Accounting-based Valuation with Inter-firm Information Transfers

Daphne Hart

University of Illinois at Chicago

Bjorn N. Jorgensen

Copenhagen Business School

Abstract

We extend the Ohlson (1995) accounting-based valuation model with linear information dynamics (LID) to a setting with two firms. We model LID with direct inter-firm information transfers as well as other information that is either common or firm-specific. We show how inter-firm information transfers in stock prices and returns rely on the underlying LID parameters. We then consider two cases. First, we demonstrate how stock prices and returns reflect information transfers between two firms that report financial statements with differing frequency, such as when one firm reports quarterly and the other reports semi-annually. Second, we derive accounting-based valuation for a nondisclosing firm based on inter-firm information transfers from a publicly disclosing peer. All our results extend to a setting with multiple firms. Finally, we analyze portfolio allocation guided by LID parameters. With multiple stocks, the return on the market portfolio depends on news from aggregate abnormal earnings and common other information.

Keywords: Fundamental Analysis, Valuation Modeling
JEL Classifications: M41 G12

We thank seminar participants at Alliance Manchester Business School, Cambridge Judge Business School, BI Norwegian Business School, DAR&DART Accounting Theory Seminar, and Vasiliki Athanasakou, Brian Burnett, Colin Clubb, Emmanuel De George, Art Kraft and Matt Lyle for helpful comments and suggestions. Current draft: October 2021. The authors' e-mails are: hartd@uic.edu and bnj.acc@cbs.dk.

1. Introduction

In an influential paper, Ohlson (1995) studies a single firm and demonstrates how clean surplus accounting and linear information dynamics allow valuation based on *book value of equity* and current *abnormal earnings*. Bernard (1995) notes that Ohlson (1995) shifts attention from analyzing prices to forecasting abnormal earnings and that this approach to estimating intrinsic values comports with fundamental analysis. To date, accounting-based valuation models confine their analysis to a single firm, while other asset pricing models have been extended to multiple security settings (e.g., Garman and Ohlson 1980, Admati 1985, Holthausen and Verrecchia 1988). The lack of an analytical foundation linking accounting fundamentals of multiple firms to stock prices and returns is surprising, given the empirical literature documenting the importance and prevalence of inter-firm information.

The empirical literature examines intra-industry information transfers (Foster 1981, Baginski 1987, Han, Wild, and Ramesh 1989, Han and Wild 1990, Dontoh and Ronen 1993, Wang 2014) and the market's and analysts' reactions to peer firms' disclosures (Freeman and Tse 1992, Ramnath 2002, Thomas and Zhang 2008, Shroff, Verdi, and Yost 2017). However, these studies lack a rigorous analytical framework. We fill this gap by generalizing accounting-based valuation settings to two (or more) related firms and providing analytical benchmarks for assessing information transfers from two sources: abnormal earnings and other information.

The U.S. Securities and Exchange Commission (SEC) requires that firms disclose relative information about total shareholder returns in the financial statements (Ma, Shin, and Wang 2021). Analytical analysis of this and similar disclosure regulations requires valuation settings with two or more firms. Multi-firm accounting-based valuation settings allow calculation of portfolio returns from trading strategies in an efficient market with multiple securities.

We initially extend the Ohlson (1995) model to a two-firm setting by specifying various linear information dynamics (LID) that permit inter-firm information transfers and closed form solutions when each firm's abnormal earnings are directly affected by lagged abnormal earnings of the other firm and affected by other information. Since prior analytical accounting-based valuation models consider a single firm, they cannot distinguish the roles of common and firm-specific sources of information. We consider LID that encompass common or firm-specific sources of other information and demonstrate that the nature of other information, common or firm-specific, affects how market values and stock returns respond to contemporaneous financial statement disclosures from a related firm. We focus on the simplest setting with two firms, which allows separate analyses and comparisons of common versus firm-specific other information. We then demonstrate that our results extend to a setting with multiple firms.

We further consider inter-firm information transfers in two disclosure settings: (i) when firms report financial statements with differing frequency and (ii) when only one firm discloses financial statements. First, we extend accounting-based valuation to allow for two related firms with the same fiscal year-end to report interim financial statements with differing frequency. For example, the SEC requires that US domestic filers disclose financial statements quarterly. In contrast, Australia and the European Union (EU), among others, require that publicly traded firms disclose semi-annually. Arif and De George (2020) find weaker information transfers arising for quarters in which only a US firm discloses, relative to quarters in which a related, semi-annually disclosing non-US firm also discloses. Kajuter, Klassmann, and Nienhaus (2018) document information transfers within a single country. They show information spillover from large public Singaporean firms that must report quarterly to small firms that need only disclose semi-annually.[1]

Relative to the benchmark case of two related firms that disclose with the same frequency, we find that stock *prices* reflect inter-firm information transfers differently only in periods where one firm is not disclosing. In contrast, stock *returns* reflect inter-firm information transfers

[1] Different mandatory disclosure frequencies arise across and within countries. First, large (small) public firms in Singapore with market capitalization above (below) S$75 million must report quarterly (semi-annually). Second, public (larger private) EU firms are required to disclose financial statements semi-annually (annually). Third, U.S. firms must report quarterly, while foreign private issuers registered with the SEC are required to report with only the same frequency as in their home country. Finally, we acknowledge that the variation in reporting frequency may be voluntary or mandatory around a policy change (Butler, Kraft, and Weiss 2007, Fu, Kraft, and Zhang 2012, Kraft, Vashishtha, and Venkatachalam 2018).

differently in every period. Therefore future empirical-archival analyses of information transfers between firms with different disclosure frequency should rely on stock prices when the more frequently disclosing firm is used as a control.

Second, we derive accounting-based valuation of a nondisclosing private firm based on inter-firm information transfers from a disclosing publicly traded firm. Valuation of private firms that do not disclose financial statements is notoriously difficult. In addition, some public US firms are exempt from the periodic reporting, such as those with securities trading on the Pink Sheets that are below the SEC's size or shareholders-of-record thresholds (Brüggemann et al. 2018).[2] A proposed SEC rule suggests that regulators remain concerned about firms with securities trading on the Pink Sheets (SEC Release No. 34-87115, 2019). Our model assists in valuation of nondisclosing firms and more generally suggests that the effect of removing disclosure exemptions varies with the extent of inter-firm information transfers.[3]

We make three main contributions. First, we present accounting-based valuation models with LID to value multiple firms. We consider three settings with two firms that (i) incorporate information about their peers' abnormal earnings directly, (ii) have common other information, and (iii) have firm-specific other information. These models provide an analytical foundation for studying information transfers and the effects of common and firm-specific other information on prices and returns. We show how portfolio selection based on accounting-based valuation allows

[2] SEC Rule 15c2-11 requires that only the initial broker-dealer obtain financial statement information before starting to quote these securities in the market. Under the piggyback exception, subsequent broker-dealers rely on the initial filings reviewed by the initial broker-dealer. Thus disclosure of financial statements is voluntary for a substantial number of firms trading on the Pink Sheets and a significant proportion elect not to disclose financial statements (Brüggemann et al. 2018).

[3] Bushee and Leuz (2005) study U.S. firms that were not disclosing financial statements to investors but nonetheless were actively quoted on the Over-the-Counter Bulletin Board (OTCBB). They find that the eligibility rule in 1999, which introduced compliance with the periodic reporting requirements of the 1934 Securities Exchange Act for these firms to remain trading on the OTCBB, causes a negative market reaction but higher liquidity, consistent with both costs and benefits of mandatory disclosure. They also document evidence consistent with positive externalities to the firms that were already disclosing financial statements with the SECs prior to the eligibility rule. These firms experience positive stock returns and increases in liquidity after the eligibility rule took effect. Moreover, Burnett (2020) shows that stock prices reflect undisclosed financial statement information for OTCBB firms immediately before the eligibility rule. These findings are generally consistent with externalities from information transfers that we predict. Inter-firm information transfers is one channel through which firms benefit from their peers' disclosures and through which undisclosed information is revealed.

investors to neutralize shocks in returns. Specifically, we provide an analytical foundation for the different relations between returns and earnings surprises at the firm level and at the aggregate, industry level, or economy level.

Second, we develop a benchmark for the degree of inter-firm information transfers between firms that report financial statements with differing frequency. We show that financial statement information from more frequently reporting firms spills over into stock prices and returns of less frequently reporting firms but that this relation is not one sided. The prices of more frequent reporters are also altered in the absence of financial statement information from the less frequent reporters. Furthermore, firms' returns differ, relative to a setting where firms disclose financial statement with the same frequency.

Third, we derive an accounting-based valuation of firms that do not publicly disclose financial statements based on information transfers from related firms that do. This allows valuation of previously public U.S. firms that went private and therefore are exempt from reporting. We show that, in the presence of a related "private" firm that does not provide financial statements, returns may exhibit correlation over time, even in an informationally efficient market. Our findings should be of interest to investors and researchers, as they model information flows between firms and offer a benchmark for the degree of inter-firm information transfers in stock market values and returns.

The paper proceeds as follows. Section 2 reviews the Ohlson (1995) accounting-based valuation model without other information and describes its extension to a two-firm setting with direct information transfers as a benchmark, before introducing two types of other information, common and firm-specific. Section 3 considers two settings where firms disclose with differing frequency. In the first setting, one firm discloses twice as frequently as the other. In the second setting, one firm must disclose every period, while the other never discloses after its initial financial statements. Section 4 concludes. Notation and variable names are defined in Appendix A. The remaining appendices present the proofs.

2. Accounting-based Valuation Model

Ohlson (1995) presents a valuation model for companies that engage exclusively in operating activities, so that future abnormal earnings are correlated with current abnormal earnings.[4] Ohlson (1995) shows that accounting-based valuation anchors on book value, such that a firm's market value equals book value and the expected discounted sum of future abnormal earnings. Ohlson (1995) also assumes LID, such that the most recent abnormal earnings suffice for predicting all abnormal earnings, which simplifies the expression of firm value.

2.1. Review of Accounting-based Valuation for One Firm

Ohlson (1995) considers a single firm (denoted by j) and risk neutral investors. At the beginning of each period t, the firm has book value of equity denoted by $B_j(\mathrm{t}-1)$. By the end of period t, firm j reports earnings of $x_j(\mathrm{t})$ and pays dividends (net of capital contributions) of $d_j(\mathrm{t})$. At the end of period t, book value of equity follows from *Clean Surplus Relation* (CSR):

$$B_j(\mathrm{t}) = B_j(\mathrm{t}-1) + x_j(\mathrm{t}) - d_j(\mathrm{t}).$$

CSR merely describes how book value evolves over time, it increases with earnings, $x_j(\mathrm{t})$, and decreases with dividends paid, $d_j(\mathrm{t})$.[5]

Prior literature assumes that firm j's market value at time t, $P_j(\mathrm{t})$, equals the *Present Value of Expected Dividends* (PVED):

$$P_j(\mathrm{t}) = \sum_{\tau=1}^{\infty} E_t\left[R_F^{-\tau} d_j(\mathrm{t}+\tau)\right],$$

where E_t denotes expectations taken using information available at time t and $R_F = 1 + r > 1$ is one plus the risk-free interest rate, r. Interest rates remain constant over time for simplicity; that is, the term structure of interest rates is deterministic and flat. We also abstract from risk

[4] Feltham and Ohlson (1995) articulate that retained earnings are reinvested in risk-free (government) bonds, which do not generate future abnormal earnings.
[5] The implications, necessity, and validity of the CSR assumption are discussed by Preinreich (1936, 1938), Peasnell (1982), Ohlson (1995), and O'hanlon and Peasnell (2002), among others.

adjustments. These assumptions simplify exposition and are not critical, and the literature shows how to relax them.[6]

Using PVED, we can value the firm based on CSR. Defining abnormal earnings for the firm as: $x_j^a(\mathrm{t}) = x_j(\mathrm{t}) - (R_F - 1)B_j(\mathrm{t}-1)$, CSR can be written as:

(1) $$d_j(\mathrm{t}) = R_F B_j(\mathrm{t}-1) - B_j(\mathrm{t}) + x_j^a(\mathrm{t}).$$

Using PVED yields:

(2) $$P_j(\mathrm{t}) = B_j(\mathrm{t}) + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t\left[x_j^a(\mathrm{t}+\tau)\right].$$

Alternatively, Penman (1992) expresses the market-to-book ratio, $MB_j(\mathrm{t}) = \frac{P_j(\mathrm{t})}{B_j(\mathrm{t})}$, based on its future abnormal return-on-equity ratios, $\left(ROE_j(\mathrm{t}+\tau) - r\right)$, as: $MB_j(\mathrm{t}) = 1 + \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t\left[\frac{B_j(\mathrm{t}+\tau-1)}{B_j(\mathrm{t})}\left(ROE_j(\mathrm{t}+\tau) - r\right)\right]$.

Ohlson (1995) introduces LID, and, without other information, it reduces to:

(3) $$\tilde{x}_j^a(\mathrm{t}+1) = \omega x_j^a(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1),$$

where the parameter is bounded with $-1 \leq \omega \leq 1$ and the disturbance term, $\tilde{\varepsilon}_j(\mathrm{t}+1)$, has zero mean and is independent over time. This simplified LID assumption reduces goodwill (the infinite sum in the second term of Equation 2) to the product $\alpha x_j^a(\mathrm{t})$, where the constant $\alpha = \frac{\omega}{(R_F - \omega)}$ represents the sensitivity of price to abnormal earnings, which is increasing in ω. We next extend this special case of LID without other information to a setting with two firms denoted by j and k.

2.2. Two Firms Without Other Information

As a benchmark, we initially mute the role of other information. We also assume firms must disclose their financial statements simultaneously at the end of each period. This benchmark

[6] See Feltham and Ohlson (1999), Gode and Ohlson (2004), Christensen and Feltham (2009), Nekrasov and Shroff (2009), Clubb (2013), and Lyle, Callen, and Elliott (2013), among others.

allows us to characterize the effect of two firms' simultaneous disclosure of their earnings and book values in the absence of other information. Formally, this benchmark involves LID for two firms where:

(4) $\tilde{x}_j^a(\mathrm{t}+1) = \omega_{jj} x_j^a(\mathrm{t}) + \omega_{jk} x_k^a(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1);$

(5) $\tilde{x}_k^a(\mathrm{t}+1) = \omega_{kj} x_j^a(\mathrm{t}) + \omega_{kk} x_k^a(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1).$

In the setting presented by Equations (4) and (5), the two firms are related, as captured by the parameters ω_{jk} and ω_{kj}, which permit direct inter-firm information transfers. These parameters might be nonzero, because the two firms share customers, suppliers, or engage in transactions, such as a sale from one to the other. Throughout the paper, all parameters are common knowledge and the disturbance terms, $\tilde{\varepsilon}_j$ and $\tilde{\varepsilon}_k$, have mean zero, variances $\sigma_j^2 = VAR\left[\tilde{\varepsilon}_j(\mathrm{t}+1)\right]$ and $\sigma_k^2 = VAR[\tilde{\varepsilon}_k(\mathrm{t}+1)]$, and are independent of each other and independent over time.[7]

THEOREM 1: Given PVED and the LID presented in equations (4) and (5), firm values are determined by

(6) $P_j(\mathrm{t}) = B_j(\mathrm{t}) + \alpha_{jj} x_j^a(\mathrm{t}) + \alpha_{jk} x_k^a(\mathrm{t});$

(7) $P_k(\mathrm{t}) = B_k(\mathrm{t}) + \alpha_{kj} x_j^a(\mathrm{t}) + \alpha_{kk} x_k^a(\mathrm{t});$

where the alpha coefficients are:

$$\alpha_{jj} = \Delta^{-1}\{\omega_{jj}(R_F - \omega_{kk}) + \omega_{jk}\omega_{kj}\} \qquad \alpha_{jk} = \Delta^{-1} R_F \omega_{jk}$$

$$\alpha_{kj} = \Delta^{-1} R_F \omega_{kj} \qquad \alpha_{kk} = \Delta^{-1}\{\omega_{kk}(R_F - \omega_{jj}) + \omega_{kj}\omega_{jk}\}$$

and $\Delta = (R_F - \omega_{jj})(R_F - \omega_{kk}) - \omega_{jk}\omega_{kj} \neq 0.$

[7] We assume the parameters $\omega_{jj}, \omega_{jk}, \omega_{kj}$, and ω_{kk} are bounded between -1 and 1. We refer to *direct* inter-firm information transfer when at least one off-diagonal LID coefficient is nonzero; that is, $\omega_{jk} \neq 0$, for $j \neq k$.

Our proof of Theorem 1 follows Ohlson (1995) closely, although it involves solving a two-dimensional system with slightly more general coefficient (see Appendix B). Consistent with the single firm setting without information transfers and without other information, the sensitivity of firm j value to its own abnormal earnings, α_{jj}, is increasing in ω_{jj}. Similarly, the sensitivity of firm k value to its own abnormal earnings, α_{kk}, is increasing in ω_{kk}.

As for the inter-firm information transfers, the sensitivity of firm j to firm k's abnormal earnings is captured by α_{jk}, while the sensitivity of firm k to firm j's abnormal earnings is captured by α_{kj}. When α_{jk} (α_{kj}) is positive, the abnormal earnings of firm j (k) have a positive association with the lagged abnormal earnings of firm k (j).

When both α_{jk} and α_{kj} are positive, the firms' abnormal earnings have a complementary effect over time: when one has higher abnormal earnings, the other is expected to have higher abnormal earnings in the next period, ceteris paribus. However, when both α_{jk} and α_{kj} are negative, the firms' abnormal earnings have a substitution effect over time: when one firm has lower abnormal earnings, the other is expected to have higher abnormal earnings in the next period, ceteris paribus.

In the special case of absence of direct inter-firm information transfers, that is, $\omega_{jk} = \omega_{kj} = 0$, firms are valued independently, equivalent to Ohlson (1995) without other information. In another special case, when direct inter-firm information transfers are one-sided, such as when $\omega_{jk} = 1$ and $\omega_{kj} = 0$, firm j's valuation is equivalent to Ohlson (1995), where abnormal earnings of firm k could be considered as other information (for example, from a bellwether firm), that affects firm j's next period's abnormal earnings.

Theorem 1 can be used to characterize the relation between market-to-book ratio and return-on-equity ratios. We extend Penman (1992) to a setting with LID and inter-firm information transfers.

COROLLARY 1: Given PVED and the LID presented in equations (4) and (5), market-to-book ratio of firm j can be expressed as:

$$MB_j(\mathrm{t}) = 1 + \alpha_{jj}\frac{B_j(\mathrm{t}-1)}{B_j(\mathrm{t})}\left(ROE_j(\mathrm{t}) - \mathrm{r}\right) + \alpha_{jk}\frac{B_k(\mathrm{t}-1)}{B_j(\mathrm{t})}\left(ROE_k(\mathrm{t}) - \mathrm{r}\right).$$

Firm j's market-to-book ratio depends on the return-on-equity ratios of both firms, where Firm j's own return-on-equity ratio is adjusted by its book value growth, $\frac{B_j(\mathrm{t}-1)}{B_j(\mathrm{t})}$, consistent with Penman (1992), and where firm k's return-on-equity ratio has two multiplicative adjustments, due to (1) its book value growth, $\frac{B_k(\mathrm{t}-1)}{B_k(\mathrm{t})}$, and (2) the difference in scale between the two firms, $\frac{B_k(\mathrm{t})}{B_j(\mathrm{t})}$.

While Theorem 1 and Corollary 1 characterize the relation between market values and accounting variables, some papers characterize the relation between stock returns and accounting variables, including Ohlson (1995), Easton and Harris (1991) and Easton and Pae (2004). Following Ohlson (1995), we define one-period-ahead excess stock returns of firm j for as:

$$\widetilde{Ret}_j(\mathrm{t}+1) = \frac{\left(\tilde{P}_j(\mathrm{t}+1) + d_j(\mathrm{t}+1) - R_F P_j(\mathrm{t})\right)}{P_j(\mathrm{t})}.$$

THEOREM 2: Given PVED and the LID presented in equations (4) and (5), excess stock returns of firm j are given by:

$$\widetilde{Ret}_j(\mathrm{t}+1) = \left(1 + \alpha_{jj}\right)\frac{\tilde{\varepsilon}_j(\mathrm{t}+1)}{P_j(\mathrm{t})} + \alpha_{jk}\frac{\tilde{\varepsilon}_k(\mathrm{t}+1)}{P_j(\mathrm{t})}.$$

Theorem 2 shows that each firm's stock returns are independent over time. However, for any period t, two firms' stock returns are correlated through both firms' abnormal earnings disturbance terms.

Theorem 2 allows constructing a portfolio that exclusively exposes investors to one firm's abnormal earnings disturbance terms. By appropriately choosing two portfolio weights, which add up to one, the disturbance term of one firm can be fully diversified (see proof in Appendix C.1).

Moreover, Easton, Harris, and Ohlson (1992) investigate how simultaneous aggregation over time of both firm-level stock returns and earnings affect the returns–earnings relation and changes its goodness-of-fit. Theorem 2 allows us to characterize aggregate stock returns. Appendix C.2 proves how to incorporate aggregation from one to two periods in the presence of direct inter-firm information transfers.

2.3. Two Firms with Other Information

2.3.1. Common Other Information

When other information is common for both firms, we assume that:

(8) $$\tilde{x}_j^a(\mathrm{t}+1) = \omega_{jj} x_j^a(\mathrm{t}) + \omega_{jk} x_k^a(\mathrm{t}) + v(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1);$$

(9) $$\tilde{x}_k^a(\mathrm{t}+1) = \omega_{kj} x_j^a(\mathrm{t}) + \omega_{kk} x_k^a(\mathrm{t}) + v(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1);$$

(10) $$\tilde{v}(\mathrm{t}+1) = \gamma v(\mathrm{t}) + \tilde{\varepsilon}_v(\mathrm{t}+1);$$

where $v(\mathrm{t})$ denotes common other information (COI) with persistence γ that is bounded between -1 and 1.

Both firms directly incorporate information about their own abnormal earnings as well as the other firm's abnormal earnings. In addition, COI is a leading indicator of future abnormal earnings for both firms.

THEOREM 3: Given PVED and LID with COI as presented in equations (8)–(10), firm values are determined by:

(11) $$P_j(\mathrm{t}) = B_j(\mathrm{t}) + \alpha_{jj} x_j^a(\mathrm{t}) + \alpha_{jk} x_k^a(\mathrm{t}) + \beta_j v(\mathrm{t});$$

(12) $$P_k(\mathrm{t}) = B_k(\mathrm{t}) + \alpha_{kj} x_j^a(\mathrm{t}) + \alpha_{kk} x_k^a(\mathrm{t}) + \beta_k v(\mathrm{t});$$

with the four alpha coefficients (α_{jj}, α_{jk}, α_{kk}, and α_{kj}) are as stated in Theorem 1 for the setting without other information and:

$$\beta_j = \Delta^{-1} R_F \frac{\left(R_F - \omega_{kk} + \omega_{jk}\right)}{(R_F - \gamma)};$$

$$\beta_k = \Delta^{-1} R_F \frac{\left(R_F - \omega_{jj} + \omega_{kj}\right)}{(R_F - \gamma)}.$$

The four alpha coefficients are the same with and without COI, because LID assumes that abnormal earnings are determined by lagged COI and that COI's effect is independent of each firm's abnormal earnings disturbance terms.[8]

Theorem 3 suggests that the firm's price is shaped by its own and its peer's abnormal earnings as well as by COI. This relationship represents inter-firm information transfers. However, for each firm, COI may be priced differently, as the beta coefficients (β_j and β_k) reflect both the firm's and its peer's underlying information dynamic. COI has the same valuation effect on both firms (i.e., $\beta_j = \beta_k$), when the sum of the parameters is the same across firms (i.e., $\omega_{jj} + \omega_{jk} = \omega_{kk} + \omega_{kj}$).

COI still affects the valuation of both firms in the special case without direct inter-firm information transfers (i.e., $\omega_{jk} = \omega_{kj} = 0$). Specifically, when $\alpha_{jk} = \alpha_{kj} = 0$, then:

$$\beta_j = \frac{R_F}{(R_F - \gamma)\left(R_F - \omega_{jj}\right)};$$

$$\beta_k = \frac{R_F}{(R_F - \gamma)(R_F - \omega_{kk})}.$$

THEOREM 4: Given PVED and the LID with COI as presented in equations (8)–(10), excess stock returns of firm j are:

---

[8] This result might not hold if LID were replaced with nonlinear information dynamics, following Finn and Ye (1999).

$$\widetilde{Ret}_j(\mathrm{t}+1) = (1+\alpha_{jj})\frac{\tilde{\varepsilon}_j(\mathrm{t}+1)}{P_j(t)} + \alpha_{jk}\frac{\tilde{\varepsilon}_k(\mathrm{t}+1)}{P_j(\mathrm{t})} + \beta_j\frac{\tilde{\varepsilon}_v(\mathrm{t}+1)}{P_j(\mathrm{t})}.$$

When firm j and firm k have COI, the returns of the firms equal the weighted sum of firm j's and firm k's disturbance terms and the disturbance term of the common information.

2.3.2. Firm-specific Other Information

When other information is firm-specific, we assume the following.

(13) $$\tilde{x}_j^a(\mathrm{t}+1) = \omega_{jj}x_j^a(\mathrm{t}) + \omega_{jk}x_k^a(\mathrm{t}) + v_j(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1);$$

(14) $$\tilde{x}_k^a(\mathrm{t}+1) = \omega_{kj}x_j^a(\mathrm{t}) + \omega_{kk}x_k^a(\mathrm{t}) + v_k(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1);$$

(15) $$\tilde{v}_j(\mathrm{t}+1) = \gamma_j v_j(\mathrm{t}) + \tilde{\varepsilon}_{vj}(\mathrm{t}+1);$$

(16) $$\tilde{v}_k(\mathrm{t}+1) = \gamma_k v_k(\mathrm{t}) + \tilde{\varepsilon}_{vk}(\mathrm{t}+1);$$

where v_j (v_k) represents other information, specific to firm j (k) with persistence γ_j (γ_k) that is bounded between -1 and 1. Firm-specific other information (FOI) evolves over time based on prior period FOI and disturbance terms.

THEOREM 5: Given PVED and LID with FOI as presented in equations (13)–(16), market prices are:

(17) $$P_j(\mathrm{t}) = B_j(\mathrm{t}) + \alpha_{jj}x_j^a(\mathrm{t}) + \alpha_{jk}x_k^a(\mathrm{t}) + \beta_{jj}v_j(\mathrm{t}) + \beta_{jk}v_k(\mathrm{t});$$

(18) $$P_k(\mathrm{t}) = B_k(\mathrm{t}) + \alpha_{kj}x_j^a(\mathrm{t}) + \alpha_{kk}x_k^a(\mathrm{t}) + \beta_{kj}v_j(\mathrm{t}) + \beta_{kk}v_k(\mathrm{t}).$$

The four alpha coefficients (α_{jj}, α_{jk}, α_{kk}, and α_{kj}) are as stated in Theorem 1 for the setting without other information, and the beta coefficients (β_{jj}, β_{jk}, β_{kk}, and β_{kj}) are given by:

$$\beta_{jj} = \Delta^{-1}R_F\frac{(R_F-\omega_{kk})}{(R_F-\gamma_j)} \qquad \beta_{jk} = \Delta^{-1}R_F\frac{\omega_{jk}}{(R_F-\gamma_k)}$$

$$\beta_{kj} = \Delta^{-1}R_F\frac{\omega_{kj}}{(R_F-\gamma_j)} \qquad \beta_{kk} = \Delta^{-1}R_F\frac{(R_F-\omega_{jj})}{(R_F-\gamma_k)}.$$

The four alpha coefficients are unchanged, relative to the setting without other information presented in Theorem 1, and the setting with COI presented in Theorem 3, because LID renders FOI independent of each firm's abnormal earnings disturbance terms.

Theorem 5 shows that the firm's own other information and its peer's other information are priced. The sensitivity of firm j's value to its own other information (β_{jj}) depends on firm k's abnormal earnings response to its lagged abnormal earnings (ω_{kk}). This indicates that the sensitivity of firm j's value to its own other information is shaped by the inter-firm information transfers.

In the special case where $R_F = \omega_{kk} + \omega_{kj} = \omega_{jj} + \omega_{jk}$, it follows that $\beta_{jj} = \beta_{kj}$ and $\beta_{kk} = \beta_{jk}$, such that both firms have the same sensitivity to firm-specific other information. Moreover, FOI of a peer firm does not affect valuation in the special case where $\omega_{jk} = \omega_{kj} = 0$, since $\beta_{jk} = \beta_{kj} = 0$ and

$$\beta_{jj} = \frac{R_F}{(R_F - \gamma_j)(R_F - \omega_{jj})};$$

$$\beta_{kk} = \frac{R_F}{(R_F - \gamma_k)(R_F - \omega_{kk})}.$$

Lastly, comparing the settings where other information is common and firm-specific, the sensitivity of market prices to other information remains the same when $\omega_{jk} = \omega_{kj} = 0$ and $\gamma_j = \gamma_k$, that is, when information transfer does not occur and when FOI persistence parameters are the same across firms.

THEOREM 6: Given PVED and the LID with FOI as presented in equations (13)–(16), excess stock returns of firm j are:

$$\widetilde{Ret}_j(\mathrm{t}+1) = (1 + \alpha_{jj})\frac{\tilde{\varepsilon}_j(\mathrm{t}+1)}{P_j(\mathrm{t})} + \alpha_{jk}\frac{\tilde{\varepsilon}_k(\mathrm{t}+1)}{P_j(\mathrm{t})} + \beta_{jj}\frac{\tilde{\varepsilon}_{vj}(\mathrm{t}+1)}{P_j(\mathrm{t})} + \beta_{jk}\frac{\tilde{\varepsilon}_{vk}(\mathrm{t}+1)}{P_j(\mathrm{t})}.$$

Firm j's stock returns reflect the disturbance terms of abnormal earnings and FOI of both firms. The price and the returns of firm j reflect both the inter-firm information transfers and the different sources of other information.

The literature classifies disclosure effects as competitive or contagious (Wang 2014). The competitive (contagious) effect arises when favorable news from firm k triggers a negative (positive) abnormal stock return for firm j. Theorem 6 shows that these effects could arise from either direct information transfers, where a negative (positive) α_{jk} induces a competitive (contagious) effect, or from firm-specific other information, where a negative (positive) β_{jk} induces a competitive (contagious) effect.

2.4. Discussion

Our model offers analytical foundations and guidance for studying information transfers and correlations across firms. Specifically, we characterize how inter-firm information transfers vary cross-sectionally with accounting fundamentals and how the underlying LID parameters have nonlinear moderating effects on firm values and stock returns. Our characterization of nonlinearities in the relation to underlying LID parameters could guide the empirical specifications used for studying inter-firm information transfers.

Moreover, our analysis implies that, if COI is observable, the LID parameters can be estimated in a single-stage approach. However, if COI is unobservable, a two-stage approach might first estimate LID parameters, based on observable abnormal earnings, and then estimate the effect of unobservable common other information from stock prices or stock returns (as explained below). More generally, method of moments estimation can exploit data from abnormal earnings and stock prices simultaneously.

However, when LID parameters are known, our results can be used to mute the effect of individual components of stock returns. In section 2.2, we discuss how to construct a portfolio that eliminates a firm's abnormal earnings disturbance terms. Nevertheless, the disturbance terms of

COI and FOI can also be eliminated. Theorem 4 implies that investors can construct a portfolio using two firms such that the portfolio returns are unaffected by COI (see Appendix C.3). Theorem 6 implies that investors can construct a portfolio using two firms such that the portfolio returns are unaffected by firm-specific other information (see Appendix C.4). Similarly, investors can also construct a portfolio that fully diversifies other information using many firms. Overall, our model permits constructing diversifying portfolios based on accounting fundamentals and LID.

Our results generalize to a multiple firm setting with two sources of unobservable other information, that is, common and firm-specific. Even absent direct inter-firm information transfers between firms (i.e., $\omega_{jk} = 0$, for $j \neq k$), researchers may not be able to separate the two sources of unobservable common information based on two firms. However, a value-weighted portfolio constructed using n firms will have portfolio returns that follow COI in the limit for $n \to \infty$ (see Appendix C.5). As the number of firms in the portfolio grows, disturbance terms of firms' abnormal earnings are fully diversified, and the value-weighted portfolio only captures disturbance terms to COI. Furthermore, even in the presence of FOI, the value-weighted portfolio is a COI-mimicking portfolio. This portfolio's returns allow the identification of COI and can be used to empirically identify observable variables that are leading indicators of future abnormal earnings.[9]

Lastly, empirical research aggregates stock returns and earnings across firms to investigate the relation between the returns on an aggregate market portfolio and cross-sectional aggregate earnings surprise (Kothari, Lewellen and Warner 2006, Sadka 2007, Ball and Sadka 2015). Consistent with the literature, the analysis presented in Appendix C.5 shows that, when firm value is affected by both COI and FOI, the returns on the aggregate market portfolio depend only on COI. Our analysis also reconciles prior firm-level and aggregate-level findings. When estimating the cross-sectional association between value-weighted market returns and aggregate earnings

[9] Early empirical tests of Ohlson (1995) include the work of Frankel and Lee (1998) and Dechow, Hutton, and Sloan (1999). Myers (1999) investigates observable other information. Also, recent empirical studies that decompose common and firm-specific other information include those by Bhojraj, Mohanram, and Zhang (2020) and Jackson, Plumlee, and Rountree (2018).

surprises, we expect variables that are correlated with COI (e.g., unemployment) to have a significant explanatory power, while variables that are correlated with firm-specific other information (e.g., order backlog; see Myers 1999) likely have lower explanatory power, relative to firm-level estimation.

3. Accounting-based Valuation of Two firms Reporting with Different Frequency

Thus far we have assumed all firms disclose their financial information simultaneously at the end of each period. Nevertheless, two related firms may have different disclosure requirements and disclose with different frequencies. Therefore we relax this assumption and examine two settings. First, we consider a setting with two firms, where one provides financial statements twice as often as the other. Second, we analyze a setting where one firm discloses financial statements every period, while another discloses only once.

3.1. Different Reporting Frequency

We derive the accounting-based valuation of two firms that disclose their financial statements with different frequencies. For example, publicly traded firms in the US (registered with the SEC) must disclose quarterly financial statements. In contrast, publicly traded firms in the EU must disclose semi-annually. As a result, when comparing two firms with the same fiscal year-end, one listed in US and the other in EU, both will disclose financial statement information semi-annually. However, at the end of the first and third fiscal quarters, only the US firm discloses its financial statements. Our model also applies to valuation of private EU firms that are required to file their financial statements only annually and not as often as public EU firms.[10]

We consider a setting where firms have direct inter-firm information transfers, and, for simplicity, we abstract from common and firm-specific other information. We assume that both

[10] Larger private EU firms must publicly disclose financial statements annually, while public EU firms must publicly disclose financial statements semi-annually; see Kausar, Shroff, and White (2016) and Bernard, Burgstahler, and Kaya (2018).

firm j and firm k disclose at time $2t$, $\forall t$, such that, by the end of the period, investors know the firms' book values and abnormal earnings as disclosed and the total dividends paid by the firms, which is public knowledge.[11] However, at the end of the next period, $2t + 1$, $\forall t$, only firm j discloses, and investors update their information using only the information disclosed by firm j and the total dividends paid by both firms. Since firm k does not provide information about its current book value and abnormal earnings, investors use the available information to form expectations about firm k's book value and abnormal earnings. In the following period, $2t + 2, \forall t$, when both firms disclose again, the information is updated using both firms' disclosures and dividends paid.[12] In period $2t + 3, \forall t$, when only firm j discloses, the information set is updated again using only the disclosed information from firm j and dividends paid, and so on from one period to the next.

Overall, the information available in even periods (i.e., periods where both firms disclose, denoted by $2t\ \forall t$) differs from that available in odd periods (i.e., periods where only firm j discloses, denoted by $2t + 1\ \forall t$); therefore the information used to value the firms differs in even periods and odd periods. At the end of any even period, both firms j and k disclose and thus, as in the benchmark setting, firms' values are based on the most recently disclosed information: $\{B_j(2\mathrm{t}); x_j^a(2\mathrm{t}); B_k(2\mathrm{t}); x_k^a(2\mathrm{t})\}$. At the end of any odd period, only firm j discloses, and investors fill the informational gap by forming expectations based on LID and CSR regarding firm k's book value and abnormal earnings. Overall, in odd periods, firms' values are based on the most recently

[11] At the end of each period, investors know how much dividends each firm paid. This assumption is based on the fact that current investors know their percentage ownership as well as how much dividends they received, and hence they know the total dividends paid by the firm.

[12] To derive firm values in this setting, we assume investors can perfectly infer the book values and the abnormal earnings of firm k in each of the last two quarters from the firm's semi-annual disclosures. Even if investors are not presented with information concerning each of the two last quarters, they can use prior information disclosed and other sources of information to assess the firm's performance in each of the two quarters separately. Alternatively, we assume that the semi-annual disclosures provide financial information pertaining to the two quarters it covers separately. This assumption essentially requires that the two firms provide comparable information, information relating to the same period, even if the disclosure of one firm may not be as timely as the disclosure of its peer.

disclosed information by firm j and the information disclosed in the previous period by both firms: $\{B_j(2\mathrm{t}+1); x_j^a(2\mathrm{t}), x_j^a(2\mathrm{t}+1); B_k(2\mathrm{t}); x_k^a(2\mathrm{t}); d_k(2t+1)\}$.[13]

THEOREM 7: Given PVED and the LID presented in equations (4)–(5):

(i) When both firms j and k disclose at time $2t$ $\forall t$, firm values are determined by:

(19) $$P_j(2t) = B_j(2t) + \alpha_{jj} x_j^a(2t) + \alpha_{jk} x_k^a(2t);$$

(20) $$P_k(2t) = B_k(2t) + \alpha_{kj} x_j^a(2t) + \alpha_{kk} x_k^a(2t);$$

where the alpha coefficients (α_{jj}, α_{jk}, α_{kk}, and α_{kj}) are the same as in Theorem 1.

(ii) When only firm j discloses at time $2t+1$ $\forall t$ (i.e., firm k does not disclose), firm values are determined by:

(21) $$P_j(2\mathrm{t}+1) = B_j(2\mathrm{t}+1) + \delta_j x_j^a(2\mathrm{t}+1) + \delta_{jj} x_j^a(2\mathrm{t}) + \delta_{jk} x_k^a(2\mathrm{t});$$

(22) $$P_k(2\mathrm{t}+1) = R_f B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + \delta_k x_j^a(2\mathrm{t}+1) + \delta_{kj} x_j^a(2\mathrm{t}) + \delta_{kk} x_k^a(2\mathrm{t});$$

where:

$\delta_j = \alpha_{jj}$ $\quad$ $\delta_{jj} = \alpha_{jk}\omega_{kj}$ $\quad$ $\delta_{jk} = \alpha_{jk}\omega_{kk}$

$\delta_k = \alpha_{kj}$ $\quad$ $\delta_{kj} = \Delta^{-1}\omega_{kj}\{R_F(R_F - \omega_{jj})\}$ $\quad$ $\delta_{kk} = \Delta^{-1}\omega_{kk}\{R_F(R_F - \omega_{jj})\}$.

Investors do not observe firm k's book value or abnormal earnings at $2\mathrm{t}+1$, as firm k does not provide financial statements in odd periods. Theorem 7 uses CSR to express investors' beliefs about firm k's undisclosed book value and LID to express their beliefs about undisclosed abnormal earnings:

[13] Firm k does not provide financial statement information at time $2t+1$, and thus investors do not observe $B_k(2\mathrm{t}+1)$. Instead investors use CSR and information about the dividends paid by firm k at $2t+1$ to estimate $B_k(2\mathrm{t}+1)$. Using CSR we can express the expected value of firm k's book value at $2t+1$ as $E_{2t+1}[B_k(2\mathrm{t}+1)] = B_k\widehat{(2\mathrm{t}+1)} = R_F B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + x_k^a\widehat{(2\mathrm{t}+1)}$, where $x_k^a\widehat{(2\mathrm{t}+1)} = E_{2t+1}[x_k^a(2\mathrm{t}+1)]$.

$$E_{2t+1}[\tilde{B}_k(2\mathrm{t}+1)] = R_f B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)];$$

$$E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)] = \omega_{kj} x_j^a(2t) + \omega_{kk} x_k^a(2t).$$

The valuation formulas presented in Theorem 7 oscillate between even and odd periods. In even periods, both firms provide financial statement information, and Equations (19) and (20) characterize prices that rely only on the most recently disclosed information by both firms. In odd periods, only firm j provides financial statement information, and Equations (23) and (24) characterize prices that use the most recently disclosed information by firm j and the information disclosed by both firms j and k in the previous period. Because the available information differs between even and odd periods, the pricing functions along with the pricing coefficients differ as well. In particular, the pricing functions and pricing coefficients differ not only for firm k but also for firm j, as, in the odd periods, investors use expectations about the nondisclosing firm (firm k) to update firm j's price. In periods where only firm j discloses, investors base prices on the best estimate of the unobservable information, i.e., expectations about firm k's undisclosed financial information.

THEOREM 8: Given PVED and the LID presented in equations (4) and (5):

(i) Firms' excess stock returns from period $2t$, where both firms disclose, to period $2t+1$, where only firm j discloses, are given by:

$$\widetilde{Ret}_j(2\mathrm{t}+1) = (1+\alpha_{jj})\frac{\tilde{\varepsilon}_j(2\mathrm{t}+1)}{P_j(2\mathrm{t})};$$

$$\widetilde{Ret}_k(2\mathrm{t}+1) = \alpha_{kj}\frac{\tilde{\varepsilon}_j(2\mathrm{t}+1)}{P_k(2\mathrm{t})}.$$

(ii) Firms' excess stock returns from period $2t+1$, where only firm j discloses, to period $2t+2$, where both firms disclose, are given by:

$$\widetilde{Ret}_j(2t+2) = (1+\alpha_{jj})\frac{\tilde{\varepsilon}_j(2t+2)}{P_j(2t+1)} + \alpha_{jk}\frac{\tilde{\varepsilon}_k(2t+2)}{P_j(2t+1)};$$

$$\widetilde{Ret}_k(2\mathrm{t}+2) = \alpha_{kj}\frac{\tilde{\varepsilon}_j(2\mathrm{t}+2)}{P_k(2\mathrm{t}+1)} + (1+\alpha_{kk})\frac{\tilde{\varepsilon}_k(2\mathrm{t}+2)}{P_k(2\mathrm{t}+1)} + R_F\frac{\tilde{\varepsilon}_k(2\mathrm{t}+1)}{P_k(2\mathrm{t}+1)}.$$

Theorem 8 indicates that the firms' excess stock returns also change with publicly available information. In odd periods, the excess returns of both firms depend only on the disturbance term of firm j. Nevertheless, in even periods, following disclosures of financial information from both firms, the returns depend on the disturbance terms of both firms. The returns of firm k, $\widetilde{Ret}_k(2\mathrm{t}+2)$, also depend on prior disturbance terms, since the realization of these disturbance terms becomes observable only in even periods, when firm k's current and prior financial information is disclosed. While the firms' returns exhibit correlation in even and in odd periods, firm k's returns in even periods also exhibit autocorrelation, since they are correlated with historical information (i.e., earnings information from the previous period). This occurs as additional information about firm $k's$ past performance is revealed in even periods.

Overall, our analysis indicates that different disclosure frequencies alter the pricing coefficients of firms that provide more frequent disclosures as well as those that disclose less frequently. Arif and De George (2020) compare US firms that disclose every quarter to non-US firms that disclose semi-annually. They document that information transfers from US firms' announcements to non-US firms' stock returns are larger for quarters in which only the US firms disclose.

Our analytical results are consistent with Arif and De George's (2020) findings. Nevertheless, we also show that US firms' pricing may use abnormal earnings differently in periods where non-US firms disclose, relative to periods where information from non-US firms is unavailable. Therefore inter-firm information transfers may also arise for US firms in periods where non-US firms do not disclose, as, in these periods, prices for US firms reflect expectations about the undisclosed information by non-US firms. Consequently, returns may be altered for both US and non-US firms, relative to the hypothetical benchmark where all firms disclose with the same frequency. Our results suggest that empirical research should consider the role of information disclosure dynamics in firms' valuation by financial statements users. Furthermore, since returns may be shaped by disclosure frequency, returns in periods where all firms disclose may not be an appropriate control group for evaluating information transfers.

3.2. Valuation When One Firm Does Not Disclose

Not all firms publicly disclose financial statement information. For example, firms listed on the Pink Sheets below the SEC's size or shareholder-of-record thresholds are exempt from the 1934 Exchange Act's periodic reporting requirement. Furthermore, other firms that had publicly disclosed financial statement information may cease to do so, e.g., firms that abandon an IPO or public firms that were acquired and privatized. Givoly, Hayn, and Katz (2010) note that US firms with private equity but public debt also need to comply with the periodic reporting requirement. But these firms are exempt from periodic reporting to the SEC after their public debt is retired.

Investors can value firms that discontinue periodic reporting based on financial statement information of related firms. We propose an accounting-based valuation model that relies on the firm's last available financial statement disclosure before it ceased to provide financial statements to the public and subsequent financial statement information disclosed by related firms. We consider a setting with two firms, where firm j discloses every period and firm k only disclosed initially at time 0 and never thereafter. To illustrate, consider firm j as a public firm that discloses every period (e.g., every quarter or every year) and firm k as either a firm that abandoned an IPO or a public company that has gone private and last provided financial information at time $t = 0$.

In this setting, we assume the firms have direct inter-firm information transfer and no common or firm-specific other information. The available information for investors at time 0 is given by $\left\{B_j(0); x_j^a(0); d_j(0); B_k(0); x_k^a(0); d_k(0)\right\}$, and, in each subsequent period $t > 0$, investors' information is expanded with the additional information disclosed by firm j and with the observed dividends paid: $\{B_j(\mathrm{t}); x_j^a(\mathrm{t}); d_j(\mathrm{t}); d_k(\mathrm{t})\}$.[14] Although firm k does not provide financial statement information after time $t > 0$, investors perfectly infer total dividends from the dividend payments they receive, provided they know the fraction of the firm they own.[15]

Both firms disclose at $t = 0$, and therefore prices are characterized by Theorem 1. In the next period, $t = 1$, only firm j discloses, and prices are described by Theorem 7(ii). Moreover, following Theorem 8, excess stock returns at time $t = 1$ are given by:

$\widetilde{Ret}_j(1) = (1 + \alpha_{jj})\tilde{\varepsilon}_j(1)/P_j(0)$ and $\widetilde{Ret}_k(1) = \alpha_{kj}\tilde{\varepsilon}_j(1)/P_k(0)$.

In contrast to Theorem 2, in this setting with asymmetric disclosures, stock returns at time $t = 1$ simplify and do not depend on $\tilde{\varepsilon}_k(1)$. Moreover, since investors know the realizations of $x_j^a(0)$, $x_j^a(1)$ and $x_k^a(0)$, at the end of period $t = 1$, investors perfectly infer the realization of the disturbance term in the abnormal earnings of firm j: $\varepsilon_j(1) = x_j^a(1) - \omega_{jj}x_j^a(0) - \omega_{jk}x_k^a(0)$. For ease of presentation, we use the realizations of the disturbance terms and the posterior beliefs about disturbance terms to value the firms.

In period $t = 2$, firm j continues to disclose but firm k does not disclose for two consecutive periods. Although investors do not know the realizations of $\tilde{\varepsilon}_k(1)$, $\tilde{\varepsilon}_j(2)$ and $\tilde{\varepsilon}_k(2)$, investors do know the realizations of $x_j^a(0)$, $x_j^a(1)$, $x_j^a(2)$, and $x_k^a(0)$, and thus they update their beliefs regarding $\tilde{x}_k^a(1)$, $\tilde{\varepsilon}_k(1)$, and $\tilde{\varepsilon}_j(2)$ conditional on the realizations of the abnormal earnings. Using Equation (4), firm j's abnormal earnings at $t = 2$ can be expressed as: $x_j^a(2) = \omega_{jj}x_j^a(1) + \omega_{jk}\left(\omega_{jj}x_j^a(0) + \omega_{jk}x_k^a(0) + \tilde{\varepsilon}_k(1)\right) + \tilde{\varepsilon}_j(2)$.

To price the firms, we use the posterior distributions of $\tilde{\varepsilon}_k(1)$ and $\tilde{\varepsilon}_j(2)$ at time $t = 2$. We assume that investors hold the belief that disturbance terms are normally distributed to derive

[14] That is, investors' information set at time t is: $\{B_j(0), B_j(1), \dots, B_j(\mathrm{t}-1), B_j(\mathrm{t}); x_j^a(0), x_j^a(1), \dots, x_j^a(\mathrm{t}-1), x_j^a(\mathrm{t}); d_j(0), d_j(1), \dots, d_j(\mathrm{t}-1), d_j(\mathrm{t}); B_k(0); x_k^a(0); d_k(0), d_k(1), \dots, d_k(\mathrm{t}-1), d_k(\mathrm{t})\}$.

[15] For example, all UK private firms must file annually a return statement with a statement of capital and the names of the firm's directors. These firms also submit a confirmation statement with details of shareholders' names and holdings.

prices at time $t = 2$ and the resulting stock returns for period 2 (see Appendix D). In each period, investors update their beliefs, using the information disclosed by firm j, and price the realized and expected disturbance terms.

Overall, while at $t = 0$, the firm values are given by Theorem 1, in later periods, firm values depend on the initially disclosed information at $t = 0$ and the realizations of the abnormal earnings of firm j. As time passes, investors continue to use the information that was initially disclosed ($x_j^a(0)$, $x_k^a(0)$, and $\varepsilon_j(1)$) as well as new information that becomes available, i.e., more recent disturbance terms. See Appendix D for the firms values at $t > 0$.

In each period $t > 0$, the disturbance terms may have a nonzero expected value, given the realizations of the abnormal earnings of firm j, and thus investors also price these conditional disturbance terms. The effect of the conditional disturbance terms on the price appears to persist, suggesting that, while new information is disclosed by firm j, information pertaining to the disturbance terms remains relevant for valuation.

While a firm may not provide public disclosures, it may still have a stock price that reflects new information. For example, some firms listed on the Pink Sheets have stock prices without public disclosure of abnormal earnings. If privately informed traders possess information about the undisclosed abnormal earnings, then stock prices partially reflect that information.[16] Uninformed investors and researchers may use our model to infer the undisclosed abnormal earnings from stock prices. Furthermore, if stock prices are also affected by COI, researchers can infer multiple sources of undisclosed information (corresponding to firm k's abnormal earnings in section 3.2 along with unobservable COI), using stock returns for two or more firms.

[16] Burnett (2020) provides empirical-archival evidence that stock returns reflect financial statement information, even when not publicly disclosed. For OTCBB firms where stock returns are observable, researchers can infer undisclosed abnormal earnings by relying on LID.

COROLLARY 2: Assume that the disturbance terms in abnormal earnings are normally distributed with variances σ_j^2 and σ_k^2, respectively. If $\frac{\sigma_j^3}{\sigma_k^3} \neq -\omega_{jk}{}^2 \frac{(1+\alpha_{jj})}{(1+\alpha_{jj}\omega_{kj})}$, then

(i) $\widetilde{Ret}_j(1)$ and $\widetilde{Ret}_j(2)$ are correlated, and

(ii) $\widetilde{Ret}_k(1)$ and $\widetilde{Ret}_j(2)$ are correlated.

Further, if $\frac{\sigma_j^3}{\sigma_k^3} \neq -\omega_{jk} \frac{R_f+\omega_{kk}+\alpha_{kj}\omega_{jk}}{\alpha_{kj}}$, then

(iii) $\widetilde{Ret}_k(1)$ and $\widetilde{Ret}_k(2)$ are correlated, and

(iv) $\widetilde{Ret}_j(1)$ and $\widetilde{Ret}_k(2)$ are correlated.

Given that the firms' valuation depends on prior disturbance terms, excess stock returns may exhibit correlation across firms and over time. Corollary 2 illustrate this for $t = 1{,}2$ and shows that the returns of firm j and firm k are autocorrelated. Even though stock returns exhibit correlation, the stock markets are informationally efficient. In each period following the last disclosure by firm k, investors learn about the previous periods through the disclosures of firm j. While the firms' valuations anchor on the initially disclosed information and are updated using new financial information disclosed by firm j, as we move away from the last date when firm k provided financial information, investors price more disturbance terms, which may create correlation in returns.

According to Corollary 2, returns may exhibit patterns that are consistent with post-earnings announcement drift. Empirical studies document that, in periods following an earnings announcement, abnormal returns tend to be positively correlated with subsequent earnings surprises, implying a correlation between returns and earnings information that was disclosed in prior periods. Post-earnings announcement drift is often interpreted as a market anomaly, that is, a mispricing by investors who do not fully incorporate all the available earnings information into returns in a timely manner (Bernard and Thomas 1989, Abarbanell and Bernard 1992).

Nevertheless, Dontoh, Ronen, and Sarath (2003) show that patterns consistent with post-earnings announcement drift may arise in a rational expectations model, due to variation in share supply. Our results show that post-earnings announcement drift may also be driven by the information available in each period to rational investors operating in an informationally efficient market.

When some firms do not provide financial statements, investors react to prior periods' disturbance terms. This may explain the observed correlation between historical earnings information and current returns. As more information is revealed with respect to firm j, investors updated the posteriors of the disturbance terms of both firms, such that it appears *as if* historical earnings information is reflected in current returns with a delay. Thus the observed earnings drift is not due to mispricing or nontimely incorporation of information but rather arises from updates of investors' beliefs about undisclosed financial information. Overall, our result is consistent with information revelation and does not violate semi-strong market efficiency, as in each period all publicly available information is priced.

4. Conclusion

We extend Ohlson's (1995) accounting-based valuation setting from one firm to two related firms. This allows us to study inter-firm information transfers arising directly from another firm's abnormal earnings and indirectly from other information. We consider both common and firm-specific other information and show their effects on accounting-based valuation. We demonstrate that these results extend to a setting with multiple firms.

We evaluate inter-firm information transfers in three settings. In the first benchmark setting, two firms disclose with the same frequency at the end of each period. In the second setting, two firms with the same fiscal year-end disclose their interim financial statements with different frequency, such as semi-annually and quarterly. We show that prices of both are altered in periods where only one firm discloses, relative to the benchmark where both disclose in all periods. Moreover, returns change in all periods when one firm discloses less frequently (semi-annually instead of quarterly). In the third setting, we characterize information transfers between a disclosing firm and a nondisclosing one. This third setting permits valuation of firms listed on Pink Sheets, public firms that were privatized, and private firms that abandoned IPOs, among others. We show that, in a setting with only one disclosing firm, returns exhibit correlation across firms and over time, because subsequent announcements revise investors' information about previous announcements.

Following the analytical accounting-based valuation literature, we consider stock returns for longer time intervals, from instantaneously after an announcement of financial statements to instantaneously after the following announcement. We show how stock returns depend on the disturbance terms of the firm, its peer, and other information. However, stock returns can also be calculated for instantaneous short periods, from immediately before an announcement of financial statements to immediately afterward. This approach would allow the comparison of short-horizon stock returns around consecutive announcements of financial statements, as considered in recent research by Noh, So, and Verdi (2021).

Finally, we characterize various portfolios with portfolio weights selected based on LID parameters. These portfolios involve muting the shocks from each firm's abnormal earnings and other information as well as mimicking the effect of common other information. In sum, accounting-based valuation of multiple firms has different implications for portfolio selection in the presence of both common and firm-specific other information. Future research on portfolio selection might follow Feltham and Ohlson (1999) to explicitly incorporate risk aversion in accounting-based valuation through a change in probability measure. Lyle and Yohn (2021) consider risk aversion and portfolio selection articulating the link between accounting fundamentals and stock returns but do not model information trasfers. Future analytical research might also use accounting fundamentals to select firms for an equity portfolio with no initial cash outlay, taking a long position in some stocks and a short position in others.

References

Abarbanell, J. and Bernard, V. L. 1992. Tests of analyst overreaction/underreaction to earnings as an explanation for anomalous stock price behavior. *The Journal of Finance,* 47(3): 1181–1207.

Admati, A. R. 1985. A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica*, 53(3): 629–657.

Arif, S., and De George, E. T. 2020. The dark side of low financial reporting frequency: Investors' reliance on alternative sources of earnings news and excessive information spillovers. *The Accounting Review*, 95(6): 23-49.

Baginski, S. 1987. Intraindustry information transfer associated with management forecasts of earnings. *Journal of Accounting Research*, 25(2): 196–216.

Ball, R. and Sadka, G., 2015. Aggregate earnings and why they matter. *Journal of Accounting Literature*, 34: 39-57.

Bernard, D., Burgstahler, D., and Kaya, D. 2018. Size management by European private firms to minimize proprietary costs of disclosure. *Journal of Accounting and Economics*, 66(1): 94-122.

Bernard, V. L. 1995. The Feltham-Ohlson framework: Implications for empiricists. *Contemporary Accounting Research*, 11(2): 733–747.

Bernard, V. L., and Thomas, J. K. 1989. Post-earnings-announcement drift: Delayed price response or risk premium?. *Journal of Accounting Research*, 27(Supplement): 1-36.

Bhojraj, S., Mohanram, P. S., and Zhang, S. 2020. ETFs and information transfer across firms. *Journal of Accounting and Economics*, 70(2-3).

Brüggemann, U., Kaul, A., Leuz, C., and Werner, I. M. 2018. The twilight zone: OTC regulatory regimes and market quality. *Review of Financial Studies*, 31(3): 898-942.

Burnett, B. M. 2020. Do stock prices reflect undisclosed financial statement information? Evidence from the OTCBB. *Journal of Accounting and Public Policy*, 39(5): 1-18.

Bushee, B. J., and Leuz, C. 2005. Economic consequences of SEC disclosure regulation: Evidence from the OTC bulletin board. *Journal of Accounting and Economics,* 39(2): 233–264.

Butler, M., Kraft, A., and Weiss, I. S. 2007. The effect of reporting frequency on the timeliness of earnings: The cases of voluntary and mandatory interim reports. *Journal of Accounting and Economics*, 43(2): 181–217.

Christensen, P. O., and Feltham, G. A. 2009. Equity valuation. *Foundations and Trends® in Accounting,* 4(1): 1-112.

Clubb, C. D. B. 2013. Information dynamics, dividend displacement, conservatism, and earnings measurement: A development of the Ohlson (1995) valuation framework. *Review of Accounting Studies,* 18(2): 360-385.

Dechow, P.M., Hutton, A.P. and Sloan, R.G., 1999. An empirical assessment of the residual income valuation model. *Journal of Accounting and Economics,* 26(1-3): 1-34.

Dontoh, A., and Ronen, J. 1993. Information content of accounting announcements. *The Accounting Review*, 68(4): 857–869.

Dontoh, A., Ronen, J., and Sarath, B. 2003. On the rationality of the post-announcement drift. *Review of Accounting Studies,* 8(1): 69-104.

Easton, P. D., and Harris, T. S. 1991. Earnings as an explanatory variable for returns. *Journal of Accounting Research*, 29(1): 19–36.

Easton, P. D., Harris, T. S., and Ohlson, J. A. 1992. Aggregate accounting earnings can explain most of security returns: The case of long return intervals. *Journal of Accounting and Economics* 15(2-3): 119-142.

Easton, P. D., and Pae, J. 2004. Accounting conservatism and the relation between returns and accounting data. *Review of Accounting Studies,* 9(4): 495–521.

Feltham, G. A., and Ohlson, J. A. 1995. Valuation and clean surplus accounting for operating and financial activities. *Contemporary Accounting Research*, 11(2): 689–731.

Feltham, G. A., and Ohlson, J. A. 1999. Residual earnings valuation with risk and stochastic interest rates. *The Accounting Review*, 74(2): 165–183.

Finn, M. W., and Ye, J. 1999. Nonlinear accounting-based equity valuation models. *Proceedings of the 31st Symposium on the Interface: Computing Science and Statistics*, 317–326.

Foster, G. 1981. Intra-industry information transfers associated with earnings release. *Journal of Accounting and Economics*, 3(3): 201–232.

Frankel, R. M., and Lee, C. M., 1998. Accounting valuation, market expectation, and cross-sectional stock returns. *Journal of Accounting and Economics*, *25*(3): 283-319.

Freeman, R., and Tse, S. 1992. An earnings prediction approach to examining intercompany information transfers. *Journal of Accounting and Economics*, 15(4): 509–523.

Fu, R., Kraft, A., and Zhang, H. 2012. Financial reporting frequency, information asymmetry, and the cost of equity. *Journal of Accounting and Economics*, 54(2): 132–149.

Garman, M. B., and Ohlson, J. A. 1980. Information and the sequential valuation of assets in arbitrage-free economies. *Journal of Accounting Research,* 18(2): 420–440.

Givoly, D., Hayn, C. K., and Katz, S P. 2010. Does public ownership of equity improve earnings quality?. *The Accounting Review,* 85(1): 195-225.

Gode, D., and Ohlson, J. A. 2004. Accounting-based valuation with changing interest rates. *Review of Accounting Studies,* 9(4): 419–441.

Han, J. C. Y., and Wild, J. J. 1990. Unexpected earnings and intraindustry information transfers: Further evidence. *Journal of Accounting Research,* 28(1): 211–219.

Han, J. C. Y., Wild, J. J., and Ramesh. K. 1989. Managers' earnings forecasts and intra-industry information transfer. *Journal of Accounting and Economics*, 11(1): 3–33.

Holthausen, R. W., and Verrecchia, R. E. 1988. The effect of sequential information releases on the variance of price changes in an intertemporal multi-asset market. *Journal of Accounting Research*, 26(1): 82-106.

Jackson, A. B., Plumlee, M. A., and Rountree, B. R. 2018. Decomposing the market, industry, and firm components of profitability: Implications for forecasts of profitability. *Review of Accounting Studies*, 23(3): 1071-1095.

Kajuter, P., Klassmann, F., and Nienhaus, M. 2018. The effect of mandatory quarterly reporting on firm value. *The Accounting Review,* 94(3): 251–277**.**

Kausar, A., Shroff, N., and White, H. 2016. Real effects of the audit choice. *Journal of Accounting and Economics*, 62(1): 157-181.

Kothari, S. P., Lewellen, J. and Warner, J. B. 2006. Stock returns, aggregate earnings surprises, and behavioral finance. *Journal of Financial Economics*, 79(3): 537-568.

Kraft, A. G., Vashishtha, R., and Venkatachalam, M. 2018. Frequent financial reporting and managerial myopia. *The Accounting Review*, 93(2): 249–275.

Lyle, M. R., Callen, J. L. and Elliott, R. J. 2013. Dynamic risk, accounting-based valuation and firm fundamentals. *Review of Accounting Studies*, 18(4): 899-929.

Lyle, M. R., and Yohn, T. L. 2021. Fundamental analysis and mean-variance optimal portfolios. *The Accounting Review* (forthcoming).

Ma, P., Shin, J.-E., and Wang, C. C. Y. 2021. rTSR: When do relative performance metrics capture relative performance?. Working Paper, Harvard Business School.

Myers, J. N. 1999. Implementing residual income valuation with linear information dynamics. *The Accounting Review,* 74(1): 1–28.

Nekrasov, A., and Shroff, P. K. 2009. Fundamentals-based risk measurement in valuation. *The Accounting Review*, 84(6): 1983-2011.

Noh, S., So, E. C., and Verdi, R. S. 2021. Calendar rotations: A new approach for studying the impact of timing using earnings announcements. *Journal of Financial Economics*, 143(3): 865-893.

O'hanlon, J., and Peasnell, K. V. 2002. Residual income and value-creation: The missing link. *Review of Accounting Studies* 7(2-3): 229-245.

Ohlson, J. A. 1995. Earnings, book values, and dividends in equity valuation. *Contemporary Accounting Research*, 11(2): 661–687.

Peasnell, K. V. 1982. Some formal connections between economic values and yields and accounting numbers. *Journal of Business Finance and Accounting*, 9(3): 361–381.

Penman, S. H. 1992. Return to fundamentals. *Journal of Accounting, Auditing, and Finance*, 7(4): 465–483.

Preinreich, G. A. D. 1936. The law of goodwill. *The Accounting Review*, 11(4): 317–329

Preinreich, G. A. D. 1938. Annual survey of economic theory: The theory of depreciation. *Econometrica*, 6(3): 219-241.

Ramnath, S. 2002. Investor and analyst reactions to earnings announcements of related firms: An empirical analysis. *Journal of Accounting Research*, 40(5): 1351–1376.

Sadka, G. 2007. Understanding stock price volatility: The role of earnings. *Journal of Accounting Research*, 45(1): 199-228.

Shroff, N., Verdi, R. S., and Yost. B. 2017. When does peer-firm information matter? *Journal of Accounting and Economics*, 64(2-3): 183–214.

Thomas, J., and Zhang, F. 2008. Overreaction to intra-industry information transfers?. *Journal of Accounting Research,* 46(4): 909-940.

U. S. Securities and Exchange Commission. 2019. Publication or submission of quotations without specified information. Release No. 34-87115; File No. S7-14-19.

Wang, C. 2014. Accounting standards harmonization and financial statement comparability: Evidence from transnational information transfer. *Journal of Accounting Research*, 52(4): 955-992.

Appendix A: Variable Definitions

n	Number of firms
j, k	Firm indicators in two-firm setting
t	Time indicator
$B_j(\mathrm{t})$	Firm j's book value of equity at date t
$x_j(\mathrm{t})$	Firm j's earnings at date t
$x_j^a(\mathrm{t})$	Firm j's abnormal earnings at date t
$d_j(\mathrm{t})$	Dividends paid by firm j at date t
$P_j(\mathrm{t})$	Ex-dividend market price of firm j at date t
R_F	One plus the risk-free interest rate $(1 + r)$
$MB_j(\mathrm{t})$	Market-to-book ratio of firm j at date t
$ROE_j(\mathrm{t})$	Return on equity of firm j at date t
$\tilde{\varepsilon}_j(\mathrm{t}+1)$	Disturbance term to firm j's abnormal earnings
$\widetilde{Ret}_j(\mathrm{t}+1)$	One-period ahead excess stock returns of firm j
$v(\mathrm{t})$	Common other information at date t
$\tilde{\varepsilon}_v(\mathrm{t}+1)$	Disturbance term to common other information
$v_j(\mathrm{t})$	Firm j's firm-specific other information at date t
$\tilde{\varepsilon}_{vj}(\mathrm{t}+1)$	Disturbance term to firm j's firm-specific other information

APPENDIX B: PROOFS

PROOF OF THEOREM 1

We initially follow Ohlson (1995) and assume that full information set, $\Omega_1(\mathrm{t})$, is common knowledge at any time t:

$$\Omega_1(\mathrm{t}) = \{B_j(0), x_j^a(0), B_k(0), x_k^a(0), B_j(1), x_j^a(1), B_k(1), x_k^a(1), \ldots, B_j(\mathrm{t}), x_j^a(\mathrm{t}), B_k(\mathrm{t}), x_k^a(\mathrm{t})\}.$$

We express LID without other information as: $X_2(\mathrm{t}+1) = R_F M_2 X_2(\mathrm{t}) + \begin{pmatrix} \tilde{\varepsilon}_j(\mathrm{t}+1) \\ \tilde{\varepsilon}_k(\mathrm{t}+1) \end{pmatrix}$

Where $X_2(\mathrm{t}) = \begin{pmatrix} x_j^a(\mathrm{t}) \\ x_k^a(\mathrm{t}) \end{pmatrix}$ is a 1 × 2-vector and $M_2 = R_F^{-1} \begin{pmatrix} \omega_{jj} & \omega_{jk} \\ \omega_{kj} & \omega_{kk} \end{pmatrix}$ is an invertible 2 × 2-matrix with non-zero determinant: $|M_2| = R_F^{-2}(\omega_{jj}\omega_{kk} - \omega_{jk}\omega_{kj})$. The LID assumption implies that the most recent information, $\{B_j(\mathrm{t}); x_j^a(\mathrm{t}); B_k(\mathrm{t}); x_k^a(\mathrm{t})\}$, forms a sufficient statistic for all prior periods information with regards to valuation of both firms. Following Appendix 1 in Ohlson (1995):

$$R_F^{-1} E_t[\tilde{x}_j^a(\mathrm{t}+1)] = (1 \ \ 0) M_2 X_2(\mathrm{t})$$

and for $\tau > 1$:

$$R_F^{-\tau} E_t[\tilde{x}_j^a(\mathrm{t}+\tau)] = (1 \ \ 0) M_2^\tau X_2(\mathrm{t}).$$

We evaluate Ohlson's goodwill expression:

$$g_j(\mathrm{t}) = P_j(\mathrm{t}) - B_j(\mathrm{t}) = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t[\tilde{x}_j^a(\mathrm{t}+\tau)] = (1 \ \ 0)\left(\sum_{\tau=1}^{\infty} M_2^\tau\right) X_2(\mathrm{t})$$
$$= (1 \ \ 0)(M_2 + M_2^2 + M_2^3 + \cdots) X_2(\mathrm{t}) = \alpha_{jj} x_j^a(\mathrm{t}) + \alpha_{jk} x_k^a(\mathrm{t}).$$

Assuming that the maximum characteristic root of the matrix M_2 is less than one, it follows that the infinite sum of the matrix series $(M_2 + M_2^2 + M_2^3 + \cdots)$ converges to the 2 × 2-matrix $M_2(I_2 - M_2)^{-1}$ where $I_2 = diag(1\,,1)$ is the 2 × 2-identity matrix. Thus, the vector of valuation coefficients is: $(\alpha_{jj} \ \ \alpha_{jk}) = (1 \ \ 0) M_2 (I_2 - M_2)^{-1}$.

Proceeding similarly for firm k, we find that:

$$g_k(\mathrm{t}) = P_k(\mathrm{t}) - B_k(\mathrm{t}) = (M_2 + M_2^2 + M_2^3 + \cdots) X_2(\mathrm{t}) = \alpha_{kj} x_j^a(\mathrm{t}) + \alpha_{kk} x_k^a(\mathrm{t})$$

Such that: $(\alpha_{kj} \ \ \alpha_{kk}) = (0 \ \ 1) M_2 (I_2 - M_2)^{-1}$

The comparative statics for Theorem 1 are as follows:

$$\frac{\partial \alpha_{jj}}{\partial \omega_{jj}} = \Delta^{-2}(R_F - \omega_{kk})^2 > 0 \qquad \frac{\partial \alpha_{jj}}{\partial \omega_{kk}} = \Delta^{-2} R_F \omega_{jk} \omega_{kj}$$

$$\frac{\partial \alpha_{jj}}{\partial \omega_{jk}} = \Delta^{-2} R_F (R_F - \omega_{jj}) \omega_{kj} \qquad \frac{\partial \alpha_{jj}}{\partial \omega_{kj}} = \Delta^{-2} R_F (R_F - \omega_{jj}) \omega_{jk}$$

$$\frac{\partial \alpha_{jk}}{\partial \omega_{jj}} = \Delta^{-2} R_F (R_F - \omega_{kk}) \omega_{jk} \qquad \frac{\partial \alpha_{jk}}{\partial \omega_{kk}} = \Delta^{-2} \omega_{jk} R_F (R_F - \omega_{jj})$$

$$\frac{\partial \alpha_{jk}}{\partial \omega_{jk}} = \Delta^{-2} R_F (R_F - \omega_{jj})(R_F - \omega_{kk}) > 0 \qquad \frac{\partial \alpha_{jk}}{\partial \omega_{kj}} = \Delta^{-2} R_F \omega_{jk}^2 > 0$$

We note that the proof of Theorem 1 generalizes to multiple firms without other information. Consider an n-firm setting with LID is given by:

$$\begin{pmatrix} x_1^a(\mathrm{t}+1) \\ x_2^a(\mathrm{t}+1) \\ \vdots \\ x_n^a(\mathrm{t}+1) \end{pmatrix} = R_F M_{n,G} \begin{pmatrix} x_1^a(\mathrm{t}) \\ x_2^a(\mathrm{t}) \\ \vdots \\ x_n^a(\mathrm{t}) \end{pmatrix} + \begin{pmatrix} \tilde{\varepsilon}_1(\mathrm{t}+1) \\ \tilde{\varepsilon}_2(\mathrm{t}+1) \\ \vdots \\ \tilde{\varepsilon}_n(\mathrm{t}+1) \end{pmatrix},$$

where $M_{n,G}$ is a $n \times n$-matrix. The vector of valuation coefficients of any firm j is:

$$\left(\alpha_{j1}\ \ \alpha_{j2} \dots \alpha_{jj} \dots\ \alpha_{jn}\right) = (0\ \ 0 \dots\ 1 \dots\ 0) M_{n,G}\left(I_n - M_{n,G}\right)^{-1},$$

where $I_n = diag(1\ \dots\ 1)$ is the $n \times n$-identity matrix. In the absence of direct inter-firm information transfers, $M_{n,G} = R_F^{-1} diag(\omega_{11}\ \ \dots \dots\ \omega_{nn})$.

PROOF OF THEOREM 2

Following Ohlson (1995), we define one-period ahead excess stock returns of firm j as:

$$\widetilde{Ret}_j(\mathrm{t}+1) = \left(\tilde{P}_j(\mathrm{t}+1) + d_j(\mathrm{t}+1) - R_F P_j(\mathrm{t})\right) / P_j(\mathrm{t})$$

Proceeding as in Easton and Pae (2004), we rewrite the numerator of excess returns of firm j as:

$B_j(\mathrm{t}+1) + d_j(\mathrm{t}+1) - R_F B_j(\mathrm{t}) + \alpha_{jj} x_j^a(\mathrm{t}+1) + \alpha_{jk} x_k^a(\mathrm{t}+1) - R_F \alpha_{jj} x_j^a(\mathrm{t}) - R_F \alpha_{jk} x_k^a(\mathrm{t})$

The first three terms on the right-hand side reduce by CSR to abnormal earnings, $x_j^a(\mathrm{t}+1)$. By substitution of the definition of abnormal earnings for both firms, the remaining terms can be rewritten as:

$$\left(1+\alpha_{jj}\right) x_j^a(\mathrm{t}+1) + \alpha_{jk} x_k^a(\mathrm{t}+1) - R_F \alpha_{jj} x_j^a(\mathrm{t}) - R_F \alpha_{jk} x_k^a(\mathrm{t}) = \left(1+\alpha_{jj}\right)\left(\omega_{jj} x_j^a(\mathrm{t}) + \omega_{jk} x_k^a(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1)\right) + \alpha_{jk}\left(\omega_{kj} x_j^a(\mathrm{t}) + \omega_{kk} x_k^a(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1)\right) - R_F \alpha_{jj} x_j^a(\mathrm{t}) - R_F \alpha_{jk} x_k^a(\mathrm{t}) = \left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1) + \zeta_{jj} x_j^a(\mathrm{t}) + \zeta_{jk} x_k^a(\mathrm{t})$$

Given the alpha coefficients from Theorem 1 we get:

$$\zeta_{jj} = \left(1+\alpha_{jj}\right)\omega_{jj} + \alpha_{jk}\omega_{kj} - R_F \alpha_{jj} = 0$$

$$\zeta_{jk} = \left(1+\alpha_{jj}\right)\omega_{jk} + \alpha_{jk}\omega_{kk} - R_F \alpha_{jk} = 0$$

Therefore, $\widetilde{Ret}_j(\mathrm{t}+1) = \left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1)/P_j(\mathrm{t}) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1)/P_j(\mathrm{t})$ and similarly, $\widetilde{Ret}_k(\mathrm{t}+1) = (1+\alpha_{kk})\tilde{\varepsilon}_k(\mathrm{t}+1)/P_k(\mathrm{t}) + \alpha_{kj}\tilde{\varepsilon}_j(\mathrm{t}+1)/P_k(\mathrm{t})$.

We next present the variances of the firms' returns and the covariance of their returns:

$$VAR\left[\widetilde{Ret}_j(\mathrm{t}+1)\right] = \left(\left(1+\alpha_{jj}\right)^2 \sigma_j^2 + \alpha_{jk}^2 \sigma_k^2\right) / \left(P_j(\mathrm{t})\right)^2$$

$$VAR\left[\widetilde{Ret}_k(\mathrm{t}+1)\right] = \left((1+\alpha_{kk})^2 \sigma_k^2 + \alpha_{kj}^2 \sigma_j^2\right) / \left(P_k(\mathrm{t})\right)^2$$

$$COV\left[\widetilde{Ret}_j(\mathrm{t}+1), \widetilde{Ret}_k(\mathrm{t}+1)\right] = \left(\left(1+\alpha_{jj}\right)\alpha_{kj}\sigma_j^2 + (1+\alpha_{kk})\alpha_{jk}\sigma_k^2\right) / \left(P_j(\mathrm{t}) P_k(\mathrm{t})\right)$$

Where $\sigma_j^2 = VAR\left[\tilde{\varepsilon}_j(\mathrm{t}+1)\right]$ and $\sigma_k^2 = VAR[\tilde{\varepsilon}_k(\mathrm{t}+1)]$. Hence, the correlation between the returns of firm j and the returns of firm k is:

$$\rho = Corr\left[\widetilde{Ret}_j(\mathrm{t}+1), \widetilde{Ret}_k(\mathrm{t}+1)\right] = \frac{\left((1+\alpha_{jj})\alpha_{kj}\sigma_j^2 + (1+\alpha_{kk})\alpha_{jk}\sigma_k^2\right)}{\left((1+\alpha_{jj})^2\sigma_j^2 + \alpha_{jk}^2\sigma_k^2\right)^{1/2}\left((1+\alpha_{kk})^2\sigma_k^2 + \alpha_{kj}^2\sigma_j^2\right)^{1/2}}$$

Theorem 2 shows that the stock returns are independent over time. Further, stock returns for any given period t are correlated between firms j and k, through the firms' abnormal earnings disturbance terms. The variance of returns and the covariance between the firms' returns can be expressed as the weighted sum of the disturbance terms' variances.

PROOF OF THEOREM 3

We define LID with common other information as follows: $X_3(\mathrm{t}+1) = R_F M_3 X_3(\mathrm{t}) + \begin{pmatrix} \tilde{\varepsilon}_j(\mathrm{t}) \\ \tilde{\varepsilon}_k(\mathrm{t}) \\ \tilde{\varepsilon}_v(\mathrm{t}) \end{pmatrix}$,

where $X_3(\mathrm{t}) = \begin{pmatrix} x_j^a(\mathrm{t}) \\ x_k^a(\mathrm{t}) \\ v(\mathrm{t}) \end{pmatrix}$ is a 3 × 1-vector, and $M_3 = R_F^{-1}\begin{pmatrix} \omega_{jj} & \omega_{jk} & 1 \\ \omega_{kj} & \omega_{kk} & 1 \\ 0 & 0 & \gamma \end{pmatrix}$, is an invertible 3 × 3-matrix with non-zero determinant: $|M_3| = R_F^{-2}\left(\omega_{jj}\omega_{kk} - \omega_{jk}\omega_{kj}\right)\gamma$. Following Appendix 1 in Ohlson (1995):

$$R_F^{-1}E_t\left[\tilde{x}_j^a(\mathrm{t}+1)\right] = (1\ \ 0\ \ 0)M_3X_3(\mathrm{t})$$

and for $\tau > 1$:

$$R_F^{-\tau}E_t\left[\tilde{x}_j^a(\mathrm{t}+\tau)\right] = (1\ \ 0\ \ 0)M_3^{\tau}X_3(\mathrm{t})$$

As in proof of proposition 1, we evaluate the Ohlson's goodwill expression:

$$g_j(\mathrm{t}) = \sum_{\tau=1}^{\infty} R_F^{-\tau}E_t\left[\tilde{x}_j^a(\mathrm{t}+\tau)\right] = (1\ \ 0\ \ 0)\left(\sum_{\tau=1}^{\infty} M_3^{\tau}\right)X_3(\mathrm{t}) = \alpha_{jj}x_j^a(\mathrm{t}) + \alpha_{jk}x_k^a(\mathrm{t}) + \beta_j v(\mathrm{t}).$$

Goodwill for firm k is derived similarly as:

$$g_k(\mathrm{t}) = (0\ \ 1\ \ 0)\left(\sum_{\tau=1}^{\infty} M_3^{\tau}\right)X_3(\mathrm{t}) = \alpha_{kj}x_j^a(\mathrm{t}) + \alpha_{kk}x_k^a(\mathrm{t}) + \beta_k v(\mathrm{t}).$$

To extend to the proof for the common other information setting with 3 firms denoted by j, k, and l, we would define the invertible 4 × 4-matrix, $R_F^{-1}\begin{pmatrix} \omega_{jj} & \omega_{jk} & \omega_{jl} & 1 \\ \omega_{kj} & \omega_{kk} & \omega_{kl} & 1 \\ \omega_{lj} & \omega_{lk} & \omega_{ll} & 1 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$, with non-zero determinant:

$$Det = R_F^{-3}\left(\omega_{jj}\omega_{kk}\omega_{ll} - \omega_{jj}\omega_{kl}\omega_{lk} - \omega_{jk}\omega_{kj}\omega_{ll} + \omega_{jk}\omega_{kl}\omega_{lj} + \omega_{jl}\omega_{kj}\omega_{lk} - \omega_{jl}\omega_{lj}\omega_{kk}\right)\gamma.$$

The generalized setting with n firms and common other information would require a $(\mathrm{n}+1)$ × $(\mathrm{n}+1)$-matrix. In the absence of direct inter-firm information transfers, this matrix reduces to a diagonal matrix $R_F^{-1}diag(\omega_{11} \ldots\ldots \omega_{nn}\ \gamma)$ with non-zero determinant $Det = R_F^{-n}(\omega_{11}\omega_{22} \ldots\ldots \omega_{nn})\gamma$.

PROOF OF THEOREM 4

To derive the stock returns with common information, we proceed as the proof of proposition 2 based on Easton and Pae (2004), we use CSR to rewrite the numerator in stock returns of firm j as:

$P_j(\mathrm{t}+1) + d_j(\mathrm{t}+1) - R_F P_j(\mathrm{t}) = (1+\alpha_{jj})x_j^a(\mathrm{t}+1) + \alpha_{jk}x_k^a(\mathrm{t}+1) + \beta_j v(\mathrm{t}+1) - R_F\alpha_{jj}x_j^a(\mathrm{t}) - R_F\alpha_{jk}x_k^a(\mathrm{t}) - R_F\beta_j v(\mathrm{t})$

Substitution of abnormal earnings for both firms and common other information, the right-hand side becomes:

$(1+\alpha_{jj})\left(\omega_{jj}x_j^a(\mathrm{t}) + \omega_{jk}x_k^a(\mathrm{t}) + v(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1)\right) + \alpha_{jk}\left(\omega_{kj}x_j^a(\mathrm{t}) + \omega_{kk}x_k^a(\mathrm{t}) + v(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1)\right) + \beta_j\left(\gamma v(\mathrm{t}) + \tilde{\varepsilon}_v(\mathrm{t}+1)\right) - R_F\alpha_{jj}x_j^a(\mathrm{t}) - R_F\alpha_{jk}x_k^a(\mathrm{t}) - R_F\beta_j v(\mathrm{t}) = (1+\alpha_{jj})\tilde{\varepsilon}_j(\mathrm{t}+1) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1) + \beta_j\tilde{\varepsilon}_v(\mathrm{t}+1) + \zeta_{jj}x_j^a(\mathrm{t}) + \zeta_{jk}x_k^a(\mathrm{t}) + \zeta_{jv}\mathrm{v}(\mathrm{t})$

Where given the alpha coefficients are the same as in Theorem 1, it follows from proof of Theorem 2 that $\zeta_{jj} = \zeta_{jk} = 0$, and we verify that: $\zeta_{jv} = (1+\alpha_{jj}) + \alpha_{jk} + \beta_j\gamma - R_F\beta_j = 0$.

Hence, stock returns for firm j is:

$$\mathrm{Ret}_j(\mathrm{t}+1) = (1+\alpha_{jj})\tilde{\varepsilon}_j(\mathrm{t}+1) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1) + \beta_j\tilde{\varepsilon}_v(\mathrm{t}+1)$$

Stock returns for firm k follows immediately by symmetry.

Moreover, let $\sigma_v^2 = VAR[\tilde{v}(\mathrm{t}+1)]$, then the variance of returns of firm j is $VAR\left[\widetilde{Ret}_j(\mathrm{t}+1)\right] = \left((1+\alpha_{jj})^2\sigma_j^2 + \alpha_{jk}^2\sigma_k^2 + \beta_j^2\sigma_v^2\right) / \left(P_j(\mathrm{t})\right)^2$ and the covariance between firms' returns is: $COV\left[\widetilde{Ret}_j(\mathrm{t}+1), \widetilde{Ret}_k(\mathrm{t}+1)\right] = \left((1+\alpha_{jj})\alpha_{kj}\sigma_j^2 + (1+\alpha_{kk})\alpha_{jk}\sigma_k^2 + \beta_j\beta_k\sigma_v^2\right) / \left(P_j(\mathrm{t})P_k(\mathrm{t})\right)$.

PROOF OF THEOREM 5

LID with firm-specific other information can be written as:

$$X_4(\mathrm{t}+1) = R_F M_4 X_4(\mathrm{t}) + \begin{pmatrix} \tilde{\varepsilon}_j(\mathrm{t}+1) \\ \tilde{\varepsilon}_k(\mathrm{t}+1) \\ \tilde{\varepsilon}_{vj}(\mathrm{t}+1) \\ \tilde{\varepsilon}_{vk}(\mathrm{t}+1) \end{pmatrix},$$

where $X_4(\mathrm{t}) = \begin{pmatrix} x_j^a(\mathrm{t}) \\ x_k^a(\mathrm{t}) \\ v_j(\mathrm{t}) \\ v_k(\mathrm{t}) \end{pmatrix}$ is a 4 × 1-vector, and $M_4 = R_F^{-1}\begin{pmatrix} \omega_{jj} & \omega_{jk} & 1 & 0 \\ \omega_{kj} & \omega_{kk} & 0 & 1 \\ 0 & 0 & \gamma_j & 0 \\ 0 & 0 & 0 & \gamma_k \end{pmatrix}$ is an invertible 4 × 4-matrix with non-zero determinant: $|M_4| = R_F^{-2}(\omega_{jj}\omega_{kk} - \omega_{jk}\omega_{kj})\gamma_j\gamma_k$.

Following Appendix 1 in Ohlson (1995):

$$R_F^{-1} E_t\left[\tilde{x}_j^a(\mathrm{t}+1)\right] = (1\ \ 0\ \ 0\ \ 0) M_4 X_4(\mathrm{t})$$

and for $\tau > 1$:

$$R_F^{-\tau} E_t\left[\tilde{x}_j^a(\mathrm{t}+\tau)\right] = (1\ \ 0\ \ 0\ \ 0) M_4^\tau X_4(\mathrm{t})$$

As in proof of proposition 1, we evaluate the Ohlson's goodwill expression:

$$g_j(\mathrm{t}) = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_t\left[\tilde{x}_j^a(\mathrm{t}+\tau)\right] = (1\ \ 0\ \ 0\ \ 0)\left(\sum_{\tau=1}^{\infty} M_4^\tau\right) X_4(\mathrm{t})$$
$$= \alpha_{jj} x_j^a(\mathrm{t}) + \alpha_{jk} x_k^a(\mathrm{t}) + \beta_{jj} v_j(\mathrm{t}) + \beta_{jk} v_k(\mathrm{t})$$

As in the proofs of Theorem 1, the solution is symmetric for firm k and yields:

$$g_k(\mathrm{t}) = (0\ \ 1\ \ 0\ \ 0)\left(\sum_{\tau=1}^{\infty} M_4^\tau\right) X_4(\mathrm{t}) = \alpha_{kj} x_j^a(\mathrm{t}) + \alpha_{kk} x_k^a(\mathrm{t}) + \beta_{kj} v_j(\mathrm{t}) + \beta_{kk} v_k(\mathrm{t}).$$

To extend to the proof for a common other information setting with 3 and n firms, we would need 6×6-matrix and $(2\mathrm{n}) \times (2\mathrm{n})$-matrices, respectively.

PROOF OF THEOREM 6

To derive the returns with common information, we proceed as the proof of proposition 2 based on Easton and Pae (2004), we use CSR to rewrite the numerator in returns of firm j as:

$P_j(\mathrm{t}+1) + d_j(\mathrm{t}+1) - R_F P_j(\mathrm{t}) = \left(1+\alpha_{jj}\right) x_j^a(\mathrm{t}+1) + \alpha_{jk} x_k^a(\mathrm{t}+1) + \beta_{jj} v(\mathrm{t}+1) + +\beta_{jk} v_k(\mathrm{t}+1) - R_F \alpha_{jj} x_j^a(\mathrm{t}) - R_F \alpha_{jk} x_k^a(\mathrm{t}) - R_F \beta_{jj} v(\mathrm{t}) - R_F \beta_{jk} v_k(\mathrm{t})$

Substitution of abnormal earnings for both firms and common information, the right-hand side becomes: $\left(1+\alpha_{jj}\right)\left(\omega_{jj} x_j^a(\mathrm{t}) + \omega_{jk} x_k^a(\mathrm{t}) + v_j(\mathrm{t}) + \tilde{\varepsilon}_j(\mathrm{t}+1)\right) + \alpha_{jk}\left(\omega_{kj} x_j^a(\mathrm{t}) + \omega_{kk} x_k^a(\mathrm{t}) + v_k(\mathrm{t}) + \tilde{\varepsilon}_k(\mathrm{t}+1)\right) + \beta_{jj}\left(\gamma_j v_j(\mathrm{t}) + \tilde{\varepsilon}_{vj}(\mathrm{t}+1)\right) + \beta_{jk}\left(\gamma_k v_k(\mathrm{t}) + \tilde{\varepsilon}_{vk}(\mathrm{t}+1)\right) - R_F \alpha_{jj} x_j^a(\mathrm{t}) - R_F \alpha_{jk} x_k^a(\mathrm{t}) - R_F \beta_{jj} v(\mathrm{t}) - R_F \beta_{jk} v_k(\mathrm{t}) = \left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1) + \beta_{jj}\tilde{\varepsilon}_{vj}(\mathrm{t}+1) + \beta_{jk}\tilde{\varepsilon}_{vk}(\mathrm{t}+1) + \zeta_{jj} x_j^a(\mathrm{t}) + \zeta_{jk} x_k^a(\mathrm{t}) + \zeta_{j,vj} v_j(\mathrm{t}) + \zeta_{j,vk} v_k(\mathrm{t})$

It follows from proof of Theorem 2 that $\zeta_{jj} = \zeta_{jk} = 0$, and we verify that:

$$\zeta_{j,vj} = \left(1+\alpha_{jj}\right) + \beta_{jj}\gamma_j - R_F \beta_{jj} = 0$$
$$\zeta_{j,vk} = \alpha_{jk} + \beta_{jk}\gamma_k - R_F \beta_{jk} = 0$$

Hence, stock returns for firm j is:

$\widetilde{Ret}_j(\mathrm{t}+1) = \left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1)/P_j(\mathrm{t}) + \alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1)/P_j(\mathrm{t}) + \beta_{jj}\tilde{\varepsilon}_{vj}(\mathrm{t}+1)/P_j(\mathrm{t}) + \beta_{jk}\tilde{\varepsilon}_{vk}(\mathrm{t}+1)/P_j(\mathrm{t})$

Let $\sigma_{vj}^2 = VAR\left[\tilde{v}_j(\mathrm{t}+1)\right]$ and $\sigma_{vk}^2 = VAR[\tilde{v}_k(\mathrm{t}+1)]$ then the variance of returns of firm j is $VAR\left[\widetilde{Ret}_j(\mathrm{t}+1)\right] = \left(\left(1+\alpha_{jj}\right)^2\sigma_j^2 + \alpha_{jk}^2\sigma_k^2 + \beta_{jj}^2\sigma_{vj}^2 + \beta_{jk}^2\sigma_{vk}^2\right) / \left(P_j(\mathrm{t})\right)^2$, and the covariance between firms' returns is $COV\left[\widetilde{Ret}_j(\mathrm{t}+1), \widetilde{Ret}_k(\mathrm{t}+1)\right] = \left((1+\alpha_{jj})\alpha_{kj}\sigma_j^2 + (1+\alpha_{kk})\alpha_{jk}\sigma_k^2 + \beta_{jj}\beta_{kj}\sigma_{vj}^2 + +\beta_{jk}\beta_{kk}\sigma_{vk}^2\right) / \left(P_j(\mathrm{t})P_k(\mathrm{t})\right)$.

By symmetry, the results follow for firm k.

PROOF OF THEOREM 7

For any number of periods in the future, τ, we need to evaluate $E_t\left[\tilde{x}_j^a(2\mathrm{t}+\tau)\right]$ and $E_t[\tilde{x}_k^a(2\mathrm{t}+\tau)]$. Assume that in period 0 both firm j and k disclose, thus, the information set is given by: $\Omega_7(0) = \left\{B_j(0); x_j^a(0); B_k(0); x_k^a(0)\right\}$. In the next period, only firm j discloses, therefore the information set is given by:

$$\Omega_7(1) = \left\{B_j(0), B_j(1); x_j^a(0), x_j^a(1); B_k(0); x_k^a(0)\right\} = \Omega_7(0)U\left\{B_j(1); x_j^a(1)\right\}.$$

That is, in period 1, the information set is updated using the disclosed information by firm j, while in the following period, period 2, when both firms disclose again, the information set is updated using both firms' disclosures and is given by: $\Omega_7(2) = \Omega_7(1)U\left\{B_j(2); x_j^a(2); B_k(2); x_k^a(2)\right\}$. In period 3, when again, only firm j discloses, the information set is updated again using only the disclosed information from firm j: $\Omega_7(3) = \Omega_7(2)U\left\{B_j(3); x_j^a(3)\right\}$. For any future date $\tau > 3$, the same dynamics proceed, and the information sets are updated given the available disclosures, using financial statements information from both firms in even periods, and using financial statements information from firm j only in odd periods.

Under these assumptions, the information sets are different in "even" periods (i.e., period where both firms disclose) and "odd periods" (i.e., periods where only firm j discloses). At the end of any even period, denoted by $2t$, both firms j and k have disclosed and thus as in the Theorem 1, the most recently disclosed information, $\left\{B_j(2\mathrm{t}); x_j^a(2\mathrm{t}); B_k(2\mathrm{t}); x_k^a(2\mathrm{t})\right\}$, forms a sufficient statistics for all prior periods information and the pricing coefficients are as in Theorem 1. At the end of any odd period, denoted by $(2t+1)$, only firms j disclosed and thus to value the firms we require the most recently disclosed information by firm j, as well as the information discloses in the previous period by both firms: $\left\{B_j(2\mathrm{t}), B_j(2\mathrm{t}+1); x_j^a(2\mathrm{t}), x_j^a(2\mathrm{t}+1); B_k(2\mathrm{t}); x_k^a(2\mathrm{t})\right\}$.

In period $2t\ \forall t$, when both firms disclose their book values and earnings, the expected abnormal earnings are formed using all the available current information and are given by:

$$E_{2t}\left[\tilde{x}_j^a(2\mathrm{t}+\tau)\right] = E_{2t}\left[\tilde{x}_j^a(2\mathrm{t}+\tau) | x_j^a(2\mathrm{t}), x_k^a(2\mathrm{t})\right]$$

$$E_{2t}[\tilde{x}_k^a(2\mathrm{t}+\tau)] = E_{2t}\left[\tilde{x}_k^a(2\mathrm{t}+\tau) | x_j^a(2\mathrm{t}), x_k^a(2\mathrm{t})\right]$$

And we assume LID without other information is given by:

$$E_{2t}\left[\tilde{x}_j^a(2\mathrm{t}+1)\right] = \omega_{jj}x_j^a(2\mathrm{t}) + \omega_{jk}x_k^a(2\mathrm{t})$$

$$E_{2t}[\tilde{x}_k^a(2\mathrm{t}+1)] = \omega_{kj}x_j^a(2\mathrm{t}) + \omega_{kk}x_k^a(2\mathrm{t})$$

In period $2t+1\ \forall t$ where only firm j discloses, investors believe the non-disclosed report from firm k to be: $E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)] = \omega_{kj}x_j^a(2t) + \omega_{kk}x_k^a(2t)$, and the expected abnormal earnings are formed using all the available current and prior information and are given by:

$$E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+1+\tau)\right] = E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+1+\tau)\middle| x_j^a(2\mathrm{t}+1), x_j^a(2t), x_k^a(2t)\right]$$

$$E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1+\tau)] = E_{2t+1}\left[\tilde{x}_k^a(2\mathrm{t}+1+\tau)\middle| x_j^a(2\mathrm{t}+1), x_j^a(2t), x_k^a(2t)\right]$$

Thus, LID without other information can be written as follows:

$E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+2)\right] = \omega_{jj}x_j^a(2\mathrm{t}+1) + \omega_{jk}E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)] = \omega_{jj}x_j^a(2t+1) + \omega_{jk}\omega_{kj}x_j^a(2\mathrm{t}) + \omega_{jk}\omega_{kk}x_k^a(2t)$

$E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+2)] = \omega_{kj}x_j^a(\mathrm{t}) + \omega_{kk}E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)] = \omega_{kj}x_j^a(2t+1) + \omega_{kk}\omega_{kj}x_j^a(2t) + \omega_{kk}\omega_{kk}x_k^a(2t)$

The available information oscillate between periods in which both firms disclose and periods in which only firm j discloses. This suggests that the pricing coefficients oscillate as well. Therefore, to value the firm, we need to consider two scenarios: (i) the valuation in periods when both firms disclose and (ii) the valuation in periods when only firm j discloses.

In periods when both firms disclose, the solution and proof follow Theorem 1. In periods when only firm j discloses, we evaluate the Ohlson's goodwill expression at $2\mathrm{t}+1\ \forall\mathrm{t}$ as: $P_j(2\mathrm{t}+1) - B_j(2\mathrm{t}+1) = \sum_{\tau=1}^{\infty} R_F^{-\tau} E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+1+\tau)\right]$

To assess firm values, we consider the expected abnormal earnings for both firms simultaneously in periods where only firm j discloses as well as in periods where both firm j and k disclose. We can now express the Ohlson's goodwill expression as the sum of expected abnormal earnings in even and odd periods:

$$\begin{pmatrix}\sum_{\tau=1}^{\infty} R_F^{-\tau}E_{2t}\left[\tilde{x}_j^a(2\mathrm{t}+1+\tau)\right]\\ \sum_{\tau=1}^{\infty} R_F^{-\tau}E_{2t}[\tilde{x}_k^a(2\mathrm{t}+1+\tau)]\end{pmatrix} = R_F^{-1}\begin{pmatrix}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+2)\right]\\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+2)]\end{pmatrix} + R_F^{-2}(I + M_2(I_2 - M_2)^{-1})\begin{pmatrix}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+3)\right]\\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+3)]\end{pmatrix}$$

$$\begin{pmatrix}\sum_{\tau=1}^{\infty} R_F^{-\tau}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+1+\tau)\right]\\ \sum_{\tau=1}^{\infty} R_F^{-\tau}E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1+\tau)]\end{pmatrix} = R_F^{-1}\begin{pmatrix}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+2)\right]\\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+2)]\end{pmatrix} + R_F^{-2}\begin{pmatrix}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+3)\right]\\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+3)]\end{pmatrix} + R_F^{-2}\sum_{\tau=3}^{\infty} R^{-\tau+2}\begin{pmatrix}\omega_{jj} & \omega_{jk}\\ \omega_{kj} & \omega_{kk}\end{pmatrix}^{\tau-2} * \begin{pmatrix}E_{2t+1}\left[\tilde{x}_j^a(2\mathrm{t}+3)\right]\\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+3)]\end{pmatrix}$$

It follows from LID that:

$$\begin{pmatrix} E_{2t+1}[\tilde{x}_j^a(2\mathrm{t}+2)] \\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+2)] \end{pmatrix} = \begin{pmatrix} \omega_{jj}x_j^a(2\mathrm{t}+1) + \omega_{jk}\omega_{kj}x_j^a(2\mathrm{t}) + \omega_{jk}\omega_{kk}x_k^a(2\mathrm{t}) \\ \omega_{kj}x_j^a(2\mathrm{t}+1) + \omega_{kk}\omega_{kj}x_j^a(2\mathrm{t}) + \omega_{kk}\omega_{kk}x_k^a(2\mathrm{t}) \end{pmatrix},$$ and

$$\begin{pmatrix} E_{2t+1}[\tilde{x}_j^a(2\mathrm{t}+3)] \\ E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+3)] \end{pmatrix} =$$

$$\begin{pmatrix} (\omega_{jj}\omega_{jj} + \omega_{jk}\omega_{kj})x_j^a(2\mathrm{t}+1) + \omega_{jk}(\omega_{kj}\omega_{jj} + \omega_{kk}\omega_{kj})x_j^a(2\mathrm{t}) + \omega_{jk}(\omega_{kk}\omega_{jj} + \omega_{kk}\omega_{kk})x_k^a(2\mathrm{t}) \\ (\omega_{kj}\omega_{jj} + \omega_{kk}\omega_{kj})x_j^a(2\mathrm{t}+1) + \omega_{kj}(\omega_{kj}\omega_{jk} + \omega_{kk}\omega_{kk})x_j^a(2\mathrm{t}) + \omega_{kk}(\omega_{kj}\omega_{jk} + \omega_{kk}\omega_{kk})x_k^a(2\mathrm{t}) \end{pmatrix}$$

Substituting and rearranging, we get that for firm *j* Ohlson's goodwill expression is given by:

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_{2t+1}[\tilde{x}_j^a(2\mathrm{t}+1+\tau)] = \frac{\omega_{jk}\omega_{kj}+\omega_{jj}(R_F-\omega_{kk})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}} x_j^a(2\mathrm{t}+1) +$$

$$\omega_{jk}\omega_{kj}\frac{\omega_{kk}+(R_F-\omega_{kk})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}} x_j^a(2\mathrm{t}) + \omega_{kk}\omega_{jk}\frac{\omega_{kk}+(R_F-\omega_{kk})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}} x_k^a(2\mathrm{t})$$

which we can write as: $\sum_{\tau=1}^{\infty} R_F^{-\tau} E_{2t+1}[\tilde{x}_j^a(2\mathrm{t}+1+\tau)] = \delta_j x_j^a(2\mathrm{t}+1) + \delta_{jj}x_j^a(2\mathrm{t}) + \delta_{jk}x_k^a(2\mathrm{t})$

where:

$$\delta_j = \frac{\omega_{jk}\omega_{kj}+\omega_{jj}(R_F-\omega_{kk})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}};\ \delta_{jj} = \frac{R_F\omega_{jk}\omega_{kj}}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}};\ \delta_{jk} = \frac{R_F\omega_{kk}\omega_{jk}}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$$

To complete the proof, we proceed similarly for firm k, and find that:

$$\sum_{\tau=1}^{\infty} R_F^{-\tau} E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1+\tau)] = \delta_k x_j^a(2\mathrm{t}+1) + \hat{\delta}_{kj}x_j^a(2\mathrm{t}) + \hat{\delta}_{kk}x_k^a(2\mathrm{t})$$

where: $\delta_k = \frac{R_F\omega_{kj}}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$, $\hat{\delta}_{kj} = \omega_{kj}\frac{\omega_{jk}\omega_{kj}+\omega_{kk}(R_F-\omega_{jj})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$, and

$\hat{\delta}_{kk} = \omega_{kk}\frac{\omega_{kj}\omega_{jk}+\omega_{kk}(R_F-\omega_{jj})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$.

Since investors do not know firm k's book value at time $2t+1$, we use *CSR* to express the expected value using the information previously disclosed: $E_{2t+1}[\tilde{B}_k(2\mathrm{t}+1)] = R_F B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + E_{2t+1}[\tilde{x}_k^a(2\mathrm{t}+1)] = R_F B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + \omega_{kj}x_j^a(2\mathrm{t}) + \omega_{kk}x_k^a(2\mathrm{t})$. The firm value is then given by: $P_k(2\mathrm{t}+1) = R_F B_k(2\mathrm{t}) - d_k(2\mathrm{t}+1) + \delta_k x_j^a(2\mathrm{t}+1) + \delta_{kj}x_j^a(2\mathrm{t}) + \delta_{kk}x_k^a(2\mathrm{t})$

where:

$$\delta_{kj} = \omega_{kj}\frac{R_F(R_F-\omega_{jj})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$$

$$\delta_{kk} = \omega_{kk}\frac{R_F(R_F-\omega_{jj})}{(R_F-\omega_{jj})(R_F-\omega_{kk})-\omega_{jk}\omega_{kj}}$$

PROOF OF THEOREM 8

We can express excess returns in odd periods, when only firm *j* discloses as:

$$\widetilde{Ret}_j(2\mathrm{t}+1) = \left(\tilde{P}_j(2\mathrm{t}+1) + d_j(2\mathrm{t}+1) - R_F P_j(2\mathrm{t})\right)/P_j(2\mathrm{t})$$

$$\widetilde{Ret}_k(2\text{t}+1) = \left(\tilde{P}_k(2\text{t}+1) + d_k(2\text{t}+1) - R_F P_k(2\text{t})\right)/P_k(2\text{t})$$

and in even periods, when both firms disclose as:

$$\widetilde{Ret}_j(2\text{t}+2) = \left(\tilde{P}_j(2\text{t}+2) + d_j(2\text{t}+2) - R_F P_j(2\text{t}+1)\right)/P_j(2\text{t}+1)$$

$$\widetilde{Ret}_k(2\text{t}+2) = \left(\tilde{P}_k(2\text{t}+2) + d_k(2\text{t}+2) - R_F P_k(2\text{t}+1)\right)/P_k(2\text{t}+1)$$

Given Theorem 7, excess returns for firm j at $2\text{t}+1$ is given by:

$\widetilde{Ret}_j(2\text{t}+1) = \Big((1+\delta_j)\tilde{\varepsilon}_j(2\text{t}+1) + \big((1+\delta_j)\omega_{jj} + \delta_{jj} - R_F\alpha_{jj}\big)x_j^a(2t) + \big((1+\delta_j)\omega_{jk} + \delta_{jk} - R_F\alpha_{jk}\big)x_k^a(2t)\Big)/P_j(2\text{t})$

Since $(1+\delta_j)\omega_{jj} + \delta_{jj} - R_F\alpha_{jj} = 0$, and $(1+\delta_j)\omega_{jk} + \delta_{jk} - R_F\alpha_{jk} = 0$, we get that: $\widetilde{Ret}_j(2\text{t}+1) = (1+\delta_j)\tilde{\varepsilon}_j(2\text{t}+1)/P_j(2\text{t})$.

Similarly, for firm k, we express excess returns as: $\widetilde{Ret}_k(2\text{t}+1) = \Big(\delta_k\tilde{\varepsilon}_j(2\text{t}+1) + \big(\delta_k\omega_{jj} + \delta_{kj} - R_F\alpha_{kj}\big)x_j^a(2t) + \big(\delta_{kk} + \delta_k\omega_{jk} - R_F\alpha_{kk}\big)x_k^a(2t)\Big)/P_k(2\text{t})$

Since $\delta_k\omega_{jj} + \delta_{kj} - R_F\alpha_{kj} = 0$, and $\delta_{kk} + \delta_k\omega_{jk} - R_F\alpha_{kk} = 0$, we get that: $\widetilde{Ret}_k(2\text{t}+1) = \Big(\delta_k\tilde{\varepsilon}_j(2\text{t}+1)\Big)/P_k(2\text{t})$.

Given Theorem 7, excess returns for firm j at $2\text{t}+2$ is given by: $\widetilde{Ret}_j(2\text{t}+2) = \Big((1+\alpha_{jj})\tilde{\varepsilon}_j(2\text{t}+2) + \alpha_{jk}\tilde{\varepsilon}_k(2\text{t}+2) + \big((1+\alpha_{jj})\omega_{jj} + \alpha_{jk}\omega_{kj} - R_F\delta_j\big)x_j^a(2\text{t}+1) + \big((1+\alpha_{jj})\omega_{jk}\omega_{kj} + \alpha_{jk}\omega_{kk}\omega_{kj} - R_F\delta_{jj}\big)x_j^a(2\text{t}) + \big((1+\alpha_{jj})\omega_{jk}\omega_{kk} + \alpha_{jk}\omega_{kk}\omega_{kk} - R_F\delta_{jk}\big)x_k^a(2\text{t})\Big)/P_j(2\text{t}+1)$.

Since $(1+\alpha_{jj})\omega_{jj} + \alpha_{jk}\omega_{kj} - R_F\delta_j = 0$, $(1+\alpha_{jj})\omega_{jk}\omega_{kj} + \alpha_{jk}\omega_{kk}\omega_{kj} - R_F\delta_{jj} = 0$, and $(1+\alpha_{jj})\omega_{jk}\omega_{kk} + \alpha_{jk}\omega_{kk}\omega_{kk} - R_F\delta_{jk} = 0$, we get that excess returns for firm j is: $\widetilde{Ret}_j(2t+2) = \Big((1+\alpha_{jj})\tilde{\varepsilon}_j(2t+2) + \alpha_{jk}\tilde{\varepsilon}_k(2t+2)\Big)/P_j(2t+1)$

Similarly, for firm k at $2\text{t}+2$, excess returns are given by:

$\widetilde{Ret}_k(2\text{t}+2) = \Big(B_k(2t+2) + d_k(2\text{t}+2) - R_F R_f B_k(2\text{t}) + R_F d_k(2\text{t}+1) + \alpha_{kj}x_j^a(2t+2) + \alpha_{kk}x_k^a(2t+2) - R_F\delta_k x_j^a(2\text{t}+1) - R_F\delta_{kj}x_j^a(2\text{t}) - R_F\delta_{kk}x_k^a(2\text{t})\Big)/P_k(2\text{t}+1)$

We know from CSR that: $x_k^a(2t+2) + R_F x_k^a(2t+1) = B_k(2t+2) - R_F R_F B_k(2\text{t}) + R_F d_k(2\text{t}+1) + d_k(2\text{t}+2)$, therefore: $\widetilde{Ret}_k(2\text{t}+2) = \Big(\alpha_{kj}x_j^a(2t+2) + (1+\alpha_{kk})x_k^a(2t+2) -$

$R_F \delta_k x_j^a(2t+1) + R_F x_k^a(2t+1) - R_F \delta_{kj} x_j^a(2t) - R_F \delta_{kk} x_k^a(2t)\Big) / P_k(2t+1)$, which can be simplified and presented as follows:[17]

$$\widetilde{Ret}_k(2t+2) = \Big(\alpha_{kj}\tilde{\varepsilon}_j(2t+2) + (1+\alpha_{kk})\tilde{\varepsilon}_k(2t+2) + R_F\tilde{\varepsilon}_k(2t+1) + \big(\alpha_{kj}\omega_{jj} + (1+\alpha_{kk})\omega_{kj} - R_F\delta_k\big)x_j^a(2t+1) + \big(R_F\omega_{kj} + (1+\alpha_{kk})\omega_{kk}\omega_{kj} + \alpha_{kj}\omega_{jk}\omega_{kj} - R_F\delta_{kj}\big)x_j^a(2t) + \Big((1+\alpha_{kk})\omega_{kk}\omega_{kk} + \alpha_{kj}\omega_{jk}\omega_{kk} + R_F\omega_{kk} - R_F\delta_{kk}\Big)x_k^a(2t)\Big) / P_k(2t+1)$$

Since $\alpha_{kj}\omega_{jj} + (1+\alpha_{kk})\omega_{kj} - R_F\delta_k = 0$,$R_F\omega_{kj} + (1+\alpha_{kk})\omega_{kk}\omega_{kj} + \alpha_{kj}\omega_{jk}\omega_{kj} - R_F\delta_{kj} = 0$, and$(1+\alpha_{kk})\omega_{kk}\omega_{kk} + \alpha_{kj}\omega_{jk}\omega_{kk} + R_F\omega_{kk} - R_F\delta_{kk} = 0$, we get that: $\widetilde{Ret}_k(2t+2) = \Big(\alpha_{kj}\tilde{\varepsilon}_j(2t+2) + (1+\alpha_{kk})\tilde{\varepsilon}_k(2t+2) + R_F\tilde{\varepsilon}_k(2t+1)\Big) / P_k(2t+1)$.

[17] Not that at t=2, the realization of $x_k^a(2t+1)$ is known, hence we can replace $x_k^a(2t+1)$ with: $\omega_{kj}x_j^a(2t) + \omega_{kk}x_k^a(2t) + \tilde{\varepsilon}_k(2t+1)$.

APPENDIX C: Proofs of Portfolio Results Discussed in Text

C.1 Diversifying each firm's abnormal earnings

Based on Theorem 2, we consider a portfolio that invests a fraction of wealth, y_j, in the shares of firm j and the remaining fraction $\left(1-y_j\right)$ in the shares of firm k. The single period excess returns on this portfolio are: $y_j\widetilde{Ret}_j(\mathrm{t}+1)+\left(1-y_j\right)\widetilde{Ret}_k(\mathrm{t}+1)$. This portfolio's returns are unaffected by the disturbance terms to the abnormal earnings of firm k provided that: $y_jP_k(\mathrm{t})\alpha_{jk}+\left(1-y_j\right)P_j(\mathrm{t})(1+\alpha_{kk})=0$, such that $y_j=\frac{P_j(\mathrm{t})(1+\alpha_{kk})}{P_j(\mathrm{t})(1+\alpha_{kk})-P_k(\mathrm{t})\alpha_{jk}}=\frac{P_j(\mathrm{t})(R_F-\omega_{jj})}{P_j(\mathrm{t})(R_F-\omega_{jj})-P_k(\mathrm{t})\omega_{jk}}$ and $1-y_j=\frac{-P_k(\mathrm{t})\omega_{jk}}{P_j(\mathrm{t})(R_F-\omega_{jj})-P_k(\mathrm{t})\omega_{jk}}$.

Similarly, we can construct a portfolio that invests a fraction of wealth, y_k, in the shares of firm j and the remaining fraction $(1-y_k)$ in the shares of firm k such that the portfolio's returns are unaffected by the disturbance terms to the abnormal earnings of firm j: $y_kP_k(\mathrm{t})\left(1+\alpha_{jj}\right)+(1-y_k)P_j(\mathrm{t})\alpha_{kj}=0$, such that $y_k=\frac{-P_j(\mathrm{t})\alpha_{kj}}{P_k(\mathrm{t})(1+\alpha_{jj})-P_j(\mathrm{t})\alpha_{kj}}=\frac{-P_j(\mathrm{t})\omega_{kj}}{P_k(\mathrm{t})(R_F-\omega_{kk})-P_j(\mathrm{t})\omega_{kj}}$ and $1-y_k=\frac{P_k(\mathrm{t})(R_F-\omega_{kk})}{P_k(\mathrm{t})(R_F-\omega_{kk})-P_j(\mathrm{t})\omega_{kj}}$.

C.2 Aggregation over time of stock returns and abnormal earnings disturbance terms

Based on Theorem 2, we extend Easton, Harris and Ohlson (1992) to allow for direct inter-firm information transfers. We define one-period ahead excess returns firm j as: $\widetilde{Ret}_j(\mathrm{t}+1)=\left(\tilde{P}_j(\mathrm{t}+1)+d_j(\mathrm{t}+1)-R_FP_j(\mathrm{t})\right)/P_j(\mathrm{t})$, and two-period ahead excess returns as : $\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+2)=\left(\tilde{P}_j(\mathrm{t}+2)+d_j(\mathrm{t}+2)+R_Fd_j(\mathrm{t}+1)-{R_F}^2P_j(\mathrm{t})\right)/P_j(\mathrm{t})$. Adding and subtracting $R_F\tilde{P}_j(\mathrm{t}+1)$, $\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+2)$ reduces to $\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+2)=\left(\tilde{P}_j(\mathrm{t}+2)+d_j(\mathrm{t}+2)-R_F\tilde{P}_j(\mathrm{t}+1)\right)/P_j(\mathrm{t})+R_F\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+1)=\widetilde{Ret}_j(\mathrm{t}+1,\mathrm{t}+2)\left(\tilde{P}_j(\mathrm{t}+1)/P_j(\mathrm{t})\right)$

By the definition of excess returns, it follows that:

$\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+1)=\left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1)/P_j(\mathrm{t})+\alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1)/P_j(\mathrm{t})$

$\widetilde{Ret}_j(\mathrm{t}+1,\mathrm{t}+2)=\left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+2)/P_j(\mathrm{t}+1)+\alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+2)/P_j(\mathrm{t}+1)$

$\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+2)=\left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+2)/P_j(\mathrm{t})+\alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+2)/P_j(\mathrm{t})+R_F\left(1+\alpha_{jj}\right)\tilde{\varepsilon}_j(\mathrm{t}+1)/P_j(\mathrm{t})+R_F\alpha_{jk}\tilde{\varepsilon}_k(\mathrm{t}+1)/P_j(\mathrm{t})$

Thus: $\widetilde{Ret}_j(\mathrm{t},\mathrm{t}+2)=\left(1+\alpha_{jj}\right)\left(\tilde{\varepsilon}_j(\mathrm{t}+2)+R_F\tilde{\varepsilon}_j(\mathrm{t}+1)\right)/P_j(\mathrm{t})+\alpha_{jk}\left(\tilde{\varepsilon}_k(\mathrm{t}+2)+R_F\tilde{\varepsilon}_k(\mathrm{t}+1)\right)/P_j(\mathrm{t})$

Aggregate disturbance terms (sum of disturbance terms over time) for both firms are needed to model stock returns.

C.3 Diversifying common other information

Based on Theorem 4 we construct a portfolio such that the portfolio returns are unaffected by disturbance terms to common other information, $\tilde{\varepsilon}_v(\mathrm{t}+1)$, provided that:

$y_j P_k(\mathrm{t})\beta_j + (1-y_j)P_j(\mathrm{t})\beta_k = 0$ then $P_j(\mathrm{t})\beta_k = y_j\big(P_j(\mathrm{t})\beta_k - P_k(\mathrm{t})\beta_j\big)$ or $y_j = \frac{P_j(\mathrm{t})\beta_k}{(P_j(\mathrm{t})\beta_k - P_k(\mathrm{t})\beta_j)} = \frac{P_j(\mathrm{t})(R_F+\omega_{kj}-\omega_{jj})}{P_j(\mathrm{t})(R_F+\omega_{kj}-\omega_{jj}) - P_k(\mathrm{t})(R_F+\omega_{jk}-\omega_{kk})}$

C.4 Diversifying k's firm-specific other information

Based on Theorem 6, we construct a portfolio with returns that are unaffected by disturbance terms in firm-specific other information, $\tilde{\varepsilon}_{vk}(\mathrm{t}+1)$, provided that:

$y_j P_k(\mathrm{t})\beta_{jk} + (1-y_j)P_j(\mathrm{t})\beta_{kk} = 0$ then $P_j(\mathrm{t})\beta_{kk} = y_j\big(P_j(\mathrm{t})\beta_{kk} - P_k(\mathrm{t})\beta_{jk}\big)$ or $y_j = \frac{P_j(\mathrm{t})\beta_{kk}}{(P_j(\mathrm{t})\beta_{kk} - P_k(\mathrm{t})\beta_{jk})} = \frac{P_j(\mathrm{t})(R_F-\omega_{jj})}{P_j(\mathrm{t})(R_F-\omega_{jj}) - P_k(\mathrm{t})\omega_{jk}}$.

This portfolio weight also eliminates inter-firm information transfers as the portfolio diversifies disturbance terms to firm k's abnormal earnings (see Appendix C.1).

The same approach can be applied in order to diversify firm j's firm-specific other information

C.5 Value-weighted market returns and aggregate earnings

Consider a setting with two firms and no other information. We construct the value-weighted portfolio such that investors invest a fraction $f_j = P_j(\mathrm{t})/\big(P_j(\mathrm{t}) + P_k(\mathrm{t})\big)$ in firm j and the remaining fraction $f_k = P_k(\mathrm{t})/\big(P_j(\mathrm{t}) + P_k(\mathrm{t})\big)$ in firm k. These weights add up to one: $f_j + f_k = 1$. The return on this portfolio is: $\widetilde{Ret}_{VW}(t+1) = f_j\widetilde{Ret}_j(t+1) + f_k\widetilde{Ret}_k(t+1) = \Big(\big(1+\alpha_{jj}+\alpha_{kj}\big)\tilde{\varepsilon}_j(t+1) + \big(1+\alpha_{kk}+\alpha_{jk}\big)\tilde{\varepsilon}_k(t+1)\Big) / \Big(P_j(t) + P_k(t)\Big)$

Assume that the firms are identical, such that $\omega_{jj} = \omega_{kk}$ and $\omega_{jk} = \omega_{kj}$, then: $\widetilde{Ret}_{VW}(t+1) = \big(1+\alpha_{jj}+\alpha_{kj}\big)\frac{\big(\tilde{\varepsilon}_j(t+1)+\tilde{\varepsilon}_k(t+1)\big)}{\big(P_j(t)+P_k(t)\big)}$.

Under this assumption the returns on the value-weighted portfolio depends only on aggregate abnormal earnings disturbance terms scaled by the market value of the value-weighted portfolio.

Consider next a setting with n firms setting and without other information. With n stocks we construct the value-weighted portfolio such that investors invest a fraction in firm j: $f_j = P_j(\mathrm{t})/P_m(\mathrm{n},\mathrm{t})$, where $P_m(n,t) = P_1(t) + \dots + P_n(t)$ is the market value of n stocks. These weights add up to one: $f_1 + \cdots + f_n = 1$. If firms are identical such that $\omega_{jj} = \omega_{kk}$ and $\omega_{jk} = \omega_{kj}$ for any $j \neq k$, then: $\widetilde{Ret}_{VW}(t+1) = \left(1 + \alpha_{jj} + \alpha_{kj}\right) \frac{\left(\tilde{\varepsilon}_1(t+1) + \cdots \ldots + \tilde{\varepsilon}_n(t+1)\right)}{P_m(n,t)}$.

With identical firms, $P_m(n,t)$ grows in the number of firms. If all disturbance terms, $\tilde{\varepsilon}_j(t+1)$, are independent normally distributed, then value-weighted portfolio returns are also normally distributed with a variance that converges to zero in the number of firms.

Lastly, consider a setting with n firms and COI, but without direct inter-firm information transfers (i.e., $\omega_{jk} = 0$ for any $j \neq k$), then: $\widetilde{Ret}_{VW}(\mathrm{t}+1) = \left((1 + \alpha_{11})\tilde{\varepsilon}_1(\mathrm{t}+1) + \cdots + (1 + \alpha_{nn})\tilde{\varepsilon}_n(\mathrm{t}+1)\right)/P_m(\mathrm{n},\mathrm{t}) + B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t})$, where $B_n = \beta_1 + \beta_2 + \cdots + \beta_n$.

If firms are identical such that $\omega_{jj} = \omega_{kk}$ and $\omega_{jk} = \omega_{kj}$ for any $j \neq k$, it follows that: $\widetilde{Ret}_{VW}(\mathrm{t}+1) = \left(1 + \alpha_{\mathrm{jj}}\right)\left(\tilde{\varepsilon}_1(\mathrm{t}+1) + \cdots + \tilde{\varepsilon}_n(\mathrm{t}+1)\right)/P_m(\mathrm{n},\mathrm{t}) + B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t})$

where $B_n = \mathrm{n}\beta_{\mathrm{j}}$. Furthermore, assume that $P_m(\mathrm{n},\mathrm{t})$ grows with n, meaning that the total market value increases with the number of firms. If all disturbance terms are normally distributed then the first term vanishes (goes to zero with probability one). In the limit, the portfolio's return is: $Plim_{n\to\infty}\,\widetilde{Ret}_{VW}(\mathrm{t}+1) = B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t})$. Thus, the portfolio's return is the product of (i) the disturbance term in common other information, $\tilde{\varepsilon}_v(\mathrm{t}+1)$, and (ii) a scalar, $B_n/P_m(\mathrm{n},\mathrm{t})$= $\beta_1/(P_m(\mathrm{n},\mathrm{t})/\mathrm{n})$.

If instead we consider a setting with n firms and both COI and FOI, but without direct inter-firm information transfers, then:

$\widetilde{Ret}_{VW}(\mathrm{t}+1) = \left((1 + \alpha_{11})\tilde{\varepsilon}_1(\mathrm{t}+1) + \cdots + (1 + \alpha_{nn})\tilde{\varepsilon}_n(\mathrm{t}+1)\right)/P_m(\mathrm{n},\mathrm{t}) + B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t}) + (B_{11}\tilde{\varepsilon}_{v1}(\mathrm{t}+1) + \cdots + B_{nn}\tilde{\varepsilon}_{vn}(\mathrm{t}+1))/P_m(\mathrm{n},\mathrm{t})$

If firms are identical such that $\omega_{jj} = \omega_{kk}$, $\omega_{jk} = \omega_{kj}$ and $\gamma_j = \gamma_k$ for any $j \neq k$, it follows that:

$\widetilde{Ret}_{VW}(\mathrm{t}+1) = \left(1 + \alpha_{\mathrm{jj}}\right)\left(\tilde{\varepsilon}_1(\mathrm{t}+1) + \cdots + \tilde{\varepsilon}_n(\mathrm{t}+1)\right)/P_m(\mathrm{n},\mathrm{t}) + B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t}) + \mathrm{n}\beta_{\mathrm{jj}}((\tilde{\varepsilon}_{v1}(\mathrm{t}+1) + \cdots + \tilde{\varepsilon}_{vn}(\mathrm{t}+1))/P_m(\mathrm{n},\mathrm{t})$

If all disturbance terms are normally distributed then $Plim_{n\to\infty}\,\widetilde{Ret}_{VW}(\mathrm{t}+1) = B_n\tilde{\varepsilon}_v(\mathrm{t}+1)/P_m(\mathrm{n},\mathrm{t})$. As the number of firms in the portfolio grow, disturbance terms of firms' abnormal earnings and FOI are fully diversified and the value-weighted portfolio only captures disturbance terms to COI.

Appendix D: Valuation When One Firm Does Not Disclose

Assume that firm j always discloses while firm k disclose only once, at $t = 0$. In this setting, the initial information set at $t = 0$ is given by: $\Omega(0) = \{B_j(0); x_j^a(0); d_j(0); B_k(0); x_k^a(0); d_k(0)\}$, and in every period after $t > 0$, investors' information set is given by: $\Omega(t) = \{B_j(0), B_j(1), \dots, B_j(\mathrm{t}-1), B_j(\mathrm{t}); x_j^a(0), x_j^a(1), \dots, x_j^a(\mathrm{t}-1), x_j^a(\mathrm{t}); d_j(0), d_j(1), \dots, d_j(\mathrm{t}-1), d_j(\mathrm{t}); B_k(0); x_k^a(0); d_k(0), d_k(1), \dots, d_k(\mathrm{t}-1), d_k(\mathrm{t})\}$.

In period $t = 0$, both firms provide financial disclosers and firm values are determined by Theorem 1. At this point, book values and abnormal earnings are available for both firms. However, in period $t = 1$, the first period where only firm j discloses, current book values and abnormal earnings are available only for firm j. Since firm k does not provide financial statement information, investors use CSR and LID to derive the firm's price. Specifically, firm k's expected book value, $\tilde{B}_k(1)$, can be expressed as: $E_1[\tilde{B}_k(1)] = R_F B_k(0) - d_k(1) + E_1[\tilde{x}_k^a(1)] = R_F B_k(0) - d_k(1) + \omega_{kj} x_j^a(0) + \omega_{kk} x_k^a(0)$ and firm values are as characterized by Theorem 7.[18]

In period $t = 1$, investors know the realizations of $x_j^a(1)$, $x_j^a(0)$ and $x_k^a(0)$, thus, they can infer the realization of the disturbance term in the abnormal earnings of firm j: $\varepsilon_j(1) = x_j^a(1) - \omega_{jj} x_j^a(0) - \omega_{jk} x_k^a(0)$. Therefore, we can present the firm values at $t = 1$, as:

(23) $$P_j(1) = B_j(1) + \lambda_{jj1}(1)\varepsilon_j(1) + \lambda_{jj}(1) x_j^a(0) + \lambda_{jk}(1) x_k^a(0)$$

(24) $$P_k(1) = R_F B_k(0) - d_k(1) + \lambda_{kj1}(1)\varepsilon_j(1) + \lambda_{kj}(1) x_j^a(0) + \lambda_{kk}(1) x_k^a(0)$$

where the pricing coefficients for $t = 1$ are given by:

$\lambda_{jj1}(1) = \alpha_{jj}$ $\quad \lambda_{jj}(1) = \alpha_{jj}\omega_{jj} + \alpha_{jk}\omega_{kj}$ $\quad \lambda_{jk}(1) = \alpha_{jj}\omega_{jk} + \alpha_{jk}\omega_{kk}$

$\lambda_{kj1}(1) = \alpha_{kj}$ $\quad \lambda_{kj}(1) = \alpha_{kj}\omega_{jj} + \alpha_{kk}\omega_{kj} + \omega_{kj}$ $\quad \lambda_{kk}(1) = \alpha_{kj}\omega_{jk} + \alpha_{kk}\omega_{kk} + \omega_{kk}$

At $t = 1$ 1, $\tilde{\varepsilon}_k(1)$ and $\tilde{\varepsilon}_j(2)$ are independent random variables with normal prior distribution, where $\tilde{\varepsilon}_k(1) \sim N(0, \sigma_k^2)$ and $\tilde{\varepsilon}_j(2) \sim N(0, \sigma_j^2)$, while $\tilde{x}_k^a(2)$ has a normal prior distribution with mean $\mu_1\{x_j^a(2)\} = \omega_{jj} x_j^a(1) + \omega_{jk}\left(\omega_{kj} x_j^a(0) + \omega_{kk} x_k^a(0)\right)$ and variance $\sigma_1{}^2\{x_j^a(2)\} = \omega_{jj}{}^2\sigma_j^2 + \omega_{jk}^2\sigma_k^2$. At $t = 2$, investors do not know the realizations of $\tilde{\varepsilon}_k(1)$, $\tilde{\varepsilon}_j(2)$ and $\tilde{\varepsilon}_k(2)$. However, investors do know the realization of $x_j^a(2)$ and they hold the prior that $\tilde{\varepsilon}_k(2) \sim N(0, \sigma_k^2)$, and knowing the realizations of $x_j^a(1)$ and $x_j^a(2)$, investors update their prior beliefs regarding $\tilde{\varepsilon}_k(1)$ and $\tilde{\varepsilon}_j(2)$ conditional on these realizations. Specifically, the posterior of $\tilde{\varepsilon}_k(1)$ is conditional on the realization

[18] The discounted sum of the firms' future abnormal earnings are given by: $\begin{pmatrix} \sum_{\tau=1}^{\infty} R_F^{-\tau} E_1[x_j^a(1+\tau)] \\ \sum_{\tau=1}^{\infty} R_F^{-\tau} E_1[x_k^a(1+\tau)] \end{pmatrix} = M_2(I_2 - M_2)^{-1} \begin{pmatrix} x_j^a(1) \\ E_1[x_k^a(1)] \end{pmatrix}$.

of $x_j^a(1)$ and $x_j^a(2)$, while the posterior of $\tilde{\varepsilon}_j(2)$ is conditional on the realization of $x_j^a(2)$, and both posteriors are normally distributed with $\mu_2\{\varepsilon_k(1)|x_j^a(2)\} = \frac{COV[x_j^a(2),\varepsilon_k(1)]*\sigma_k}{\sigma_1\{x_j^a(2)\}}\left(x_j^a(2) - \mu_1\{x_j^a(2)\}\right)$ and $\mu_2\{\varepsilon_j(2)|x_j^a(2)\} = \frac{COV[x_j^a(2),\varepsilon_j(2)]*\sigma_j}{\sigma_1\{x_j^a(2)\}}\left(x_j^a(2) - \mu_1\{x_j^a(2)\}\right)$.

Using CSR, LID, and the posteriors $\mu_2\{\varepsilon_k(1)|x_j^a(2)\}$ and $\mu_2\{\varepsilon_j(2)|x_j^a(2)\}$, we can present the firm values at $t = 2$ as:[19]

(25) $$P_j(2) = B_j(2) + \lambda_{jj1}(2)\varepsilon_j(1) + \lambda_{jj2}(2)\mu_2\{\varepsilon_j(2)|x_j^a(2)\} + \lambda_{jk1}(2)\mu_2\{\varepsilon_k(1)|x_j^a(2)\} + \lambda_{jj}(2)x_j^a(0) + \lambda_{jk}(2)x_k^a(0)$$

(26) $$P_k(2) = R_f{}^2B_k(0) - R_Fd_k(1) - d_k(2) + \lambda_{kj1}(2)\varepsilon_j(1) + \lambda_{kj2}(2)\mu_2\{\varepsilon_j(2)|x_j^a(2)\} + \lambda_{kk1}(2)\mu_2\{\varepsilon_k(1)|x_j^a(2)\} + \lambda_{kj}(2)x_j^a(0) + \lambda_{kk}(2)x_k^a(0)$$

where the pricing coefficients are given by:

$\lambda_{jj1}(2) = \alpha_{jj}\omega_{jj} + \alpha_{jk}\omega_{kj}$ $\quad\lambda_{jj2}(2) = \alpha_{jj}\omega_{kj}$ $\quad\lambda_{jk1}(2) = \alpha_{jj}\omega_{jk}$

$\lambda_{jj}(2) = \alpha_{jj}\left(\omega_{jj}{}^2 + \omega_{jk}\omega_{kj}\right) + \alpha_{jk}\left(\omega_{jj}\omega_{kj} + \omega_{kk}\omega_{kj}\right)$

$\lambda_{jk}(2) = \alpha_{jj}\left(\omega_{jj}\omega_{jk} + \omega_{jk}\omega_{kk}\right) + \alpha_{jk}\left(\omega_{kk}{}^2 + \omega_{jk}\omega_{kj}\right)$

$\lambda_{kj1}(2) = \alpha_{kj}\omega_{jj} + (1 + \alpha_{kk})\omega_{kj}$ $\quad\lambda_{kj2}(2) = \alpha_{kj}$ $\quad\lambda_{kk1}(2) = R_f + \omega_{kk} + \alpha_{kj}\omega_{jk}$

$\lambda_{kj}(2) = \left(R_F + \omega_{jj}\right)\omega_{kj} + \omega_{kk}\omega_{kj} + \alpha_{kj}\left(\omega_{jj}{}^2 + \omega_{jk}\omega_{kj}\right) + \alpha_{kk}\left(\omega_{jj}\omega_{kj} + \omega_{kk}\omega_{kj}\right)$

$\lambda_{kk}(2) = (R_F + \omega_{kk})\omega_{kk} + \omega_{jk}\omega_{kj} + \alpha_{kj}\left(\omega_{jj}\omega_{jk} + \omega_{jk}\omega_{kk}\right) + \alpha_{kk}\left(\omega_{kk}{}^2 + \omega_{jk}\omega_{kj}\right)$

At $t = 2$, $\tilde{x}_k^a(3)$ is unknown and has a normal prior distribution with mean $\mu_2\{x_j^a(3)\} = \omega_{jj}x_j^a(2) + \omega_{jk}\omega_{kj}x_j^a(1) + \omega_{jk}\omega_{kk}\left(\omega_{kj}x_j^a(0) + \omega_{kk}x_k^a(0)\right)$ and variance $\sigma_2{}^2\{x_j^a(3)\} = \omega_{jj}{}^2\sigma_j^2 + \omega_{jk}^2\sigma_k^2$. At time $t = 3$, the posteriors of $\tilde{\varepsilon}_k(1)$, $\tilde{\varepsilon}_k(2)$, $\tilde{\varepsilon}_j(2)$ and $\tilde{\varepsilon}_j(3)$ are conditional on the realization of $x_j^a(1)$, $x_j^a(2)$ and $x_j^a(3)$, and are also normally distributed, with $\mu_3\{\varepsilon_k(1)|x_j^a(3)\} = \frac{COV[x_j^a(3),\varepsilon_k(1)]*\sigma_k}{\sigma_2\{x_j^a(3)\}}\left(x_j^a(3) - \mu_2\{x_j^a(3)\}\right)$, $\mu_3\{\varepsilon_k(2)|x_j^a(3)\} = \frac{COV[x_j^a(3),\varepsilon_k(2)]*\sigma_k}{\sigma_2\{x_j^a(3)\}}\left(x_j^a(3) - \mu_2\{x_j^a(3)\}\right)$, $\mu_3\{\varepsilon_j(2)|x_j^a(3)\} = \frac{COV[x_j^a(3),\varepsilon_j(2)]*\sigma_j}{\sigma_2\{x_j^a(3)\}}\left(x_j^a(3) - \mu_2\{x_j^a(3)\}\right)$ and $\mu_3\{\varepsilon_j(3)|x_j^a(3)\} =$

---

[19] The sum of the firms' abnormal earnings is given by: $\begin{pmatrix}\sum_{\tau=1}^{\infty} R_F^{-\tau}E_2[x_j^a(2+\tau)]\\ \sum_{\tau=1}^{\infty} R_F^{-\tau}E_2[x_k^a(2+\tau)]\end{pmatrix} = M_2(I_2 - M_2)^{-1}\begin{pmatrix}x_j^a(2)\\ E_2[x_k^a(2)]\end{pmatrix}$; and firm k's book value is conditional on the realization $x_j^a(2)$ and is given by: $E_2[B_k(2)|x_j^a(2)] = R_f{}^2B_k(0) - R_fd_k(1) - d_k(2) + \omega_{kj}\varepsilon_j(1) + (R_F + \omega_{kk})\mu_2\{\varepsilon_k(1)|x_j^a(2)\} + \left(R_F\omega_{kj} + \omega_{jj}\omega_{kj} + \omega_{kk}\omega_{kj}\right)x_j^a(0) + (R_f\omega_{kk} + \omega_{kk}{}^2 + \omega_{kj}\omega_{jk})x_k^a(0)$.

$\frac{COV\left[x_j^a(3),\varepsilon_j(3)\right]*\sigma_j}{\sigma_2\left\{x_j^a(3)\right\}}\left(x_j^a(3) - \mu_2\left\{x_j^a(3)\right\}\right)$ respectively. Investors still hold the prior that $\tilde{\varepsilon}_k(3)\sim N(0,\sigma_k^2)$. Investor can follow the process described for $t = 2$, and use CSR, LID and the posteriors $\mu_3\left\{\varepsilon_k(1)|x_j^a(2)\right\}$, $\mu_3\left\{\varepsilon_k(2)|x_j^a(3)\right\}$, $\mu_3\left\{\varepsilon_j(2)|x_j^a(2)\right\}$, $\mu_3\left\{\varepsilon_j(3)|x_j^a(3)\right\}$ and $E_2\left[B_k(2)|x_j^a(2)\right]$ to value the firms at $t = 3$,

Given CSR, PVED and the LID presented in equations (3) – (4), the generalized form of firms' values at time t are determined by:

(27) $$P_j(\mathrm{t}) = B_j(\mathrm{t}) + \sum_{\tau=1}^{t} \lambda_{jj\tau}(t)\, \mu_t\left\{\varepsilon_j(\tau)|x_j^a(\mathrm{t})\right\} + \sum_{\tau=1}^{t} \lambda_{jk\tau}(t)\, \mu_t\left\{\varepsilon_k(\tau)|x_j^a(\mathrm{t})\right\} + \lambda_{jj}(t)x_j^a(0) + \lambda_{jk}(t)x_k^a(0)$$

(28) $$P_k(\mathrm{t}) = {R_f}^t B_k(0) - \sum_{\tau=1}^{t} {R_F}^{t-\tau} d_k(\tau) + \sum_{\tau=1}^{t} \lambda_{kj\tau}(t)\mu_t\left\{\varepsilon_j(\tau)|x_j^a(\mathrm{t})\right\} + \sum_{\tau=1}^{t} \lambda_{kk\tau}(t)\mu_t\left\{\varepsilon_k(\tau)|x_j^a(\mathrm{t})\right\} + \lambda_{kj}(t)x_j^a(0) + \lambda_{kk}(t)x_k^a(0)$$

The information initially disclosed at $t = 0$ by both firms, along with the conditional disturbance terms are priced. Moreover, the effect of the conditional prior disturbance terms on the price persists, suggesting that while new information is disclosed by firm j, prior information pertaining to the disturbance terms remains relevant for valuation.

PROOF OF COROLLARY 2

Using the firm value at $t = 1{,}2$, we can now express the firms' excess returns as:

$$\widetilde{Ret}_j(1) = \frac{\widetilde{\varepsilon_j}(1)+\lambda_{jj1}(1)\widetilde{\varepsilon_j}(1)}{P_j(0)} \qquad \widetilde{Ret}_j(2) = \frac{(1+\lambda_{jj2}(2))\mu_2\left\{\varepsilon_j(2)|x_j^a(2)\right\}+(\omega_{jk}+\lambda_{jk1}(2))\mu_2\left\{\varepsilon_k(1)|x_j^a(2)\right\}}{P_j(1)}$$

$$\widetilde{Ret}_k(1) = \frac{\lambda_{kj1}(1)\widetilde{\varepsilon_j}(1)}{P_k(0)} \qquad \widetilde{Ret}_k(2) = \frac{\lambda_{kj2}(2)\mu_2\left\{\varepsilon_j(2)|x_j^a(2)\right\}+\lambda_{kk1}(2)\mu_2\left\{\varepsilon_k(1)|x_j^a(2)\right\}}{P_k(1)}$$

We can express $\widetilde{Ret}_j(2)$ as:

$$\widetilde{Ret}_j(2) = \frac{(1+\lambda_{jj2}(2))}{P_j(1)}\frac{COV\left[x_j^a(2),\varepsilon_j(2)\right]*\sigma_j}{\sigma_1\left\{x_j^a(2)\right\}}\left(x_j^a(2) - \omega_{jj}x_j^a(1) + \omega_{jk}\left(\omega_{kj}x_j^a(0) + \omega_{kk}x_k^a(0)\right)\right) + \frac{(\omega_{jk}+\lambda_{jk1}(2))}{P_j(1)}\frac{COV\left[x_j^a(2),\varepsilon_k(1)\right]*\sigma_k}{\sigma_1\left\{x_j^a(2)\right\}}\left(x_j^a(2) - \omega_{jj}x_j^a(1) + \omega_{jk}\left(\omega_{kj}x_j^a(0) + \omega_{kk}x_k^a(0)\right)\right)$$

Replacing $x_j^a(1) = \omega_{jj}x_j^a(0) + \omega_{jk}x_k^a(0) + \tilde{\varepsilon}_j(1)$, we get:

$$\widetilde{Ret}_j(2) = \frac{(1+\lambda_{jj2}(2))}{P_j(1)}\frac{COV\left[x_j^a(2),\varepsilon_j(2)\right]*\sigma_j}{\sigma_1\left\{x_j^a(2)\right\}}\left(x_j^a(2) - \omega_{jj}\tilde{\varepsilon}_j(1) - \omega_{jj}\left(\omega_{jj}x_j^a(0) + \omega_{jk}x_k^a(0)\right) + \omega_{jk}\left(\omega_{kj}x_j^a(0) + \omega_{kk}x_k^a(0)\right)\right) + \frac{(\omega_{jk}+\lambda_{jk1}(2))}{P_j(1)}\frac{COV\left[x_j^a(2),\varepsilon_k(1)\right]*\sigma_k}{\sigma_1\left\{x_j^a(2)\right\}}\left(x_j^a(2) - \omega_{jj}\tilde{\varepsilon}_j(1) + \omega_{jj}\left(\omega_{jj}x_j^a(0) + \omega_{jk}x_k^a(0)\right) - \omega_{jk}\left(\omega_{kj}x_j^a(0) + \omega_{kk}x_k^a(0)\right)\right).$$

$\widetilde{Ret}_j(1)$ and $\widetilde{Ret}_j(2)$ are correlated if $COV\left[\widetilde{Ret}_j(1), \widetilde{Ret}_j(2)\right] \neq 0$:

$$\frac{(1+\lambda_{jj2}(2))}{P_j(1)} \frac{COV\left[x_j^a(2), \varepsilon_j(2)\right] * \sigma_j}{\sigma_1\left\{x_j^a(2)\right\}}\left(-\omega_{jj}\widetilde{\varepsilon}_j(1)\right) + \frac{(\omega_{jk}+\lambda_{jk1}(2))}{P_j(1)} \frac{COV\left[x_j^a(2), \varepsilon_k(1)\right] * \sigma_k}{\sigma_1\left\{x_j^a(2)\right\}}\left(-\omega_{jj}\widetilde{\varepsilon}_j(1)\right) \neq 0$$

$\frac{COV\left[x_j^a(2), \varepsilon_j(2)\right] * \sigma_j}{COV\left[x_j^a(2), \varepsilon_k(1)\right] * \sigma_k} \neq -\frac{\left(\omega_{jk}+\lambda_{jk1}(2)\right)}{\left(1+\lambda_{jj2}(2)\right)}$, which simplifies to: $\frac{\sigma_j^3}{\sigma_k^3} \neq -\omega_{jk}^2 \frac{(1+\alpha_{jj})}{(1+\alpha_{jj}\omega_{kj})}$.

Furthermore, $\widetilde{Ret}_k(1)$ and $\widetilde{Ret}_j(2)$ are correlated if and $COV\left[\widetilde{Ret}_k(1), \widetilde{Ret}_j(2)\right] \neq 0$:

$\frac{\lambda_{kj2}(2)\mu_2\left\{\varepsilon_j(2)|x_j^a(2)\right\}+\lambda_{kk1}(2)\mu_2\left\{\varepsilon_k(1)|x_j^a(2)\right\}}{P_k(1)} \neq 0$, which simplifies to: $\frac{\sigma_j^3}{\sigma_k^3} \neq -\omega_{jk} \frac{R_F+\omega_{kk}+\alpha_{kj}\omega_{jk}}{\alpha_{kj}}$.