Facilitating the Search for Partners on Matching Platforms[*]

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Two-sided matching platforms can control and optimize over many aspects of the search for partners. To understand how matching platforms should be designed, we introduce a dynamic two-sided search model with strategic agents who must bear a cost to discover their value for each potential partner, and can do so non-simultaneously. We characterize evolutionarily stable stationary equilibria and find that, in many settings, the platform can mitigate wasted search effort by imposing suitable restrictions on agents. In unbalanced markets, the platform should force the short side of the market to initiate contact with potential partners, by disallowing the long side from doing so. This allows the agents on the long side to exercise more choice in equilibrium. When agents are vertically differentiated, the platform can significantly improve welfare even in the limit of vanishing screening costs by forcing the shorter side of the market to propose and by hiding information about the quality of potential partners. Furthermore, a Pareto improvement in welfare is possible in this limit.

Keywords: matching markets, market design, search frictions, sharing economy, platforms.

1 Introduction

During the past decade, the use of online matching platforms for dating, labor markets, and more, has grown rapidly. In fact, over 50 million Americans have used online dating services (which allow individuals to match with prospective romantic partners). Similarly, online labor markets (where businesses and individuals can hire freelancers) have experienced a collective annual growth of over 20% in the past ten years. A common characteristic of these platforms is that agents on *both* sides of the market (e.g., men and women in (heterosexual) dating markets, workers and employers in labor markets) have *heterogeneous* preferences over potential partners, with a significant *idiosyncratic* component ("beauty lies in the eye of the beholder") that the platform cannot easily uncover.

This feature plays an important role in the platform's choice of "search environment" design, that is, the framework that allows agents to match with each other. Most platforms for dating

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and online labor currently use a decentralized search environment, where agents search for a match using the information about individual potential partners provided by the platform, and both parties must agree to the match. Compared to a setting where the platform centrally decides on the matches, a decentralized environment allows agents to obtain matches that better take into account their idiosyncratic preferences, which are not fully known to the platform. However, decentralized markets suffer from search frictions, as agents might waste effort screening potential partners who do not reciprocate their interest. As the effort needed in the online world to acquire more information about each potential partner is typically small, one might be tempted to believe that the impact of search frictions in these markets is negligible. However, empirical evidence suggests that search frictions significantly impact agent welfare, a phenomenon that has been documented in the context of Airbnb (Fradkin 2018), Upwork (Horton 2019), and TaskRabbit (Cullen and Farronato 2014).

Even though many platforms have adopted a decentralized search environment, they still differ in their choices on how agents meet each other and gather information to decide whether to pursue a match, with differences arising even between platforms operating in the same space. To illustrate, in most heterosexual dating platforms (including Tinder and OkCupid), both men and women can initiate conversations. By contrast, Bumble, a dating platform launched in 2015, requires women to initiate conversations, and prevents men from doing so. Similarly, while potential clients reach out to freelancers in many labor makets (including Fiverr and Handy), TaskRabbit opted, until 2014, for a design in which freelancers bid on the jobs posted by clients. Moreover, platforms also differ in the information about a potential partner that can be acquired at first sight, and the information that can be learned only after entering a specific profile. For instance, Upwork makes freelancers' job success rate prominently visible, whereas this information is less prominent on TaskRabbit. Dating platforms internally calculate a quality score for each user profile (e.g., Tinder's Elo score; see Cook 2017), but do not reveal this score to users. One might ask whether the design choices on how agents meet and what information is displayed have any impact on the market outcome. Indeed, the empirical evidence highlights the importance of these choices by the platform: Cullen and Farronato (2014) find a significant reduction in search costs and an increase in the number of matches formed as a consequence of the redesign of TaskRabbit in 2014, a key component of which was the change from having freelancers bid on jobs to having clients reach out to freelancers.

Therefore, the objective of this paper is to provide a theoretical framework to study the impact that different search design choices have on the equilibria that arise, and hence guide, market design. Our contribution is threefold: we provide a novel dynamic model of a two-sided search-and-matching platform with strategic agents, we characterize equilibrium structure and the main equilibrium inefficiencies arising in these markets, and we show how simple interventions can alleviate these inefficiencies. We next discuss each of these contributions in turn.

Our analytical framework. We introduce a dynamic fluid model of two-sided matching without transfers, mediated by a platform (Section 2). Our model, while stylized, captures distinctive features of these markets: strategic agents on both sides of the market have (partly) idiosyncratic preferences for partners that can be discovered only by the agents themselves after exerting screening effort, and agents on both sides of the market can *choose* whether to issue proposals, whether to consider incoming proposals, and also whether to screen before issuing or accepting proposals. If a match occurs, both agents leave the market. Agents unable to match for an extended period leave the market (exogenously). To focus on search frictions arising from heterogeneous private match values and costly screening, we consider markets without prices, such as dating markets. We briefly discuss how some of our insights may extend to markets with prices in Section 5.

We study the resulting evolutionarily stable stationary equilibria, where agents' best responses are utility-maximizing solutions to the optimal stopping problem they face in the steady state of the market. We characterize the equilibria arising in the *no platform intervention* setting, in which both sides are allowed to propose, to decide whether to uncover pair-specific match values via screening, and to learn the quality of potential partners at no cost. We next characterize the equilibria that arise under *simple platform interventions*. Motivated by the examples discussed earlier, the platform is allowed: (1) to block either side from proposing (as Bumble does) and, (2) when vertical differentiation is present, to choose whether to hide the information about quality (as dating platforms do).

While the present paper makes specific modeling choices and focuses only on two platform interventions, our broader goal is to provide a flexible framework to study the effect of design choices on the equilibrium structure in search-and-matching platforms, where contact may be initiated by either side of the market and idiosyncratic match values can only be discovered by screening.

Main insights. We study two settings: one with no vertical differentation and then one with vertical differentiation. In each setting, we start by characterizing the equilibria that arise under no platform intervention. Taking average social welfare as the platform's objective, we characterize the social planner's constrained-efficient solution and compare it with the equilibrium outcome in order to reveal the inefficiencies that arise in two-sided markets with search. We then study when these inefficiencies can be mitigated through simple design choices. Average welfare is a relevant objective for many online platforms, as user retention and acquisition crucially depend on it. However, our equilibrium analysis is independent of the choice of the platform's objective; thus, our framework and analysis would easily allow us to derive insights for other performance metrics.

Markets with no vertical differentiation are studied in Section 3. We find that both in the no-intervention equilibria and in the planner's solution, three regimes arise: when screening costs are small relative to idiosyncratic variation in match values, one side screens and proposes, and the other side screens and accepts/rejects incoming proposals; with medium-sized screening costs, one side proposes without screening and the other side screens and accepts/rejects; finally, with large screening costs, both sides propose and accept proposals without screening. This finding is interesting on its own: although agents on both sides have the ability to propose, in equilibrium agents on only one side of the market proactively reach out (except for large screening costs). Intuitively, an agent receiving proposals faces no risk of rejection (if he accepts the proposal, a match occurs), and hence he does not want to proactively reach out and risk being rejected. Moreover, equilibrium welfare is lower than the planner's solution for small screening costs, because *recipients of proposals are too selective*: their acceptance threshold is higher than the socially optimal one as they do not internalize the adverse effect that their selectivity (high threshold) has on proposers.

This phenomenon is even more striking in unbalanced markets: when there is a moderate difference in the arrival rates of agents on both sides of the market, the only equilibria that arise are the ones where the agents on the faster-arriving side (long side) are proactive so as to increase their chances of matching. As the long side always proposes in equilibrium and, due to a high risk of their proposals being rejected by the short side, *long-side agents can screen only for very small screening costs and, even then, they use a low threshold.* This inefficiency can be mitigated by preventing the long side from proposing, thereby creating new equilibria in which the short side proposes. Agents on the long side no longer face rejection and so they can be somewhat selective and, moreover, their increased selectivity has a *positive same-side effect* because it increases the effective availability of options to other long-side agents. Thus, this intervention provides a significant welfare boost to the long side at a small cost to the short side, which has its proposals infrequently rejected.

We then study markets with vertical differentiation in Section 4, by allowing the long side of the market (call them workers) to have two quality levels, high and low, where high workers are fewer than (ex-ante identical) employers, and employers are fewer than workers overall. The platform knows the quality of each worker and, under no intervention, makes this information visible to the employers. In the no-intervention equilibrium, employers endogenously split into two groups, "reachers", who wait for a chance to propose to a high worker while ignoring low workers, and "settlers", who are willing to consider incoming proposals from low workers. Reachers are overrepresented in steady state as they hold out for a dream match. This causes two inefficiencies, which persist even as screening costs vanish: a significant fraction of reacher employers *leave unmatched*, reducing the number of employers available to match with low workers and, further, low workers propose *without screening* as most of their proposals are *ignored* by reacher employers. Our dynamic model, with populations of different agents whose presence in the market is *endogenously* determined, enables us to identify the latter phenomenon. The platform can significantly increase welfare by blocking the workers (the long side) from proposing (thus allowing low workers to be somewhat selective), and by *hiding the qualities of workers* from employers, to prevent them from wastefully leaving while waiting to match with a high worker. Importantly, in the limit of vanishing screening costs, a significant increase in the welfare of the low workers is possible without hurting any other type of agent; i.e., a Pareto improvement of welfare occurs in this limit.

1.1 Related literature

We study a two-sided matching market with two-sided heterogeneity in preferences, two-sided search, and costly information acquisition. Our sequential search model, while inspired by the search-theoretic models extensively studied in the context of labor markets (see the excellent survey of Rogerson et al. 2005), differs from these in several important aspects. First, in the literature on search frictions, the protocol by which agents meet is given: agents either meet *randomly*,[1] or one side of the market posts information[2] and the other side, observing this information, chooses which agents to target. In our model, meetings occur as a consequence of a sequence of (equilibrium) choices by agents: an agent chooses whether to request a candidate and, if a proposal is issued, the candidate chooses whether to meet the proposer. Second, most search-theoretic models assume that when a meeting occurs, learning the information about the potential partner is costless. By contrast, we model a setting where acquiring information about a potential partner is costly, and agents can decide whether to incur the cost to learn the information or not. Thus, our action space is significantly larger than that of traditional search models (where agents choose a threshold and possibly a market to target), as agents must also choose a set of actions (whether to propose or not and to whom, whether to accept incoming proposals or not, when to screen and with what threshold), making the analysis more involved. The fact that information acquisition is costly and that recipients act only *after* they receive a proposal are two novel features of our model that play a fundamental role in our results. Third, while other search models endogenize entry while

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[1] Often as indicated by an aggregate matching function introduced in the much-celebrated Diamond–Mortensen–Pissarides framework (Diamond 1981, 1982a,b, Mortensen 1982a,b, Pissarides 1984, 1985).

[2] Typically firms post wages, in what is known as the directed search paradigm (Moen 1997).

simplifying other aspects, we instead assume that the arrival rates of populations are given and we are interested in studying how these imbalances in populations affect agents' choices.

The directed search literature assumes that workers reach out to firms and has, for the most part, omitted market design questions. An exception is Herreiner (1999), who considers an urn-ball matching model to study how the number of matches depends on the side of the market making proposals (similar to our "block one side from proposing" intervention), and finds that the long side should propose as this causes fewer lost matches due to coordination failures (multiple agents propose to the same potential partner, while other agents get no proposals). Shi and Delacroix (2018) extend the finding of Herreiner (1999) to a setting with costs to participate and organize trade. Frictions are due to coordination failures specified by an exogenous matching function. By contrast, our model is designed such that there are no coordination failures, as we assume instantaneous screening, proposals, and acceptances. Instead, frictions arise due to costly information acquisition per potential partner, and agents have a rich action space that gives rise endogenously to equilibrium match rates, which we believe better captures the reality of many online matching platforms. These modeling differences explain the opposite conclusions reached.

The stable marriage literature (e.g., Gale and Shapley 1962, Roth 1982) has focused on two-sided heterogeneity in preferences and market design issues but has traditionally suppressed search-related concerns. Recently, there has been a growing interest in models combining matching markets and search theory (see the survey of Chade et al. 2017). These models mostly assume a common evaluation of agents, and study whether sorting can be recovered in the presence of search frictions. Our stylized model combines a common evaluation of agents and a "beauty lies in the eye of the beholder" component that is costly to learn, and it is tailored to explore platform design questions.

Recent papers explore how a platform should make matching decisions in dynamic settings (e.g., Gurvich and Ward 2014, Banerjee et al. 2015, Hu and Zhou 2018, Ashlagi et al. 2019, Banerjee et al. 2016); however, these papers assume that matching is centralized and thus they suppress the role of search frictions. Recent works also identify platform designs to improve efficiency in markets where frictions are caused by lack of coordination, such as limiting the number of applications an agent can submit (Arnosti et al. 2015), restricting choice (Halaburda et al. 2017), preference signaling (e.g., Coles et al. 2013, 2010, Lee and Niederle 2015), and grouping similar sellers and enabling communication (Allon et al. 2012), among others.

One of our proposed interventions involves hiding information. Using the Bayesian persuasion framework (Rayo and Segal 2010, Kamenica and Gentzkow 2011), recent papers in operations showcase the benefits of carefully designing an information disclosure policy to induce exploration in an online recommendation system (Papanastasiou et al. 2018), to increase throughput (Lingenbrink and Iyer 2017), to increase revenue in e-tail (Drakopoulos et al. 2019, Lingenbrink and Iyer 2018), among others. Hiding information is a simple information disclosure policy, which has been shown to mitigate inefficiencies by preventing the market from unraveling (Ostrovsky and Schwarz 2010) and reducing cherry-picking (Levin and Milgrom 2010). A concurrent paper (Romanyuk and Smolin 2019) explores the benefit of hiding information, albeit in a one-sided buyer-seller setting. Our findings also highlight the importance of the information design: we find that the platform can induce agents to consider a larger set of potential partners by hiding information about quality.

2 Model

This section introduces a model of a dynamic two-sided matching market without transfers, mediated by a platform. We refer to the two sides of the market using the gender-neutral terminology "workers" and "employers" (though we draw parallels with online dating and other types of matching platforms throughout the paper). A probability space (which encompasses all relevant randomness and information) and a nonatomic measure space of agents are fixed. We rely throughout on applications of the exact law of large numbers (LLN) for a continuum of random variables, a precise version of which can be found in Sun (2006).[3] As in Duffie et al. (2009) and in the related search literature, we also rely formally on a continuous-time LLN for random search and matching that has only been rigorously established in discrete-time settings (Duffie et al. 2018).[4]

Agent types and arrivals. All employers are of the same quality, whereas workers can be divided into at most two quality tiers, High (H) and Low (L). On each side and tier, there is a continuum of agents (workers or employers); workers in tier t arrive at rate λ_w^t, and employers at rate λ_e. (Whenever there is only one quality tier of workers, references to tiers are dropped to ease notation; the roles of workers and employers are interchangeable in our model in this case.) When present in the market, agents will get *opportunities* to (propose to and) match with specific agents on the other side according to endogenous dynamics that we describe below. When a match forms, the concerned agents leave the market immediately. Agents in the platform depart unmatched with an exogenous (small) hazard rate $\mu > 0$, which is common across all agents.

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[3] In more detail, let $(\Omega, \mathcal{F}, P)$ and $(I, \mathcal{I}, \alpha)$ denote the probability space and the nonatomic measure space of agents, respectively. Sun (2006) establishes technical conditions on the measurable subsets $\Omega \times I$ of such that the LLN holds. Informally, Sun (2006) defines a "Fubini extension" product measure space which is, roughly, a rich enough product space of $(\Omega, \mathcal{F}, P)$ and $(I, \mathcal{I}, \alpha)$ that the Fubini property holds.

[4] An alternative, which we avoid for simplicity, would be to describe limiting results for a sequence of models with finitely many agents as the number of agents gets large.

Match utilities and screening. As described in the Introduction, there are two novel features in our model. The first is that agents have idiosyncratic values for matching with each potential partner that the platform cannot uncover, but that the agents can discover after incurring a screening cost. In more detail, each agent i has a match value v_{ij} for every agent j on the other side, which is the sum of two terms $v_{ij} \triangleq q_j + u_{ij}$. The term q_j represents the *quality* of agent j, which is determined by the side of the market and the tier of the agent. For simplicity, we assume that all employers and L-workers (or all workers, in case of one tier of workers) have quality 0. In case of two worker tiers, all H-workers are assumed to have quality $q > 0$.

The quality term is known to the platform and to agent j himself and, unless otherwise stated, is also known to agent i at no cost. On the other hand, the term u_{ij} is an (i, j)-specific *idiosyncratic utility*. We assume that the u_{ij}'s are independent and identically distributed (i.i.d.) Uniform$(0, 1)$ across every ordered pair of worker and employer for simplicity. The fact that the u_{ij}'s are i.i.d. Uniform$(0, 1)$ and the quality of each tier are assumed to be common knowledge among agents and the platform. We highlight that the value u_{ij} is different from u_{ji}, the latter being the idiosyncratic value that j would derive if matched with i; for simplicity, we model u_{ij} as being independent of u_{ji}. Unlike qualities, u_{ij}'s are unknown a priori to both agents and the platform. An agent i can *privately* learn her u_{ij} for agent j by incurring a *screening cost* $c > 0$.

Match opportunities and proposals. The second novel feature of our model, with the goal of capturing the dynamics of online platforms, is that agents on both sides of the market can *choose* whether to issue proposals, whether to consider incoming proposals, and also whether to screen before issuing or accepting proposals. We assume that each agent is endowed with an independent Poisson "opportunity" clock of rate 1. When worker i's opportunity clock rings, he can costlessly request to view a *candidate*, in which case the platform shows i a uniformly random employer, if any is available. Upon receiving a candidate (call her j), worker i can either incur the screening cost to learn his idiosyncratic utility u_{ij} for employer j or not screen. Either way, i then decides whether to propose to j. If i proposes, his proposal is conveyed to j. When an employer receives a proposal, she learns the tier of the proposer at no cost (unless otherwise stated). Knowing i's tier, agent j decides whether to incur the screening cost to learn u_{ji}, and then whether to accept i's proposal. If j accepts, a match occurs and the pair leaves the market; otherwise, both agents stay. All these events occur instantaneously. The events that occur when an employer's opportunity clock rings are analogous. The only difference is that, when there are two tiers of workers, employer j can costlessly request to view a candidate of a given tier. The platform will show j a uniformly random candidate of the preferred tier, if available; if none is available, j can request a candidate from the complementary tier. We summarize the sequence of events and decisions after an agent's opportunity clock rings in Figure 1.

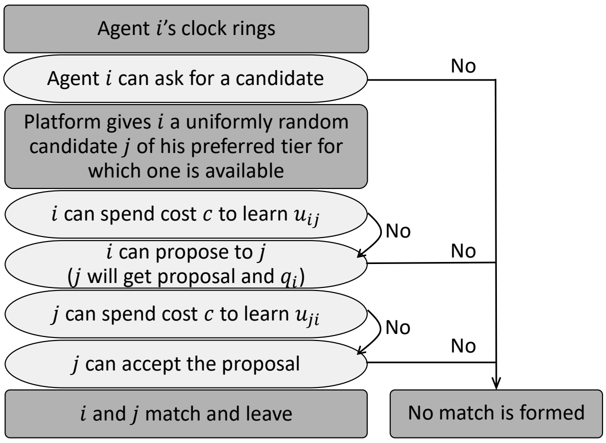

Figure 1: Schematic showing the (instantaneous) sequence of events and decisions after an agent i's opportunity clock rings.

Strategy space. We now define the agents' strategy space. Agents select a deterministic strategy upon arrival, which is assumed to be fixed throughout their lifetime.

Definition 1 (Strategies with one quality tier on the other side)**.** *When there is one quality tier on the other side, we consider agents' strategies* $s = (a, \underline{\theta})$ *defined by*

1. A deterministic selection of actions $a = (a^{\mathrm{o}}, a^{\mathrm{i}})$, *where*

- *(i) The selection* $a^{\mathrm{o}} \in \mathcal{A}^{\mathrm{o}} \triangleq \{\mathrm{DN}, \mathrm{P\ w/o\ S}, \mathrm{S{+}P}\}$ *specifies how opportunities are handled: whether the agent foregoes the opportunity, i.e., does nothing* (DN)*, proposes without screening* (P w/o S)*, or screens and then decides whether to propose or not* (S+P)*.*
- *(ii) The selection* a^{i} *where* $a^{\mathrm{i}} \in \mathcal{A}^{\mathrm{i}} \triangleq \{\mathrm{I}, \mathrm{A}, \mathrm{S{+}A/R}\}$ *specifies how incoming proposals are handled: whether the agent ignores them* (I)*, accepts without screening* (A)*, or screens and then chooses to accept/reject* (S+A/R)*.*

2. Deterministic acceptability thresholds $\underline{\theta} = (\theta^{\mathrm{o}}, \theta^{\mathrm{i}}) \in \mathbb{R}^2$ *used after screening: if the agent screens a candidate, he proposes if and only if the match value he learns from screening exceeds* θ^{o}*; if the agent screens an incoming proposal, he accepts it if and only if the match value he learns from screening exceeds* θ^{i}*.*

Definition 2 (Employers' strategies with two tiers of workers)**.** *When there are two tiers of workers, the employers' strategy space consists of the following expanded deterministic selection of actions:*

(i) The selection $a^{\mathrm{o}} = (\succ^{\mathrm{o}}, a^{\mathrm{o}}_{\mathrm{H}}, a^{\mathrm{o}}_{\mathrm{L}}) \in \{\mathrm{H} \succ \mathrm{L}, \mathrm{L} \succ \mathrm{H}, \mathrm{H}, \mathrm{L}, \phi\} \times \mathcal{A}^{\mathrm{o}} \times \mathcal{A}^{\mathrm{o}}$ *specifies how opportunities are handled: the employer has a preferred order* $\succ^{\mathrm{o}}$ *over worker tiers (unacceptable tiers are excluded from* $\succ^{\mathrm{o}}$*) and receives a candidate from the topmost-ranked tier for which a worker is available; actions* $a^{\mathrm{o}}_{\mathrm{H}}, a^{\mathrm{o}}_{\mathrm{L}} \in \mathcal{A}^{\mathrm{o}}$ *indicate how candidates from each tier are evaluated.*[5]

(ii) The selection $a^{\mathrm{i}} = (a^{\mathrm{i}}_{\mathrm{H}}, a^{\mathrm{i}}_{\mathrm{L}}) \in \mathcal{A}^{\mathrm{i}} \times \mathcal{A}^{\mathrm{i}}$ *specifies how incoming proposals from each tier of workers are handled.*

and a deterministic selection of thresholds $\underline{\theta} = (\theta^{\mathrm{H,o}}, \theta^{\mathrm{L,o}}, \theta^{\mathrm{H,i}}, \theta^{\mathrm{L,i}})$ *to screen opportunities/incoming proposals from each tier, respectively.*

Thus, besides the choice of acceptability thresholds, a strategy involves a choice of actions a from a finite set of options. An intervention in our setting is a restriction of the allowed a's on one or both sides of the market, as formally defined later in this section. We denote the finite sets of *allowed actions* for workers and employers (under the chosen platform intervention, if any) by $\mathcal{A}_w$ and $\mathcal{A}_e$, respectively. For workers, $\mathcal{A}_w \subseteq \mathcal{A}^{\mathrm{o}} \times \mathcal{A}^{\mathrm{i}}$ (e.g., if workers are not allowed to propose then $\mathcal{A}_w = \{\mathrm{DN}\} \times \mathcal{A}^{\mathrm{i}}$), and similarly for employers. We denote the set of *allowed strategies* (that is, those strategies whose actions are in the allowed set) by $\mathcal{S}_w$ and $\mathcal{S}_e$, respectively.

System state. Fix a time τ. Let the current *agent mix* $\bar{N} = (\bar{N}_e, \bar{N}^{\mathrm{H}}_w, \bar{N}^{\mathrm{L}}_w)$ be the measure over agent strategies and tiers present in the system.[6] Here, $\bar{N}_e$ is the measure over employer strategies; i.e., $\bar{N}_e(S)$ is the mass of employers following strategies in $S \subseteq \mathcal{S}_e$, and so on. Suppose that upon entering the system, each employer picks a strategy drawn independently from a probability measure f_e over $\mathcal{S}_e$. We define f^{H}_w and f^{L}_w analogously. Let $\bar{f} = (f_e, f^{\mathrm{H}}_w, f^{\mathrm{L}}_w)$ be the current *strategy distributions for new arrivals.* (We later endogenize $\bar{f}$ by allowing agents to pick utility-maximizing strategies.) Finally, $\bar{N}$ and $\bar{f}$ together constitute the system state.

The reader can preview the system dynamics that arise given the system state in Figure 2. Due to the exact LLN, system evolution is deterministic. Under our equilibrium notion formalized in Section 2.1 below, the system will further be in steady state.

Consistency requirements. Because we have a continuum model, we need to add two formal requirements to ensure that our model correctly captures system search and matching dynamics, given the system state. The first requirement, which we call *mass balancing*, ensures that if a certain mass of workers get matched, the total mass of their (distinct) matched partners is also the

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[5] For example, if $a^{\mathrm{o}} = (\mathrm{H} \succ \mathrm{L}\,, a^{\mathrm{o}}_{\mathrm{H}} = \mathrm{P\ w/o\ S}\,, a^{\mathrm{o}}_{\mathrm{L}} = \mathrm{S{+}P})$, the employer prefers to propose without screening to a H-worker; if none is available, he requests a L-worker, and screens and proposes if such a worker is available. One may think of the preference order over tiers $\succ^{\mathrm{o}}$ as capturing how an employer directs his search to a preferred tier.

[6] For the sake of readability, we suppress the dependence of $\bar{N}$ and $\bar{f}$ (defined later) on τ.

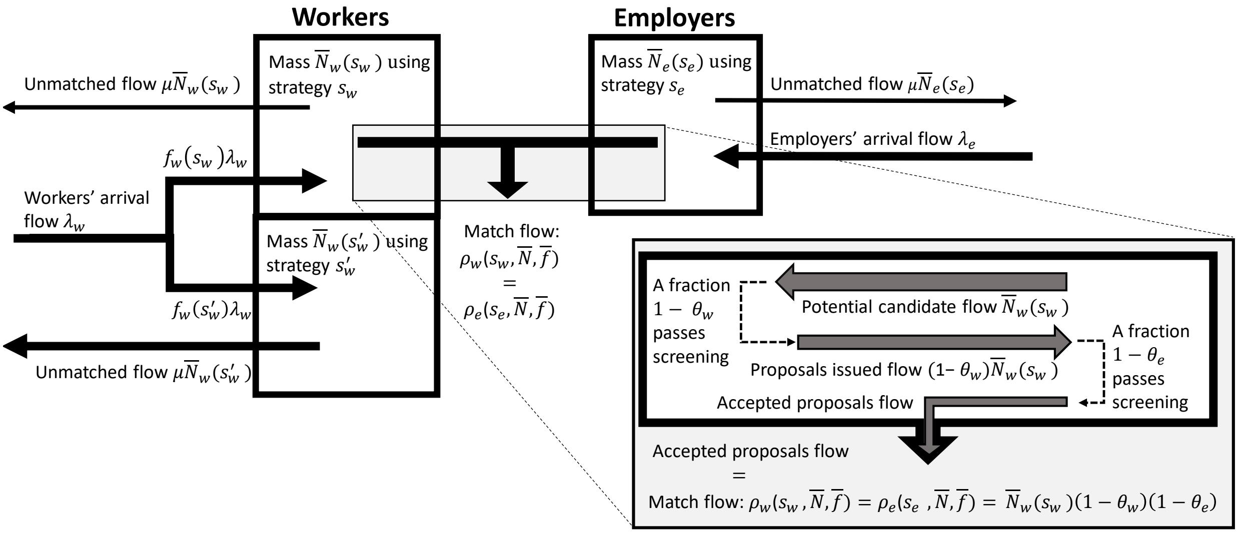

Figure 2: Schematic diagram of system dynamics. Employers all follow strategy $s_e = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_e)$ such that they do not propose and they screen and then accept/reject incoming proposals. A fraction $f_w(s_w)$ of workers follow strategy $s_w = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{S{+}P}, \theta_w)$ such that they screen and propose and a fraction $f_w(s'_w) = 1 - f_w(s_w)$ of workers follow strategy $s'_w = (a^{\mathrm{i}} = \mathrm{I}, a^{\mathrm{o}} = \mathrm{DN}, \theta'_w)$ such that they do not propose. The dynamics leading to the match formation are pictured in the lower right corner of the figure. Workers following strategy s_w see candidates at a flow rate $\bar{N}_w(s_w)$ and, after screening, they issue proposals to a fraction $1 - \theta_w$, leading to a proposal flow $(1 - \theta_w)\bar{N}_w(s_w)$. These proposals are screened by employers and a fraction $(1 - \theta_e)$ are accepted, thus determining the match flow. We highlight that the agents' strategies here do not constitute an equilibrium, as all workers who select s'_w leave unmatched.

same. Since matching is a consequence of the search process, we impose the analogous mass-balance requirement at the first stage of the search, i.e., for candidates being displayed, and the equality of the masses of matched partners will then follow. The second requirement ensures that for a tier whose agents match as soon as they arrive, the likelihood of a candidate from that tier being available (if their opportunity clock rings) is the same for all agents requesting such a candidate; in other words, *scarce candidates are spread equally.* We remark here that if the agents in our model were discrete instead of continuous, both of these properties would hold automatically. These two assumptions are formally stated in Appendix B.2, and remain in force throughout the paper.

It now remains to define our equilibrium concept and specify the allowed set of platform interventions. Along the way, we will define and partially characterize agent best responses.

Agent anticipated utilities and the agent's Markov decision process (MDP). For each agent, we define the *anticipated utility*, based on the system state $\bar{N}$ and $\bar{f}$ at the time of the agent's arrival, as the expected utility of the agent under a given strategy assuming stationarity, i.e., that $\bar{N}$ and $\bar{f}$ remain constant during their lifetime. (Our equilibrium concept will require the system to be in steady state.) The following observation is immediate.

Observation 2.1. *If $\bar{N}$ and $\bar{f}$ are invariant over time, the anticipated utility is identical to the expected utility realized.*

We denote the anticipated utility by $U_w^t(s_w, \bar{N}, \bar{f})$ for a tier-t worker following strategy s_w, and by $U_e(s_e, \bar{N}, \bar{f})$ for an employer following strategy s_e.

The calculation of the anticipated utility contains no surprises; the net (anticipated) utility is simply the difference between the expected match utility and the expected screening cost spent. Also, we must take into account both possible routes to match formation for an agent, i.e., via incoming proposals and via opportunities to see candidates. Consider a particular worker. The system state $\bar{N}$ and $\bar{f}$ determines the rate at which a given worker receives proposals, and the potential arrival rate of candidates following each strategy (these rates are computed explicitly in Proposition 11 in Appendix B.2). As the worker assumes stationarity of $\bar{N}$ and $\bar{f}$ in calculating anticipated utility, the calculation takes these rates to be time invariant.

Using these rates, one could write an expression for the anticipated utility. However, we present an alternative way of computing the anticipated utility, which will be instrumental in establishing our equilibrium characterization results described in subsequent sections. From the point of view of a worker who assumes stationarity, choosing a strategy is equivalent to choosing a control to a MDP where he starts at a state of "waiting" for one of three things to happen: (i) a candidate arrives, (ii) a proposal arrives, or (iii) he departs unmatched, and all three occur independently at time-invariant rates. When a candidate or a proposal arrives it is handled according to the worker's strategy and, if it doesn't result in a match, the worker returns to the state of waiting. In fact, the anticipated utility $U_w^t(s_w, \bar{N}, \bar{f})$ is nothing other than *the expected value of the MDP under the control corresponding to the worker strategy* s_w. Furthermore, as the MDP is time invariant, it has an optimal policy that is time invariant, deterministic, and Markovian, i.e., there is an optimal policy that lies in the strategy space we have specified earlier (Definitions 1 and 2). This will facilitate our definition of best response below.

We formally define the MDP in Appendix B.3. Immediately after our first equilibrium characterization result (Theorem 3), we informally describe the corresponding MDP for one side (employers).

Agent best responses. Consider a tier-t worker i, and let $\bar{N}$ be the agent mix and $\bar{f}$ be the distribution over strategies of arriving agents at the time i arrives. A worker strategy s_w^* is said to be a *weak best response* for worker i if it maximizes the anticipated utility, i.e.,

$$s_w^* \in \underset{s_w \in \mathcal{S}_w}{\arg\max}\, U_w^t(s_w, \bar{N}, \bar{f}), \tag{1}$$

where $U^t_w(s_w, \bar{N}, \bar{f})$ is the anticipated utility defined in the previous subsection. The key tradeoffs an agent faces in picking a utility-maximizing strategy are: (i) whether to incur the cost to screen and what threshold to use (a higher threshold means higher match utility but also higher cost as more prospective partners will be screened), (ii) whether to forego opportunities to propose, which eliminates the risk of rejection, but increases the risk of leaving unmatched, (iii) if there are two worker tiers, an employer must choose whether to exclusively pursue H-workers to obtain higher match utility but at an increased risk of failing to match.

Note that a redundant multiplicity of weak best responses may arise when either (i) the worker receives incoming proposals from employers at rate 0, or (ii) incoming proposals arrive at rate ∞ and the worker matches immediately upon arrival due to an incoming proposal (the latter situation indeed arises for high workers). In the former (latter) case, all choices of an incoming proposal strategy $(a^{\mathrm{i}}, \theta^{\mathrm{i}})$ (a strategy for opportunities $(a^{\mathrm{o}}, \theta^{\mathrm{o}})$) result in the same anticipated utility for the worker. To rule out such multiplicity, we say that a strategy s^*_w is a *best response* if it is a weak best response and, moreover, if it is still a weak best response (i) in a setting where the worker receives a "bonus" incoming proposal immediately before joining the system, and (ii) in a setting where, immediately before joining the system, he receives a "bonus" opportunity to propose and an employer is available. These requirements serve to eliminate spurious equilibria, such as the one where agents do not propose and ignore incoming proposals. The interested reader can find a more extensive discussion in Appendix B.4 culminating in a formal definition of our best response notion (Definition 6, summarized above).

The following proposition characterizes the acceptability thresholds used when an agent screens in any best response (and hence in equilibrium). The proof is in the online appendix.

Proposition 1 (Equality of screening thresholds for candidates and for incoming proposals)**.** *Fix any* $\bar{N}$ *and* $\bar{f}$*. In any best response* s^*_w*, a worker (or employer) uses the same acceptability threshold* θ^* *whenever he screens (whether it is a candidate, or an incoming proposal, and irrespective of the tier of the other agent) and, moreover,* θ^* *is equal to his best-response anticipated utility, i.e.,* $\theta^* = \max_{s_w \in \mathcal{S}_w} U^t_w(s_w, \bar{N}, \bar{f})$.

Proposition 1 follows from noting that a best response must be an optimal control to the agent's MDP described above. In any optimal control (best response), whenever the worker screens, he should use a threshold of match utility equal to his (optimal) continuation value from passing on the present opportunity/proposal in order to return to the state of "waiting" for a new opportunity/proposal: if the match utility exceeds this threshold he should propose/accept; otherwise he should pass. Moreover, this threshold is the best-response anticipated utility of the agent, since he starts in this same state of waiting for a candidate or a proposal upon entry.

The equilibrium notion we will define and use is "stationary equilibrium"; informally, this will be a system steady state where all agents are playing best responses. Now Proposition 1 implies that, for any time-invariant $\bar{N}$ and $\bar{f}$ where all agents are playing best responses, all agents in the same tier use the same threshold (equal to the best-response expected utility) whenever they screen. This motivates us to hereafter restrict all agents in a given tier to use the same fixed screening threshold for both opportunities and incoming proposals, irrespective of arrival time. We denote these fixed thresholds by $(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e)$.

Definition 3. *Given thresholds* $(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e) \in \mathbb{R}^3$*, the set of* threshold-consistent *strategies for tier-t workers,* $\mathcal{S}_w(\theta_w^t)$*, is the discrete finite set*

$$\mathcal{S}_w(\theta_w^t) \triangleq \{s = (a, \theta) : \ a \in \mathcal{A}_w \ \textit{ and } \ \theta^{\mathrm{i}} = \theta_w^t \text{ if } a^{\mathrm{i}} = \mathrm{S{+}A/R} \text{ and } \theta^{\mathrm{o}} = \theta_w^t \text{ if } a^{\mathrm{o}} = \mathrm{S{+}P}\} \, ,$$

and the set of threshold-consistent strategies for employers $\mathcal{S}_e(\theta_e)$ *is defined analogously.*

Restriction 1. *For some fixed thresholds* $(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e) \in \mathbb{R}^3$*, agent mix* $\bar{N}$ *and strategy distributions for incoming arrivals* $\bar{f}$ *are supported on threshold-consistent strategies at all times. In particular,* $\bar{N}$ *and* $\bar{f}$ *have discrete finite support.*

Accordingly, given an agent tier with fixed threshold θ, we identify strategy $s = (a, \theta)$ with the associated vector of actions a.

Dynamic evolution of the system. With Restriction 1 in place, it is straightforward to write down the dynamic evolution of the agent mix $\bar{N}$ given an initial agent mix. Let $\rho_w^t(s_w, \bar{N}, \bar{f})$ be the flow (mass per unit time) of matches involving workers in tier t following strategy s_w, and define $\rho_e(s_e, \bar{N}, \bar{f})$ analogously. (Appendix B.1 shows how to compute $\rho_w^t(s_w, \bar{N}, \bar{f})$.) Then we can express the rate of change of $\bar{N}_w^t(s_w)$ with respect to time τ as

$$\frac{\mathrm{d}\bar{N}_w^t(s_w)}{\mathrm{d}\tau} \quad = \quad f_w^t(s_w)\lambda_w^t - \bar{N}_w^t(s_w)\mu - \rho_w^t(s_w, \bar{N}, \bar{f}) \qquad \forall s_w^t \in \mathcal{S}_w(\theta_w^t) \, , \tag{2}$$

where the first term on the right captures the arrival flow of tier-t workers following strategy s_w, the second term is the departure-without-matching flow of such workers, and the third term is the flow of matches formed involving such workers. Analogously, we have

$$\frac{\mathrm{d}\bar{N}_e(s_e)}{\mathrm{d}\tau} \quad = \quad f_e(s_e)\lambda_e - \bar{N}_e(s_e)\mu - \rho_e(s_e, \bar{N}, \bar{f}) \qquad \forall s_e \in \mathcal{S}_e(\theta_e) \, . \tag{3}$$

2.1 Equilibrium concept: Evolutionarily stable stationary equilibria

Our equilibrium notion is closely related to the stationary equilibrium introduced by Hopenhayn (1992), which considers game-theoretic equilibria corresponding to a dynamical steady state in a large market.[7] Fix the sets of allowed actions $\mathcal{A}_w$, $\mathcal{A}_e$ under the platform's intervention (if any). The following is our definition of stationary equilibrium.

Definition 4 (Stationary equilibrium)**.** *Fix thresholds* $\left(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e\right)$*, and time-invariant distributions over threshold-consistent strategies for new arrivals* $\bar{f}$*, and suppose that the agent mix* $\bar{L} = (\bar{L}_w^{\mathrm{H}}, \bar{L}_w^{\mathrm{L}}, \bar{L}_e)$ *is a* steady state*, i.e., an attractive fixed point of the dynamical equations* (2) *and* (3) *under* $\bar{f}$*. Then* $\left(\left(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e\right), \bar{f}, \bar{L}\right)$ *is a* stationary equilibrium (SE) *if, for each* $t \in \{\mathrm{H}, \mathrm{L}\}$*, the threshold* θ_w^t *is the best-response expected utility*[8] *for tier-t workers under system state* $\bar{L}$ *and* $\bar{f}$*, and the distribution* f_w^t *is supported on best response strategies, and similarly for employers.*

Notice that, leveraging the characterization of best responses in Proposition 1, the definition of SE requires screening thresholds equal to the best-response expected utilities. In addition, $\bar{f}$ must be supported on best response strategies. Also note that SE requires a steady state, i.e., an attractive fixed point of the dynamical equations (2) and (3). Thus, each of our results claiming a stationary equilibrium characterizes the corresponding steady state as part of the proof.[9]

We refine our equilibrium concept to focus on the subset of stationary equilibria that are *evolutionarily stable* (a classic reference is Rohlfs 1974). Intuitively, an evolutionarily stable equilibrium is robust in the sense that if the mix of agents $\bar{N}$ deviates slightly from the steady state value of $\bar{L}$ (slightly changing the utility derived from different strategies) and *incoming agents choose their strategies as a best response to the current agent mix* $\bar{N}$ *(and unchanged* $\bar{f}$*)*, this reaction should push the system back toward $\bar{L}$. This refinement crucially serves to rule out implausible equilibria. We formalize the notion of evolutionary stability in Appendix B.5.

2.2 Performance metric and platform interventions permitted

The performance metric used in this paper is the *average welfare in steady state*. Fix the sets of allowed actions $\mathcal{A}_w$, $\mathcal{A}_e$ under the platform intervention (if any), and the strategy distributions for

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[7] This notion is closely related to that of mean field equilibrium, which has been effectively employed in the operations literature to study complex dynamic games with many players (e.g., Iyer et al. (2014), Balseiro et al. (2015)). What makes these equilibrium concepts appealing and tractable is that they relax the informational requirements of agents, requiring them to know only the aggregate description of the system (in our case, the steady-state mix of agents and the strategy distributions for new arrivals).

[8] Recall Observation 2.1: in steady state, the anticipated utility is identical to the expected utility.

[9] In fact, for each equilibrium we find in this paper, we prove that there is a unique steady state for the given $\bar{f}$.

new arrivals $\bar{f}$. (We will provide a general definition of average welfare, not just in equilibria, and hence do not force threshold consistency.) Assume that $\bar{L}$ is the resulting steady state. The average welfare in steady state is simply the weighted average across agent tiers of the expected utility

$$\text{Avg-welf}(\bar{L}, \bar{f}) \triangleq \frac{\lambda_e \int_{\mathcal{S}_e} U_e(s_e; \bar{L}, \bar{f}) df_e(s_e) + \sum_{t \in \{\text{H},\text{L}\}} \lambda_w^t \int_{\mathcal{S}_w} U_w^t(s_w; \bar{L}, \bar{f}) df_w^t(s_w)}{\lambda_e + \lambda_w^{\text{H}} + \lambda_w^{\text{L}}} . \tag{4}$$

We will compare the average welfare in equilibrium with that achievable by a planner who can choose the strategy distributions for new arrivals. Our planner will not be constrained by Restriction 1, which is the reason we have stated our definition of average welfare allowing f_e, f_w^{H}, and f_w^{L} to have arbitrary support. Note that although we use average welfare as the performance metric, our equilibrium characterizations (which include the welfare obtained by each type of agent) can equally be used to obtain platform design recommendations under alternate platform objectives.

We allow the platform to only make the following interventions, through Section 4:

(i) *Prevent agents on one side from proposing*: If (say) workers are prevented from proposing, this simply enforces that the set of allowed actions for workers of all tiers t is $\mathcal{A}_w = \{a = (a^{\text{o}}, a^{\text{i}}) : a^{\text{o}} = \text{DN}\}$; i.e., the workers cannot propose to employers.

(ii) *Hide quality information from employers*: If quality information is hidden from employers, then the employer strategies $\mathcal{S}_e$ specify a single strategy a^{o} for proposing to workers (irrespective of tier) and a single strategy a^{i} for incoming proposals (irrespective of tier) and a single threshold to screen candidates/proposals. If an employer asks for a candidate and one is available, he is given a uniformly random one from among those present across both tiers.

We briefly discuss a few additional interventions in Section 5. Exploring a larger design space for the platform is an interesting direction for future work.

A note on the analysis. We focus the analysis in the next sections on the case where μ, the rate at which agents leave unmatched, is small, i.e., $\mu \ll 1$. As the opportunity clock of the agents rings at rate 1, assuming $\mu \ll 1$ implies that agents have a large number of clock rings during their lifetime. When μ is relatively large, an agent who has just received a candidate or a proposal will not be selective in proposing/accepting (even if c is very small) as this may be his last chance to match. Therefore, focusing on the small μ regime allows us to focus squarely on the welfare impact of screening costs, rather than on the consequences of the difficulty of meeting potential partners.

While our equilibrium characterizations are established for small enough $\mu > 0$, several of our theorem statements provide exact expressions for utilities in the limit $\mu \to 0$ for the convenience of the reader, and leave out the unwieldy expressions or implicit characterizations we obtain for

$\mu > 0$. The latter can be found in the proofs, and leave our findings unaffected.

Terminology and notation. Throughout the paper, we use the term equilibrium to refer to an evolutionarily stable stationary equilibrium, and utility to refer to expected utility. A list of notation is provided in Appendix A, divided based on the section in which it is introduced. Note that much of the notation introduced so far was essential to set up our formal framework but does not reappear in the main paper: From here on we will frequently reference the primitives λ's, c and μ, the equilibrium θ's and $\bar{f}$, and, for the planner's solution will reference a^{o}, a^{i}, $\mathcal{A}$, θ^{o}, θ^{i} and $\bar{L}$.

3 Ex-ante homogeneous agents: making the correct side propose

This section demonstrates two fundamental sources of equilibrium inefficiencies arising in two-sided markets with search frictions: recipients of proposals are too selective, and the long side of the market makes the proposals and is thus unable to be selective. We illustrate these inefficiencies in the simplest setting in which they occur: in markets with ex-ante homogeneous agents on each side, who can be distinguished only by screening. We characterize the agents' strategies in the planner's solution (the constrained-efficient benchmark) and then leverage this characterization to showcase the inefficiencies in equilibrium. As we shall later see in Section 4, (i) the characterization of the planner's solution with ex-ante homogeneous agents is the basic building block for the characterization of the planner's solution in vertically differentiated markets, and (ii) the inefficiencies studied in this section arise, even more strikingly, in markets with vertical differentiation.

3.1 The planner's solution in markets with ex-ante homogeneous agents

Consider a central planner who chooses the distribution over strategies[10] adopted by arriving agents on each side of the market so as to maximize the average welfare. The planner is constrained in that agents must wait for opportunities to arise and must exert screening effort to discover idiosyncratic match utility terms, as in the decentralized market. The planner's problem is defined as follows.

Definition 5 (Planner's problem for ex-ante homogeneous agents)**.** *Suppose that all quality terms are zero. Then, the set of strategies $\mathcal{P}$ available to the planner is defined as*

$$\mathcal{P} = \left\{ \ (a, \theta^{\mathrm{o}}, \theta^{\mathrm{i}}) : \quad a = (a^{\mathrm{o}}, a^{\mathrm{i}}) \in \mathcal{A}, \ \theta^{\mathrm{o}} \in [0,1], \ \theta^{\mathrm{i}} \in [0,1] \ \right\}$$

where $\mathcal{A}$ is the possible set of actions and θ^{o} (θ^{i}) denotes the threholds used to screen opportunities (incoming proposals) if a^{o} (a^{i}) includes screening, consistent with Definition 1. The planner chooses

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[10] The planner can assign strategies deterministically or at random; the welfare is unaffected due to the exact LLN.

a pair of probability measures (h_w, h_e) *over strategies in* $\mathcal{P}$*. Arriving workers and employers are assigned strategies drawn i.i.d. from* h_w *and* h_e*, respectively. The planner's objective is to maximize the average welfare in steady state, defined using* (h_w, h_e) *instead of* (f_w, f_e) *as per* (4)*.*

We next derive an upper bound to the average welfare achievable by any pair of (h_w, h_e), and show that the welfare described by the bound can be achieved by the planner in the limit as $\mu \to 0$.

Proposition 2 (Planner's solution)**.** *Consider a market with only one tier of agents on each side. Let* h_w, h_e *denote probability measures over* $\mathcal{P}$*, representing the distribution over strategies adopted by workers and employers upon entering, and let* $\bar{L}(h_w, h_e)$ *be the induced steady-state agent mix. Then, for every fixed* $c > 0$ *and* $\mu > 0$*, we have the upper bound*

$$\sup_{h_w \in \Delta(\mathcal{P}), h_e \in \Delta(\mathcal{P})} \text{Avg-welf}(\bar{L}(h_w, h_e), (h_w, h_e)) \leq \frac{\min(\lambda_w, \lambda_e)\mathcal{W}(c)}{\lambda_w + \lambda_e}\,, \qquad \textit{where}$$

$$\mathcal{W}(c) \triangleq \begin{cases} g(c) & \textit{if } c \in (0, \tilde{c})\,, \\ 3/2 - \sqrt{2c} & \textit{if } c \in \left[\tilde{c}, \frac{1}{8}\right)\,, \\ 1 & \textit{if } c \geq \frac{1}{8}\,, \end{cases} \tag{5}$$

$$g(c) \triangleq \sup_{(\theta^{\text{o}}, \theta^{\text{i}}) \in (0,1)^2} \frac{1 + \theta^{\text{o}}}{2} - \frac{c}{(1 - \theta^{\text{o}})(1 - \theta^{\text{i}})} + \frac{1 + \theta^{\text{i}}}{2} - \frac{c}{(1 - \theta^{\text{i}})}, \tag{6}$$

and $\tilde{c}$ *is defined as the unique solution to* $3/2 - \sqrt{2\tilde{c}} = g(\tilde{c})$ *(numerically,* $\tilde{c} \approx 0.060$*).*[11]

Moreover, the following policy (as a function of c*) achieves a limiting average welfare equal to the upper bound* $\frac{\min(\lambda_w, \lambda_e)}{\lambda_w + \lambda_e}\mathcal{W}(c)$ *as* $\mu \to 0$*:*

(i) $\mathbf{c} \in (\mathbf{0}, \tilde{\mathbf{c}})$*: All agents screen and propose with threshold* $\theta^{\text{o}}(c)$*, and screen and accept/reject with threshold* $\theta^{\text{i}}(c)$*, where* $(\theta^{\text{o}}(c), \theta^{\text{i}}(c)) \in (0,1)^2$ *is the unique solution to the following equations:*

$$\big(1 - \theta^{\text{o}}(c)\big)\big(1 + \big(1 - \theta^{\text{o}}(c)\big)\big) = \big(1 - \theta^{\text{i}}(c)\big) \quad \textit{and} \quad \big(1 - \theta^{\text{i}}(c)\big)^3\big(1 + \big(1 - \theta^{\text{i}}(c)\big)\big) = 2c\,. \tag{7}$$

(ii) $\mathbf{c} \in [\tilde{\mathbf{c}}, \mathbf{1/8})$*: All agents propose without screening, and screen + accept/reject incoming proposals with threshold* $\theta^{\text{i}}(c) = 1 - \sqrt{2c}$*.*

(iii) $\mathbf{c} \geq \mathbf{1/8}$*: All agents propose and accept without screening.*

Furthermore, for any $\lambda_e \neq \lambda_w$*, for each* $c > 0$ *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$ *the above policy produces average welfare exactly equal to the upper bound.*

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[11] Recall that we defined average welfare *in steady state* via (4), and have stated Proposition 2 accordingly for simplicity. However, our proof of the upper bound does not require the existence of a steady state, and even allows h_w and h_e to vary over time: for any $\mu > 0$ and any (possibly time-varying) h_w and h_e, we show that the long-run average welfare (defined in the natural manner) cannot exceed $\frac{\min(\lambda_w, \lambda_e)}{\lambda_w + \lambda_e}\mathcal{W}(c)$. Then for every fixed c, we show that for the specific h_w and h_e defined in the second part of the proposition (in each of the three cases), a steady state exists and the upper bound is achieved as $\mu \to 0$.

The proof can be found in the online appendix. Intuitively, $\min(\lambda_w, \lambda_e)$ upper bounds the average match flow as the flow of agents leaving via matching cannot exceed their arrival rate. The function $\mathcal{W}(c)$ represents the maximum per-match utility that the planner can achieve given c. That is, if we let $V_w(s_w, s_e)$ denote the expected per-match utility of a match resulting from a worker following strategy s_w proposing to an employer following s_e (the sum of the two expected match values minus the expected screening cost incurred en route to the formation of one match) and define $V_e(s_e, s_w)$ analogously, then $\mathcal{W}(c) = \sup_{s_w \in \mathcal{P}, s_e \in \mathcal{P}} \{\max(V_w(s_w, s_e), V_e(s_e, s_w))\}$. Therefore, $\frac{\min(\lambda_w, \lambda_e)\mathcal{W}(c)}{\lambda_w + \lambda_e}$ upper bounds the average welfare. Finally, $g(c)$ represents, for a given screening cost c, an upper bound of the per-match utility that can be achieved when the agents issuing proposals do so after screening, and those receiving proposals also screen before accepting.

The proposition establishes that three regimes arise in the planner's solution. *For large screening costs $c \geq 1/8$, no agent should screen,* as the potential gain from a better match is not worth the cost. The expected per-match utility is simply $\mathcal{W}(c) = 2 \times \mathrm{E}[v_{ij}] = 2 \times \frac{1}{2} = 1$. *For intermediate costs $c \in [\tilde{c}, 1/8)$, agents screen only when receiving proposals*, with welfare-maximizing threshold $\theta^{\mathrm{i}} = 1 - \sqrt{2c}$. Here the expected utility generated per match is 1/2 from the proposer plus $\max_{\theta^{\mathrm{i}} \in [0,1]} \mathrm{E}[v_{ji} | v_{ji} > \theta^{\mathrm{i}}] - \frac{c}{1-\theta^{\mathrm{i}}} = \frac{1+\theta^{\mathrm{i}}}{2} - \frac{c}{1-\theta^{\mathrm{i}}} = 1 - \sqrt{2c}$ from the recipient, where $\frac{c}{1-\theta^{\mathrm{i}}}$ is the expected screening cost incurred per match, as on average $\frac{1}{1-\theta^{\mathrm{i}}}$ proposals are screened for each one accepted. *For small costs $c \in (0, \tilde{c})$, agents screen both when issuing and receiving proposals* as this significantly increases the sum of expected match values while causing a smaller increase in the expected screening cost incurred. The thresholds are given by (7), which are the first-order conditions for maximizing per-match utility as per (6). Notably, in the planner's solution the threshold θ^{o} used for opportunities is *different* from the threshold θ^{i} for incoming proposals. From the first part of (7), we can deduce that $\theta^{\mathrm{o}} > \theta^{\mathrm{i}}$, an observation we highlight in the following remark.

Remark 1. *In the planner's solution, agents are less selective when they receive proposals than when they issue proposals.*

To see why, consider the net per-match utility (given by (6)). The value an agent obtains by matching is his expected value of matching with an agent on the other side conditional on her exceeding the threshold; thus, the expected sum of match values conditional on matching $\frac{1+\theta^{\mathrm{i}}}{2} + \frac{1+\theta^{\mathrm{o}}}{2}$ depends only on the *unordered* pair of thresholds used, and not on which of the two thresholds is used by the recipient and which by the proposer. By contrast, the expected screening cost depends not only on the thresholds but also on the *order* in which they are used: in expectation, proposers screen $\frac{1}{(1-\theta^{\mathrm{o}})(1-\theta^{\mathrm{i}})}$ candidates per match—in expectation, $\frac{1}{1-\theta^{\mathrm{o}}}$ candidates are screened per proposal issued and (independently) $\frac{1}{1-\theta^{\mathrm{i}}}$ proposals are issued before one is accepted—whereas recipients screen only $\frac{1}{1-\theta^{\mathrm{o}}}$ proposers per match. Therefore, from a welfare perspective, recipients should be using the lower threshold, to mitigate wasted screening effort on the part of proposers.

3.2 Equilibrium structure and inefficiencies

We will now characterize equilibria and find that, remarkably, *agents on only one side of the market propose in equilibrium.* Furthermore, we will identify that the following two inefficiencies arise:

1. *Recipients are too selective.* Recipients of proposals use a threshold higher than the socially optimal one, as they do not internalize the adverse effect their selectivity has on proposers.
2. *In markets where one side arrives at a faster rate (unbalanced markets), the faster-arriving side (the long side) always proposes and cannot afford to be selective in equilibrium.*

We start by characterizing the equilibria in the simplest possible setting of a symmetric market where both sides have the same arrival rates, and observe that the first inefficiency arises.[12]

Theorem 3 (Equilibria in symmetric markets)**.** *Consider a market with one tier on each side where each of the two sides has an arrival rate of* 1*. For each* c *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$*, all stable stationary equilibria as a function of* c *are characterized as follows:*

1. $\mathbf{c} \in \left(\mathbf{0}, \frac{\mathbf{1}}{\mathbf{32}}\right)$*: There is a unique equilibrium where (workers screen + propose, employers screen + accept/reject) and employers do not propose. Agents have an expected utility identical to the screening threshold they employ, and thresholds satisfy* $\lim_{\mu\to 0}\theta_w = 1-(2c)^{1/4}$ *and* $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$*. The symmetric counterpart equilibrium, where the roles of workers and employers are reversed, is the only other equilibrium in this regime.*

2. $\mathbf{c} \in \left(\frac{\mathbf{1}}{\mathbf{32}}, \frac{\mathbf{1}}{\mathbf{8}}\right)$*: There is a unique equilibrium where (workers propose without screening, employers screen + accept/reject) and employers do not propose. Workers' expected utility satisfies* $\lim_{\mu\to 0}\theta_w = 1/2$*, and employers' expected utility (and threshold) satisfies* $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$*. The symmetric counterpart equilibrium is the only other equilibrium in this regime.*

3. $\mathbf{c} \geq \frac{\mathbf{1}}{\mathbf{8}}$*: There is a unique equilibrium where all agents propose without screening, and accept all incoming proposals without screening. Both sides earn an expected utility of* $\theta_e = \theta_w = \frac{1}{2+\mu} \xrightarrow{\mu\to 0} \frac{1}{2}$*.*

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[12] For the sake of readability, we omit the following technical details in the theorem statements in the main text: (i) if one side of the market never proposes in equilibrium, then we do not state what an agent on the other side will do upon receiving a proposal and (ii) we omit specifying the steady-state masses of different types of agents. These details can be found in the detailed theorem statements in the corresponding appendices. In the interest of brevity, we also omit the tedious and unilluminating exercise of characterizing equilibria at screening costs in the boundaries between regimes. Finally, in this section we impose a mild technical constraint on agents strategies to facilitate complete characterizations of *all* equilibria for *all* possible screening costs; we state the restriction (Restriction 4) and discuss its implications at the beginning of Appendix D. It is worth noting that this is not needed to establish existence of the equilibria characterized in this section, nor to establish our results for two tiers of workers in Section 4, where we consider only small screening costs. A detailed statement and proof of Theorem 3 are in Appendix D.2.

Equilibrium structure. Theorem 3 reveals that a remarkable equilibrium structure arises even in the simplest setting. As in the planner's solution, we find three regimes distinguished by magnitude of c: all agents screen for small c, only proposal recipients screen for intermediate c, and no one screens for large c. Notably, in the small and medium-cost regimes there is *spontaneous breaking of symmetry between the two sides in equilibrium*: one side of the market proposes (henceforth, the "active" side) while the other side waits for incoming proposals (the "passive" side).

To explain this phenomenon, it is key to convey why passive-side agents (say, employers) don't want to propose when active-side agents (workers) are proposing. To do so, we first sketch the utility calculation for an employer (which also provides the reader with a flavor of the agent's MDP), in the medium-c regime.[13] Fix any $c \in (\frac{1}{32}, \frac{1}{8})$ and consider the situation of an employer when all other employers ignore opportunities ($a^{\mathrm{o}} = \mathrm{DN}$) and screen incoming proposals ($a^{\mathrm{i}} = \mathrm{S{+}A/R}$). As the arrival rates of agents on the two sides are the same, the steady-state masses of agents on the two sides are equal. It then follows that the rate at which the employer receives proposals is equal to the rate of making a proposal for a worker, i.e., a rate of 1. Let's calculate an employer's highest achievable expected utility (call it V_e) by optimizing over the threshold θ_e under the actions $a^{\mathrm{i}} = \mathrm{S{+}A/R}$ and $a^{\mathrm{o}} = \mathrm{DN}$. A comparison of hazard rates reveals that an employer receives a proposal before leaving unmatched with probability $\frac{1}{1+\mu}$, in which case he screens it at a cost c, and accepts if the match value exceeds θ_e, i.e., with probability $1 - \theta_e$. Conditional on accepting, he earns expected match utility $\frac{1+\theta_e}{2}$. If he rejects the proposal, he returns to the initial state and has future expected utility V_e. Combining, we obtain the Bellman equation

$$V_e = \max_{\theta_e \in [0,1]} \frac{1}{1+\mu} \left(-c + (1-\theta_e) \left(\frac{1+\theta_e}{2} \right) + \theta_e V_e \right) . \tag{8}$$

As captured in Proposition 1, the optimal choice of θ_e is clearly $\theta_e = V_e$. Plugging in this value of θ_e, solving the quadratic equation, and noting that the agent utility cannot exceed 1, we obtain

$$\theta_e = V_e = 1 + \mu - \sqrt{2c + 2\mu + \mu^2} \xrightarrow{\mu \to 0} 1 - \sqrt{2c}\,. \tag{9}$$

which agrees with the limiting value of θ_e specified in Equilibrium 2 of Theorem 3. Also, observe that the probability the employer leaves without matching is $\frac{\mu}{\mu+\sqrt{2c}} \xrightarrow{\mu \to 0} 0$ under the above strategy.

We can now argue why employers do not want to propose. Proposing without screening would yield expected match utility 1/2 if the proposal is accepted, but this is an unattractive alternative for small enough μ because $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c} > 1/2$ for $c < 1/8$. What about screening

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[13] The calculation for the small-c regime is identical, except for the fact that active-side agents propose only with probability $1 - \theta_w$, and thus proposals arrive to passive-side agents at rate $1 - \theta_w$.

and proposing? The drawback here is that, as we formalize in the detailed version of Theorem 3 in Appendix D.2, workers screen hypothetical incoming proposals with threshold θ_w (the theorem tells us that $\lim_{\mu \to 0} \theta_w = 1/2$). Hence, *passive-side agents do not want to propose because proposing risks rejection* (this is true also for the small-c regime). As the risk of an employer leaving without matching when ignoring opportunities is small (recall that it is $\frac{\mu}{\mu+\sqrt{2c}} \xrightarrow{\mu \to 0} 0$) it follows that, for small enough μ, ignoring opportunities is a best response for employers. Note that as μ increases, the risk of leaving without matching increases, and there is some threshold value μ_0 such that for $\mu > \mu_0$, the employer would rather take advantage of opportunities. For example, for $c = 0.05 \in (1/32, 1/8)$, it turns out that $\mu_0 \approx 0.065$, and moreover that for $\mu < \mu_0$ the specified equilibrium actions are best responses for workers as well, i.e., for $c = 0.05$, Equilibrium 2 exists for all $\mu < 0.065$.

We conclude by briefly explaining why there are no symmetric equilibria. Given that our setting is symmetric, one might conjecture the existence of symmetric equilibria, for instance an equilibrium where both sides mix between proposing and not proposing. Evolutionary stability plays a crucial role in ruling out such equilibria: the conjectured mixed equilibrium is not evolutionarily stable because a small increase in the fraction of workers proposing leads to a decrease in the fraction of employers proposing, which in turn leads to an increase in the fraction of workers proposing, ultimately resulting in all employers not proposing while all workers propose; see Lemma D.2.

Equilibrium inefficiency: Recipients are too selective. The left-hand side of Figure 3 compares the equilibrium average welfare in the symmetric market with the planner's solution. The threshold between the small-c and intermediate-c regimes is lower in equilibrium ($1/32 < \tilde{c} \approx 0.060$). When at most one side screens in the planner's solution, then the welfare in the planner's solution coincides with that in equilibrium. (In particular, when only one side screens in equilibrium, that side uses the socially efficient threshold.) However, when both sides screen in the planner's solution (small c), the welfare in the planner's solution is higher than in equilibrium.

The reason behind the difference in welfare is that, in equilibrium, *recipients do not internalize the adverse effect of their selectivity on proposers* and thus use a threshold higher than the socially optimal one. If a proposer screens before issuing a proposal and the proposal is then rejected, that screening cost is wasted. However, when a recipient chooses his threshold to screen proposals, he trades off his expected match value with his own expected total screening cost, but he ignores the harm caused to a proposer when he rejects a proposal (see (8)). As a result, in equilibrium, recipients have a higher threshold and a higher utility than proposers, whereas the opposite is true in the planner's solution (see Remark 1). As a consequence of the high threshold used by

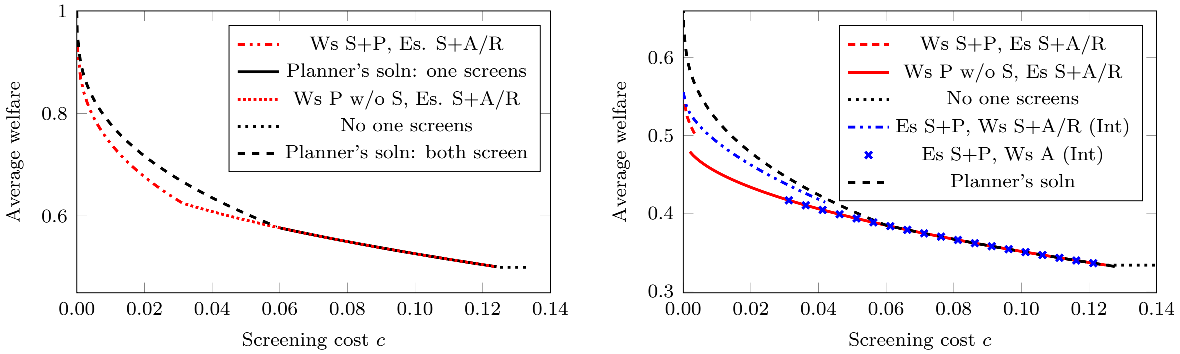

Figure 3: Average welfare in a symmetric market with ex-ante homogeneous agents (*left*) and in an unbalanced market with $R = 2$ under no intervention and under intervention (*right*) as $\mu \to 0$. In both cases, we display the average welfare in the planner's solution for comparison. In the legends, Ws and Es denote workers and employers, respectively, S+P = screen and propose, S+A/R = screen and then accept/reject, A = accept, and (Int) denotes that the equilibrium *exists only under the proposed intervention*. The "no one screens" regime is both for the planner's solution and for the equilibrium.

recipients, proposers *are unable to be sufficiently selective*; they use a low threshold and quit screening altogether for relatively small values of c. Let θ_r denote the thresholds used by recipients. As each issued proposal has a probability $1/(1-\theta_r)$ of being accepted, proposers face an effective screening cost of $c/(1-\theta_r)$ per "useful" opportunity (one where the candidate would accept a proposal), which explains why proposers are unable to be selective.

To conclude our analysis of the symmetric setting, we note that the platform *cannot increase average welfare* by blocking one side from proposing: this intervention would only force the symmetric counterpart equilibrium to arise, and due to market symmetry, the average welfare would remain unchanged. However, this case is an exception: we find next that blocking one side from proposing is helpful when the two sides of the market are asymmetric even with ex-ante homogeneous agents.

In particular, we next focus on unbalanced markets, where the second inefficiency arises. Assume, without loss of generality, that workers arrive faster than employers; we refer to the side that arrives faster as the *long side* and to the other side as the *short side*. Our main finding is that, if the market imbalance is not too small, *the long side proposes in all equilibria* under no intervention.[14]

Theorem 4 (Equilibria in unbalanced markets under no intervention)**.** *Consider a market with one tier on each side where workers arrive faster than employers ($\lambda_w > \lambda_e$) and let $R = \lambda_w/\lambda_e \geq 1.25$ denote the market imbalance. Then for any screening cost $c > 0$ there exists $\mu_0 = \mu_0(c) > 0$ such*

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[14] Theorem 14 in Appendix D.3 is a detailed version of Theorem 4. Lemma D.6 and Lemma D.7 there introduce the quantities $\xi(R, c)$ (for utility) and $\underline{c}$ and $\bar{c}$ (for equilibrium existence boundaries) and clarify how they arise as in the theorem statement for equilibria where workers propose. The proof of the theorem uses Lemma D.7.

that for all $\mu < \mu_0$ the following occurs. The following are all the equilibria:

1. For very small $c \in (0, 2\bar{c}^2)$, there is an equilibrium where (workers screen + propose, employers screen + accept/reject); the thresholds are $\theta_w = \xi(R, \sqrt{\frac{c}{2}})$ and $\theta_e = 1 - \sqrt{2c}$, where

$$\xi(R,c) \triangleq \frac{R - \sqrt{(R-1)^2 + 2c(2R-1)}}{2R-1}\, , \ \bar{c} \triangleq \frac{1}{4}\left(1 - \frac{R}{1 + \sqrt{1 + (R-1)^2}}\right)\, , \ \underline{c} \triangleq \frac{1}{8R^2}\, . \tag{10}$$

2. If $c \in \left(2\underline{c}^2, \frac{1}{8}\right)$, there is an equilibrium where (workers propose without screening, employers screen + accept/reject) with threshold $\theta_e = 1 - \sqrt{2c}$. Workers' expected utility is $1/(2R)$.

3. For $c > \frac{1}{8}$, agents on both sides propose without screening, and accept all incoming proposals without screening. Employers' expected utility is $1/2$ and workers' expected utility is $1/(2R)$.

Note that $2\underline{c}^2 < 2\bar{c}^2 \leq 1/32$, and thus Equilibria 1 and 2 coexist for[15] *$c \in (2\underline{c}^2, 2\bar{c}^2)$. For all other c there is a unique equilibrium.*

Here, Equilibrium 1 breaks down for $c > 2\bar{c}^2$ because workers prefer to propose without screening, while Equilibrium 2 breaks down for $c < 2\underline{c}^2$ because workers prefer to screen and propose.

Note that for $c < 1/8$, workers propose (and employers do not propose) in *all* equilibria.

Equilibrium inefficiency: The long side always proposes and cannot be selective. The theorem tells us that the expected utility of the employers is greater than the expected utility of the workers. An obvious reason for this is that all employers match (as $\mu \to 0$), while only a fraction $1/R \leq 0.8$ of workers match. The risk a worker faces of leaving unmatched has the additional consequence that, for $R \geq 1.25$, *there is no equilibrium where the short side (employers) proposes* even though both sides are allowed to propose, as it is always a best response for a long-side agent (a worker) to act on opportunities to propose so as to increase his chances of matching before leaving. Given that workers are thus active, employers would rather passively wait for incoming proposals than screen and propose and face potential rejection. Moreover, competition for scarce employers prevents workers from being selective in equilibrium, which is further exacerbated by the fact that workers are proposing and thus facing rejection. As a result, for all but very small values of c (i.e., for $c \geq 2\bar{c}^2$), workers propose *without screening* and match with a random employer when they are lucky enough to match. The welfare loss due to workers being unable to be selective is substantial (see comparison with the planner's solution on the right-hand side of Figure 3).

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[15] This relates to the positive spillover effect of screening by the long side; see Remark 2 below. Moreover, for such c, there are stationary equilibria where the long side mixes between proposing with and without screening, but these equilibria are not evolutionarily stable (again, related to the effect in Remark 2 below) as established in the appendix.

3.3 Intervention to improve welfare in unbalanced markets

Note that the only allowed platform intervention with ex-ante homogeneous agents is to block one side from proposing. In contrast to the case of the fully symmetric market, the platform *is* able to alleviate the inefficiency resulting from the long side (workers) proposing in all equilibria by *blocking the long side from proposing*, thus forcing the short side (employers) to propose in an unbalanced market. The following theorem formalizes this.[16]

Theorem 5 (Equilibria when workers are not allowed to propose)**.** *Consider the market described in the statement of Theorem 4 and suppose that the workers are not allowed to propose. Recall* $\xi(R,c)$*,* $\bar{c}$ *and* $\underline{c}$ *defined in* (10)*, and let* $\hat{c} \triangleq \frac{8R-7}{32(2R-1)}$*. Then, for each* $c \in (0, \min(\bar{c}, \hat{c}))$ *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for* $\mu < \mu_0$ *there is an equilibrium where (employers screen + propose, workers screen + accept/reject), where thresholds (and expected utilities) satisfy* $\lim_{\mu\to 0} \theta_w = \xi(R,c)$ *and* $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c/(1-\xi(R,c))}$*. Moreover, this is the unique equilibrium for* $c \in (0, \min(\hat{c}, \underline{c}))$*.*

The intervention *creates* new equilibria when $R \geq 1.25$, and increases the interval of screening costs for which both sides screen. The idea behind the intervention is to be able to apply one of the fundamental lessons of the planner's solution: the side with the lower threshold should be the one receiving proposals since the resulting negative effect on the proposers is smaller (see Remark 1). Even though equilibrium thresholds are endogenously determined, the long-side agents are natural candidates for having a lower threshold[17] (they can't be very selective as they must account for the risk of leaving unmatched), and thus the long side should be receiving the proposals. In fact, the welfare of employers slightly decreases under our intervention, as they must now propose while risking (infrequent) rejection. On the other hand, the welfare of workers increases dramatically, as they no longer face rejection (previously they were frequently rejected) and can thus be more selective. In turn, the selectivity of workers has a positive same-side spillover effect on other workers as it makes scarce employers more available to other workers.

Remark 2. *We find that the* selectivity of long-side agents has a positive same-side spillover effect*: when a worker rejects an employer he makes the employer available for matching with other workers. This effect leads to a virtuous cycle: the selectivity of other workers increases the availability of options to a particular worker and hence allows him to be more selective, thus boosting the benefit*

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[16] Theorem 15 in Appendix D.3 is a more detailed version of Theorem 5. Appendix A summarizes the semantic origins of the quantities $\xi(R,c)$, $\hat{c}$, $\bar{c}$, and $\underline{c}$ that appear in the theorem statement.

[17] In fact, in markets with $R \geq 1.25$ and $c < \min(\bar{c}, \hat{c})$, workers have a lower threshold than employers in all equilibria with and without intervention.

of our proposed intervention.[18]

Due to the positive same-side effect, numerics show that our intervention performs close to the planner's solution for intermediate values of c (see Figure 3). Yet, despite the intervention, the welfare is significantly lower than that in the planner's solution for small values of c. The reason is that regardless of whether agents on the long side are proposers or recipients, they use a low threshold to account for the chances of leaving unmatched before receiving the next proposal (or the next opportunity to propose). By contrast, in the planner's solution, agents on the long side do not account for the risk of leaving unmatched and utilize a higher threshold. Finally, we note that blocking the long side from proposing increases welfare for small enough[19] c.

Corollary 6 (Welfare impact of intervention)**.** *Fix any* $R > 1.25$*. There exists* $c_{\max} > 0$ *such that for all* $c \in (0, c_{\max})$ *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$ *the intervention of preventing the long side from proposing strictly increases average welfare in equilibrium (the equilibrium is unique both under this intervention and under no intervention).*

We focus on the small-c regime in the next section, where we show that with two tiers of workers, blocking the long side from proposing significantly increases welfare even as screening costs vanish.

4 The effect of multiple agent quality tiers

We now incorporate vertical differentiation into our model, study the inefficiencies that arise in equilibrium, and propose interventions to mitigate them. For ease of exposition, we consider the simplest case in which these inefficiencies arise: a market with one quality tier of employers and two tiers of workers (high and low), where employers arrive faster than high workers but slower than workers overall. (Below we justify why this is the interesting parameter regime.) We find that, in equilibrium, agents lacking market power (low workers) propose without screening even as screening costs vanish because most of their proposals are not merely rejected but completely *ignored*; thus the inefficiencies discussed in Section 3.2 (namely, that agents lacking market power can't be selective) are even more stark in this setting. Also, a new source of inefficiency arises: some agents on the short side leave unmatched, reducing the *number* of matches formed. We identify suitable platform interventions that improve average welfare even in the limit of vanishing screening costs, motivating

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[18] Formally, the selectivity of workers has a positive spillover by increasing the steady-state probability p that a worker will get a proposal before leaving unmatched: we have $\mathrm{p} = 1/R$ if all workers accept proposals without screening and $\mathrm{p} = 1/\big(\theta_w + R(1-\theta_w)\big)$ if all workers screen and accept/reject with threshold θ_w; see Lemma I.1 in the online appendix.

[19] Corollary 6 is established in the online appendix (there we also discuss larger c).

us to study the small-c regime. The new intervention introduced in this section is that the platform should hide quality information from agents on the short side to prevent a fraction of these agents from leaving unmatched. As $c \to 0$, our interventions substantially benefit the weakest agents (low workers) without hurting other agents, thus achieving a Pareto improvement in welfare.

Parameter regime studied. We briefly describe the parameter regime considered in this section. See Appendix E for a more detailed discussion.

We fix the quality $q \in (0,1)$ of high workers. The constraint $q \in (0,1)$ results in distributional "overlap" in the sense that, with positive probability, a particular employer prefers a particular low worker to a particular high worker. (The case $q \geq 1$ produces no new insights so we skip it.)

For simplicity, we focus on small c (we allow larger c in Appendix E). We employ the order notations $o_c(\cdot)$ and $\Theta_c(\cdot)$ to capture scaling as $c \to 0$ (of quantities that do not vary with μ).

Finally, we focus on the regime where employers arrive faster than high workers but slower than workers overall. Specifically, all results in this section hold for $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$ (or a superset of this range). This is the "interesting" range for λ_e: briefly, for smaller λ_e all low workers remain unmatched under no intervention, whereas for larger λ_e, the market resembles that of Section 3 with workers (high + low) being on the short side.

Planner's solution. We now briefly discuss the planner's solution for the markets considered in this section. The set of strategies available to the planner for the workers is $\mathcal{P}_w = \mathcal{P}$ (where $\mathcal{P}$ is defined as in Definition 5), and the set of strategies available for employers, $\mathcal{P}_e$, is defined as

$$\mathcal{P}_e \triangleq \left\{(a, \theta^{\mathrm{H,o}}\theta^{\mathrm{L,o}}, \theta^{\mathrm{H,i}}, \theta^{\mathrm{L,i}}) : \; a = (a^{\mathrm{o}}, a^{\mathrm{i}}) \in \mathcal{A}\,, \; (\theta^{t,\mathrm{o}}, \theta^{t,\mathrm{i}}) \in [q_t, q_t + 1]^2 \text{ for } t \in \{\mathrm{H}, \mathrm{L}\}\right\}\,.$$

That is, $\mathcal{P}_e$ extends $\mathcal{P}$ by allowing employers to use different thresholds for different worker tiers.

Proposition 7 (Planner's solution)**.** *Fix high-worker quality $q \in (0,1)$ and $\lambda_w^{\mathrm{H}} > 0$, $\lambda_w^{\mathrm{L}} > 0$, and $\lambda_e \in (\lambda_w^{\mathrm{H}}, \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$. Let h_w^{H}, h_w^{L}, and h_e denote probability measures over $\mathcal{P}_w$, $\mathcal{P}_w$, and $\mathcal{P}_e$, respectively, chosen by the planner; the planner assigns a strategy to each agent drawn (independently) from the distribution for that tier. Let $\bar{L}(h_w^{\mathrm{H}}, h_w^{\mathrm{L}}, h_e)$ be the induced steady-state agent mix. Consider $c_0(q)$ defined in* (40) *and $g(\cdot)$ defined in* (6) *(Proposition 2). Fix any $c \in (0, c_0(q))$. For any $\mu > 0$ we have the upper bound*

$$\sup_{h_w^{\mathrm{H}}, h_w^{\mathrm{L}} \in \Delta(\mathcal{P}_w), h_e \in \Delta(\mathcal{P}_e)} \text{Avg-welf}\left(\bar{L}(h_w^{\mathrm{H}}, h_w^{\mathrm{L}}, h_e), (h_w^{\mathrm{H}}, h_w^{\mathrm{L}}, h_e)\right) \leq \frac{\lambda_w^{\mathrm{H}} q + \lambda_e g(c)}{\lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}} + \lambda_e}\,. \tag{11}$$

Moreover, the planner can achieve this upper bound as follows. Let $\theta^{\mathrm{i}}(c)$ and $\theta^{\mathrm{o}}(c)$ be defined as in the statement of Proposition 2. Then, agents using the following strategies (as a function of c)

produces a limiting average welfare as $\mu \to 0$ *equal to the upper bound in* (11)*:*

(i) All workers screen and accept/reject with threshold $\theta^{\mathrm{i}}(c)$*. All employers screen and accept/reject with threshold* $\theta^{\mathrm{i}}(c)$ *for low workers and threshold* $q + \theta^{\mathrm{i}}(c)$ *for high workers.*

(ii) High workers screen and propose using threshold $\theta^{\mathrm{o}}(c)$*. Employers screen and propose with a preference for a candidate from the high tier, using thresholds* $q + \theta^{\mathrm{o}}(c)$ *and* $\theta^{\mathrm{o}}(c)$ *to screen high and low workers, respectively. Low workers do not propose.*

The intuition for the upper bound (11) resides in the following two facts. First, to maximize the average welfare, all employers should match with an equal volume of workers, and the workers leaving unmatched should be the low-quality ones as they produce less valuable matches. In other words, there should be a flow (i.e., mass per unit time) λ_w^{H} of matches between employers and high workers, and a total match flow λ_e. Second, the maximum per-match utility between a worker of quality q_w and an employer that the planner can achieve when both proposers and recipients screen is $q_w + g(c)$, and both sides screening is indeed planner-optimal for small enough c (this follows from Proposition 2 and its proof). These two facts together lead to the upper bound (11).

The stated planner policy is one way to achieve this upper bound in the limit $\mu \to 0$. To ensure that all H-workers are matched, the policy makes employers propose with a preference for high-tier candidates. To ensure a per-match utility of $q+g(c)$ for matches involving quality-q workers, agents screen using the thresholds *on the idiosyncratic part of the utility* described in Proposition 2. Thus, while workers use the same thresholds as in Proposition 2, employers use different thresholds for the two worker tiers. See the online appendix for the proof of Proposition 7.

4.1 Equilibrium under no intervention

We now describe the equilibrium in the absence of any intervention.

Theorem 8 (Equilibria under no intervention)**.** *Fix high-worker quality* $q \in (0, 1)$ *and* $\lambda_w^{\mathrm{H}} > 0$*,* $\lambda_w^{\mathrm{L}} > 0$*, and* $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}}(1 + q/2) + \lambda_w^{\mathrm{L}})$*. Then there exists* $c_{\max} > 0$ *such that for all* $c < c_{\max}$*, there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$ *there is a stable equilibrium with the following description:*

(i) Employers do not propose to low workers. When the opportunity arises, employers request a high worker and, if one is available, screen and propose with a threshold equal their expected utility θ_e *that satisfies* $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c}$*. The employers split into two types based on how they respond to proposals from low workers.*

- *Reachers: Reachers use strategy* s^{re} *that ignores proposals from low workers. The fraction of reachers* $f_e(s^{\mathrm{re}})$ *satisfies* $\lim_{\mu\to 0} f_e(s^{\mathrm{re}}) = \frac{\lambda_w^{\mathrm{H}}+\delta^{\mathrm{H}}}{\lambda_e}$ *for* $\delta^{\mathrm{H}} \triangleq \lambda_w^{\mathrm{H}} q/2(1+o_c(1))$*.*
- *Settlers: All other employers use strategy* s^{se}*, which screens incoming proposals from low workers using threshold* θ_e*.*

(ii) High workers do not propose, and screen incoming proposals using a threshold equal to their expected utility $\theta_w^{\mathrm{H}} = 1-\sqrt{2c}$*.*

(iii) Low workers propose without screening and earn expected utility θ_w^{L}*. We have* $\lim_{\mu\to 0}\theta_w^{\mathrm{L}} = \frac{\lambda_e-\lambda_w^{\mathrm{H}}-\delta^{\mathrm{H}}}{2\lambda_w^{\mathrm{L}}}$*.*

The equilibrium structure is similar to the one described in Theorem 4 for unbalanced markets, as agents propose to those with more market power (i.e., low workers propose to employers and employers propose only to high workers). However, there are important differences, which we now explain.[20] Note that, in equilibrium, the expected utility of reachers and settlers must be equal. However, reachers earn a higher match utility of $(1+q+\theta_e)/2$ when they match (and spend less screening effort on average) than the limiting match utility of $(1+\theta_e)/2$ for settlers. This higher match utility is compensated by a significant likelihood of leaving without matching in equilibrium: a limiting fraction[21] $\frac{\delta^{\mathrm{H}}}{\delta^{\mathrm{H}}+\lambda_w^{\mathrm{H}}} = \Theta(1)$ of reachers leave unmatched, and so reachers have an $\Theta(1/\mu)$ expected lifetime in the system, whereas the expected lifetime of settlers in the system is $\Theta(1)$ as each employer receives proposals from low workers at rate $\Theta(1)$, and settlers match after screening $\Theta(1)$ proposals. Thus, a stark *inspection paradox* arises in the equilibrium steady state in our dynamic model: there is a large steady-state mass $\Theta(1/\mu)$ of reachers in the market, in contrast to a much smaller mass $\Theta(1)$ of settlers. As a result of the preponderance of reachers among employers present in the system, there are effectively two independent submarkets in the limit $\mu \to 0$:

1. *"High submarket":* High workers match immediately and leave, and earn expected utility $1-\sqrt{2c}$. All but a vanishing fraction match to reachers. A limiting fraction $\frac{\lambda_w^{\mathrm{H}}}{\lambda_w^{\mathrm{H}}+\delta^{\mathrm{H}}}$ of reachers match with H-workers, and the rest leave unmatched. Reachers have a threshold and expected utility of $\theta_e \xrightarrow{\mu\to 0} 1-\sqrt{2c}$.
2. *"Low submarket":* Settler employers earn the same utility of θ_e, but all but a vanishing fraction match with a low worker. Only a fraction $\frac{\lambda_e-\lambda_w^{\mathrm{H}}-\delta^{\mathrm{H}}}{\lambda_w^{\mathrm{L}}}$ of low workers match, earning an expected match utility of 1/2 each (as they propose without screening), and the rest leave without matching. Thus, the overall expected utility of low workers is $\frac{\lambda_e-\lambda_w^{\mathrm{H}}-\delta^{\mathrm{H}}}{2\lambda_w^{\mathrm{L}}}$.

Both submarkets are unbalanced markets, with reacher employers being on the long side in the high

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[20] Our explanation draws upon Theorem 16, a stronger and more detailed version of Theorem 8; see Appendix E.1.

[21] The notation $\Theta(\cdot)$ captures scaling as $\mu \to 0$.

submarket, and the low workers being on the long side in the low submarket. Thus, the equilibrium behavior of agents in the high submarket resembles that in Equilibrium 1 of Theorem 4, where both sides screen, whereas in the low submarket it resembles that in Equilibrium 2 of Theorem 4, where the long-side agents do not screen; we discuss why in point 2 below.

Equilibrium inefficiencies. By comparing the equilibrium outcome with the planner's solution we observe that, in equilibrium, there is an equal flow λ_w^{H} of matches involving high workers, but producing a smaller welfare per unit of matches, and a smaller flow $\lambda_e - \lambda_w^{\mathrm{H}} - \delta^{\mathrm{H}}$ of matches involving low workers (compared to a flow $\lambda_e - \lambda_w^{\mathrm{H}}$ in the planner's solution), producing a smaller welfare per unit of matches. Thus, the equilibrium just described has the following major inefficiencies:

1. *Employers (short-side agents) leave without matching:* In contrast to the planner's solution, where all employers match, a flow δ^{H} of employers leave without matching. As a consequence, the flow at which low workers match is δ^{H} smaller than that in the planner's solution. This new inefficiency, arising only because workers are vertically differentiated, persists even as $c \to 0$.
2. *Low workers (the weakest long-side agents) cannot be selective* at all*:* This phenomenon occurs due to the above-described inspection paradox that arises in our dynamic setup. As the vast majority of employers present in the market are reachers and reachers *ignore* low-worker proposals, low workers propose without screening. This inefficiency resembles the one described in Section 3.2, where agents on the long side are not selective as they can screen only for small c. However, it is much more striking in this setting: low workers cannot screen even as $c \to 0$.
3. *Employers are not selective enough when considering* H*-workers but are too selective when considering* L*-workers, and* H*-workers are too selective in screening incoming proposals:* Compared to the socially optimal threshold of θ^{i} on idiosyncratic utilities for accepting proposals, both high workers and employers screen incoming proposals using a higher threshold of $1 - \sqrt{2c}$ on idiosyncratic utilities, and hence are too selective. Further, relative to the planner's solution employers use a lower threshold to screen and propose to H-workers; i.e., they are not selective enough, and this inefficiency persists as $c \to 0$, even under platform interventions. Informally, this happens because there are not enough H-workers for all employers to earn utility $\theta_e > 1$ by matching with them (if $\theta_e > 1$, all employers will ignore L-workers). Therefore, it follows that the employer equilibrium utility and threshold must satisfy $\theta_e \leq 1$ whereas in the planner's solution employers use a threshold converging to $1 + q$ as $c \to 0$ when considering H-workers.

The above inefficiencies have a similar flavor to certain phenomena observed in online platforms. Fradkin (2018) finds that, similarly to reacher employers, searchers on Airbnb often leave the market although they could have found a suitable partner (cf. Inefficiency 1), and a similar effect has been uncovered in Upwork (Horton 2019). One-third of men on Tinder report that they casually "Like" most profiles (cf. the equilibrium behavior of low workers, Inefficiency 2) and women on Tinder are much more selective than men: 59% of women (compared to just 9% of men) report that they "Like" less than 10% of all profiles that they encounter (Tyson et al. 2016).

4.2 Interventions to mitigate inefficiencies

Above, we characterized three equilibrium inefficiencies under no intervention. As in Section 3, the third inefficiency cannot be eliminated using the simple interventions we consider. However, our interventions will prove useful in eliminating Inefficiency 1 and mitigating Inefficiency 2.

Blocking workers from proposing. Inefficiency 2 (low workers cannot be selective at all) is an extreme version of the one uncovered in Section 3.2, where long-side agents screen only for small c and, even then, use an inefficiently low threshold. It is natural to ask whether the intervention proposed there (block the long side from proposing) can also alleviate the inefficiency in the current setup. Indeed, we find[22] that when the platform blocks all workers from proposing (regardless of their tier), low workers who receive a proposal do not fear rejection and can be somewhat selective, thus mitigating the inefficiency. In contrast to Section 3, forcing the short side to propose improves the limiting average welfare as $c \to 0$: low workers earn a larger limiting expected utility per match than in the no-intervention case, whereas employers and high workers continue to earn a limiting expected utility of 1 (see Table 1). Therefore, *this intervention produces a Pareto improvement in the limit* $c \to 0$. However, as in Section 3, the inefficiency is not eliminated: low workers are still less selective than in the planner's solution because they are concerned about leaving unmatched.[23]

This intervention resembles the design of Bumble where only *women* are allowed to send the first message, and the revamped design of Coffee Meets Bagel (CMB), where *men* typically first like or pass over candidates ("Bagels"), and then women receive proposals from the men who liked them. Note that Bumble has more male users, whereas CMB has more female users.[24]

Blocking workers from proposing and hiding quality information from employers. Even though blocking workers from proposing mitigates Inefficiency 2 in the resulting equilibrium employers still split into reachers (who only propose to high workers) and settlers (who propose to

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[22] In the interest of space, we defer our formal characterization of equilibrium to Appendix E.2 (Theorem 17).

[23] Note also that the selectivity of low workers has a positive spillover effect on other low workers, similar to our finding with one tier on each side (Remark 2). As in footnote 18, selectivity by low workers has a positive spillover by increasing the probability that a low worker will get a proposal before he leaves without matching.

[24] Source: Business Insider, 2016, `https://www.businessinsider.com/dating-apps-that-have-highest-percentage-of-women-2016-6`.

Intervention	θ_w^{H}	θ_e	θ_w^{L}	Employers leave w/o matching or not	Low workers can choose or not
No intervention	1	1	$1/(2R_\delta^{\mathrm{L}})$	Yes	Can't choose
Workers can't propose	1	1	$1/(2R_\delta^{\mathrm{L}}-1)$	Yes	Can choose
Hide quality of workers, and workers can't propose	1	1	$1/(2R^{\mathrm{L}}-1)$	No	Can choose

Table 1: Expected utility for each agent type, as $\mu \to 0$ and then $c \to 0$, under interventions in markets with vertical differentiation, parameterized by $\lambda_w^{\mathrm{H}}, \lambda_w^{\mathrm{L}}, \lambda_e$, and $q \in (0,1)$, where $\lambda_e \in (\lambda_w^{\mathrm{H}}(1+q/2), \lambda_w^{\mathrm{H}}+\lambda_w^{\mathrm{L}})$, and using $R_\delta^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}}(1+q/2))$ and $R^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}})$. Note that $R_\delta^{\mathrm{L}} > R^{\mathrm{L}} > 1$, and so $1/(2R_\delta^{\mathrm{L}}) < 1/(2R_\delta^{\mathrm{L}}-1) < 1/(2R^{\mathrm{L}}-1)$, i.e., each intervention increases the utility of low workers.

both tiers, with a preference for high workers), and a fraction of reachers still leave unmatched (Inefficiency 1). To address this inefficiency, note that being a reacher is a feasible strategy because employers can observe the quality of the workers and make their actions depend on these qualities, thus treating workers in different tiers asymmetrically. Suppose that, in addition to blocking workers from proposing, the platform does not reveal to employers the tier of a candidate worker (employers are still able to find this out by screening). As workers can't propose, employers will always request to see a candidate in equilibrium and, for small enough c, they will also screen them prior to proposing. As candidates from a specific tier can't be requested and employers are on the short side, candidates are always available. Moreover, as we argued while discussing Inefficiency 3, the equilibrium threshold for employers cannot exceed 1. Thus, after screening a number of candidates a proposal is issued and, after a number of proposals are issued, one gets accepted. Hence, an equilibrium arises in which (as $\mu \to 0$) all employers match before leaving and Inefficiency 1 is eliminated. Moreover, workers cannot propose and the gain from that intervention is still preserved; i.e., Inefficiency 2 is alleviated as workers can be somewhat selective.

We formalize this discussion by characterizing the intervention equilibrium as follows.

Theorem 9 (Equilibria when workers are blocked from proposing and quality information is hidden from employers)**.** *Fix high-worker quality* $q \in (0,1)$ *and* $\lambda_w^{\mathrm{H}} > 0$, $\lambda_w^{\mathrm{L}} > 0$, *and* $\lambda_e \in (\lambda_w^{\mathrm{H}}, \lambda_w^{\mathrm{H}}+\lambda_w^{\mathrm{L}})$. *Define* $R^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}})$. *Then there exists* $c_{\max} > 0$ *such that for any* $c \le c_{\max}$, *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$, *the following unique equilibrium arises (quantified to leading order in small* c*):*

(i) Employers screen and propose with threshold (and expected utility) θ_e, *where* $\lim_{\mu\to 0}\theta_e = 1 - K_e c(1+o_c(1))$ *for* $K_e \triangleq \frac{2(\lambda_e-\lambda_w^{\mathrm{H}})}{q\lambda_w^{\mathrm{H}}} \cdot \frac{2R^{\mathrm{L}}-1}{2(R^{\mathrm{L}}-1)}$.

(ii) High workers screen and accept/reject with threshold (and expected utility) θ_w^{H}, *where* $\lim_{\mu\to 0}\theta_w^{\mathrm{H}} = 1 - K_{\mathrm{H}}\sqrt{c}\,(1+o_c(1))$ *for* $K_{\mathrm{H}} \triangleq \sqrt{\frac{4(\lambda_w^{\mathrm{H}}+\lambda_w^{\mathrm{L}}-\lambda_e)}{q^2\lambda_w^{\mathrm{H}}}+2}$.

(iii) Low workers screen and accept/reject with threshold (and expected utility) θ_w^{L}*, where* $\lim_{\mu\to 0}\theta_w^{\mathrm{L}} = \xi(R^{\mathrm{L}},0)\,(1+o_c(1)) = \frac{1}{2R^{\mathrm{L}}-1}\,(1+o_c(1))$ *for* $\xi(\cdot,\cdot)$ *is defined as in* (10).

The limiting utilities are 1 each for high workers and employers (as has been the case in all settings discussed so far). The low workers' utility is greater than under the "block workers from proposing" intervention as not only are they now able to screen but also, as no employer leaves unmatched, a larger fraction of them find a match. (See the comparison of the limiting utilities in Table 1.) As we previously argued, Inefficiency 3 persists. Relative to the planner's solution, the threshold $\theta_e = 1 - \Theta_c(c)$ is not selective enough for screening H-workers (leading to a reduction in match utility) and too selective for L-workers (leading to wasted screening effort). The loss in welfare from the latter is exacerbated by the fact that L-workers are overrepresented in steady state. This inspection paradox arises because, as employers use the same screening threshold for all workers, given a candidate, an employer is $\frac{1-\theta_e}{1+q-\theta_e} = \Theta_c(c)$ times less likely to propose if the candidate is an L-worker than if it is an H-worker.

The "hiding quality information" intervention fits well with what dating platforms do. For instance, Tinder learns the attractiveness of a user's profile and encodes this in an internal vertical "Elo" rating used to guide its recommendations (Cook 2017), which is not revealed to its users.

5 Discussion

Throughout the paper we focused only on two simple platform interventions, namely, preventing one side of the market from proposing, and hiding quality information of agents on the other side. We also considered some other design levers that may be available to the platform, leveraging our equilibrium characterizations to study their effectiveness:

1. **Artificially inflating screening costs:** Matching platforms may be able to increase screening costs for some agent types, for instance by reducing the ease of viewing profile information of potential matches. We found numerically (using our equilibrium characterizations as a function of c) that this intervention can at best produce only a minimal welfare improvement. These (negative) results are omitted in the interest of space.
2. **Admission control:** Platforms may deny or slow down admission to agents on one side of the market. For example, some dating apps do not grant immediate access; instead, interested users may join a waiting list. We find that in (sufficiently) unbalanced markets with one tier on each side, suitable admission control on the long side together with blocking long-side agents from proposing *eliminates* equilibrium inefficiency in the limit $\mu \to 0$. The following corollary

of Proposition 2 and Theorem 5 formalizes the attractiveness of admission control.

Corollary 10. *Consider a market with one tier on each side where workers arrive faster than employers with* $R = \lambda_w/\lambda_e \geq 1.28$ *and any screening cost* $c < 1/32$. *There exists a unique* $R^* = R^*(c) \in (1, 1.28)$ *which solves* $\xi(\cdot, c) = \theta^{\mathrm{i}}(c)$. *(Recall* $\xi(\cdot, c)$ *defined in* (10) *and* $\theta^{\mathrm{i}}(c)$ *and* $\theta^{\mathrm{o}}(c)$ *defined in* (7)*.) The dual platform intervention of blocking workers from proposing and limiting the flow of admitted workers to* $R^*\lambda_e$ *leads to the following unique equilibrium for small enough* μ*: (employers screen + propose, workers screen + accept/reject) with thresholds which satisfy* $\lim_{\mu\to 0} \theta_w = \theta^{\mathrm{i}}(c)$, *and* $\lim_{\mu\to 0} \theta_e = \theta^{\mathrm{o}}(c)$. *The limiting average welfare in equilibrium as* $\mu \to 0$ *is equal to that under the planner's solution.*

The intuition for this result is as follows. When workers are admitted at rate $R^*\lambda_e$ and employers are forced to propose, for workers, the following two factors perfectly offset each other in equilibrium in the limit $\mu \to 0$: (i) the tendency to be too selective because they only *receive* proposals and hence face no risk of rejection (see inefficiency 1 in Section 3.2), (ii) the risk that they will be left unmatched. As a result, workers use an equilibrium screening threshold for incoming proposals equal to the efficient threshold $\theta^{\mathrm{i}}(c)$. Employers (short-side agents) face vanishing risk of being left unmatched as $\mu \to 0$, and their selectivity while proposing has no spillover effect on other agents, hence their best response screening threshold converges to the planner's screening threshold for outgoing proposals $\theta^{\mathrm{o}}(c)$. The proof of Corollary 10 is in the online appendix.

Throughout the paper we focused on a setting without prices, which captures many platforms of interest (such as dating platforms). Our model can also be used to study a setting with prices where the prices are chosen by the platform. In such settings, one could think of a price as being part of the "quality" term (that is, prices act by shifting the value distribution of agents), and our equilibria characterization (with and without intervention) can be leveraged to study other objectives such as maximizing revenue. We leave the question of analyzing markets where agents can also choose prices as an interesting avenue for future research.

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Appendix to "Facilitating the Search for Partners on Matching Platforms"

Roadmap for the appendix. The organization of the appendix is as follows.

Appendix A provides a list of the notation introduced throughout the paper.

Appendix B contains supplementary material to the model section (Section 2), primarily some formal definitions (complementing the descriptions provided in Section 2).

Appendix C provides some useful lemmas that are used repeatedly throughout the proofs.

Appendix D provides the proofs for the equilibrium results in Section 3: equilibria in balanced markets, no-intervention equilibria in unbalanced markets, and equilibria under the "block workers from proposing" intervention.

Finally, Appendix E supplies the proofs for the equilibrium results in Section 4 which considers the setting with two tiers of workers, high workers and low workers, and a single tier of employers.

The online appendix. In the interest of space, we have relegated some proofs to a online appendix. Whenever a proof can be found in the online appendix, we make this clear before or after stating the result. Each proof we have chosen to relegate is either similar to some other proof in the main appendix, or follows a standard argument, or deals with a specific technical aspect that does not directly contribute to the main message of the paper. We also opted to put our characterizations of the planner's solutions in the online appendix.

A List of notation and key quantities

Notation introduced in Section 2 (model)

- H (high) and L (low): worker quality tiers, when there are two tiers of workers.
- λ_w^t (λ_e): arrival rate of tier-t workers (employers).
- μ: (exogenous) rate at which agents leave without matching ($\mu > 0$).
- τ: used to denote time, whenever needed.
- v_{ij}: match value that agent i has for agent j on the other side. We assume $v_{ij} \triangleq q_j + u_{ij}$, where q_j is the quality of agent j.
- u_{ij}: idiosyncratic utility that i derives from j.
- q: The quality of H-workers. All other agents have quality 0.
- c: screening cost ($c > 0$).
- $s \triangleq (a, \underline{\theta})$: Agent strategies defined by:
  1. $a \triangleq (a^{\mathrm{o}}, a^{\mathrm{i}})$: deterministic set of actions.
  2. $\underline{\theta} \triangleq (\theta^{\mathrm{o}}, \theta^{\mathrm{i}})$ deterministic acceptability thresholds used to screen candidates (θ^{o}) and incoming proposals (θ^{i}). Employers have one such pair of thresholds for each tier of workers.
- $\mathcal{A}_w$, $\mathcal{A}_e$: set of allowed actions a's for workers and employers, respectively, under the chosen platform intervention (if any).
- $\mathcal{S}_w^t$, $\mathcal{S}_e$: set of allowed strategies for tier-t workers and employers, respectively, under the chosen platform intervention (if any). All allowed strategies must use actions from the allowed action sets $\mathcal{A}_w$, $\mathcal{A}_e$.
- $\bar{N}_e(S')$: the measure over employer following strategies in $S' \subseteq \mathcal{S}_e$ at a given time. (We omit dependence on time to avoid cumbersome notation). The notation $\bar{N}_w^t(S)$ for $t \in \{\mathrm{H}, \mathrm{L}\}$ and $S \subseteq \mathcal{S}_w$ is defined similarly.
- $\bar{N} = (\bar{N}_e, \bar{N}_w^{\mathrm{H}}, \bar{N}_w^{\mathrm{L}})$ (*agent mix*): measure over agent strategies and tiers present in the system at a given time (e.g, when an agent i enters).

- $\bar{f} = (f_e, f_w^{\mathrm{H}}, f_w^{\mathrm{L}})$ (*strategy distributions for new arrivals*): distributions over strategies at a given time such that, each employer arriving at the given time picks a strategy drawn independently from a probability measure f_e over $\mathcal{S}_e$, and analogously for workers.
- $U_w^t(s_w, \bar{N}, \bar{f})$ (*anticipated utility*): expected lifetime utility of a tier-t worker following s_w assuming that $\bar{N}$ and $\bar{f}$ at the time of his arrival remain constant during his lifetime. The anticipated utility will be identical to the realized expected utility of the agent in steady state.
- $\rho_w^t(\mathcal{S}, \bar{N}, \bar{f})$ (*match flow*): flow (mass per unit time) of matches involving workers in tier t following strategy $s_w \in \mathcal{S}$, for any measurable subset $\mathcal{S} \subseteq \mathcal{S}_w$,
- $\mathcal{S}_w(\theta_w^t)$ (*set of threshold-consistent strategies*): for a given screening threshold for tier-t workers θ_w^t, $\mathcal{S}_w(\theta_w^t)$ is discrete finite set defined as

$$\mathcal{S}_w(\theta_w^t) \triangleq \{s = (a, \theta) : \ a \in \mathcal{A}_w \ \text{ and } \ \theta^{\mathrm{i}} = \theta_w^t \text{ if } a^{\mathrm{i}} = \mathrm{S{+}A/R} \ \text{ and } \ \theta^{\mathrm{o}} = \theta_w^t \text{ if } a^{\mathrm{o}} = \mathrm{S{+}P}\},$$

- $\bar{L} \triangleq \left(\bar{L}_w^{\mathrm{H}}, \bar{L}_w^{\mathrm{L}}, \bar{L}_e\right)$ (*steady-state agent mix*): measure over agent strategies and tiers present in the system in steady state. Given a time invariant $\bar{f}$, a steady state is an attractive fixed point of the dynamics (2) and (3).
- Avg-welf$(\bar{L}, \bar{f})$: average welfare in steady state.

Notation introduced in the appendix to the model section (specifically, in Appendix B.1), and used in the appendices only

- G: cumulative distribution of the idiosyncratic utility term of workers over employers (and employers over workers), i.e., (the cumulative distribution of) Uniform$(0,1)$.
- $\kappa_w^t\left(\cdot\,; s_w, \bar{N}, \bar{f}\right)$ (*candidate arrival rates*): arrival-rate measure over candidate strategies s_e in the arrival of candidates to a tier-t worker following strategy s_w. That is, for any measurable subset $\mathcal{S}' \subseteq \mathcal{S}_e$, $\kappa_w^t\left(\mathcal{S}'; s_w, \bar{N}, \bar{f}\right)$ denotes the hazard rate that the worker is presented with a candidate employer with strategy $s_e \in \mathcal{S}'$.
- $\hat{\kappa}_w^t\left(\cdot\,; s_w, \bar{N}, \bar{f}\right)$ (*rates of being shown as a candidate*): show-as-candidate-rate measure over employer strategies s_e in the *display* of a tier-t worker following strategy s_w as a candidate to employers; i.e., for any measurable subset $\mathcal{S}' \subseteq \mathcal{S}_e$, $\hat{\kappa}_w^t\left(\mathcal{S}'; s_w, \bar{N}, \bar{f}\right)$ is the hazard rate at which such a worker is shown as a candidate to employers with strategy $s_e \in \mathcal{S}'$.
- $\eta_w^t(s_w, s_e)$(*match success probability*): probability that, when a tier-t worker following s_w is presented with a candidate employer following s_e, this results in a match.
- $\nu_w^t(\bar{N}, \bar{f})$ (*proposal arrival rate*): rate at which a tier-t worker receives proposals.
- N_e: the total mass of employers present, i.e., $N_e \triangleq \int_{\mathcal{S}_e} d\bar{N}_e(s)$. Similarly, N_w^t is the total mass of tier $t \in \{\mathrm{H}, \mathrm{L}\}$ workers present, and N_w is the total mass of workers present (irrespective of tier).
- L_e: the total steady-state mass of employers present, i.e., $L_e \triangleq \int_{\mathcal{S}_e} d\bar{L}_e(s)$. Similarly, L_w^t is the total steady-state mass of tier $t \in \{\mathrm{H}, \mathrm{L}\}$ workers present.

We omit from this list the notation used locally in Appendix B.3 to formally specify the agent's MDP, since that notation is not reused elsewhere.

Notation and key quantities in Section 3 (ex-ante homogeneous agents on each side)

- $\mathcal{P}$: the set of planner strategies as given in Definition 5.
- $g(c)$: Welfare per match achievable by the planner, skipping the quality terms in the match utility (see Proposition 2).
- h_w, h_e: probability measures over $\mathcal{P}$ chosen by the planner, representing the fraction of the mass of workers and employers who adopt each strategy upon entering (see Proposition 2).
- $\tilde{c}$: defined in Proposition 2. Interpretation: largest c for which the planner wants both proposer and recipient to screen.
- $R \triangleq \lambda_w/\lambda_e \geq 1$: The "imbalance", i.e., ratio of arrival rates on the two sides.

The next three quantities are conceptually introduced in Lemma D.6 in Appendix D.1, to characterize equilibria between workers (long side) in a hypothetical setting where the employers (short side) mechanically accept proposals without screening and do not propose. We include their interpretation in this hypothetical setting:

- $\underline{c}$: defined in (10). Interpretation: smallest c for which there is an equilibrium where the long side proposes without screening.
- $\xi(R, c)$: defined in (10). Interpretation: screening threshold on the long side, given imbalance R and screening cost c.
- $\bar{c}$: defined in (10). Interpretation: largest c for which there is an equilibrium where the long side screens and proposes.

The above three quantities appear in the statements of Theorem 4 (where the quantities are suitably adapted, as per Lemma D.7, to account for the employers screening incoming proposals with threshold $\theta_e = 1 - \sqrt{2c}$) and Theorem 5 (the connection is that as $\mu \to 0$, the setting of Theorem 5 where employers are forced to propose closely resembles the aforementioned hypothetical setting from the perspective of workers).

- $\hat{c}$: defined in Theorem 5. Interpretation: largest c for which the short side is willing to screen before proposing, given that the long side will screen and accept/reject. Lemma D.8 formalizes the origins of $\hat{c}$ and simultaneously formalizes why $\lim_{\mu \to 0} \theta_e = 1 - \sqrt{2c/(1 - \xi(R, c))}$ in equilibrium.

Key quantities in Section 4 (vertical differentiation)

- δ^{H}: flow (i.e., mass per unit time) of Reachers who will leave without matching in equilibrium.
- $R_\delta^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}}(1 + q/2))$. Interpretation: imbalance in the average submarket as $c \to 0$ when worker quality information is visible.
- $R^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}})$. Interpretation: imbalance in the average submarket as $c \to 0$ when the platform hides worker quality information from employers.
- K_e and K_{H}: defined in Theorem 9. (Constants that capture the leading order c dependence of the equilibrium match utility thresholds under the proposed dual intervention.)

B Appendix to Section 2: Model

Roadmap for Appendix B. This appendix contains the supplementary material to the model section, Section 2, primarily some formal definitions (complementing the descriptions provided in Section 2). It is divided into subsections as follows.

Appendix B.1 introduces notation for some rates which quantify the search for partners on the platform, including the rates at which an agent receives proposals and sees candidates.

Appendix B.2 formally specifies the consistency requirements, and then shows that with these requirements in place, the aforementioned rates are uniquely determined. The latter fact is established by computing the aforementioned rates explicitly. The calculations are not particularly involved, but the cases where the total mass of one agent type (i.e., low workers, high workers, or employers) is zero need to be handled carefully.

Appendix B.3 shows that the agent faces a Markov decision process (MDP) and provides a full specification of this MDP. The specification is routine, but somewhat lengthy and tedious. We leverage the agent's MDP to prove Proposition 1 (agents use screening thresholds equal to their anticipated utility in any best response) in the online appendix.

Appendix B.4 provides a formal definition of our notion of best response.

Finally, Appendix B.5 formally defines and discusses evolutionary stability of equilibria.

B.1 Rates quantifying the search process

In this section we introduce rates quantifying the search for partners on the platform. To be clear, these rates are not model primitives but rather are system-level derived quantities. We will formally specify two consistency requirements in Appendix B.2, and with these requirements in place we will be able to explicitly express these rates in terms of model primitives. These rates will be used to define the agent's Markov decision process in Appendix B.3, and will feature in many of our computations throughout the paper, starting with the proof of Proposition 1. Note that since the idiosyncratic utilities $u_{ij} \sim \text{Uniform}(0,1)$, it holds that $\Pr(u_{ij} < \theta) = \theta$ for $\theta \in (0,1)$, which can potentially obscure the fact that a particular term in a calculation or definition captures $\Pr(u_{ij} < \theta)$ and not something else. To make it completely clear when we are referring $\Pr(u_{ij} < \theta)$, in this appendix section we use G to denote the cumulative distribution of u_{ij}, i.e., $G(\theta) = \theta$ for $\theta \in [0,1]$.

We now define measures κ and $\hat{\kappa}$ which allow us to capture the progress of the search process: the rates at which an agent sees candidates who are following different strategies, and the rates at which an agent is shown as a candidate to others, respectively. Consider a tier-t worker following strategy s_w. Let $\kappa_w^t\left(\,\cdot\,; s_w, \bar{N}, \bar{f}\right)$ be the arrival-rate measure over candidate strategies s_e in the arrival of candidates to a tier-t worker following strategy s_w. That is, for any measurable subset $\mathcal{S}' \subseteq \mathcal{S}_e$, $\kappa_w^t\left(\mathcal{S}'; s_w, \bar{N}, \bar{f}\right)$ denotes the hazard rate that the worker is presented with a candidate employer with strategy $s_e \in \mathcal{S}'$. We refer to the κs as the *candidate arrival rates.* Let $\hat{\kappa}_w^t\left(\,\cdot\,; s_w, \bar{N}, \bar{f}\right)$ be the show-as-candidate-rate measure over employer strategies s_e in the *display* of a tier-t worker following strategy s_w as a candidate to employers; i.e., for any measurable subset $\mathcal{S}' \subseteq \mathcal{S}_e$, $\hat{\kappa}_w^t\left(\mathcal{S}'; s_w, \bar{N}, \bar{f}\right)$ is the hazard rate at which a worker following s_w is shown as a candidate to employers with strategy $s_e \in \mathcal{S}'$. In words, κ captures the measure-rate at which a given agent sees candidates and $\hat{\kappa}$ the measure-rate as which he is shown as a candidate. Analogously, for an employer following s_e, let $\kappa_e^t\left(\,\cdot\,; s_e, \bar{N}, \bar{f}\right)$ be the arrival-rate measure over candidate strategies in the arrival of candidates from worker-tier t and $\hat{\kappa}_e^t\left(\,\cdot\,; s_e, \bar{N}, \bar{f}\right)$ be the show-as-candidate-rate measure over worker strategies in being displayed as a candidate to a tier-t worker.

Let $\eta_w^t(s_w, s_e)$ be the *match success probability* defined as the probability that, when a tier-t worker following s_w is presented with a candidate employer following s_e, this results in a match:

$$\eta_w^t(s_w, s_e) = \begin{cases} 0 \qquad \text{if } s_w \text{ does not propose or } s_e \text{ ignores proposals}\,, \\ \big(1 - G(\theta^{\mathrm{o}}(s_w))\mathbb{I}(s_w \text{ involves S+P})\big)\big(1 - G(\theta^{t,\mathrm{i}}(s_e) - q_w^t)\mathbb{I}(s_e \text{ involves S+A/R tier t})\big) \\ \qquad\qquad\qquad \text{otherwise}\,. \end{cases} \tag{12}$$

That is, a match can occur only if the worker proposes (with or without screening) and the employer does not ignore the incoming proposal. In that case, the probability of a match is the product of (i) the probability that, when presented with a candidate, a worker following strategy s_w proposes and (ii) the probability that an employer following s_e accepts such a proposal (recall that we have assumed u_{ij} independent of u_{ji}). Analogously, we define $\eta_e^t(s_e, s_w)$ as the probability that when an employer following s_e is presented with a tier-t worker following s_w, this results in a match.

Let $\nu_w^t(\bar{N}, \bar{f})$ denote the rate at which a worker in tier t receives proposals. Incoming proposals result from being shown as a candidate, upon which the employer proposes with probability $1 - G(\theta^{\mathrm{o}}(s_e) - q_w^t)\mathbb{I}(s_e \text{ involves S+P})$ if her strategy is s_e. Hence, the rate of receiving incoming proposals is

$$\nu_w^t(\bar{N}, \bar{f}) = \int_{\mathcal{S}_e} \big(1 - G(\theta^{\mathrm{o}}(s_e) - q_w^t)\mathbb{I}(s_e \text{ involves S+P})\big)\, d\hat{\kappa}_w^t(s_e; s_w, \bar{N}, \bar{f})\,, \tag{13}$$

where the integral is with respect to s_e. Similarly, let $\nu_e^t(\bar{N}, \bar{f})$ denote the rate at which an employer receives proposals from tier-t workers. We have

$$\nu_e^t(\bar{N}, \bar{f}) = \int_{\mathcal{S}_w} (1 - G(\theta^{\text{o}}(s_w))\mathbb{I}(s_w \text{ involves S+P}))\, d\hat{\kappa}_e^t(s_w; s_e, \bar{N}, \bar{f})\,, \tag{14}$$

Note that the flow (mass per unit time) of matches $\rho_w^t(\mathcal{S}, \bar{N}, \bar{f})$ involving workers in tier t following strategy $s_w \in \mathcal{S}$, for any measurable subset $\mathcal{S} \subseteq \mathcal{S}_w$, can be written as

$$\rho_w^t(\mathcal{S}, \bar{N}, \bar{f}) = \int_{\mathcal{S}_e} \int_{\mathcal{S}} \big(\kappa_w^t(s_e; s_w, \bar{N}, \bar{f})\eta_w^t(s_w, s_e) + \kappa_e^t(s_w; s_e, \bar{N}, \bar{f})\eta_e^t(s_e, s_w)\big) d\bar{N}_w^t(s_w) d\bar{N}_e(s_e) \tag{15}$$

where the first term in the integral captures the match flow due to outgoing proposals and the second term captures the match flow due to incoming proposals. Similarly, the flow of matches $\rho_e(\mathcal{S}', \bar{N}, \bar{f})$ involving employers following strategy $s_e \in \mathcal{S}'$, for any measurable subset $\mathcal{S}' \subseteq \mathcal{S}_e$ is

$$\begin{aligned} &\rho_e(\mathcal{S}', \bar{N}, \bar{f}) = \\ &\sum_{t \in \{\text{H}, \text{L}\}} \int_{\mathcal{S}'} \int_{\mathcal{S}_w} \big(\kappa_w^t(s_e; s_w, \bar{N}, \bar{f})\eta_w^t(s_w, s_e) + \kappa_e^t(s_w; s_e, \bar{N}, \bar{f})\eta_e^t(s_e, s_w)\big) d\bar{N}_w^t(s_w) d\bar{N}_e(s_e) \end{aligned} \tag{16}$$

Finally, we use N_e to denote the total mass of employers present, i.e., $N_e \triangleq \int_{\mathcal{S}_e} d\bar{N}_e(s)$. Similarly, N_w^t will denote the total mass of tier $t \in \{\text{H}, \text{L}\}$ workers present, and $N_w = N_w^{\text{H}} + N_w^{\text{L}}$ will denote the total mass of all workers present. We use L_e (and similarly, L_w^t and L_w) to denote the total steady-state mass of employers (tier-t workers and all workers) present, i.e., $L_e \triangleq \int_{\mathcal{S}_e} d\bar{L}_e(s)$.

B.2 Consistency Requirements

For our model to be fully specified, the rates quantifying the search and matching process on the platform (these rates were introduced in the previous subsection) must be uniquely determined by the system state $\bar{N}$ and $\bar{f}$. In our continuum model we need two additional consistency requirements to complete our model specification and uniquely determine these rates.

We now formally state the consistency requirements introduced informally in Section 2. These requirements are in force throughout the paper.

Requirement 2 (Mass balancing)**.** *Fix any measure $\bar{N}$ of agents in the system, and any distributions $\bar{f}$ over strategies picked by entering agents. For any worker tier t and any measurable subsets $\mathcal{S} \subseteq \mathcal{S}_w$ and $\mathcal{S}' \subseteq \mathcal{S}_e$, it must be that the flow of workers in tier t following a strategy in $\mathcal{S}$* being shown *(as per $\hat{\kappa}$) to employers following a strategy in $\mathcal{S}'$ equals the flow of workers in tier t following a strategy in $\mathcal{S}$* seen *(as per κ) by employers following a strategy in $\mathcal{S}'$, i.e.,*

$$\int_{\mathcal{S}} \hat{\kappa}_w^t(\mathcal{S}'; s_w, \bar{N}, \bar{f}) d\bar{N}_w^t(s_w) = \int_{\mathcal{S}'} \kappa_e^t(\mathcal{S}; s_e, \bar{N}, \bar{f}) d\bar{N}_e(s_e)\,. \tag{17}$$

Analogously for employers being shown to workers of any tier t, and any measurable $\mathcal{S} \subseteq \mathcal{S}_w$ and $\mathcal{S}' \subseteq \mathcal{S}_e$,

$$\int_{\mathcal{S}'} \hat{\kappa}_e^t(\mathcal{S}; s_e, \bar{N}, \bar{f}) d\bar{N}_e(s_e) = \int_{\mathcal{S}} \kappa_w^t(\mathcal{S}'; s_w^t, \bar{N}, \bar{f}) d\bar{N}_w^t(s_w^t)\,. \tag{18}$$

Requirement 3 (Scarce candidates are spread equally)**.** *Fix time τ, $\bar{N}$, $\bar{f}$ and a tier t of workers such that $N_w^t = 0$ while all other agent tiers (including employers) have a strictly positive mass of* *agents.*[25] *For any worker i in tier t, the hazard rate of i being shown as a candidate to an employer following a strategy $s_e \in \mathcal{S}' \subseteq \mathcal{S}_e$ is proportional to $\bar{N}_e(\mathcal{S}' \cap \mathcal{S}_e(t,1))$, where $\mathcal{S}_e(t,1)$ is the set of employer strategies which ask for a tier-t worker candidate as the first choice.*[26]

We make an analogous assumption for the case $N_e = 0$ and $N_w > 0$, about employers being shown as candidates to workers.

Informally, the above requirements ensure that our continuum model correctly captures the limit of a large market with discrete agents. The reasoning is that in a finite market mass balance obviously holds, whereas scarce candidates will be spread equally if the opportunity clocks of agents requesting them are Poisson with identical rates. We remark that consistency requirements are typically needed in continuum models in the search and matching literature (e.g., see Eq. (2) in Duffie et al. 2018).

We now return to our formal development. With Requirements 2 and 3 in place, the rates $(\kappa_w^t, \kappa_e^t, \hat{\kappa}_w^t, \hat{\kappa}_e^t)$ and also the νs and ρs are uniquely determined by agent strategies. This fact is formalized in the following proposition.[27] The proof of the proposition obtains these rates explicitly as a function of $\bar{N}, \bar{f}$.

Proposition 11. *Under Requirements 2 and 3, for any given $\bar{N}, \bar{f}$ such that at most one tier of agents has mass 0, for each tier t of workers the rates $(\kappa_w^t, \kappa_e^t, \hat{\kappa}_w^t, \hat{\kappa}_e^t)$ are uniquely determined. The rates ν_w^t and ν_e^t for each tier t, and the match flows ρ_w^{H}, ρ_w^{L} and ρ_e are also uniquely determined.*

The proof of the proposition is fairly straightforward and can be found in the online appendix. We show that $(\kappa_w^t, \kappa_e^t, \hat{\kappa}_w^t, \hat{\kappa}_e^t)$ are uniquely determined, by establishing this in each of the two relevant cases —when there is a positive mass of agents in each tier, and when one of the tiers has a mass zero of agents. We then deduce that νs and ρs are also uniquely determined.

B.3 The agent's Markov decision process

In this subsection, we show that the agent's problem (under the assumption that $\bar{N}$ and $\bar{f}$ are time invariant) can be written as a standard Markov decision process (MDP), and provide a full description of this MDP. This transformation is not particularly involved but it does require several steps, which makes this a lengthy subsection. We consider a worker in tier t (the MDP for an employer is analogous), and opt to specify the MDP for a *fixed* vector of actions $a_w \in \mathcal{A}_w$ chosen by the worker[28] since the resulting action space is simpler, and this prevents the cumbersome MDP definition from being even more unwieldy. In proving our results, e.g., Proposition 1, we consider (or find) the $a_w \in \mathcal{A}_w$ such that the value of the corresponding MDP is the largest. A viable alternative would have been to incorporate a_w in the action space of the MDP defined.

Fix the vector of actions $a_w = (a_w^{\mathrm{i}}, a_w^{\mathrm{o}}) \in \mathcal{A}_w$ used by the worker to handle incoming proposals and opportunities.[29] This fixes whether the worker considers opportunities and, if so, whether he screens them, and whether he considers proposals and, if so, whether he screens them. We now observe the following.

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[25] We have avoided formalizing the appropriate version of Assumption 3 for the case where multiple tiers of agents have zero mass since such a case does not arise in the present paper.

[26] Note that this condition already holds by definition in our model in the case $N_w^t > 0$, since the opportunity clock of each employer rings at the same rate of 1, and the employer is presented with a uniformly random candidate from tier t. Assumption 3 eliminates ambiguity in the case $N_w^t = 0$, where our continuum model would otherwise have allowed employers with different strategies to see different measures over candidate strategies (and availability).

[27] The proposition covers only the relevant cases, i.e., where at most one agent tier has zero mass, see footnote 25.

[28] See Definition 1 part 1; $a_w = (a_w^{\mathrm{i}}, a_w^{\mathrm{o}})$ captures how the worker handles incoming proposals and opportunities.

[29] In the construction for employers, the preference order $\succ$ over tiers when an opportunity arises is also fixed.

Observation B.1. *Consider two strategies* $s_w = (a_w, \Theta_w)$ *and* $s'_w = (a_w, \Theta'_w)$ *that have the same set of actions but may differ on the thresholds used. Then, for any given* $\bar{N}$ *and* $\bar{f}$*, the candidate arrival-rate measures* $\kappa^t_w\left(\,\cdot\,; s_w, \bar{N}, \bar{f}\right)$ *and* $\kappa^t_w\left(\,\cdot\,; s'_w, \bar{N}, \bar{f}\right)$ *are identical, i.e., for any measurable subset* $\mathcal{S} \subseteq \mathcal{S}_e$*, we have* $\kappa^t_w\left(\,\cdot\,; s_w, \bar{N}, \bar{f}\right) = \kappa^t_w\left(\,\cdot\,; s'_w, \bar{N}, \bar{f}\right)$*.*

This can be readily seen from the definition of κs in Appendix B.2. Intuitively, having fixed $\bar{N}$ and $\bar{f}$, the rate at which an agent sees candidates will depend only on whether he requests to see candidates or not. Hence, throughout the rest of the proof, we slightly abuse notation and write the candidate arrival rate measures using the action a_w instead of the whole strategy s_w, i.e., we write $\kappa^t_w\left(\,\cdot\,; a_w, \bar{N}, \bar{f}\right)$.

Note that, fixing $\bar{N}$, $\bar{f}$ and a_w also fixes the arrival-rate measure $\kappa^t_w\left(\,\cdot\,; a_w, \bar{N}, \bar{f}\right)$ over candidate strategies s_e, and the rate $\nu^t_w(\bar{N}, \bar{f})$ of receiving proposals as described in Appendix B.1.

We now show how to formulate the worker's problem (for a worker who assumes stationarity) as an infinite horizon, discrete time, time-invariant Markov decision process (MDP), where his decisions are whether to propose/accept each time he receives an opportunity or proposal and its value is revealed by screening. The states, actions, payoffs, and state transition probabilities for the worker's MDP are as follows.

Epochs and states. Consider the discrete epochs $n = 1, 2, \ldots$ when the worker receives an opportunity or a proposal. For each epoch n, the state x_n belongs to the (epoch) state space

$$\mathcal{X} = \{(\mathrm{IP}, v_{ij}) : v_{ij} \in \mathbb{R} \cup \{\mathrm{NULL}\} \cup \{(\mathrm{O}, v_{ij}) : v_{ij} \in \mathbb{R} \cup \{\mathrm{NULL}\}\} \cup \{\mathrm{Matched}\}\,,$$

where IP denotes incoming proposal, O denotes an opportunity, and $v_{ij} \in \mathbb{R}$ is the match value if the proposer/candidate was screened under a_w and NULL otherwise.[30] Given a state $x \in \mathcal{X}$, let $\Pi(x)$ denote the set of actions available to the worker under that state. The worker's action π_n at epoch n must be chosen from those available in the current state x_n, that is, $\pi_n \in \Pi(x_n)$. We define $\Pi(x)$ as follows:

$$\Pi(x) = \begin{cases} \{\mathrm{Accept}, \mathrm{Reject}\} \qquad \text{if } x = (\mathrm{IP}, v_{ij}) \text{ for some } v_{ij} \in \mathbb{R} \text{ (screened IP)}\,, \\ \{\mathrm{Propose}, \text{Do Not Propose (DNP)}\} \quad \text{if } x = (\mathrm{O}, v_{ij}) \text{ for some } v_{ij} \in \mathbb{R} \text{ (screened O)} \\ \emptyset \qquad \text{otherwise} \end{cases},$$

That is, if an incoming proposal was received there are two possibilities: (i) if $a^{\mathrm{i}}_w = \mathrm{S{+}A/R}$ the proposal was automatically screened revealing the value v_{ij}, and the worker can choose whether to accept/reject ; (ii) otherwise, if $a^{\mathrm{i}}_w \neq \mathrm{S{+}A/R}$ the proposal was not screened and no action is available to him as acceptance/rejection is fully specified by a^{i}_w. Similarly, if an opportunity was just received: (i) if $a^{\mathrm{o}}_w = \mathrm{S{+}P}$ the opportunity was automatically screened and the worker can choose whether to propose or not; (ii) if $a^{\mathrm{o}}_w \neq \mathrm{S{+}P}$, he will automatically propose or not without screening as per a^{o}_w. Finally, if the worker is already Matched (or if his action has been determined by a_w), then he cannot choose an action. In all cases that $\Pi(x_n) = \emptyset$, we write that $\pi_n = \mathrm{NULL}$.

Post-epoch states. We find it convenient to also define "post-epoch" states that serve as a tool for specifying the state transition probabilities. This approach also allows our proof to illuminate the fundamental reason that the proposition holds —*the system regenerates each time the worker returns to waiting for an epoch.* An equivalent proof can be written without ever defining these states, but may be harder to parse.

Formally, to every epoch n, we associate a"post-epoch" state x_{n+} which belongs to the set $\{\mathrm{Waiting}, \mathrm{Matched}\}$. We specify that x_{n+1} is conditionally independent of (x_n, π_n) (and everything

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[30] In the case of employers, the state also includes the tier of proposer/candidate.

else up to epoch n) given x_{n+}. It thus suffices to specify the distributions $\Pr(x_{n+}|x_n, \pi_n)$ as well as the probability measure $\Pr(\cdot|x_{n+})$ of the state $x_{n+1} \in \mathcal{X}$. We start by specifying the latter.

Probability measure $\Pr(\cdot|x_{n+})$ *of the state* $x_{n+1} \in \mathcal{X}$. First, if $x_{n+} = \text{Matched}$ then, deterministically, $x_{n+1} = \text{Matched}$, i.e., $\Pr(x_{n+1} = \text{Matched}|x_{n+} = \text{Matched}) = 1$.

Second, towards specifying the distribution over x_{n+1} when $x_{n+} = \text{Waiting}$, recall that in the original system we had that (1) candidates arrive to worker i at rate $\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)$; (2) Proposals arrive to worker i at rate $\nu_w^t(\bar{N}, \bar{f})$; and (3) The worker leaves without matching at hazard rate μ. In defining our MDP, rather than introducing a state Left-without-matching to which the worker transitions from the Waiting state with probability $\frac{\mu}{\mu+\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)+\nu_w^t(\bar{N}, \bar{f})}$, we adopt the equivalent formulation by defining a discount factor

$$r \triangleq 1 - \frac{\mu}{\mu + \kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right) + \nu_w^t(\bar{N}, \bar{f})} \tag{19}$$

per epoch (equal to the probability of not leaving without matching before the next proposal/candidate) and define the distribution of x_{n+1} given $x_{n+} = \text{Waiting}$ as follows:[31]

1. x_{n+1} is an incoming proposal epoch: With probability $\frac{\nu_w^t(\bar{N})}{\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)+\nu_w^t(\bar{N}, \bar{f})}$, we have $x_{n+1} = (\text{IP}, v_{ij})$ for some $v_{ij} \in \mathbb{R} \cup \{\text{NULL}\}$. If a_w does not specify to screen incoming proposals, then $v_{ij} = \text{NULL}$ with probability 1. If a_w specifies to screen incoming proposals, then the second state coordinate v_{ij} is drawn from the distribution $\text{Uniform}(0, 1)$.
2. x_{n+1} is an opportunity epoch: With the remaining probability $\frac{\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)}{\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)+\nu_w^t(\bar{N}, \bar{f})}$, we have $x_{n+1} = (\text{O}, v_{ij})$ for some $v_{ij} \in \mathbb{R} \cup \{\text{NULL}\}$. If a_w does not specify to screen candidates, then $v_{ij} = \text{NULL}$ with probability 1. If a_w does specify to screen incoming proposals, then the second state coordinate v_{ij} is drawn from the distribution $\text{Uniform}(0, 1)$.

We make here the key observation that will lead to the characterization of acceptability thresholds in the proposition: *the Waiting state causes the system to regenerate.* As a result, the worker's best response will be to attempt to match (Accept/Propose) if the match utility exceeds the continuation value from Waiting, and to pass otherwise (hence transitioning to Waiting).

The above completes our specification of the probability measure $\Pr(\cdot|x_{n+})$ of the epoch state x_{n+1}. We further define $x_{0+} = \text{Waiting}$ as the initialization, which automatically specifies the distribution x_1 in the correct fashion. We return to the pending specification of the distributions $\left(\Pr(x_{n+}|x_n, \pi_n)\right)_{x_n \in \mathcal{X}, \pi_n \in \Pi(x_n)}$ after a brief detour to discuss discounting.

Discounting. Payoff in the n-th epoch is discounted by a factor r^n, with $n = 1$ being the first epoch, and r given by (19). Thus, within our MDP the worker never leaves without matching, but discounting exactly captures the payoff impact of the likelihood r^n that the worker will have left without matching before the n-th epoch. In particular, under any worker strategy, the expected discounted sum of payoffs of the worker in our MDP will be identical to the expected net utility of the worker in the original system.

We now proceed to specify the transition probabilities from x_n to x_{n+} as well as the per-epoch payoffs in our MDP to correspond exactly what occurs in the original system (conditioned on the worker not leaving without matching). The specification is routine, though somewhat tedious. If $x_n = \text{Matched}$ then nothing happens, i.e., the epoch payoff is zero and the post-epoch state is again $x_{n+} = \text{Matched}$. We now provide the specification for proposal epochs and opportunity epochs,

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[31] The transition probabilities for our discrete-time MDP are written in terms of transition rates for the underlying continuous time model as in the uniformization method (see, e.g., Grassmann 1977): the distribution of x_{n+1} is given by the arrival rates of incoming proposals and opportunities in continuous time scaled by the sum of arrival rates.

and we use $\Upsilon(x_n, \pi_n, z_n)$ to denote the (expected) payoff as a function of the state x_n, the action π_n, and, possibly, a random disturbance z_n. This random disturbance has support $\{0, 1\}$ and will be defined appropriately when needed. In particular, we will use it to model whether a proposal issued by the agent was accepted (1) or rejected (0). Therefore, the payoffs in most epoch states are independent of z_n.

Transition probabilities and payoffs from Incoming Proposal epochs. If an epoch is an incoming proposal, the state is $x_n = (\text{IP}, v_{ij})$. The following are the possibilities with respect to the handling of proposals under a_w:

1. If a_w specifies to accept the proposal without screening, i.e. $a_w^{\text{i}} = \text{A}$, the worker automatically matches and leaves. (Recall that, within our MDP, the worker has no ability to choose an action as the action space associate to such a state is $\emptyset$, forcing $\pi_n = \text{NULL}$). From the point of view of the agent, the (expected) epoch payoff is the expected match value of 1/2, that is $\Upsilon(x_n = (\text{IP}, \text{NULL}), \pi_n = \text{NULL}, \cdot) = 1/2$ (the payoff in this state is independent of the random disturbance). The $x_{n+} = \text{Matched}$ state results deterministically (and nothing happens in further epochs), i.e., $\Pr(x_{n+} = \text{Matched} | x_n = (\text{IP}, \text{NULL}), \pi_n = \text{NULL}) = 1$.
2. If a_w specifies to reject the proposal without screening, i.e. $a_w^{\text{i}} = \text{I}$, the worker has no ability to choose an action, i.e., $\pi_n = \text{NULL}$ and he transitions deterministically to the $x_{n+} = \text{Waiting}$ post-epoch state, i.e., $\Pr(x_{n+} = \text{Waiting} | x_n = (\text{IP}, \text{NULL}), \pi_n = \text{NULL}) = 1$. The epoch payoff is $\Upsilon(x_n = (\text{IP}, \text{NULL}), \pi_n = \text{NULL}, \cdot) = 0$.
3. If a_w specifies to screen the proposal $a_w^{\text{i}} = \text{S+A/R}$, screening has occurred, and the worker sees the state (IP, v_{ij}). The worker chooses between Accept and Reject.
   If the worker chooses $\pi_n = \text{Reject}$ then he returns deterministically to $x_{n+} = \text{Waiting}$, and the epoch payoff is $-c$. Formally, $\Pr(x_{n+} = \text{Waiting} | x_n = (\text{IP}, v_{ij}), \pi_n = \text{Reject}) = 1$, and $\Upsilon(x_n = (\text{IP}, v_{ij}), \pi_n = \text{Reject}, \cdot) = -c$.
   If the worker chooses $\pi_n = \text{Accept}$, then the $x_{n+} = \text{Matched}$ state results deterministically, and the epoch payoff is $v_{ij} - c$. Formally, $\Pr(x_{n+} = \text{Matched} | x_n = (\text{IP}, v_{ij}), \pi_n = \text{Accept}) = 1$, and $\Upsilon(x_n = (\text{IP}, v_{ij}), \pi_n = \text{Accept}, \cdot) = v_{ij} - c$.

Transition probabilities and payoffs from Opportunity epochs. If an epoch is an opportunity, the state is $x_n = (\text{O}, v_{ij})$. The following possibilities arise, depending on the action specified by a_w:

1. If a_w specifies to do nothing with the opportunity, i.e., $a_w^{\text{o}} = \text{DN}$, then the worker has no ability to choose an action, i.e., $\pi_n = \text{NULL}$ and the worker goes deterministically to the post-epoch state $x_{n+} = \text{Waiting}$. Also, there is zero epoch payoff. That is, $\Pr(x_{n+} = \text{Waiting} | x_n = (\text{O}, \text{NULL}), \pi_n = \text{DN}) = 1$, and $\Upsilon(x_n = (\text{O}, \text{NULL}), \pi_n = \text{DN}, \cdot) = 0$.
2. If a_w specifies to propose to candidates without screening, i.e., $a_w^{\text{o}} = \text{P w/o S}$, the worker automatically makes a proposal. (Recall that, within our MDP, the worker has no ability to choose an action, i.e., $\pi_n = \text{NULL}$). However, both the post epoch-state and the payoff will depend on the random event of whether his proposal was accepted or not. Let z_n be the indicator variable that the proposal at epoch n was accepted. Note that the proposal results in a match with probability[32]

   $$p^w_{\text{acc},t} = \frac{\int_{\mathcal{S}_e} \Pr(\text{Employer following } s_e \text{ accepts proposal from tier } t \text{ worker}) d\kappa_w^t\left(s_e; a_w, \bar{N}\right)}{\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}\right)},$$

   and therefore, conditional on $(x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL})$, z_n is Bernoulli$(p^w_{\text{acc},t})$.
   If the proposal is accepted, then $x_{n+} = \text{Matched}$ and the (expected) epoch payoff is the expected match value of 1/2. That is, $\Pr(z_n = 1 | x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}) = p^w_{\text{acc},t}$,

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[32] Here we use that the probability measure over candidate strategies s_e is $\kappa_w^t\left(\cdot\,; a_w, \bar{N}, \bar{f}\right)/\kappa_w^t\left(\mathcal{S}_e; a_w, \bar{N}, \bar{f}\right)$.

$\Pr(x_{n+} = \text{Matched}|x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}, z_n = 1) = 1$, and $\Upsilon(x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}, z_n = 1) = 1/2$.

If the proposal is rejected, then $x_{n+} = \text{Waiting}$ and the epoch payoff is zero, i.e., $\Pr(z_n = 0|x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}) = 1 - p^w_{\text{acc},t}$, $\Pr(x_{n+} = \text{Waiting}|x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}, z_n = 0) = 1$, and $\Upsilon(x_n = (\text{O}, \text{NULL}), \pi_n = \text{NULL}, z_n = 0) = 0$.

3. If a_w specifies to screen the candidate, i.e., $a^{\text{o}}_w = \text{S+P}$, screening has occurred, and the worker sees the state (O, v_{ij}). The worker chooses between Propose and DNP.

   If the worker chooses $\pi_n = \text{DNP}$ then he returns to $x_{n+} = \text{Waiting}$ with probability 1, and the epoch payoff is $-c$, i.e., $\Pr(x_{n+} = \text{Waiting}|x_n = (\text{O}, v_{ij}), \pi_n = \text{DNP}) = 1$, and $\Upsilon(x_n = (\text{O}, v_{ij}), \pi_n = \text{DNP}, \cdot) = 0$.

   If the worker chooses $\pi_n = \text{Propose}$, then again the resulting payoff and post-epoch transition depend on whether the proposal is accepted or not (i.e., the realization of z_n) :

   - With probability $p^w_{\text{acc},t}$ specified above the proposal is accepted, so $x_{n+} = \text{Matched}$ and the epoch payoff is $v_{ij} - c$, i.e., $\Pr(z_n = 1|x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}) = p^w_{\text{acc},t}$, $\Pr(x_{n+} = \text{Matched}|x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}, z_n = 1)$, and $\Upsilon(x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}, z_n = 1) = v_{ij} - c$.
   - With probability $1 - p^w_{\text{acc},t}$ the proposal is rejected, in which case $x_{n+} = \text{Waiting}$ and the epoch payoff is $-c$, i.e., $\Pr(z_n = 0|x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}) = 1 - p^w_{\text{acc},t}$, $\Pr(x_{n+} = \text{Waiting}|x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}, z_n = 0) = 1$, and $\Upsilon(x_n = (\text{O}, v_{ij}), \pi_n = \text{Propose}, z = 0) = -c$.

This completes our specification of the MDP, which has been defined to capture exactly the worker's problem in the original system.

B.4 Formal definition of best response

To eliminate redundant multiplicity of best responses (and consequent spurious equilibria), we assume agents best respond in a robust sense, as we describe in this section.

First, to see why this multiplicity might arise, consider a worker i who sees an agent mix $\bar{N}$ and strategy distributions for new arrivals $\bar{f}$ when he enters the system. Fixing $\bar{N}, \bar{f}$ also fixes the rate $\nu^t_w(\bar{N}, \bar{f})$ of receiving proposals (as described in Section 2 and Appendix B.2). Recall that the worker treats this rate as time invariant when calculating his anticipated (expected) utility for any given strategy. We term a strategy s_w satisfying (1) a *weak best response*. A redundant multiplicity of weak best responses may arise when $\nu^t_w(\bar{N}, \bar{f}) = \infty$ or when $\nu^t_w(\bar{N}, \bar{f}) = 0$ as follows:

(i) No incoming proposals: Suppose that $\nu^t_w(\bar{N}, \bar{f}) = 0$. Consider any weak best response $s_w = (a_w, \Theta)$ for the worker with $a_w = (a^{\text{i}}_w, a^{\text{o}}_w)$ and $\Theta = (\theta^{\text{i}}_w, \theta^{\text{o}}_w)$. Then, any strategy $s'_w = (a'_w, \Theta')$ with $a_w = (\hat{a}^{\text{i}}_w, a^{\text{o}}_w)$ and $\Theta = (\hat{\theta}^{\text{i}}_w, \theta^{\text{o}}_w)$ with $\hat{a}^{\text{i}}_w \in \mathcal{A}^t_w$ and arbitrary $\hat{\theta}^{\text{i}}_w$, is also a weak best response. This is because the worker will never receive an incoming proposal, so the strategy he adopts to deal with such proposals does not impact his utility.

(ii) No opportunities during lifetime: Suppose $\nu^t_w(\bar{N}, \bar{f}) = \infty$. Consider any weak best response $s_w = (a_w, \Theta)$ with $a_w = (a^{\text{i}}_w, a^{\text{o}}_w)$ and $\Theta = (\theta^{\text{i}}_w, \theta^{\text{o}}_w)$ and $a^{\text{i}}_w \neq I$, that is, the worker does not ignore incoming proposals. That means that the worker will match immediately[33] and hence will never have an opportunity to propose. Then, any strategy $s'_w = (a'_w, \Theta')$ with $a'_w = (a^{\text{i}}_w, \hat{a}^{\text{o}}_w)$ and $\Theta = (\theta^{\text{i}}_w, \hat{\theta}^{\text{o}}_w)$ with $\hat{a}^{\text{o}}_w \in \mathcal{A}^t_w$ and arbitrary $\hat{\theta}^{\text{o}}_w$, is also a weak best response since his strategy for handling opportunities does not impact his utility.

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[33] If $a^{\text{i}}_w = \text{A}$ then the worker will match with the first incoming proposal. Alternately, $a^{\text{i}}_w = \text{S+A/R}$ then it must be that θ^{i}_w is such that the worker accepts each proposal with a positive probability (i.i.d.), else s_w cannot be a weak best response. In either case, the worker matches immediately since $\nu^t_w(\bar{N}, \bar{f}) = \infty$.

This multiplicity of weak best responses results in implausible equilibria. The simplest of these implausible equilibria is the equilibrium (in weak best responses) where all agents do not propose and ignore all incoming proposals. We will rule out such equilibria by refining our notion of best response to incorporate a certain form of robustness.

Definition 6 (Best Response)**.** *Consider a tier t-worker and let $\bar{N}$ and $\bar{f}$ be the agent mix and the distribution over strategies of arrivals at the time the agent arrives. Then, a worker strategy s_w^* is said to be a best response if it maximizes the anticipated utility (i.e., it satisfies* (1)*) and further satisfies the following:*

1. *The strategy s_w^* also satisfies* (1) *in a setting where, immediately before joining the system, he receives a "bonus" proposal from an employer (with match value drawn independently from the same distribution* Uniform(0,1)*) and responds according to s_w^*. If he screens the bonus proposal this costs c as usual, and if the worker rejects the proposal, he proceeds to enter the original system as usual.*

2. *The strategy s_w^* also satisfies* (1) *in a setting where, immediately before joining the system, he receives a "bonus" opportunity and a candidate employer is available (with match value drawn independently from the same distribution* Uniform(0,1) *and probability of accepting a proposal identical to that in the original system under $\bar{N}$ and $\bar{f}$) and the worker acts as indicated by s_w^*. If he screens the candidate this costs c as usual, and if the worker does not propose or the candidate rejects the proposal, the worker proceeds to enter the original system as usual.*

*The definition for employers is analogous. In case of two worker tiers, part 1 above now applies to a bonus proposal from tier t, for each $t \in \{\mathrm{H}, \mathrm{L}\}$, and part 2 above applies to a bonus opportunity for each of three cases: (a) only a tier-*H *candidate is available, (b) only a tier-*L *candidate is available, (c) both tier-*H *and tier-*L *candidates, with attributes independent of each other, are available.*

Part 1 of the definition handles the multiplicity of weak best responses described under (i) above, by requiring the agent to be rational if a proposal were, hypothetically, to arrive. Part 2 of the definition handles the multiplicity of weak best responses described under (i) above, by requiring the agent to be rational if an opportunity were, hypothetically, to arrive.

Note that the definition may not completely eliminate multiplicity of best responses. For example, if $\nu_w^t(\bar{N}, \bar{f}) = \infty$, then a worker might decide to "do nothing" and pass on the opportunity to propose. However, it disallows other "irrational" strategies such as proposing with a very high threshold or, in a setting with small c, proposing without screening.

B.5 Evolutionarily stable equilibrium

Consider a stationary equilibrium $\left(\bar{f}, (\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e), \bar{L}\right)$. We now define the additional test that the SE must pass in order for it to qualify as *evolutionarily stable.* The high level idea is that even if the agent mix $\bar{N}$ deviates slightly from $\bar{L}$, if the arriving agents play a best response to $\bar{f}$ and the current agent mix $\bar{N}$, the system should return to the stationary equilibrium.

We start by defining the *evolutionary dynamics* that result when arriving agents play a best response to $\bar{f}$ and the current agent mix $\bar{N}$. When a new tier-t worker enters, he considers the anticipated utility $U_w^t(s_w^t; \bar{N}, \bar{f})$ that would result from using strategy s_w^t. We hold the thresholds $(\theta_w^{\mathrm{H}}, \theta_w^{\mathrm{L}}, \theta_e)$ fixed[34] and the worker chooses a threshold-consistent strategy

$$\tilde{s}_w^t(\bar{N}, \bar{f}) \in \operatorname*{arg\,max}_{s_w^t \in \mathcal{S}_w^t(\theta_w^t)} U_w^t(s_w^t; \bar{N}, \bar{f})$$

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[34] We expect that holding these thresholds fixed generally should not impact whether an equilibrium classifies as evolutionarily stable or not, and we find that our "weak" notion of evolutionary stability with fixed thresholds suffices to eliminate implausible equilibria.

to maximize his anticipated utility, and retains this strategy for his lifetime. (When there are ties, we allow them to be broken arbitrarily, including possible mixing. This will ensure that all stationary equilibria are captured as fixed points of the differential equations below.) This leads to the following coupled ODEs capturing evolutionary dynamics in the system:

$$\begin{aligned}
\frac{\mathrm{d}\bar{N}_w^t(s)}{\mathrm{d}\tau} &= \mathbb{I}\big(s = \tilde{s}_w^t(\bar{N},\bar{f})\big)\lambda_w^t - \bar{N}_w^t(s_w)\mu - \rho_w^t(s_w,\bar{N},\bar{f}) \qquad \forall s \in \mathcal{S}_w(\theta_w^t)\,, \\
\frac{\mathrm{d}\bar{N}_e(s)}{\mathrm{d}\tau} &= \mathbb{I}\big(s = \tilde{s}_e^t(\bar{N},\bar{f})\big)\lambda_e^t - \bar{N}_e(s_e)\mu - \rho_e(s_e,\bar{N},\bar{f}) \qquad \forall s \in \mathcal{S}_e(\theta_e)\,,
\end{aligned} \tag{20}$$

where we abuse notation and allow the indicator to represent any arbitrary distribution over the strategies that maximize the agents' utilities, if more than one such strategy exists. Note the difference between the dynamical system defined by (20) on one hand and the dynamical system in Eqs. (2) and (3) for time invariant $\bar{f}$ on the other. The strategy distribution of arriving agents in the former system may vary over time; in particular, the arriving agents pick a best responses to the current agent mix $\bar{N}$ and the $\bar{f}$ in the test equilibrium.

As mentioned in Section 2.1, all stationary equilibria correspond to fixed points of the dynamical system described by Eqs. (2) and (3). We focus on the subset of stationary equilibria that are plausible from an evolutionary/dynamical standpoint.

Definition 7. *Each stationary equilibrium* $\Big(\big(\theta_w^{\mathrm{H}},\theta_w^{\mathrm{L}},\theta_e\big),\bar{f},\bar{L}\Big)$ *corresponds to a fixed point of the evolutionary dynamics* (20) *for the given* $\big(\theta_w^{\mathrm{H}},\theta_w^{\mathrm{L}},\theta_e\big)$ *and* $\bar{f}$*. A stationary equilibrium is* evolutionarily stable *if the corresponding fixed point is attractive under the evolutionary dynamics* (20)*. We sometimes refer to this simply as a* stable equilibrium*.*

C Some useful lemmas

In this appendix section, we provide some lemmas that are used repeatedly throughout the proofs.

C.1 Effective screening cost

Recall from our discussion in Section 3 that proposers can internalize the cost imposed by the other side rejecting them by making decisions based on what we call the *effective screening cost.* The following simple lemma formalizes this idea and is used throughout the paper:

Lemma C.1 (Effective screening cost)**.** *Consider the following two systems. In each case, the rate of leaving without matching is* μ*, and the value of an item to an agent is drawn i.i.d. from some distribution* F*. Any incoming option is screened (at some cost) to reveal the true value of the option, and then accepted/requested if this value exceeds a threshold* θ*. We assume* $F(\theta) < 1$*.*

(1) *System 1: "Potential options" arise according to a Poisson point process of rate* η*. Each potential option is screened at a cost* c *to reveal its value, and requested if the value exceeds* θ*. The request is approved i.i.d. with probability* q*, in which case the agent obtains the item and leaves. If there is no request or the request is denied, the agent remains active.*

(2) *System 2: Options arise according to a point process of rate* ηq*. Each option is screened at a cost* c/q*, to reveal its value. The agent chooses to obtain the item if the value exceeds* θ*.*

Then, the two systems produce the same expected value.

The proof is routine, and can be found in the online appendix.

C.2 Upper bound on expected utility

The following lemma applies generally, whether there is one worker tier or two. Though it is stated for employers, the symmetric counterpart bound holds for workers.

Lemma C.2 (Upper bound on expected utility)**.** *Consider any employer, and let* q_w *be the highest worker quality. Then, for any agent mix* $\bar{N}$ *and distribution over strategies of incoming agents* $\bar{f}$*, the anticipated utility (and also the actual expected utility) of the employer is bounded above as*

$$\sup_{s \in \mathcal{S}_e} U_e(s, \bar{N}, \bar{f}) \leq q_w + \max(1/2, 1 - \sqrt{2c})\,. \tag{21}$$

Analogously, the anticipated utility (and actual expected utility) of a worker cannot exceed $q_e + \max(1/2, 1 - \sqrt{2c})$.

Moreover, if the employer leaves without matching with positive probability $\epsilon > 0$ *(e.g., because they do not not consider incoming proposals), or if the employer (screens + proposes + is rejected) with positive probability* $\epsilon > 0$ *then the inequality is strict:*

$$\sup_{s \in \mathcal{S}_e} U_e(s, \bar{N}, \bar{f}) \leq q_w + \max(1/2, 1 - \sqrt{2c}) - \epsilon \min(1, c)\,. \tag{22}$$

Note that the upper bound in the lemma holds irrespective of the strategy adopted by the agent (and is strict for any strategy which does not consider incoming proposals).

Proof of Lemma C.2. We show the claimed bounds for the actual expected utility, with no assumptions on how the system state evolves. The same bounds then clearly hold also for the anticipated utility (which is nothing but the expected utility assuming the system state remains the same).

Consider arbitrary $\bar{N}$, $\bar{f}$ and s. In the best case, the employer has probability zero of leaving without matching, and never spends screening effort on candidates who later reject him (this utopian situation can indeed occur if the employer receives incoming proposals at an infinite rate, and accepts each proposal with a positive probability). Taking this *hypothetical utopian situation* as a given, the employer would no longer distinguish between candidates and incoming proposals, since there is no risk of rejection. So henceforth, we just use the term "incoming proposal" to refer to either situation. If the employer accepts incoming proposals without screening, his utopian expected utility is $q_w + 1/2$. If the employer screens and accepts/rejects incoming proposals with threshold θ, his utopian expected utility

$$\frac{1 + q_w + \theta}{2} - \frac{c}{1 + q_w - \theta}$$

where the first term captures the largest possible expected match value (based on the threshold θ and a Uniform$(q_w, q_w + 1)$ distribution of match values for the highest quality worker), and the second term captures the smallest possible expected screening cost, since the expected number of proposals screened is

$$\frac{1}{\text{Likelihood of accepting a proposal}(\theta)} \geq \frac{1}{1 + q_w - \theta}\,.$$

This upper bound on utopian expected utility is maximized for $\theta = 1 + q_w - \sqrt{2c}$, yielding an upper bound of $q_w + \max(1/2, 1 - \sqrt{2c})$ on the utopian expected utility and, therefore, on the anticipated utility. The upper bound for the expected utility of a worker is merely the symmetric counterpart.

It remains to establish the last sentence of the lemma. If the employer (worker) leaves without matching with positive probability, the utopian expected utility is unachievable, and the upper bound we have obtained above is strict. Note that if an employer (worker) does not consider incoming proposals, then the probability of leaving without matching is strictly positive (opportunities arrive only at rate 1, so there is a chance at least $1/(1+\mu)$ of leaving without matching). $\square$

C.3 Dynamics when agents on each side follow a single strategy

We now analyze the system dynamics and the resulting steady state when there is one tier of workers and all agents on the same side use the same strategy (i.e., the same actions and thresholds). We find that the corresponding dynamical system always has a *unique* steady state/fixed point $\bar{L}$, that is always stable. When the fixed strategies employed are the unique best responses on each side of the market to $\bar{L}$, they are also best responses in a neighborhood of $\bar{L}$; hence the system dynamics (2) and (3) precisely matches the evolutionary dynamics (20) in a neighborhood of $\bar{L}$, implying that $\bar{L}$ corresponds to a stable equilibrium. As we will argue later, in all the settings we consider in Section 3, in each stable equilibrium, all agents on the same side of the market do, in fact, use the same strategy. In other words, there is no mixed equilibrium that is stable.

Proposition 12. *Consider a system with only one worker tier, where workers arrive at rate λ_w and employers at rate λ_e. Suppose that, upon arrival, all workers choose strategy s_w and all employers choose strategy s_e. To ease notation, denote the match success rates as $\eta_w \triangleq \eta_w(s_w, s_e)$ and $\eta_e \triangleq \eta_e(s_e, s_w)$, where $\eta_w(s_w, s_e)$ is as defined in (12). Suppose that the following conditions hold:*

$$\lambda_w \neq \frac{\eta_e \lambda_e}{\mu+\eta_e}, \tag{23}$$

$$\lambda_e \neq \frac{\eta_w \lambda_w}{\mu+\eta_w}. \tag{24}$$

Then, unique steady state of the system, $\bar{L} = [\bar{L}_e(s_e), \bar{L}_w(s_w)] = [L_e, L_w]$, is characterized as follows:

1. *When $\lambda_w < \frac{\eta_e \lambda_e}{\mu+\eta_e}$, the system converges to a steady state of*

$$\bar{L} = \begin{bmatrix} \frac{\lambda_e - \lambda_w}{\mu} \\ 0 \end{bmatrix}. \tag{25}$$

2. *When $\lambda_w > \frac{\eta_e \lambda_e}{\mu+\eta_e}$ and $\lambda_e > \frac{\eta_w \lambda_w}{\mu+\eta_w}$, the system converges to a steady state of*

$$\bar{L} = \begin{bmatrix} \frac{\lambda_e(\mu+\eta_w) - \lambda_w \eta_w}{\mu(\mu+\eta_e+\eta_w)} \\ \frac{\lambda_w(\mu+\eta_e) - \lambda_e \eta_e}{\mu(\mu+\eta_e+\eta_w)} \end{bmatrix}. \tag{26}$$

Note that as $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_+$ we have $L_w \to 0$ and $L_e \to \frac{\lambda_e}{\mu+\eta_e}$. These limiting values of L_w and L_e match the limiting values that arise when $\lambda_w \to \left(\frac{\eta_e \lambda_e}{\mu+\eta_e}\right)_-$.

The proof of this proposition follows from standard arguments, and can be found in the supplementary appendix.

Remark 3. *Proposition 12 is also applicable (for small enough μ) in the case where $\lambda_w > \lambda_e$, all employers use the same strategy to handling incoming proposals resulting in $\eta_w > 0$, and employers have an arbitrary mixture of strategies in their handling of opportunities.*

As $\lambda_e < \lambda_w$ and $\eta_w > 0$, for μ small enough we have that $\lambda_e < \frac{\eta_w \lambda_w}{\mu+\eta_w}$. This means that the characterization in case 1 holds. To see why, note that this characterization uses the fact that employers match as soon as they arrive. Therefore, how they handle opportunities plays no role in the system dynamics, as they will always match before their opportunity clock rings.

D Appendix to Section 3: ex-ante homogeneous agents

This appendix provides the proofs for the results in Section 3 which considers the setting with ex-ante homogeneous agents on each side. (The planner's solution in this setting, Proposition 2, is proved in the online appendix.)

To obtain the results in this section we impose a mild technical constraint on agent strategies to facilitate complete characterizations of *all* equilibria for *all* possible screening costs c. (This constraint is not needed to establish equilibrium existence, nor to establish our results with multiple worker tiers in Section 4 which only consider small screening costs c.) Because agents return to their original state each time an opportunity or incoming proposal does not produce a match (see Appendix B.3), agents can reason separately about best response handling of opportunities, and best response handling of incoming proposals (formalized in Lemma F.1 in online appendix F). This motivates us to impose the following restriction.

Restriction 4. *We require that* $f_w, f_e \in \Delta(\mathcal{A}^{\mathrm{i}}) \times \Delta(\mathcal{A}^{\mathrm{o}})$*, i.e., agents choose their handling of incoming proposals* a^{i} *independently of their handling of opportunities* a^{o}*.*

Roadmap for Appendix D. This appendix is divided into the following sections:

Appendix D.1 states and proves some useful lemmas that will help in characterizing the equilibria in markets with one tier on each side.

In Appendix D.2, we state and prove a detailed version of Theorem 3, which characterizes the equilibria in symmetric markets.

In Appendix D.3, we state detailed versions of Theorems 4 and 5, which characterize the equilibria in unbalanced markets without and with intervention, respectively. The proofs of these theorems can be found in the online appendix, as the arguments followed are similar to those used in the proof of Theorem 3.

In the online appendix (Section J.3.1) we investigate when the intervention of blocking the long side from proposing is welfare enhancing, including a detailed version and proof of Corollary 6.

In some of the proofs, we will rely on the notation introduced in Appendix B. For a quick reference, the reader can find a list of such notation in Appendix A.

D.1 Some useful lemmas to characterize equilibria

In this subsection, we state and prove two auxiliary lemmas to help us to characterize equilibria and to rule out other equilibria besides the ones whose existence we establish. In the online appendix, we state and prove additional lemmas that will help us to characterize equilibria in strictly unbalanced markets. Without loss of generality, we assume $\lambda_w \geq \lambda_e$ throughout this subsection.

Lemma D.1. *Consider one tier on each side of the market and any* $R = \lambda_w/\lambda_e \geq 1$*. Then, in any equilibrium, all agents consider incoming proposals, and moreover accept an incoming proposal with probability at least* $\sqrt{2c}$*.*

Proof. We prove the result by contradiction. Suppose an agent does not consider incoming proposals in equilibrium. Then, by Lemma C.2, the continuation value V is strictly less than $\max(1/2, 1-\sqrt{2c}) + q$ where q is the quality of agents on the other side. Consider the decision problem the agent faces when an incoming proposal arrives. By ignoring the incoming proposal, the agent returns to waiting and earns expected utility V. On the other hand, suppose the agent chooses to consider the incoming proposal as follows: if $c < 1/2$, he screens with threshold $\theta = V$, whereas if $c \geq 1/2$, he accepts the proposal without screening. Under this strategy, for $c \geq 1/2$ we have that $V < 1/2 + q$, the agent earns expected utility $1/2 + q > V$, which contradicts our assumption that he does not consider incoming proposals in equilibrium. For $c < 1/2$ we have that $V = \theta < 1 - \sqrt{2c} + q$, the agent earns expected utility

$$(1 + q - \theta)\frac{1 + q + \theta}{2} + \theta(\theta - q) - c = \frac{(1 + q)^2 - 2q\theta + \theta^2}{2} - c = (1 + q - \theta)^2/2 - c + \theta > \theta\,, \quad (27)$$

where we used $1 + q - \theta > \sqrt{2c}$. Once again, this inequality contradicts our assumption that he does not consider incoming proposals in equilibrium. □

Lemma D.2. *Consider one tier on each side of the market and any* $R = \lambda_w/\lambda_e \geq 1$. *In any equilibrium where one side of the market mixes between proposing and not proposing, all agents on the other side of the market propose.*

Proof. **Both sides cannot mix between proposing and not proposing.** Suppose there is an equilibrium where both sides mix between proposing and not proposing. We first show that both sides must have a positive mass of agents in steady state. Suppose not; since the mass of workers in the system is at least as large as the mass of employers given $R \geq 1$, it must be that there is a zero mass of employers in the system. Then workers who do not propose receive proposals at a rate zero, and so have an expected utility of zero. But they can obtain a positive expected utility by proposing without screening: when an opportunity arises, with positive probability an employer will be available and will accept the proposal (we know that there is a positive overall match flow in the system equal to the arrival rate of employers, and is entirely the result of workers proposals being accepted by employers). Thus we have a contradiction.

Having shown that both sides must have a positive mass of agents, we now show that such an equilibrium cannot be evolutionarily stable. Suppose the proportion of workers who are proposing is slightly larger than in steady state whereas the distribution of a^{i} among workers is unchanged (we retain our assumption that f_w is a product distribution at all times). Then employers are no longer indifferent between proposing and not proposing: since more workers are proposing, the utility of employers under not proposing has increased more than the utility of employers under proposing since the hazard rate of matching via proposing under the latter strategy (which is positive as per Lemma D.1) is unchanged. As a result, entering employers all prefer to not propose under the evolutionary dynamics (20), and in turn, the new workers all prefer to propose, i.e., the equilibrium is not stable.

It cannot be that one side mixes, and no one proposes on the other side. Next, suppose one side mixes between proposing and not proposing, but the other side does not propose. This is ruled out because all agents on the first side will want to propose (not proposing yields utility zero, whereas proposing without screening yields a positive utility since the other side accepts incoming proposals with positive probability). Then, it follows that if one side is mixing between proposing and not proposing, all agents on the other side propose. □

D.2 Proof of Theorem 3: Equilibria in symmetric markets

We state a detailed version of Theorem 3 and then proceed to prove it.

Theorem 13. *Consider a market with one tier on each side where each of the two sides has an arrival rate of* 1. *For each* c *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$, *all stable stationary* *equilibria as a function of c (with the exception*[35] *of $c = 1/32$) are characterized as follows:*

1. **$\mathbf{c} \in \left(\mathbf{0}, \frac{\mathbf{1}}{\mathbf{32}}\right)$**: *There is a unique equilibrium where all employers use $a_e = (a^{\mathrm{o}} = \mathrm{DN}, a^{\mathrm{i}} = \mathrm{S{+}A/R})$ and all workers use $a_w = (a^{\mathrm{o}} = \mathrm{S{+}P}, a^{\mathrm{i}} = \mathrm{S{+}A/R})$, with thresholds (and expected utilities) $\theta_e = \theta_e(\mu)$ and $\theta_w = \theta_w(\mu)$, respectively, and steady state given by $\bar{L} = \left[\frac{1}{\mu+\eta},\ \frac{1}{\mu+\eta}\right]'$ where $\eta = (1-\theta_e)(1-\theta_w)$. The thresholds in this equilibrium satisfy $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$ and $\lim_{\mu\to 0}\theta_w = 1-(2c)^{1/4}$. The symmetric counterpart equilibrium where the roles of workers and employers are reversed is the only other equilibrium in this regime.*

2. **$\mathbf{c} \in \left(\frac{\mathbf{1}}{\mathbf{32}}, \frac{\mathbf{1}}{\mathbf{8}}\right)$**: *There is a unique equilibrium where all employers use $a_e = (a^{\mathrm{o}} = \mathrm{DN}, a^{\mathrm{i}} = \mathrm{S{+}A/R})$ and all workers use $a_w = (a^{\mathrm{o}} = \mathrm{P\ w/o\ S}, a^{\mathrm{i}} = \mathrm{S{+}A/R})$, with expected utilities (and threshold) $\theta_e = \theta_e(\mu)$ and $\theta_w = \theta_w(\mu)$, respectively, and steady state given by $\bar{L} = \left[\frac{1}{\mu+1-\theta_e},\ \frac{1}{\mu+1-\theta_e}\right]'$. The expected utilities in this equilibrium satisfy $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$ and $\lim_{\mu\to 0}\theta_w = 1/2$. The symmetric counterpart equilibrium is the only other equilibrium in this regime.*

3. **$\mathbf{c} \geq \frac{\mathbf{1}}{\mathbf{8}}$**: *Agents on both sides propose without screening, and accept all incoming proposals without screening $s_w = s_e = (a^{\mathrm{o}} = \mathrm{P\ w/o\ S}, a^{\mathrm{i}} = \mathrm{A})$, and earn expected utility $\theta_w = \theta_e = \frac{1}{\mu+2} \xrightarrow{\mu\to 0} 1/2$. The steady state $\bar{L} = [L_e, L_w]'$ is given by $\bar{L} = \left[\frac{1}{\mu+2}, \frac{1}{\mu+2}\right]'$.*

The average welfare in equilibrium remains unchanged under the platform intervention of preventing one side from proposing.

Roadmap for the proof of Theorem 13. We now prove the proposition via a series of lemmas.

In three successive lemmas we establish the existence of the three equilibria stated in the theorem. In particular, Lemma D.3 establishes the existence of the equilibrium described in the regime $c < 1/32$. Lemma D.4 establishes the existence of the equilibrium described in the regime $c \in (1/32, 1/8)$. Finally, Lemma D.5 establishes the existence of the equilibrium described in the regime $c \geq 1/8$. All steady state characterizations follow from Proposition 12.

In addition, each of these lemmas will also rule out the corresponding equilibrium outside the specified interval of c values, which will then assist us in showing that there are no other equilibria.

In the proof of the theorem, we use the aforementioned lemmas to establish existence, and we formally establish that there are no other equilibria.

One important thing to note is that the proofs of the lemmas characterize the *exact expected utilities on both sides of the market as functions of μ for all small positive μ*; in the theorem, we state only the limiting values for equilibria 1 and 2 in the interest of having a cleaner statement.

The following lemma characterizes equilibria in which all workers use actions $a_w = (\mathrm{S{+}P}, \mathrm{S{+}A/R})$ and all employers use actions $a_e = (\mathrm{DN}, \mathrm{S{+}A/R})$. We find that there is a unique such equilibrium for $c < 1/32$, and no such equilibrium for $c > 1/32$. The lemma does not comment on whether there are equilibria with other actions (a_w, a_e) or mixed equilibria.

Lemma D.3 (Pure equilibrium where workers S+P and employers S+A/R)**.** *The following is a characterization of equilibria where all workers use $a_w = (\mathrm{S{+}P}, \mathrm{S{+}A/R})$ and all employers use $a_e = (\mathrm{DN}, \mathrm{S{+}A/R})$.*

(i) For any $c \in (0, \frac{1}{32})$ there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, there is a unique such equilibrium. The thresholds in this equilibrium satisfy $\lim_{\mu\to 0}\theta_w = 1-(2c)^{1/4}$ and $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$, the steady state is $\bar{L} = \left[\frac{1}{\mu+(1-\theta_e)(1-\theta_w)}, \frac{1}{\mu+(1-\theta_e)(1-\theta_w)}\right]'$, and the equilibrium is stable.

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[35] In the interest of brevity, we omit the tedious and unilluminating exercise of characterizing equilibria at the boundary 1/32 between regimes.

(ii) For any $c > 1/32$ there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$ there is no such equilibrium.

Proof. **Existence, uniqueness and limiting thresholds for $c < 1/32$.**

Fix $c \in (0, \frac{1}{32})$. Our high level approach will be as follows. We first constrain all workers to use $a_w = (\text{S+P}, \text{S+A/R})$ and all employers to use $a_e = (\text{DN}, \text{S+A/R})$ and characterize (i) the equilibrium θ_w between workers for given θ_e, (ii) the equilibrium θ_e between employers for given θ_w, and then to show that the composition of the two single-side equilibrium operators is contractive for small enough μ, which will imply equilibrium existence, uniqueness as well as the limiting equilibrium description as $\mu \to 0$. At the end we will check that a_w and a_e constitute best responses and show stability.

For now we constrain workers to use $a_w = (\text{S+P}, \text{S+A/R})$ and employers to use $a_e = (\text{DN}, \text{S+A/R})$. We will characterize the resulting equilibrium thresholds on both sides of the market. From Proposition 1, we know that in any such equilibrium, all workers use the same threshold θ_w equal to their expected utility and similarly for employers with threshold θ_e. Using Lemma C.2 we further know that $\theta_w \le 1 - \sqrt{2c}$ and $\theta_e \le 1 - \sqrt{2c}$. Thus, we are searching for equilibria with $(\theta_w, \theta_e) \in [0, 1 - \sqrt{2c}]^2$.

We start by characterizing the equilibrium θ_w between workers for a given employer threshold θ_e. Let V_w be the expected utility for a worker. Now a worker receives an candidate before leaving without matching with probability $1/(1+\mu)$ (his opportunity clock rings at rate 1 and, as both sides arrive at the same rate, there are always agents on the other side available, whereas he leaves without matching at rate μ), in which case he screens it at a cost c, and proposes (if the match value exceeds θ_w) with probability $1 - \theta_w$. His proposal is accepted with probability $1 - \theta_e$, and if so, produces an expected match value of $(1 + \theta_w)/2$. If he does not propose or the proposal is rejected, he returns to the initial state and has future expected utility V_w. Hence,

$$V_w = \frac{1}{1+\mu}\left(-c + (1-\theta_w)(1-\theta_e)\left(\frac{1+\theta_w}{2}\right) + \big(1 - (1-\theta_w)(1-\theta_e)\big)V_w\right).$$

Substituting $V_w = \theta_w$, solving the quadratic equation for θ_w and excluding the root which exceeds $1 > 1 - \sqrt{2c}$, we obtain

$$\theta_w = l_w(\theta_e) \quad \text{where } l_w(\theta) \triangleq 1 + \frac{\mu}{1-\theta} - \sqrt{\frac{2c}{1-\theta} + \frac{2\mu}{1-\theta} + \left(\frac{\mu}{1-\theta}\right)^2}. \tag{28}$$

Observe that for all $\mu > 0$ we have $l_w : [0, 1 - \sqrt{2c}] \to [0, 1 - \sqrt{2c}]$, i.e., l_w maps valid θ_e to valid θ_w, since

$$l_w(\theta) = \frac{1 - 2c/(1-\theta)}{1 + \frac{\mu}{1-\theta} + \sqrt{\frac{2c}{1-\theta} + \frac{2\mu}{1-\theta} + \left(\frac{\mu}{1-\theta}\right)^2}} \in (0, 1 - \sqrt{2c/(1-\theta)})$$

by multiplying and dividing $l_w(\theta)$ by $1 + \frac{\mu}{1-\theta} + \sqrt{\frac{2c}{1-\theta} + \frac{2\mu}{1-\theta} + \left(\frac{\mu}{1-\theta}\right)^2}$ and then noting that the denominator exceeds $1 + \sqrt{2c/(1-\theta)}$, and that the numerator is positive since $\theta \le 1 - \sqrt{2c}$ and $c < 1/32$. A routine calculation of $l_w'(\theta)$ suffices to establish that $|l_w'(\theta)| < c^{-1/4}$ for small enough μ and all $\theta \in [0, 1 - \sqrt{2c}]$, implying that l_w is $(c^{-1/4})$-Lipschitz on its domain for small enough μ. (The value of the Lipschitz constant will not matter for our argument except that there is a Lipschitz constant that applies uniformly for all small enough μ. The latter fact is not surprising; we may expect it given that $\lim_{\mu \to 0} l_w(\theta) = 1 - \sqrt{\frac{2c}{1-\theta}}$ is clearly a Lipschitz function on its domain

$[0, 1-\sqrt{2c}]$.)

Now let us similarly characterize the equilibrium θ_e between employers for a given worker threshold θ_w. First note that in steady state, the mass of employers must be equal to the mass of workers: the arrival rates are equal, and the rates of leaving by matching are equal, so the rates of leaving without matching must be equal, meaning their masses must be equal. It then follows that the hazard rate of receiving a proposal for an employer is equal to the hazard rate of making a proposal for a worker, i.e., $(1-\theta_w)$. Let V_e be the expected utility for a employer. Now a employer receives a proposal before leaving without matching with probability $(1-\theta_w)/(1-\theta_w+\mu)$, in which case he screens it at a cost c, and accepts (if the match value exceeds θ_e) with probability $1-\theta_e$. If he rejects the proposal, he returns to the initial state and has future expected utility V_e. Hence,

$$V_e = \frac{1-\theta_w}{1-\theta_w+\mu}\left(-c+(1-\theta_e)\left(\frac{1+\theta_e}{2}\right)+\theta_e V_e\right). \tag{29}$$

Substituting $V_e = \theta_e$, solving the quadratic equation for θ_e and excluding the root which exceeds $1 > 1-\sqrt{2c}$, we obtain

$$\theta_e = l_e(\theta_w) \quad \text{where } l_e(\theta) \triangleq 1 + \frac{\mu}{1-\theta} - \sqrt{2c + \frac{2\mu}{1-\theta} + \left(\frac{\mu}{1-\theta}\right)^2}\,. \tag{30}$$

Notice that l_e is a close cousin of l_w with only the term involving c being different by a factor $1-\theta$. As above we find that for all $\mu > 0$, we have $l_e : [0, 1-\sqrt{2c}] \to [0, 1-\sqrt{2c}]$, i.e., l_e maps valid θ_w to valid θ_e. A routine calculation of $l'_e(\theta)$ suffices to establish that $|l'_e(\theta)| < \mu/c^{3/2}$ for small enough μ and all $\theta \in [0, 1-\sqrt{2c}]$, implying that l_w is $(\mu/c^{3/2})$-Lipschitz on its domain for small enough μ. (The exact Lipschitz constant will not matter for our argument except that it is decaying as $\mu \to 0$.) Crucially, the Lipschitz constant goes to 0 as $\mu \to 0$, which captures the intuition that the side which receives proposals (employers) is only affected by the threshold used by the proposing side (workers) to the extent that it increases the likelihood of leaving without matching (note that $\lim_{\mu\to 0} l_e(\theta) = 1-\sqrt{2c}$ does not depend on θ_e).

Now consider the composition of the two operators $l = l_e \circ l_w$, $l : [0, 1-\sqrt{2c}] \to [0, 1-\sqrt{2c}]$. The θ_e in any equilibrium must be fixed point of l. Now, for small enough μ, since each of the individual operators is Lipschitz with the given constants, the composition l is Lipschitz with constant equal to the product $(\mu/c^{7/4})$ of the constants. It follows that for μ small enough, the operator l is (1/2)-Lipschitz, i.e., it is contractive since its Lipschitz constant is less than 1. Since the range of l is identical to its domain, we deduce that it has a unique fixed point.

Next we characterize this fixed point. Note that $\lim_{\mu\to 0} l_w(1-\sqrt{2c}) = 1-(2c)^{1/4}$, and

$$\lim_{\mu\to 0} l(1-\sqrt{2c}) = 1-\sqrt{2c}\,. \tag{31}$$

For clarity we use $\theta_e^* = \theta_e^*(\mu)$ to denote the unique fixed point of l. We will now show that $\lim_{\mu\to 0}\theta_e^*(\mu) = 1-\sqrt{2c}$. Fix any $\epsilon > 0$. Using (31), for small enough μ we have that $|l(1-\sqrt{2c}) - (1-\sqrt{2c})| < \epsilon/4$. Further, since l is (1/2)-Lipschitz it then follows that the fixed point satisfies

$$|\theta_e^*(\mu) - (1-\sqrt{2c})| \le \epsilon/4 \times 1/(1-1/2) = \epsilon/2 < \epsilon$$

for small enough μ, as needed.

Finally, note that $\lim_{\mu\to 0}\theta_e^*(\mu) = 1-\sqrt{2c}$ implies also that the equilibrium threshold $\theta_w^* = \theta_w^*(\mu)$ of workers satisfies $\lim_{\mu\to 0}\theta_w^*(\mu) = \lim_{\mu\to 0} l_w(\theta_e^*(\mu)) = 1-(2c)^{1/4}$. Thus we have obtained the required limiting characterizations of equilibrium thresholds.

It remains to check that a_e and a_w are best responses to complete the proof that we have indeed found an equilibrium where all workers use $s_w = (a_w, \theta_w^*)$ and all employers use $s_e = (a_e, \theta_e^*)$. First consider the workers. Consider a worker's options for handling opportunities a^{o}. The payoff from screening and proposing is θ_w, the payoff from doing nothing is 0 and the payoff from proposing without screening is no more than 1/2. Clearly $\theta_w > 1/2$ for small enough μ since $\lim_{\mu\to 0}\theta_w = 1-(2c)^{1/4} > 1/2$ for $c < 1/32$. Hence the worker screens and proposes. Now consider the worker's choice of a^{i}. Workers receive no incoming proposals in equilibrium. Hypothetically, if a worker were to receive an incoming proposal just before entering (see Definition 6 of a best response), he would then choose how to handle it by comparing the expected (continuation) utility resulting from different action choices: ignoring it would pay θ_w, the expected utility from accepting without screening would be 1/2, and the expected utility from screening with threshold θ_w would be $(1-\theta_w)(1+\theta_w)/2 + \theta_w\theta_w - c = (1+\theta_w^2)/2 - c$. Since $\theta_w > 1/2$, this eliminates the option of accepting without screening. The expected utility from the third option minus that from the first option is $(1+\theta_w^2)/2 - \theta_w - c = (1-\theta_w)^2/2 - c \xrightarrow{\mu\to 0} \sqrt{c/2} - c > 0$ since $c < 1/32 < 1/2$, so the worker prefers the third option, i.e., screen incoming proposals with threshold θ_w for small enough μ. Thus, we have established that $a_w = (\mathrm{S{+}P}, \mathrm{S{+}A/R})$ constitutes a worker best response.

Now consider the employers. We showed above that using $a_e = (\mathrm{DN}, \mathrm{S{+}A/R})$, the employers are able to earn an expected utility $\theta_e \xrightarrow{\mu\to 0} 1-\sqrt{2c}$, which matches the upper bound in Lemma C.2 as $\mu \to 0$. It is easy to verify that for any other choice of a_e, the resulting expected utility is bounded away from $1-\sqrt{2c}$: Since $L_w = L_e$, the likelihood that an opportunity arises before an incoming proposal is over 1/2, and an employer's choice of a_e materially affects the expected utility he earns. Since workers screen incoming proposals, the employers face an effective screening cost of $c/(1-\theta_w) > c$ in proposing, which implies that their expected utility is strictly below $1-\sqrt{2c}$ even as $\mu \to 0$ if they deploy $a^{\mathrm{o}} = \mathrm{S{+}P}$, whereas $a^{\mathrm{o}} = \mathrm{P\ w/o\ S}$ leads to expected utility $1/2 < 1-\sqrt{2c}$ with probability $1-\theta_w > 0$ if the proposal is accepted (and less than $1-\sqrt{2c}$ if it is rejected). Thus, employers prefer to use $a^{\mathrm{o}} = \mathrm{DN}$. Now consider a^{i}. Clearly, $a^{\mathrm{i}} = \mathrm{A\ w/o\ S}$ leads to expected utility less than $1/2 < 1-\sqrt{2c}$, whereas $a^{\mathrm{i}} = \mathrm{I}$ leads to an expected utility of 0, so the employers prefer $a^{\mathrm{i}} = \mathrm{S{+}A/R}$.

The steady state agent mix $\bar{L} = \left[\frac{1}{\mu+(1-\theta_e)(1-\theta_w)}, \frac{1}{\mu+(1-\theta_e)(1-\theta_w)}\right]'$ is immediate from Proposition 12, which tells us that $\bar{L}$ is the unique attractive fixed point of (2) and (3). In showing that a_e and a_w are best responses, we in fact showed that they are strict best responses. As a result, they are also best responses for any agent mix $\bar{N}$ that is a small perturbation of the steady state $\bar{L}$, which implies evolutionary stability (Definition 7).

This completes our proof that for small enough μ there is a unique equilibrium with the given description, and that the equilibrium is stable.

No equilibrium with a_w and a_e for $c > 1/32$. Fix any $c > 1/32$. Then if a worker proposes without screening his expected utility is $1/2 > 1-(2c)^{1/4}$ in the limit $\mu \to 0$, meaning that the worker would prefer to screen and propose for small enough μ. ☐

The following lemma characterizes equilibria in which all workers use $a_w = (\mathrm{P\ w/o\ S}, \mathrm{S{+}A/R})$ and all employers use $a_e = (\mathrm{DN}, \mathrm{S{+}A/R})$.

Lemma D.4 (Pure equilibrium where workers P w/o S and employers S+A/R)**.** *The following is a characterization of equilibria where all workers use* $a_w = (\mathrm{P\ w/o\ S}, \mathrm{S{+}A/R})$ *and all employers use* $a_e = (\mathrm{DN}, \mathrm{S{+}A/R})$.

(i) For any $c \in (\frac{1}{32}, \frac{1}{8})$ *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$*, there is a unique such equilibrium. The expected utilities/thresholds in this equilibrium satisfy* $\lim_{\mu\to 0}\theta_w = 1/2$ *and* $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c}$*, the steady state is* $\bar{L} = \left[\frac{1}{\mu+(1-\theta_e)}, \frac{1}{\mu+(1-\theta_e)}\right]'$ *and the equilibrium is stable.*

(ii) For any $c \notin [1/32, 1/8)$ there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$ there is no such equilibrium.

Proof. **Existence, uniqueness and limiting expected utilities for $c \in (1/32, 1/8)$.**

Fix $c \in (1/32, 1/8)$. Our proof approach will be similar to that used in the previous lemma. We first constrain all workers to use $a_w = (\text{P w/o S}, \text{S+A/R})$ and all employers to use $a_e = (\text{DN}, \text{S+A/R})$ and compute the equilibrium θ_e between employers. We then check that a_w and a_e constitute best responses and show stability.

For now we constrain workers to use $a_w = (\text{S+P}, \text{S+A/R})$ and employers to use $a_e = (\text{DN}, \text{S+A/R})$. Recall that in steady state with $R = 1$, the mass of employers is equal to the mass of workers. It then follows that the hazard rate of receiving a proposal for an employer is equal to the hazard rate of making a proposal for a worker, i.e., the hazard rate is 1. Let V_e be the expected utility for a employer. Similar to (29) we obtain

$$V_e = \frac{1}{1+\mu}\left(-c + (1-\theta_e)\left(\frac{1+\theta_e}{2}\right) + \theta_e V_e\right).$$

Substituting $V_e = \theta_e$ based on the equilibrium thresholds characterization in Proposition 1, solving the quadratic equation for θ_e and excluding the root which exceeds $1 > 1 - \sqrt{2c}$, we obtain

$$\theta_e = 1 + \mu - \sqrt{2c + 2\mu + \mu^2} \xrightarrow{\mu \to 0} 1 - \sqrt{2c}. \tag{32}$$

Thus, we have shown that there is a unique equilibrium threshold θ_e and established its limiting value. A straightforward calculation tells us that the workers earn expected utility

$$\theta_w = \frac{1-\theta_e}{2(1-\theta_e+\mu)} \xrightarrow{\mu \to 0} \frac{1}{2}. \tag{33}$$

We now check that a_e and a_w are best responses to complete the proof that we have indeed found an equilibrium: where all workers use $s_w = (a_w, \theta_w^*)$ and all employers use $s_e = (a_e, \theta_e^*)$. The proof that a_e is a best response for an employer in the proof of Lemma D.3 applies verbatim here. It remains to show that a_w is a best response for a worker. Consider the worker's options for handling opportunities a^{o}. The payoff from screening and proposing is $l_w(\theta_e)$ where l_w was defined in (28). In particular, as $\mu \to 0$, we have $\theta_e \xrightarrow{\mu \to 0} 1 - \sqrt{2c}$ and so the limiting payoff is $1 - (2c)^{1/4} < 1/2$, i.e., the workers prefer $a^{\text{o}} = \text{P w/o S}$ for small enough μ. The payoff from doing nothing is 0. Hence the worker proposes without screening for small enough μ. Now consider the worker's choice of a^{i}. Workers receive no incoming proposals in equilibrium. Hypothetically, if a worker were to receive an incoming proposal just before entering (see Definition 6 of a best response), he would then choose how to handle it by comparing the expected (continuation) utility resulting from different action choices: ignoring it would pay $\theta_w < 1/2$, the expected utility from accepting without screening would be $1/2$, and the expected utility from screening with threshold θ_w would be $-c + (1-\theta_w)(1+\theta_w)/2 + \theta_w\theta_w = (1+\theta_w^2)/2 - c$. The first option is dominated by the second and so we compute the expected utility from the third option minus that of the second option. This difference is $(1+\theta_w^2)/2 - c - 1/2 \xrightarrow{\mu \to 0} 1/8 - c > 0$, so the worker prefers the third option, i.e., screen incoming proposals with threshold θ_w. Thus, we have established that $a_w = (\text{S+P}, \text{S+A/R})$ constitutes a worker best response.

The steady state agent mix $\bar{L} = \left[\frac{1}{\mu+(1-\theta_e)}, \frac{1}{\mu+(1-\theta_e)}\right]'$ is immediate from Proposition 12. Since we showed that a_e and a_w are *strict* best responses, evolutionary stability follows as in the proof of Lemma D.3.

This completes our proof that for small enough μ there is a unique equilibrium with the given description, and that the equilibrium is stable.

No equilibrium with a_w and a_e for $c \notin [1/32, 1/8)$. Fix any $c < 1/32$. Then if a worker screens and proposes his expected utility as $\mu \to 0$ is $1 - (2c)^{1/4} > 1/2$. Hence, for small enough μ, the worker can earn utility larger than θ_w, which breaks the equilibrium.

Fix any $c \geq 1/8$. Then if an employer accepts without screening his expected utility is $\frac{1}{2(1+\mu)} > \theta_e$. To see why the inequality holds, notice that θ_e is monotone decreasing in c, so for $c \geq 1/8$ we have $\theta_e \leq 1 + \mu - \sqrt{1/4 + 2\mu + \mu^2} = \frac{3/4}{1+\mu+\sqrt{1/4+2\mu+\mu^2}} < \frac{1}{2(1+\mu)}$ (the last inequality is easy to verify via routine simplification). So the employer would prefer to accept without screening for small enough μ. $\square$

The following lemma characterizes equilibria in which all agents use $a = (\text{P w/o S}, \text{A})$.

Lemma D.5 (Pure equilibrium where all agents propose and accept w/o screening)**.** *The following is a characterization of equilibria where all agents use* $a = (\text{P w/o S}, \text{A})$.

(i) For any $c \geq \frac{1}{8}$ *there is a unique such equilibrium. The expected utilities in this equilibrium are* $\theta_w = \theta_e = \frac{1}{\mu+2} \xrightarrow{\mu\to 0} \frac{1}{2}$, *the steady state is* $\bar{L} = \left[\frac{1}{\mu+2}, \frac{1}{\mu+2}\right]'$ *and the equilibrium is stable.*

(ii) For any $c < 1/8$ *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$ *there is no such equilibrium.*

Proof. **Existence and limiting expected utilities for $c \geq 1/8$.**

Fix $c \geq 1/8$. Suppose all agents use $a = (\text{P w/o S}, \text{A})$. Recall that in steady state with $R = 1$, the mass of employers is equal to the mass of workers (see proof of Lemma D.3). It then follows that the hazard rate of receiving a proposal for an agent and the hazard rate of making a proposal for the agent are each equal to 1. It follows that all agents earn expected utility

$$\theta = \theta_w = \theta_e = \frac{1}{2}\frac{2}{(2+\mu)} = \frac{1}{2+\mu} \xrightarrow{\mu\to 0} \frac{1}{2}\,. \tag{34}$$

We now check that action a is a best response for an agent: Note that an opportunity is equivalent to an incoming proposal, since an outgoing proposal is sure to be accepted given that other agents are all using a. By (Bertsekas 2007, Proposition 1.2.5), a is a (strict) best response if and only if, when an opportunity/incoming proposal occurs, screening (with threshold θ) and reverting to a if no match occurs (because the match value is below the threshold), leads to a (strictly) smaller payoff than sticking with a and hence matching without screening immediately leading to an expected payoff of 1/2. (It is clear that ignoring the opportunity/proposal is not competitive, since the continuation value is $\theta < 1/2$.) But this is true because the expected payoff of the former approach is

$$(1-\theta)\frac{1+\theta}{2} + \theta^2 - c = \frac{1+\theta^2}{2} - c < 5/8 - c \leq 1/2 \tag{35}$$

where we considered the possibilities of matching now (which occurs with probability $1-\theta$) or not, along with the screening cost c, and used $\theta < 1/2$ and $c \geq 1/8$.

The steady state agent mix $\bar{L} = \left[\frac{1}{\mu+2}, \frac{1}{\mu+2}\right]'$ is immediate from Proposition 12. Since we showed that a is a *strict* best response for all agents, evolutionary stability follows as in the proof of Lemma D.3.

This completes our proof that all agents using a is a stable equilibrium, and has limiting expected utility $1/2$ for all agents.

All agents using a is not an equilibrium for $c < 1/8$. Fix any $c < 1/8$. Then the equality in (35) still holds. But now $\frac{1+\theta^2}{2} - c \xrightarrow{\mu \to 0} 5/8 - c > 1/2$ using $\lim_{x\to 0} \theta = 1/2$ and $c < 1/8$, i.e., the inequality is reversed. It follows that the agent would prefer to screen for small enough μ. ☐

We are now ready to provide a formal proof of Theorem 13.

Proof of Theorem 13. Lemmas D.3, D.4, D.5 establish the equilibria stated in the proposition, including the range of c values for which they exist. It remains to show uniqueness.

Uniqueness for $c \geq 1/8$. First we consider $c \geq 1/8$ and establish that the equilibrium in Lemma D.5 is the only one. Consider any equilibrium. We start by showing that all agents accept incoming proposals without screening. We saw that in steady state for $R = 1$, there must be an equal mass of agents on each side of the market. As a result, these masses are strictly positive and for any agent there is a strictly positive probability of leaving without matching. Lemma C.2 then tells us that for any agent the expected utility θ is strictly less than $1/2$. When an incoming proposal arrives, screening it (with threshold θ as per Proposition 1) yields expected utility

$$(1-\theta)\frac{1+\theta}{2} + \theta^2 - c < 1/2\,. \tag{36}$$

It follows that the agent prefers to accept without screening, since this yields a larger expected utility of $1/2$. Since this is true on both sides, the immediate consequence is that opportunities are equivalent to incoming proposals, and furthermore, all agents propose without screening whenever an opportunity arises. We have shown that all agents must use $a = (\text{P w/o S}, \text{A})$ in any equilibrium. Thus, there are no other equilibria for $c \geq 1/8$.

No other equilibria for $c < 1/8$. By Lemma D.2, we know that all agents on one side must propose in any equilibrium. Without loss of generality since $R = 1$, we assume that all workers propose (the complete set of equilibria will be the equilibria we find along with their symmetric counterparts). We now establish a series of useful facts that hold for all such equilibria. We implicitly appeal throughout to Lemma D.1 which says that all agents consider incoming proposals in any equilibrium.

Since $c < 1/8$, all employers prefer to screen and accept/reject: Since agent masses on the two sides are equal, and all workers propose, and an opportunity for a worker translates into a proposal with probability at least $\sqrt{2c}$, it follows that each employer receives proposals at a rate at least $\sqrt{2c}$. As a result, employers can earn limiting utility $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c} > 1/2$ (equal to the largest possible) by screening and accepting/rejecting, and hence prefer to do so over accepting without screening.

Workers have expected utility bounded below as $\liminf_{\mu\to 0} \theta_w \geq 1/2$ since they can earn $1/2$ by proposing and accepting without screening. As a result, workers screen incoming proposals. The reason is that accepting without screening would give payoff $1/2$ whereas screening the proposal (with threshold θ_w as per Proposition 1) gives expected utility

$$-c + (1-\theta_w)\frac{1+\theta_w}{2} + \theta_w^2 > 1/2\,,$$

where the inequality holds for small enough μ using $c < 1/8$ and $\liminf_{\mu\to 0} \theta_w \geq 1/2$.

As a consequence of the risk of rejection, employers do not propose: By ignoring an opportunity they earn the upper bound utility of $1-\sqrt{2c} > 1/2$ in the limit, whereas proposing without screening incurs a risk of earning just $1/2$ (if the proposal is accepted), and screening and proposing earns expected utility

$$\begin{aligned}&-c + (1-\theta_e)(1-\theta_w)\frac{1+\theta_e}{2} + \theta_e(1-(1-\theta_e)(1-\theta_w))\\&= -c + \theta_e + (1-\theta_w)(1-\theta_e)^2/2 \xrightarrow{\mu\to\infty} 1-\sqrt{2c} - c\theta_w \ < \ 1-\sqrt{2c}\end{aligned}$$

which is again smaller (after all, the screening effort c is wasted with probability equal to the risk of rejection θ_w).

In the previous three paragraphs we have shown that employers necessarily use $a = (a^{\text{o}} = \text{DN}, a^{\text{i}} = \text{S+A/R})$ and workers necessarily use $a^{\text{i}} = \text{S+A/R}$ in any equilibrium. It remains to consider the two possibilities for the a^{o} used by workers, which is what we executed in Lemmas D.3 and D.4. A little detail remains: we need to rule out the possibility of an equilibrium where workers mix between screening and not screening while proposing. This is straightforward. For $c < 1/32$, *no* worker wants to propose without screening, because the limiting payoff as $\mu \to 0$ is just $1/2$, whereas screening and proposing yields limiting expected payoff $1-(2c)^{1/4} > 1/2$. Similarly, for $c < (1/8, 1/32)$, *no* worker wants to screen and propose, because the limiting payoff is $1-(2c)^{1/4} < 1/2$. $\square$

D.3 Detailed versions of Theorems 4 and 5

In this subsection we state more detailed versions of Theorems 4 and 5. In particular, we first state Theorem 14, which is detailed version of Theorem 4. Next, we state Theorem 15, which is a detailed version of Theorem 4. Immediately after stating each theorem, we describe the semantic origins of the key quantities that appear in the theorem statement. Both theorems are proved in the online appendix.

D.3.1 Statement of Theorem 14 (detailed version of Theorem 4)

Theorem 14 (No-intervention equilibria)**.** *Consider a market with one tier on each side where workers arrive faster than employers ($\lambda_w > \lambda_e$), $\lambda_e = 1$ and let $R = \lambda_w/\lambda_e = \lambda_w \geq 1.25$ denote the market imbalance.*[36] *Then for each $c > 0$ there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, a subset of stable stationary equilibria is as follows for $\bar{c}$ and $\underline{c}$ defined in* (38)*:*

1. $\mathbf{c} \in (\mathbf{0}, \mathbf{2\bar{c}^2})$: *All workers, upon arrival, choose strategy $s_w = (a^{\text{o}} = \text{S+P}, a^{\text{i}} = \text{S+A/R}, \theta_w = \xi(R, \sqrt{\frac{c}{2}}))$, i.e., workers screen and propose and screen and accept/reject proposals using threshold θ_w, where $\xi(\cdot,\cdot)$ was defined in* (37)*. All employers, upon arrival, select strategy $s_e = (a^{\text{o}} = \text{DN}, a^{\text{i}} = \text{S+A/R}, \theta_e = 1-\sqrt{2c})$, i.e., employers do no propose, and screen and accept/reject incoming proposals with threshold θ_e.*

   The steady state $\bar{L} = [L_e, L_w]'$ is given by $\bar{L} = [0, (R-1)/\mu]'$.

2. $\mathbf{c} \in \left(\mathbf{2\underline{c}^2}, \mathbf{\frac{1}{8}}\right)$: *All workers, upon arrival, choose strategy $s_w = (a^{\text{o}} = \text{P w/o S}, a^{\text{i}} = a^{\text{i}}_w, \theta_w = \frac{1}{2R})$, i.e., workers propose without screening and handle incoming proposals as per a^{i}_w (specified below; no incoming proposals arrive in equilibrium), with threshold $\theta_w = \frac{1}{2R}$ equal to their expected utility.*

   All employers, upon arrival, select strategy $s_e = (a^{\text{o}} = a^{\text{o}}_e, a^{\text{i}} = \text{S+A/R}, \theta_e = 1-\sqrt{2c})$, i.e., employers handle opportunities as per a^{o}_e (specified below; the mass of employers present in

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[36] Note that $\lambda_e = 1$ and $\lambda_w = R$ is just a normalization to reduce the notational burden. Given R, the same results hold for any arbitrary λ_e with $\lambda_w = R\lambda_e$ by scaling the system accordingly. The steady state masses scale up in proportion to λ_e, whereas thresholds, expected utilities and screening cost regimes are unaffected.

steady state is zero) and screen and accept/reject incoming proposals, with threshold $\theta_e = 1 - \sqrt{2c}$ *equal to their expected utility.*

If $c \in (2\underline{c}^2, \underline{c})$ *then* $a^{\mathrm{i}}_w = \mathrm{S{+}A/R}$ *and* $a^{\mathrm{o}}_e = \mathrm{DN}$*. Otherwise if* $c \in (\underline{c}, 1/8)$*, then* $a^{\mathrm{i}}_w = \mathrm{A}$ *and* $a^{\mathrm{o}}_e \in \{\mathrm{DN}, \mathrm{S{+}P}\}$*. This completes the specification of strategies.*[37]

The steady state is given by $\bar{L} = [L_e, L_w]' = [0, (R-1)/\mu]$*.*

3. $\mathbf{c > \frac{1}{8}}$ *: All workers, upon arrival, choose strategy* $s_w = (a^{\mathrm{o}} = \mathrm{P\ w/o\ S}, a^{\mathrm{i}} = \mathrm{A})$*, i.e., they propose without screening and accept any incoming proposal without screening, and earn expected utility* $\theta_w = 1/(2R)$*. All employers, upon arrival, choose strategy* $s_e = (a^{\mathrm{o}} = a^{\mathrm{o}}_e, a^{\mathrm{i}} = \mathrm{A})$ *with* $a^{\mathrm{o}}_e \in \{\mathrm{P\ w/o\ S}, \mathrm{DN}\}$*, and accept any incoming proposal without screening, and earn expected utility* $\theta_e = 1/2$*. The steady state is given by* $\bar{L} = [L_e, L_w]' = [0, (R-1)/\mu]'$*,*

Furthermore, there are no other stable equilibria (we do not characterize the equilibria at the boundaries $c \in \{2\bar{c}^2, 2\underline{c}^2, \underline{c}, 1/8\}$*).*

Lemma D.6 formalizes the semantic origins of the quantities $\xi(R,c)$, $\bar{c}$ and $\underline{c}$ in the context of equilibria in a certain *hypothetical setting*, defined as follows.

Hypothetical setting. The *hypothetical setting* considered is one where all employers (short-side agents) do not propose and accept all incoming proposals without screening, and workers (long-side agents) propose (with or without screening).

The following lemma (proved in the online appendix) characterizes equilibria between workers when employers accept without screening and do not propose. It introduces the crucial quantities $\xi(R,c)$, $\bar{c}$ and $\underline{c}$ and formalizes their semantic origins. (The proof appeals to evolutionary stability to rule out other equilibria.)

Lemma D.6. *Fix* $R = \lambda_w/\lambda_e > 1$*. Define*

$$\xi(R,c) \triangleq \frac{R - \sqrt{R^2 - 2R(1-2c) + (1-2c)}}{2R-1} = \frac{R - \sqrt{(R-1)^2 + 2c(2R-1)}}{2R-1}, \quad \textit{and} \tag{37}$$

$$\bar{c} \triangleq \frac{1}{4}\left(1 - \frac{R}{1 + \sqrt{1 + (R-1)^2}}\right), \ \textit{and} \ \ \underline{c} \triangleq 1/(8R^2)\,. \tag{38}$$

Suppose all employers do not propose and accept incoming proposals without screening. Then there exists $\mu_0 = \mu_0(c) > 0$ *such that for any* $\mu < \mu_0$*, the following are the equilibria between workers (here we ignore how workers deal with incoming proposals since employers do not propose):*

1. *If* $c < \bar{c}$*, there is an equilibrium where all workers screen and propose with threshold (and expected utility)* $\theta_w = \xi(R,c)$*.*
2. *If* $c > \underline{c}$*, there is an equilibrium where all workers propose without screening and earn expected utility* $\frac{1}{2R}$*.*

Moreover there are no other equilibria. (We omit to characterize whether Equilibrium 1 exists at the boundary $c = \bar{c}$ *and whether Equilibrium 2 exists at the boundary* $c = \underline{c}$*.)*

We note the following:

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[37] Note that employers have two choices of a^{o}_e in equilibrium for $c \in [\underline{c}, 1/8)$. However, the *total* steady state mass L_e of employers (the first coordinate of $\bar{L}$) is zero, and hence the mass of employers following each of the two possible strategies is zero.

Remark 4. *It is easy to verify that $\bar{c} > \underline{c}$. One simply uses*

$$\sqrt{1+(R-1)^2} = R\sqrt{1-\frac{2(R-1)}{R^2}} > R\left(1-\frac{(R-1)}{R^2}\right) = R-1+1/R\,,$$

to obtain $\bar{c} > \frac{1}{4(R^2+1)} > \frac{1}{8R^2} = \underline{c}\,$.

We now informally describe the findings of the above lemma.

Semantic origins of $\xi(R,c)$. The function $\xi(R,c)$ captures the threshold for screening a candidate used by the workers in the hypothetical setting, in the equilibrium where all workers screen.

Semantic origins of $\bar{c}$. In the hypothetical setting, a worker must decide between screening or not the candidates. Then $\bar{c}$ is the supremum of screening costs such that (for small enough μ) there exists an equilibrium between workers where all workers screen and propose using a threshold of $\theta_w = \xi(R,c)$.

Semantic origins of $\underline{c}$. The quantity $\underline{c}$ is the smallest value of c such that (for small enough μ) there is an equilibrium between workers in the hypothetical setting where all workers issue proposals without screening.

Note that the semantic descriptions of $\xi(R,c)$, $\bar{c}$ and $\underline{c}$ are based on the hypothetical setting where employers do not screen. The bridge between the theorem (Equilibria 1 and 2) and the hypothetical setting is provided by the effective screening cost $c_{\text{eff}} = \sqrt{c/2}$ that workers face while proposing when employers screen with threshold $\theta_e = 1-\sqrt{2c}$ (see Lemma C.1), as formalized in Lemma D.7 (proved in the online appendix).

Lemma D.7. *Fix $R > 1$. Consider $\xi(\cdot,\cdot)$ defined in* (37) *and $\bar{c}$ and $\underline{c}$ defined in* (38).

Suppose all employers do not propose and accept incoming proposals with probability $\sqrt{2c}$. Then there exists $\mu_0 = \mu_0(c) > 0$ such that for any $\mu < \mu_0$, the following are the equilibria between workers (we ignore how workers deal with incoming proposals since employers do not propose):

1. *If $c < 2\bar{c}^2$, there is an equilibrium where all workers screen and propose with threshold (and expected utility) $\theta_w = \xi(R, \sqrt{c/2})$.*
2. *If $c > 2\underline{c}^2$, there is an equilibrium where all workers propose without screening and earn expected utility $\frac{1}{2R}$.*

Moreover there are no other equilibria. (We omit to characterize whether Equilibrium 1 exists at the boundary $c = 2\bar{c}^2$ and whether Equilibrium 2 exists at the boundary $c = 2\underline{c}^2$.)

D.3.2 Statement of Theorem 15 (detailed version of Theorem 5)

Theorem 15. *Consider a market with arrival rates $\lambda_e = 1$ and $\lambda_w = R > 1$. Suppose that workers are not allowed to propose. For any $c \in (0, \min(\bar{c}, \hat{c}))$ there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, there is an equilibrium where all employers use $a_e = (\text{S+P}, \text{S+A/R})$ and all workers use $a_w = (\text{DN}, \text{S+A/R})$, with thresholds $\theta_e = \theta_e(\mu)$ and $\theta_w = \theta_w(\mu)$, respectively, and steady state given by $\bar{L} = \left[\frac{1}{\mu+\eta_e},\ \frac{R-1}{\mu} + \frac{1}{\mu+\eta_e}\right]$ where $\eta_e = (1-\theta_e)(1-\theta_w)$. Furthermore, the equilibrium is stable, the thresholds in this equilibrium satisfy $\lim_{\mu\to 0}\theta_w = \xi(R,c)$ and $\lim_{\mu\to 0}\theta_e = 1-\sqrt{2c/(1-\xi(R,c))}$. Moreover, for every $c \in (0, \min(\bar{c}, \underline{c}, \hat{c}))$ and every $\mu < \mu_0(c)$, this is the unique equilibrium.*
Here $\bar{c}$ and $\underline{c}$ are as per (38), *$\xi(R,c)$ is defined in* (37), *and*

$$\hat{c} \triangleq \frac{8R-7}{32(2R-1)}\,. \tag{39}$$

We now describe why the value $\hat{c}$ arises.

Semantic origins of $\hat{c}$. Consider the setting where employers screen and propose and workers screen and accept/reject. Workers are not permitted to propose. Then $\hat{c}$ is the screening cost (as $\mu \to 0$) at which employers are indifferent between screening and not screening before they propose, assuming workers are screening with threshold $\xi(R, c)$. (We show via Lemma I.2 in the online appendix that workers indeed employ a screening threshold $\theta_w \xrightarrow{\mu\to 0} \xi(R, c)$ in any equilibrium where all employers propose. Lemma I.2 is a corollary of Lemma D.6.)

Note that if workers are screening with threshold $\xi(R, c)$, then by Lemma C.1, the effective cost for employers is equal to $c_{\text{eff}} = c/(1 - \xi(R, c))$. Now the value and threshold for employers when they screen before proposing is $\theta_e(c) = 1 - \sqrt{2c_{\text{eff}}}$. The value when employers propose without screening is $1/2$. It follows that employers want to screen before proposing (for small enough μ) if and only if $1 - \sqrt{2c_{\text{eff}}} > 1/2$, and it turns out this occurs if and only if $c < \hat{c}$ where $\hat{c}$ given by (39) is the value of c at which $1 - \sqrt{2c_{\text{eff}}} = 1/2$. The next lemma, which is proved in the online appendix, formalizes this claim.

Lemma D.8. *Consider a setting with $R > 1$ in which workers mechanically screen and accept proposals $a^{\text{i}} = \text{S+A/R}$ with threshold $\theta_w = \theta_w(\mu)$ such that $\lim_{\mu\to 0} \theta_w = \xi(R, c)$ and do not propose $a^{\text{o}} = \text{DN}$. Consider the equilibrium proposal behavior of employers. In any equilibrium all employers propose. Furthermore:*

1. *For any $c < \hat{c}$, there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, all employers screen and propose $a^{\text{o}} = \text{S+P}$ with threshold and expected utility θ_e satisfying*

$$\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c/(1 - \xi(R, c))} > 1/2\,.$$

2. *For any $c > \hat{c}$, there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, all employers propose without screening $a^{\text{o}} = \text{P w/o S}$ and earn expected utility $\theta_e = \frac{1-\theta_w}{2(\mu+1-\theta_w)} \xrightarrow{\mu\to 0} 1/2$.*

E Appendix to Section 4: vertical differentiation

This appendix supplies the proofs for the equilibrium results in Section 4 which considers the setting with two tiers of workers, high workers and low workers, and a single tier of employers. (The proof of the planner's solution in this setting, Proposition 7, can be found in the online appendix.)

We start by describing and justifying the parameter regime studied.

In Appendix E.1, we prove a stronger version of Theorem 8 (Theorem 16) which describes the no-intervention equilibrium. The proof is far more intricate than the case of one tier on each side, and is divided into five claims.

Appendix E.2 provides detailed statements of the theorems characterizing the equilibria under the "block workers from proposing" and "block workers from proposing and hide quality information" interventions, respectively. The proofs can be found in the online appendix.

Parameter regime studied. We now describe and justify the parameter regime we focus on, and discuss what happens in other regimes. We consider one tier of employers and two tiers of workers, high (H) and low (L), where employers arrive faster than high workers but slower than workers overall.

First, we fix the quality $q \in (0,1)$ of high workers. The constraint $q \in (0,1)$ results in distributional "overlap" in the sense that, with positive probability, a particular employer prefers a particular low worker to a particular high worker.[38]

Second, for a given q, in Theorem 16 we characterize no-intervention equilibria for c's satisfying

$$c < c_0(q) \triangleq \begin{cases} \frac{1}{2}\left(\frac{1-\sqrt{1-4q}}{2}\right)^4 & \text{if } q \in (0, 1/4] \\ \frac{1}{2}\left(\frac{-1+\sqrt{5-4q}}{2}\right)^4 & \text{if } q \in (1/4, 1). \end{cases} \tag{40}$$

Remark 5. *If $c > c_0(q)$ for $q \in (1/4, 1)$, then employers propose without screening to high workers. If $c > c_0(q)$ for $q \leq 1/4$, then high workers are not "sufficiently attractive" and high workers screen and propose to employers in equilibrium instead of employers screening and proposing to high workers. In both cases the qualitative findings in this section are not affected and the same set of inefficiencies arise. We skip these cases in the interest of space.*

Remark 6. *All our results hold for $\lambda_e \in (\lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}, \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$ or a superset of this interval, where δ^{H} is defined as*[39]

$$\delta^{\mathrm{H}} \triangleq \lambda_w^{\mathrm{H}} \left(\frac{q - \sqrt{2c}\left(\frac{1}{q+\sqrt{2c}} - 1\right)}{2(1-\sqrt{2c})} \right) > 0 \qquad \text{for } c < c_0(q)\,. \tag{41}$$

Note that this definition of δ^{H} is consistent with the specification of δ^{H} in Theorem 8 item (i), where, for simplicity, we only specify δ^{H} to leading order as $c \to 0$.

This is the "interesting" range for λ_e:

(i) If $\lambda_e < \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}$, then employers do not match with low workers at all in the no-intervention equilibrium; thus, only high workers and employers interact, and this interaction is analogous to that in an unbalanced market with ex-ante homogeneous agents (see Section 3).

(ii) If employers arrive faster than workers overall ($\lambda_e > \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}}$), then the market resembles that of Section 3 with workers (high + low) being on the short side.[40]

Note also that since we consider small c throughout Section 4 and $\lim_{c\to 0} \delta^{\mathrm{H}} = \lambda_w^{\mathrm{H}} q/2$, the range $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$ is the one of interest there.

Equilibrium characterization turns out to be (even) more challenging under platform interventions. Hence, motivated by online matching platforms we characterize equilibria under intervention in the small-c regime, which allows us to provide analytic expressions for agent utilities. Since $\lim_{c\to 0} \delta^{\mathrm{H}} = \lambda_w^{\mathrm{H}} q/2$, our results under platform interventions hold for $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$ or a superset of this interval. We find that our interventions boost welfare even as $c \to 0$.

E.1 No-intervention equilibrium

Consider the setting specified in Section 4 with no intervention. The following theorem is a stronger version of Theorem 8, in that it considers arbitrary $c < c_0(q)$. We provide a detailed statement. (Before proving it, we provide a short formal argument deducing Theorem 8 from it.)

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[38] We omit the case $q \geq 1$ as it produces no new insights. If q is large enough, employers are interested in matching only with high workers, and the market resembles the unbalanced market in Section 3. Otherwise, the equilibrium structure is the same as for $q \in (1/4, 1)$ and $c \geq c_0(q)$ (see Remark 5) and our qualitative findings below are unaffected.

[39] Since $c < c_0(q)$ implies $q > (2c)^{1/4} - (2c)^{1/2}$ (see Lemma E.1 in Appendix E.1), this ensures $\delta^{\mathrm{H}} > 0$.

[40] For instance, similarly to what is described in Theorem 4, if the imbalance is sufficiently large, then only employers propose in equilibrium: they propose to both tiers, with a preference for high workers. This resemblance is closest when the platform hides quality information from employers, an intervention we later recommend.

Theorem 16. *Fix high-worker quality* $q \in (0,1)$ *and* $\lambda_w^{\mathrm{H}} > 0$, $\lambda_w^{\mathrm{L}} > 0$, *and* $\lambda_e \in (\lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}, \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}} + \delta^{\mathrm{H}})$. *Then for* $c_0(q)$ *given by* (40), *for all* $c < c_0(q)$, *there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$ *there is a stable equilibrium with the following description:*

(i) High workers do not propose, and screen incoming proposals using a threshold equal to their expected utility $\theta_w^{\mathrm{H}} = 1 - \sqrt{2c}$, *i.e., they use strategy* $(a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_w^{\mathrm{H}} = 1 - \sqrt{2c})$.

(ii) Low workers propose without screening and earn expected utility θ_w^{L}, *i.e., they use strategy* $(a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{P\ w/o\ S}, \theta_w^{\mathrm{L}})$. *We have* $\lim_{\mu\to 0} \theta_w^{\mathrm{L}} = (\lambda_e - \lambda_w^{\mathrm{H}} - \delta^{\mathrm{H}})/(2\lambda_w^{\mathrm{L}})$.

(iii) Employers do not propose to low workers. When the opportunity arises, employers request a high worker and, if one is available, screen and propose with threshold equal their expected utility θ_e *which satisfies* $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c}$. *The employers split into two types based on how they respond to proposals from low workers.*

  - *Reachers: Reachers ignore proposals from low workers, i.e, they use strategy* $s^{\mathrm{re}} = (a_{\mathrm{H}}^{\mathrm{i}} = \mathrm{S{+}A/R}, a_{\mathrm{L}}^{\mathrm{i}} = \mathrm{I}, \succ^{\mathrm{o}} = \mathrm{H}, a_{\mathrm{H}}^{\mathrm{o}} = \mathrm{S{+}P}, a_{\mathrm{L}}^{\mathrm{o}} = \mathrm{DN}, \theta_e)$. *The fraction of reachers* $f_e(s^{\mathrm{re}})$ *satisfies* $\lim_{\mu\to 0} f_e(s^{\mathrm{re}}) = \frac{\lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}}{\lambda_e}$.
  - *Settlers: All other employers screen incoming proposals from low workers using threshold* θ_e, *i.e., settlers use strategy* $s^{\mathrm{se}} = (a_{\mathrm{H}}^{\mathrm{i}} = \mathrm{S{+}A/R}, a_{\mathrm{L}}^{\mathrm{i}} = \mathrm{S{+}A/R}, \succ^{\mathrm{o}} = \mathrm{H}, a_{\mathrm{H}}^{\mathrm{o}} = \mathrm{S{+}P}, a_{\mathrm{L}}^{\mathrm{o}} = \mathrm{DN}, \theta_e)$.

The steady state is as follows in the limit $\mu \to 0$: *There is a flow* λ_w^{H} *of matches between reacher employers and high workers, a limiting flow* δ^{H} *of reacher employers leaving without matching, whereas all settler employers match with low workers, producing a limiting flow* $\lambda_e - \lambda_w^{\mathrm{H}} - \delta^{\mathrm{H}}$ *of such matches. There is a mass* $\bar{L}_e(s^{\mathrm{re}})$ *of reachers satisfying* $\lim_{\mu\to 0} \mu \bar{L}_e(s^{\mathrm{re}}) = \delta^{\mathrm{H}}$, *a limiting mass* $\lim_{\mu\to 0} \bar{L}_e(s^{\mathrm{se}}) = \frac{(\lambda_e - \lambda_w^{\mathrm{H}} - \delta^{\mathrm{H}})\delta^{\mathrm{H}}}{(\lambda_w^{\mathrm{L}} + \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}} - \lambda_e)\sqrt{2c}}$ *of settlers, and a mass* L_w^{L} *of low workers satisfying* $\lim_{\mu\to 0} \mu L_w^{\mathrm{L}} = \lambda_w^{\mathrm{L}} + \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}} - \lambda_e$, *and the mass of high workers in the system is always* $L_w^{\mathrm{H}} = 0$ *(they match as soon as they arrive). It equilibrium is evolutionarily stable, i.e., it satisfies Definition 7.*

We can define an "*effective arrival flow*" of $\tilde{\lambda}^{\mathrm{H}} \triangleq \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}$ for reacher employers (as $\mu \to 0$). Then, the equilibrium captured in the theorem has an incoming flow $\tilde{\lambda}^{\mathrm{H}}$ of reacher employers, who wait to propose and match to a high worker whom they like (or else leave unmatched), and an incoming flow of settler employers arriving at the residual rate of $(\lambda_e - \tilde{\lambda}^{\mathrm{H}}) < \lambda_w^{\mathrm{L}}$, who consider incoming proposals from low workers, and typically match with a low worker whom they like. There is effectively a "high submarket," consisting of high workers and reacher employers with imbalance $\tilde{\lambda}^{\mathrm{H}}/\lambda_w^{\mathrm{H}}$ and employers being on the long side; and a "low submarket," consisting of settler employers and low workers with imbalance $\lambda_w^{\mathrm{L}}/(\lambda_e - \tilde{\lambda}^{\mathrm{H}}) > 1$ and low workers being on the long side. The behavior of agents under equilibrium in the high submarket resembles that in Theorem 4 Equilibrium 1, and in the low submarket it resembles that in Theorem 4 Equilibrium 2.

Proof of Theorem 8. We show that Theorem 8 follows from Theorem 16.

Comparing the statement of Theorem 8 with that of Theorem 16, we observe only the following differences (besides the fact that the statement is less detailed):

1. The range of λ_e is $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}}(1 + q/2) + \lambda_w^{\mathrm{L}})$.
2. The maximum allowed value of c (denoted by $c_{\max}$) is not quantified.
3. δ^{H} is specified only to leading order as per $\delta^{\mathrm{H}} = \lambda_w^{\mathrm{H}} q/2(1 + o_c(1))$.

Point 3 is a non-issue since (41) implies $\delta^{\mathrm{H}} = \lambda_w^{\mathrm{H}} q/2(1 + o_c(1))$. Point 2 notes that the statement of Theorem 8 is weaker in the sense that it makes an assertion only for small enough c. In fact, this deviation allows us to take care of point 1 and hence deduce the theorem: consider any $\lambda_e \in (\lambda_w^{\mathrm{H}}(1 + q/2), \lambda_w^{\mathrm{H}}(1 + q/2) + \lambda_w^{\mathrm{L}})$. Since $\lim_{\mu\to 0} \delta^{\mathrm{H}} = \lambda_w^{\mathrm{H}} q/2$, it follows that for small enough c, $\lambda_e \in (\lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}, \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}} + \delta^{\mathrm{H}})$. The theorem then follows from Theorem 16. □

Roadmap for the proof of Theorem 16. The formal proof will proceed in four steps, each of which will be formalized and proved in a different claim. We will first establish the existence of a fixed point, with the given limiting description, of the dynamical system (2) and (3) under a fixed mix of agent strategies (the mix of strategies will itself have the given limiting description); this is achieved in Claim 1. This fixed point will have the additional property that reacher and settler employers obtain exactly the same utility. Second, we will show that the fixed point is attractive in Claim 2 (proved in the online appendix). Third, we will show that each category of agent (by category we mean high workers, low workers, reacher employers, and settler employers) is playing a best response in Claim 3. This will establish that we have found a stationary equilibrium. Fourth, we will show that the equilibrium is also evolutionarily stable in Claim 4 (proved in the online appendix).

We make use of the following fact throughout this section.

Lemma E.1. *The pair of assumptions* $q \in (0,1)$ *and* $c < c_0(q)$ *in Theorem 16 is equivalent to the pair of assumptions* $c < 1/32$ *and* $q \in ((2c)^{1/4} - (2c)^{1/2}, 1 - (2c)^{1/4} - (2c)^{1/2})$*.*

Proof of Lemma E.1. We first show the forward implication and then the reverse implication.

Forward implication. Assume $q \in (0,1)$ and $c < c_0(q)$. Clearly from the definition (40), we see that $c_0(q)$ is increasing in $(0, 1/4]$ and decreasing in $(1/4, 1)$. The largest possible value is $c_0(1/4) = 1/32$, so $c < c_0(q) \leq 1/32$. It remains to show that $q \in ((2c)^{1/4} - (2c)^{1/2}, 1 - (2c)^{1/4} - (2c)^{1/2})$.

If $q \leq 1/4$, we have $c < c_0(q) = \frac{1}{2}\big(\frac{1-\sqrt{1-4q}}{2}\big)^4$ which implies $q > (2c)^{1/4} - (2c)^{1/2}$. Meanwhile, $1 - (2c)^{1/4} - (2c)^{1/2} > 1/4 \geq q$ using $c < 1/32$, as desired.

On the other hand if $q > 1/4$, we have $c < c_0(q) = \frac{1}{2}\big(\frac{-1+\sqrt{5-4q}}{2}\big)^4$ which implies $q < 1 - (2c)^{1/4} - (2c)^{1/2}$. Meanwhile, $(2c)^{1/4} - (2c)^{1/2} < 1/4 < q$ using $c < 1/32$, as desired.

Reverse implication. Assume $c < 1/32$ and $q \in ((2c)^{1/4} - (2c)^{1/2}, 1 - (2c)^{1/4} - (2c)^{1/2})$. Clearly we have $q \in (0,1)$ since $0 < (2c)^{1/2} < (2c)^{1/4}$. It remains to show $c < c_0(q)$.

If $q \leq 1/4$, it follows from $q > (2c)^{1/4} - (2c)^{1/2}$ that $c < \frac{1}{2}\big(\frac{1-\sqrt{1-4q}}{2}\big)^4 = c_0(q)$.

If $q > 1/4$, if follows from $q < 1 - (2c)^{1/4} - (2c)^{1/2}$ that $c < \frac{1}{2}\big(\frac{-1+\sqrt{5-4q}}{2}\big)^4 = c_0(q)$. □

Throughout this section we find it convenient to the *arrival rate of reacher employers* $\lambda_e^{\mathrm{re}} \triangleq \lambda_e f_e(s^{\mathrm{re}})$ and the *arrival rate of settler employers* $\lambda_e^{\mathrm{se}} \triangleq \lambda_e f_e(s^{\mathrm{se}}) = \lambda_e(1 - f_e(s^{\mathrm{re}})) = \lambda_e - \lambda_e^{\mathrm{re}}$.

Fixed point of dynamical system. High workers arrive at rate λ_w^{H} and low workers arrive at rate λ_w^{L}. Employers arrive at rate λ_e. Let agent strategies be as per the theorem statement, including thresholds such that $\lim_{\mu\to 0} \theta_e = \theta_w^{\mathrm{H}} = 1 - \sqrt{2c}$, where $\theta_e = \theta_e(\mu)$ is as specified below.

Claim 1. *There exists* $\epsilon > 0$*, an arrival rate of reacher employers* $\lambda_e^{\mathrm{re}} = \lambda_e^{\mathrm{re}}(\mu) \in (\lambda_w^{\mathrm{H}} + \epsilon, \lambda_e - \epsilon)$ *such that* $\lim_{\mu\to 0} \lambda_e^{\mathrm{re}} = \tilde{\lambda}^{\mathrm{H}} \triangleq \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}}$*, and a threshold* $\theta_e = \theta_e(\mu) \in (\epsilon, 1-\epsilon)$ *such that* $\lim_{\mu\to 0} \theta_e = 1 - \sqrt{2c}$*, such that under this fixed mix of agent strategies, there is a fixed point of* (2) *and* (3) *for* $\mu \in (0, \epsilon)$ *with the following properties:*

1. *High workers match as soon as they arrive* $L_w^{\mathrm{H}} = 0$*.*
2. *The utility of reacher and settler employers is exactly the same, and equal to* θ_e*.*
3. *The mass of reacher employers* $\bar{L}_e(s^{\mathrm{re}})$ *satisfies* $\lim_{\mu\to 0} \mu \bar{L}_e(s^{\mathrm{re}}) = \delta^{\mathrm{H}}$*.*
4. *The limiting mass of settler employers is* $\lim_{\mu\to 0} \bar{L}_e(s^{\mathrm{se}}) = \frac{\delta^{\mathrm{H}}(\lambda_e - \tilde{\lambda}^{\mathrm{H}})}{\sqrt{2c}(\lambda_w^{\mathrm{L}} + \tilde{\lambda}^{\mathrm{H}} - \lambda_e)}$*.*
5. *The limiting mass of low workers* L_w^{L} *satisfies* $\lim_{\mu\to 0} \mu L_w^{\mathrm{L}} = \lambda_w^{\mathrm{L}} + \tilde{\lambda}^{\mathrm{H}} - \lambda_e$*.*

The intuition for the claim is as follows. (We present a formal proof below.) First, the masses of Reachers and of low workers in the system should be such that, when multiplied by μ, each gives the rate at which agents in that category leave unmatched (points 3 and 5). In turn, these masses give the rate at which these categories (want to) propose to the others. We expect that all high workers and settler employers match in the limit $\mu \to 0$, and so this allows us to calculate their expected number in the system in order to achieve this rate, given the rate of incoming proposals. Next, we verify that for the given masses the rate of high workers and settler employers leaving without matching is 0 as $\mu \to \infty$, and in fact the high workers match as soon as they arrive (point 1). Finally, we can choose λ_e^{re} such that incoming employers are indifferent between being Reachers and Settlers (point 2), and in the limit $\mu \to 0$, this condition implies that a fraction $\delta^{\mathrm{H}}/(\lambda_w^{\mathrm{H}}+\delta^{\mathrm{H}})$ of reacher employers must leave without matching, leading to point 3. The mass of settler employers is such as to ensure that the flow of matches is the same as the arrival flow as $\mu \to 0$, since there is an excess of low workers.

Proof of Claim 1. In this proof, to make the notation lighter, we make use of the following shorthand notations: $N_e^{\mathrm{re}} \triangleq \bar{N}_e(s^{\mathrm{re}})$, $L_e^{\mathrm{re}} \triangleq \bar{L}_e(s^{\mathrm{re}})$, $N_e^{\mathrm{se}} \triangleq \bar{N}_e(s^{\mathrm{se}})$, $L_e^{\mathrm{se}} \triangleq \bar{L}_e(s^{\mathrm{se}})$.

Let us write the DEs capturing the system's dynamics. Fix a time τ. Fix the incoming flow of reacher employers $\lambda_e^{\mathrm{re}} \in (\lambda_w^{\mathrm{H}}+\epsilon, \lambda_e-\epsilon)$ for any $\epsilon > 0$. Settler employers arrive at rate $\lambda_e^{\mathrm{se}} = \lambda_e - \lambda_e^{\mathrm{re}}$. Let N_e^{re} be the mass of reacher employers in the market, N_e^{se} be the mass of settler employers, N_e be the total mass of employers, and let N_w^{L} be the mass of low workers in the market. If we begin with all three masses positive and a zero mass of high workers in the system, we have the following dynamical equations:

$$\begin{aligned}
\frac{\mathrm{d}N_w^{\mathrm{L}}}{\mathrm{d}\tau} &= -\left(\frac{N_e^{\mathrm{se}}\eta_{\mathrm{L}}}{N_e} + \mu\right) N_w^{\mathrm{L}} + \lambda_w^{\mathrm{L}} \\
\frac{\mathrm{d}N_e^{\mathrm{re}}}{\mathrm{d}\tau} &= -\lambda_w^{\mathrm{H}} \frac{N_e^{\mathrm{re}}}{N_e} - \mu N_e^{\mathrm{re}} + \lambda_e^{\mathrm{re}} \\
\frac{\mathrm{d}N_e^{\mathrm{se}}}{\mathrm{d}\tau} &= -\lambda_w^{\mathrm{H}} \frac{N_e^{\mathrm{se}}}{N_e} - \frac{N_e^{\mathrm{se}}\eta_{\mathrm{L}} N_w^{\mathrm{L}}}{N_e} - \mu N_e^{\mathrm{se}} + \lambda_e^{\mathrm{se}},
\end{aligned} \tag{42}$$

where $\eta_{\mathrm{L}} \triangleq 1 - \theta_e = \sqrt{2c} + O(\mu)$ is the fraction of low workers that employers find acceptable (we will choose the $O(\mu)$ term later; for now our analysis will work for an arbitrary term of this form). To obtain the equations in (42) we have used that matches form between high workers and employers at rate λ_w^{H} (these matches form with each type of employer in proportion to their respective mass present in the system). Matches form between settler employers and low workers (as a result of low workers proposing without screening) at rate $N_w^{\mathrm{L}} \frac{N_e^{\mathrm{se}}}{N_e} \eta_{\mathrm{L}}$ (low workers at rate N_w^{L}, a fraction $\frac{N_e^{\mathrm{se}}}{N_e}$ of these proposals go to settlers and, from this, a fraction η_{L} is accepted).

We now show that there is a (stable) fixed point whose limiting description as $\mu \to 0$ is

$$L_w^{\mathrm{L}} = \frac{\lambda_w^{\mathrm{L}} - \lambda_e^{\mathrm{se}}}{\mu} \qquad L_e^{\mathrm{re}} = \frac{\lambda_e^{\mathrm{re}} - \lambda_w^{\mathrm{H}}}{\mu} \qquad L_e^{\mathrm{se}} = \frac{\lambda_e^{\mathrm{se}}(\lambda_e^{\mathrm{re}} - \lambda_w^{\mathrm{H}})}{(\lambda_w^{\mathrm{L}} - \lambda_e^{\mathrm{se}})\sqrt{2c}}, \tag{43}$$

as claimed. Note that this fixed point has all three masses positive, and hence the dynamical equations (42) remain valid (including that high workers match as soon as they arrive, and hence the mass of high workers in the system remains zero). To that end, define

$$z_1 \triangleq \mu(N_w^{\mathrm{L}} - L_w^{\mathrm{L}}) \qquad z_2 \triangleq \mu(N_e^{\mathrm{re}} - L_e^{\mathrm{re}}) \qquad z_3 \triangleq N_e^{\mathrm{se}} - L_e^{\mathrm{se}}. \tag{44}$$

Substituting (44) and (43) in (42), and considering $\|z\| = O(1)$, we get

$$\begin{aligned}
\frac{1}{\mu}\frac{\mathrm{d}z_1}{\mathrm{d}\tau} &= -\left(1+\frac{\lambda_e^{\mathrm{se}}}{\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}}}\right)z_1 + \frac{\lambda_e^{\mathrm{se}}}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}z_2 - \frac{\eta_{\mathrm{L}}(\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}})}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}z_3 + O(\mu)\,, \\
\frac{1}{\mu}\frac{\mathrm{d}z_2}{\mathrm{d}\tau} &= -z_2 + O(\mu)\,, \\
\frac{\mathrm{d}z_3}{\mathrm{d}\tau} &= -\frac{\lambda_e^{\mathrm{se}}}{\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}}}z_1 + \frac{\lambda_e^{\mathrm{se}}}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}z_2 - \frac{\eta_{\mathrm{L}}(\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}})}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}z_3 + O(\mu)\,.
\end{aligned} \tag{45}$$

Here the $O(\mu)$ terms are, in fact, Lipschitz continuous in z and λ_e^{se} with a Lipschitz constant that is $O(\mu)$. Let A be the coefficient matrix for the linear terms above. We have

$$A \triangleq \begin{bmatrix} -1-\xi & \beta & -\gamma \\ 0 & -1 & 0 \\ -\xi & \beta & -\gamma \end{bmatrix} \quad \text{where} \quad \xi \triangleq \frac{\lambda_e^{\mathrm{se}}}{\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}}}\,, \quad \beta \triangleq \frac{\lambda_e^{\mathrm{se}}}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}\,, \quad \gamma \triangleq \frac{\eta_{\mathrm{L}}(\lambda_w^{\mathrm{L}}-\lambda_e^{\mathrm{se}})}{\lambda_e^{\mathrm{re}}-\lambda_w^{\mathrm{H}}}\,. \tag{46}$$

It is easy to verify that A has full rank with singular values bounded away from 0. Hence, there is a fixed point z^* of this set of equations satisfying

$$z^* = O(\mu)\,.$$

Thus, (43) gives the correct fixed point to within $O(\mu)$ in relative terms (also, the relative errors are Lipschitz with a Lipschitz constant that is $O(\mu)$).

The steady-state utility of reacher employers is decreasing in L_e^{re}, as more of them leave unmatched as this value increases. In fact, this utility is a constant plus $(\lambda_e^{\mathrm{re}} - \lambda_w^{\mathrm{H}})/\lambda_e^{\mathrm{re}} = \text{constant} - \lambda_w^{\mathrm{H}}/\lambda_e^{\mathrm{re}}$ up to $O(\mu)$, which is decreasing in λ_e^{re}. In comparison, the settler employers have a fixed utility in the limit that $\mu \to 0$. We deduce that there is a unique value of λ_e^{re} such that reacher and settler employers have the same utility. Using (43) and $\xi_{\mathrm{L}} = \sqrt{2c}$ yields that this uniquely determined rate of arrival of reacher employers is $\lambda_e^{\mathrm{re}} = \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}} + O(\mu)$.

We now need to set θ_e appropriately; we have left the $O(\mu)$-sized correction term ambiguous until now. Suppose we start with $\theta_e = 1 - \sqrt{2c}$ exactly and do the above process. Then we correct θ_e, for now holding λ_e^{re} fixed, so that it matches the utility employers were getting. This correction in θ_e will be $O(\mu)$, because the utility of settler employers is $1 - \sqrt{2c} - O(\mu)$. This change in θ_e will change the fixed point by $O(\mu)$. As a result λ_e^{re} will need to be adjusted by $O(\mu)$ to make utilities equal for reacher and settler employers. However, the change in (settler) employer utility is only

$$O\big(\mu|(\text{Change in steady-state parameters})| + (\text{Change in } \theta_e)^2\big) = O(\mu^2) + O(\mu^2) = O(\mu^2).$$

Then, the next time, when we again adjust θ_e, then λ_e^{re}, and then calculate the change in utility, all changes will be a factor $O(\mu)$ less than in the previous iteration. In other words, this iterative process converges rapidly for small enough μ and, upon convergence, produces $\lambda_e^{\mathrm{re}} = \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}} + O(\mu)$ and $\theta_e = 1 - \sqrt{2c} - O(\mu)$ such that the utilities of reacher and settler employers are both exactly equal to θ_e. For a quick sanity check, note that the utility of settlers as $\mu \to 0$ is

$$\underbrace{\frac{\lambda_{\mathrm{H}}}{\lambda_{\mathrm{H}} + \delta_{\mathrm{H}}}}_{\substack{\text{Fraction of}\\ \text{settlers who match}}} \left(\underbrace{\frac{2 + q - \sqrt{2c}}{2}}_{\substack{\text{Expected value}\\ \text{cond. on matching}}} - c \underbrace{\frac{1}{\sqrt{2c}(q + \sqrt{2c})}}_{\substack{\text{Expected \# high}\\ \text{workers screened}\\ \text{per match}}} \right) = 1 - \sqrt{2c}.$$

Using (43) and $\xi_{\mathrm{L}} = \sqrt{2c}$, yields that this uniquely determined arrival rate of reacher employers is $\lambda_e^{\mathrm{re}} = \lambda_w^{\mathrm{H}} + \delta^{\mathrm{H}} + O(\mu)$. Substituting in (43) and using $\lambda_e^{\mathrm{se}} = \lambda_e - \lambda_e^{\mathrm{re}}$ completes the claim. ☐

The fixed point is stable.

Claim 2. *The fixed point of* (2) *and* (3) *found in Claim 1 is attractive.*

To check dynamical stability (under a fixed mix of agent strategies), we need to investigate the eigenvalues of

$$A_1 \triangleq \begin{bmatrix} -\mu(1+\xi) & \mu\beta & -\mu\gamma \\ 0 & -\mu & 0 \\ -\xi & \beta & -\gamma \end{bmatrix}, \qquad \text{since } \frac{\mathrm{d}z}{\mathrm{d}\tau} = A_1 z + \begin{bmatrix} \epsilon_1(z) \\ \epsilon_2(z) \\ \epsilon_3(z) \end{bmatrix}, \tag{47}$$

where ϵ_1 and ϵ_2 are $O(\mu^2)$ and Lipschitz continuous in z with a constant that is $O(\mu^2)$, whereas ϵ_3 is $O(\mu)$ and Lipschitz continuous in z with a constant that is $O(\mu)$. The eigenvalues turn out to be

$$\begin{aligned}
\lambda_1 &= -\mu && \text{with eigenvector} \quad v_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}' \\
\lambda_2 &= \frac{-\phi + \sqrt{\phi^2 - 4\mu\gamma}}{2} = -\mu - O(\mu^2) && \text{with eigenvector} \quad v_2 = \begin{bmatrix} \gamma & 0 & -(\lambda_2 + \mu(1+\xi))/\mu \end{bmatrix}' \\
& && \phantom{\text{with eigenvector} \quad v_2} = \begin{bmatrix} \gamma & 0 & -\xi + O(\mu) \end{bmatrix}' \\
\lambda_3 &= \frac{-\phi - \sqrt{\phi^2 - 4\mu\gamma}}{2} = -\gamma + O(\mu) && \text{with eigenvector} \quad v_3 = \begin{bmatrix} \mu & 0 & -\big(\lambda_3 + \mu(1+\xi)\big)/\gamma \end{bmatrix}' \\
& && \phantom{\text{with eigenvector} \quad v_3} = \begin{bmatrix} \mu & 0 & 1 - O(\mu) \end{bmatrix}' \\
&\text{where } \phi \triangleq \mu(1+\xi) + \gamma\,.
\end{aligned} \tag{48}$$

(Note that all eigenvectors have been scaled to a magnitude of order 1.) Clearly, all eigenvalues have a negative real part. However, we still need to deal with the error terms in (47). Notice that each of the three error terms is only order μ times the size of the corresponding leading term (when the leading term is non-zero). Hence, we expect to obtain stability of the fixed point for small μ. We formally prove Claim 2 in the online appendix, where also provide an elegant interpretation of the eigendecomposition above.

All agent categories are playing a best response.

Claim 3. *For the choice of* $\lambda_e^{\mathrm{re}} = \lambda_e^{\mathrm{re}}(\mu) > 0$ *and the fixed point (steady state) in the statement of Claim 1, with* $\theta_w^{\mathrm{H}} = 1 - \sqrt{2c}$ *and a suitable choice of* $\theta_e = \theta_e(\mu)$ *satisfying* $\lim_{\mu\to 0} \theta_e(\mu) = 1 - \sqrt{2c}$, *the following holds for* $\mu \in (0, \epsilon)$: *High workers, low workers, and employers are all playing a best response. High workers earn expected utility* θ_w^{H}, *employers earn expected utility* θ_e, *and low workers earn limiting expected utility* $(\lambda_e - \lambda_w^{\mathrm{H}} - \delta^{\mathrm{H}})/(2\lambda_w^{\mathrm{L}})$ *as* $\mu \to 0$.

Proof of Claim 3. We consider each category of agent in turn.

1. High workers: They are earning the highest possible utility of $1-\sqrt{2c}$ and cannot do any better by proposing or using a different threshold. This follows from the fact that this threshold solves $\theta_w^{\mathrm{H}} = \frac{1+\theta_w^{\mathrm{H}}}{2} - \frac{c}{(1-\theta_w^{\mathrm{H}})}$, the optimal stopping problem faced by an agent who gets an infinite stream of potential partners whose utility it uniformly distributed in $(0, 1)$ and must pay a cost c to learn each utility. It can be readily checked that they cannot do better by proposing, even if they other side is not screening. Note that the threshold is optimal for any μ without taking the limit, since high workers match instantly and depart, and hence a 0 fraction of them die.

2. Employers: Employers will not want to propose to low workers if μ is small enough, since they are better off waiting for (frequent) incoming proposals from them and avoiding wasted screening effort. Recall Lemma E.1, which tells us that $q \in ((2c)^{1/4} - (2c)^{1/2}, 1 - (2c)^{1/4} - (2c)^{1/2})$ under the assumptions of Theorem 16. The assumption $q < 1-(2c)^{1/4}-(2c)^{1/2}$ implies that employers prefer to screen and propose to high workers over proposing without screening. The assumption $q > (2c)^{1/4} - (2c)^{1/2}$ implies that high workers are sufficiently attractive relative to waiting for an incoming proposal from a low worker, and that an employer would like to screen and propose to a high worker when an opportunity arises. Note that the best response argument in the previous two sentences is closely related to Remark 5.

   It remains to choose between being a reacher and a settler and to choose a threshold. By Claim 1, Reachers and Settlers earn exactly the same utility, and θ_e matches this utility, and is hence an optimal threshold. We deduce that employers, both Reachers and Settlers, are playing a best response and earning utility θ_e.
3. Low workers: Fixing the strategies of the other agents, low workers have no hope of matching unless they propose. Only a vanishing fraction of employers who are present in the system will even consider their proposal, and hence the effective screening cost they face (see Lemma C.1) is diverging; thus, it is a best response for them to propose without screening. Low workers form matches only at a flow rate of $\lambda_e - \tilde{\lambda}^{\mathrm{H}}$ as $\mu \to 0$, since almost all arriving settler employers match with them. Hence, the fraction of low workers who match is $(\lambda_e - \tilde{\lambda}^{\mathrm{H}})/\lambda_w^{\mathrm{L}}$, and we compute the expected utility earned by low workers as $(\lambda_e - \tilde{\lambda}^{\mathrm{H}})/(2\lambda_w^{\mathrm{L}})$ as those who match obtain a random partner.

□

Evolutionary stability. Evolutionary stability is formalized in the following claim, which is proved in the online appendix.

Claim 4. *The steady state in Claim 1 is an attractive fixed point of the evolutionary dynamics (106).*

Proof of Theorem 16. The theorem follows immediately from Claims 1, 2, 3 and 4. □

E.2 Detailed statements of the intervention theorems

We now provide detailed statements of the theorems characterizing the equilibria under the proposed interventions. The corresponding proofs can be found in the online appendix.

Theorem 17 (Block workers from proposing)**.** *Fix high-worker quality $q \in (0,1)$ and $\lambda_w^{\mathrm{H}} > 0$, $\lambda_w^{\mathrm{L}} > 0$, and*[41] $\lambda_e \in (\lambda_w^{\mathrm{H}}(1+q/2), \lambda_w^{\mathrm{H}}(1+q/2)+\lambda_w^{\mathrm{L}})$. *Define $R_\delta^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}}(1+q/2))$. Then there exists $c_{\max} > 0$ such that for any $c \le c_{\max}$, there exists $\mu_0 = \mu_0(c) > 0$ such that for all $\mu < \mu_0$, the following equilibrium arises (quantified to leading order in small c):*

(i) High workers screen incoming proposals with threshold (and expected utility) $\theta_w^{\mathrm{H}} = 1 - \sqrt{2c}$, i.e., they use strategy $s_w^{\mathrm{H}} = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_w^{\mathrm{H}} = 1 - \sqrt{2c})$.

(ii) Low workers screen and accept/reject incoming proposals with threshold (and expected utility) θ_w^{L}, i.e., they use strategy $s_w^{\mathrm{L}} = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_w^{\mathrm{L}})$. We have $\lim_{\mu\to 0}\theta_w^{\mathrm{L}} = \xi(R_\delta^{\mathrm{L}}, 0)(1 - o_c(1)) = \frac{1}{2R_\delta^{\mathrm{L}}-1}\big(1 - o_c(1)\big)$ where $\xi(\cdot,\cdot)$ is as defined in (10).

(iii) All employers screen and propose to a high worker candidate if one is available. They use a threshold (and earn expected utility) θ_e which satisfies $\lim_{\mu\to 0}\theta_e = 1 - \sqrt{\frac{c(2R_\delta^{\mathrm{L}}-1)}{R_\delta^{\mathrm{L}}-1}}\big(1 + o_c(1)\big)$.

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[41] Note that δ_{H} defined by (41) tends to $\lambda_w^{\mathrm{H}} q/2$ as $c \to 0$. Hence, the lower limit in the range of allowed λ_e here is consistent with that in Theorem 8.

- *Reachers: Reachers ignore opportunities to propose to low workers, i.e., reachers use strategy*[42] $s^{\mathrm{re}} = (\succ^{\mathrm{o}} = \mathrm{H}, a^{\mathrm{o}}_{\mathrm{H}} = \mathrm{S{+}P}, a^{\mathrm{o}}_{\mathrm{L}} = \mathrm{DN}, \theta_e)$*. There is a fraction* $f_e(s^{\mathrm{re}})$ *of reachers where* $\lim_{\mu\to 0} f_e(s^{\mathrm{re}}) = \frac{\lambda_w^{\mathrm{H}}(1+q/2)}{\lambda_e}\big(1+o_c(1)\big)$.
- *Settlers: The remaining employers are Settlers who are willing to screen and propose to low workers, i.e., settlers use strategy* $s^{\mathrm{se}} = (\succ^{\mathrm{o}} = (\mathrm{H} \succ \mathrm{L}), a^{\mathrm{o}}_{\mathrm{H}} = \mathrm{S{+}P}, a^{\mathrm{o}}_{\mathrm{L}} = \mathrm{S{+}P}, \theta_e)$.

Moreover, if $\lambda_e < \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}}$ *(employers are on the short side), then this is the unique equilibrium.*

As $\mu \to 0$*, there is a flow* λ_w^{H} *of matches between reacher employers and high workers, a flow* $\frac{\lambda_w^{\mathrm{H}} q}{2}\big(1+o_c(1)\big)$ *of reacher employers leaving without matching, whereas all settler employers match with low workers, producing a flow* $(\lambda_e - \lambda_w^{\mathrm{H}}(1+q/2))(1+o_c(1))$ *of such matches. The steady state masses are given by*

$$\lim_{\mu\to 0} \mu \bar{L}_e(s^{\mathrm{re}}) = \tfrac{\lambda_w^{\mathrm{H}} q}{2}(1+o_c(1)) \text{ for reachers, } \lim_{\mu\to 0} \bar{L}_e(s^{\mathrm{se}}) = (\lambda_e - \lambda_w^{\mathrm{H}}(1+q/2))\sqrt{\tfrac{2R^{\mathrm{L}}_{\delta}-1}{4c(R^{\mathrm{L}}_{\delta}-1)}}(1+o_c(1)) \text{ for settlers,}$$
$$\lim_{\mu\to 0} \mu L_w^{\mathrm{L}} = (\lambda_w^{\mathrm{L}} + \lambda_w^{\mathrm{H}}(1+q/2) - \lambda_e)(1+o_c(1)) \text{ for low workers, and } L_w^{\mathrm{H}} = 0 \text{ for high workers.}$$

We state and prove a more detailed version (that includes steady state masses and match flows) of Theorem 9, about the equilibrium when workers are not allowed to propose and worker quality information is hidden from employers.

Theorem 18 (Block workers from proposing and do not reveal quality information to employers)**.** *Fix high-worker quality* $q \in (0,1)$ *and* $\lambda_w^{\mathrm{H}} > 0$, $\lambda_w^{\mathrm{L}} > 0$, *and* $\lambda_e \in (\lambda_w^{\mathrm{H}}, \lambda_w^{\mathrm{H}} + \lambda_w^{\mathrm{L}})$*. Define* $R^{\mathrm{L}} \triangleq \lambda_w^{\mathrm{L}}/(\lambda_e - \lambda_w^{\mathrm{H}})$*. Then there exists* $c_{\max} > 0$ *such that for any* $c \le c_{\max}$*, there exists* $\mu_0 = \mu_0(c) > 0$ *such that for all* $\mu < \mu_0$*, the following unique equilibrium arises (quantified to leading order in small* c*):*

1. *High workers screen and accept/reject with threshold (and expected utility)* θ_w^{H}*, i.e., they use strategy* $s_w^{\mathrm{H}} = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_w^{\mathrm{H}})$*. Here*
$$\lim_{\mu\to 0} \theta_w^{\mathrm{H}} = 1 - K_{\mathrm{H}}\sqrt{c}\,(1+o_c(1)) \quad \text{for } K_{\mathrm{H}} \triangleq \sqrt{\tfrac{4(\lambda_w^{\mathrm{H}}+\lambda_w^{\mathrm{L}}-\lambda_e)}{q^2\lambda_w^{\mathrm{H}}} + 2}\,.$$
2. *Low workers screen and accept/reject with threshold (and expected utility)* θ_w^{L}*, i.e., they use strategy* $s_w^{\mathrm{L}} = (a^{\mathrm{i}} = \mathrm{S{+}A/R}, a^{\mathrm{o}} = \mathrm{DN}, \theta_w^{\mathrm{L}})$*. Here*
$$\lim_{\mu\to 0} \theta_w^{\mathrm{L}} = \xi(R^{\mathrm{L}}, 0)\,(1+o_c(1)) = \tfrac{1}{2R^{\mathrm{L}}-1}\,(1+o_c(1)) \quad \text{for } \xi(\cdot,\cdot) \text{ defined in (10).}$$
3. *Employers screen and propose with threshold (and expected utility)* θ_e*, i.e., they use strategy* $s_e = (a^{\mathrm{o}} = \mathrm{S{+}P}, \theta_e)$*. Here*
$$\lim_{\mu\to 0} \theta_e = 1 - K_e c(1+o_c(1)) \quad \text{for } K_e \triangleq \tfrac{2(\lambda_e-\lambda_w^{\mathrm{H}})}{q\lambda_w^{\mathrm{H}}} \cdot \tfrac{2R^{\mathrm{L}}-1}{2(R^{\mathrm{L}}-1)}\,.$$

As $\mu \to 0$*, in steady state there is a flow* $\lambda_w^{\mathrm{H}} - o_c(1)$ *of matches between high workers and employers, and a flow* $\lambda_e - \lambda_w^{\mathrm{H}} + o_c(1)$ *of matches between low workers and employers. A flow* $\lambda_w^{\mathrm{L}} + \lambda_w^{\mathrm{H}} - \lambda_e - o_c(1)$ *of low workers leave without matching, and a flow* $\lambda_w^{\mathrm{H}}(K_{\mathrm{H}}/2 - 1/K_{\mathrm{H}})\sqrt{c}\,(1+o_c(1))$ *of high workers leave without matching. The limiting steady state masses are given by*

$$\lim_{\mu\to 0} \mu L_w^{\mathrm{H}} = \lambda_w^{\mathrm{H}}(K_{\mathrm{H}}/2 - 1/K_{\mathrm{H}})\sqrt{c}\,(1+o_c(1)) \textit{ for high workers,}$$
$$\lim_{\mu\to 0} \mu L_w^{\mathrm{L}} = (\lambda_w^{\mathrm{L}} + \lambda_w^{\mathrm{H}} - \lambda_e)(1-o_c(1)) \textit{ for low workers, and } \lim_{\mu\to 0} L_e = \tfrac{q\lambda_w^{\mathrm{H}}}{2c}(1+o_c(1)) \textit{ for employers.}$$

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[42] Since workers are not allowed to propose, we do not specify how employers handle incoming proposals.