Growth and Associated Capital Market Anomalies

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Abstract

In this paper, we develop an analytic accounting-based asset-pricing model. It establishes a nonlinear relationship between stock returns and accounting fundamentals including earnings systematic risk, asset growth, company profitability, the book-to-market ratio and earnings components. This is in contrast to traditional factor models that augment the CAPM with empirically determined accounting variables. Directly relating stock returns to accounting fundamentals facilitates the exploration of many empirical conundrums. It enables us to reconcile several empirical puzzles into a parsimonious unified framework and to provide insights into the structuring of future empirical investigations. We find that the interaction between risky asset growth and future corporate earnings plays a central role in predicting stock returns. Our analysis casts light on investment and accrual anomalies and further clarifies the role of the book-to-market factor in asset pricing.

Keywords: Growth, Stock returns, Accruals, Investment, Anomalies

1. Introduction

The increasing availability of data and the sophistication of accompanying econometric packages have generated an abundance of empirical research identifying statistical relationships between accounting variables and stock returns. However, this approach frequently results in findings that appear to be contradictory or for which explanations are many and even somewhat tenuous. Many such capital market ‘‘anomalies’’ involve the relationship between growth and accounting fundamentals. In contrast to the vast empirical literature, there exist relatively few formal analytical models that encompass the relationship between stock returns, earnings systematic risk, asset growth, company profitability, the book-to-market ratio and accruals. In this paper, we aim to fill this gap by adopting a more rigorous analytical approach. This enables us to reconcile several empirical puzzles into a parsimonious unified framework and to provide insights into the structuring of future empirical investigations.

We find that several investment and accrual anomalies are closely linked to an embedded risky growth factor. For example, in a seminal paper Sloan (1996) finds a strong negative relationship between firm level future stock returns and current accruals. In contrast, Hirshleifer et al. (2009) find an even stronger positive relationship between aggregate accruals and aggregate stock returns. Our analytical modeling shows that differing (implicit) underlying assumptions made in empirical investigations about the co-variability of risky growth with accruals affects both the sign and size of the accrual coefficient. We find what turns out to be a related body of empirical studies in finance that document a negative relationship between various forms of corporate investment and future stock returns (e.g., Cooper et al. (2008), Watanabe et al. (2013)) can be explained by their failure to adequately incorporate a growth factor. The importance of including such a growth factor finds empirical support in the work of Fairfield et al. (2003) and Zhang (2007). Although much of the analysis of this paper is on the impact of growth on the pricing of earnings components, our theoretical modeling casts light on several other important issues. We are able to explore the role of book-to-market, the assumed linearity of factor models and other limitations of market-based accounting research.

We start by assuming that scaled earnings and accruals follow joint mean-reverting processes. This facilitates an independent treatment of the risks inherent in earnings and accruals. It is particularly important in exploring accrual risk in characteristic models since the income component as represented by accruals is largely contractual. Throughout our analysis, we recognize the key role played by book value growth embedded in the intertemporal scaling, where growth is related to macro-economic factors. We emphasize that accruals are more than naturally occurring random variables. We argue that their values arise from careful analysis and decision making by individual firms operating in a constrained environment. Integral to this is the management of accounts payable and receivable, as well as the chosen method of depreciation as such the non-contextual treatment of accruals as merely constituting a convenient set of random variables is not tenable. Indeed, the management of working capital including the accrual component probably occupies more time of practicing accountants in industry and commerce more than do the dividend decision and the capital structure decision combined. The deliberated nature of accruals has two implications. First, the description of high and low accruals judged relative to other firms may not provide a meaningful basis for the investigation of the association between stock returns and accruals. Accruals should be judged high or low relative to an individual firm's own accruals policy, where this policy is constrained by the norms of the industrial sector in which the firm operates. Thus, we posit that high and low accruals need to be defined in terms of deviation from the firm's long-run mean levels. Second, such deviations are dependent on economic conditions, giving rise to a positive correlation between growth and aggregate accruals. In periods of favorable economic conditions, firms are likely to have a higher level of earnings and accruals relative to their long-run means and vice versa. The most obvious measure of economic activity is consumption and thus our analysis exploits the framework afforded by the consumption CAPM.

To establish a theoretical link between stock returns and earnings (components), we first develop a valuation model based on the present value of future dividends using the risk-free rate as a discount rate, and adjusting for correlation risk between market information and accounting fundamental variables (Feltham and Ohlson (1999)). Earnings and accruals information dynamics and the assumption of clean surplus are used to articulate future financial statements and their embedded risk structures. To overcome a potential problem where the nominal growth rate is greater than the risk-free rate in the long run, we introduce a concept of certainty equivalent growth rate. This in turn generates a growth premium, defined as the difference between the expected long-run growth rate and the certainty equivalent growth rate. Equity values are then expressed in terms of accounting fundamentals including the book value of equity and the deviation of (scaled) earnings and accruals from their long-run means, with a risk adjusted growth value premium and an earnings risk adjusted value premium. Based on our equity valuation model, we produce theoretical values of stock returns in consecutive periods. This return model takes one of two algebraically equivalent forms. The first of these is a CAPM style model where returns in excess of the risk-free rate are expressed as a function of earnings and accrual market risk together with a growth risk premium. The second takes the form of a nonlinear characteristic model relating future stock returns to fundamental accounting ratios such as return on equity, book-to-market and scaled accruals and asset growth, and market covariance terms. These two forms for the equity return facilitate the exploration of a number of anomalous results in asset pricing.

Our modeling implies that any accounting or non-accounting variable that can be used to forecast future earnings can be informative in predicting future stock returns in a setting with a growth risk premium. In particular, the accounting variable and growth risk premium are interactive in predicting future stock returns. If a variable is positively (negatively) related to future earnings-to-price, then it will have a negative (positive) growth risk premium associated with future returns. The so-called investment anomaly can be explained by investigating the relative magnitudes of the accounting rate of return (ROE) and the cost of equity capital after controlling for the growth risk premium in our model. When ROE is larger than the cost of capital or the economic spread is greater than zero, asset growth will increase firm values. Investors need a smaller growth risk premium and hence a lower return for higher growth firms. Therefore, the coefficient attached to a growth proxy that is positively related to future earnings in stock return regressions is expected to be negative in cross sectional analysis given the fact that the economic spread is expected to be greater than zero for the majority of companies in the market in a normal year.

Accruals are associated with stock returns due to asset growth being interlinked with future earnings, with accruals being useful in forecasting future earnings when cash flows and accruals have different degrees of persistence. Accruals interact with risky growth in predicting stock returns. The different degree of persistence between cash flows and accruals is a necessary but not sufficient condition in explaining the “accrual anomaly”. If we control for the growth risk premium, then there is no abnormal return associated with a separate accrual component. Depending on whether controlling for earnings or cash flows in regression analysis, accruals can be negatively or positively related to stock returns. We show that our analysis offers a rational explanation of the “accrual anomaly” and whether accrual risk is appropriately priced in the model. Existing studies frequently do not consider the correlation risk between future asset growth and earnings. Sloan's firm level results hold since growth is effectively held constant after controlling for earnings in cross sectional analysis. When growth is implicitly viewed as a constant, our model suggests that firms' returns should be negatively related to accruals. In contrast, Hirshleifer et al. (2009) results prevail in their time series analysis mainly because future growth is positively related to aggregate market values of weighted accruals.

Our model also helps to explain why accruals remain highly significant in explaining returns with or without controlling for the proxy for the accrual risk factor loading as documented in Hirshleifer et al. (2012). We argue this is what we would expect under our modeling of the accruals process even when we do not control for growth risk. Problems arise because most empirical investigations invariably use scaled accruals as the basis of classification of high and low accrual firms and choose it as the variable of interest, rather than the deviations from the long-run means, which we identify in our model as a key determinant of returns. The importance of this is that high and low accruals should imply high and low relative to a firm's long-run mean, not as used in cross-sectional analysis where high and low are measured relative to the annual cross-sectional mean.

We believe our paper makes several significant contributions to the literature. We establish a formal analytical relationship between stock returns, earnings systematic risk, asset growth, company profitability, the book-to-market ratio and earnings components, focusing on accruals. We identify the paramount role of the interaction between future asset growth and earnings in asset pricing offering a rational explanation of investment and apparent accrual anomalies. While a resolution of the investment and accrual anomalies is our principal focus, the resulting development of a nonlinear accounting-based characteristic model gives rise to several further contributions to the literature. We introduce the concept of certainty equivalent growth rate. This concept ensures convergence in risk-free valuation analysis even where average long-run growth is greater than the risk-free rate used in terminal value estimation. Importantly, it can be used to characterize the growth risk premium in determining stock returns. Finally, our analysis casts light on the pervasive role of, and the positive coefficient associated with the book-to-market in factor analysis in asset pricing, which we attribute to the mapping of conservative accounting valuations to real monetary valuations. Ball et al. (2020) argue that book-to-market predicts stock returns since it is subsumed by retained earnings-to-market and the latter is a better proxy for underlying earnings yield. In our model, book-to-market emerges from the implicit assumption within our theoretical modeling that, as in practice, equity valuation is largely based on processing accounting information. Hence book-to-market not only converts ROE to future earnings-yield but occurs naturally as a convenient summary of accumulated transactions.[2] In contrast to its use in factor analysis, it occurs in a multiplicative form and not as a separate additive factor.

The rest of the paper is organized as follows. Section 2 presents the motivation and relation to prior literature. Section 3 shows the basic structure of our model. Section 4 develops the valuation model in terms of accounting fundamentals, earnings systematic risk, and a value premium when incorporating future growth. Section 5 derives our expression for stock returns and offers a rational explanation on the investment anomaly. In section 6, we show how our model can be used to explain contradictory empirical results on accrual related anomalies. Section 7 concludes the paper.

2. Relation to prior literature and motivational issues

The development of our valuation models starts from information dynamics of fundamental accounting attributes: earnings, accruals and the book value of equity. These dynamics are

[2] It serves to convert units of accounting dollars into units of monetary dollars.

consistent with evidence that firms' accounting rates of return follow well-established mean-reverting processes[3] together with the notion that the differential persistence of earnings, cash flows and accruals is a firm-specific phenomenon (Francis and Smith (2005), Call et al. (2016)). We employ a set of nonlinear mean-reverting simultaneous equations to model earnings and the accrual component. Our earnings dynamic is also consistent with the measurement of earnings quality as proxied by the extent to which accruals relate to future, present and past cash flows (Dechow and Dichev (2002)). We emphasize that the classification of high or low accruals is relative to the established accounting policy of the firm as is evidenced in the income smoothing literature and the literature on accrual-based earnings management (e.g., Defond and Park (1997), Tucker and Zarowin (2006)). Penman (2016) offers an explanation of the risk role played by high and low accruals: "high current accruals mean that cash that will be earned in the future (but is not yet realized) contains little risk and lower investors' expectations of future income, because future income has been booked." This argument is of course based on accruals returning to their long-run means and not a cross-sectional mean. Because of the paucity of long-run time series for individual firms, most empirical investigations invariably use cross-sectional analysis and scaled accruals as the basis of classification of high and low accrual firms, rather than their deviation from their long-run means.

Our information dynamics reflect the role of assets and the interaction between growth of assets and the current state of earnings (and its components) as well as aggregate market condition in generating future earnings. They imply that accrual-to-book equity can be used to explain return on equity (ROE) (e.g., Fama and French (2006)).[4] With the evolution of assets, we extend Feltham

[3] see for example, Beaver (1970), Freeman et al. (1982), Nissim and Penman (2001), Moehrle (2002), Thomas and Zhang (2002), Chan et al. (2006), Dechow et al. (2011), and Allen et al. (2013), Evans et al. (2017).

[4] Sloan (1996) documents that asset scaled accruals are useful in forecasting asset scaled earnings (ROA).

and Ohlson (1999) into a nonlinear valuation framework in which equity market value is anchored on the scaled book value and is adjusted by deviations of scaled earnings and accruals. We also incorporate the effects of risky growth of assets and economy-wide risk measured by the correlation between future earnings (components) and aggregate consumption. This is in line with prior studies that we use accounting earnings changes rather than stock returns to estimate firm-level systematic risk (Beaver et al. (1970), Fama and French (1995), Ball et al. (2009), Da and Warachka (2009), Lyle et al. (2013), Ellahie (2020), Penman and Zhang (2020), Ball et al. (2022)).[5]

Factor models have grown out of a series of anomalies where existing equilibrium asset pricing theory fails to explain the cross-section of stock returns. For example, the Fama-French (1992, 1993) 3-factor model was a response to the failure of the CAPM to explain the role of size and book-to-market in empirical observations of stock returns. Since then much effort has been devoted to resolving a number of other anomalies. Fama and French (2015) have produced a 5-factor model adding profitability and investment to the original set, while Hou et al. (2015) have also included investment and profitability in their q-model. Unlike Fama and French (2006, 2015) who justify profitability (ROE), book-to-market and investment growth in their 5-factor model construction from a comparative analysis on the residual income valuation model, our partial equilibrium model implies a nonlinear or multiplicative relationship between stock returns and these value attributes. Again, in contrast to Hou et al. (2015) who develop the linear q-factor model underpinning the factors on profitability and contemporaneous growth of assets on a static investment framework, and Hou et al. (2021)'s extension to a dynamic setting to motivate a

[5] Ellahie (2020) constructs earnings betas and finds that an earnings beta based on price-scaled expectations shocks performs consistently better in explaining the cross-section of stock returns.

separate expected growth factor, we show directly how the expected investment growth interacts with profitability and book-to-market to determine the expected return.

Our return expressions can be also used to explain why numerous empirical studies investigating the relation between firms' investment growth and future stock returns document a negative association between the two. For example, capital investment, changes in net operating assets, net share issuance and an increase in accruals all appear to be negatively related to subsequent stock returns (Fairfield et al. (2003), Titman et al. (2004), and Hirshleifer et al. (2004), Fama and French (2006), Penman and Zhu (2014, 2022)). These findings are said to be robust when fine-tuning measures of growth are applied (e.g., Richardson et al. (2006), Anderson and Garcia-Feijoo (2006), Cooper et al., (2008)). Behavioral mispricing-based explanations include overreaction to firm investments by investors (e.g., Lakonishok et al. (1994)), excessive investments by empire-building managers (e.g., Titman et al. (2004)), and earnings management prior to acquisitions (e.g., Teoh et al. (1998a; 1998b)). Risk-based explanations include the negative discount rate effect of investments (e.g., Cochrane (1991, 1996), Liu et al. (2009), Wu et al. (2010)), decreasing return to scale (Lyandres et al. (2008) and Li et al. (2009)), and reduced risk after exercising growth options (Berk et al. (1999), Carlson et al. (2004)). The investment anomaly is also said to be persistent in international markets (Watanabe et al. (2013), Titman et al. (2013)). Lam and Wei (2011) argue that mispricing and rational pricing may coexist. Our analysis suggests that a negative sign is likely to be attached to the growth risk premium for firms with accounting rates of return being greater than the cost of capital.

Sloan's (1996) findings that firm level accounting accruals are negatively associated with future stock returns have inspired a large volume of empirical research in both accounting and finance over the last two decades. The findings are argued to have implications in that a piece of public information can be used to generate abnormal returns.[6] The puzzle is further confounded by another influential empirical investigation in Hirshleifer et al. (2009) whose evidence appears to contradict Sloan's findings. Analyzing aggregate stock returns on aggregate accruals in successive periods, they find that future returns are positively related to the current level of accruals. The obvious explanation is that a variable that is interacted with or related to accruals represents a type of risk that is not captured by known risk factors and it correlates with other variables differently in their respective approaches of cross sectional and time series analysis. Dechow et al. (2011) suggest that accruals represent growth in management's estimates of the future benefits that will accrue to a firm and accruals and growth proxies are positively correlated across years. We believe that our analysis provides a framework for the resolution of these contradictory findings. Consistent with Guo and Jiang (2011), our model shows accruals correlate with the determinants of the conditional equity risk premium. While they argue the common component of firm-level accruals explains the positive relation between aggregate accruals and future stock market returns and the residual component is responsible for the negative cross-sectional relation between firm-level accruals and future stock returns, we offer a different interpretation. Aggregate accruals are positively related to future market returns since accruals and asset growth both are correlated with aggregate economic activity, but the negative cross-sectional relation between firm-level accruals and future stock returns is because of the absence of, or deliberate omission of, any significant

[6] A large body of follow-up empirical research provide supporting evidence, see for example, Bradshaw et al. (2001), Xie (2001), Richardson et al. (2005, 2006), Chan et el. (2006), Lev and Nissim (2006), Mashruwala et al. (2006), Pincus et al. (2007), Zhang (2007), Dechow et al. (2011), Allen et al. (2013). Fama and French (2008) suggest that the accrual anomaly is one of the most pervasive return anomalies. Note that Green et al. (2011) states that the accrual anomaly has largely disappeared in their later sample period from 2004 to 2010.

correlation between growth and accruals. Our analysis indicates that reported accrual associated abnormal returns in the existing literature is likely because growth risk is not fully accounted for.[7] Sloan (1996) attributes the "accrual anomaly" to the lower persistence of the accrual component of earnings. We find that the lower persistence is a necessary but not sufficient condition for the role of accruals in explaining returns. The existence of a deviation between book value scaled accruals and its long-run mean, and a growth risk premium are also necessary conditions for accruals in predicting future returns in cross sectional studies. Our analysis indicates that the deviation of the scaled accruals from its long-run mean interacts with risky growth and explains why Hirshleifer et al. (2012) find that accruals remain highly significant after controlling for the proxy for their accrual risk factor. It also explains why the "accrual anomaly" cannot be diversified away. Although the Mishkin (1983) test used in Sloan (1996) is commonly employed to identify mispricing in a cross sectional analysis, our analysis suggests that we must be cautious in interpreting the testing results. We provide a formal analytical relation between risky growth, earnings surprise and stock returns.[8]

3. Model setup

To establish an analytical relationship between stock value, returns and earnings components, we assume that economic activity or consumption affects both aggregate returns and individual firm's earnings growth. We emphasize that our model, although based on a theoretical development of valuation, is designed to capture and explain many aspects of empirical research. Of necessity, we

[7] It echoes to Khan (2008) who suggests that a large portion of the cross-sectional variation in stock returns to high and low accrual firms can be explained by risk.
[8] Kraft et al. (2007) and Penman and Zhu (2014) also argue that the Mishkin test is not appropriate for companies with growth unless variables that forecast growth are included.

make several simplifying assumptions to ensure tractability.[9] Even then the development involves substantial algebraic manipulation. Thus, for ease of readership we largely confine this manipulation of the mathematical development to the appendices.

A natural starting point for identifying an appropriate form for the discount function is the consumption CAPM. We assume that fluctuations in capital market returns ε_m about the level of economic activity follow an independent and identically distributed (i.i.d) normal distribution with mean of zero and variance of σ_m^2.

Assumption 1. The capital market is absent of arbitrage opportunities. There exists a stochastic discount factor that prevails for the period $t+\tau-1$ to $t+\tau$ ($\tau=1,2,...$) of the form:

$$m_{t+\tau} = e^{-\lambda^2\sigma_m^2/2-\lambda\varepsilon_{m,t+\tau}} / R_f \,, \tag{1}$$

where $R_f = 1+r_f$ is one plus the constant risk-free rate, the parameter λ represents investors' degree of risk aversion and $\varepsilon_{m,t+\tau} \sim N(0,\sigma_m^2)$. It follows that the market value of equity: $MVE_t = E_t\left[\sum_{\tau=1}^{\infty}\left(M_{t+\tau}d_{t+\tau}\right)\right]$, where d_t is dividends (net of new capital contribution) at time t, $M_{t+\tau} = \prod_{n=1}^{\tau} m_{t+n}$ and $E_t[.]$ is expectation operator based on time t information.

To express the value distribution (or dividends) in terms of accounting fundamentals, we assume:

Assumption 2. The clean surplus accounting relation holds: $B_t = B_{t-1} + e_t - d_t$, where B_t and e_t are book value of equity and earnings at time *t* respectively.

Based on Assumptions 1 and 2, the market value of equity can be written as:

[9] These mainly involve assumptions in the use of normal and lognormal distributions. It should be borne in mind that the econometric sophistication employed in empirical research largely arises from attempts to correct estimation errors caused by the non-normality of real data and for the limitations of estimation procedures largely based on the first two statistical moments of the data. As such, normality can be considered as a robust first-order approximation to empirical statistical relationships.

$$MVE_t = E_t\left[\sum_{\tau=1}^{\infty}\left(M_{t+\tau}\left(ROE_{t+\tau} - \frac{\Delta B_{t+\tau}}{B_{t+\tau-1}}\right)B_{t+\tau-1}\right)\right], \tag{2}$$

where $\Delta B_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ and $ROE_{t+\tau} = e_{t+\tau} / B_{t+\tau-1}$.

Equation (2) suggests that the fundamental accounting determinates of equity value are future return on equity and the evolution of book value. Our immediate task is to assume the underlying dynamics of future (scaled) earnings. Our focus is on the following two aspects. First, prior literature documents that accruals as an earnings component have incremental information in predicting future earnings after controlling for current earnings. Earnings components, such as cash flows and accruals, have different predictability of future earnings (e.g., Sloan, 1996). Second, there is a large body of evidence in which accounting rates of return follow a mean reverting process due to market competition and the application of accrual accounting. Therefore, we assume the book value deflated earnings (ROE) follow a parsimonious mean-reverting process:[10]

$$\begin{aligned} \frac{e_{t+1}}{B_t} - \mu_e &= \alpha_{11}\left(\frac{e_t}{B_{t-1}} - \mu_e\right) + \alpha_{12}\left(\frac{ac_t}{B_{t-1}} - \mu_a\right) + \varepsilon_{e/B,t+1}, \\ &= \alpha_{11}\left(\frac{cf_t}{B_{t-1}} - \mu_c\right) + (\alpha_{11} + \alpha_{12})\left(\frac{ac_t}{B_{t-1}} - \mu_a\right) + \varepsilon_{e/B,t+1}, \end{aligned} \tag{3}$$

where $e_t = cf_t + ac_t$, cf_t and ac_t are cash flows from operations and total accruals at time t. μ_e, μ_c and μ_a are the expected long-run means of the return on equity, cash flows-to-book and accruals-to-book respectively on a firm-by-firm basis, $\mu_e = \mu_c + \mu_a$. The parameter α_{11} measures

[10]Dynamic (3) is consistent with earnings forecasting model (e.g., Hou et al. (2012)). We can also incorporate a variable that describes investor sentiment about future prospects of the firm into our dynamics. Objectively it can be thought of as representing changes in the long-term profitability as measured by the changes in the mean ROE in response to changing economic circumstances. We ignore such a variable for the parsimony of model development so we can focus on variables of our interest. However, all our main results hold if we include the variable in the information dynamics. Unlike Larson et al. (2018), we do not consider the role of additional leads and lags of earnings beyond one year.

the speed of convergence of ROE and α_{12} term represents the interaction between future earnings and current accruals. The smaller persistence of accruals relative to cash flows in predicting future earnings (as documented in Sloan (1996) and others) implies that $\alpha_{12} < 0$.

Deviations from the mean reverting path are reflected in the error term $\varepsilon_{e/B,t+1}$ and are attributable to changing economy wide and firm specific factors.[11] For mathematical convenience we will assume linearity such that $\varepsilon_{e/B,t+1} = k^e \varepsilon_{m,t+1} + \varepsilon^e$, where the time-varying component is only related to market risk with a firm specific constant $k^e > 0$. ε^e is independent of $\varepsilon_{m,t}$ and normally distributed with mean of zero and variance of σ_e^2. It also follows that ε^e or any function of ε^e are also independent of $\varepsilon_{m,t}$. We also assume the absence of serial correlation. Dynamic (3) implies that future earnings are a non-linear function of current book value, earnings, accruals, and growth in book value.[12]

While it is not necessary to specify a corresponding dynamic of accruals in empirical studies, we require such to derive a closed form valuation model. Specifically, we assume that the deviation of book value deflated accruals in t+1 depends on the t+1 deviation in earnings $\left(\frac{e_{t+1}}{B_t} - \mu_e\right)$ together with a legacy contribution from the previous periods accruals. Using equation (3) to replace the $\left(\frac{e_{t+1}}{B_t} - \mu_e\right)$ term we are able to write our second key dynamic in a symmetric form as in equation (4):

[11] Ball et al. (2022) argue that firms' earnings comove significantly due to aggregate economic forces.
[12] We can show that dynamic (3) is also consistent with the measurement of accounting quality as proxied by the extent to which accruals relate to future, present and past earnings (cash flows) (Dechow and Dichev 2002, Francis et al. (2005), Aboody et al. 2005).

$$\frac{ac_{t+1}}{B_t} - \mu_a = \alpha_{21}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{22}(\frac{ac_t}{B_{t-1}} - \mu_a) + \varepsilon_{a/B,t+1}. \tag{4}$$

As in the earnings information dynamic (3), we assume that fluctuations in capital market returns are reflected in the accrual error term with $\varepsilon_{a/B,t+1} = k^a \varepsilon_{m,t+1} + \varepsilon^a$. Hence we decompose accruals innovations into market innovations $\varepsilon_{m,t+1}$ and time-invariant firm-specific innovations ε^a. The more volatile the market, the more volatile the future accruals, $k^a > 0$. Assume ε^a is independent of $\varepsilon_{m,t}$ and follows a normal distribution with mean of zero and variance of σ_a^2. We also assume the absence of serial correlation. To interpret the empirical evidence on earnings components in prior literature, we denote $k^c \equiv k^e - k^a$ and $\varepsilon^c \equiv \varepsilon^e - \varepsilon^a$. The separate identification of the accruals component gives us the ability to distinguish between the characteristics of the components of earnings. By including earnings in dynamic (4), we can also show that current earnings (cash flows) are related to future, present and past accruals (e.g., deferred expenses).

In line with empirical evidence and to ensure convergence of our model, we impose the condition that the maximum eigenvalue of $\mathrm{A} \equiv \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$ is less than one plus the risk-free rate. While empirical statistical analysis broadly supports the relationship between earnings in one period to earnings and accruals in the previous period, the underlying mechanism is that changes in earnings (cash flows) are related to economic activity or consumption. The actual split between earnings and accruals is largely governed by the firm's working capital management and depreciation policies. In turn, this split is constrained by the industrial sector in which the firm operates. In essence $\alpha_{i,j}$ $(i, j = 1, 2)$ are parameters determined by the firm and its operating environment.[13]

The earnings and accrual information dynamics (3) and (4) involve the evolution of book value. We now introduce the dynamic for the evolution of book value and the concept of certainty equivalent (long-run) growth rate. We assume that book value growth is stochastic and is affected by general economic factors as well as firm specific factors. Specifically, book value dynamic satisfies

$$B_{t+1} = \tilde{\delta}_{t+1} B_t , \tag{5}$$

where $\tilde{\delta}_{t+1} = e^{\varepsilon_{\delta,t+1}}$ and $\varepsilon_{\delta,t+1} = k^{\delta}\varepsilon_{m,t+1} + \varepsilon^{\delta}$. ε^{δ} is independent of $\varepsilon_{m,t+\tau}$. Again, we assume ε^{δ} follows a normal distribution with mean of zero and variance of σ_{δ}^2 implying that book value growth $\tilde{\delta}_{t+1}$ follows a lognormal distribution.[14] It follows that investment growth is determined by the market conditions and firm-specific innovations. We allow for the correlation between the firm's asset growth and earnings ($cov(\varepsilon^{e}, \varepsilon^{\delta})$), and growth and accruals ($cov(\varepsilon^{a}, \varepsilon^{\delta})$) to be nonzero since accounting accruals may affect future growth (Fairfield et al. (2003)).

It follows that the expected book value at t+1 is

$$E_t[B_{t+1}] = \bar{\delta}\, B_t \ , \tag{6}$$

where $\bar{\delta} \equiv E_t[\tilde{\delta}_{t+1}] = e^{(k^{\delta}\sigma_m)^2/2 + \sigma_{\delta}^2/2}$.

The evaluation of the market value of equity (MVE_t) involves computing the adjusted present value of the growth terms in earnings, accruals and book values. Adjusting book growth for market risk introduces the useful concept of certainty equivalent growth δ_{ce}, defined as below:

[13] It is worth mentioning that our theoretical results apply to any two earnings components that satisfy the connections represented in the assumed information dynamics and clean surplus relation, though our analysis focuses on the joint time series properties of operating cash flows and accruals.

[14] All results hold if we assume $\varepsilon^{\delta} \sim N(g_0, \sigma_{\delta}^2)$, where g_0 is a constant.

$$\delta_{ce} \equiv E_t[e^{-\lambda^2\sigma_m^2/2-\lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}] = \bar{\delta} e^{-\lambda k^\delta \sigma_m^2}. \tag{7}$$

We assume $\delta_{ce} < R_f$ and call the difference between the long-run asset growth rate and its certainty equivalent rate, $(\bar{\delta} - \delta_{ce}) = E_t[\tilde{\delta}_{t+1}] - \delta_{ce}$, the growth premium. Since our aim is to establish explicitly the relation between stock return and earnings components, we assume for the purpose of presentation that the correlation risks between the error terms in our information dynamics, risky growth and stochastic discount factor: $E_t[\varepsilon_{e/B,t+1} e^{-\lambda^2\sigma_m^2/2-\lambda\varepsilon_{m,t+1}}]$, $E_t[\varepsilon_{e/B,t+1} e^{\varepsilon_{\delta,t+1}} e^{-\lambda^2\sigma_m{}^2/2-\lambda\varepsilon_{m,t+1}}]$ and $E_t[\varepsilon_{a/B,t+1} e^{\varepsilon_{\delta,t+1}} e^{-\lambda^2\sigma_m{}^2/2-\lambda\varepsilon_{m,t+1}}]$ are all time-invariant.[15] At this stage all the parameters and variables are firm specific but for ease of presentation we omit subscripts relating to individual firms in the subsequent analysis, unless specifically required for clarity.

4. Valuation of equity when incorporating future growth and accounting accruals

In this section, we show that equity values can be written in terms of accounting fundamentals including book values, asset growth and components of earnings as well as risk adjustment terms in a nonlinear fashion with the risk-free rate used for discounting. All proofs are in Appendix B.

Proposition 1**:** Assume Assumptions 1-2, and information dynamics (3)-(5) hold. The market value of equity can be written as

$$MVE_t = (1+\beta_0)B_t + \beta_1(\frac{e_t}{B_{t-1}} - \mu_e)B_t + \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)B_t + Growthadj_t - Riskadj_t, \tag{8}$$

where

[15] It is also consistent with constant time-invariant betas as is normally assumed in standard estimations of the CAPM beta (Nekrasov and Shroff (2009)). Evaluation of these expressions can be found in Appendix B.

$\beta_0 = \dfrac{\mu_e - (R_f - 1)}{(R_f - \delta_{ce})}$, $\beta_1 = \dfrac{\alpha_{11}(R_f - \delta_{ce}\alpha_{22}) + \delta_{ce}\alpha_{12}\alpha_{21}}{\Delta}$, $\beta_2 = \dfrac{\alpha_{12}R_f}{\Delta}$,

$\Delta \equiv (R_f - \delta_{ce}\alpha_{11})(R_f - \delta_{ce}\alpha_{22}) - \alpha_{12}\alpha_{21}(\delta_{ce})^2 > 0$,

$Growthadj_t = \dfrac{\delta_{ce}}{R_f - \delta_{ce}}[(\beta_1 k^e + \beta_2 k^a)k^\delta \sigma_m^2 + \beta_1 cov(\varepsilon^e, \varepsilon^\delta) + \beta_2 cov(\varepsilon^a, \varepsilon^\delta)]B_t$ is growth adjustment, and

$Riskadj_t = \dfrac{\lambda\sigma_m^2}{R_f - \delta_{ce}}[(1+\beta_1\delta_{ce})k^e + \beta_2\delta_{ce}k^a]B_t$ is the earnings risk adjustment.

A number of observations follow. Firstly, balance sheet items and income statements jointly determine the market value of equity. Equity market value is anchored on the book value of equity. β_0 in Proposition 1 is the capitalized value of growing excess accounting return over the risk-free rate. The first term $(1+\beta_0)B_t = \dfrac{\mu_e B_t - (\delta_{ce} - 1)B_t}{(R_f - \delta_{ce})}$ represents the steady-state risk neutral present values of earnings less retentions (i.e. long-run future dividends). We note that discounting at the risk free rate the dividend contribution is evaluated as though they grow at the certainty equivalent rate of $\delta_{ce} - 1\ (< \bar{\delta} - 1)$. In our parsimonious form, market value of equity is a nonlinear function of book value and earnings and earnings components. Accruals play a role if accruals have incremental effect in forecasting future earnings after controlling for earnings ($\alpha_{12} \neq 0$). It is not earnings (and components) per se, but the deviation of scaled earnings (cash flows) and accruals from their long-run means that make the contribution to the value after controlling for book value and the expected long-run mean of the return on book equity (μ_e). After controlling for book value, deviations of scaled earnings from their long-run mean and value adjustment components, equity value is negatively related to accruals and the deviation of scaled accrual from its long-run mean when accruals are less persistent in forecasting of future earnings ($\alpha_{12} < 0$). Examination of the structure of the value multiples of earnings (β_1) and accruals (β_2) terms reveals that their value reflect the product of attenuation terms (α_{ij}) and growth δ_{ce} .

Secondly, the valuation model includes a growth premium term, $Growthadj$, since a risk-neutral growth rate is applied to earnings (cash flows) and accruals. It changes with the market volatility (σ_m) and the correlations between earnings (components) and investments. $cov(\varepsilon^e, \varepsilon^\delta)$ and $cov(\varepsilon^a, \varepsilon^\delta)$ are determined by the firm's investment policy and accounting policy.

Finally, the earnings risk adjustment term reduces the certainty equivalent valuation in which accounting fundamentals are discounted using a risk-free rate. If all investors are risk neutral $(\lambda = 0)$, the $Riskadj$ term disappears. The risk adjustment term effectively converts earnings (cash flows) and accruals to their certainty equivalent numbers. Collectively, the risk neutral valuation attributable to earnings (cash flows) and accruals is augmented by the present value of "abnormal" risky growth and adjusted by earnings risk whose values are growing in line with the scaling factor of book values.

In terms of cash flows and accruals, we have

$$MVE_t = (1+\beta_0)B_t + \beta_1(\frac{cf_t}{B_{t-1}} - \mu_c)B_t + (\beta_1 + \beta_2)(\frac{ac_t}{B_{t-1}} - \mu_a)B_t + Growthadj_t - Riskadj_t . \quad (9)$$

It suggests that, after controlling for book value, deviations of scaled cash flows from their long-run mean and growth and risk adjustments, equity value can be positively related to accruals and the deviation of scaled accrual from its long-run mean even if accruals are less persistent in forecasting of future earnings. The sign depends on whether $(\beta_1 + \beta_2) > 0$.

5. Stock returns when incorporating asset growth and accounting accruals

Changes in consumption may affect both the earnings of individual firms and the general level of capital market returns generating the observed statistical relationship between stock returns and earnings components. The following proposition formally analyzes the association between equity book value growth and future stock returns. In our model setup, growth risk interacts with the deviation of the components of earnings from their long-run means. Lemma 1 and Proposition 2 below identify the important role played by growth in capital market theory and a potential explanation of several existing empirical puzzles

Lemma 1: Assume Assumptions 1-2, and information dynamics (3)-(5) hold. The total stock return at *t+1*, $R_{t+1} \equiv \frac{MVE_{t+1} + d_{t+1}}{MVE_t}$, can be written as

$$R_{t+1} = R_f + \frac{(\tilde{\delta}_{t+1} - \delta_{ce})}{\delta_{ce}}[R_f - (1 + \frac{e_{t+1}}{B_t})\frac{B_t}{MVE_t}] + \tilde{\delta}_{t+1}\{\lambda\sigma_m^2(\frac{k^e}{\delta_{ce}} + \beta_1 k^e + \beta_2 k^a) + \frac{1}{\delta_{ce}}\varepsilon_{e/B,t+1}$$
$$+\beta_1[\varepsilon_{e/B,t+1} - \mathrm{cov}_t(\varepsilon_{e/B,t+1}, \varepsilon_{\delta,t+1})] + \beta_2[\varepsilon_{a/B,t+1} - \mathrm{cov}_t(\varepsilon_{a/B,t+1}, \varepsilon_{\delta,t+1})]\}\frac{B_t}{MVE_t}. \tag{10}$$

Note that we have used tilde to emphasize the random nature of growth. Lemma 1 shows the importance of the interaction between future investment growth and earnings in determining future stock returns. Taking expectation on equation (10), we have the following proposition.[16]

Proposition 2: Assume Assumptions 1-2, and information dynamics (3)-(5) hold. The expected stock return for firm *i* at time *t* can be written as

$$E_t[R_{i,t+1}] = R_f + ERP_{i,t} + GRP_{i,t} \tag{11}$$

[16] We temporarily reintroduce the i-subscript to remind readers that this proposition applies to individual firms as well as portfolios.

where

$$ERP_{i,t}=\lambda\sigma_m^2[(1+\beta_{i,1}\delta_{i,ce})k_i^c+(1+(\beta_{i,1}+\beta_{i,2})\delta_{i,ce})k_i^a]\frac{B_{i,t}}{MVE_{i,t}}, \tag{12}$$

and

$$GRP_{i,t}=\frac{(\bar{\delta}_i-\delta_{i,ce})}{\delta_{i,ce}}[R_f+ERP_{i,t}-(1+E_t[\frac{e_{i,t+1}}{B_{i,t}}])\frac{B_{i,t}}{MVE_{i,t}}]. \tag{13}$$

$ERP_{i,t}$ is the earnings risk premium attributable to the earnings (yield) risk priced by the market. The form of the expression in square brackets in equation (12) accommodates the differential risk characteristics of the components of earnings. $GRP_{i,t}$ in equation (13) is the growth risk premium representing the impact of the growth premium, $(\bar{\delta}_i-\delta_{i,ce})$, on equity returns. It is distinct from, but interacts with, ERP. It exists even if ERP = 0. $GRP_{i,t}$ can be rewritten as

$$GRP_{i,t}=\frac{(\bar{\delta}_i-\delta_{i,ce})}{\bar{\delta}_i}[E_t[R_{i,t+1}]\frac{MVE_{i,t}-B_{i,t}}{MVE_{i,t}}-(1+E_t[ROE_{i,t+1}]-E_t[R_{i,t+1}])\frac{B_{i,t}}{MVE_{i,t}}]. \tag{14}$$

We see that $GRP_{i,t}$ is composed of three elements: $(\bar{\delta}_i-\delta_{i,ce})/\bar{\delta}_i\approx\lambda k^{\delta}\sigma_m^2$, capturing the covariability of growth with the market, the expected stock return on the goodwill proportion of market value, less book values scaled by market value multiplied by the economic spread. The last element is the scaled economic profit if the expected return is a good proxy for the cost of capital. Proposition 2 presents an accounting-based asset pricing model, demonstrating that the expected stock return can be expressed as the risk-free rate, adjusted by the earnings risk premium (ERP) and firm's growth risk premium (GRP).

We observe that when the expected ROE is higher than the market risk adjusted return or cost of capital, the firm's return component GRP decreases with investment, consistent with valuation theory where growth in assets will increase the firm value, and hence with investors requiring lower risk premium. Accordingly, our model shows a negative relationship between investment growth and stock returns. For firms with the anticipated ROE less than the market risk adjusted return, growth in assets will destroy the firm value, hence investors will require higher risk premiums and higher returns.[17] That is, the firm's return component GRP increases with investment. Our model then shows a positive relation between investment growth and stock returns. Therefore, it follows that the coefficient attached to a growth proxy that is positively related to future earnings in cross-sectional regressions of stock returns is expected to be negative since the accounting rate of return is expected to be greater than cost of capital for the majority of companies in the market. Indeed, empirical evidence frequently documents a negative relationship between various forms of corporate investment and future stock returns (e.g., Cooper et al. (2008), Richardson et al. (2010)).

Since expected earnings can be expressed in terms of current earnings and earnings components as dynamic (3), Proposition 2 suggests that earnings yield and scaled earnings components can be useful in predicting future returns since they are associated with the growth risk premium. Individual earnings components such as accruals have no separate role to play when controlling for book value and future aggregate earnings in GRP.[18] As an extension of this argument, it follows that any accounting or non-accounting variables that can be used to forecast future earnings (or cum-dividend book value) can be informative in predicting future stock returns in a setting with a growth risk premium. The variable acts together with the growth risk premium in predicting future stock returns. The negative sign attached to the anticipated ROE implies that if a variable is positively (negatively) related to future earnings-to-price, then it will have a negative (positive) growth risk premium associated with future returns. It suggests that any "anomaly" related to any

[17] It is consistent with empirical evidence on size effect. Small (large) firms are less (more) profitable (ROE) implying large (small) GRP and should have high (low) returns controlling for growth rate premium.

[18] Hou et al. (2021) suggest that their q5 model, which augments the Hou et al. (2015) q-factor model with the expected growth factor, "largely explains the accruals anomaly".

variables that can be used to forecast future earnings-to-price can be part of a more general "growth anomaly".

The implications of the effect of growth on the accrual component of earnings and its sign on returns can be further illustrated by recalling the relationship in dynamic (3) between expected (scaled) earnings and accruals:

$$
\begin{aligned}
E_t[\frac{e_{i,t+1}}{B_{i,t}}] &= \alpha_{i,11}(\frac{e_{i,t}}{B_{i,t-1}} - \mu_{i,e}) + \alpha_{i,12}(\frac{ac_{i,t}}{B_{i,t-1}} - \mu_{i,a}) \\
&= \alpha_{i,11}(\frac{cf_{i,t}}{B_{i,t-1}} - \mu_{i,c}) + (\alpha_{i,11} + \alpha_{i,12})(\frac{ac_{i,t}}{B_{i,t-1}} - \mu_{i,a}).
\end{aligned}
\tag{15}
$$

Equation (15) together with equation (13) emphasize that both deviations of scaled earnings (cash flows) and accruals from their long-run means interact with the growth risk premium. Controlling for the growth risk premium, both coefficients of scaled earnings (cash flows) and book-to-market are negative as they are positively associated with future earnings yield. Equations (13) then together with return model (11) are consistent with Penman and Yehuda (2019) and Penman and Zhang (2020) who suggest that the recognition of current cash flows (earnings) implies lower risk, and similarly booked assets have lower risk. If we hold the growth risk premium constant, a lower (higher) cash flows realization implies higher (lower) expected returns. Current cash flows are negatively related to macroeconomic fluctuations since large cash flows provide a means of hedging risk. Accounting fundamentals governed by conservative principles effectively convey the risk of future expected growth. However, for a given growth risk premium, the sign of accrual and its deviation term are determined by the coefficients $-(\alpha_{11} + \alpha_{12})$ when controlling for cash flows, and $-\alpha_{12}$ when controlling for earnings. Existing empirical evidence shows the observed values of these coefficients to be negative and positive respectively, as does our own investigations reported in Appendix A, Table 1. Therefore, inference of returns on accruals in a univariate regression can be misleading. Collectively, the role of accruals in predicting returns is determined by the growth premium, $(\bar{\delta}-\delta_{ce})$, the persistence of accruals α_{12} or $(\alpha_{11}+\alpha_{12})$ in predicting future earnings, and the ratio of book-to-market. These three factors interact and jointly show the effect of accruals on future stock returns. The apparently contradictory signs on accruals suggest that regressions of returns on accruals without incorporating growth risk may constitute a correlated omitted variable problem, an issue that we will explore in more depth in section 6. If the persistence of cash flows is equal to that of accruals in predicting future earnings (i.e., $\alpha_{12}=\beta_2=0$), then it is clear that separate accruals will have no role to play in expected returns after controlling for aggregate earnings and book equity. This is consistent with Sloan (1996) that the differential persistence ($\alpha_{12}\neq 0$) between accruals and cash flows is a necessary (but not sufficient) condition for the role of accruals in explaining future stock returns after controlling for aggregate earnings. $\bar{\delta}\neq\delta_{ce}$ and $ac_t/B_{t-1}\neq\mu_a$ are also necessary conditions for accruals in predicting future returns. Therefore, it is not surprising that empirical evidence shows a connection between accruals and future stock returns, and why the "accrual anomaly" cannot be simply arbitraged away in the presence of (risky) growth. Evidence in Guo and Jiang (2011) broadly supports our proposition: accruals predict stock returns because accruals are correlated with determinants of the conditional equity premium presented in GRP.

When $k^{\delta}=0$ in equation (7) or investment policy is independent of the state of economy, $\delta_{ce}=\bar{\delta}$, the growth risk premium equals zero. We have mapped uncertainty in cash flows fluctuations ($k^c\sigma_m^2$) and accrual fluctuations ($k^a\sigma_m^2$) to uncertainty in price fluctuations in equation (11). The return equation (11) can then be simplified to

$$\frac{E_t[MVE_{t+1}+d_{t+1}]}{MVE_t}=R_f+\lambda\sigma_m^2((1+\beta_1\delta_{ce})k^c+(1+(\beta_1+\beta_2)\delta_{ce})k^a)\frac{B_t}{MVE_t}. \qquad (16)$$

The corresponding earnings risk premium terms associated with cash flows $\lambda\sigma_m^2(1+\beta_1\delta_{ce})k^c \frac{B_t}{MVE_t}$ and accruals $\lambda\sigma_m^2(1+(\beta_1+\beta_2)\delta_{ce})k^a \frac{B_t}{MVE_t}$ are both positive. One implication of Proposition 2 is that these positive risk premiums should not be overlooked in empirical studies when investigating the impact of current accruals on the magnitude of future returns. They have implications to offset the hedge returns in the accrual-based trading strategies. The fundamental growth part of the model as described in equation (13) is often incorrectly specified or inadequately parameterized in empirical studies of capital markets. This essentially reflects the development of factor models where the empirical limitations of the simple CAPM have been augmented by accounting fundamentals to produce a factor model.

For convenience to explore these ideas in greater depth, we rewrite the expression for the total stock return at *t+1* in the following corollary of Proposition 2.

Corollary 1: Assume Assumptions 1-2, and information dynamics (3)-(5) hold. The total stock return at *t+1* can be rewritten as

$$\begin{aligned}
R_{i,t+1} = {} & \tilde{\delta}_{i,t+1} + (1+\mu_{i,e}-\tilde{\delta}_{i,t+1})\frac{B_{i,t}}{MVE_{i,t}} \\
& +[\alpha_{i,11} + \frac{(R_f-\delta_{i,ce})\beta_{i,1}-\alpha_{i,11}}{\delta_{i,ce}}\tilde{\delta}_{i,t+1}](\frac{e_{i,t}}{B_{i,t-1}}-\mu_{i,e})\frac{B_{i,t}}{MVE_{i,t}} \\
& +[\alpha_{i,12} + \frac{(R_f-\delta_{i,ce})\beta_{i,2}-\alpha_{i,12}}{\delta_{i,ce}}\tilde{\delta}_{i,t+1}](\frac{ac_{i,t}}{B_{i,t-1}}-\mu_{i,a})\frac{B_{i,t}}{MVE_{i,t}} \\
& +\varepsilon_{i,e/B,t+1}\frac{B_{i,t}}{MVE_{i,t}} + (\beta_{i,1}\varepsilon_{i,e/B,t+1}\tilde{\delta}_{i,t+1} + \beta_{i,2}\varepsilon_{i,a/B,t+1}\tilde{\delta}_{i,t+1})\frac{B_{i,t}}{MVE_{i,t}}.
\end{aligned} \tag{17}$$

Taking expectation, we can write the expected stock return as:

$$E_t[R_{i,t+1}] = \bar{\delta}_{i,t} + (1+\mu_{i,e} - \bar{\delta}_{i,t})\frac{B_{i,t}}{MVE_{i,t}} + [\alpha_{i,11} + \frac{(R_f - \delta_{i,ce})\beta_{i,1} - \alpha_{i,11}}{\delta_{i,ce}}\bar{\delta}_{i,t}](\frac{e_{i,t}}{B_{i,t-1}} - \mu_{i,e})\frac{B_{i,t}}{MVE_{i,t}}$$
$$+[\alpha_{i,12} + \frac{(R_f - \delta_{i,ce})\beta_{i,2} - \alpha_{i,12}}{\delta_{i,ce}}\bar{\delta}_{i,t}](\frac{ac_{i,t}}{B_{i,t-1}} - \mu_{i,a})\frac{B_{i,t}}{MVE_{i,t}}$$
$$+\bar{\delta}_{i,t}[(\beta_{i,1}k_i^e + \beta_{i,2}k_i^a)k_i^\delta \sigma_m^2 + \beta_{i,1}cov(\varepsilon_i^e, \varepsilon_i^\delta) + \beta_{i,2}cov(\varepsilon_i^a, \varepsilon_i^\delta)]\frac{B_{i,t}}{MVE_{i,t}}. \tag{18}$$

In implementation, the estimated CAPM beta is largely based on the correlation between the growth in the individual stock values and growth in the aggregate stock market values. It fails to fully incorporate the significance by analysis of the public information embedded in accounting and economic data. This lack of detail has thus been added empirically to improve the explanatory power and forecasting qualities of such models. In contrast, our model adopts an analytical approach by using accounting data to produce a model of firm value and its role in predicting stock returns. For example, both Fama and French (2015) and Hou et al. (2015) add risk factors associated with ROE and asset growth in their models. Hou et al. (2021) claim their factor model can explain “accrual anomaly” when including a risk factor formed from a proxy for the expected growth, an explanation with which we concur when we explore this issue at a theoretical level in more depth in subsequent sections.

A common feature of factor models is the inclusion of a book-to-market factor. We also see in Proposition 2 and its corollary the pervasive role of book-to-market. Returns are related via book-to-market to return on book equity, together with market-determined factors. However, a fundamental difference from most factor models is that our model is nonlinear and factors are in an integrated term in addition to a growth factor. In contrast, their models, which for reasons of empirical expediency, are linear. The multiplicative nonlinearity in our models as in equations (10)

-(18) suggests that returns are related to the product of the ratios such as $\frac{e_t}{B_{t-1}} \times \frac{B_t}{MVE_t}$ (return on equity at t multiplied by book-to-market) or equally as involving $\frac{e_t}{MVE_t} \times \frac{B_t}{B_{t-1}}$, i.e. earnings yield times growth in book values. This ambiguity of "risk structure" finds an expedient resolution in the linearization implicit in the statistical methodology of factor analysis.

Our derivation also implies a somewhat different interpretation of this factor. The ratio of book-to-market is often argued as a proxy for growth, but in our model it effectively converts dollars of fluctuations in accounting items such as earnings and accruals into prices (monetary dollars). Thus, although accounting values and stock prices are both measured in dollars, a change in one dollar of book value or accounting earnings is not easily mapped into a corresponding dollar change in stock prices. Accounting procedures are essentially conservative and each firm makes their own risk assessments (Penman and Zhang (2020)). This generates a need for a convenient firm specific mapping between accounting dollars and monetary dollars. The ratio (B_t/MVE_t) provides a simple and convenient summary mapping specific to individual firms. Since the degree of conservatism is not an unfettered choice and is related to the operating structure of the industrial sector, we argue that book-to-market is not so much a risk factor as a convenient link between accounting uncertainty and valuations. Thus, the apparent positive relation between book-to-market and future returns is not necessarily causal but is rather a reflection of an accounting-based valuation process. Of course, conservative accounting may shift earnings across reporting periods and induce short-term earnings growth, and the ratio of price-to-book (the inverse of book-to-price) amplifies the growth risk premium.

Finally, our model emphasizes that it is scaled accruals from their long-run means (i.e. deviation), rather than accruals per se represents risk associated with risky growth when controlling for the growth risk premium and book value. It has an important implication to explain an anomaly on the accruals-based risk factor documented in Hirshleifer et al. (2012). We will explore this in subsection 6.3.

6. Interpretation of existing literature on "accrual anomaly"

In this section, we explore the use of our theoretical model to interpret and understand the accrual anomaly and the apparently conflicting empirical results found in the literature. We mainly focus on two papers. The first of these is the seminal paper by Sloan (1996) who finds a strong negative relationship between future stock returns and accruals. The second paper is an empirical investigation by Hirshleifer et al. (2009) which appears to contradict Sloan's findings. The major difference between the two empirical studies is the econometric methods they employ: the former is a cross-sectional study whilst the latter is a time series study.

What is common in the two empirical studies, however, is that growth as an earnings-correlated variable is explicitly omitted or implicitly treated as if it were independent of earnings. To examine the role of the correlation between risky growth and future earnings and the components of earnings, we apply the functional dependency of return equation (17). It draws attention to the fact that returns are both firm/portfolio specific and time dependent. We will use this relationship in a theoretical replication of different empirical approaches to measuring the coefficient of accruals. Our principal aim is to shed light on the reasons for the apparently conflicting findings.

It is noteworthy to mention that the above two papers base their analysis on operating income and assets at the firm level, while our theoretical model is based on net income and book equity. This subtle difference between definitions of earnings allows us to ignore the leverage effects on equity returns in developing our model.[19] Nevertheless, when accruals are similarly estimated using the indirect balance sheet method as the above papers, we show our model predicts both the sign and magnitude of empirical observations.[20] Our numerical analysis serves a secondary purpose as a check on the plausibility of our model. It helps to explain why accruals might be observed to be both negatively and positively associated with future stock returns.

6.1.Cross-sectional regressions of stock returns on accruals

We start by comparing our theoretical predictions of the effect of accruals on stock returns with the findings of Sloan (1996). Sloan investigates the role of accruals scaled by average assets using a cross-sectional analysis of 10 portfolios ranked on the accrual component. He repeats this process for each year of his data set and reports the average values of the coefficients from regressing future returns on accruals together with several risk proxies. Based on this analysis, he reports a significant negative one-year ahead return to accruals.

Based on equation (17), we carry out a theoretical replication of this cross-sectional regression study. Since time is a common constant for any particular year, $\tilde{\delta}_{i,t+1}$ can be regarded as independent of time in each cross-sectional study. The regression analysis involves computing the sum of cross-product terms over firms (not time) which we equate to the expected values of $E_i\left[\tilde{\delta}_{i,t+1}(\frac{e_{i,t}}{B_{i,t-1}}-\mu_{i,e})\right]$ and $E_i\left[\tilde{\delta}_{i,t+1}(\frac{ac_{i,t}}{B_{i,t-1}}-\mu_{i,a})\right]$. In addition, in a given year t, such studies

[19] Other papers (e.g., Hribar and Collins (2002), Pincus et al. (2007)) use net income not operating incomes in their empirical studies. If we model operating income and net operating assets, our return expression would be the weighted average cost of capital.

[20] As expected, it does limit on our ability to reproduce the exact numerical value reported in the last two cited papers. Magnitude here refers to the correct power in the normal exponential representation of the number.

ignore the correlations between book value growth $\tilde{\delta}_{i,t+1}$ and deviations $(\frac{e_{i,t}}{B_{i,t-1}} - \mu_{i,e})$ and $(\frac{ac_{i,t}}{B_{i,t-1}} - \mu_{i,a})$. It implicitly assumes that small noisy correlations can be averaged away in the cross section.[21] Theoretically, it is equivalent to the assumption that all the variables are dependent on the (same) market return together with an uncorrelated random noise term. Equation (18) then gives a theoretical cross-sectional model of next periods expected return as a function of deviations in earnings and accruals as:

$$\bar{\delta} + \left\{ \begin{array}{l} (1+\mu_e - \bar{\delta}) + \left[\alpha_{11} + \dfrac{\bar{\delta}\left(\beta_1(R_f - \bar{\delta}_{ce}) - \alpha_{11}\right)}{\delta_{ce}} \right] (\dfrac{e_t}{B_{t-1}} - \mu_e) \\ \qquad + \left[\alpha_{12} + \dfrac{\bar{\delta}\left(\beta_2(R_f - \delta_{ce}) - \alpha_{12}\right)}{\delta_{ce}} \right] (\dfrac{ac_t}{B_{t-1}} - \mu_a) \end{array} \right\} \frac{B_t}{MVE_t}. \tag{19}$$

The coefficients of earnings and accruals in equation (19) represents the net result of a cross-sectional analysis of individual firms in year t and whose coefficients are then average over all years. Since we are interested in the aggregate coefficients over several years we have dropped t subscript in growth and replaced it by its expected value. In this replication of a cross-sectional analysis we have ignored the error term in equation (18). It contains a risk adjustment. Throughout our analysis, our modeling is carried out assuming the absence of arbitrage opportunities. For this reason, we do not explore the hedge returns associated with growth risk which justifies the omission of the last line in equation (18), in our theoretical replication of Sloan's approach.

[21] Growths across firms *i* and *j* have a common market factor and firm specific noisy term. At firm level time series analysis, the correlation between growth and earnings is volatile, and the correlation between aggregate firm characteristics and market becomes important. Firm specific risk in a portfolio is diversified away.

Our initial interest as much of existing literature is with the sign of the accrual component $(\alpha_{12}+\frac{\bar{\delta}(\beta_2(R_f-\delta_{ce})-\alpha_{12})}{\delta_{ce}})$ in equation (19). A critical issue is the parameters emerging from our earnings dynamics equation (3), where we were to use parameters for $\alpha_{11}=0.838$ and $\alpha_{12}=-0.273$ as reported in Sloan (1996). We similarly estimate parameters from accrual dynamic equation (4) using the average asset as a deflator for $\alpha_{21}=0.08$ and $\alpha_{22}=0.202$ from the same period. We assume $\delta_{ce}=1.03$, $r_f=0.08$ and $\bar{\delta}-1=0.1$ (the average 10-year US government bond yield and asset growth over the same period) and obtain a theoretical value for the regression coefficient of return on accruals to average asset of -0.065.[22] However, because of the book-to-market factor or average asset-to-firm value, this is effectively the regression coefficient of return on accruals scaled by firm market values. Using the sum of market capitalization and long-term debt as a proxy for firm value, we estimate the standard deviations of accrual-to-average asset and accrual-to-firm value of 0.077 and 0.136 over the sample period. We then have the theoretical conversion of returns to average assets of 1.77 (=0.136/0.077) as identified in Appendix B2. We get the corresponding coefficient of -11.5% (=-0.065$\times$0.177). This is comparable to Sloan's findings in sign and magnitude. Our estimate of the regression coefficient to earnings-to-market is 0.253, which again is comparable to the figure reported in Table 7 in Sloan (1996). In comparing our theoretical predictions with the results reported by Sloan (1996), however, it should be remembered that equation (19) has no direct equivalent to the empirical regression equations used by Sloan, with the closest structure being that reported in his Table 7. Even then the structural differences are significant, book-to-market is an integral part of our theoretical structure and serves

[22] We find this value is relatively robust to our estimates of α_{21} and α_{22} as well as $\delta_{ce}(<R_f)$.

as multiplier of other factors, while it is linearized as a control variable or separate factor by Sloan and indeed nearly by almost all empirical research in this field. Neither are we able to include size as a control variable since this variable is outside the scope of our theoretical models.

We also explore the consistency of our model with existing empirical findings over the longer time period 1963-2019, summary data for which can be found in Appendix A. Our results confirm that accruals are less persistent than cash flows in predicting future earnings ($\alpha_{11} + \alpha_{12} < \alpha_{11}$). Accruals play a role in valuation because accruals have incremental role ($\alpha_{12} \neq 0$) in forecasting of future aggregate earnings after controlling for the current earnings. We also find that the coefficient accrual predicted in our equation (19) is negative for 55 out of the 56 years and the predicted coefficient of earnings is positive for 44 out of the 56 years. However, the magnitudes of accrual coefficients are noticeably smaller over the longer and more recent period, which supports evidence in Green et al. (2011) who find that the accrual effect has slowly been eradicated over their sample period. We repeat this analysis in terms of cash flows and accruals via $e_{i,t} = cf_{i,t} + ac_{i,t}$. When we use cash flows and accruals to predict future returns, then all but 12 of the predicted accrual coefficients are positive.

Sloan (1996) uses the Mishkin test (1983) to infer investors' expectations of the earnings process from the behavior of stock returns by regressing the risk-adjusted stock returns at t+1 on the unexpected earnings at t+1. The methodology has been used extensively by follow up studies. Based on equations (17), (18) and (3), the unexpected returns are related to the unexpected earnings as following:

$$R_{t+1} - E_t[R_{t+1}] = (1+\beta_1\tilde{\delta}_{t+1})\frac{e_{t+1} - E_t[e_{t+1}]}{MVE_t} - \beta_1\frac{\text{cov}_t[\tilde{\delta}_{t+1}, e_{t+1}]}{MVE_t} + (\tilde{\delta}_{t+1} - \bar{\delta})\{\frac{MVE_t - B_t}{B_t}$$
$$-\frac{1}{\delta_{ce}}((\alpha_{11} - \beta_1(R_f - \delta_{ce}))(\frac{e_t}{B_{t-1}} - \mu_e) + (\alpha_{12} - \beta_2(R_f - \delta_{ce}))(\frac{ac_t}{B_{t-1}} - \mu_a))\}\frac{B_t}{MVE_t}$$
$$+\beta_2\{\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1} - E_t[\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}]\}\frac{B_t}{MVE_t}. \tag{20}$$

Indeed, $R_{t+1} - E_t[R_{t+1}] = (1+\beta_1\bar{\delta})\frac{e_{t+1} - E_t[e_{t+1}]}{MVE_t} + \beta_2\varepsilon_{a/B,t+1}\bar{\delta}\frac{B_t}{MVE_t}$ holds if growth is assumed to be constant ($\bar{\delta}$) following equation (20). While the greater persistence of cash flows over accruals in their relationship with future earnings is attributable to the fact that in a simple bivariate regression cash flows are more positively correlated with future earnings than are accruals, it is not necessarily true if future return is the dependent variable. In fact, Sloan finds that returns are more positively correlated with scaled accruals than with scaled cash flows. Since the earnings response coefficient $1+\beta_1\tilde{\delta}_{t+1}$ in (20) is a stochastic variable changing with investment growth, any inference based on earnings surprise must consider this correlated variable. Lewellen (2010) also points out "the impact of correlated omitted variables on the slopes carries over directly to Mishkin tests." When regressing time t+1 stock returns on t+1 earnings, empiricists often implicitly omit the correlated growth variable ($\tilde{\delta}_{t+1}$) or view it as an independent variable. The impact of which should become cleared in the next section.

6.2.Time series regressions of stock returns on accruals

In contrast to the cross-sectional analysis by Sloan (1996), Hirshleifer et al. (2009) (hereafter HHTa) adopt a time series approach. This has important implications in interpreting our growth variables. As we have seen in a cross-sectional study, the growth variable $\tilde{\delta}_{t+1}$ is treated as a constant at time t and uncorrelated at a firm level with scaled earnings and accruals. However, in a time series analysis, $\tilde{\delta}_{t+1}$ is stochastic and potentially correlated with aggregate accruals and cash flows at time t. In contrast, HHTa implicitly treat growth as if it were independent of accruals and cash flows.

We modify our model (17) in Corollary 1 in line with the reported results in HHTa by relating returns to current levels of accruals and cash flows via $e_t = cf_t + ac_t$, where earnings, accruals, cash flow and dividends now represent the market weighted aggregate values at time t. They constitute a series of random observations alongside $\tilde{\delta}_{t+1}$, the growth in the book asset of the aggregate market. The dependent variable is the market wide stock returns R_{t+1}, where

$$\begin{aligned} R_{t+1} &= \tilde{\delta}_{t+1} + (1+\mu_e - \tilde{\delta}_{t+1})\frac{B_t}{MVE_t} + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t} \\ &+ \tilde{\delta}_{t+1}[\frac{(R_f - \delta_{ce})\beta_1 - \alpha_{11}}{\delta_{ce}}(\frac{cf_t}{B_{t-1}} - \mu_c) + \frac{(R_f - \delta_{ce})(\beta_1 + \beta_2) - (\alpha_{11} + \alpha_{12})}{\delta_{ce}}(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t} \\ &+ \varepsilon_{e/B,t+1}\frac{B_t}{MVE_t} + \beta_1\varepsilon_{e/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t} + \beta_2\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t}. \end{aligned} \quad (21)$$

The subsequent analysis is simplified by a key observation that the first line of the right-hand side of equation (21) reduces to our growth term $\tilde{\delta}_{t+1}$ plus the dividend yield, dy_{t+1}.[23] As we shall see this observation has important implications in the interpretation of both the sign and the magnitude of the observed dependency between market returns and accruals. However before we can offer such an interpretation, we need first to validate our theoretical model. Again, we carry out a

[23] Details of the mathematical development can be found in Appendix A as equation (26).

theoretical replication of the methodology adopted in HHTa using a simple univariate regression[24] and estimate the theoretical values of the coefficient of accruals from a regression of R_{t+1} on $\frac{ac_t}{B_{t-1}}$.

This involves evaluating the value of $\operatorname{cov}(R_{t+1}, \frac{ac_t}{B_{t-1}} - \mu_a) / \sigma_{ac}^2$, where σ_{ac}^2 is the variance of $\frac{ac_t}{B_{t-1}}$.

In effect this gives us a value for the LHS of equation (21). Evaluation of the RHS of equation (21) requires the computation of :

$$
\begin{aligned}
&\frac{\operatorname{cov}(\tilde{\delta}_{t+1}, \frac{ac_t}{B_{t-1}} - \mu_a)}{\sigma_{ac}^2} + \frac{\operatorname{cov}(dy_{t+1}, \frac{ac_t}{B_{t-1}} - \mu_a)}{\sigma_{ac}^2} \\
&+ \frac{(\beta_1 + \beta_2)(R_f - \delta_{ce}) - (\alpha_{11} + \alpha_{12})}{\delta_{ce}} \frac{\operatorname{cov}\left(\tilde{\delta}_{t+1}(\frac{ac_t}{B_{t-1}} - \mu_a)\frac{B_t}{MVE_t}, \frac{ac_t}{B_{t-1}} - \mu_a\right)}{\sigma_{ac}^2} \\
&+ \frac{\beta_1(R_f - \delta_{ce}) - \alpha_{11}}{\delta_{ce}} \frac{\operatorname{cov}\left(\tilde{\delta}_{t+1}(\frac{cf_t}{B_{t-1}} - \mu_c)\frac{B_t}{MVE_t}, \frac{ac_t}{B_{t-1}} - \mu_a\right)}{\sigma_{ac}^2} \\
&+ \operatorname{cov}\left(\left(\varepsilon_{e/B,t+1} + \beta_1 \varepsilon_{e/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t} + \beta_2 \varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t}\right), \frac{ac_t}{B_{t-1}} - \mu_a\right) / \sigma_{ac}^2.
\end{aligned}
\tag{22}
$$

Note in expression (22), we have replaced the first line of equation (21) by growth plus dividend yield. As we show in appendix A, if we treat all the key variables as approximately normal for a numerical evaluation, we merely need the means and standard deviations of aggregate variables together with their correlation matrix. Again, we collect this data over the sample period 1963-2018 and report it in Table 2 Panels A and B in Appendix A. Following HHTa, we select firms with December fiscal year ends. However consistent with our model development, our earnings

[24] Though a univariate regression in this context is not usually applicable, it works approximately in their study since cash flows and accruals are almost independent (with a correlation of -0.03) as reported in their Table 1 (p394). We make no such assumption of finding.

are net income. We also note in contrast to HHTa, our resulting aggregate cash flows are highly negatively correlated to aggregate accruals. We make estimates of the values of α_{ij} (i, j =1,2) together with the long-run means of the scaled aggregate earnings, accruals and cash flows: μ_e, μ_a and μ_c. We estimate $r_f = 6\%$, the average of 10-year US government bond yield over 1963-2018 and $\delta_{ce} - 1$ of 3%.[25] From this data, we then calculate β_1 and β_2 whose values we also report in Table 2 Panel C. This enables an estimate of size and sign of the predicted theoretical coefficient of accruals together with a breakdown of the contributions made by the individual components forming expression (22).[26]

Applying simple linear regression to the LHS of equation (21), we estimate the coefficient of accruals to be +0.52 . Application of the Iserrilis' Theorem enables us to calculate the RHS values of expression (22) based on the data in Panels A and B in Appendix A in Table 2. We find these values sum to +0.46, which is mainly attributable to items in the first line in expression (22), namely correlation between growth and accruals (+0.244) and the correlation between dividend yield and accruals (+0.216). The combined effect of the contribution from the second and third lines in expression (22) is negligible (-0.003) with the individuals contribution being -0.024 and +0.021 respectively. Finally, the combined contribution of the errors terms in expression (22) is also negligible being only 1.5×10^{-7}.

Analysis, similar to the foregoing, carried out on cash flows reveals that the negative regression coefficient of cash flows (-0.307) is also due to the strong negative correlation between the cash flow measure and growth (-0.147) and between the cash flow measure and dividend yields (-0.154), again with the combined effects of the remaining terms being insignificant.

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[25] We find our results are not sensitive to these assumptions.
[26] Evaluating the breakdown is based on equations (27) and (28) in appendix A.

Hence in our analysis of a time series formulation of the relationship between accruals and future returns, we find it is the growth and associated dividend elements that leads to the observed positive (negative) high dependency with accruals (cash flows).[27] This underlines the importance of the need to treat accruals (cash flows) and the growth of assets as jointly interdependent through the roles they play in stock returns.

6.3. Accruals and accrual risk factor

While a large body of empirical studies seem to support Sloan's original explanation on capital market inefficiency, Hirshleifer et al. (2012) (hereafter HHTb) examine whether accruals represent a type of risk that is not captured by known risk factors. Following Fama and French (1993), they construct a long-short factor mimicking portfolio to measure accrual risk to proxy for the underlying unknown fundamental risk factor. However, when applying their factor analysis, HHTb find that accruals remain highly significant with or without controlling for the proxy for the accrual risk factor loading in their cross-sectional analysis. Furthermore, the accrual factor loading becomes insignificant after controlling for accruals. We argue this is what we would expect under our modeling of the accruals process even if one does not control for growth risk. Problems arise because most empirical investigations invariably use scaled accruals as the basis of classification of high and low accrual firms and choose it, i.e. $ac_{i,t} / B_{i,t-1}$ as the variable of interest, rather than its deviation from its long-run mean $(\mu_{a,i} - \dfrac{ac_{i,t}}{B_{i,t-1}})$, which we identify in our model as a key

[27] If we calculate the expected value regression coefficient of market return on standardized accruals using the data published in HHTa, we get the reported value of 0.068 (HHTa Table 3, p395). However, if we calculate the regression coefficient of market return on the standardized value of dividend yield, we get 0.0062. This suggest that 91% of the value reported by HHTa is attributable to the growth element in market return and only 9% attributable to the income portion of return.

determinant of returns. Note that $\mu_{a,i}$ is firm-i specific. The importance of this is that high and low accruals should imply high and low relative to a firm's long-run mean, not as used in cross-sectional analysis where high and low are measured relative to the annual cross-sectional mean.

Factor analysis identifies a hyperplane in factor space that relates factors in a linear fashion to expected returns. This factor hyperplane is determined by risk return points based on groupings of extreme portfolios. These risk return points when combined with the risk-free rate define this hyperplane in factor space where the expected return on an individual firm is based on its projection onto this hyperplane. Hence the contribution to the return on an individual firm attributable to a particular characteristic is based on the relative value or co-ordinate of that characteristic. Our theoretical model suggests that it is the deviation of accruals from their long-run mean that is relevant for the prediction of stock returns. Thus a hyperplane based on accruals rather than the deviation of accruals from their long-run mean, implies that the projection of accruals on to a pure accrual determined hyperplane, rather than an accrual deviation hyperplane will be dependent on the actual accrual. We can see the effect of this assumption and its implications for HHTb analysis schematically in the following discussion.[28]

Following our arguments as in equation (18), suppose the portion of the return relating to accruals for firm *i* can be written in the following form:

$$r_i = \gamma_1 + \gamma_2(\mu_{a,i} - \frac{ac_i}{B_i}), \tag{23}$$

where γ_1 is the intercept, and $\gamma_2 > 0$ is the slope, $\mu_{a,i}$ is the long-run mean level (relative to book equity) of accruals for firm *i*. At time t, we form a (cross-sectional) portfolio on firms with high

[28] The argument is schematic in that we ignore risk under the assumption that it can be diversified away and concentrate solely on the equilibrium expected values.

values of $(\mu_{a,i} - \frac{ac_i}{B_i})$ firms, i.e., low accrual firms, where the return on this portfolio is $r_L \equiv \gamma_1 + \gamma_2 \left[\bar{\mu}_a - \left(\frac{ac}{B} \right)_L \right]$, where $\bar{\mu}_a$ is the cross-sectional mean. Similarly, we can form a portfolio on firms with a low value of $(\mu_{a,i} - \frac{ac_i}{B_i})$ firms, i.e., high accrual firms, where the return on this portfolio is $r_H = \gamma_1 + \gamma_2 \left[\bar{\mu}_a - \left(\frac{ac}{B} \right)_H \right]$, where $r_L > r_H$.

The high minus low (return) portfolio defines a co-ordinate combination in factor space $r_L - r_H = \gamma_2 \left[\left(\frac{ac}{B} \right)_H - \left(\frac{ac}{B} \right)_L \right]$, which when combined with the risk-free rate defines a line in the factor hyperplane. Hence for any arbitrary portfolio, consistent with our theoretical model we expect the return to take the form

$$r_i = \gamma_1 + \gamma_2 (\mu_{a,i} - \frac{ac_i}{B_i}) = \gamma_1 + \frac{(\mu_{a,i} - \frac{ac_i}{B_i})}{\left[\left(\frac{ac}{B} \right)_H - \left(\frac{ac}{B} \right)_L \right]} (r_L - r_H) . \tag{24}$$

In HHTb, the accrual factor is still based on the difference $(r_L - r_H)$ between portfolios of high and low accruals, but in our context, the accrual factor loading is a function of μ_a and $\frac{ac_i}{B_i}$, and stock return can be rewritten as

$$r_i = \gamma_1 + \frac{\mu_{a,i}}{\gamma} (r_L - r_H) - \frac{1}{\gamma} \frac{ac_i}{B_i} (r_L - r_H) \tag{25}$$

where $\gamma \equiv \left[\left(\frac{ac}{B} \right)_H - \left(\frac{ac}{B} \right)_L \right] > 0$.

We can view $\frac{\mu_i}{\gamma}$ as the accrual risk loading in estimations in HHTb. Therefore, both accruals and long-run means can be useful in explaining stock returns. Thus the evidence in HHTb is in fact consistent with our theoretical predictions and provides empirical support for our modeling. Furthermore, the variation of mean $\mu_{a,i}$ related to $\frac{ac_i}{B_i}$ is small, hence $\frac{ac_i}{B_i}(r_L - r_H)$ is expected to have more power to explain cross-sectional expected returns confirming their findings.

7. Conclusion

In this paper, we develop an analytic model that connects stock returns to fundamental determinants including earnings systematic risk, asset growth, company profitability, the book-to-market ratio and earnings components. By starting from an accounting-based valuation model we avoid some of the problems of traditional factor models that augment the CAPM by adding empirically determined accounting variables. Our approach facilitates an understanding of many empirical conundrums. Notably, we find that asset growth interacts with future earnings yield and book-to-market. Our model indicates a need to treat earnings components and growth of assets as jointly interdependent in asset pricing. This helps to explain why empirical evidence shows a negative relationship between stock returns and investment growth and why accruals can be negatively or positively related to stock returns. It also shows the role of deviation of accrual from its long-run mean in predicting returns and demonstrates why the Hirshleifer et al. (2012) accrual-based risk factor cannot explain the "accrual anomaly".

Throughout the paper, a recurrent theme of our analysis is the distorting effect of the role played by risky growth as potentially a correlated confounding variable. In developing our model, we also cast light on several related theoretical constructs. These include the differential risk of the components of earnings, the concept of certainty equivalent growth rate, and the role of book-to-market in asset pricing. Indeed, our resulting model could be viewed as a nonlinear characteristic model where the characteristics identified include earnings systematic risk, asset growth, profitability and book-to-market. However, in our model we find that book-to-market appears as a pervasive multiplier of other factors. This is because book-to-market serves as a convenient proxy for capturing the conservative nature of accounting transactions and their fluctuations mapping these into their equivalent market valuations. In a similar vein, the concept of certainty equivalent growth rate enables us to overcome the problem in discounting formulations where the nominal growth rate is greater than the risk-free rate.

Existing empirical work often implicitly omit the correlation between risky growth and future accruals or view growth as an independent variable. Our theory implies that the sign and size attached to accruals in its relationship to stock returns not only depends on the choice of control variables but also on the choice of empirical methodology, in particular the choice between time series analysis at an aggregate level and cross-sectional analysis at a firm level. In the case of cross-sectional analysis of the role of current accruals, growth is implicitly assumed to be unrelated to accruals. We show this results in a negative relationship between a firm's future returns and current accruals when accruals are less persistent than cash flows in predicting earnings. In the case of a time series analysis, uncertain future growth is correlated with aggregate current accruals. We demonstrate how this is likely to lead to a positive relationship between aggregate future returns and the scaled aggregate current accruals.

While ours is clearly not an empirical paper, we hope our theoretical analysis provides insights into empirical findings. Like all theoretical papers, we have had to make a number of assumptions among these are assumptions of normality and log-normality to simplify the model development.

We argue that assumptions of normality and log-normality can be considered as robust approximations to assumed distributions used in empirical work. As such we have tested our assumptions numerically by comparing our predictions of returns by carrying out a theoretical replication of cross-sectional and time series approaches. Our aim has been to develop and use our model to explain and understand existing empirical puzzles, not to produce revised estimates. We leave empirical implications arising from our theoretical modeling for future research.

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Appendix A.

To calibrate our theoretical valuation model, we use net income (before extraordinary item) and lagged book values as deflators to estimate parameters in information dynamics (3) and (4). We collect accounting data from Compustat's entire dataset for the US firms from 1963–2019. Data includes total book value (ceq) and earnings (ib). Total operating accruals are computed as the change in non-cash current assets less the change in current liabilities, excluding short-term debt and taxes payable, minus depreciation and amortization expense. Firms with stock price less than 0.5 dollars are deleted. 1% outliners of the scaled earnings and scaled accruals, book-to-market, total accrual-to-market capitalization and total accrual-to-asset are dropped.

A1. Theoretical replication of cross-sectional regressions of stock returns on accruals

Table 1 Panel A reports the sample description of lagged book value deflated earnings and accruals (current and forward), accrual-to-market capitalization, accrual-to-asset, book-to-market and earnings-to-price. The number of observations, mean and standard deviation are reported.

Panel B of Table 1 presents the statistics of the information parameters in dynamics (3) and (4) on a year-by-year basis using the cross-sectional regressions. Parameters α_{11}, α_{12}, α_{21} and α_{22} are estimated based on seemingly unrelated regressions. We assume the certainty equivalent growth rate $\delta_{ce} - 1 = (1/3)r_f$ in each of the 56 years to ensure $\delta_{ce} < 1 + r_f$, where r_f is 10-year US government bond yield as a proxy of risk-free rate. We use the average of one-year ahead log book value growth rate as a proxy of $\bar{\delta}$ in each cross-section. On substitution into equation (19), we get values of the accruals coefficient and earnings coefficient. The statistics for these coefficients, $\bar{\delta}$ and δ_{ce} are also reported.

Table 1: Data for a theoretical replication of cross-sectional regressions of stock returns on accruals

Panel A: Sample statistics

	e_{t+1}/B_t	ac_{t+1}/B_t	e_t/B_{t-1}	ac_t/B_{t-1}	ac_t/MVE_t	ac_t/A_t	B_t/MVE_t	e_t/MVE_t
N	139933	139933	139933	139933	141638	142448	142735	142735
Mean	0.070	-0.101	0.086	-0.088	-0.086	-0.038	0.892	0.036
StDev	0.191	0.184	0.175	0.183	0.378	0.099	1.403	0.154

Panel B: The time series average of annual parameters

	α_{11}	α_{12}	α_{21}	α_{22}	δ_1	δ_{ce}	Accrual coefficient	Earnings coefficient
N	56	56	56	56	56	56	56	56
Mean	0.726	-0.114	0.077	0.335	0.099	1.021	-0.027	0.085
StDev	0.066	0.061	0.093	0.114	0.039	0.009	0.021	0.085
p10	0.645	-0.205	-0.037	0.166	0.051	1.008	-0.051	-0.026
p25	0.668	-0.167	0.004	0.262	0.074	1.014	-0.038	0.020
p50	0.721	-0.123	0.043	0.329	0.102	1.020	-0.024	0.092
p75	0.784	-0.058	0.170	0.405	0.123	1.026	-0.011	0.137
p90	0.817	-0.028	0.199	0.511	0.144	1.034	-0.005	0.200

Notes: ac_t, cf_t and e_t are accruals, cash flows and earnings at time t. A_t is book value of assets at time t. B_t is book value of equity and MVE_t is market value of equity at time t. δ_1 is the one-year ahead log book value growth. $\delta_{ce} - 1 = (1/3)r_f$, where r_f is 10-year US government bond yield.

A2. Theoretical Replication of time series regressions of stock returns on accruals

We first note that the first line in equation (21) can be rewritten as

$$\tilde{\delta}_{t+1} + (1+\mu_e - \tilde{\delta}_{t+1})\frac{B_t}{MVE_t} + (\frac{e_{t+1}}{B_t} - \mu_e)\frac{B_t}{MVE_t} = \tilde{\delta}_{t+1} + \frac{e_{t+1} - (\tilde{\delta}_{t+1} - 1)B_t}{MVE_t} = \tilde{\delta}_{t+1} + \frac{d_{t+1}}{MVE_t}. \qquad (26)$$

Also note future dividends are associated with future growth $\tilde{\delta}_{t+1}$. The remaining terms in equation (22) can be evaluated by applying the Isserlis' theorem.

Isserlis' Theorem: If (X_1, X_2, …X_n) is a zero-mean multivariate normal random vector, then $E[X_1X_2X_3]=0$ and $E[X_1X_2X_3X_4]= E[X_1X_2]E[X_3X_4]+E[X_1X_3]E[X_2X_4]+E[X_1X_4]\, E[X_2X_3]$.

Application of the Isserlis' theorem then gives

$$\begin{aligned}
\text{cov}\left(\frac{ac_t}{B_{t-1}} - \mu_a, \tilde{\delta}_{t+1}(\frac{cf_t}{B_{t-1}} - \mu_c)_t(\frac{B_t}{MVE_t})\right) &= \text{cov}(\tilde{\delta}_{t+1}, \frac{B_t}{MVE_t})cov(\frac{cf_t}{B_{t-1}} - \mu_c, \frac{ac_t}{B_{t-1}} - \mu) \\
&+ c\,\text{ov}(\frac{cf_t}{B_{t-1}} - \mu_c, \frac{B_t}{MVE_t})\,\text{cov}(\tilde{\delta}_{t+1}, \frac{ac_t}{B_{t-1}} - \mu_a) \\
&+ \text{cov}(\tilde{\delta}_{t+1}, \frac{cf_t}{B_{t-1}} - \mu_c)\,\text{cov}(\frac{B_t}{MVE_t}, \frac{ac_t}{B_{t-1}} - \mu_a) \\
&+ \bar{\delta}\overline{(\frac{B_t}{MVE_t})}cov(\frac{cf_t}{B_{t-1}} - \mu_c, \frac{ac_t}{B_{t-1}} - \mu_a).
\end{aligned} \qquad (27)$$

where $\bar{\delta}$ is one plus the average growth rate of book assets, and $\overline{(\frac{B_t}{MVE_t})}$ is the average value of book-to-market.

Similarly, we have

$$\begin{aligned}
Cov(\tilde{\delta}_{t+1}(\frac{ac_t}{B_{t-1}} - \mu_a)(\frac{B_t}{MVE_t}), (\frac{ac_t}{B_{t-1}} - \mu_a)) &= \text{cov}(\tilde{\delta}_{t+1}, \frac{B_t}{MVE_t})\sigma_{ac}^2 \\
&+ 2\,\text{cov}(\frac{B_t}{MVE_t}, \frac{ac_t}{B_{t-1}} - \mu_a)\,\text{cov}(\tilde{\delta}_{t+1}, \frac{ac_t}{B_{t-1}} - \mu_a) + \bar{\delta}\overline{(\frac{B_t}{MVE_t})}\sigma_{ac}^2.
\end{aligned} \qquad (28)$$

In order to evaluate the covariance in equation (28), we report the means and standard deviations of aggregate variables in Table 2 Panel A over sample period 1963-2018. Their correlations are reported in Table 2 Panel B. Consistent with our model development, our earnings are net income. Applying the scaled aggregate earnings and accruals, we use seemingly unrelated regressions to estimate the values of α_{ij} (i, j =1,2) and the long-run means of the scaled earnings, accruals and cash flows: μ_e, μ_a and μ_c. We report these in Table 2 Panel C. We also report β_1 and β_2.

Table 2: Data for a theoretical replication of time series regressions of stock returns on accruals

Panel A: Descriptive statistics of aggregate variables

	Ret1	δ_1	dy	ac_t/B_{t-1}	cf_t/B_{t-1}	e_t/B_{t-1}	B_t/MVE_t	e_t/MVE_t
N	56	56	56	56	56	56	56	56
Mean	0.052	0.091	0.033	-0.090	0.248	0.161	0.571	0.066
SD	0.181	0.045	0.015	0.034	0.034	0.020	0.185	0.029
p50	0.083	0.099	0.030	-0.100	0.259	0.162	0.539	0.058

Panel B: Correlation

	Ret1	δ_1	dy	ac_t/B_{t-1}	cf_t/B_{t-1}	e_t/B_{t-1}	B_t/MVE_t	e_t/MVE_t
Ret1	1							
δ_1	0.230	1						
dy	0.347	0.055	1					
ac_t/B_{t-1}	0.097	0.180	0.486	1				
cf_t/B_{t-1}	-0.115	-0.111	-0.354	-0.824	1.000			
e_t/B_{t-1}	-0.016	0.093	0.170	0.218	0.370	1		
B_t/MVE_t	0.342	0.150	0.930	0.589	-0.520	0.061	1	
e_t/MVE_t	0.308	0.140	0.934	0.555	-0.299	0.389	0.924	1

Panel C: LID parameters and valuation parameters

	α_{11}	α_{12}	μ_e	μ_a	μ_c	β_1	β_2	R_f
	0.490	-0.006	0.149	-0.086	0.235	0.902	-0.034	1.060

Notes: Ret1 is one-year ahead log returns. The 12-month ahead realized returns for all firms in the sample are collected from CRSP. δ_1 is one-year ahead log book value growth. dy is the dividend yield. ac_t, cf_t and e_t are accruals, cash flows and earnings at time t. B_t is book value of equity and MVE_t is market value of equity at time t. Earnings, accruals, cash flow, dividends and book value growth are the market weighted aggregate values. $R_f = 1 + r_f$, where r_f is the average of 10-year US government bond yield over the sample period.

Appendix B:

B1. The discount factor

Assume a constant relative risk aversion (CRRA) utility function: $u(c_t) = c_t^{1-\lambda}/(1-\lambda)$, where λ measures the degree of relative risk aversion. Following Cochrane (2005), the discount factor between $t+\tau-1$ and $t+\tau$, $\tau = 1,2,3...$, can be written as:

$$df_{t+\tau} = \beta \frac{u'(c_{t+\tau})}{u'(c_{t+\tau-1})} = \beta \left(\frac{c_{t+\tau}}{c_{t+\tau-1}} \right)^{-\lambda} , \tag{29}$$

where β captures impatience, a subjective discount factor.

Assume that the logarithmic growth in consumptions follows a random walk with drift,

$$\ln\left(\frac{c_{t+\tau}}{c_{t+\tau-1}} \right) = \text{g} + \varepsilon_{m,t+\tau}, \text{ or } \; c_{t+\tau} = c_{t+\tau-1} e^{g+\varepsilon_{m,t+\tau}},$$

where innovations of consumption growth or market innovations follow a normal distribution, $\varepsilon_{m,t+\tau} \sim N(0, \sigma_m^2)$. Then

$$df_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\lambda} = \beta e^{-\lambda g - \lambda \varepsilon_{m,t+1}} . \tag{30}$$

Now assuming a constant risk-free rate, $r_f \equiv R_f - 1$, we have $E_t[df_{t+1}] = \beta e^{-\lambda g} e^{\lambda^2 \sigma_m^2 / 2}$. Let $E_t[df_{t+1}] = \frac{1}{R_f}$ we have $\beta = \frac{1}{R_f} e^{\lambda g - \lambda^2 \sigma_m^2 / 2}$.

It follows from (30) that

$$df_{t+1} = \frac{1}{R_f} e^{\lambda g - \lambda^2 \sigma_m^2 / 2} e^{-\lambda g - \lambda \varepsilon_{m,t+1}} = \frac{e^{-\lambda^2 \sigma_m^2 / 2}}{R_f} e^{-\lambda \varepsilon_{m,t+1}} . \tag{31}$$

The discounting factor measures investors' impatience and risk aversion.

B2. Conversion ratio: accruals-to-market value vs. accruals-to-total assets

Our return expression (18) is effectively in terms of accrual-to-market value of equity, but empirical studies often use ratio of accrual-to-book value of assets (or equity) as an independent variable in regressions. We need to estimate a conversion ratio when we compare the theoretical coefficient and regression coefficient of accrual terms.

Consider $\tilde{y} = \gamma_0(\tilde{x}_0 - \mu_0) + \tilde{\varepsilon}_0$, where $\tilde{y}, \tilde{x}_0, \tilde{\varepsilon}_0$ represent vectors of observations. Suppose $\tilde{y} = \gamma_1(\tilde{x}_1 - \mu_1) + \tilde{\varepsilon}_1$ and $(\tilde{x}_1 - \mu_1) = S(\tilde{x}_0 - \mu_0)$, where S is a constant. Then we have the relation on two standard deviations: $\sigma(\tilde{x}_1) = S\sigma(\tilde{x}_0)$ and

$$\gamma_1 = \frac{\text{cov}(\tilde{y}, \tilde{x}_1 - \mu_1)}{\sigma^2(\tilde{x}_1 - \mu_1)} = \frac{\rho(\tilde{y}, \tilde{x}_1 - \mu_1)\sigma(\tilde{y})}{\sigma(\tilde{x}_1 - \mu_1)} = \frac{\rho(\tilde{y}, S(\tilde{x}_0 - \mu_0))\sigma(\tilde{y})}{\sigma(S(\tilde{x}_0 - \mu_0))} = \frac{\rho(\tilde{y}, (\tilde{x}_0 - \mu_0))\sigma(\tilde{y})}{S\sigma(\tilde{x}_0 - \mu_0)} = \frac{\gamma_0}{S}.$$

Therefore, we have the following conversion ratio:

$$\frac{\gamma_0}{\gamma_1} = S = \frac{\sigma(\tilde{x}_1)}{\sigma(\tilde{x}_0)}. \quad (32)$$

B3. Evaluation of expectations

We evaluate the covariance risk and summarize them as below:

$$\psi(e,m) \equiv E_t[\varepsilon_{e/B,t+1} e^{-\lambda^2\sigma_m^2/2-\lambda\varepsilon_{m,t+1}}] = e^{-\lambda^2\sigma_m^2/2} E_t[k^e \varepsilon_{m,t+1} e^{-\lambda\varepsilon_{m,t+1}}] = k^e e^{-\lambda^2\sigma_m^2/2} \operatorname{cov}(\varepsilon_{m,t+1}, e^{-\lambda\varepsilon_{m,t+1}}).$$

The Stein's lemma implies $\operatorname{cov}(\varepsilon_{mt+1}, e^{-\lambda\varepsilon_{m,t+1}}) = -\lambda e^{\lambda^2\sigma_m^2/2}\sigma_m^2$.[29] Hence

$$\psi(e,m) = k^e e^{-\lambda^2\sigma_m^2/2}(-\lambda e^{\lambda^2\sigma_m^2/2}\sigma_m^2) = -\lambda k^e \sigma_m^2. \quad (33)$$

Similarly, we have

$$\psi(a,m) \equiv E_t[\varepsilon_{a/B,t+1} e^{-\lambda^2\sigma_m^2/2-\lambda\varepsilon_{m,t+1}}] = -\lambda k^a \sigma_m^2.$$

$$\begin{aligned} \psi(e,\delta,m) &\equiv E_t[\varepsilon_{e/B,t+1} e^{\varepsilon_{\delta,t+1}} e^{-\lambda^2{\sigma_m}^2/2-\lambda\varepsilon_{m,t+1}}] \\ &= e^{-\lambda^2{\sigma_m}^2/2}\{k^e \operatorname{cov}(\varepsilon_{mt+1}, e^{(k^\delta-\lambda)\varepsilon_{m,t+1}}) e^{\sigma_\delta^2/2} + e^{\frac{(k^\delta-\lambda)^2}{2}\sigma_m^2} e^{\sigma_\delta^2/2} \operatorname{cov}(\varepsilon^e, \varepsilon^\delta)\} \\ &= e^{-\lambda^2{\sigma_m}^2/2} e^{\sigma_\delta^2/2} e^{\frac{(k^\delta-\lambda)^2}{2}\sigma_m^2}\{k^e(k^\delta-\lambda)\sigma_m^2 + \operatorname{cov}(\varepsilon^e,\varepsilon^\delta)\} \\ &= -\delta_{ce}\{k^e(\lambda-k^\delta)\sigma_m^2 - \operatorname{cov}(\varepsilon^e,\varepsilon^\delta)\} = -\lambda\delta_{ce}k^e\sigma_m^2 + \delta_{ce}\operatorname{cov}_t(e,\delta), \end{aligned} \quad (34)$$

where $\operatorname{cov}_t(e,\delta) \equiv cov_t(\varepsilon_{e/B,t+1}, \varepsilon_{\delta,t+1}) = k^e k^\delta \sigma_m^2 + cov(\varepsilon^e, \varepsilon^\delta)$.

Similarly, we have

$$\psi(a,\delta,m) \equiv E_t[\varepsilon_{a/B,t+1} e^{\varepsilon_{\delta,t+1}} e^{-\lambda^2{\sigma_m}^2/2-\lambda\varepsilon_{m,t+1}}] = -\lambda\delta_{ce}k^a\sigma_m^2 + \delta_{ce}\operatorname{cov}_t(a,\delta), \quad (35)$$

where $\operatorname{cov}_t(a,\delta) \equiv cov_t(\varepsilon_{a/B,t+1}, \varepsilon_{\delta,t+1}) = k^a k^\delta \sigma_m^2 + cov(\varepsilon^a, \varepsilon^\delta)$.

We also have

$$\begin{aligned} \psi(e,\delta) &\equiv E_t[\varepsilon_{e/B,t+1} e^{\varepsilon_{\delta,t+1}}] = \operatorname{cov}_t(\varepsilon_{e/B,t+1}, e^{\varepsilon_{\delta,t+1}}) = E_t[e^{\varepsilon_{\delta,t+1}}]\operatorname{cov}_t(\varepsilon_{e/B,t+1}, \varepsilon_{\delta,t+1}) \\ &= \bar{\delta}\operatorname{cov}_t(k^e\varepsilon_{mt+1}+\varepsilon^e, k^\delta\varepsilon_{mt+1}+\varepsilon^\delta) = \bar{\delta}[k^e k^\delta {\sigma_m}^2 + \operatorname{cov}_t(\varepsilon^e,\varepsilon^\delta)] = \bar{\delta}\operatorname{cov}_t(e,\delta). \end{aligned} \quad (36)$$

$$\psi(a,\delta) \equiv E_t[\varepsilon_{a/B,t+1} e^{\varepsilon_{\delta,t+1}}] = \bar{\delta}\operatorname{cov}_t(a,\delta). \quad (37)$$

B4. Proofs of propositions

Proof of Proposition 1.

Assume Assumptions 1 and 2. Equation (2), information dynamics (3), (4) and (5) together imply that value of equity can be written in a form as

[29] Stein's Lemma: if Y and Z are bivariate normal random variables and *f(.)* is a differentiable function, then $cov(Y, f(Z)) = E[f'(Z)]cov(Y, Z)$.

$$MVE_t = (1+\beta_0)B_t + \beta_1(\frac{e_t}{B_{t-1}} - \mu_e)B_t + \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)B_t$$
$$+\beta_3 B_t \psi(e,m) + \beta_4 B_t \psi(e,\delta,m) + \beta_5 B_t \psi(a,\delta,m), \tag{38}$$

where $\psi(e,m)$, $\psi(e,\delta,m)$ and $\psi(a,\delta,m)$ are defined in (33), (34) and (35) respectively. We need to identify the valuation multiples $\beta_j (j = 1-5)$.

Equation (38), the clean surplus relation: $B_{t+1} + d_{t+1} = B_t + e_{t+1}$ and equation (3): $e_{t+1} = \mu_e B_t + \alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e)B_t + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)B_t + B_t \varepsilon_{e/B,t+1}$ together imply that

$$MVE_{t+1} + d_{t+1} = B_t + e_{t+1} + \beta_0 B_{t+1} + \beta_1(\frac{e_{t+1}}{B_t} - \mu_e)B_{t+1} + \beta_2(\frac{ac_{t+1}}{B_t} - \mu_a)B_{t+1}$$
$$+\beta_3 B_{t+1}\psi(e,m) + \beta_4 B_{t+1}\psi(e,\delta,m) + \beta_5 B_{t+1}\psi(a,\delta,m)$$
$$= (1+\mu_e)B_t + \beta_0 B_{t+1} + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a) + \varepsilon_{e/B,t+1}](B_t + \beta_1 B_{t+1})$$
$$+\beta_2[\alpha_{21}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{22}(\frac{ac_t}{B_{t-1}} - \mu_a) + \varepsilon_{a/B,t+1}]B_{t+1} + [\beta_3\psi(e,m) + \beta_4\psi(e,\delta,m) + \beta_5\psi(a,\delta,m)]B_{t+1}.$$

It follows that

$$MVE_{t+1} + d_{t+1} = (1+\mu_e)B_t + \beta_0 e^{\varepsilon_{\delta,t+1}} B_t + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]B_t$$
$$+\varepsilon_{e/B,t+1}B_t + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\beta_1 e^{\varepsilon_{\delta,t+1}} B_t + \varepsilon_{e/B,t+1}\beta_1 e^{\varepsilon_{\delta,t+1}} B_t \tag{39}$$
$$+\beta_2[\alpha_{21}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{22}(\frac{ac_t}{B_{t-1}} - \mu_a)]e^{\varepsilon_{\delta,t+1}} B_t + \beta_2 \varepsilon_{a/B,t+1} e^{\varepsilon_{\delta,t+1}} B_t$$
$$+[\beta_3\psi(e,m) + \beta_4\psi(e,\delta,m) + \beta_5\psi(a,\delta,m)]e^{\varepsilon_{\delta,t+1}} B_t.$$

Assumption 1 implies

$$E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}}(MVE_{t+1} + d_{t+1})] = R_f \times MVE_t \ . \tag{40}$$

Note that $E_t[\varepsilon_{m,t+1}] = E_t[\varepsilon_{\delta,t+1}] = E_t[\varepsilon_{a/B,t+1}] = E_t[\varepsilon_{e/B,t+1}] = 0$ and $E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}}] = 1,$ we have

$$(1+\mu_e)B_t + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]B_t + \beta_0 E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}]B_t$$
$$+E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}}\varepsilon_{e/B,t+1}]B_t + [\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\beta_1 E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}]B_t$$
$$+\beta_1 E_t[\varepsilon_{e/B,t+1} e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}]B_t + \beta_2[\alpha_{21}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{22}(\frac{ac_t}{B_{t-1}} - \mu_a)]E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}]B_t$$
$$+\beta_2 E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}}\varepsilon_{a/B,t+1} e^{\varepsilon_{\delta,t+1}}]B_t + [\beta_3\psi(e,m) + \beta_4\psi(e,\delta,m) + \beta_5\psi(a,\delta,m)]E_t[e^{-\lambda^2\sigma_m^2/2 - \lambda\varepsilon_{m,t+1}} e^{\varepsilon_{\delta,t+1}}]B_t$$
$$= R_f \times MVE_t.$$

The above can be written as

$$(1+\mu_e)B_t+[\alpha_{11}(\frac{e_t}{B_{t-1}}-\mu_e)+\alpha_{12}(\frac{ac_t}{B_{t-1}}-\mu_a)]B_t+\beta_0\delta_{ce}B_t$$

$$+\psi(e,m)B_t+[\alpha_{11}(\frac{e_t}{B_{t-1}}-\mu_e)+\alpha_{12}(\frac{ac_t}{B_{t-1}}-\mu_a)]\beta_1\delta_{ce}B_t$$

$$+\beta_1\psi(e,\delta,m)B_t+\beta_2[\alpha_{21}(\frac{e_t}{B_{t-1}}-\mu_e)+\alpha_{22}(\frac{ac_t}{B_{t-1}}-\mu_a)]\delta_{ce}B_t$$

$$+\beta_2\psi(a,\delta,m)B_t+[\beta_3\psi(e,m)+\beta_4\psi(e,\delta,m)+\beta_5\psi(a,\delta,m)]\delta_{ce}B_t$$

$$=R_f\{(1+\beta_0)B_t+\beta_1(\frac{e_t}{B_{t-1}}-\mu_e)B_t+\beta_2(\frac{ac_t}{B_{t-1}}-\mu_a)B_t+[\beta_3\psi(e,m)+\beta_4\psi(e,\delta,m)+\beta_5\psi(a,\delta,m)]B_t\}.$$

Compare coefficients $B_t, (\frac{e_t}{B_{t-1}}-\mu_e)B_t$, $(\frac{ac_t}{B_{t-1}}-\mu_a)B_t, \psi(e,m)B_t, \psi(e,\delta,m)B_t$, and $\psi(a,\delta,m)B_t$ in the above equation, we have

$$\begin{aligned}
&1+\mu_e+\beta_0\delta_{ce}=R_f(1+\beta_0),\\
&\alpha_{11}+\alpha_{11}\beta_1\delta_{ce}+\beta_2\alpha_{21}\delta_{ce}=R_f\beta_1,\\
&\alpha_{12}+\alpha_{12}\beta_1\delta_{ce}+\beta_2\alpha_{22}\delta_{ce}=R_f\beta_2,\\
&1+\beta_3\delta_{ce}=R_f\beta_3,\\
&\beta_1+\beta_4\delta_{ce}=R_f\beta_4,\\
&\beta_2+\beta_5\delta_{ce}=R_f\beta_5.
\end{aligned} \tag{41}$$

Solving the above equation system, we have

$$\beta_0=\frac{\mu_e-(R_f-1)}{(R_f-\delta_{ce})},\ \beta_1=\frac{\alpha_{11}(R_f-\delta_{ce}\alpha_{22})+\delta_{ce}\alpha_{12}\alpha_{21}}{\Delta},\ \beta_2=\frac{\alpha_{12}R_f}{\Delta},\ \beta_3=\frac{1}{R_f-\delta_{ce}},$$

$$\beta_4=\frac{\beta_1}{R_f-\delta_{ce}},\beta_5=\frac{\beta_2}{R_f-\delta_{ce}},\ \text{where}\ \ \Delta\equiv(R_f-\delta_{ce}\alpha_{11})(R_f-\delta_{ce}\alpha_{22})-\alpha_{12}\alpha_{21}(\delta_{ce})^2>0.$$

Therefore, we have

$$\begin{aligned}
MVE_t=&\frac{1+\mu_e-\delta_{ce}}{R_f-\delta_{ce}}B_t+\beta_1(\frac{e_t}{B_{t-1}}-\mu_e)B_t+\beta_2(\frac{ac_t}{B_{t-1}}-\mu_a)B_t\\
&+\frac{B_t}{R_f-\delta_{ce}}[\psi(e,m)+\beta_1\psi(e,\delta,m)+\beta_2\psi(a,\delta,m)]
\end{aligned} \tag{42}$$

From equations (33), (34) and (35) , we have

$$MVE_t=(1+\beta_0)B_t+\beta_1(\frac{e_t}{B_{t-1}}-\mu_e)B_t+\beta_2(\frac{ac_t}{B_{t-1}}-\mu_a)B_t+Growthadj_t-Riskadj_t,$$

where $Growthadj_t=\frac{B_t\delta_{ce}}{R_f-\delta_{ce}}[(\beta_1k^e+\beta_2k^a)k^\delta\sigma_m^2+\beta_1cov(\varepsilon^e,\varepsilon^\delta)+\beta_2cov(\varepsilon^a,\varepsilon^\delta)]$, and

$$Riskadj_t=\frac{B_t}{R_f-\delta_{ce}}\lambda\sigma_m^2[(1+\beta_1\delta_{ce})k^e+\beta_2\delta_{ce}k^a].$$

Proof of Lemma 1.

From Proposition 1, the sum of the growth adjustment and risk adjustment terms can be written as:

$$\begin{aligned} & MVE_t - (1+\beta_0)B_t - \beta_1(\frac{e_t}{B_{t-1}} - \mu_e)B_t - \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)B_t \\ & = -[\lambda\sigma_m^2(k^e + \beta_1\delta_{ce}k^e + \beta_2\delta_{ce}k^a) - \beta_1\delta_{ce}\operatorname{cov}(e,\delta) - \beta_2\delta_{ce}\operatorname{cov}(a,\delta)]\frac{B_t}{R_f - \delta_{ce}}. \end{aligned} \tag{43}$$

It follows from (39), we have

$$\begin{aligned} & MVE_{t+1} + d_{t+1} = MVE_t e^{\varepsilon_{\delta,t+1}} + (1+\mu_e - e^{\varepsilon_{\delta,t+1}})B_t + [\alpha_{11} + (\alpha_{11}-1)\beta_1 e^{\varepsilon_{\delta,t+1}} + \alpha_{21}\beta_2 e^{\varepsilon_{\delta,t+1}}](\frac{e_t}{B_{t-1}} - \mu_e)B_t \\ & +[\alpha_{12} + \alpha_{12}\beta_1 e^{\varepsilon_{\delta,t+1}} + (\alpha_{22}-1)\beta_2 e^{\varepsilon_{\delta,t+1}}](\frac{ac_t}{B_{t-1}} - \mu_a)B_t + \varepsilon_{e/B,t+1}B_t + \varepsilon_{e/B,t+1}\beta_1 e^{\varepsilon_{\delta,t+1}}B_t + \beta_2\varepsilon_{a/B,t+1}e^{\varepsilon_{\delta,t+1}}B_t. \end{aligned} \tag{44}$$

Note equation system (41) gives

$\alpha_{11} + (\alpha_{11}-1)\beta_1 e^{\varepsilon_{\delta,t+1}} + \alpha_{21}\beta_2 e^{\varepsilon_{\delta,t+1}} = \alpha_{11} + \dfrac{(R_f - \delta_{ce})\beta_1 - \alpha_{11}}{\delta_{ce}} e^{\varepsilon_{\delta,t+1}}$, and

$\alpha_{12} + \alpha_{12}\beta_1 e^{\varepsilon_{\delta,t+1}} + (\alpha_{22}-1)\beta_2 e^{\varepsilon_{\delta,t+1}} = \alpha_{12} + \dfrac{(R_f - \delta_{ce})\beta_2 - \alpha_{12}}{\delta_{ce}} e^{\varepsilon_{\delta,t+1}}$.

Therefore, it follows that

$$\begin{aligned} \frac{MVE_{t+1} + d_{t+1}}{MVE_t} = {} & e^{\varepsilon_{\delta,t+1}} + (1+\mu_e - e^{\varepsilon_{\delta,t+1}})\frac{B_t}{MVE_t} + [\alpha_{11} + \frac{(R_f - \delta_{ce})\beta_1 - \alpha_{11}}{\delta_{ce}} e^{\varepsilon_{\delta,t+1}}](\frac{e_t}{B_{t-1}} - \mu_e)\frac{B_t}{MVE_t} \\ & +[\alpha_{12} + \frac{(R_f - \delta_{ce})\beta_2 - \alpha_{12}}{\delta_{ce}} e^{\varepsilon_{\delta,t+1}}](\frac{ac_t}{B_{t-1}} - \mu_a)\frac{B_t}{MVE_t} + \varepsilon_{e/B,t+1}\frac{B_t}{MVE_t} \\ & +\beta_1\varepsilon_{e/B,t+1}e^{\varepsilon_{\delta,t+1}}\frac{B_t}{MVE_t} + \beta_2\varepsilon_{a/B,t+1}e^{\varepsilon_{\delta,t+1}}\frac{B_t}{MVE_t}. \end{aligned} \tag{45}$$

or

$$\begin{aligned} R_{t+1} = {} & e^{\varepsilon_{\delta,t+1}} + \frac{B_t}{MVE_t} + [\frac{e_{t+1}}{B_t} - \varepsilon_{e/B,t+1}]\frac{B_t}{MVE_t} - e^{\varepsilon_{\delta,t+1}}\frac{B_t}{MVE_t} \\ & + \frac{R_f - \delta_{ce}}{\delta_{ce}} e^{\varepsilon_{\delta,t+1}}[\beta_1(\frac{e_t}{B_{t-1}} - \mu_e) + \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t} \\ & - \frac{e^{\varepsilon_{\delta,t+1}}}{\delta_{ce}}[\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t} + \varepsilon_{e/B,t+1}\frac{B_t}{MVE_t} \\ & +\beta_1\varepsilon_{e/B,t+1}e^{\varepsilon_{\delta,t+1}}\frac{B_t}{MVE_t} + \beta_2\varepsilon_{a/B,t+1}e^{\varepsilon_{\delta,t+1}}\frac{B_t}{MVE_t}. \end{aligned} \tag{46}$$

Note equation (42) implies that

$$\beta_1(\frac{e_t}{B_{t-1}}-\mu_e)B_t+\beta_2(\frac{ac_t}{B_{t-1}}-\mu_a)B_t = MVE_t-\frac{1+\mu_e-\delta_{ce}}{R_f-\delta_{ce}}B_t$$

$$-\frac{B_t}{R_f-\delta_{ce}}[\psi(e,m)+\beta_1\psi(e,\delta,m)+\beta_2\psi(a,\delta,m)].$$

By reorganizing terms, (46) gives

$$R_{t+1}=R_f+\frac{(e^{\varepsilon_{\delta,t+1}}-\delta_{ce})}{\delta_{ce}}\{R_f-(1+\frac{e_{t+1}}{B_t})\frac{B_t}{MVE_t}\}$$

$$+\{-\frac{e^{\varepsilon_{\delta,t+1}}}{\delta_{ce}}(\psi(e,m)+\beta_1\psi(e,\delta,m)+\beta_2\psi(a,\delta,m))+\frac{e^{\varepsilon_{\delta,t+1}}}{\delta_{ce}}\varepsilon_{e/B,t+1}+\beta_1\varepsilon_{e/B,t+1}e^{\varepsilon_{\delta,t+1}}+\beta_2\varepsilon_{a/B,t+1}e^{\varepsilon_{\delta,t+1}}\}\frac{B_t}{MVE_t},$$

or

$$R_{t+1}=R_f+\frac{(e^{\varepsilon_{\delta,t+1}}-\delta_{ce})}{\delta_{ce}}\{R_f-(1+\frac{e_{t+1}}{B_t})\frac{B_t}{MVE_t}\}+e^{\varepsilon_{\delta,t+1}}\{\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}}+\beta_1k^e+\beta_2k^a]+\frac{1}{\delta_{ce}}\varepsilon_{e/B,t+1}$$

$$+\beta_1[\varepsilon_{e/B,t+1}-\mathrm{cov}_t(\varepsilon_{e/B,t+1},\varepsilon_{\delta,t+1})]+\beta_2[\varepsilon_{a/B,t+1}-\mathrm{cov}_t(\varepsilon_{a/B,t+1},\varepsilon_{\delta,t+1})]\}\frac{B_t}{MVE_t}.$$

This is equation (10), where $\delta_{t+1}=e^{\varepsilon_{\delta,t+1}}$.

Proof of Proposition 2.

Taking expectation on both sides of (10) and note $E_t[e^{\varepsilon_{\delta,t+1}}]\equiv\overline{\delta}$, we have

$$E_t[R_{t+1}]=R_f+E_t[\frac{(e^{\varepsilon_{\delta,t+1}}-\delta_{ce})}{\delta_{ce}}\{R_f-(1+\frac{e_{t+1}}{B_t})\frac{B_t}{MVE_t}\}]+E_t[\{e^{\varepsilon_{\delta,t+1}}\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}}+\beta_1k^e+\beta_2k^a]+\frac{e^{\varepsilon_{\delta,t+1}}\varepsilon_{e/B,t+1}}{\delta_{ce}}$$

$$+\beta_1e^{\varepsilon_{\delta,t+1}}[\varepsilon_{e/B,t+1}-\mathrm{cov}_t(\varepsilon_{e/B,t+1},\varepsilon_{\delta,t+1})]+\beta_2e^{\varepsilon_{\delta,t+1}}[\varepsilon_{a/B,t+1}-\mathrm{cov}_t(\varepsilon_{a/B,t+1},\varepsilon_{\delta,t+1})]\}]\frac{B_t}{MVE_t}$$

$$=R_f+\frac{\overline{\delta}-\delta_{ce}}{\delta_{ce}}[R_f-(1+\frac{E_t[e_{t+1}]}{B_t})\frac{B_t}{MVE_t}]-\frac{1}{\delta_{ce}}\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{e/B,t+1})\frac{B_t}{MVE_t}$$

$$+\{\overline{\delta}\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}}+\beta_1k^e+\beta_2k^a]+\frac{1}{\delta_{ce}}\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{e/B,t+1})+\beta_1\,\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{e/B,t+1})-\beta_1\overline{\delta}\,\mathrm{cov}_t(\varepsilon_{e/B,t+1},\varepsilon_{\delta,t+1})$$

$$+\beta_2\,\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{a/B,t+1})-\beta_2\overline{\delta}\,\mathrm{cov}_t(\varepsilon_{a/B,t+1},\varepsilon_{\delta,t+1})\}\frac{B_t}{MVE_t}$$

$$=R_f+\frac{\overline{\delta}-\delta_{ce}}{\delta_{ce}}[R_f-(1+\frac{E_t[e_{t+1}]}{B_t})\frac{B_t}{MVE_t}]-\frac{1}{\delta_{ce}}\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{e/B,t+1})\frac{B_t}{MVE_t}$$

$$+\{\overline{\delta}\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}}+\beta_1k^e+\beta_2k^a]+\frac{1}{\delta_{ce}}\mathrm{cov}(e^{\varepsilon_{\delta,t+1}},\varepsilon_{e/B,t+1})\}\frac{B_t}{MVE_t}$$

$$=R_f+\frac{\overline{\delta}-\delta_{ce}}{\delta_{ce}}[R_f-(1+\frac{E_t[e_{t+1}]}{B_t})\frac{B_t}{MVE_t}]+\overline{\delta}\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}}+\beta_1k^e+\beta_2k^a]\frac{B_t}{MVE_t}.$$

It can be reorganized as

$$E_t[R_{t+1}] = R_f + \frac{\bar{\delta} - \delta_{ce}}{\delta_{ce}}[R_f + MRP_t - (1 + \frac{E_t[e_{t+1}]}{B_t})\frac{B_t}{MVE_t}]$$

$$+\bar{\delta}\lambda\sigma_m^2[\frac{k^e}{\delta_{ce}} + \beta_1 k^e + \beta_2 k^a]\frac{B_t}{MVE_t} - \frac{\bar{\delta} - \delta_{ce}}{\delta_{ce}}[\lambda\sigma_m^2[(1+\beta_1\delta_{ce})k^c + (1+(\beta_1+\beta_2)\delta_{ce})k^a]\frac{B_t}{MVE_t}]$$

$$= R_f + \lambda\sigma_m^2[(1+\beta_1\delta_{ce})k^e + \beta_2\delta_{ce})k^a]\frac{B_t}{MVE_t} + \frac{\bar{\delta} - \delta_{ce}}{\delta_{ce}}[R_f + MRP_t - (1 + \frac{E_t[e_{t+1}]}{B_t})\frac{B_t}{MVE_t}].$$

This is equation (11), where

$$MRP_{i,t} = \lambda\sigma_m^2[(1+\beta_{i,1}\delta_{i,ce})k_i^c + (1+(\beta_{i,1}+\beta_{i,2})\delta_{i,ce})k_i^a]\frac{B_{i,t}}{MVE_{i,t}}, \text{ and}$$

$$GRP_{i,t} = \frac{(\bar{\delta}_i - \delta_{i,ce})}{\delta_{i,ce}}[R_f + MRP_{i,t} - (1 + E_t[\frac{e_{i,t+1}}{B_{i,t}}])\frac{B_{i,t}}{MVE_{i,t}}]$$

Alternatively, from equation (3), we have

$$GRP_t = \frac{(\bar{\delta} - \delta_{ce})}{\delta_{ce}}[R_f + MRP_t - (1+\mu_e)\frac{B_t}{MVE_t} - \alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e)\frac{B_t}{MVE_t} - \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)\frac{B_t}{MVE_t}].$$

Proof of Corollary 1.

Note that equation (17) in Corollary 1 is equation (45) in different form.

Taking expectation on both sides of equation (45), and applying equations (36) and (37), we have equation (18) in Corollary 1.

Proof of equation (20).

From equation (46), we have

$$R_{t+1} = (1+\mu_e)\frac{B_t}{MVE_t} + \tilde{\delta}_{t+1}(1 - \frac{B_t}{MVE_t}) + (1 - \frac{\tilde{\delta}_{t+1}}{\delta_{ce}})(\frac{e_{t+1}}{B_t} - \mu_e)\frac{B_t}{MVE_t}$$

$$+\frac{R_f - \delta_{ce}}{\delta_{ce}}\tilde{\delta}_{t+1}[\beta_1(\frac{e_t}{B_{t-1}} - \mu_e) + \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t}$$

$$+(\frac{1}{\delta_{ce}} + \beta_1)\tilde{\delta}_{t+1}[\frac{e_{t+1}}{B_t} - \mu_e - \alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) - \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t} + \beta_2\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t}.$$

Reorganizing terms, we have

$$R_{t+1} = (1+\mu_e)\frac{B_t}{MVE_t} + \tilde{\delta}_{t+1}(1 - \frac{B_t}{MVE_t}) + (1 + \beta_1\tilde{\delta}_{t+1})(\frac{e_{t+1}}{B_t} - \mu_e)\frac{B_t}{MVE_t}$$

$$+\{[\frac{R_f - \delta_{ce}}{\delta_{ce}}\beta_1 - (\frac{1}{\delta_{ce}} + \beta_1)\alpha_{11}](\frac{e_t}{B_{t-1}} - \mu_e) + [\frac{R_f - \delta_{ce}}{\delta_{ce}}\beta_2 - (\frac{1}{\delta_{ce}} + \beta_1)\alpha_{12}](\frac{ac_t}{B_{t-1}} - \mu_a)\}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t}$$

$$+\beta_2\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}\frac{B_t}{MVE_t}.$$

Taking expectation, we have

$$E_t[R_{t+1}] = (1+\mu_e)\frac{B_t}{MVE_t} + \bar{\delta}(1-\frac{B_t}{MVE_t}) + (1+\beta_1\bar{\delta})(\frac{E_t[e_{t+1}]}{B_t} - \mu_e)\frac{B_t}{MVE_t} + \beta_1 \operatorname{cov}_t(\tilde{\delta}_{t+1}, \frac{e_{t+1}}{B_t})\frac{B_t}{MVE_t}$$
$$+\{[\frac{R_f - \delta_{ce}}{\delta_{ce}}\beta_1 - (\frac{1}{\delta_{ce}} + \beta_1)\alpha_{11}](\frac{e_t}{B_{t-1}} - \mu_e) + [\frac{R_f - \delta_{ce}}{\delta_{ce}}\beta_2 - (\frac{1}{\delta_{ce}} + \beta_1)\alpha_{12}](\frac{ac_t}{B_{t-1}} - \mu_a)\}\bar{\delta}\frac{B_t}{MVE_t}$$
$$+\beta_2 E_t[\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}]\frac{B_t}{MVE_t}.$$

Subtracting the above two and reorganizing terms, we have

$$R_{t+1} - E_t[R_{t+1}] = (1+\beta_1\tilde{\delta}_{t+1})\frac{e_{t+1} - E_t[e_{t+1}]}{MVE_t} - \beta_1 \operatorname{cov}_t(\tilde{\delta}_{t+1}, \frac{e_{t+1}}{MVE_t})$$
$$+(\tilde{\delta}_{t+1} - \bar{\delta})\{1 - \frac{B_t}{MVE_t} + \beta_1(\frac{E_t[e_{t+1}]}{B_t} - \mu_e)\frac{B_t}{MVE_t} - (\frac{1}{\delta_{ce}} + \beta_1)[\alpha_{11}(\frac{e_t}{B_{t-1}} - \mu_e) + \alpha_{12}(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t}$$
$$+\frac{R_f - \delta_{ce}}{\delta_{ce}}[\beta_1(\frac{e_t}{B_{t-1}} - \mu_e) + \beta_2(\frac{ac_t}{B_{t-1}} - \mu_a)]\frac{B_t}{MVE_t}\} + \beta_2[\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1} - E_t[\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}]]\frac{B_t}{MVE_t}.$$

Applying equation (15), we have

$$R_{t+1} - E_t[R_{t+1}] = (1+\beta_1\tilde{\delta}_{t+1})\frac{e_{t+1} - E_t[e_{t+1}]}{MVE_t} - \beta_1\frac{\operatorname{cov}_t[\tilde{\delta}_{t+1}, e_{t+1}]}{MVE_t} + (\tilde{\delta}_{t+1} - \bar{\delta})\{\frac{MVE_t - B_t}{B_t}$$
$$-\frac{1}{\delta_{ce}}((\alpha_{11} - \beta_1(R_f - \delta_{ce}))(\frac{e_t}{B_{t-1}} - \mu_e) + (\alpha_{12} - \beta_2(R_f - \delta_{ce}))(\frac{ac_t}{B_{t-1}} - \mu_a))\}\frac{B_t}{MVE_t}$$
$$+\beta_2\{\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1} - E_t[\varepsilon_{a/B,t+1}\tilde{\delta}_{t+1}]\}\frac{B_t}{MVE_t}.$$