Dynamic Information Acquisition and Optimal Disclosure Frequency *

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Abstract

We develop a model where information acquisition and disclosure are jointly chosen over time. The manager seeks to maximize future stock prices, and collects information privately about underlying firm fundamentals. Information acquisition increases the arrival rate of private information signals, but it is costly. The manager can choose to make public disclosures about his information if any. Our model can characterize the trade-offs in the dual information acquisition/disclosure decision when such decisions have to be made over time and the manager has reputational concerns. We consider the impact of the information acquisition and disclosure activities upon the firm's endogenous productivity investments. Furthermore, we analyze the optimal mandatory disclosure frequency from the perspective of a regulator aiming to maximize firm value.

Keywords— Information acquisition, investment, disclosure, learning, reputation, dynamics

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1 Introduction

The frequency of mandatory disclosure in the United States has increased over time, from annual, to semi-annual to quarterly, and it is possible that this trend will continue.[1] Indeed, market pundits have speculated that mandatory disclosure will eventually happen in real/continuous time. At the same time, important technological innovations (i.e., Twitter, social networks, robot-journalism) have increased firms' ability to obtain and disseminate information on a voluntary basis,[2] potentially making mandatory announcements less necessary and relevant. In this paper, we develop a theory to connect both facts to help us understand the optimal frequency of mandatory disclosures when firms can strategically release their information voluntarily, but may conceal unfavorable information.

We consider a dynamic model of voluntary disclosure with endogenous information acquisition and hidden investment. We study what drives firms to collect private information that they may or may not disclose voluntarily and how this affects the firm's incentive to invest in the first place. In our benchmark, firm value is uncertain and exogenous. At the start, firm value is unknown to both the manager and investors. Over time, the manager may receive information at a constant Poisson rate. Furthermore, the manager can exert effort continuously to increase the rate of information arrivals. The manager seeks to maximize the present value of future stock prices net of information acquisition costs. Investors are Bayesian and they update the firm's price continuously based on the manager's disclosures or lack thereof. Since the likelihood the manager observes information grows over time, in the absence of a disclosure, investors become skeptical about firm value, and the stock price drifts down, further motivating the manager to acquire information to correct mispricing.

The equilibrium has a simple structure whereby the manager resorts to costly information acquisition after some time, when the market is sufficiently pessimistic relative to the manager's beliefs. At the start, the manager abstains from acquiring information because investor beliefs are relatively similar to those of the manager. Over time, however, if the manager fails to disclose good news, then in the spirit of Dye 1985 the market becomes increasingly suspicious that the manager is concealing bad news to avoid a drop in the stock price. This process drives a growing wedge between the market and the manager's beliefs. If the manager is not hiding bad news, then there is a point at which he decides to start acquiring information, hoping to be able to receive good information and disclose it to the market to correct the mispricing. In equilibrium, however, the market anticipates the manager's information acquisition activities, which further accelerates the stock price decline in

[1] See, for instance, Gigler, Kanodia, Sapra, and Venugopalan (2014).

[2] See Blankespoor, deHaan, and Zhu (2018).

the absence of disclosure.

Our benchmark model assumes that the firm value is exogenous. We then study a setting where firm value is endogenous, influenced by the manager's initial private investment. In particular, we assume that at the outset the manager makes an unobservable investment that increases the probability that the firm value is high. The manager does so in anticipation of the trajectory of future prices and his own incentives to acquire and disclose any information he may receive. Investment tends to be higher when investors expect that the manager will become privately informed, because in general this will trigger more information acquisition and more voluntary disclosure, allowing the manager to internalize the benefits of his unobservable investments. In that sense, a greater arrival of private information leads to a higher investment in quality in our model.

We investigate the influence of the information environment (e.g., the cost of obtaining information, the rate of news arrival, and the strength of agency conflicts) on the inclination to acquire and reveal information, as well as the overall level of investment. Although the capacity to acquire and disclose information reduces the issue of underinvestment, it also leads to excessive deadweight costs. In our context, information acquisition can be a double-edged sword: while it encourages disclosure, which is beneficial for investment, the firm may end up doing too much of it, thus diminishing the profitability of investment.

We then study the role of mandatory disclosure as a means of increasing firm value. We model the mandatory disclosure policy as a fixed deadline T at which the manager is obligated to reveal any information he may have received, whether positive or negative. In this context, the equilibrium structure changes: as before the manager waits until the price has dropped sufficiently before acquiring any information. He then starts to collect information for a period of time, but there is a moment, just before T, when the manager ceases to collect information, because the benefit of acquiring information disappears as the company approaches the mandatory reporting date T.

In this context, we ask whether a mandatory disclosure deadline can increase firm value.

Our findings indicate that when the focus is on investment efficiency, it is beneficial to reduce the frequency of mandatory disclosure in order to promote the acquisition and voluntary disclosure of information and thus enhance investment incentives. On the other hand, when the regulator is mainly concerned with cutting down on information acquisition costs, a higher frequency of mandatory disclosure is the most suitable regulatory action.

By limiting the firm's ability to withhold bad news, mandatory disclosure reduces the information asymmetry between the firm and the market, but it also reduces the incentive to engage in information acquisition, as in Shavell 1994. Under some conditions, this exacerbates the underinvestment problem. Hence, mandatory disclosure can sometimes reduce the amount of information available to investors and induce less investment. When the cost of investment is high, and thus investment is relatively insensitive to information, mandatory disclosure both curbs excessive information acquisition and boosts the firm's investment.

We present a unified theoretical framework that combines voluntary disclosure, information acquisition, investment, and mandatory disclosure. Our model provides new explanations for observed phenomena and suggests empirical implications for the real effects of mandatory disclosure regulations. Our results suggest that a high frequency of disclosure is beneficial in situations where the investment friction (cost of investment) is either very low or very high. In both cases, investment is extreme, and the initial level of uncertainty is relatively low, which reduces the need for disclosure and thus the acquisition of information. As such, frequent mandatory disclosures are used to reduce the information asymmetry and boost investment. By contrast, for intermediate investment costs, the level of initial uncertainty is large, the price is very sensitive to the lack of disclosure, which exacerbates the tendency to excessively acquire information. Mandatory disclosures are infrequent in this context. Both effort and information costs are then relatively sensitive to T. An intermediate frequency is then optimal so to mitigate information acquisition costs while still inducing information acquisition.

Related Literature This paper builds on vast literature on voluntary disclosure with uncertain information endowments, in particular Dye (1985) and Jung and Kwon (1988). However, most literature is static in nature.[3] Furthermore, a relatively standard assumption in this literature is that the manager's information endowment is exogenous. In our paper, the manager controls the likelihood that he learns the state by deciding to invest in acquiring information, and can choose to disclose his information once he gets it. Farrell (1986), Shavell (1994), and Pae (1999) study the relation between information acquisition and voluntary disclosure in static settings. By contrast, in our model, the manager's information acquisition decision is a dynamic decision; specifically, it is the solution to an optimal stopping problem. The manager chooses when to invest in an information acquisition technology that increases the probability of learning the state per unit time. The manager's voluntary disclosure decision then depends on the information that he learns as a result.

Our paper is related to Gigler et al. (2014).[4] They consider a real earnings management model a la Stein (1989) to highlight the idea that frequent reporting may exacerbate

[3]Exceptions include Dye (2010), Acharya, DeMarzo, and Kremer (2011), Guttman, Kremer, and Skrzypacz (2014), Marinovic and Varas (2016), Aghamolla and An (2021), Gietzmann and Ostaszewski (2023), and Kremer, Schreiber, and Skrzypacz (2023).

[4]See also Gigler and Hemmer (1998), who show that increasing the frequency of mandatory disclosure harms information quality by reducing the scope for voluntary disclosures.

managerial myopia. In particular, they show that the price pressure created by high reporting frequency induces managers to adopt a short-term perspective (myopia) in choosing the firm's investments.

Crucially, we also allow the manager to make productive investments in firm quality. That is, the manager can not only control *when* he learns the state, but also *what* he learns. The relationship between voluntary disclosure and investment is examined by Beyer and Guttman (2012), Ben-Porath, Dekel, and Lipman (2018), DeMarzo, Kremer, and Skrzypacz (2019), Guttman and Meng (2021), and Migrow and Severinov (2022). Our paper departs from the extant literature in that the manager's productive investments directly affect the distribution of underlying firm quality, which is precisely what he is acquiring information about. Furthermore, the manager can choose his information acquisition and productive effort levels separately due to the multitasking nature of our model. We also address the optimal mandatory disclosure frequency when the manager can make investments in quality and acquire information which is novel to the literature.

Finally, our paper also speaks to the literature on dynamic reputations and investment, particularly Board and Meyer-ter-Vehn (2013, 2022), Marinovic, Skrzypacz, and Varas (2018), Dilmé (2019), Thomas (2019), Varas, Marinovic, and Skrzypacz (2020), and Hauser (2022a, 2022b). In these models, the firm can undertake effort to improve the state, but information arrivals are exogenous.

2 Model

2.1 Setup

Time is continuous and indexed by $t \in [0, \infty)$. A strategic manager of a firm and a market interact over time. Unobservable firm fundamentals are represented by a random variable $\theta \in \{L, H\}$, where $L < H$. Without loss of generality, set $L = 0$ and $H = 1$. Suppose θ is fixed at the beginning of the game. We first consider the case where θ is exogenous. Both parties are assumed to be risk-neutral.

Information arrives over time. The manager observes private information regarding firm fundamentals at an exponentially distributed random time $T_I > 0$. Let $N^I = \{N^I_t\}_{t \geq 0}$ denote the manager's private information arrival process, where $dN^I_t = 1_{\{t=T_I\}}$. Without any additional effort on the manager's part, N^I has arrival rate $\lambda \in (0, 1)$, which is common knowledge.

There is information acquisition over time. The manager can undertake costly information acquisition to increase the arrival rate to λ^*, where $\Delta = \lambda^* - \lambda > 0$. Let $\alpha_t \in [0, 1]$ be the manager's information acquisition strategy at time t. The cost of information acquisition is linear in the manager's acquisition policy: $c\alpha_t$, where $c > 0$. Thus, information acquisition increases the probability that the manager is privately informed in any given time interval. The actual time-t arrival rate of private information is a process $\alpha_t\lambda^* + (1-\alpha_t)\lambda = \lambda + \alpha_t\Delta$.

When the manager observes information about fundamentals, he has the option to make a disclosure to the market. Disclosures are verifiable so if the manager chooses to disclose his private information, then he must do so truthfully; he cannot lie to the market. The manager's disclosure strategy is $D_t = D_\theta 1_{\{t=T_I\}}$, where $D_\theta \in \{0,1\}$. $D_t = 1$ if the manager chooses to disclose θ at time $t = T_I$ and $D_t = 0$ if he chooses to withhold this information. When the market sees nondisclosure at any time $t \geq 0$, because it cannot observe the manager's information acquisition strategy, it is unsure if the nondisclosure was due to strategic withholding on the manager's part ($D_t = 0$) or the manager simply being uninformed ($dN_t^I = 0$).

Upon receiving private information at T_I, the manager's disclosure strategy can be represented as follows. Let $F_t^x(\alpha) = (1 - e^{-\int_0^t (\lambda+\Delta\alpha_s)ds})1_{\{\theta=x\}}F^D(x)$ denote the cumulative distribution function (CDF) of a disclosure of $x \in \{0,1\}$ at time $t = T_I$, where $F^D(x) \in [0,1]$ is the probability with which the manager will disclose the signal realization $\theta = x$. A jump in F_t^x is indicative of a positive probability that the manager has received information at time t and will disclose it to the market. Let $H_t^x(\alpha)$ denote the corresponding time-t hazard rate.

Let $D = \{D_t\}_{t>0}$ and $\alpha = \{\alpha_t\}_{t\geq 0}$ be the disclosure and information acquisition strategies chosen by the manager, respectively. Also, let $\tilde{p}_0 = E[\theta]$ denote the manager's beliefs over fundamentals at the start of the game.

Analogously, let $\hat{D} = \{\hat{D}_t\}_{t>0}$ and $\hat{\alpha} = \{\hat{\alpha}_t\}_{t\geq 0}$ be the disclosure and information strategies conjectured by the market. Furthermore, let $p_t = E_t^{(\hat{D},\hat{\alpha})}[\theta]$ denote the market's beliefs over firm fundamentals conditional on nondisclosure until time t, where $E^{(\hat{D},\hat{\alpha})}[\cdot]$ is the expectation taken under the probability measure induced by the the market's conjecture of the manager's disclosure/information acquisition strategies $(\hat{D}, \hat{\alpha})$.

Consider an interval of time $[t, t+dt]$, where $dt > 0$. Absent disclosure in this interval, by Bayes' rule,

$$p_{t+dt} = \frac{p_t\left(1 - H_t^1(\hat{\alpha})dt\right)}{p_t\left(1 - H_t^1(\hat{\alpha})dt\right) + (1-p_t)\left(1 - H_t^0(\hat{\alpha})dt\right)} + o(dt).$$

Taking the limit as $dt \downarrow 0$, the market's beliefs absent disclosure follow the law of motion

$$\dot{p}_t = -p_t\left(1 - p_t\right)\left(H_t^1(\hat{\alpha}) - H_t^0(\hat{\alpha})\right)\left(\lambda + \Delta\hat{\alpha}_t\right). \tag{1}$$

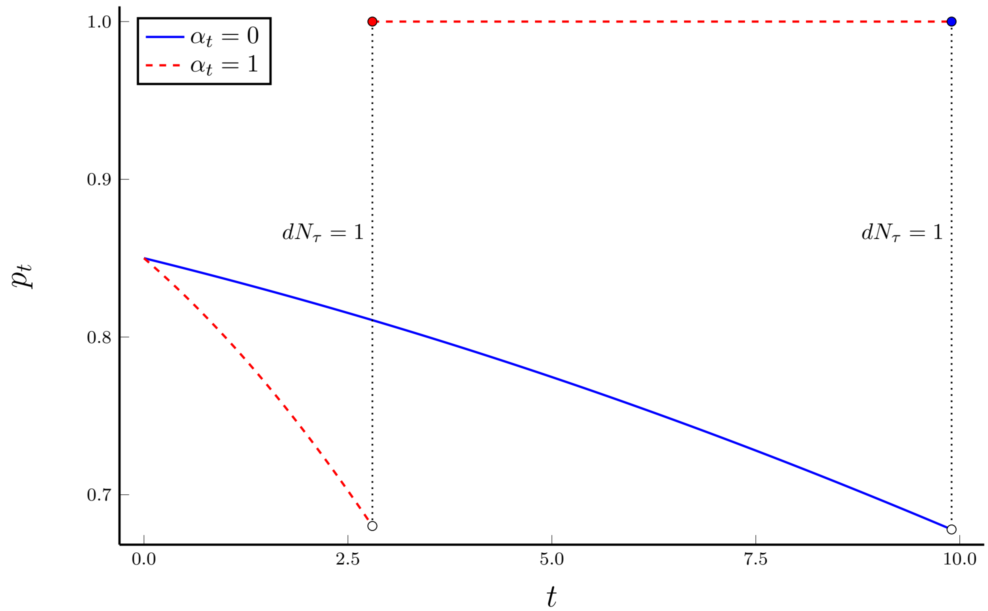

Figure 1: Example sample paths of the firm's reputation when $\alpha_t = 0$ and $\alpha_t = 1$ for all $t \geq 0$, for fixed initial beliefs of the manager $\tilde{p}_0$ and initial market beliefs $p_0 = \tilde{p}_0 = 0.85$. The arrival of information is assumed to contain good news and therefore induces immediate disclosure by the firm. The baseline parameters are $r = 0.1$, $c = 0.25$, $\lambda = 0.1$, and $\Delta = 0.25$.

Equation (1) illustrates the determinants of the price drift in the absence of disclosure: given no disclosure, the price falls more strongly when the manager is assumed to have acquired more information. Figure 1 depicts sample paths of the firm's reputation for different choices of α.

2.2 Payoffs and Equilibrium

Given the market's conjecture of firm fundamentals p, the manager chooses the firm's disclosure strategy and the information acquisition strategy to maximize his expected payoff:

$$\sup_{\alpha, D} E^{(\alpha,D)} \left[\int_0^\infty e^{-rt} \left(p_t - c\alpha_t \right) dt \,\middle|\, \tilde{p}_0 \right], \tag{2}$$

subject to the law of motion (1), where $r > 0$ is the firm's discount rate and $E^{(\alpha,D)}$ is the expectation under the probability measure induced by the information acquisition/disclosure strategies (α, D).

Here, the market's conjecture of fundamentals p serves as the manager's reputation, and his expected payoff is driven by this value. The assumption that the manager maximizes a weighted average of future prices is relatively standard in the literature.[5]

[5]See, for instance, Acharya et al. (2011) and Marinovic and Varas (2016).

Definition 1. *An equilibrium consists of a pair of strategies* $\{D_t, \alpha_t\}_{t\geq 0}$ *chosen by the manager and market conjectures* $\{\hat{D}_t, \hat{\alpha}_t\}_{t\geq 0}$ *such that the manager's choices are optimal given the market's conjectures and the market's conjectures are correct on the equilibrium path.*

Definition 2. *The equilibrium* (D, α) *is Markov if*

$$\begin{cases} F_t^0(\alpha) = 0, \\ F_t^1(\alpha) = \left(1 - e^{-\int_0^t (\lambda + \Delta\alpha_s) ds}\right). \end{cases}$$

That is, $F^D(0) = 0$ *and* $F^D(1) = 1$.

The solution to the law of motion (1) is (potentially) discontinuous, meaning there may exist points in time where $\dot{p}$ is not well-defined. To address this, we impose a mild condition on the market's conjectured strategies $\hat{\alpha}$.[6]

Definition 3. *Let* $\mu(p_t) = -(\lambda + \Delta\hat{\alpha}_t)p_t(1-p_t)$ *denote the drift of the firm's reputation under Definition 2 and let* $\{p_i^\dagger\}_{i\geq 1}$ *be an increasing finite sequence with* $p_i^\dagger \in [0,1]$. *Suppose that at any* $p_i^\dagger$, *either* $\mu(p_i^\dagger) = 0$, *or* $\hat{\alpha}$ *and therefore* $\mu(p)$ *is left-continuous at* $p_i^\dagger$. $\hat{\alpha}$ *is admissible if it is Lipschitz-continuous over any* $(p_i^\dagger, p_{i+1}^\dagger)$ *and* $\mu(\cdot)$ *satisfies the above conditions at any* $p_i^\dagger$.

Under the conditions in Definition 3, a unique solution $\dot{p}$ to the law of motion for beliefs exists almost-surely.[7]

The disclosure strategy is trivial. If the private information arrival reveals good news, then the manager immediately discloses it; otherwise, he withholds the information forever. The manager has no incentives to delay the good news disclosure because the price drifts downwards over time; delaying good news can only reduce the manager payoffs.

Remark 1. *The restriction of the manager's disclosure strategy to being immediate disclosure or perpetual withholding is without loss of generality here. Delayed disclosure is not optimal in our setting because, by equation* (1), *withholding only causes the firm's reputation to drift downwards. On the other hand, immediate disclosure at time* t *causes the firm's reputation to jump from* $p_{t-} \in (0,1)$ *to* $\{0,1\}$, *which are absorbing states.*[8]

Remark 2. *Bad news is always withheld by the manager here due to the binary nature of the state space and the nature of the manager's objective function* (2). *With a continuous*

[6] This condition essentially is identical to the condition in Board and Meyer-ter-Vehn (2013), who utilize the concept of admissible beliefs from Klein and Rady (2011).

[7] This follows by the Picard-Lindelöf theorem. See Section A.1 in Board and Meyer-ter-Vehn (2013).

[8] Delay may be optimal in a setting with social learning, whereby delaying disclosure can enable the firm to learn about the actions of other firms. See, for instance, Aghamolla and Hashimoto (2020).

state space, bad news disclosures may arise here if the manager is particularly risk-averse or has state-dependent preferences (e.g., Hummel, Morgan, and Stocken 2023).

3 Analysis

3.1 Benchmark: Exogenous effort

The model assumes that the manager cannot make investments in quality to increase p_0. In this section, we analyze this benchmark. We endogenize p_0 in Section 3.2.

Given the disclosure strategy in Definition 2, we turn to the incentives of the manager to acquire information. Information acquisition is a stopping time.

It is intuitive to assume that at the outset, when beliefs are symmetric, information acquisition is unproductive. Conjecture an equilibrium where the manager starts acquiring information at time $t = \tau > 0$. That is, the manager's information acquisition strategy is given by

$$\alpha_t = \begin{cases} 0 & \text{for } t < \tau \text{ or } t > T_I, \\ 1 & \text{for } t \in (\tau, T_I]. \end{cases} \tag{3}$$

This conjecture is natural: the manager acquires information to hopefully find and disclose favorable information, and thus correct what he perceives as mispricing. From the perspective of an uninformed manager, mispricing arises over time because absent disclosure the market will become increasingly suspicious that the manager is withholding negative information, causing the price to drift down. If the discrepancy between the manager's belief and the stock price is sufficiently large, the manager has an incentive to acquire information. At the outset, there is no discrepancy (unless we assume heterogeneous priors), and thus the manager has little incentive to acquire information.

When the manager observes an arrival at T_I, he chooses to disclose or withhold that information from the market and naturally stops acquiring information. Let $U : [0,1]^2 \to \mathbb{R}$ represent the time-t continuation value of an uninformed manager given his reputation and own initial beliefs about fundamentals.

If the manager receives positive information at time t, then he immediately discloses it to the market and obtains a continuation value $U(1, \tilde{p}_0) = \frac{1}{r}$. The manager's continuation value upon receiving bad news at time t is

$$U^0(p_t) \equiv U(0, \tilde{p}_0) = \int_t^\infty e^{-r(s-t)} p_s ds, \tag{4}$$

which reflects the fact that when the manager has bad news, he knows that he will not be disclosing in the future, so the price path is deterministic from the manager's standpoint.

Therefore, the manager's continuation value prior to being informed at time $t < T_I$ can be written as

$$U(p_t, \tilde{p}_0) = \sup_{\alpha} E_t^{\alpha}\left[\int_t^{\infty} e^{-\int_t^s (r+\lambda+\Delta\alpha_u)du}\left(p_s - c\alpha_s + (\lambda + \Delta\alpha_s)\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_s)\right)\right) ds\right],$$

where $E^{\alpha}[\cdot]$ is the expectation under the probability measure induced by the manager's choice of α. It satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} rU(p,\tilde{p}_0) = \sup_{\alpha}\; & p - c\alpha + \dot{p}U_p(p,\tilde{p}_0) \\ & + (\lambda + \Delta\alpha)\left(\tilde{p}_0\left(\frac{1}{r} - U(p,\tilde{p}_0)\right) - (1-\tilde{p}_0)\left(U(p,\tilde{p}_0) - U^0(p)\right)\right), \end{aligned} \tag{5}$$

subject to the law of motion (1), where $U_p(p, \tilde{p}_0) = \partial U(p, \tilde{p}_0)/\partial p$.

The right-hand side of equation (5) captures the rewards to the manager and the left-hand side represents the cost of capital. The manager's value is given by a flow consisting of prices net of information acquisition costs and by capital gains arising from the reputational drift (that is, $\dot{p}$) or the possibility of an information arrival.

Differentiating the HJB equation (5), the optimal information acquisition strategy when the manager is uninformed is

$$\alpha(p_t, \tilde{p}_0) = \begin{cases} 1 & \text{if } \tilde{p}_0\left(\frac{1}{r} - U(p_t,\tilde{p}_0)\right) > c/\Delta + (1-\tilde{p}_0)\left(U(p_t,\tilde{p}_0) - U^0(p_t)\right), \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The manager acquires information when the marginal benefit from doing so, given by the potential capital gains arising from a positive disclosure, is greater than the marginal cost of acquiring information c.

Proposition 1. *The equilibrium is characterized as follows. Let p_{τ}^* denote the optimal information acquisition threshold and τ^* the corresponding information acquisition time. Define the initial belief thresholds for the manager $\tilde{p}_0^-$ and $\tilde{p}_0^+$ as*

$$\tilde{p}_0^- = \frac{c\,(r+\lambda)}{\Delta}, \qquad \tilde{p}_0^+ = \frac{c\,(r+\lambda)}{(1-p_0)\,\Delta},$$

where $\tilde{p}_0^+ > \tilde{p}_0^-$. Then, the following hold.

- *If $\tilde{p}_0 < \tilde{p}_0^-$, then $\alpha_t = 0$ for all $t \geq 0$.*

- *If* $\tilde{p}_0 \in (\tilde{p}_0^-, \tilde{p}_0^+)$*, then* $\tau > 0$*:* $\alpha_t = 1$ *if* $p_t < p_\tau^*$ *and* $\alpha_t = 0$ *otherwise,* $p_\tau^* \in (0,1)$*, and*

$$p_\tau^* = 1 - \frac{c}{\tilde{p}_0}\left(\frac{r+\lambda}{\Delta}\right).$$

- *Finally, if* $\tilde{p}_0 \in [\tilde{p}_0^+, 1]$*, then* $\alpha_t = 1$ *for all* $t \geq 0$*.*

The equilibrium derived in Proposition 1 is intuitive: the manager's decision to acquire information depends on the size of the discrepancy between his beliefs and those of the market, $\tilde{p}_0 - p_t$. The higher the discrepancy, the more eager the manager is to acquire information to correct the mispricing. However, when the manager's beliefs are low enough, acquiring information is not profitable because most likely the arrival will yield bad news anyways, and the manager will thus withhold it. To acquire information, the manager must therefore be sufficiently optimistic about the state. In fact, the manager acquires information immediately at $t = 0$ if he is optimistic enough, even when the market is more optimistic than the manager. Of course, the fact that the manager can withhold information if it is unfavorable drives this result.

The general form of the information acquisition strategy in Proposition 1 is depicted in Figure 2 for $\tilde{p}_0 \in (\tilde{p}_0^-, 1]$. The comparative statics for p_τ are intuitive: the manager reduces information acquisition if λ is higher and increases it when Δ is higher or c is lower.

Figure 3 depicts the effect of information acquisition skill Δ and the manager's initial beliefs $\tilde{p}_0$. When the beliefs of the manager are higher than the market's, increasing Δ increases the firm's value if $\tilde{p}_0$ is high and decreases value if $\tilde{p}_0$ is low. This is due to the increased probability of obtaining good news in the former case, and the greater negative price drift due to nondisclosure in the latter (in that case, the manager is likely to receive bad news, and the stock price decreases faster if the market expects the manager to become informed more quickly).

A higher Δ increases the marginal impact of $\tilde{p}_0$ on firm value due to the increased likelihood of learning good news and disclosing it. This effect is especially strong when the discrepancy between the manager's and market's beliefs over quality is strong, namely when $\tilde{p}_0$ is high but p_0 is low. When the market's and manager's initial beliefs over quality are equal, however, the discrepancy effect is eliminated.

Before we endogeneize effort, we verify some intuitive results arising when information acquisition is observable.

Observable Information Acquisition Assume the market can observe the manager's information set at each point in time. Then, under common priors ($\tilde{p}_0 = p_0$) the manager never acquires information: $\alpha_t = 0$ at all $t \geq 0$.

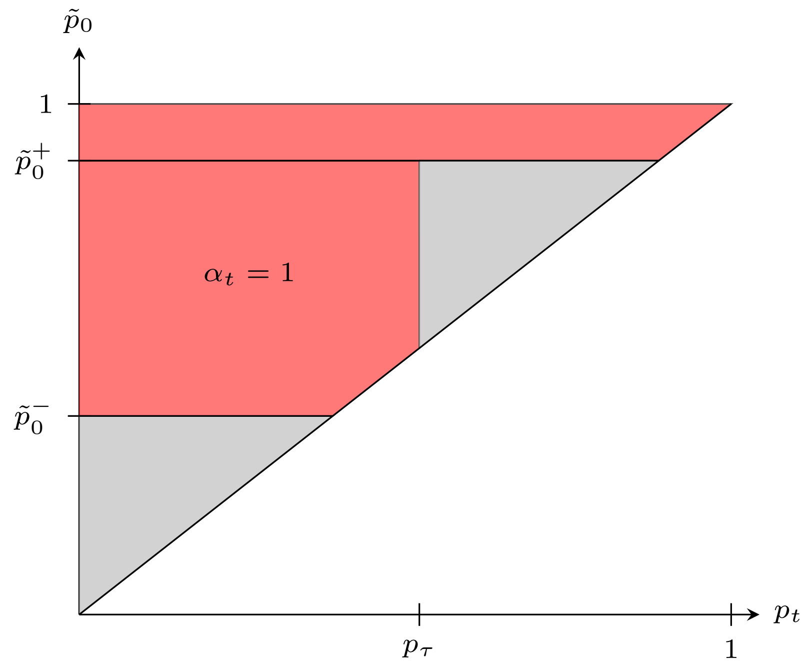

Figure 2: The general form of the equilibrium derived in Proposition 1, where $\tilde{p}_0 > p_0$. The manager acquires information for $(\tilde{p}_0, p_t)$ values in the shaded red area and does not acquire information for $(\tilde{p}_0, p_t)$ values in the shaded gray area. The manager does not acquire information when $\tilde{p}_0 < \tilde{p}_0^-$ due to the risk of learning that the firm is a low type. On the other hand, the manager does not acquire information when $p_t > p_\tau$ due to his high reputation and the personal cost of information acquisition.

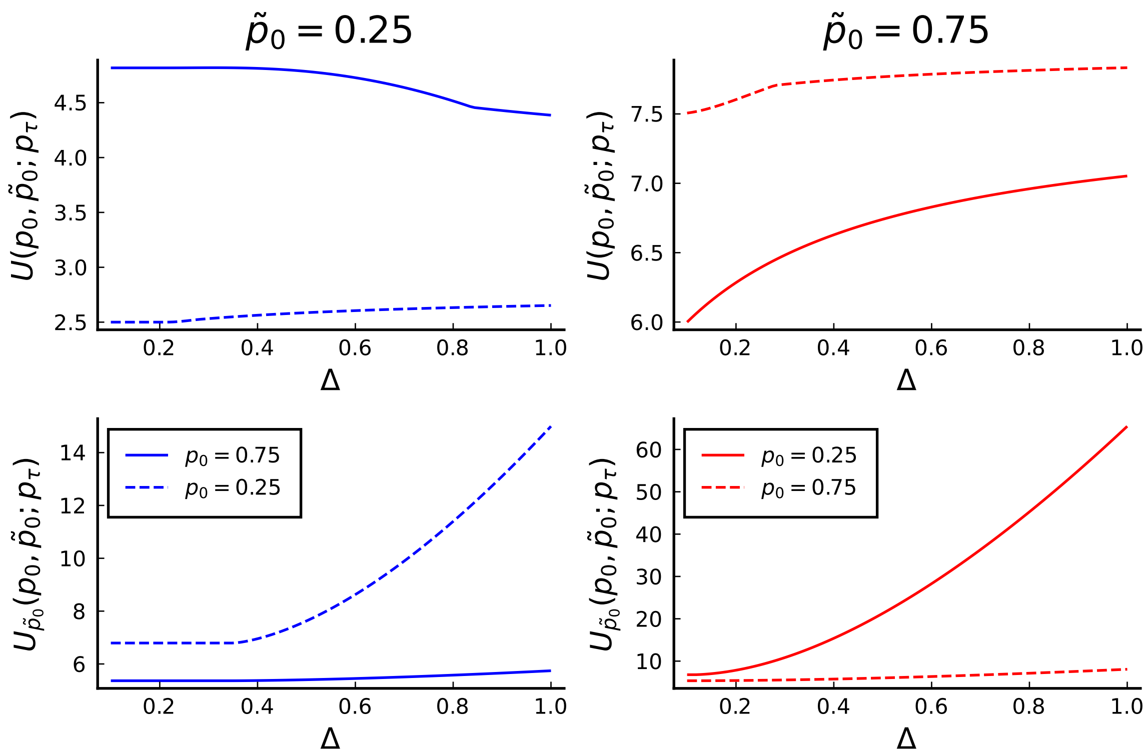

Figure 3: The top two plots depict the effect of Δ on the manager's equilibrium value $U(p_0, \tilde{p}_0; p_\tau)$ for different values of the manager's initial beliefs $\tilde{p}_0$. The bottom two plots depict the effect of Δ on the marginal impact of the manager's initial beliefs $\tilde{p}_0$, $U_{\tilde{p}_0}(p_0, \tilde{p}_0; p_\tau)$. The dashed lines set the market's initial beliefs at $p_0 = \tilde{p}_0$, while the solid lines allow the market's initial beliefs to differ from the manager's initial beliefs.

Now suppose that the market can observe the manager's information acquisition effort, α_t, but not the actual information arrival. In this case, the manager can still withhold bad news, since the market does not observe the actual endowment of information the manager has. The following result characterizes the information acquisition strategy in this case.

Corollary 1. *Suppose the market can observe the manager's decision to begin acquiring information, and let $p_\tau^{obs} \in [0,1]$ be the corresponding information acquisition threshold. Then, $p_\tau^{obs} < p_\tau^*$. If the manager and the market share common initial beliefs, $p_0 = \tilde{p}_0$, then $p_\tau^{obs} = 0$.*

Hence, under common priors, the manager never acquires information if information acquisition effort is observable. This is intuitive and consistent with static models. Under common priors, the manager has no incentive to acquire information when acquisition is observable because, by the law of iterated expectations, information acquisition can only affect the variance of prices but not the mean. Therefore, there is no benefit in acquiring information.

3.2 Endogenous Effort

To understand the "real effects" of information acquisition and disclosure, we now allow the manager to affect the distribution of the state θ by exerting a "quality-enhancing" effort. We model the effort as a private investment, taken at $t = 0$, that determines the probability of the high state, $\tilde{p}_0$.

Specifically, we model effort as an unobservable investment that increases the chances of the state being high. Specifically, let $A \in [0, \overline{A}]$ denote the probability that $\theta = 1$, where $\overline{A} \leq 1$. The manager chooses A, at the start, at cost $\kappa h(A)$, where $\kappa > 0$ and $h : [0, \overline{A}] \to \mathbb{R}_+$ is an twice continuously differentiable function with $h'(\cdot) > 0$ and $h''(\cdot) \geq 0$. The manager's belief about quality is $\tilde{p}_0 = A$. The market does not observe A. But, as in the previous section, the manager acquires information about the state of nature and eventually discloses it, unless the information is unfavorable, in which case he conceals it. Anticipating this, the price drops in the absence of disclosure (see Figure 4). The more quality-enhancing effort the manager is believed to have exerted, the more information he will be expected to acquire thereafter. As a result, the price drifts down faster in the absence of disclosure.

Notice also that, given the binary support of θ, the chosen effort level A changes the amount of initial uncertainty, which provides countervailing incentives. When the incentives for productive investment are either too high or too low, there is relatively little uncertainty. In these cases, the price drifts relatively slowly thereby reducing the incentives to acquire information. The level of initial uncertainty is maximized when $A = \frac{1}{2}$.

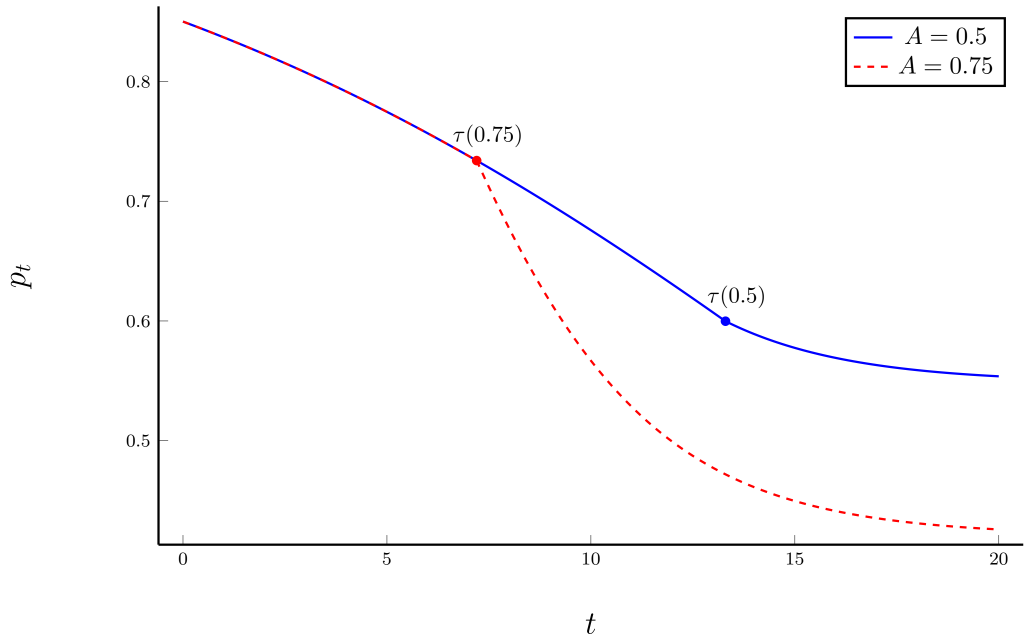

Figure 4: Example sample paths of the firm's reputation prior to any disclosure for various levels of productive effort $A \in [0, \overline{A}]$ and fixed initial market beliefs $p_0 \in (0, 1)$. The endogenous information acquisition time is $\tau(A)$.

Let $\hat{A} \in [0, \overline{A}]$ denote the market's conjecture of the manager's effort choice. Given the market's conjectures, the manager chooses investment and information acquisition strategies to solve

$$U(p_0) = \sup_{\alpha, A} E^{(\alpha,A)} \left[\int_0^\infty e^{-rt} \left(p_t - c\alpha_t \right) dt - \kappa h(A) \,\middle|\, p_0 = \hat{A} \right], \tag{7}$$

subject to the law of motion (1), where $E^{(\alpha,A)}[\cdot]$ is the expectation under the probability measure induced by the manager's strategies (α, A).

Given the information acquisition strategy (3), the manager's total expected payoff can be written as

$$\begin{aligned} U(p_0) = \sup_A \int_0^{\tau(p_0,A)} & e^{-(r+\lambda)t} \left(p_t + \lambda \left(\frac{A}{r} + (1-A)\, U^0(p_t) \right) \right) dt - \kappa h(A) \\ & + e^{-(r+\lambda)\tau(p_0,A)} U^\Delta(p_\tau(A)), \end{aligned}$$

where $p_\tau(A) \in [0, 1]$ is the information acquisition threshold as a function of A, $U^\Delta : [0, 1] \to \mathbb{R}$ is the manager's continuation value after deciding to acquire information:

$$U^\Delta(p_\tau(A)) = \int_{\tau(p_0,A)}^\infty e^{-(r+\lambda+\Delta)(t-\tau(p_0,A))} \left(p_t - c + (\lambda + \Delta) \left(\frac{A}{r} - (1-A) U^0(p_t) \right) \right) dt,$$

and $\tau(p_0, A) \geq 0$ is the time it takes for the market beliefs to reach the acquisition threshold $p_\tau(A)$ given the market's initial beliefs p_0:

$$\tau(p_0, A) = -\frac{1}{\lambda} \log \left(\left(\frac{1-p_0}{p_0} \right) \frac{p_\tau(A)}{1-p_\tau(A)} \right).$$

Notice again that effort affects the manager's subsequent incentives to acquire and disclose information. A higher effort level makes the manager more optimistic about the state, and thus more prone to acquiring information to convey good news to the market. Also, when the market anticipates higher effort, this may lead to greater uncertainty (if effort is less than 1/2), and to a faster drift, which again reinforces the incentives to acquire information sooner.

The following result derives information acquisition and productive effort strategies when the effort is endogenous.

Proposition 2. *When the manager is allowed to choose effort, the equilibrium information acquisition threshold is*

$$p^*_\tau(A) = \begin{cases} 0 & \text{if } A < \frac{c(r+\lambda)}{\Delta}, \\ 1 - \frac{c(r+\lambda)}{\Delta A} & \text{otherwise.} \end{cases} \tag{8}$$

The equilibrium level of productive effort A^* satisfies

$$A^* = \left\{ \min \left\{ h'^{-1} \left(\frac{1}{\kappa} \left(\psi(A^*) + \frac{\omega(A^*, A^*)}{r+\lambda+\Delta} \right) \right), \overline{A} \right\} \right\}^+, \tag{9}$$

where $\{x\}^+ = \max\{0, x\}$. The functions $\omega : (0,1) \times [0, \overline{A}] \to \mathbb{R}_+$ and $\psi : [0,1) \to \mathbb{R}_+$ are defined by

$$\omega(p_0, A) = \frac{\Delta e^{-(r+\lambda)\tau^*(p_0, A)}}{r+\lambda},$$
$$\psi(p_0) = \frac{\lambda}{r(r+\lambda)} - \int_0^\infty e^{-rt} \left(1 - e^{-\lambda t}\right) p^\dagger_t dt,$$

where $p(t, p_0) = p^\dagger_t$ for $t \geq 0$, $p^\dagger$ is the solution to the differential equation $\dot{p} = -\lambda p(1-p)$, and $\psi'(p_0) \leq 0$ for all p_0.

Equation (8) shows that the structure of the information acquisition strategy is the same as in the case without endogenous effort and can be characterized as a threshold τ. The threshold is increasing in A for the reasons mentioned previously.

The following corollary to Proposition 2 considers the relationship between the equilibrium effort and the information acquisition skill, Δ.

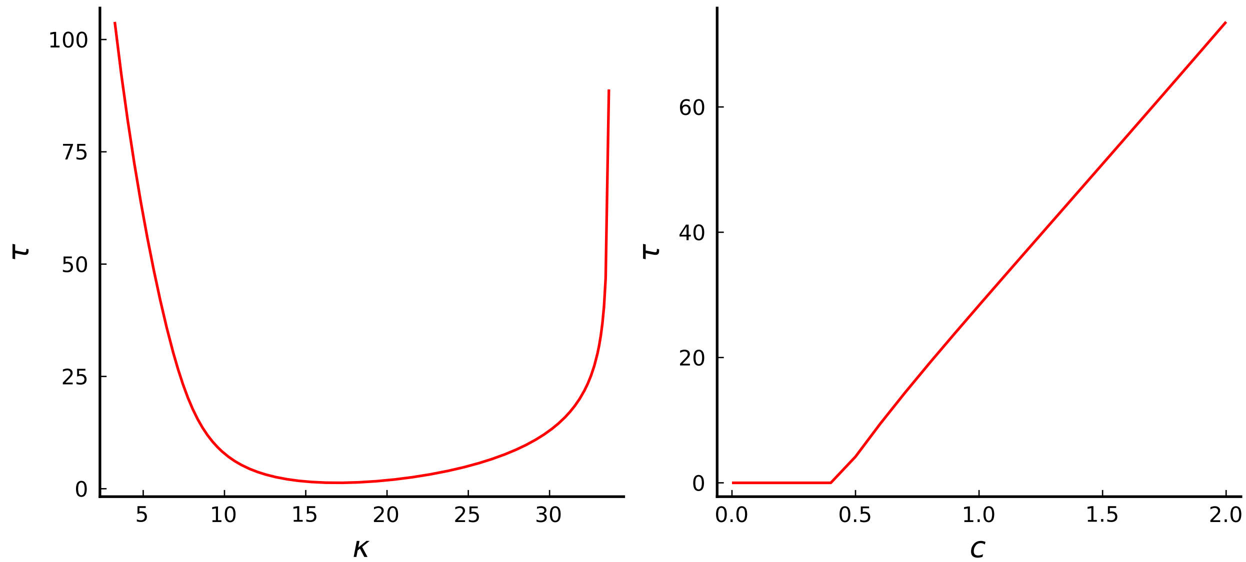

Figure 5: Comparative statics of the optimal information acquisition time τ with respect to the cost of investment κ and the flow cost of information acquisition c.

Corollary 2. *If A^* is the equilibrium level of the productive effort of Proposition 2, then A^* increases in Δ.*

Corollary 2 is verified by our numerical results and shows that effort increases when the manager is more able to obtain information (i.e., Δ is higher), regardless of the cost of investment (κ). Greater access to information allows the manager to disclose more information in the event of good news, thus mitigating the moral hazard problem. The price becomes more informative and the manager gets to better internalize the output of his effort. A higher Δ makes it easier for the manager to prove high quality and thus internalize the benefits of effort.

Notice there is a potential countervailing effect here because a greater Δ may induce the manager to spend more resources/time acquiring information, if this leads to a lower threshold τ. On the other hand, a higher Δ also may reduce the overall cost of information acquisition, because it shortens the expected time the manager spends acquiring information, for a given threshold.

3.3 The Optimal Frequency of Mandatory Disclosure

So far, we have examined the effects of voluntary disclosure in the absence of mandatory disclosures. In this section, we add mandatory disclosures to the model. In particular, we study the optimal mandatory disclosure frequency from the perspective of a regulator seeking to maximize firm value (or reduce the cost of capital). As usual, one can also think of mandatory disclosure as a firm ex-ante commitment. CITES

We model mandatory disclosure as a fixed deadline at which the manager must disclose any information that he has received. Specifically, at time $T \geq 0$, any information that the manager may have observed prior to T must be revealed to investors. Thus, the manager can only conceal negative information until T. After that, the information becomes symmetric. If the manager is not informed at T, then the game restarts until a disclosure (voluntary or mandatory) is made, in which case no further disclosures are needed, since the state is fixed.

Formally, suppose that a regulator can design a mandatory disclosure policy that specifies ex ante a sequence of disclosure dates. Formally, the regulator chooses an increasing sequence of measurable stopping times $\{T_i\}_{i\geq 0}$, where $T_0 = 0$, $T_i \in (0, \infty]$ for all $i \geq 1$, and $\lim_{i\to\infty} T_i = \infty$. At time T_i, the manager must disclose any information he has received and has not voluntarily disclosed previously. If the manager is not informed at time T_i, then nothing is disclosed, and the manager continues to learn, and can make voluntary disclosures aft.er T_i. He must make a disclosure by time T_{i+1} if he receives information in the interval $(T_i, T_{i+1}]$.

As before, in equilibrium, the manager discloses positive arrivals voluntarily and delays disclosing negative arrivals until the mandatory deadline. Mandatory disclosures are thus more informative when they are not preceded by voluntary disclosures. This is consistent with the empirical finding that markets are more sensitive to mandatory earnings announcements when they convey negative news (see Kothari 2001).

At any deadline $T_i > 0$, if the manager has nothing to disclose, then the market learns that the manager is uninformed. As such, by Bayes' rule, the market's beliefs jump back from p_{T_i-} to p_0. [9]

Thus, given an exponentially distributed information arrival time $T_I > 0$, the price satisfies the boundary condition

$$p_{T_i} = \begin{cases} \theta & \text{if } T_I \leq T_i, \\ p_0 & \text{if } T_I > T_i, \end{cases} \tag{10}$$

for any $i \geq 1$. Absent disclosure, during the interval (T_i, T_{i+1}), p drifts down deterministically according to the law of motion (1).

We look for mandatory disclosure times $\mathcal{T} = \{T_i\}_{i\geq 1}$ that maximize ex-ante firm value.

Given a mandatory disclosure strategy $\mathcal{T}$, let $A(\mathcal{T})$ be the manager's productive effort choice, which will be a function of $\mathcal{T}$, and let $\hat{A}(\mathcal{T})$ denote the market's conjecture of the manager's underlying effort.

[9] If the manager's effort cost κ was private information, then the lack of disclosure would lead the market to update negatively the manager's effort in the absence of disclosures.

Then, given the manager's disclosure, information acquisition, and productive effort strategies, the mandatory disclosure times are chosen to solve

$$\sup_{\mathcal{T}} \left\{ E^{(\hat{\alpha},\hat{A}(\mathcal{T}))} \left[\sum_{i\geq 0} \int_{T_i}^{T_{i+1}} e^{-rt} \left(p_t - c\hat{\alpha}_t\right) dt \,\middle|\, p_0 = \hat{A}(\mathcal{T}) \right] - \kappa h(A(\mathcal{T})) \right\}, \tag{11}$$

subject $A(\mathcal{T}) \in [0, \overline{A}]$, the law of motion (1), and the boundary condition (10). The manager's problem is to choose the information acquisition and productive effort strategies to solve

$$\sup_{\alpha, A(\mathcal{T})} \left\{ E^{(\alpha,A(\mathcal{T}))} \left[\sum_{i\geq 0} \int_{T_i}^{T_{i+1}} e^{-rt} \left(p_t - c\alpha_t\right) dt \,\middle|\, p_0 = \hat{A}(\mathcal{T}) \right] - \kappa h(A(\mathcal{T})) \right\}, \tag{12}$$

subject to equations (1) and (10).

With some abuse of notation, let $U : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}$ denote the manager's value function and $W^{\alpha,A} : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}$ the manager's objective given his information acquisition and initial investment decisions. Equation (10) implies that the environment is stationary given the manager's initial investment decision. Therefore, by the principle of optimality, $W^{\alpha,A}$ can be written recursively as

$$\begin{aligned} W^{\alpha,A}(p_0, T) = &\int_0^T e^{-\int_0^t (r+\lambda+\Delta\alpha_s)ds} \left(p_t - c\alpha_t + (\lambda + \Delta\alpha_t) \left(\frac{A}{r} + (1 - A)\, U^0(p_t, T) \right) \right) dt \\ &+ e^{-\int_0^T (r+\lambda+\Delta\alpha_t)dt} W^{\alpha,A}(p_0, T), \end{aligned}$$

where A is the effort choice and $U^0 : [0, 1] \times \mathbb{R}_+ \to \mathbb{R}_+$ denotes the manager's continuation value when he learns bad news at time $t \in (0, T)$:

$$U^0(p_t, T) = \int_t^T e^{-r(s-t)} p_s ds.$$

Then, the manager's objective (12) can be written as

$$U(p_0, T) = \sup_{\alpha, A} \left\{ W^{\alpha,A}(p_0, T) - \kappa h(A) \right\}. \tag{13}$$

Similarly, the regulator's objective (11) can be written as

$$\sup_{T\geq 0} U(p_0, T), \tag{14}$$

Thus, $T \equiv T_{i+1} - T_i$, $i \geq 0$, captures the frequency with which the manager has to make an audited disclosure about the firm's value.

With some abuse of notation, let $p : [0,T) \times [0,1] \to [0,1]$ denote the firm's reputation at time $t \in [0,T)$ starting from initial market beliefs $p_0 \in [0,1]$. The following proposition derives the optimal information acquisition strategy with a finite mandatory disclosure frequency T.

Proposition 3. *Let $T \geq 0$ be the mandatory disclosure frequency. If T is nonzero and finite, then there exist two thresholds $\tau^*(A,T)$ and $\tau_S(A,T)$, $0 \leq \tau^*(A,T) \leq \tau_S(A,T) \leq T$, such that the manager's optimal information acquisition strategy under T satisfies*

$$\alpha_t^*(A,T) = \begin{cases} 0 & \text{if } t \in [0, \tau^*(A,T)), \\ 1 & \text{if } t \in [\tau^*(A,T), \tau_S(A,T)), \\ 0 & \text{if } t \in [\tau_S(A,T), T). \end{cases}$$

Furthermore, if $\tau^*(A,T) \notin \{0,T\}$, then these information acquisition thresholds are unique and are related via

$$e^{-\lambda\tau_S(A,T) - \Delta(\tau_S(A,T) - \tau^*(A,T))} = \frac{1-A}{A}\left(1 - \frac{c(r+\lambda)}{\Delta A}\right). \tag{15}$$

The firm optimally chooses the level of productive effort, $A^* \in [0, \overline{A}]$, that satisfies

$$A^* = \left\{\min\left\{h'^{-1}\left(\frac{\lambda\Psi(A^*,T) + (\lambda+\Delta)\Omega(A^*,T)}{\kappa\left(1 - e^{-(r+\lambda)T - \Delta(\tau_S(A^*,T) - \tau^*(A^*,T))}\right)}\right), \overline{A}\right\}\right\}^+, \tag{16}$$

where

$$\Psi(A,T) = \int_0^{\tau^*(A,T)} e^{-(r+\lambda)t}\beta_t^A(A,T)dt + e^{-\Delta(\tau_S(A,T)-\tau^*(A,T))}\int_{\tau_S(A,T)}^T e^{-(r+\lambda)t}\beta_t^A(A,T)dt,$$
$$\Omega(A,T) = \int_{\tau^*(A,T)}^{\tau_S(A,T)} e^{-(r+\lambda+\Delta)t}\beta_t^A(A,T)dt,$$

and $\beta_t^A(A,T) = \frac{1}{r} - U^0(p_t,T)$ is the marginal value of investment.

$\Psi(A,T)$ is the marginal value of investment in the region without information acquisition $\{p : p \in [p_{T-}, p_{\tau_S}(A,T)] \cup (p_{\tau^*}(A,T), 1)\}$ and $\Omega(A,T)$ is the marginal value in the information acquisition region $\{p : p \in (p_{\tau_S}(A,T), p_{\tau^*}(A,T)]\}$, where $p_{T-} = \lim_{\epsilon \downarrow 0} p_{T-\epsilon}$.

Note that as $T \to \infty$, equation (16) approaches equation (9). Comparing Propositions 2 and 3 (with $\tilde{p}_0 = A$), the manager is only able to fully internalize the benefits of quality investments if and only if he receives a positive signal before T. Otherwise, he is forced to disclose being uninformed at time T.

Let $p_\tau(A) \in [0,1]$ denote the information acquisition threshold without mandatory disclosure (i.e., $T = \infty$). Proposition 4 shows that when $T < \infty$, $p_\tau(A,T) < p_\tau(A)$: namely, mandatory disclosure reduces the manager's incentives to acquire information, which is consistent with Shavell (1994). The obligation to disclose arrivals, including negative ones, weakens the incentives to acquire information because it removes the option to withhold bad news. Ex-ante, this has some benefits though, as it allows the firm to reduce information acquisition costs.

The following proposition characterizes the mandatory disclosure frequency T^* that maximizes firm value.

Proposition 4. *Let A^* and α^* be the agent's optimal investment and information acquisition strategies from Proposition 3, with corresponding stopping times (τ^*, τ_S). Furthermore, let T^* denote the mandatory disclosure frequency that maximizes expected firm value. If $|\tau_S - \tau^*| < T^*$ and $A^* \in (0, \overline{A})$, then T^* satisfies*

$$T^* = \left\{ -\frac{1}{\lambda} \log\left(\frac{1-A^*}{A^*} \frac{\chi(A^*,T^*)}{1-\chi(A^*,T^*)} \right) - \frac{\Delta}{\lambda}(\tau_S - \tau^*) \right\}^+,$$

where

$$\chi(A,T) = \left((r+\lambda+\Delta\alpha_T)^2 - (\lambda + \Delta\alpha_T) \right) \frac{A}{r} - (r+\lambda+\Delta\alpha_T - 1)\, c\alpha_T.$$

Furthermore, there exists a $\bar{c} > 0$ such that $T^* = 0$ if $c \geq \bar{c}$. Otherwise, $T^* \geq 0$.

Proposition 4 shows that optimal frequency of disclosure depends on the cost of information acquisition, the cost of effort, and the rate of information arrivals. When information acquisition and productive effort are interior, the optimal mandatory disclosure frequency is (potentially) finite and nonzero.

We now consider the interaction between the incentives for information acquisition and investment in determining the optimal mandatory disclosure frequency. Let $A^*(\kappa)$ denote the equilibrium level of effort from Proposition 3 as a function of κ. The following corollaries to Proposition 4 characterize the comparative statics for τ^* and T^* for two different investment cost functions.

Corollary 3. *Suppose $c < \bar{c}$ and set $h(A) = \frac{1}{2}A^2$, so that the relation between $A^*(\kappa)(1 - A^*(\kappa))$ and κ exhibits an inverse-U shape.*

- *There exist $\underline{\kappa}$ and $\overline{\kappa}$, $0 \leq \underline{\kappa} \leq \overline{\kappa}$, such that $\tau^* = \infty$ for $\kappa \leq \underline{\kappa}$ or $\kappa \geq \overline{\kappa}$. For $\kappa \in (\underline{\kappa}, \overline{\kappa})$, τ^* is U-shaped in κ.*

- $T^* = 0$ *for all* $\kappa \leq \kappa_1$ *and* $\kappa \geq \kappa_2$*, where* $\kappa_1 \geq \underline{\kappa}$ *and* $\kappa_2 \leq \overline{\kappa}$*. Finally, for* $\kappa \in (\kappa_1, \kappa_2)$*,* T^* *is U-shaped in* κ*.*

Corollary 4. *Suppose* $c < \bar{c}$ *and set* $h(A) = \frac{1}{2}\left(A - A^\dagger\right)^2$*, where* $A^\dagger = \lim_{\kappa \to \infty} A^*(\kappa)$ *is the cost-minimizing level of effort and is set such that* $A^*(\kappa)(1 - A^*(\kappa))$ *is increasing in* κ*.*

- *There exists a* $\underline{\kappa}^\dagger \geq 0$ *such that* $\tau^* = \infty$ *for* $\kappa \leq \underline{\kappa}^\dagger$*. Otherwise,* τ^* *is decreasing in* κ*.*

- *There exists a* $\kappa_1^\dagger \geq \underline{\kappa}^\dagger$ *such that* $T^* = 0$ *for* $\kappa \leq \kappa_1^\dagger$*. For* $\kappa > \kappa_1^\dagger$*,* T^* *is (weakly) decreasing in* κ*.*

Figure 6 shows the sensitivity of the equilibrium to the main parameters, as described in Corollaries 3 and 4. Two things are worth noting. First, τ is non-monotonic in κ. That is, information acquisition slows down when the effort cost is extreme. This is due to the lower uncertainty arising when effort is extremely low or extremely high. Under little uncertainty, the negative priced drift is smaller, and then the incentive of the manager to acquire information and correct the mispricing become weaker.

As we see in Figure 6, the optimal T is zero at extreme levels of effort: since the information acquisition friction is mild, then the incentive provision motive dominates: frequent mandatory disclosure mitigates its under-investment problem without affecting information acquisition incentives.

For intermediate levels of effort, initial uncertainty is relatively large. The market grows skeptical very fast about firm value, given no disclosure, which triggers information acquisition and disclosure. The need to moderate information acquisition costs, leads to a finite T. T mirrors the behavior of τ^* over this region.

The optimal mandatory disclosure frequency strikes a balance between lowering information acquisition costs (which for a fixed effort are deadweight costs) and mitigating the moral hazard problem. In effect, by increasing the frequency of mandatory disclosures, the deadline policy reduces the incentives to engage in costly information acquisition. However, it also reduces the ability to obtain information and disclose it voluntarily, potentially weakening the incentives to exert effort in the first place.

Figure 6 shows the sensitivity of T^* with respect to the main parameters. First it shows that T^* is hill-shaped in the cost of effort, κ. This is intuitive: as the cost of effort becomes extreme, the policy's main objective is to reduce information acquisition costs, since effort becomes relatively insensitive to disclosures in this case.

The frequency of mandatory disclosure T^* also is hill-shaped in the cost of information acquisition c. When c is very low, the information acquisition threshold τ is zero, so the manager always acquires information. In that context, the regulator's main objective is to

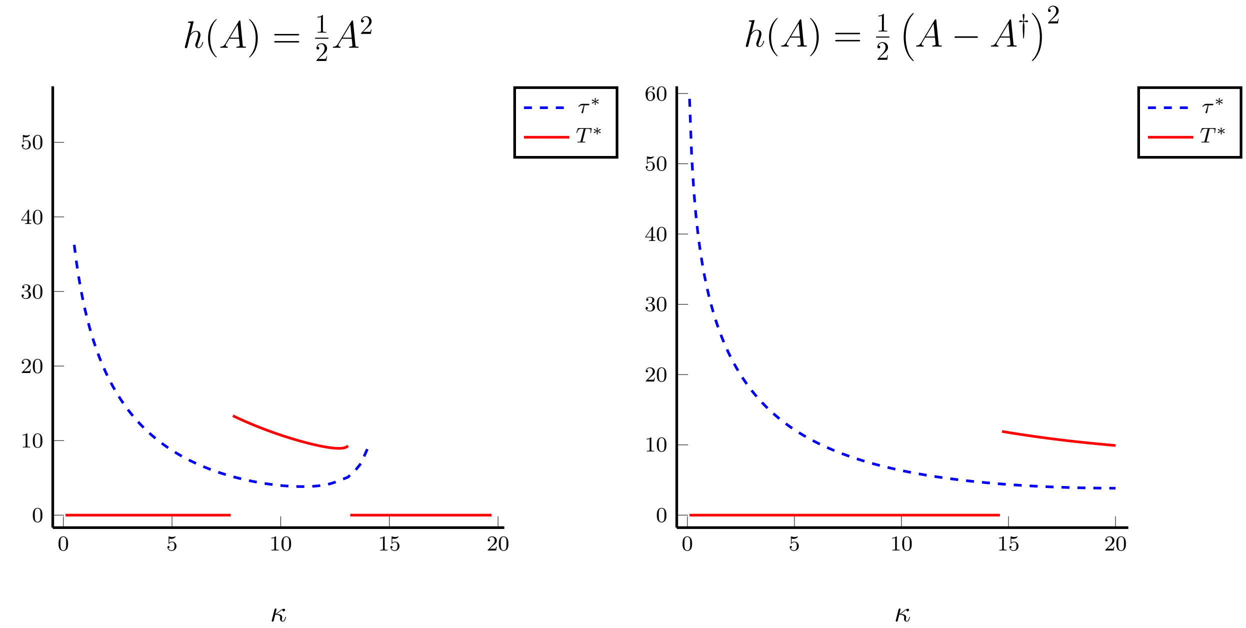

Figure 6: Comparative statics of the equilibrium information acquisition time τ^* and the equilibrium mandatory disclosure frequency T^* with respect to κ. In the left panel, the cost-minimizing level of effort is 0. In the right panel. the cost-minimizing level of effort is set to $A^\dagger = 0.25$.

increase effort, leading to a high frequency of disclosure, that is, a low T^*. A the other extreme, when c is very large, there is no information acquisition. Once again, the main objective is to increase effort, thereby leading to a high frequency of mandatory disclosure.

For low c, τ increases in c. In response, the deadline T is extended to accommodate higher acquisition costs and not further discourage information acquisition by the manager. On the other hand, when c are relatively large, incentives to acquire information vanish and moral hazard concerns become dominant; leading to a lower T^* to compensate for a lower frequency of voluntary disclosure.

The following corollary to Proposition 3 compares the equilibrium effort levels with and without mandatory disclosure.

Corollary 5. *Let $A^*(T^*)$ and A^∞ denote the optimal effort levels with disclosure frequencies T^* and ∞, respectively. Then, there exists a threshold cost of investment $\underline{\kappa}$ such that $A^*(T^*) > A^\infty$ for $\kappa < \underline{\kappa}$ and $A^*(T^*) < A^\infty$ for $\kappa > \underline{\kappa}$ for a nonempty range of (Δ, c).*

Corollary 5 shows that, under certain conditions, imposing a mandatory disclosure frequency T^* can actually reduce investment/quality. Under some conditions (e.g., when the effort cost is large), the mandatory disclosure regime will lead to less effort than a no-mandatory-disclosure regime. Therefore, we find that a higher frequency of mandatory disclosure could lead to lower effort. However, this is still optimal because mandatory disclosure here is not aimed at maximizing effort, but firm value net of information acquisition costs. Under some conditions, the regulator's priority is not to boost quality, but to reduce costly information acquisition, which can be accomplished by increasing the frequency of mandatory disclosure. This is particularly important when the manager is likely to become informed spontaneously/for free, even in the absence of information acquisition (i.e., when λ is high).

4 Empirical Implications

As Gigler et al. (2014) documents, the frequency of financial reporting has increased in the USA from annual, to semi annual to quarterly. It is likely that this regulatory pressure towards higher frequency will continue. It is thus important to better understand the economic benefits of more frequent disclosure. Some of this pressure is driven by the fact that information technologies have made it less expensive to acquire relevant information, and mandatory disclosure frequency must reflect this (see Blankespoor et al. 2018). Our analysis suggests that the effect of lowering the cost of acquiring information has ambiguous effects on the optimal frequency of mandatory disclosures.

The empirical literature has documented a negative relation between earnings guidance and mandatory 8K disclosures (see Noh et al. 2019). Our model yields a theoretical explanation for this phenomenon by linking mandatory disclosure and incentives for acquiring information. Mandatory disclosure regulations discourage information acquisition by reducing the availability of private information that can be released via voluntary channels. This is a consequence of comparing Propositions 1 and 4. Mandatory disclosure regulation limits the time horizon over which the manager can internalize the benefits of acquiring information.

Roychowdhury et al. (2019) review the empirical literature on the effect of disclosure on corporate investment decisions and broadly find that financial reporting affects investments through two channels: learning about future growth opportunities and reduced adverse selection/moral hazard frictions. They note that the empirical literature in these two streams has evolved somewhat independently. Our model combines several pertinent elements of both channels. In particular, the relation between financial reporting and investment depends on the extent of moral hazard and the incentives to acquire information.

Empirical Implication 1. *Investment is particularly sensitive to mandatory disclosure regulation for firms with relatively mild agency problems and inefficient monitoring technologies.*

Empirical Implication 1 is a result of Proposition 4 and in particular, the dual role of mandatory disclosure regulation in our model: inducing efficient investment and mitigating inefficient information acquisition. For instance, firms with relatively inefficient monitoring technologies are those characterized by a high c. In this case, because the firm does not have much incentive to engage in information acquisition in the first place (see Proposition 1), the regulator will choose T to induce a higher level of effort.

Jayaraman and Wu (2019) show that mandatory disclosure reduces investment efficiency because it crowds out information acquisition by traders and learning from prices. Our model shows that this story is somewhat incomplete: it ignores the interaction between learning and moral hazard.

Empirical Implication 2. *Mandatory disclosure regulation will reduce investment-q sensitivity for firms with relatively efficient monitoring technologies and severe agency problems. However, such regulation will increase investment efficiency for firms with comparatively severe agency problems, but less so for firms with relatively efficient monitoring technologies.*

Empirical Implication 2 is a consequence of Corollary 5 and our numerical results. Consider the disaggregated segment disclosures mandated under SFAS 131, which mandates that managers gather information and evaluate the performance of their business units. Empirical Implication 2 suggests that the decrease in investment efficiency following SFAS 131 documented by Jayaraman and Wu (2019) is concomitant with a decrease in inefficient information acquisition. Furthermore, Empirical Implication 2 predicts that firms that are characterized by more severe moral hazard problems (e.g., firms that are financially constrained) will tend to exhibit a positive investment-mandatory disclosure relationship. This effect, however, depends on the incentives for information acquisition. The role of mandatory disclosure in mitigating the effect of moral hazard is magnified for firms that do not find it difficult to acquire private information.

Empirically, Bourveau et al. (2022) document that regulators have demanded more information about human capital investments in firm disclosures. In our setting, this is equivalent to an increasing disclosure frequency. Proposition 4 suggests that such increased transparency can have adverse effects on the incentives to invest in human capital in the first place. Specifically, Corollary 5 shows that increased disclosure frequency can lead to less investment for firms or initiatives that are costly to implement and/or relatively easy to gather information about.

In general, our model can be used to understand how the information environment (e.g, cost of investment, rate of information arrivals, cost of information acquisition) affect the incremental informativeness of mandatory disclosures. The information environment affects the information available to the manager, as well as the possibility that the manager releases his information prior to the mandatory announcement date. It also has real effects, changing the level of investment chosen by the firm. These three channels interact to determine the amount of information contained in mandatory announcements.

5 Conclusion

In this paper, we study dynamic disclosure when firm quality is endogenous and information acquisition takes place over time. Managers can conceal bad news, and this option leads them to acquire too much private information, potentially reducing the firm's productivity.

We use our setting to understand the optimal frequency of mandatory disclosure when firm's have the ability to preempt mandatory announcements via voluntary disclosures. Mandatory disclosure is particularly effective as a means to reduce wasteful information acquisition costs. This is the case especially when effort is relatively insensitive to subsequent disclosures. However, when moral hazard is an important concern, the optimal regulation prescribes relatively infrequent mandatory disclosures, as a way to stimulate information acquisition and voluntary disclosure.

We find that as the cost of acquiring private information goes down, the effect on the frequency of mandatory disclosure is ambiguous. Sometimes, cheaper voluntary disclosures may lead to excessive information acquisition ex post, calling for more frequent mandatory disclosure as a means to reduce the market pressure for firms to acquire and disclose information. In our setting, mandatory disclosure is used to moderate market pressures.

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Appendices

A Proofs

The following results characterize the manager's value function U in the equilibrium characterized by equation (6), and will be useful in the subsequent analysis.

Lemma A1. *The manager's total expected payoff from following the information acquisition strategy* $\alpha_t = 1_{\{t\geq\tau\}}$ *can be written as*

$$U(p_0,\tilde{p}_0) = \sup_{\tau}\left\{\int_0^{\tau} e^{-(r+\lambda)t}\left(p_t^{\dagger} + \lambda\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^{\dagger})\right)\right)dt + e^{-(r+\lambda)\tau}U(p_\tau^{\dagger},\tilde{p}_0)\right\},$$

where $p^{\dagger}$ *is the solution to the ODE* $\dot{p} = -(\lambda+\Delta\hat{\alpha})\lambda p(1-p)$ *with* $p_0\in(0,1)$ *given admissible beliefs* $\hat{\alpha}$ *and conditional on a history of no disclosure.*

Proof. For all initial beliefs $p_0\in(0,1)$, the manager's total expected payoff at the outset can be written as

$$\begin{aligned}U(p_0,\tilde{p}_0) = &\underbrace{\int_0^{\tau} e^{-\lambda t}\lambda\left(\int_0^t e^{-rs}p_s^{\dagger}ds + e^{-rt}\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^{\dagger})\right)\right)dt}_{\text{News arrives before }\tau}\\ &+\underbrace{e^{-\lambda\tau}\left(\int_0^{\tau} e^{-rt}p_t^{\dagger}dt + e^{-r\tau}U(p_\tau^{\dagger},\tilde{p}_0)\right)}_{\text{News arrives after }\tau},\end{aligned} \tag{17}$$

for an arbitrary information acquisition time $\tau>0$. By the HJB equation (5),

$$U(p_\tau^{\dagger},\tilde{p}_0) = \int_\tau^{\infty} e^{-(r+\lambda+\Delta)(t-\tau)}\left(p_t^{\dagger} - c + (\lambda+\Delta)\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^{\dagger})\right)\right)dt.$$

Substituting this into equation (17),

$$\begin{aligned}U(p_0,\tilde{p}_0) = &\int_0^{\tau} e^{-(r+\lambda)t}\left(p_t^{\dagger} + \lambda\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^{\dagger})\right)\right)dt\\ &+\int_\tau^{\infty} e^{-(r+\lambda)t-\Delta(t-\tau)}\left(p_t^{\dagger} - c + (\lambda+\Delta)\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^{\dagger})\right)\right)dt,\end{aligned}$$

by changing the order of integration. □

Lemma A2. *The marginal impact of* p *on the manager's value satisfies*

$$U_p(p_t, \tilde{p}_0) = \frac{1}{p_t(1-p_t)} \int_t^{\infty} e^{-\int_t^s (r+(\lambda+\Delta\alpha_{t'}))dt'} p_s(1-p_s)\, ds,$$

Proof. By the HJB equation (5), the manager's value function at any time $t \geq 0$ can be written as

$$U(p_t, \tilde{p}_0) = \int_t^{\infty} e^{-r(s-t)} \left(p_s - c\alpha_s + (\lambda + \Delta\alpha_s) \left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\, U^0(p_s) - U(p_s, \tilde{p}_0) \right) \right) ds.$$

Note that for all times $s > t$,

$$\frac{dp_s}{dp_t} = e^{-\int_t^s (\lambda+\Delta\alpha_{t'})(1-2p_{t'})dt'} = \frac{p_s(1-p_s)}{p_t(1-p_t)}.$$

Differentiating $U(p_t, \tilde{p}_0)$ with respect to p_t,

$$U_p(p_t, \tilde{p}_0) = \int_t^{\infty} e^{-\int_t^s (r+(\lambda+\Delta\alpha_{t'})(1-2p_{t'}))dt'} \left(1 + (\lambda+\Delta\alpha_s) \left((1-\tilde{p}_0)\, U^{0'}(p_s) - U_p(p_s, \tilde{p}_0) \right) \right) ds,$$

where

$$U^{0'}(p_t) = \int_t^{\infty} e^{-r(t-s)} \left(\frac{p_s(1-p_s)}{p_t(1-p_t)} \right) ds.$$

This implies that $U_p(p, \tilde{p}_0)$ satisfies

$$rU_p(p, \tilde{p}_0) = 1 + \dot{p} U_{pp}(p, \tilde{p}_0) + (\lambda + \Delta\alpha) \left((1-\tilde{p}_0)\, U^{0'}(p) - U_p(p, \tilde{p}_0) \right).$$

Solving for $U_p(p_t, \tilde{p}_0)$,

$$\begin{aligned}
U_p(p_t, \tilde{p}_0) &= \int_t^{\infty} e^{-\int_t^s (r+2(\lambda+\Delta\alpha_{t'})(1-p_{t'}))dt'} \left(1 + (\lambda+\Delta\alpha_s)(1-\tilde{p}_0)\, U^{0'}(p_s) \right) ds \\
&= \int_t^{\infty} e^{-r(s-t)} \left(\frac{p_0 + e^{\int_0^t (\lambda+\Delta\alpha_s)ds}(1-p_0)}{p_0 + e^{-\int_0^s (\lambda+\Delta\alpha_t)dt}(1-p_0)} \right)^2 ds \\
&= \int_t^{\infty} e^{-\int_t^s (r+\lambda+\Delta\alpha_{t'})dt'} \left(\frac{p_s(1-p_s)}{p_t(1-p_t)} \right) ds,
\end{aligned}$$

where the second and third equalities follow since $p_s = \frac{p_t}{p_t + (1-p_t)e^{\int_t^s (\lambda+\Delta\alpha_{t'})dt'}}$ for any times $s > t$. □

Proof of Proposition 1

Proof. To pin down the information acquisition time τ, we need to rule out profitable deviations. In particular, at time τ the manager must be indifferent between acquiring information and not. By equation (6), this means that τ must satisfy

$$\tilde{p}_0 \left(\frac{1}{r} - U^0(p_\tau) \right) = \frac{c}{\Delta} + \left(U(p_\tau, \tilde{p}_0) - U^0(p_\tau) \right). \tag{18}$$

By the HJB equation (5),[10] $U(p_\tau, \tilde{p}_0)$ can be written as

$$\begin{aligned} U(p_\tau, \tilde{p}_0) &= \int_\tau^\infty e^{-(r+\lambda+\Delta)(t-\tau)} \left(p_t^\dagger - c + (\lambda + \Delta) \left(\frac{\tilde{p}_0}{r} + (1 - \tilde{p}_0) U^0(p_t^\dagger) \right) \right) dt \\ &= \frac{1}{r + \lambda + \Delta} \left((\lambda + \Delta) \frac{\tilde{p}_0}{r} - c \right) + \tilde{p}_0 \int_\tau^\infty e^{-(r+\lambda+\Delta)(t-\tau)} p_t^\dagger dt + (1 - \tilde{p}_0) U^0(p_\tau), \end{aligned}$$

where the second equality follows by changing the order of integration. Therefore, equation (18) can be written as

$$\int_\tau^\infty e^{-(r+\lambda+\Delta)(t-\tau)} p_t^\dagger dt = \frac{1}{r + \lambda + \Delta} - \frac{c}{\tilde{p}_0} \left(\frac{1}{\Delta} - \frac{1}{r + \lambda + \Delta} \right).$$

The solution to this integral equation is

$$p_\tau = 1 - \frac{c(r + \lambda)}{\Delta \tilde{p}_0},$$

for any $\tau \geq 0$,[11] as desired.

For $p \in (0, 1)$, let $D(p, \tilde{p}_0)$ be the difference in value to the manager between remaining uninformed and withholding bad news:

$$D(p, \tilde{p}_0) = U(p, \tilde{p}_0) - U^0(p). \tag{19}$$

The following lemma verifies the existence of a threshold equilibrium.

Lemma A3. *$U^0(p)$ and $D(p, \tilde{p}_0) + \tilde{p}_0 U^0(p)$ are increasing in reputation p.*

[10] See Theorem 32.10 in Davis (1993).
[11] See Polyanin and Manzhirov (2008), pg. 118.

Proof. $U^0(p)$ and $D(p,\tilde{p}_0)$ satisfy the HJB equations

$$rU^0(p) = p - (\lambda + \Delta\hat{\alpha})\, p\,(1-p)\, U^{0'}(p), \tag{20}$$

$$\begin{aligned} rD(p,\tilde{p}_0) &= -c\alpha - (\lambda+\Delta\hat{\alpha})\, p\,(1-p)\, D_p(p,\tilde{p}_0) \\ &\quad + (\lambda+\Delta\alpha)\left(\tilde{p}_0\left(\frac{1}{r} - U^0(p)\right) - D(p,\tilde{p}_0)\right), \end{aligned} \tag{21}$$

respectively. Denote $\mathcal{U}(p,\tilde{p}_0) = D(p,\tilde{p}_0) + \tilde{p}_0 U^0(p)$ for all p. By equations (20) and (21), $\mathcal{U}(p,\tilde{p}_0)$ satisfies the HJB equation

$$r\mathcal{U}(p,\tilde{p}_0) = -c\alpha + \tilde{p}_0 p - (\lambda+\Delta\hat{\alpha})\, p\,(1-p)\,\mathcal{U}_p(p,\tilde{p}_0) + (\lambda+\Delta\alpha)\left(\frac{\tilde{p}_0}{r} - \mathcal{U}(p,\tilde{p}_0)\right). \tag{22}$$

Differentiating the HJB equations (20) and (22),

$$rU^{0'}(p) = 1 - (\lambda+\Delta\hat{\alpha})\left((1-2p)\, U^{0'}(p) + p\,(1-p)\, U^{0''}(p)\right),$$

$$r\mathcal{U}_p(p,\tilde{p}_0) = \tilde{p}_0 - (\lambda+\Delta\hat{\alpha})\left((1-2p)\,\mathcal{U}_p(p,\tilde{p}_0) + p\,(1-p)\,\mathcal{U}_{pp}(p,\tilde{p}_0)\right) - (\lambda+\Delta\alpha)\,\mathcal{U}_p(p,\tilde{p}_0).$$

Solving these equations yields

$$\begin{aligned} U^{0'}(p_t) &= \int_t^\infty e^{-\int_t^s (r+(\lambda+\Delta\hat{\alpha}_u)(1-p_u))du} ds = \int_t^\infty e^{-r(t-s)} \frac{p_s\,(1-p_s)}{p_t\,(1-p_t)} ds > 0, \\ \mathcal{U}_p(p_0,\tilde{p}_0) &= \int_t^\infty e^{-\int_t^s (r+\lambda+\Delta\alpha_u+(\lambda+\Delta\hat{\alpha}_u)(1-2p_u))du} \tilde{p}_0 ds \\ &= \tilde{p}_0 \int_t^\infty e^{-\int_t^s (r+\lambda+\Delta\alpha_u)du} \frac{p_s\,(1-p_s)}{p_t\,(1-p_t)} ds \geq 0, \end{aligned}$$

where the inequalities follow by construction. □

Since p has negative drift, $\alpha_t = 1$ for $p_t < p_\tau$ and $\alpha_t = 0$ for $p_t \geq p_\tau$. Because p^*_τ is a reputational threshold, □

Proof of Corollary 1

Proof. If information acquisition is observable by the market, then the manager's HJB equation becomes

$$\begin{aligned} rU(p,\tilde{p}_0) = \sup_\alpha\; & p - c\alpha - (\lambda+\Delta\alpha)\, p\,(1-p)\, U_p(p,\tilde{p}_0) \\ & + (\lambda+\Delta\alpha)\left(\tilde{p}_0\left(\frac{1}{r} - U(p,\tilde{p}_0)\right) - (1-\tilde{p}_0)\left(U(p,\tilde{p}_0) - U^0(p)\right)\right), \end{aligned} \tag{23}$$

subject to the law of motion

$$\dot{p}_t = -\left(\lambda + \Delta\alpha_t\right) p_t \left(1 - p_t\right).$$

Differentiating equation (23) with respect to α, the optimal information acquisition strategy in this case satisfies

$$\alpha_t^{obs} = \begin{cases} 1 & \text{if } \tilde{p}_0 \left(\frac{1}{r} - U(p_t, \tilde{p}_0)\right) > \frac{c}{\Delta} + (1 - \tilde{p}_0)\left(U(p_t, \tilde{p}_0) - U^0(p_t)\right) + p_t\left(1 - p_t\right) U_p(p_t, \tilde{p}_0), \\ 0 & \text{otherwise.} \end{cases}$$

Let p_τ^{obs} be the reputation threshold level satisfying the indifference condition

$$\tilde{p}_0 \left(\frac{1}{r} - U(p_\tau^{obs}, \tilde{p}_0)\right) = \frac{c}{\Delta} + (1 - \tilde{p}_0)\left(U(p_\tau^{obs}, \tilde{p}_0) - U^0(p_\tau^{obs})\right) + p_\tau^{obs}\left(1 - p_\tau^{obs}\right) U_p(p_\tau^{obs}, \tilde{p}_0),$$

which can be written as

$$U(p_\tau^{obs}, \tilde{p}_0) - (1 - \tilde{p}_0)\, U^0(p_\tau^{obs}) = \frac{\tilde{p}_0}{r} - \frac{c}{\Delta} - p_\tau^{obs}\left(1 - p_\tau^{obs}\right) U_p(p_\tau^{obs}, \tilde{p}_0), \tag{24}$$

and let τ^{obs} be the corresponding information acquisition time.

By Lemma A2 and rearranging terms,

$$\begin{aligned} U_p(p_\tau, \tilde{p}_0) &= \frac{1}{p_\tau^{obs}\left(1 - p_\tau^{obs}\right)} \left(\tilde{p}_0 \int_{\tau^{obs}}^{\infty} e^{-(r+\lambda+\Delta)\left(t - \tau^{obs}\right)} p_t \left(1 - p_t\right) dt \right. \\ &\qquad \left. + (1 - \tilde{p}_0) \int_{\tau^{obs}}^{\infty} e^{-r\left(t - \tau^{obs}\right)} p_t \left(1 - p_t\right) dt \right) \\ &= \frac{1}{p_\tau^{obs}\left(1 - p_\tau^{obs}\right)} (1 - \tilde{p}_0) \int_{\tau^{obs}}^{\infty} e^{-r\left(t - \tau^{obs}\right)} \left(1 + \left(\frac{p_0 - p_\tau^{obs}}{(1 - p_0)\, p_\tau^{obs}}\right) p_t\right) p_t dt. \end{aligned}$$

The indifference condition (24) can therefore be written as

$$\begin{aligned} &\tilde{p}_0 \int_{\tau^{obs}}^{\infty} e^{-(r+\lambda+\Delta)\left(t - \tau^{obs}\right)} p_t \left(2 - p_t\right) dt + (1 - \tilde{p}_0) \int_{\tau^{obs}}^{\infty} e^{-r\left(t - \tau^{obs}\right)} p_t \left(1 - p_t\right) dt \\ &\quad + \frac{1}{r + \lambda + \Delta} \left(\frac{(\lambda + \Delta)\tilde{p}_0}{r} - c\right) = \frac{\tilde{p}_0}{r} - \frac{c}{\Delta}, \end{aligned}$$

or equivalently,

$$\int_{\tau^{obs}}^{\infty} e^{-(r+\lambda+\Delta)\left(t-\tau^{obs}\right)} p_t\,(2-p_t)\,dt + \frac{(1-\tilde{p}_0)\,p_0}{\tilde{p}_0\,(1-p_0)} e^{-\lambda\tau^{obs}} \int_{\tau^{obs}}^{\infty} e^{-(r+\lambda+\Delta)\left(t-\tau^{obs}\right)}\,(1-p_t)^2\,dt$$
$$= \frac{p_\tau^{unobs}}{r+\lambda+\Delta}, \tag{25}$$

where $p_\tau^{unobs} = 1 - \frac{c(r+\lambda)}{\Delta\tilde{p}_0}$ is the reputation threshold under unobservable information acquisition. Setting $p_0 = \tilde{p}_0$ in equation (18), the left-hand side of equation (25) is weakly less than $\frac{1+p_\tau^\circ}{p_\tau^\circ}\int_{\tau^{obs}}^{\infty} e^{-(r+\lambda+\Delta)(t-\tau^{obs})}dt$ for some $p_\tau^\circ \in (0,1]$. Integrating and equating this expression with the right-hand side of equation (25), $p_\tau^\circ = -\frac{1}{1-p_\tau^{unobs}} \leq 0$, since $p_\tau^{unobs} \in [0,1]$ by Proposition 1. By Lemma A3, the left-hand side of equation (25) is increasing in p. Therefore, by the inverse function theorem, $p_\tau^{obs} \leq \max\{0, p_\tau^\circ\}$, meaning $p_\tau^{obs} = 0$. □

Proof of Proposition 2

Proof. The following technical result is a consequence of the equilibrium information acquisition strategy and will be useful for the subsequent analysis.

Lemma A4. *Let $p_\tau^*(p_0,\tilde{p}_0)$ denote the equilibrium information acquisition threshold derived in Proposition 1. Then, in the equilibrium characterized by an information acquisition strategy of the form* (3),

$$U^0(p_\tau^*(p_0,\tilde{p}_0)) = \frac{p_\tau^*(p_0,\tilde{p}_0)}{r+\lambda+\Delta},$$

where

$$\tau^*(p_0,\tilde{p}_0) = -\frac{1}{\lambda}\ln\left(\frac{(1-p_0)\,p_\tau^*(p_0,\tilde{p}_0)}{p_0\,(1-p_\tau^*(p_0,\tilde{p}_0))}\right).$$

Proof. By the HJB equation (5), the manager's value at the threshold information acquisition time τ can be written as

$$U(p_\tau^*,\tilde{p}_0) = \int_{\tau^*}^{\infty} e^{-(r+\lambda+\Delta)(t-\tau^*)}\left(p_t^\dagger - c + (\lambda+\Delta)\left(\frac{\tilde{p}_0}{r} + (1-\tilde{p}_0)\,U^0(p_t^\dagger)\right)\right)dt$$
$$= \frac{1}{r+\lambda+\Delta}\left(\frac{(\lambda+\Delta)\,\tilde{p}_0}{r} - c\right) + \tilde{p}_0\int_{\tau^*}^{\infty} e^{-(r+\lambda+\Delta)(t-\tau^*)} p_t^\dagger dt + (1-\tilde{p}_0)\,U^0(p_\tau^*),$$

where the second equality follows from changing the order of integration. Because $p_\tau^*(p_0,\tilde{p}_0) = 1 - \frac{c(r+\lambda)}{\Delta\tilde{p}_0}$, the indifference condition for information acquisition can be written as

$$\int_{\tau^*(p_0,\tilde{p}_0)}^{\infty} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,\tilde{p}_0))} p_t^\dagger dt = \frac{p_\tau^*(p_0,\tilde{p}_0)}{r+\lambda+\Delta}. \tag{26}$$

For $t \geq \tau^*(p_0, \tilde{p}_0)$, because $\alpha_t = 1$ and $t - \tau^*(p_0, \tilde{p}_0)$ is the time that it takes for beliefs to reach $p_t^\dagger$ starting from $p_\tau^*(p_0, \tilde{p}_0)$, given $\tau^*(p_0, \tilde{p}_0)$, $e^{-(\lambda+\Delta)(t-\tau^*(p_0,\tilde{p}_0))} = \frac{(1-p_\tau^*(p_0,\tilde{p}_0))p_t^\dagger}{p_\tau^*(p_0,\tilde{p}_0)(1-p_t^\dagger)}$ for any $t > \tau^*(p_0, \tilde{p}_0)$, where $p_t^\dagger < p_\tau^*(p_0, \tilde{p}_0)$. Therefore,

$$\int_{\tau^*(p_0,\tilde{p}_0)}^{\infty} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,\tilde{p}_0))} \left(1 - p_t^\dagger\right) dt = \frac{1 - p_\tau^*(p_0, \tilde{p}_0)}{p_\tau^*(p_0, \tilde{p}_0)} \int_{\tau^*(p_0,\tilde{p}_0)}^{\infty} e^{-r(t-\tau^*(p_0,\tilde{p}_0))} p_t^\dagger dt. \quad (27)$$

Equations (26) and (27) yield the result. □

We now continue the proof of Proposition 2. We first consider the manager's choice of information acquisition. When the manager can endogenously choose the level of productive effort, the indifference condition for information acquisition becomes

$$\Delta A \left(\frac{1}{r} - U(p_\tau)\right) = c + \Delta\left(1 - A\right)\left(U(p_\tau) - U^0(p_\tau)\right). \quad (28)$$

The indifference condition (28) can be written as an integral equation in terms of p. Solving this integral equation as in the proof of Proposition 1 yields the result.

We next consider the manager's choice of productive effort. By Lemma A1 and changing the order of integration, the manager's total expected value can be written as

$$\begin{aligned} U(p_0) = &\int_0^\tau \left\{ e^{-(r+\lambda)t}\left(p_t^\dagger + \lambda\frac{A}{r}\right) + (1-A)\, e^{-rt}\left(1 - e^{-\lambda t}\right) p_t^\dagger \right\} dt \\ &+ e^{-(r+\lambda)\tau} \int_\tau^\infty \left\{ e^{-(r+\lambda+\Delta)(t-\tau)} \left(p_t^\dagger - c + (\lambda+\Delta)\frac{A}{r}\right) \right. \\ &\qquad\qquad \left. + (1-A) e^{-r(t-\tau)}\left(1 - e^{-(\lambda+\Delta)(t-\tau)}\right) p_t^\dagger \right\} dt - \frac{\kappa}{2}A^2. \end{aligned}$$

By the envelope theorem, the first-order condition with respect to A is

$$\begin{aligned} \kappa A &= \frac{\lambda\left(1 - e^{-(r+\lambda)\tau^*(p_0,A)}\right)}{r\,(r+\lambda)} - \int_0^{\tau^*(p_0,A)} e^{-rt}\left(1 - e^{-\lambda t}\right) p_t^\dagger dt \\ &\quad + e^{-(r+\lambda)\tau^*(p_0,A)} \left(\frac{\lambda+\Delta}{r\,(r+\lambda+\Delta)} + \int_{\tau^*(p_0,A)}^\infty e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A))} p_t^\dagger dt - U^0(p_\tau^*(p_0,A)) \right) \\ &= \frac{\lambda\left(1 - e^{-(r+\lambda)\tau^*(p_0,A)}\right)}{r\,(r+\lambda)} - \int_0^{\tau^*(p_0,A)} e^{-rt}\left(1 - e^{-\lambda t}\right) p_t^\dagger dt + \frac{(\lambda+\Delta)\, e^{-(r+\lambda)\tau^*(p_0,A)}}{r\,(r+\lambda+\Delta)}, \end{aligned}$$

where $\tau^*(p_0, A) = -\frac{1}{\lambda}\log\left(\frac{(1-p_0)p_\tau^*(p_0,A)}{p_0(1-p_\tau^*(p_0,A))}\right)$, the first equality follows from changing the order of integration and the second equality follows from Lemma A4. Setting $\psi(p_0) = \frac{\lambda}{r(r+\lambda)} -$

$\int_0^\infty e^{-rt}(1-e^{-\lambda t})p_t^\dagger dt$, straightforward integration yields

$$\psi(p_0) = \frac{1}{r} - \frac{1}{(r+\lambda)(1-p_0)} G\left(1, \frac{r+\lambda}{\lambda}, 2+\frac{r}{\lambda}, -\frac{p_0}{1-p_0}\right),$$

where G is the Gauss hypergeometric function. Evaluating at $p_0 = A$ yields the condition for the equilibrium level of A. Differentiating ψ with respect to p_0, it follows that

$$\psi'(p_0) = \frac{1}{p_0}\left(\frac{1}{r} - \psi(p_0)\left(1 - \frac{r}{\lambda(1-p_0)}\right)\right) < 0,$$

by the properties of the Gauss hypergeometric function (Abramowitz and Stegun 1964). □

Proof of Corollary 2

Proof. Let A^* be the equilibrium level of productive effort from Proposition 2. Define the function $\Upsilon : [0, \overline{A}] \times \mathbb{R}_+ \to \mathbb{R}$ as

$$\Upsilon(A, \Delta) = \kappa A - \frac{\omega(A, A)}{r+\lambda+\Delta} - \psi(A),$$

where the functions ω and ψ are defined in Proposition 2. Let $\tau^*(A)$ be the time it takes for reputation to hit the information acquisition threshold $p_\tau^*(A)$ defined by equation (8). By equation (9), A^* is the solution to $\Upsilon(A, \Delta) = 0$. Differentiating Υ with respect to Δ,

$$\frac{\partial \Upsilon(A, \Delta)}{\partial \Delta} = \frac{e^{-(r+\lambda)\tau^*(A)}}{(r+\lambda+\Delta)^2}\left(1 + \frac{r+\lambda+\Delta}{\lambda p_\tau^*(A)}\right) > 0.$$

By Theorem 1 in Milgrom and Roberts (1994), this is sufficient to ensure that A^* is increasing in Δ. □

Proof of Proposition 4

Proof. Given effort $A \in (0,1)$, the manager's value function follows a regenerative process, meaning it satisfies the HJB equation

$$\begin{cases} (r+\lambda+\Delta\alpha)\, U(p^\dagger, T) = p^\dagger - c\alpha + (\lambda+\Delta\alpha)\left(\frac{A}{r} + (1-A)\, U^0(p^\dagger, T)\right), \\ U(p_T^\dagger, 0) = U(p_0, T). \end{cases} \tag{29}$$

Equation (29) implies that the optimal information acquisition strategy is

$$\alpha_t^* = \begin{cases} 1 & \text{if } A\left(\frac{1}{r} - U(p_t^\dagger, T)\right) > c/\Delta + (1-A)\left(U(p_t^\dagger, T) - U^0(p_t^\dagger, T)\right), \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the optimal information acquisition threshold $p_\tau^* \in [0,1]$ must satisfy the indifference condition

$$U(p_\tau^*, T) - (1-A)\, U^0(p_\tau^*, T) = \frac{A}{r} - \frac{c}{\Delta}. \tag{30}$$

By the definitions of $U(p_\tau, T)$ and $U^0(p_\tau, T)$, condition (30) can be written as

$$\int_{\tau^*}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*)} p_t^\dagger dt = \frac{1}{r} - \frac{c}{\Delta A} - \frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*)}}{r+\lambda+\Delta}\left(\frac{\lambda+\Delta}{r} - \frac{c}{A}\right),$$

which is equivalent to

$$\begin{aligned} &\int_{\tau^*}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*)}\left(p_t^\dagger - \frac{r+\lambda+\Delta}{1-e^{-(r+\lambda+\Delta)(T-\tau^*)}}\left(\frac{1}{r} - \frac{c}{\Delta A}\right)\right) dt \\ &= -\frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*)}}{r+\lambda+\Delta}\left(\frac{\lambda+\Delta}{r} - \frac{c}{A}\right) \\ &= \int_{T}^{\tau^*} e^{(r+\lambda+\Delta)(\tau^*-t)}\left(\frac{r+\lambda+\Delta}{1-e^{-(r+\lambda+\Delta)(T-\tau^*)}}\left(\frac{1}{r} - \frac{c}{\Delta A}\right) - p_t\right) dt. \end{aligned} \tag{31}$$

Equation (31) is a Volterra integral equation of the first kind. Differentiating both sides by τ^*, equation (31) reduces to a Volterra integral equation of the second kind:

$$\frac{r+\lambda+\Delta}{1-e^{-(r+\lambda+\Delta)(T-\tau^*)}}\left(\frac{1}{r} - \frac{c}{\Delta A}\right) - p_\tau = \frac{\lambda+\Delta}{r} - \frac{c}{A},$$

which can be simplified to

$$p_\tau^* - (r+\lambda+\Delta)\int_{\tau^*}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*)} p_t dt = e^{-(r+\lambda+\Delta)(T-\tau^*)}\left(\frac{\lambda+\Delta}{r} - \frac{c}{A}\right).$$

By Polyanin and Manzhirov (2008), the solution to this integral equation is $p_\tau^* = \frac{\lambda+\Delta}{r} - \frac{c}{A}$. The belief thresholds ϕ^- and ϕ^+ follow because $p_\tau^*(A) = \max\{0, \min\{p_\tau^*(A), 1\}\}$ by construction.

We now consider the manager's choice of A under the mandatory disclosure regime. The following technical result is the concomitant result to Lemma A4 with a mandatory disclosure time T.

Lemma A5. *Let $p^*_\tau(A,T)$ denote the optimal information acquisition threshold derived in Proposition 4 with mandatory disclosure frequency T and $\tau^*(p_0,A,T)$ the corresponding information acquisition time. Then,*

$$U^0(p^*_\tau(A,T),T) = \mathscr{L}^*_\tau(A,T)\left\{\frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*(p_0,A,T))}}{r+\lambda+\Delta}\left(1+\frac{\lambda+\Delta}{r}-\frac{c}{A}\right)-\left(\frac{1}{r}-\frac{c}{\Delta A}\right)\right\},$$

where $\mathscr{L}^*_\tau(A,T) = \frac{p^*_\tau(A,T)}{1-p^*_\tau(A,T)}$ *and* $\tau^*(p_0,A,T) = -\frac{1}{\lambda}\ln\left(\frac{(1-p_0)p^*_\tau(A,T)}{p_0(1-p^*_\tau(A,T))}\right)$.

Proof. By Proposition 4, the indifference condition (30) can be written as

$$\begin{aligned}&\int_{\tau^*(p_0,A,T)}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A,T))}p^\dagger_t dt\\ &\quad= \frac{1}{r}-\frac{c}{\Delta A}-\int_{\tau^*(p_0,A,T)}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A,T))}\left(-\frac{c}{A}+\frac{\lambda+\Delta}{r}\right)dt.\end{aligned}$$

Straightforward integration yields

$$\begin{aligned}&\int_{\tau^*(p_0,A,T)}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A,T))}\left(1-p^\dagger_t\right)dt\\ &\quad= \frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*(p_0,A,T))}}{r+\lambda+\Delta}\left(1+\frac{\lambda+\Delta}{r}-\frac{c}{A}\right)-\left(\frac{1}{r}-\frac{c}{\Delta A}\right)\\ &\quad= \mathscr{L}^*_\tau(A,T)\int_{\tau^*(p_0,T)}^{T} e^{-r(t-\tau^*(p_0,A,T))}p^\dagger_t dt,\end{aligned}$$

where the last equality follows by the definition of $\tau^*(p_0,A,T)$. Matching and rearranging terms,

$$\int_{\tau^*(p_0,A,T)}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A,T))}p_t dt = \frac{1}{r}-\frac{c}{\Delta A}-\frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*(p_0,A,T))}}{r+\lambda+\Delta}\left(\frac{\lambda+\Delta}{r}-\frac{c}{A}\right),$$

which implies that

$$\begin{aligned}&\int_{\tau^*(p_0,A,T)}^{T} e^{-r(t-\tau^*(p_0,A,T))}p^\dagger_t dt-\int_{\tau^*(p_0,A,T)}^{T} e^{-(r+\lambda+\Delta)(t-\tau^*(p_0,A,T))}p^\dagger_t dt\\ &\quad= \frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*(p_0,A,T))}}{(r+\lambda+\Delta)\left(1-p^*_\tau(A,T)\right)}\left(p^*_\tau(A,T)+\frac{\lambda+\Delta}{r}-\frac{c}{A}\right)-\frac{1}{1-p^*_\tau(A,T)}\left(\frac{1}{r}-\frac{c}{\Delta A}\right).\end{aligned}$$

□

Differentiating the manager's value function $U(p_0,T)$ with respect to A using the envelope theorem yields the first-order condition

$$\int_0^{\tau^*(p_0,A,T)} e^{-rt}\left(e^{-\lambda t}\frac{\lambda}{r}-\left(1-e^{-\lambda t}\right)p_t^\dagger\right)dt+e^{-(r+\lambda)\tau^*(p_0,A,T)}\mathcal{W}(p_0,A,T)$$
$$=\left(1-e^{-(r+\lambda)T-\Delta(T-\tau^*(p_0,A,T))}\right)\kappa A, \tag{32}$$

where

$$\mathcal{W}(p_0,A,T)=\int_{\tau^*(p_0,A,T)}^T e^{-r(t-\tau^*(p_0,A,T))}\left(e^{-(\lambda+\Delta)(t-\tau^*(p_0,A,T))}\frac{\lambda+\Delta}{r}-\left(1-e^{-(\lambda+\Delta)(t-\tau^*(p_0,A,T))}\right)p_t^\dagger\right)dt.$$

By value-matching using Lemma A5,

$$\mathcal{W}(p_0,A,T)=\frac{1-e^{-(r+\lambda+\Delta)(T-\tau^*(p_0,A,T))}}{r+\lambda+\Delta}\left(\frac{\lambda+\Delta}{r}-\frac{2p_\tau^*(A,T)}{1-p_\tau^*(A,T)}\right)+\frac{1}{1-p_\tau^*(A,T)}\left(\frac{1}{r}-\frac{c}{\Delta A}\right),$$

where

$$e^{-\Delta(T-\tau^*(p_0,A,T))}=\left(\frac{1-p_\tau^*(A,T)}{p_\tau^*(A,T)}\left(\frac{p_T}{1-p_T}\right)\right)^{\frac{\Delta}{\lambda+\Delta}}.$$

Evaluating at $p_0=A$, setting $\tau^*(A,T)=\tau^*(A,A,T)$, and rearranging terms gives the result. □

Proof of Proposition 3

Proof. Because the mandatory disclosure time is chosen to maximize expected firm value, it solves the problem

$$\max_T E^{\hat\alpha,\hat A}\left[\left(\frac{1}{1-e^{-\int_0^T(r+\lambda+\Delta\hat\alpha_t)dt}}\right)V^{\hat\alpha,\hat A}(p_0,T)-\frac{\kappa}{2}\hat A^2\,\middle|\,p_0=\hat A\right], \tag{33}$$

where

$$V^{\hat\alpha,\hat A}(p_0,T)=\int_0^T e^{-\int_0^t(r+\lambda+\Delta\hat\alpha_s)ds}\left(p_t-c\hat\alpha_t+(\lambda+\Delta\hat\alpha_t)\left(\frac{A}{r}+(1-A)\,U^0(p_t,T)\right)\right)dt.$$

The first-order condition of problem (33), evaluated at $p_0=\hat A=A^*$ and $\hat\alpha=\alpha^*$, is

$$-\,e^{-(r+\lambda)T-\Delta(T-\tau^*)}\,(r+\lambda+\Delta)\,V^{\alpha^*,A^*}(A^*,T)$$
$$+\left(1-e^{-(r+\lambda)T-\Delta(T-\tau^*)}\right)\frac{\partial V^{\alpha^*,A^*}(A^*,T)}{\partial T}=0. \tag{34}$$

Let τ^* denote the optimal stopping time such that $\alpha_t^* = 1_{\{t \geq \tau^*\}}$. Then, define

$$
\begin{aligned}
V^{\hat{\alpha}^*,\hat{A}^*}(A^*,T) &= \int_0^{\tau^*} \left\{ e^{-(r+\lambda)t} \left(p_t + \lambda \frac{A^*}{r} \right) + (1-A^*)\, e^{-rt} \left(1 - e^{-\lambda t} \right) p_t \right\} dt + e^{-(r+\lambda)\tau^*} \mathcal{U}(p_\tau^*, T) \\
&= (1-A) \left(\int_0^{\tau} e^{-rt} \left(1 - e^{-\lambda t} \right) p_t dt + e^{-(r+\lambda)\tau} \int_\tau^T e^{-r(t-\tau)} \left(1 - e^{-(\lambda+\Delta)(t-\tau)} \right) p_t dt \right) \\
&\quad + \int_0^{\tau} e^{-(r+\lambda)t} \left(p_t + \lambda \frac{A}{r} \right) dt + e^{-(r+\lambda)\tau} \int_\tau^T e^{-(r+\lambda+\Delta)(t-\tau)} \left(p_t - c + (\lambda+\Delta) \frac{A}{r} \right) dt,
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{U}(p_\tau^*, T) &= \int_{\tau^*}^T e^{-(r+\lambda+\Delta)(t-\tau^*)} \left(p_t - c + (\lambda+\Delta) \left(A \frac{1}{r} + (1-A) U^0(p_t, T) \right) \right) dt \\
&= \frac{A}{r} - \frac{c}{\Delta} + (1-A)\, U^0(p_\tau^*, T).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
V &= -\left(1 - \mathcal{E}(T)\right) \kappa A \left(1-A\right) + \int_0^{\tau} e^{-(r+\lambda)t} \left(p_t + \frac{\lambda}{r} \right) dt \\
&\quad + e^{-(r+\lambda)\tau} \left\{ \frac{1}{r} - \frac{c}{\Delta A} + \frac{1 - e^{-(r+\lambda+\Delta)(t-\tau)}}{r+\lambda+\Delta} \left(\frac{1-A}{A} \right) c \right\} \\
&= -\left(1 - \mathcal{E}(T)\right) \kappa A \left(1-A\right) + \int_0^{\tau} e^{-(r+\lambda)t} \left(p_t + \frac{\lambda}{r} \right) dt \\
&\quad + e^{-(r+\lambda)\tau} \left(\frac{1}{r} - \frac{c}{\Delta A} + \frac{1}{r+\lambda+\Delta} \left(\frac{1-A}{A} \right) c \right) - \frac{\mathcal{E}(T)}{r+\lambda+\Delta} \left(\frac{1-A}{A} \right) c.
\end{aligned}
$$

Furthermore, differentiating $V^{\hat{\alpha},\hat{A}}(p_0, T)$ with respect to T yields

$$
\begin{aligned}
\frac{\partial V^{\hat{\alpha},\hat{A}}(p_0,T)}{\partial T} &= e^{-(r+\lambda)T - \Delta(T-\tau)} \left(p_T - c + A \frac{\lambda+\Delta}{r} + (1-A) \left(\frac{p_\tau - p_T}{p_T(1-p_\tau)} \right) p_T \right) \\
&= e^{-(r+\lambda)T - \Delta(T-\tau)} \left(A \left(1 + \frac{\lambda+\Delta}{r} \right) + (1-A) \left(\frac{p_\tau - p_T}{p_T(1-p_\tau)} \right) p_T - c \right).
\end{aligned}
$$

Evaluating this at $p_0 = A = A^*$ and $\tau = \tau^*$ and rearranging terms,

$$
\frac{\partial V^{\alpha^*,A^*}(A^*,T)}{\partial T} = e^{-(r+\lambda)T - \Delta(T-\tau^*)} \left(A^* \left(1 + \frac{\lambda+\Delta}{r} \right) + (1-A^*) \left(\frac{p_\tau^* - A^*}{1 - p_\tau^*} \right) - c \right).
$$

Given τ^*, define the random variable $\mathcal{E}(T) = e^{-(r+\lambda)T-\Delta(T-\tau^*)}$. Then, the first-order condition (34) can be written in terms of $\mathcal{E}(T)$:

$$
\begin{aligned}
&-\mathcal{E}(T)(r+\lambda+\Delta)V^{\alpha^*,A^*}(A^*,T^*) + (1-\mathcal{E}(T))\,\mathcal{E}(T)\left(A^*\left(1+\frac{\lambda+\Delta}{r}+\frac{p^*_\tau - A^*}{p^*_\tau}\right)-c\right)\\
&=\mathcal{E}(T)\left\{(r+\lambda+\Delta)\left(\int_0^\tau e^{-(r+\lambda)t}\left(p_t+\frac{\lambda}{r}\right)+e^{-(r+\lambda)\tau}\left(\frac{1}{r}-\frac{c}{\Delta A}\right)\right)\right.\\
&\qquad\left. -\Xi(A^*)+\Xi(A^*)\mathcal{E}(T)\right\} = 0, \qquad (35)
\end{aligned}
$$

where

$$\Xi(A) = (r+\lambda+\Delta)\kappa A(1-A) - \frac{c}{A} + A\left(1+\frac{\lambda+\Delta}{r}\right) + (1-A)\left(\frac{p_\tau - A}{1-p_\tau}\right).$$

Solving the quadratic equation (35) for $\mathcal{E}(T)$,

$$\mathcal{E}(T) = 1-\frac{X(A^*)}{\Xi(A^*)} = 1-\frac{1}{\kappa A(1-A)+Y(A^*)}\left(\int_0^\tau e^{-(r+\lambda)t}\left(p_t+\frac{\lambda}{r}\right)+e^{-(r+\lambda)\tau}\left(\frac{1}{r}-\frac{c}{\Delta A}\right)\right),$$

where

$$
\begin{aligned}
X(A) &= (r+\lambda+\Delta)\left(\int_0^\tau e^{-(r+\lambda)t}\left(p_t+\frac{\lambda}{r}\right)+e^{-(r+\lambda)\tau}\left(\frac{1}{r}-\frac{c}{\Delta A}\right)\right)\\
Y(A) &= \frac{1}{r+\lambda+\Delta}\left(A\left(1+\frac{\lambda+\Delta}{r}\right)+(1-A)\left(\frac{p_\tau - A}{1-p_\tau}\right)-\frac{c}{A}\right)\\
&= \frac{1}{r+\lambda+\Delta}\left\{A\left(A+\frac{\lambda+\Delta}{r}+(1-A)\left(1-e^{-\lambda\tau^*(A)}\right)\right)-\frac{c}{A}\right\}.
\end{aligned}
$$

Therefore,

$$T^* = -\log\left(\left(\frac{(1-A^*)\,p^*_\tau}{A^*\,(1-p^*_\tau)}\right)^{\frac{1}{\lambda}}\xi(A^*)^{\frac{1}{(1-A^*)(r+\lambda+\Delta)}}\right).$$

The following result shows that if A is large enough, then the solution to equation (34) is interior and unique.

Lemma A6. *If $A > \frac{rc}{\lambda+\Delta}$, then an interior solution $p_{T^*} \in (0,1)$ to equation* (34) *exists and is unique.*

Proof. Let $y(p_T)$ denote the left-hand side of equation (34), where y is continuous. The support of y is $(0,1]$, where $\lim_{p_T\to 0} y(p_T) = 0$ and $y(1) = \frac{1}{r}$. Furthermore, $y'(p_T) = \frac{1}{r} + 1 - 2p_T$ and $y''(p_T) = -2$. Therefore, y has a unique maximum at $p^m \equiv \min\{\frac{1+r}{2r}, 1\}$. We have to consider two cases. First, suppose $r \leq 1$. Then, $p^m = 1$. In this case, $y(p_T)$ increases monotonically as $p_T \to 1$. Therefore, $p^m = 1$ and $y(p^m) = \frac{1}{r}$. Because $\frac{A}{r} - \frac{c}{\lambda+\Delta} \in [0, \frac{1}{r})$, by the continuity of y and the intermediate value theorem, there exists a unique $p_T \in (0,1)$ such that $y(p_T) = \frac{A}{r} - \frac{c}{\lambda+\Delta}$. Next, suppose $r > 1$. Then, $p^m = \frac{1+r}{2r} \in (0,1)$. In this case, $y(p^m) = \left(\frac{1+r}{2r}\right)^2 > \frac{1}{r}$. Since $\frac{A}{r} - \frac{c}{\lambda+\Delta} \in [0, \frac{1}{r})$, the relevant support of y is $[0, \frac{1+r}{2r}]$. Again by the intermediate value theorem, there exists a unique $p_T \in (0,1)$ such that $y(p_T) = \frac{A}{r} - \frac{c}{\lambda+\Delta}$. Therefore, as long as $A > \frac{rc}{\lambda+\Delta}$, there exists a $p_{T^*} \in (0,1)$ that satisfies the first-order condition (34). □

Next, suppose $A < \frac{rc}{\lambda+\Delta}$. Since $p_T < 1$, equation (34) implies that $p_T = 0$ in this case. Suppose $\tau^*(p_0, A) < T$. Let $T(p_T)$ denote the mandatory disclosure time as a function of p_T. Then, $T(p_T)$ can be written relative to $\tau^*(p_0, A)$ as

$$T(p_T) = -\frac{1}{\lambda+\Delta} \ln \left(\frac{(1 - p^*_\tau(A,T))\, p_T}{p^*_\tau(A,T)\,(1 - p_T)} \right),$$

because $\alpha_t = 1$ for $t \in [\tau^*(p_0, T), T]$. It follows that $\lim_{p_T \to 0} T(p_T) = \infty$. An analogous argument shows that $\lim_{p_T \to 0} T(p_T) = \infty$ for $\tau^* \geq T$. It follows that

$$T^* = \frac{1}{\lambda+\Delta} \left(\Delta \tau^*(p_0, A) - \ln \left(\frac{(1 - p_0)\, p_T}{p_0\,(1 - p_T)} \right) \right).$$

Substituting the definition of $\tau^*(p_0, A)$ and simplifying gives the result. □

Proof of Corollary 5

Proof. Let $T^*(A)$ denote the optimal mandatory disclosure time in Proposition 4 as a function of A. Define the mappings $\chi : [0, \overline{A}] \to [0, \overline{A}]$ and $\chi_T : [0, \overline{A}] \to [0, \overline{A}]$ as

$$\chi(A) = \frac{1}{\kappa} \left(\psi(A) + \frac{\omega(A,A)}{r + \lambda + \Delta} \right),$$
$$\chi_T(A) = \frac{1}{\kappa} \left(\Psi(A, T^*(A)) + \frac{\Omega(A, A, T^*(A))}{r + \lambda + \Delta} \right),$$

respectively, where ψ and ω are defined in Proposition 2, and Ψ and Ω are defined in Proposition 3. Therefore, the equilibrium effort levels are such that A^* is the fixed point of $\chi(A)$ and $A^*(T)$ is the fixed point of $\chi_T(A)$. The difference between the maps χ and χ_T can be expressed as

$$\begin{aligned}\chi_T(A) - \chi(A) \ &= \frac{1}{\kappa}\left\{\Psi(A, T^*(A)) - \psi(A) + \frac{e^{-(r+\lambda)\tau(A,T(A))}}{r+\lambda+\Delta}\left(\frac{\Delta\left(1-e^{(r+\lambda)(\tau(A,T(A))-\tau(A))}\right)}{r+\lambda}\right.\right.\\ &\left.\left.\quad -\frac{\lambda+\Delta}{r}e^{-(r+\lambda+\Delta)(T(A)-\tau(A,T(A)))} + \delta(A, T(A))\right)\right\}.\end{aligned}$$

Since $p_0 < 1$ and beliefs always drift downward,

$$\Psi(A, T^*(A)) - \psi(A) = \int_T^\infty e^{-rt}\left(1-e^{-\lambda t}\right)p_t dt < \frac{1}{r}.$$

By Proposition 4, in equilibrium, $p^*_\tau(A^*(T^*), T^*) > p^*_\tau(A^*)$, which implies that $e^{-(r+\lambda)\tau(A^*)} > e^{-(r+\lambda)\tau(A^*(T^*),T^*)}$. Therefore, $1-e^{-(r+\lambda)(\tau(A^*)-\tau(A^*(T^*),T^*))} < 0$. As such, a sufficient condition for the equilibrium difference $\chi_T(A^*(T^*))-\chi(A^*)$ to be strictly negative is $\delta(A^*(T^*), T^*)+\frac{1}{r} \leq 0$. This would imply that the largest fixed point of χ is below the smallest fixed point of χ_T.[12]

We now show the conditions under which $\delta(A^*(T^*), T^*) + \frac{1}{r} \leq 0$, which is a sufficient condition for $A^*(T^*) < A^*$. Note that

$$\begin{aligned}\delta(A^*(T^*), T^*) &= \frac{(r+\lambda+\Delta)\left(\Delta\frac{A^*(T)-p_{T^*}}{r} - c\right)}{\Delta\left((r+\lambda+\Delta)\frac{A^*(T)-p_{T^*}}{r} - c\right)}\\ &= \frac{(r+\lambda+\Delta)\left(\Delta\left(\phi(A^*(T))\left(1-\phi(A^*(T))\right)+\frac{c}{\lambda+\Delta}\right)-c\right)}{\Delta\left((r+\lambda+\Delta)\left(\phi(A^*(T))\left(1-\phi(A^*(T))\right)+\frac{c}{\lambda+\Delta}\right)-c\right)},\end{aligned} \tag{36}$$

since $\phi(A^*(T)) = p_{T^*}$ by Proposition 3, where the second equality follows from the proof of Proposition 3. In an abuse of notation, let $\delta(\phi(A^*(T)))$ denote the right-hand side of equation (36). Then,

$$\delta'(\phi(A^*(T))) = \frac{c(r+\lambda)(\lambda+\Delta)^2(r+\lambda+\Delta)(1-2\phi(A^*(T)))}{\Delta\left(rc+(\lambda+\Delta)\phi(A^*(T))\left(1-\phi(A^*(T))\right)(r+\lambda+\Delta)\right)^2},$$

which is positive if and only if $p_{T^*} \leq \frac{1}{2}$ and is negative otherwise. Furthermore,

$$\begin{aligned}\delta''(\phi(A^*(T))) \propto &-\left(rc+(\lambda+\Delta)\phi(A^*(T))\left(1-\phi(A^*(T))\right)(r+\lambda+\Delta)\right)\\ &-\left(1-2\phi(A^*(T))\right)^2(\lambda+\Delta)(r+\lambda+\Delta) < 0.\end{aligned}$$

[12]See Theorem 1 in Villas-Boas (1997).

Therefore, $\delta(\cdot)$ is strictly concave over $[0,1]$ and has a unique maximum at $\delta(\frac{1}{2}) = \frac{1}{2} + r\left(\frac{1}{4} + \frac{c}{\lambda+\Delta}\right) > 0$. Note that $\phi(A^*(T))$ is continuous and monotonically increasing in $A^*(T)$:

$$\phi'(A^*(T)) = \frac{1}{r\sqrt{\left(1+\frac{1}{r}\right)^2 - 4\left(\frac{A^*(T)}{r} - \frac{c}{\lambda+\Delta}\right)}} = \frac{1}{1 + r\left(1 - 2\phi(A^*(T))\right)} > 0, \tag{37}$$

Taking the limits of $\delta(\phi(A^*(T)))$ as $\phi(A^*(T))$ approaches the boundaries of the domain,

$$\lim_{\phi(A^*(T))\to 0} \delta(\phi(A^*(T))) = \lim_{\phi(A^*(T))\to 1} \delta(\phi(A^*(T))) = -\frac{\lambda\left(r+\lambda+\Delta\right)}{r\Delta} < 0.$$

Since the domain of $\delta(\cdot)$ is a closed compact set, if $\delta(A^*(T)) + \frac{1}{r} < 0$, then it must be that $\lim_{\phi(A^*(T))\to 0} \delta(\phi(A^*(T))) = \lim_{\phi(A^*(T))\to 1} \delta(\phi(A^*(T))) < -\frac{1}{r}$. Rearranging terms gives the condition in the statement of the corollary.

Suppose this condition holds, so that the domain under which $\delta(A^*(T)) + \frac{1}{r} < 0$ is nonempty. Then, the solutions to $\delta(\phi(A^*(T))) + \frac{1}{r} = 0$ are

$$\phi^{\pm}(A^*(T)) = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4rc}{(1+r)(\lambda+\Delta)}\left(\frac{\lambda}{\Delta} - \frac{\Delta}{r+\lambda+\Delta}\right)}\right), \tag{38}$$

such that $\delta(\phi(A^*(T))) < -\frac{1}{r}$ if $\phi(A^*(T)) < \phi^-(A^*(T))$ or $\phi(A^*(T)) > \phi^+(A^*(T))$. By Proposition 3,

$$\phi(A^*(T)) = \frac{1}{2}\left(1 + \frac{1}{r} - \sqrt{\left(1+\frac{1}{r}\right)^2 - 4\left(\frac{A}{r} - \frac{c}{\lambda+\Delta}\right)}\right). \tag{39}$$

Matching equations (38) and (39), let $A^-(T)$ denote the solution to $\phi(A^*(T)) = \phi^-(A^*(T))$ and $A^+(T)$ the solution to $\phi(A^*(T)) = \phi^+(A^*(T))$. It is straightforward to show that they satisfy

$$A^{\pm}(T) = \frac{rc}{1+r}\left(\frac{r}{\Delta} + \frac{1}{r+\lambda+\Delta}\right) + \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4rc}{(1+r)(\lambda+\Delta)}\left(\frac{\lambda}{\Delta} - \frac{\Delta}{r+\lambda+\Delta}\right)}\right).$$

Then, equation (37) implies that $\delta(\phi(A^*(T))) < -\frac{1}{r}$ if $A^*(T) < A^-(T)$ or $A^*(T) > A^+(T)$. However, as long as $\lambda(r+\lambda+\Delta) > \Delta$, which holds by assumption, $A^+(T) > 1$, which is outside of the feasible domain, since $\overline{A} \leq 1$. Therefore, the only feasible domain over which $\delta(\phi(A^*(T))) < -\frac{1}{r}$ is $[0, A^-(T)]$. □