## 7

## A SEQUENTIAL MECHANISM FOR DIRECT PRICE REGULATION

Peter B. Linhart<br>Roy Radner<br>Frank W. Sinden

## 1. Introduction

In this article, we describe a mathematical model of price-cap regulation, in a simplified institutional setting. We show that, even in the presence of uncertainty as to the success of actions taken to improve productivity, and even in the presence of moral hazard, such a regulatory method can achieve its aim-namely, decreasing output prices and increasing productivity. ${ }^{1}$ Thus we consider the problem faced by a regulator who wants to provide incentives for a firm to effect cost reductionsand hence price reductions-through technological change or by other means. We model the manager of the firm as facing constraints imposed by the shareholders and other providers of capital, by the customers, and by the regulator. The regulator's ultimate objective is a secular real decrease in the firm's prices. However, the manager's private utility may not be maximized by activities that are maximally cost-reducing. Moreover, the regulator cannot directly observe all of the manager's actions, the outcomes of which are also influenced by random exogenous events. Hence a problem of moral hazard arises.

We propose a regulatory policy in which the regulator directly requires the firm to lower its real prices at (or faster than) some prescribed target annual rate. We suppose that the manager is replaced when he can no longer simultaneously repay the cost of capital, lower the prices at the rate prescribed by the regulator, and satisfy the market demand at those prices. Whenever a manager is replaced, the regulator reverts to conventional rate-of-return regulation for a period sufficient to enable the firm to build up a new cash reserve.

The resulting situation leads naturally to a model of a sequential principal-agent relationship, in which the regulator is the principal and the manager is the agent. This is not a repeated game, however, because both the firm's prices and its
productivity are changing through time, endogenously and stochastically. Using new techniques for the analysis of this nonstationary process, we (1) derive a lower bound on the expected length of tenure of a manager, and (2) show that if the manager does not discount future utility too much, then the realized long-run rate of price decrease will be correspondingly close to the target rate (if indeed this target is technically achievable).

### 1.1 General and Historical Remarks

It is commonly believed that regulation should simulate competition in order to maximize social welfare. This is sometimes taken to imply that regulators should select prices so that the firm's economic profits are always zero. If, however, this rule were actually followed precisely, the firm would be deprived of all economic incentive to minimize cost. In particular, the firm would have no incentive to select an optimal mix of factor inputs, to obtain these inputs from the lowest-cost suppliers, or to invest in cost-reducing innovation.

In fact, the record of productivity improvement in regulated industries compares favorably on the whole with that of the entire economy (Houthakker, 1979, 1981). It would appear, then, that real regulation provides stronger incentives for efficiency than the above simple description suggests.

Both positive and normative theories of regulation have already received considerable attention in the economics literature. In this introductory section, we indicate briefly why there is a need for further study of efficiency incentives under regulation, and sketch several properties that a feasible incentive mechanism should have.

Real competitive markets provide incentives for innovation through temporary monopoly profits, as described by Schumpeter (1942). More or less unintentionally, real regulation provides a similar incentive. Regulation is not exact and continuous, but takes the form of a discrete series of reviews, at which (in a simplified picture of the process) the prices are reset so that economic profit is zero. Between reviews, if the firm can reduce its costs, it can earn a positive economic profit. It has been pointed out (Baumol, 1968) that the imperfection of this process (loosely called "regulatory lag"), far from being a drawback, confers a positive benefit on society. Baumol recommends that it be institutionalized.

It is, of course, difficult to say how efficient or close to the social optimum the incentives provided by regulatory lag are; one of the aims of the present article is to provide a framework within which the efficiency of such regulatory mechanisms can be discussed more precisely. In order to do this, we have found it desirable to go beyond consideration of the firm as an abstraction, and deal with the incentives experienced by the firm's managers. In this respect our analysis is in the spirit of the "managerial theory of the firm;" see, for example, Williamson (1968).

When inflation raises the cost of the firm's inputs, regulatory lag as an incentive mechanism no longer works so well. This comes about as follows: In any industry, given its technological possibilities, there is an economically optimal rate of cost reduction; beyond this rate, the cost of developing and installing new cost-reducing
technology exceeds the cost reduction. Suppose the rate of inflation exceeds this optimal rate of productivity increase, as has arguably been the case for most American industry in some recent years. Then if prices are fixed in nominal terms at regulatory reviews, as they customarily are, the firm will necessarily lose money between reviews no matter what cost-reducing measures it takes.

One would expect, therefore, that unusually rapid inflation would cause firms to apply for rate reviews with unusual frequency in order to reduce their losses. The data bear this out. Figure 1, for example, shows the remarkable correlation between the rate of inflation and the frequency of general rate orders for Bell Operating Companies in the years 1955-81.

RATE ORDERS FOLLOW INFLATION


Frequent rate reviews have two social costs: the direct cost of the extra hearings and the indirect cost of reducing regulatory lag, which weakens incentives to improve productivity. In the limit, as reviews become continuous, productivity incentives approach zero, and the rational firm invests nothing in productivity improvement.

In times of rapid inflation, then, conventional regulation fails in two ways: it leads to losses and it leads to weakened incentives. These problems have led both regulators and utilities to seek formulas ("inflation adjustment clauses") that correct for inflation and at the same time preserve productivity incentives. ${ }^{2}$ This search has in turn led some investigators to examine more closely the game that underlies regulation.

### 1.2 The Present Approach

In this article we present a new approach to regulation that is suggested by the theory of sequential principal-agent games. A number of principal-agent relations (among shareholders, managers, regulators, and customers, and others) can be found in the situation of the regulated firm, as discussed in Section 2, and each such relation has its incentive problems. In the present article, we deal only with the relation between the regulator and manager as principal and agent. Our model is sequential and emphasizes productivity improvement (hence costs are endogenously determined).

The model takes account of the following fundamental characteristics, among others, of the regulatory situation:
1.The regulator and the firm's manager have different information. In particular, the manager has more information about the possibilities for productivity improvement than the regulators. In fact, one of the manager's options is to invest in research in order to obtain more of this information. In principle, the regulator could also obtain more information at some cost, but matching the manager's information seldom appears to be part of the regulator's strategy. In the present model, the regulator does not even try to elicit information about the firm's costs, hence misrepresentation is not a problem.
2. The regulator and the firm's manager to some extent have different goals. The regulator may strive to provide incentives strong enough to overcome the difference, but in general we would not expect an equilibrium outcome to meet the regulator's goals entirely.
3. The service is deemed essential, so that its continued availability must be assured in spite of possible mismanagement and/or bankruptcy.
4. To be acceptable in the real world, a regulatory mechanism must not differ too radically from those that already exist. The strategies we discuss resemble conventional regulation in that periods of regulatory inaction alternate with periods of action that are intended to be corrective.

The essence of the regulator's problem is that he cannot directly observe the manager's actions, nor can he observe the exogenous random events that also affect productivity. He can, however, observe the consequences of those actions and events, namely the realized profits of the firm, and whether or not demand is met. (He may also, with additional effort, be able to observe productivity changes, but we do not in our model rely on this possibility.) This situation, combined with the divergence of the regulator's and the managers' goals, leads to a problem of moral hazard. (See, for example, Radner, 1981.)

Suppose that the regulator provisionally fixes a sequence of prices that declines in real terms at a fixed "target rate" (which must be suitably chosen). If this sequence of prices is beyond the firm's control, then it has, essentially, the desirable incentive property of a lumpsum payment. Suppose further that the regulator requires the firm to meet demand at the given prices, as long as it is feasible to do so, and that the shareholders and directors require the manager to pay out the cost of capital at a given rate, again as long as this is feasible. These two requirements can be met as long as the firm's cash reserve is positive. However, through bad luck or bad management, the cash reserve can become negative. This event we call a crisis; when a crisis occurs, the manager is fired and replaced. The regulator must now provide some way for the firm to get back on its feet. Thus time is divided into alternating segments: incentive phases and recovery phases.

In the context of a particular formal model of a single-product firm, we have shown that, under this class of regulatory strategies, the management of the regulated firm will have an incentive to engage in productivity improvement. Furthermore, if the management's behavior is optimal from its own point of view, then the incentive phases will be long relative to the recovery phases, and the resulting long-run average rate of actual price decrease will be close to the regulator's target rate of price decrease, provided the management does not discount its own future benefits too strongly.

Thus, under suitable conditions, this class of regulatory strategies induces approximately efficient behavior on the part of the manager, without placing a large informational burden on the regulators and their staff, and in particular without requiring the regulators to monitor the firm's rate of return.

Several features of our approach should be emphasized. First, as mentioned above, we model the firm's manager as the active decision-maker in the firm, optimizing his own utility subject to constraints imposed by shareholders, customers, and regulators.

Second, we portray the regulators as seeking a mechanism that is easy to administer and that gives "satisfactory" results. In this case, satisfactory means achieving a target rate of price reduction, perhaps only approximately. Thus the regulator does not seek an "optimal" mechanism in any precise sense.

Third, the regulatory mechanism we describe replaces explicit rate-of-return regulation. We are interested in alternatives to rate-of-return regulation because (1) we are concerned about the weakness of its incentive properties, as described above, and (2) its informational requirements are heavy. Rate-of-return regulation is also difficult to administer if some of the firm's activities are regulated and others are not, as in the case of telecommunications today; see Linhart and Radner (1983).

Fourth, from a technical point of view, our model requires an analysis that goes substantially beyond currently available results for repeated principal-agent games. The reason for this is that both the firm's productivity and its prices are changing from period to period, and these changes are both endogenous and stochastic. Thus our model leads to a sequential-but not repeated-principal-agent relationship, with endogenous state variables, which requires new techniques of analysis.
(Further discussion of these features, including how they relate to the previous and current literature on the subject, will be found in Sections 4 and 5.)

Although inflation has played a role in stimulating the present interest in incentives, inflation is not essential to the theoretical structure of the underlying game. We sidestep inflation problems by assuming that prices are always measured in real dollars. We do not address the nontrivial econometric problem of defining a suitable price index.

In Section 2, we discuss in general terms the relations between the so-called "principal-agent" problem and the problem of providing incentives for efficiency in a regulatory situation. In Section 3, we describe a particular formal model that embodies some of the considerations sketched in Section 2 and prove several theorems about efficiency for a class of regulatory strategies. Section 4 provides some remarks on possible generalizations and extensions of the model. Section 5 locates the present model with respect to the previous literature.

## 2. Regulation and the Principal-Agent Problem

### 2.1 Principal-Agent Games

In a principal-agent situation, a principal hires an agent to act on his behalf, generally because the agent has better information about some enterprise of interest to the principal. The resulting outcome depends on a random state of the environment as well as on the agent's action. After observing the outcome, the principal makes a payment to the agent according to a pre-announced reward function, which depends directly only on the observed outcome. (This last restriction expresses the fact that the principal cannot directly observe the agent's action, nor can the principal observe the information on which the agent bases his action.)

A short-run principal-agent relationship can be naturally modeled as a two-move game, in which the principal first announces a reward function to the agent, and then the agent chooses an action (or decision function if he has prior information about the environment). An equilibrium of such a game is a reward function (by the principal) and a decision function (by the agent), each of which is optimal given the other. The equilibria of such a game are typically inefficient (unless the agent is neutral towards risk), in the sense that there will typically be another (but nonequilibrium) reward-decision pair that yields higher expected utilities to both players.

If such a game is repeated infinitely many times, the whole set of repetitions is said to constitute a "supergame." In such a supergame, more efficient equilibria exist, in the sense that both "players" are on the average better off than in the short-run relationship. This is essentially because in the supergame, the principal can design his reward strategy in such a way as to punish noncooperative behavior by the agent.

Of course, since the principal cannot observe the agent's action, but only the outcome of that action, he cannot be sure in case of a bad outcome whether the agent acted in bad faith or was the victim of bad luck (i.e., the random state of the
environment was unfavorable). However, after many repetitions the principal can distinguish statistically between bad faith and bad luck and impose (or withhold) penalties accordingly.

Finally, if the agent's objective is to maximize the discounted sum (present worth) of future rewards, there is some loss in the efficiency of equilibria, since future penalties lose some of their force compared to present gains. But as the discount rate approaches zero, this loss in efficiency disappears. Repeated prin-cipal-agent games, including those with discounting, are discussed in Radner (1981, 1985, 1986).

### 2.2 Principal-Agent Relationships in Regulation

When we begin to look for principal-agent relationships in the regulatory scene, several immediately spring to mind. First of all, the regulatory body can be seen as an agent of society or the electorate. Regulators may have-doubtless do havetheir own personal goals (see, for example, Stigler, 1971). The problem for society, acting through the political process, is then to provide a system of rewards and penalties such that regulators will best further their personal interests by acting in the general interest.

Second, in a simplified picture, the firm can be seen as an agent of the regulators. If incentives for the regulators to act in the interest of society are correct, and if incentives for the firm to act in the interest of the regulators are correct, then the chain will hold and the firm will act in the interest of society. The actions of the firm in which we are most interested (in this article) consist in undertaking productivity-improving projects, typically research and development (R\&D). Such projects cost money, involve the expenditure of effort, and may or may not result in lowering the firm's production costs. The regulators can observe the cost reduction, if any, and the expenditure of dollars on $\mathrm{R} \& \mathrm{D}$, but not the expenditure of effort. Rewards to the firm might take the form of increased allowed profit.

In reality, however, the firm is not a monolith. A principal-agent relation exists between the firm's owners and its managers. It is reasonable to assume that the owners want to maximize the firm's profit, but there is no reason to think that the managers internalize this objective; their direct interests may include salary and bonuses, longevity of employment, span of control, effort (to be avoided), and research interests. The owners' well-known problem is then to provide incentives for managers to act in the direction of profit maximization. In this connection, we may take the owners (shareholders) to be risk-neutral and the managers risk-averse. In the case of a regulated firm, the owners may also be thought of as transmitting to the managers the constraints (such as the requirement that demand be met at specified prices) imposed by the regulators.

Neither the regulators nor the owners can discriminate between bad luck and bad faith on the part of the managers, except statistically, so both regulatory rewards and penalties to the firm (in terms of profit) and the owners' rewards and penalties
to the management (in terms of bonuses, promotions, etc.) will have to be based on statistical evidence.

Other principal-agent relationships can be discerned. For example, a board of directors may be interposed between the owners (shareholders) and the managers. Also, we have said nothing about those of the firm's employees who are not managers; a whole hierarchy of principal-agent relationships exists here. Moreover, these workers, if they are unionized, may employ union officials as their agents in negotiations with management.

For our present purposes, we concentrate on a single principal-agent relationship, that between the regulator and the manager. The owners are represented by their power to fire the manager, and by the requirement that the firm earn enough to pay the cost of capital. The electorate is represented in the regulator's desire to reduce prices. Furthermore, we do not explicitly model the relationship as a game, in the sense that (1) we do not formally describe the entire set of alternative strategies available to the regulator, and (2) we do not completely specify the goals of the regulator. What we do analyze are the implications of a class of simple strategies for the regulator, under the hypothesis that the manager will respond optimally (in his own interest) to any particular regulator's strategy. A full-blown game-theoretic analysis might be desirable, and in Section 4 we make some suggestions about the directions such an analysis might take.

## 3. Description of a Formal Model

We have claimed that a certain class of regulatory strategies (sketched in Section 1), which is consistent with the informational asymmetry between the manager and the regulator, can induce economically efficient behavior on the part of the manager. We now substantiate this claim by showing that it is so, and in what sense it is so, in the context of a particular model. This model, although much simplified for purposes of tractability, is plausibly representative of a wide class of regulatory models.

### 3.1 Technological Model

We assume that all the firm's inputs, including capital, can be adjusted in each period so that the firm can just meet demand in that period at minimum cost. Thus capital investment is reversible, and questions of excess capacity do not arise.

We assume that the real cost, per unit of output, to the firm of each of its factor inputs, including capital, is the same in each period. We take the unit cost of capital to be in a fixed ratio to the cost of all other inputs plus the depreciation of capital; this would be the case, for example, if the depreciation schedule were unchanging and if capital and all other inputs were optimally used in fixed proportions (technically, if the production function were homothetic).

We assume Hicks-neutral technical change, i.e., that changes in productivity affect all inputs proportionally, so that we may take as a measure of productivity either (1) the ratio of output to capital (where "output" is an index of the outputs
of the firm and "capital" is an index of the capital of the firm, measured in constant dollars), or (2) equivalently, total factor productivity.

We think of changes in productivity as brought about by research and development (R\&D). We suppose, for simplicity, that the R\&D budget is fixed, so that the manager's choices lie in how he manages the R\&D budget and how he implements any resulting productivity improvement. Although in reality the results of R\&D activities in any one period may only be realized after some lags, we suppose here that these results are entirely realized after one period.

We recognize that increases in productivity are affected by the manager's actions, as just described, but are also affected by exogenous or random factors beyond the manager's control (e.g., the unpredictability of the outcomes of research projects). We assume the technology of productivity change to be stationary, in that given managerial actions combined with given exogenous conditions will achieve the same productivity change in any period. Thus, let
$F_{t}=$ total factor productivity during period $t=1,2, \ldots$, measured as output per unit of capital.
$r=$ cost of capital,
$c=$ all other costs per unit capital,
$C=c+r$.
$Q_{t}=$ output during period $t$ (or, in the case of a multiproduct firm, an index of output).
Then,

$$
\text { total } \operatorname{cost}=\frac{Q_{t} C}{F_{t}}
$$

Further, let
$A_{t}=$ actions taken by manager during period $t$ to improve productivity (the manager may use his knowledge of the past history of productivity change in choosing $A_{t}$ ),
$X_{t}=$ random factors beyond the manager's control,
$G_{t}=\ln \left(F_{t} / F_{t-1}\right)$, the logarithmic productivity gain during period $t$, i.e., from $(t-1)$ to $t$,
$H_{t}=G_{1}+\ldots+G_{t}$, the total (logarithmic) productivity gain from 0 to $t$.
Then

$$
F_{t}=F_{0} e^{H_{t}}
$$

We assume that

$$
\begin{equation*}
G_{t}=\Gamma\left(A_{t-1}, X_{t-1}\right), \tag{1}
\end{equation*}
$$

and that the random variables ( $X_{t}$ ) are independent and identically distributed with distribution known to the manager. We further assume that the function $\Gamma$ is bounded below, i.e., arbitrarily large productivity losses cannot occur in a single period.

### 3.2 Demand Model

With respect to demand, we assume exogenous growth in demand at a constant rate:

$$
\begin{equation*}
Q_{t}=e^{\tilde{\rho} t} D\left(P_{t}\right) \tag{2}
\end{equation*}
$$

where $P_{t}$ is the price (or, in the case of a multiproduct firm, a price index) of output in period $t$. We further assume constant price-elasticity (less than unitary) of demand, so that

$$
\begin{equation*}
D(P)=e^{\alpha} P^{-\eta}, 0 \leq \eta<1 \tag{3}
\end{equation*}
$$

Thus,

$$
Q_{t}=e^{\alpha+\tilde{\rho} t} P_{t}^{-\eta}
$$

### 3.3 The Cash Reserve

We assume that the firm's net profits (i.e., revenues minus costs, including capital costs) are accumulated in a cash reserve, which is initially positive. This reserve earns zero real return. Since net profits can be negative, the cash reserve can decrease and even become negative. Each time the cash reserve becomes negative, we say a crisis has occurred. The occurrence of a crisis says something, statistically, about the manager's performance during the periods since the preceding crisis. We shall have more to say below on how the regulator deals with a crisis.

It turns out that this device, of hinging the regulator's action solely on the sign of the firm's cash reserve, is the key simplification that makes the model tractable.

Thus, let
$s=$ initial value of cash reserve, assume positive,
$S_{t}=$ value of cash reserve at date $t$ (the end of period $t$ ), so that $S_{0}=s$,
$l_{t}=$ net loss in period $t$ (i.e., the negative of profit, net of the cost of capital) $=$ decrease in cash reserve during period $t$.
All dollar magnitudes are in constant dollars, and we assume that the real rate of interest on the cash reserve is zero. Then the decrease in the cash reserve in period $t$ is cost less revenue:

$$
\begin{equation*}
l_{t}=Q_{t}\left(\frac{C}{F_{t}}-P_{t}\right) \tag{4}
\end{equation*}
$$

### 3.4 Distribution of Information

We assume that the regulator cannot directly observe the manager's actions, nor can he observe the exogenous random events that also affect productivity changes. This limitation on the regulator's information constitutes the essence of his problem. (He may, with effort, be able to observe the resulting productivity changes, but we do not in our model rely on this possibility.) The regulator can, however, observe the consequences of these managerial actions and random events, namely, the realized profits of the firm and whether or not demand is met.

The manager is of course aware of his own past actions, and of the past history of prices and realized productivity changes. Importantly, he is also aware of the regulator's policy, which the regulator has announced. Thus, if it is part of the regulator's policy to reduce (real) prices by a fixed percentage each period (until the next crisis), the manager is aware of this projected schedule of prices. He can base his actions in each period on all this information. A managerial strategy is a rule (sequence of functions) according to which the manager determines his action in each period as a function of his information.

It remains only to add that we assume that both the manager and the regulator know the price elasticities of demand for the firm's products.

### 3.5 Objectives

We assume that the regulator wants low prices for the firm's outputs. Because of increasing productivity, the firm can afford to lower its prices over time and still remain solvent. Thus, more precisely, the regulator's objective is to maximize the long-run average rate of actual price decrease.

With respect to the manager, we assume that his wage is fixed, and we make the perhaps rather Draconian assumption that at each crisis, when the firm first becomes insolvent, the manager is fired and replaced by the owners; the regulator, knowing this will occur, can use the manager's tenure as an incentive tool. Thus, one component of the manager's objective is to maximize his tenure, i.e., the period during which he receives his (fixed) wage.

On the other hand, we admit the possibility that the manager's preferences among the alternative actions he can take in one period would not, other things being equal, lead to maximum productivity increase (and hence maximum price decrease) in the long run. It is this possibility, of course, that motivates the regulator to devise a policy that will enhance the manager's incentive to increase productivity. These preferences form the other component of the manager's objective.

Both sorts of consideration-his desire to continue receiving his salary and his preference for certain managerial actions-will contribute to determining the manager's optimal strategy. For example, we would expect that when the cash reserve gets close to zero, the manager would act so as to increase productivity very rapidly, thus reducing costs more than the regulator is reducing prices. On the other hand, when the cash reserve is very large, we would expect him to act in a way closer to his one-period preference.

Formally, if the manager receives a salary $w$ per period and chooses action $A_{t}$ in period $t$, then his utility for period $t$ is

$$
\begin{equation*}
U_{t}=U\left(w, A_{t}\right) . \tag{5}
\end{equation*}
$$

Let $\delta$ be the manager's discount rate, and $T$ his tenure in his job (a random variable). Then we assume that he will choose his strategy to maximize his expected discounted utility for the duration of his employment, i.e.,

$$
\begin{equation*}
\nu=(1-\delta) E \sum_{t=1}^{T} \delta^{t-1} U_{t} \tag{6}
\end{equation*}
$$

where $E$ denotes expectation.
Thus we make the convention that after he is fired the manager's utility per period is zero (i.e., this is the origin from which the manager's utility is measured). The normalization factor $(1-\delta)$ has the effect of keeping $v$ uniformly bounded in $\delta(0 \leq \delta 1)$.

We assume that there exists a managerial action $\hat{A}$ such that

$$
\begin{gather*}
\hat{u} \equiv U(w, \hat{A})>0  \tag{7}\\
\hat{\gamma} \equiv E \Gamma(\hat{A}, X)>0 \tag{8}
\end{gather*}
$$

This assumption states that it is possible for the manager to achieve a positive expected rate of productivity growth and still be better off during employment than after being fired.

Define $A^{*}$ to be the management action that maximizes his one-period utility in (5), given $w$; let $\gamma^{*}$ be the corresponding expected logarithmic rate of change of productivity, namely,

$$
\begin{equation*}
\gamma^{*}=E \Gamma\left(A^{*}, X\right), \tag{9}
\end{equation*}
$$

and let $u^{*}$ be the manager's corresponding one-period utility,

$$
\begin{equation*}
u^{*} \equiv \boldsymbol{U}\left(w, A^{*}\right) \tag{10}
\end{equation*}
$$

Finally, without loss of generality, we assume that a managerial action $\hat{A}$ exists so that not only (7) and (8) are satisfied, but moreover

$$
\begin{align*}
& \hat{\gamma}>\gamma^{*}  \tag{11}\\
& \hat{u}<u^{*} \tag{12}
\end{align*}
$$

(If (11) could not be satisfied, i.e., if $\gamma^{*}$ were the maximum feasible expected rate of growth of productivity, then there would be no divergence between the manager's short-term goals (as represented by his one-period utility) and the goal of maximizing the long-run rate of price decrease.) We shall assume that some $\hat{A}$ satisfying (7), (8), (11) and (12) has been chosen.

### 3.6 Strategies

As we have said, the manager is required to adjust the firm's output to meet demand exactly, and to pay all costs including the cost of capital. He is able to do these things as long as the firm is solvent, i.e., as long as its cash-reserve is positive. Through bad luck or bad management, the cash reserve can become negative. This event we call a crisis; when a crisis occurs, the manager is fired and replaced. The regulator must now provide some way for the firm to get back on its feet; this part
of the regulator's strategy will be described below. Thus time is divided into alternating segments: incentive phases and recovery phases.

During each incentive phase, the regulator requires that prices decrease at a fixed percentage rate-say $\varphi$-in each period. For the scheme to work, $\varphi$ must be set at a level not greater than the greatest achievable level of productivity increase. The regulator will want to meet this condition, since he wants the firm to be solvent most of the time. We do not go into the problem of how the regulator can elicit true cost information from the firm, but simply assume that his information on the firm's past productivity behavior enables him to choose a $\varphi$ that is not too high. Thus during an incentive phase

$$
P_{t}=P_{0} e^{-\phi t}
$$

whence, from (2) and (3),

$$
\begin{equation*}
Q_{t}=Q_{0} e^{\mathrm{p} t} \tag{13}
\end{equation*}
$$

where

$$
\begin{aligned}
\rho & =\tilde{\rho}+\eta \varphi \\
Q_{0} & =e^{\alpha} P_{0}^{-\eta}
\end{aligned}
$$

We can now describe completely the behavior of the cash reserve during an incentive phase. From (4), the decline in period $t$ is

$$
l_{t}=Q_{0} e^{\rho t}\left(\frac{C}{F_{0}} e^{-H_{t}}-P_{0} e^{-\varphi t}\right)
$$

We assume that the regulator chooses $P_{0}$ so that at the beginning of the incentive phase profit is zero, $l_{0}=0$, i.e.,

$$
P_{0}=\frac{C}{F_{0}}
$$

Then

$$
l_{t}=Q_{0} P_{0} e^{\rho t}\left(e^{-H_{t}}-e^{-\phi t}\right)
$$

We may choose units such that the initial revenue flow is unity:

$$
Q_{0} P_{0}=1
$$

Denote the growth rate of revenue during the incentive phase by

$$
\sigma=\rho-\varphi=\widetilde{\rho}-\varphi(1-\eta),
$$

and assume $\sigma$ positive, i.e.,

$$
\tilde{\rho}>\varphi(1-\eta) .
$$

Let

$$
\begin{equation*}
W_{t}=\varphi t-H_{t} . \tag{14}
\end{equation*}
$$

$W_{t}$ might be called the (logarithmic) "productivity gap" at $t$. Then the law of motion of the cash reserve during the incentive phase may be stated briefly as

$$
\left\{\begin{array}{l}
S_{t}=S_{t-1}-l_{t}  \tag{15}\\
l_{t}=e^{\alpha t}\left(e^{W}-1\right)
\end{array}\right.
$$

with initial conditions $W_{0}=0$, and $S_{0}=s>0$. Let $T$ be the first $t$ such that $S_{t} \leq 0$; the periods $t=1, \ldots, T$ constitute the first incentive phase. The first crisis is said to occur at the end of period $T$; the first manager is fired at this time, and a new manager is appointed.

The first crisis is followed by a recovery phase, $t=T+1, \ldots, T+T^{*}$, where $T^{*}$ is exogenously determined and is the same for every recovery phase. Thus, to summarize, the parameters which the regulator can manipulate are $\varphi, s$, and $T^{*}$. (He may also be able to choose, or at least cap, the manager's salary; see equation (7), above.)

During the recovery phase, the regulator sets prices in such a way that dollar profit is the same in each period, and that at the end the cash reserve is back up to its initial value, $s$. (Note that to achieve this result some ex post adjustment of the price will in general be required after the value of the random variable $X_{t}$ is realized.) Thus the dollar profit, $n^{*}$, in each recovery period is determined by

$$
\begin{equation*}
S_{T}+T^{*} n^{*}=s \tag{16}
\end{equation*}
$$

Here $S_{T}$, the value of the cash reserve at the beginning of the recovery phase, is non-positive. With constant demand elasticity, $\eta$, which is strictly less than unity, the condition (15) can always be met. This becomes clear when we notice that revenues,

$$
P_{t} Q_{t}=e^{\alpha+\tilde{\rho} t} P_{t}^{1-\eta}, \text { with } \eta<1,
$$

can be given any arbitrary positive value by choice of $P_{t}$, while (for any given level of productivity) an increase in price lowers total costs.

Following the end of the first recovery phase, a second incentive phase is started, with $P_{T+T^{*+1}}$ set so that profit is zero, and thereafter prices decline at the same incentive rate $\varphi$ until the next crisis, etc.

It is clear that prices are not apt to fall as rapidly as the rate $\varphi$ during the recovery phase; they may even increase. Thus if the system is to work at all well, the recovery phases must be infrequent, and short relative to the average length of an incentive phase. That this can indeed be the case is one of our results, which we shall prove below (Theorem 3).

We turn now to the study of the optimal strategy of the manager during the incentive phase.

Theorem 1. Let $\widetilde{T}(\delta)$ be the length of the incentive phase, given that the manager's discount factor is $\delta$ and he follows an optimal strategy; then if $\varphi<\hat{\gamma}$,

$$
\lim _{\delta \rightarrow 1} E \tilde{T}(\delta)=+\infty
$$

Proof. Recall that we have assumed that $s>0$. One strategy available to the manager is to use the action $\hat{A}$ throughout the incentive phase. By assumption, this strategy allows the firm to meet the regulator's target rate of price decrease, $\varphi$, and remain solvent, in an average sense. But of course it does not rule out the possibility of insolvency in some particular period. Let $z_{t}=\ln \left(P_{t} F_{t}\right)$; with this strategy $\left(z_{t}\right)$ is a random walk with increments $\left(G_{t}-\varphi\right)$ which are bounded from below and which are independent and identically distributed with mean $\hat{\gamma}-\varphi$. Since $\hat{\gamma}-\varphi>0$, if such a random walk were to continue indefinitely,

$$
\begin{equation*}
\hat{\pi}=\operatorname{Prob}\left\{\sum_{c=1}^{t}\left(G_{n}-\varphi\right) \geq 0 \text { for all } t \geq 1\right\}>0 . \tag{17}
\end{equation*}
$$

Recall that $P_{0}$ is chosen so that profit is initially zero, i.e., so that

$$
P_{0} F_{0}=C
$$

Hence

$$
\begin{equation*}
\operatorname{Prob}\left\{P_{t} F_{t} \geq C \text { for all } t>1\right\}=\hat{\pi} \tag{18}
\end{equation*}
$$

and hence, a fortiori,

$$
\begin{equation*}
\operatorname{Prob}\left\{S_{t} \geq s \text { for all } t\right\} \geq \hat{\pi} \tag{19}
\end{equation*}
$$

This is more than sufficient to guarantee that

$$
\begin{equation*}
\operatorname{Prob}\{\hat{f}=+\infty\} \geq \hat{\pi}, \tag{20}
\end{equation*}
$$

where $\hat{r}_{\text {is }}$ the length of the incentive phase of this strategy. The expected total discounted utility for the manager from this strategy is

$$
\begin{align*}
\hat{\mathrm{v}} & =(1-\delta) E \sum_{i=1}^{\hat{T}} \delta^{t-1} \hat{u} \\
& =\hat{u}\left(1-E \delta^{\hat{T}}\right) \tag{21}
\end{align*}
$$

Now consider an optimal strategy for the manager; let $\tilde{T}$ denote the corresponding length of the incentive period, and $\widetilde{v}$ the corresponding expected total discounted utility. By the definition of $u^{*}$,

$$
\begin{equation*}
u_{t} \leq u^{*} \tag{22}
\end{equation*}
$$

for all $u_{t}$, and hence

$$
\widetilde{\mathrm{v}} \leq(1-\delta) E \sum_{t=1}^{\widetilde{T}} \delta^{t-1} u^{*}
$$

$$
\begin{equation*}
=u^{*}\left(1-E \delta^{\tilde{T}}\right) \tag{23}
\end{equation*}
$$

Since $\tilde{v}$ is optimal, $\tilde{v} \geq \hat{v}$, so by (21) and (23),

$$
u^{*}\left(1-E \delta^{\widetilde{T}}\right) \geq \hat{u}\left(1-E \delta^{\widetilde{T}}\right)
$$

or

$$
\begin{equation*}
E \delta^{\widetilde{T}} \leq 1-\left(\frac{\hat{u}}{u^{*}}\right)\left(1-E \delta^{\widetilde{T}}\right) \tag{24}
\end{equation*}
$$

Note that, since $\delta^{t}$ is convex in $t$,

$$
\delta^{E \widetilde{T}} \leq E \delta^{\widetilde{T}}
$$

so that by (24)

$$
\begin{align*}
\delta^{E \tilde{T}} & \leq 1-\left(\frac{\hat{u}}{u^{*}}\right)\left(1-E \delta^{\tilde{T}}\right) \\
E \tilde{T} & \geq \frac{\ln \left[1-\left(\frac{\hat{u}}{u^{*}}\right)\left(1-E \delta^{\tilde{T}}\right)\right]}{\ln \delta}  \tag{25}\\
& \geq \frac{\ln \left(1-\frac{\hat{u} \hat{\pi}}{u^{*}}\right)}{\ln \delta}
\end{align*}
$$

Recall that the optimal strategy of the manager, and hence the distribution of $T$, depends on $\delta$. From (12) and (17), the right-hand side of (25) tends to infinity as $\delta$ tends to 1 . This completes the proof of Theorem 1.

Corresponding to Theorem 1 is a result for the case in which there is an exogenous upper bound on the length of the incentive phase, say $\tau$. Thus let $\tau$ be a positive number, define

$$
\begin{equation*}
\hat{T}_{\tau}=\min (\hat{T}, \tau) \tag{26}
\end{equation*}
$$

and let $\tilde{T_{\tau}}$ be the (random) length of the incentive phase for the policy of the manager that is optimal given $\tau$. (Recall that $\widetilde{T_{\tau}}$ depends on $\delta$, although this is suppressed for the time being in the notation.) It is straightforward to verify that equation (25) is also valid for $\widetilde{\tau_{\tau}}$, i.e., that

$$
\begin{equation*}
E \widetilde{T}_{\tau} \geq \frac{\ln \left[1-\left(\frac{\hat{u}}{u^{*}}\right)\left(1-E \delta^{\hat{T}_{\tau}}\right)\right]}{\ln \delta} \tag{27}
\end{equation*}
$$

The limit of the right-hand side of (27) is, by an application of l'Hôpital's Rule,

$$
\begin{equation*}
\left(\frac{\hat{u}}{u^{*}}\right)\left[\operatorname{Pr}\left(\hat{\zeta}_{\tau}=1\right)+\ldots+\tau \operatorname{Pr}\left(\hat{\zeta}_{\tau}=\tau\right)\right] \tag{28}
\end{equation*}
$$

On the other hand,

$$
\begin{equation*}
\operatorname{Pr}\left(\hat{T}_{\tau}=\tau\right) \geq \operatorname{Pr}(\hat{T}=\infty) \geq \hat{\pi} \tag{29}
\end{equation*}
$$

Hence (27)-(29) imply:
Theorem 2. Let $T \tilde{\tau}(\delta)$ be the length of the incentive phase, given that it is truncated at $\tau$, that the manger's discount factor is $\delta$, and that he follows an optimal strategy; then if $\varphi<\hat{\gamma}$,

$$
\begin{equation*}
\lim _{\delta \rightarrow 1} \inf E \tilde{T}_{\tau}(\delta) \geq \tau \hat{\pi}\left(\frac{\hat{u}}{u^{*}}\right) \tag{30}
\end{equation*}
$$

We turn next to a study of the long-run average rate of price decrease. For this study, we need to characterize the optimal behavior of managers during the recovery phases. We could assume either that (1) an "acting" manager is appointed during the recovery phase, and a new regular manager is appointed during the subsequent incentive phase, or (2) a new regular manager is appointed at the beginning of each recovery phase, and he continues his assignment to the end of the next incentive phase. In the present analysis, we make the first assumption; the qualitative nature of the results would be the same under the second assumption, but the optimal strategies during the recovery phases would be different.

We are interested in the average rate of price decrease over a sequence of cycles. By a cycle, we mean an incentive phase and the following recovery phase. The behavior of price during the first cycle is shown in figure 2 .


- Price is initially $P_{0}$, leading to zero profit when productivity is $F_{0}$ and demand is $Q_{0}$.
- Price then falls exponentially at rate $\varphi$ for $T$ periods to $P_{0} e^{-\varphi T}$. Cash reserve is now non-positive.
- Price is raised to $P_{T}$. This increase can be thought of as taking place in two parts: from $P_{0} e^{-\varphi T}$ to $P_{T}^{0}$, the break-even price at $T$, and from $P_{T}^{0}$ to $P_{T}$, the price which leads to a profit $n^{*}$ at $T$.
- Price decreases at average rate $\varphi^{*}$, which may be negative, during recovery phase.
- At the start of the next incentive phase price is adjusted downward to $P_{T+T^{*}}^{0}$, the break-even price under the conditions that then prevail.
Thus we are interested in the long-run average over cycles of the average rate of price decrease which is, for the first cycle,

$$
\begin{equation*}
\bar{\varphi}_{1}=\frac{\varphi T}{T+T^{*}}+\frac{1}{T+T^{*}}\left[\ln \left(\frac{P_{0} e^{-\varphi T}}{P_{T}^{0}}\right)+\ln \left(\frac{P_{T}^{0}}{P_{T+T^{*}}^{0}}\right)\right] \tag{31}
\end{equation*}
$$

Our strategy will be, very roughly speaking, to show that the terms in the square bracket are uniformly bounded in $T$, and hence when $T$ is large the long-run average rate of price decrease is approximately $\varphi T /\left(T+T^{*}\right)$, which in turn is approximately $\varphi$.

Let successive cycles (where a cycle consists of an incentive phase followed by a recovery phase) be indexed by $m$. Then the average rate of price decreasing during the first $M$ cycles is

$$
\begin{equation*}
\bar{\varphi}_{M}=\frac{\sum_{m=1}^{M}\left(\varphi T_{m}+a_{m} T^{*}\right)}{\sum_{m=1}^{M}\left(T_{m}+T^{*}\right)} \tag{32}
\end{equation*}
$$

where the $a_{m}$ are random variables defined by

$$
\begin{equation*}
a_{m} T^{*}=-W_{T}+\ln \left(\frac{P_{T}^{0}}{P_{T+T^{*}}^{0}}\right) \tag{33}
\end{equation*}
$$

In what follows, we shall assume that the length of the incentive phase is also exogenously bounded, say by $\tau$, as in Theorem 2.

Theorem 3. Under the conditions of Theorem 2,

$$
\begin{aligned}
& \bar{\varphi}(\delta, \tau) \equiv \liminf _{M \rightarrow \infty} \bar{\varphi}_{M} \geq \frac{\varphi-\frac{A}{h(\delta, \tau)}}{1+\frac{T^{*}}{h(\delta, \tau)}}, \text { where } \\
& h(\delta, \tau) \equiv \frac{\ln \left[1-\left(\frac{\hat{u}}{u^{*}}\right)\left(1-E \delta^{\hat{T}_{\tau}}\right)\right]}{\ln \delta} .
\end{aligned}
$$

## Corollary.

$$
\lim _{\delta \rightarrow 1} \bar{\varphi}(\delta, \tau) \geq \varphi
$$

Proof. Consider the optimal strategy of an acting manger during a recovery phase. Since $T^{*}$, the length of the phase, is not affected by his actions, his optimal strategy is to choose the action $A^{*}$ in each period $t$ to maximize his one-period utility (5). This may result in decreasing productivity, i.e., increasing unit costs. But, because of the assumption that $\Gamma$ is bounded below, the total increase in unit costs over the recovery phase is bounded; hence, the change in break-even price is bounded; hence the term

$$
\ln \left(\frac{P_{T}^{0}}{P_{T+T^{*}}^{0}}\right)
$$

in (31) is bounded, and bounded uniformly over the sequence of cycles.
With respect to the term $\ln \left(\frac{P_{0} e^{-\varphi T}}{P_{T}^{0}}\right)$, note that

$$
P_{T}^{0}=\frac{C}{F_{T}}=\frac{C}{F_{0}} e^{-H_{T}}=P_{0} e^{-H_{T}},
$$

so that

$$
\ln \left(\frac{P_{0} e^{-\varphi T}}{P_{T}^{0}}\right)=H_{T}-\varphi T=-W_{T}
$$

We shall now show that $W_{T}$, the productivity gap at the first crisis, is also bounded.
Suppose, as before, that the cash reserve first becomes non-positive at $T$, and let

$$
\begin{equation*}
\hat{W}(T, s) \equiv \operatorname{Max} W_{T} \text { subject to } \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
L_{t} \equiv \sum_{t=1}^{\tau} l_{s}=\sum_{t=1}^{\tau} e^{\alpha t}\left(e^{W_{t}}-1\right)<s \text { for } t=1, \ldots, T-1 \tag{35}
\end{equation*}
$$

$$
\begin{equation*}
\text { and } L_{\tau} \geq s \tag{36}
\end{equation*}
$$

We want to show that $\hat{W}(T, s)$ is bounded above by a bound independent of $T$. In fact, it will suffice to produce such a bound subject to the condition, weaker than (33),

$$
\begin{equation*}
L_{T-1}<s, \tag{35}
\end{equation*}
$$

and (34).
Lemma. Let $\hat{W}_{T}=$ Max $W_{T}$ subject to (35') and (36), where $\sigma>0$ and

$$
\begin{equation*}
w_{t}=W_{t}-W_{t-1} \leq w \text { for } t=2, \ldots, T \tag{37}
\end{equation*}
$$

(This last is a restatement of the condition that $\Gamma$ is bounded below.)
Then $\hat{W}_{T}$ is uniformly bounded with respect to $T$.
Proof of Lemma. We first show that $\hat{W}_{T-1}$ is uniformly bounded because of (35') alone. (35') reads

$$
\begin{equation*}
\sum_{t=1}^{T-1} e^{\sigma t}\left(e^{W}-1\right)<s \tag{38}
\end{equation*}
$$

Move all terms but the last to the right-hand side:

$$
\begin{gather*}
e^{\sigma(T-1)}\left(e^{\left.W_{T-1}-1\right)<s-} \sum_{t=1}^{T-2} e^{\sigma t}\left(e^{W_{t}}-1\right),\right.  \tag{39}\\
e^{W_{T-1}<1+s e^{-\sigma(T-1)}-e^{-\sigma(T-1)} \sum_{t=1}^{T-2} e^{\sigma t}\left(e^{W_{t}}-1\right)} . \tag{40}
\end{gather*}
$$

Choose $W_{1}, \ldots, W_{T-2}$ to maximize the right-hand side, i.e., to minimize the sum. The sum is minimal when

$$
\begin{equation*}
W_{1}=W_{2}=\ldots=W_{T-2}=-\infty . \tag{41}
\end{equation*}
$$

Then the inequality becomes

$$
\begin{align*}
& e^{W_{t-1}}<1+s e^{-\sigma(T-1)}+e^{-\sigma(T-1)} \sum_{t=1}^{T-2} e^{\sigma t}  \tag{42}\\
& e^{W_{T-1}}<1+s e^{-\sigma(T-1)}+\frac{1-e^{-\sigma(T-2)}}{e^{\sigma}-1}<1+s+\frac{1}{e^{\sigma}-1} \tag{43}
\end{align*}
$$

Since the right-hand side is independent of $T, e^{W_{T-1}}$, hence $W_{T-1}$, is uniformly bounded. (Note that the bound is finite when $\sigma>0$.)

So far we have not used (37), the upper bound on the $w_{t}$. We now invoke it to conclude that, even though the cash reserve goes negative in period $T, W_{T}$ is uniformly bounded (by the implied bound on $W_{T-1}$, above, plus $w$ ). This finishes the proof of the Lemma.

Returning now to (31), we can rewrite this equation as

$$
\begin{equation*}
\bar{\varphi}_{1}=\frac{\varphi T}{T+T^{*}}+\frac{1}{T+T^{*}}\left[-W_{T}+\ln \left(\frac{P_{T}^{0}}{P_{T}^{0}+T}\right)\right] \tag{31}
\end{equation*}
$$

and we have now shown that the second and third terms on the right are bounded. Hence the random variables $a_{m}$ in (33) are uniformly bounded.

The random variables in successive cycles are typically neither mutually independent nor identically distributed, since successive cycles will typically have different initial conditions. Nevertheless, we can obtain a renewal-theorem-like result on the asymptotic behavior of $\bar{\varphi}_{M}$. First, let

$$
\begin{equation*}
\varphi_{m}^{\prime}=\min \left(a_{m}, \varphi\right) \tag{44}
\end{equation*}
$$

then

$$
\begin{equation*}
\bar{\varphi}_{M} \geq \frac{\sum_{n=1}^{M}\left(T_{m} \varphi+\varphi_{m}^{\prime} T^{*}\right)}{\sum_{m=1}^{M}\left(T_{m}+T^{*}\right)} \equiv P_{M} . \tag{45}
\end{equation*}
$$

Second, let $E_{m}$ denote the operation of mathematical expectation conditioned on the history of the exogenous random variables $X_{t}$ through cycle ( $m-1$ ). By Levy's Strong Law of Large Numbers for dependent variables, ${ }^{4}$

$$
\begin{gather*}
\lim _{M \rightarrow \infty} \frac{\sum_{m=1}^{M}\left(T_{m} \varphi+\varphi_{m}^{\prime} T^{*}\right)}{\sum_{m=1}^{M} E_{m}\left(T_{m} \varphi+\varphi_{m}^{\prime} T^{*}\right)}=1 \\
\lim _{M \rightarrow \infty} \frac{\sum_{m=1}^{M}\left(T_{m}+T^{*}\right)}{\sum_{m=1}^{M} E_{m}\left(T_{m}+T^{*}\right)}=1
\end{gather*}
$$

both limits holding almost surely. Hence

$$
\begin{equation*}
\lim _{M \rightarrow \infty} \frac{P_{M}}{y_{M}}=1, \tag{47}
\end{equation*}
$$

where

$$
\begin{equation*}
y_{M}=\frac{\sum_{m=1}^{M} E_{m}\left(T_{m} \varphi+\varphi_{m}^{\prime} T^{*}\right)}{\sum_{m=1}^{M} E_{m}\left(T_{m}+T^{*}\right)} . \tag{48}
\end{equation*}
$$

Since the $a_{m}$ are bounded, there exists $A$ such that

$$
\begin{equation*}
\left|E_{m} \varphi_{m}^{\prime}\right| \leq A \tag{49}
\end{equation*}
$$

Recall that $h(\delta, \tau)$ denotes the right-hand side of (27); thus (27) implies that

$$
\begin{equation*}
E_{m} T_{m} \geq h(\delta, \tau) \tag{50}
\end{equation*}
$$

Hence, from (44) and (48)-(50),

$$
\begin{align*}
y_{M} & \geq \frac{\sum_{m=1}^{M}\left[h(\delta, \tau) \varphi+E_{m} \varphi_{m}^{\prime} T^{*}\right]}{\sum_{m=1}^{M}\left[h(\delta, \tau)+T^{*}\right]} \\
\geq & \frac{\varphi-\frac{A}{h(\delta, \tau)}}{1+\frac{T^{*}}{h(\delta, \tau)}} .
\end{align*}
$$

The conclusion of Theorem 3 now follows from (45), (47), (48), and (51).
The Corollary obtains when the two limits are taken in either order. That is, it can be shown that the numerator of the expression for $h(\delta, \tau)$ approaches a strictly positive number when the limits are taken in either order. We omit the details.

We have now shown that the long-run average rate of price decrease approaches the target rate, $\varphi$, provided that the latter is technologically feasible. The equilibrium described might still not be acceptable if prices could increase without limit during recovery phases, even though such increases are offset when profit is set equal to zero at the end of the recovery phase. It is straightforward to show, however, that for a constant-elasticity demand function, and with $\Gamma$ bounded, the maximum price increase during a recovery phase, i.e. the increase from $P_{T}^{0}$ to point $A$ in figure 2, is also bounded.

## 4. Remarks

In the model of Section 3, we have simplified the real regulatory situation in many ways. Some of these simplifications were made for ease of exposition, others for mathematical tractability, and still others to avoid problems of a more fundamental nature. In this concluding section, we comment on several possible generalizations and extensions of the model.

Two extensions which seem not to be fundamental are the following:

1. We have supposed that the productivity consequences of $R \& D$ are all realized with a one-period lag. Actually, it is characteristic of R\&D efforts that their benefits are realized after long and distributed lags. Inclusion of this complication, although perhaps difficult, would not entail a basic change in the model.
2. We have assumed that the firm's output just meets demand. The firm may provide several products, in which case we speak of output and price indices. It is easy to see how, by counting outputs of different quality as different outputs, quality can be included in the output index; thus when we assume demand is met we also assume quality is maintained.

In reality, of course, utilities may fail to meet demand, in a quantitative sense, and may fail to meet standards of quality. It would be reasonable to let such failures trigger a recovery phase, just as cash insolvency triggers it in our example. In fact, in one sense our model may be reinterpreted as already having this feature: namely, if there is in the model no reason other than insolvency for the firm to fail to meet demand or quality standards, then such failures are subsumed in the triggering mechanism we have already described.

Three much more fundamental extensions are the following:
3. We have included in the model the action the regulators take if the firm runs out of cash, but have not mentioned various possibilities for regulatory action if the firm prospers too greatly, i.e., if the cash reserve becomes very large. The regulator will be tempted to take some action in this case, because enormous profits for the firm will be politically unpopular, even if the firm's prices are dropping rapidly. For instance, it would clearly be possible to distribute part of a large cash reserve to the customers either in the form of still further reduced prices or as a lumpsum refund. To complete the specification of an equilibrium of this regulatory game, it would be necessary to describe the regulator's announced strategy when the cash reserve becomes very large and to say what deterrent the manager provides to prevent the regulator from reneging on this strategy.
4. We have assumed that amounts of capital and other inputs can be optimally adjusted in each period. This ignores any irreversibility of capital investment by a regulated firm, as well as difficulties associated with laying off employees. Actually, the problem of how to best perform even "conventional" rate-of-return regulation when demand falls off and there is irreversible investment may not have been treated in the literature. To deal with this requires a more general sequential principal-agent model.
5. We have not included in the model the possibility of treating the distribution of information as endogenous. That is, we could include as a parameter of the regulators' strategy the amount of information they obtain, at a cost, about the firm's technological possibilities, and perhaps even about the manager's "private" utility.

In addition:
6. We have not attempted numerical estimates of the magnitude of the effects we are discussing. We have shown, in a simplified example, that there are equilibria of the sequential game (corresponding to productivity incentive clauses) that are more efficient than short-term equilibria (corresponding to conventional rate-ofreturn regulation), but how much more efficient are they?

Such questions are difficult to answer, for two reasons. First, the technological possibilities open to the firm will (in the real world) usually be known to the managers only very incompletely, and will be even less well known to the regulators. Thus it is hard to estimate the maximum achievable growth in productivity, and hence the maximum achievable rate of price decrease. Second, it is hard to know how to estimate the manager's private utility (or disutility) for various actions, and hence how great a share of the cost-savings the managers must receive in order to be persuaded to act in a cost-minimizing way. (The managers may, for example, receive this share in the form of increased longevity in office, which requires a larger cash reserve, undistributed to customers.) Still, examination of the productivity growth of utilities as a function of the intervals between regulatory reviews might yield some empirical evidence on these points.
7. Finally, if we were to analyze this problem explicitly as a sequential game, we would have to specify the regulator's objectives and the set of regulatory strategies that are available. In doing so, we would try to take into account the issues raised in point (3) above. In addition, such sequential games typically have a multiplicity of equilibria. If one of these is better than all others for both the firm and society, it may well be adopted. Otherwise, the choice of an equilibrium strategy pair will have to be established by bargaining between the regulators and the firm. Such bargaining mechanisms are not discussed in this article.

## 5. The Literature

Without attempting to be exhaustive, we comment on the relationship of the present article to some of the more recent literature dealing with incentives in regulation.

The modern literature on this subject may be said to start with Averch and Johnson (1962). The Averch-Johnson model is a static model in which the regulator's function is to impose an upper bound on a single-product monopoly firm's rate of return on capital, while the firm acts to maximize profit subject to this constraint. Both the regulator and the firm have complete knowledge of cost and demand, and there is no stochastic element in the situation. A thorough critique of this model is given in Klevorick (1973). In that study, Klevorick introduces a multi-period model that recognizes the regulator's role in setting price as well as
bounding the rate of return, and in which regulatory review is occasional and uncertain (although exogenously determined). Hence, he is able to consider the incentive effects of regulatory lag, discussed in Section 1 above. In particular, he considers, as we do, the incentives for investment in technological progress. Informational asymmetry is recognized in this model: the regulator has complete knowledge of the firm's current parameters (cost, output, capital stock, technological level), but cannot forecast the effects of technological and demand changes as accurately as can the firm itself.

This multiperiod or sequential line of modeling is continued in studies by Bawa and Sibley (1980) and by Vogelsang and Finsinger (1979). In Bawa-Sibley, the probability of regulatory review is no longer exogenous; it depends on the firm's financial results in the previous period. These results are known to the regulatori.e., the firm's past costs are known to the regulator. In Vogelsang-Finsinger, the discussion is extended to a multiproduct firm; here the regulator no longer sets the firm's prices, but constrains them in such a way that the profit-maximizing firm is induced to converge to Ramsey pricing. The firm's incentives to subvert such a scheme by deliberate waste are discussed in Sappington (1980).

All of these sequential models exist in the context of rate-base, rate-of-return regulation. In all of them except Klevorick's, the firm's cost function is assumed stationary. In general, except for Sappington's comment (1980), the emphasis is not on possible strategic behavior-misrepresentation, waste, etc.-on the part of the firm.

A somewhat different line of thought starts with Demsetz (1968). Here the firm's costs are not assumed known to the regulator, even retrospectively (in fact, these are single-period or stationary models), and the context is not that of rate-base rate-of-return regulation. Demsetz' suggestion is that a monopoly franchise be awarded to the bidding firm that undertakes to supply service at the lowest price; this will lead essentially to the competitive price. It is not said how the price is to be adjusted as costs change. A second study in this line is that of Loeb and Magat (1979), in which it is proposed that the monopoly firm set its own price (in a manner which could be responsive to changing cost and demand conditions). The firm would then receive a subsidy, so designed as a function of its price, that the profit-maximizing firm would set price equal to marginal cost. Unfortunately, this subsidy would leave the firm with all the surplus associated with its operation. Loeb and Magat propose removing part or all of this surplus from the firm by prefixing a process of competitive bidding for the franchise.

But what if there is only one bidder? As Baron and Myerson (1982) remark, a lumpsum tax might be levied on the firm; however, in the absence of cost information the regulator might set the tax too high, in which case the firm would decline to produce. (This is equivalent to the event, in our model, that the regulator sets the rate of price decrease, $\varphi$, beyond the technological capability of the firm. We have not analyzed this problem in the present article.)

The regulator's objective function, for Baron-Myerson, is maximization of an arbitrarily weighted sum of profit and consumer surplus; when these weights are
equal, so that total surplus is being maximized, their solution reduces to that of Loeb-Magat. Their solution is explicitly incentive-compatible, i.e., it offers the firm no incentive to misrepresent its costs.

Finally, in a further extension of this line of thought, Sappington (1983) extends the analysis to a multiproduct firm; a single parameter of this firm's cost function is unknown to the regulator. In Sappington's model, the regulator presents the firm with a menu of a finite number of regulatory regimes, from which it makes a binding choice.

The present article differs from all of the above most saliently in that we focus on the incentives of the manager, rather than the more abstract "firm." (Thus profit maximization now appears indirectly in the form of the manager's desire to extend his tenure by keeping the cash reserve high.) We also differ from all of the studies mentioned above, except Klevorick's, in that we emphasize technological charge, i.e., the firm's costs are endogenously determined. Because the manager's "tastes" differ from the regulator's and because the regulator cannot observe the manager's cost-reducing efforts but only their outcome (which also depends on an unobserved random variable), we deal with the problem of moral hazard. The sequential nature of our model enables us to design a regulatory mechanism that leads to improved efficiency in the face of moral hazard. We have not claimed that this regulatory mechanism is optimal; we are satisficing rather than optimizing.

## Notes


#### Abstract

The views expressed here are those of the authors, and not necessarily those of AT\&T Bell Laboratories. 1. A version of the ideas in the present article was published-without the mathematics-in the proceedings of an "Airlie House" conference that took place in 1982 (see Linhart, Radner, and Sinden, 1983). The term "price-cap" was not yet in use at that time. 2. One such clause had a three-year trial for Michigan Bell. The Michigan plan required that a price index of the firm's output drop in real terms by a fixed percentage every year (subject to a rate-of-return ceiling) to allow for expected productivity improvement. This allowance (initially $4 \% /$ year), along with the rest of the formula, was to be reviewed every three years. Incentive clauses of this general sort are discussed in Baumol (1982) and in Linhart and Sinden (1982). In the Michigan trial, the Commission may have set the productivity growth target unachievably high. 3. We do not explore in this article the complex econometric problems associated with the construction of appropriate quantity and price indices, or with the conversion of prices from nominal to real terms. 4. See Sec. 1 of D. Freedman, "Another Note on the Borel-Cantelli Lemma and the Strong Law," Annals of Probability, 1 (1973).


## References

Averch, H., and J. L. Johnson. 1962. "Behavior of the Firm under Regulatory Constraint." American Economic Review 52:1052-1069.
Bawa, V. S., and D. S. Sibley. 1980. "Dynamic Behavior of a Firm Subject to Stochastic Regulatory Review." International Economic Review 21:627-642.
Baron, D. P., and R. B. Meyerson. 1982. "Regulating a Monopolist with Unknown Costs." Econometrica 50:911-930.

Baumol, W. J. 1968. "Reasonable Rules for Rate Regulation: Plausible Policies for an Imperfect World." In Prices: Issues in Theory, Practice, and Public Policy, edited by Almarin Phillips. Philadelphia, PA: University of Pennsylvania Press.
Baumol, W. J. 1982. "Productivity Incentive Clauses and Rate Adjustments for Inflation." Public Utilities Fortnightly 110:11-18.
Demsetz, H. 1968. "Why Regulate Utilities." Journal of Law and Economics 11:55-65.
Houthakker, H. S. 1979. "Growth and Inflation: Analysis by Industry." Brookings Papers on Economic Activity 1:241-256.
Houthakker, H. S. 1981. "Competition, Regulation and Efficiency." In Regulation and Deregulation, edited by Jules Backman. Indianapolis: Bobbs-Merrill.
Klevorick, A. K. 1973. "The Behavior of a Firm Subject to Stochastic Regulatory Review." Bell Journal of Economics and Management Science 4:57-88.
Linhart, P. B., and F. W. Sinden. 1982. "Productivity Incentives Under Rate Regulation." Bell Labs Economic Discussion Paper 236.
Linhart, P. B., R. Radner, and F. W. Sinden. 1983. "A Sequential Principal-Agent Approach to Regulation." In Proceedings from the Tenth Annual Telecommunications Policy Research Conference, edited by O. H. Gandy, Jr., P. Espinosa, and J. A. Ordover. Norwood, NJ: Ablex Publishing Corp.
Linhart, P. B., and R. Radner. 1984. "Deregulation of Long-Distance Telecommunications." In Policy Research in Telecommunications, edited by V. Mosco. Norwood, NJ: Ablex Publishing Corp.
Loeb, M., and W. Magat. 1979. "A Decentralized Method for Utility Regulation." Journal of Law and Economics 22:399-404.
Radner, R. 1981. "Monitoring Cooperative Agreements in a Repeated Principal-Agent Relationship." Econometrica 49:1127-1148.
Radner, R. 1985. "Repeated Principal-Agent Games with Discounting." Econometrica 53:1173-1197.
Radner, R. 1986. "Repeated Moral Hazard with Low Discount Rates." In Uncertainty, Information, and Communication, edited by W. P. Heller, R. M. Starr, and D. Starrett. Cambridge: Cambridge University Press, pp. 25-64.
Sappington, D. 1980. "Strategic Firm Behavior under a Dynamic Regulatory Adjustment Process." Bell Journal of Economics 11:360-372.
Sappington, D. 1983. "Optimal Regulation of a Multiproduct Monopoly with Unknown Technological Capabilities." Bell Journal of Economics 14:453-463.
Schumpeter, J. A. 1942. Capitalism, Socialism, and Democracy. New York: Harper and Row.
Stigler, G. T. 1971. "The Theory of Economic Regulation." Bell Journal of Economics 2:3-21.
Williamson, O. 1968. "A Dynamic Stochastic Theory of Managerial Behavior." In Prices: Issues in Theory, Policy, and Practice, edited by A. Phillips and O. Williamson. Philadelphia, PA: University of Pennsylvania Press.

