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Real Options: A Primer[1]

Lenos Trigeorgis
University of Cyprus

Abstract - This paper serves to introduce the basic ideas and valuation principles for corporate real options, and basic concepts related to growth options, competition and strategy. It first uses an example to motivate the discussion of various real options and presents practical principles for valuing several common real options, such as the options to defer investment, expand capacity, abandon the project, or switch uses. It then presents a conceptual discussion of growth options, competition and strategy, proposing strategic questions and a new project classification scheme as a practical aid for option-based analysis. Finally, it discusses various applications and notes areas for future research.

Many academics and practicing managers now recognize that the net present value (NPV) rule and other discounted cash flow (DCF) approaches to capital budgeting are inadequate in that they cannot properly capture management's flexibility to adapt and revise future decisions in response to unexpected market developments. Traditional NPV makes implicit assumptions about an "expected scenario" of cash flows and presumes management's passive commitment to a certain "operating strategy" (for example, to initiate the project immediately and operate it continuously at base scale until the end of its pre-specified expected useful life).

The real-world marketplace, however, is characterized by change, uncertainty and competitive interactions, where the realization of cash flows will probably differ from what management expected initially. As new information arrives and uncertainty about market conditions and future cash flows is gradually resolved, management may have valuable flexibility to alter its operating strategy in order to capitalize on favorable future opportunities or mitigate downside losses. For example, management may be able to defer, expand, contract, abandon, or otherwise alter a project at different stages during its useful operating life.

Management's flexibility to adapt its future actions in response to altered future market conditions expands an investment opportunity's value by improving its upside potential. It can also limit downside losses relative to management's initial expectations under passive management. The resulting asymmetry (skewness) caused by managerial adaptability calls for an *expanded NPV* rule that reflects both value components: the traditional (static or passive) NPV of expected cash flows, and the option value of operating and strategic adaptability. This does not mean that traditional NPV should be scrapped; rather, it should be seen as a crucial and necessary input to an options-based *expanded NPV* analysis, i.e.:

expanded (strategic) NPV = static (passive) NPV of expected cash flows (1)
+ value of options from active management

An options approach to capital budgeting has the potential to conceptualize, and even quantify, the value of options from active management. This value is manifest as a collection of corporate real (call or put) options embedded in capital investment opportunities, having as the underlying asset the gross project value of expected operating cash flows (ignoring capital costs and any embedded options). Many of these real options occur naturally (e.g., to defer, contract, shut down or abandon a project), while others may be planned and built-in at some cost (e.g., to expand capacity or build growth options, to abandon during construction when investment is staged sequentially, to switch between alternative inputs or outputs).

This paper provides an overview of real options, describing the basic principles for quantifying their value as well as thinking conceptually about the important competitive/strategic dimensions. An oil extraction and refinery project is used as an example to introduce the basic nature of various real options. The paper then presents, through simple numerical examples, useful principles for valuing various upside-potential operating options, such as to defer an investment or expand production, as well as various downside-protection options, such as to abandon for salvage value, switch among alternative uses (e.g., inputs or outputs), or abandon a project midstream during construction. Finally, a new options-based project classification scheme and strategic questions for capital budgeting analysis are proposed.

Section 1 of this paper uses an example to motivate the discussion of various real options and presents practical principles for valuing several such options. Section 2 presents a conceptual discussion of growth options, competition and strategy. Section 3 discusses applications and notes areas for future research. The paper's conclusions are presented in Section 4.

1. AN EXAMPLE AND BASIC VALUATION PRINCIPLES

This section discusses conceptually the basic nature and types of real options through a comprehensive example, and then illustrates some basic principles for valuing such options. A summary of the most common types of real options, the industries in which they are important, and related literature is provided in Table 1.

Table 1: Common Real Options

Category	Description	Important in	References
Option to defer	Management holds a lease on (or an option to buy) valuable land or resources. It can wait *x* years to see if output prices justify constructing a building or a plant or developing a field.	All natural-resource-extraction industries; real-estate development; farming; paper products.	McDonald and Siegel 1986; Paddock et al. 1988; Tourinho 1979; Titman 1985; Ingersoll and Ross 1992
Time-to-build option (staged investment)	Staging investment as a series of outlays creates the option to abandon the enterprise in midstream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequent stages and valued as a compound option.	All R&D-intensive industries, especially pharmaceuticals; long-development capital-intensive projects (e.g., large-scale construction or energy-generating plants); startup ventures.	Majd and Pindyck 1987; Carr 1988; Trigeorgis 1993
Option to alter operating scale (e.g., to expand; to contract; to shut down and restart)	If market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favorable than expected, it can reduce the scale of operations. In extreme cases, production may be halted and restarted.	Natural-resource industries (e.g., mining); facilities planning and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate.	Trigeorgis and Mason 1987; Pindyck 1988; Mcdonald and Siegel 1985; Brennan and Schwartz 1985
Option to abandon	If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets on secondhand markets.	Capital-intensive industries (e.g., airlines, railroads); financial services; new-product introductions in uncertain markets.	Myers and Majd 1990
Option to switch (e.g., outputs or inputs)	If prices or demand change, management can change the output mix of the facility (product flexibility). Alternatively, the same outputs can be produced using different types of inputs (process flexibility).	*Output shifts:* Any good sought in small batches or subject to volatile demand (e.g., consumer electronics); toys; specialty paper; machine parts; autos. *Input shifts:* All feedstock-dependent facilities; electric power; chemicals; crop switching; sourcing.	Margrabe 1978; Kensinger 1987; Kulatilaka 1988; Kulatilaka and Trigeorgis 1994
Growth options	An early investment (e.g., R&D, lease on undeveloped land or oil reserves, strategic acquisition, information network) is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities (e.g., new product or process, oil reserves, access to new market, strengthening of core capabilities). Like interproject compound options.	All infrastructure-based or strategic industries--esp. high tech, R&D, and industries with multiple product generations or applications (e.g., computers, pharmaceuticals); multinational operations; strategic acquisitions.	Myers 1977; Brealey and Myers 1991; Kester 1984, 1993; Trigeorgis 1988; Pindyck 1988; Chung and Charoenwong 1991
Multiple interacting options	Real-life projects often involve a collection of various options. Upward-potential-enhancing and downward-protection options are present in combination. Their combined value may differ from the sum of their separate values, i.e., they interact. They may also interact with financial flexibility options.	Real-life projects in most industries listed above.	Trigeorgis 1993; Brennan and Schwartz 1985; Kulatilaka 1994

1.1 An Oil Extraction and Refinery Project

A large oil company has a one-year lease to start drilling on undeveloped land with potential oil reserves. Initiating the project may require certain exploration costs, to be followed by the construction of roads and other infrastructure outlays, I_1. This would be followed by outlays for the construction of a new processing facility, I_2. Extraction can begin only after construction is completed, i.e., cash flows are generated only during the "operating stage" that follows the last outlay.

During construction, if market conditions deteriorate, management can choose to abandon the project midstream and forego any future planned outlays. It could also choose to reduce the scale of operation by c%, saving a portion of the last outlay, I_C, if the market is weak. Also, the processing plant can be designed up front so that, if oil prices turn out higher than expected, the rate of production can be enhanced by x% with a follow-up outlay of I_E. At any time, management may salvage a portion of its investment by selling the plant and equipment for their salvage value or switching them to an alternative use value, A. An associated refinery plant – which may be designed to operate with alternative sources of energy inputs – can convert crude oil into a variety of refined products. This type of project presents the following collection of *real options*:

- **The option to defer investment.** The lease enables management to defer investment for up to one year and benefit from the resolution of uncertainty about oil prices during this period. Management would invest I_1 (i.e., exercise its option to extract oil) *only if* oil prices increase sufficiently, but would not commit to the project, saving the planned outlays, if prices decline. Just before the lease expires, the value creation will be $\max(V - I_1, 0)$. The option to defer is thus analogous to a U.S. call option on the gross present value of the completed project's expected operating cash flows, V, with the exercise price being equal to the required outlay, I_1. Because early investment implies sacrificing the option to wait, this option value loss is like an additional investment opportunity cost, justifying investment only if the value of cash benefits, V, actually exceeds the initial outlay by a substantial premium. As noted in Table 1, the option to wait is particularly valuable in resource extraction industries, farming, paper products, and real estate development due to high uncertainties and long investment horizons.
- **The option to abandon during construction (or the time-to-build option).** In most real-life projects, the required investment is not incurred as a single up-front outlay. The staging of capital investment as a series of outlays over time creates valuable options to "default" at any given stage (e.g., after explo-

ration if the reserves or oil prices are determined to be very low). Thus, each stage (e.g., building necessary infrastructure) can be viewed as an option on the value of subsequent stages by incurring the installment cost outlay (e.g., I_1) required to proceed to the next stage, and can therefore be valued in a manner similar to compound options. This option is valuable in all R&D-intensive industries, especially pharmaceuticals; in highly uncertain, long-development, capital-intensive industries, such as energy-generating plants or large-scale construction; and in venture capital.

- **The option to expand.** If oil prices or other market conditions turn out more favorable than expected, management can accelerate the rate or expand the scale of production (by x%) by incurring a follow-up cost outlay (I_E). This is similar to a call option to acquire an additional part (x%) of the base-scale project, paying I_E as the exercise price. The investment opportunity with the option to expand can be viewed as the base-scale project plus a call option on future investment, i.e., $V + \max(xV - I_E, 0)$. Given an initial design choice, management may deliberately favor a more expensive technology because of the built-in flexibility to expand production if and when it becomes desirable. As discussed further below, the option to expand may also be of strategic importance, especially if it enables the firm to capitalize on future growth opportunities. When the firm buys vacant undeveloped land, or when it builds a small plant in a new geographic location (domestic or overseas) to position itself to take advantage of a developing large market, it essentially installs an expansion/growth option. This option, which will be exercised only if future market developments turn out to be favorable, can make a seemingly unprofitable (based on static NPV) base-case investment worth undertaking.

- **The option to contract.** If market conditions are weaker than originally expected, management can operate below capacity or even reduce the scale of operations (by c%), thereby saving part of the planned investment outlays (I_C). This flexibility to mitigate loss is analogous to a put option on part (c%) of the base-scale project, with the exercise price equal to the potential cost savings (I_C), giving $\max(I_C - cV, 0)$. The option to contract, just as the option to expand, may be particularly valuable in the case of new product introductions in uncertain markets. This option may also be important, for example, in choosing among technologies or plants with a different construction-to-maintenance cost mix, where it may be preferable to build a plant with lower initial construction costs and higher maintenance expenditures in order to acquire the flexibility to contract operations by cutting down on maintenance if market conditions turn out to be unfavorable.

- **The option to shut down (and re-start) operations.** In real life, the plant does not have to operate (i.e., extract oil) in each and every period automatically. In fact, if oil prices are such that cash revenues are not sufficient to cover variable operating costs (e.g., maintenance), it might be better not to operate temporarily, especially if the costs of switching between the operating and idle modes are relatively small. If prices rise sufficiently, operations can start again. Thus, operation in each year can be seen as a call option to acquire that year's cash revenues (C) by paying the variable costs of operating (I_V) as the exercise price, i.e., $\max(C - I_V, 0)$.[2] Options to alter the operating scale (i.e., expand, contract, or shut down) are typically found in natural resource industries, such as mine operations, facilities planning and construction in cyclical industries, fashion apparel, consumer goods, and commercial real estate.

- **The option to abandon for salvage value.** If oil prices suffer a sustainable decline or the operation does poorly for some other reason, management does not have to continue incurring the fixed costs. It may instead have a valuable option to abandon the project permanently in exchange for its salvage value (i.e., the resale value of its capital equipment and other assets in second-hand markets). As noted, this option can be valued as a U.S. put option on current project value (V) with the exercise price being the salvage or best alternative use value (A), entitling management to receive $V + \max(A - V, 0)$ or $\max(V, A)$. Naturally, more general-purpose capital assets would have a higher salvage and option abandonment value than special-purpose assets. Valuable abandonment options are generally found in capital-intensive industries, such as in airlines and railroads, in financial services, and in new product introductions in uncertain markets.

- **The option to switch use (e.g., inputs or outputs).** Suppose the associated oil refinery operation can be designed to use alternative forms of energy inputs (e.g., fuel oil, gas, electricity) to convert crude oil into a variety of output products (e.g., gasoline, lubricants, polyester). This would provide valuable built-in flexibility to switch from the current input to the cheapest future input, or from the current output to the most profitable future product mix, as the relative prices of the inputs or outputs fluctuate over time. In fact, the firm should be willing to pay a certain positive premium for such a flexible technology over a rigid alternative that confers no or less choice. If the firm can in this way develop extra uses for its assets over its competitors, it may be at a significant advantage.

  Generally, *process* flexibility can be achieved not only via technology (e.g., by building a flexible facility that can switch among alternative energy *inputs*) but also by maintaining relationships with a variety of suppliers and changing the mix as their relative prices change. Subcontracting policies may allow further flexibility to contract the scale of future operations at a low cost in case of unfavorable market developments. A multinational oil company may locate production facilities in various countries, allowing it to shift production to the lowest-cost producing facilities, as the relative costs, other local market conditions, or exchange rates change over time. Process flexibility is valuable in feedstock-dependent facilities, such as oil, electric power, chemicals, and crop switching. *Product* flexibility, which enables the firm to switch among alternative *outputs*, is more valuable in industries such as automobiles, consumer electronics, toys or pharmaceuticals, where product differentiation and diversity are important and/or product demand is volatile. In such cases, it may be worthwhile to install a more costly flexible capacity that gives the company the ability to alter product mix or production scale in response to changing market demands.

- **Corporate growth options.** Another version of the earlier option to expand that is of considerable strategic importance is *corporate growth options* that set the path of future opportunities. Suppose, in the above example, that the proposed refinery facility is based on a new, technologically superior *process* for oil refinement that has been developed and tested internally on a pilot plant basis. Although the proposed facility may appear unattractive in isolation, it could be only the first in a series of similar facilities if the process is successfully developed and commercialized, and may even lead to entirely new oil by-products. More generally, many early investments (e.g., R&D, a lease on undeveloped land or a tract with potential oil reserves, a strategic acquisition, an information technology network) can be seen as prerequisites or links in a chain of interrelated projects. The value of these projects may derive not so much from their expected directly measurable cash flows, but rather from unlocking future growth opportunities (e.g., a new-generation product or process, oil reserves, access to a new or expanding market, strengthening of the firm's core capabilities or strategic positioning). An opportunity to invest in a first generation high-tech product, for example, is analogous to an option on options (an inter-project compound option). Despite a seemingly negative NPV, the infrastructure, experience, and potential by-products generated during the development of the first-generation product may serve as spring-boards for developing lower-cost or improved-quality future generations of that product, or even for generating new applications into other areas. But unless the firm makes that initial investment, subsequent generations or other applications would not even be feasible. The infrastructure and experience gained can be proprietary and can place the firm at a competitive advantage, which may even reinforce itself if learning cost curve effects are present. Growth options are found in all infrastructure-based or strategic industries, especially in high-tech, R&D, or industries with multiple product generations or applications (e.g., semiconductors, computers, pharmaceuticals), in multi-national operations, and in strategic acquisitions.

In a more general context, the operating and strategic adaptability represented by corporate real options can be achieved at various stages during the value chain, from switching the factor input mix among various suppliers and subcontracting practices, to rapid product design (e.g., computer-aided design) and modularity in design, and to shifting production among various products rapidly and cost-efficiently in a flexible manufacturing system. The next section illustrates, through simple numerical examples, basic practical principles for valuing several of the above real options. For expositional simplicity, any return shortfall or other dividend-like effects are ignored.

1.2 Principles for Valuing Real Options

Consider, as in Trigeorgis and Mason (1987),[3] valuing a generic investment opportunity (e.g., similar to the above oil extraction project). Specifically, suppose a company is faced with an opportunity to invest I_0 = \$104 (in millions) in an oil project whose (gross) value in each period will either move up by 80% or down by 40%, depending on oil price fluctuations: a year later, the project will have an expected value (from subsequent cash flows) of \$180 (million) if the oil price moves up ($C^+ = 180$) or \$60 if it moves down ($C^- = 60$).[4] There is an equal probability ($q = .5$) that the price of oil will move up or down in any year. Let S be the price of oil, or generally of a *twin security* that is traded in the financial markets and has the same risk characteristics as (i.e., is perfectly correlated with) the real project under consideration (such as the stock price of a similar operating unlevered oil company). Both the project and its twin security (or oil prices) have an expected rate of return (or discount rate) of $k = 20$ percent, while the risk-free interest rate is $r = 8$ percent.

In what follows assume throughout that the value of the project (i.e., the value, in millions of dollars, in each year, t, of its subsequent expected cash flows appropriately discounted back to that year), V_t, and its twin security price (e.g., a twin oil stock price in \$ per share, or simply, the price of oil in \$ per barrel), S_t, move through time as follows:

```
                                        (324, 64.88)
                                       /
                        (180, 36)
                       /         \
          (100, 20)                     (108, 21.6)
                       \         /
                        (60, 12)
                                 \
                                        (36, 7.2)
Year  0                           1          2
```

For example, the pair (V_0, S_0) above represents a current gross project value of \$100 million, and a spot oil price of \$20 a barrel (or a \$20 a share twin oil stock price). Under traditional (passive) NPV analysis, the current gross project value would be obtained first by discounting the project's end-of-period values (derived from subsequent cash flows), using the expected rate of return of the project's twin security (or, here, of oil prices) as the appropriate discount rate, i.e., $V_0 = (.5 \times 180 + .5 \times 60)/1.20 = 100$. Note that this gross project value is, in this case, exactly proportional to the twin security price (or the spot oil price). After subtracting the current investment costs, $I_0 = 104$, the project's NPV is finally given by:

$$NPV = V_0 - I_0 = 100 - 104 = -4 \ (< 0). \qquad (2)$$

In the absence of managerial flexibility or real options, traditional DCF analysis would have *rejected* this project based on its negative NPV. However, passive DCF cannot properly capture the value of embedded options because of their discretionary asymmetric nature and dependence on future events that are uncertain at the time of the initial decision. The fundamental problem, of course, lies in the valuation of investment opportunities whose claims are not symmetric or proportional and whose discount rates vary in a complex way over time.

Nevertheless, such real options can be properly valued using contingent claims analysis (CCA) within a backward risk-neutral valuation process.[5] Essentially, the same solution can be obtained in the actual risk-averse world as in a *risk-neutral* world in which the current value of any contingent claim could be obtained from its expected future values — with expectations taken over the *risk-neutral probabilities*, p, imputed from the twin security's (or oil) prices — discounted at the *riskless* rate, r. In such a risk-neutral world, the current (beginning of the period) value of the project (or of equityholders' claim), E, is given by:

$$E = [pE^+ + (1 - p)E^-]/(1 + r),$$
$$\text{with } p = [(1 + r)S - S^-]/(S^+ - S^-). \qquad (3)$$

The probability, p, can be estimated from the price dynamics of the twin security (or of oil prices):

$p = [1.08 \times 20 - 12]/(36 - 12) = 0.4$ (as distinct from the actual probability, $q = 0.5$),
and can then be used to determine "certainty-equivalent" values (or expected cash flows), which can be properly discounted at the risk-free rate. For example,

$$V_0 = [pC^+ + (1 - p)C^-]/(1 + r) = [.4 \times 180 + .6 \times 60]/1.08 = 100.[6] \qquad (4)$$

In what follows, it is assumed that if any part of the required investment outlay (having a present value of $104 million) is not going to be spent immediately but in future installments, that amount is placed in an escrow account earning the riskless interest rate.[7] The next section illustrates how various kinds of both upside-potential options (such as to defer or expand) and downside-protection options (such as to abandon for salvage or default during construction) can enhance the value of the *opportunity* to invest (i.e., the value of equity or NPV) in the above generic project, under the standard assumption of all-equity financing. The focus here is on basic practical principles for valuing one kind of operating option at a time.

1.2.1 The Option to *Defer* Investment

The company has a one-year lease that gives it a proprietary right to defer undertaking the project (i.e., extracting the oil) for a year, thus benefiting from the resolution of uncertainty about oil prices over this period. Athough undertaking the project *immediately* has a negative NPV (of -4), the *opportunity* to invest afforded by the lease has a positive worth because management would invest *only* if oil prices and project value rise sufficiently, while it has no obligation to invest under unfavorable developments. Since the option to wait is analogous to a call option on project value, V, with an exercise price equal to the required outlay next year, $I_1 = 112.32$ (= 104 x 1.08):

$$E^+ = \max(V^+ - I_1, 0) = \max(180 - 112.32, 0) = 67.68,$$
$$E^- = \max(V^- - I_1, 0) = \max(60 - 112.32, 0) = 0. \qquad (5)$$

The project's total value (i.e., the *expanded NPV*, which includes the value of the option to defer) from Equation (3) is:

$$E_0 = [pE^+ + (1 - p)E^-]/(1 + r) = [.4 \times 67.68 + .6 \times 0]/1.08 = 25.07. \quad (6)$$

From Equation (1), the value of the option to defer provided by the lease itself is thus given by:

$$\textit{Option to defer} = \text{expanded NPV} - \text{passive NPV} = 25.07 - (-4) = 29.07, \quad (7)$$

which, incidentally, is equal to almost one-third of the project's gross value.[8]

1.2.2 The Option to *Expand* (Growth Option)

Once the project is undertaken, any necessary infrastructure is completed, and the plant is operating, management may have the option to accelerate the rate or expand the scale of production by, say, 50% ($x = 0.50$) by incurring a follow-on investment outlay of $I_E = 40$, provided oil prices and general market conditions turn out better than originally expected. Thus, in year 1 management can choose either to maintain the base scale operation (i.e., receive project value, V, at no extra cost) or expand by 50% the scale and project value by incurring the extra outlay. That is, the original investment opportunity is seen as the initial-scale project plus a call option on a future opportunity, or $E = V + \max(xV - I_E, 0) = \max(V, (1+x)V - I_E)$:

$$E^+ = \max(V^+, 1.5V^+ - I_E) = \max(180, 270 - 40) = 230 \text{ (expand)};$$
$$E^- = \max(V^-, 1.5V^- - I_E) = \max(60, 90 - 40) = 60 \text{ (maintain base scale)}. \quad (8)$$

The value of the investment opportunity (including the value of the option to expand if market conditions turn out better than expected) then becomes:

$$E_0 = [pE^+ + (1 - p)E^-]/(1 + r) - I_0 = [.4 \times 230 + .6 \times 60]/1.08 - 104 = 14.5, \quad (9)$$

and thus the value of the *option to expand* = 14.5 - (-4) = 18.5, (10)

or 18.5% of the gross project value.

1.2.3 Options to Abandon for *Salvage* Value or *Switch Use*

In terms of downside protection, management has the option to abandon the oil extraction project at any time in exchange for its *salvage value* or value in its best alternative use, if oil prices suffer a sustainable decline. The associated oil refinery plant also can use alternative energy inputs and has the flexibility to convert crude oil into a variety of products. As market conditions change and the relative prices of inputs, outputs or the plant resale value in a second-hand market fluctuate, equityholders may find it preferable to abandon the current project's use by switching to a cheaper input, a more profitable output, or simply selling the plant's assets to the second-hand market. Let the project's value in its best alternative use, A (or the *salvage value* for which it can be exchanged) fluctuate over time as:

```
                   230.4
                  /
            144
          /     \
     90           115.2
          \     /
            72
                  \
                   57.6
Year 0      1      2
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Note that the project's current salvage or alternative use value ($A_0 = 90$) is below the project's value in its present use ($V_0 = 100$) – otherwise management would have switched use immediately – and has the same expected rate of return (20%). It nevertheless has a smaller variance so that if the market keeps moving up it would not be optimal to abandon the project early for its *salvage value*, but if it moves down management may find it desirable to *switch use* (e.g., in year 1 exchange the present use value of $V_1 = 60$ for a higher alternative use value of $A_1 = 72$).[9] Thus, equityholders can choose the maximum of the project's value in its present use, V, or its value in the best alternative use, A, i.e., E = max(V, A):

$$E^+ = \max(V^+, A^+) = \max(180, 144) = 180 = V^+ \text{ (continue);}$$
$$E^- = \max(V^-, A^-) = \max(60, 72) = 72 = A^- \text{ (switch use).} \quad (11)$$

The value of the investment (including the option to abandon early or switch use) is then:

$$E_0 = [pE^+ + (1 - p)E^-]/(1 + r) - I_0 = [.4 \times 180 + .6 \times 72]/1.08 - 104 = +2.67, \quad (12)$$

so that the project with the option to switch use is now desirable. The value of the option itself is:

$$\textit{Option to switch use} = 2.67 - (-4) = 6.67, \quad (13)$$

or almost 7 percent of the project's gross value. This value is clearly dependent on the schedule of salvage or alternative use values.

1.2.4 The Option to *Default* During Construction

Even during the construction phase, management may abandon a project to save any subsequent investment outlays, if the coming required investment exceeds the value from continuing the project (including any future options). Suppose that the investment (of $104 present value) necessary to implement the oil extraction project can be staged as a series of "installments:" I_0 = $44 out of the $104 allocated amount will need to be paid out immediately (in year 0) as a start-up cost for infrastructure, with the $60 balance placed in an escrow account (earning the risk-free rate) planned to be paid as a I_1 = $64.8 follow-up outlay for constructing the processing plant in year 1. Next year management will then pay the investment cost "installment" as planned only in return for a higher project value from continuing; otherwise, it will forego the investment and receive nothing. Thus, the option to default when investment is staged sequentially during construction translates into $E = \max(V - I_1, 0)$:

$$E^+ = \max(V^+ - I_1, 0) = \max(180 - 64.8, 0) = 115.2 \text{ (continue)};$$
$$E^- = \max(V^- - I_1, 0) = \max(60 - 64.8, 0) = 0 \text{ (default)}. \quad (14)$$

The value of the investment opportunity (with the option to default on future outlays) is given by:

$$E_0 = [pE^+ + (1 - p)E^-]/(1 + r) - I_0 = [.4 \times 115.2 + .6 \times 0]/1.08 - 44 = -1.33, \quad (15)$$

and the *option to abandon by defaulting* during construction = -1.33 - (-4) = 2.67, (16)

or about 3 percent of project value. This value is, of course, dependent on the staged cost schedule.

For simplicity, the above examples were based on a one-period risk neutral, backward valuation procedure. This procedure can be easily extended to a discrete multiperiod setting with any number of stages. Starting from the terminal values, the process would move backwards calculating option values one step earlier (using the up and down values obtained in the preceding step), and so on. As the number of steps increases, the discrete-time solution would approach its continuous (Black-Scholes type) equivalent (with appropriate adjustments).

2. GROWTH OPTIONS, COMPETITION AND STRATEGY

The most significant decisions in many cases, of course, are not so much the operating ones but those involving growth options, competition and strategy. For these dimensions to be properly captured, it must first be explicitly recognized that there are certain important differences between financial and real options. This section describes a general conceptual framework for viewing real investment opportunities as collections of real options (an *expanded or strategic NPV* framework) that integrates the important operating options (e.g., the options to defer or abandon a project early) with competitive/strategic interactions. Specifically, Section 2.1 presents an alternative, options-based project classification scheme, while Section 2.2 discusses strategic questions for capital budgeting analysis. To motivate the new options-based classification scheme, it is useful to first discuss some of the important differences between real and financial options. These include:

- **(Non)proprietary ownership/competitive impact.** A standard call option on common stock is "*proprietary*" in that it gives its owner an exclusive right of whether and when to exercise, i.e., the option holder does not have to worry about competition for the underlying investment. Similarly, some real options are proprietary in that they provide their holder with such exclusive rights of exercise, uninhibited by competitive threats. Investment opportunities with high barriers of entry for competitors such as a patent for developing a product having no close substitutes, or a unique knowhow of a technological process or market conditions that competitors are unable to duplicate for at least some time, are but a few examples of such real proprietary options.

  Other types of investment opportunities, however, may be jointly held by more than a single competitor. These real options are "*shared*" in that, as collective opportunities of the industry, they can be exercised by any one of the participants. Examples of such *shared real options* include the opportunity to introduce a new product unprotected by the possible introduction of close substitutes, or to penetrate a new geographic market without barriers to competitive entry. The nature of competitive reaction may, of course, be different if the investment opportunity is proprietary or shared.

- **Non-tradeability and preemption.** Standard call options on stocks, like stocks themselves, can be traded frequently in efficient financial markets at minimal costs. Real options, however, like most investment projects, are not generally tradeable.[10] Some proprietary real options – such as investment opportunities related to patents or licensing agreements – may be traded, although possibly at substantial costs in imperfect markets. Of course, certain proprietary projects may be abandoned before the end of their useful lives and traded for their *salvage value.*

Other real options may inseparably depend on other real or intangible assets with which they may be sold only as a package. On the other hand, shared real options may not be salable at all because they are already a collective or "public good" of the whole industry; a firm holding a real option shared by competitors cannot easily avoid even anticipated losses in value resulting from competitive entry by just turning around to sell the option. In many cases, the only available protection against such value losses is an early investment on its part, if it can, by so doing, preempt competitors from exercising their shared rights (e.g., see Spence 1979 and Dixit 1980 for various treatments of preemptive investments). For example, a firm anticipating an increase in demand – and hence subsequent competitive entry – may rush to expand its own production capacity early in order to preempt competition, whereas in the absence of such competition, it might have preferred to wait until the uncertainty surrounding future demand would resolve itself.

- **Across-time (strategic) interdependencies/compoundness.** Standard call options on common stock are *simple* in the sense that their value upon exercise derives entirely from the received shares of stock. Similarly, some real options (such as maintenance or standard replacement projects) are "*simple*" in that their value upon exercise is limited to the value of the underlying project's cash flows in themselves.

  Other real options, however, lead to further discretionary investment opportunities when exercised. In essence, they are options on options, or *compound options* (i.e., options whose payoff is another option).[11] Research and development (R&D) investments, a lease for an undeveloped tract with potential oil reserves, or an acquisition of an unrelated company are not undertaken just for the sake of the underlying asset alone, but also (or perhaps primarily) for the new opportunities that they may open up (a new technological breakthrough, large reserves of oil, or access to a new market).

  Real compound options may have a more *strategic* impact on a firm and are more complicated to analyze. They can no longer be looked at as independent investments, but rather as links in a chain of interrelated projects, the earlier of which may be prerequisites for the ones to follow. Again, the nature of compound real options that may invite competitive reaction (e.g., shared) may involve a more complicated (game-theoretic) analysis than proprietary ones.

2.1 Dimensions of Real Options Analysis: Toward a New Project Classification

In practice, firms often classify projects according to risk or functional characteristics (e.g., replacement or new product introduction) to simplify the capital budgeting process. These schemes are incomplete, however, in that they often overlook the option aspects of projects described earlier. To motivate a new options-based classification and be better able to appreciate the various dimensions that it encompasses, this discussion starts from simple NPV and gradually builds up the framework highlighting one aspect at a time. After discussing the flexibility to defer or abandon a project, it then focuses on the dimension of *compoundness* first within and later among projects, and finally highlights interactions introduced by competition.

2.1.1 Commitment to Invest: Static (Passive) NPV

Traditional NPV typically ignores strategic competitive interactions. But even in dealing with *games against nature*, naively applied NPV is further limited in that it implicitly presumes that management is *passive*, i.e., that all decisions are unequivocally taken upfront as if management does not have the flexibility to review its original plans in response to nature's deviation from its expected scenario of cash flows. As explained earlier, in the absence of such managerial flexibility, static or passive NPV would be correct: management would make an *immediate* investment outlay, I (considering for now the simplest case of a *single* one-time expenditure), only in return for a higher present value of expected cash inflows, V. The difference, i.e., NPV = V - I, is of course the current value of the *investment* (i.e., of an installed or completed project), provided the manager had no other choice but to "take it immediately, or leave it."

Note that mere delay in undertaking an investment does not *necessarily* confer flexibility to a project. Suppose that the firm has a *commitment* (e.g., due to environmental regulations) to make an investment, I, in the future (τ years from now). If the investment is traded and involves no intermediate cash flows, this delayed commitment value, as given by the value of a *forward contract* on a (non-dividend paying) asset of value V (assuming the investment cost does not escalate), would be: $V - I\,e^{-r\tau}$.

2.1.2 The *Opportunity* to Invest (Flexibility to Defer)

What is really of interest, however, is not the value of the immediate investment per se (or of the delayed commitment), but rather the value of the investment *opportunity*. As explained earlier, in a world of uncertainty where nature can "play games" (V may fluctuate randomly) the *opportunity to invest* can be more valuable than *immediate investment* (or a delayed commitment) because it gives management the *flexibility to defer* undertaking the investment until circumstances turn more favorable, or back out altogether if they become unsatisfactory.[12] The value of this opportunity to invest therefore exceeds the static NPV of cash flows from immediate investment (V - I) by the value of the flexibility to defer the investment. It also exceeds the value of a delayed commitment due to the future *choice* to avoid potentially unfavorable outcomes.

Such an investment opportunity may thus be economically desirable, even if the investment itself may have a *negative* NPV (i.e., $V < I$). It would therefore be very useful to distinguish between opportunities that allow management the *flexibility* to defer their undertaking and make the *choice* later after receiving additional information (such as projects with patents or leases), and projects that involve a *commitment* (such as an expiring offer to immediately expand capacity to meet extra demand by an impatient client or a required outlay to meet environmental regulations in the future).

Even if management lacks the flexibility to defer the undertaking of a project when faced with an immediate accept/reject decision, it may still have the flexibility to abandon a once-undertaken project for its *salvage value* before the end of its expected life if it turns out to perform worse than expected.[13] The flexibility to abandon a project early should therefore be explicitly accounted for in the investment decision whenever appropriate.

2.1.3 Multi-staged Projects (Intra-project Compoundness)

For now, assume that the flexibility to defer undertaking the project or abandon it for its salvage value is suppressed. Consider, however, the investment outlay, I, no longer as a single one-time expenditure at the outset, but rather as a sequence of investment cost "installments" starting immediately and extending throughout much of the life of the investment. In such a case the investment can actually be seen as a *compound option*, where an earlier investment cost installment represents the exercise price needed to acquire a subsequent option to continue operating the project until the next installment comes due, and so on. This is the idea of compoundness within the same multi-staged project – an intra-project interaction. If managerial flexibility is considered, intra-project compoundness highlights a series of distinct points in time (or *decision nodes*) – just before a subsequent investment installment comes due – when the project might be better discontinued if it turns out not to perform satisfactorily. DCF techniques, and particularly

NPV, that deal with the sequence of investment installments simply by subtracting their present value from that of the expected cash inflows (as if they were a commitment) or even by including all but the first investment installment costs in the so called "net cash flows," clearly undervalue such compound investments.

2.1.4 Project Interdependence (Inter-project Compoundness)

Return to the simple case of a single one-time investment outlay at the start of each project. Consider, however, the case of *contingent or interdependent projects* where undertaking the first is a prerequisite for the next, or provides the opportunity to acquire at maturity the benefits of the new investment by making a new investment. For example, a research project provides at completion the opportunity to acquire the revenues of the developed, commercialized product upon incurring a production outlay. This idea of *inter-project compoundness* is remarkably similar in structure when looking at a sequence of projects to the intra-project compoundness described above, with the difference that each investment "installment" now provides the opportunity to begin a new project rather than continue (another phase of) the same one. Compoundness between projects is an interaction of considerable *strategic* importance because it may justify the undertaking of projects with a *negative* NPV of direct cash flows on the basis of opening up subsequent future investment opportunities (or *growth options*).

2.1.5 Competition

Another dimension to the valuation of investment opportunities is introduced by competitive interaction. Here it is possible to distinguish between two forms of analysis depending on the type of interaction between competitors. If the impact of competitive entry can be considered *exogenous* and pertains basically to the threat of capturing part of the value of the investment away from the incumbent firm, then its management still faces an *optimization problem* – although a more complex one – in that it must incorporate the impact of competition in its own investment decision but can ignore any reciprocal effects of that decision on competitors' actions. If, however, each competitor's investment decisions are contingent upon and sensitive to the others' moves, as in most oligopolistic industries, then a more complex *game-theoretic* treatment becomes necessary. Investing earlier than one otherwise would to preempt competitive entry is a simple case of such *strategic games against competition*. Competitive strategy can be analyzed using a combination of option valuation principles with industrial organization (game-theoretic) concepts (e.g., see Smit and Trigeorgis, 1993).

2.2 Strategic Questions and an Options-based Project Classification

Based on the above dimensions of real options analysis, it is important for management to address a number of strategic questions in the investment evaluation process. The first refers to the *exclusiveness of option ownership* and the *effect of competition* on the firm's ability to fully appropriate the option value for itself. If the firm retains an exclusive right as to whether and when to invest, unaffected by competitive initiatives, then its investment opportunity is classified as a *proprietary option*. Investment opportunities with high barriers of entry for competitors such as a patent for developing a product having no close substitutes, or a unique knowhow of a technological process, or market conditions that competitors are unable to duplicate for at least some time, are just a few examples of such proprietary real options. In such cases, management may have the flexibility to abandon a project early (i.e., the project has additional *abandonment value*), or even temporarily interrupt the project's operation in certain "unprofitable" periods.[14] If, however, competitors share the right to exercise and may be able to take part (or all) of the project's value away from the firm, then the option is *shared*.[15] Shared real options can be seen as jointly held opportunities of a number of competing firms or of a whole industry, and can be exercised by any one of their collective owners. Such shared real options are, for example, the opportunity to introduce a new product unprotected by the possible introduction of close substitutes, or to penetrate a new geographic market without barriers to competitive entry. The loss in value suffered by a firm as a result of competitive interaction when a competitive firm exercises its shared rights is called *competitive loss*.[16]

The second strategic question concerns *inter- (or intra-) project interactions*, specifically compoundness.[17] Is an investment opportunity valuable in and by itself, or is it a prerequisite for subsequent investment opportunities? If the opportunity is a real option leading, upon exercise, to further discretionary investment opportunities, or an option whose payoff is another option, then it is classified as a *compound* option. Such real options on options may have a more *strategic* impact on a firm and are more complicated to analyze. They can no longer be looked at as *independent* investments, but rather as *links in a chain of interrelated projects*, the earlier of which are prerequisites for the ones to follow. An R&D investment, a lease for an undeveloped tract with potential oil reserves, or an acquisition of an unrelated company are just a few examples of such compound real options that may be undertaken, not just for their direct cash flows but also (or perhaps primarily) for the new opportunities that they may open up (a new technological breakthrough, large reserves of oil, or access to a new market). On the other hand, if the project can be evaluated as a *stand-alone* investment opportunity, it is referred to as a *simple* option. Such independent opportunities, whose value upon exercise is limited only to the underlying project in and of itself, are, for instance, standard replacement or maintenance projects.

The last *strategic question* refers to the committal or discretionary nature of the decision, focusing specifically on the *urgency of the decision.* Management must distinguish between those projects that need an immediate accept/reject decision (i.e., *expiring* investment opportunities) and those that can be deferred for future action (i.e., *deferrable* real options).[18] It would also be useful to further distinguish between deferrable investments that merely represent delayed commitments versus future decision (choice) opportunities. The value of the flexibility to defer undertaking a project is referred to here as the project's *deferrability value.* Discretionary deferrable projects require a more extensive analysis of the optimal timing of investment because management must compare the net value of taking the project today with the net value of taking it in all possible future years. Thus, management must analyze the relative benefits and costs of waiting in association with other strategic considerations (e.g., the threat of competitive entry in a shared-deferrable option may justify early capital commitment for preemptive purposes). This mode of analysis leads to the real options-based *classification scheme* shown in Figure 1.[19]

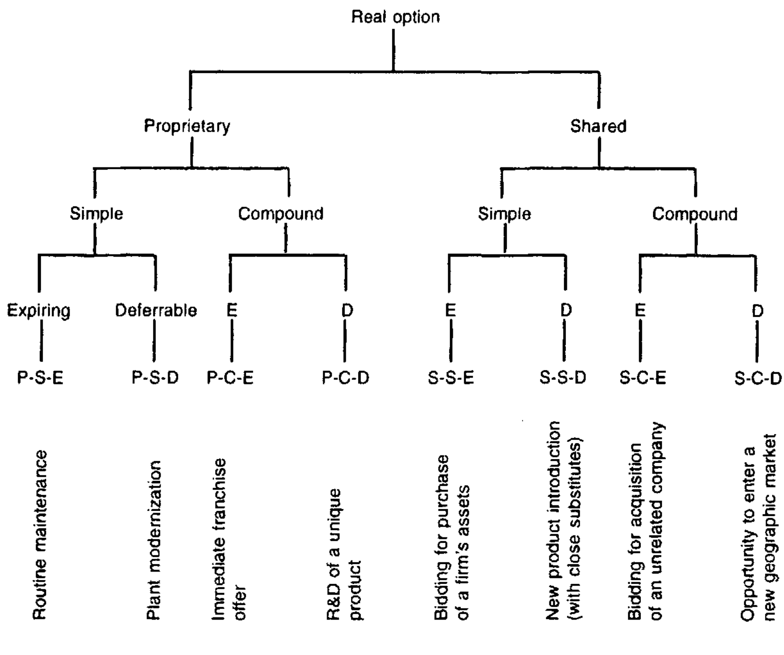

Figure 1: A Conceptual Options Framework for Capital Budgeting

This eight-fork classification scheme is intended to focus management's attention on the important characteristics of investment opportunities as options on real assets, as described above. Although the distinctions between the various categories may at times be more relative rather than absolute, most real investment opportunities, including strategic ones, can find a place in one of the eight branches of the options-based classification tree. For example, routine maintenance could be classified and analyzed as a proprietary-simple-expiring (P-S-E) option, plant modernization as proprietary-simple-deferrable (P-S-D), bidding for the purchase of assets as shared-simple-expiring (S-S-E), a new product introduction with close substitutes as shared-simple-deferrable (S-S-D), an immediate franchise offer as proprietary-compound-expiring (P-C-E), research and development of a unique product as proprietary-compound-deferrable (P-C-D), bidding for the acquisition of an unrelated company as shared-compound-expiring (S-C-E), and the opportunity to enter a new geographic market as shared-compound-deferrable (S-C-D).[20] It is clear that real options provides an intuitive way of thinking that is useful in most (especially the strategic) situations, as well as a way of quantifying the value of flexibility in specific (mostly operational) decision problems.

3. CURRENT AND FUTURE APPLICATIONS

Real options has been applied in a variety of contexts, such as in natural resource investments, land development, leasing, flexible manufacturing, government subsidies and regulation, R&D, new ventures and acquisitions, and foreign investment and strategy. Early applications naturally arose in the area of *natural resource investments* due to the availability of traded resource or commodity prices, high volatilities and long durations, resulting in higher and better option value estimates. Brennan and Schwartz (1985) first utilized the convenience yield derived from futures and spot prices of a commodity to value the options to shut down or abandon a mine. Paddock, Siegel and Smith (1988) valued options embedded in undeveloped oil reserves and provided the first empirical evidence that option values are better than actual DCF-based bids in valuing offshore oil leases. Trigeorgis (1990) values an actual minerals project considered by a major multinational company involving options to cancel during construction, expand production, and abandon for salvage. Bjerksund and Ekern (1990) value a Norwegian oil field with options to defer and abandon. Morck, Schwartz and Stangeland (1989) value forestry resources under stochastic inventories and prices. Laughton and Jacoby (1993) examine biases in the valuation of real options and long-term decision making when a mean-reversion price process is more appropriate. Kemna (1993) analyzes cases at Shell involving the timing of developing an offshore oil field, valuing a growth option in a manufacturing venture and the abandonment decision of a refining production unit.

In the area of *land development*, Titman (1985), Capozza and Sick (1994), and Quigg (1995) show that the value of vacant land should reflect not only its value based on its best immediate use (e.g., from constructing a building now) but also its option value if development is delayed and the land is converted into its best alternative use in the future. It may thus pay to hold land vacant for its option value, even in the presence of currently thriving real estate markets. Quigg (1993) reports empirical results indicating that option-based land valuation that incorporates the option to wait to develop land provides better approximations of actual market prices. In a different context, McLaughlin and Taggart (1992) view the opportunity cost of using excess capacity as the change in the value of the firm's options caused by diverting capacity to an alternative use. In *leasing*, Copeland and Weston (1982), McConnel and Schallheim (1983), Trigeorgis (1995b), and Grenadier and Weiss (1997) value various operating options embedded in leasing contracts.

In the area of *flexible manufacturing*, the flexibility provided by flexible manufacturing systems, flexible production technology or other machinery having multiple uses has been analyzed from an options perspective by Kulatilaka (1993, 1995), Triantis and Hodder (1990), and Kulatilaka and Trigeorgis (1994), among others. Baldwin and Clark (1993) study the flexibility created by modularity in design that connects components of a larger system through standard interfaces.

In the area of *government subsidies and regulation*, Mason and Baldwin (1988) value government subsidies to large-scale energy projects as put options, while Teisberg (1994) provides an option valuation analysis of investment choices by a regulated firm. In *research and development*, Kolbe, Morris and Teisberg (1991) discuss option elements embedded in R&D projects. Option elements involved in the staging of *start-up ventures* are discussed in Sahlman (1988) and Trigeorgis (1993b). Strategic *acquisitions* of other companies also often involve a number of growth, divestiture, and other flexibility options, as discussed by Smith and Triantis (1995). On the empirical side, Kester (1984) estimates that the value of a firm's growth options is more than half the market value of equity for many firms, even 70-80% for more volatile industries. Similarly, Pindyck (1988) suggests that growth options represent more than half of firm value if demand volatility exceeds 20%. In *foreign investment*, Baldwin (1987) discusses the various location, timing and staging options present when firms scan the global marketplace. Bell (1995) and Kogut and Kulatilaka (1994), among others, examine entry, capacity, and switching options for firms with multinational operations under exchange rate volatility. Hiraki (1995) suggests that the Japanese bank-oriented corporate governance system serves as the basic infrastructure that enables companies to jointly develop corporate real options. Various other option applications can be found in areas ranging from *shipping* (Bjerksund and Ekern, 1995) to *environmental pollution and global warming* (e.g., Hendricks, 1991). The potential for future applications itself seems like a growth option.

Other comprehensive treatments of real options can be found in articles by Mason and Merton (1985) and Trigeorgis and Mason (1987), a monograph by Sick (1989), an economics review article by Pindyck (1991), as well as edited volumes by Trigeorgis (1995a), Brennan and Trigeorgis (1999), and Trigeorgis (2000). The Spring 1987 issue of the *Midland Corporate Finance Journal*, a 1991 special issue of *Managerial Finance* (vol. 17, number 2/3), a special issue of *Financial Management* (Fall 1993), and a special issue of *The Quaterly Review of Economics and Finance* (vol. 38) have also been devoted to real options and capital budgeting. An Annual International Conference on Real Options also promotes current research and applications (www.realoptions.org). Clearly, an increased attention to application and implementation issues is the next stage in the evolution of real options.

Future applications are expected in the following areas:

- Focusing more on investments (such as in R&D, pilot or market tests, or excavations) that can *generate* information and learning (e.g., about the project's prospects) by extending/adjusting option pricing and risk-neutral valuation with Bayesian analysis or alternative (e.g., jump) processes.
- Exploring in more depth endogenous competitive counteractions and a variety of competitive/market structure and strategic issues using a combination of game theoretic industrial organization with option valuation tools.
- Modelling better the various strategic and growth options.
- Extending real options in an agency context, recognizing that the potential (theoretical) value of real options may not be realized in practice if managers, in pursuing their own agendas (e.g., expansion or growth, rather than firm value maximization), misuse their discretion and do not follow the optimal exercise policies implicit in option valuation. This raises the need for the firm to design proper corrective incentive contracts (taking also into account asymmetric information).
- Recognizing better that real options may interact not only among themselves but also with financial flexibility options, and understanding the resulting implications for the combined, interdependent corporate investment and financing decisions.
- On the practical side, applying real options to the valuation of flexibility in related areas, such as in competitive bidding, information technology or other

platform investments, energy and R&D problems, international finance options, and so on.

- Using real options to explain empirical phenomena that are amenable to observation or statistical testing, such as examining empirically whether the managements of firms that are targets for acquisition may sometimes turn down tender offers in part due to the option to wait in anticipation of receiving better future offers.
- Doing more field, survey, or empirical studies to test the conformity of theoretical real options valuation and its implications with management's intuition and experience, as well as with actual price data, when available.
- Revising current compensation and control systems to reflect the value of corporate real options and encouraging their proper exercise and management over time.
- Linking natural risk management through the exercise of the firm's real options in the capital budgeting area, with the broader risk management of the firm's other (financial) exposures in an holistic way as part of a total-enterprise package solution.

4. CONCLUSION

This paper provided a primer on real options, both describing the basic principles for quantifying their value as well as thinking conceptually about the important competitive/strategic dimensions. It described, through simple examples, practically useful principles for valuing various upside-potential operating options, such as to defer an investment or expand production, as well as various downside-protection options, such as to abandon for salvage value, switch among alternative uses (e.g., inputs or outputs), or abandon a project midstream during construction. It also sought to describe qualitatively a conceptual framework (an expanded or strategic NPV approach) for thinking about capital investment opportunities as collections of corporate real options, with emphasis on the important competitive/strategic dimensions that are typically left out of conventional DCF analyses. A new options-based project classification scheme and several strategic questions for capital budgeting analysis were proposed. This conceptual framework is intended as a practical aid in recognizing and understanding the frequently encountered collections of real options and the competitive/strategic dimensions of many investment opportunities.

NOTES

[1] This paper partly draws on work published by the author in *Financial Management, Advances in Options and Futures Research*, and elsewhere.

[2] Alternatively, management has an option to obtain project value V (net of fixed costs, I_F) minus variable costs (I_V), or shut down and receive the project value minus that year's foregone cash revenue (C), i.e., $\max(V - I_V, V - C) - I_F = (V - I_F) - \min(I_V, C)$. The latter expression implies that the option not to operate enables management to acquire project value (net of fixed costs) by paying the minimum of variable costs (if the project does well and management decides to operate) or the cash revenues (that would be sacrificed if the project does poorly and it chooses not to operate).

[3] Trigeorgis and Mason (1987) use a similar example to show how options-based valuation can be seen operationally as a special, although economically-corrected, version of decision tree analysis that recognizes open-market opportunities to trade and borrow.

[4] All project values are subsequently assumed to be in millions of dollars (with "millions" dropped).

[5] As noted, the basic idea is that management can replicate the payoff to equity by purchasing a specified number of shares of the "twin security" and financing the purchase in part by borrowing a specific amount at the riskless interest rate, r. This ability to construct a *synthetic* claim or an equivalent/replicating portfolio (from the "twin security" and riskless bonds) based on no arbitrage equilibrium principles enables the solution for the current value of the equity claim to be independent of the actual probabilities (in this case .5) or investors' risk attitudes (the twin security's expected rate of return or discount rate, k = .20).

[6] This confirms the gross project value, $V_0 = 100$, obtained earlier using traditional DCF with the actual probability (q = 0.5) and the risk-adjusted discount rate (k = 0.20).

[7] This assumption is intended to make the analysis somewhat more realistic and invariant to the cost structure make-up, and is not at all crucial to the analysis.

[8] The above example confirms that CCA is *operationally* identical to decision tree analysis (DTA), with the key difference that the probabilities are transformed so as to allow the use of a risk-free discount rate. Note, however, that the DCF/DTA value of waiting may differ from that given by CCA. The DCF/DTA approach in this case will overestimate the value of the option if it discounts at the constant 20% rate required of securities comparable in risk to the *naked* (passive) project:

$E_0 = [qE^+ + (1 - q)E^-]/(1 + k) = [.5 \times 67.68 + .5 \times 0]/1.20 = 28.20.$

Again, the error in the traditional DTA approach arises from the use of a single (or constant) risk-adjusted discount rate. Asymmetric claims on an asset do not have the same riskiness (and hence, expected rate of return) as the underlying asset itself. CCA corrects for this error by transforming the probabilities.

[9] For simplicity, it is assumed here that the project's value in its current use and in its best alternative use (or salvage value) are perfectly positively correlated. Of course, the option to switch use would be even more valuable when the correlation between V and A is lower.

[10] The possibility that the option to take a project may not be tradable may necessitate dividend-like adjustments and justify preemptive investments, thus indirectly affecting the timing of exercise and value of a real option.

[11] There are, of course, examples of compound options in traded financial securities as well, such as callable convertible bonds.

[12] The *opportunity to invest* is thus formally equivalent to a *call option* on the value of a completed project, V, with exercise price the one-time investment outlay, I.

[13] "*Salvage value,*" or value in the best alternative use, may come from the value of expected cash flows from switching use (or inputs/outputs), a market price for which the project may sell in a second-hand market or, in situations where subsequent expenditures are due, the value of subsequent cost savings from discontinuing the project.

[14] To simplify exposition, the rest of this section ignores the option (not) to operate, as well as the options to expand or contract the scale of operation and various other options.

[15] As pointed out earlier, shared options can be differentiated further depending on whether the impact of competition is taken as exogenous or causes endogenous strategic counteractions. The latter can be further differentiated depending on the nature of competitive reaction (contrarian or reciprocating).

[16] Normally, "competitive loss" has a negative value (i.e., it is a loss), especially if competitors enter after the firm has undertaken the project. In some cases, however, it may actually be a gain (i.e., a negative "loss"). One example is an R&D investment that develops a new technology that competitors can easily imitate, resulting in lower production costs and higher profits for all competitors. Another example is when a competitor's investment, such as advertising expenditures, promotes the whole product category and not just the competitor's particular brand (e.g., "buy liquid soap," rather than "Jergen's Liquid Soap is the Best"), thus increasing the total "market pie" for all, or reducing the need for advertising expenditures by the particular firm. In this case, a competitor's investment is like a public service benefiting all (a shared investment). As a third example, consider the effect of competition on the value of the option to introduce a new product when acceptance by the market is highly uncertain. An introduction of a substitute product by a competitor may on the one hand take some market share away from the firm, while on the other it may resolve uncertainty about the market's reception of that type of product. It is conceivable that the "*learning effect*" for the firm may be more valuable than the direct market share loss, so that the firm may obtain a net gain from such competitive entry.

[17] There are, of course, other forms of interproject dependence such as "mutually exclusive" projects where undertaking one project precludes undertaking the other, or "synergistic" projects that enhance each other's cash flows when taken together. These interactions are ignored here; compoundness is the focus instead.

[18] The distinction between "expiring" and "deferrable" investment opportunities is one of degree. It is also in a sense related to the distinction between shared and proprietary options, in that in a shared option the threat of competition may, for preemptive reasons, effectively turn a "deferrable" option into an "expiring" one (although, in this case, management still has a choice as to whether or not to make an immediate preemptive investment, whereas in a strictly expiring option such a choice is precluded entirely).

Also, the horizon of a deferrable real option is a relative notion compared to contractual financial options. In the case of real options, it may be useful to analyze whether the expiration of the option (end of the waiting horizon) is brought about by abrupt versus incremental changes. An abrupt event such as the termination of a patent for producing a new product or of a lease for oil drilling can be treated as an exogenously determined point in time when the deferrable option expires. On the other hand, incremental changes in value resulting from the introduction of substitute products in a shared-deferrable option can be treated as endogenous effects analogous to dividends in call options (although in the extreme case where the substitute product is a technological breakthrough causing an abrupt project value drop to zero, its introduction may effectively be treated as the expiration time for the horizon of the incumbent firm's real option).

[19] The basic form of this classification is similar to that first proposed by Kester (1984).

[20] It is worth noting that, under this real options classification scheme, conventional (static) NPV investments are properly seen as a special case under the leftmost branch of proprietary-simple-expiring options because such investments are typically evaluated as if they were exclusively owned (i.e., ignoring competitive interaction, hence *proprietary*), independent (hence *simple*), and immediate (hence *expiring*) opportunities.

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