta)\big)+\delta\big(V\big(b(\theta)\big)-P(\theta)\big)\big]\\&\text{subject to (3), (4), and (5).}\end{aligned}\tag{6}$$

Throughout our analysis, we assume that the solution to (6) admits a piecewise continuously differentiable function $b(\theta)$, which allows us to establish our results using perturbations.[17]

3. OPTIMAL FISCAL RULE

We characterize the optimal fiscal rule by solving program (6). We will establish conditions under which the unique solution is a deficit limit with maximal enforcement.

Let $b^r(\theta)$ denote type θ's flexible level of debt conditional on no penalty:

$$b^r(\theta)\in\arg\max_{b\in[\underline{b},\overline{b}]}\big\{\theta U(\omega+b)+\beta\delta V(b)\big\}.$$

We define the following:

[17]If the program admits multiple solutions that differ only on a countable set of types, we select the solution that maximizes social welfare for those types.

DEFINITION 1: $(b(\theta), P(\theta))$ is a *maximally enforced deficit limit* if there exist $\theta^* \in [0, \overline{\theta})$ and finite $\theta^{**} > \max\{\theta^*, \underline{\theta}\}$ such that

$$\big(b(\theta), P(\theta)\big) = \begin{cases} \big(b^r(\theta), 0\big) & \text{if } \theta < \theta^*, \\ \big(b^r(\theta^*), 0\big) & \text{if } \theta \in [\theta^*, \theta^{**}], \\ \big(b^p(\theta), \overline{P}(b^p(\theta))\big) & \text{if } \theta > \theta^{**}, \end{cases} \tag{7}$$

where

$$\begin{aligned} &\theta^{**}U\big(\omega + b^r(\theta^*)\big) + \beta\delta V\big(b^r(\theta^*)\big) \\ &\quad = \theta^{**}U\big(\omega + b^p(\theta^{**})\big) + \beta\delta\big(V\big(b^p(\theta^{**})\big) - \overline{P}\big(b^p(\theta^{**})\big)\big). \end{aligned} \tag{8}$$

Figure 1 provides an example in which we take $\overline{P}(b) = \overline{P}$ for all $b \in [\underline{b}, \overline{b}]$. We depict the borrowing schedule (top panel) and the penalty schedule (bottom panel) under a maximally enforced deficit limit with $\theta^* > \underline{\theta}$ and $\theta^{**} < \overline{\theta}$. Under this rule, types $\theta \in [\underline{\theta}, \theta^*)$ choose their flexible debt levels conditional on no penalty, $b^r(\theta)$; types $\theta \in [\theta^*, \theta^{**}]$ choose type θ^*'s flexible debt level conditional on no penalty, $b^r(\theta^*)$; and types $\theta \in (\theta^{**}, \overline{\theta}]$ choose their flexible debt levels conditional on the maximum penalty, $b^p(\theta)$. Types $\theta \leq \theta^{**}$ receive no penalty whereas types $\theta > \theta^{**}$ receive the maximum penalty. By (8), the enforcement constraint holds with equality for type θ^{**}.

We can verify that the fiscal rule described in Definition 1 is incentive compatible:

LEMMA 1: *If* $(b(\theta), P(\theta))$ *is a maximally enforced deficit limit, then it satisfies the truthtelling constraint* (3) *and the enforcement constraint* (4).

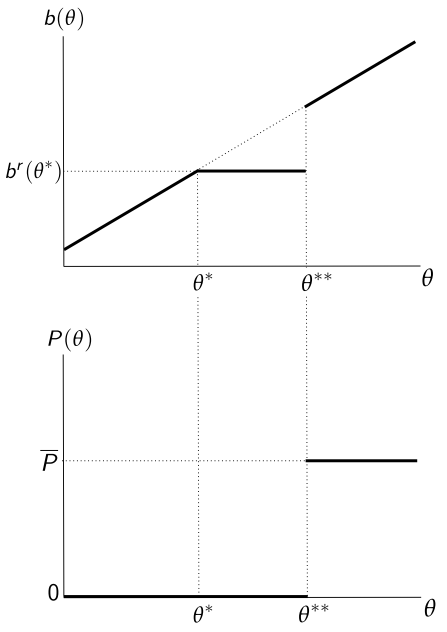

FIGURE 1.—Example of a maximally enforced deficit limit under $\overline{P}(b) = \overline{P}$ for all $b \in [\underline{b}, \overline{b}]$.

In terms of implementation, this fiscal rule can be implemented using a maximum deficit limit, spending limit, or debt limit, where this limit would be associated with the borrowing level $b^r(\theta^*)$. If the government respects the limit, it receives no penalty; if it breaches the limit, it receives the maximum penalty given its level of debt, $\overline{P}(b)$. Note that the limit is breached on path if and only if $\theta^{**} < \overline{\theta}$; we will provide conditions under which this inequality holds in an optimal maximally enforced deficit limit.

Our analysis proceeds as follows. Section 3.1 presents some preliminary results which yield a convenient formulation of the objective in program (6). Section 3.2 shows that any (interior) solution to this program must feature bang-bang penalties. Section 3.3 establishes that under a distributional assumption, optimal bang-bang penalties are monotonic. Section 3.4 uses this result to characterize the optimal borrowing schedule, showing that any solution to (6) is a maximally enforced deficit limit. We further establish that the optimal limit is unique, and provide a necessary and sufficient condition for the government to violate the limit following high enough shocks. Finally, Section 3.5 shows that the distributional assumption introduced in Section 3.3 is not only sufficient but also necessary for any solution to (6) to be a maximally enforced deficit limit.

3.1. *Preliminaries*

The next lemma follows from standard arguments; see Fudenberg and Tirole (1991):

LEMMA 2: $(b(\theta), P(\theta))$ *satisfies the truthtelling constraint* (3) *if and only if*: (*i*) $b(\theta)$ *is nondecreasing*, *and* (*ii*) *the following local truthtelling constraints are satisfied*:

1. *At any point* θ *at which* $b(\cdot)$, *and thus* $P(\cdot)$, *are differentiable*,

$$\frac{db(\theta)}{d\theta}\big(\theta U'\big(\omega + b(\theta)\big) + \beta\delta V'\big(b(\theta)\big)\big) - \beta\delta P'(\theta) = 0.$$

2. *At any point* θ *at which* $b(\cdot)$ *is not differentiable*,

$$\lim_{\theta'\uparrow\theta}\big\{\theta U\big(\omega + b(\theta')\big) + \beta\delta\big(V\big(b(\theta')\big) - P(\theta')\big)\big\}$$
$$= \lim_{\theta'\downarrow\theta}\big\{\theta U\big(\omega + b(\theta')\big) + \beta\delta\big(V\big(b(\theta')\big) - P(\theta')\big)\big\}.$$

Observe that since $b(\theta)$ is nondecreasing in θ, satisfaction of (3) requires that $V(b(\theta)) - P(\theta)$ be nonincreasing in θ.

The local truthtelling constraints imply that the derivative of government welfare with respect to θ is $U(\omega + b(\theta))$. Hence, in an incentive compatible rule, government welfare for type $\theta \in \Theta$ satisfies

$$\begin{aligned}&\theta U\big(\omega + b(\theta)\big) + \beta\delta\big(V\big(b(\theta)\big) - P(\theta)\big)\\ &\quad = \underline{\theta} U\big(\omega + b(\underline{\theta})\big) + \beta\delta\big(V\big(b(\underline{\theta})\big) - P(\underline{\theta})\big) + \int_{\underline{\theta}}^{\theta} U\big(\omega + b(\widetilde{\theta})\big)d\widetilde{\theta}.\end{aligned} \tag{9}$$

Following Amador, Werning, and Angeletos (2006), we can substitute (9) into the social welfare function in (6) to rewrite social welfare as

$$\frac{1}{\beta}\underline{\theta} U\big(\omega + b(\underline{\theta})\big) + \delta\big(V\big(b(\underline{\theta})\big) - P(\underline{\theta})\big) + \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} U\big(\omega + b(\theta)\big)Q(\theta)\, d\theta, \tag{10}$$

where

$$Q(\theta) \equiv 1 - F(\theta) - \theta f(\theta)(1-\beta).$$

This formulation will be useful for our characterization of the optimal fiscal rule, which will appeal to properties of the function $Q(\theta)$. To interpret $Q(\theta)$, observe that the term $1 - F(\theta)$ resembles that in a *virtual surplus* formulation in mechanism design (Myerson (1981)).[18] This term captures the fact that if society prescribes more borrowing for a type θ, then it must prescribe more borrowing for higher types so that their welfare increases at the same rate as required by incentive compatibility (see (9)). In a standard mechanism design problem, $1 - F(\theta)$ enters the virtual surplus negatively, as a higher allocation for an agent type θ entails a cost to the principal. In our problem, instead, this term enters positively as society values borrowing: a higher borrowing allocation for a type θ entails a benefit to society that is proportional to the mass of types higher than θ.

The term $-\theta f(\theta)(1-\beta)$ in $Q(\theta)$ reflects the fact that society and the government do not value borrowing equally. As discussed in Section 2.1, since $\beta < 1$, society weighs current borrowing relative to the future continuation value comparatively less than the government. Hence, a higher borrowing allocation for type θ reduces social welfare relative to government welfare by $-\theta f(\theta)(1-\beta)$. Note that $\theta f(\theta)$ is the weight that social welfare (1) places on the current utility from borrowing by type θ, and $(1-\beta)$ is the extent of the disagreement between society and the government.

In sum, $Q(\theta)$ represents the social value of increasing the borrowing allocation for type θ. The shape of the function $Q(\theta)$ will therefore tell us how society wishes to allocate borrowing across different government types.

3.2. *Bang-Bang Incentives*

Society uses penalties to discipline the government and limit overborrowing. The next proposition shows that in any optimal rule, these penalties are bang-bang.

PROPOSITION 1—Necessity of Bang-Bang: *Assume $Q(\theta)$ satisfies the generic property that $Q'(\theta) \neq 0$ almost everywhere. If $(b(\theta), P(\theta))$ is an optimal rule with $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$, then $P(\theta) \in \{0, \overline{P}(b(\theta))\}$ for all $\theta \in \Theta$.*

This result shows that the bang-bang property is *necessary* for social welfare maximization. An optimal fiscal rule using only extreme penalty values always exists in our framework; this is true simply because an interior penalty $P(\theta) \in (0, \overline{P}(b(\theta)))$ can be assigned in expectation by randomizing over 0 and $\overline{P}(b(\theta))$. Proposition 1 proves a stronger result: any rule with interior penalties is strictly dominated by one with high-powered incentives.

For intuition, recall that $Q(\theta)$ represents the weight that society assigns to increasing borrowing by type θ. The condition in Proposition 1 says that the set of types θ for which $Q'(\theta) = 0$ is nowhere dense,[19] which implies that $Q(\theta)$ is either strictly decreasing or strictly increasing over any sufficiently small interval. Given the representation in

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[18]Similarly to the standard virtual surplus expression, we can rewrite (10) using the inverse hazard rate:

$$\frac{1}{\beta}\underline{\theta}U\big(\omega + b(\underline{\theta})\big) + \delta\big(V\big(b(\underline{\theta})\big) - P(\underline{\theta})\big) + \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} U\big(\omega + b(\theta)\big)\left(\frac{1-F(\theta)}{f(\theta)} - \theta(1-\beta)\right) f(\theta)\, d\theta.$$

[19]Given $f(\theta)$ continuously differentiable, this condition holds generically. Specifically, this condition fails only if $\theta f'(\theta)/f(\theta) = -(2-\beta)/(1-\beta)$ for a positive mass of types θ, but then any arbitrarily small perturbation of β would render the condition true.

(10), it follows that society benefits from concentrating borrowing on either lower types or higher types in the interval, and thus spreading out incentives by using extreme penalties always allows to increase social welfare. Penalties, however, must be assigned in an incentive feasible manner, satisfying the constraints in (9) together with the monotonicity of borrowing.

It is worth pointing out the scope of Proposition 1. This result does not rely on any non-generic properties of the distribution function $f(\theta)$. Moreover, the result holds regardless of the tightness of enforcement constraints and thus even absent constraint (4). Proposition 1 therefore applies more generally to other delegation problems with money burning. One example is the widely studied delegation setting in which the agent's bias and private information take a multiplicative form; see Amador and Bagwell (2013). In our context, that coincides with the case in which the maximum penalty is $\overline{P}(b) = \overline{P}$ for all $b \in [\underline{b}, \overline{b}]$ and some $\overline{P} > 0$. Proposition 1 implies that in any interior solution to this delegation problem, each agent type is assigned either no money burning or maximal money burning.[20]

We next provide a summary of the proof of Proposition 1. The proof uses perturbation arguments and proceeds in three steps.[21] Step 1 shows that in any optimal rule, $V(b(\theta)) - P(\theta)$ is left-continuous at each $\theta \in (\underline{\theta}, \overline{\theta}]$, a fact that we utilize for the rest of our analysis. We also show that $P(\underline{\theta}) = 0$ must hold in any optimal rule.

Step 2 rules out incentive provision via locally increasing penalties. We show that given an optimal rule, there is no interval $[\theta^L, \theta^H]$ over which $V(b(\theta)) - P(\theta)$ is continuously strictly decreasing in θ with interior penalties $P(\theta) \in (0, \overline{P}(b(\theta)))$. Suppose by contradiction that such an interval exists. By Lemma 2, $b(\theta)$ must be strictly increasing over $[\theta^L, \theta^H]$, and by the generic property in Proposition 1, we can take an interval with either $Q'(\theta) < 0$ or $Q'(\theta) > 0$ for all $\theta \in [\theta^L, \theta^H]$. We then show that there is an incentive feasible perturbation that strictly increases social welfare. The idea is to compress borrowing over the interval when $Q'(\theta) < 0$, and to spread out borrowing when $Q'(\theta) > 0$.

More precisely, if $Q'(\theta) < 0$, we construct a *flattening perturbation* that rotates the increasing $b(\theta)$ schedule clockwise over $[\theta^L, \theta^H]$. Since $P(\theta)$ is interior, it can be adjusted so that the rule remains incentive feasible; moreover, since $P(\theta)$ does not appear in the formulation in (10), we can evaluate the change in social welfare independent of such adjustment. We then obtain that this perturbation is socially beneficial because, given $Q'(\theta) < 0$, society prefers to concentrate borrowing on lower rather than higher types. Analogously, if $Q'(\theta) > 0$, we construct a *steepening perturbation* that drills a hole in the $b(\theta)$ schedule by making allocations in (θ^L, θ^H) no longer available. This perturbation is socially beneficial because, given $Q'(\theta) > 0$, society prefers to concentrate borrowing on higher rather than lower types. Figure 2 illustrates the perturbations.

Step 3 completes the proof by ruling out interior penalties. Suppose $P(\theta) \in (0, \overline{P}(b(\theta)))$ at some $\theta \in \Theta$. By the previous steps and Lemma 2, type θ must belong to a stand-alone segment $(\theta^L, \theta^H]$ such that $b(\theta) = b$ and $V(b(\theta)) - P(\theta) = V(b) - P$ for all $\theta \in (\theta^L, \theta^H]$, $b \in (\underline{b}, \overline{b})$ and $P \in (0, \overline{P}(b))$, and $b(\theta)$ jumps at each boundary unless $\theta^H = \overline{\theta}$. We then show that there is an incentive feasible perturbation that strictly increases social welfare. The idea again builds on the shape of the function $Q(\theta)$. If

[20] Ambrus and Egorov (2013) provided examples in the setting of Amador, Werning, and Angeletos (2006) in which borrowing is at a corner and, as a result, interior punishments can be optimal.

[21] Some of the arguments that we use when $Q(\theta)$ is decreasing are similar to those employed by Athey, Atkeson, and Kehoe (2005) in their analysis of optimal monetary rules. Unlike in their work, where rules are perfectly enforced, our arguments take into account the constraints due to limited enforcement.

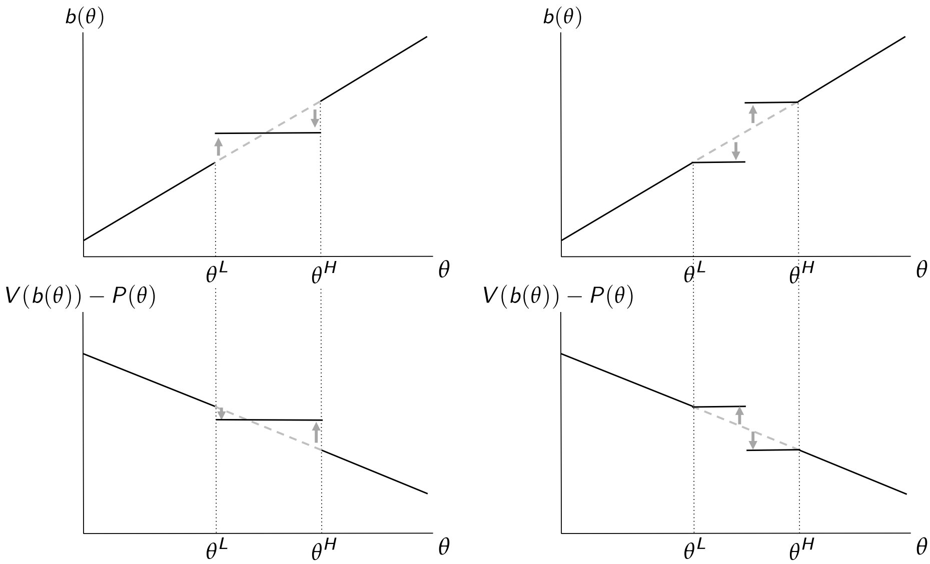

FIGURE 2.—Examples of a flattening perturbation (left panel) and a steepening perturbation (right panel), as used in Step 2 of the proof of Proposition 1.

$\int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta > \int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta$, we perform a *segment-shifting steepening perturbation* over $(\theta^L, \theta^H]$ as illustrated in Figure 3: we marginally increase b and P so as to leave the government welfare of type θ^H unchanged, thus letting types arbitrarily close to θ^L jump down to a lower debt level. This perturbation is socially beneficial because, given $\int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta > \int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta$, society prefers to concentrate borrowing on $(\theta^L, \theta^H]$ compared to θ^L.[22] If $\int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta \leq \int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta$, the generic property in Proposition 1 ensures that $\int_{\theta^L}^{\theta^h} Q(\theta)\,d\theta < \int_{\theta^L}^{\theta^h} Q(\theta^L)\,d\theta$ for some $\theta^h \in (\theta^L, \theta^H]$. We then show that an analogous perturbation over $(\theta^L, \theta^h]$ but in the opposite direction, namely, a *segment-shifting flattening perturbation*, is socially beneficial.

3.3. *Monotonic Incentives*

We make the following assumption for the remainder of our analysis:

ASSUMPTION 1: *There exists $\widehat{\theta} \in \Theta$ such that $Q'(\theta) < 0$ if $\theta < \widehat{\theta}$ and $Q'(\theta) > 0$ if $\theta > \widehat{\theta}$.*

This assumption states that $Q(\theta)$ has a minimum value $Q(\widehat{\theta}) \leq Q(\overline{\theta}) < 0$, being strictly decreasing for $\theta < \widehat{\theta}$ and strictly increasing for $\theta > \widehat{\theta}$. For intuition, recall that $Q(\theta)$ represents the social value of increasing the borrowing allocation for type θ, and is given by $Q(\theta) = 1 - F(\theta) - \theta f(\theta)(1 - \beta)$. The term $1 - F(\theta)$ is decreasing in θ as assigning higher borrowing to a type θ entails a social benefit that is proportional to the mass of types above θ. The term $-\theta f(\theta)(1 - \beta)$ may be increasing or decreasing in θ as assigning

[22]The change in social welfare from the types whose borrowing jumps down is of the same order as the change in social welfare from the types whose borrowing is increased. The shape of the function $Q(\theta)$ is then what establishes the sign of the perturbation.

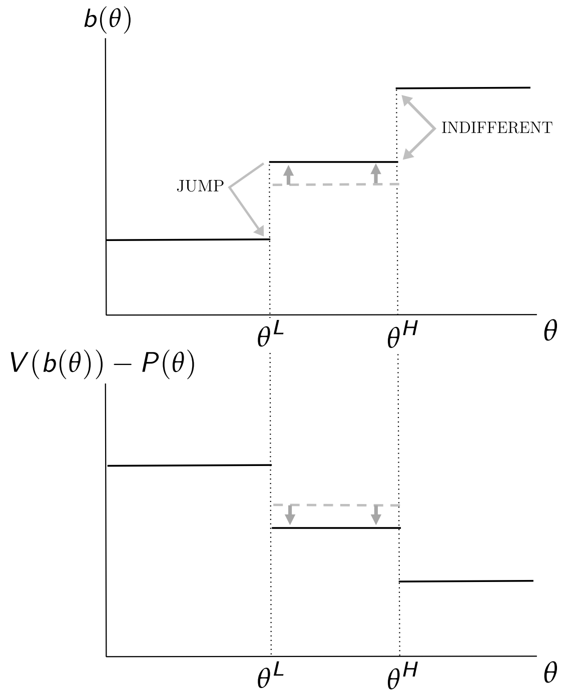

FIGURE 3.—Example of a segment-shifting steepening perturbation, as used in Step 3 of the proof of Proposition 1.

higher borrowing to a type θ entails a social cost that is proportional to the probability of type θ. This term is increasing in θ if $f'(\theta)$ is sufficiently negative, in which case increasing borrowing for higher types is less distortionary than for lower types. Assumption 1 says that the declining $1 - F(\theta)$ effect dominates for $\theta < \widehat{\theta}$, whereas a rising $-\theta f(\theta)(1-\beta)$ effect dominates for $\theta > \widehat{\theta}$. Note that the assumption allows for $Q(\theta)$ to be strictly decreasing or strictly increasing over the whole set Θ; in this case, $\widehat{\theta}$ is defined as either the upper bound or the lower bound of the set Θ.

Assumption 1 holds for a broad range of distribution functions, including uniform, exponential, log-normal, gamma, and beta for a subset of its parameters. This assumption implies the generic property required in Proposition 1 and is similar to, but stronger than, the distributional assumption used in Amador, Werning, and Angeletos (2006). We show in Section 3.5 that Assumption 1 is necessary for our characterization of optimal rules.

Using Assumption 1, the next result shows that optimal penalties are monotonic.

PROPOSITION 2: *If* $(b(\theta), P(\theta))$ *is an optimal rule with* $b(\theta) \in (\underline{b}, \overline{b})$ *for all* $\theta \in \Theta$, *then either* $P(\theta) = 0$ *for all* $\theta \in \Theta$, *or there exists* $\theta^{**} \in (\underline{\theta}, \overline{\theta})$ *such that* $P(\theta) = 0$ *for all* $\theta \in [\underline{\theta}, \theta^{**}]$ *and* $P(\theta) = \overline{P}(b(\theta))$ *for all* $\theta \in (\theta^{**}, \overline{\theta}]$.

The intuition is related to the shape of the function $Q(\theta)$, which tells us how society wishes to allocate borrowing across types. For types $\theta > \widehat{\theta}$, where $\widehat{\theta}$ is defined in Assumption 1, society prefers to concentrate borrowing on relatively high types. This is achieved by using high-powered incentives that specify the maximum penalty $\overline{P}(b(\theta))$ for high levels of borrowing. In contrast, for types $\theta < \widehat{\theta}$, society prefers to concentrate borrowing on relatively low types. This is achieved by using flat incentives that impose no penalties.

The proof of Proposition 2 consists of three steps. Step 1 shows that any interval of types receiving the maximum penalty $\overline{P}(b(\theta))$ must lie above $\widehat{\theta}$. Step 2 then establishes that incentives are monotonic: if $P(\theta^{**}) = \overline{P}(b(\theta^{**}))$ for some type $\theta^{**} \geq \widehat{\theta}$ in an optimal rule, then $P(\theta) = \overline{P}(b(\theta))$ for all types $\theta \geq \theta^{**}$. Both of these steps use perturbation arguments similar to those used in the proof of Proposition 1. Observe that together with that proposition and Step 1 of its proof, these two steps imply that if $P(\theta) = \overline{P}(b(\theta))$ for some type $\theta \in \Theta$, then there exists $\theta^{**} \in [\underline{\theta}, \overline{\theta})$ such that $P(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta^{**}]$ and $P(\theta) = \overline{P}(b(\theta))$ for all $\theta \in (\theta^{**}, \overline{\theta}]$. Step 3 completes the proof of Proposition 2 by establishing that $\theta^{**} > \underline{\theta}$ must hold in any optimal rule.

3.4. *Maximally Enforced Deficit Limit*

Proposition 1 and Proposition 2 characterize the penalty schedule in any optimal rule. The next proposition characterizes the borrowing schedule and states our main result.

PROPOSITION 3—Optimal Rule: *If $(b(\theta), P(\theta))$ is an optimal rule with $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$, then it satisfies (7)–(8) for some $\theta^* \in [0, \overline{\theta})$ and finite $\theta^{**} > \max\{\theta^*, \underline{\theta}\}$. Hence, any interior solution is a maximally enforced deficit limit.*

Recall from Proposition 2 that in any optimal rule, either no government type $\theta \in \Theta$ is penalized, or the assigned penalty jumps from 0 to the maximum penalty $\overline{P}(b(\theta))$ at a point $\theta^{**} \in (\underline{\theta}, \overline{\theta})$. To establish Proposition 3, we take θ^{**} as given and solve for the optimal borrowing schedule above and below this point. If $\theta \in (\theta^{**}, \overline{\theta}]$, borrowing is pinned down by the binding enforcement constraint with $(b(\theta), P(\theta)) = (b^p(\theta), \overline{P}(b^p(\theta)))$. If $\theta \in [\underline{\theta}, \theta^{**}]$, we use perturbation arguments similar to those used in our other proofs to show that the borrowing schedule $b(\theta)$ must be continuous. Thus, the allocation over $[\underline{\theta}, \theta^{**}]$ takes the form of bounded discretion, and since a minimum borrowing requirement would reduce social welfare (given the government's bias towards overborrowing), a maximum borrowing limit $b^r(\theta^*)$ is optimal for types in this range. Such a borrowing limit necessarily keeps type θ^{**} indifferent between $(b^r(\theta^*), 0)$ and $(b^p(\theta^{**}), \overline{P}(b^p(\theta^{**})))$.

Proposition 3 shows that any interior solution must be a maximally enforced deficit limit, but it is silent on whether this limit is violated on path, namely, whether $\theta^{**} < \overline{\theta}$. To address this issue, consider first the problem under perfect enforcement, as in the work of Amador, Werning, and Angeletos (2006). If the enforcement constraint (4) can be ignored (and Assumption 1 is maintained), then the optimal rule solves

$$\max_{\theta^* \in [0, \overline{\theta})} \left\{ \int_{\underline{\theta}}^{\theta^*} \big(\theta U\big(\omega + b^r(\theta)\big) + \delta V\big(b^r(\theta)\big)\big) f(\theta)\, d\theta \right.$$
$$\left. + \int_{\theta^*}^{\overline{\theta}} \big(\theta U\big(\omega + b^r(\theta^*)\big) + \delta V\big(b^r(\theta^*)\big)\big) f(\theta)\, d\theta \right\}.$$

Denote the solution by $\theta_e \in [0, \overline{\theta})$. Using the definition of $b^r(\theta)$ and the welfare representation in (10), the first-order condition can be shown to be equivalent to[23]

$$\int_{\theta_e}^{\overline{\theta}} Q(\theta)\, d\theta = 0. \tag{11}$$

[23]Note that $Q(\theta) = 1$ for $\theta < \underline{\theta}$.

The perfect-enforcement deficit limit is such that the average social value of increasing borrowing for types constrained by the limit is zero. Clearly, if this limit can be enforced given $\overline{P}(b)$, then it is also optimal under limited enforcement:

COROLLARY 1: *Suppose*

$$\overline{\theta}U(\omega + b^r(\theta_e)) + \beta\delta V(b^r(\theta_e)) \geq \overline{\theta}U(\omega + b^p(\overline{\theta})) + \beta\delta(V(b^p(\overline{\theta})) - \overline{P}(b^p(\overline{\theta}))). \quad (12)$$

Then the enforcement constraint does not bind and any optimal rule $(b(\theta), P(\theta))$ *with* $b(\theta) \in (\underline{b}, \overline{b})$ *for all* $\theta \in \Theta$ *coincides with the perfect-enforcement deficit limit, with* $\theta^* = \theta_e$ *and* $\theta^{**} \geq \overline{\theta}$.

When condition (12) holds, the highest type $\overline{\theta}$, and thus all types $\theta \in \Theta$, prefer to respect the perfect-enforcement limit and receive no penalty rather than breach the limit and receive the maximum penalty. Hence, the enforcement constraint is non-binding and the optimal rule corresponds to that under perfect enforcement, with no on-path penalties.

Our interest is in characterizing the optimal rule when condition (12) does not hold and thus the enforcement constraint binds, that is, the perfect-enforcement limit $b^r(\theta_e)$ is not enforceable given $\overline{P}(b)$. To this end, we can define a unique type θ_c as corresponding to the tightest deficit limit that all types $\theta \in \Theta$ would be willing to respect:

$$\overline{\theta}U(\omega + b^r(\theta_c)) + \beta\delta V(b^r(\theta_c)) = \overline{\theta}U(\omega + b^p(\overline{\theta})) + \beta\delta(V(b^p(\overline{\theta})) - \overline{P}(b^p(\overline{\theta}))). \quad (13)$$

Note that $\theta_c \leq \theta_e$ whenever the perfect-enforcement limit is enforceable, and $\theta_c > \theta_e$ otherwise. Importantly, the value of θ_c is only a function of preferences, the maximum feasible penalty $\overline{P}(b)$, and the value of $\overline{\theta}$, and is thus independent of the distribution of types over the support $[\underline{\theta}, \overline{\theta}]$. We will use these properties in Section 4 to derive comparative statics. Given this definition of θ_c, the next proposition provides a necessary and sufficient condition for penalties to be optimally used on path.

PROPOSITION 4—Use of Penalties: *If* $(b(\theta), P(\theta))$ *is an optimal rule with* $b(\theta) \in (\underline{b}, \overline{b})$ *for all* $\theta \in \Theta$, *then it is the unique such rule. Moreover, if*

$$\int_{\theta_c}^{\overline{\theta}} (Q(\theta) - Q(\overline{\theta}))\, d\theta \geq 0, \quad (14)$$

this rule is a maximally enforced deficit limit with $\theta^* = \max\{\theta_c, \theta_e\}$ *and* $\theta^{**} \geq \overline{\theta}$. *Otherwise, this rule is a maximally enforced deficit limit with* $\theta^* \in (\theta_e, \theta_c)$ *and* $\theta^{**} < \overline{\theta}$.

Whenever the perfect-enforcement limit $b^r(\theta_e)$ is enforceable, that is, $\theta_c \leq \theta_e$, Assumption 1 guarantees that $\int_{\theta_c}^{\overline{\theta}} Q(\theta)\, d\theta \geq \int_{\theta_e}^{\overline{\theta}} Q(\theta)\, d\theta = 0$. Hence, condition (14) holds and the optimal rule coincides with that under perfect enforcement, as noted in Corollary 1.

If instead the perfect-enforcement limit $b^r(\theta_e)$ is not enforceable, that is, $\theta_c > \theta_e$, then society faces the following tradeoff. On the one hand, society can raise the value of θ^* to the point that the associated limit $b^r(\theta^*)$ satisfies the enforcement constraint of type $\overline{\theta}$ and thus of all types $\theta \in \Theta$. This option entails setting $\theta^* = \theta_c$ and $\theta^{**} = \overline{\theta}$ and has the benefit of avoiding socially costly penalties on path, albeit at the cost of potentially allowing significant overborrowing within the relaxed deficit limit. On the other hand, society can impose a tighter limit $b^r(\theta^*)$ which does not satisfy the enforcement constraint of all types. This option sets $\theta^* < \theta_c$ and $\theta^{**} < \overline{\theta}$ and induces higher discipline on types $\theta \leq \theta^{**}$, but at the cost of imposing penalties whenever a shock $\theta > \theta^{**}$ is realized.

Proposition 4 shows that which of these two options is optimal for society depends on whether the inequality in (14) holds or not. The proof of this proposition uses the properties of $Q(\theta)$ to show that there exist a unique threshold θ^* and associated θ^{**} satisfying (8) that optimally resolve the tradeoff between imposing fiscal discipline and avoiding on-path penalties. The intuition for condition (14) is familiar by now: it tells us how society wishes to allocate borrowing, and in particular whether society prefers to concentrate borrowing on types $[\theta_c, \overline{\theta})$ versus $\overline{\theta}$. A relaxed deficit limit that avoids on-path penalties concentrates borrowing on $[\theta_c, \overline{\theta})$, whereas tightening the deficit limit by the use of penalties increases borrowing by type $\overline{\theta}$ who then breaches the limit.

3.5. *Discussion of Distributional Assumption*

Our characterization in Section 3.4 shows that Assumption 1 is sufficient to obtain the unique optimality of maximally enforced deficit limits among interior solutions. In this section, we explore the necessity of Assumption 1 for our findings.

DEFINITION 2: Assumption 1 is *weakly violated* if there exist $\theta^L, \theta^H \in \Theta$, $\theta^H > \theta^L$, and $\Delta > 0$ such that (i) $Q'(\theta) \geq 0$ for $\theta \in [\theta^L, \theta^L + \Delta]$ and (ii) $Q'(\theta) \leq 0$ for $\theta \in [\theta^H - \Delta, \theta^H]$. Assumption 1 is *strictly violated* if the inequalities in (i)–(ii) are strict.

Both weak and strict violations of Assumption 1 would affect our results. The next proposition considers functions $V(b)$ which are continuously differentiable and strictly concave and functions $\overline{P}(b)$ which are continuous and bounded, as assumed in Section 2.

PROPOSITION 5—Necessity of Distributional Assumption: *If Assumption 1 is weakly violated, then for any function $V(b)$, there exists a function $\overline{P}(b)$ under which not every optimal rule with $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$ is a maximally enforced deficit limit. Moreover, if Assumption 1 is strictly violated, then for any function $V(b)$, there exists a function $\overline{P}(b)$ under which no such optimal rule is a maximally enforced deficit limit.*

Assumption 1 is necessary for maximally enforced deficit limits to be uniquely optimal (among interior solutions) given any functions $V(b)$ and $\overline{P}(b)$. In this sense, our analysis identifies the minimal structure that guarantees the unique optimality of this class of rules, a class that resembles fiscal rules commonly used in practice in the form of deficit, spending, and debt limits. Note that both weak and strict violations of Assumption 1 are possible even when the generic property in Proposition 1 is satisfied. Under these violations, an optimal rule under limited enforcement would feature bang-bang incentives yet induce an allocation that is not implementable by a deficit limit. We prove the first part of Proposition 5 by construction and the second part by contradiction.[24]

[24] As noted in Section 3.3, Assumption 1 is stronger than the distributional assumption used in Amador, Werning, and Angeletos (2006). Define θ_a as the lowest value such that $\int_{\theta}^{\overline{\theta}} Q(\widetilde{\theta})\, d\widetilde{\theta} \leq 0$ for all $\theta \geq \theta_a$. Then using our notation, their paper assumes $Q'(\theta) \leq 0$ for all $\theta \leq \theta_a$. One can verify that there are distribution functions satisfying this assumption for which Assumption 1 is strictly violated.

4. COMPARATIVE STATICS

In this section, we study how the optimal fiscal rule depends on features of the environment. Throughout, we assume that the optimal rule admits $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$, so it takes the form of a maximally enforced deficit limit as established in Proposition 3.

4.1. *Penalties*

We begin by studying comparative statics with respect to the severity of penalties that are available to society. The next proposition studies how penalty severity affects the tightness of enforcement constraints and the use of penalties on path.

PROPOSITION 6—Penalty Severity on Enforcement and Use of Penalties: *Consider shifting $\overline{P}(b)$ to $\overline{P}(b) + k$ for a constant $k > 0$.*

1. *Suppose the enforcement constraint binds under $\overline{P}(b)$. There exists finite $k' > 0$ such that the enforcement constraint does not bind under $\overline{P}(b) + k$ if and only if $k \geq k'$.*
2. *Suppose the enforcement constraint binds and on-path penalties are optimal under $\overline{P}(b)$. There exist finite $k''' > k'' > 0$ such that the enforcement constraint binds and on-path penalties are suboptimal under $\overline{P}(b) + k$ if and only if $k \in [k'', k''')$.*

The first part of the proposition shows that enforcement is not a binding constraint if (and only if) the penalties available to society are sufficiently severe. The perfect-enforcement deficit limit is enforceable, and thus optimal by Corollary 1, provided that the government can be punished harshly enough for violating it. Our environment under sufficiently severe penalties therefore coincides with that of Amador, Werning, and Angeletos (2006), which is nested as a special case of our model.

The second part of the proposition shows that, given a binding enforcement constraint, the optimality of on-path penalties depends on their severity. Recall that by Proposition 4, the optimal rule prescribes penalties on path if and only if condition (14) is not satisfied. Proposition 6 tells us that this can only happen if penalties are relatively mild. Intuitively, society uses penalties on path only if avoiding them is too costly, that is, only if a too lax deficit limit would have to be specified for all government types to respect it. As penalties become more severe, the tightest deficit limit that all types are willing to respect becomes tighter (i.e., the value of θ_c defined in (13) declines), so eventually setting this limit and avoiding on-path punishments becomes optimal.

The next proposition studies how penalty severity affects the cutoffs $\{\theta^*, \theta^{**}\}$ specified by the optimal deficit limit.

PROPOSITION 7—Penalty Severity on Deficit Limit: *Consider shifting $\overline{P}(b)$ to $\overline{P}(b) + k$ for a constant k such that $\overline{P}(b) + k > 0$ for all $b \in [\underline{b}, \overline{b}]$. Suppose the enforcement constraint binds before and after the shift and the optimality/suboptimality of on-path penalties is preserved.*

1. *The optimal value of θ^* decreases (increases) with the shift if $k > 0$ ($k < 0$).*
2. *If on-path penalties are optimal, the optimal value of θ^{**} increases (decreases) with the shift if $k > 0$ ($k < 0$).*

Proposition 7 considers changes in penalty severity that are local in that they preserve a binding enforcement constraint as well as the optimality/suboptimality of on-path penalties. To understand the results, take first the case in which on-path penalties are suboptimal. The optimal deficit limit is then $\theta^* = \theta_c$, namely, the tightest limit that all government types are willing to respect. If penalties become less severe, society must relax this deficit limit for no type to want to breach it; thus, θ^* increases when $\overline{P}(b)$ declines. Analogously, if penalties become more severe, society can tighten the deficit limit without any type wanting to breach it; thus, θ^* declines when $\overline{P}(b)$ increases.

Take next the case in which on-path penalties are optimal. We can show that the optimal deficit limit $\{\theta^*, \theta^{**}\}$ is then pinned down by the indifference condition (8) and the following first-order condition:

$$\int_{\theta^*}^{\theta^{**}} \big(Q(\theta) - Q(\theta^{**})\big)\, d\theta = 0. \tag{15}$$

This condition says that the average social value of increasing borrowing for constrained types in $[\theta^*, \theta^{**}]$ equals the social value of increasing borrowing for type θ^{**}. Given Assumption 1, (15) requires $Q(\theta^*) > 0 > Q(\theta^{**})$ and thus $\theta^* < \widehat{\theta} < \theta^{**}$. Using this, we can show that the left-hand side of (15) is decreasing in both θ^* and θ^{**}, which allows us to establish how these values change with $\overline{P}(b)$. For example, suppose that penalties become less severe. Given θ^*, the indifference condition (8) is then satisfied at a lower value of θ^{**}, so (15) requires that θ^* increase and θ^{**} decrease. Intuitively, holding the deficit limit fixed, a larger set of government types want to break the limit if their punishment for doing so becomes milder. Moreover, since $Q'(\theta) > 0$ at θ^{**} (by $\widehat{\theta} < \theta^{**}$), having these additional lower types violate the limit comes at an increased marginal social cost, so it is optimal for society to relax the limit in response. The decline in $\overline{P}(b)$ therefore results in both more lax policies and more frequent punishment. The opposite occurs following an increase in $\overline{P}(b)$, which results in tighter policies and less frequent punishment.

EXAMPLE 1—Constant Maximum Penalty: As an illustration of the results above, consider a setting in which $\overline{P}(b) = \overline{P}$ for all $b \in [\underline{b}, \overline{b}]$ and some $\overline{P} > 0$. Then there exist finite $\overline{P}'' > \overline{P}' \geq 0$ such that the optimal rule is the perfect-enforcement deficit limit if $\overline{P} \geq \overline{P}''$, the tightest enforceable deficit limit with no on-path penalties if $\overline{P}' \leq \overline{P} < \overline{P}''$, and a deficit limit with on-path penalties if $\overline{P} < \overline{P}'$. Moreover, $\overline{P}' > 0$ if and only if $\widehat{\theta} < \overline{\theta}$; that is, penalties are used on path for $\overline{P}$ low enough if and only if $Q'(\theta) > 0$ for high values of θ.

If $Q'(\theta) \leq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, high types occur with relatively high probability, so punishing them for violating a tight deficit limit is too costly to society. The optimal deficit limit sets $\theta^* = \theta_c$ for all $\overline{P} < \overline{P}''$, and the value of θ^* increases as $\overline{P}$ declines below $\overline{P}''$. In the limit as $\overline{P}$ approaches 0, the value of θ^* approaches $\overline{\theta}$, thus converging to full flexibility.

If $Q'(\theta) > 0$ for θ high enough, high types occur with relatively small probability, so for sufficiently low $\overline{P}$ it is optimal to specify a tight deficit limit and punish the high types who violate it. The optimal deficit limit sets $\theta^* \in (\theta_e, \theta_c)$ and $\theta^{**} < \overline{\theta}$ for all $\overline{P} < \overline{P}'$, and the value of θ^* increases and that of θ^{**} decreases as $\overline{P}$ declines below $\overline{P}'$. In the limit as $\overline{P}$ approaches 0, these values converge to a single point $\theta^* = \theta^{**} < \overline{\theta}$, which is equivalent to full flexibility.

The left panel of Figure 4 illustrates how the optimal deficit limit changes in response to an increase in the severity of penalties. We take a constant maximum penalty $\overline{P} > 0$, as in Example 1 above, and consider a setting in which penalties are optimally used on path both before and after $\overline{P}$ increases. The figure shows that a harsher maximum penalty benefits society via two effects: a tighter deficit limit (lower value of θ^*) that improves fiscal

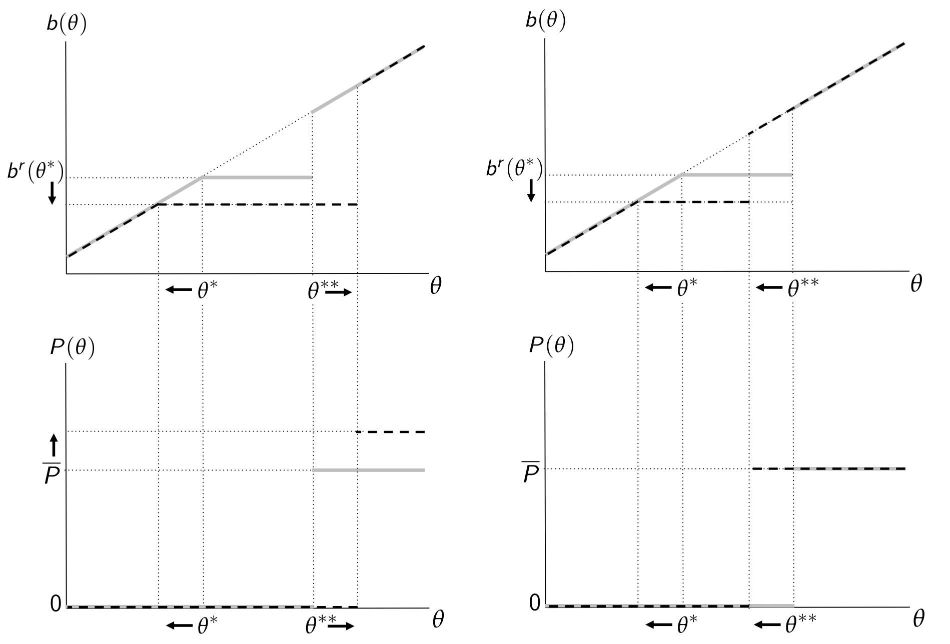

FIGURE 4.—Comparative statics with respect to penalty severity (left panel) and distribution of types (right panel). The solid gray lines depict the optimal rule before the change and the dashed black lines after the change.

discipline, and increased enforceability (higher value of θ^{**}) that reduces the frequency with which penalties are used on path.

The results of this section are consistent with evidence that fiscal limits are more frequently breached in countries with weaker enforcement regimes (e.g., Reuter (2017)). We find that such countries should impose more relaxed fiscal limits yet experience more frequent violations than countries with stronger regimes. Our findings also highlight the importance of calibrating fiscal rules to the institutional environment. In particular, improvements in enforcement mechanisms, such as by introducing a fiscal council (e.g., Eyraud et al. (2018b)), should not only reduce violations, but should also be accompanied by a tightening of fiscal limits to further increase fiscal discipline.

4.2. *Distribution of Types*

We next study comparative statics with respect to the distribution of types. Take a distribution function $f(\theta)$ with full support over $\Theta = [\underline{\theta}, \overline{\theta}]$. We consider perturbations that yield a new distribution function $\widetilde{f}(\theta)$ with full support over $\widetilde{\Theta} \equiv [\widetilde{\underline{\theta}}, \widetilde{\overline{\theta}}]$. As for the original function $f(\theta)$, we require that $\widetilde{f}(\theta)$ be continuously differentiable in θ, and that the associated cumulative distribution function $\widetilde{F}(\theta)$ and weight function $\widetilde{Q}(\theta)$ satisfy Assumption 1. We will focus on a class of perturbations that we call *Q-monotonic*:

DEFINITION 3: A distribution function perturbation that yields $\widetilde{f}(\theta)$ over $\widetilde{\Theta}$ is *Q-monotonic* if $\Theta \cap \widetilde{\Theta} \neq \emptyset$ and $\widetilde{Q}(\theta) - Q(\theta)$ is strictly monotonic for all $\theta \in \Theta \cap \widetilde{\Theta}$. The perturbation is *Q-increasing* (*Q-decreasing*) if $\widetilde{Q}(\theta) - Q(\theta)$ is strictly increasing (decreasing) for all $\theta \in \Theta \cap \widetilde{\Theta}$.

As we discuss below, Q-monotonic distribution function perturbations include simple perturbations that may change the mean and/or variance of θ. The next proposition studies how this class of perturbations affect the use of penalties on path.

PROPOSITION 8—Type Distribution on Use of Penalties: *Consider a Q-monotonic distribution function perturbation with $\widetilde{\Theta} = \Theta$.*

1. *If on-path penalties are suboptimal under $f(\theta)$ and the perturbation is Q-decreasing, on-path penalties remain suboptimal under $\widetilde{f}(\theta)$.*
2. *If on-path penalties are optimal under $f(\theta)$ and the perturbation is Q-increasing, on-path penalties remain optimal under $\widetilde{f}(\theta)$.*

For intuition, observe first that if the support $[\underline{\theta}, \overline{\theta}]$ is kept unchanged, then a Q-increasing (Q-decreasing) perturbation decreases (increases) the mean of θ by increasing (decreasing) the mass of types below some interior $\theta \in (\underline{\theta}, \overline{\theta})$ while decreasing (increasing) the mass of types above.[25] Hence, if on-path penalties are suboptimal under $f(\theta)$, they remain suboptimal following a support-preserving Q-decreasing perturbation: this perturbation makes it less appealing for society to impose penalties on high types (whose probability rises) in order to tighten the deficit limit for low types (whose probability declines).[26] Similarly, if on-path penalties are optimal under $f(\theta)$, they remain optimal following a support-preserving Q-increasing perturbation: this perturbation makes it less appealing for society to relax the deficit limit for low types (whose probability rises) in order to avoid penalties on high types (whose probability declines). To prove this result, we show that the left-hand side of condition (14) in Proposition 4 increases (decreases) with Q-decreasing (Q-increasing) perturbations that keep the support unchanged.

The next proposition studies how Q-monotonic distribution function perturbations affect the cutoffs $\{\theta^*, \theta^{**}\}$ specified by the optimal deficit limit.

PROPOSITION 9—Type Distribution on Deficit Limit: *Consider a Q-monotonic distribution function perturbation. Suppose the enforcement constraint binds before and after the perturbation and the optimality/suboptimality of on-path penalties is preserved.*

1. *Suppose on-path penalties are suboptimal. Then the optimal value of θ^* increases (decreases) with the perturbation if $\widetilde{\overline{\theta}} > \overline{\theta}$ ($\widetilde{\overline{\theta}} < \overline{\theta}$).*
2. *Suppose on-path penalties are optimal and the optimal deficit limit has $\theta^*, \theta^{**} \in \widetilde{\Theta} \cap \Theta$ before and after the perturbation. Then the optimal values of θ^* and θ^{**} both decrease (increase) with the perturbation if it is Q-increasing (Q-decreasing).*

Proposition 9 considers perturbations that are local in that they preserve a binding enforcement constraint as well as the optimality/suboptimality of on-path penalties. For

[25] By definition, a Q-increasing perturbation must satisfy

$$-(2-\beta)\big(\widetilde{f}(\theta) - f(\theta)\big) - (1-\beta)\theta\big(\widetilde{f}'(\theta) - f'(\theta)\big) > 0.$$

This condition requires $\widetilde{f}'(\theta) - f'(\theta) < 0$ at any θ at which $\widetilde{f}(\theta) - f(\theta) \geq 0$. Hence, for a support-preserving Q-increasing perturbation, there must be $\theta' \in (\underline{\theta}, \overline{\theta})$ such that $\widetilde{f}(\theta) - f(\theta) > 0$ for $\theta < \theta'$ and $\widetilde{f}(\theta) - f(\theta) < 0$ for $\theta > \theta'$. It follows that the perturbation decreases the mean of θ. Analogous arguments imply that a support-preserving Q-decreasing perturbation increases the mean of θ.

[26] We can also see the intuition by recalling that the weight $Q(\theta)$ represents the social value of increasing borrowing by type θ. A Q-decreasing perturbation increases the social weight on borrowing by low types relative to high types. Thus, a tightening of the deficit limit that reduces borrowing by low types and increases borrowing by high types (who then violate the limit) becomes less appealing to society following this perturbation.

the first part, recall that if the enforcement constraint binds and on-path penalties are suboptimal, then the optimal deficit limit sets $\theta^* = \theta_c$, namely, the tightest limit that all government types are willing to respect. By (13), this limit corresponds to the tightest limit that the highest type $\overline{\theta}$ would respect, as this type has the highest marginal utility of spending. Hence, if the value of $\overline{\theta}$ increases, this limit must be relaxed, whereas if the value of $\overline{\theta}$ declines, the limit can be tightened.

The second part of the proposition studies the case in which on-path penalties are optimal. As discussed in the previous subsection, the optimal deficit limit $\{\theta^*, \theta^{**}\}$ is then pinned down by the indifference condition (8) and the first-order condition (15), where the left-hand side of (15) is decreasing in both θ^* and θ^{**}. We show that, keeping θ^* and θ^{**} unchanged, the left-hand side of (15) decreases (increases) following a Q-increasing (Q-decreasing) perturbation; hence, to preserve condition (15), the values of θ^* and θ^{**} must both decrease (increase) following a Q-increasing (Q-decreasing) perturbation. For intuition, take a Q-increasing perturbation and recall that the weight $Q(\theta)$ represents the social value of increasing borrowing by type θ. Hence, by definition, a Q-increasing perturbation increases the social weight on borrowing by high types relative to the social weight on borrowing by low types. This means that following the perturbation, society benefits from tightening the deficit limit: this reduces borrowing by relatively low types while increasing borrowing by relatively high types who then violate the limit.

EXAMPLE 2—Support-Preserving Perturbations: As an illustration of the results above, consider a support-preserving Q-monotonic distribution function perturbation. As noted, given a fixed support, the perturbation reduces the mean of θ if it is Q-increasing, and it increases this mean if it is Q-decreasing. In the case that the enforcement constraint binds and on-path penalties are suboptimal (before and after the perturbation), the perturbation does not affect the optimal deficit limit, as this limit is pinned down by the support. Instead, in the case that on-path penalties are optimal, the perturbation affects both the tightness of the optimal deficit limit as well as the range of types that breach it. For example, if the perturbation is mean-decreasing, it results in a tighter limit (lower value of θ^*) which is violated by a larger set of types (lower value of θ^{**}). Intuitively, since the perturbation adds to the mass of relatively low types while decreasing that of relatively high types (see footnote 25), the expected benefit of disciplining low types increases, while the expected cost of punishing high types goes down.

EXAMPLE 3—Mean-Preserving Perturbations: For another illustration, consider mean-preserving Q-monotonic distribution function perturbations that expand or contract the support of the distribution. Specifically, consider a variance-increasing perturbation with $\underline{\widetilde{\theta}} < \underline{\theta}$, $\widetilde{\overline{\theta}} > \overline{\theta}$, and $\widetilde{f}(\theta) = f(\theta) - m$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and a constant $m > 0$, and consider a variance-decreasing perturbation with $\underline{\widetilde{\theta}} > \underline{\theta}$, $\widetilde{\overline{\theta}} < \overline{\theta}$, and $\widetilde{f}(\theta) = f(\theta) + m$ for all $\theta \in [\underline{\widetilde{\theta}}, \widetilde{\overline{\theta}}]$ and a constant $m > 0$. We can show that the variance-increasing perturbation is Q-increasing and the variance-decreasing perturbation is Q-decreasing. If the enforcement constraint binds and on-path penalties are suboptimal (before and after the perturbation), the optimal deficit limit is pinned down by the support, so the variance-increasing perturbation relaxes this limit whereas the variance-decreasing perturbation tightens it. Interestingly, this prediction is reversed when on-path penalties are optimal (and the conditions in Proposition 9 hold). For example, the variance-increasing perturbation in this case results in a tighter optimal deficit limit that is violated by a larger set of types. Intuitively, the perturbation extends the upper tail by reducing the mass of relatively high types and introducing even higher types which occur with sufficiently small probability. The marginal cost of punishing high types therefore declines and it is beneficial for society to tighten the limit for lower types. The opposite holds for the variance-decreasing perturbation.

The right panel of Figure 4 illustrates how the optimal deficit limit changes in response to a Q-increasing distribution function perturbation. This perturbation may be a support-preserving, mean-decreasing perturbation as in Example 2, or a mean-preserving, variance-increasing perturbation as in Example 3. We consider a setting in which penalties are optimally used on path both before and after the perturbation. The figure shows that the perturbation results in a tighter deficit limit (lower value of θ^*), thus improving discipline for relatively low types, but also results in reduced enforceability (lower value of θ^{**}), thus allowing overborrowing and imposing penalties for a larger range of relatively high types.

The results of this section highlight the importance of calibrating fiscal rules to forecasts of macroeconomic and fiscal conditions. Rule calibration is a central recommendation of the International Monetary Fund with regards to the design of fiscal rules; together with rule selection, it is viewed as one of the two key steps to building a fiscal framework (Eyraud et al. (2018a)). Our analysis elucidates how fiscal limits should be adjusted according to the government's expected fiscal needs and their volatility. Importantly, we find that calibrations should seek not only to regulate the degree of fiscal discipline, but also to ensure the right balance with the use of costly penalties.

5. CONCLUDING REMARKS

We have studied the optimal design of fiscal rules when enforcement is limited. Under perfect enforcement, the optimal rule is a deficit limit which is never breached. Under limited enforcement, the optimal rule is a maximally enforced deficit limit, which, if violated, leads to the maximum feasible penalty for the government. We established necessary and sufficient conditions under which the optimal deficit limit is violated following high enough shocks to the value of spending, and we provided comparative statics describing how the limit and violations vary with features of the environment.

We believe there are potentially interesting directions for future research. For example, one could explore the generality of our result that optimal incentive provision takes a bang-bang form. As we discussed, this result does not rely on the presence of enforcement constraints and may apply to other models of delegation and adverse selection under joint penalties. It would be useful to understand whether this result is robust to different specifications of punishment: while joint penalties (as we have assumed) are natural given our focus on fiscal policy, and are also consistent with the formalization of money burning in delegation models, one could consider settings in which penalties are experienced asymmetrically by a principal and an agent.

Another possible direction for future work would be to explore environments where penalties are not externally enforced but emerge endogenously from strategic play. For example, in Halac and Yared (2022b), we consider an infinite horizon setting in which fiscal rules are self-enforced by the interaction of a sequence of governments. We find that self-enforcing penalties take the form of temporary overborrowing, with the optimal deficit limit being reinstated only once borrowing becomes high enough. Future research could also explore the possibility of multilateral enforcement, such as when a group of countries or subnational regions are subject to a coordinated fiscal rule. The properties of an optimal common rule would depend on governments' enforcement constraints and on the nature of feasible collective punishments.

Finally, while we have focused on fiscal policy, the insights of this paper may be applied to other settings featuring a commitment-versus-flexibility tradeoff and limited enforcement. For example, consider an individual who suffers from a self-control problem and establishes rules for herself to curb her consumption of a temptation good such as alcohol. The individual values discipline but also benefits from having the flexibility to increase her consumption when highly valuable. Moreover, any rule she imposes on herself must be enforced by some form of costly penalties which are naturally limited. We find that the optimal rule is a consumption threshold, and that the individual may violate this threshold when her value of the temptation good is high enough.

APPENDIX A

This appendix provides the proofs of Proposition 1, Proposition 2, and Proposition 3. See the Supplemental Material (Halac and Yared (2022c)) for the proofs of our other results.

A.1. *Proof of Proposition 1*

Take any solution to (6) with $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$. We proceed in three steps.

STEP 1: We show that $V(b(\theta)) - P(\theta)$ is left-continuous at each $\theta \in (\underline{\theta}, \overline{\theta}]$ and $P(\underline{\theta}) = 0$.

For the first claim, suppose by contradiction that there is $\theta \in (\underline{\theta}, \overline{\theta}]$ at which $V(b(\theta)) - P(\theta)$ is not left-continuous. Denote $(b(\theta^-), P(\theta^-)) \equiv \lim_{\theta' \uparrow \theta}(b(\theta'), P(\theta'))$. By Lemma 2,

$$0 < \theta\big(U\big(\omega + b(\theta)\big) - U\big(\omega + b\big(\theta^-\big)\big)\big) = \beta\delta\big(V\big(b\big(\theta^-\big)\big) - P\big(\theta^-\big) - V\big(b(\theta)\big) + P(\theta)\big).$$

Given $\beta \in (0, 1)$, this implies

$$\theta\big(U\big(\omega + b(\theta)\big) - U\big(\omega + b\big(\theta^-\big)\big)\big) < \delta\big(V\big(b\big(\theta^-\big)\big) - P\big(\theta^-\big) - V\big(b(\theta)\big) + P(\theta)\big).$$

It follows that a perturbation that assigns $(b(\theta^-), P(\theta^-))$ to type θ is incentive feasible, strictly increases social welfare from type θ, and does not affect social welfare from types other than θ. This contradicts the optimality of the original rule, proving the claim.

For the second claim, suppose by contradiction that $P(\underline{\theta}) > 0$. We perform a perturbation where we change $b(\underline{\theta}) \in (\underline{b}, \overline{b})$ by $-db(\underline{\theta}) < 0$ arbitrarily close to zero and change $P(\underline{\theta})$ so as to keep type $\underline{\theta}$ equally well off:

$$db(\underline{\theta})\big(\underline{\theta}U'\big(\omega + b(\underline{\theta})\big) + \beta\delta V'\big(b(\underline{\theta})\big)\big) - \beta\delta\, dP(\underline{\theta}) = 0.$$

This perturbation is incentive feasible and does not affect social welfare from types $\theta \in (\underline{\theta}, \overline{\theta}]$. The change in social welfare from type $\underline{\theta}$ is equal to

$$-\big[db(\underline{\theta})\big(\underline{\theta}U'\big(\omega + b(\underline{\theta})\big) + \delta V'\big(b(\underline{\theta})\big)\big) - \delta\, dP(\underline{\theta})\big] = db(\underline{\theta})\underline{\theta}U'\big(\omega + b(\underline{\theta})\big)\left(\frac{1}{\beta} - 1\right) > 0.$$

This contradicts the optimality of the original rule, proving the claim.

STEP 2: We show that $V(b(\theta)) - P(\theta)$ is a step function over any interval $[\theta^L, \theta^H]$ with $P(\theta) \in (0, \overline{P}(b(\theta)))$ for $\theta \in [\theta^L, \theta^H]$.

By the truthtelling constraints, $V(b(\theta)) - P(\theta)$ is piecewise continuously differentiable and nonincreasing. Suppose by contradiction that there is an interval $[\theta^L, \theta^H]$ over which $V(b(\theta)) - P(\theta)$ is continuously strictly decreasing in θ with $0 < P(\theta) < \overline{P}(b(\theta))$. By Lemma 2, $b(\theta)$ must be continuously strictly increasing over the interval, and without loss we can take an interval over which $b(\theta)$ is continuously differentiable. Moreover, by the generic property in Proposition 1, we can take an interval with either $Q'(\theta) > 0$ or $Q'(\theta) < 0$ for all $\theta \in [\theta^L, \theta^H]$. We consider each possibility in turn.

Case 1: Suppose $Q'(\theta) < 0$ for all $\theta \in [\theta^L, \theta^H]$. We show that there is an incentive feasible flattening perturbation that rotates the increasing borrowing schedule $b(\theta)$ clockwise over $[\theta^L, \theta^H]$ and strictly increases social welfare. Define

$$\overline{U} = \frac{1}{(\theta^H - \theta^L)} \int_{\theta^L}^{\theta^H} U(\omega + b(\theta))\, d\theta.$$

For given $\kappa \in [0, 1]$, let $\widetilde{b}(\theta, \kappa)$ be the solution to

$$U(\omega + \widetilde{b}(\theta, \kappa)) = \kappa \overline{U} + (1 - \kappa) U(\omega + b(\theta)), \tag{A.1}$$

which clearly exists. Define $\widetilde{P}(\theta, \kappa)$ as the solution to

$$\begin{aligned} &\theta U(\omega + \widetilde{b}(\theta, \kappa)) + \beta\delta(V(\widetilde{b}(\theta, \kappa)) - \widetilde{P}(\theta, \kappa)) \\ &\quad = \theta^L U(\omega + b(\theta^L)) + \beta\delta(V(b(\theta^L)) - P(\theta^L)) + \int_{\theta^L}^{\theta} U(\omega + \widetilde{b}(\widetilde{\theta}, \kappa))\, d\widetilde{\theta}. \end{aligned} \tag{A.2}$$

The original allocation corresponds to $\kappa = 0$. We consider a perturbation where we increase κ marginally above zero if and only if $\theta \in [\theta^L, \theta^H]$. Note that differentiating (A.1) and (A.2) with respect to κ yields

$$\frac{d\widetilde{b}(\theta, \kappa)}{d\kappa} = \frac{\overline{U} - U(\omega + b(\theta))}{U'(\omega + \widetilde{b}(\theta, \kappa))}, \tag{A.3}$$

$$\begin{aligned} &\frac{d\widetilde{b}(\theta, \kappa)}{d\kappa}(\theta U'(\omega + \widetilde{b}(\theta, \kappa)) + \beta\delta V'(\widetilde{b}(\theta, \kappa))) - \beta\delta \frac{d\widetilde{P}(\theta, \kappa)}{d\kappa} \\ &\quad = \int_{\theta^L}^{\theta} \frac{d\widetilde{b}(\widetilde{\theta}, \kappa)}{d\kappa} U'(\omega + \widetilde{b}(\widetilde{\theta}, \kappa))\, d\widetilde{\theta}. \end{aligned} \tag{A.4}$$

Substituting (A.3) in (A.4) yields that for a type $\theta \in [\theta^L, \theta^H]$, the change in government welfare from a marginal increase in κ, starting from $\kappa = 0$, is equal to

$$D(\theta) \equiv \int_{\theta^L}^{\theta} (\overline{U} - U(\omega + b(\widetilde{\theta})))\, d\widetilde{\theta}.$$

We begin by showing that the perturbation satisfies constraints (3)–(5). For (3), note that $D(\theta^L) = D(\theta^H) = 0$, so the perturbation leaves the government welfare of types θ^L

and θ^H (and that of types $\theta < \theta^L$ and $\theta > \theta^H$) unchanged. Using Lemma 2 and the representation in (9), it then follows from (A.2) and the fact that $\widetilde{b}(\theta, \kappa)$ is nondecreasing that the perturbation satisfies (3) for all $\theta \in \Theta$ and any $\kappa \in [0, 1]$.

To prove that the perturbation satisfies (4), we show that the government welfare of types $\theta \in [\theta^L, \theta^H]$ weakly rises when κ increases marginally. Since $D(\theta^L) = D(\theta^H) = 0$, it is sufficient to show that $D(\theta)$ is concave over (θ^L, θ^H) to prove that $D(\theta) \geq 0$ for all θ in this interval. Indeed, we can verify that $D''(\theta) = -U'(\omega + b(\theta))\frac{db(\theta)}{d\theta} < 0$.

Last, observe that (5) is satisfied for $\kappa > 0$ small enough. This follows from $\overline{P}(b(\theta))$ being continuous and from $P(\theta) \in (0, \overline{P}(b(\theta)))$ for $\theta \in [\theta^L, \theta^H]$ in the original allocation.

We next show that the perturbation strictly increases social welfare. Using the representation in (10), the change in social welfare from an increase in κ is equal to

$$\frac{1}{\beta}\int_{\theta^L}^{\theta^H} \frac{d\widetilde{b}(\theta, \kappa)}{d\kappa} U'\big(\omega + \widetilde{b}(\theta, \kappa)\big)Q(\theta)\, d\theta.$$

Substituting with (A.3) yields that at $\kappa = 0$, this is equal to

$$\frac{1}{\beta}\int_{\theta^L}^{\theta^H} \big(\overline{U} - U\big(\omega + b(\theta)\big)\big)Q(\theta)\, d\theta.$$

This is an integral over the product of two terms. The first term is strictly decreasing in θ since $b(\theta)$ is strictly increasing over $[\theta^L, \theta^H]$. The second term is also strictly decreasing in θ; this follows from $Q'(\theta) < 0$ for all $\theta \in [\theta^L, \theta^H]$. Therefore, the two terms are positively correlated with one another, and thus the change in social welfare is strictly greater than

$$\frac{1}{\beta}\int_{\theta^L}^{\theta^H} \big(\overline{U} - U\big(\omega + b(\theta)\big)\big)\, d\theta \int_{\theta^L}^{\theta^H} Q(\theta)\, d\theta,$$

which is equal to 0. Hence, we obtain that if $P(\theta)$ is strictly interior and $Q'(\theta) < 0$ over a given interval, then $V(b(\theta)) - P(\theta)$ must be a step function over the interval.

Case 2: Suppose $Q'(\theta) > 0$ for all $\theta \in [\theta^L, \theta^H]$. Recall that $b(\theta)$ is continuously strictly increasing over $[\theta^L, \theta^H]$. We begin by showing that the enforcement constraint cannot bind for all $\theta \in [\theta^L, \theta^H]$. Suppose by contradiction that it does. Using the representation of government welfare in (9), this implies

$$\int_{\theta}^{\theta^H} \big(U\big(\omega + b^p(\widetilde{\theta})\big) - U\big(\omega + b(\widetilde{\theta})\big)\big)\, d\widetilde{\theta} = 0$$

for all $\theta \in [\theta^L, \theta^H]$, which requires $(b(\theta), P(\theta)) = (b^p(\theta), \overline{P}(b^p(\theta)))$ for all $\theta \in (\theta^L, \theta^H)$. However, this contradicts the assumption that $P(\theta) \in (0, \overline{P}(b(\theta)))$ for all $\theta \in [\theta^L, \theta^H]$. Hence, the enforcement constraint cannot bind for all types in the interval, and without loss we can take an interval with this constraint being slack for all $\theta \in [\theta^L, \theta^H]$.

We next show that there is a steepening perturbation that is incentive feasible and strictly increases social welfare. Consider drilling a hole around a type θ^M within $[\theta^L, \theta^H]$ so that we marginally remove the allocation around this type. That is, θ^M can no longer choose $(b(\theta^M), P(\theta^M))$ and is indifferent between jumping to the lower or upper limit of the hole. With some abuse of notation, denote the limits of the hole by θ^L and θ^H, where the perturbation marginally increases θ^H from θ^M. Since the enforcement constraint is slack for all $\theta \in [\theta^L, \theta^H]$, the perturbation is incentive feasible. The change in social welfare is

$$\frac{1}{\beta}\int_{\theta^M}^{\theta^H} \frac{db(\theta^H)}{d\theta^H} U'(\omega + b(\theta^H))Q(\theta)\,d\theta + \frac{1}{\beta}\frac{d\theta^M}{d\theta^H}\bigl(U(\omega + b(\theta^L)) - U(\omega + b(\theta^H))\bigr)Q(\theta^M). \tag{A.5}$$

By indifference of type θ^M,

$$\theta^M U(\omega + b(\theta^L)) + \beta\delta\bigl(V(b(\theta^L)) - P(\theta^L)\bigr) = \theta^M U(\omega + b(\theta^H)) + \beta\delta\bigl(V(b(\theta^H)) - P(\theta^H)\bigr).$$

Differentiating this indifference condition with respect to θ^H yields

$$\frac{d\theta^M}{d\theta^H} = \frac{db(\theta^H)}{d\theta^H} U'(\omega + b(\theta^H)) \frac{(\theta^H - \theta^M)}{U(\omega + b(\theta^H)) - U(\omega + b(\theta^L))},$$

where we have used the truthtelling constraint $\frac{db(\theta^H)}{d\theta^H}(\theta^H U'(\omega + b(\theta^H)) + \beta\delta V'(b(\theta^H))) - \beta\delta P'(\theta^H) = 0$. Substituting back into (A.5), the change in social welfare is

$$\frac{1}{\beta}\frac{db(\theta^H)}{d\theta^H} U'(\omega + b(\theta^H)) \int_{\theta^M}^{\theta^H} \bigl(Q(\theta) - Q(\theta^M)\bigr)\,d\theta.$$

Since $\frac{db(\theta^H)}{d\theta^H} > 0$, $U'(\omega + b(\theta^H)) > 0$, and $Q'(\theta) > 0$, this expression is strictly positive. Hence, we obtain that if $P(\theta)$ is strictly interior and $Q'(\theta) > 0$ over a given interval, then $V(b(\theta)) - P(\theta)$ must be a step function over the interval.

STEP 3: We show that $P(\theta) \in \{0, \overline{P}(b(\theta))\}$ for all $\theta \in \Theta$.

Suppose by contradiction that $P(\theta) \in (0, \overline{P}(b(\theta))$ for some $\theta \in \Theta$. By the previous steps and Lemma 2, type θ belongs to a stand-alone segment $(\theta^L, \theta^H]$, such that $b(\theta) = b$ and $P(\theta) = P$ for all $\theta \in (\theta^L, \theta^H]$, $b \in (\underline{b}, \overline{b})$ and $P \in (0, \overline{P}(b))$ (by assumption), $b(\theta)$ jumps at θ^L, and $b(\theta)$ jumps at θ^H unless $\theta^H = \overline{\theta}$.

We first show that the enforcement constraint must be slack for all $\theta \in (\theta^L, \theta^H)$. Express this constraint as the difference between the left-hand and right-hand sides of (4), so that it must be weakly positive and it equals zero if it binds. By the truthtelling constraints, the derivative of the enforcement constraint with respect to θ is $U(\omega + b(\theta)) - U(\omega + b^p(\theta))$. Since $b(\theta)$ is constant over $(\theta^L, \theta^H]$ and $b^p(\theta)$ is nondecreasing, it follows that the enforcement constraint is weakly concave over the interval. Then, if the constraint binds at any interior point $\theta' \in (\theta^L, \theta^H)$, it must bind at all $\theta \in (\theta^L, \theta^H)$. However, by the arguments used in Case 2 in Step 2 above, that would require $b = b^p(\theta)$ and $P = \overline{P}(b)$ for $\theta \in (\theta^L, \theta^H)$, contradicting the assumption that P is strictly interior.

We next show that there is an incentive feasible perturbation that strictly increases social welfare. We consider segment-shifting perturbations that marginally change the constant borrowing level b and penalty P. There are two cases:

Case 1: Suppose $\int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta < \int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta$. Consider a perturbation that marginally changes the borrowing level by $db > 0$ and changes P in order to keep type θ^H equally well off. For arbitrarily small $db > 0$, this perturbation makes the lowest types in $(\theta^L, \theta^H]$, arbitrarily close to θ^L, jump either to the allocation of type θ^L or to their flexible allocation under the maximum penalty $(b^p(\theta), \overline{P}(b^p(\theta)))$, where we let the perturbation introduce the latter. In the limit as db goes to zero, the change in social welfare is[27]

$$\frac{1}{\beta}\int_{\theta^L}^{\theta^H} U'(\omega+b)Q(\theta)\,d\theta + \frac{1}{\beta}\frac{d\theta^L}{db}\big(U\big(\omega+b\big(\theta^L\big)\big) - U(\omega+b)\big)Q\big(\theta^L\big). \tag{A.6}$$

The perturbation satisfies

$$db\big(\theta^H U'(\omega+b) + \beta\delta V'(b)\big) - \beta\delta\, dP = 0, \tag{A.7}$$

and the following indifference condition for type θ^L:

$$\theta^L U(\omega+b) + \beta\delta\big(V(b) - P\big) = \theta^L U\big(\omega+b\big(\theta^L\big)\big) + \beta\delta\big(V\big(b\big(\theta^L\big)\big) - P\big(\theta^L\big)\big).$$

To verify that the perturbation is incentive feasible for db arbitrarily close to zero, note that the enforcement constraint is slack for all $\theta \in (\theta^L, \theta^H)$, P is strictly interior, and the government welfare of types θ^L and θ^H remains unchanged with the perturbation.

To verify that the perturbation strictly increases social welfare, note that differentiating the indifference condition of type θ^L and substituting with (A.7) yields

$$\frac{d\theta^L}{db} = -U'(\omega+b)\frac{\big(\theta^H - \theta^L\big)}{U\big(\omega+b\big(\theta^L\big)\big) - U(\omega+b)}.$$

Substituting back into (A.6), the change in social welfare is

$$\frac{1}{\beta}U'(\omega+b)\int_{\theta^L}^{\theta^H}\big(Q(\theta) - Q\big(\theta^L\big)\big)\,d\theta.$$

Since $U'(\omega+b) > 0$ and by assumption $\int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta < \int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta$, the above expression is strictly positive. The claim follows.

Case 2: Suppose $\int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta \geq \int_{\theta^L}^{\theta^H} Q(\theta)d\theta$. By the generic property in Proposition 1, there exists $\theta^h \in (\theta^L, \theta^H]$ such that $\int_{\theta^L}^{\theta^h} Q(\theta^L)\,d\theta > \int_{\theta^L}^{\theta^h} Q(\theta)\,d\theta$. Then consider a perturbation where, for $\theta \in (\theta^L, \theta^h]$, we change the borrowing level by $-db < 0$ arbitrarily close to zero and change P in order to keep type θ^h equally well off. This perturbation makes types arbitrarily close to θ^L jump up to the allocation of the stand-alone segment. Arguments analogous to those in Case 1 above imply that the perturbation is incentive feasible. Moreover, following analogous steps as in that case yields that the implied change in social welfare is

$$-\frac{1}{\beta}U'(\omega+b)\int_{\theta^L}^{\theta^h}\big(Q(\theta) - Q\big(\theta^L\big)\big)\,d\theta.$$

Since $U'(\omega+b) > 0$ and by assumption $\int_{\theta^L}^{\theta^h} Q(\theta^L)\,d\theta > \int_{\theta^L}^{\theta^h} Q(\theta)\,d\theta$, the above expression is strictly positive. The claim follows.

[27]The arguments that follow are unchanged if $(b(\theta^L), P(\theta^L))$ is replaced with $(b^p(\theta^L), \overline{P}(b^p(\theta^L)))$ for the cases where the enforcement constraint binds.

A.2. *Proof of Proposition 2*

Take any solution to (6) with $b(\theta) \in (\underline{b}, \overline{b})$ for all $\theta \in \Theta$. We proceed in three steps.

STEP 1: We show that if $P(\theta^{**}) = \overline{P}(b(\theta^{**}))$, then $\theta^{**} \geq \widehat{\theta}$.

By Proposition 1 and Step 1 in the proof of that proposition, if $P(\theta^{**}) = \overline{P}(b(\theta^{**}))$ for some $\theta^{**} \in \Theta$, then $P(\theta) = \overline{P}(b(\theta))$ over an interval $(\theta^L, \theta^H]$ that contains θ^{**}. Take the largest such interval. We establish that $\theta^L \geq \widehat{\theta}$. Suppose by contradiction that $\theta^L < \widehat{\theta}$. Note that constraint (4) requires $b(\theta) = b^p(\theta)$ for all $\theta \in (\theta^L, \theta^H]$. There are two cases:

Case 1: Suppose $b^p(\theta)$ is strictly increasing over a subset of $(\theta^L, \theta^H]$ below $\widehat{\theta}$, and without loss take a subset over which $b^p(\theta)$ is continuously differentiable. Then we can perform a flattening perturbation that rotates the borrowing schedule clockwise over this subset, analogous to the perturbation used in Step 2 in the proof of Proposition 1. By the arguments in that step, this perturbation is incentive feasible. In particular, note that since the perturbation weakly increases the government welfare of all types θ in the subset while simultaneously changing their borrowing allocation, it follows from the definition of $b^p(\theta)$ that the perturbation must necessarily decrease $P(\theta)$ below $\overline{P}(b(\theta))$. Moreover, by $Q'(\theta) < 0$ for all types θ in the subset (by the subset being below $\widehat{\theta}$ and Assumption 1), the perturbation strictly increases social welfare, yielding a contradiction.

Case 2: Suppose $b^p(\theta)$ is constant for $\theta \in (\theta^L, \theta^M]$, where $\theta^M \equiv \min\{\theta^H, \widehat{\theta}\}$. Then we can perform an incentive feasible segment-shifting perturbation analogous to that described in Step 3 in the proof of Proposition 1: for $\theta \in (\theta^L, \theta^M]$, we marginally reduce the constant borrowing level and change the penalty so as to keep the government welfare of type θ^M unchanged. Since $Q'(\theta) < 0$ over $(\theta^L, \theta^M]$ implies $\int_{\theta^L}^{\theta^M} Q(\theta^L)\,d\theta > \int_{\theta^L}^{\theta^M} Q(\theta)\,d\theta$, this perturbation strictly increases social welfare, yielding a contradiction.

STEP 2: We show that if $P(\theta^{**}) = \overline{P}(b(\theta^{**}))$, then $P(\theta) = \overline{P}(b(\theta))$ for all $\theta \geq \theta^{**}$.

Suppose by contradiction that $P(\theta^{**}) = \overline{P}(b(\theta^{**}))$ for $\theta^{**} \in \Theta$ and $P(\theta) < \overline{P}(b(\theta))$ for some $\theta > \theta^{**}$. By Step 1, $\theta^{**} \geq \widehat{\theta}$. Moreover, by Proposition 1 and Step 1 in the proof of that proposition, there exist $\theta^H > \theta^L \geq \theta^{**}$ such that $P(\theta) = 0$ for all $\theta \in (\theta^L, \theta^H]$.

We begin by establishing that $b(\theta) = b$ for all $\theta \in (\theta^L, \theta^H]$ and some $b \in (\underline{b}, \overline{b})$. Suppose by contradiction that $b(\theta)$ is strictly increasing at some point $\theta' \in (\theta^L, \theta^H]$. Note that the truthtelling constraint (3) implies $b(\theta) = b^r(\theta)$, and thus a slack enforcement constraint, in the neighborhood of such a type θ'. Then we can perform an incentive feasible steepening perturbation that drills a hole in the $b(\theta)$ schedule in this neighborhood, as that described in Step 2 (Case 2) in the proof of Proposition 1. By the arguments in that step, this perturbation strictly increases social welfare, yielding a contradiction.

We next show that a segment $(\theta^L, \theta^H]$ with $b(\theta) = b$ and $P(\theta) = P = 0$ for all $\theta \in (\theta^L, \theta^H]$ and $\theta^L \geq \theta^{**}$ cannot exist. Suppose by contradiction that it does. Take θ^L to be the lowest point weakly above θ^{**} at which $P(\theta)$ jumps, and take θ^H to be the lowest point above θ^L at which $P(\theta)$ jumps again, or $\theta^H = \overline{\theta}$ if $P(\theta)$ does not jump above θ^L. Then $(\theta^L, \theta^H]$ is a stand-alone segment with constant borrowing b and no penalty. By arguments analogous to those in Step 3 of the proof of Proposition 1, the enforcement constraint must be slack for all $\theta \in (\theta^L, \theta^H)$. We then show that there is an incentive feasible segment-shifting perturbation that is socially beneficial. There are three cases to consider:

Case 1: Suppose $\theta^H U(\omega+b)+\beta\delta V(b)\leq\theta^H U(\omega+b')+\beta\delta V(b')$ for $b'=b+\varepsilon$, $\varepsilon>0$ arbitrarily small. Then we perform a segment-shifting perturbation as that in Step 3 in the proof of Proposition 1, where we marginally increase b and increase P marginally above 0 so as to keep type θ^H's welfare unchanged. This perturbation is incentive feasible. Moreover, since $\theta^L\geq\theta^{**}$ and Assumption 1 imply $\int_{\theta^L}^{\theta^H} Q(\theta^L)\,d\theta<\int_{\theta^L}^{\theta^H} Q(\theta)\,d\theta$, this perturbation strictly increases social welfare, yielding a contradiction.

Case 2: Suppose $\theta^H U(\omega+b)+\beta\delta V(b)>\theta^H U(\omega+b')+\beta\delta V(b')$ for $b'=b+\varepsilon$, $\varepsilon>0$ arbitrarily small, and $\theta^H<\overline{\theta}$. Then we perform a segment-shifting perturbation that marginally changes the borrowing level by $-db<0$ and increases P marginally above 0 so as to keep type θ^L's welfare unchanged. This perturbation is incentive feasible. Denote by $(b(\theta^h),P(\theta^h))$ the allocation above θ^H over which type θ^H is initially indifferent. Note that analogously to the perturbation in Step 3 in the proof of Proposition 1, this perturbation makes the highest types in $(\theta^L,\theta^H]$, arbitrarily close to θ^H, jump either to $(b(\theta^h),P(\theta^h))$ or to their flexible allocation under the maximum penalty $(b^p(\theta),\overline{P}(b^p(\theta)))$, where we let the perturbation introduce the latter. In the limit as db goes to zero, the change in social welfare is[28]

$$-\frac{1}{\beta}\int_{\theta^L}^{\theta^H} U'(\omega+b)Q(\theta)\,d\theta+\frac{1}{\beta}\frac{d\theta^H}{db}\big(U\big(\omega+b\big(\theta^h\big)\big)-U(\omega+b)\big)Q\big(\theta^H\big). \tag{A.8}$$

The perturbation satisfies

$$db\big(\theta^L U'(\omega+b)+\beta\delta V'(b)\big)-\beta\delta\,dP=0, \tag{A.9}$$

and the following indifference condition for type θ^H:

$$\theta^H U(\omega+b)+\beta\delta\big(V(b)-P\big)=\theta^H U\big(\omega+b\big(\theta^h\big)\big)+\beta\delta\big(V\big(b\big(\theta^h\big)\big)-P\big(\theta^h\big)\big).$$

Differentiating this indifference condition and substituting with (A.9) yields

$$\frac{d\theta^H}{db}=U'(\omega+b)\frac{\big(\theta^H-\theta^L\big)}{U\big(\omega+b\big(\theta^h\big)\big)-U(\omega+b)}.$$

Substituting back into (A.8), the change in social welfare is

$$-\frac{1}{\beta}U'(\omega+b)\int_{\theta^L}^{\theta^H}\big(Q(\theta)-Q\big(\theta^H\big)\big)\,d\theta.$$

Since $U'(\omega+b)>0$ and $Q'(\theta)>0$ over $[\theta^L,\theta^H]$ given $\theta^L\geq\theta^{**}$, this expression is strictly positive. Thus, the perturbation strictly increases social welfare, yielding a contradiction.

Case 3: Suppose $\theta^H U(\omega+b)+\beta\delta V(b)>\theta^H U(\omega+b')+\beta\delta V(b')$ for $b'=b+\varepsilon$, $\varepsilon>0$ arbitrarily small, and $\theta^H=\overline{\theta}$. Then we perform a segment-shifting perturbation as that in Case 2 above, where we marginally reduce b and increase P marginally above 0 so as to keep type θ^L's welfare unchanged. This perturbation is incentive feasible. Note that analogously to Case 2, this perturbation makes the highest types in $(\theta^L,\theta^H]$, arbitrarily close to θ^H, either jump to their flexible allocation under the maximum penalty

[28]The arguments that follow are unchanged if $(b(\theta^h),P(\theta^h))$ is replaced with $(b^p(\theta^H),\overline{P}(b^p(\theta^H)))$ for the cases where the enforcement constraint binds.

$(b^p(\theta), \overline{P}(b^p(\theta)))$ or remain with the perturbed allocation. In the former case, the same arguments as in Case 2 apply, yielding that the perturbation strictly increases social welfare by $-\frac{1}{\beta}U'(\omega+b)\int_{\theta^L}^{\theta^H}(Q(\theta)-Q(\theta^H))>0$. In the latter case, those arguments imply that the change in social welfare is equal to $-\frac{1}{\beta}U'(\omega+b)\int_{\theta^L}^{\theta^H}Q(\theta)\,d\theta$, which is strictly positive since $Q(\overline{\theta})\leq 0$ and $Q'(\theta)>0$ over $[\theta^L,\theta^H]$. Hence, the perturbation strictly increases social welfare, yielding a contradiction.

STEP 3: We show that $V(b(\theta))-P(\theta)$ is right-continuous at $\underline{\theta}$.

Suppose by contradiction that this is not the case. Then by the previous steps, Proposition 1, and Step 1 in the proof of Proposition 1, $P(\theta)=\overline{P}(b(\theta))$ for all $\theta\in(\underline{\theta},\overline{\theta}]$ and $P(\theta)$ jumps up at $\underline{\theta}$ from 0. Note that constraint (4) implies $b(\theta)=b^p(\theta)$ for all $\theta\in(\underline{\theta},\overline{\theta}]$, and indifference of $\underline{\theta}$ requires

$$\underline{\theta}U(\omega+b(\underline{\theta}))+\beta\delta V(b(\underline{\theta}))=\lim_{\theta\downarrow\underline{\theta}}\{\underline{\theta}U(\omega+b^p(\theta))+\beta\delta(V(b^p(\theta))-\overline{P}(b^p(\theta)))\}.$$

Take $\Delta\in(0,\min_{\theta\in\Theta}\{\overline{P}(b(\theta))\})$. Consider a global perturbation that assigns $P(\theta)=\overline{P}(b(\theta))-\Delta$ to all $\theta\in(\underline{\theta},\overline{\theta}]$ and assigns type $\underline{\theta}$ the limit allocation to its right. This perturbation keeps borrowing unchanged for types $\theta\in(\underline{\theta},\overline{\theta}]$ and is incentive feasible. Moreover, using the representation in (10), the change in social welfare from this perturbation is equal to $\delta\Delta$. Thus, the perturbation strictly increases social welfare, yielding a contradiction.

A.3. *Proof of Proposition 3*

Take any solution to (6) with $b(\theta)\in(\underline{b},\overline{b})$ for all $\theta\in\Theta$. By Proposition 2 and the enforcement constraint (4), there exists $\theta^{**}>\underline{\theta}$ such that $(b(\theta),P(\theta))=(b^p(\theta),\overline{P}(b^p(\theta)))$ for all $\theta>\theta^{**}$ and $P(\theta)=0$ for all $\theta\leq\theta^{**}$ (where it is possible that $\theta^{**}>\overline{\theta}$). Moreover, since the enforcement constraint holds with equality at θ^{**}, this type's allocation satisfies

$$\begin{aligned}&\theta^{**}U(\omega+b(\theta^{**}))+\beta\delta V(b(\theta^{**}))\\&\quad=\theta^{**}U(\omega+b^p(\theta^{**}))+\beta\delta(V(b^p(\theta^{**}))-\overline{P}(b^p(\theta^{**}))).\end{aligned}\tag{A.10}$$

These results characterize the allocation for types $\theta\geq\theta^{**}$. To characterize the allocation for types $\theta<\theta^{**}$, we proceed in three steps.

STEP 1: We show that $b(\theta)$ is continuous over $[\underline{\theta},\theta^{**}]$.

By Step 1 in the proof of Proposition 2, $\theta^{**}\geq\widehat{\theta}$. There are two cases to consider:

Case 1: Suppose by contradiction that $b(\theta)$ has a point of discontinuity below $\widehat{\theta}$: there is a type $\theta^M<\widehat{\theta}$ which is indifferent between choosing $\lim_{\theta\uparrow\theta^M}b(\theta)$ and $\lim_{\theta\downarrow\theta^M}b(\theta)>\lim_{\theta\uparrow\theta^M}b(\theta)$. Note that given $P(\theta)=0$ for all $\theta\in[\underline{\theta},\theta^{**}]$ and $\theta^{**}\geq\widehat{\theta}$, there must be a hole with types $\theta\in[\theta^L,\theta^M)$ bunched at $b^r(\theta^L)$ and types $\theta\in(\theta^M,\theta^H]$ bunched at $b^r(\theta^H)$, for some $\theta^L<\theta^M<\theta^H$. Now consider perturbing the rule by marginally increasing θ^L, in an effort to slightly close the hole. This perturbation leaves the government welfare of types strictly above θ^M unchanged and is incentive feasible. The change in social welfare is[29]

$$\frac{1}{\beta}\int_{\theta^L}^{\theta^M}\frac{db^r(\theta^L)}{d\theta^L}U'(\omega+b^r(\theta^L))Q(\theta)\,d\theta$$
$$+\frac{1}{\beta}\frac{d\theta^M}{d\theta^L}\big(U(\omega+b^r(\theta^L))-U(\omega+b^r(\theta^H))\big)Q(\theta^M). \tag{A.11}$$

By indifference of type θ^M,

$$\theta^M U(\omega+b^r(\theta^L))+\beta\delta V(b^r(\theta^L))=\theta^M U(\omega+b^r(\theta^H))+\beta\delta V(b^r(\theta^H)).$$

Differentiating this indifference condition with respect to θ^L yields

$$\frac{d\theta^M}{d\theta^L}=\frac{db^r(\theta^L)}{d\theta^L}U'(\omega+b^r(\theta^L))\frac{(\theta^M-\theta^L)}{U(\omega+b^r(\theta^H))-U(\omega+b^r(\theta^L))},$$

where we have used the fact that $\theta^L U'(\omega+b^r(\theta^L))=-\beta\delta V'(b^r(\theta^L))$. Substituting back into (A.11), the change in social welfare is

$$\frac{1}{\beta}\frac{db^r(\theta^L)}{d\theta^L}U'(\omega+b^r(\theta^L))\int_{\theta^L}^{\theta^M}(Q(\theta)-Q(\theta^M))\,d\theta.$$

Since $\frac{db^r(\theta^L)}{d\theta^L}>0$, $U'(\omega+b^r(\theta^L))>0$, and $Q'(\theta)<0$ over $\theta\in[\theta^L,\theta^M]$ given $\theta^M<\widehat{\theta}$, this expression is strictly positive. Thus, the perturbation strictly increases social welfare, showing that $b(\theta)$ cannot jump at a point below $\widehat{\theta}$.

Case 2: Suppose by contradiction that $b(\theta)$ is discontinuous at a point $\theta\in[\widehat{\theta},\theta^{**}]$. Note that since $P(\theta)=0$ for all $\theta\in[\widehat{\theta},\theta^{**}]$, we can apply the same logic as in Step 2 in the proof of Proposition 2 to show that $\frac{db(\theta)}{d\theta}=0$ over any continuous interval in $[\widehat{\theta},\theta^{**}]$. Hence, if $b(\theta)$ jumps at a point $\theta\in[\widehat{\theta},\theta^{**}]$, then there exists a stand-alone segment $(\theta^L,\theta^H]$ with constant borrowing $b\in(\underline{b},\overline{b})$ and penalty $P=0$, satisfying $\theta^L\geq\widehat{\theta}$. However, using again the arguments in Step 2 in the proof of Proposition 2, we can then perform an incentive feasible segment-shifting perturbation that strictly increases social welfare. Thus, $b(\theta)$ cannot jump at a point $\theta\in[\widehat{\theta},\theta^{**}]$.

STEP 2: We show that $b(\theta)\leq b^r(\theta)$ for all $\theta\in[\underline{\theta},\theta^{**}]$.

By Step 1 above, the allocation over $[\underline{\theta},\theta^{**}]$ must be bounded discretion, with either a minimum borrowing level or a maximum borrowing level or both. We next show that a binding minimum borrowing requirement is strictly suboptimal. Suppose by contradiction that this is not the case, namely, there exist $\theta^\star>\underline{\theta}$ and an optimal rule prescribing $(b(\theta),P(\theta))=(b^r(\theta^\star),0)$ for all $\theta\in[\underline{\theta},\theta^\star]$, where $b^r(\theta)<b^r(\theta^\star)$ for all $\theta\in[\underline{\theta},\theta^\star)$. Consider a perturbation where we remove this minimum borrowing requirement, that is, we set $(b(\theta),P(\theta))=(b^r(\theta),0)$ for all $\theta\in[\underline{\theta},\theta^\star)$. Clearly, this perturbation is incentive feasible, and it keeps the allocation of types $\theta\in[\theta^\star,\overline{\theta}]$, and thus the social welfare from these

[29]Note that this welfare representation is valid even if $\theta^L<\underline{\theta}$, as we can apply the envelope condition in (9) from any positive $\theta'<\underline{\theta}$. For $\theta<\underline{\theta}$, we have $Q(\theta)=1$.

types, unchanged. The change in social welfare from each type $\theta \in [\underline{\theta}, \theta^{\star})$ is

$$\theta U(\omega + b^r(\theta)) + \delta V(b^r(\theta)) - \theta U(\omega + b^r(\theta^{\star})) - \delta V(b^r(\theta^{\star})).$$

Note that by the definition of $b^r(\theta)$,

$$\delta V(b^r(\theta)) - \delta V(b^r(\theta^{\star})) \geq \frac{1}{\beta}\big(\theta U(\omega + b^r(\theta^{\star})) - \theta U(\omega + b^r(\theta))\big).$$

Substituting back into the previous expression, we obtain that the change in social welfare from each $\theta \in [\underline{\theta}, \theta^{\star})$ is greater than

$$\left(\frac{1}{\beta} - 1\right)\big(\theta U(\omega + b^r(\theta^{\star})) - \theta U(\omega + b^r(\theta))\big),$$

which is strictly positive. Thus, the perturbation strictly increases social welfare, implying that a binding minimum borrowing requirement is strictly suboptimal.

STEP 3: We show that $b(\theta) < b^r(\theta)$ for some $\theta \in \Theta$.

By Step 1 and Step 2, the allocation for types $\theta \in [\underline{\theta}, \theta^{**}]$ is as described in Definition 1 for some $\theta^* \geq 0$. That is, equation (A.10) necessarily holds for $b(\theta^{**}) = b^r(\theta^*)$. All that remains to be shown is that $\theta^* < \overline{\theta}$. Suppose by contradiction that this is not true, which implies $(b(\theta), P(\theta)) = (b^r(\theta), 0)$ for all $\theta \in \Theta$. Consider an incentive feasible perturbation that assigns $(b(\theta), P(\theta)) = (b^r(\overline{\theta} - \varepsilon), 0)$ to all $\theta \in [\overline{\theta} - \varepsilon, \overline{\theta}]$, where $\varepsilon > 0$ is chosen to be small enough as to continue to satisfy the enforcement constraint (4) for all types $\theta \in \Theta$. Using the representation in (10), the change in social welfare is

$$\frac{1}{\beta}\int_{\overline{\theta}-\varepsilon}^{\overline{\theta}} \big(U(\omega + b^r(\overline{\theta} - \varepsilon)) - U(\omega + b^r(\theta))\big) Q(\theta)\, d\theta.$$

For $\varepsilon > 0$ arbitrarily small, $b^r(\overline{\theta} - \varepsilon) < b^r(\theta)$ and $Q(\theta) < 0$ for all $\theta \in (\overline{\theta} - \varepsilon, \overline{\theta})$. Thus, the perturbation strictly increases social welfare, proving the claim.

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