# Blockchain, Cryptocurrencies \& Digital Tokens Demystified 

Prof. R.A. Farrokhnia<br>Columbia Business School<br>Fall 2023 (EMBA)

## Welcome \& Agenda

## About the Course Faculty

- Prof. R.A. Farrokhnia (far.oak.nia)
- Teaching at Columbia Business \& Engineering Schools
- Recipient of Dean's Award for Teaching Excellence



## About the Course Faculty

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- Teaching at Columbia Business \& Engineering Schools
- Recipient of Dean's Award for Teaching Excellence
- Executive Director (Dean's Office) of "Advanced Projects and Applied Research in Fintech" at Columbia Business School
- Board Member \& Senior Lecturer: Columbia Journalism School KB Program
- Building a next-gen DevLab


## fintech.gsb.columbia.edu

## Columbia Business School

AT THE VERY CENTER OF BUSINESS"*

## Advanced Projects and Applied Research in Fintech

```
About Projects \& Research Courses Fellowship Events Contact
```


## The Future of Financial Services

```
Advanced Projects and Applied Research in Fintech ("APAR") is a multidisciplinary initiative at the intersection of business and engineering. Its two primary goals are:
1. to research the innovative forms and functions of new enterprise and consumer financial services products, and
2. to explore the development of novel technological solutions and oversee their industry implementation.
```

Before we begin ...

## farrokhnia@gsb.columbia.edu

## Class Schedule - Nov 4, Nov 18, Dec 2, Dec 9

## Class Plan

Nov 4
Nov 18
Dec 2
Dec 9

08:30 am to 6:45 pm (K-440)Module $1+2$
08:30 am to 6:45 pm (K-440)Module $3+4$
08:30 am to 6:45 pm (K-440)Midterm Project + 5 \& $6+$ Guest Speaker
08:30 am to 6:45 pm (K-440)Module 7 \& $8+$ Guest Speaker + final presentations

Daily Schedule
8:30-9:45 am
9:45-10:00 am
10:00-11:15 pm
11:15 am-12:30 pm
12:30-2:00 pm 2:00-2:15 pm
2:15-3:30 pm
3:30-3:45 pm
3:45-5:00 pm
5:00-5:15 pm
5:15-6:45 pm

## Lecture

Break
Lecture
Lunch (1h15min) - Kravis 2nd floor (Smith Dining)

## Lecture

Break

## Lecture

Break
Lecture
Break
Lecture

## Curriculum Roadmap

| Morning | Nov 4 | Nov 18 | Dec 2 | Dec 9 |
| :---: | :---: | :---: | :---: | :---: |
|  | Networks \& Protocols | Hashing, Hashing Tables \& One- Way Functions \& a few more tech | Bitcoin + other forms of crypto payments and store of value mechanisms and media | DeFi \& Other Applications (Digital Tokens, CBDC, etc.) + Speaker: Future of Finance + Discussion Forum |
|  | Lunch | Lunch | Lunch | Lunch |
| Afternoon | Encryption \& Cryptography (plus some math!) | Bring it All <br> Together: Let's build a blockchain \& discuss variety of cases | Ethereum \& Other Digital Tokens + Speaker: Regulatory \& Legal Considerations in Blockchain \& Digital Assets | Governance, <br>  <br> More; Final Lecture on <br> How the Future May Play <br> Out + Final <br> Presentations |

## Administrative Requirements

- Please be on time and present for the duration of the class
- Class content is sequential. Don't miss class sessions (and watch recordings if you do)
- Lots of technical topics, but I won't use ANY code or much math (only 2-3 parts might be tough - l'll give you the heads-up when we reach these points in our curriculum), so don't worry :-)
- I can explain it to you, but I cannot understand it for you! So be sure to ask questions
- Your breaks are my breaks too! l'll provide ample opportunities for Q\&A in class though
- Office hours by appointment (just email me)
- Make sure to read the syllabus
- CBS code of conduct, incl. during guest speaker presentations
- Team formations: finalized by Nov 18 no later than 3:30 pm ET (today is even better!)
- Midterm Project
- Final Papers and deliverables: all the details
- Final Papers due on Monday Dec 18 at 5 pm ET
- This is a demanding class, and we are all in it together. Let's make it the best class we can
- My promise to you all + let's have a fun, productive course ... worthy of a 5 out of 5


## DISCLAIMER

## One more thing ... <br> Digital Device Policy Recommendation + Sharing of Class Slides

## Also a reminder of a good practice

# The Pen Is Mightier Than the Keyboard: Advantages of Longhand Over Laptop Note Taking 

## Psychological Science

2014, Vol. 25(6) 1159-1168
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sagepub.com/journalsPermissions.nav DOI: $10.1177 / 0956797614524581$
pss.sagepub.com
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Pam A. Mueller ${ }^{1}$ and Daniel M. Oppenheimer ${ }^{2}$

${ }^{1}$ Princeton University and ${ }^{2}$ University of California, Los Angeles


#### Abstract

Taking notes on laptops rather than in longhand is increasingly common. Many researchers have suggested that laptop note taking is less effective than longhand note taking for learning. Prior studies have primarily focused on students' capacity for multitasking and distraction when using laptops. The present research suggests that even when laptops are used solely to take notes, they may still be impairing learning because their use results in shallower processing. In three studies, we found that students who took notes on laptops performed worse on conceptual questions than students who took notes longhand. We show that whereas taking more notes can be beneficial, laptop note takers' tendency to transcribe lectures verbatim rather than processing information and reframing it in their own words is detrimental to learning.


## Also a reminder of a good practice



## Abstract

Taking notes on laptops rather than in longhand is increasingly common. Many researchers have suggested that laptop note taking is less effective than longhand note taking for learning. Prior studies have primarily focused on students' capacity for multitasking and distraction when using laptops. The present research suggests that even when laptops are used solely to take notes, they may still be impairing learning because their use results in shallower processing. In three studies, we found that students who took notes on laptops performed worse on conceptual questions than students who took notes longhand. We show that whereas taking more notes can be beneficial, laptop note takers' tendency to transcribe lectures verbatim rather than processing information and reframing it in their own words is detrimental to learning.

## Also a reminder of a good practice

n Pr WNYCRADIO news arts \& life music programs


Class is mostly slides for Day 1 and $2+$ we'd switch to discussions \& whiteboarding (no slides) on subsequent days

## All done? Then let's go ... but first, a little fun!



## I. A Series of Tubes

How does the internet work? Why do we need to protect it?


## An ideal network



## An ideal network



## An ideal network



## An ideal network



## An ideal network



## An ideal network



## An ideal network



## The real world



## The real world: Routers \& Switches




## The real world ... as it was!



## The real world ... with PROTOCOLS!



## The real world



## The real world



## The real world



## The real world



## A few words on networks ... in the context of order, complexity, and resiliency

Networks: a collection of connected nodes


Networks: a collection of connected nodes


Networks: a collection of connected nodes

-
$\bullet$

Networks: a collection of connected nodes

-
-


## Centralized (vs. Decentralized vs. Distributed)

## Centralized (vs. Decentralized vs. Distributed)



## Centralized vs. Decentralized (vs. Distributed)



## Centralized vs. Decentralized Networks



## Centralized vs. Decentralized Networks



If this node is compromised, the whole network goes down!

## Centralized vs. Decentralized Networks



A

## Decentralized Networks



A


B

## Decentralized Networks



A


B

## Real Decentralized Technologies



Internet


## Real Decentralized Technologies



Internet

## Real Decentralized Technologies



Internet


Bitcoin

## Centralized vs. Decentralized vs. Distributed



## A few words on how internet works as a network ...

## A few words on Internet

- Billions of connected (computing) hosts/end-systems - mobile devices now outnumber others by a large margin


## A few words on Internet

- Billions of connected (computing) hosts/end-systems (mobile devices now outnumber others by a large margin)
- laptops
- smartphones, tablets
- TVs
- Gaming consoles
- Webcams
- Automobiles,
- Environmental sensing devices,
- Picture frames
- Home electrical
- Security systems
- And more ...


## A few words on Internet

- Billions of connected (computing) hosts/end-systems - mobile devices now outnumber others by a large margin
- laptops, smartphones, tablets, TVs, gaming consoles, Webcams, automobiles, environmental sensing devices, picture frames, and home electrical, security systems, ...
- Other constituents of the network (mobile, enterprise, home, ISPs, etc.):
- Servers
- Routers
- Link-layer Switches
- Modems
- Base Stations
- Cell Towers
- And more ...


## A few words on Internet

- Billions of connected (computing) hosts/end-systems + other constituents (mobile devices now outnumber others by a large margin)
- These devices and hosts/end-systems run network apps
- They are all connected via communication links (fiber, copper, radio, satellite, etc.) and packet switches with various transmission rates (i.e. bandwidth)
- Packet Switches such as routers and switches send around and forward data packets (i.e. chunks of data) throughout the network
- In essence, you have decentralized network of networks (e.g. ISPs) + protocols + internet standards


## A few words on Protocols

- TCP/IP
- SMTP
- IMAP
- POP
- FTP
- HTTP
- HTTPS/TLS
- UDP
- WLAN
- DNS .... and many more!


## A few words on Protocols

```
    Internet protocol suite
    Application layer
    BGP DHCP - DNS FTP - HTTP ·IMAP .
    LDAP - MGCP - NNTP - NTP • POP .
ONC/RPC · RTP · RTSP · RIP · SIP · SMTP .
    SNMP}\cdot\mathrm{ SSH · Telnet · TLS/SSL ·MPP .
                more...
            Transport layer
TCP - UDP - DCCP - SCTP · RSVP - more...
                    Internet layer
    IP (IPv4 - IPv6) - ICMP | ICMPv6 - ECN -
            IGMP}\cdot|Psec · more...
            Link layer
ARP - NDP - OSPF · Tunnels (L2TP) · PPP ·
MAC (Ethernet · DSL · ISDN · FDDI) - more...
```


## "The Unsung Heros"

## "The Unsung Heros"

Ebye Aew Hillocimes

Daniel Kaminsky, Internet Security Savior, Dies at 42

If you are reading this obituary online, you owe your digital safety to him.


## All Communication needs protocols!

## All Communication needs protocols!



## All Communication needs protocols!



## All Communication needs protocols!



## A few (more) words on Protocols

- Protocols are standardized methods that facilitate communication between and across different "things," creating a common framework
- In short, Protocols define how data should be "packetized," addressed, transmitted, routed, and received $\rightarrow$ examples to follow
- Let's use the example of exchanging messages: first with humans (asking for time, exchanging business cards, mailing a letter), then machines - all communications are in essence governed by protocols
- Protocols help manage complexity across various building blocks of the internet (hosts, routers, switches, applications, hardware, software, etc.) ... BUT ... how do we organize them and the structure of our network?
- [by the way, it was mostly a volunteer effort, with no possibility for monetization by the makers]


## Sample Computer Network Protocol (signals \& msgs)


case comparison: how do you ask questions in class?

## How a web page is rendered (put simplistically)

Meanwhile, it's instructive that now that we're all locked up at home, video calls have become a huge consumer phenomenon, but VR has been not. This should have been a VR moment, and it isn't.


## Master Definition of a Protocol

"A protocol defines the format and the order of messages exchanged between two or more communicating entities, as well as the actions taken on the transmission and/or receipt of a message or other event."

- James Kurose, Keith Ross


## Let's organize a flight ... through a series of steps

ticket (purchase)
baggage (check)
gates (load)
runway takeoff
airplane routing
airplane routing
ticket (complain)
baggage (claim)
gates (unload)
runway landing
airplane routing
airplane routing

## Organizing a flight ... through functionality layers

| ticket (purchase) |  | ticket (complain) | ticket |
| :---: | :---: | :---: | :---: |
| baggage (check) |  | baggage (claim | baggage |
| gates (load) |  | gates (unload) | gate |
| runway (takeoff) |  | runway (land) | takeofflanding |
| airplane routing | airplane routing airplane routing | airplane routing | airplane routing |
| departure airport | intermediate air-traffic control centers | arrival airport |  |

## Organizing a flight ... through functionality layers

| ticket (purchase) |  |  | ticket (complain) | ticket |
| :---: | :---: | :---: | :---: | :---: |
| baggage (check) |  |  | baggage (claim | baggage |
| gates (load) |  |  | gates (unload) | gate |
| runway (takeoff) |  |  | runway (land) | takeofflanding |
| airplane routing | airplane routing | airplane routing | airplane routing | airplane routing |

departure
airport
intermediate air-traffic
control centers
arrival
airport

- Each LAYER implements a service ... via its own internally-layer processes ... and relying on the services provided by layer below


## Internet Protocol Stack

Application: support and enable end-user apps
Transport: process data transfer
Network: routing of data from source to destination
Link: data transfer between neighboring network elements (e.g. WiFi)

Physical: bits "on the wire" (hardware)

| application |
| :---: |
| transport |
| network |
| link |
| physical |

Why is all this important?!

## "Fat Protocols" (by Joel Monegro, USV)

The Web


Blockchain


## "Fat Protocols \& Value Capture " (Johnson Nakano)

Proportional Value Capture in the Blockchain Ecosystem over time


## BLOCKCHAIN TECHNOLOGY STACK

## Application Layer

Acts as the User Interface that combines
business logic and customer interactions.
dApp Browsers


Decentralized Applications


Application Hosting


Programming Languages

Services and Optional Components
Serves to enable application operations with a view to connecting with other technologies and platforms.



Oracles



Protocol Layer
Decides the methods of consensus
network participation.


Consensus Algorithms


Permissioned and
Permissionless


EVMs

Network Layer
Acts as a transportation medium and
interface for the Peer-to-Peer interface for the Peer-to-Peer
network and decides how data is packetized, addressed, transmitted, routed and received.


RPLX


Roll Your Own

$\square: \square$
Block Delivery Networks


Irusted Execution Environment


Peer-to-Peer

Infrastructure Layer In-house infrastructure or Blockchain as a Service (BaaS) to control the nodes.

Mining
Network


101 Blockchains

## Back to networks ... and security

## So what do we need to protect?



## So what do we need to protect?



## So what do we need to protect?



## So what do we need to protect?



## So what do we need to protect?



## II. The Bad Guys

Types of attackers, and the cryptographic techniques we can use to circumvent them.


## The Snooper



## The Snooper



## The Snooper



## The Snooper



## The Snooper


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## The Snooper



## The Snooper



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## The Snooper



## The Snooper



| $\rightarrow$ ~ traceroute google.com |  |
| :---: | :---: |
| traceroute to google.com (216.58.219.206), 64 hops max, 52 byte packets |  |
| 1 | cc-wlan-1-vlan3562-1.net.columbia.edu (160.39.252.2) 2.698 ms 2.311 ms 1.555 ms |
| 2 | phi-core-1-x-cc-wlan-1.net.columbia.edu (128.59.255.225) 1.683 ms 1.698 ms 1.653 ms |
| 3 | nyser111-gw-1-x-phi-core-1.net.columbia.edu (128.59.255.14) 2.106 ms 2.007 ms 1.816 ms |
| 4 | nyser32-gw-1-x-nyser111-gw-1.net.columbia.edu (128.59.255.9) 8.161 ms 2.492 ms 3.124 ms |
| 5 | be4222.rcr24.jfk01.atlas.cogentco.com (38.122.8.209) 2.472 ms 2.381 ms 2.582 ms |
| 6 | be2897.ccr42.jfk02.atlas.cogentco.com (154.54.84.213) 2.725 ms 2.260 ms 2.754 ms |
| 7 | be2061.ccr21.jfk05.atlas.cogentco.com (154.54.3.70) 2.898 ms 4.139 ms 2.952 ms |
| 8 | tata.jfk05.atlas.cogentco.com (154.54.12.18) 3.705 ms 2.854 ms 2.881 ms |
| 9 | if-ae-12-2.tcore1.n75-new-york.as6453.net (66.110.96.5) 2.821 ms 2.897 ms 3.346 ms |
| 10 | 72.14 .214 .68 ( 72.14 .214 .68$) 3.015 \mathrm{~ms}$ |
|  | $72.14 .195 .232(72.14 .195 .232) 3.461 \mathrm{~ms}$ |
|  | $72.14 .218 .224(72.14 .218 .224) 3.865 \mathrm{~ms}$ |
| 11 | 209.85 .248 .242 (209.85.248.242) 3.952 ms 3.901 ms |
|  | 216.239.50.106 (216.239.50.106) 4.658 ms |
| 12 | 209.85 .253 .111 (209.85.253.111) 3.984 ms 4.066 ms 4.171 ms |
| 13 | lga25s40-in-f206.1e100.net (216.58.219.206) 3.642 ms 3.851 ms 3.591 ms |

$\rightarrow$ ~ traceroute www.columbia.edu
traceroute to www-ltm.cc.columbia.edu (128.59.105.24), 64 hops max, 52 byte packets 1 cc-wlan-1-vlan3562-1.net.columbia.edu (160.39.252.2) 14.735 ms 2.005 ms 1.733 ms 2 phi-core-1-x-cc-wlan-1.net.columbia.edu (128.59.255.225) 2.264 ms 1.882 ms 3.439 ms 3 cc-conc-1-x-phi-core-1.net.columbia.edu (128.59.255.214) $1.956 \mathrm{~ms} \quad 1.706 \mathrm{~ms} \quad 2.532 \mathrm{~ms}$ 4 columbia.university (128.59.105.24) 1.833 ms 34.477 ms 2.024 ms
$\rightarrow$ ~ traceroute cam.ac.uk
traceroute to cam.ac.uk (131.111.150.25), 64 hops max, 52 byte packets
1 cc-wlan-1-vlan3562-1.net.columbia.edu (160.39.252.2) 31.050 ms 3.855 ms 7.104 ms
2 phi-core-1-x-cc-wlan-1.net.columbia.edu (128.59.255.225) 6.714 ms 8.490 ms 3.632 ms
3 nyser111-gw-1-x-phi-core-1.net.columbia.edu (128.59.255.14) 434.333 ms 314.247 ms 6.011 ms
4 nyser32-gw-1-x-nyser111-gw-1.net.columbia.edu (128.59.255.9) 13.434 ms 3.637 ms 5.680 ms
5 nyc-9208-columbia.nysernet.net (199.109.4.13) 38.134 ms 2.071 ms 1.959 ms
6 i2-newy-nyc-9208.nysernet.net (199.109.5.2) 2.150 ms 2.233 ms 2.052 ms
7 internet2.mx1.ams.nl.geant.net ( 62.40 .124 .46 ) 80.376 ms 85.414 ms 85.330 ms
8 ae2.mx1.lon.uk.geant.net (62.40.98.80) 86.359 ms 84.861 ms 89.197 ms
9 janet-gw.mx1.lon.uk.geant.net (62.40.124.198) 88.979 ms 101.630 ms 90.211 ms
10 ae28.lowdss-sbr1.ja.net (146.97.33.18) 101.747 ms 88.167 ms 105.850 ms
$11 \quad 146.97 .38 .10(146.97 .38 .10) \quad 110.981 \mathrm{~ms} 93.755 \mathrm{~ms} 98.262 \mathrm{~ms}$
12146.97 .65 .106 (146.97.65.106) $92.656 \mathrm{~ms} 91.787 \mathrm{~ms} \quad 131.232 \mathrm{~ms}$

13 university-of-cambridge.cambab-rbr1.eastern.ja.net (146.97.130.2) 90.627 ms 96.497 ms 98.185 ms
14 d-dw.s-dw.net.cam.ac.uk (193.60.88.2) 91.853 ms 102.465 ms 163.091 ms
15 d-dw.s-dw.net.cam.ac.uk (193.60.88.2) 91.447 ms 91.686 ms 92.272 ms
16 outside.fw-srv.net.cam.ac.uk (128.232.128.6) 90.952 ms 92.305 ms 127.274 ms
17 link-srv.uis.fw-srv.net.cam.ac.uk (128.232.129.2) 94.121 ms 90.736 ms 91.246 ms
18 primary.admin.cam.ac.uk (131.111.150.25) $91.621 \mathrm{~ms} \quad 101.475 \mathrm{~ms} 93.549 \mathrm{~ms}$

$\rightarrow$ ~ traceroute cam.ac.uk
traceroute to cam.ac.uk (131.111.150.25), 64 hops max, 52 byte packets
1 cc-wlan-1-vlan3562-1.net.columbia.edu (160.39.252.2) 31.050 ms 3.855 ms 7.104 ms
2 phi-core-1-x-cc-wlan-1.net.columbia.edu (128.59.255.225) 6.714 ms 8.490 ms 3.632 ms
3 nyser111-gw-1-x-phi-core-1.net.columbia.edu (128.59.255.14) 434.333 ms 314.247 ms 6.011 ms
4 nyser32-gw-1-x-nyser111-gw-1.net.columbia.edu (128.59.255.9) 13.434 ms 3.637 ms 5.680 ms
5 nyc-9208-columbia.nysernet.net (199.109.4.13) 38.134 ms 2.071 ms 1.959 ms
6 i2-newy-nyc-9208.nysernet.net (199.109.5.2) 2.150 ms 2.233 ms 2.052 ms
7 internet2.mx1.ams.nl.geant.net (62.40.124.46) 80.376 ms 85.414 ms 85.330 ms
8 ae2.mx1.lon.uk.geant.net ( 62.40 .98 .80 ) 86.359 ms 84.861 ms 89.197 ms
9 janet-gw.mx1.lon.uk.geant.net (62.40.124.198) 88.979 ms 101.630 ms 90.211 ms
10 ae28.lowdss-sbr1.ja.net (146.97.33.18) 101.747 ms 88.167 ms 105.850 ms
11 146.97.38.10 (146.97.38.10) 110.981 ms 93.755 ms 98.262 ms
12 146.97.65.106 (146.97.65.106) $92.656 \mathrm{~ms} 91.787 \mathrm{~ms} \quad 131.232 \mathrm{~ms}$
13 university-of-cambridge.cambab-rbr1.eastern.ja.net (146.97.130.2) 90.627 ms 96.497 ms 98.185 ms
14 d-dw.s-dw.net.cam.ac.uk (193.60.88.2) 91.853 ms 102.465 ms 163.091 ms
15 d-dw.s-dw.net.cam.ac.uk (193.60.88.2) 91.447 ms 91.686 ms 92.272 ms
16 outside.fw-srv.net.cam.ac.uk (128.232.128.6) 90.952 ms 92.305 ms 127.274 ms
17 link-srv.uis.fw-srv.net.cam.ac.uk (128.232.129.2) 94.121 ms 90.736 ms 91.246 ms
18 primary.admin.cam.ac.uk (131.111.150.25) 91.621 ms 101.475 ms 93.549 ms

## The Snooper



How to send messages and information securely, knowing any info transmitted over the internet can be stolen and we cannot trust anyone?

## 0. Please Don't Tell

A brief primer on the codes and ciphers used throughout history to protect information.


## Plaintext vs. Ciphertext

Plaintext
Ciphertext

I love the sun
w jd7h bmg vns

## Cipher Shift (or substitution), aka Caesar Cipher

| Plaintext | I love the sun |
| :--- | :--- |
| Ciphertext | ? ???? ??? ??? |

## Cipher Shift (zero or no shift)

| Plaintext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ciphertext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | N |


| Plaintext Alphabet | N | O | P | Q | R | S | T | U | V | W | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ciphertext Alphabet | N | C | P | Q | R | S | T | U | V | W | X | Y | Z |

## Cipher Shift (zero or no shift)

| Plaintext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ciphertext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | N |
| Plaintext Alphabet | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| Ciphertext Alphabet | N | C | P | Q | R | S | T | U | V | V | X | Y | Z |

## Cipher Shift (shift of one)

| Plaintext Alphabet |  |  | C | D | E | F | G | H | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | A | B | c | D | E | F | G | H | 1 | J | K | L | N |


| Plaintext Alphabet | N | O | P | Q | R | S | T | U | V | W | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ciphertext Alphabet | N | C | P | Q | R | S | T | U | V | W | X | Y | Z |

## Cipher Shift (shift of one)

| Plaintext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ciphertext Alphabet | B | C | D | E | F | G | H | I | J | K | L | N |  |

## Cipher Shift (+1)

| Plaintext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | B | C | D | E | F | G | H | I | J | K | L | N | N |


| Plaintext Alphabet | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | C | P | G | R | S | T | U | V | W | X | Y | Z | A |

Plaintext
i love the sun
Ciphertext
??????????????

## Cipher Shift (+1)

| Plaintext Alphabet | A | B | C | D | E | F | G | H | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | B | c | D | E | F | G | H | 1 | J | K | L | N | N |


| Plaintext Alphabet | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | C | P | G | R | S | T | U | V | W | X | Y | Z | A |

Plaintext i love the sun

Ciphertext

## Cipher Shift (+1)



## Cipher Shift (+1)

| Plaintext Alphabet | A | B | c | D | E | G | H | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ciphertext Alphabet | в | c | - | E | F | H | 1 | J | k | L | N | N |
| Plaintext Alphabet | N | 0 | P | Q | R | T | $u$ | v | w | x | Y | z |
| Ciphertext Alphabet | - | P | G | R | s | u | v | $n$ | x | Y | z | A |
| Plaintext |  |  |  | V |  |  |  |  |  |  |  |  |
| Ciphertext |  |  |  |  |  |  |  |  |  |  |  |  |

## Cipher Shift Wheel



# A brief history of how Caesar Cipher was broken ... and rest is history 

## Cipher Shift Decoded (or rather, decrypted!)

## Cipher Shift Decoded (or rather, decrypted!)



## An in-class exercise ... time to become Code Breakers

## Cipher Shift Decoded (or rather, decrypted!)

Can you guess?

$$
\underline{B}
$$

## Cipher Shift Decoded (or rather, decrypted!)

Can you guess?
$B \quad \mathrm{E}$

## Cipher Shift Decoded (or rather, decrypted!)

Can you guess?
$B \quad \mathrm{E} \quad \mathrm{R}$

## Cipher Shift Decoded (or rather, decrypted!)

Can you guess?
$B \quad \mathrm{E} \quad \mathrm{L}$

## Cipher Shift Decoded (or rather, decrypted!)

Can you guess?
$\begin{array}{lll}\mathrm{B} & \mathrm{R} \quad \mathrm{L} \quad \mathrm{I}\end{array}$

## Cipher Shift Decoded (or rather, decrypted!)



## Another Cipher Shift Decoded (with numbers)

Can you guess?
2

## Another Cipher Shift Decoded (with numbers)

Can you guess?
$2 \quad 5$

## Another Cipher Shift Decoded (with numbers)

Can you guess?


But wait a minute!?

## Another exercise ... time to become REAL Code Breakers!

## Let's try to break a coded message

MPQZCP HP NLY ELWV LMZFE ESP DAPNTQTND ZQ XZOPCY NCJAEZRCLASJ, MWZNVNSLTYD, ZC MTENZTY, HP XFDE QTCDE ELWV LMZFE ESP CZWP ZQ XLESPXLETND, FYOPCDELYOTYR SZH TE TD LAAWTPO LYO SZH TE TD QFYOLXPYELW EZ LWW ESLE EPNSYZWZRJ LTXD EZ LNSTPGP.

## Let's try to break a code by hand (you have 10 mins)

| E: 21 | D: 10 | A: 5 |
| :--- | :--- | :--- |
| L: 18 | W: 9 | H: 4 |
| Z: 17 | C: 8 | V: 3 |
| P: 16 | Q: 6 | J: 3 |
| T: 15 | X: 6 | R: 3 |
| Y: 11 | O: 6 | G: 1 |
| N: 10 | M: 5 |  |
| S: 10 | F: 5 |  |

## Let's try to break a coded message (key=11)

Before we can talk about the specifics of modern cryptography, blockchains, or bitcoin, we must first talk about the role of mathematics, understanding how it is applied and how it is fundamental to all that technology aims to achieve.

## Let's talk about DATA ...

## A little "bit" of data

## Unit

Bit (b) $\quad 1$ or 0
Byte (B) 8 bits

## Comments

Short for Binary Digit, after the binary code
WHY 8?

# American Standard Code for Information Interchange 

## ASCII (character encoding standard/protocol)

## Binary Decoding (8-bit)

| 0 | 00110000 | C | 0100 | 0011 | $P$ | 0101 | 0000 | c | 0110 | 0011 | p | 0111 | 0000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 00110001 | D | 0100 | 0100 | $Q$ | 0101 | 0001 | d | 0110 | 0100 | a | 0111 | 0001 |
| 2 | 00110010 | E | 0100 | 0101 | R | 0101 | 0010 | e | 0110 | 0101 | r | 0111 | 010 |
| 3 | 00110011 | $F$ | 0100 | 0110 | S | 0101 | 0011 | f | 0110 | 0110 |  |  |  |
|  | 0 | G | 0100 | 0111 | T | 0101 | 0100 | $g$ | 0110 | 0111 | 5 | 111 | 001 |
| 4 | 00110100 | H | 0100 | 1000 | U | 0101 | 0101 | h | 0110 | 1000 | t | 0111 | 10 |
| 5 | 00110101 | I | 0100 | 1001 | v | 0101 | 0110 | I | 0110 | 1001 | u | 0111 | 010 |
| 6 | 00110110 | J | 0100 | 1010 | W | 0101 | 0111 | j | 0110 | 1010 | v | 0111 |  |
| 7 | 00110111 | K | 0100 | 1011 | x | 0101 | 1000 | k | 0110 | 1011 | w |  |  |
| 8 | 00111000 | L | 0100 | 1100 | Y | 0101 | 1001 | 1 | 0110 | 1100 |  |  |  |
| 9 | 00111001 | M | 0100 | 1101 | z | 0101 | 1010 | m | 0110 | 1101 | $x$ | 0111 |  |
| A | 01000001 | N | 0100 | 1110 | a | 0110 | 0001 | n | 0110 | 1110 | Y | 0111 |  |
|  | 01000010 | $\bigcirc$ | 0100 | 1111 | b | 0110 | 0010 | $\bigcirc$ | 011 | 1111 | $z$ | 0111 | 101 |

Decimal - Binary - Octal - Hex - ASCII
Conversion Chart

| Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 00000000 | 000 | 00 | NUL | 32 | 00100000 | 040 | 20 | SP | 64 | 01000000 | 100 | 40 | @ | 96 | 01100000 | 140 | 60 | - |
| 1 | 00000001 | 001 | 01 | SOH | 33 | 00100001 | 041 | 21 | ! | 65 | 01000001 | 101 | 41 | A | 97 | 01100001 | 141 | 61 | a |
| 2 | 00000010 | 002 | 02 | STX | 34 | 00100010 | 042 | 22 | * | 66 | 01000010 | 102 | 42 | B | 98 | 01100010 | 142 | 62 | b |
| 3 | 00000011 | 003 | 03 | ETX | 35 | 00100011 | 043 | 23 | \# | 67 | 01000011 | 103 | 43 | C | 99 | 01100011 | 143 | 63 | c |
| 4 | 00000100 | 004 | 04 | EOT | 36 | 00100100 | 044 | 24 | \$ | 68 | 01000100 | 104 | 44 | D | 100 | 01100100 | 144 | 64 | d |
| 5 | 00000101 | 005 | 05 | ENQ | 37 | 00100101 | 045 | 25 | \% | 69 | 01000101 | 105 | 45 | E | 101 | 01100101 | 145 | 65 | e |
| 6 | 00000110 | 006 | 06 | ACK | 38 | 00100110 | 046 | 26 | \& | 70 | 01000110 | 106 | 46 | F | 102 | 01100110 | 146 | 66 | $f$ |
| 7 | 00000111 | 007 | 07 | BEL | 39 | 00100111 | 047 | 27 |  | 71 | 01000111 | 107 | 47 | G | 103 | 01100111 | 147 | 67 | 9 |
| 8 | 00001000 | 010 | 08 | BS | 40 | 00101000 | 050 | 28 | ( | 72 | 01001000 | 110 | 48 | H | 104 | 01101000 | 150 | 68 | h |
| 9 | 00001001 | 011 | 09 | HT | 41 | 00101001 | 051 | 29 | ) | 73 | 01001001 | 111 | 49 | 1 | 105 | 01101001 | 151 | 69 | i |
| 10 | 00001010 | 012 | OA | LF | 42 | 00101010 | 052 | 2A | * | 74 | 01001010 | 112 | 4A | J | 106 | 01101010 | 152 | 6A | j |
| 11 | 00001011 | 013 | OB | VT | 43 | 00101011 | 053 | 2 B | + | 75 | 01001011 | 113 | 4B | K | 107 | 01101011 | 153 | 6 B | k |
| 12 | 00001100 | 014 | 0 C | FF | 44 | 00101100 | 054 | 2 C | , | 76 | 01001100 | 114 | 4C | L | 108 | 01101100 | 154 | 6 C | 1 |
| 13 | 00001101 | 015 | OD | CR | 45 | 00101101 | 055 | 2D | - | 77 | 01001101 | 115 | 4D | M | 109 | 01101101 | 155 | 6 D | m |
| 14 | 00001110 | 016 | OE | SO | 46 | 00101110 | 056 | 2 E | . | 78 | 01001110 | 116 | 4 E | N | 110 | 01101110 | 156 | 6 E | n |
| 15 | 00001111 | 017 | OF | SI | 47 | 00101111 | 057 | 2 F | 1 | 79 | 01001111 | 117 | 4 F | 0 | 111 | 01101111 | 157 | 6 F | 0 |
| 16 | 00010000 | 020 | 10 | DLE | 48 | 00110000 | 060 | 30 | 0 | 80 | 01010000 | 120 | 50 | P | 112 | 01110000 | 160 | 70 | p |
| 17 | 00010001 | 021 | 11 | DC1 | 49 | 00110001 | 061 | 31 | 1 | 81 | 01010001 | 121 | 51 | Q | 113 | 01110001 | 161 | 71 | q |
| 18 | 00010010 | 022 | 12 | DC2 | 50 | 00110010 | 062 | 32 | 2 | 82 | 01010010 | 122 | 52 | R | 114 | 01110010 | 162 | 72 | r |
| 19 | 00010011 | 023 | 13 | DC3 | 51 | 00110011 | 063 | 33 | 3 | 83 | 01010011 | 123 | 53 | S | 115 | 01110011 | 163 | 73 | s |
| 20 | 00010100 | 024 | 14 | DC4 | 52 | 00110100 | 064 | 34 | 4 | 84 | 01010100 | 124 | 54 | T | 116 | 01110100 | 164 | 74 | t |
| 21 | 00010101 | 025 | 15 | NAK | 53 | 00110101 | 065 | 35 | 5 | 85 | 01010101 | 125 | 55 | U | 117 | 01110101 | 165 | 75 | $u$ |
| 22 | 00010110 | 026 | 16 | SYN | 54 | 00110110 | 066 | 36 | 6 | 86 | 01010110 | 126 | 56 | v | 118 | 01110110 | 166 | 76 | v |
| 23 | 00010111 | 027 | 17 | ETB | 55 | 00110111 | 067 | 37 | 7 | 87 | 01010111 | 127 | 57 | W | 119 | 01110111 | 167 | 77 | w |
| 24 | 00011000 | 030 | 18 | CAN | 56 | 00111000 | 070 | 38 | 8 | 88 | 01011000 | 130 | 58 | X | 120 | 01111000 | 170 | 78 | x |
| 25 | 00011001 | 031 | 19 | EM | 57 | 00111001 | 071 | 39 | 9 | 89 | 01011001 | 131 | 59 | Y | 121 | 01111001 | 171 | 79 | y |
| 26 | 00011010 | 032 | 1A | SUB | 58 | 00111010 | 072 | 3A | : | 90 | 01011010 | 132 | 5A | Z | 122 | 01111010 | 172 | 7A | z |
| 27 | 00011011 | 033 | 1B | ESC | 59 | 00111011 | 073 | 3 B | , | 91 | 01011011 | 133 | 5B | [ | 123 | 01111011 | 173 | 7 B | \{ |
| 28 | 00011100 | 034 | 1 C | FS | 60 | 00111100 | 074 | 3 C | < | 92 | 01011100 | 134 | 5C | 1 | 124 | 01111100 | 174 | 7 C | 1 |
| 29 | 00011101 | 035 | 1D | GS | 61 | 00111101 | 075 | 3D | $=$ | 93 | 01011101 | 135 | 5D | ] | 125 | 01111101 | 175 | 7D | \} |
| 30 | 00011110 | 036 | 1E | RS | 62 | 00111110 | 076 | 3 E | > | 94 | 01011110 | 136 | 5E | $\wedge$ | 126 | 01111110 | 176 | 7E | $\sim$ |
| 31 | 00011111 | 037 | 1F | US | 63 | 00111111 | 077 | 3 F | ? | 95 | 01011111 | 137 | 5 F | - | 127 | 01111111 | 177 | 7 F | DEL |

## A little "bit" of data

## Unit

| Bit (b) | 1 or 0 |
| :--- | :--- |
| Byte (B) | 8 bits |

Kilobyte (KB) $\quad 1,000$ B or $2^{10}$ bytes
Megabyte (MB) $\quad 1,000 \mathrm{~KB}$ or $2^{20}$ bytes
Gigabyte (GB) $\quad 1,000 \mathrm{MB}$ or $2^{30}$ bytes

## Size

1 or 0
Byte (B)
8 bits

## Comments

Short for Binary Digit, after the binary code
Enough info to create letters and numbers (basic unit of computing)
"thousands" in Greek
"large" in Greek
"giant" in Greek

## A little "bit" of data (cont'd)

## Unit <br> Size

Terabyte (TB) $\quad 1,000$ GB or $2^{40}$ bytes
"monster" in Greek, about 2 billion credit card transactions

## AWS Snowball (up to 80 TB, 72 TB usable)



## A little "bit" of data (cont'd)

Unit
Size
Terabyte (TB) $\quad 1,000 \mathrm{~GB}$ or $2^{40}$ bytes

Petabyte (PB) 1,000 TB or $2^{50}$ bytes
Exabyte (EB) 1,000 PB or $2^{60}$ bytes In 2009, the entire internet was estimated at $\sim 500$ EB. In 2013, annual internet traffic flow surpassed 667 EB (Cisco)

## AWS Snowmobile!



## A little "bit" of data (cont'd)

Unit
Size
Zettayte (ZB) $1,000 \mathrm{~EB}$ or $2^{70}$ bytes

Yottabyte (YB) $\quad 1,000 \mathrm{ZB}$ or $2^{80}$ bytes

Comments
About 615 billion newspapers ( 88 copies for every human being)

Waaaay too big! Currently, all the combined hard-drives and storage capacity in the world are estimated at <0.0004 YB!

## Plaintext vs. Binary Ciphertext (in "old"ASCII)

Plaintext
Binary

$K e y=$ ?

## Plaintext vs. Binary Ciphertext (in "old"ASCII)

| Plaintext | $\overbrace{1001000}^{H E E L E O}$ | 1000101 | 1001100 | 1001100 |
| :--- | :--- | :--- | :--- | :--- |
| Binary | 1001111 |  |  |  |

## Plaintext vs. Binary Ciphertext (in "old"ASCII)

Plaintext
Binary
Key = David
Ciphertext (xor)

```
H E L L O
```

1 | 1001000 | 1000101 | 1001100 | 1001100 | 1001111 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1000100 | 1000001 | 1010110 | 1001001 | 1000100 |

## Boolean Logic \& Logic Gates

## AND

OR

NOT

NOR

NAND

XNOR

XOR

## Conjunction (AND), a logical operation



## Conjunction (AND), a logical operation



## Conjunction (AND), a logical operation



## Conjunction (AND), a logical operation



## Conjunction (AND), a logical operation



## Disjunction (OR), a logical operation

| OR | $A \longrightarrow Q$ | $B-\geq 1-Q$ | $A+B$ or $A \vee B$ | Input |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | Q |
|  |  |  |  | 0 | 0 | 0 |
|  |  |  |  | 0 | 1 | 1 |
|  |  |  |  | 1 | 0 | 1 |
|  |  |  |  | 1 | 1 | 1 |

## Exclusive Disjunction (XOR), a logical operation

| XOR | $B \rightarrow Q$ | $B-=1-Q$ | $A \oplus B$ or $A \bigvee B$ | Input |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | Q |
|  |  |  |  | 0 | 0 | 0 |
|  |  |  |  | 0 | 1 | 1 |
|  |  |  |  | 1 | 0 | 1 |
|  |  |  |  | 1 | 1 | 0 |

## Exclusive Disjunction (XOR), a logical operation

| Input |  | Output |
| :---: | :---: | :---: |
|  | B | $(\mathrm{A} \oplus \mathrm{B})$ |
| 0 | 0 | 0 |
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |

## Plaintext vs. Binary Ciphertext (in "old"ASCII)

Plaintext
Binary
Key = David
Ciphertext (xor)

```
H E L L O
```

```
1001000 1000101 10011001001100 1001111
1000100 1000001 10101101001001 1000100
0 0 0 1 1 0 0 ~ 0 0 0 0 1 0 0 ~ 0 0 1 1 0 1 0 ~ 0 0 0 0 1 0 1 ~ 0 0 0 1 0 1 1
```


## Plaintext vs. Binary Ciphertext (in "old"ASCII)

| Plaintext | $\overbrace{1001000}^{H E E L E O}$ | 1000101 | 1001100 | 1001100 | 1001111 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Binary | 1000100 | 1000001 | 1010110 | 0110010 | 1000100 |

## Let's discuss in the context of a case model...

## Securing a connection



## Securing a connection


$\stackrel{\ominus}{-1}$

## Securing a connection



## Securing a connection



## Some "Key" Definitions!



OR


Open padlock (unlocked) OR decrypted


## Securing a connection


$\stackrel{\ominus}{-1}$

## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Securing a connection



## Remember these?!

## Key <br> OR <br> 

Closed padlock (locked)
Open padlock (unlocked)


## But how to safely and securely transmit the cipher-shift "key"?

# A clever thought-experiment to transmit key, esp to those you haven't met before! 

How it works? Well, ...

$\square$









## 






















# Isn't that cool? We exchanged a secret (encrypted) message without having to agree to and exchange keys beforehand! 

## Digital Cryptography



## One encryption on top of another! Remember LIFO?



# A clever way to transmit key, in particular to those you haven't met before! 

## How we do this in practice?

## A clever way to transmit key, in particular to those you haven't met before!

"irreversible" solution = Public + Private Key Pairs


## "irreversible" solution = Public + Private Key Pairs



Main Key Pair Attributes:

- Related, but separate (each unique on its own)
- They are unique to each person/user
- When one locks, only the other one can unlock
- Do NOT share private key ... ever!


## Let's see how it all work ...

## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Isn't that super cool? <br> But how about the following scenario ...

## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



## Who can decrypt this message? What do you need to do it?

## Sending an Encrypted Message with Key Pairs



## Sending an Encrypted Message with Key Pairs



# Who can decrypt this message? EVERYONE What do you need to do it? MY PUBLIC KEY 

## Wouldn't that be stupid?

# Who can decrypt this message? EVERYONE What do you need to do it? MY PUBLIC KEY 

## Wouldn't that be stupid? OR WOULD IT?!

# Digital Signatures ... Proving Authorship 

## Do you know these gentlemen?



## Pioneers in Cryptography

Hellman says of Merkle:


Whitfield Diffie Martin Hellman Ralph Merkle
"Ralph, like us, was willing to be a fool, and the way to get to the top of the heap in terms of developing original [thought] is to be a fool, because only fools keep trying. You have idea number 1 , you get excited and it flops. Then you have idea number 2, you get excited and it flops. Then you have idea number 99, you get excited and it flops. Only a fool would be excited by the 100th idea, but it might take 100 ideas before one really pays off. Unless you're foolish enough to be continually excited, you won't have the motivation and the energy to carry it through. And God rewards fools."

## Remember safe deposit boxes?



## Remember safe deposit boxes?



## How to encrypt, sign, transmit, and decrypt a msg



## An in-class exercise ... some simple math ;-)

## Multiplying

## 294 * 992 = ? (by hand)

You have 5 minutes!

## Multiplying

## 294 * $992=291,648$

## Multiplying vs. Factoring

$$
294 * 992=291,648
$$

Now factor 938,081 (by hand)
You have 10 minutes!

## Multiplying vs. Factoring

$$
294 * 992=291,648
$$

Now factor 938,081 1087 * 863 (two primes)

# Use of Prime Numbers and Modular Arithmetics 

There are
1,925,320,391,606,803,968,923
prime numbers below $10^{23}$ alone

## Largest prime number discovered yet ...

INDEPENDEN'T News Voices Sports Culture Indy/Life Video Daily E Q = = =

News Science

## Largest known prime number discovered with over 23 million digits

Discovery made on computer belonging to electrical engineer who searched for the elusive number for 14 years

Josh Gabbatiss Science Correspondent | @josh_gabbatiss |Friday 5 January 2018 18:00| 2 comments

If two plus three equals five $(2+3=5)$ and two plus eleven equals one ( $2+11=1$ ), then what is five plus eleven? ( $5+11=$ ??)

## Hint ...



If two plus three equals five $(2+3=5)$ and two plus eleven equals one ( $2+11=1$ ), then what is five plus eleven? ( $5+11=$ ??)

## Let's review some (simple) math ... sorry!!

A few words on (math) functions

$$
f(x)=x^{2}+8
$$

## Functions in Math

- Simply put, a function is a (mathematical) operation ...
- ... one input equals to one output
- $f(x)$ where $x$ is the input value
- Example:
- our function is "Doubling" $\rightarrow$
- $f(x)=2 x \rightarrow$
- Take an input, then double it (or multiply by 2 )
- For $x=4$ (i.e. input is 4 ), then the output is 8
- But then a funny thing happens ...


## Functions in Math

- But then a funny thing happens ...
- ... our function is still "Doubling" $\rightarrow$
- So what if I give you the output only? Can you figure out the input?
- OF COURSE ... we'll just reverse the function
- Example:
- our function is "Doubling" $\rightarrow$
- $f(x)=2 x \rightarrow$
- If the output is $\mathbf{4 4}$, then the input is ...
- 22 ;-)
- Most functions in math are Two-way Functions (reversible)
- But then ...


## Modular (or clock) Arithmetics



## Start at 5, then jump 11 units ...



## If $(2+3=5)$ and $(2+11=1)$, <br> then ( $5+11=\underline{4}$ )

## $2+3=5(\bmod 12)$ <br> $2+11=1(\bmod 12)$

$5+11=4(\bmod 12)$

## Let's calculate $11 \times 9(\bmod 13)=$ ?

# Let's calculate $11 x 9(\bmod 13)=$ ? <br> First, let's use "regular" math: 11x9=99 <br> Then, let's divide: $99 \div 13=7$, with remainder 8 

> So
> $11 \mathrm{x} 9=8(\bmod 13)$

## In-Class Exercise (you can use calculators only)

| for $\mathrm{x}=$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $3^{\mathrm{x}}$ |  |  |  |  |  |  |
| $3^{x}(\bmod 7)$ |  |  |  |  |  |  |

## Homework for Next Class (can use calculator

| for $x=$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $3^{x}$ | 3 | 9 | 27 | 81 | 243 | 729 |
| $3^{x}(\bmod 7)$ | 3 | 2 | 6 | 4 | 5 | 1 |

## Let's consider this special one-way function ...

$Y^{x}(\bmod P) \ldots$ with $Y<P$ as two prime numbers
$Y$ and $P$ are NOT secrets and can be shared

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

Alice
Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )

Bob
Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

## Alice

Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Alice picks a secret number $\mathbf{A}$ (e.g. 3)

Bob
Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Bob picks a secret number $\mathbf{B}$ (e.g. 6)

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

## Alice

Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Alice picks a secret number $\mathbf{A}$ (e.g. 3)
Plug 3 as X into our function to get $\boldsymbol{a}$, so $7^{3}$ $(\bmod 11) \rightarrow 343(\bmod 11)=2$

## Bob

Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Bob picks a secret number $\mathbf{B}$ (e.g. 6)
Plug 6 as $X$ into our function to get $\boldsymbol{\beta}$, so $7^{6}$ $(\bmod 11) \rightarrow 117,649(\bmod 11)=4$

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

## Alice

Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Alice picks a secret number $\mathbf{A}$ (e.g. 3)
Plug 3 as $X$ into our function to get $\boldsymbol{a}$, so $7^{3}$ $(\bmod 11) \rightarrow 343(\bmod 11)=2$

Send a (or 2 ) to Bob

## Bob

Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Bob picks a secret number B (e.g. 6)
Plug 6 as $X$ into our function to get $\boldsymbol{\beta}$, so $7^{6}$
$(\bmod 11) \rightarrow 117,649(\bmod 11)=4$
Send $\boldsymbol{\beta}$ (or 4) to Alice

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

## Alice

Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Alice picks a secret number $\mathbf{A}$ (e.g. 3)
Plug 3 as X into our function to get $\boldsymbol{a}$, so $7^{3}$ $(\bmod 11) \rightarrow 343(\bmod 11)=2$

Send $\boldsymbol{a}$ to Bob
Plug $\boldsymbol{\beta}$ into $\boldsymbol{\beta}^{A}(\bmod 11) \rightarrow 4^{3}(\bmod 11) \rightarrow 64$ $(\bmod 11)=9$

## Bob

Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Bob picks a secret number $\boldsymbol{B}$ (e.g. 6)
Plug 6 as $X$ into our function to get $\boldsymbol{\beta}$, so $7^{6}$ $(\bmod 11) \rightarrow 117,649(\bmod 11)=4$

Send $\boldsymbol{\beta}$ to Alice
Plug $\boldsymbol{a}$ into $\boldsymbol{a}^{\mathrm{B}}(\bmod 11) \rightarrow 2^{6}(\bmod 11) \rightarrow 64$ $(\bmod 11)=9$

## Our function is $Y^{x}(\bmod P) \ldots$ with $Y<P$

## Alice

Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Alice picks a secret number $\mathbf{A}$ (e.g. 3)
Plug 3 as X into our function to get $\boldsymbol{a}$, so $7^{3}$ $(\bmod 11) \rightarrow 343(\bmod 11)=2$

Send $\boldsymbol{a}$ to Bob
Plug $\beta$ into $\beta^{A}(\bmod 11) \rightarrow 4^{3}(\bmod 11) \rightarrow 64$ $(\bmod 11)=9$

How cool! Alice has the same KEY as Bob without exchange of the actual key!

## Bob

Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Bob picks a secret number $\boldsymbol{B}$ (e.g. 6)
Plug 6 as $X$ into our function to get $\boldsymbol{\beta}$, so $7^{6}$ $(\bmod 11) \rightarrow 117,649(\bmod 11)=4$

Send $\beta$ to Alice
Plug $\boldsymbol{a}$ into $\boldsymbol{a}^{\mathrm{B}}(\bmod 11) \rightarrow 2^{6}(\bmod 11) \rightarrow 64$ $(\bmod 11)=9$

How cool! Bob has the same KEY as Alice without exchange of the actual key!

## If you are Eve (snooper), can you figure out the key?

Alice
Agree \& share on Y \& P (e.g. Y=7 \& P=11)
Alice picks a secret number $\mathbf{A}$ (e.g. $\boldsymbol{\square}$ )
Plug as X into our function to get $\boldsymbol{a}$, so 7] $(\bmod 11)=2$

Send $\boldsymbol{a}$ to Bob
Plug $\boldsymbol{\beta}$ into $\boldsymbol{\beta}^{A}(\bmod 11) \rightarrow 4 \(\bmod 11)=\square$ $\left[7^{\times}(\bmod 11)\right]$ and both $\boldsymbol{a}$ and $\boldsymbol{\beta}$, but neither A nor B!

## Bob

Agree \& share on $Y$ \& $P$ (e.g. $Y=7$ \& $P=11$ )
Bob picks a secret number $\mathbf{B}$ (e.g. )
Plug as X into our function to get $\boldsymbol{\beta}$, so 7 $(\bmod 11)=4$

Send $\boldsymbol{\beta}$ to Alice
Plug $\boldsymbol{a}$ into $\boldsymbol{a}^{B}(\bmod 11) \rightarrow 2 \|(\bmod 11)=\square$
What is the KEY? Eve knows the function, Y, P What is the KEY? Even knows the function, Y ,
$P\left[7^{\times}(\bmod 11)\right]$ and both $\boldsymbol{a}$ and $\boldsymbol{\beta}$, but neither A nor B!

## We'll come back to one-way functions later on, ... so stay tuned ;-)

Now back to our Public-Private Key Pair model

## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption: Ease Computational Reqs.



## Public Key Encryption

## -

$\left.)_{3}\right)_{3}$

## Public Key Encryption

## -8 気

Q Q


## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## Public Key Encryption



## HTTPS Adoption (link)

Percentage of pages loaded over HTTPS in Chrome by platform
——Windows - Android - Chrome - Linux - Mac
100\%

90\%

80\%
$70 \%$

60\%
50\%

40\%


Fragment navigations, history push state navigations, and all schemes besides HTTP/HTTPS (including new tab page navigations) are not included.

## A little fun learning ... with movies!








## One of the greatest movies of all time ... if not THE greatest is ...

BALE CANE LEDGER OLOMMAN ECK゙मART GYLELENHAAL FRREEMAN
THE DARK KNIGH T





The database is null-key encrypted.
$\square$

It can only be accessed by one person.

# Aren't movies fun?! Back to our impostor ... 

## The Impostor (or Impersonator)



## The Impostor



## The Impostor

Alice


## The Impostor



## A Real-Life Impostor


> Someone just used your password to try to sign in to your Google Account

```
> john.podesta@gmail.com.
```

$>$
> Details:
> Saturday, 19 March, 8:34:30 UTC
> IP Address: 134.249.139.239
> Location: Ukraine
$>$
$>$ Google stopped this sign-in attempt. You should change your password
> immediately.
$>$
> CHANGE PASSWORD [https://bit.ly/1PibSU0](https://bit.ly/1PibSU0)
$>$
$>$ Best,
> The Gmail Team
> You received this mandatory email service announcement to update you
about
> important changes to your Google product or account.
$>$

```
> Someone just used your password to try to sign in to your Google
Account
> john.podesta@gmail.com.
>
> Details:
> Saturday, 19 March, 8:34:30 UTC
> IP Address: 134.249.139.239
> Location: Ukraine
>
> Google stopped this sign-in attempt. You should change your password
> immediately.
>
    CHANGE PASSWORD <https://bit.ly/1PibSUO>
> Best,
> The Gmail Team
> You received this mandatory email service announcement to update you
about
> important changes to your Google product or account.
>
```


## Digital Signatures



## Digital Signatures



## Digital Signatures



## Digital Signatures

## Sending Side

## Digital Signatures



Private Key

## Sending Side

## Digital Signatures



## Sending Side

## Digital Signatures



## Digital Signatures



Message


## Sending Side

## Digital Signatures



## Digital Signatures



Images: Wikimedia

## Digital Signatures

Receiving Side

## Digital Signatures



Public Key
of Sender
Receiving Side

## Digital Signatures



Public Key
of Sender


Message

## Digital Signatures



## Receiving Side

## Digital Signatures



## Digital Signatures



## Receiving Side

## Digital Signatures



## Receiving Side

## Digital Signatures



Images: Wikimedia

## Digital Signatures



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates: An Analogy



## Certificates Authorities



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates

VERISIGN


## TLS Handshake

- Secure
https://www.google.com/


## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates: A Bar Analogy



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \& Certificates



## Digital Signatures \＆Certificates


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## 目。

## Digital Signatures \& Certificates



## Digital Signatures \& Certificates




Images: Wikimedia

## One more thing ... best password?

## GMw89\#hUPn_d>k

horse_correct_bat

## One more thing ... best password?

GMw89\#hUPn_d>k
horse_correct_bat



## One more thing ... best password?

GMw89\#hUPn_d>k (72.0 bits of entropy)
horse_correct_bat (74.3 bits of entropy)

Horse_correct_bat (82.6 bits of entropy)


## III. Too Big to Fail

What happens when the attacker is someone we're supposed to trust?

## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Can't Trust the Banks

## Can't Trust the Banks

## $\equiv \quad$ The Alewhork wimes $Q$ <br> ASIA PACIFIC <br> Bangladesh Bank Chief Resigns After Cyber Theft of $\$ 81$ Million

By RICK GLADSTONE MARCH 15, 2016





## Things We’ve Been Trusting

- The banks with which we store our money.
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- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Can't Trust the Tech Companies

## Can't Trust the Tech Companies

| - $0, \cdots$ | theguardian |  |
| :---: | :---: | :---: |
| home ) US | politics world opinion | 三 all |
| US national security <br> Glenn Greenwald on security and liberty |  |  |
| NSA Prism program taps in to user data of Apple, Google and others |  |  |
| Top-secret Prism program claims direct access to servers of firms including Google, Apple and Facebook - Companies deny any knowledge of program in operation since 2007 |  |  |

## Can't Trust the Tech Companies



## Can't Trust the Tech Companies



## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Can't Trust the Stores

## Can't Trust the Stores

## Target: 40 million credit cards compromised

by CNNMoney Staff @CNNMoney<br>(L) December 19, 2013: 4:41 PM ET

## Can't Trust the Stores

|  |  |
| :---: | :---: |
| () © (ㅌ) (9) | manimaes smazaz un |
| Home Depot Hackers | edit cards compromised |
| Email Addresses | $\square 000{ }^{\circ}$ |
| Hackers Used Password Stolen From Vendor to Gain Access to Retailer's Systems By Shelly banjo |  |

## Can't Trust the Stores

| $\equiv$ | THEW W | FOXNEWS Food \& Drink | Q |
| :--- | :--- | :--- | :--- |

## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Can't Trust the CAs

## Can't Trust the CAs

## 

## Hacking in the Netherlands Took Aim at Internet Giants

By THE ASSOCIATED PRESS SEPT. 5, 2011


AMSTERDAM (AP) - Attackers who hacked into a Dutch Web security firm have issued hundreds of fraudulent security certificates for intelligence agency Web sites, including the C.I.A., as well as for Internet giants like Google, Microsoft and Twitter, the Dutch government said on Monday.

## Can't Trust the CAs

|  | cs |
| :---: | :---: |
| ((cc.e.) The Securit ina serious way ${ }^{\text {Sews }}$ | s |
| Chinese Certificate Authority 'mistakenly' gave out SSL Certs for GitHub Domains |  |

## Can't Trust the CAs

( ( (

## Things We’ve Been Trusting

- The banks with which we store our money.
- The tech companies with which we send messages and share files with friends/family.
- The stores we shop from.
- The list of trusted certificate authorities from our web browser.


## Things We’ve Been Trusting

- The banks with whir tore our mone
- The tech companies friends/family.
- The stores we shop from.
- The list of trusted



## Decentralized Networks



A


B

## Real Decentralized Technologies



Internet


Bitcoin

## End of First Session (yay

## Thank you for your attention and participation

Get some rest, review the material, and we'll see you in our next class

# Blockchain, Cryptocurrencies \& Digital Tokens Demystified <br> Fall 2023 (EMBA) <br> Columbia Business School 

## Welcome Back to Session 2

## Curriculum Roadmap

| Morning | Nov 4 | Nov 18 | Dec 2 | Dec 9 |
| :---: | :---: | :---: | :---: | :---: |
|  | Networks \& Protocols | Hashing, Hashing Tables \& One- Way Functions \& a few more tech | Bitcoin + other forms of crypto payments and store of value mechanisms and media | DeFi \& Other Applications (Digital Tokens, CBDC, etc.) + Speaker: Future of Finance + Discussion Forum |
|  | Lunch | Lunch | Lunch | Lunch |
| Afternoon | Encryption \& Cryptography (plus some math!) | Bring it All <br> Together: Let's build a blockchain \& discuss variety of cases | Ethereum \& Other Digital Tokens + Speaker: Regulatory \& Legal Considerations in Blockchain \& Digital Assets | Governance, <br>  <br> More; Final Lecture on <br> How the Future May Play <br> Out + Final <br> Presentations |

## Class Schedule - Nov 4, Nov 18, Dec 2, Dec 9

## Class Plan

Nov 4
Nov 18
Dec 2
Dec 9

08:30 am to 6:45 pm (K-440)Module $1+2$
08:30 am to 6:45 pm (K-440)Module $3+4$
08:30 am to 6:45 pm (K-440)Midterm Project + 5 \& $6+$ Guest Speaker
08:30 am to 6:45 pm (K-440)Module 7 \& $8+$ Guest Speaker + final presentations

Daily Schedule
8:30-9:45 am
9:45-10:00 am
10:00-11:15 pm
11:15 am-12:30 pm
12:30-2:00 pm 2:00-2:15 pm
2:15-3:30 pm
3:30-3:45 pm
3:45-5:00 pm
5:00-5:15 pm
5:15-6:45 pm

## Lecture

Break
Lecture
Lunch (1h15min) - Kravis 2nd floor (Smith Dining)

## Lecture

Break

## Lecture

Break
Lecture
Break
Lecture

## Important Admin Items for the Day

- Team formations finalized today, ideally by 3:30 pm and no later than end of day today
- Details on your midterm project
- Thoughts on "Blockchain Killer App" for Sessions 3 and/or 4
- Make sure not to fall behind as Sessions $1 \& 2$ are foundational
- Watch lecture recordings and email me for office hours
- I REALLY enjoyed our first session, and thank you VERY much for the amazing level of participation and engagement. Let's hope today would be equally fun, if not more :)
- ... btw, did you watch The Simpsons episode right after our first class session? It was about blockchain \& NFTs!! Check out S35E5.


## THE MOST Important Admin Item for the Day

## THE MOST Important Admin Item for the Day

Catering today is by Dinosaur BBQ:

- Mac \& cheese
- Turkey
- Beef brisket
- BBQ Salmon
- Portabella Mushrooms w/ peppers \& onions
- Simmered Greens
- Sweet Potatoes


# Before we begin, any interesting points or lessons from our first session you'd like to share? 

## Let's start our Session 2

## History of Cryptographically-based e-Currencies:

It's nothing new:

- Remember Error 402?


## History of Cryptographically-based e-Currencies:

It's nothing new:

- Remember Error 402?
- DigiCash: proposed in 1983 by David Chaum, set up eCash, launched in 1989, declared bankruptcy in 1998
- CyberCash: payment service founded in 1994, IPO in 1996, set up CyberCoin for micro-payments (through NetBill at CMU), went bankrupt in 2001
- Hashcash: proposed in 1997 by Adam Back,
- BitGold: proposed by Nick Szabo in 1998 (he coined "Smart Contracts.") Although never implemented, it has many similarities to Bitcoin!
- ... and others (Hashcash, B-Money, First Virtual, etc.)

Why did these early forms of digital currencies fail?

# Double-Spending, Trust, and Consensus are amongst the top reasons ... 

Speaking of consensus ...

# Byzantine Generals Problem \& <br> the question of Byzantine Fault Tolerance 

## The Byzantine Generals Problem

LESLIE LAMPORT, ROBERT SHOSTAK, and MARSHALL PEASE SRI International

Reliable computer systems must handle malfunctioning components that give conflicting information to different parts of the system. This situation can be expressed abstractly in terms of a group of generals of the Byzantine army camped with their troops around an enemy city. Communicating only by messenger, the generals must agree upon a common battle plan. However, one or more of them may be traitors who will try to confuse the others. The problem is to find an algorithm to ensure that the loyal generals will reach agreement. It is shown that, using only oral messages, this problem is solvable if and only if more than two-thirds of the generals are loyal; so a single traitor can confound two loyal generals. With unforgeable written messages, the problem is solvable for any number of generals and possible traitors. Applications of the solutions to reliable computer systems are then discussed.
Categories and Subject Descriptors: C.2.4. [Computer-Communication Networks]: Distributed Systems—network operating systems; D.4.4 [Operating Systems]: Communications Managementnetwork communication; D.4.5 [Operating Systems]: Reliability-fault tolerance

ACM Transactions on Programming Languages and Systems, Vol. 4, No. 3, July 1982, Pages 382-401.

# In a distributed network, how many node failures can the system tolerate and still function as intended in delivering consensus? 

## IV. Building the Blockchain

Using cryptography to build decentralized technologies.

## Blockchains



Alice

Edith

## Central Ledger Ledger



## Central Ledger Ledger



## Central Ledger Ledger



## Central Ledger Ledger



Central Ledger $\times$ Ma Ledger


## Central Ledger Ledger




Blockchains x han Ledger


## Blockchains



## Blockchains



## Blockchains: everyone updates on their own asap!



## Blockchains



## Blockchains: stay in sync with code and NO trust



## Blockchains: store in blocks chained together



## Blockchain: a cryptographically-verifiable Tx chain

Everyone gets $\$ 100$

## Blockchain: a cryptographically-verifiable Tx chain

Everyone gets $\$ 100$

Alice gives Bob \$5

## Blockchain: a cryptographically-verifiable Tx chain

Everyone gets $\$ 100$


## Blockchain: a cryptographically-verifiable Tx chain



## Blockchain: a cryptographically-verifiable Tx chain



Edith gives Carol \$25


How to Verify? How to Encrypt?

> Bob gives Edith \$10

## Once again, we need some simple math (don't we love math by now?!)

Remember functions?

$$
f(x)=x^{2}+8
$$

## Functions in Math

- Simply put, a function is a (mathematical) operation ...
- ... one input equals to one output
- $f(x)$ where $x$ is the input value
- Example:
- our function is "Doubling" $\rightarrow$
- $f(x)=2 x \rightarrow$
- Take an input, then double it (or multiply by 2 )
- For $\mathbf{x = 4}$ (i.e. input is $\mathbf{4}$ ), then the output is $\mathbf{8}$
- But then a funny thing happens ...


## Functions in Math

- But then a funny thing happens ...
- ... our function is still "Doubling" $\rightarrow$
- So what if I give you the output only? Can you figure out the input?
- OF COURSE ... we'll just reverse the function
- Example:
- our function is "Doubling" $\rightarrow$
- $f(x)=2 x \rightarrow$
- If the output is $\mathbf{4 4}$, then the input is ...
- 22 ;-)
- Most functions in math are Two-way Functions (reversible)
- But then ...


## Hashing (One Way Functions)



## Hashing (One Way Functions)



## Hashing (One Way Functions)

## Some Input $\rightarrow \begin{gathered}\text { Hash } \\ \text { Function }\end{gathered} \rightarrow$ Output

## Hashing (One Way Functions)

## Some Input <br> Hash <br> Function <br> Output

## Output

## Hashing (One Way Functions)

## Some Input <br> Hash <br> Function <br> Output

## Output

## Hash

Reverser

## Hashing (One Way Functions)

## Some Input <br> Hash <br> Function <br> Output

Doesn't Exist

A great example of a One-way Function ...


## Another great example of a One-way Function

## Real-World One-Way Function (Hashing Function)



## Real-World One-Way Function (Hashing Function)

| SuperPages.com | 195 |
| :---: | :---: |
| Cartage New England Inc <br> 26 Allen In Inswich 01938. <br> 978 356-9960 | Carter F 24 Hillock Ros 02131.............. 617 327-1105 Faye \& Ricky |
| Cartagema Lydia | 357 Columbus Av Bos 02116........... 617 437-7331 |
| 18 Jewett Ros $02131 . . . . . . . . . . . . . . . . . . . ~$ <br> Cartagena Avith | Franklin \& Anne |
| 98 Bancoft Rox 02119................ 617 442-9780 |  |
| Hyd 02136........................... $617361-5253$ |  |
| Jessica 50 Decatur Cha 02129......... 617 241-0152 | Fred 96 Hinckley Rd Mi $02186 . . . . . . . . . . .617$ 698-1343 |
| Lucilla 174 Harard Cam 02139....... $6174391-5621$ | G \& R 8 Verdun Dor 02124.............617 436-8906 |
| M 95 R Rowe Ros 02131................. 617 | G T 27 franklin Av Som 02145........... 617 623-7121 |
| Melvin 5016 Green Cam | Gayle 25 frontenac Dor 02224.......... 617825 -0322 |
| arte Nicholas | Geo S 15 Moss Hill R d Jam 02230......617 522-3215 |
| 18 Appleton Boston 02116.............617 695-6996 |  |
| Cartegena 044 Milford Bos 02118 .......617 338-8219 | Carter Halliday Associate |
| Carten Thos J Sr \& Claire <br> IParadise Rd Mil 02186................ 617 698-6163 | Carter Harry F |
| omas \& Kathl | 26 Runng Brik Rd W Rox $02132 . . . . . . .617$ 325-5465 |
| 50 Thompson LL Mil 02186............617 696-6919 | Carter Hide Co Inc |
| rter A Ros 02131...................... 617 327-2257 | 146 Summer Bos 02110..................... 617 542-7987 |
| A Roxbury.............................. 617 442-5230 | Carter Hilary 61 Harve Cam 02140..... 617 876-2750 |
| A 31 Bethune Wy Rorbury 0219........ 617 442-1219 | Horace |
| A 260 Putuam ar Cambridge 02139.....617 492-4174 | Howard Jr 26 Notre Dme Rox 02119.617 445-5552 |
| A M 255 Masclsts Av Bos 02115........617 266-7153 |  |

## Real-World One-Way Function (Hashing Function)

## SuperPages.com

| Cartage New England Inc 26 Allen Ln Ipswich 01938......... | 978 356-9960 |
| :---: | :---: |
| Cartagema Lydia |  |
| 18 Jewett Ros 02131 | 617 323-7639 |
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| B Hyd 02136. | 617 361-5253 |
| Jessica 50 Decatur Cha 0212 | 617 241-0152 |
| Lucilla 174 Harvard Cam 0213 | 617 491-5621 |
| M- 95 Rowe Ros 02131. | 617 323-9713 |
| Melvin 501 Green Cam 02139 | 576-1061 |
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| Cartegena 04 Milford Bos 021 | 8....... 617 338-8219 |
| Carten Thos J Sr \& Claire 1 Paradise Rd Mil 02186..... | 617 698-6163 |
| Thomas \& Kathleen |  |
| 50 Thompson Ln Mill 02186. | 617 696-6919 |
| arter A Ros 02131. | 617 327-2257 |
| A Roxbury... | 617 442-5230 |
| A 31 Bethune Wy Roxbury 0211 | ........ 617 442-1219 |
| A 260 Putnam Av Cambridge 02 | 39..... 617 492-4174 |
| A M 255 Maschsts Ar Bos 0271 | 617 266-7153 |



## Our Function is = <br> for a given input, find the output

## Our Function is = for a given input (name) $\rightarrow$

## find the output (corresponding phone number)

## A Real-World Hashing Function

SuperPages.com

| Cartage New England Inc |  |
| :---: | :---: |
| Cartagema Lydia |  |
| 18 Jewett Ros 02 |  |
| Cartagena Avith |  |
| 9 Bancroft Rox 021 |  |
| Hyd 02136 | 617 361-5253 |
| Jessica 50 Decatur Cha $02129 \quad 617$ 241-0152 |  |
| Lucilla 174 Harvard Cam 02139........617 491-5621 |  |
| M-95 Rowe Ros 02131....................617 323-9713 |  |
| Melvin 501 Green Cam |  |
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| 18 Appleton Boston 02116.............617 695-6996 |  |
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| 1 Paradise Rd Mil 02186.............. 617 698-6163 |  |
| Thomas \& Kathleen |  |
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| A Roxbury................................ 617 442-5230 |  |
| A 31 Bethune Wy Roxbury 02119.........617 442-1219 |  |
| A 260 Putnam Av Cambridge 02139...... 617 492-4174 |  |
| 255 Maschsts Av Bos $02115 . . . . . . .617$ 266-7 |  |


| Carter F 24 Hillock Ros 02131............... 6 Faye \& Ricky |
| :---: |
| 357 Columbus Av Bos 02116..........617 437-7331 |
| Francis S 134 Temple W Rox 02132.. 617 323-6781 |
| Franklin \& Anne |
| 221 Mt Auburn Cam 02138........... 617 354-0798 |
| Fred 42 Haverford Jàm 02130..........6617 524-3078 |
| Fred 96 Hinckley RdMin 02186...........617 698-1343 |
| G \& R 8 Verdun Dor 02124...............617 436-8906 |
| G T 27 Frankin Av Som 02145...........617 623-7121 |
| Gayle 25 Frontenac Dor 02124..........617 825-0322 |
| Geo S 115 Moss Hill Rd Jam 02130.....617 522-3215 |
| George 125 Nashua Bos 02114.......617 367-9548 |
| Carter Halliday Associate |
| 107 S Street Bos 02111.....................617 456-1689 |
| arter Harry F |
| 26 Runng Brk Rd W Rox 02132........617 325-5465 |
| arter Hide Co Inc |
| 146 Summer Bos 02110.....................617 542-7987 |
| Carter Hilary 61 Harvey Cam 02140.....617 876-2750 |
| Horace |
| 241 Walnut Av Roxbury 02119.........617 442-5307 |
| Howard Jr 26 Notre Dme Rox 02119.617 445-5552 |

357 Columbus Av Bos 02116...........617 437-7331
Francis S 134 Temple W Rox 02132.. 617 323-6781 ranklin \& Anne
221 Mt Auburn Cam 02138.............617 354-0798
Fred 96 Hinckley Rd MiI $02186 . . . . . . . . . .617$ 698-1343
G \& R 8 Verdun Dor 02124................617 436-8906
G T 27 Frankin Av Som 02145............ 617 623-7121
Gayle 25 Frontenac Dor 02124.......... 617 825-0322
Geo S 115 Moss Hill Rd Jam 02130..... 617 522-3215
George 125 Nashua Bos 0214......... 617 367-9548
107 S Street Bos 02111........................617 456-1689
26 Runng Brk Rd W Rox 02132........ 617 325-5465
146 Summer Bos 02110........................ 617 542-7987
Carter Hilary 61 Harvey Cam 02140..... 617 876-2750

Howard Jr 26 Notre Dme Rox 02119.617 445-5552

## A Real-World Hashing Function



## A Real-World Hashing Function



## A Real-World Hashing Function



## A Real-World Hashing Function

Columbia
Business

(212) 854-1100

## A Real-World Hashing Function

(212) 854-5553


## A Real-World Hashing Function

(212) 854-5553





## So that you know what one-way functions are, let's continue to learn more about hashing and hashing tables ...

Imagine we have a database of over 50 million phone numbers of our customers in the United States. My database does not allow sorting, so how do I find the name of a business associated with a phone number in our database?

| Business Name | Phone Number |
| :--- | :--- |
| Stone Rock Capital LLC | $212-854-3487$ |
| Simple Basic Partners LLP | $213-718-1696$ |
| Blue Pebble Capital LLC | $212-376-3900$ |
| Navy Rock Ventures LLC | $323-839-1748$ |
| Sky Limit Venture Partners LLP | $650-337-6291$ |


| Business Name | Phone Number |
| :--- | :--- |
| Stone Rock Capital LLC | $212-854-3487$ |
| Simple Basic Partners LLP | $213-718-1696$ |
| Blue Pebble Capital LLC | $212-376-3900$ |
| Navy Rock Ventures LLC | $323-839-1748$ |
| Sky Limit Venture Partners LLP | $650-337-6291$ |

# Let's develop a method (Protocol or Algorithm) to simplify these phone numbers and be able to create sub-categories for storing in our database ... 

212-854-3487 (take a number from our directory)

2128543487 (separate into two-digit numbers)
$2+12+85+43+48+7$ (add up the digits of each tw-digit pair until you get a single-digit number)
$\begin{array}{lllll}3 & 10 & 9 & 7 & 15\end{array}$
$\begin{array}{llll}3 & 1+0 & 9 & 7\end{array}$
$\begin{array}{llll}3 & 1 & 9 & 6 \text { (Done! Then combine to form a } 5 \text {-digit category number for storing) }\end{array}$
31976

| Business Name | Phone Number | Category |
| :--- | :--- | :--- |
| Stone Rock Capital LLC | $212-854-3487$ | 31976 |
| Simple Basic Partners LLP | $213-718-1696$ |  |
| Blue Pebble Capital LLC | $212-376-3900$ |  |
| Navy Rock Ventures LLC | $323-839-1748$ |  |
| Sky Limit Venture Partners LLP | $650-337-6291$ |  |

213-718-1696 (take a number from our directory)

2137181696 (separate into two-digit numbers)
$2+13+71+81+69+6$ (add up the digits of each tw-digit pair until you get a single-digit number)

## $\begin{array}{lllll}3 & 10 & 9 & 7 & 15\end{array}$

$\begin{array}{llll}3 & 1+0 & 9 & 7\end{array}$
$\begin{array}{llll}3 & 1 & 9 & 6 \text { (Done! Then combine to form a } 5 \text {-digit category number for storing) }\end{array}$
31976

| Business Name | Phone Number | Category |
| :--- | :--- | :--- |
| Stone Rock Capital LLC | $212-854-3487$ | 31976 |
| Simple Basic Partners LLP | $213-718-1696$ | 31976 (is this a problem?!) |
| Blue Pebble Capital LLC | $212-376-3900$ |  |
| Navy Rock Ventures LLC | $323-839-1748$ |  |
| Sky Limit Venture Partners LLP | $650-337-6291$ |  |

## 78742938817753999196055303459477291037892373684068

78742938817753999196055303459477291037892373684068
$7+87+42+9$ 3+8 ...
15131111 ...
$1+51+31+11+1 \ldots$

6422 ...

## Let's continue to learn more about hashing and hashing tables ...

| Name | ID Codes |
| :--- | :--- |
| Dara | 330 i |
| Cara | X7 |
| Bea | X3 |
| Alice | M4 |
| Ella | 128 i |


| Name | V |
| :--- | :--- |
| Alice | ID Codes |
| Bea | X3 |
| Cara | X7 |
| Dara | 330 i |
| Ella | 128 i |


| Name | ID Codes |
| :--- | :--- |
| Ella | 128 i |
| Dara | 330 i |
| Alice | M4 |
| Bea | X3 |
| Cara | X7 |

## Again, imagine no sorting is allowed ...

 or the table has tens of thousands of rows and hundred of columns (big data)| Name | Codes |
| :--- | :--- |
| Stadtverordnetenversammlung | 2840 |
| KraftfahrzeugHaftpflichtversicherung | 9508 |
| Siebentausendzweihundertvierundfünfzig | 7254 |
| Rechtsschutzversicherungsgesellschaften | 3126 |
| Rindfleischetikettierungsüberwachungsaufgabenübertragungsgesetz | 5434 |
| Donaudampfschifffahrtselektrizitätenhauptbetriebswerkbauunterbeamtengesellschaft | 8923 |


| Item Number | Tariff Code |
| :--- | :--- |
| 78742938817753999196055303459477291037892373684068 | z9m0 |
| 76539710192327255231902237652982747470592661143566 | $0 h 23$ |
| 88984727710651739231245830019043173775547558023984 | 3 f 26 |
| 77603278128172851537873810966507560948211829756526 | 787 y |
| 46527684654614009996682441601858375203324083908888 | $8 \mathrm{nc6}$ |


| Item Number | Tariff Code |
| :--- | :--- |
| 78742938817753999196055303459477291037892373684068 | z9m0 |
| 76539710192327255231902237652982747470592661143566 | 0 h 23 |
| 88984727710651739231245830019043173775547558023984 | $3 f 26$ |
| 77603278128172851537873810966507560948211829756526 | 787 y |
| 46527684654614009996682441601858375203324083908888 | $8 \mathrm{nc6}$ |

## 78742938817753999196055303459477291037892373684068

78742938817753999196055303459477291037892373684068
$7+87+42+9$ 3+8 ...
15131111 ...
$1+51+31+11+1 \ldots$

6422 ...

| Hash | Item Number | Tariff <br> Code |
| :--- | :--- | :--- |
| 1458 | 78742938817753999196055303459477291037892373684068 | z9m0 |
| 5624 | 76539710192327255231902237652982747470592661143566 | Oh23 |
| 4548 | 88984727710651739231245830019043173775547558023984 | $3 f 26$ |
| 4465 | 77603278128172851537873810966507560948211829756526 | 787 y |
| 2677 | 46527684654614009996682441601858375203324083908888 | 8nc6 |

# We need to REALLY minimize the chance of two items having the same hash ... 

SHA to the rescue!

## Bitcoin's Hashing Function



## Bitcoin's Hashing Function

## Arbitrary Data

## SHA-256

## Bitcoin's Hashing Function



## Bitcoin's Hashing Function


$0 \rightarrow \mathbf{2}^{\wedge} \mathbf{2 5 6}$

## SHA-256

SHA-256 hash: a number with the range:

$$
0 \rightarrow 2^{256}
$$

## SHA-256

## 2^256 equals to:

115792089237316195423570985008687907853269984665640564039457584007913129639936

## SHA-256

## SHA-256 hash: a number with the range:

$$
0 \rightarrow 2^{256}
$$

## SHA-256: Using an unimaginably large number!

Note that $2^{256}$ is approximately $10^{77}$
The sum of all the atoms in the universe are estimated to be $10^{80}$ (or between $10^{78}$ and $10^{82}$ )

## SHA-256 Hash: a continuous number line

## SHA-256 Hash: a continuous number line


$\mathbf{2}^{\wedge} \mathbf{2 5 6}=115,792,089,237,316,195,423,570,985,008,687,907,853,269,984,665,640,564,039,457,584,007,913,129,639,936$

## SHA-256: points on the long line

## Each point would be consisting of many digits:

```
0
1
2
3
4
8
25
```

9387
23430174432
57098500868790785
7316195423570985008687907853269984665640
4853269984665907859895748813748971384798546645240492
115792089237316195423570985008687907853269984665640564039457584007913129639

## Numerical Encoding

|  | Example | Digits Used |
| :--- | :---: | :---: |
| Binary Number | 11011000 | 01 |
| Decimal Number | 2128541100 | 0123456789 |
| Hexadecimal Number | 7 edef5ac | $0123456789 a b c d e f$ |

## Odometer (mileage count)



## Odometer (mileage count)



## Odometer (mileage count)



## Numerical Encoding

|  | Example | Digits Used |
| :--- | :---: | :---: |
| Binary Number | 11011000 | 01 |
| Decimal Number | 2128541100 | 0123456789 |
| Hexadecimal Number | 7 edef5ac | $0123456789 a b c d e f$ |

## SHA-256 Hash

## Decimal HexaDecimal

O 000000000000000000000000000000000000000000000000000000000000000

## SHA-256 Hash

## Decimal HexaDecimal

O 000000000000000000000000000000000000000000000000000000000000000
1000000000000000000000000000000000000000000000000000000000000001

## SHA-256 Hash

## Decimal HexaDecimal

O 000000000000000000000000000000000000000000000000000000000000000
$1 \quad 000000000000000000000000000000000000000000000000000000000000001$
2000000000000000000000000000000000000000000000000000000000000002

## SHA-256 Hash

## Decimal HexaDecimal

## 0 000000000000000000000000000000000000000000000000000000000000000 <br> 1000000000000000000000000000000000000000000000000000000000000001 <br> 2000000000000000000000000000000000000000000000000000000000000002 <br> 9 000000000000000000000000000000000000000000000000000000000000009

## SHA-256 Hash

## Decimal HexaDecimal

( 000000000000000000000000000000000000000000000000000000000000000
$1 \quad 000000000000000000000000000000000000000000000000000000000000001$ 2000000000000000000000000000000000000000000000000000000000000002

9 00000000000000000000000000000000000000000000000000000000000009
100000000000000000000000000000000000000000000000000000000000000 a

## SHA-256 Hash

## Decimal HexaDecimal

0 000000000000000000000000000000000000000000000000000000000000000
100000000000000000000000000000000000000000000000000000000000000120000000000000000000000000000000000000000000000000000000000000029 000000000000000000000000000000000000000000000000000000000000009
$10 \quad 00000000000000000000000000000000000000000000000000000000000000$ ..... a
1100000000000000000000000000000000000000000000000000000000000000 ..... b
1500000000000000000000000000000000000000000000000000000000000000 ..... f

## SHA-256 Hash

Decimal HexaDecimal

> 0 000000000000000000000000000000000000000000000000000000000000000 1000000000000000000000000000000000000000000000000000000000000001 2000000000000000000000000000000000000000000000000000000000000002 9 000000000000000000000000000000000000000000000000000000000000009 $10 \quad 00000000000000000000000000000000000000000000000000000000000000$ a

## SHA-256 Hash

## Decimal HexaDecimal

( 000000000000000000000000000000000000000000000000000000000000000
$1 \quad 000000000000000000000000000000000000000000000000000000000000001$ 2000000000000000000000000000000000000000000000000000000000000002
9 000000000000000000000000000000000000000000000000000000000000009
1000000000000000000000000000000000000000000000000000000000000000 a
1100000000000000000000000000000000000000000000000000000000000000 b
1500000000000000000000000000000000000000000000000000000000000000 £
16000000000000000000000000000000000000000000000000000000000000010
17000000000000000000000000000000000000000000000000000000000000011

## SHA-256 Hash

## Decimal HexaDecimal

000000000000
1000000000000000000000000000000000000000000000000000000000000001 2000000000000000000000000000000000000000000000000000000000000002 9 000000000000000000000000000000000000000000000000000000000000009 1000000000000000000000000000000000000000000000000000000000000000 a
11 000000000000000000000000000000000000000000000000000000000000 bb

16000000000000000000000000000000000000000000000000000000000000010
17000000000000000000000000000000000000000000000000000000000000011
1,0000000000000000000000000000000000000000000000000000000000000003 e8 $1,000,000 \quad 0000000000000000000000000000000000000000000000000000000000$ f4240

## Numerical Encoding

|  | Example | Digits Used |
| :--- | :---: | :---: |
| Binary Number | 11011000 | 01 |
| Decimal Number | 2128541100 | 0123456789 |
| Hexadecimal Number | 7 edef5ac | $0123456789 a b c d e f$ |




# Decimal - Binary - Octal - Hex - ASCII 

Conversion Chart

| Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 00000000 | 000 | 00 | NUL | 32 | 00100000 | 040 | 20 | SP | 64 | 01000000 | 100 | 40 | @ | 96 | 01100000 | 140 | 60 | - |
| 1 | 00000001 | 001 | 01 | SOH | 33 | 00100001 | 041 | 21 | ! | 65 | 01000001 | 101 | 41 | A | 97 | 01100001 | 141 | 61 | a |
| 2 | 00000010 | 002 | 02 | STX | 34 | 00100010 | 042 | 22 | * | 66 | 01000010 | 102 | 42 | B | 98 | 01100010 | 142 | 62 | b |
| 3 | 00000011 | 003 | 03 | ETX | 35 | 00100011 | 043 | 23 | \# | 67 | 01000011 | 103 | 43 | C | 99 | 01100011 | 143 | 63 | c |
| 4 | 00000100 | 004 | 04 | EOT | 36 | 00100100 | 044 | 24 | \$ | 68 | 01000100 | 104 | 44 | D | 100 | 01100100 | 144 | 64 | d |
| 5 | 00000101 | 005 | 05 | ENQ | 37 | 00100101 | 045 | 25 | \% | 69 | 01000101 | 105 | 45 | E | 101 | 01100101 | 145 | 65 | e |
| 6 | 00000110 | 006 | 06 | ACK | 38 | 00100110 | 046 | 26 | \& | 70 | 01000110 | 106 | 46 | F | 102 | 01100110 | 146 | 66 | f |
| 7 | 00000111 | 007 | 07 | BEL | 39 | 00100111 | 047 | 27 | - | 71 | 01000111 | 107 | 47 | G | 103 | 01100111 | 147 | 67 | 9 |
| 8 | 00001000 | 010 | 08 | BS | 40 | 00101000 | 050 | 28 | ( | 72 | 01001000 | 110 | 48 | H | 104 | 01101000 | 150 | 68 | h |
| 9 | 00001001 | 011 | 09 | HT | 41 | 00101001 | 051 | 29 | ) | 73 | 01001001 | 111 | 49 | 1 | 105 | 01101001 | 151 | 69 | i |
| 10 | 00001010 | 012 | 0A | LF | 42 | 00101010 | 052 | 2A | * | 74 | 01001010 | 112 | 4A | J | 106 | 01101010 | 152 | 6 A | j |
| 11 | 00001011 | 013 | OB | VT | 43 | 00101011 | 053 | 2 B | + | 75 | 01001011 | 113 | 4B | K | 107 | 01101011 | 153 | 6 B | k |
| 12 | 00001100 | 014 | OC | FF | 44 | 00101100 | 054 | 2 C | , | 76 | 01001100 | 114 | 4 C | L | 108 | 01101100 | 154 | 6C | 1 |
| 13 | 00001101 | 015 | OD | CR | 45 | 00101101 | 055 | 2D | - | 77 | 01001101 | 115 | 4D | M | 109 | 01101101 | 155 | 6 D | m |
| 14 | 00001110 | 016 | OE | SO | 46 | 00101110 | 056 | 2E | . | 78 | 01001110 | 116 | 4E | N | 110 | 01101110 | 156 | 6 E | n |
| 15 | 00001111 | 017 | OF | SI | 47 | 00101111 | 057 | 2 F | 1 | 79 | 01001111 | 117 | 4F | 0 | 111 | 01101111 | 157 | 6 F | 0 |
| 16 | 00010000 | 020 | 10 | DLE | 48 | 00110000 | 060 | 30 | 0 | 80 | 01010000 | 120 | 50 | P | 112 | 01110000 | 160 | 70 | p |
| 17 | 00010001 | 021 | 11 | DC1 | 49 | 00110001 | 061 | 31 | 1 | 81 | 01010001 | 121 | 51 | Q | 113 | 01110001 | 161 | 71 | q |
| 18 | 00010010 | 022 | 12 | DC2 | 50 | 00110010 | 062 | 32 | 2 | 82 | 01010010 | 122 | 52 | R | 114 | 01110010 | 162 | 72 | r |
| 19 | 00010011 | 023 | 13 | DC3 | 51 | 00110011 | 063 | 33 | 3 | 83 | 01010011 | 123 | 53 | S | 115 | 01110011 | 163 | 73 | s |
| 20 | 00010100 | 024 | 14 | DC4 | 52 | 00110100 | 064 | 34 | 4 | 84 | 01010100 | 124 | 54 | T | 116 | 01110100 | 164 | 74 | t |
| 21 | 00010101 | 025 | 15 | NAK | 53 | 00110101 | 065 | 35 | 5 | 85 | 01010101 | 125 | 55 | U | 117 | 01110101 | 165 | 75 | u |
| 22 | 00010110 | 026 | 16 | SYN | 54 | 00110110 | 066 | 36 | 6 | 86 | 01010110 | 126 | 56 | v | 118 | 01110110 | 166 | 76 | v |
| 23 | 00010111 | 027 | 17 | ETB | 55 | 00110111 | 067 | 37 | 7 | 87 | 01010111 | 127 | 57 | W | 119 | 01110111 | 167 | 77 | w |
| 24 | 00011000 | 030 | 18 | CAN | 56 | 00111000 | 070 | 38 | 8 | 88 | 01011000 | 130 | 58 | X | 120 | 01111000 | 170 | 78 | x |
| 25 | 00011001 | 031 | 19 | EM | 57 | 00111001 | 071 | 39 | 9 | 89 | 01011001 | 131 | 59 | Y | 121 | 01111001 | 171 | 79 | y |
| 26 | 00011010 | 032 | 1A | SUB | 58 | 00111010 | 072 | 3 A | : | 90 | 01011010 | 132 | 5A | Z | 122 | 01111010 | 172 | 7A | z |
| 27 | 00011011 | 033 | 1B | ESC | 59 | 00111011 | 073 | 3 B | ; | 91 | 01011011 | 133 | 5B | [ | 123 | 01111011 | 173 | 7 B | \{ |
| 28 | 00011100 | 034 | 1C | FS | 60 | 00111100 | 074 | 3 C | < | 92 | 01011100 | 134 | 5C | 1 | 124 | 01111100 | 174 | 7 C | 1 |
| 29 | 00011101 | 035 | 1D | GS | 61 | 00111101 | 075 | 3 D | = | 93 | 01011101 | 135 | 5D | 1 | 125 | 01111101 | 175 | 7D | \} |
| 30 | 00011110 | 036 | 1E | RS | 62 | 00111110 | 076 | 3 E | > | 94 | 01011110 | 136 | 5E | $\wedge$ | 126 | 01111110 | 176 | 7E | $\sim$ |
| 31 | 00011111 | 037 | 1F | US | 63 | 00111111 | 077 | 3F | ? | 95 | 01011111 | 137 | 5F | - | 127 | 01111111 | 177 | 7 F | DEL |

Decimal - Binary - Octal - Hex - ASCII
Conversion Chart

| Decimal | Binary | Octal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII | Decima | Binary | Dctal | Hex | ASCII | Decimal | Binary | Octal | Hex | ASCII |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 00000000 | 000 | 00 | NUL | 32 | 00100000 | 040 | 20 | SP | 64 | 01000000 | 100 | 40 | @ | 96 | 01100000 | 140 | 60 | , |
| 1 | 00000001 | 001 | 01 | SOH | 33 | 00100001 | 041 | 21 | ! | 65 | 01000001 | 101 | 41 | A | 97 | 01100001 | 141 | 61 | a |
| 2 | 00000010 | 002 | 02 | STX | 34 | 00100010 | 042 | 22 | * | 66 | 01000010 | 02 | 42 | B | 98 | 01100010 | 142 | 62 | b |
| 3 | 00000011 | 003 | 03 | ETX | 35 | 00100011 | 043 | 23 | \# | 67 | 01000011 | 03 | 43 | C | 99 | 01100011 | 143 | 63 | c |
| 4 | 00000100 | 004 | 04 | EOT | 36 | 00100100 | 044 | 24 | \$ | 68 | 01000100 | 04 | 44 | D | 100 | 01100100 | 144 | 64 | d |
| 5 | 00000101 | 005 | 05 | ENQ | 37 | 00100101 | 045 | 25 | \% | 69 | 01000101 | 05 | 45 | E | 101 | 01100101 | 145 | 65 | e |
| 6 | 00000110 | 006 | 06 | ACK | 38 | 00100110 | 046 | 26 | \& | 70 | 01000110 | 06 | 46 | F | 102 | 01100110 | 146 | 66 | f |
| 7 | 00000111 | 007 | 07 | BEL | 39 | 00100111 | 047 | 27 | ' | 71 | 01000111 | 07 | 47 | G | 103 | 01100111 | 147 | 67 | g |
| 8 | 00001000 | 010 | 08 | BS | 40 | 00101000 | 050 | 28 | ( | 72 | 01001000 | 10 | 48 | H | 104 | 01101000 | 150 | 68 | h |
| 9 | 00001001 | 011 | 09 | HT | 41 | 00101001 | 051 | 29 | ) | 73 | 01001001 | 11 | 49 | 1 | 105 | 01101001 | 151 | 69 | i |
| 10 | 00001010 | 012 | OA | LF | 42 | 00101010 | 052 | 2A | * | 74 | 01001010 | 12 | 4A | J | 106 | 01101010 | 152 | 6A | j |
| 11 | 00001011 | 013 | OB | VT | 43 | 00101011 | 053 | 2 B | + | 75 | 01001011 | 13 | 4 B | K | 107 | 01101011 | 153 | 6B | k |
| 12 | 00001100 | 014 | OC | FF | 44 | 00101100 | 054 | 2 C | , | 76 | 01001100 | 14 | 4 C | L | 108 | 01101100 | 154 | 6C | I |
| 13 | 00001101 | 015 | 0D | CR | 45 | 00101101 | 055 | 2D | - | 77 | 01001101 | 15 | 4D | M | 109 | 01101101 | 155 | 6D | m |
| 14 | 00001110 | 016 | OE | So | 46 | 00101110 | 056 | 2E | . | 78 | 01001110 | 16 | 4 E | N | 110 | 01101110 | 156 | 6 E | n |
| 15 | 00001111 | 017 | OF | SI | 47 | 00101111 | 057 | 2 F | 1 | 79 | 01001111 | 17 | 4F | 0 | 111 | 01101111 | 157 | 6 F | 0 |
| 16 | 00010000 | 020 | 10 | DLE | 48 | 00110000 | 060 | 30 | 0 | 80 | 01010000 | 20 | 50 | P | 112 | 01110000 | 160 | 70 | p |
| 17 | 00010001 | 021 | 11 | DC1 | 49 | 00110001 | 061 | 31 | 1 | 81 | 01010001 | 21 | 51 | Q | 113 | 01110001 | 161 | 71 | q |
| 18 | 00010010 | 022 | 12 | DC2 | 50 | 00110010 | 062 | 32 | 2 | 82 | 01010010 | 22 | 52 | R | 114 | 01110010 | 162 | 72 | r |
| 19 | 00010011 | 023 | 13 | DC3 | 51 | 00110011 | 063 | 33 | 3 | 83 | 01010011 | 23 | 53 | S | 115 | 01110011 | 163 | 73 | s |
| 20 | 00010100 | 024 | 14 | DC4 | 52 | 00110100 | 064 | 34 | 4 | 84 | 01010100 | 24 | 54 | T | 116 | 01110100 | 164 | 74 | t |
| 21 | 00010101 | 025 | 15 | NAK | 53 | 00110101 | 065 | 35 | 5 | 85 | 01010101 | 25 | 55 | U | 117 | 01110101 | 165 | 75 | u |
| 22 | 00010110 | 026 | 16 | SYN | 54 | 00110110 | 066 | 36 | 6 | 86 | 01010110 | 26 | 56 | v | 118 | 01110110 | 166 | 76 | v |
| 23 | 00010111 | 027 | 17 | ETB | 55 | 00110111 | 067 | 37 | 7 | 87 | 01010111 | 27 | 57 | w | 119 | 01110111 | 167 | 77 | w |
| 24 | 00011000 | 030 | 18 | CAN | 56 | 00111000 | 070 | 38 | 8 | 88 | 01011000 | 30 | 58 | X | 120 | 01111000 | 170 | 78 | x |
| 25 | 00011001 | 031 | 19 | EM | 57 | 00111001 | 071 | 39 | 9 | 89 | 01011001 | 31 | 59 | Y | 121 | 01111001 | 171 | 79 | y |
| 26 | 00011010 | 032 | 1A | SUB | 58 | 00111010 | 072 | 3 A | : | 90 | 01011010 | 32 | 5A | Z | 122 | 01111010 | 172 | 7A | z |
| 27 | 00011011 | 033 | 1B | ESC | 59 | 00111011 | 073 | 3 B | ; | 91 | 01011011 | 133 | 5B | [ | 123 | 01111011 | 173 | 7B | \{ |
| 28 | 00011100 | 034 | 1 C | FS | 60 | 00111100 | 074 | 3 C | < | 92 | 01011100 | 134 | 5C | 1 | 124 | 01111100 | 174 | 7C | I |
| 29 | 00011101 | 035 | 1D | GS | 61 | 00111101 | 075 | 3D | = | 93 | 01011101 | 135 | 5D | 1 | 125 | 01111101 | 175 | 7D | \} |
| 30 | 00011110 | 036 | 1E | RS | 62 | 00111110 | 076 | 3E | > | 94 | 01011110 | 136 | 5E | $\wedge$ | 126 | 01111110 | 176 | 7E | $\sim$ |
| 31 | 00011111 | 037 | 1F | US | 63 | 00111111 | 077 | 3 F | ? | 95 | 01011111 | 137 | 5F | - | 127 | 01111111 | 177 | 7F | DEL |




# A few thoughts on "Collisions"... 

## Remember these?



## Brute force the unlocking of this briefcase ...

# Brute force the unlocking of this briefcase ... ... it will take you 3 seconds per each try. 

How long will it take to "hack" the briefcase open without knowing the secret lock code?

## Combination Lock (3 rotary dials)



## Combination Lock (3 rotary dials)

$10 \times 10 \times 10=1,000=10^{3}$ total possible combinations
With 3 seconds per each combination, we will need: $3 \times 10^{3}$ second (or 3,000 seconds)

There are 60 seconds in each minutes, so:
$\left(3 \times 10^{3}\right) \div 60=50$ minutes max to open each lock OR
$3,000 \div 60=50$ minutes max to open each lock

## Combination Lock (3 rotary dials)

$10 \times 10 \times 10=1,000=10^{3}$ total possible combinations
3 seconds per each combination
$1 \mathrm{~min}=60$ seconds
So $60 \div 3=20$ combinations per minute $1,000 \div 20=50$ minutes to open each lock

## Combination Lock (3 rotary dials)



| 3 | O | $\bigcirc$ | $\square$ | 8 | 0 | $\bigcirc$ | 1 | 8 | 0 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Digits | Digits | Digits | Digits | Digits | Digits | Digits | Digits | Digits | Digits | Digits |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| a | a | a | a | a | a | a | a | a | a | a |
| b | b | b | b | b | b | b | b | b | b | b |
| c | c | c | c | c | c | c | c | c | c | c |
| d | d | d | d | d | d | d | d | d | d | d |
| e | e | e | e | e | e | e | e | e | e | e |
| f | f | f | f | f | f | f | f | f | f | f |
| 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |

## SHA-256 Hash: Why 64 characters?

0
$2^{\wedge} 256$

Text SHA-256 Hash (HexaDecimal)
hello 2cf24dba5fb0a30e26e83b2ac5b9e29e1b161e5c1fa7425e73043362938b9824
$16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 x . . \quad[16$ multiplied by itself 64 times]

## SHA-256 Hash: Why 64 characters?

0

Text SHA-256 Hash (HexaDecimal)
hello 2cf24dba5fb0a30e26e83b2ac5b9e29e1b161e5c1fa7425e73043362938b9824 $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 x . . \quad[16$ multiplied by itself 64 times]
$16^{64}=\left(2^{4}\right)^{64}=2^{4 \times 64}=$ $2^{256}$

## SHA-256

## SHA-256 hash: a number with the range:

$$
0 \rightarrow 2^{256}
$$

## SHA-256: Using an unimaginably large number!

Note that $2^{256}$ is approximately $10^{77}$
The sum of all the atoms in the universe are estimated to be $10^{80}$ (or between $10^{78}$ and $10^{82}$ )

## Let's brute-force a SHA-256 collision by using state-of-the-art machine:

## Some of the Fastest Machines

## Bitmain

Antminer S21 Hyd (335Th)


## Description

Model Antminer S21 Hyd (335Th) from Bitmain mining SHA-256 algorithm with a maximum hashrate of $335 \mathrm{Th} / \mathrm{s}$ for a power consumption of 5360W.

Bitmain
Antminer S19 XP Hyd (255Th)


Description
Model Antminer S19 XP Hyd (255Th) from Bitmain mining SHA-256 algorithm with a maximum hashrate of $\mathbf{2 5 5} \mathbf{T h} / \mathbf{s}$ for a power consumption of 5304 W .

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## Let’s find a SHA-256 collision

1 Terahash = 1 trillion hashes per second $=10^{12} \mathrm{~h} / \mathrm{s}$ SHA-256 is appx. $10^{77}$ total possible numbers (i.e. hashes)

So, how long will it take with one machine at $255 \mathrm{Th} / \mathrm{s}$ to run through all numbers between 0 and 10 ${ }^{77}$ ?

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$$
\left(10^{77}\right) \div\left(255 \times 10^{12}\right)=3.92 \times 10^{62} \text { seconds }
$$

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So, how long will it take with one machine at $255 \mathrm{Th} / \mathrm{s}$ to run through all numbers between 0 and 1077?

$$
\left(10^{77}\right) \div\left(255 \times 10^{12}\right)=3.92 \times 10^{62} \text { seconds }
$$

How many years will that be?
Well, there are appx ( 365 days $\times 24$ hrs x 60 mins $\times 60$ secs) seconds per year, so there are appx $31,536,000$ seconds per year, OR $3.15 \times 10^{7}$ secs/year

## Let’s find a SHA-256 collision

1 Terahash = 1 trillion hashes per second $=10^{12} \mathrm{~h} / \mathrm{s}$ SHA-256 is appx. $10^{77}$ total possible numbers (i.e. hashes)

So, how long will it take with one machine at $255 \mathrm{Th} / \mathrm{s}$ to run through all numbers between 0 and 107?

$$
\left(10^{77}\right) \div\left(255 \times 10^{12}\right)=3.92 \times 10^{62} \text { seconds }
$$

How many years will that be?
With $31,536,000$ seconds per year, OR $3.15 \times 10^{7}$ secs/year $\rightarrow$
$\left(3.92 \times 10^{62}\right.$ seconds $) \div\left(3.15 \times 10^{7}\right.$ secs/year $)=1.24 \times 10^{55}$ years
Let's get 1 billion $\left(10^{9}\right)$ of these machines, so:
$\left(1.24 \times 10^{55}\right.$ years $) \div 10^{9}=1.24 \times 10^{46}$ years

## Let's find a SHA-256 collision

So, how long will it take with one billion machines at $255 \mathrm{Th} / \mathrm{s}$ to run through all numbers between 0 and $10^{77}$ ?
$\left(1.24 \times 10^{55}\right.$ years $) \div 10^{9}=1.24 \times 10^{46}$ years
Age of the Universe: Age of Earth:
$13.8 \times 10^{9}$ years
$4.5 \times 10^{9}$ years
We would need these many universe lifetimes to make it:
$\left(1.24 \times 10^{46} \mathrm{yrs}\right) \div\left(13.8 \times 10^{9} \mathrm{yrs}\right)=8.99 \times 10^{35}$
898,550,724,637,681,159,420,289,855,072,463,768

899,000,000,000,000,000,000,000,000,000,000,000

## Let's find a SHA-256 collision

1 Terahash = 1 trillion hashes per second $=10^{12} \mathrm{~h} / \mathrm{s}$ SHA-256 is appx. $10^{77}$ total possible numbers (i.e. hashes)

So, how long will it take with one machine at $255 \mathrm{Th} / \mathrm{s}$ to run through all numbers between 0 and 10 ${ }^{77}$ ?

$$
\left(10^{77}\right) \div\left(255 \times 10^{12}\right)=3.92 \times 10^{62} \text { seconds }
$$

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Let's get 1 billion $\left(10^{9}\right)$ of these machines, so:
$\left(1.24 \times 10^{55}\right.$ years $) \div 10^{9}=1.24 \times 10^{46}$ years
At $\$ 3,000$ a machine, we'd need $\$ 3,000,000,000,000$ just to buy them (3 trillion dollars ... annual GDP of France!)

## SHA-256 Hash

## Decimal HexaDecimal

000000000000

## SHA-256 Hash

## Decimal HexaDecimal

000000000000000000000000000000000000000000000000000000000000010
17000000000000000000000000000000000000000000000000000000000000011
10000000000000000000000000000000000000000000000000000000000000003 e 8
(2^256) - $\mathbf{~ ( 2 f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f f ~}$
2^25 100000000000000000000000000000000000000000000000000000000000000

## SHA-256 Hex Encoding

## Instead of a long hash consisting of many digits:

```
0
1
2
3
4
8
25
9387
23430174432
57098500868790785
7316195423570985008687907853269984665640
4853269984665907859895748813748971384798546645240492
115792089237316195423570985008687907853269984665640564039457584007913129639
```


## SHA-256 Hex Encoding

## Instead of a long hash consisting of many digits:

1
25
23430174432
57098500868790785
7316195423570985008687907853269984665640
4853269984665907859895748813748971384798546645240492
115792089237316195423570985008687907853269984665640564039457584007913129639

## SHA-256 Hex Encoding

## Instead of a long hash consisting of many digits:

1
25
23430174432
57098500868790785
7316195423570985008687907853269984665640
4853269984665907859895748813748971384798546645240492
1157920892373161954235709850086879078532699846656405640394575840079131296399

## We have (a fixed string of 64 characters ... always):

fd04788626e5f87b3b22b2b855bddaae2f1ee43956232d2fa57c5afa7d3f09b9
4faa640f3077ded9d2b7fc6f429050defc5d26e08e5b241edadd39a49e56af51
933e1c934309c9d942921fcebcd8fc398553f2c39ccb162cb53bd998149b042b

## SHA-256


$8 £ 434346648 £ 6 b 96 d f 89 d d a 901 c 5176 b 10 a 6 d 83961 d d 3 c 1 a c 88 b 59 b 2 d c 327 a a 4$
$79 f 5 c 65 f e 815417 f e 2 d c 3 f d b f b d a 9 d b f f 7 e 0 e c f 63 d e a 6162 d 4339546 e 7 a a 4 d 49$
fd04788626e5f87b3b22b2b855bddaae2f1ee43956232d2fa57c5afa7d3f09b9
d38b38a2dd476e045c299e8ee5d6466834456d97bd592a71746b423a6a05f386

DEMO: Hash (SHA-256)

## SHA-256 Hash: Remember why 64 characters?

## Text SHA-256 Hash (HexaDecimal)

hello 2cf24dba5fb0a30e26e83b2ac5b9e29e1b161e5c1fa7425e73043362938b9824

## SHA-256 Hash: setting thresholds



Numbers with one leading zero

## SHA-kespeare

Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of Shakespeare | Expected Number of Lines | Actual \% of lines (out of $\sim 110,968$ ) |
| :---: | :---: | :---: | :---: |
| 4 Leading Zeros | Calculate the expected number of lines from no leading zero to 4 leading zeros. You have 5 minutes. |  |  |
| 3 Leading Zeros |  |  |  |
| 2 Leading Zeros |  |  |  |
| 1 Leading Zero |  |  |  |
| No Leading Zeros |  |  |  |

## SHA-kespeare

Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of <br> Shakespeare | Expected <br> Number of Lines | Actual \% of lines <br> (out of ~110,968) |
| :--- | :--- | :--- | :--- |
| 4 Leading Zeros |  |  |  |
| 3 Leading Zeros |  |  |  |
| 2 Leading Zeros |  |  |  |
| 1 Leading Zero |  |  |  |
| No Leading Zeros |  |  |  |

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Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of <br> Shakespeare | Expected <br> Number of Lines | Actual \% of lines <br> (out of ~110,968) |
| :--- | :--- | :--- | :--- |
| 4 Leading Zeros |  |  |  |
| 3 Leading Zeros |  |  |  |
| 2 Leading Zeros |  |  |  |
| 1 Leading Zero |  | $\sim 6,935$ |  |
| No Leading Zeros |  | $\sim 104,033$ |  |

## SHA-kespeare

Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of <br> Shakespeare | Expected <br> Number of Lines | Actual \% of lines <br> (out of ~110,968) |
| :--- | :--- | :--- | :--- |
| 4 Leading Zeros |  |  |  |
| 3 Leading Zeros |  |  |  |
| 2 Leading Zeros |  |  |  |
| 1 Leading Zero |  | $-6,935$ |  |
| No Leading Zeros |  | $\sim 104,033$ |  |

## SHA-256 Hash: setting thresholds



Numbers with AT LEAST one leading zero

## SHA-256 Hash: setting thresholds


$\left(\frac{1}{16}\right) \times 2^{256}$


Numbers with AT LEAST
one leading zero

## SHA-256 Hash: setting thresholds


$\left(\frac{1}{16}\right) \times 2^{256}$


Numbers with AT LEAST
one leading zero
110,968-104,033 = 6,935.5

## SHA-256 Hash: setting thresholds

Value Value(15/16) Value-Value $(15 / 16)$

|  | A | B | c |
| :---: | :---: | :---: | :---: |
| 1 | 110,968 | 104,033 | 6,936 |
| 2 | 6,936 | 6,502 | 433 |
| 3 | 433 | 406 | 27 |
| 4 | 27 | 25 | 2 |


| 110,968 | =A1*15/16 | =A1-B1 |
| :--- | :--- | :--- |
| =C1 | =A2 ${ }^{*} 15 / 16$ | =A2-B2 |
| =C2 | =A3*15/16 | =A3-B3 |
| =C3 | =A4*15/16 | =A4-B4 |

## SHA-kespeare

Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of <br> Shakespeare | Expected <br> Number of Lines | Actual \% of lines <br> (out of ~110,968) |
| :--- | :--- | :--- | :--- |
| 4 Leading Zeros |  | $\sim 2$ | $0.0018 \%$ |
| 3 Leading Zeros |  | $\sim 25$ | $0.0160 \%$ |
| 2 Leading Zeros |  | $\sim 406$ | $0.3740 \%$ |
| 1 Leading Zero |  | $\sim 6,502$ | $6.0044 \%$ |
| No Leading Zeros |  | $\sim 104,033$ | $93.603 \%$ |

## SHA-kespeare

Approx 110,968 lines of Shakespeare written (all works)


| Hash Criteria | Lines of <br> Shakespeare | Expected <br> Number of Lines | Actual \% of lines <br> (out of ~110,968) |
| :--- | :--- | :--- | :--- |
| 4 Leading Zeros | 2 | $\sim 2$ | $0.0018 \%$ |
| 3 Leading Zeros | 18 | $\sim 25$ | $0.0160 \%$ |
| 2 Leading Zeros | 415 | $\sim 406$ | $0.3740 \%$ |
| 1 Leading Zero | 6,663 | $\sim 6,502$ | $6.0044 \%$ |
| No Leading Zeros | 103,870 | $\sim 104,033$ | $93.603 \%$ |

## SHA-kespeare



## Hamlet, Act I, Scene 2:

## King. Have you your father's leave?

 What says Polonius?

00005779 d9bda7accb203c8256e6106e 2d44d68025b83624af59e31c3527275

## Blockchain: a cryptographically-verifiable Tx chain



Edith gives Carol \$25


How to make the chain secured?

> Bob gives Edith \$10




## IV. Building the Blockchain

Using all we've learned to build an immutable chain of "digital assets" (and more)

## Exercise: let's do a (theoretical) deal!

Parties involved (client wants to use their own legal \& accounting)
1.
2.
3.
4.

## Exercise: let's do a deal!

## Parties involved (client wants to use their own legal \& accounting)

1. Bridget Fonda (BF); Commercial Bank Corp (CBC); IB

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3. Jeff Dewey (JD); Dewey, Cheatem \& Howe (DCH); Law

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1. Bridget Fonda (BF); Commercial Bank Corp (CBC); IB
2. Robert Farrokhnia (RF); Columbia University (COL); Advisor
3. Jeff Dewey (JD); Dewey, Cheatem \& Howe (DCH); Law
4. Alex Runne (AR); Steel, Runne \& Hyde (SRH): Accounting

We will have lots of documents going back and forth.

## Exercise: let's do a deal!

## Our document naming convention, or protocol:

## [type of doc]_[company name]_[author's initials][author's employer]_[date: mm/dd/yy]_[version number: v\#]

## [type of doc]_[company name]_[author's initials]_[author's employer]_[date: mm/dd/yy]_[version number: v\#]

PPM_Newco_RF_COL_041523_v1
[type of doc]_[company name]_[author's initials]_[author's employer]_[date: mm/dd/yy]_[version number: v\#]

# PPM_Newco_RF_COL_041523_v1 PPM_Newco_BF_CBC_041623_v2 

## Exercise: naming protocol sorted by "Name"

| Name | Date Modified | Size | Kind |
| :---: | :---: | :---: | :---: |
| (a) PPM_Newco_AR_SRH_072817_v9 | Today, 4:20 PM | 23 KB | Microsoft Word document |
| (a) PPM_Newco_BF_CBC_072717_v8 | Today, 4:20 PM | 23 KB | Microsoft Word document |
| a PPM_Newco_RF_COL_072717_v5 | Today, 4:20 PM | 23 KB | Microsoft Word document |
| (a) PPM_Newco_AR_SRH_072717_V4 | Today, 5:44 PM | 23 KB | Microsoft Word document |
| ( ${ }_{\text {a }}$ PPM_Newco_JD_DCH_072617_v3 | Today, 4:19 PM | 22 KB | Microsoft Word document |
| (a) PPM_Newco_BF_CBC_072617_v2 | Today, 4:19 PM | 22 KB | Microsoft Word document |
| (a) PPM_Newco_RF_COL_072517_v1 | Today, 3:49 PM | 22 KB | Microsoft Word document |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  | 3 |  |

## Exercise: naming protocol sorted by "Date Modified"



What can do wrong? How to fix the system?

Let's build a blockchain, connecting and linking verified digital files in an immutable way with a shared ledger to keep track of it all that every party can see.








# One of the earliest papers on "Blockchain" 

# How to Time-Stamp a Digital Document* 

Stuart Haber<br>stuart @bellcore.com

W. Scott Stornetta<br>stornetta@bellcore.com

Bellcore<br>445 South Street<br>Morristown, N.J. 07960-1910


#### Abstract

The prospect of a world in which all text, audio, picture, and video documents are in digital form on easily modifiable media raises the issue of how to certify when a document was created or last changed. The problem is to time-stamp the data, not the medium. We propose computationally practical procedures for digital time-stamping of such documents so that it is infeasible for a user either to back-date or to forward-date his document, even with the collusion of a time-stamping service. Our procedures maintain complete privacy of the documents themselves, and require no record-keeping by the time-stamping service.


[^0]
# One of the earliest papers on "Blockchain" 

How to Time-Stamp a Digital Document*

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#### Abstract

The prospect of a world in which all text, audio, picture, and video documents are in digital form on easily modifiable media raises the issue of how to certify when a document was created or last changed. The problem is to time-stamp the data, not the medium. We propose computationally practical procedures for digital time-stamping of such documents so that it is infeasible for a user either to back-date or to forward-date his document, even with the collusion of a time-stamping service. Our procedures maintain complete privacy of the documents themselves, and require no record-keeping by the time-stamping service.


[^1]
## Let's power on ...

## Blockchain



## Assume all

 transactions here are signed, and the creator of the hash verified that the sender had the necessary funds
## Edith gives Carol \$25



## Blockchain: hash each block

Everyone gets $\$ 100$ Hash: 09592b438bfe8ac1fd


## Blockchain (link each block)

Prev Hash: 0000000000000 Everyone gets \$100 Block Hash: 09592b438bfe8ac1fd


## Blockchain: out-of-sync hashes?

Prev Hash: 0000000000000
Everyone gets \$100
Block Hash: 09592b438bfe8ac1fd

Prev Hash: fa1960e7a6b57ee967
Edith gives Carol \$25 Hash: 7ed2db73b7921eebed

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5 Hash: fa1960e7a6b57ee967

## Blockchain: re-calculate hashes

Prev Hash: 0000000000000
Everyone gets \$100
Block Hash: 09592b438bfe8ac1fd

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5 Hash: 1a19dbada78ed53aa6b3851


## Blockchain

Prev Hash: 0000000000000
Everyone gets \$100
Block Hash: 09592b438bfe8ac1 fd

Prev Hash: 1a19dbada78ed53aa6b3851 Edith gives Carol \$25 Hash: 9f6f9cfc699cc4fcbd3375da0e9c

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5
Hash: 1a19dbada78ed53aa6b3851

## Blockchain

Prev Hash: 0000000000000 Everyone gets \$100 Block Hash: 09592b438bfe8ac1fd

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5
Hash: 1a19dbada78ed53aa6b3851

Prev Hash: 1a19dbada78ed53aa6b3851 Edith gives Carol \$25
Hash: 9f6f9cfc699cc4fcbd3375da0e9c


## Blockchain

Prev Hash: 0000000000000 Everyone gets \$100 Block Hash: 09592b438bfe8ac1fd

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5
Hash: 1a19dbada78ed53aa6b3851


## Blockchain

Prev Hash: 0000000000000 Everyone gets \$100 Block Hash: 09592b438bfe8ac1fd

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5 Hash: 1a19dbada78ed53aa6b3851

Prev Hash: 1a19dbada78ed53aa6b3851 Edith gives Carol \$25 Hash: 9f6f9cfc699cc4fcbd3375da0e9c


## Blockchain

Prev Hash: 0000000000000 Everyone gets \$100 Block Hash: 09592b438bfe8ac1fd

Prev Hash: 09592b438bfe8ac1fd Alice gives Bob \$5
Hash: 1a19dbada78ed53aa6b3851

Prev Hash: 1a19dbada78ed53aa6b3851 Edith gives Carol \$25
Hash: 9f6f9cfc699cc4fcbd3375da0e9c


DEMO: Blockchain

## Blockchains



## Blockchains



## Blockchains



## Blockchains



## Blockchains



## Blockchains



## Blockchains



## Blockchain Recap

1. The transaction is broadcasted to the world.
2. Each node that receives the broadcast verifies via the signature and their copy of the ledger that the sending party has the funds to send that amount of money, and that the transaction actually came from the sending party.
3. Each updates their ledger in a cryptographically consistent and verifiable way, forever cementing the transaction as part of the chain.
4. Once the majority of nodes have updated their ledger with the valid transaction, the recipient of the money effectively "has" the new money because they now, according to the ledger shared by the majority, have the funds they need to send a new, valid transaction with the funds they received.

DEMO: Distributed

## V. Bitcoin

Leveraging the blockchain to create a decentralized digital crypto-currency.


## SHA-256 Hash

0

## $\left(\frac{1}{16}\right) \times 2^{256}$

0000000000000000000000000000000000000000000000000000000000000000 0000000000000000000000000000000000000000000000000000000000000001 0000000000000000000000000000000000000000000000000000000000000002 00000000000000000000000000000000000000000000000000000000000009 000000000000000000000000000000000000000000000000000000000 a a 0000000000000000000000000000000000000000000000000000000000000 ab 000000000000000000000000000000000000000000000000000000000000000 f 0000000000000000000000000000000000000000000000000000000000000010 03b0c44298fc1c149afbf4c8996fb92427ae41e4649b934ca495991b7852b855 1000000000000000000000000000000000000000000000000000000000000000

## Bitcoin: a shared Blockchain (cooperative)



## Bitcoin: change USD to Bitcoin



## Bitcoin: no names, just (public) keys



## Bitcoin: keys also on the Tx's, no names



## Bitcoin: multiple keys are allowed



## Bitcoin: wallets (or keychains)



## Wallet



## Bitcoin: cryptographic puzzle



## Bitcoin:"computational puzzle"



## Sample attributes verified by nodes in each Tx:

1. The transaction's syntax and data structure must be correct.
2. Neither lists of inputs or outputs are empty.
3. The transaction size in bytes is less than MAX_BLOCK_SIZE.
4. Each output value, as well as the total, must be within the allowed range of values (less than 21 m coins, more than 0 ).
5. None of the inputs have hash=0, $\mathrm{N}=-1$ (coinbase transactions should not be relayed).
6. nLockTime is less than or equal to INT_MAX.
7. The transaction size in bytes is greater than or equal to 100.
8. The number of signature operations contained in the transaction is less than the signature operation limit.
9. The unlocking script (scriptSig) can only push numbers on the stack, and the locking script (scriptPubkey) must match isStandard forms (this rejects "nonstandard" transactions).
10. A matching transaction in the pool, or in a block in the main branch, must exist.
11. For each input, if the referenced output exists in any other transaction in the pool, the transaction must be rejected.
12. For each input, look in the main branch and the transaction pool to find the referenced output transaction. If the output transaction is missing for any input, this will be an orphan transaction. Add to the orphan transactions pool, if a matching transaction is not already in the pool.
13. For each input, if the referenced output transaction is a coinbase output, it must have at least COINBASE_MATURITY (100) confirmations.
14. For each input, the referenced output must exist and cannot already be spent.
15. Using the referenced output transactions to get input values, check that each input value, as well as the sum, are in the allowed range of values (less than 21 m coins, more than 0 ).
16. Reject if the sum of input values is less than sum of output values.
17. Reject if transaction fee would be too low to get into an empty block.
18. The unlocking scripts for each input must validate against the corresponding output locking scripts.

The Bitcoin "Puzzle"


## The Bitcoin "Puzzle"



Prev Hash: 8a7b6618e714c6a 5 BTC
For: b197be
*From: a519f8...

## The Bitcoin "Puzzle"



Prev Hash: 8a7b6618e714c6a 5 BTC
For: b197be
*From: a519f8...
Hash:

## The Bitcoin "Puzzle"

$\times$ Man
Prev Hash: 8a7b6618e714c6a 5 BTC
For: b197be
*From: a519f8...
Nonce:
Nonce Solver:
Hash:

## The Bitcoin "Puzzle"



## The Bitcoin "Puzzle"



## The Bitcoin "Puzzle": example of how miners mine



## The Bitcoin "Puzzle"



## The Bitcoin "Puzzle"



## The Bitcoin "Puzzle"



## Bitcoin



## Bitcoin: Tx done



## Bitcoin: Tx distributed



## Bitcoin: funds transferred



## The Bitcoin "Puzzle": can you steal the nonce?



## The Bitcoin "Puzzle": nonce is block-specific



## Bitcoin



## Bitcoin: one Tx per block? Not really!



## Bitcoin



## Bitcoin



## Bitcoin



## Bitcoin



## Bitcoin



## Bitcoin



## ~ Every

Bitcoin

~ Every

## 10 min



## Calibrating The Bitcoin "Puzzle" w/ "Difficulty"



## Calibrating The Bitcoin "Puzzle" w/ Difficulty



## Calibrating The Bitcoin "Puzzle" w/ Difficulty



## Calibrating The Bitcoin "Puzzle"...



# Blockchain, Cryptocurrencies \& Digital Tokens Demystified <br> Fall 2023 (EMBA) <br> Columbia Business School 

## Welcome Back to Session 3

## Curriculum Roadmap

| Morning | Nov 4 | Nov 18 | Dec 2 | Dec 9 |
| :---: | :---: | :---: | :---: | :---: |
|  | Networks \& Protocols | Hashing, Hashing Tables \& One- Way Functions \& a few more tech | Bitcoin + other forms of crypto payments and store of value mechanisms and media | DeFi \& Other Applications (Digital Tokens, CBDC, etc.) + Speaker: Future of Finance + Discussion Forum |
|  | Lunch | Lunch | Lunch | Lunch |
| Afternoon | Encryption \& Cryptography (plus some math!) | Bring it All <br> Together: Let's build a blockchain \& discuss variety of cases | Ethereum \& Other Digital Tokens + Speaker: Regulatory \& Legal Considerations in Blockchain \& Digital Assets | Governance, <br>  <br> More; Final Lecture on <br> How the Future May Play <br> Out + Final <br> Presentations |

## Class Schedule - Nov 4, Nov 18, Dec 2, Dec 9

## Class Plan

Nov 4
Nov 18
Dec 2
Dec 9

08:30 am to 6:45 pm (K-440)Module $1+2$
08:30 am to 6:45 pm (K-440)Module $3+4$
08:30 am to 6:45 pm (K-440)Midterm Project + 5 \& $6+$ Guest Speaker
08:30 am to 6:45 pm (K-440)Module 7 \& $8+$ Guest Speaker + final presentations

Daily Schedule
8:30-9:45 am
9:45-10:00 am
10:00-11:15 pm
11:15 am-12:30 pm
12:30-2:00 pm 2:00-2:15 pm
2:15-3:30 pm
3:30-3:45 pm
3:45-5:00 pm
5:00-5:15 pm
5:15-6:45 pm

## Lecture

Break
Lecture
Lunch (1h15min) - Kravis 2nd floor (Smith Dining)

## Lecture

Break

## Lecture

Break
Lecture
Break
Lecture

## Important Admin Items for the Day

- Note last class is on Dec 9 (next week, not in two weeks)
- Final projects assigned already
- Details on your final projects (presentation \& paper)
- Final presentations next week
- Final papers due on Dec 18
- Thoughts on "Blockchain Killer App" for today and/or 4
- Discussion Forum next class
- Watch lecture recordings and email me for office hours


## THE MOST Important Admin Item for the Day

## THE MOST Important Admin Item for the Day

Catering today is by Dig Inn:

- Brown Rice
- Lemon \& Herb Farro
- Maple Glazed Crispy Tofu
- Herb Roasted Chicken
- Beef \& Chicken Meatballs
- Wild Salmon
- Broccoli
- Brussels Sprouts
- Sweet Potatoes


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- Sweet Potatoes


## THE MOST Important Admin Item for the Day

Catering today is by Dig Inn:

- Brown Rice
- Lemon \& Herb Farro (Farrokhnia!!)
- Maple Glazed Crispy Tofu
- Herb Roasted Chicken
- Beef \& Chicken Meatballs
- Wild Salmon
- Broccoli
- Brussels Sprouts
- Sweet Potatoes


# Before we begin, any interesting points or lessons from our prior session you'd like to share? 

## Let's start our Session 3

## Why the Puzzle?

Normal Miner's Blockchain:


## Why the Puzzle?

Normal Miner's Blockchain:


## Why the Puzzle?

Normal Miner's Blockchain:


## Why the Puzzle?

Normal Miner's Blockchain:


## Malicious Miner's Blockchain:



## Why the Puzzle? Let's spam!

Normal Miner's Blockchain:


Malicious Miner's Blockchain:


## Why the Puzzle?

Normal Miner's Blockchain:


## Malicious Miner's Blockchain:



## Why the Puzzle?

Normal Miner's Blockchain:


## Malicious Miner's Blockchain:



## Why the Puzzle?

Normal Miner's Blockchain:


## Malicious Miner's Blockchain:



## Why the Puzzle?

Normal Miner's Blockchain:


Malicious Miner's Blockchain:


## Why the Puzzle?

Normal Miner's Blockchain:


## Bitcoin: other topics



## Bitcoin



## Bitcoin



## Bitcoin



## Bitcoin






## Funny Story: guard your wallet (Dec 2013)!!



## Bitcoin Questions

- Is Bitcoin anonymous?
- Is Bitcoin really invulnerable to compromise?


## Bitcoin Questions

- Is Bitcoin anonymous?
- Is Bitcoin really invulnerable to compromise?


## Home Welcome to Blockchain

| Height | Age | Transactions | Total Sent | Relayed By | Size (kB) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 453057 | 10 minutes | 1884 | $15,911.31$ BTC | Bitcoin.com | 998.11 |  |
| 453056 | 16 minutes | 1834 | $27,998.50$ BTC | ViaBTC | 999.16 |  |
| 453055 | 38 minutes | 2331 | $17,512.90$ BTC | BitFury | 998.18 |  |
| 453054 | 48 minutes | 2524 | $17,116.92$ BTC | F2Pool | 999.91 |  |
| 453053 | 59 minutes | 2321 | 2096 | $15,615.56$ BTC | AntPool | 998.09 |
| 453052 | 1 hour 15 minutes | $9,727.30$ BTC | BTCC Pool | 998.12 |  |  |

## Latest Transactions

9d92666f7028abe9860f7235f.
$<1$ minute

## Search

You may enter a block height, address, block hash, transaction hash, hash160, or ipv4 address...
Address / ip / SHA hash Search

## Block \#453054

| Summary |  |
| :--- | :--- |
| Number Of Transactions | 2524 |
| Output Total | $17,116.9190252$ BTC |
| Estimated Transaction Volume | $3,595.87663859$ BTC |
| Transaction Fees | 1.24502972 BTC |
| Height | 453054 (Main Chain) |
| Timestamp | $2017-02-14$ 17:06:39 |
| Received Time | $2017-02-14$ 17:06:39 |
| Relayed By | F2Pool |
| Difficulty | $422,170,566,883.84$ |
| Bits | 402823865 |
| Size | 999.913 KB |

## Hashes

Hash 000000000000000001a13b341900b61b36ad8664ceae30da3cc0c52d9faa0b99

Previous 000000000000000000734158f091f9918677ccdc9e50281794c5f4f433ec582a Block

## Next

Block(s)
$000000000000000000 e e 1 b 60 d 8 f d e 428589 e 7 a c 37 e c 35 b 3 c 7 d e 9119040047043$

Merkle
759f9e4d7c7266410e5b 40a7f245e757b9eb69873ecdf7f0e3f45a25b2467467
Root

Network Propagation


## Block \#453054



## Block \#453054



## Block \#453054



432b41f520dd8806531db5bcd1 bc418e9cfdbe9653f16d6579fa9f26962f6215

1LzqQ7oj49pwr6pDbNUT4usxt9C4qxAr7v 1NfC2rPsdMUfabCZ7D1VjfYJp5crqt1F6f
0.01017799 BTC
0.29887622 BTC

Protecting the integrity of digital assets.

OVER \$15 BILLION WORTH OF BITCOIN TRANSACTIONS CHECKED BY CHAINALYSIS ON BEHALF OF OUR CUSTOMERS

01017799 BTC 29887622 BTC


## Bitcoin Nodes Log List of bitcoin nodes blockchain.info has connected to in the past.

## Total Unique Ip Addresses: 16,043



## Bitcoin Questions

- Is Bitcoin anonymous?
- Is Bitcoin really invulnerable to compromise?


## Bitcoin Questions

- Is Bitcoin anonymous?

Yes and No! It is pseudonymous.

- Is Bitcoin really invulnerable to compromise?


## Bitcoin Questions

- Is Bitcoin anonymous?

Yes and No! It is pseudonymous.

- Is Bitcoin really invulnerable to compromise?


## Bitcoin 51\% Attack



## Bitcoin 51\% Attack



## Bitcoin 51\% Attack



## Bitcoin 51\% Attack



## Could this actually happen?

## Could this actually happen?

## ars TECHNICA $\quad$ ミ scill.

## RISK ASSESSMENT -

## After reaching 51\% network power, Bitcoin mining pool says "trust us"

GHash notes it has never attacked, double-spent against Bitcoin.
CYRUS FARIVAR - 6/16/2014, 4:50 PM

## Could this actually happen?

## (ars TECHNICA a E SIGNIN•

Topic: GHASH.IO IS NEARING 51\% - LEAVE THE POOL (Read 2078 times)

## GHASH.IO IS NEARING 51\% - LEAVE THE POOL

January 09, 2014, 10:39:25 AM

GHash notes it has never attacked, double-spent against Bitcoin.
CYRUS FARIVAR - 6/16/2014, 4:50 PM

## Could this actually happen?



## Could this actually happen?



## Quantum Computing \& Cryptography

## Newsweek

## TECH \& SCIENCE IS BITCOIN DOOMED?

BY ANTHONY CUTHBERTSON ON 10/12/16 AT 10:08 AM

## Quantum Computing \& Cryptography



## Quantum Computing \& Cryptography

# NSA working on quantum computer to break any encryption 

The spy agency is reportedly in a race to build its own quantum computer to stay ahead of others seeking to own the mother of all decryption machines.

## Bitcoin Questions

- Is Bitcoin anonymous?

Yes and No! It is pseudonymous.

- Is Bitcoin really invulnerable to compromise?


## Bitcoin Questions

- Is Bitcoin anonymous?

Yes and No! It is pseudonymous. Is Bitcoin really invulnerable to compromise?
Probably not, at least in the short term.



## Latest blocks

| Height | Hash Mined | Miner | Size |
| :---: | :---: | :---: | :---: |
| 588944 | 0000000000000000001be255f513d686d195ebf... 17:30 PM | BTC.com | 1,274,162 bytes |
| 588943 | 00000000000000000017db20ea351d2d0e425fb... 17:18 PM | BTC.com | 1,104,880 bytes |
| 588942 | 00000000000000000008208c98e373a2db0542... 17:14 PM | AntPool | 1,335,648 bytes |
| 588941 | 000000000000000000051c28e6ec6e1b7b30920...17:04 PM | F2Pool | 1,249,497 bytes |
| 588940 | 00000000000000000004ca36580eaee738367d... 16:59 PM | F2Pool | 1,366,541 bytes |
| 588939 | 000000000000000000067173d7fe7c2ba7bcdec... 16:37 PM | Unknown | 1,076,107 bytes |
| 588938 | 00000000000000000003ea92f022801fce2965d... 16:37 PM | SlushPool | 1,223,835 bytes |
| 588937 | 00000000000000000015aaaef25cd813d574781e...16:24 PM | AntPool | 1,212,669 bytes |
| 588936 | 0000000000000000000f793419624af43f22c9d... 16:24 PM | Unknown | 1,237,063 bytes |
| 588935 | 00000000000000000004349f5e9d4140247cae6...16:22 PM | AntPool | 1,268,144 bytes |
| 588934 | 0000000000000000000ea4d6189877fe9048da7... 16:18 PM | Unknown | 1,242,545 bytes |
| 588933 | 00000000000000000004092599b25e760c74b6... 16:08 PM | Unknown | 1,179,208 bytes |
| 588932 | 00000000000000000003011df1e9bc109685206... 16:05 PM | Unknown | 1,188,151 bytes |
| 588931 | 00000000000000000000e066ff0d28f32d86478... 16:02 PM | Unknown | 1,248,546 bytes |
| 588930 | 000000000000000000090eObef88087be6b787a...15:58 PM | BTC.TOP | 1,181,402 bytes |

## VI. Beyond Bitcoin

What applications does the blockchain have beyond cryptocurrencies like Bitcoin?


## Other Cryptocurrencies

## Other Cryptocurrencies

逢 litecoin

## Other Cryptocurrencies

祭 litecoin

## (4) Primecoin

Other Cryptocurrencies

## 楿) litecoin

 (4) PrimecoinNnamecoin

## VI. Beyond Bitcoin

Ethereum


## Other Cryptocurrencies



## What is Ethereum?

- Simply put, it is an "open-source and globally decentralized computing infrastructure that executes programs called Smart Contracts. It uses blockchain to synchronize and store system's state changes, using a cryptocurrency called Ether (ETH) to meter [or measure] and constrain execution resource costs."


## What is Ethereum?

- It shares many similarities \& common elements with Bitcoin or other cryptocurrencies (P2P network connecting participants, Byzantine Fault Tolerant consensus algos, proofs, hashes, sigs)
- But it's also different in other aspects, esp in having Utility Functions ("world computer, virtual machine") + "general purpose blockchain" \& Decentralized Applications (dApps or DApps!)


## "Smart" Contracts: records of prog. agreements

- Ethereum contracts are programs that control money, running inside Ethereum VM
- Once created, they have an Ethereum address, just like wallets (say, belonging to a person)
- Transactions sent to an address may have ether, data, or both $\rightarrow$ ethers get "deposited" to the contract balance; data can specify a named functions (in the contract) and call it


## "Smart" Contracts




## "Smart" Contracts





## "Smart" Contracts


ethereum


## "Smart" Contracts

## $\vartheta$ <br> ethereum



## "Smart" Contracts



○, - 三

IBM Blockchain

## Quickly run a blockchain network in a secure Cloud environment

Spin up a blockchain network on a private, virtualized environment; create and secure digital assets in test applications to trade over a permissioned network


$$
\dot{z}^{2}-\frac{0}{2}=
$$

$$
\dot{z}^{2} \frac{2}{2}
$$

## The DAO \& "Forking"



## The DAO \& "Forking"



## The DAO \& "Forking"



The DAO \& "Forking"


The DAO \& "Forking"


The DAO \& "Forking"


## The DAO \& "Forking" (2016/2017)



## The DAO \& "Forking" (4/2023) <br>  <br> cap, up from <br> \$110m

A quick regulatory lesson

## The "Howey Test"

- It is an investment of money
- There is an expectation of profits from the investment
- The investment of money is in a common enterprise
- Any profit comes from the efforts of a promoter or third party


## Class Discussion

## Easter Egg ... for those with a Mac ;-)

> Open either 1) Finder OR 2) click on Go, then Computer ... then click on Macintosh HD at the bottom of the window, then System $\rightarrow$ Library $\rightarrow$ Image Capture $\rightarrow$ Devices. Once there, right click on VirtualScanner.app and choose "Show Package Contents." Open Resources, and click on "simpledoc.pdf." What do you see?!



# Congratulations! <br> You made it to the end of slides ... almost! We still have a few more days to go 

Thank You!

## End of Slides


[^0]:    *Appeared, with minor editorial changes, in Journal of Cryptology, Vol. 3, No. 2, pp. 99-111, 1991.

[^1]:    ${ }^{*}$ Appeared, with minor editorial changes, in Josrnal of Cryptology, Vol. 3, No. 2, pp. 99-111, 1991.

