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Competition and Integration Among Complements, and Network Market Structure

Nicholas Economidas
and
Steven C. Salop

Columbia Institute for Tele-Information
Graduate School of Business
809 Uris Hall
Columbia University
New York, New York 10027
(212) 854-4222

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by

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I. Introduction

Recent advances in the analysis of issues of product compatibility and networks have focused renewed attention on complementary goods. Production and distribution networks often are comprised of both competing and complementary brands of components. The complementary components then can be combined to produce composite products or systems, which are, of course, substitutes for one another.

This formulation applies to complementary components such as mutually compatible hardware and software. For example, personal computers, VCRs, automobiles and gasoline, and audio recording (records, cassettes and CDs) obviously all fit this model. Many electronic communication networks also can be analyzed in the same fashion. For example, an Automatic Teller Machine (ATM) network is comprised of ATMs and bankcards. The consumer combines the use of an ATM terminal owned by one member bank and the use of a bankcard (possibly) issued by another member to complete a cash withdrawal. The ATMs and bankcards are complementary products. However, ATMs are substitutes for one another, as are different bankcards. Credit card networks like Visa or Mastercard have a similar structure, as modelled by Baxter (1983) and Phillips (1987). So do real estate multiple listing services and other electronic and product networks.[1] Networks

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[1] Similarly, a floral ordering network like FTD allows a consumer to combine two components — a florist in the consumer's home city from which to order and a florist in the destination city to fill the order and deliver it — to create a delivered floral bouquet composite product. The Western Union money

vary in the way in which market competition is structured and the degree of integration among component producers.

Cournot (1838) considered the merger of two monopolists that produce complementary goods (zinc and copper) into a single (fused) monopolist who produces the combination of the two complementary goods (brass). He showed that joint ownership by a single integrated monopolist reduces the sum of the two prices, relative to the equilibrium prices of the independent monopolists. This is because the two independent firms ignore the effect of their individual markups on each other, while the integrated monopolist internalizes this externality.

We generalize the Cournot model to the case of multiple producers of differentiated brands of each component, under the assumption that components are fully compatible and the number of brands of each component is endogenous. We derive and compare the equilibrium prices under a varied set of assumptions regarding market structure. Following Cournot, we compare independent and joint ownership (i.e., full integration) of component producers. We also analyze a number of other market structures involving partial integration that are adopted be various networks. Thus, we provide a basic model in which a variety of networking and product compatibility issues can be easily analyzed.

The paper is organized in follows. Section II briefly reviews the Cournot complements model. In section III, we generalize the model to multiple producers of each component. In section IV we analyze the two market structures considered by Cournot -- <u>independent ownership</u>, that is, oligopoly among independently-owned price-setting component producers; and <u>joint ownership</u>, that is, full integration by all component producers into a single jointly owned monopolist. For example, independent ownership generally characterizes computer hardware and software.

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transfer network is equivalent, except that cash replaces flowers.

Some firms produce hardware while different firms produce software. Each sets price independently. Similarly, in national credit card networks like VISA and MASTERCARD, the merchant's bank sets a fee to the merchant for processing its transactions (and the merchant sets the price of credit, i.e., the cash discount), while the consumer's bank sets a fee (or rebate) for using the card and the interest rate for credit card balances.

Joint ownership characterizes some networks like Western Union money transfer. Western Union sets the total price for the service and the division of this price between the originating and terminating agent. Similarly, Nintiendo sets the price of its hardware and the price of its software. It indirectly controls the price of licensees' software by its license fee, and its control over cartridge manufacturing.

Our analysis highlights the tradeoff between the gains from "vertical" integration and the losses from "horizontal" integration. Joint ownership internalizes two externalities, the "vertical" externality among complements identified by Cournot, and the "horizontal" externality among competing products. Thus, prices may rise or fall according to the relationship between the own and cross elasticities of demand for the composite products. If the own elasticity is large relative to the cross elasticities, then joint ownership reduces prices, and vice versa.

In section V we consider two benchmark cases. First we briefly describe "first best" or <u>optimality</u>. This structure can result from perfect price regulation (i.e., price equal to marginal cost) of the markets for both complementary components. We also consider a market structure in which each composite good is sold by a different independent firm. This <u>composite goods competition</u> structure involves competition among N composite goods packagers, instead of competition among component producers. In this structure, different packagers sell differentiated complete computer systems, rather than the consumers building their own systems from components they purchase. This structure leads to the lowest prices of any of the structures analyzed. This is because it fully internalizes all the vertical externalities even while maintaining full horizontal competition. Thus, it is a useful "second best" benchmark.

In Sections VI and VII, we analyze alternative forms of partial integration. Parallel vertical integration perhaps is the most common network market structure. This structure has joint ownership of pairs of complementary components, along with continued competition among substitute compatible components. For example, a consumer can withdraw cash either from an "on us" ATM owned by his own bank or a "foreign" ATM owned by another network member. An airline traveler on a one-stop itinerary may use the same airline ("on line") or change airlines ("interline") for the second leg of the trip. In this market structure, the gains from the (vertical) integration of complementary components are partially achieved even while maintaining competition among substitute products. Parallel vertical integration is the appropriate generalization of vertical integration in Cournot's duopoly of complementary goods. It leads to lower prices than does independent ownership.

Section VII analyzes the case of one-sided regulation. In this market structure, the price of one component is set at marginal cost and there is independent competition among producers of the other component. For example, some ATM networks jointly set the price (called the interchange fee) received by the ATM owner while card issuers continue to set transaction fees independently (Salop (1990)). A multiple listing services (MLS) might set the commission to the "selling" agent while permitting price competition among "listing" agents.

Credit card networks like VISA have a similar interchange fee system (Baxter (1983)). This structure may be implemented in practice by requiring the regulated component to be sold at a regulated "wholesale" price to the producers of the other component, who then compete at retail by constructing and selling composite products. One-sided regulation always results in lower prices for the composite goods than either parallel vertical integration or independent ownership. This is because one-sided regulation forces the price of one of the complements down to marginal cost.[2,3] However, one-sided regulation leads to a higher equilibrium price than both composite goods competition and optimal regulation. We also identify which component's price it is better to regulate, if only one price can be regulated. (If both prices can be regulated, the first best can obviously be achieved.) Lower prices are achieved when the less competitive of the two markets is regulated, and we characterize the measure of competitiveness. The conclusion makes some suggestions for further work.

II. Cournot's Model of Complementary Goods Duopoly

Cournot's (1838) model of complementary duopoly provides the a simple introduction to complementary products. Firms A and B produce components A and B at zero marginal cost and sell these components at prices p and q respectively. Consumers combine these two components in fixed proportions (i.e.,

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[2] As a reaction to the low price of the regulated component, the non-regulated component's price is higher than in parallel vertical integration and independent ownership. However the response by the non-regulated firms is smaller than the original decrease in the price of the regulated firms. Thus the total effect of one-sided regulation is negative on prices.

[3] Issues of optimal product variety are not addressed because we assume an exogenous number of brands of each component. Permitting variety to be determined endogenously may change these results and their welfare implications. See Salop (1991).

one unit of each) to form a composite product AB. Demand for the composite product is denoted by $D(s)$ and depends on the sum of the two component prices, $s = p + q$.[4] Each firm chooses price to maximize profits, taking the price of the complementary component as given. Thus, in modern terminology, we solve for the non-cooperative equilibrium (i.e., the Nash equilibrium in prices).

Under <u>independent ownership</u>, the two firms choose prices to maximize profits, given by,

$$\Pi_A = pD(s) = pD(p + q), \tag{1}$$

$$\Pi_B = qD(s) = qD(p + q). \tag{2}$$

Differentiating with respect to the own price and noting that $s = p + q$, we have the two first order conditions,

$$\partial\Pi_A/\partial p = pD'(s) + D(s) = 0, \tag{3}$$

$$\partial\Pi_B/\partial q = qD'(s) + D(s) = 0. \tag{4}$$

These two equations define best-response functions $p = R_A(q)$ and $q = R_B(p)$ for the two components. These can be solved for the Nash equilibrium. Summing equations (3) and (4) to define the equilibrium price s^I of the composite good AB, we have

$$s^I D'(s^I) + 2D(s^I) = 0. \tag{5}$$

<u>Joint ownership</u> of the two components (i.e., vertical integration) involves maximization of joint profits, or

$$\Pi(s) = sD(s), \tag{6}$$

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[4] We assume D(s) is continuous with a declining marginal revenue function.

Differentiating (6) with respect to s, we have

$$\partial\Pi/\partial s = s^J D'(s^J) + D(s^J) = 0. \tag{7}$$

Comparing equations (5) and (7), it follows that the price for the composite good is lower under joint ownership rather than independent ownership, or $s^I > s^J$. Thus, we have the now standard result that joint ownership or integration by complementary products firms raises welfare.[5] Of course, it should be emphasized that this is a second-best result. The joint ownership price exceeds the optimal (marginal cost) price $s^0 = 0$ that would be determined by optimal regulation.

III. The General Model

Suppose there are multiple differentiated brands of each of two components A and B. Formally, suppose there are m differentiated brands of component A, where brand A_i has price p_i, $i = 1, 2, \ldots, m$. Similarly, suppose there are n differentiated brands of component B, where brand B_j has price q_j, $j = 1, 2, \ldots, n$.[6] We assume zero marginal costs for all components.[7] We also assume full compatibility among components. Thus, brands of each component may be combined to form all $m \times n$ composite products, such as A_iB_j available at a

[5] Allen (1938) p. 361 also noted that for complementary goods the merger of two independent monopolists into a single one can reduce prices. We are grateful to Larry White for this observation.

[6] Brands of the same component are substitutes among themselves, while brands of different components are complements.

[7] Of course, the results are identical for positive constant marginal costs with "prices" reinterpreted as differences between prices and marginal costs.

total price $s_{ij} = p_i + q_j$. The various composite goods are substitutes for one another.[8]

Consider the simple case where there are two brands of component A and two brands of component B. The four composite goods A_1B_1, A_1B_2, A_2B_1 and A_2B_2 are available at prices s_{11}, s_{12}, s_{21} and s_{22}. For example, the demand function for composite good A_1B_1 depends on the prices of the four composite goods.

$$D^{11} = D^{11}(s_{11}, s_{12}, s_{21}, s_{22}).$$

Because the various composite goods are substitutes for one another, demand for A_1B_1 is decreasing in its own price, s_{11}, and increasing in the prices of the three substitute composite goods, s_{12}, s_{21}, and s_{22}. Denoting by D_k^{ij} the derivative of the demand for product A_iB_j with respect to the kth argument, $D_1^{11} < 0$ and $D_k^{11} > 0$, $k \neq 1$.

We follow the notational convention of reserving the first argument of the demand functions for the own price, the second for the price of the composite good that differs in the B component, the third for the price of the composite good that differs in the A component, and the fourth for the composite good that differs in both components. Because the arguments are arranged in this way, the signs of the partial derivatives for each argument are identical for each composite good.[9]

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[8] This setting is similar to Matutes and Regibeau (1988) and Economides (1989a,b).

[9] Thus the demand for the other composite goods can be written as follows: A_1B_2: $D^{12} = D^{12}(s_{12}, s_{11}, s_{22}, s_{21})$; A_2B_1: $D^{21} = D^{21}(s_{21}, s_{22}, s_{11}, s_{12})$; A_2B_2: $D^{22} = D^{22}(s_{22}, s_{21}, s_{12}, s_{11})$.

We assume all components are compatible, that is, that components can be combined to form all $m \times n$ composite goods.[10] We derive the demand functions for the components from these composite good demand functions. Since component A_1 is sold as a part of composite goods A_1B_1 and A_1B_2, the demand for component A_1 is given by[11]

$$D^{A_1} = D^{11} + D^{12}.$$

We further assume that the demand system is symmetric. In this case, the demand system can be represented by a single demand function, D(.). This implies, in particular, that when the prices for all four composite goods are equal, the demand for each of them is the same, or

<u>Assumption A1:</u> The demand functions for all composite goods are identical, $D^{11}(s) = D^{12}(s) = D^{21}(s) = D^{22}(s)$, where $s = (u, v, w, x)$ is the vector of the

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[10] As a general matter, compatibility depends on technical feasibility, along with the technological and contractual decisions of component producers. In the related literature on compatibility it has been established that if the demand for hybrid composite goods (e.g. A_1B_2) are as large as the demand for single-producer goods, then independent vertically integrated firms (parallel vertical integration, in our jargon), will choose full compatibility of their components [Carmen Matutes and Pierre Regibeau (1988), Nicholas Economides (1989a), (1989b), (1991)]. We assume that the demand for hybrids is the same size as the demand for single-producer composites, and, therefore, there are strong incentives for full compatibility. Under other organizational structures that we will consider in this paper, such as independent ownership and joint ownership, provided that the number of brands is exogenous, firms have even stronger incentives to avoid incompatibilities.

[11] Similarly, the demands for components A_2, B_1, and B_2 respectively are given by

$$D^{A_2} = (D^{21} + D^{22});\ D^{B_1} = (D^{11} + D^{21});\ D^{B_2} = (D^{12} + D^{22}).$$

prices of the four composite substitute goods, ordered according to the notational convention set out previously.

We assume that composite goods are gross substitutes. Therefore, an equal increase in the prices of all composite goods reduces the demand of each composite good, or

<u>Assumption A2</u>: An equal increase in the prices of all components (or equivalently in the prices of all composite goods) decreases the demand of every composite good, that is

$$\sum_{k=1}^{4} D_k^{ij} < 0.$$

It follows immediately from Assumption A2 that an equal increase in the prices of all components implies a decrease in the demand for each component.[12]

To illustrate our results for different market structures, we will analyze the case of linear demand, or

$$D^{11}(s_{11}, s_{12}, s_{21}, s_{22}) = a - b \cdot s_{11} + c \cdot s_{12} + d \cdot s_{21} + e \cdot s_{22},$$

where $a, b, c, d, e > 0$ and $b > c + d + e$.

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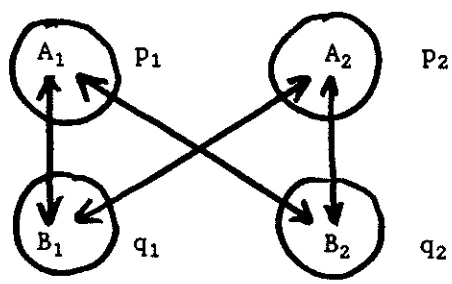
[12] For example, for component A_1 the effect of such a price change is $\partial D^{A_1}/\partial p_1 + \partial D^{A_1}/\partial p_2 + \partial D^{A_1}/\partial q_1 + \partial D^{A_1}/\partial q_2 = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} < 0$.

IV. Equilibrium Pricing

In this section, we analyze the two basic market structures considered by Cournot, independent ownership and joint ownership. In this and in subsequent sections, the firms are assumed not to price discriminate in favor of customers who purchase both components (i.e., who purchase the firm's own composite product.

A. Independent Ownership (I)

Suppose that all component brands A_i and B_j are independently owned, as illustrated in Figure 1.[13]



Independent Ownership

Figure 1

The profit functions of the four firms are given by,

$$\Pi_{A_1} = p_1 D^{A_1} = p_1(D^{11} + D^{12}); \qquad \Pi_{A_2} = p_2 D^{A_2} = p_2(D^{21} + D^{22}),$$
$$\Pi_{B_1} = q_1 D^{B_1} = q_1(D^{11} + D^{21}); \qquad \Pi_{B_2} = q_2 D^{B_2} = q_2(D^{12} + D^{22}).$$

[13] We denote ownership patterns by circling commonly owned components.

For example, profit maximization by firm A_1 is characterized by[14]

(9) $\partial\Pi_{A_1}(p_1, p_2, q_1, q_2)/\partial p_1 = D^{11} + D^{12} + p_1(D_1^{11} + D_2^{11} + D_1^{12} + D_2^{12}) = 0.$

The solution of the four first order conditions like (9) for the four components defines the equilibrium prices $(p_1^I, p_2^I, q_1^I, q_2^I)$ under independent ownership. We solve for the symmetric equilibrium, that is, $q_1 = q_2 = q$ and $p_1 = p_2 = p$ as follows. We first define a two dimensional representation of four dimensional best-response functions. Assuming identical price q for both brands of component B, we define $p_1 = p_2 = p^*(q)$ to be the solution of[15]

(9a) $$\partial\Pi_{A_1}(p^*(q), p^*(q), q, q)/\partial p_1 = 0.$$

Profit maximization by firm B_1 is characterized by

(10) $$\partial\Pi_{B_1}(p_1, p_2, q_1, q_2)/\partial q_1 = 0.$$

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[14] Recall that D_k^{ij} denotes the derivative of D^{ij} (the demand for composite good A_iB_j) with respect to its kth argument. Also note that $D_1^{ij} < 0$ because the first argument of each demand function is the own price, and each demand function is downward sloping. Further note that $D_k^{ij} > 0$ for $k \neq 1$, since all the other arguments of the demand function are the prices of the other composite goods and all composite goods are gross substitutes.

[15] By the same reasoning, it is also true that

(9b) $$\partial\Pi_{A_2}(p^*(q), p^*(q), q, q)/\partial p_2 = 0.$$

Note also that $p^*(q)$ is the best response by firm A_1 to $(p_1, q_1, q_2) = (p^*(q), q, q)$. Let $p_1 = R_{A_1}(p_2, q_1, q_2)$ and $p_2 = R_{A_2}(p_1, q_1, q_2)$ be the best response functions of firms A_1 and A_2 respectively. By its definition,

$$p^*(q) = R_{A_1}(p^*(q), q, q) = R_{A_2}(p^*(q), q, q).$$

A similar argument applies to $q^*(p)$.

Assuming identical price p for all brands of component A, we similarly define $q_1 = q_2 = q^*(p)$ to be the solution of[16]

$$\partial\Pi_{B_1}(p,\ p,\ q^*(p),\ q^*(p))/\partial q_1 = 0. \tag{10a}$$

Then the non-cooperative equilibrium $(p^I,\ q^I)$ occurs at the intersection of the lines $p^*(q)$ and $q^*(p)$, and fulfills $p^I = p^*(q^I)$ and $q^I = q^*(p^I)$ as illustrated in Figure 2.[17]

<<< Insert Figure 2 >>>

In Figure 2, the best-response curves $p^*(q)$ and $q^*(p)$ are downward sloping. Totally differentiating the first order conditions, it follows that the slope of $p^*(q)$ is given by,

$$dp^*/dq = -\,(\sum_i \partial^2\Pi_{A_j}/\partial p_j\partial q_i)/(\sum_i \partial^2\Pi_{A_j}/\partial p_j\partial p_i). \tag{11}$$

The function $p^*(q)$ is decreasing in q if component A_j is a strategic substitute with the collection of the A_is and with the collection of the B_is, as assumed below.

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16 By the same reasoning, it is also true that

$$\partial\Pi_{B_2}(p,\ p,\ q^*(p),\ q^*(p))/\partial q_2 = 0. \tag{10b}$$

17 More formally, the equilibrium is the fixed point of the mapping $(p^*(q),\ q^*(p))$ from $[0,\ k] \times [0,\ k]$ into $[0,\ k] \times [0,\ k]$.

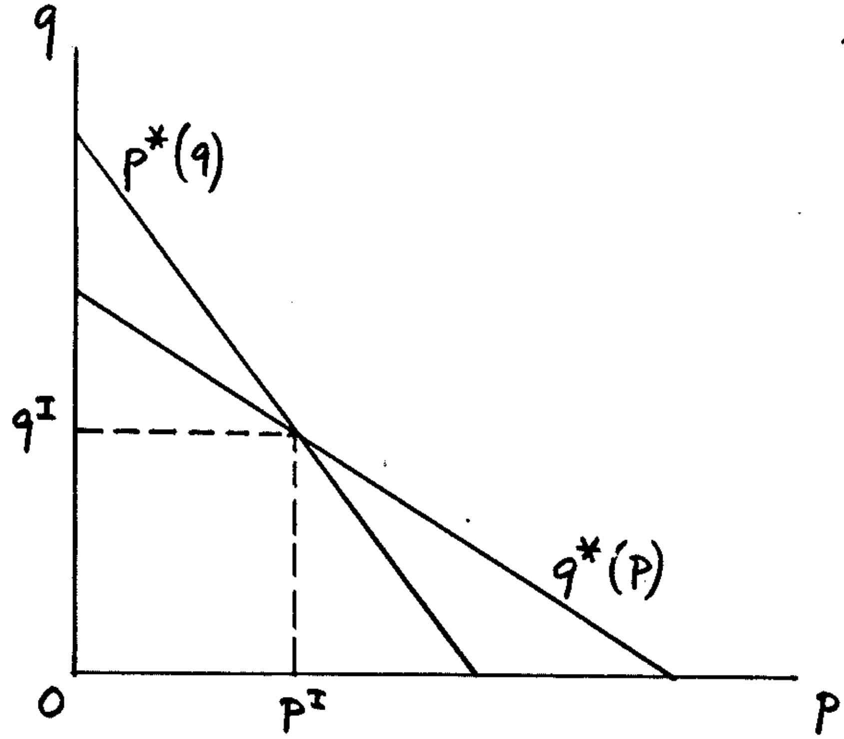

Figure 2.

Assumption A3: Component A_i is a strategic substitute with the collection of components of type A,[18]

$$\partial^2\Pi_{A_i}/\partial p_i^2 + \partial^2\Pi_{A_i}/\partial p_i\partial p_j < 0.$$

Strategic substitutability of A_j with the collection A_is ensures that the denominator in (11) is negative. Strategic substitutability of A_j with the collection of B_is implies that the sum in the numerator of (11) negative. Thus, $dp^*/dq < 0$. A similar argument shows that $q^*(p)$ is downward-sloping.

We now illustrate these results for the case of linear demand functions. For linear demand, Assumption A2 (gross substitutability) implies Assumption A3 (strategic substitutability).[19] Best responses $p^*(q)$ and $q^*(p)$ are derived by substitution in equations (9a) and (10a) as follows:

(9L) $$p^*(q) = [a - 2(b - c - d - e)q]/[2(2b - 2c - d - e)].$$

(10L) $$q^*(p) = [a - 2(b - c - d - e)p]/[2(2b - c - 2d - e)].$$

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[18] Strategic substitutability with the collection of type B components is defined analogously.

[19] Gross substitutability (assumption A2) implies that a component, say good A_1, is a strategic substitute with the collection of goods of the same type A. It is easy to check that $\partial^2\Pi_{A_1}/\partial p_1^2 = 4(c - b) < 0$, $\partial^2\Pi_{A_1}/\partial p_1\partial p_2 = 2(d + e) > 0$, and therefore

$$\partial^2\Pi_{A_1}/\partial p_1^2 + \partial^2\Pi_{A_1}/\partial p_1\partial p_2 = 2(- b + c) + (- b + c + d + e) < 0$$

by gross substitutability. This means that A_1 is a strategic complement of good A_2 but a strategic substitute of goods A_1 and A_2 taken together. It is also easy to check that

$$\partial^2\Pi_{A_1}/\partial p_1\partial q_1 = - b + c + d + e < 0, \quad \partial^2\Pi_{A_1}/\partial p_1\partial q_2 = - b + c + d + e < 0,$$

by gross substitutability. This means that A_1 is a strategic substitute of good B_1 and of good B_2 and with the collection of "B"-type goods.

Solving (9L) and (10L) we have the equilibrium prices under independent ownership,

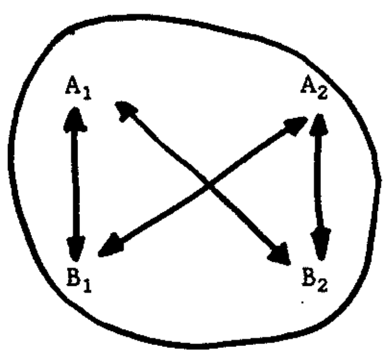
$$p^I = a(b - d)/F; \quad q^I = a(b - c)/F; \qquad (12L)$$
$$s^I = a(2b - c - d)/F,$$

where

$$F = (b - c)(b - d) + (2b - c - d)(b - c - d - e) > 0.$$

B. Joint Ownership (J)

We now consider the market structure of joint ownership, or full integration, of all component producers. Compared to independent ownership, joint ownership creates some downward pressure on prices because of the "vertical" integration of complementary complements (A_i merging with B_j) and some upward pressure on prices because of the "horizontal" integration of substitutes (e.g. A_i merging with A_j). Figure 3 illustrates the joint ownership structure.



Joint Ownership

Figure 3

The jointly owned firm or network maximizes the sum of the profits of the four component producers, or

$$\Pi = \Pi_{A_1} + \Pi_{B_1} + \Pi_{A_2} + \Pi_{B_2},$$

Differentiating, we have

$$(13)\quad d\Pi/dp_1 = D^{11} + D^{12} + p_1(D_1^{11} + D_2^{11} + D_1^{12} + D_2^{12}) + q_1(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) + \\ + p_2(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) + q_2(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) = 0.$$

Similar conditions may be derived for the other prices. Let the resulting symmetric equilibrium be denoted (p^J, q^J).

To compare the joint ownership equilibrium with the independent ownership equilibrium we use the same method of representation of the equilibrium. Assuming $q_1 = q_2 = q$, we define $p_1 = p_2 = p^{**}(q)$ to be the solution of

$$(13a)\qquad \partial\Pi(p^{**}(q),\ p^{**}(q),\ q,\ q)/\partial p_1 = 0.$$

Assuming $p_1 = p_2 = p$, we similarly define $q_1 = q_2 = q^{**}(p)$ to be the solution of

$$(13b)\qquad \partial\Pi(p,\ p,\ q^{**}(p),\ q^{**}(p))/\partial q_1 = 0.$$

Then the equilibrium under joint ownership (p^J, q^J) occurs at the intersection of $p^{**}(q)$ and $q^{**}(p)$, and fulfills $p^J = p^{**}(q^J)$ and $q^J = q^{**}(p^J)$.

Comparing equation (13) with equation (9), note that the first two terms as well as the first parenthesis are identical in both equations, and equation (9) contains no other terms. These represent the effect of changes of p_1 on Π_{A_1}. Of the remaining terms of equation (13), the second and the fourth parenthesis represent the effects of the <u>vertical</u> mergers of firm A_1 with firms B_1 and B_2. They place a <u>negative</u> influence on prices. However, the third parenthesis in equation (13) is positive. It is a summation of the effects on demand of increases in the price of a substitute. It represents the effects of the <u>horizontal</u> merger between firms A_1 and A_2 and places a positive influence on prices. Thus, the overall effect of the full merger is ambiguous, since it depends on the relative magnitudes of the second and fourth parentheses compared to the third one.

When the composite goods are very close substitutes, that is, when the cross elasticities of demand among <u>composite</u> goods outweigh the own elasticity of demand, the horizontal effects of the merger dominate. Thus an increase in the price of good A_1 increases the total sales of A_2, B_1, and B_2. This is simply another way of saying that the sum of the cross elasticities of the demand for <u>component</u> A_1 with respect to components A_2, B_1, and B_2 is positive. It is shown in the appendix that close substitution implies that

$$p^{**}(q) > p^{*}(q). \tag{14a}$$

Similarly, it can be shown that

$$q^{**}(p) > q^{*}(p). \tag{14b}$$

From (14b) and (14c) it is immediate that

$$s^{J} = p^{J} + q^{J} > p^{I} + q^{I} = s^{I}. \tag{14c}$$

Thus, when composite goods are close substitutes, prices rise from the integration of the four independent firms.

Note that the joint ownership equilibrium for a linear demand system is the solution of (13), or

$$p^{J} = q^{J} = a/4(b - c - d - e), \tag{13L}$$
$$s^{J} = a/2(b - c - d - e).$$

Thus, for linear demand, comparing equations (13L) and (9L), $s^J > s^I$ if and only if

$$(b - c - d - e)(b - c) < (b - d)(d + e).\text{[20]} \tag{14d}$$

When $c = d$, the prices of the complementary components are equal, $p^I = q^I$, and (14d) simplifies to the sum of the three parenthesis above being positive, or

$$b < 3c + 2e. \tag{14e}$$

Thus, joint ownership raises prices when the cross-elasticities of demand for composite products (and the corresponding derivatives of the demand, c, d, e) are high, relative to the own elasticity (and the corresponding derivative of the demand, -b). This is because those own and cross-elasticities define the degree of competition among substitute components under independent ownership that is eliminated by joint ownership relative to the size of the "vertical" externality among complementary components that joint ownership internalizes. Joint ownership raises prices when there was intense competition to begin with under independent ownership, that is, when composite goods, and different brands of the same component, are close substitutes for one another.

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[20] For linear demand, the effect of an increase in the price of A_1 on the demand for complement B_1, represented by the second parenthesis of (13) is

$$(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) = -(b - c - d - e) < 0.$$

The effect of an increase in the price of A_1 on the demand for substitute A_2, represented by the third parenthesis of (13) is

$$(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) = 2(c + d) > 0.$$

The effect on the demand for complement B_2, represented by the fourth parenthesis in (13) is

$$(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) = -(b - c - d - e) < 0.$$

These terms are weighted by the prices in (14).

Proposition 1: Prices are higher in joint ownership than with independent ownership if and only if the composite goods are close substitutes.

V. Benchmarks

In this section we analyze two market structures that can serve as benchmarks, optimal regulation and composite goods competition.

A. Optimal Regulation (O)

Consider the first best outcome, where, a regulator imposes marginal cost pricing on all components.[21] We denote the marginal cost price of the composite goods as $s^O = 0$. Thus, we have

$$s^O < \min (s^I, s^J).$$

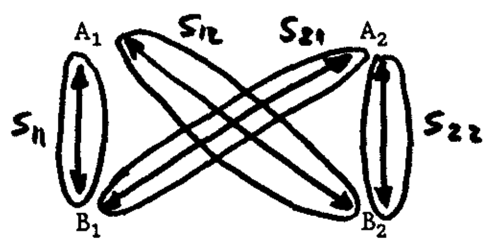
B. Composite Goods Competition (C)

As a second benchmark, consider the composite goods competition market structure in which each of the four composite goods is produced by a different firm $i = 11, 12, 21, 22$. For example, consider the market for vacations, where the vacation composite good package is comprised of two components, airline transportation and resort hotel stay. Suppose there are two airlines and two hotels and marginal costs are zero. In independent ownership, airlines and

[21] This result obviously depends on the assumption of an exogenous number of brands. If the number of brand is determined by free entry in equilibrium, optimal prices exceed marginal cost. Similarly, all the comparisons that follow depend on this assumption of a fixed number of brands. This clearly restricts the policy implications that flow directly from this model. See Spence (1976), Salop (1979, 1991), Economides (1989).

hotels set prices and the consumers purchase components to create their own vacation package. In contrast, composite goods competition would involve competition among four travel agents, with zero production costs, each of whom sells one of the four different vacation packages, as illustrated in Figure 4. Thus, in composite goods competition, there are still four sellers, but the products sold differ.

One might expect prices to be the same under composite goods competition as under independent ownership, since there are four sellers in both structures. In fact, we can show that prices are always lower in composite goods competition. This is because that structure internalizes all the vertical externalities while maintaining horizontal competition.



Composite Goods Competition

Figure 4.

The profit function of firm 11 is given by

$$\Pi^{11} = s_{11}D^{11}(s_{11},\ s_{12},\ s_{21},\ s_{22}).$$

Differentiating with respect to s_{11}, we have the first order condition,

$$D^{11} + s_{11}D^{11}_{1} = 0. \tag{16}$$

For linear demand, the equilibrium price is given by

(16L) $$s^C = a/(2b - c - d).$$

From direct comparison of (12L), (13L) and (16L) in the linear case, it follows that the equilibrium price for composite goods is lower in composite goods competition than the prices of both independent ownership and parallel vertical integration,[22]

$$s^C < \min (s^I, s^J).$$

VI. Partial Integration

There are a variety of market structures in which there is partial integration. We analyze two important ones in this section and the next one.

A. Parallel Vertical Integration, (V)

The case of joint ownership does not reflect Cournot's (1838) result that prices necessarily fall because of integration. Joint ownership involves both vertical and horizontal effects. The parallel vertical integration structure separates these effects. Parallel vertical integration involves the integration of compatible complementary components while maintaining competition among substitute components.

Formally, let $m = n$ and suppose that each A_i and B_i integrate to form firm i, $i = 1, 2, \ldots, m$. Firm-i continues to sell its compatible

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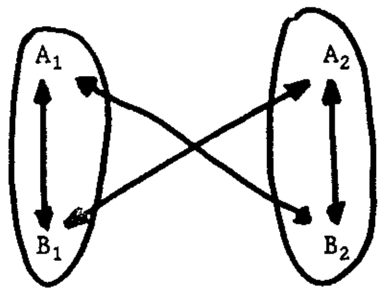
[22] This is also true for general demand functions, but economy of exposition dictates that it should not be proved at this stage. The result follows indirectly from comparisons in Propositions 7, 8, and 9 of prices in composite goods competition with another market structure (one-sided regulation) introduced later.

components separately, however, as well as composite product A_iB_i. Thus, consumers can still purchase components from different firms to produce hybrid composites like A_iB_j. Figure 5 illustrates the ownership structure of parallel vertical integration (V).



Parallel Vertical Integration

Figure 5.

Parallel vertical integration is common in networks. Many firms produce and sell compatible complementary components in addition to a composite product composed of its components. For example, in ATM networks, a consumer can obtain a cash withdrawal from an ATM at its own bank or from an ATM owned by another bank. A PC user can mix spreadsheets and word-processing programs of different companies or choose programs of the same software producer. In airline travel, both on-line and inter-line one-stop trips are often possible.

Formally, the profit function for firm 1 is given by

$$\Pi^1 = \Pi_{A_1} + \Pi_{B_1} = p_1(D^{11} + D^{12}) + q_1(D^{11} + D^{21}).$$

Maximizing with respect to p_1, we have[23]

[23] The analysis with respect to q_1 is similar.

(17) $d\Pi^1/p_1 = D^{11} + D^{12} + p_1(D_1^{11}+D_2^{11}+D_2^{12}+D_1^{12}) + q_1(D_1^{11}+D_2^{11}+D_3^{21}+D_4^{21}) = 0.$

To compare parallel vertical integration with independent ownership, we compare equations (17) with (9). These equations differ only in the last parenthesis, which is negative, from Assumptions A1 and A2. This is the negative vertical effect on price p_1 from the merger of A_1 with B_1. Thus, the best response of the merged entity (A_1+B_1) is shifted to the left compared to the best response of A_1. Similarly, the best response of the merged entity (A_2+B_2) is shifted to the left compared to the best response of A_2. Therefore, the parallel merger equilibrium will result in lower prices for all composite goods, or

$$s^V < s^I.$$

Formally, we apply the same technique used earlier. Assuming symmetry in the market for the "B"-type components, i.e., $q_1 = q_2 = q$, we define $p_1 = p_2 = p^{***}(q)$ to be the solution of

(17a) $$\partial\Pi^1(p^{***}(q), p^{***}(q), q, q)/\partial p_1 = 0.$$

Assuming now symmetry in the market for "A"-type components, $p_1 = p_2 = p$, we similarly define $q_1 = q_2 = q^{***}(p)$ to be the solution of the first order condition for firm 2,

(18a) $$\partial\Pi^2(p, p, q^{***}(p), q^{***}(p))/\partial q_1 = 0.$$

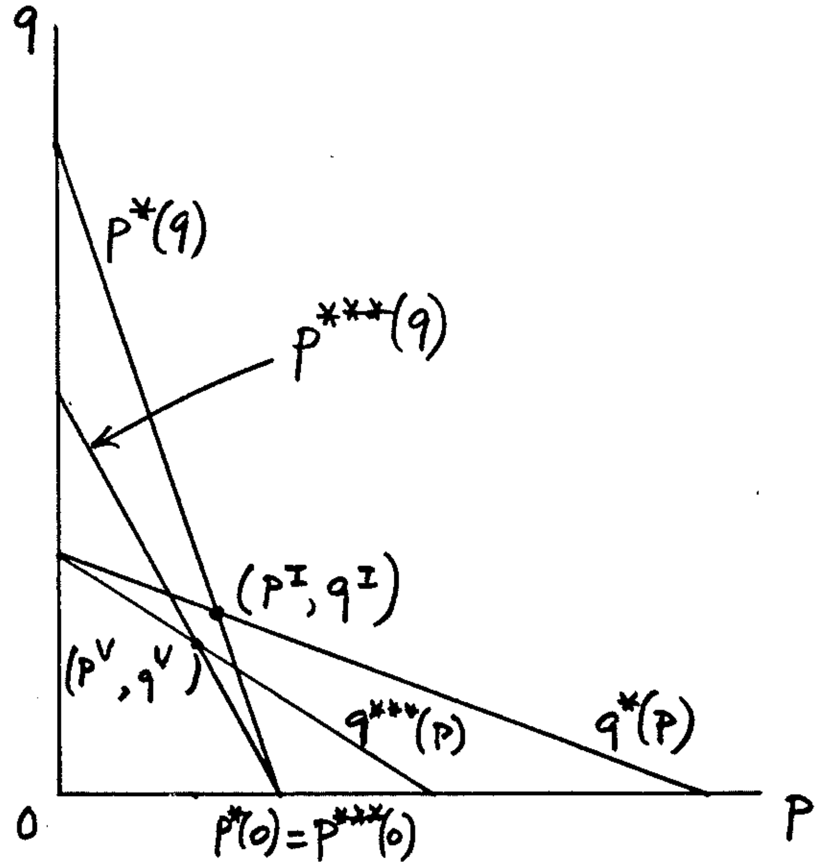


Figure 6.

Then the non-cooperative equilibrium under parallel vertical integration (p^V, q^V) occurs at the intersection of $p^{***}(q)$ and $q^{***}(p)$, as illustrated in Figure 6.[24]

<<<<INSERT FIGURE 6>>>>

It is shown in the Appendix that for $q = 0$, $p^{***}(0) = p^*(0)$, but for $q > 0$,

(19a) $$p^{***}(q) < p^*(q),$$

and similarly, for $p > 0$, that

(19b) $$q^{***}(p) < q^*(p).$$

From (19a) and (19b) it follows that

(19c) $$s^V = p^V + q^V < p^I + q^I = s^I.$$

<u>Proposition 2:</u> **Prices are always lower in parallel vertical integration than in independent ownership.**

The price comparison between parallel vertical integration and joint ownership is ambiguous. This is because the parallel vertical integration does not eliminate all the vertical externalities. In particular, parallel vertical integration leaves uninternalized externalities between the prices of components A_1 and B_2, and components A_2 and B_1. Thus full integration of the two pair-wise integrated firms into joint ownership may lower prices.

This integration has both vertical and horizontal effects. Integration of (A_1+B_1) with A_2 has a positive influence on the price of A_1, while integration

[24] Formally, the equilibrium is the fixed point of the mapping $(p^{***}(q), q^{***}(p))$ from $[0, k]\times[0, k]$ to $[0, k]\times[0, k]$.

of (A_1+B_1) with B_2 has a negative influence on the price of A_1.[25] Therefore, the comparison is ambiguous, depending on the relative magnitudes of the own and cross elasticities.

If the composite goods are close substitutes, the cross elasticities term will dominate the own elasticity of demand. An increase in the price of component A_1 increases total sales of A_2 and B_2. This means that the sum of the cross elasticities of demand of component A_1 with respect to components A_2 and B_2 is positive. Then,

(20a) $$p^{**}(q) > p^{***}(q),$$

(20b) $$q^{**}(p) > q^{***}(p).$$

It then follows immediate that prices will rise as a result of the merger of the pair-wise integrated firms,

(20c) $$s^J > s^V.$$

In contrast, if the four composite goods are not close substitutes, the own price elasticity will dominate and prices will fall as a result of the horizontal merger of the pair-wise integrated firms, or $s^J < s^V$.

[25] Equations (13) and (17) differ in two ways — the third parenthesis in (13), which is positive; and the fourth parenthesis in (13), which is negative. Thus the total effect of p_1 on the profits of firm 2 (that produces A_2 and B_2) is given by the sum of the parentheses below,

$$\partial\Pi_{A_2}/\partial p_1 + \partial\Pi_{B_2}/\partial p_1 = p_2(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) + q_2(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) > 0.$$

Proposition 3: Prices are higher in joint ownership than in parallel vertical integration if and only if the composite goods are close substitutes.[26,27]

For linear demand, equations (17a) and (17b) are solved as follows:

(21a) $$p^V = 2a(b + c - d + e)/F',$$

(21b) $$q^V = 2a(b - c + d + e)/F',$$

(21c) $$s^V = 4a(b + e)/F',$$

---

[26] The proof of Proposition 3 is very similar to the proof of Proposition 1 and is not repeated.

[27] Note that the possibility of lower prices in joint ownership than in parallel vertical integration is dependent on the existence of hybrid composite goods. Without the hybrids, a merger of two firms producing differentiated substitutes typically will increase prices. To show this, assume that only goods A_1B_1 and A_2B_2 exist, and are sold at prices s_{11} and s_{22} respectively. Demand functions are $D^{11}(s_{11}, s_{22})$ and $D^{22}(s_{11}, s_{22})$ respectively. Profit functions for firms 11 and 22 are $\Pi^{11} = s_{11}D^{11}$ and $\Pi^{22} = s_{22}D^{22}$. The profit function for the fully integrated firm is

$$\Pi = \Pi^{11} + \Pi^{22} = s_{11}D^{11} + s_{22}D^{22}.$$

Profit maximization under independence implies

$$D^{11} + s_{11}D^{11}_1 = 0,$$

whereas under joint ownership it implies

$$D^{11} + s_{11}D^{11}_1 + s_{22}D^{22}_2 = 0.$$

The only difference between these equations is the last term in the second equation, which is positive. The result follows immediately.

where $F' = 4(2b - 2c - d - e)(2b - 2d - c - e) - 9(b - c - d - e)^2 > 0$. If $c = d$, then $p^V = q^V$, and price will rise as a result of the merger of the pair-wise integrated firms if and only if

(20d) $$b < 4c + 3e$$

Summarizing Propositions 1 and 3, when composite goods are not close substitutes, joint ownership results in lower prices than parallel vertical integration. When goods are moderately close substitutes, full integration results in a higher price than parallel vertical integration but a lower price than independent ownership. For very close substitutes, full integration results in a price even higher than independent ownership.[28]

The comparison between composite goods competition and parallel vertical integration is straightforward. Comparing (16) and (17) and noting that D^{11} and D^{12} are of the same size by Assumption A1, it is clear that the price of the composite good $s_{11} = s^C$ is lower than $p_1 + q_1 = s^V$ under parallel vertical integration, $s^C < s^V$.

<u>Proposition 4</u>: **Prices are higher in parallel vertical integration than in composite goods competition.**

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[28] From (14e) and (20d) we have the following price comparisons for linear demand and $c = d$:

$$s^J > s^I \iff b < 3c + 2e,$$

$$s^J > s^V \iff b < 4c + 3e.$$

Thus when the composite goods not close substitutes, $4c + 3e < b$, full integration results in lower prices for composite goods than both independent ownership and parallel vertical integration. When the composite goods are moderately close substitutes, i.e., when $3c + 2e < b < 4c + 3e$, prices increase as a result of a merger of two pair-wise integrated firms but not by the merger of four independent firms. When the composite goods are very close substitutes, $b < 3c + 2e$, both mergers increase prices.

Intuitively, composite goods competition internalizes all the vertical externalities but none of the horizontal externalities. Thus, the maximum degree of competition results.[29]

VII. One-Sided Regulation (R)

In some networks, network rules require the producers of components to sell their component to the producers of the other complementary component at a "wholesale" price regulated by the network. These latter producers then package the components into composite goods and sell them at retail to consumers. We denote this structure as <u>one-sided regulation</u>. In many ATM networks, for example, a network-determined "interchange" fee is paid to the ATM owner by the card-issuing bank for each transaction. The ATM owner may not supplement (or reduce) this fee by setting a consumer surcharge or rebate. Thus, in effect, the ATM owners sell their component at wholesale to the card issuers. These card issuers then charge consumers competitively determined "foreign" fees for each cash withdrawal. Credit card networks have similar interchange fee systems.[30]

In our simple model, the controlling component producers obviously have an incentive to set the price of the "regulated" component at marginal cost.[31] In

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[29] The proof of Proposition 4 follows closely the proofs of Propositions 1 and 3 and therefore is omitted.

[30] Some real estate multiple listing services apparently used to mandate that the selling agent receive a fixed percentage share of the listing agent's commission. This percentage sharing structure has somewhat different results.

[31] Our assumption of an exogenous number of brands limits the application of this model to network policy. If the number of component producers is instead determined by a process of free entry then the number and variety of brands of the regulated component will depend on the regulated component price. As a result, consumers and the controlling component producers both would benefit by a regulated price above marginal cost. Unfortunately, there is no reason to expect that consumers and the controlling side of the market prefer the same price–variety combination. Instead, the controlling component producers act as

this way, the regulated component producers obtain no surplus and the vertical externality is internalized. Of course, the unregulated components (and the composite goods) continue to sell above marginal cost. Thus, prices would fall further if both component prices were regulated. And in this model, there is no reasón why both prices could not be regulated.[32] To explore this issue further, we derive the optimal one-sided regulation. That is, assuming that it is feasible to regulate only one component, we show which component it is better to regulate.

A. Equilibrium with One-Sided Regulation of Component A

Formally, one-sided regulation of component A sets the prices of A-brands at marginal cost, or $p_i = 0$, $i = 1, 2, \ldots, m$. At the same time, the prices of brands of component B are set independently and non-cooperatively.

At the resulting equilibrium, each component B producer receives a price of $q^*(0)$ since $p = 0$, that also is the total price $s^{RA} = q^*(0)$. We now compare this equilibrium independent ownership (p^I, q^I). Recall that the price of a composite good at the independent ownership equilibrium can be written as

$$p^I + q^I = p^I + q^*(p^I)$$

where $q^*(p)$ was defined in equation (9a) and illustrated in Figure 2. Figure 2 is redrawn as Figure 7 with two new curves, for the "total" (composite good)

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a buyer cartel in a way that does not maximize consumer welfare. See Salop (1991) for further analysis of this structure in a somewhat different model.

[32] For example, this is a peculiarity of ATM network self-regulation. Only the ATM owners' fees are regulated, and the card issuers' foreign fees are not, even though the externalities apply to both components.

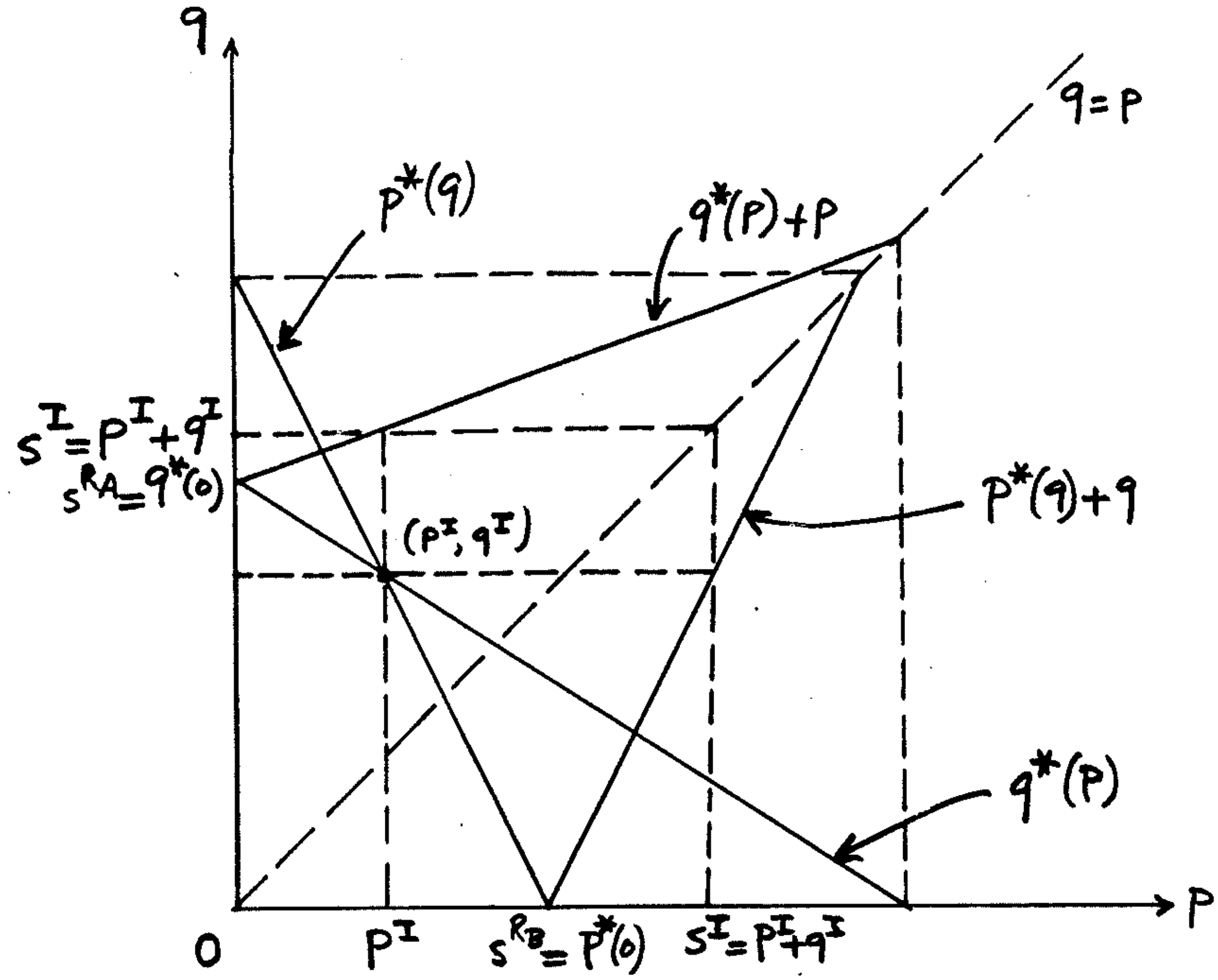


Figure 7.

price, $q^*(p) + p$, and $p^*(q) + q$. In Figure 7, by drawing the vertical line that passes through the equilibrium point (p^I, q^I) we define $p^I + q^I = p^I + q^*(p^I)$ on the "total" price line $q^*(p) + p$. The composite good price in the regime of one-sided regulation also lies on the "total" price line $q^*(p) + p$, at $q^*(0) + 0$. Now the comparison between one-sided regulation and independent ownership is easy, since it depends only on the slope of the "total" price line $q^*(p) + p$. This is guaranteed by Lemma 1, which is proved in the appendix.

<u>Lemma 1</u>: **The "total" price lines $q^*(p) + p$ and $p^*(q) + q$ have positive slope.**

Therefore one-sided regulation of component A results in a decrease in the price of composite goods.

<u>Proposition 5</u>: **Composite goods prices are lower in the regime of one-sided regulation than under independent ownership,**

(22) $$\max\,(s^{R_A},\ s^{R_B}) < s^I.$$

This result that the price of a composite good s falls is not surprising. By setting the regulated price at marginal cost while maintaining duopoly competition among producers of the other component, the total amount of market power is reduced, relative to independent ownership.

We can also show that the producers of the unregulated component gain from the regulation of the other side of the market. This follows directly from the fact that the best response function of the unregulated component declines with increases in the price of the regulated component, that is $q^*(p)$ and $p^*(q)$

are decreasing functions of their arguments. This implies that producers of one component have the incentive to engage in collective action to force down the price of the other component, even if they continue to compete among themselves.

B. Optimal One-Sided Regulation

One-sided regulation results in a lower price than independent ownership no matter which component is regulated. This raises the issue of which side should be regulated if only one component can be regulated.

Intuition suggests that regulation of the less competitive market will result in the lowest price for the composite good. The less competitive market is characterized by a smaller number of components and/or smaller cross elasticities of demand between the components. When the less competitive price is set equal to marginal cost, a larger price decrease is achieved. Then, continued competition among the brands of the unregulated component will maintain a low price in that market.

We have analyzed optimal one-sided regulation for the case of linear demand but for any number of brands $n \geq 2$, $m \geq 2$. The proofs are in the Appendix. For a linear demand system of m brands of component A and n brands of component B, where $\partial D^{ij}/\partial s_{kj} = d$, $k \neq i$, $\partial D^{ij}/\partial s_{ik} = c$, $k \neq j$, the unregulated prices of component A, when component B is regulated, is given by

$$p^*(0) = a/(2b - 2C - D - E) \tag{23L}$$

or, if A is regulated and B is not,

$$q^*(0) = a/(2b - C - 2D - E), \tag{24L}$$

where $C = (n - 1)c$; $D = (m - 1)d$; and $E = (m - 1)(n - 1)e$.

Comparing equations (23L) and (24L), it is easy to show that

$$\text{(25L)} \qquad s^{R_B} - p^*(0) > s^{R_A} - q^*(0) \iff C > D,$$

where C and D are defined above.

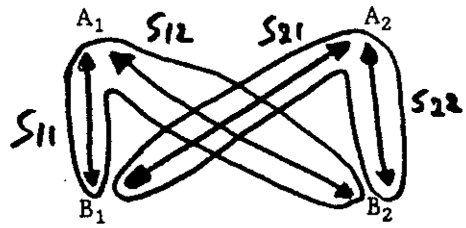
Thus, "competitiveness" in the market for component A_1 is measured by the index $D - (m - 1)d$, comprised of the number $(m-1)$ of firms selling substitutes to A_1, multiplied with the degree of substitution (d) between composite goods that differ only in component A, such as A_1B_1 and A_2B_1. Similarly, competition in the component B market is characterized by the index $(n - 1)c$. Thus, in equation (26L), if the index for component A is lower, it is the less competitive side of the market, and its price p should be regulated, as stated below.

<u>Proposition 6</u>: Lower prices of composite goods are achieved when the less competitive of the two markets is regulated, where competitiveness is measured by the index $(n-1)c$, where there are n brands of a component and c is the cross-elasticity of demand of composite goods differing by that component.

C. <u>Comparison to Composite Goods Competition</u>

The equilibrium prices arising in the one-sided regulation structure also obtain in a market structure involving partial integration of composite goods competitors, as follows. Recall that the four producers in the composite goods competition structure sell the four goods A_1B_1, A_1B_2, A_2B_2, and A_2B_1 respectively. Suppose that the two firms that use component A_1 (i.e., A_1B_1 and A_1B_2) integrate into a firm that we denote as IA_1. Similarly, suppose that the two firms that use component A_2 (i.e., A_2B_2 and A_2B_1) integrate into a firm that we denote as IA_2. Suppose that these two integrated firms IA_1 and IA_2

continue to compete non-cooperatively. We denote this market structure as partially integrated composite goods competition, as illustrated in Figure 8.



Partially Integrated Composite Goods Competition

Figure 8.

Partially integrated composite goods competition yields the same equilibrium prices as does one-sided regulation of component B.[33] This can be seen as follows. The profit function of firm IA_1 is

$$\Pi^1 = s_{11}D^{11} + s_{12}D^{12}.$$

Differentiating with respect to s_{11} and s_{12} we have the first order conditions

$$(26) \qquad \partial\Pi^1/\partial s_{11} = D^{11} + s_{11}D_1^{11} + s_{12}D_2^{12} = 0.$$

$$(27) \qquad \partial\Pi^1/\partial s_{12} = D^{11} + s_{12}D_1^{12} + s_{11}D_2^{11} = 0.$$

Recall equation (9), the first order condition for independent ownership, is

$$(9) \qquad \partial\Pi_{A_1}(p_1, p_2, q_1, q_2)/\partial p_1 = D^{11} + D^{12} + p_1(D_1^{11} + D_2^{11} + D_1^{12} + D_2^{12}) = 0.$$

[33] Similarly, a merger of composite goods produces A_1B_1 and A_2B_1 into firm IB_1 and produces A_1B_2 and A_2B_2 into firm IB_2 imply the same equilibrium prices as one-sided regulation of component A.

Setting $p_1 = s_{11} = s_{12} = s^{R_B}$, $p_2 = s_{21} = s_{22} = s^{R_B}$, and $q_1 = q_2 = 0$, it follows that the first order conditions of partially integrated composite goods competition as shown in equations (26) and (27) and one-sided regulation of component B coincide. That is,

$$\partial\Pi^1/\partial s_{11} = \partial\Pi_{A_1}(s^{R_B}, s^{R_B}, 0, 0) = 0.$$

Therefore the two structures result in the same composite goods prices.

Of course, the partially integrated composite goods competition structure results from horizontal mergers of composite goods producers. Since these are mergers of substitutes, it follows that partially integrated composite goods competition (and, equivalently, one-sided regulation) yield higher composite goods prices than does pure composite goods competition (C).

<u>Proposition 7</u>: Composite goods competition results in lower prices than one-sided regulation

$$s^C < \min(s^{R_A}, s^{R_B}).$$

C. <u>Comparison to Partial and Full Integration Structures</u>

We can also show that one-sided regulation results in composite goods prices that are lower than in parallel vertical integration, and joint ownership[34]

$$\max(s^{R_A}, s^{R_B}) < s^V.$$

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[34] These Propositions are proved in the Appendix.

Proposition 8: Composite goods prices are lower in the regime of one-sided regulation than under parallel vertical integration.

We can also similarly show that for linear demand prices are lower in one-sided regulation than in joint ownership.

Proposition 9: Composite good prices are lower in one-sided regulation than in joint ownership,

$$\max(s^{R_A}, s^{R_B}) < s^J.$$

To summarize, we have shown that the prices of the composite goods under one-sided regulation are less than the prices in parallel vertical integration, independent ownership and joint ownership, but above the prices in composite goods competition and optimal regulation, i.e.,

$$s^o < s^C < s^R < s^V < s^I.$$

and

$$s^R < s^J.$$

VI. Conclusion

We have analyzed competition and integration among complementary products in networks by examining a variety of alternative market structures. We have shown that different market structures internalize "vertical" and "horizontal" externalities in various ways. Aside from regulation of both component prices at marginal cost, the optimal market structure is composite goods competition, where a different firm produces each differentiated composite good. In that way, externalities among complementary components are fully internalized while maintaining competition among substitute composite goods.

We have also shown that parallel pair-wise vertical integration generalizes Cournot's (1838) result that mergers among complements reduce prices. However, we noted that a merger of all firms in the industry may or may not increase prices, depending on the relative sizes of the own and cross elasticities of demand. We also showed that one-sided regulation, which in effect limits monopoly power to one side of the market only, results in lower prices for composite goods than independent ownership, and we have characterized which component it is better to regulate.

The analysis in this paper can be extended and generalized in a variety of ways. First, many of our results pertain to the case of only two brands of each component. We believe that they can be extended to the case where each component has many brands. Some of our results assumed linear demand. Thus, our analysis can be extended to a broader range of cases. Second, we assumed that the number of brands is exogenous. This limits the generality of the model as well as its applicability to some network policy issues. When the number of brands is endogenous, the analysis is complicated by issues of product variety. As a result, the welfare implications of price comparisons are less clear and optimal network self-regulation is far more complicated. Third, we assumed that integrated firms do not price discriminate in favor of customers who purchase both components (i.e., who purchase the firm's own composite product.) By relaxing these assumptions, a richer set of strategies that may be important in certain product networks, can be analyzed.

Appendix

Proof of Proposition 1:

To show that the value of the second parenthesis of equation (13) is negative, first we use the symmetry assumption A1. At a symmetric equilibrium all composite goods are available at the same price $s = p + q$. The vector of the prices of the four composite goods is $s = (s, s, s, s)$. Using assumption A1, by the symmetry of the demand functions, $D_3^{21}(s) = D_3^{11}(s)$, $D_4^{21}(s) = D_4^{11}(s)$. The second parenthesis of equation (13) now becomes

$$(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) = \sum_{k=1}^{4} D_k^{11} < 0.$$

Using assumption A2, we note that this sum is negative, since it is equivalent to the effect on demand of good A_1B_1 of an equal price increase of the prices of all goods. Thus, the second parenthesis in equation (13) is negative. Using a similar argument, the fourth parenthesis in equation (13) is also negative and the third parenthesis is positive. The overall effect depends on the balance of the horizontal and vertical effects.

When the composite goods are close substitutes

$$\partial\Pi_{B_1}/\partial p_1 + \partial\Pi_{A_2}/\partial p_1 + \partial\Pi_{B_2}/\partial p_1 = q_1(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) +$$
$$+ p_2(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) + q_2(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) > 0,$$

and therefore,

$$\partial\Pi_{A_1}(p^{**}(q), p^{**}(q), q, q)/\partial p_1 = - (\partial\Pi_{B_1}/\partial p_1 + \partial\Pi_{A_2}/\partial p_1 + \partial\Pi_{B_2}/\partial p_1) < 0.$$

The parentheses above represent the effect of changes in p_1 on sales of B_1, A_2, and B_2 respectively. Thus the horizontal effects dominate.

By Assumption A3, the marginal profits of firm A_1 are decreasing with equal increases in prices p_1 and p_2. Since $p^*(q)$ was defined so that $\partial\Pi_{A_1}/\partial p_1 = 0$ in equation (9a), it follows that

$$p^{**}(q) > p^*(q).$$

Similarly,

$$q^{**}(p) > q^*(p),$$

and therefore

$$s^J = p^J + q^J > p^I + q^I = s^I$$

When the own elasticity of demand strongly outweighs the cross elasticities,

$$\partial\Pi_{B_1}/\partial p_1 + \partial\Pi_{A_2}/\partial p_1 + \partial\Pi_{B_2}/\partial p_1 =$$
$$q_1(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) + p_2(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) + q_2(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) < 0,$$

then the vertical effects will dominate. Then

$$\partial\Pi_{A_1}(p^{**}(q),\ p^{**}(q),\ q,\ q)/\partial p_1 = -(\partial\Pi_{B_1}/\partial p_1 + \partial\Pi_{A_2}/\partial p_1 + \partial\Pi_{B_2}/\partial p_1) > 0.$$

It follows that

$$p^{**}(q) < p^*(q),$$
$$q^{**}(p) < q^*(p).$$

and the overall effect of the full merger is going to be negative on prices,

$$s^J = p^J + q^J < p^I + q^I = s^I.$$

Proof of Proposition 2:

For $q > 0$, in comparison with equation (9) of independent ownership, equation (17) has an extra term (the last parenthesis) that is negative since it represents the effects of changes in the price of A_1 on the profits generated by the sales of the complementary good B_1,

$$\partial\Pi_{A_1}(p^{***}(q), p^{***}(q), q, q)/\partial p_1 = -\partial\Pi_{B_1}(p^{***}(q), p^{***}(q), q, q)/\partial p_1 > 0.$$

By Assumption A3, the marginal profits of firm 1 are decreasing with equal increases in prices p_1 and p_2. It follows then from equation (16) that for $q > 0$,

$$p^{***}(q) < p^*(q).$$

Similarly, it can be shown that

$$q^{***}(p) < q^*(p).$$

From the two equations above it is immediate that

$$s^V = p^V + q^V < p^I + q^I = s^I,$$

and Proposition 2 follows.

Proof of Lemma 1:

Through total differentiation of the first order condition of firm A_1, $\partial\Pi_{A_1}(p_1, p_2, q_1, q_2)/\partial p_1 = 0$, equation (9), the slope of $q + p^*(q)$ is found to be

$$\text{(A1.1)} \qquad d(q+p^*(q))/dq = \left(\sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial p_j\partial p_1 - \sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial q_j\partial p_1\right)/\left(\sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial p_1\partial p_j\right).$$

Direct computation reveals that

$$\sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial p_j \partial p_1 = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} + \sum_{j=1}^{2} (D_j^{11} + D_j^{12}) + p_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{1k}^{12}+D_{2k}^{12})$$

and

$$\sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial q_j \partial p_1 = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} + p_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{1k}^{12}+D_{2k}^{12}).$$

Therefore the numerator in (A1.1),

$$\sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial p_j \partial p_1 - \sum_{j=1}^{2} \partial^2\Pi_{A_1}/\partial q_j \partial p_1 = \sum_{j=1}^{2} (D_j^{11} + D_j^{12}) < 0,$$

is negative,

since $\sum_{k=1}^{4} D_k^{ij} < 0$, $D_1^{ij} < 0$, and $D_k^{ij} > 0$ for $k \neq 1$.

By Assumption A3, the numerator is positive. Therefore, $d(q+p^*(q))/dq > 0$. QED.

Proof of Proposition 6:

There are $i = 1, \ldots, m$ components A_i of type "A" and $j = 1, \ldots, n$ components B_j of type "B". Good A_iB_j is available at price s_{ij}. For the case of independent ownership, let p_i be the price of A_i and q_j be the price of B_j. Then $s_{ij} = p_i + q_j$. Table 1 shows the available products and their prices.

Table 1

$q \setminus p$	A_1	A_2	.	.	.	A_m
B_1	s_{11}	s_{21}	.	.	.	s_{m1}
B_2	s_{12}	s_{22}	.	.	.	s_{m2}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
B_n	s_{1n}	s_{2n}	.	. . .	.	s_{mn}

Assuming a linear demand system, let $\partial D^{11}/\partial s_{11} = -b < 0$, $\partial D^{11}/\partial s_{1j} = c > 0$, $\partial D^{11}/\partial s_{i1} = d > 0$, and $\partial D^{11}/\partial s_{ij} = e$ for $i \neq 1$, $j \neq 1$. Demand for goods A_1B_1 to A_1B_n is then written as

$$D^{11} = a - b(p_1 + q_1) - c\cdot\sum_{j=2}^{n} (p_1 + q_j) + d\cdot\sum_{i=2}^{m} (p_i + q_1) + e\cdot\sum_{\substack{i\neq 1\\ j\neq 1}} (p_i + q_j)$$

$$\vdots$$

$$D^{1n} = a - b(p_1 + q_n) - c\cdot\sum_{j\neq n} (p_1 + q_j) + d\cdot\sum_{i=2}^{m} (p_i + q_n) + e\cdot\sum_{\substack{i\neq 1\\ j\neq n}} (p_i + q_j)$$

The profits of firm A_1 are

$$\Pi_{A_1} = p_1\cdot \sum_{k=1}^{n} D^{1k},$$

where

$$\sum_{k=1}^{n} D^{1k} = na - nbp_1 - b\cdot\sum_{j=1}^{n} q_j + cn(n-1)p_1 + c(n-1)\cdot\sum_{j=1}^{n} q_j + d(m-1)\cdot\sum_{j=1}^{n} q_j + nd\cdot\sum_{i=2}^{m} p_i +$$

$$+ e(m-1)\cdot\sum_{i\neq 1, j\neq 1} \sum^{n} (p_i+q_j)\,.$$

At the symmetric equilibrium $p_i = p$, $q_j = q$, the optimization condition $d\Pi_{A_1}/dp_1 = 0$, simplifies to

(A2.1) $$a + p(-2b + 2C + D + E) + q(-b + C + D + E) = 0,$$

where $C = (n - 1)c$, $D = (m - 1)d$, and $E = (m - 1)(n - 1)e$. This can be solved as $p = p^*(q)$. Similarly, $d\Pi_{B_1}/dq_1 = 0$ simplifies to

(A2.2) $$a + p(-b + C + D + E) + q(-2b + 2C + D + E) = 0.$$

This can be solved as $q = q^*(p)$. The common solution of (A2.1) and (A2.2) defines the equilibrium under independent ownership,

$$p^I = a(b - C)/Det, \; q^I = a/(b - D)/Det,$$

where

$$Det = (b - C)(b - D) + (2b - C - D)(b - C - D - E).$$

The price of a composite good at equilibrium is

$$s^I = p^I + q^I = a(2b - C - D)/Det.$$

The price of composite goods when the "A" (respectively "B") market is regulated regimes is found as $q^*(0)$ by setting $p = 0$ in (A2.2) (respectively as $p^*(0)$ by setting $q = 0$ in (A2.1)). Direct substitution shows

$$p^*(0) = a/(2b - 2C - D - E), \; q^*(0) = a/(2b - C - 2D - E).$$

It is then immediate that

$$p^*(0) > q^*(0) \iff C > D \iff (n - 1)c > (m - 1)d. \quad \text{QED.}$$

<u>Proof of Proposition 8:</u>

Note first that the first order condition of parallel vertical integration (equation (17) coincides with the first order condition of independent ownership (equation (9)) for $q = 0$. Thus, $p^{***}(0) = p^*(0) = s^{R_B}$. For $q > 0$, as discussed in section VI.A on parallel vertical integration,

$$p^{***}(q) < p^*(q).$$

Similarly, $q^{***}(0) = q^*(0) = s^{R_A}$, and for $p > 0$,

$$q^{***}(p) < q^*(p).$$

Define the line of "total" prices under parallel vertical integration,

$$p + q^{***}(p).$$

The price of a composite good under parallel vertical integration lies on this line since it can be written as

$$s^V = p^V + q^V = p^V + q^{***}(p^V).$$

Similarly, the price of a composite good in the one-sided regulation regime lies on this line since

$$q^{***}(0) = q^*(0) = s^{R_A}.$$

Therefore the comparison between s^{R_A} and s^V depends on the slope of

$$p + q^{***}(p).$$

Lemma 2 below establishes that $p + q^{***}(p)$ is increasing in p. Proposition 8 immediately follows.

We now state and prove Lemma 2.

Lemma 2: The "total price" lines $p + q^{***}(p)$ and $q + p^{***}(q)$ have positive slope.

Proof of Lemma 2:

From total differentiation of the first order condition of the first firm in parallel vertically integration, $\partial\Pi^1(p_1, p_2, q_1, q_2)/\partial p_1 = 0$, equation (14), the slope of $q + p^{***}(q)$ is found to be

$$\text{(A3.1)} \qquad d(q+p^{***}(q))/dq = \left(\sum_{j=1}^{2} \partial^2\Pi^1/\partial p_j\partial p_1 - \sum_{j=1}^{2} \partial^2\Pi^1/\partial q_j\partial p_1\right)/\left(\sum_{j=1}^{2} \partial^2\Pi^1/\partial p_1\partial p_j\right).$$

Direct computation reveals that

$$\sum_{j=1}^{2} \partial^2\Pi^1/\partial p_j\partial p_1 = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} + \sum_{j=1}^{2} (D_j^{11} + D_j^{12}) + p_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{1k}^{12}+D_{2k}^{12}) +$$
$$+ q_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{3k}^{21}+D_{4k}^{21})$$

and

$$\sum_{j=1}^{2} \partial^2\Pi^1/\partial q_j\partial p_1 = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} + p_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{1k}^{12}+D_{2k}^{12}) + \sum_{j=1}^{2} D_j^{11} + \sum_{j=3}^{4} D_j^{21} +$$
$$+ q_1 \sum_{k=1}^{4} (D_{1k}^{11}+D_{2k}^{11}+D_{3k}^{21}+D_{4k}^{21}).$$

Therefore the numerator in (A3.1),

$$\sum_{j=1}^{2} \partial^2\Pi^1/\partial p_j \partial p_1 - \sum_{j=1}^{2} \partial^2\Pi^1/\partial q_j \partial p_1 = \sum_{j=1}^{2} D_j^{12} - \sum_{j=3}^{4} D_j^{21} < 0,$$

is negative since the first sum is negative and the second sum is positive. To complete the proof we also assume that

$$\sum_{j=1}^{2} \partial^2\Pi^1/\partial p_j \partial p_1 < 0.$$

Then $d(q+p^{***}(q))/dq > 0$. QED.

Proof of Proposition 9:

For linear demand, the prices of composite goods in one-sided regulation and joint ownership are

(23L) $s^{R_B} = p^*(0) = a/(2b - 2c - d - e)$, $s^{R_A} = q^*(0) = a/(2b - c - 2d - e)$,

(13L) $$s^J = a/2(b - c - d - e).$$

Direct comparison reveals

$$s^{R_B} < s^J \iff 0 < d + e, \text{ true}.$$

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