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REGULATING BY CAPPING PRICES

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1. Introduction

The purpose of this article is to look at the theoretical underpinnings and properties of price caps, specifically, the rules governing how a multiproduct regulated firm may adjust its prices. To identify a standard for implementing price caps, we begin by identifying the constrained optimization to which "price caps" are the solution. Since price caps are at heart a policy in which a firm can do what it likes as long as prices do not rise, the relevant optimization is maximization of welfare or profit subject to a constraint that aggregate consumer welfare must not fall below a given level. Permitting prices to be flexible, subject to maintaining a constant weighted average where the weights are based only upon the quantities sold for each service, can approximate the behavior of a firm maximizing profit subject to an aggregate consumer surplus constraint. Flexibility to change prices is in this sense "optimal," subject to the acceptability of the consumer welfare achieved under the original prices.

These virtues become less apparent over time. As the regulatory periods lengthen, non-marginal changes in price or demand raise the question of whether to base the flexibility constraint on prior or current quantities. Interpreting the weights becomes more difficult, as the regulator has to estimate how much of the shift in sales is exogenous and how much is due to changes in the regulated firm's prices. This will reintroduce some of the information requirements that make conventional cost-of-service regulation so burdensome. We will model both the incentives to supply false information to the regulator and potentially welfare-reducing adaptation by the regulated firm. Finally, legal or political inability of the government to commit over time to permit the regulated firm to either lose money or earn supranormal profits will lead to price caps becoming more like conventional regulation, with an institutionalized regulatory lag. While conditions for an optimal lag can be derived in theory, they are likely to be of little practical use. For all of these reasons, the advantages of a price cap appear greater the more likely it is that regulation will cease in the near future.

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2. Welfare-Constrained Profit Maximization and Pricing Flexibility Rules

Consider a regulated multiproduct firm, with monopolies in each of its markets.[1] Let $i = 1, \ldots, n$ index the markets, q_i indicate the output in market i, and $p_i(q_i)$ be the demand price in market i for q_i. For computational convenience, we assume demand in one market is independent of demand in others.[2] Let $G_i(q_i)$ be the gross consumer welfare generated in market i from selling q_i goods; $G_i'(q_i) = p_i(q_i)$. Net consumer welfare or consumer surplus, S_i, is $G_i - p_i q_i$. Let S be the sum of S_i over all n markets. The cost of producing the vector of output $q = (q_1, \ldots, q_n)$ is $c(q)$, with $\partial c/\partial q_i = c_i$. Profit, $\Pi(q)$, equals $p \bullet q - c(q)$, where $p = (p_1(q_1), \ldots, p_n(q_n))$.

Suppose the regulated firm maximizes profit subject to the constraint that aggregate consumer surplus not fall below $S°$. The Lagrangian is

$$\max_q L(q) = p \bullet q - c(q) - \lambda(S - S°),$$

and the associated first-order conditions for each q_i are

$$\frac{\partial L}{\partial q_i} = p_i + p_i' q_i - c_i - \lambda(p_i - p_i - p_i' q_i) = 0.$$

Rearranging terms gives

$$\frac{p_i - c_i}{p_i} = \frac{1 + \lambda}{\varepsilon_i}$$

where $\varepsilon_i = -p_i/p_i' q_i$, the elasticity of demand in market i at q_i.[3]

As with Ramsey prices, the price-cost margins are inversely proportional to the elasticity of demand. If we let $\Pi°$ equal the firm's profits when maximizing profit subject to this constraint, we obtain the market quantities and prices that maximize welfare subject to permitting the firm to earn $\Pi°$. These prices and quantities reflect "second best efficiency," in that the firm cannot get more than $\Pi°$ without consumers getting less than $S°$. Moreover, costs are minimized since inefficient production only reduces profits without generating additional consumer surplus to meet the $S°$ constraint. Nothing is gained by fabricating cost data, since prices are limited only by the consumer surplus constraint, not by reported cost. A regulatory scheme that would replicate this effect will therefore eliminate the incentives to engage in the inefficient production, cross-subsidization, or transfer pricing to which cost-of-service regulation is prone.[4]

If the regulator identifies a vector of prices $p°$, e.g., current prices, which generate an acceptable level of consumer surplus $S°$, economic welfare will be maximized if the regulated firm is permitted deviations from $p°$ that do not reduce welfare. Looking at marginal deviations ∂p_i from $p°$, we find that aggregate consumer welfare S is not reduced if

$$dS = \sum_{i=1}^{n} \frac{\partial S_i}{\partial p_i} dp_i \geq 0.$$

Since

$$\frac{\partial S_i}{\partial p_i} = \frac{\frac{\partial S_i}{\partial q_i}}{\frac{\partial p_i}{\partial q_i}} = \frac{-p_i' q_i}{-p_i'} = -q_i\,,$$

we obtain the result that all price deviations from $p°$ should be permitted as long as

$$-dS = \sum_{i=1}^{n} q_i\,(dp_i) \leq 0. \tag{1}$$

In other words, we should permit price changes where the net average of the changes, weighted by the quantity of output, is negative. A price increase in one market is acceptable as long as price is reduced in other markets to keep (1) satisfied.[5] Note that $p°$ need not initially maximize profits subject to the consumer welfare constraint, especially if $p°$ were the vector of prices inherited from an inefficient regulatory scheme. If $p°$ is not already profit-maximizing, the firm will have an incentive to change prices, even where constrained by (1).

3. Problems of Implementation: Non-Marginal Changes and Time

The flexibility rule in equation (1) involves only marginal price changes, where market demands and fringe supply functions are fixed. Non-marginality of price changes and time can lead to divergences from the welfare maxima and impose greater information requirements on the regulator.

3.1 Non-Marginal Price Changes

If the regulated firm elects to adjust p from the prescribed level by more than an infinitesimal amount, the welfare changes will not be accurately represented by (1). Let t_0 and t_1 represent the time before and after the regulated firm's price adjustment, and let superscript j represent time period j. In market i, a change $\Delta p_i = p_i^1 - p_i^0$ will induce changes in sales from q_i^0 to q_i^1. The change in consumer surplus will be

$$\Delta S = S(q(p^1)) - S(q(p^0)).$$

Using standard consumer surplus measures,[6] the net consumer welfare change in market i from a change in prices from p_i^0 to p_i^1 will be

$$\Delta S_i = -\int_{p_i^0}^{p_i^1} q_i(z)\, dz \,,$$

and the aggregate change in consumer surplus ΔS is given by

$$\Delta S = -\sum_{i=1}^{n} \int_{p_i^0}^{p_i^1} q_i(z)\, dz \,.$$

The "no consumer surplus reduction" condition becomes

$$\sum_{i=1}^{n} \int_{p_i^0}^{p_i^1} q_i(z)\, dz \le 0 \,. \tag{2}$$

Implementing an accurate price flexibility rule requires estimating or approximating these integrals. As a practical matter, pre-change quantities will probably be used since, unlike post-change quantities, information on them is available to the regulator without ex ante prediction.[7] Since $q_i(z)$ is not constant between p_i^0 and p_i^1, this will introduce variations from Ramsey-like pricing rules. Condition (2) becomes equivalent to a rule in which the regulated firm's post-adjustment price average must be less than the adjusted average of pre-adjustment prices, where both are weighted by pre-cap quantities:[8]

$$p^1 \times q^0 \le p^0 \times q^0.$$

Let $R = p^0 \times q^0$. R is independent of the firm's post-adjustment output choices. The regulated firm's problem will be to choose q^1 to maximize constrained profit:

$$L(q^1) = p^1 \times q^1 - c(q^1) - \lambda(p^1(q^1) \times q^0 - R).$$

First-order conditions imply that in each market i,

$$\frac{p_i^1 - c_i^1}{p_i^1} = \frac{1}{\varepsilon_i}\left(1 - \frac{\lambda q_i^0}{q_i^1}\right).$$

Since λ, the "shadow value" to the regulated firm of increasing the constraint, is positive, pre-cap quantity weighting will induce it to act as a monopolist with more elastic demands. Only if the percentage change in quantity is equal across all markets will the short-term result be a welfare optimum for the profit level permitted under the cap. However, we can show that (a) has the following properties:

Property 1. Let p^t be the price vector selected by the firm at time t. If $S(p^t)$ = consumer surplus evaluated at p^t, $S(p^{t+1}) \ge S(p^t)$ for all t.

Property 2. Assume the price vector p that maximizes aggregate consumer surplus subject to $\Pi(p) = \Pi°$ is unique for any $\Pi°$. As $t \rightarrow \infty$, the sequence $\{p^t\}$ converges to a point where consumer surplus and profit satisfy the Ramsey condition, i.e., consumer surplus is maximized subject to the profit received by the firm.[9]

If one is willing to wait, "prior period" weighting may be preferable to a regulatory method that moves directly to a "Ramsey point" where profits are maximized subject to only the original level of consumer surplus.

3.2 Changes in Demand: Uncertainty and Intertemporal Manipulation

We assume the regulator elects to calculate "average" prices using last period's quantities, and investigate what may happen when demand may change between periods. That the regulator may choose "wrong" weights using data derived from prior period demands goes without saying; in addition, there are ways the regulated firm can take advantage of uncertainty regarding changes in demand over time.

Changing the weights. One way the regulator may account for changes in demand is to weight price changes by what last period's quantities would have been had the current period's demand curve been in effect. This would give the regulated firm correct credit for the welfare gains attributable to its price decreases and correct penalty for the losses attributable to its price increases. For example, if demand exogenously declines in a market in which the regulated firm raised price, it will be excessively penalized if its increase in price is weighted by last period's actual sales, rather than what last period's sales would have been had this period's demand been in place.[10] Predicting current demand at past prices to generate weights, though, will invite debate over what the sales at the old price would have been. To maximize the credit it gets for reducing some prices and minimize the penalty for increasing others, the regulated firm will argue that demand curves are

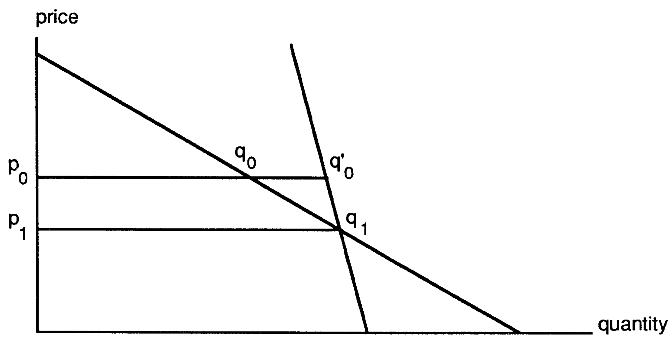

Figure 1. Price Decreases

inelastic, implying that changes in its sales, whether from increased or decreased prices, are attributable to outside factors.

Consider price reductions, as portrayed in figure 1.

Let q_0 be the quantity sold in period 0 at price p_0, and q_1 be the quantity sold in period 1 at price p_1. For illustrative purposes, assume the demand curve does not change from period 0 to period 1, hence that the demand curve in both periods goes through the points labeled q_0 and q_1 in figure 1. If the regulator is open to the possibility that demand may have changed, however, the regulated firm may try to persuade the regulator that demand would have been q_0', rather than q_0, had it not reduced price. This would imply a greater welfare gain to consumers from its price reduction than was in fact the case.

A similar story may be told for price increases, as illustrated in figure 2.

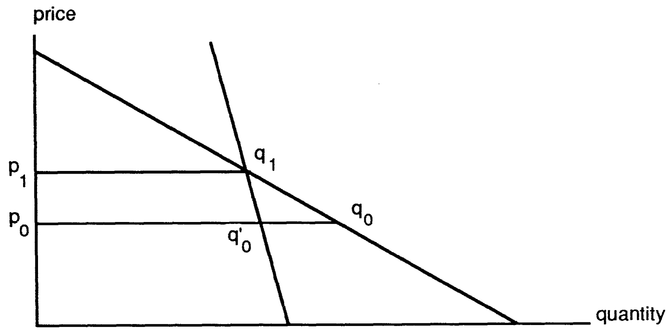

Figure 2. Price Increases

The points are labeled as before. Here, price goes up from p_0 to p_1; demand goes down from q_0 to q_1; the demand curve is assumed not to shift. Here, the regulated firm wants to minimize its penalty for raising price. If it can persuade the regulator that period 1 demand goes through q'_0, exogenous demand shifts rather than the price increase will be "blamed" for the reduction in sales. Generally, since q'_0 is unobservable, the firm may be able to make this argument to the regulator repeatedly.

Similar strategies may be taken by other parties with interests in the regulated firm's pricing policies. Competitors will generally want to discourage price decreases and encourage price increases in their markets. If the regulated firm attempts to cut price in their markets, competitors will argue that demand facing the regulated firm is more elastic in its markets than past data suggest, e.g., that q'_0 is smaller than q_0, to minimize the reward the regulated firm gets for cutting price. On the other hand, competitors are likely to support the regulated firm's claims of inelastic demand when it elects to raise price in the competitive firm's markets. Consumers will have interests divergent from competitors; if they expect a price cut, they will claim demand is less elastic than it appears; if they expect a price increase, they will claim the opposite. In this way, distributional pressures between various parties will invite debate over the relevant standard for price adjustments, increasing the regulator's administrative burden.

If demand is likely to change during the price-cap period, the regulator can either use prior period quantities, with certain error, or it can attempt to ascertain what current sales would have been at the past period price, in which case it will have to resolve contending views and evidence as to whether shifts in sales are due to exogenous shifts in demand or changes in price. We may note, though, that while the regulated firm has a short-run interest in arguing that demand for its products is inelastic, such claims may run against a competition-based case for eventual deregulation.[11]

Reactions to anticipated shifts in demand. That the regulator may make a mistake in using old demand data to evaluate current price changes is obvious. What is less obvious is that doing so can induce a regulated firm, who can forecast demand accurately, to act in advance of price changes in ways that may reduce aggregate economic welfare over time. The possibility can be illustrated by looking at a regulated firm in two markets, *X* and *Y*.

Consider figure 3, in the space of combinations of p_X and p_Y.

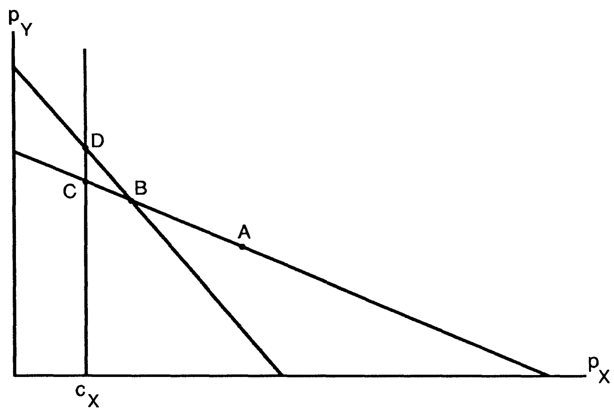

Figure 3. p_X-p_Y Space

Assume at period 0 the X and Y markets are in a long-run equilibrium with prices (p_X^A, p_Y^A) at point A, where the firm's profits are maximized over the set of prices (p_X, p_Y) in p_X-p_Y space such that

$$p_X X + p_Y Y \leq p_X^A X(p_X^A) + p_Y^A Y(p_Y^A).$$

These points are indicated by the straight line going through points A and C, with slope equal to $-X(p_X^A)/Y(p_Y^A)$. Suppose that before prices are set in period 1, news comes that firms are going to enter market X, with marginal cost c_X. Thus, the regulated firm knows that in and after period 2 it will not be able to set p_X above c_X. Absent any reaction to entry, it would continue to choose point A in period 1, followed by point C in period 2.[12]

In period 2, the regulated firm would rather be at a point directly above C, with a higher price in market Y and the competitive price c_X in market X. It can achieve this goal by choosing a point such as B between A and C. Since p_Y is greater and p_X is smaller at B than at A,

$$\frac{X(p_X^B)}{Y(p_Y^B)} = \frac{X(p_X^A)}{Y(p_Y^A)}.$$

The line demarcating the set of points available to the regulated firm in period 2 going through B will be steeper than the line of period 1 options going through point A, enabling the firm to set prices at D in subsequent periods. The closer point B is to point C, the steeper is the line going through B and the higher will be the price and, up to a certain point, profits in market Y. On the other hand, since A was a long-run equilibrium, the regulated firm will lose profits in period 1 (before demand has changed) by moving away from point A, and will lose more profits the farther away point B is from point A.[13] Assume points B and D in figure 2 reflect the optimum choices for the regulated firm, given its time rate of discount.[14]

If we let Φ and $1 - \Phi$ reflect the relative weights given to welfare in period 1 and in period 2 and subsequent periods, and let $W(\cdot)$ refer to net economic welfare, we find that welfare is reduced if

$$\frac{1-\Phi}{\Phi} > \frac{W(A) - W(B)}{W(D) - W(C)},$$

i.e., any gains to consumers net of producer losses in period 1 from moving to B are outweighed by the net losses in subsequent periods from moving to D. Since p_Y at D exceeds p_Y at C, $W(D) < W(C)$. While no general conclusion can be reached, if $W(B) < W(A)$ or Φ is sufficiently small, the firm will adjust to foreseen price changes in ways that lead to overall welfare reductions.[15]

4. The Long Term

Over time, inflation or increased resource scarcity may cause cost to rise; technological change or declining input prices may cause it to fall. These changes in costs call into question the regulator's commitment not to tie prices to costs and profits. The Supreme Court's standard that a regulator must provide "just and reasonable" returns to regulated firms itself keep a government agency from committing to set prices regardless of the firm's profits. Political impetus to adjust the price cap may be forthcoming if it is perceived that the regulated firm's profits constitute too great a fraction of the available social surplus if costs decline without rates declining.[16] The most a regulator may be able to achieve is an institutionalized "regulatory lag," in which a firm can reduce its costs without fearing imminent price reductions from the regulator, but the regulator will subject the firm to periodic review with action taken on the basis of achieved profit.[17] However, to plan "lags" efficiently, the regulator must know the relationship between investment and productivity, but lags are needed to encourage cost-reducing investment only when the regulator cannot monitor it (Brennan, 1988). While some investments by regulated firms may fit these conditions of non-monitorability but with predictable benefits, it does not seem that the benefits of regulatory lag are widespread.

Divorcing price from cost can preserve a regulated firm's incentives to innovate and minimize costs, and eliminate the incentive to cross-subsidize. Allowing pricing flexibility so long as the average of price changes, weighted by quantity sold, is negative will guarantee that social welfare is maximized given the acceptability of either the welfare achieved by the firm's consumers and competitors, or the acceptability of its eventual profit level. Over time, though, non-infinitesimal price changes and changes in demand will confront the regulator with decisions regarding the use of past quantities, estimated "past quantities" under current demand curves, or predicted demands, all of which invite misinformation on demand elasticities and intertemporal strategic behavior. Since these problems, as well as doubt regarding the government's commitment to separate prices from costs, appear to grow more severe as time goes on, the advantages price caps will be greatest as part of a regulatory regime designed for elimination in the near future.

Appendix

Proof of Property 1. For any point $p°$ in price space, $\partial S(p)/\partial p_i = -q_i$. Therefore, the aggregate consumer surplus "indifference surface" ($\{p \in P \mid S(p) = S(p°)\}$) is always tangent to the hyperplane of possibilities available to the regulated firm in the next period ($\{p \in P \mid p \bullet q(p°) = p° \bullet q(p°)\}$). Because of the convexity of the consumer surplus indifference surface,[18] this point of tangency is a minimum. Since this tangency point is always the next period "starting point" for the regulated firm, consumer surplus must be at least as great after the firm's choice of price in period $t+1$, p^{t+1}, than before, i.e., $S(p^{t+1}) \geq S(p^t)$.

Proof of Property 2. The conclusion would follow immediately from the analysis in section 3.1 if the sequence $\{p^t\}$ converges to a single limit point. We show that for all limit points of $\{p^t\}$, consumer surplus and profit are equal and each is the maximum it can be given the value of the other. The "uniqueness" assumption thus implies that all limit points of $\{p^t\}$ are identical, hence that $\{p^t\}$ converges to a point satisfying the Ramsey condition. The proof that each limit point of $\{p^t\}$ meets the Ramsey condition proceeds as follows:[19]

Assume a finite maximum reservation price for each good, and that profits for the regulated firm in the initial equilibrium, Π_0, are positive. These, plus continuity of the profit function, ensure that the set of prices in price space such that $\Pi(p) \geq \Pi_0$ is compact. Call that set Γ. Since $\{p^t\}$ is a subset of Γ, it must have at least one limit point. Let Λ be the set of limit points. Since $\{S(p^t)\}$ and $\{\Pi(p^t)\}$ are bounded nondecreasing sequences,[20] they have unique limits S^* and Π^*. It must be the case that for any $\lambda \varepsilon \Lambda$, $S(\lambda) = S^*$ and $\Pi(\lambda) = \Pi^*$.

Let $H^t = \{p \varepsilon P \mid p \bullet q(p^t) = p^t \bullet q(p^t)\}$. The firm chooses $p^{t+1} \varepsilon H^t$ such that $\Pi(p^{t+1}) \geq \Pi(p^t)$ for all $p \varepsilon H^t$. Thus, the profit indifference surface for $\Pi(p^{t+1})$must be tangent to H^t. From the above, $S(p^t) \leq S(p)$ for all $p \varepsilon H^t$, implying the consumer surplus indifference surface for $S(p^t)$ is tangent to H^t. Choose some $\lambda \varepsilon \Lambda$. Let $\{p^t(j)\}$ be a sequence of choices that converges to λ, where $t(j+1) > t(j)$ for all j. Assuming continuity of derivatives, the gradients of the hyperplanes $H^t(j)$ converge to the gradients of the consumer surplus indifference curves of $S = S(p^t(j))$ at $p^t(j)$ and the gradients of the profit indifference curves for $\Pi = (p^{t(j)+1})$ at $p^{t(j)+1}$.

By construction, $\{p^{t(j)}\}$ converges to λ. The sequence $\{p^{t(j)+1}\}$, being bounded, must have limit points. Suppose there exists one different from λ; call it λ'. The gradient of $\{p \varepsilon P \mid S(p) = S(\lambda)\}$ at λ is the same as the gradient of $\{p \varepsilon P \mid \Pi(p) = \Pi(\lambda')\}$ at λ', since both equal the limit of the sequence of gradients of $\{H^{t(j)}\}$. Let H_λ and $H_{\lambda'}$ be the hyperplanes respectively tangent to the consumer surplus surface at λ and the profit surface at λ'. Since consumer surplus and profit are identical for any limit point, $S(\lambda) = S(\lambda') = S^*$ and $\Pi(\lambda) = \Pi(\lambda') = \Pi^*$. If $\lambda \neq \lambda'$, convexity of the consumer surplus indifference curves implies $H_{\lambda'}$ must lie strictly "above" H_λ in the sense of being on the other side of H_λ from the origin. By similar argument, convexity of profit surfaces says H_λ must lie strictly "above" $H_{\lambda'}$. Since this cannot happen, λ must equal λ'.

If $\lambda = \lambda'$, the consumer surplus and profit indifference surfaces are tangent at λ. Hence, λ meets the Ramsey condition. Since λ was chosen from Λ arbitrarily, the Ramsey condition must be met for all limit points. If the set of such points is unique for Π^*, as assumed, λ must be the limit point for all subsequences of $\{p^t\}$, hence $\{p^t\}$ converges to λ.

Property 1 clearly holds even if the firm acts nonmyopically. Since the regulated firm may take future effects of its actions into account, though, we cannot assume $\Pi(p^t)$ is increasing in t. However, it must be the case that all limit points of the sequence $\{\Pi(p^t)\}$ must be the same; otherwise at some time t' the firm would be better off staying with $p^{t'}$ in perpetuity than by continuing the sequence $\{p^t\}$, contradicting the assumption that $\{p^t\}$ maximizes discounted profits. Let Π' be the limit of $\{\Pi(p^t)\}$. Since Property 1 holds, $\{S(p^t)\}$ is increasing; being bounded, it has a unique limit point S'. If there were two price vectors p^0 and p^1 such that $S(p^0) = S(p^1) = S'$ and $\Pi(p^0) = \Pi(p^1) = \Pi'$, the uniqueness assumption and convexity of the isoprofit and indifference surfaces imply that there is a price vector p^* with $\Pi(p^*) > \Pi'$ and $S(p^*) \geq S'$. Continuity therefore implies that $\{p^t\}$ could not be profit-maximizing for the "nonmyopic" firm.

Notes

The author is Associate Professor, Public Policy, Communication and Economics, George Washington University. This chapter is adapted from versions of his paper, "Regulating by Capping Prices," Antitrust Division Discussion Paper 88-11 and in the *Journal of Regulatory Economics* 1 (1989): 133-47. Discussions with Richard Clarke, Michael Einhorn, Bert Smiley, Thomas Spavins, Ingo Vogelsang, Robert Willig, Michael Williams, Glenn Woroch, and especially Mary Fitzpatrick are most appreciated. The author retains sole responsibility for errors.

1. The model extends easily to the case where there is a competitive fringe, where the "market" demand curves are reinterpreted as those facing the regulated firm. The extension requires that fringe firms' profits are included in the aggregate welfare constraint.

2. This is a reasonable assumption in many cases. To take long-distance telephony as an example, the demand to call city A from city B is not likely to be too sensitive to the price of calling city C. It will be a less realistic assumption in other cases, for example, when one "service" is a volume discount version of another or where one price covers initial hookup and another price covers usage, as in a two-part tariff. However, with the exception of the conclusion regarding demand elasticities, these results can be duplicated without the independence assumption. If demands are interdependent, the elasticities can be replaced with "superelasticities" (Brock, 1983) that embody the non-zero cross-elasticities.

3. The model can be generalized to include dynamic Ramsey-like prices, by allowing the index to refer to markets over time and permitting cost complementarities, in that changes in output in a given period may affect marginal cost in subsequent periods. I thank Glenn Woroch for that suggestion.

4. Riordan and Cabral (1988) show that the incentive to innovate may be greater if the regulator holds price low, since the amount of output over which the benefits of innovation would be spread is greater.

5. Where markets include a price-taking competitive fringe, the constraint on marginal price changes to ensure no loss in consumer and fringe welfare is essentially the same, where q_i is the output of just the regulated firm rather than total market quantity.

Including fringe profits in the welfare constraint is not only consistent with methodological indifference regarding the distribution of economic gains and the influence both consumer groups and fringe firms may have on the regulator, but it also leads to more efficient welfare standards. If fringe profits are not to be considered in the welfare constraint, price changes are weighted by total supply in the market.

If the fringe are not price takers, the marginal welfare effect of a change in price will be affected by changes in the output of the fringe on marginal output. As in other contexts (Stiglitz, 1981; Schwartz, 1984; Mankiw and Whinston, 1986), a more complex oligopoly theory makes general results harder to come by.

6. Subject, of course, to the usual qualifications (e.g., no income effects) on consumer surplus estimates (Willig, 1976).

7. See Brennan (1989) for discussion of adjustment rules employing post-change quantities.

8 This is equivalent to requiring that the "Laspeyres index" of price changes be less than one.

9. I thank Robert Willig for suggesting these intertemporal properties to me; proofs are in the Appendix. Vogelsang (1988) provides an additional illustration of the proposition that regulation may perform less well as the mechanisms more closely correspond to the regulator's goals.

10. Suppose in market j fringe supply becomes perfectly elastic at price p_j, forcing the regulated firm to match that price. From the post-adjustment perspective, there is no welfare gain from the regulated firm's price reduction. If it were to raise price above p_j there would be no welfare loss; consumers would simply turn to the competitors. Brennan (1987b) shows how entry into a subset of regulated markets may reduce overall economic welfare, even when the entrants can serve this market at lower cost than can the regulated firm, if prices in less competitive markets are allowed to rise as other markets become more competitive.

11. Where the regulator elects to base price on predictions of period 1 quantities, the regulated firm will want to convince the regulator that demand is more elastic than is the case, to exaggerate the welfare contributions of its price reductions and to understate the welfare reductions from its price increases.

12. We assume p_Y^m, the monopoly price in market Y, is above the price in market Y that would be set at point C. If not, the regulated firm would stay at point A in period 1 and move to a point on the vertical line below point C in period 2.

13. This follows from convexity of the profit function, independence of demand functions for X and Y, and that the monopoly price in market X is above the price in market X that would obtain at point C.

14. Since A is a period 1 optimum, the marginal loss of profit of moving away from A is zero, while there is a marginal gain in profit from moving up from C. Thus, the optimum B will not coincide with A. But since B is not a period 1 optimum, the marginal profit of moving B incrementally away from A is negative, implying the marginal profit from moving D up must be positive. Thus, the price in market Y at D will still be less than the monopoly price in Y.

15. It may also be noted that shifts in demand for the firm's products may keep it from recovering its costs under a price-cap mechanism, leading to either exit or modification of the caps.

16. See Fitzpatrick (1987) for a discussion and empirical test of this mechanism.

17. FCC (1988, par. 128). Note that the FCC regards "regulatory lag" disparagingly at par. 105.

18. Convexity of this surface for individuals follows from Varian (1984, 139-140). Since the aggregate surface is the sum of the individuals' surfaces, it too is convex. This is obvious if demands are independent, for $\partial^2 S(p)/\partial p_i^2 = -q_i' > 0$ and $\partial^2 S(p)/\partial p_i \partial p_j = 0$.

19. The proof is similar in character to a convergence proof of Proposition 1 in Vogelsang and Finsinger (1979).

20. A virtue of an "optimal control" proof based on the properties of a function $p(t)$ that maximizes discounted profits subject to the price-cap adjustment constraint is that it would cover strategic behavior by the firm in which short-term profits may be sacrificed to put the firm in a position to earn longer term gains. Such behavior is predictable when demand changes, as illustrated in section 3.2. If such a strategy is profitable, we cannot assume that $\{\Pi(p_t)\}$ is a nondecreasing sequence with only one limit point.

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