Reconciling Macroeconomics and Finance for the U.S. Corporate Sector: 1929 - Present*

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Abstract

We examine how to reconcile, quantitatively, the high volatility of market valuations of U.S. corporations with the relative stability of macroeconomic quantities since 1929. Macroeconomic and financial variables are measured in a consistent fashion using the Integrated Macroeconomic Accounts (IMA) of the United States. We first use a finance-style valuation model that builds on Campbell and Shiller (1987) to interpret fluctuations in the market value of U.S. corporations from 1929 to 2023 using these IMA data. We find that fluctuations in expected cash flows to firm owners have been the dominant driver of those fluctuations in value, with fluctuations in expected rates of return playing a smaller role. We then develop a stochastic growth model, extended to incorporate factorless income, which we use to decompose corporate cash flows and associated valuations into income and value due to physical capital and factorless income. Finally, we ask whether expected returns to investing in capital in our macroeconomic model are consistent with the series for expected returns estimated from our finance-style valuation model. We find that they are. In this sense, we reconcile volatile market valuations and stable capital output ratios.

JEL Classification Numbers: E44, G12

Key Words: Valuation, Free Cash Flow, Investment, Integrated Macroeconomic Accounts, Tobin's Q

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1 Introduction

How can the volatile market valuations of U.S. corporations manifest in public equity markets be reconciled with the relatively smooth evolution of most macroeconomic variables observed in data from the National Income and Product Accounts (NIPA)? There are two reasons why a reconciliation is challenging.

First, the most basic stochastic growth model offers the prediction that the market valuation of U.S. corporations should coincide in equilibrium with the quantity of capital owned by those corporations. But the data on the market valuation of U.S. corporations and their measured holdings of capital, at least since World War II, are far from this simple model benchmark, indicating that the simplest macro models cannot explain volatile valuations.[1]

Second, reasearch based on finance-style valuation models going back to Shiller (1981), Campbell and Shiller (1987) and Campbell and Shiller (1988) leads to the view that volatile valuations primarily reflect volatile fluctuations in expected returns required by investors. That explanation poses a challenge for macro models. If required returns for firm owners are very volatile, why does that not translate into a volatile capital stock, with investment surging dramatically when required returns are temporarily low? (See, for example, Section 4 of Cochrane 2017).

Our goal in this paper is to build a macroeconomic model that can simultaneously generate (i) volatile valuations, (ii) a relatively stable capital stock, and (iii) expected returns to investment in measured capital that are consistent with those emerging from a conventional asset pricing model. It is in this sense that we aim to reconcile macroeconomics and finance.

One dimension in which our work differs from the prior literature is in the data used. We will exploit a new unified data set known as the Integrated Macroeconomic Accounts (henceforth IMA), which has been developed as a joint project between the Bureau of Economic Analysis and the Federal Reserve Board. The IMA data combines NIPA data on macroeconomic flows and stocks with comprehensive data drawn from the Financial Accounts of the United States on financial flows and balance sheets with equity measured at market value.[2] These data are of great use to us because they offer an internally consistent set of income statements, cash flow statements, and market value balance sheets for major sectors of the U.S. economy. In this paper, we focus on these data for the U.S. corporate sector.

We will explore these IMA data through two lenses. First, we explore valuations of the corporate sector through a relatively standard reduced-form asset pricing model in the style

[1] We discuss below that in the data from 1929 through World War II, valuation data appear to be much closer to this simple macro benchmark.

[2] The Financial Accounts of the United States produced by the Federal Reserve Board were formerly known as the Flow of Funds. See Cagetti et al. (2013) for an introduction to the construction of the IMA data set.

of Campbell and Shiller (1987) and Campbell and Kyle (1993). Here we use the IMA data on the market valuation of U.S. corporations and the cash flows available to owners of these corporations to estimate the sequence of expected returns and expected future cash flows on the U.S. corporate sector as a whole needed to account for the aggregate market valuation of that sector, year-by-year for the period 1929-2023.[3]

Second, we consider a macro model which is an modification of the standard stochastic growth model in that we assume that firms face a time-varying wedge between total revenue and total costs that leads to a pure rent for firm owners that we refer to as *factorless income* following Karabarbounis and Neiman (2019). The valuation of this factorless income drives a gap in the model between the enterprise value of the corporate sector and the stock of measured capital held by firms in that sector.[4]

If firm owners are directing investment decisions, then the expected returns that our asset pricing model identifies as being consistent with observed valuations and cash flows should be closely connected to the expected returns that drive capital investment. We will ask whether we can reconcile the observed path for investment in the IMA data within our macro model over the period 1929 through 2023 when the path for the expected rate of return to those investments is exactly the one we identify from our asset pricing model estimated on the same IMA data. We find that the answer to this question is yes. Thus, we see our model as offering a reconciliation of observed market valuations of the U.S. corporate sector and observed investments in measured capital over the period 1929-2023.

How is this reconciliation achieved? Largely through the finding that the IMA data offer a different picture of the drivers of the volatile valuation of the U.S. corporate sector than is offered by standard public firm data on price per share and dividends per share for the aggregate stock market.[5] We find with IMA data that our finance-style valuation exercise attributes most of the fluctuations in the valuation of the corporate sector relative to corporate output to low-frequency fluctuations in the share of corporate output flowing to firm owners, with only modest variation over time in expected returns.[6] In particular, the valuation to cash flow spread defined as in Campbell and Shiller (1987) but applied to IMA data strongly predicts future IMA cash flow growth, but much more weakly predicts future

[3]We use a linear valuation model rather than the log-linear valuation framework of Campbell and Shiller (1988) to allow for negative cash flows.

[4]Our model also features an explicit model of corporate taxes which have important time-varying impacts on after-tax cash flows and valuations.

[5]See Campbell (2018) Chapter 5 for a summary of prior work with public firm data.

[6]In this regard, our results confirm prior findings by Larraine and Yogo (2008). See also Davydiuk et al. (2023). We go beyond these papers to examine whether observed capital stocks can be rationalized at the expected returns found from the finance-style valuation exercise. We see the question of exactly why one would find different drivers of volatile valuations with different data sets both consistent with the same annualized returns as an important area for further research.

returns. We obtain these findings despite the fact that the time series for realized annual returns on claims to the corporate sector in the IMA data are close to those found with the CRSP Value Weighted Total Market Index.

Once we estimate a time series of annual expected returns to aggregate claims on the U.S. corporate sector in our finance-style valuation exercise, we then use our macro model to ask whether this series for expected returns is consistent with observed aggregate investment in measured capital. In our modified stochastic growth model, investment is governed by a standard capital Euler equation. The expected return on capital investment will depend on a range of factors. A key one is the quantity of capital actually put in place, which we can measure in our IMA data. Other factors include expected tax rates and depreciation rates and the expected future wedge between revenue and cost, which we model in a simple way. One important latent driver of expected returns is the expected growth rate of productivity, or equivalently value-added.

We use the macro model to make the following calculation. In every year from 1929 through 2023, we compute the the expected growth in value-added that is required to equate the expected return to investment in measured capital in our macroeconomic model with the expected return to enterprise value from our asset pricing model, when investment (and capital) are identical to their observed values in the IMA data. Our principal finding is that this exercise delivers a path for expected growth in value-added that appears quite plausible. Based on this finding, we conclude that volatile valuations for the corporate sector as a whole are consistent with a relatively stable capital stock. The key to this reconciliation of macroeconomics with finance is that we attribute most of the fluctuations in valuations to fluctuations in the share of corporate output flowing to owners of corporations in the form of factorless income, with only a modest role for fluctuations in expected returns. These modest fluctuations in expected returns do not do violence to the dynamics of investment or the capital stock in our macroeconomic model. At the same time our macroeconomic model generates large and volatile fluctuations in the valuation of the corporate sector relative to its stock of capital driven by shocks to the share of corporate output that flows to firm owners as a pure rent.

Our paper is organized as follows.

In Section 2, we place our work in context of a large prior literature on this topic.

In Section 3, we use the IMA data to construct measures of aggregate cash flows to firm owners and firm valuation consistent with the definition of these concepts in a standard stochastic growth model. We then demonstrate that the realized annual returns for the U.S. corporate sector over the period 1929-present constructed using these macro measures of cash flow and corporate value look remarkably similar to measures of returns obtained from the

CRSP Value-Weighted Total Stock Market Index. Based on these observations we argue that the IMA are a useful unified data set for macrofinance.

In Section 4, we conduct our finance-style valuation exercise following the framework laid out in Campbell and Shiller (1987) and Campbell and Kyle (1993) but using the IMA data on valuations and cash flows.

In Section 5 we present the modification of the standard stochastic growth model that we use as a quantitative accounting framework to connect standard macroeconomic flows and stock to financial measures of valuations and returns.

In Section 6 we conduct our macro-model based valuation exercise evaluating whether the series of expected returns that we estimate in Section 4 are consistent with the capital-Euler equation of our macro model.

Finally, in Section 7, we conclude.

2 Related Literature

We aim to simultaneously account for the observed fluctuations in the value of U.S. corporations year-by-year since 1929 and for measured U.S. corporate holdings of capital and investment in capital over this time period.

As discussed in Gomme, Ravikumar, and Rupert (2011) and Cochrane (2017), and as implied by the work of Tobin (1969) and Hayashi (1982), one of the challenges to accounting for the volatility of market valuations and returns of U.S. corporations together with the relatively smooth data on measured capital-output ratios and accounting returns to capital in a standard stochastic growth model is that, in such a model, the value of firms is always equal to the value of their installed capital. In our model, this link is broken by the introduction of factorless income and considerations of corporate taxation.

In contrast, much of the macro-finance literature has taken a different approach, relying on time-variation in the risk premium on investment in the capital stock together with adjustment costs for that investment to reconcile the high volatility of corporate valuations and the relatively smooth evolution of the stock of measured capital. Jermann (1998), Gourio (2012), Ilut and Schneider (2014), Basu and Bundick (2017), Hall (2017), Cambell, Pflueger, and Viceira (2020), and Basu et al. (2023) are examples of stochastic growth models with time-varying risk premia on capital arising from a variety of different sources. See also Cochrane (1991), Merz and Yashiv (2007), Philippon (2009), and Jermann (2010). Here we do not include adjustment costs to investment to keep our model as parsimonious as possible.

We depart from this literature in accounting for much of the volatility of corporate valuations based on a model of fluctuations in expected cash flows to owners of firms rather than variation in discount rates.[7] To provide such an accounting, our model combines two key ingredients. First, we include a time-varying wedge between corporate revenue and costs that generates factorless income, as in Karabarbounis and Neiman (2019). Second, we use a model of the impact of rates of return and taxes on the cash flow to owners of capital and the valuation of that cash flow based on the framework of Hall and Jorgenson (1967). We model taxes in a similar way to Gravelle (1994), Gravelle (2006), and Barro and Furman (2018). As in McGrattan and Prescott (2005) and McGrattan (2023) we find that taxes play an important role in shaping our model's implications for the valuation of and marginal returns to measured capital.[8]

In our accounting of the data since World War II, the high volatility in the valuation of U.S. corporations is driven primarily by shifts in investors' expectations of the share of factorless income in corporate output in the long run. Investment in measured capital, on the other hand, is driven by more near-term considerations such as one-year interest rates, growth rates, corporate tax rates, depreciation rates, and changes in the relative price of capital goods. Thus, investment and measures of Tobin's Q are only weakly connected in our model (see Abel and Eberly (2012) for a related argument).[9]

In that vein, our accounting for the stability of the ratio of physical capital to output in the face of falling risk free interest rates, changing tax rates, and rising depreciation rates in the past several decades is related to that in Gutiérrez and Philippon (2017), Crouzet and Eberly (2018), and Crouzet and Eberly (2023). That is, on the one hand, a declining share of income accruing to physical capital coupled with higher average depreciation have been important forces tending to depress investment. But these forces in our accounting have been offset by declining corporate tax rates and declining expected returns. Thus, we see this stable capital to output outcome as largely coincidental rather than some fundamental property of the model. In particular, we note that we do see large changes in the ratio of measured capital to corporate gross value added in the data prior to World War II. Moreover, these large changes in the capital-output ratio accounted for large changes in both the ratios of enterprise value and free cash flow to gross value added in that time period.

Our focus on shocks to current and future factorless income is closely related to the arguments of Lustig and Van Nieuwerburgh (2008) and Greenwald, Lettau, and Ludvigson (2023)

[7]We pursue this theme further in a companion paper Atkeson, Heathcote, and Perri (2024) that models the volatility of stock prices based on fluctuations in expected cash flows using CRSP data on price per share and dividends per share.

[8]As of yet, we have not included consideration of the impact of taxes on corporate distributions on the level of corporate valuations as in McGrattan and Prescott (2005) and McGrattan (2023). We plan to do so going forward.

[9]We note that our findings about the volatility of cash flows to owners of U.S. corporations differ from the findings in Hall (2003). We believe that this is due to our use of free cash flow rather than EBITDA and to the time periods considered.

that shocks to the distribution of income between workers and owners of firms have been an important driver of fluctuations in the valuation of U.S. corporations. Our principal contribution relative to these papers is to add consideration of physical capital and investment. We follow a large recent literature in macro-finance that builds on these ideas. See, for example, Caballero, Farhi, and Gourinchas (2017), Farhi and Gourio (2018), Philippon (2019), and Eggertsson, Robbins, and Wold (2021). With the notable exception of Crouzet and Eberly (2023), these papers do not account year-by-year for both corporate valuations and changes in capital investment over a long time period.

In building our accounting model, we make the stark assumption that the production function relating measured capital and labor to aggregate output has remained stable over the past 100 years. It is this assumption that allows us, through the model, to measure the share of factorless income in corporate gross value added year-by-year from data on tax rates and the share of labor compensation in corporate gross value added. In this regard, our work is closely related to recent work by Barkai (2020) and Karabarbounis and Neiman (2019) who both use a Hall and Jorgenson (1967) style measurement framework to estimate the rental rate on measured capital and the corresponding share of rental income on measured capital in gross value added. This prior work differs from ours in that it starts with data on risk free rates and an estimate of the risk premium on capital to estimate the rental rate on measured capital without imposing restrictions on the production function. If the specification of our model is correct (and measurement is without error) then the estimates of rental income on measured capital obtained from our framework and their framework should coincide. It appears that our results do roughly coincide for the period after the late 1980's in that we find that the expected excess returns to investment in physical capital is roughly constant at five percentage points above a risk-free rate, in line with the assumption in Barkai (2020) that the cost of equity financing is five percentage points above a risk free rate.[10]

In the data, as noted by Gomme, Ravikumar, and Rupert (2015), Reis (2022), Harper and Retus (2022), and others, the accounting returns to capital in the corporate sector (measured by the ratio of net operating surplus pre and post tax to installed capital) have remained remarkably constant since at least 1960, even as measures of the risk free interest rates have fallen quite sharply. Our accounting model is consistent with these accounting returns data. And yet we find a falling return to measured capital in recent decades, a fall of similar magnitude to the decline in risk free rates. In fact, we find that in recent years, the return to measured capital is falling close to the threshold for dynamic inefficiency of Abel et al. (1989). In our model, these differential trends between accounting returns and true returns to capital

[10]For the period prior to 1980, there is a discrepancy between our results and those of Karabarbounis and Neiman (2019) regarding returns to capital and the amount of factorless income. We aim to explore this discrepancy in greater detail in future work.

emerge because only a (declining) portion of after-tax net operating surplus is compensation of measured capital, while the remaining (rising) portion contributes to factorless income. Once we differentiate appropriately between free cash flow to physical capital and free cash flow associated with factorless income, we find that expected returns to all assets appear to declining over time, and at similar rates.

In our measurement, we have abstracted from the role of unmeasured intangible capital in accounting for fluctuations in the value of the U.S. corporate sector. Many papers consider the role of unmeasured intangible capital in driving the boom in the market valuation of U.S. firms in recent decades. See, for example, Hall (2001), McGrattan and Prescott (2010) and Crouzet et al. (2022).[11] We see this as a fruitful avenue for future research, but we see two hurdles that should be overcome in developing this hypothesis.

First, the aggregate data on unmeasured capital cited in Corrado et al. (2022) are not favorable to the hypotheses that changes in the stock of unmeasured capital have contributed importantly to fluctuations in the value of the U.S. corporate sector because these data exhibit no trend in the stock of this unmeasured capital relative to value added over the past decade or more.[12]

Second, a model of the variability of the market valuation of the U.S. corporate sector over the past century based on fluctuations in the stock of unmeasured capital held by U.S. corporations should also account for observed free cash flow to owners of these corporations, as this measure of cash flow is invariant to failure to measure investment; see Atkeson (2020). A question for research going forward is whether the fluctuations we see in free cash flow in the data are consistent with large fluctuations over time in unmeasured investment in intangible capital.

We now turn to our discussion of the IMA data.

3 Measuring Values, Cash Flows, and Returns

In this paper, we focus on valuation and cash flow measures in the data from the Integrated Macroeconomic Accounts (IMA) closest to those concepts in a standard macroeconomic stochastic growth model. We refer to the concept of the value of the U.S. corporate sector corresponding to this baseline model as *enterprise value*. We refer to the corresponding concept of cash flows for the U.S. corporate sector as *free cash flow from operations*, or free

[11] Eisfeldt and Papanikolaou (2014), Belo et al. (2022), Eisfeldt, Kim, and Papanikolaou (2022) and the papers cited therein argue that measured of intangible capital drawn from firms' accounting statements that is not included in the National Income and Product Accounts help account for the valuation of firms in the cross section.

[12] These data are available at http://www.intaninvest.net/.

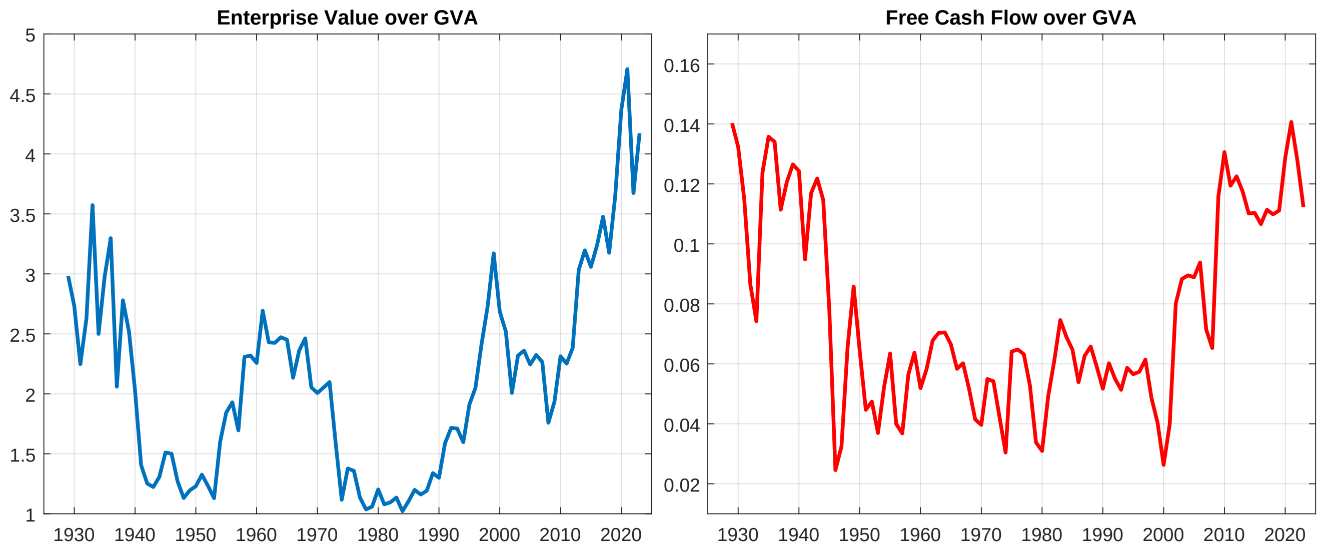

Figure 1: Left Panel: Enterprise Value of U.S. Corporations over Corporate Gross Value Added. Right Panel: Free Cash Flow from U.S. Corporations over Corporate Gross Value Added. 1929-2023

cash flow for short. We give a detailed description of the series we compute from the IMA data in Appendix A.

We use the IMA data to construct a measure of enterprise value for the U.S. corporate sector as the sum of the market value of the equity and financial liabilities less the financial assets of U.S. corporations.[13] Our measure of free cash flow in the IMA data is equal to after-tax gross operating surplus less investment expenditures of U.S. corporations. These valuation and cash flow measures are similar to those used in Hall (2001).

We plot our valuation and cash flow measures relative to the gross value added of the U.S. corporate sector in Figure 1. We show enterprise value in the left panel in blue and free cash flow in the right panel in red. We see that both enterprise value and free cash flow are quite volatile relative to the gross value added of the U.S. corporate sector. In Appendix Section B.4, we decompose fluctuations in the ratio of free cash flow to corporate gross value added into the contributions of changes in labor compensation, changes in investment, and changes in taxes. The decline in free cash flow at the start of the sample period primarily reflects rising corporate taxes. The rise at the end of the sample period primarily reflects a decline in labor's share of value added.

To what extent do the fluctuations in the ratio of enterprise value to GVA and free cash

[13]This measure of enterprise value for the Financial and Non-Financial corporate sectors is reported on Table B1 "The Derivation of U.S. Net Wealth" of the Financial Accounts of the United States. See https://www.federalreserve.gov/econresdata/notes/feds-notes/2015/us-net-wealth-in-the-financial-accounts-of-the-united-states-20151008.html

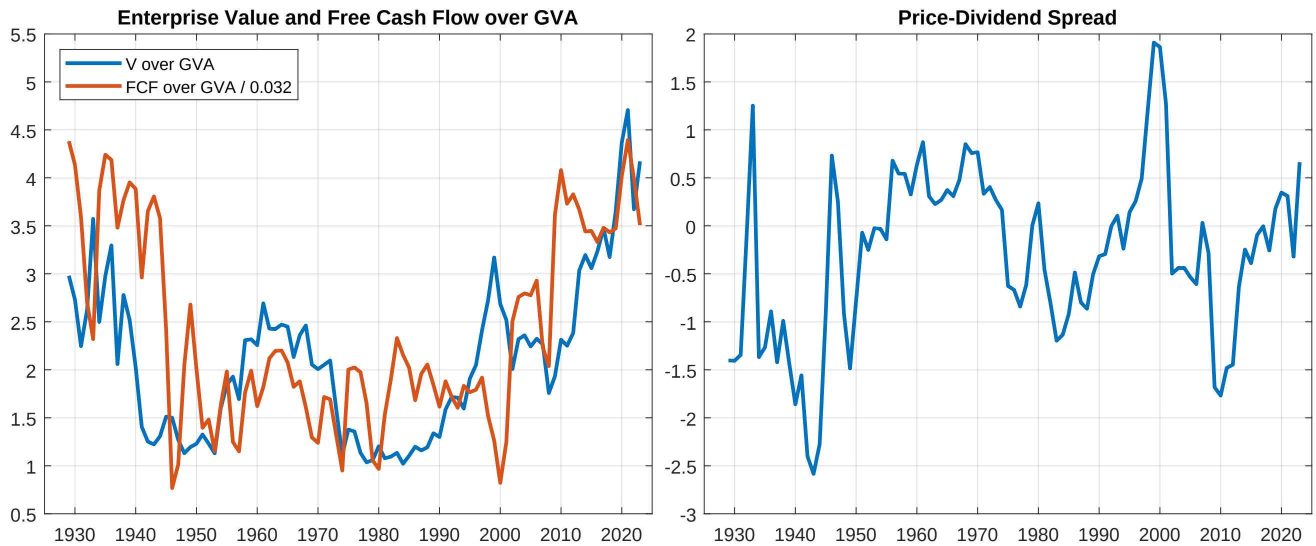

Figure 2: Left panel: Enterprise value over gross value added actual (in blue) and predicted from corporate free cash flow (in red) using a valuation multiple of $1/0.032 = 31.25$, 1929-2023. Right panel: The price-dividend spread computed as the difference between the blue and red lines as in Campbell and Shiller (1987) and Campbell and Kyle (1993)

flow to GVA line up with a very simple model of valuation? To address this question, the left panel of Figure 2 plots the ratio of enterprise value to gross value added in blue and a predicted value of this ratio if enterprise value were a fixed multiple (31.25) of free cash flow in red. We see in this panel that the low frequency fluctuations in the ratio of enterprise value to gross value added appear to be fairly well accounted for by low frequency fluctuations in the ratio of free cash flow to gross value added, when those are valued at a constant price dividend ratio.

The right panel of Figure 2 shows a valuation statistic suggested by Campbell and Shiller (1987) that we refer to as the *price-dividend spread*. This is computed as the difference between the blue line and the red line in the left panel of this figure. That is, it is the difference between the ratio of enterprise value to corporate GVA and the predicted value of this ratio if enterprise value were a fixed multiple (31.25) of free cash flow.

We see in the right panel of Figure 2 that the price-dividend spread shows sizable transitory fluctuations, but it does not show any trend over the past century. This observation further corroborates the view from the left panel of this figure that, at low frequencies, changes in the ratio of free cash flow to GVA at a constant valuation multiple account for much of the fluctuations in enterprise value over GVA.

We now consider properties of the annual returns on enterprise value implied by the IMA data. We compute the returns on enterprise value from the perspective of a household in a stochastic growth model that owns the entire corporate sector and receives all cash paid out by that sector. Using that perspective, we denote enterprise value at the end of period t as V_t, free cash flow in period $t+1$ as FCF_{t+1}, and construct realized returns on enterprise value each year as

$$1 + r_{t+1}^V = \frac{FCF_{t+1} + V_{t+1}}{V_t}.$$

We deflate these and all nominal returns by the growth in the PCE deflator to compute realized real returns.

In Figure 3 we examine the extent to which this measure of realized real returns on enterprise value lines up with realized real returns on publicly traded equities computed using the CRSP Value-Weighted Total Market portfolio. The correlation of returns on enterprise value with those on the value-weighted CRSP portfolio is 0.943 for the period 1929-2023. In Appendix B we construct a measure of returns to equity (as opposed to enterprise value) in the IMA data, and show that this series aligns even more closely with the series for CRSP returns (see Figure B.3).

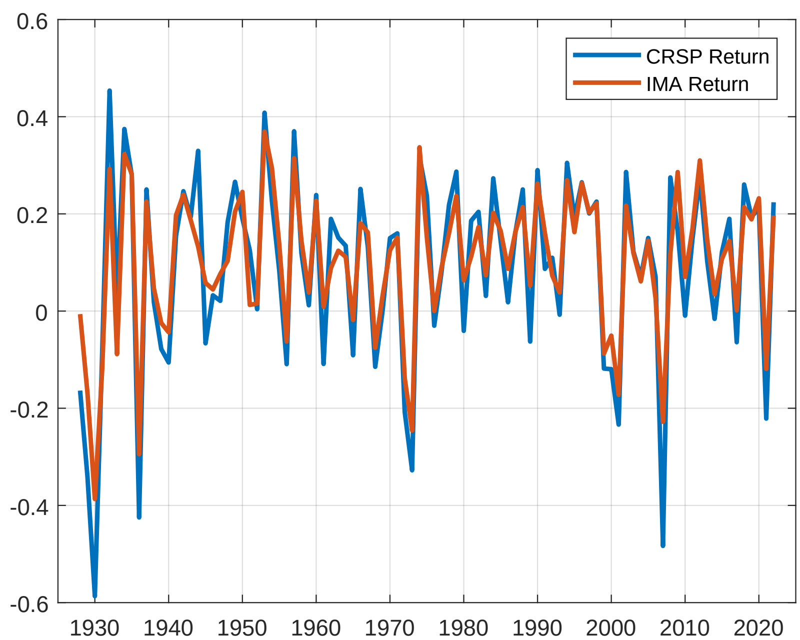

Figure 3: Realized Returns on IMA Enterprise Value, r_{t+1}^V, Versus CRSP Value-Weighted Total Market Return, 1929-2023.

We report some basic statistics of the mean and standard deviations of log real returns using this IMA return to enterprise value concept as well as analogous return and dividend growth statistics computed using CRSP returns on the Total Value-Weighted Market portfolio in Table 1. We see in this table that these two measures of returns have similar means and standard deviations. Note that returns to enterprise value are slightly less volatile than CRSP returns to equity, perhaps reflecting the fact that equity is a leveraged claim on value.

Table 1: Mean and Standard Deviation of Real Log Returns and Log Cash Flow Growth on Enterprise Value and the CRSP Total Market Value-Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std Cash Flow growth
Enterprise Value	1929-2023	0.073	0.146	0.280
CRSP VW	1929-2023	0.062	0.193	0.138

While CRSP and IMA returns look very similar, the dynamics of IMA free cash flow and CRSP dividends look very different. Free cash flow growth is more volatile than CRSP dividends (Table 1), which reflects the fact that firms seek to smooth dividend payments. In addition, the two series exhibit different trends. Figure 4 plots the dynamics of the CRSP dividend to price ratio dividend spread against the IMA ratio of Free Cash Flow to Enterprise Value. While the latter appears stationary, the former does not, reflecting the fact that over time entry rates of new firms into public markets have changes and many incumbent firms have stopped paying dividends and turned to other ways to return free cash flow to their owners (see, e.g., Fama and French 2001). Evidence of a non-stationary dividend to price ratio poses challenges for estimating valuation models using CRSP data (see, e.g., Lettau and Van Nieuwerburgh 2008) which is another advantage of focusing on our free cash flow income measure.

We now conduct one final comparison of our measures of free cash flow and enterprise value in the IMA data with analogous measures obtained from Compustat data on the financial accounting statements of publicly-traded firms.[14] We expect to see differences in these measures of cash flows and valuation from these two data sets for many reasons, two of which stand out.

First, the IMA data are constructed to cover both publicly-traded and closely-held corporations, while the Compustat data cover only publicly-traded corporations.[15] This conceptual distinction between the two data sets should act to make measures of free cash flow and enterprise value larger in the IMA data than corresponding estimates from Compustat data.

[14] In the Compustat data, our measure of free cash flow is computed (following Adame et al. 2023) as Operating Activities–Net Cash Flow (OANCF) minus capital expenditures (CAPX). Enterprise value is computed as Total Market Value (MKVALT) plus Total Liabilities (LT) minus current assets total (ACT), which includes cash and other short term investments, receivables, inventories, and other current assets. Further details are given in Appendix A

[15] For a discussion of the methodology used in the IMA to value closely held corporate equities see `https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/corporate-equities-by-issuer-in-the-financial-accounts-of-the-united-states-20160329.html`

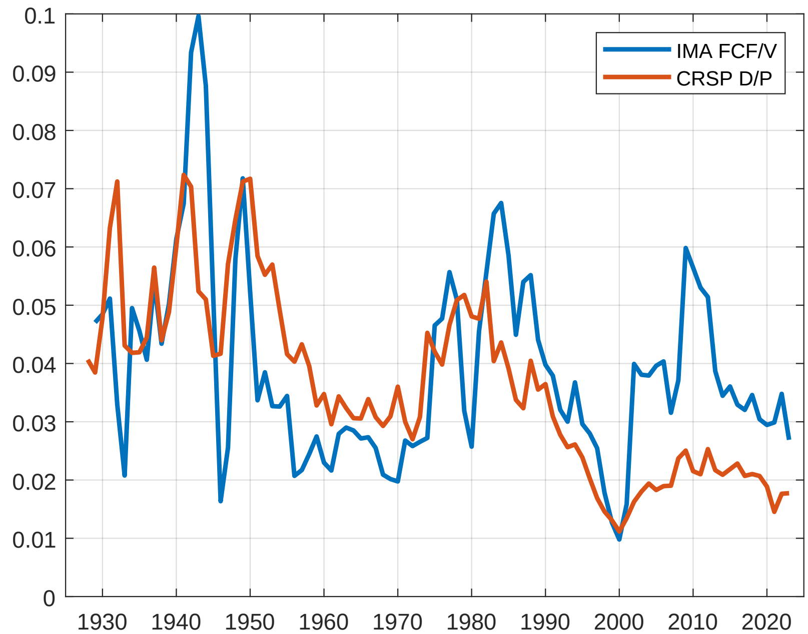

Figure 4: Free Cash Flow to Enterprise Value in the IMA Versus Dividends Per Share to Price Per Share in CRSP, 1929-2023.

Second, the IMA data are constructed to cover only U.S. resident corporations. A U.S. resident corporation is an entity incorporated in the United States. Thus, these corporations include the U.S. subsidiaries of foreign multinational corporations but exclude the foreign subsidiaries of U.S. multinational corporations. In contrast, Compustat data covers the worldwide operations of a list of public companies that are determined to be U.S. corporations in terms of the entity listing equity on U.S. markets. (See Atkeson, Heathcote, and Perri 2023 for further discussion of this point.) To the extent that the foreign subsidiaries of U.S. multinational corporations generate more free cash flow and contribute more to enterprise value than the U.S. subsidiaries of foreign multinationals, this conceptual distinction between these two data sets should act to make measures of free cash flow and enterprise value smaller in the IMA data than corresponding estimates from Compustat data.

In Figure 5 we plot free cash flow (left panel) and enterprise value (right panel) in the IMA and in Compustat, both divided by same denominator, which is gross value added of the corporate sector from the IMA. The left panel shows that in both Compustat and the IMA data the share of free cash flow in GVA roughly doubles from the early 1990s to the late 2000s. The right panel shows that enterprise value relative to IMA GVA increases by a similar amount in both data sets.

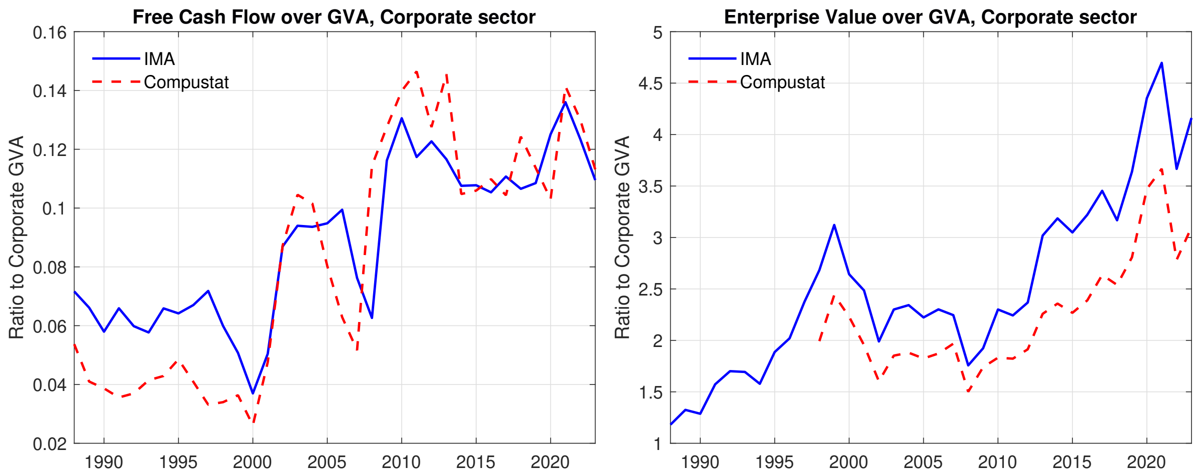

Figure 5: Free Cash Flow and Enterprise Value in the IMA and in Compustat

The close correspondence between measures of value and returns for claims on the U.S. corporate sector from the Integrated Macroeconomic Accounts with measures of value and returns constructed from CRSP and Compustat data on public firms gives us some confidence that the Integrated Macroeconomic Accounts are a useful data source for further work in macrofinance aimed at offering an integrated account of aggregate corporate valuations and cash flows.

4 A Finance-Style Valuation Exercise

We start with a finance-style valuation exercise that is not based on a particular macroeconomic model. Instead, it is based on the valuation framework laid out in Campbell and Shiller (1987) and Campbell and Kyle (1993). Our goal is to estimate the extent to which fluctuations in the price-dividend spread measured using the ratio of enterprise value to corporate GVA and free cash flow to corporate GVA as shown in the right panel of Figure 2 are due to changes in the rate of return investors expect to earn on enterprise value versus changes in the expected ratio of future free cash flow to corporate gross value added. We use this valuation model to construct an estimated series for expected returns on enterprise value every year from 1929-2023. This series will be a key driver of investment choices in the macroeconomic model that we will explore in Section 5.

Our finance-style valuation model is based on the following identity. For any observed sequence of prices $\{p_t\}$ for an asset with observed dividends (cash flows) $\{d_t\}$, we have the following valuation identity

$$p_t = \underbrace{p_t^\star}_{fundamental\ price} + \underbrace{\phi_t}_{residual}, \tag{1}$$

where we define the *fundamental price* $p_t^\star$ as the discounted expected value of dividends at constant discount rate β:

$$p_t^\star \equiv \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t d_{t+k}. \tag{2}$$

By definition, the residual ϕ_t represents all other influences on price. The value of the parameter β and the stochastic process for dividends that defines conditional expectations $\mathbb{E}_t d_{t+k}$ are inputs into the model.

In the general specification of this valuation framework, we assume that dividends follow an ARIMA process, that is, they are integrated of order one. Under this assumption, following Beveridge and Nelson (1981), we can decompose the dynamics of dividends d_t into a trend component x_t and a transitory component y_t. We define the trend component as the expected value of dividends in the long run:

$$x_t \equiv \lim_{k\to\infty} \mathbb{E}_t d_{t+k}.$$

By construction, we have that

$$\mathbb{E}_t x_{t+k} = x_t$$

for all k, that is, x_t should be a Martingale. The transitory component of dividends is then given by $y_t = d_t - x_t$, which is assumed to be stationary and mean zero.

Our general valuation model is a joint process for the two unobserved components of dividends $\{x_t, y_t\}$ and the third unobserved residual term $\{\phi_t\}$. We describe the specific assumptions we use to estimate these three unobserved components of valuation below. Before doing do, we first describe some useful properties of the general specification of the model that will inform the set of moments we will use in estimation.

First, note that our valuation expression can be written as

$$\begin{aligned} p_t &= \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t[x_{t+k} + y_{t+k}] + \phi_t \\ &= \frac{\beta}{1-\beta} x_t + \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t y_{t+k} + \phi_t. \end{aligned}$$

From this expression it is clear that innovations to the expected long run value for free cash flow, x_t, will change market valuations in proportion to the factor $\beta/(1-\beta)$.

Second, let

$$pd_t \equiv p_t - \frac{\beta}{1-\beta} d_t$$

denote the price-dividend spread. Substituting in the expression for p_t from the above expression one can see that

$$pd_t = \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t \left[y_{t+k} - y_t \right] + \phi_t. \tag{3}$$

Thus, a high price-dividend spread can reflect either a high value for the residual term ϕ_t in valuations, or an expectation that future dividends will exceed current dividends. Note that the price-dividend spread is independent of the long-run expected value for dividends x_t.

Third, let

$$r_{t,t+1} \equiv \beta \left(p_{t+1} + d_{t+1} \right) - p_t$$

denote what we will label the *quasi-return* between t and $t+1$. And similarly let

$$r_{t,t+s} = \beta^s p_{t+s} + \sum_{j=1}^{s} \beta^j d_{t+j} - p_t = \sum_{k=0}^{s-1} \beta^k r_{t+k,t+k+1}$$

denote quasi-returns between t and $t+s$.

In this framework, the dynamics of expected quasi-returns are driven by the dynamics of ϕ_t. In particular, one can show that

$$\mathbb{E}_t r_{t,t+1} \equiv \beta \mathbb{E}_t \left[p_{t+1} + d_{t+1} \right] - p_t = \beta \mathbb{E}_t \phi_{t+1} - \phi_t. \tag{4}$$

This equation holds regardless of the dynamics of dividends.[16] Thus, the general valuation model features time-varying expected returns to the extent that it features time variation in $\beta \mathbb{E}_t \phi_{t+1} - \phi_t$. A similar result applies at longer horizons: $\mathbb{E}_t r_{t,t+s} = \beta^s \mathbb{E}_t \phi_{t+s} - \phi_t$.

[16]To see this result, note that

$$\beta \mathbb{E}_t (p_{t+1} + d_{t+1}) - p_t = \beta \mathbb{E}_t (p^\star_{t+1} + d_{t+1}) - p^\star_t + \beta \mathbb{E}_t \phi_{t+1} - \phi_t$$

Next, observe that

$$\beta p^\star_{t+1} - p^\star_t = \sum_{k=2}^{\infty} \beta^k \left[\mathbb{E}_{t+1} d_{t+k} - \mathbb{E}_t d_{t+k} \right] - \beta \mathbb{E}_t d_{t+1}$$

Since, by the Law of Iterated Expectations,

$$\mathbb{E}_t \mathbb{E}_{t+1} d_{t+k} = \mathbb{E}_t d_{t+k}$$

we have

$$\mathbb{E}_t \beta p^\star_{t+1} - p^\star_t = -\beta \mathbb{E}_t d_{t+1}$$

From equation (4), together with the assumption that $\lim_{k\to\infty}\beta^k\mathbb{E}_t\phi_{t+k}=0$,[17] we can solve for the level of ϕ_t from

$$\phi_t = -\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t r_{t+k,t+k+1}. \tag{5}$$

We seek to estimate processes for $\{x_t, y_t, \phi_t\}$ such that $d_t = x_t + y_t$ and equations (3) and (5) hold. We use the following two assumptions in our estimation.

First, we model the deviation of the price-dividend spread from its unconditional mean $\bar{\phi}$ as an AR(1) process with persistence ρ:[18]

$$(pd_{t+1}-\bar{\phi}) \;=\; \rho(pd_t-\bar{\phi})+\varepsilon_{pd,t+1}, \tag{6}$$

Second, we assume that both expected changes in dividends and expected quasi-returns are given as linear functions of the price dividend spread:

$$\mathbb{E}_t d_{t+1}-d_t=\mathbb{E}_t y_{t+1}-y_t \;=\; \alpha_d+\gamma_d\times pd_t, \tag{7}$$

$$\mathbb{E}_t r_{t,t+1}=\beta\mathbb{E}_t\phi_{t+1}-\phi_t \;=\; \alpha_r+\gamma_r\times pd_t. \tag{8}$$

If expected quasi-returns are a linear function of pd_t (equation 8) then from (5) and (6), ϕ_t must also be a linear function of pd_t, and must inherit the same persistence ρ. A similar argument can be made for y_t based on this result for ϕ_t and equations (3), (6) and (7). This implies that the price-dividend spread (equation 3) can be written as

$$pd_t=\phi_t-\Gamma y_t, \tag{9}$$

where

$$\Gamma=\frac{\beta}{1-\beta}-\frac{\beta\rho}{1-\beta\rho}.$$

These results imply that in our model to be estimated, price p_t is given by

$$p_t=\phi_t+\frac{\beta\rho}{1-\beta\rho}y_t+\frac{\beta}{1-\beta}x_t \tag{10}$$

Equation (9) and the assumptions that the average values of y_t and ϕ_t are zero and $\bar{\phi}$ respectively together imply the following restrictions on the coefficients in equations (7) and

[17] Here we assume that $\lim_{k\to\infty}\beta^k\mathbb{E}_t\phi_{t+k}=0$, thus ruling out explosive bubbles in $p_t-p_t^\star$.

[18] Recall that the time series for pd_t in Figure 2 appears to be stationary.

(8):

$$\alpha_d = -\gamma_d\bar{\phi},$$
$$\alpha_r = (\beta - 1)\bar{\phi} - \gamma_r\bar{\phi},$$
$$\frac{-\gamma_r}{1-\beta\rho} = 1 - \Gamma\frac{\gamma_d}{1-\rho}.$$

The logic for this last restriction is that if quasi-returns and expected dividend growth are both forecast using the same observable (pd_t) then any observed value for pd_t must map to a linear combination of y_t and ϕ_t. At the average value of pd_t, that combination must be the average value for y_t (zero) and the average value for ϕ_t ($\bar{\phi}$). Higher observed values of pd_t signal a mix of lower expected returns (higher ϕ_t) and stronger expected dividend growth (lower y_t), but that mix must add up to support the observed pd_t. Thus, for a given pd_t, weaker expected returns (a more negative γ_r) must coincide with weaker expected dividend growth (a smaller γ_d).[19]

We let $\psi \equiv \frac{-\gamma_r}{1-\beta\rho}$ index the extent to which investors interpret a higher pd_t as reflecting a higher value for ϕ_t versus a lower value for y_t. Thus, we take β as given and we parameterize our model to be estimated with $\bar{\phi}$, ρ, and ψ such that

$$\phi_t - \bar{\phi} = \psi\left(pd_t - \bar{\phi}\right), \tag{11}$$
$$-\Gamma y_t = (1-\psi)\left(pd_t - \bar{\phi}\right). \tag{12}$$

and a covariance matrix Σ of the innovations to the price dividend spread $\varepsilon_{pd,t+1}$ and innovations to the trend x_t of dividends $\varepsilon_{x,t+1}$. We assume that these innovations are mean zero and jointly normal.

We apply this valuation framework to our data on enterprise value and free cash flow as follows. We let p_t denote enterprise value over corporate gross value added ($p_t = V_t/GVA_t$). We let d_t correspond to free cash flow over value added ($d_t = FCF_t/GVA_t$).

There are two implications of defining prices and cash flows relative to gross value added. First, the cash flow shocks that matter for valuation here are shocks that change the *share* of cash flow in value added. That is, shocks that move free cash flow and GVA in proportion (and thus do not change x_t or y_t) do not impact enterprise value relative to GVA in this model. The second implication is that returns measured as $(p_{t+1} + d_{t+1})/p_t$ are returns *in excess of growth* in GVA. Thus, when we estimate the extent to which changes in valuations are driven by shocks to expected quasi-returns, these should be interpreted as shocks to expected returns net of expected growth.

[19] This argument follows Cochrane (2008) applied to this linear valuation model.

We discuss the relationship between the valuation framework given by equations 1 and 2 and more standard asset pricing equations as well as our motivation for choosing the scale price and cash flow by corporate GVA in Appendix E.

Our motivation for exploring this tightly parameterized model specification is as follows. First, Figure 2 showed a strong low frequency correlation between enterprise value and free cash flow, suggesting that these price movements might be driven by changes in long run expected cash flow, x_t.

Second, there is a long tradition in asset pricing of exploring whether price-dividend spreads forecast future returns or future dividend growth. We have run forecasting regressions using our data on enterprise value and free cash flow of the form

$$r_{t,t+s} = \alpha_{r,s} + \gamma_{r,s} \times pd_t + \epsilon_{r,t+s}, \tag{13}$$

$$d_{t+s} - d_t = \alpha_{d,s} + \gamma_{d,s} \times pd_t + \epsilon_{d,t+s}, \tag{14}$$

where s denotes the forecasting horizon. These regressions yield estimates of the slope coefficients $\gamma_{r,s} < 0$ and $\gamma_{d,s} > 0$, indicating that a high price-dividend spread forecasts both low future quasi-returns and high future dividend growth. From equation (4), evidence of quasi-return predictability indicates that fluctuations in the residual term ϕ_t play a role in driving fluctuations in the price-dividend spread. Likewise, evidence of dividend growth predictability suggests that fluctuations in the transitory component of dividends y_t also play a role in driving fluctuations in the spread.

The restricted model makes stark predictions about the extent of this predictability at different future horizons s. In particular, the estimated slope coefficients $\hat{\gamma}_{r,s}$ in regressions (13) should be given by $(\beta^s \rho^s - 1)\psi$ and the estimated slope coefficients $\hat{\gamma}_{d,s}$ in regressions (14) should be given by $(1 - \rho^s)(1 - \psi)/\Gamma$.

We choose our model parameters as follows. We choose $\beta/(1-\beta) = 31.25$ so that the price-dividend spread pd_t corresponds to the series shown in the right panel of Figure 2. We then set $\bar{\phi}$ equal to the average value for pd_t in our sample, which implies $\bar{\phi} = -0.2985$.

We estimate the remaining parameters (ρ, ψ, Σ) using the method of simulated moments. Here we target five sets of moments involving dividend growth and quasi returns at horizons up to 15 years, for a total of $5 \times 15 = 75$ moments. Those moments are: (i) the variance of quasi-returns $r_{t,t+s}$ for horizons $s = 1$ to 15, (ii) the variance of dividend growth $d_{t+s} - d_t$ for horizons $s = 1$ to 15, (iii) the covariance between $r_{t,t+s}$ and $d_{t+s} - d_t$ for $s = 1$ to 15, (iv) the covariance between pd_t and $r_{t,t+s}$ for $s = 1$ to 15, and (v) the covariance between pd_t and $d_{t+s} - d_s$ for $s = 1$ to 15. We measure the distance between model and data as the weighted sum of squared percentage differences between the simulated model and data moments, where the optimal weighting matrix is constructed using the standard two-stage approach. We let σ_x, σ_{pd}, and $\rho_{x,pd}$ denote, respectively, the standard deviation of innovations to x_t, the standard deviation of innovations to $(pd_t - \bar{\phi})$, and the correlation between those two innovations. The parameter values we estimate are $\sigma_x = 0.00931$, $\sigma_{pd} = 0.544$, $\rho_{x,pd} = 0.498$, $\rho = 0.835$, and $\psi = 0.255$.

An advantage of a specification in which there are only two shocks is that given two observables (p_t and d_t) we can uniquely invert the model to identify the sequences for x_t and pd_t that perfectly replicate the data. Of course, the fact that there exist sequences for shocks that replicate the observed data does not mean that this is a good model – the shocks required might be wildly inconsistent with our distributional assumptions. We assess model performance by drawing serially independent normal innovations to x_t and pd_t according to the estimated joint distribution, and simulating many artificial histories for valuations and cash flows. We can then see whether the valuation moments we can measure in the data are typical of those that emerge from our simulations, or whether some data moments are highly improbable given the distribution of simulated model moments.

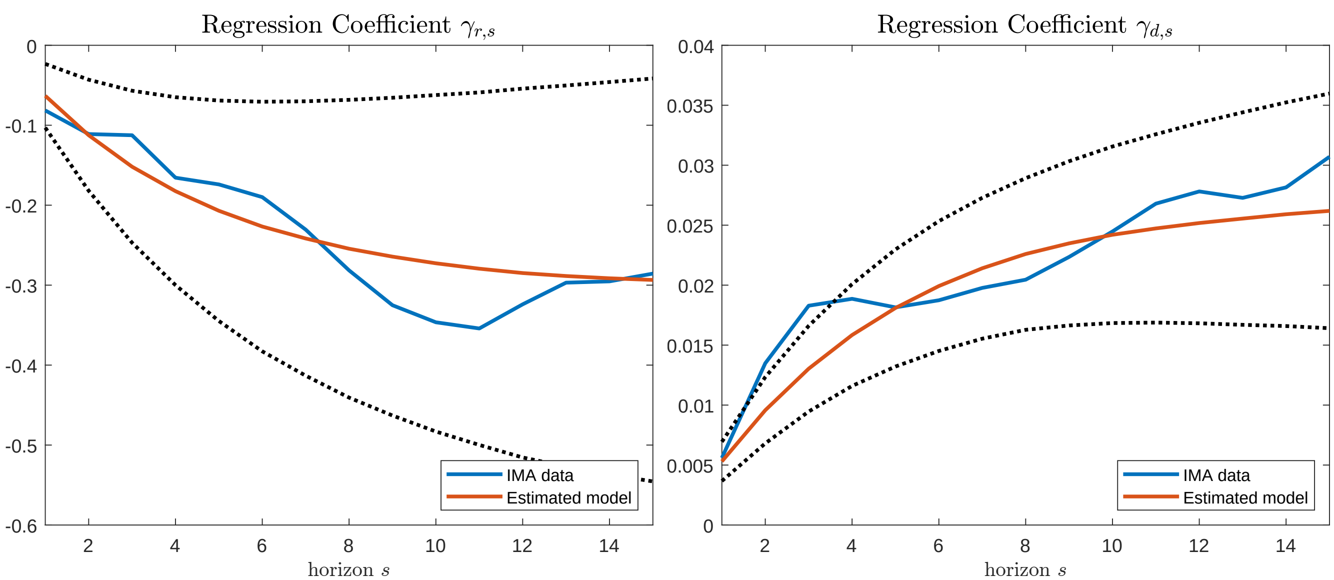

Figure 6: Left Panel: Estimated Slope Coefficients from Regressing Quasi-Returns $r_{t,t+s}$ at Horizon s on the Price-Dividend Spread pd_t. Right Panel: Estimated Slope Coefficients from Regressing Dividend Growth $d_{t+s} - d_t$ at Horizon s on the Price-Dividend Spread pd_t. The dotted lines show one standard error bands around the average model moments.

Figure 6 plots estimated regression coefficients for $\gamma_{r,s}$ and $\gamma_{d,s}$ from regressions of the form (13) and (14).[20] The blue lines show the coefficients estimated given our series for

[20]The regression coefficient $\gamma_{r,s}$ is equal to $\mathbb{C}ov(pd_t, r_{t,t+s})/\mathbb{V}ar(pd_t)$. Our estimation routine targets the

$r_{t,t+s}$, $d_{t+s} - d_t$, and pd_t constructed from the Integrated Macroeconomic Accounts. The red lines show averages across $1,000$ simulations of our restricted model, given the estimated parameter values described above. The dotted lines show one standard error bands around the model averages constructed from the 1000 model simulations. Our simple restricted model is broadly successful in replicating these moments.

Figure 7 plots the variances of quasi returns and of dividend growth at different horizons. Again, simulations of the restricted model generate similar moments, though the model appears to deliver too little high frequency volatility in dividend growth. In Appendix F we report the model fit for a range of additional moments. In Figure F.2 in particular, we confirm that our model of the price dividend spread as an AR1 as in equation (6) fits the data well.

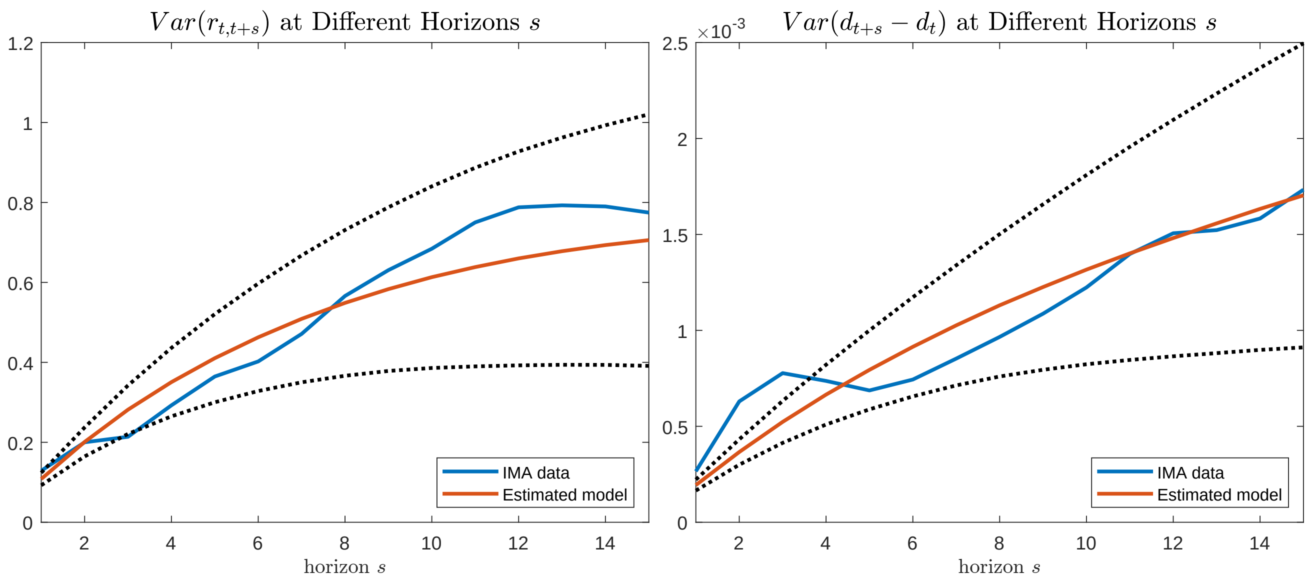

Figure 7: Left Panel: Variance of Quasi-Returns $r_{t,t+s}$ at Horizon s. Right Panel: Variance of Dividend Growth $d_{t+s} - d_t$ at Horizon s.

Next we invert the model at the estimated parameter values to identify time series for $\{x_t\}$, $\{y_t\}$ and $\{\phi_t\}$ that reproduce observed valuations and dividends.

The left panel of Figure 8 plots the contributions of different terms to enterprise value according to equation (10). The stark message from the figure is that the model attributes almost all of the movements in observed valuations to fluctuations in the long run expected value for dividends, x_t.[21] The shocks that drive fluctuations in the price-dividend spread (shocks to y_t and to ϕ_t) are, taken together, almost irrelevant for understanding fluctuations

numerator but not the denominator.

[21] To check that the series for $\{x_t\}$ implied by our model is indeed a Martingale, we have run regressions

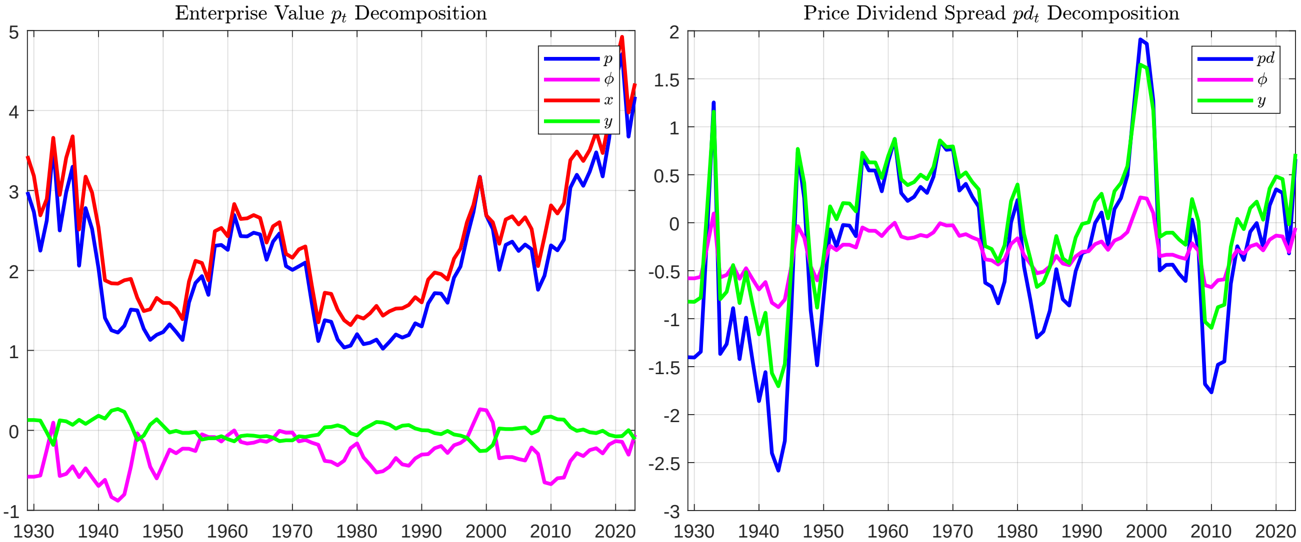

Figure 8: Left Panel: Enterprise Value over GVA p_t, and Model-Inferred Contributions of Fluctuations in x_t (red), y_t (green) and ϕ_t (magenta). Right Panel: Enterprise Value Minus Free Cash Flow Spread $p_t - \frac{\beta}{1-\beta} d_t$, and Model-Inferred Contributions of Fluctuations in y_t (green) and ϕ_t (magenta).

in valuations.

There are two reasons for our finding that fluctuations in valuations are predominantly driven by shocks to x_t. The most important reason is that the estimation procedure delivers a small value for ψ of 0.255, attributing most of observed fluctuations in the price dividend spread to Γy_t rather than to ϕ_t. Fluctuations in ϕ_t generate equal-sized movements in equilibrium prices p_t, while fluctuations in Γy_t generate much smaller price movements of size $\beta\rho/(1-\beta\rho) \times (1/\Gamma) = 0.157$. Intuitively, transitory shocks to dividends generate large movements in the price dividend spread, but only small movements in equilibrium prices. The second reason why the estimation identifies a small collective role for y_t and ϕ_t is that our two modeling assumptions imply that innovations to y_t are perfectly negatively correlated with innovations to ϕ_t, so the impact of the two shocks on price fluctuations partially offset.

The right panel of Figure 8 indicates that our estimates point to transitory shocks to dividends as playing the dominant role in explaining fluctuations in the price-dividend spread. This observation follows immediately from our estimate of the parameter ψ. Thus, we identify

to predict changes in x_t of the form

$$x_{t+s} - x_t = \alpha_{x,s} + \gamma_{x,s} \times pd_t + \epsilon_{x,t+s}$$

and

$$x_{t+s} - x_t = \alpha_{dx,s} + \gamma_{dx,s} \times (x_t - x_{t-1}) + \epsilon_{dx,t+s}.$$

We find that these estimated coefficients are not statistically different from zero.

shocks to cash flow as the key driver of fluctuations in valuation, irrespective of whether valuation is measured as price relative to value added or as the price-dividend spread. But permanent shocks drive the former valuation measure, while transitory shocks drive the latter.

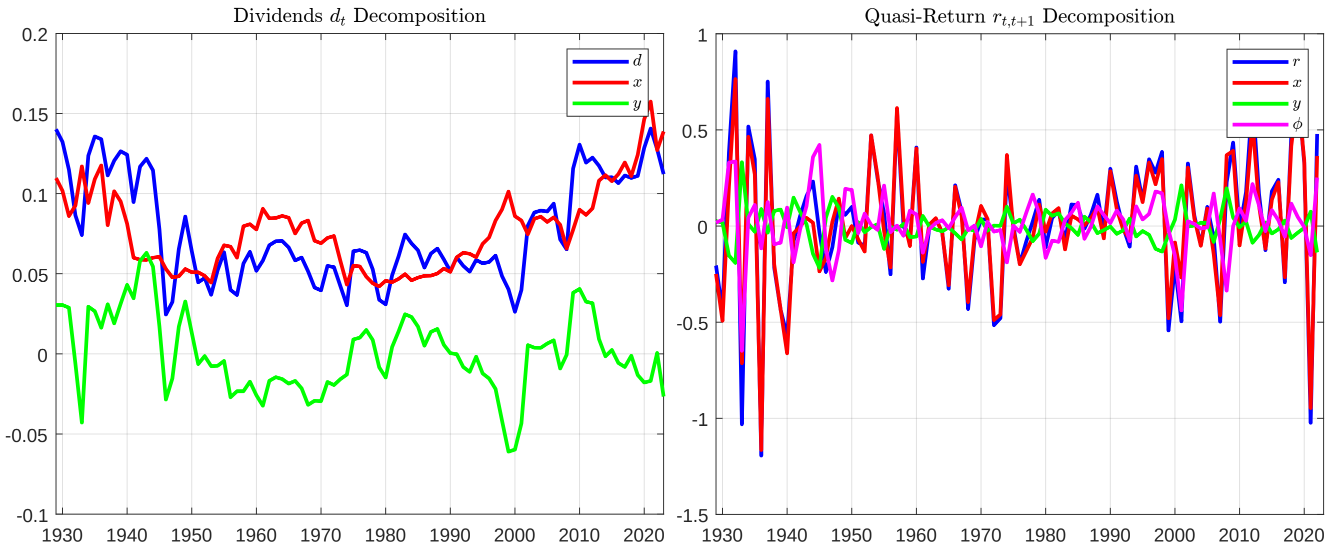

Figure 9: Left Panel: Free Cash Flow over GVA d_t, and Model-Inferred Contributions of Fluctuations in x_t (red) and y_t (green). Right Panel: Realized Quasi-Returns $\beta(p_{t+1} + d_{t+1}) - p_t$, and Model-Inferred Contributions of Fluctuations in x_t (red), y_t (green), and ϕ_t (magenta).

The left panel of Figure 9 plots the decomposition of the ratio of free cash flow to corporate gross value added, d_t, into permanent and transitory components x_t and y_t. The model infers a lot of high frequency volatility in the transitory component y_t – which drives (inversely) most of the observed volatility in the price-dividend spread in the right panel of Figure 8. The permanent trend component x_t tracks low frequency movements in dividends. Note that the series for expected long-run free cash flow x_t is less volatile (at high frequency) than the series for actual free cash flow. But fluctuations in x_t translate into fluctuations in p_t with a large valuation multiple of $\beta/(1-\beta) = 31.25$, which allows the model to replicate the large observed fluctuations in enterprise value.

The right panel of Figure 9 decomposes realized one-year quasi-returns into contributions from the three shocks. The message from this figure is that movements in realized returns are almost entirely driven by the red line, which shows the contribution from innovations to the expected long run value for dividends, $\frac{\beta}{1-\beta}\varepsilon_{x,t+1}$. Fluctuations in y_t and ϕ_t do impact realized returns, but their effects nearly exactly offset.

Figure 10 plots expected returns to enterprise value in excess of expected growth in value-added. This measure of expected returns is given by

$$\mathbb{E}_t\left[\left(\frac{V_{t+1}+FCF_{t+1}}{V_t}\right)\frac{GVA_t}{GVA_{t+1}}\right]=\mathbb{E}_t\left[\frac{p_{t+1}+d_{t+1}}{p_t}\right]=\frac{\frac{(p_t-\phi_t)}{\beta}+\rho\phi_t+(1-\rho)\bar{\phi}}{p_t}, \tag{15}$$

where the first equality follows from our definitions for p_t and d_t, and the second from model equations. Note that the model identifies no long-run trend in expected returns in excess of expected growth in value-added. This series for expected returns will be an important input for investment decisions in the macroeconomic model we will turn to shortly.

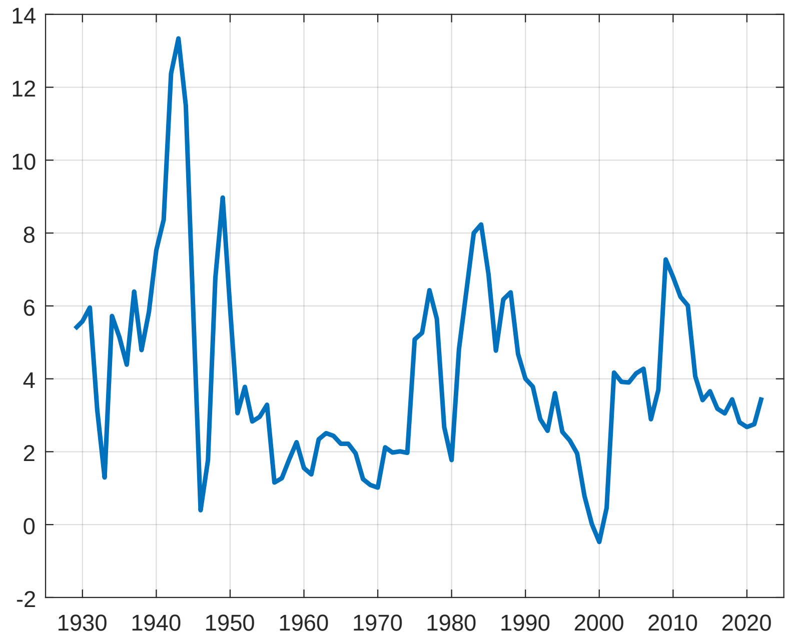

Figure 10: Expected Returns Net of Expected Growth in GVA. The object plotted corresponds to net expected returns, in percent, where gross expected returns are defined in equation (15).

5 A Macroeconomic Model

We now introduce the macroeconomic model we use to interpret the IMA data. Our model of production and income in the corporate sector is a modified version of the standard stochastic growth model. The main modification is that we assume that firms' total revenue includes a time-varying wedge relative to the cost of hiring physical capital and labor. With this wedge in our model, a portion of value added corresponds to a pure rent paid to the owners of firms. Following Karabarbounis and Neiman (2019), we refer to this rent as *factorless* *income*. We also explicitly model corporate taxation, and how it impacts cash flow, returns, and valuations.

We note that this factorless income can be positive or negative. To the extent that firms have power to charge a markup over the costs of labor and measured capital, factorless income is positive. To the extent that managers of firms fail to earn surplus sufficient to cover the opportunity cost of the measured capital owned by these firms, factorless income is negative.

Our analysis of the IMA data through the lens of our macroeconomic model will proceed in steps.

First, we show that with a minimal set of assumptions, we can use the model structure and the Integrated Macroeconomic Accounts to decompose the series for corporate free cash flow described earlier into a portion of cash flow accruing to owners of measured capital, and a portion accruing to owners of claims to factorless income.

Second, we show how our model can be used to decompose aggregate enterprise value into the market value of cash flows to the owners of the measured capital stock and the market value of claims to factorless income. We show that in the data prior to World War II, much of the fluctuations in enterprise value relative to corporate output correspond fluctuations in the stock of measured capital relative to corporate output, while after World War II, fluctuations in the market value of measured capital account for little of the large observed swings in enterprise value. Given times series for the value of capital and for cash flows to capital we can compute a sequence of realized returns to capital.

Third, we turn to the capital Euler equation implied by our model to consider the model's implications for *expected* returns to capital. We show that these expected returns depend on the path for the capital-output ratio – which we take from the IMA data – and on expectations for tax rates, the depreciation rate, the revenue-cost wedge, the change in the price of investment goods, and the expected growth rate for value-added. We ask whether the expected returns to capital in our macroeconomic model track the time series for expected returns to enterprise value that we estimated in our finance valuation model, as one might expect if investors are both pricing firms and dictating investment choices.

Recall that in our valuation model we estimated a time series for expected returns *in excess of* the expected growth rate of value added. For a given path for the expected growth rate of value added, we can translate that path into a path for expected returns. We compute the expected growth rate of value added, year-by-year from 1929 through 2023, that equates the expected return to capital computed from our macroeconomic model with expected returns in our valuation model. We find that the resulting series for expected growth in value added appears plausible, indicating that firm valuation and capital investment are jointly consistent with the same path for expected returns.

The macro model is as follows.

5.1 Technology

Aggregate output, corresponding to gross value added of the corporate sector, GVA_t, is given by a Cobb-Douglas production function,

$$GVA_t = K_t^{\alpha}(Z_t L)^{1-\alpha}, \tag{16}$$

where K_t is the stock of physical capital in units of capital services, L is labor, which is inelastically supplied, and Z_t is a shock to aggregate productivity. We will assume that the share of capital services in production, denoted by α, is constant over time.

The evolution of the stock of capital services is given by

$$K_{t+1} = (1 - \delta_t)K_t + I_t,$$

where δ_t is a time-varying physical depreciation rate for capital services and I_t is investment in new capital services. Note that we assume here that there are no investment adjustment costs.

The terms K_t and I_t are not directly measured in the data. Instead, the IMA report end of period nominal values for the stock of capital at replacement cost, nominal investment expenditures, nominal consumption of fixed capital, and nominal revaluations of the stock of capital carried into the period due to changes in the replacement cost of that capital. We write the nominal end-of-period t replacement cost of capital as $Q_t K_{t+1}$, nominal investment expenditure in period t as $Q_t I_t$, nominal consumption of fixed capital as $\delta_t Q_t K_t$, and nominal revaluations of the replacement value of capital carried into period t as $(Q_t - Q_{t-1})K_t$.[22]

Thus, the accounting for the evolution of the replacement value of the capital stock in the IMA data is

$$\underbrace{Q_t K_{t+1}}_{ReplacementCost_{t+1}} = \underbrace{Q_{t-1} K_t}_{ReplacementCost_t} + \underbrace{(Q_t - Q_{t-1})K_t}_{Reval_t + Other_t} - \underbrace{\delta_t Q_t K_t}_{CFC_t} + \underbrace{Q_t I_t}_{Investment_t}$$

The growth rate for Q_t can be directly inferred by dividing the reported revaluation value by the reported end of t replacement cost. One can then construct a series for $Q_t K_t$ and from that infer a series for δ_t given reported consumption of fixed capital.

For the purposes of interpreting valuations, it is helpful to conceptualize two types of firms operating in the economy. One type, which we call investment firms, holds measured

[22]We include the category *other volume changes* in the IMA in this revaluation category in our model.

capital, makes investment decisions, and earns income by renting out this capital at a rental rate per unit of capital services R_t. The second type of firm, which we call factorless income firms, rent capital and labor, whose wage rate is W_t, and use these inputs to produce the final good according to equation (16). These factorless income firms earn this income by selling output with a wedge μ_t between revenue and the cost of measured capital and labor in production. As discussed above, this wedge can be greater or less than one.

We assume that both types of firms are 100 percent equity financed, and that both pay out all the free cash flow they generate as model dividends. We also assume that both types of firms seek to maximize the present value of model dividends payable to shareholders, where these dividends at date $t + k$ are discounted back to date t according to a common pricing kernel, $M_{t,t+k}$.

5.2 Corporate Taxation

To construct measures of free cash flow for firms we need to specify how they are taxed. We model two sorts of taxes paid by corporations. First, we assume factorless income firms pay a proportional tax at a time-varying rate τ_t^s that applies to their gross value added, GVA_t. This tax in the model corresponds to indirect business taxes in the data. Thus, we estimate τ_t^s by dividing the sum of "taxes on production and imports less subsidies" plus "business current transfer payments" from NIPA Table 1.14 by corporate gross value-added.

Second, we model corporate income taxes as follows, building on Gravelle (1994) and Barro and Furman (2018). We assume corporate income is taxed at a proportional rate τ_t^c. We assume that investment firms can fully expense economic depreciation and can also expense a constant fraction λ of net new investment. Given these assumptions, the effective tax rate on capital is approximately equal to $\tau_t^c(1 - \lambda)/(1 - \lambda\tau_t^c)$.[23] Factorless income firms pay the corporate income tax on their factorless income, but are entitled to a time-varying lump-sum tax credit, T_t^L. We use this credit to reconcile marginal tax rates with total corporate income tax revenue as reported in the IMA data.

Given this model, total corporate income taxes paid are given by

$$Taxes_t^c = \tau_t^c \left[(1 - \tau_t^s)GVA_t - W_t L - \delta_t Q_t K_t - \lambda Q_t \left(K_{t+1} - K_t\right)\right] - T_t^L \tag{17}$$

We set the value for τ_t^c in each year t equal to corresponding value for the top rate of federal corporate income tax. We set $\lambda = 0.2$. These choices imply a time path for the effective tax rate on capital income similar to the one estimated by Gravelle (2006). Given those choices, we set the time path for T_t^L so that implied total corporate income tax revenue

[23]See Appendix C for the derivation and for an exact expression.

(equation 17) matches the series for "taxes on corporate income" in NIPA Table 1.14. Figure 11 plots corporate income tax revenue as a share of gross value added, and the time paths for the statutory and effective tax rates that we use in our model.

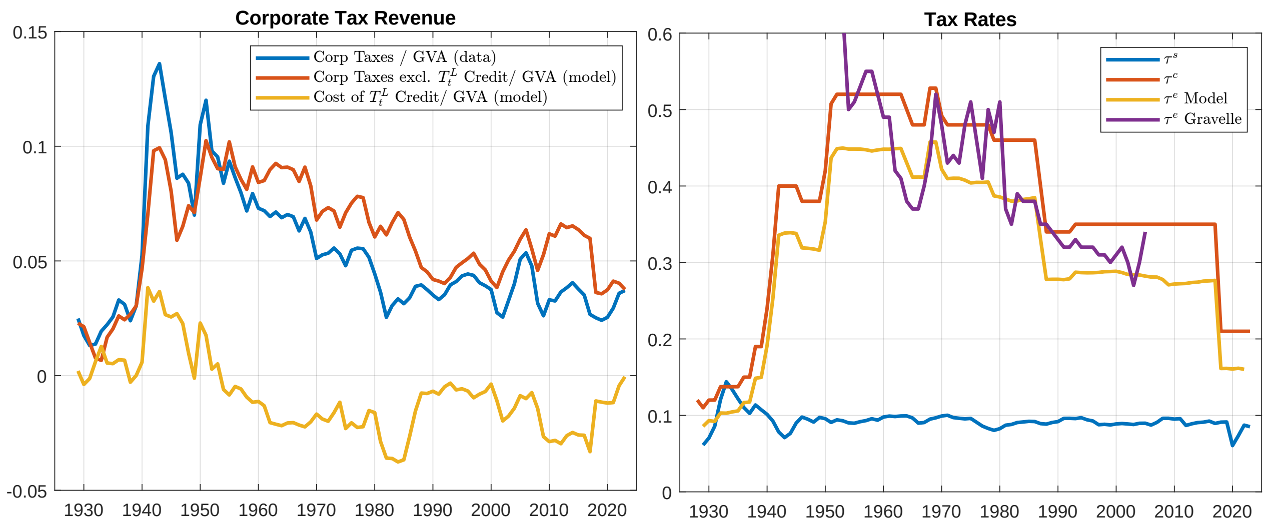

Figure 11: Left Panel: Corporate Income Tax Revenue Divided by GVA, 1929-2023. Blue line is data. Red line is model revenue excluding T_t^L credit. Yellow line is model T_t^L credit which reconciles tax collections implied by our model tax rates and tax collections in the data (yellow = blue - red). Right Panel: Model Tax Rates, 1929-2023. τ_t^s is the value-added tax rate, τ_t^c is the statutory corporate income tax rate, and τ_t^e is the marginal effective tax rate on capital.

5.3 Income Shares

We use our model to split total free cash flow into a component going to owners of measured capital, and income to owners of claims to factorless income. After-tax free cash flows from investment-producing and factorless income firms are given, respectively, by

$$FCF_t^K = R_t K_t - Q_t I_t - \tau_t^c \left[R_t K_t - \delta_t Q_t K_t - \lambda Q_t \left(K_{t+1} - K_t \right) \right] \tag{18}$$

and

$$FCF_t^{\Pi} = (1 - \tau_t^c)\Pi_t + T_t^L,$$

where

$$\Pi_t = (1 - \tau_t^s) GVA_t - W_t L - R_t K_t \tag{19}$$

denotes pre-corporate-tax factorless income.

To construct these free cash flow series requires an estimate of capital rental income $R_t K_t$ (or equivalently for Π_t.) We now use our model structure to construct such a series.

In our model, factorless income firms solve static problems, choosing how much capital and labor to rent each period to minimize costs. Given the Cobb-Douglas production function, the optimal ratio of capital relative to labor services is given by

$$\frac{K_t}{L} = \frac{\alpha}{(1-\alpha)} \frac{W_t}{R_t} \tag{20}$$

These firms set prices net of value-added tax with a time-varying wedge μ_t over unit cost:[24]

$$(1-\tau_t^s) GVA_t = \mu_t (W_t L + R_t K_t) \tag{21}$$

Given, equations (20) and (21), our model's implications for the division of gross value added into income shares is as follows. Share τ_t^s, accrues to the government as taxes on production and imports less subsidies. The remainder is divided according to

$$\frac{W_t L}{GVA_t} = (1-\tau_t^s)(1-\alpha)\frac{1}{\mu_t}, \tag{22}$$

$$\frac{R_t K_t}{GVA_t} = (1-\tau_t^s)\alpha\frac{1}{\mu_t}, \tag{23}$$

$$\frac{\Pi_t}{GVA_t} = (1-\tau_t^s)\frac{(\mu_t - 1)}{\mu_t}, \tag{24}$$

where $\frac{W_t L}{GVA_t}$ is the model equivalent of compensation of employees and the sum $\frac{R_t K_t}{GVA_t} + \frac{\Pi_t}{GVA_t}$ is the model equivalent of gross operating surplus. The model equivalent of taxes on corporate income and wealth is given by (17) and is not considered an income share in NIPA.

We let κ_t denote free cash flow to owners of factorless income firms at date t, relative to gross value added:

$$\kappa_t \equiv \frac{FCF_t^{\Pi}}{GVA_t} = (1-\tau_t^c)\frac{\Pi_t}{GVA_t} + \frac{T_t^L}{GVA_t} = (1-\tau_t^c)(1-\tau_t^s)\frac{\mu_t - 1}{\mu_t} + \tau_t^L \tag{25}$$

where $\tau_t^L = \frac{T_t^L}{GVA_t}$.

Equation (22) implies a tight link between fluctuations in the price-cost wedge μ_t and fluctuations in labor's share of income. Using that relationship one can express free cash flow

[24] In Atkeson, Heathcote, and Perri (2023) we show how such wedges can be micro-founded as arising from Bertrand competition between more and less productive potential producers.

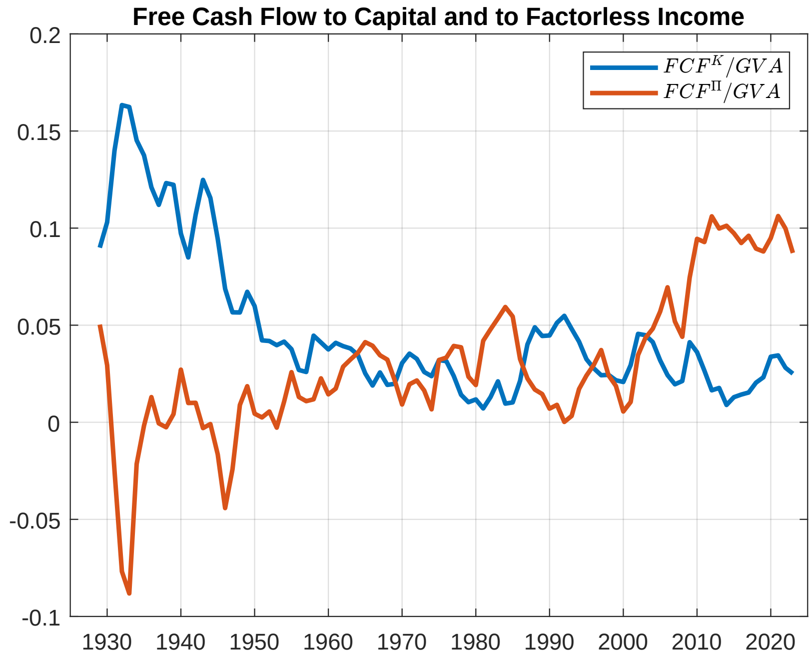

Figure 12: Decomposition of Free Cash Flow into Free Cash Flow to Capital (blue) and Free Cash Flow to Factorless Income (red).

to factorless income firms κ_t as a function of tax rates and labor's share:

$$\kappa_t = (1 - \tau_t^s)(1 - \tau_t^c) + \tau_t^L - \frac{(1 - \tau_t^c)}{(1 - \alpha)} \frac{W_t L_t}{GVA_t}. \tag{26}$$

Thus, in this model, given tax parameters and a choice for the share parameter α, the path for factorless income as a share of corporate GVA can be identified given a path for compensation to labor as a share of gross value added, which we take straight from the IMA. Of course, given total free cash flow, and free cash flow to factorless income firms, we also have free cash flow to owners of capital.

Figure 12 plots free cash flow to factorless income κ_t and free cash flow to capital as ratios to corporate GVA, given a choice of $\alpha = 0.29$ (we discuss this choice below).

The ratio of factorless income to GVA is quite volatile. In addition, it generally appears to trend upward over time, and has risen sharply since 2000, from a share near two percent, to around 10 percent of corporate GVA. Mechanically, this is largely driven by the decline in labor's share of income over this period: see equation (26) and Appendix Figure B.6.

Free cash flow to capital as a share of gross-value added declines quite dramatically in the early decades of our sample, but appears relatively stable from around 1970 onward at a fairly low level. Cash flow to capital was high during the Great Depression because investment and corporate taxes were both very low. Over time, the main driving of declining cash flow to capital has been rising investment as a share of gross value-added (see Figure B.5). This in turn reflects an upward trend in the depreciation rate δ. Rental income from capital $R_t K_t$ in the model moves in lockstep with compensation of employees (see equations 22 and 23), so the declining labor share post 2000 also worked to reduce cash flow to capital. In this sense our results here are similar to those in Barkai (2020).

Note that the path for free cash flow to capital that we find reflects a minimum of model structure: our assumption of a Cobb-Douglas production function with a constant capital share in costs α. Higher values for α shift the series for free cash flow to capital (factorless income) up (down) without changing the trend.

5.4 Firm Valuation

Enterprise value in our model is the expected discounted present value of free cash flow to owners of firms, with those present values computed using the model's pricing kernel. Given our division of free cash flow into a component that is factorless income, $FCF_t^{\Pi} = \kappa_t GVA_t$, and a component that is free cash flow to capital, FCF_t^K, it is natural to decompose enterprise value, denoted by V_t as the sum of the values of these two cash flows,

$$V_t = V_t^K + V_t^{\Pi}, \tag{27}$$

where V_t^K denotes the value of future free cash flow to capital and V_t^{Π} denotes the value of future factorless income.

The firm that owns and manages the physical capital stock takes as given an initial capital stock K_t and chooses future capital $\{K_{t+k}\}$ and after-tax free cash flow payable to owners $\{FCF_{t+k}^K\}$ for $k \geq 1$ to maximize

$$FCF_t^K + V_t^K$$

where

$$V_t^K = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} FCF_{t+k}^K \right]$$

and FCF_t^K is given by equation (18).

The first-order condition with respect to K_{t+1} is

$$\mathbb{E}_t \left[M_{t,t+1} \left[(1 - \tau_{t+1}^c) \left(R_{t+1} - Q_{t+1} \delta_{t+1} \right) + (1 - \lambda \tau_{t+1}^c) Q_{t+1} \right] \right] = (1 - \lambda \tau_t^c) Q_t. \tag{28}$$

If investment firms choose investment according to the capital Euler equation (28) at every date, then one can show that the value of future free cash flow to capital is given by

$$V_t^K = (1 - \lambda\tau_t^c)Q_t K_{t+1}. \tag{29}$$

This result is independent of the specification for the pricing kernel $M_{t,t+k}$, but it does rely on the assumption that the production function is constant returns to scale, and that there are no investment adjustment costs.[25] Given this result, we measure the value of factorless income using the IMA data using the difference between enterprise value and the value of the claims to capital:

$$V_t^\Pi = V_t - (1 - \lambda\tau_t^c)Q_t K_{t+1}. \tag{30}$$

Note that if $\lambda = 1$ (full expensing of net investment) then a constant corporate tax rate does not distort investment: all the tax terms in equation (28) cancel out. However, firm value is depressed relative to the replacement cost of capital by a factor $(1-\tau_t^c)$. The intuition is that the capital tax depresses income to capital – and thus the market value of capital – which reduces the incentive to invest. But full expensing allowance sufficiently subsidizes the cost of new investment to exactly offset that effect. Conversely, if $\lambda_t = 0$, then investment and capital are depressed when $\tau_t^c > 0$, but the value of the firm is equal to the replacement cost of its capital.[26]

We show the breakdown of enterprise value relative to gross value added into these two components in Figure 13. In the left panel of this figure, we show enterprise value (in blue) and the market value of the capital stock (in red). In the right panel of this figure, we again show enterprise value (in blue) and the value of claims to factorless income (in red). We see in the left panel of this figure that between 1929 and World War II (WWII), fluctuations in the value of capital account for much of the fluctuations in enterprise value, but that after WWII, the ratio of the value of capital to value added has remained remarkably stable. In the right panel of Figure 13, we see that it is fluctuations in the value of claims to factorless income that account for the majority of fluctuations in enterprise value after WWII.

6 Connecting the Finance and Macroeconomic Models

We now measure realized returns to capital and construct an expression for expected returns to capital in our macroeconomic model.

[25]See Hayashi (1982). We provide a proof of this result in Appendix D.

[26]See Abel (1982) and McGrattan and Prescott (2005) for a related discussion.

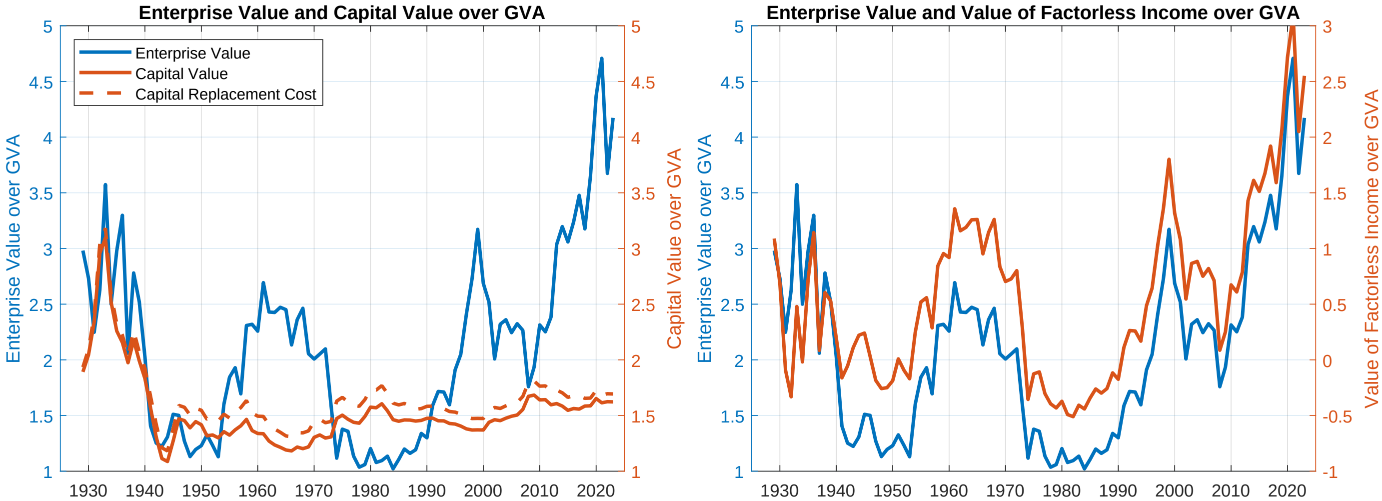

Figure 13: Left Panel: Enterprise Value (left axis) and Value of Capital Stock (right axis) over U.S. Corporate Gross Value Added, 1929-2023. Right Panel: Enterprise Value (left axis) and Value of Factorless Income (right axis) over U.S. Corporate Gross Value Added, 1929-2023.

6.1 Expected Returns

The gross realized return to capital can be measured given the series for value and cash flow described above as

$$1 + r^K_{t+1} \equiv \frac{V^K_{t+1} + FCF^K_{t+1}}{V^K_t} \tag{31}$$

To construct a series for expected returns we turn to the macroeconomic model expression for returns to capital (see equation 28)

$$1 + r^K_{t+1} = \frac{1}{(1-\lambda\tau^c_t)}\left[(1-\tau^c_{t+1})\left(\frac{\frac{R_{t+1}K_{t+1}}{GVA_{t+1}}\frac{GVA_{t+1}}{GVA_t}}{\frac{Q_tK_{t+1}}{GVA_t}} - \frac{Q_{t+1}}{Q_t}\delta_{t+1}\right) + (1-\lambda\tau^c_{t+1})\frac{Q_{t+1}}{Q_t}\right], \tag{32}$$

where from equations (23) and (25) the share of capital rental income in value added is given by

$$\frac{R_{t+1}K_{t+1}}{GVA_{t+1}} = \alpha\left[(1-\tau^s_{t+1}) - \frac{(\kappa_{t+1} - \tau^L_{t+1})}{(1-\tau^c_{t+1})}\right].$$

These returns can be interpreted as nominal or real, depending on whether GVA_t and the price of capital Q_t are measured in nominal terms or after deflating by a measure of the general price level. We will deflate by the NIPA deflator for Personal Consumption Expenditures (PCE).

Computing *expected* real returns, $1 + \mathbb{E}_t\left[r_{t+1}^K\right]$, requires specifying a joint stochastic process for $\{\tau_t^c, \tau_t^s, \tau_t^L, \delta_t, \kappa_t, \frac{Q_{t+1}}{Q_t}, \frac{GVA_{t+1}}{GVA_t}\}$. We assume all these variables are independent, and that most of them follow unit root processes. Thus, $\mathbb{E}_t\left[\tau_{t+1}^c\right] = \tau_t^c$, $\mathbb{E}_t\left[\tau_{t+1}^s\right] = \tau_t^s$, $\mathbb{E}_t\left[\tau_{t+1}^L\right] = \tau_t^L$, $\mathbb{E}_t\left[\delta_{t+1}\right] = \delta_t$, $\mathbb{E}_t\left[\kappa_{t+1}\right] = \kappa_t$, and $\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right] = \bar{g}_Q$, where $\bar{g}_Q$ is the sample average gross growth rate for the price of investment goods relative to the PCE deflator. The one conditional expectation we do not model is the expected growth rate for real value added, which in our macroeconomic model maps to an expected growth rate for productivity Z_t.

Given these assumptions, we have

$$1 + \mathbb{E}_t[r_{t+1}^K] = \frac{\alpha\left[(1-\tau_t^c)(1-\tau_t^s) - \kappa_t + \tau_t^L\right]}{(1-\lambda\tau_t^c)\frac{Q_t K_{t+1}}{GVA_t}}\mathbb{E}_t\left[\frac{GVA_{t+1}}{GVA_t}\right] - \frac{(1-\tau_t^c)}{(1-\lambda\tau_t^c)}\delta_t\bar{g}_Q + \bar{g}_Q. \tag{33}$$

We will explore a simple theory for investment, which is that firms invest up to the point that the expected return to buying one more unit of capital is equal to the expected return to the enterprise value the corporate sector. The expected return to enterprise value from the finance-style valuation model is

$$1 + \mathbb{E}_t[r_{t+1}^V] = \mathbb{E}\left[\frac{V_{t+1} + FCF_{t+1}}{V_t}\right] = \mathbb{E}\left[\frac{p_{t+1} + d_{t+1}}{p_t}\frac{GVA_{t+1}}{GVA_t}\right].$$

Our valuation model gives expected values for p_{t+1} and d_{t+1} that are independent of expected growth in value added, so using equation (15) expected returns can be written as

$$1 + \mathbb{E}_t[r_{t+1}^V] = \frac{\frac{p_t - \phi_t}{\beta} + \rho\phi_t + (1-\rho)\bar{\phi}}{p_t}\mathbb{E}_t\left[\frac{GVA_{t+1}}{GVA_t}\right]. \tag{34}$$

We now compute, year by year, the value for expected growth in value added that equates expected returns to enterprise value and to capital. Equating the expressions for $\mathbb{E}_t[r_{t+1}^K]$ and $\mathbb{E}_t[r_{t+1}^V]$ (equations 33 and 34) gives the following solution for that expected growth rate.

$$\mathbb{E}_t\left[\frac{GVA_{t+1}}{GVA_t}\right] = \bar{g}_Q\left(1 - \frac{1-\tau_t^c}{1-\lambda\tau_t^c}\delta_t\right)\left[\frac{\frac{p_t-\phi_t}{\beta} + \rho\phi_t + (1-\rho)\bar{\phi}}{p_t} - \frac{\alpha\left[(1-\tau_t^c)(1-\tau_t^s) - \kappa_t + \tau_t^L\right]}{(1-\lambda\tau_t^c)\frac{Q_t K_{t+1}}{GVA_t}}\right]^{-1} \tag{35}$$

Given this series for expected growth in value added, we can immediately construct expected returns. Note that higher values for α increase expected returns to capital (equation 33) which in turn increase expected growth via equation 35. Our chosen value of $\alpha = 0.29$ implies an average expected growth rate for real value added of 2 percent.

There are two ways in which we can test whether the resulting series for expected growth in value added is plausible. First, does the resulting series for expected returns to capital

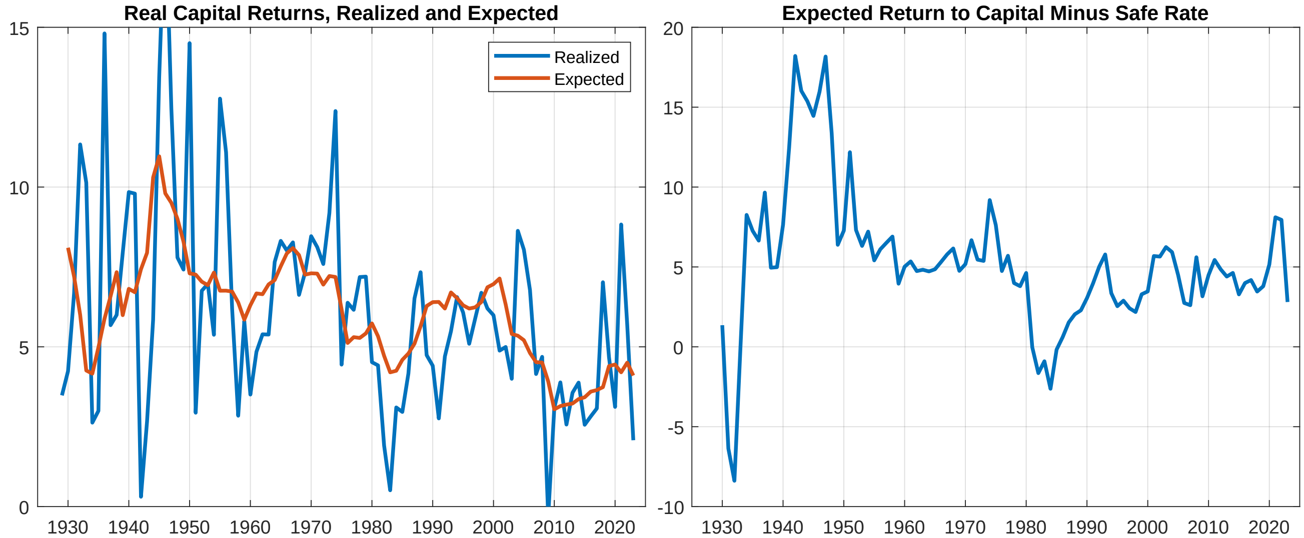

Figure 14: Left Panel: Realized Real Returns to Capital and Expected Real Returns to Capital, 1929-2023. For each year t we plot the realized return between $t-1$ and t and the return expected at $t-1$. Expected returns to enterprises are identical, year by year, to expected returns to capital. Right Panel: Expected Returns to Capital (Enterprises) Minus One-Year Treasury Rate. The gross nominal return on Treasuries between $t-1$ and t is divided by the gross growth rate of the PCE deflator between $t-1$ and t.

(equivalently to enterprises) appear plausible given the time path for realized returns to capital? Second, does expected growth in value-added look plausible given the path for realized growth in value added?

The left panel of Figure 14 plots realized real returns to capital (equation 31) alongside expected returns, computed by substituting equation (35) into (33). Expected returns track realized returns quite closely and have generally declined over time,

The right panel of Figure 14 plots the difference between the expected real return to capital (which is identical to our estimate return to enterprise value) and the inflation-adjusted rate of return on one-year Treasury Securities. Expected returns to the U.S. corporate sector appear very volatile relative to risk free rates during the Great Depression and World War II, but the return differential appears fairly stable at around 5 percentage points during the 1960s and 70s. In the early 1980s the differential fell, as the Federal Reserve pushed up short term rates to combat inflation. But from the 1990s onward, the differential again appears fairly stable at around 5 percent.

We conclude that our model yields a plausible time path for realized and expected after-tax returns to capital, with a downward trend in expected real returns over time that broadly tracks measured declines in safe rates.

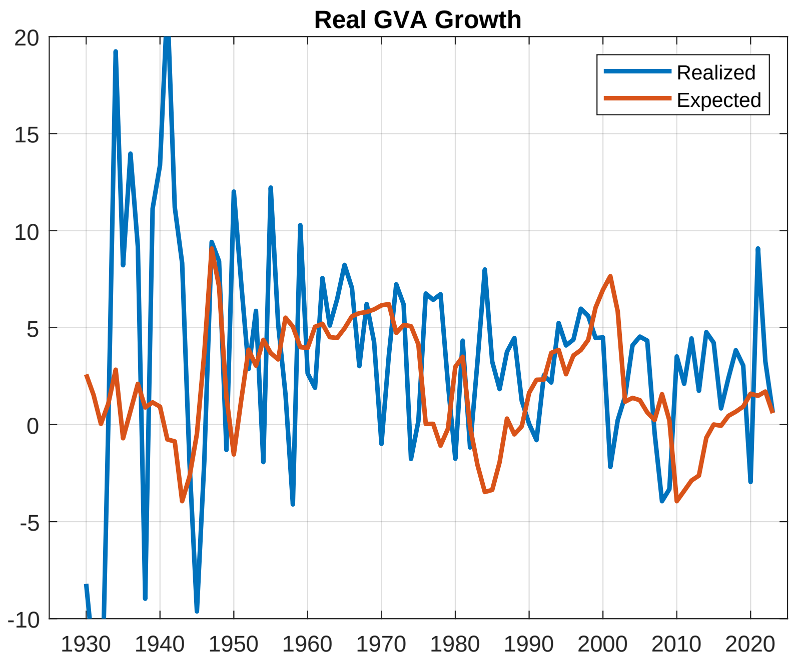

Figure 15: Right Panel: Realized and Expected Growth in Real Gross Value Added. For each year t we plot the realized growth rate between $t-1$ and t and the growth rate expected at $t-1$.

Figure 15 plots expected real growth in gross value added against realized growth. Actual growth rates have been volatile, especially in the early part of our sample period. The path for expected growth inferred through the model points to strong expected growth from the mid 1950s through the 1960s, followed by a slowdown in expected growth in the 1970s and early 1980s. Expected growth rises again from the mid 1980s through the 1990s, before slowing in the 2000s.

To provide some intuition regarding how expected returns and expected growth are connected, consider the year 2000, just before the dot-com bubble burst. At that time, enterprise values were high relative to free cash flow (see the right panel of Figure 8). Through the lens of our finance valuation model, that is interpreted, in part, as reflecting a high value for the residual component of price ϕ_t, and thus a low value for expected enterprise returns in excess of expected growth in GVA (Figure 10). Low expected returns in excess of expected growth could, in turn, reflect either low expected returns, or strong expected growth. Equating expected returns to capital and enterprise value within the model resolves this decomposition in favor of a high expected growth rate for value added in 2000 (Figure 15).[27] The model

[27]Low expected enterprise returns in excess of expected growth require equally low expected returns to capital in excess of expected growth. Faster expected growth narrows the gap between expected returns to capital and expected growth because faster expected growth does not impact the (large) component of

favors that interpretation because low expected returns to capital are inconsistent with a relatively stable observed capital stock (Figure 13). In fact, the model infers such strong expected growth in value added in 2000 that expected real returns (the left panel of Figure 14) are actually quite high in 2000 relative to what came after that date.

Given paths for expected returns and expected output growth, we can invert equation (33) to compute the impact on the equilibrium capital stock of time variation in taxes, in the depreciation rate δ_t, and in the factorless income share κ_t. We conduct such an exercise in Appendix B.5, where we show that time variation in these parameters have large impacts on the equilibrium capital stock. These calculations suggest that the relative stability of the capital to output ratio in the data reflects the fact that different factors have tended to work in offsetting directions. For example, since 2000 lower expected returns have boosted the desired capital stock, but that effect was offset by faster depreciation and a rising share of factorless income.

6.2 Counterfactuals

A key reason why we are able to reconcile expected returns from our valuation model with the dynamics of investment in our macroeconomic model is that the valuation model attributes a relatively small share of the observed fluctuations in the value to cash flow spread to fluctuations in the ϕ_t parameter, which drives fluctuations in expected returns. To illustrate the importance of this finding of relatively little time variation in expected returns for our reconciliation of returns to enterprise value and returns to capital, we now explore two counterfactuals. In the first, we set $\psi = 0$, thereby attributing all fluctuations in the enterprise value to free cash flow spread for the U.S. corporate sector to time-varying expected cash flow growth. In the second, we set $\psi = 1$, thus attributing all fluctuations in the enterprise value to free cash flow spread to time-varying expected returns. From equation (11) these assumptions translate into different paths for ϕ_t and thus into alternative paths for expected growth in real value added and for expected returns.

In the model with $\psi = 0$, the path for expected returns is similar to the one in our baseline model, and the path for expected growth in value added tracks expected returns closely. Note that if we were in addition to impose $\bar{\phi} = 0$, then equation (34) would collapse to $1 + \mathbb{E}_t\left[r^V_{t+1}\right] = \frac{1}{\beta}\mathbb{E}_t\left[\frac{GVA_{t+1}}{GVA_t}\right]$ so that the two expectations would be perfectly correlated. The $\psi = 0$ series for expected returns and for expected growth appear plausible. But recall that the data exhibit some predictable variation in returns in excess of growth in GVA, which is inconsistent with $\psi = 0$.

returns to capital that reflects the value of undepreciated capital.

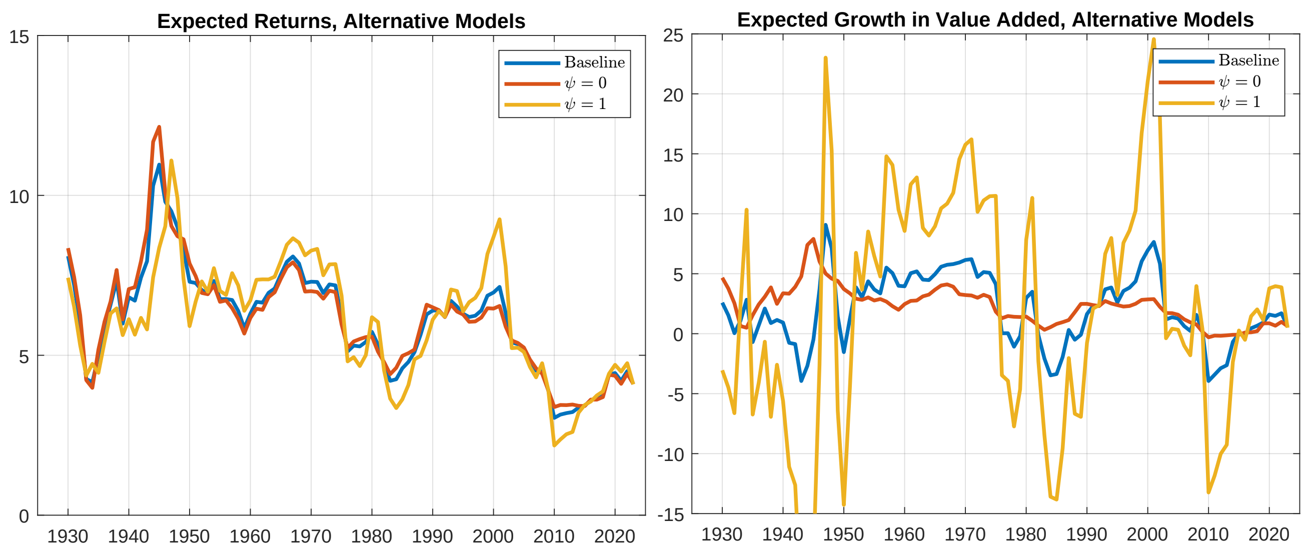

Figure 16: Left Panel: Expected Real Returns to Capital Under Alternative Valuation Models. Right Panel: Expected Growth for Real Gross Value Added Under Alternative Valuation Models. The blue lines correspond to our baseline model. The red lines correspond to a model in which fluctuations in the spread between enterprise value and free cash flow are entirely driven by fluctuations in expected cash flow growth. The yellow lines correspond to a model in which fluctuations in the spread are entirely driven by fluctuations in expected returns in excess of growth in value added.

The model with $\psi = 1$ attributes all fluctuations in the value to cash flow spread and all fluctuations in expected quasi-returns to fluctuations in ϕ_t. This model – which might be thought of as approximating the conventional view in asset pricing – translates to a very volatile series for expected returns in excess of expected growth in value added, and thus either expected returns or expected growth (or both) must be volatile to reconcile the observed stability of the capital output ratio with the Euler equation for capital with these returns. When we identify the relative contributions of these two sources of fluctuations by equating expected returns to capital and enterprise value, the model points to extremely volatile expected growth. It is easy to understand why. If expected growth did not vary much, then the model with $\psi = 1$ would point to high volatilty in expected returns. But high volatility in expected returns is inconsistent, within the macroeconomic model, with a relatively stable observed ratio of capital to output.

We view the yellow time series for expected growth as implausible. Thus, we view the alternative model in which fluctuations in the value to cash flow spread are driven by expected returns as being inconsistent with our macroeconomic model – in addition to its being inconsistent with the strong predictability of cash flow growth observed in the IMA data.

7 Conclusion

How are the dynamics of corporate valuations and returns connected to the dynamics of value added, investment and the capital stock? The asset pricing literature has generally attributed most of high observed volatility in corporate valuations to time-varying expected returns to buying corporate equity. But the corporate capital stock is famously stable, suggesting a relatively smooth path for expected returns to investment in physical capital.

In this paper we have interpreted aggregate financial and macroeconomic data for the U.S. corporate sector using two models: a variant of a standard asset pricing model, and a variant of a standard stochastic growth model. The key finding from our valuation model is that changes in expected cash flow – not changes in expected returns – have been the key driver of the observed time path for corporate valuations over the period 1929 to 2023. The key finding from our macroeconomic model is that the observed time path for investment in physical capital (and thus the observed path for the capital stock) is consistent with the path for expected returns inferred from our asset pricing model. Thus, there is no inconsistency between the high observed volatility of valuations, and the stable time path for the capital to output ratio.

We emphasize that we were able to reconcile macroeconomics and finance in this way thanks to special features of both our macroeconomic and asset pricing models.

With respect to our valuation model, an important reason why we end up interpreting the source of pricing volatility differently to much of the previous literature is that we emphasize a broad free cash flow measure of income to asset owners. This measure maps naturally into dividend income in our macroeconomic model, and it is not distorted by historical changes in the share of firms choosing to pay dividends. Free cash flow is strongly correlated with corporate valuation at low frequencies, and we show that a simple model in which changing expectations of long-run free cash flow are the main driver of valuations is consistent with a rich set of moments involving the joint dynamics of returns and cash flow growth.

Our macroeconomic model introduces a wedge between firm revenue and costs that delivers factorless income that accrues neither to labor nor capital. Fluctuations in the share of factorless income generate fluctuations in firm value that do not coincide with fluctuations in the value of capital in place.

There are two related reasons why we find volatile valuations and a relatively stable capital to be jointly consistent with the same time path for expected returns. First, our valuation model implies a relatively smooth path for expected returns, and small changes in required expected returns translate into small changes in desired investment. Second, shocks to the share of factorless income in our model generate large percentage changes in valuation, but only small percentage changes in capital's share of income and in desired investment.

One contribution of our paper has been to construct time series for expected rates of return, for expected growth in value added, and for the shares of income accruing to labor, capital and factorless income that are consistent with the observed time series for valuations and for a range of macro aggregates. Given all these inputs, we can use our models to interpret historical developments.

In the post Great Recession period, corporate valuations have soared. Over the same period, free cash flow has also risen sharply, and our model attributes the observed rise in valuations primarily to a similar-sized rise in long run expected free cash flow, coupled with a modest decline in expected returns in excess of expected output growth.

The model identifies a stable capital to output ratio in recent decades as reflecting the largely offsetting impacts of different factors. Two factors boosting desired capital have been declining expected returns, which have fallen in tandem with safe rates, and lower corporate tax rates, which were cut in 2018. Two factors working in the opposite direction were a rising average depreciation rate for capital, and a rising share of factorless income in value added, which depressed capital's share of income.

Prior to World War II, the ratio of measured capital to value added moved by large amounts. In fact, the observed movements in the stock of measured capital relative to GVA in this early period were so large that they account for much of the observed variation in the ratio of enterprise value to value added, as would be indicated by standard theory. However, our estimates of expected excess returns during the period before the end of World War II are quite volatile. Is this finding due to estimation error in the turbulent periods of the Great Depression and World War II? Or did expected excess returns actually vary a great deal during this time period? We see the resolution of these questions as a subject for future research.

While we believe our paper has made useful progress in integrating our understanding of valuations and returns with the dynamics of investment, a broader challenge is to simultaneously integrate those same valuations with the dynamics of consumption. It is possible that our finding that changes in valuations are primarily driven by changing expectations about future cash flow (rather than a time-varying risk premium) might prove helpful in the long-standing research agenda of reconciling volatile valuations with relatively stable aggregate consumption growth, just as we have found that narrative to be consistent with a relatively stable capital stock.

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Appendices

A Data Appendix

In this appendix we list the sources for the data used in this paper. We make reference to four main sources of information. There is often considerable overlap between these data sources.

A.1 Aggregate data

- Integrated Macroeconomic Accounts (IMA) Tables S5 and S6 for the Nonfinancial Corporate and Financial Sectors respectively. These tables present flows from 1946-2022 and end of year balance sheets from 1945 to 2022. These tables can be found towards the back of the publication Z1 *Financial Accounts of the United States* (previously known as the Flow of Funds). They can also be found on the website of the Bureau of Economic Analysis.

- National Income and Product Accounts (NIPA) Table 1.14. This table offers annual data on gross value added and the breakdown of income into components for the corporate sector from 1929 through 2022. We also refer to other tables from the NIPA and refer to them with this abbreviation.

- Fixed Assets (FA) Tables 6.1, 6.4, and 6.7 which offer data on investment, consumption of fixed capital (depreciation), and year-end capital stocks for the non-financial corporate and financial sectors from 1929 to the present.

- Various tables from the *Financial Accounts of the United States* which we refer to by the abbreviation FOF and the table number.

We now describe our specific data sources.

The following series for the corporate sector 1929-2023 are taken from NIPA Table 1.14. These data series are also available broken down for the non-financial corporate and financial sectors separately on NIPA Table 1.14 and on IMA Tables S5 and S6. These tables are updated on different schedules, so the source with the most up to date data depends on the time of year. Small differences between these two data sources may exist due to different accounting standards for the NIPA and the IMA. We list the line and table numbers for these series below.

- Gross Value Added. NIPA Table 1.14 Line 1. See also IMA Tables S5 FA106902501.A and S6 FA796902505.A

- Tax Payments are the sum of three lines from NIPA Table 1.14. These are line 7, Taxes on production and imports less subsidies, line 10, Business current transfer payments (net), and line 12, Taxes on corporate income. See also IMA Tables S5 FA106240101.A, FA106403001.A, FA106220001.A and S6 FA796240101.A, FA796403005.A, FA796220001.A

- Compensation of Employees. NIPA Table 1.14 Line 4. See also IMA Tables S5 FA106025005.A, FA796025005.A

- Consumption of Fixed Capital. NIPA Table 1.14 Line 2. See also IMA Tables S5 FA106300003.A, FA796330081.A

We obtain data on investment expenditures (gross fixed capital formation) by the corporate sector from two sources listed below. Small differences between the data on Fixed Assets Table 6.7 and IMA Tables S5 and S6 for the period for which they overlap are due to different accounting standards for the two accounts.

- Investment 1929 - 1945. Fixed Assets Table 6.7 line 2

- Investment 1946 - 2022. The sum of IMA Tables S5 line FA105019085.A and S6 line FA795013005.A

We obtain data on the reproduction value of the capital stock in the corporate sector from two sources listed below. It is important to note that the value of nonfinancial assets listed on the balance sheets of Tables S5 and S6 include measures of the value of land, which we exclude from our model. Thus, we do not use those measures. Instead we use the following sources that are restricted to fixed assets.

- Capital 1929 - 1944. Fixed Assets Table 6.1 line 2

- Capital 1945 - 2023. FOF Table L4. Sum of lines FL105015085.A and FL795013865.A

We obtain data on enterprise value of the corporate sector from two sources.

- For the period 1945 - 2023, we use balance sheet data from IMA Tables S5 and S6. These series are constructed for the Nonfinancial corporate sector as the difference between the line Total Liabilities and Net Worth minus the line titled Financial Assets. The construction is the same for the Financial Sector using Table S6. These series for enterprise value are reported on FOF Table B1 in lines LM102010405.A and LM792010405.A. We use these series from B1.

- For the period 1929 - 1944 we use data from the 1945 Statistics of Income Part 2 available here `https://www.irs.gov/pub/irs-soi/45soireppt2ar.pdf`. We use data from Table 20 on page 420 of this document (page 425 of the PDF). For financial assets, we use Total Assets on line 9 less Capital Assets on line 7. For liabilities, we use Total Liabilities on line 21 less Capital Stock Common on line 17. For the market value of equity, we use the total market capitalization for the CRSP Value Weighted Index.

We use two sources of data on the market value of corporate equities. These are

- 1929-1944: CRSP Value Weighted Index Total Market Capitalization

- 1929-1944: FOF Table L224 Nonfinancial Corporate Equities LM103164105.A for both public and closely held and LM103164115.A for public alone plus Financial Sector Corporate Equities LM793164105.A less equities issued by Closed End Funds from FOF Table L123 LM554090005.A and by ETFs from FOF Table L124 FA564090005.A.

We use two sources of data on dividend payments. We pay particular attention to distinguishing between dividends as reported in the IMA (and other places) and monetary dividends paid. One of the big distinctions between these two concepts concerns the treatment of dividends on foreign direct investment which are an accounting entry and not a measure of dividends paid. Note, however, that our measure of dividends does include cash dividends paid on foreign direct investment in the U.S. Our sources are

- 1929 - 1945. NIPA Table 7.16 line 31 (Dividends paid in cash or assets, IRS), plus line 32 (Post tabulation amendments and revisions), less line 33 (Dividends paid by Federal Reserve Banks).

- 1946 - 2022. NIPA Table 7.10 line 2 (Monetary Dividends Paid Domestic Corporate Business) less line 4 (Paid by Federal Reserve Banks).

We use two sources of data for returns on corporate equities.

- 1929-1945. Returns without dividends are set equal to returns without dividends on the CRSP Value Weighted Total Market Index. The dividend return is computed as the ratio of dividends paid in year $t+1$ to the value of corporate equities in year t.

- 1946 - 2022. Returns without dividends are set equal to the ratio of the sum from Table S5 of revaluations of nonfinancial corporate equities FR103164105.A and non financial foreign direct investment in the U.S. FR103192105.A and from Table S6 revaluations of financial corporate equities FR793164105.A and financial foreign direct investment in the U.S. FR793192105.A to the sum of the corresponding levels of these variables at the end of the previous year: from S5 LM103164105.A + FL103192105.A and from S6 LM793164105.A + LM793192105.A. Returns with dividends adds to this return the ratio of dividends in year t to the value of corporate equities plus FDI equity in the U.S. in year $t-1$.

A.2 Compustat data

For producing the Compustat lines in Figure 5 we restrict the Compustat sample as follows. First we select only firms incorporated in the United States, we then drop observations for firms which report their 10k in financial services format, observations for which there is no year information or for which a firm year is duplicated. That leaves us with a sample of 316562 firm/year observations over the 1988-2023 period. Free cash flow is computed (following Adame et al. 2023) as Operating Activities–Net Cash Flow (OANCF) minus capital expenditures (CAPX). Enterprise value is computed as Total Market Value (MKVALT) plus Total Liabilities (LT) minus current assets total (ACT), which includes cash and other short term investments, receivables, inventories, and other current assets.

B Additional Data Plots

In this appendix, we include additional data plots.

B.1 Equity Valuation in the IMA Data

We can use the IMA to construct a market valuation of the equity of U.S. corporations (both publicly traded and closely held corporations) and a corresponding cash flow measure of monetary dividends paid to the owners of these corporations. We include these alternative measures of valuation and cash flow to facilitate comparisons between the IMA data and work using data from CRSP and Compustat for publicly traded firms. We show the IMA measures for the value of equity and for dividends relative to gross value added for the U.S. corporate sector in red in the left and right panels of Figure B.1. Our measures of enterprise value and free cash flow are in blue.

We see in the left panel of Figure B.1 that the fluctuations in the market value of equity and enterprise value for U.S. corporations are tightly linked. By comparing the different scales for enterprise value (left axis) and equity (right axis), we see that enterprise value is consistently about 50 percentage points of gross value added larger than the market value of equity. This difference between enterprise value and equity value corresponds to net debt of the U.S. corporate sector.

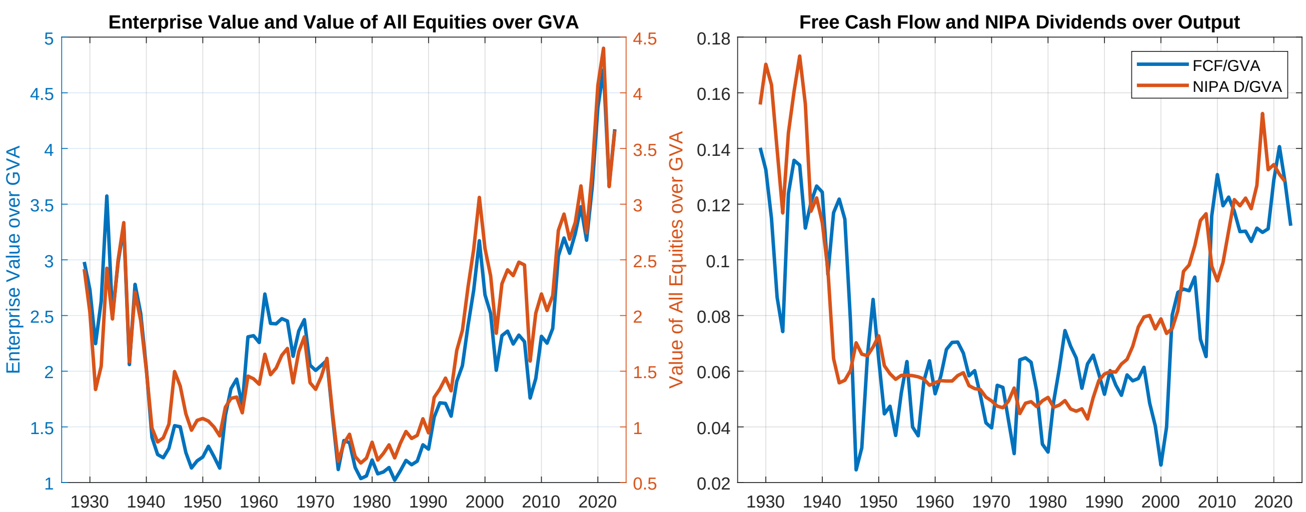

Figure B.1: Left Panel: Enterprise Value (left axis) and Equity Value (right axis) of U.S. Corporations over Corporate Gross Value Added. Right Panel: Free Cash Flow and NIPA Monetary Dividends Paid over Corporate Gross Value Added. 1929-2023. NIPA Dividends Paid data are not yet available for 2023

The IMA imputes the market value of closely held equity from data on the market value of public equities. In Figure B.2, we show the IMA measures of Enterprise Value over corporate GVA and the Market Value of Public Equities over corporate GVA.

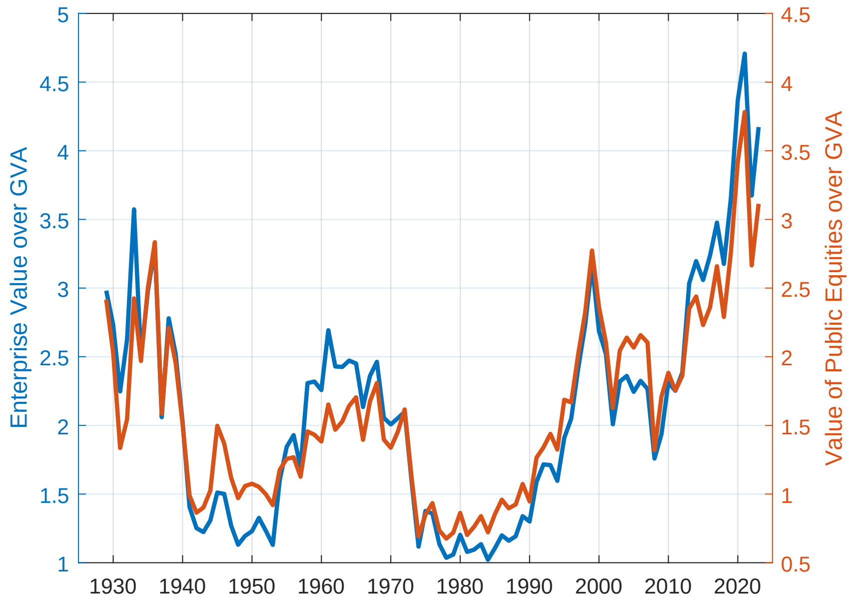

Figure B.2: Enterprise Value (left axis) and the Market Value of Corporate Public Equities (right axis) over Gross Value Added. 1929-2023

B.2 Equity Returns in the IMA Data

We can use the IMA data to compute realized returns on equity from the perspective of a household that purchases equity at the end of period t at price V_t^E, collects dividend payments in year $t+1$, D_{t+1}^{IMA}, and sells that equity realizing a capital gain corresponding to the IMA reported revaluation of outstanding equity at $t+1$, $REVAL_{t+1}^E$. We compute this realized return as

$$\exp(r_{t+1}^E) = \frac{D_{t+1}^{IMA} + REVAL_{t+1}^E + V_t^E}{V_t^E}$$

This calculation of returns is closer to what is done in CRSP or Standard and Poors' data for public firms. We report basic statistics for these IMA returns to equity in Table B.1

In Figure B.3 we examine the extent to which these measures of realized real returns on enterprise value and on IMA equity line up with measures of realized real returns computed using the CRSP value-weighted portfolio. In the left panel, we show a scatter plot of realized annual returns on the CRSP portfolio on the x-axis and returns on enterprise value on the y-axis. The red line is the 45 degree line. We show the corresponding scatter plot for CRSP returns and realized returns on IMA equity in the right panel. We show the same scatter plots using data from the 1946-2022 time period in Appendix Figure B.4. The correlation of returns on enterprise value with those on the value-weighted CRSP portfolio is 0.943 for the period 1929-2023. The corresponding correlation for IMA equity returns with CRSP returns is 0.981 for 1929-2022. Note that one would expect some deviation of returns on enterprise value from returns on equity given the presence of net debt documented in the left panel of Figure B.1.

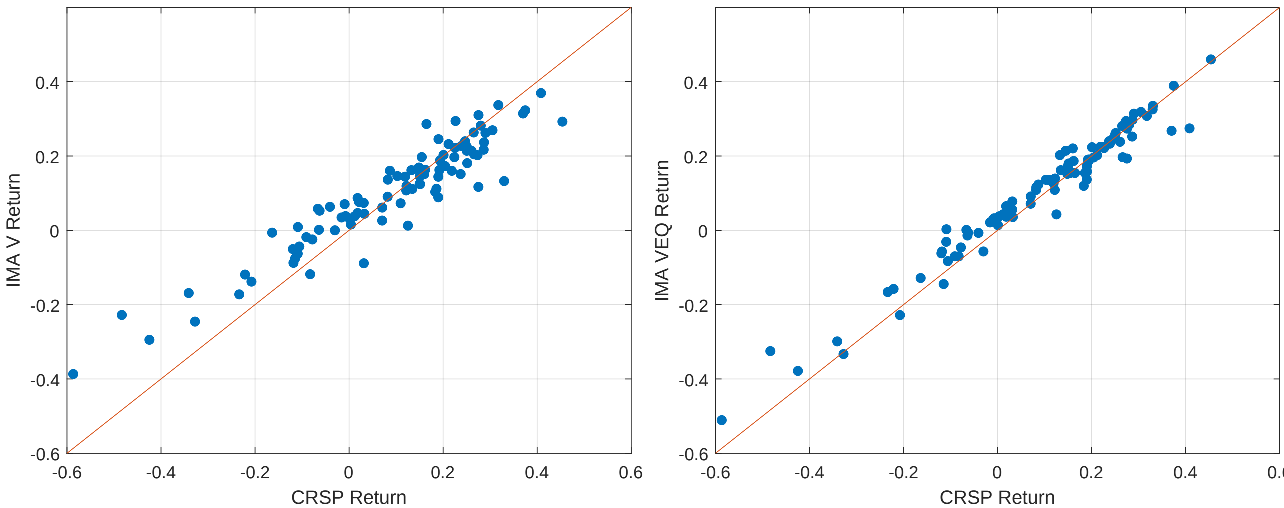

Figure B.3: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value-Weighted Return 1929-2023. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value-Weighted Return: 1929-2022

Table B.1: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value-Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1929-2023	0.073	0.146	0.280
IMA Equity	1929-2022	0.076	0.173	0.073
CRSP VW	1929-2023	0.062	0.193	0.138

The Integrated Macroeconomic Accounts start with measures of end of year balance sheet items in 1945. In this section, we report statistics computed using only the data from these accounts.

Table B.2: Mean and Standard Deviation of Real Log Returns and Log Dividend Growth on Enterprise Value, IMA Equity, and CRSP Value Weighted Portfolio

Return	Time Period	Mean Return	Std Return	Std D growth
Enterprise Value	1946-2022	0.08	0.132	0.279
IMA Equity	1946-2022	0.082	0.15	0.15
CRSP VW	1946-2022	0.069	0.172	0.132

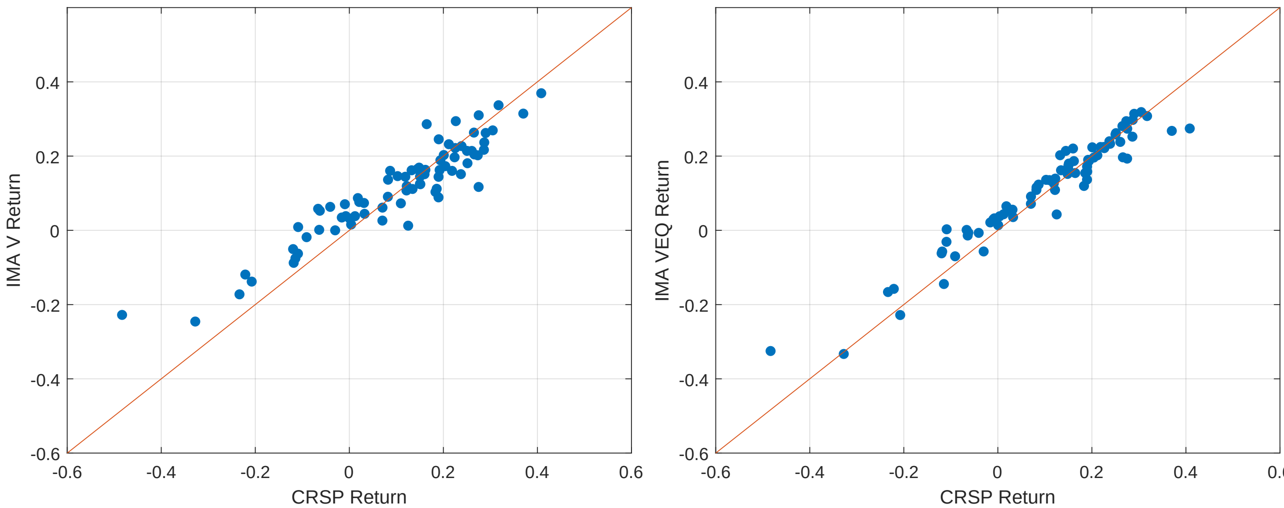

Figure B.4: Left Panel: Realized Returns on Enterprise Value vs. CRSP Value Weighted Return 1946-2023. Right Panel: Realized Returns on IMA Equity Value vs. CRSP Value Weighted Return: 1946-2022

B.3 Drivers of Free Cash Flow to Capital

Figure B.5 plots free cash flow to capital as a share of gross value added and the (negative) contribution of capital investment to free cash flow to capital. There is a strong correlation between these two series, indicating that rising investment (as a share of value added) can mechanically account for most of the observed decline in free cash flow to capital. This rising investment has not translated into an upward trend in the replacement value of the capital stock as a share of value added because the depreciation rate for capital has also been rising over time (see the dashed line in the figure).

B.4 Drivers of Total Free Cash Flow

Given our assumed tax structure, total corporate free cash flow is

$$FCF_t = (1 - \tau_t^s)GVA_t - W_tL - Q_tI_t - Taxes_t^c. \tag{36}$$

Measuring total free cash flow for the corporate sector using equation (36) is straightforward given the IMA series for gross-value added, compensation, investment, and corporate taxes. Figure B.6 documents the contributions of these different components in accounting for observed changes in total free cash flow. To make the plot easier to read, we measure deviations of each component from their sample average, and filter the series to remove high frequency fluctuations. The message of this figure is that changes in labor's share of income, changes in investment, and changes in corporate taxes are all important drivers of total corporate free cash flow. Note, first, that the well-documented decline in labor's share of value-added in the post 2000 period accounts for essentially all of the observed increase in free cash flow over that period. At the start of the sample period, the key reason cash flow

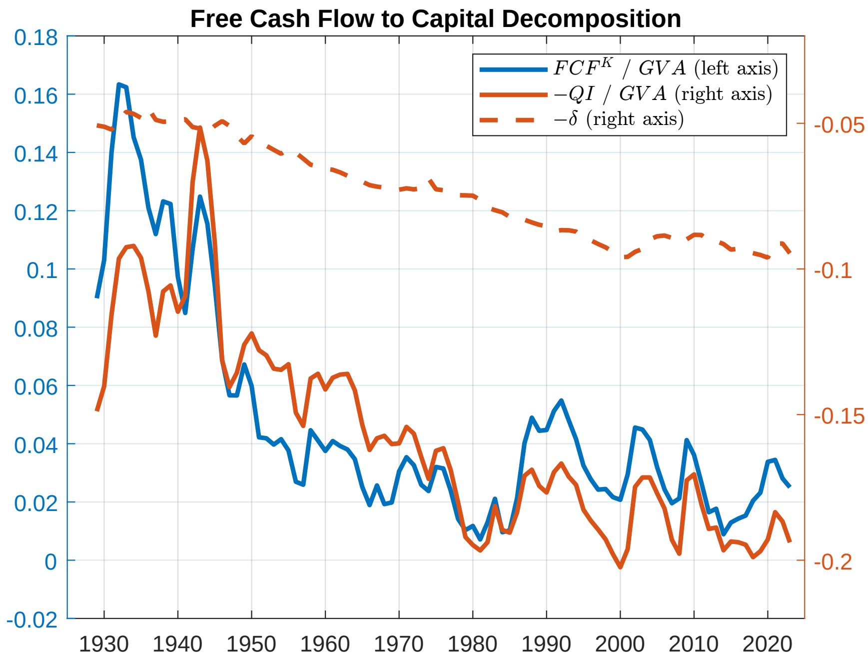

Figure B.5: Free Cash Flow to Capital and the Contribution of Investment.

declines is that corporate taxes paid rise. Between the end of World War II and 2000, labor's share of income is relatively stable. Investment rises steadily as a share of value added, reducing free cash flow, but this trend is largely offset by a declining corporate tax burden.

B.5 Equilibrium Drivers of Capital Stock

Figure B.7 plots the observed sequence for the replacement cost of capital as a share of value added, $Q_t K_{t+1}/GVA_t$ in our baseline model (which perfectly replicates the observed path in the IMA) and the equilibrium path in several counterfactuals.

In each of these counterfactuals, the time path for expected returns in excess of expected growth in value-added is identical to the path we identified from our valuation model. In each counterfactual the path for expected growth in real value-added is also identical to that in our baseline.

In the top right panel we plot the path for the capital stock that would have delivered the baseline sequence for expected returns had all tax rates (and expected tax rates) remained constant at their 1929 values. Corporate income tax rates were low in 1929, and had they (counterfactually) remained low, the model predicts we would have seen a higher capital stock throughout our simulation period.

In the bottom left panel we plot the equilibrium path for capital in a scenario in which the factorless income share κ_t remains constant at its 1929 value (which was 5.0 percent). In the post 2000 period – when the actual factorless income share was rising – this counterfactual delivers slightly more equilibrium growth in the capital stock than observed in the data.

In the bottom right panel we plot the equilibrium path for capital in a scenario in which the depreciation rate δ_t remains constant at its 1929 value. In this scenario, we observe much

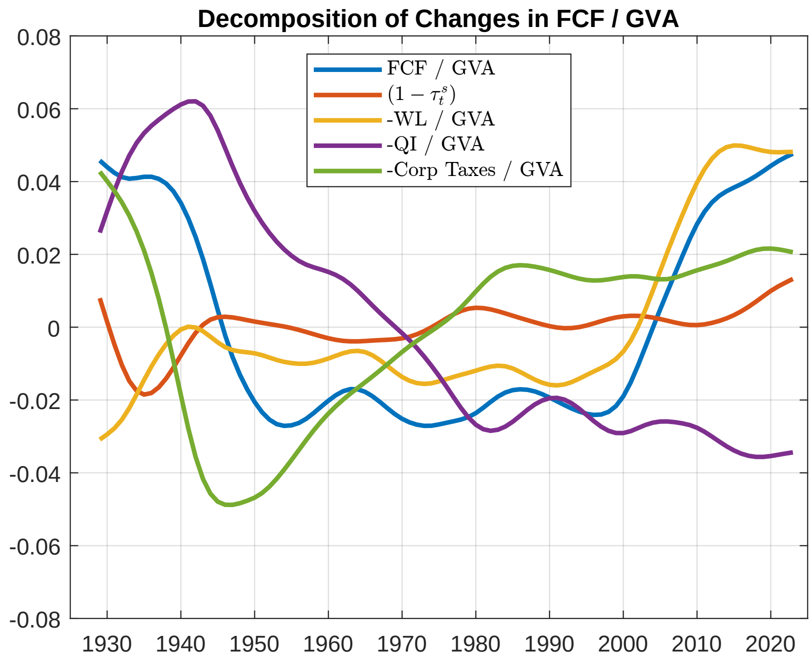

Figure B.6: Decomposition of Total Corporate Free Cash Flow. For each component, we plot the Hodrick-Prescott trend value (smoothing parameter = 100) for the difference of the component from its sample mean.

stronger growth in the equilibrium capital stock from 1960 onward. We conclude that rising depreciation rates during that period substantially dampened growth in desired capital.

C Appendix on Effective Tax Rates

Let r_{t+1}^{net} and r_{t+1}^{pretax} denote net real returns to capital between t and $t+1$ including taxes and before taxes:

$$\begin{aligned} r_{t+1}^{net} &= \frac{(1-\tau_{t+1}^c)}{(1-\lambda_t\tau_t^c)}\frac{R_{t+1}-Q_{t+1}\delta_{t+1}}{Q_t} + \frac{(1-\lambda_{t+1}\tau_{t+1}^c)}{(1-\lambda_t\tau_t^c)}\frac{Q_{t+1}}{Q_t} - 1, \\ r_{t+1}^{pretax} &= \frac{R_{t+1}-Q_{t+1}\delta_{t+1}}{Q_t} + \frac{Q_{t+1}}{Q_t} - 1. \end{aligned}$$

Define the effective tax rate on capital at t as the value for τ_t^e that satisfies

$$\mathbb{E}_t\left[M_{t,t+1}\left(1+(1-\tau_t^e)r_{t+1}^{gross}\right)\right] = \mathbb{E}_t\left[M_{t,t+1}\left(1+r_{t+1}^{net}\right)\right].$$

If we assume zero correlation between returns and the pricing kernel, we have

$$(1-\tau_t^e)\mathbb{E}_t\left[r_{t+1}^{pretax}\right] = \mathbb{E}_t\left[r_{t+1}^{net}\right]$$

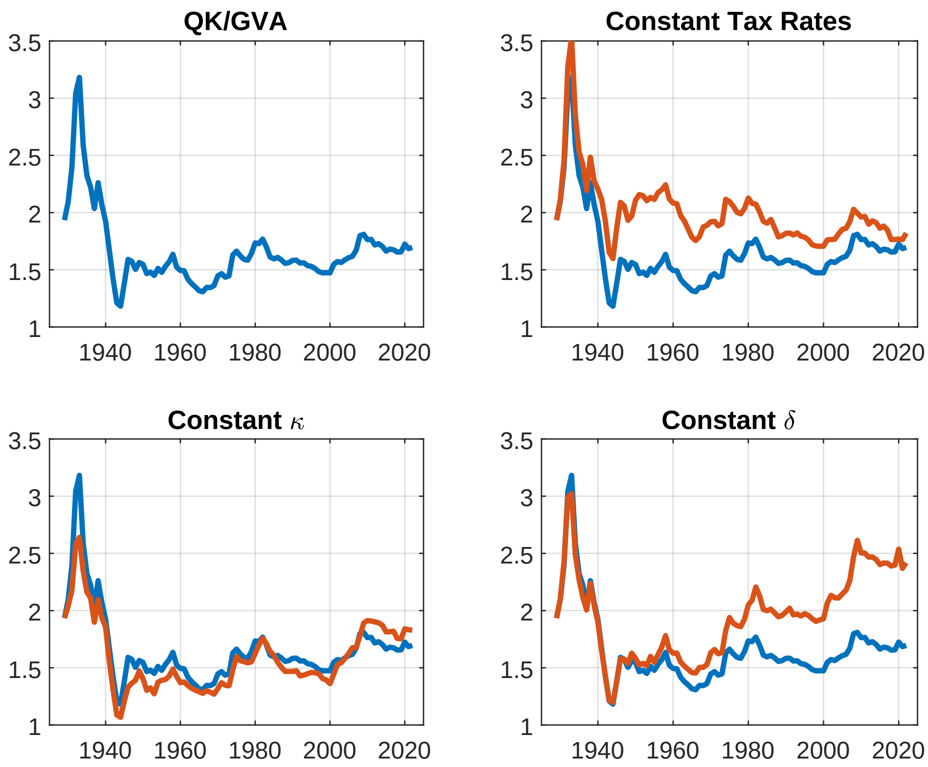

Figure B.7: Impact on the Equilibrium Capital Stock of Time Variation in Tax Rates, in the Factorless Income Share, and in the Depreciation Rate. In each panel the blue line shows the data, while the red line shows the counter-factual path had there been no time variation in the values of particular parameters.

and thus

$$\tau_t^e = \frac{\mathbb{E}_t\left[r_{t+1}^{pretax}\right] - \mathbb{E}_t\left[r_{t+1}^{net}\right]}{\mathbb{E}_t\left[r_{t+1}^{pretax}\right]},$$

which is the conventional way the marginal effective tax rate is defined (see, e.g., line 4 in Table 1 of Fullerton 1983).

In computing expected returns, we assume that all the tax parameters (τ_t^s, τ_t^c, τ_t^L and λ_t) are expected to remain unchanged between t and $t+1$.

Then

$$\begin{aligned}
\mathbb{E}_t\left[r_{t+1}^{net}\right] &= \frac{(1-\tau_t^c)}{(1-\lambda_t\tau_t^c)}\left(\frac{\mathbb{E}_t\left[\frac{R_{t+1}K_{t+1}}{GVA_{t+1}}\frac{GVA_{t+1}}{GVA_t}\right]}{\frac{Q_tK_{t+1}}{GVA_t}} - \mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\delta_{t+1}\right]\right) + \mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right] - 1, \\
\mathbb{E}_t\left[r_{t+1}^{pretax}\right] &= \left(\frac{\mathbb{E}_t\left[\frac{R_{t+1}K_{t+1}}{GVA_{t+1}}\frac{GVA_{t+1}}{GVA_t}\right]}{\frac{Q_tK_{t+1}}{GVA_t}} - \mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\delta_{t+1}\right]\right) + \mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right] - 1.
\end{aligned}$$

Plugging these expressions into the formula for the effective tax rate gives

$$\tau_t^e = \frac{\left(\frac{\tau_t^c - \lambda_t\tau_t^c}{1-\lambda_t\tau_t^c}\right)}{1 + \frac{\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]-1}{\mathbb{E}_t\left[r_{t+1}^{pretax}\right]+1-\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]}}$$

When the expected gross growth rate for the relative price of investment is equal to one, this expression simplifies to

$$\tau_t^e = \frac{\tau_t^c - \lambda_t\tau_t^c}{1-\lambda_t\tau_t^c}.$$

Note that when $\lambda = 0$ (no expensing for net investment), the effective tax rate is equal to the statutory one: $\tau_t^e = \tau_t^c$.

When $\lambda = 1$ (full expensing for net investment), the effective tax is zero.

The series for τ_t^e plotted in Figure 11 is constructed assuming that $\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right]$ is constant and equal to the average observed value over our sample period. The expected net pre-tax interest rate $\mathbb{E}_t\left[r_{t+1}^{pretax}\right]$ is computed as described in the text.

D Appendix on Value of Capital

The firm that owns and manages the physical capital stock takes as given an initial capital stock K_t and chooses future capital $\{K_{t+k}\}$ and after-tax free cash flow payable to owners $\{FCF_{t+k}^K\}$ for $k \geq 1$ to maximize

$$FCF_t^K + V_t^K$$

where

$$V_t^K = \sum_{k=1}^{\infty}\mathbb{E}_t\left[M_{t,t+k}FCF_{t+k}^K\right]$$

and

$$FCF^K_{t+k} = (1-\tau^c_{t+k})\left(R_{t+k} - Q_{t+k}\delta_{t+k}\right)K_{t+k} - (1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}\left(K_{t+k+1} - K_{t+k}\right).$$

The first-order condition with respect to K_{t+k+1} is

$$\begin{aligned}&\mathbb{E}_{t+k}\left[M_{t,t+k+1}\left[(1-\tau^c_{t+k+1})\left(R_{t+k+1} - Q_{t+k+1}\delta_{t+k+1}\right) + (1-\lambda_{t+k+1}\tau^c_{t+k+1})Q_{t+k+1}\right]\right]\\ &= M_{t,t+k}(1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}.\end{aligned} \tag{37}$$

Multiplying through by K_{t+k+1} gives

$$\begin{aligned}&\mathbb{E}_{t+k}\left[M_{t,t+k+1}\left[(1-\tau^c_{t+k+1})\left(R_{t+k} - Q_{t+k+1}\delta_{t+k+1}\right)K_{t+k+1} + (1-\lambda_{t+k+1}\tau^c_{t+k+1})Q_{t+k+1}K_{t+k+1}\right]\right]\\ &= M_{t,t+k}(1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}K_{t+k+1}.\end{aligned} \tag{38}$$

The value of the firm managing the capital stock is

$$V^K_t = \sum_{k=1}^{\infty}\mathbb{E}_t\left[M_{t,t+k}\left[(1-\tau^c_{t+k})\left(R_{t+k} - Q_{t+k}\delta_{t+k}\right)K_{t+k} + (1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}K_{t+k} - (1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}K_{t+k+1}\right]\right]$$

Using the equation (38), we can see that the term $-M_{t,t+k}(1-\lambda_{t+k}\tau^c_{t+k})Q_{t+k}K_{t+k+1}$ cancels with $\mathbb{E}_t\left[M_{t,t+k+1}\left[(1-\tau^c_{t+k+1})R_{t+k+1}K_{t+k+1} + (1-\lambda_{t+k+1}\tau^c_{t+k+1})Q_{t+k+1}(1-\delta_{t+k+1})K_{t+k+1}\right]\right]$ and so on moving up through time. The logic is that the value of free cash flow at $t+k$ that is sacrificed to increase next period capital must equal the expected value of the income plus resale value of that capital at $t+k+1$.

The only term that is left is

$$V^K_t = \mathbb{E}_t\left[M_{t,t+1}\left[(1-\tau^c_{t+1})\left(R_{t+1} - Q_{t+1}\delta_{t+1}\right)K_{t+1} + (1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}K_{t+1}\right]\right]$$

or, using equation (38) again, we have

$$V^K_t = (1-\lambda_t\tau^c_t)Q_tK_{t+1} \tag{39}$$

Note that if $\lambda_t = 1$ (full expensing) then investment is not distorted, but firm value is depressed by $(1-\tau^c_t)$.

If $\lambda_t = 0$, then investment and capital depressed when $\tau^c_t > 0$, but the value of the firm is the replacement cost of its capital.

Note that one way to check that this is the right expression for valuation is to show that it gives the correct expression for the after-tax return to capital:

$$\begin{aligned}\frac{FCF^K_{t+1} + V^K_{t+1}}{V^K_t} &= \frac{(1-\tau^c_{t+1})\left(R_{t+1} - Q_{t+1}\delta_{t+1}\right)K_{t+1} - (1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}\left(K_{t+2} - K_{t+1}\right) + (1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}K_{t+2}}{(1-\lambda_t\tau^c_t)Q_tK_{t+1}}\\ &= \frac{(1-\tau^c_{t+1})\left(R_{t+1} - Q_{t+1}\delta_{t+1}\right)K_{t+1} + (1-\lambda_{t+1}\tau^c_{t+1})Q_{t+1}K_{t+1}}{(1-\lambda_t\tau^c_t)Q_tK_{t+1}}\\ &= \frac{(1-\tau^c_{t+1})\left(\frac{R_{t+1}}{Q_t} - \frac{Q_{t+1}}{Q_t}\delta_{t+1}\right) + (1-\lambda_{t+1}\tau^c_{t+1})\frac{Q_{t+1}}{Q_t}}{(1-\lambda_t\tau^c_t)}\end{aligned}$$

which is the same as the return expression we derived earlier.

E Deriving the Valuation Model

In this appendix we derive the relationship between our valuation model in equations (1) and (2) to the more standard asset pricing equations. We use the derivation to discuss our choice of β and our choice to scale our measures of price and cash flow by corporate GVA.

We begin with the standard valuation equation for the price of an asset before rescaling:

$$P_t = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} D_{t+k} \right], \tag{40}$$

where $M_{t,t+k}$ is the nominal pricing kernel between periods t and $t+k$, P_t is the nominal asset price and D_t is the nominal cash flow to the owner of the asset.

One might consider scaling price and cash flow by a wide range of scaling variables. For example, one might scale by the price level to study the dynamics of real prices and cash flows. One might also scale by a cumulation of risk free interest rates to study returns in excess of risk free rates. We choose to scale by nominal GVA of the corporate sector. Hence we have $p_t = V_t/GVA_t$ and $d_t = FCF_t/GVA_t$. We therefore rewrite this pricing equation (40) as

$$p_t = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} \frac{GVA_{t+k}}{GVA_t} d_{t+k} \right].$$

Using the result that the expectation of a product of two random variables is the product of the expectations plus the covariance between these variables, we have

$$p_t = \sum_{k=1}^{\infty} \mathbb{E}_t \left[M_{t,t+k} \frac{GVA_{t+k}}{GVA_t} \right] \mathbb{E}_t d_{t+k} + \sum_{k=1}^{\infty} \mathbb{C}ov_t \left(M_{t,t+k} \frac{GVA_{t+k}}{GVA_t}, d_{t+k} \right). \tag{41}$$

Note that the term

$$p^{(k)}_{GVA,t} \equiv \mathbb{E}_t \left[M_{t,t+k} \frac{GVA_{t+k}}{GVA_t} \right]$$

is the price at t of a claim to aggregate corporate GVA delivered at $t+k$ relative to corporate GVA at t. We define the price at t of a claim to aggregate corporate GVA in perpetuity relative to the current level of aggregate corporate GVA as

$$p_{GVA,t} \equiv \sum_{k=1}^{\infty} p^{(k)}_{GVA,t}.$$

The terms

$$H^{(k)}_t \equiv \mathbb{C}ov_t \left(M_{t,t+k} \frac{GVA_{t+k}}{GVA_t}, d_{t+k} \right) \tag{42}$$

constitute a risk adjustment to the price of claims to dividends due to risk associated with fluctuations in the ratio d_t.

Thus, our valuation model 1 and 2 can be derived from this standard asset pricing framework as long as one sets

$$\phi_t = \sum_{k=1}^{\infty} \left[p_{GVA,t}^{(k)} - \beta^k \right] \mathbb{E}_t d_{t+k} + \sum_{k=1}^{\infty} \beta^k \mathbb{C}ov_t \left(M_{t,t+k} \frac{GVA_{t+k}}{GVA_t}, d_{t+k} \right).$$

In this sense, the residual ϕ_t captures two reasons for the deviation of observed price p_t from $p_t^\star$.

Note that a simple model with $p_t = p_t^\star$ and $\phi_t = 0$ is one in which the marginal investor has log utility with time discount factor δ and who consumes in direct proportion to nominal aggregate corporate GVA. In this case, the nominal pricing kernel is given by

$$M_{t,t+k} = \delta^k \frac{GVA_t}{GVA_{t+k}}$$

so

$$p_{GVA,t}^{(k)} = \delta^k$$

and

$$H_t^{(k)} = \mathbb{C}ov_t \left(\delta^k, d_{t+k} \right) = 0.$$

In this economy, $p_t = p_t^\star$ with $\beta = \delta$. Note that in such an economy, there may be variation over time in the risk free nominal rate given by $1/\mathbb{E}_t M_{t,t+k}$ as well as in real risk free rates, but such variation does not impact the ratio of price to GVA because of offsetting movements in the expected growth of cash flows.

F Estimation of Valuation Model by Method of Simulated Moments

We estimate the model using the Method of Simulated Moments, as described in the main text. We use model simulation to compute model moments, and compare those against their data counterparts from the Integrated Macroeconomic Accounts. Each model simulation is 95 periods long, the same length as our IMA sample. We search for a vector for the five parameters $(\sigma_x, \sigma_{pd}, \rho_{x,pd}, \rho, \psi)$ to minimize the difference, averaged across 1,000 simulations, of model moments and their empirical counterparts. We define the distance between model and data moments as the squared percentage differences for all moments, with the exception of the covariance between dividend growth and quasi-returns, for which we use the squared absolute difference (that moment crosses zero, where the percentage difference explodes).

We focus on five different sets of moments involving quasi-returns, dividend growth, and the price-dividend spread. We measure quasi returns and dividend growth at horizons from 1 to 15 years, implying $5 \times 15 = 75$ moments in total.

In an initial estimation, we search numerically (using the Matlab routine fmincon) for the parameter vector that minimizes the distance between model and data when we weight all these 75 moments equally. We then compute a variance covariance matrix of model versus data moment differences across our 1,000 simulations, and use the inverse of that matrix to construct a new weighting matrix. Given that new matrix, we re-estimate the model. We repeat this procedure until our parameter estimates (and weighting matrix) are virtually unchanged from one iteration to the next.

Below we include some additional plots illustrating the fit of our estimated valuation model. Figure F.1 plots all the five sets of moments we target. Figure F.2 shows the fit for two untargetted moments, These plots show that the data appears consistent with our assumed autoregressive process for the price-dividend spread pd_t.

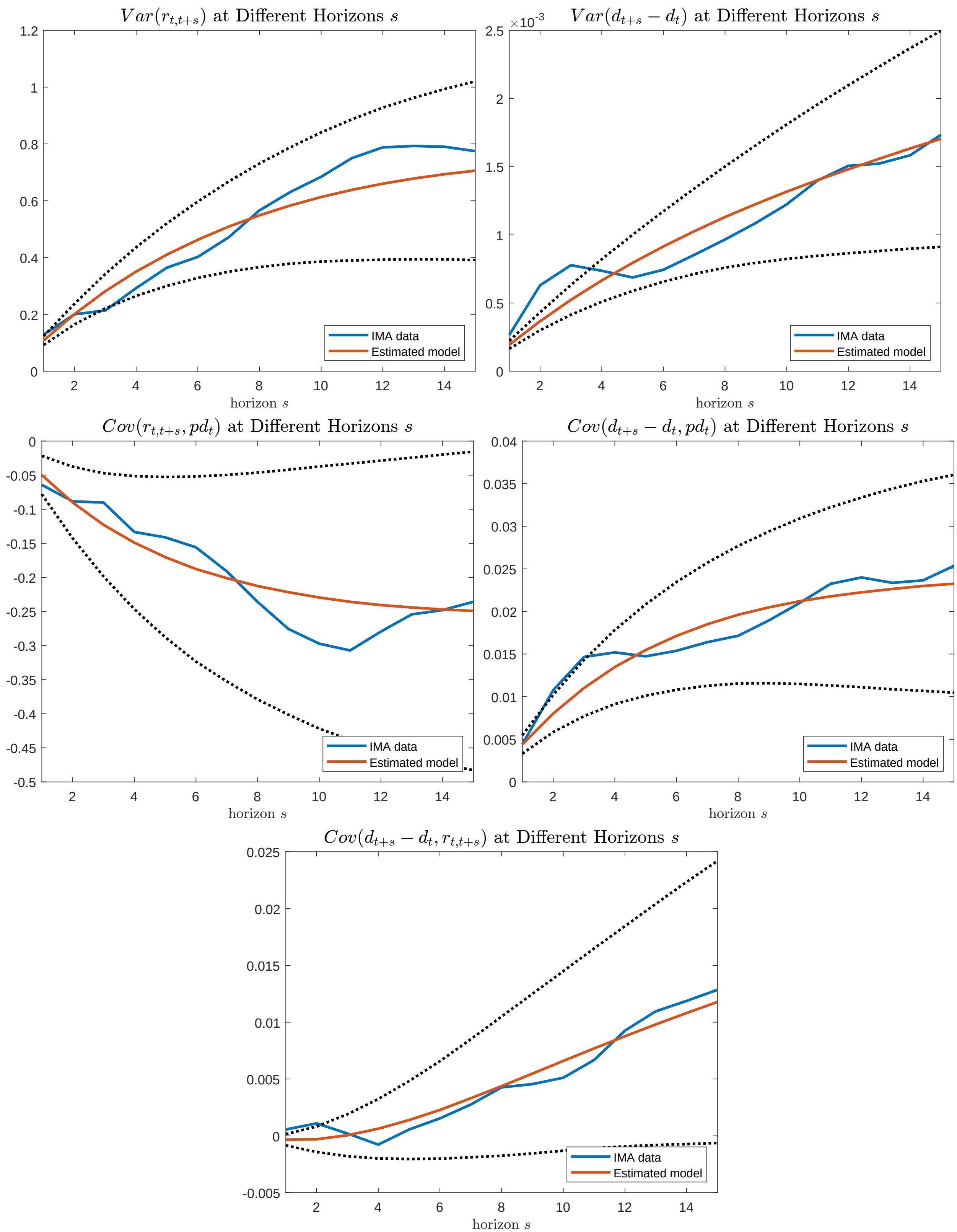

Figure F.1: Fit of Moments Targetted in Method of Simulated Moments Estimation. In each panel, the blue lines are the moment values in our IMA data. The red lines show averages across 1,000 simulations of our model, at our estimated parameter values. The dotted lines show one standard error bands around the average model moments.

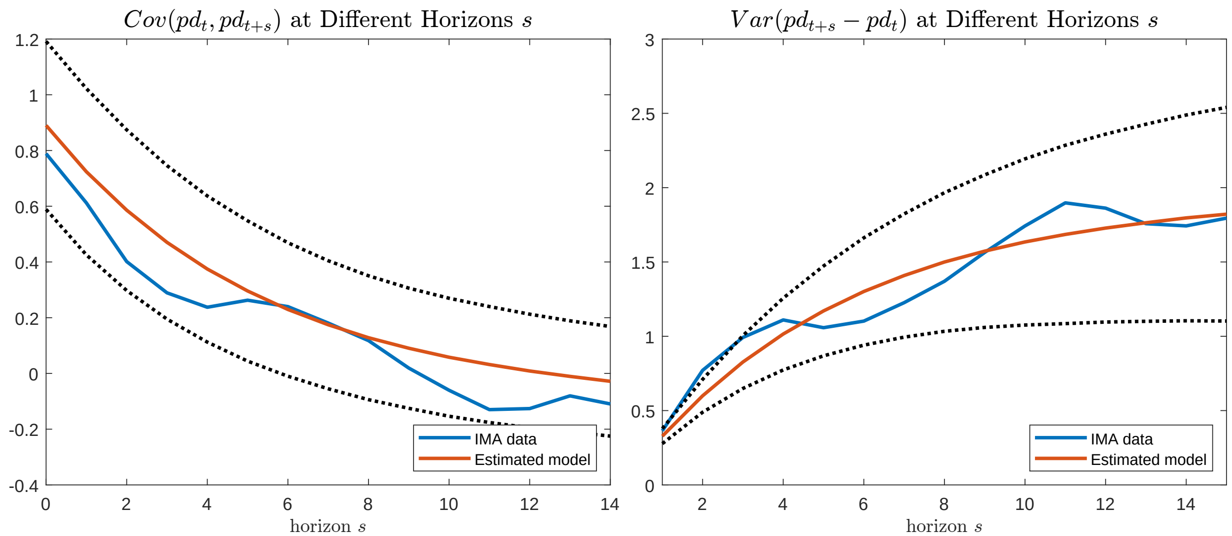

Figure F.2: Fit of Moments Not Targetted in Method of Simulated Moments Estimation. In each panel, the blue lines are the moment values in our IMA data. The red lines show averages across 1,000 simulations of our model, at our estimated parameter values. The dotted lines show one standard error bands around the average model moments.