Network Quality Choices in a Network of Networks

by William Lehr

Columbia Institute for Tele-Information
Graduate School of Business
809 Uris Hall
New York, New York 10027

(212) 854 4222

Network Quality Choices in a Network of Networks

William Lehr

Columbia Institute for Tele-Information
Graduate School of Business
809 Uris Hall
New York, New York 10027
(212) 854 4222

"Network Quality Choices in a Network of Networks"

William Lehr

Graduate School of Business

Columbia University

May 8, 1992

Presented to Columbia Institute for Tele-Information's conference on Private Networks and Public Objectives, May 15, 1992.

VERY PRELIMINARY DRAFT -- PLEASE DO NOT QUOTE OR CIRCULATE WITHOUT AUTHOR'S PERMISSION.

Introduction

Over the past year, US policy-makers have become actively concerned that the quality of our electronic communications infrastructure is threatened by trends in technology and regulatory policy (e.g., divestiture of the Bell system, increased globalization and de-regulation of communications, and advances in software-controlled, fiber optic networks)[1].

---

[1] See the Congressional Committee on Government Operations report, "Asleep at the Switch? Federal Communications Commission Efforts to Assure Reliability of the Public Telephone Network" (House Report 102-420, December, 1991) for a critique of government policies towards network reliability. The report's first three findings are as follows:

"1. The public switched networks are increasingly vulnerable to failure; and the consequences for consumers and businesses and for human safety are devastating.

2. The problem of network reliability will become increasingly acute as the telecommunications market becomes more competitive.

3. No Federal agency or industry organization is taking the steps necessary to ensure the reliability of the U.S. telecommunications network." (page 3)

Increasingly, we communicate via a network of semi-autonomous networks. Both the degree of interconnection and the quality of these networks are subject to intense public scrutiny and concern. This paper focuses on questions of network quality rather than interconnection.

Others have analyzed the question of how positive externalities associated with increased network size encourage the coalescence of smaller networks (see for example Heal (1992) or Farrell and Shapiro (1989)). These analyses rely on the increasing returns realized by larger networks to help explain why networks might be willing to incur switching costs to modify their preferred technologies to permit interconnection.

The question I address here is whether we can rely on a network of networks to adopt the "optimal" design interpreted here in terms of network quality. I begin by assuming we have a community of networks. Each sub-network represents a community of users such that quality tastes are sufficiently different that users on separate networks prefer to forego joining a common network in order to benefit from the cumulative externalities which would be realized. Therefore, I am not ignoring the usual network externalities; but rather exploring situations where either preference heterogeneity, cost dis-economies, privacy considerations or some other exogenous factor precludes existence of a single centrally-controlled network. Obviously, these two questions are not really separable since many networks may be unwilling to interconnect with lower quality networks.

This paper introduces a first attempt at a formal model of quality choice by inter-connected sub-networks to show why public concern over network reliability is justified. It attempts to address an important gap in the existing economics literature.

Most of the economists who have examined the relationship between market structure, quality and regulation have focused on the provision of quality by a multi-product monopolist (e.g., Besanko, Donnenfeld and White (1988), Laffont and Tirole (1990a, 1990b), Schmalensee (1979) and Spence (1975)). Quality is viewed as a product attribute which increases both consumer's willingness-to-pay and producer costs. As Spence demonstrated, the monopolist's optimal decision which is based on the tastes and service costs of the marginal consumer may fail to take into account the tastes and costs of inframarginal consumers whose surplus is only imperfectly captured. This approach is useful for examining the pricing/quality decisions and regulation of dominant common carriers such as AT&T or the RBOCs; but is not suitable for the quality choice faced by collections of private networks.

A somewhat newer literature has explored the effects of quality decisions on the strategic competition among oligopolists (e.g., see Motta (1991), Mussa and Rosen (1978) or Katz (1986)). Ex ante quality choices may alter subsequent competitive dynamics. Competing firms view their quality choices as a means of product differentiation.

Neither of these earlier approaches addresses the situation faced by interconnected private networks where the "goods" are calls between specific nodes. The private networks do not choose quality to price discriminate among an otherwise large and undifferentiated group of consumers[2]. Moreover, the effective quality realized by each network depends on the quality choices of all other networks.

---

[2] This paper does not consider the role of alternate common carriers.

After introducing my formal model (see Section 1), I present several propositions which demonstrate the generic inefficiency of decentralized solutions and highlight some of the problems associated with the more obvious types of regulatory interventions which have been discussed by policy-makers (see Section 2). Both the virtue and vice of these propositions are their lack of restrictive parametric assumptions. In Section 3, I extend my analysis with specialized assumptions regarding functional form and provide a brief digression on the implications of uncertainty for the earlier analyses.

As noted earlier, this represents a first attempt to analyze an extremely difficult problem. Thus, the reader is forewarned that this research is in active progress. Both conceptual and analytical errors may be present, and so critical comments are most welcome. I offer this simultaneously as an invitation, warning and apology[3].

Section 1: Model

Let there be N private networks which are interconnected via the Public Switched Telecommunications Network (PSTN). I treat each of these networks as a single agent. One may think of each network as an individual phone/PC, a business unit PBX/LAN, a corporate network, an industry association's network, a regional network, a national network, or some mixture thereof. It is also possible that the sub-networks are interconnected via some other backbone than the PSTN. Although my model can be applied to all of these myriad situations, it was developed with a view towards situations where there are a

[3] I have benefitted greatly from discussions with Bruce Greenwald, Eli Noam, Darius Palia and Paolo Siconolfi; although any errors are solely my own.

moderate number of networks among whom team cooperation cannot be presumed. Thus, it is best if one consider my model within the context of large corporate networks who use the PSTN for inter-firm telephone calling. (Figure #1 provides a diagram with three networks.)

Each of these networks has an incremental willingness-to-pay for quality which depends on the calling patterns of network members. I will focus on how heterogeneity in quality preferences and traffic patterns affects quality under a variety of decision-making scenarios.

Let each network's willingness-to-pay for incremental quality be described by the following collection of functions:

$$\{S_{ii}(q^{ii}), S_{ij}^{o}(q^{ij}), S_{ij}^{t}(q^{ji}), \text{ for } j \in N,\ j \neq i\},$$

where:

$S_{ii}(q^{ii})$	willingness-to-pay for quality for "on-net" calls which originate and terminate on network i.
$S_{ij}^{o}(q^{ji})$	willingness-to-pay for quality for "off-net" calls which originate on network i and terminate on network j.
$S_{ij}^{t}(q^{ij})$	willingness-to-pay for quality for "off-net" calls which originate on network j and terminate on network i.
q^{ij}	link-quality of calls which originate on network i and terminate on network j.

By focusing on total willingness-to-pay for quality between two nodes, I avoid the need to separately identify how quality affects the volume of calling[4]. Network i's total willingness-to-pay for improved quality is equal to:

$$S_{ii}(q^{ii})+\Sigma_{j\neq i}S_{ij}^{o}(q^{ij})+S_{ij}^{t}(q^{ji})$$

The total willingness-to-pay for quality associated with a call which originates on network i and terminates on network j is $S_{ij}^{o}(q^{ij})+S_{ji}^{t}(q^{ji})$; while the total willingness-to-pay for quality for calling between networks i and j is $S_{ij}^{o}(q^{ij})+S_{ji}^{t}(q^{ij})+S_{ji}^{o}(q^{ji})+S_{ij}^{t}(q^{ji})$. To simplify the notation, I will assume that each network values incoming and outgoing calls symmetrically so that $S_{ij}^{o}=S_{ij}^{t}=S_{ij}$; however, we would not expect $S_{ij}=S_{ji}$ in general.

The quality[5] of calls which originate on i and terminate on j, q^{ij}, will depend on the quality of the originating network, q_i; the quality of the terminating network, q_j; and the quality of intermediate networks which interconnect i and j, q_p, which is the quality of the public network for all internet calls[6]. We may think of q_p as being chosen by some centralized authority pursuant to criteria external to the model discussed here (e.g.,

4 I implicitly assume that quality never falls so low as to drive calling volume between any two nodes to zero (i.e., the networks remain interconnected). Cost implications of this approach are discussed below.

5 The assumption of a scalar representation is in keeping with the desire to see what can be done in the "best possible" decision-making environment. The results presented here should be readily extendable to cases where network quality is multi-dimensional, however, this would greatly complicate the analysis. Use of a scalar quality measure is standard in the literature.

6 Assume that $q_i, q_p \in \mathbf{R}_+$.

requirements of residential households). Let this link-quality be described by the following function:

$$q^{ij} = f(q_i, q_j, q_p) \text{ for all i and j}$$

$$(\text{Note: } q^{ii} = f(q_i, q_i, q_i))$$

Once again, to simplify the notation, I assume that the link-quality function is symmetric with respect to originating and terminating network quality, implying that:

$$f(q_i, q_j, q_p) = f(q_j, q_i, q_p)$$

This implies that $f_1(q_i,q_j,q_p) = f_2(q_j,q_i,q_p)$; but in general, $f_1(q_i,q_j,q_p) \neq f_1(q_j,q_i,q_p)$. Candidate functions include the following:

$$f(q_i, q_j, q_p) = (q_i q_j)^{\alpha} q_p^{\beta}, \text{ or}$$

$$= \alpha(q_i + q_j) + (1-\alpha)q_p$$

The choice of functional form for $f(\cdot)$ is likely to depend on our definition of quality and the network topology. Our choice is likely to differ if we are concerned with the probability of call completions versus the level of line noise.

Assume that the incremental willingness-to-pay functions, $S_{ij}(q^{ij})$, and the link-quality function, $f(q_i,q_j,q_p)$ are smooth, quasi-concave, monotonically non-decreasing functions[7] of the qualities of the constituent networks, q_i, q_j and q_p. Networks are willing to pay more for on-net and off-net calls when those calls occur over higher quality links; and, the quality of those links increases with the quality of the constituent networks.

---

[7] Formally, $S_{ij}(\cdot)$ and $f(\cdot) \in \mathbb{C}^2$. Quasiconcavity requires the first derivative of S_{ij} and the first partials $f(\cdot)$ to be non-decreasing (i.e., $f_1(\cdot)$, $f_2(\cdot)$, $f_3(\cdot)$, S_{ii}', $S_{ij}' \geq 0$); the second derivative of S_{ij} to be non-increasing (i.e., S_{ii}'',$S_{ij}'' \leq 0$); and the Hessian of $f(\cdot)$ to be negative semi-definite (or, the second own-partials to be negative and larger in absolute value than the cross-partials if the latter are positive).

There is some disagreement whether improved network quality is costly to achieve. One view is that the newer generation of digital switches and high capacity fiber optic transmission systems are more reliable and cost effective than the hybrid technologies they are replacing[8]. With enhanced options for alternate routing and software-controlled network management and fault recovery, future networks will offer higher quality. When these upgrades are justified solely on the basis of scale and scope economies, the increased quality is a freebie.

An alternate view is that the enhanced complexity and increased reliance placed on networks will lead to a deterioration in quality[9]. Recent problems with Signalling System 7 network management software deployed in the PSTN have raised doubts about the reliability of software controlled networks[10]. Although both views have merit, it is worth investigating how industry structure might affect quality decisions when improving quality is costly.

I assume that all networks possess similar technologies[11] so that we can describe the incremental costs for improving quality by the smooth, quasi-convex[12] cost function $C(q_i)$. The total cost[13] for providing quality $q = (q_1, q_2, \ldots, q_N)$ is equal to $\Sigma_i C(q_i)$. The assumption that each network's quality costs are unaffected by the quality choices of other networks

---

8 Cite source citing increased reliability at lower cost. FCC report, Bruce greenwald or Bruce Egan.

9 As we become more dependent on networks, the costs of low quality (or value of high quality) increase. Cite Government Committee report or Eli Noam testimony or Hinsdale study.

10 Cite Government committee report.

11 If desired, one can think of cost asymmetries as included in the asymmetric willingness-to-pay functions.

12 Formally, $C(\cdot) \in \mathbb{C}^2$. Quasi-convexity requires that $C'(\cdot)$ and $C''(\cdot) \geq 0$.

13 I ignore the costs associated with the PSTN, $C_p(q_p)$. Since I treat q_p as a parameter this does not change my results.

implicitly assumes that either network costs are largely fixed (and independent of calling volume) or that calling volume is not significantly affected by quality.

With these assumptions[14] and in the absence of any side-payments, we can represent network i's net willingness-to-pay for incremental quality, $w_i(q,q_p)$, and total net willingness-to-pay for incremental quality, $W(q,q_p)$ as follows:

$$w_i(q,q_p) = S_{ii}(f(q_i,q_i,q_i)) + 2\sum_{j\neq i} S_{ij}(f(q_i,q_j,q_p)) - C(q_i) \qquad (1)$$

and

$$W(q,q_p) = \sum_i w_i(q,q_p) = w_i(q,q_p) + 2\sum_{j\neq i} S_{ji}(f(q_i,q_j,q_p)) + (q_i\text{-free terms})$$

where: $q = (q_1,q_2,\ldots,q_N)$

(*Note*: $w_i(0,q_p) = 0$ *and* $W(0,q_p) = 0$)

[14] In addition, I adopt the following technical assumptions regarding boundary conditions to guarantee that the strategy space for q is compact:

i. $\forall q_i,q_j,q_p,\ 0 \leq f(q_i,q_j,q_p) \leq qmax < \infty$
ii. $\forall i,j,\ 0 \leq S_{ij}(qmax) \leq Smax < \infty$
iii. $f(0,0,0) = 0$ *and* $C(0) = 0$
iv. $\forall i,j,\ S_{ij}(0) = 0$
v. $\forall i,\ \lim_{q_i \to \infty} w_i(q,q_p) < 0$

Network j confers a positive externality on network i when it increases its quality, which at the margin, is equal to:

$$\frac{\partial w_i(q,q_p)}{\partial q_j} = 2S_{ij}^{'}f_2(q_i,q_j,q_p) > 0$$

where $2S_{ij}'$ is the marginal value which network i places on calls between networks i and j; and $f_2(q_i,q_j,q_p)$ (= $f_1(q_j,q_i,q_p)$) is the marginal improvement in link-quality when network j improves it's quality. The existence of these externalities introduces a wedge between the choice of q which maximizes total net willingness-to-pay and individual net willingness-to-pay.

Define q^* and q^c as the solutions to the centralized and decentralized problems as follows:

$$q^* \; solves \; \max_{q=(q_1,q_2,\ldots,q_N)} \sum_i w_i(q,q_p) \qquad (2a)$$

$$q^c \; solves \; \max_{q_i} w_i(q_i,q_{-i}^c,q_p) \quad \forall i \qquad (2b)$$

The decentralized Problem 2b gives rise to the following collection of First Order Necessary Conditions (FONCs):

$$\frac{\partial w_i(q,q_p)}{\partial q_i} = 0 = \frac{\partial S_{ii}}{\partial q}(2f_1(q_i,q_i,q_i)+f_3(q_i,q_i,q_i)) + 2\sum_{i\neq j}\frac{\partial S_{ij}}{\partial q}f_1(q_i,q_j,q_p) - \frac{\partial C(q_i)}{\partial q_i} \tag{3}$$

$$= S_{ii}'(2f_1(q_i,q_i,q_i)+f_3(q_i,q_i,q_i)) + 2\sum_{i\neq j} S_{ij}'f_1(q_i,q_j,q_p) - C'(q_i)$$

$$= F_i(q_i,q_{-i},q_p)$$

Each FONC implicitly defines that network's best response to quality choices made by the other networks. The decentralized solution is a Nash equilibrium of an N-player game of perfect information. A well-known result from game theory guarantees the existence of Nash equilibria for such games with convex, compact strategy spaces and quasi-concave, continuous individual payoff functions (see Friedman, 1986).

Although the assumptions made above guarantee that the strategy spaces are convex and compact and that the payoff functions are continuous in quality, they do not ensure quasi-concavity. Thus, Nash equilibria may not exist[15]. To guarantee existence, I assume that the Hessians of the individual payoff functions are negative semi-definite. An even stronger assumption is required to guarantee uniqueness[16]. When there are multiple Nash

---

[15] Quasiconcavity is a sufficient condition, not a necessary condition for existence.

[16] Uniqueness would be guaranteed if we assumed that the Jacobian of the FONCs (which implicitly define the best reply functions) is negative quasi-definite, where this is analogous to the requirement of negative definiteness for symmetric matrices (i.e., matrix A is negative quasi-definite if $B=A+A^T$ is negative definite). Since in general, $\partial^2 w_i(q,q_p)/\partial q_i\partial q_j \neq \partial^2 w_j(q,q_p)/\partial q_j\partial q_i$ the Jacobian is not symmetric. This requirement is quite strong, although may be justified under special circumstances.

equilibria, they may differ with respect to total welfare, $W(q^c,q_p)$. Designate q^{lo} and q^{hi} as members of the Nash set which minimize and maximize $W(q,q_p)$, respectively[17].

The <u>centralized</u> Problem 2a may be thought of as the socially-optimal Coase solution which would prevail if the networks could bargain efficiently and arrange unrestricted side-payments to fully internalize the quality-externalities. The <u>centralized</u> problem gives rise to the following FONCs:

$$\frac{\partial W(q,q_p)}{\partial q_i} = 0 \tag{4}$$

$$= \frac{\partial S_{ii}}{\partial q}(2f_1(q_i,q_i,q_i)+f_3(q_i,q_i,q_i)) + 2\sum_{i\neq j}\frac{\partial S_{ij}}{\partial q}f_1(q_i,q_j,q_p) - \frac{\partial C(q_i)}{\partial q_i}$$

$$+ 2\sum_{i\neq j}\frac{\partial S_{ji}}{\partial q}f_2(q_j,q_i,q_p)$$

$$= S_{ii}'(2f_1(q_i,q_i,q_i)+f_3(q_i,q_i,q_i)) \quad + 2\sum_{i\neq j}(S_{ij}'+S_{ji}')f_1(q_i,q_j,q_p)) - C'(q_i)$$

$$= F_i(q_i,q_{-i},q_p) + 2\sum_{i\neq J}S_{ji}'f_1(q_i,q_j,q_p)$$

$$= G_i(q_i,q_{-i},q_p)$$

The continuity of the individual payoff functions guarantee that aggregate payoffs, $W(q,q_p)$, are continuous, and thus, we know q^* exists[18]; however, it may not be unique. In

---

[17] When the Nash equilibrium is unique, these will be equal.

[18] A continuous function on a compact space attains a maximum.

a "first best" world, this is not an issue; but may become important in a "second best" world where the costs of getting close to different q^* may differ.

The (worst case Nash) decentralized solution and the centralized solution provide bounds for the welfare gains associated with improving the quality of the "network of networks" (i.e., $W(q^*,q_p)-W(q^{lo},q_p)$). In the next section, I discuss some general propositions which help characterize the nature of the two classes of solutions.

Section 2: Analysis of Model

In this section, I present six general propositions. The first three concern general characteristics of the centralized and decentralized solutions; the last three address different types of interventions which may be used to implement the socially efficient solution.

Proposition #1: q^* is (generally) not a Nash equilibrium

If q^* is an interior solution, then $G_i(q^*,q_p) = 0$ for all i where $q_i^*>0$. Since $F_i(q^c,q_p) = 0$ for all Nash equilibria where $q_i^*>0$, q^* is not a Nash equilibrium unless the marginal externality associated with q_i is zero, or:

$$\forall i \text{ where } q_i^*>0, \sum_{j\neq i} S_{ji}' f_1(q_i^*,q_j^*,q_p) = 0.$$

Under most reasonable traffic scenarios, this is unlikely to be the case. As long as increased quality strictly improves link-quality (i.e., $f_1(\cdot)>0$) and there is at least one network i which finds it worthwhile to improve its quality for on-net calling (i.e., $\exists i$, $S_{ii}'(0)>0$ so that $q_i^*>0$) and some other network j which values internet calling with i (i.e., $\exists j\neq i$, $S_{ji}'(f(q_i^*,q_j^*,q_p))>0$), then the marginal externality will be strictly positive.

Since $W(q^*,q_p) \geq W(q,q_p)$ for all feasible q by definition, Proposition #1 tells us that all Nash equilibria are sub-optimal. Without additional parametric assumptions regarding the form of $S_{ij}(\cdot)$, $f(\cdot)$ and $C(\cdot)$, one cannot say anything about the magnitude of the potential efficiency loss (see Section 3 for such a discussion).

Proposition #2: Generally, $q_i^c\neq q_j^c$ & $q_i^*\neq q_j^*$ for $i\neq j$

Except under unrealistically restrictive assumptions, it will not be optimal for every network to adopt the same level of quality (i.e., $q_i^*\neq q_j^*$ for $i\neq j$). Moreover, it will seldom be a Nash equilibrium (i.e., $q_i^c\neq q_j^c$ for $i\neq j$).

A sufficient condition for two networks to choose the same level of quality in the decentralized problem is if both networks have identical marginal valuations for internetwork and on-net calling, or,

$$S_{ii}' - S_{jj}' = 0 \;\; and \;\; \sum_{k \neq i} S_{ik}' - \sum_{k \neq j} S_{jk}' = 0$$

or

$$S_{ii}' - S_{jj}' = 0 \;\; and \;\; \sum_{k \neq i} (S_{ik}' + S_{ki}') - \sum_{k \neq j} (S_{jk}' + S_{kj}') = 0$$

This would be the case if the two networks had identical tastes (i.e., $q_i = q_k$ if $\forall j$, $S_{ij}' = S_{kj}'$).

If this is not the case, then a necessary condition for two networks to choose the same quality is for the differences between their marginal valuations for on-net and off-net calling to have opposite signs. One network must value on-net calling relatively more; while the other values off-net calling relatively more. It would be extremely fortuitous if this were the case and thus, in general, we should not expect to see networks adopting similar quality levels.

The proposition implies that it would not be inefficient to require every network to adopt a uniform quality standard.

Proposition #3: $q_i^c \leq q_i^* \; \forall i$ and (usually) $\exists j$ such that $q_j^c < q_j^*$

Since $F_i(q^c, q_p) = 0$ whenever $q_i^c > 0$, $G_i(x^c, q_p) \geq 0$ since $\Sigma_{j \neq i} S_{ji}' f_1(q_i, q_j, q_p) \geq 0$ so every Nash equilibrium provides insufficient quality. This implies that average quality is lower under every Nash equilibrium than is optimal. This result is intuitively obvious in light of the positive externalities and the restrictions against side-payments inherent in my characterization of the decentralized problem.

The first three propositions clearly indicate the difficulty posed by heterogeneous tastes for public policy. The following three propositions consider three interventions which might be used to improve the efficiency of the decentralized outcome.

Proposition #4: There exist penalties T^i, which support the socially efficient solution, q^*.

The following schedule of penalties assessed against each network offers one means of supporting the first-best solution, q^*:

$$T_i = 2\sum_{i \neq j} [S_{ji}(f(q_j,q_i,q_p)) - S_{ji}(f(q_j^*,q_i^*,q_p))]$$

$$w_i(q,q_p,T) = w_i(q,q_p) + T_i$$

The penalties T_i modify each network's FONC so as to internalize the externality as follows:

$$\frac{\partial w_i(q,T)}{\partial q_i} = F_i(q_i,q_{-i},q_p) + 2\sum_{j \neq i} S_{ji}'f_1(q_i,q_j,q_p) = G_i(q_i,q_{-i},q_p) = 0 \text{ @ } q^*$$

$$and, \quad W(q^*,q_p,T) = W(q^*,q_p)$$

Note that this scheme imposes penalties for $q_i < q_i^*$ and that $T_i(q_i^*)=0$ so that $\Sigma_i T(q_i^*)=0$. The scheme implicitly assumes that each network can be forced to remain connected even though its payoff may be negative (i.e., $w_i(q^*,q_p)<0$ is possible).

Since $0 \leq W(q^c,q_p) < W(q^*,q_p)$, those that benefit from the centralized solution must have gains sufficient to compensate those whose payoffs are negative[19]. Although there exist side-payment schemes (or, penalties/subsidies) which could assure each network at least a weakly positive payoff, we cannot rely on the networks voluntarily adopting such a scheme. First, the scheme involves multilateral -- not bilateral -- bargaining (which is typically more expensive and more prone to disruption). Second, there are likely to be networks which prefer the decentralized solution even when the centralized solution is augmented by side-payments[20]. Furthermore, any bargaining costs incurred in arriving at the efficient T would represent a deadweight loss (thus implying a second-best outcome[21].

The analysis above presumes perfect information regarding willingness-to-pay, link-quality and cost functions. If an efficient side-payment scheme does not arise voluntarily (i.e., is incentive compatible), then the regulatory authority would need this information to enforce the efficient outcome. It seems unlikely that regulators would possess the required information. Furthermore, if the T are based on inaccurate demand/cost information, the outcome may differ from q^* by more than even the worst Nash solution, q^{lo}. The T could lead to either excessive or insufficient quality. Although it is theoretically feasible to use

---

[19] Let L be subset of networks for which $w_i(q^c,q_p) > w_i(q^*,q_p)$, then:

$$0 < \sum_{i \in L} (w_i(q^c,q_p) - w_i(q^*,q_p)) < \sum_{i \in N-L} (w_i(q^*,q_p) - w_i(q^c,q_p))$$

It is possible that all networks are better off under q^* (or L is empty).

[20] There may not exist side-payment schemes T such that $\Sigma_i T_i = 0$ and $w_i(q^c,q_p) \leq w_i(q^*,q_p,T)\ \forall i$. Even when such schemes exist, they may be unlikely to emerge from the bargaining process. Saying more about the feasibility of various side-payment schemes would require additional parametric assumptions or assumptions on the nature of the decentralized bargaining process.

[21] $W(q^*,T) = W(q^*)$ - deadweight bargaining costs.

penalties/taxes/subsidies to support the first-best quality choice, it may be practically impossible. Thus, the implications of this proposition for public policy are cautionary.

Proposition #5: There exists a uniform pricing function, P(q), for network terminations which supports q^*.

In order to find such a function, I assume that the number of messages between nodes does not vary either with the incremental price charged for quality or link-quality. As noted earlier, this is a strong assumption which may not be satisfied in real life. Even with this assumption and perfect information, the uniform pricing schedule is quite complex and thus the conclusions drawn are analogous to those above.

Assume each network i originates n_{ij} messages which terminate on j. The total traffic originated by network i is $\Sigma_{j \neq i} n_{ij}$ and the total internet traffic is $n = \Sigma_i \Sigma_{j \neq i} n_{ij}$. Network i receives $P(q)n_{ji}$ for calls from j which terminate on i; and it pays $P(q)n_{ij}$ for calls which originate on i and terminate on j. If we assume that no network originates exactly as many messages as it terminates (or $\Sigma_{j \neq i} n_{ji} \neq \Sigma_{j \neq i} n_{ij}$), then the following pricing function supports q^*:

$$P(q) = \sum_i \frac{2\sum_{j \neq i} S_{ji}(f(q_j^*, q_i, q_p))}{\sum_{j \neq i} (n_{ji} - n_{ij})}$$

To see that this is the case, observe that:

$$w_i(q,P) = w_i(q) + P(q)\sum_{j\neq i}(n_{ji} - n_{ij})$$

$$W(q,P) = \sum_i w_i(q,P) = \sum_i w_i(q) = W(q) \quad since \sum_i \sum_{j\neq i}(n_{ji} - n_{ij}) = 0$$

$$\frac{\partial w_i(q,P)}{\partial q_i} = F_i(q_i,q_{-i},q_p) + \frac{\partial P(q)}{\partial q_i}\sum_{j\neq i}(n_{ji} - n_{ij})$$

$$= G_i(q_i,q_{-i},q_p) = 0 \ @ \ q^* \quad if,$$

$$\frac{\partial P(q)}{q_i} = \frac{2\sum_{j\neq i} S_{ji}' f_1(q_i,q_j^*,q_p)}{\sum_{j\neq i}(n_{ji} - n_{ij})}, \quad or \quad P(q) = \sum_i \frac{2\sum_{j\neq i} S_{ji}(f(q_j^*,q_i,q_p))}{\sum_{j\neq i}(n_{ji} - n_{ij})}$$

As written the equilibrium termination price $P(q^*)$ could be quite large, although the magnitude of $P(q^*)$ could be adjusted without changing P(q)'s marginal effect by subtracting a constant. Somewhat more interesting is the observation that $P(q^*)$ may be negative (or, networks pay for calls they terminate and receive payment for calls they originate). This is not unreasonable when we recognize that we may want to subsidize high quality from networks which originate a lot of calls which are highly valued by the recipients (e.g., cellular customers).

Once again, there is no a priori reason to expect this pricing function to arise naturally (e.g., via a tatonnement process). Traditional externality problems are more readily amenable to price-based solutions because the externality is associated with a commodity good. In the model presented here, incremental quality of different networks are not perfect substitutes which can be traded. It matters which network's quality is increased. The marginal externality associated with the quality of each network depends on the allocation of traffic among sub-networks (i.e., the specification of the S_{ij} functions).

Proposition #6: Although there exists a welfare enhancing minimum quality standard, qmin, such a standard (alone) cannot support q^*.

In order to support q^*, a uniform minimum quality standard cannot exceed the efficient level for the lowest quality network (i.e., qmin $\leq q_{imin}^* = \min_{i \in N} q_i^*$). From Proposition #2 we know that there is likely to be a network imin such that $0 \leq q_{imin}^* < q_j^*$ for $j \neq$ imin. These other networks are unconstrained and from the discussion of Proposition #3 we know that we should expect $F_j(q^*, q_p) < 0$ for $j \neq$ imin. These networks would choose lower-than-optimal quality levels. At best, minimum quality standards by themselves can support a second best solution.

If we let $q_{min}^c = \min_i q^c$ be the minimum quality provided by any network in the Nash equilibrium q^c, then a minimum quality standard which is lower will not affect behavior in that equilibrium (i.e., qmin $< q_{min}^c$). If there are multiple Nash equilibria, then qmin $= q_{min}^{hi}$ eliminates all socially inferior Nash equilibria[22]. The potential welfare loss is reduced by at least the difference between $W(q^{hi},q_p)$-$W(q^{lo},q_p)$. This altering of the Nash set -- a coordination function -- may very well be the greatest benefit of minimum quality standards.

We know that an infinitesimally tougher minimum quality standard would be welfare improving from the envelope theorem. At q_{hi}, forcing a small increase in the quality of q_{imin}^{hi}

[22] If the Nash equilibria are locally separated, then a slightly lower standard qmin which is not observed to be binding in equilibrium may still be regarded as welfare enhancing in an "ex ante" sense.

improves total welfare (from Proposition #3). Anything larger than an infinitesimal increase, however, may lead to shifts in the quality of unconstrained networks. Consider a very simple example with just two networks, i and j, where $q_i^{hi} < q_j^{hi}$. If the two networks regard each other's quality as substitutes, then the improvement in i's quality (from a tighter qmin) may result in a lowering of the quality of j's network[23] resulting in a net decrease in total welfare.

If the networks regard quality as complements, then welfare improves until $qmin = q_{imin}^*$, where imin=i or j. A still higher standard may be welfare improving but is certainly not a first-best optimum (i.e., $W(q^*, q_p) > W(q^c, q_p, qmin)$). Even under the most optimistic assumptions regarding strategic responses, there must exist a maximal minimum quality standard, qmin^, beyond which total welfare unambiguously decreases[24]. If costs are sufficiently convex and the divergence between q^{hi} and q^* is sufficiently small, this may happen with what appear to be relatively modest increases in minimum quality requirements.

The difficulties of calculating the optimal uniform minimum quality standard and the dangers of exceeding qmin^ should caution us against setting overly aggressive performance

---

23 If $dq_j(q^c, q_p)/dq_i < 0$ then qualities are strategic substitutes; if >0, then strategic complements. For the case of two networks this derivative is given by the implicit function theorem:

$$\frac{dq_j(q^c, q_p)}{dq_i} = -\frac{\partial^2 w_j(q^c, q_p)/\partial q_j \partial q_i}{\partial^2 w_j(q^c, q_p)/\partial q_j^2}$$

This derivative may be positive or negative under reasonable parametric assumptions.

24 The assumption that $\lim_{q \to \infty} w_i(q, q_p) < 0 \; \forall i$ guarantees that eventually welfare will decrease with higher minimum quality standards even if quality is always and for everyone a strategic complement. Note that $qmin\hat{} > \max_i q_i^*$ is possible, even if unlikely.

standards. On the other hand, the potential coordination benefits suggest that more modest standards may offer large gains which will be difficult to measure directly.

The generality of the preceding six propositions is both their greatest virtue and their greatest vice. In the next section, I discuss a variety of extensions which rely on further parameterizations of the basic model.

Section 3: Extensions and Further Questions

The following two sections discuss preliminary refinements to the results presented above. This section is subject to extensive revision, so please regard the following as tentative speculations which have not been adequately reviewed as of this draft.

Section 3.1: A Simple Example: Linear Quality and Demand

With more than a few networks and with anything but extremely simple functional forms for the willingness-to-pay, link-quality and cost functions, it is nearly impossible to obtain closed-form solutions. However, with linear quality and demand such solutions can be obtained as follows:

$\forall i,j$, let $0 \leq q_i, q_j, q_p \leq qmax < \infty$

$$f(q_i, q_j, q_p) = \alpha(q_i + q_j) + \beta q_p \text{ with } \alpha, \beta > 0$$

$$S_{ii} = Y_i q^{ii} \text{ and } S_{ij} = X_i q^{ij} \text{ with } 0 \leq Y_i, X_i \leq XYmax < \infty$$

$$\text{and, } C(q_i) = A q_i^{\delta} \text{ with } \delta > 1,\ A > 0$$

then,

$$w_i(q,q_p) = (2\alpha+\beta)Y_i q_i + 2X_i \sum_{j \neq i} (\alpha(q_i + q_j) + \beta q_p) - Aq_i^{\delta}$$

$$W(q,q_p) = w_i(q,q_p) + 2\sum_{j \neq i} (\alpha(q_i + q_j) + \beta q_p) X_j + non\text{-}q_i\ terms$$

$$\frac{\partial w_i(q,q_p)}{\partial q_i} = (2\alpha+\beta)Y_i + 2\alpha(N-1)X_i - \delta A q_i^{\delta-1}$$

$$\frac{\partial W(q,q_p)}{\partial q_i} = \frac{\partial w_i(q,q_p)}{\partial q_i} + 2\alpha \sum_{j \neq i} X_j$$

$$q_i^c = \left[\frac{(2\alpha+\beta)Y_i + 2\alpha(N-1)X_i}{\delta A}\right]^{\frac{1}{\delta - 1}}$$

$$q_i^* = \left[\frac{(2\alpha+\beta)Y_i + 2\alpha(N-1)X_i + 2\alpha \sum_{j \neq i} X_j}{\delta A}\right]^{\frac{1}{\delta - 1}}$$

or,

$$q_i^* = \left[\frac{(2\alpha+\beta)Y_i + 2\alpha(N-1)X_i + 2\alpha(N\bar{X} - X_i)}{\delta A}\right]^{\frac{1}{\delta - 1}}$$

$$where:\ \bar{X} = Xbar = \frac{1}{N}\sum_i X_i$$

The above solution makes the following points immediately clear:

i. Both the centralized and decentralized solutions are unique. Each network has a dominant strategy for its choice of q_i which is independent of other networks' choices.

As long as the taste parameters Y_i and X_i are different across networks, both the decentralized and centralized solutions call for heterogeneous quality.

ii. The decentralized solution provides too little quality to be optimal.

iii. The greater the number of networks N and the higher the average marginal valuation for internet traffic, Xbar, the greater the discrepancy between the optimal solution and the decentralized solution. Notice that the externality depends on X_i, Xbar and N.

iv. The disparity is reduced the more important on-net calling is relative to inter-net calling (i.e., ceteris paribus, Ybar increases relative to Xbar).

v. The disparity is reduced by more convex costs (i.e., larger δ). As costs get more convex, the optimal level of quality decreases and the Nash and centralized solutions are forced closer together.

vi. The optimal qualities do not depend on the quality of the intermediate network, the PSTN, q_p. Although total welfare is increasing in q_p, the public networks quality can not be used to influence the behavior of inter-connected networks in the decentralized solution.

Now, let us examine what happens if the networks have identical tastes, so $Y_i=Y$ and $X_i=X$ for all i. From Proposition #2, we know that this implies that all of the networks will adopt homogeneous quality in both solutions (i.e., $q_i^c = q^c < q^* = q_i^*$). In this case, the individual and aggregate payoff functions and the potential welfare loss are given by:

$$w_i(q,q_p) = (2\alpha+\beta)Yq + 4\alpha(N-1)Xq + 2\beta(N-1)Xq_p - Aq^\delta$$

$$W(q,q_p) = (2\alpha+\beta)NYq + 4\alpha N(N-1)Xq + 2\beta N(N-1)Xq_p - ANq^\delta$$

$$W(q^*,q_p) - W(q^c,q_p) = (2\alpha+\beta)NY(q^* - q^c) + 4\alpha N(N-1)X(q^* - q^c) + AN((q^c)^\delta - (q^*)^\delta)$$

Alternatively, if we assume that $A=1$ and $\delta=2$, then the optimal solutions simplify to:

$$q_i^c = (\alpha+\frac{1}{2}\beta)Y_i + \alpha(N-1)X_i$$

$$q_i^* = q_i^c + \alpha(N\bar{X} - X_i)$$

If we combine the assumption of homogeneous tastes and $A=1$ and $\delta=1$, the welfare loss associated with not enforcing the centralized solution is:

$$W(q^*,q_p) - W(q^c,q_p) = \frac{1}{2}\alpha\beta N(N-1)XY + 3\alpha^2 N(N-1)^2 X^2$$

(check algebra)

Section 3.2 Suppose $f(q_i,q_j,q_p) = \min(q_i,q_j,q_p)$

One functional form for link-quality which might appear reasonable is if the link-quality is the minimum of the origination, termination and intermediate network. With this functional form, the best response correspondences are no longer continuous and the analysis from Section 2 may not apply.

If the PSTN is very low quality, then the link-quality for all internet calling is limited by the quality of the PSTN. There are no quality-externalities and the centralized and decentralized solutions are identical (i.e., $q_i^c = q_i^*$).

$$if\ q_p < \min_i q_i^c,\ then$$

$$w_i(q) = S_{ii}(q_i) + \sum_{j \neq i} S_{ij}(q_p) - C(q_i) = w_i(q_i, q_p)$$

$$F_i(q, q_p) = F_i(q_i, q_p)\ \ and\ \ G_i(q, q_p) = F_i(q_i, q_p)$$

$$\Longrightarrow q_i^* = q_i^c$$

Under these circumstances, we may want to re-examine our assumption that all of the sub-networks remain inter-connected through the PSTN. If the networks find internet quality sufficiently valuable, it may be privately optimal for them to bypass the PSTN and interconnect directly. The model presented here focuses on logical interconnections and implicitly assumes that the initial topology is the most efficient possible[25]. A logical extension of the present analysis would be to consider competition among alternative routes.

---

[25] It may be the most efficient because alternative routing is sufficiently more expensive to offset quality gains relative to the low quality PSTN; or because it is globally optimal to choose q_p low and to prevent "cream-skimming" entry by bypass carriers. In general, it seems likely that allowing selected bypass would be welfare improving.

If the quality of the PSTN is very high, however, then the solution becomes more difficult to compute[26]. [BEWARE -- following subject to revision] The optimal centralized solution should be the same as discussed in the preceding sections; however the decentralized solution will generally be much worse. Consider first the case where there is no heterogeneity for on-net calling. In the absence of demand for internet calling, every network would select the same quality. Consider any equilibrium with positive demand for internet calling where some network i sets its quality above the quality called for in the "on-net-only" solution[27]. If it is gaining from this increased quality, then network i must value calling to a network j which is setting quality at least as high. At the margin, network i's net payoff increases by the cost savings from reducing its quality (i.e., by $C'(q_i)$). Therefore, it cannot be an equilibrium for a network to maintain quality higher than the "on-net-only" solution.

As long as internet calling is valuable, the only equilibrium may be for even lower quality since even the "on-net" only solution creates positive externalities for networks with lower "on-net" quality choices. An increased valuation for internet-quality may actually decrease quality, representing an especially perverse outcome. [The situation may be amenable to modelling as a Prisoner's Dilemma].

26

$$\text{if } q_p > \max_i q_i^c, \text{ then}$$

$$w_i(q) = S_{ii}(q_i) + \sum_{j \neq i} S_{ij}(\min(q_i, q_j)) - C(q_i)$$

27 The on-net only solution is the quality level chosen to maximize $S_{ii}(q_i)-C(q_i)$.

Finally, it is worthwhile noting that as the quality of the PSTN increases, quality choices by the sub-networks become more strategic and contentious. Ignoring bypass issues, when q_p is sufficiently low, everyone agrees on the optimal solution and voluntary enforcement of the efficient outcome is easily achieved. As the network quality increases, however, externality issues become more important and the threat of free-riding increases. Both the efficiency losses from failing to enforce the centralized solution and the costs of enforcement[28] are likely to be larger. It is exactly under such circumstances that specialized institutional structures (e.g., voluntary standard setting bodies) may become important, especially if information asymmetries preclude effective use of more direct regulatory interventions (e.g., the government orders everyone to adopt q^*)[29].

Section 3.3: Effect of Uncertainty

In all of the analyses above, I have assumed perfect information. As long as quality is ex post verifiable and it is possible to specify and enforce complete, contingent contracts, uncertainty should not represent a problem for implementation of the centralized solution.

Ex post verifiability and enforcement may be a reasonable assumption for agreements governing quality attributes which are based on a large sample of (at least in principle) inexpensive observations. For example, delay until dial tone is received or line noise (bit

---

[28] One might expect monitoring/enforcement costs to be larger when the private gains from deviating from q^* are larger according to the old adage "where there's a will, there's a way.."

[29] See Lehr (1992a) for a discussion of how the structure of voluntary standard setting institutions helps alleviate technology choice when viewed as a problem in collective choice.

error rates) seem amenable to low cost monitoring and successful contracting. Breach would be quickly detected.

On the other hand, network reliability -- interpreted as freedom from major disruptions -- offers a more difficult problem. Increased reliability is provided via back-up capacity. Since major network failures are (of necessity) a very infrequent occurrence, it may be much more difficult to reliably ascertain the quality of back-up systems ex ante. A moral hazard problem may arise if the probability of failure is low enough, the costs of unreliable back-up are great enough, and there are no criminal penalties available to deter breach ex ante. For example, in the absence of criminal penalties, a "fly-by-night" data base network might agree to provide high-quality database back-up services which would become operational in the event of a major system failure on connected sub-networks. The back-up provider could collect insurance premiums up front and declare bankruptcy if a major failure resulted in it breaching its contract.

Enforcement may also be difficult for international quality agreements. Would we rely on the CCITT or some other such body to enforce the centralized solution?

The effect of uncertainty on the decentralized solution will depend on the functional specification, our definition of equilibria, and the method by which beliefs are modelled. One might expect uncertainty to exacerbate the divergence between the centralized and decentralized solutions, leading to a proliferation of Nash equilibria.

Section 4: Conclusions

This paper presents a first step towards a formal model of the problem of quality choice among heterogeneous, inter-connected private networks. The principal conclusions should not be terribly surprising once one recognizes the positive externalities inherent in unilateral quality improvements. In the presence of externalities, we generally do not expect to achieve first best solutions. What is troublesome, however, are the sensitivity of the efficiency losses to parameters which one might expect to be increasingly important as private, global networks proliferate.

Although technological evolution may increase network quality, the growing public concern regarding the effects on network quality of increased decentralization, de-regulation and its attendant implications for industry structure appear warranted. Moreover, a welfare-improving regulatory response may be difficult to craft. Uniform or (too aggressive) minimal quality standards may actually leave us worse off than if nothing were done.

The present analyses should be extended in a number of directions (each of which will require more restrictive parametric assumptions along the lines of those discussed in section 3), including:

1. Introduce competition between intermediate carriers
2. Formally model effects of uncertainty
3. Consider effects of different quality-choice mechanisms (e.g., majority voting by networks)
4. More detailed examination of alternative regulatory mechanisms.

I hope to explore these and other extensions in forthcoming research.

Bibliography

Besanko, D., Donnenfeld, S. and White, L., "The Multiproduct Firm, Quality Choice and Regulation", The Journal of Industrial Economics, vol. XXXVI, no. 4, June 1988, 411-429.

Farrell, J, and C. Shapiro, "Dynamics of Standards Coalitions and Bandwagons", TOP Seminar Paper draft, 1989. New version???

Freidman, J., Game Theory with Applications to Economics, Oxford University Press, New York, 1986.

Government Printing Office, "Asleep at the Switch? Federal Communications Commission Efforts to Assure Reliability of the Public Telephone Network", Fourteenth Report by the Committee on Government Operations, House Report 102-420, 49-523-91-1, Government Printing Office, Washington, DC December 11, 1991.

Heal, G., "Economics of Networks", draft CITI Private Network Conference paper, Winter 1992.

Katz, M., "An Analysis of Cooperative Research and Development", Rand Journal of Economics, vol 31, 1986, 527-543.

Kihlstrom, R. and D. Levhari, "Quality, Regulation and Efficiency", Kyklos, vol 30, 1977, 214-234.

Laffont, J. and J. Tirole, "Provision of Quality and Power of Incentive Schemes in Regulated Industries", mimeo, April 1989.

Laffont, J. and J. Tirole, "The Regulation of Multiproduct Firms, Part I: Theory", Journal of Public Economics, vol 43 (1990a) 1-36.

Laffont, J. and J. Tirole, "The Regulation of Multiproduct Firms, Part II: Applications to Competitive Environments and Policy Analysis", Journal of Public Economics, vol 43 (1990b) 37-66.

Motta, M., "Endogenous Quality and Coordination of Decisions", Center for Operations Research and Econometrics Discussion Paper #9152, Universite Catholique de Louvain, Belgium, October 1991.

Mussa, M. and S. Rosen, "Monopoly and Product Quality", Journal of Economic Theory, vol 18., 1978.

Noam, E., "The Quality of Regulation in Regulating Quality: A Proposal for an Incentive Approach to Telephone Service Performance", in M. Einhorn (ed) Price Caps and Incentive Regulation in Telecommunications, 1991.

Organisation for Economic Co-operation and Development, Performance Indicators for Public Telecommunications Operators, Information Computer Communications Policy, OECD, Paris, 1990.

Sappington, D., "Optimal Regulation of a Multiproduct Monopoly with Unknown Technological Capabilities", Bell Journal of Economics, vol 14 (1983) 453-463.

Schmalensee, R., "Market Structure, Durability, and Quality: A Selective Survey", Economic Inquiry, vol 17, April 1979, 177-196.

Sheshinski, E., "Price, Quality and Quantity Regulation in Monopoly Situations", Economica, vol 43, 1976, 127-137.

Spence, M., "Monopoly, Quality and Regulation", Bell Journal of Economics, vol 6, no 2, Autumn 1975, 417-429.

Takahashi, K., "Transmission Quality of Evolving Telephone Networks", IEEE Communications Magazine, October 1988, 24-35.