Equilibrium Fee Schedules in a Monopolist Call Market

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Abstract

Liquidity plays a crucial role in financial exchange markets. Markets typically create liquidity through spatial consolidation with specialist/market makers matching orders arriving at different times. However, continuous trading systems have an inherent weakness in the potential for insufficient liquidity. This risk was highlighted during the 1987 market crash. Subsequent proposals suggested time consolidation in the form of call markets integrated into the continuous trading environment. This paper explores the optimal fee schedule for a monopolist call market auctioneer competing with a continuous auction market. Liquidity is an externality in that traders are not fully compensated for the liquidity they bring to the market. Thus, in the absence of differential transaction costs, traders have an incentive to delay order entry resulting in significant uncertainty in the number of traders participating at the call. A well-designed call market mechanism has to mitigate this uncertainty. The trading mechanism examined utilizes two elements: commitments to trade and discounts in fees for early commitment; thus, optimal transaction fees are time-dependent. Traders who commit early are rewarded for the enhanced liquidity that their commitment provides to the market. As participants commit earlier they pay strictly lower fees and are strictly better off by participating in the call market rather than in the continuous market. A comparison to the social welfare maximizing fee schedule shows that the monopolist does not internalize the externality completely, with the social welfare maximizing schedule offering lower fees to all traders.

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1. Introduction

Liquidity plays a crucial role in financial exchange markets. Liquidity is necessary for the existence of the market, while higher liquidity expands the set of potential counter-offers and enhances the probability of a favorable match. Thus, higher liquidity increases the expected level of utility of market participants. By participating in a market, a trader increases liquidity and benefits other traders. However, traders are not fully compensated for the liquidity they bring to the market; thus, the provision of liquidity is a positive externality. Therefore, without coordination and appropriate design of market exchange processes so as to increase liquidity, there is no guarantee of sufficient market participation or thickness of the market.[1]

Markets typically enhance liquidity through the physical gathering of orders at a single location, the trading floor. This ***spacial consolidation*** allows for the matching of orders that originate from different locations. To increase liquidity further, orders can be consolidated in time as well, and simultaneously executed in an electronic ***call market***. Following the 1987 crash a number of proposals suggested increasing liquidity through ***time consolidation*** in the form of call markets integrated into the continuous trading environment.[2] Empirical evidence

[1] A lack of liquidity also has implications for the price discovery process. A continuous trading system may experience small volume on the trading floor, especially when the specialist's limit order book is thin and there are capital constraints. This risk was highlighted during the 1987 market crash. Specialists were unable to cope with the large trading volume and order imbalances and either did not open trading in stocks or quoted prices at which little or no trading could be done. Market prices were not revealing the underlying equilibrium conditions (Edwards (1988) p. 291).

[2] Edwards (1988) suggests the addition of one or more call auctions a day. Order imbalances might elicit new bids that would balance the market without specialists or market makers having to risk capital. Cohen and Schwartz (1989) and Schwartz (1988, 1991, 1992) have called for additional electronic call markets to reopen trading after halts and at pre-specified times during the trading day, including the open and close of the market. Economides and Schwartz (1993) propose having three calls during the trading day, one at the opening, one at midday, and one

from a survey of equity managers shows that two thirds of the respondents are willing to wait until a specified time (i.e., the call) to reduce transaction costs.[3]

This paper models a call market in the presence of a continuous market that functions in parallel during the period in which orders are submitted to the call market.[4] The call market aggregates orders over time to be executed at a pre-determined instant, say 12:00. The call market opens earlier, say 11:00, for order collection, but no orders are executed until the final instant. We envision an electronic call market, where information on cumulative supply and demand of each security is disseminated throughout the period between the opening of the call market and the call. This information shows the degree of committed liquidity in the market as the call approaches.

Traders value liquidity, and want to avoid uncertainty since they are risk-averse.[5] To reduce uncertainty in the time interval between the placement of the order in the call market and the call, traders have an incentive to wait. They have an added incentive to wait if they expect the number of traders who have placed orders to increase as the call approaches. In essence, the good "placement of an order to participate in the call market," is differentiated in two dimensions of "quality": the closeness of the time of order placement to the call, as well as the committed level of liquidity in the call market at the time of placement. Thus, in the absence

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at the close. Amihud and Mendelson (1985a, 1985b, 1991) suggest a system incorporating the three trading systems (continuous auction, continuous dealer, and call auction) and allow market forces to determine under which environment a stock trades. They show that a combination of competing mechanisms enables traders to trade-off immediacy, flexibility and price.

[3] See Economides and Schwartz (1994). These survey results were based on 150 respondents representing $1.5 trillion in managed assets.

[4] Economides and Heisler (1994) discuss the theoretical equilibrium of the coexistence of call and continuous markets.

[5] See Garbade and Silber (1976, 1979) and Economides and Siow (1988) for earlier discussions of the importance of liquidity in the endogenous determination of the number and size of financial exchange markets.

of differential transaction costs, traders have incentives to wait until the last moment to place their order, thereby buying the "highest quality" good. Such behavior would result in significant uncertainty in the number of traders participating at the call.[6] A well-designed call market mechanism has to mitigate such uncertainty.

The trading mechanism we propose utilizes two elements: commitments to trade and discounts in fees and commissions. Traders are offered a discount in fees and commissions when they place an order early (to be executed at the call). There is a penalty for withdrawing an order. Thus, the auctioneer sells higher quality goods (with order entry closer to execution at the call) at higher "prices" -- transaction fees. The improved size of early entry increases traders participation and the profits of the auctioneer (or the exchange).[7] The mechanism we described increases the utility of traders by reducing transaction costs, reducing the uncertainty traders face in market interaction, and establishing market prices that reflect the underlying equilibrium prices.

Given the fee schedule offered by the monopolist, traders self-select order entry points based on their level of risk aversion. Less risk-averse, more patient, traders with low demand for immediacy commit early while more risk-averse traders commit later.[8] Very early traders

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[6] High liquidity at the call is crucial to the success of coexistence of call and continuous markets. The more liquid the call, the more attractive it is to traders. This mechanism is self-reinforcing: the more traders participate, the more liquid the call becomes. This self-reinforcing mechanism could exist in expectations of trader participation that are fulfilled at the time of the call. If a large number of traders expect that other traders will participate in the call, they themselves participate in the call, and their expectations of large participation are fulfilled. This is an equilibrium, but, as in many coordination games, it is not the only one. In fact, any size of participation is an equilibrium, including zero participation. Thus, when traders wait until the last moment, there is significant uncertainty at the call.

[7] The approach taken here is that of pricing by a multiproduct monopolist as in Mussa and Rosen (1978). For an informal description of our trading mechanism see Economides (1994).

[8] Also, traders with large orders may be at greater risk of moving the market price in less liquid markets, and therefore may prefer the call to the continuous market.

may be subsidized, i.e., charged a fee below cost, to be compensated for the very large positive externality that they create in the market. The last participant of the call, the trader who commits just prior to execution, is just indifferent between participating in the call or participating in the continuous market. All participants who commit earlier pay strictly lower fees and are strictly better off by participating in the call market rather than at the continuous market.[9]

The profit-maximizing auctioneer establishes an equilibrium fee schedule that internalizes some of the externality of liquidity provided by early traders. However, the externality is not fully internalized. A planner who tries to maximize social welfare can further internalize the externality. The planner can achieve the first best if he can implement perfect price discrimination and receive subsidies if necessary to finance deficits from market operations. We believe that the market has to be financially self-sustaining; thus, we rule out negative profits for the auctioneer. Also, arbitrage makes perfect price discrimination unfeasible. Thus, we describe a second-best social welfare maximization problem with self-selection of traders and non-negative profits for the auctioneer. Maximization of total trader surplus under these conditions requires that lower fees (in comparison with the monopolist's fee schedule) be charged to all entrants.

Section 2 outlines the model and develops the auctioneer's profit maximizing fee schedule. Section 3 examines the fee schedule that maximizes social welfare. Comparisons are then drawn between the schedules. Section 4 provides a simple example while Section 5 offers conclusions.

[9] It is also shown that there are no gaps in trader entry. That is, after the first trader commits, there will always be a trader entering at every instant up to the call market. Large participation at the call can be supported in equilibrium as traders sequentially commit to trade over time.

2. The Model

Consider a call market that offers a trading alternative to the continuous auction market. In a typical electronic call market, the auctioneer discloses to the participants in real time the magnitudes of aggregate demand and supply of those traders that have placed orders. A trader is more likely to place an order at time t if a large accumulation of orders has already been placed. The auctioneer has the opportunity to influence the order flow in the call market by offering discounts to early participants. In this section, we discuss the nature and properties of the fee schedule.

Assume investors have one share to buy or sell in the market. Let t represent the time of placing an order in the call market, where t_0 represents the opening of the call market and T the time of the call when all orders are executed. The exchange has a schedule of fees, $p(t)$, corresponding to the time that the order is placed.[10] The individual trader's problem is to maximize utility by deciding whether or not to place an order in the call market and when to place the order.

2.1 The Trader's Problem

Let $N(t)$ be the cumulative number of traders who have committed orders to the call market by time t.[11] Let $V(N(t))$ be the value associated with call market participation of N by time t, and $p(t)$ be the fee for placing an order at time t. The value function $V(N(t))$ measures the benefit obtained from trading in a more liquid market. Traders in a securities market prefer prices, all else equal, that are less volatile, i.e., they are averse to transaction

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[10] All investors are offered the same price schedule.

[11] Here $N(t)$, although observed by the trader, is viewed as being independent of the traders' actions.

price uncertainty.[12] Thus, we assume that $V(N(t))$ is increasing and concave in N, $dV/dN > 0$, $d^2V/dN^2 < 0$.

Let traders be indexed by θ according to their aversion to the risk of waiting once their order has been placed. It is assumed that there is a large number of traders and their risk aversion parameters are distributed according to a continuous density function $f(\theta)$ in the range $[\theta_\ell, \theta_h]$. The disutility of waiting is assumed linear in the time from the placement of an order to its execution at the call.[13] Thus, the utility of a trader of type θ who places an order (enters the market) at time t is given by

$$\phi(t, \theta) = V(N(t)) - p(t) - \theta(T - t)$$

The maximization problem of the trader is to enter his order at time t that maximizes his utility,

$$\text{MAX}_t \; V(N(t)) - p(t) - \theta(T - t).$$

A trader's optimal timing decision is characterized by the first order condition[14]

$$\partial\phi/\partial t = \theta + (dV/dN)(dN/dt) - dp/dt = 0,$$

i.e.,

$$dp/dt = (dV/dN)(dN/dt) + \theta \tag{1}$$

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[12] $V(N(t))$ is identical across all traders; therefore all traders value liquidity equally.

[13] This implies that the probability of some shock to the expected equilibrium price increases linearly with time. A model with a constant probability of a shock would result in an exponentially increasing risk. However, this refinement would not appear to change the form of the solution, as the essential point is that risk increases with distance between order entry and execution.

[14] The second order condition is $\partial^2\phi/\partial t^2 = d^2V/dt^2 - d^2p/dt^2 < 0$.

conditional on $\phi(t, \theta) \geq \Phi$, where Φ is the trader's reservation utility, in this case, the utility that a trader receives in a continuous auction market. We assume that the utility offered in the continuous market is constant, and normalize it by setting $\Phi = 0$.[15] Thus, the trader balances the total change in risk against the fee schedule across entry times.

The solution of this maximization problem defines (through (1)) the time $t(\theta)$ that a trader of type θ decides to place an order in the call market.[16] The entry time assignment, $t(\theta)$, is non-decreasing in θ, so that traders with greater risk aversion enter later,[17]

$$dt(\theta)/d\theta = -(\partial^2\phi/\partial t\partial\theta)/(\partial^2\phi/\partial t^2) = -1/(\partial^2\phi/\partial t^2) > 0.$$

2.2 The Auctioneer's Problem

The auctioneer cannot identify traders by risk aversion, despite knowing the distribution of risk aversion among traders. He therefore cannot price discriminate among traders; he offers the same fee $p(t)$ to all traders entering at a time t. However, equation (1) implies a ***self-selection*** by traders, that is, traders of type θ_i will all enter orders at the time $t(\theta_i)$ that maximizes their utility. The problem for the auctioneer, then, is to find a fee schedule $p(t)$ that maximizes his profits while anticipating this self-selected assignment of entry times $t(\theta)$. This is equivalent to finding the assignment of entry times, and associated fee schedule $p(\theta)$ that maximizes profit subject to the utility maximizing behavior of the traders.

The auctioneer seeks to maximize cumulative profit from traders over all entry times

$$\mathrm{MAX}_{p(t)} \ \Pi(t(\theta)) = \int_{\theta_\ell}^{\theta_h} p(\theta)f(\theta)d\theta. \tag{2}$$

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[15] This is consistent with homogenous expectations of return and constant volatility.

[16] This formulation implies an *par casiers* system, written orders only, as it does not provide any indication of the price prior to execution.

[17] The greater the aversion to waiting risk, the greater the reduction in price necessary for the trader to select an earlier entry time.

To find the fee schedule $p(\theta)$ that is optimal from the point of view of the auctioneer, define the surplus accruing to a trader of type θ as

$$z(\theta) \equiv V(N(t(\theta))) - p(\theta) - \theta(T - t(\theta)). \tag{3}$$

Since a trader has the option of using the specialist market instead of the call market, this surplus must be non-negative. Clearly the auctioneer would like to encourage traders to participate in the market, and at the same time collect as much as possible of their surplus. These goals are in partial conflict. We illustrate this by considering an example.

Suppose that there are only two types of traders θ_1 and θ_2, with $\theta_1 > \theta_2$. Given the pricing schedule $p(t)$, they assign themselves by solving (1), and enter at times $t_1 > t_2$. Their corresponding surpluses are

$$z(\theta_1) = V(N(t_1)) - \theta_1(T - t_1) - p(t_1) \equiv Z(\theta_1, t_1),$$
$$z(\theta_2) = V(N(t_2)) - \theta_2(T - t_2) - p(t_2) \equiv Z(\theta_2, t_2).$$

If perfect price discrimination and the required sorting were available, the monopolist auctioneer would be able to appropriate ***all*** the potential surplus from both types of traders by setting prices

$$p(t_1) = V(N(t_1)) - \theta_1(T - t_1),$$
$$p(t_2) = V(N(t_2)) - \theta_2(T - t_2),$$

so that the remaining surplus to each trader is zero, $Z(\theta_1, t_1) = Z(\theta_2, t_2) = 0$.

Of course, the monopolist auctioneer does not like to disturb the self-assignment of the traders on the time line. If the traders pick up the positions that maximize their utility, they realize high surplus, and then the auctioneer can extract it from them. However, the offer of both $p(t_1)$ and $p(t_2)$ creates a problem for the auctioneer. A less risk averse (type θ_2) trader is better off by entering at t_1. In this way he realizes a higher surplus than if he enters at t_2:

$$Z(\theta_2, t_1) = V(N(t_1)) - \theta_2(T - t_1) - [V(N(t_1)) - \theta_1(T - t_1)] = (\theta_1 - \theta_2)(T - t_1) > 0 = Z(\theta_2, t_2).$$

Therefore, if the auctioneer tries to extract all the surplus from all traders, the low-risk aversion traders will delay their entry into the market. This would have an added adverse effect. Since all traders value positively the cumulative extent of entry up to a certain time, delay by any trader reduces the utility of entering by all traders who have not yet entered. Ultimately, this reduces the potential trader's surplus, and the auctioneer's profits. To avoid these effects, we argue that ***the auctioneer should allow the low-risk-aversion customers to have some positive surplus, while extracting all potential surplus from high-risk-aversion customers.***

Returning to the general case, differentiating (3) with respect to θ and substituting the F.O.C. (1) yields

$$dz/d\theta = t(\theta) - T < 0. \tag{4}$$

Integrating (4), we have

$$z(\theta) = z(\theta_*) - T\theta - \int_{\theta}^{\theta_*} t(s)ds,$$

where θ_* is the trader of the highest θ who enters the call market. Clearly, $z(\theta)$, the remaining surplus of a trader of type θ, is higher for traders of lower θ, as the intuition of the example suggests. The greater the aversion to risk, the higher the price the trader is willing to pay to select a later entry time. Thus, it is best for the auctioneer to extract all the surplus from the type θ_* traders. The fee schedule makes the last participant of the call indifferent between participating in the call and participating in the continuous market.

Setting the surplus accruing to trader of type $\theta = \theta_*$ to zero yields

$$z(\theta) = T(\theta_* - \theta) - \int_{\theta}^{\theta_*} t(s)ds. \tag{5}$$

Combining (5) with (3) we find the price faced by a trader of type θ,

$$p(\theta) = V(N(t(\theta))) - T\theta_* + \theta\, t(\theta) + \int_{\theta}^{\theta_*} t(s)ds. \tag{6}$$

Equation (6) shows how the auctioneer offers lower fees to trader types with low θ.[18] Traders of type θ_i are assigned entry at $t(\theta_i)$ and would be willing to pay $V(N(t(\theta_i))) + \theta_i t(\theta_i)$ which increases with θ_i. The term $\int_{\theta}^{\theta_*} t(s)ds$ shows that the fee paid by a trader is related to the traders who have not yet entered but are anticipated to enter. Although the value function is based on the level of the precommitted participation when the trader enters, the fact that participation is expected to increase for all but θ_* type traders as the time of the call market approaches means that traders entering early should be willing to pay for the expected additional participation of traders with higher risk aversion. This is captures the self-fulfilling nature of the equilibrium.

The auctioneer chooses the assignment $t(\theta)$ to maximize profit, substituting (6) into (2)

$$\mathrm{MAX}_t\ \Pi(t(\theta)) = \int_{\theta_\ell}^{\theta_*} [V(N(t(\theta))) - T\theta_* + \theta\, t(\theta) + \int_{\theta}^{\theta_*} t(s)ds] f(\theta)d\theta \tag{7}$$

given the definition[19]

$$N(t(\theta)) = \int_{\theta_\ell}^{\theta} f(s)ds.$$

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[18] To see this, differentiate (6) with respect to θ and substitute (4),

$$dp/d\theta = dV/d\theta - T + t(\theta) + \theta(dt/d\theta) - dz/d\theta$$

$$= dV/d\theta - T + t(\theta) + \theta(dt/d\theta) - (t(\theta) + T) + \int_{\theta}^{\theta_*} dt/d\theta$$

$$= (dV/dN)(dN/dt)(dt/d\theta) + \theta(dt/d\theta) + \int_{\theta}^{\theta_*} dt/d\theta > 0,$$

since all of the derivatives and θ are positive.

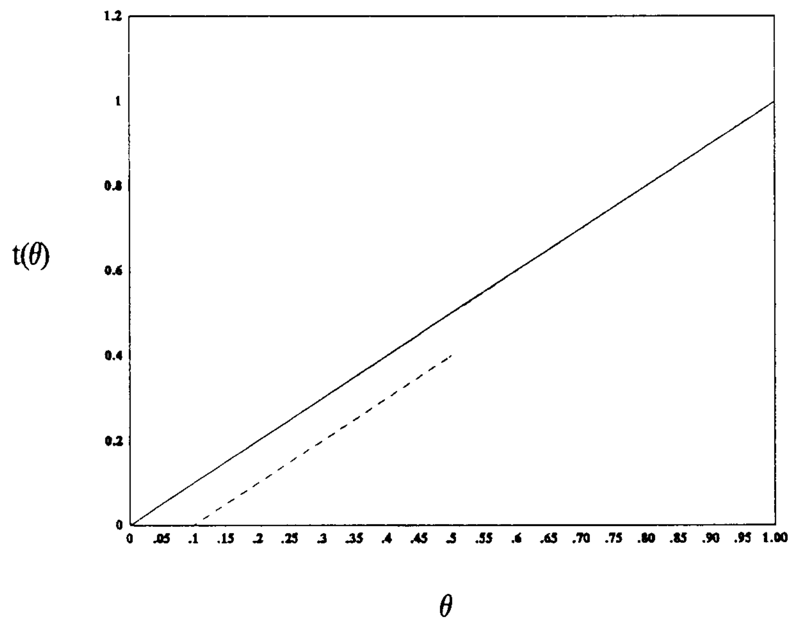
[19] The definition of participation at time t implicitly assumes that the auctioneer will induce traders with the lowest risk aversion to enter. The setting of the extensive margin at some level of risk aversion θ_* at or below θ_h insures this is the case.

2.3 The Monopolist-Auctioneer's Fee Structure

The assignment $t(\theta)$ is optimal (from the point of view of the monopolist auctioneer) if no deformation of the assignment can increase profit. Consider a deformation $h(\theta)$. The assignment $t(\theta)$ maximizes profit only if $\Pi(t + \alpha h) - \Pi(t) \leq 0$ for all $0 < \alpha \leq 1$. Now

$$\lim_{\alpha \to 0} = [\Pi(t + \alpha h) - \Pi(t)]/\alpha \equiv \Lambda(h; t) = \int_{\theta_\ell}^{\theta_h} [\theta\, h(\theta) + \int_{\theta}^{\theta_*} h(s)ds + dV/dt]\, f(\theta)d\theta \quad (8)$$

Figure 1: Deformation $v(\theta; \zeta; \Delta)$ for a linear entry assignment



Consider a deformation $v(\theta; \zeta; \Delta)$ that brings forward the entry time of traders of type $\theta \leq \zeta$ by a constant $-\Delta$ (changes entry time of these traders by $-\Delta$), while leaving the entry time of all other traders unchanged.[20] Figure 1 provides a simple example using a linear entry assignment where $\zeta = 0.5$. Substituting this deformation into (8), we see that the effect on profit is given by

[20] Thus, traders of type $\theta \leq \zeta$ have a longer time between entry and execution.

$$\Lambda(v;\, t) \;=\; \int_{\theta_\ell}^{\zeta} [-\Delta\theta - \Delta \int_{\theta}^{\zeta} ds - \Delta dV/dt] f(\theta) d\theta$$

which gives

$$= \int_{\theta_\ell}^{\zeta} [-\Delta\zeta - \Delta(dV/dt)] f(\theta) d\theta \tag{9}$$

The expression $\Delta(dV/dt)$, requires some discussion. The deformation changes the profit function in two ways. The first effect is the change in the cumulative number of traders having entered by any point in time. Now traders will observe

$$N(t(\theta)) \;=\; \int_{\theta_\ell}^{\zeta} f(s)ds + \int_{\zeta}^{\theta} f(s)ds \;=\; \int_{\theta_\ell}^{\zeta} [f(s) - \Delta f(s)]ds + \int_{\zeta}^{\theta} f(s)ds$$

$$= \int_{\theta_\ell}^{\theta} f(s)ds - \Delta \int_{\theta_\ell}^{\zeta} f(s)ds,$$

which shows the higher cumulative participation for traders over the range $[\theta_\ell,\ \zeta]$. Differentiating with respect to ζ gives

$$\Delta dN/d\zeta = \Delta f(\zeta). \tag{10}$$

where $\Delta f(\zeta)$ is the marginal increase in participation due to the advancement of entry.

The second effect will be the decrease in value for those traders who enter at earlier times, and therefore, with lower levels of participation. Differentiating (9) with respect to ζ gives

$$d\Lambda/d\zeta = -\Delta F(\zeta) - \Delta\zeta f(\zeta) - \Delta dV/dt - (d^2V/dtdN)(\Delta dN/d\zeta)F(\zeta).$$

Substituting (10) and simplifying gives

$$\Lambda(v;\, t) = -\Delta\mu(\zeta) = -\Delta[\zeta + F(\zeta)/f(\zeta) - dV/dt + (d^2V/dtdN)F(\zeta)]f(\zeta). \tag{11}$$

This can be interpreted as follows. Consider an advancement of time δ in market entry by the auctioneer for all traders of type ζ. In the neighborhood of ζ there are $f(\zeta)$ traders. The monopolist's revenue is affected in two ways. As a direct result of their advancement, the auctioneer loses total profit of $\delta f(\zeta)\zeta$, due to the increased waiting risk for type ζ traders. Now, the traders who would normally enter just prior to ζ have an incentive to delay until ζ because of the shorter waiting time and the unchanged expected level of participation by time $t(\zeta)$. Thus, all traders of type $\theta < \zeta$ must be charged a price decreased by δ. Since there are $F(\zeta)$ earlier traders, the total profit loss to the auctioneer from the foregone decrease in waiting risk to non-ζ type traders is $\delta F(\zeta)$. For the whole experiment, profit has decreased $\delta[\zeta + F(\zeta)/f(\zeta)]f(\zeta)$. We call these effects ***the marginal revenue per unit of time change for trader type*** ζ, which gives the expression

$$MR(\zeta) = \zeta + F(\zeta)/f(\zeta). \tag{12}$$

However, there are costs, associated with the changes in accumulated participation, to changing the entry assignments. For the $F(\zeta)$ traders of type $\theta \leq \zeta$, the level of participation upon entry is greater by δ, giving a total change in profit of $\delta\,(dV'/dN)f(\zeta)F(\zeta)$. For traders at ζ, there is a decrease in liquidity. They now enter at time $t(\zeta - \delta)$, with $F(\zeta - \delta)$ as the level of participation. This decrease of δ results in a total loss of $\delta(dV/dt)f(\zeta)$. All traders of type $\theta > \zeta - \delta$, see the same level of participation despite the deformation resulting in no marginal change in liquidity. We call these effects ***the marginal cost per unit of time change for trader type*** ζ,

$$MC(\zeta) = dV/dt - (dV'/dN)F(\zeta) \tag{13}$$

Then, equation (11), upon substitution of (12) and (13), can be written as

$$\mu(\theta) = \int_{\theta_\ell}^{\theta} [MR(\theta) - MC(\theta)]f(\theta)d\theta \tag{14}$$

The auctioneer's profit for a unit delay of traders $\theta \leq \zeta$ is the sum of the difference between the marginal revenue and the marginal cost of the entry assignment to all traders $\theta \leq \zeta$, weighted by their density in the market.

Since v is an admissible deformation, the optimal fee schedule and entry assignment must satisfy the condition

$$\mu(\theta) \leq 0 \qquad \text{for all } \theta. \tag{15}$$

Further, we now show that $\mu(\theta) \geq 0$ under some conditions. Consider the deformation $w(\theta; \zeta, \Delta)$ which delays the entry of traders of type $\theta \leq \zeta$ an amount Δ. Figure 2 provides an example using a linear entry assignment with $\zeta = 0.5$. Define $G(t, \Delta)$ as the set of traders in the range $t(\zeta) < t(\theta) \leq t(\zeta\text{-}) + \Delta$, where $t(\zeta\text{-})$ represents the entry time of trader type ζ approaching from the left. These traders delay for $t(\zeta\text{-}) + \Delta - t(\theta)$, while all others trade according to the original assignment. This has the effect of creating three ranges. For traders of type $\theta \leq \zeta$ and $\theta \geq \zeta + \Delta$, the result is the same as an advance in trading together with a change in the sign of the deformation. If $dt/d\theta > 0$, a linear approximation of the range of traders in $G(t, \Delta)$ is given by $\Delta t = dt/d\theta \, \Delta\theta = \Delta$, which gives $\Delta\theta = \Delta/(dt/d\theta)$. Therefore,

$$\lim_{\Delta \to 0}(1/\Delta)\, \Lambda(w; t) = -\mu(\zeta)$$

which implies that $-\mu(t) \leq 0$ at all t where $dt/d\theta \geq 0$. Combined with the optimality condition for advances in trading, it implies that the optimal pricing schedule must be consistent with the condition

$$\mu(\zeta) = 0 \quad \text{whenever } dt/d\theta \geq 0. \tag{16}$$

Over the time interval for which the call market is open, this requires that ***the optimal assignment for the monopolist-auctioneer equates marginal revenue and the marginal cost of incremental entry times***,

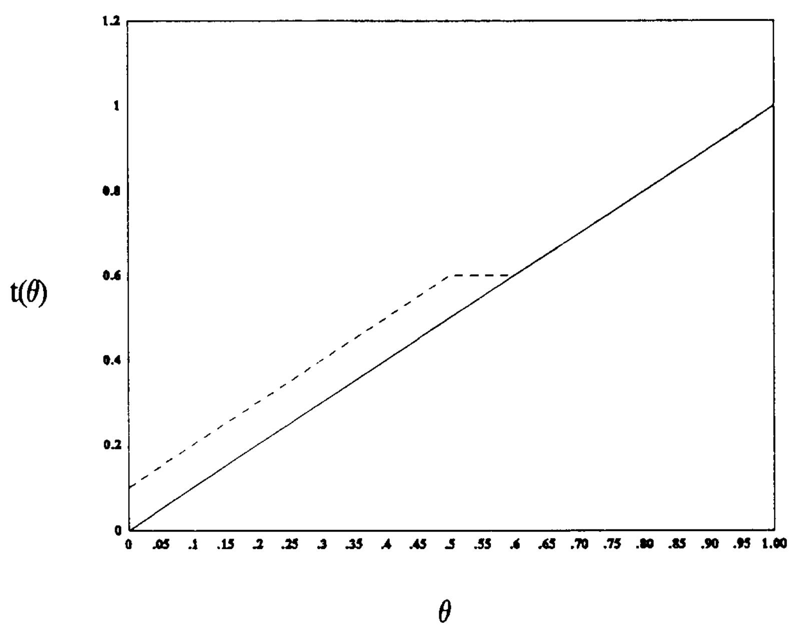
$$\theta + F(\theta)/f(\theta) = dV/dt - (dV'/dN)F(\theta). \tag{17}$$

or, equivalently (expanding the derivatives),

$$\theta + F(\theta)/f(\theta) = (dV/dN)(dN/d\theta)(d\theta/dt) - (d^2V/dN^2)(dN/d\theta)(d\theta/dt)F(\theta).$$

This expression shows that the equilibrium entry assignment depends on the ***changes*** in value given changes in accumulated participation and not on the level of value associated with a given level of participation. This is in contrast with the fee schedule, which is a function of $V(N)$ as seen in (5). Therefore, a family of functions $V(N)$ that differ from each other only by a constant leads to the same assignment. Then, reducing the constant in $V(N)$ can lead to subsidies to early traders. This is highlighted in the example of section 4.

Figure 2: Deformation $w(\theta;\ \zeta;\ \Delta)$ for a linear entry assignment



2.4 <u>The Extensive Margin</u>

The question remains as to which traders enter the call market. The fee schedule can be set by the auctioneer such that traders with sufficiently high risk aversion levels do not enter the market, that is, $\theta_* < \theta_h$. The change in profit from a change in the extensive margin is given by

$$d\Pi/d\theta_* = [V(N(t(\theta_*))) - T\theta_* + \theta_* t(\theta_*) + \int_{\theta_*}^{\theta_*} t(s)ds]f(\theta_*)$$

$$+ \int_{\theta_\ell}^{\theta_*} [dV/d\theta_* - T + \theta dt/d\theta_* + t(\theta_*) + \int_{\theta}^{\theta_*} dt/d\theta_* \, ds]f(\theta)d\theta.$$

Simplifying yields

$$d\Pi/d\theta_* = V(\theta_*)f(\theta_*) + \int_{\theta_\ell}^{\theta_*} [dV/d\theta_* + \theta dt/d\theta_* + \int_{\theta}^{\theta_*} dt/d\theta_* \, ds] \, f(\theta) \, d\theta$$

$$= V(\theta_*)f(\theta_*) + \int_{\theta_\ell}^{\theta_*} dp/d\theta_* \, f(\theta) \, d\theta = 0 \qquad (18)$$

The gain in profit from including traders with incrementally higher risk aversion is balanced by the lower price offered to less risk averse traders. Given that the time of the call does not change and that the additional, more risk averse traders will enter at the call, the less risk averse traders must enter earlier if the entry assignment is to be maintained, $dt/d\theta_* < 0$. This is accomplished by lowering the fee schedule, $dp/d\theta_* < 0$. Thus, it may be too costly to the auctioneer to include the most risk-averse traders in the market; it is possible that some traders will remain in the continuous market, i.e., that $\theta_* < \theta_h$. When this is the case depends on the traders' value function and the distribution of trader types.

2.5 Gaps

The optimal fee schedule will have no flat regions, and therefore, once the first trader has entered the market, there will be some traders entering at every instant thereafter. To show this, suppose $t(\theta)$ has a gap at $\theta = \zeta$. Then for small enough values of Δ, $G(\zeta, \Delta)$ will consist of at most one value of θ, namely ζ. Define $\bar{t}(\zeta)$ as the limit of $t(\theta)$ as θ approaches ζ from above and $\underline{t}(\zeta)$ the limit of $t(\theta)$ as θ approaches ζ from below. Then at least one of two conditions holds at ζ: $MR(\zeta) < MC(\bar{t}(\zeta))$ or $MR(\zeta) > MC(\underline{t}(\zeta))$. Any change in $t(\theta)$ to equate MR and MC results in additional profits during the gap. However, this contradicts the results of the optimization that $\mu(\theta) \leq 0$ for all θ. Therefore there are no gaps in the assignment.

3. Social Welfare Maximization

Alternatively, a market may be organized so as to maximize total surplus, i.e., the cumulative utility of all the traders minus costs. This may be done without any constraints, to achieve the first best, or may be done given that the profit to the auctioneer is zero, thus achieving a second best solution. Traders still choose their best entry time $t(\theta)$ given the fee structure by solving equation (1). If a planner were to choose a fee schedule to maximize the total surplus accruing to all traders while keeping the auctioneer at zero profits, he would maximize

$$MAX_{p(t)}\ S(t(\theta)) = \int_{\theta_\ell}^{\theta_*} [V(N(t(\theta))) - p(\theta) - \theta(T - t(\theta))]\, f(\theta)d\theta. \tag{19}$$

subject to

$$\Pi(t(\theta)) = \int_{\theta_\ell}^{\theta_*} p(\theta)f(\theta)d\theta = 0. \tag{20}$$

This is equivalent to the unconstrained maximization:

$$MAX_t\ S(t(\theta)) = \int_{\theta_\ell}^{\theta_*} [V(N(t(\theta))) - \theta(T - t(\theta))]\, f(\theta)d\theta. \tag{21}$$

Solving this gives

$$\theta = dV/dt - dV'/dN\, F(\theta). \tag{22}$$

The planner equates the marginal benefit, $MB = \theta$, to the marginal cost of changing the entry assignment. The marginal benefit differs from the marginal revenue by the absence of the term capturing the extent of downstream interference created by altering the entry of type θ traders.

We now compare this with the monopolist's solution. The range of trader types that enter the market will be greater or equal in the social welfare maximizing auction. This can be seen by solving for the extensive margin for the social welfare maximizing case

$$dS/d\theta_* = V(\theta_*)\, f(\theta_*) + \int_{\theta_\ell}^{\theta_*} [dV/d\theta_* + \theta dt/d\theta_*] f(\theta) d\theta = 0$$

Comparing this with equation (16) we see that the extensive margins are different by the amount of the change in expected participation created by a change in the last trader to enter the market. A comparison reveals that $\theta_{*m} \leq \theta_{*s}$, where θ_{*m} and θ_{*s} denote the extensive margin in the monopolist and social welfare auctions respectively. Given that $dt_m/d\theta_* \geq dt_s/d\theta_*$, it follows that

$$(dz_s - dz_m)/d\theta_* = \int_{\theta}^{\theta_*} [dt_m/d\theta_* - dt_s/d\theta_*]\, ds \geq 0,$$

and therefore

$$(dp_s - dp_m)/d\theta_* = \theta(dt_s/d\theta_* - dt_m/d\theta_*) - (dz_s/d\theta_* - dz_m/d\theta_*) \leq 0.$$

These results imply that, to satisfy the extensive margin conditions given the same value function and trader distribution, the monopolist must set the extensive margin at a lower, or equal, level of risk aversion relative to the social welfare maximizing case.

Trader entry will be delayed by the monopolist. A comparison of the optimal entry assignments (17) and (22),[21] shows that the slope of the entry assignment will always be greater for the social welfare maximizing solution. Since both assignments are defined such that $t(\theta_*)$

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[21] Expanding (17) and (22), solving for $dt_m/d\theta$ and $dt_s/d\theta$, and taking the ratio gives:

$$(dV/dN)(dN/d\theta)(d\theta/dt_m) - (d^2V)/(dN^2)(dN/d\theta)(d\theta/dt_m)F(\theta) = \theta + F(\theta)/f(\theta)$$

$$\Leftrightarrow\ (dV/dN)(dN/d\theta)(d\theta/dt_s) - (d^2V)/(dN^2)(dN/d\theta)(d\theta/dt_s)F(\theta) = \theta$$

$$\Leftrightarrow\ (dt_s/d\theta)/(dt_m/d\theta) = 1 + F(\theta)/(\theta\, f(\theta)) > 1.$$

$= T$ and it has been shown that $\theta_{*m} \leq \theta_{*s}$, this implies that the monopolist-auctioneer delays entry for all trader types $\theta < \theta_*$.

Trader surplus is greater for all trader types in the welfare maximizing solution. Trader surplus at θ_* is no longer set to zero in the social welfare maximizing auction, with the gain in surplus a positive constant for trader type θ_*. The difference in surplus is given by

$$z_m(\theta) - z_s(\theta) = z(\theta_*) + \int_{\theta}^{\theta_*} (t_m(s) - t_s(s))\, ds > 0.$$

Comparing slopes, it is clear that surplus in the social welfare maximizing case increases more quickly as trader risk aversion decreases,

$$d(z_m(\theta) - z_s(\theta))/d\theta = t_m(\theta) - T - t_s(\theta) + T > 0.$$

The price charged in the social welfare maximizing solution is lower for all θ. At θ_* the price is lower by the amount of excess surplus $z(\theta_*)$. Rearranging (3) for the monopolist and social welfare maximizing auctions results in

$$p_m(\theta) - p_s(\theta) = \theta\,(t_m(\theta) - t_s(\theta)) + (z_s(\theta) - z_m(\theta)) > 0.$$

The price difference decreases as the level of risk aversion increases

$$d(p_m(\theta) - p_s(\theta))/d\theta = \theta\,(dt_m(\theta)/d\theta - dt_s(\theta)/d\theta) + (t_m(\theta) - t_s(\theta)) < 0.$$

The monopolist delays entry resulting in higher potential surplus accruing to traders. The monopolist than extracts this surplus through higher fees at each time, and thereby, for each trader type.

4. Example

To illustrate the nature of the optimal fee schedule, suppose $f(\theta) = 6\theta(1 - \theta)$ (i.e., the distribution of types is a beta distribution with $\alpha = 2$ and $\beta = 2$) and the value function is quadratic, $V(N) = bN - aN^2 + c$.

4.1 Monopolist Auctioneer

Substituting the trader distribution and value function into the monopolist's optimal assignment (17) gives[22]

$$t = 4b\theta - 2b\theta^2 + C.$$

Setting the extensive margin at the call market, $t(\theta_*) = T$, and solving for the constant defines the trader entry schedule given the monopolist fee schedule

$$t_m(\theta) = T - 4b(\theta_* - \theta) + 2b\,(\theta_*^2 - \theta^2). \tag{i}$$

Figure 3 plots $t_m(\theta)$.

Substituting the monopolist entry assignment into the expression for the surplus accruing to traders (5) and solving for the constant by setting the surplus at the extensive margin to zero, $z(\theta_*) = 0$, gives the trader surplus function

$$z_m(\theta) = -2/3b\theta^3 + 2b\theta^2 - (4b\theta_* - 2b\theta_*^2)\theta + (2b\theta_*^2 - 4/3b\theta_*^3). \tag{ii}$$

Figure 4 plots the trader surplus function.

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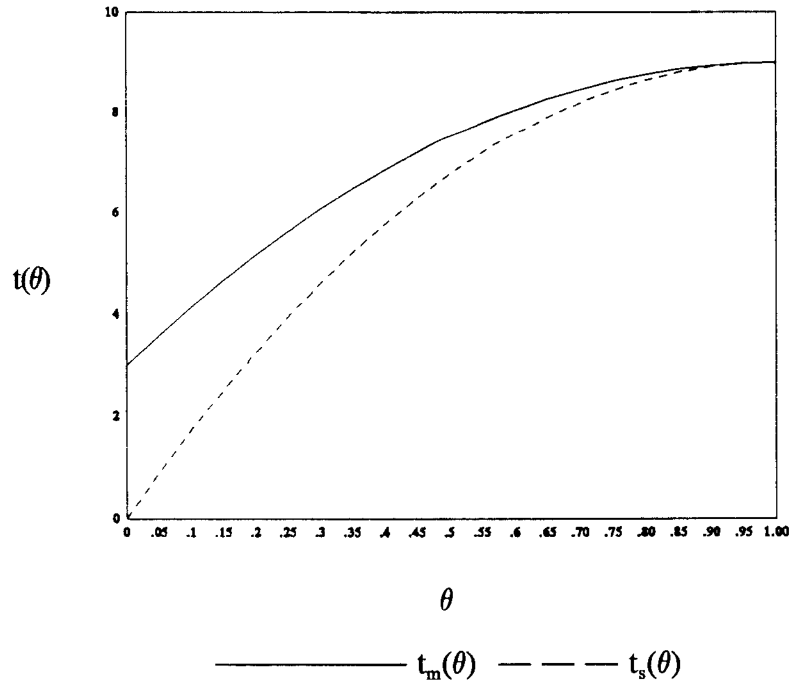
[22] This result is approximate:

$$(dV/dN)(dN/d\theta)(d\theta/dt) - (d^2V/dN^2)(dN/d\theta)(d\theta/dt)F(\theta) = \theta + F(\theta)/f(\theta)$$

$$\Leftrightarrow\ (-2aN + b)f(\theta)(d\theta/dt) - (-2a)F(\theta)f(\theta)(d\theta/dt) = \theta + F(\theta)/f(\theta)$$

$$\Leftrightarrow\ 9\theta^2 - 8\theta^3 = 36b\ \theta^2\,(1 - \theta)^2/t' \ \Leftrightarrow\ t' \approx 4b(1 - \theta).$$

Figure 3: Trader Entry Assignments



Substituting (i) and (ii) into the definition of trader surplus (3) and rearranging gives the optimal fee schedule for the monopolist auctioneer,

$$p_m(\theta) = -10/3b\theta^3 + 5b\theta^2 - (2b\theta_*^2 - 4/3b\theta_*^3) - a(3\theta^2 - 2\theta^3)^2 + c. \qquad \text{(iii)}$$

As expected the auctioneer offers lower fees to earlier entrants to build market participation. Note that by decreasing the constant, subsidies can be produced for early traders. Figure 5 presents the fee schedule.

The monopolist sets the extensive margin at

$$6\theta_*(1 - \theta_*)V(\theta_*) + 4b\theta_*(\theta_* - 1) \int_{\theta_\ell}^{\theta_*} f(\theta)\, d\theta + \int_{\theta_\ell}^{\theta_*} [4b\theta_*(\theta_* - 1)f(\theta) \int_{\theta}^{\theta_*} ds]\, f(\theta)\, d\theta = 0$$

which reduces to an expression of the form $g(b, \theta_*, \theta_\ell)(\theta_* - 1) = 0$, where $g(\cdot)$ is some polynomial function. This is clearly solved when $\theta_* = \theta_h = 1$, in which case all trader types are drawn into the call market.

Figure 4: Traders' Surplus

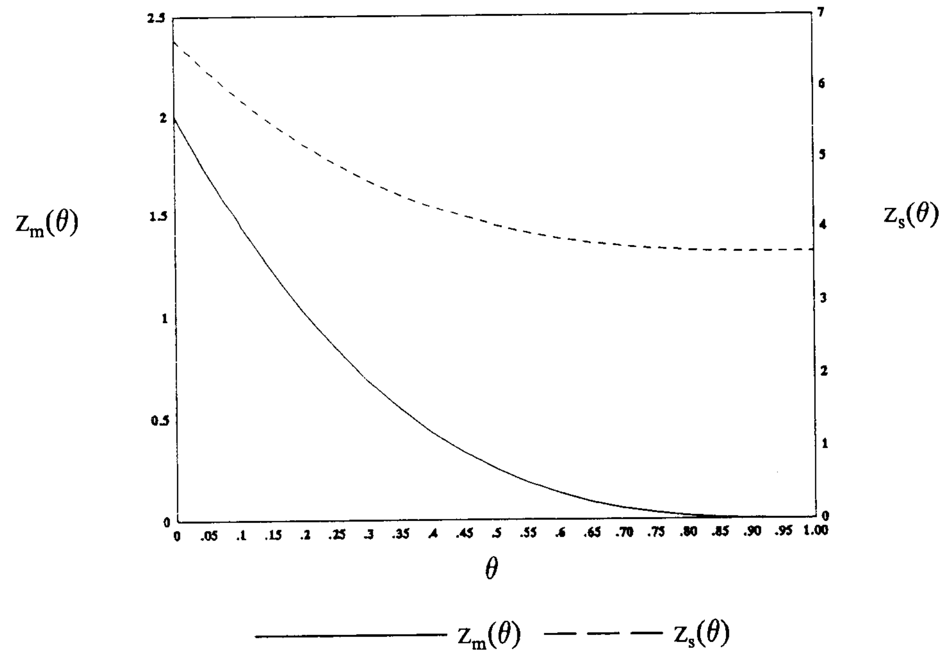
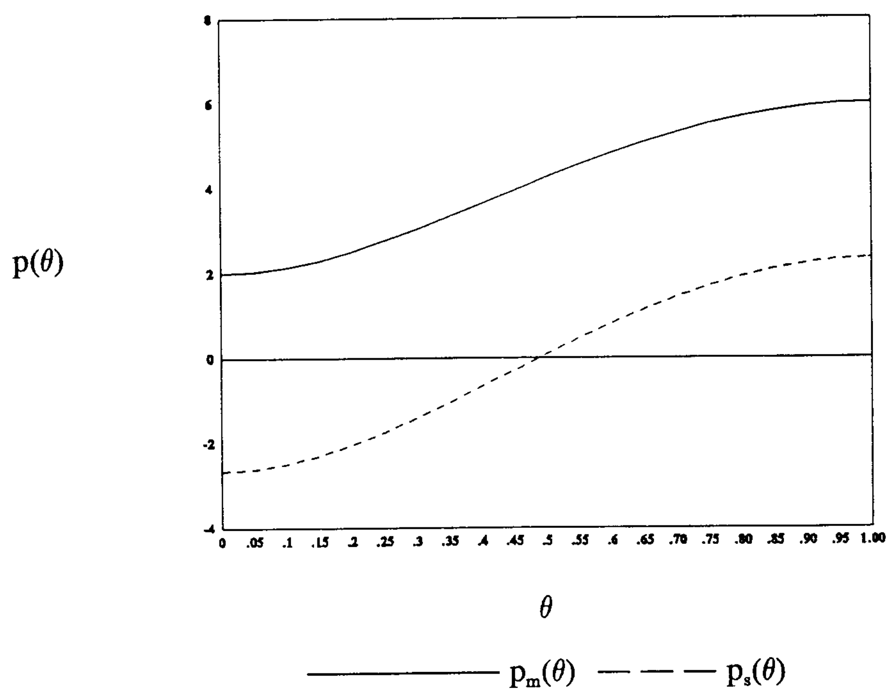


Figure 5: Fee Schedule

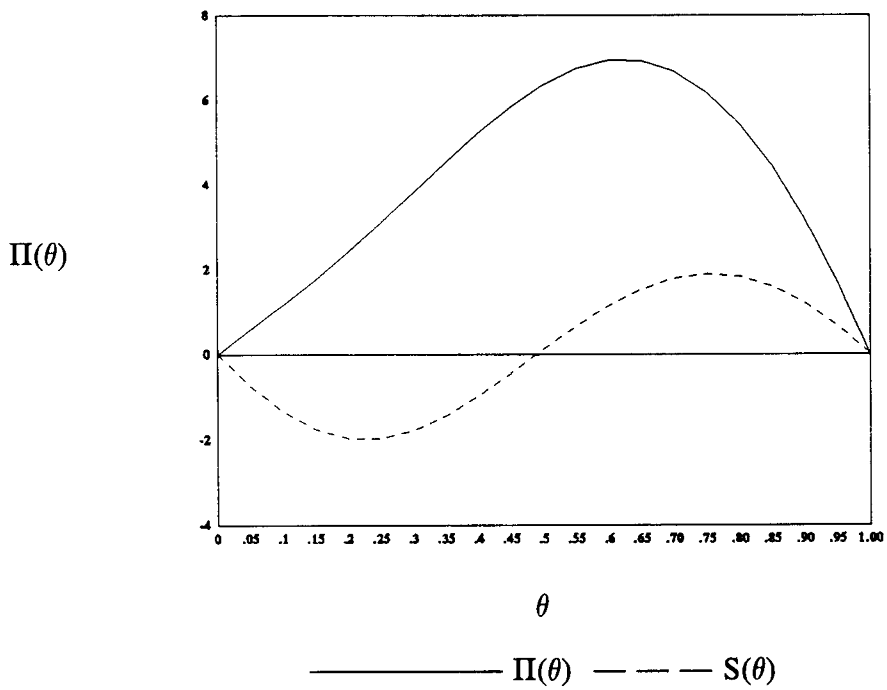


The monopolist's profit is found by substituting (iii) into (2) and setting $\theta^* = 1$. This result is plotted in Figure 6. To translate these results into natural time, solve the entry schedule (i) for t, and substitute this result into the fee schedule (iii) and profit function. Rearranging (i) gives

$$\theta = \theta^* - [(T - t)/(2\ b)]^{1/2}.$$

These results are presented in Figures 7 and 8.

Figure 6: Profits from Different Types of Traders



4.2 Social Welfare Planner

Substituting the value function and trader distribution into the social welfare maximization assignment (22) gives

$$t = 6b\theta + 3b\theta^2 + C.$$

Figure 7: Fee Schedule over Time

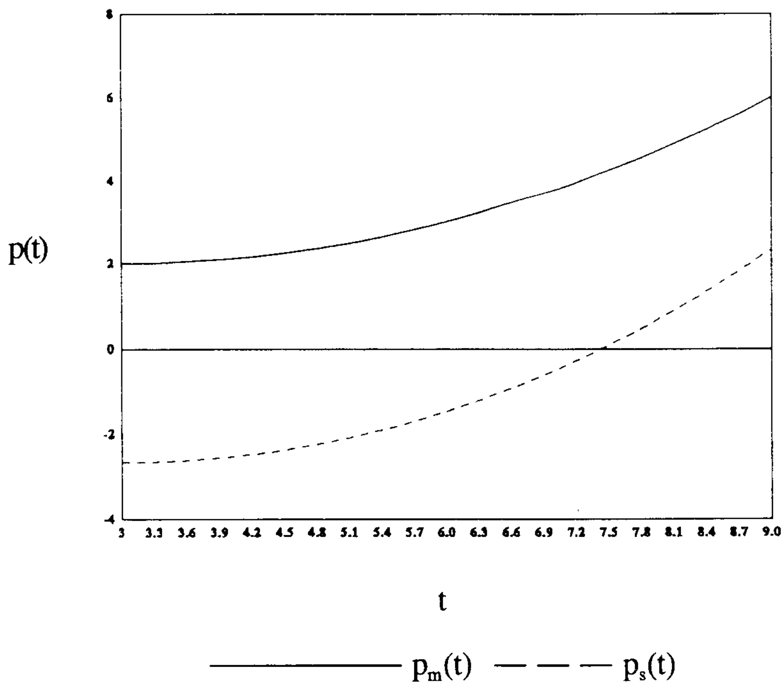
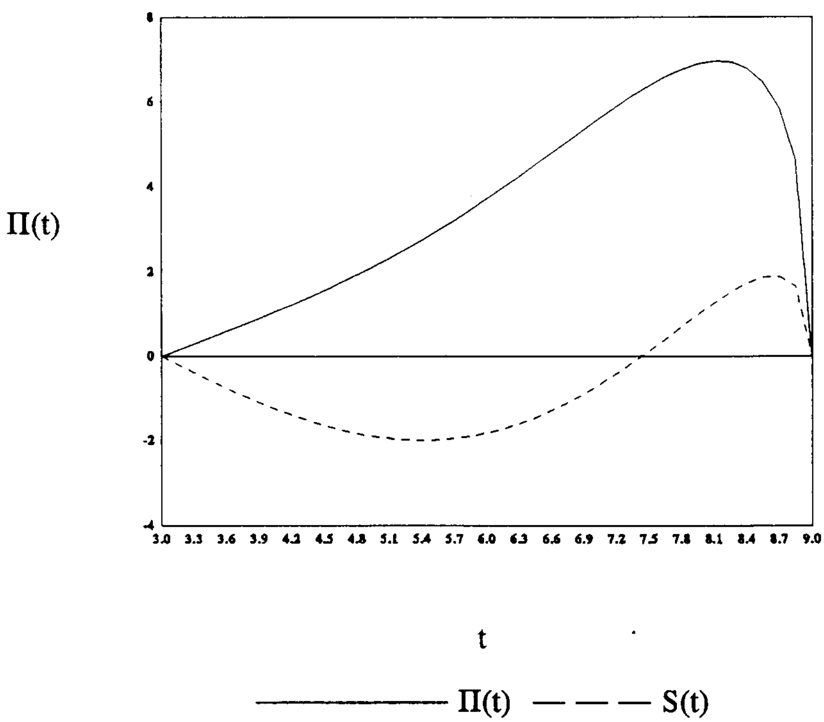


Figure 8: Profits over Time



Setting the extensive margin at the call market, $t(\theta_*) = T$, and solving for the constant defines the trader entry schedule given the social welfare maximizing fee schedule

$$t_s(\theta) = T - 6b(\theta_* - \theta) + 3b(\theta_*^2 - \theta^2). \tag{iv}$$

$t_s(\theta)$ is plotted in Figure 3.

In the social welfare maximizing market, traders will still enter at the time that maximizes their utility; therefore, the F.O.C. (1) holds. However, trader surplus at θ_* may not be zero. Substituting (iv) into (5) and solving for the constant, assuming that the surplus at θ_* is $z(\theta_*)$, gives

$$z_s(\theta) = -b\theta^3 + 3b\theta^2 - (6b\theta_* - 3b\theta_*^2)\theta + (3b\theta_*^2 - 2b\theta_*^3) + z(\theta_*).$$

Substituting (iv) and the expression for trader surplus into (3) and rearranging gives the optimal fee schedule for the welfare maximizing market

$$p_s(\theta) = V(N) - \theta(T - t) - z(\theta) = -4b\theta^3 + 6b\theta^2 - (3b\theta_*^2 - 2b\theta_*^3) - a(3\theta^2 - 2\theta^3)^2 + c - z(\theta_*).$$

In this example, the social welfare maximizing planner subsidizes earlier entrants to build market participation over the lower half of the trader distribution, and then profits from the more risk averse half that enter closer to market execution. The social welfare planner also sets the extensive margin at $\theta_* = 1$. The margin is set to satisfy

$$6\theta_*(1 - \theta_*)V(\theta_*) + 6b\theta_*(\theta_* - 1) \int_{\theta_\ell}^{\theta_*} f(\theta)\, d\theta = 0,$$

which again reduces to an expression of the form $g(b, \theta_*, \theta_\ell)(\theta_* - 1) = 0$, where $g(\cdot)$ is some polynomial function.

To solve for the trader surplus at θ_*, we apply the zero profit condition, substituting for the extensive margin at $\theta_* = 1$, and solve for $z(\theta_*)$.[23] This result is then substituted back to derive the (remaining) surplus to a trader of type θ,

$$z_s(\theta) = -b\theta^3 + 3b\theta^2 - (6b\theta_* - 3b\theta_*^2)\theta + (3b\theta_*^2 - 2b\theta_*^3) - a/3 + c, \tag{v}$$

and fee schedule

$$p_s(\theta) = -4b\theta^3 + 6b\theta^2 - (3b\theta_*^2 - 2b\theta_*^3) - a(3\theta^2 - 2\theta^3)^2 + a/3. \tag{vi}$$

The trader surplus function and the social welfare maximizing fee schedule are presented in Figures 4 and 5 respectively. Similarly, the social welfare maximizing auctioneer's profit function is found by substituting (vi) into (2); it is presented in Figure 6. To translate into natural time, we solve the entry schedule for t, and substitute into the fee schedule and profit function; rearranging (iv) gives

$$\theta = \theta^* - [(T - t)/(3b)]^{1/2}.$$

These results are plotted in Figures 7 and 8.

The comparison of the two regimes shows that the monopolist-auctioneer advances entry for all but the most risk averse trader to enter the market. This results in greater potential surplus which is extracted through higher fees, leaving all traders with lower realized surplus under monopoly. Both fee schedules are convex in time. It is interesting to note that the social welfare maximizing planner offers increasing discounts for earlier entry, resulting in a subsidy for the less risk averse half of traders.

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[23] $\Pi(t(\theta)) = \int_{\theta_\ell}^{\theta_*} p(\theta)f(\theta)d\theta = 0 \Leftrightarrow z(\theta^*) = -a/3 + c.$

5. Conclusions

In this paper we have considered the problem of establishing a call market contemporaneous to a continuous auction market. We show that a call market can attract traders despite the presence of the continuous market. Traders, except most risk averse to enter the call market, realize higher surplus in the call market than in the continuous market, $z(\theta) > 0$ for $\theta < \theta_*$. The most risk averse traders enter at the call and are made indifferent to entering the continuous market, $z(\theta_*) = 0$. Further, the call market does not replace the continuous market; thus the two types of markets co-exist . A monopolist-auctioneer running the call market will offer a time-dependent commission schedule. To increase liquidity at the call, the monopolist auctioneer offers discounts in trading costs to traders who commit early to participate in the call. This agrees with the implementation of the *Wunsch auction*, a proprietary electronic call market, where the commission schedule is time-dependant and offers discounts to traders who enter orders early in the market.[24]

The range of traders who enter the call market is a function of the distribution of their preferences regarding risk aversion and the value that they place on liquidity. Self-selection by traders, combined with the assumed absence of perfect price discrimination, places constraints on the auctioneer. It may be the case that the price necessary to induce highly risk averse traders into the market is too costly for the auctioneer to implement, as all less risk averse traders must be offered a lower price.

We also derived the social welfare maximizing fee schedule, assuming that there are no external subsidies, and that traders self-select their entry times. Compared with the market run by a monopolist-auctioneer, the social welfare-maximizing market attracts a greater or equal range of traders, with the most risk averse trader entering just prior to the call market, $t_m(\theta_*) =$

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[24] The Wunsch auction is called after the close of the NYSE. In this paper, we established the fee discounts for early commitment even when the call market operates contemporaneously with the continuous market.

$t_s(\theta_*) = T$ where $\theta_{*m} \leq \theta_{*s}$. We find that the monopolist delays entry for all other traders, $t_m(\theta) > t_s(\theta)$ for $\theta < \theta_*$. The social welfare maximizing schedule offers a lower fee to all traders, $p_m(\theta) > p_s(\theta)$. Welfare maximization, even under the constraints imposed, results in higher accrued surplus for all trader types, $z_m(\theta) < z_s(\theta)$. The loss of surplus due to monopoly decreases with θ and is smallest for type θ_* traders. To understand these results, note that the monopolist delays entry resulting in higher potential surplus accruing to traders. The monopolist then extracts this potential surplus from traders through higher fees for each trader type.

The current analysis is simplified by assuming zero costs for the operation of the call market. The fixed and variable costs associated with running the market can be modeled as a function of the time the market is open as well as a function of the size of the market based on the level of committed participation. Incorporating either cost structure does not change the results significantly.

The current formulation can also be extended to allow for changes in information occurring during the period in which as orders are taken, as in a verbal (*à la crièe*) auction. This would require multiple order entry and an explicit model of price diffusion over the period which the call market is open. This is an object of further research.

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