Mixed Bundling in Duopoly

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Abstract

We present a model where producers of complementary goods have the option to practice mixed bundling. In the first stage of a two-stage game, firms choose between a mixed bundling and a non-bundling strategy. In the second stage, firms choose prices. We show that mixed bundling is a dominant strategy for both firms. However, when the composite goods are not very close substitutes, at the bundling-bundling equilibrium both firms are worse off than when they both commit not to practice mixed bundling.

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Mixed Bundling in Duopoly

1. Introduction

Bundling, that is the sale of two or more goods in combination as a package, is a common business practice. Following Adams and Yellen (1976), two types of this practice are distinguished, pure and mixed bundling. In *pure bundling,* the individual goods are not sold separately but are sold only in combination. In *mixed bundling* the individual goods, as well as the package, are available.[1]

In some cases, bundling may provide significant economies in the joint sale of the products as a bundle. Such economies may be in packaging or alleviation of information and search costs through the sale of "matching" components in a bundle. In other cases bundling may prevent problems of adverse selection. For example, a mandatory warranty ensures no adverse selection in the population buying warranties.

Often bundling is used as an instrument of price discrimination. The traditional theory of price discrimination (Adams and Yellen (1976), Schmalensee (1984), McAfee, McMillan and Whinston (1989)) is based on the assumption of monopoly. Under fairly general conditions they show that frequently bundling increases profits for the monopolist. In this paper, we focus on the possibility of mixed bundling under conditions of oligopoly. Other studies in this area are by Einhorn (1989), Matutes and Regibeau (1992), and Wilson (1991).[2]

Suppose we have a single firm (firm 1) that produces and sells two products, A_1, and B_1. Let their prices if sold individually be p_1 and q_1 respectively. If the firm does not practice mixed bundling, the price of the combination of one unit of A_1 and one unit of B_1

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[1] Examples of pure bundling are prix-fixe menus, mandatory warranties, bed with breakfast included in hotel accommodations, uniform delivered pricing, etc. Examples of mixed bundling are season tickets, monthly passes on trains, round-trip airline tickets, all-included vacation packages, film-with-camera, quantity discounts, etc.

[2] The topic also touches issues of vertical integration and separation discussed in Economides and Salop (1992).

is the sum of their prices $p_1 + q_1$. The composite good ("bundle") A_1B_1 may also be offered separately at a price s_1. Thus, not practicing mixed bundling can be thought of as operating under the restriction $s_1 = p_1 + q_1$ on the strategies of firm 1. In general, it is to the advantage of firm 1 to set all three prices, p_1, q_1, and s_1, since the lifting of the restriction $s_1 = p_1 + q_1$ through mixed bundling cannot hurt the profits of a monopolist. If there are no restrictions in the sale of A_1 and B_1, the bundle will sell positive amounts if $s_1 < p_1 + q_1$. In most cases, introduction of mixed bundling, i.e., the relaxation of the restriction $s_1 = p_1 + q_1$, will strictly decrease the price of the bundle s_1 and increase the prices of the separately-sold goods p_1 and q_1.

The conditions of competition can be quite different when there are more than one competing firms, and therefore the analysis of the effects of mixed bundling becomes more complicated. Suppose that two vertically integrated firms, each producing one component of each type (firm i produces A_i and B_i). Consider a two-stage game where in the first stage firms commit in using or not using a mixed bundling strategy, and in the second stage firms choose prices. Starting from a situation where no firm uses mixed bundling, suppose firm 1 decides to practice mixed bundling. As the price of firm 1's bundle s_1 decreases and the prices of firm 1's components p_1 and q_1 increase from their previous levels, the competing firm (firm 2) will adjust its prices in response. Despite the adjustments of the prices of firm 2, we expect that firm 1 will still be better off by using the mixed bundling strategy. If firm 2 practices mixed bundling as well, the effects are *a priori* uncertain. It is possible that, given that firm 1 practices mixed bundling, firm 2 prefers not to bundle. For the linear demand structure employed in this paper, it is shown that firm 2 prefers to bundle as well. Therefore the practice of (mixed) bundling is a dominant strategy for both firms and it constitutes an equilibrium.

There is no guarantee that profits will be higher in the bundling-bundling equilibrium than if both firms avoided bundling. Indeed, it is shown that, for most parameter values, the equilibrium profits are lower when both firms use mixed bundling than when both firms do not use mixed bundling. This is a classic prisoners dilemma situation. *Both firms use a dominant strategy and they are worse off than if both their respective dominant strategies were not available.*

Matutes and Regibeau (1992), working in a locational setting with 0-1 choices for consumers, find that, depending on the parameters, there can be two kinds of equilibria. In the first (that occurs for competitive situations), one firm bundles, and one does not. In the second, both firms bundle. Our results, in contrast, show both firms always practice mixed bundling at equilibrium.

The rest of the paper is organized as follows. The model is set up in section 2. In sections 3, 4, and 5, we discuss the price subgames that are played under given earlier decisions of firms to use mixed bundling or not to. In section 3 we consider bundling by both firms. In section 4 we discuss the case when no firm bundles. In section 5 there is bundling by only one firm. In section 6.1 we compare prices across regimes. In section 6.2 we compare profits in the different subgames, and establish the subgame perfect equilibrium. In section 7 there are concluding remarks.

2. Set-up

There are two types of goods, A and B. There are four components, two of each type, A_i, B_i, $i = 1, 2$. Composite good (system) A_iB_j is made of one unit each of A_i and B_j. It is available at price s_{ij}. The demand system for the general model is

$$D^{11} = D^{11}(s_{11}, s_{12}, s_{21}, s_{22}),$$

$$D^{12} = D^{12}(s_{12}, s_{11}, s_{22}, s_{21}),$$

$$D^{21} = D^{21}(s_{21}, s_{22}, s_{11}, s_{12}),$$

$$D^{22} = D^{22}(s_{22}, s_{21}, s_{12}, s_{11}).$$

When demand functions are linear, and the demand system is symmetric,[3] we have

$$D^{11} = a - bs_{11} + cs_{12} + ds_{21} + es_{22},$$
$$D^{12} = a - bs_{12} + cs_{11} + ds_{22} + es_{21},$$
$$D^{21} = a - bs_{21} + cs_{22} + ds_{11} + es_{12},$$
$$D^{22} = a - bs_{22} + cs_{21} + ds_{12} + es_{11}.$$

with $a, b, c, d, e > 0$. It is assumed that an increase in the prices of all four systems will decrease the demand for each system, i.e., $b > c + d + e$.

Suppose that firm i, i = 1,2, produces components A_i and B_i. This was named *parallel vertical integration* in Economides and Salop (1992). Each firm i has three strategic variables, the price of the combination of both of each components, s_{ii}, and the two prices of the individual components it produces, p_i and q_i. In the first stage, firms choose whether or not to practice mixed bundling in the subsequent stage of price choice. They each have two strategies, "B" for (mixed) bundling and "N" for no bundling. As explained earlier, the use of strategy "N" is interpreted as imposing the restriction $s_{ii} = p_i + q_i$ on the available strategies in the subsequent price subgame. Therefore strategy "N" can be interpreted as a commitment not to give preferential treatment to customers who buy other products of the firm.[4,5]

Three types of price subgames can result. The first type, (B, B), results when both firms have used the "B" strategy in the earlier stage. The second subgame, (N, N), results when both firms have used the "N" strategy earlier. The third type of subgame results when firms have

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[3] Symmetry of the demand system implies that, when the prices of all components are equal, the realized demands for all components are also equal.

[4] In the second stage of price choice, in the absence of a commitment not to use bundling, a firm will use mixed bundling.

[5] See also Salop (1986) for a discussion of other practices that facilitate cooperation in oligopoly.

used different strategies in the earlier stage, and includes games (B, N) and (N, B). We characterize the price equilibria in all three types of subgames. Then we go back and establish the subgame-perfect equilibrium in the overall game structure.

3. Bundling by Both Firms

When both firms use mixed bundling, prices are

$$s_{11} = s_1,\ s_{12} = p_1 + q_2,\ s_{21} = p_2 + q_1,\ s_{22} = s_2.$$

The market structure is shown in Figure 1. The demand system is

$$D^{11} = D^{11}(s_1, p_1 + q_2, p_2 + q_1, s_2),$$
$$D^{12} = D^{12}(p_1 + q_2, s_1, s_2, p_2 + q_1),$$
$$D^{21} = D^{21}(p_2 + q_1, s_2, s_1, p_1 + q_2),$$
$$D^{22} = D^{22}(s_2, p_2 + q_1, p_1 + q_2, s_1).$$

The profit functions of the two firms are

$$\Pi^1 = s_1D^{11} + p_1D^{12} + q_1D^{21},\ \ \Pi^2 = s_2D^{22} + p_2D^{21} + q_2D^{12}.$$

The non-cooperative equilibrium is characterized by the following conditions:

(1) $$\partial\Pi^1/\partial s_1 = D^{11} + s_1D_1^{11} + p_1D_2^{12} + q_1D_3^{21} = 0,$$

(2) $$\partial\Pi^1/\partial p_1 = s_1D_2^{11} + D^{12} + p_1D_1^{12} + q_1D_4^{21} = 0,$$

(3) $$\partial\Pi^1/\partial q_1 = s_1D_3^{11} + p_1D_4^{12} + D^{21} + q_1D_1^{21} = 0,$$

$$\partial\Pi^2/\partial s_2 = D^{22} + s_2D_1^{22} + p_2D_2^{21} + q_2D_3^{12} = 0,$$
$$\partial\Pi^2/\partial p_2 = s_2D_2^{22} + D^{21} + p_2D_1^{21} + q_2D_4^{12} = 0,$$
$$\partial\Pi^2/\partial q_2 = s_2D_3^{22} + p_2D_4^{21} + D^{12} + q_2D_1^{12} = 0,$$

where subscript k denotes the partial derivative with respect to the kth argument of the demand function. For the linear case with c = d = e, equations (1)-(3) translate to

(1') $$a - bs + cs + 2c(p+q) - bs + c(p+q) = 0$$

(2') $$a - b(p+q) + 2cs + c(p+q) + cs - bp + cq = 0$$

(3') $$a - b(p+q) + 2cs + c(p+q) + cs - bq + cp = 0.$$

which are solved[6] for the equilibrium prices of the unbundled components, $p^{B,B}$ and $q^{B,B}$, and of the bundles, $s^{B,B}$,[7]

(4) $$p^{B,B} = q^{B,B} = 2a/[3(2b - 5c)],$$

(5) $$s^{B,B} = a/(2b - 5c).$$

It is immediate that the bundle price is lower than the sum of the prices of the unbundled components,

$$s^{B,B} < p^{B,B} + q^{B,B}.$$

Thus, as expected, bundling will be used if it is an available strategy.

The equilibrium profits in the (Bundling, Bundling) subgame are

(6) $$\Pi_1^{B,B} = \Pi_2^{B,B} = a^2(17b - 32c)/[9(2b - 5c)^2].$$

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[6] From the last two, it follows that $p = q$. Simplifying we have

$$(2b - c)s - 6cp = a, \quad 3(b - c)p - 3cs = a,$$

which are solved for (4) and (5).

[7] The realized absolute values of the elasticities of demand at equilibrium are $e_{11}^{B,B} = 3b/(3b-4c)$, $e_{12}^{B,B} = 4b/(2b-5c)$.

4. No Bundling by Both Firms

When neither of the two firms bundles, the price of any system is the sum of the prices of its component parts,

$$s_{11} = p_1 + q_1,\ s_{12} = p_1 + q_2,\ s_{21} = p_2 + q_1,\ s_{22} = p_2 + q_2.$$

The profit functions of the vertically integrated firms are

$$\Pi^1 = \Pi_{A_1} + \Pi_{B_1} = p_1(D^{11} + D^{12}) + q_1(D^{11} + D^{21}).$$
$$\Pi^2 = \Pi_{A_2} + \Pi_{B_2} = p_2(D^{21} + D^{22}) + q_2(D^{12} + D^{22}).$$

Maximizing Π^1 with respect to p_1 and q_1 we have

(7) $\partial\Pi^1/\partial p_1 = D^{11} + D^{12} + p_1(D_1^{11}+D_2^{11}+D_1^{12}+D_2^{12}) + q_1(D_1^{11}+D_2^{11}+D_3^{21}+D_4^{21}) = 0.$

(8) $\partial\Pi^1/\partial q_1 = p_1(D_1^{11}+D_3^{11}+D_2^{12}+D_4^{12}) + D^{11} + D^{21} + q_1(D_1^{11}+D_3^{11}+D_1^{21}+D_3^{21}) = 0.$

Maximizing Π^2 with respect to p_2 and q_2 we have

(9) $\partial\Pi^2/\partial p_2 = D^{21} + D^{22} + p_2(D_1^{21}+D_2^{21}+D_1^{22}+D_2^{22}) + q_2(D_1^{22}+D_2^{22}+D_3^{12}+D_4^{12}) = 0.$

(10) $\partial\Pi^2/\partial q_2 = p_2(D_1^{22}+D_3^{22}+D_2^{21}+D_4^{21}) + D^{12} + D^{22} + q_2(D_1^{22}+D_3^{22}+D_1^{12}+D_3^{12}) = 0.$

The solution of these yields the equilibrium prices[8]

(11) $$p^{N,N} = q^{N,N} = 2a/(7b - 17c).$$

The equilibrium profits in the (No Bundling, No Bundling) subgame are

(12) $$\Pi_1^{N,N} = \Pi_2^{N,N} = 8a^2(3b - 5c)/(7b - 17c)^2.$$

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[8] The absolute values of the elasticities at equilibrium are $e_{11}^{N,N} = e_{12}^{N,N} = 4b/(3b-5c)$. It is clear that $e_{11}^{B,B} < e_{11}^{N,N} = e_{12}^{N,N} < e_{12}^{B,B}$.

5. Bundling by Only One Firm

Now suppose that only firm 1 bundles its components, while firm 2 commits in stage 1 not to sell its "pure" system at a lower price than the sum of the prices of its components. This is the (Bundling, No Bundling) or (B, N) case. The four systems are offered at prices,

$$s_{11} = s_1,\ s_{12} = p_1 + q_2,\ s_{21} = p_2 + q_1,\ s_{22} = p_2 + q_2.$$

The demand system is:

$$D^{11} = D^{11}(s_1,\ p_1 + q_2,\ p_2 + q_1,\ p_2 + q_2),$$
$$D^{12} = D^{12}(p_1 + q_2,\ s_1,\ p_2 + q_2,\ p_2 + q_1),$$
$$D^{21} = D^{21}(p_2 + q_1,\ p_2 + q_2,\ s_1,\ p_1 + q_2),$$
$$D^{22} = D^{22}(p_2 + q_2,\ p_2 + q_1,\ p_1 + q_2,\ s_1).$$

The profit functions of the two firms are

$$\Pi^1 = s_1 D^{11} + p_1 D^{12} + q_1 D^{21},\ \ \Pi^2 = \Pi_{A_2} + \Pi_{B_2} = p_2(D^{21} + D^{22}) + q_2(D^{12} + D^{22}).$$

Profit maximization by firm 1 implies

(13) $$\partial\Pi^1/\partial s_1 = D^{11} + s_1 D^{11}_1 + p_1 D^{12}_2 + q_1 D^{21}_3 = 0,$$

(14) $$\partial\Pi^1/\partial p_1 = s_1 D^{11}_2 + D^{12} + p_1 D^{12}_1 + q_1 D^{21}_4 = 0,$$

(15) $$\partial\Pi^1/\partial q_1 = s_1 D^{11}_3 + p_1 D^{12}_4 + D^{21} + q_1 D^{21}_1 = 0,$$

Profit maximization by firm 2 implies

(16) $$\partial\Pi^2/\partial p_2 = D^{21} + D^{22} + p_2(D^{21}_1 + D^{21}_2 + D^{22}_1 + D^{22}_2) + q_2(D^{22}_1 + D^{22}_2 + D^{12}_3 + D^{12}_4) = 0.$$

(17) $$\partial\Pi^2/\partial q_2 = p_2(D^{22}_1 + D^{22}_3 + D^{21}_2 + D^{21}_4) + D^{12} + D^{22} + q_2(D^{22}_1 + D^{22}_3 + D^{12}_1 + D^{12}_3) = 0.$$

The solution of the system (13)-(17) gives the following equilibrium prices:

(18) $$s_1^{B,N} = a(11b - 9c)/(2F),\ p_1^{B,N} = q_1^{B,N} = a(4b - 3c)/F,$$

(19) $$p_2^{B,N} = q_2^{B,N} = 3a(b - c)/F,$$

$$\text{where } F = (11b^2 + 24c^2 - 37bc).$$

It is immediate that the bundle price is lower than the sum of the prices of the unbundled components that compose it,

$$s_1^{B,N} < p_1^{B,N} + q_1^{B,N}.$$

The interesting observation here is that the bundle price, $s_1^{B,N}$, is higher than sum of the prices of the components produced by the non-bundling firm, $p_2^{B,N} + q_2^{B,N}$, but lower than the price of a hybrid system that is composed of a component from the bundling firm and a component from the non-bundling firm, $p_i^{B,N} + q_j^{B,N}$, $i \neq j$,

$$p_2^{B,N} + q_2^{B,N} < s_1^{B,N} < p_1^{B,N} + q_2^{B,N} = p_2^{B,N} + q_1^{B,N}.$$

The equilibrium profits in the (Bundling, No Bundling) equilibrium are

(20) $$\Pi_1^{B,N} = 3a^2(83b^3 - 290b^2c + 299bc^2 - 96c^3)/[4(11b^2 + 24c^2 - 37bc)^2],$$

(21) $$\Pi_2^{B,N} = 18a^2(b - c)^2(3b - 5c)/(11b^2 + 24c^2 - 37bc)^2.$$

The firm that practices mixed bundling has higher profits,

$$\Pi_1^{B,N} > \Pi_2^{B,N}$$

Thus, price discrimination by one firm puts it in a superior position compared to the non-discriminating firm.

6. Comparisons

6.1 Price Comparisons

We now compare the equilibrium prices of the three regimes. The equilibrium prices compare as follows[9]

(22) $$s^{B,B} < s_1^{B,N} < p_2^{B,N} + q_2^{B,N} < p^{N,N} + q^{N,N} < p_1^{B,N} + q_2^{B,N} < p^{B,B} + q^{B,B} < p_1^{B,N} + q_1^{B,N}$$

Below we construct this inequality from its component parts and we give intuitive explanations.

Starting from a regime where no-one bundles (N,N), the firm that decides to bundle increases the prices of its components when sold separately and decreases their price when bundled,

$$s_1^{B,N} < p^{N,N} + q^{N,N} < p_1^{B,N} + q_1^{B,N}.$$

The opponent non-bundling firm responds by decreasing the prices of its components below the original prices before bundling, but above the price of the bundled good,

$$s_1^{B,N} < p_2^{B,N} + q_2^{B,N} < p^{N,N} + q^{N,N}.$$

The hybrid systems, which contain one component from the firm that bundles and one component from the firm that does not bundle, are sold at a higher price than in the (No-bundling, No-bundling) regime,

$$p^{N,N} + q^{N,N} < p_1^{B,N} + q_2^{B,N}.$$

Starting from the (Bundling, No-bundling) regime where only firm 1 practices mixed bundling, if firm 2 decides to bundle as well, it will charge a price for its bundle, $s^{B,B}$, below the sum of its component prices previously, $p_2^{B,N} + q_2^{B,N}$, and below the price of the opponent's bundle in the previous regime, $s_1^{B,N}$.

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[9] We introduce the convention of dropping subscripts when both firms have the same prices.

$$s^{B,B} < s_1^{B,N} < p_2^{B,N} + q_2^{B,N}.$$

When the second firm practices mixed bundling as well, it increases the price of its unbundled components, $p^{B,B} + q^{B,B}$, above their price in the (B, N) regime, $p_2^{B,N} + q_2^{B,N}$, as well as above the price of a hybrid system in the (B, N) regime, $p_1^{B,N} + q_2^{B,N}$.

$$p_2^{B,N} + q_2^{B,N} < p_1^{B,N} + q_2^{B,N} < p^{B,B} + q^{B,B}$$

However, the price of the unbundled components in the (B, B) regime, $p^{B,B} + q^{B,B}$, is below the price of the unbundled components of the bundling firm in the (B, N) regime, $p_1^{B,N} + q_1^{B,N}$.

$$p^{B,B} + q^{B,B} < p_1^{B,N} + q_1^{B,N}.$$

Firm 1, that was practicing mixed bundling in the (B, N) regime responds to mixed bundling by firm 2 by lowering the price of its bundle.

$$s^{B,B} < s_1^{B,N}.$$

Firm 1 also lowers the price of its unbundled components (as a response to firm 2's switching to a mixed bundling strategy) from $p_1^{B,N} + q_1^{B,N}$ to $p^{B,B} + q^{B,B}$, but not below the price of a hybrid system in the (B, N) regime, $p_1^{B,N} + q_2^{B,N}$.

$$p_1^{B,N} + q_2^{B,N} < p^{B,B} + q^{B,B} < p_1^{B,N} + q_1^{B,N}.$$

6.2 Comparison of Profits

Starting from a situation where no-one bundles, and assuming that the opponent does not bundle, a firm always has an incentive to bundle,

$$\Pi^{N,N} < \Pi_1^{B,N} \quad \text{and} \quad \Pi^{N,N} < \Pi_2^{N,B}. \tag{23}$$

Given that firm 1 bundles, firm 2 has higher profits if it bundles as well,

(24) $$\Pi_2^{B,N} < \Pi^{B,B}, \text{ and } \Pi_1^{N,B} < \Pi^{B,B}.$$

Thus, *"bundling" is a dominant strategy for each firm in the first stage of the game, and (Bundling, Bundling) is an equilibrium in dominant strategies.* See Figure 2.

		Player 2	
		No Bundling	(Mixed) Bundling
Player 1	No Bundling	$(\Pi^{N,N}, \Pi^{N,N})$	$(\Pi_1^{N,B}, \Pi_2^{N,B})$
	(Mixed) Bundling	$(\Pi_1^{B,N}, \Pi_2^{B,N})$	$(\Pi^{B,B}, \Pi^{B,B})$

Figure 2: The first stage of the game.

Theorem 1: In a two-stage game, where firms choose in the first stage whether they will allow themselves to use mixed bundling in the second stage in which prices are set, at the subgame-perfect equilibrium both firms use mixed bundling. Using mixed bundling is a dominant strategy for each firm in the first stage of the game.

Note that a firm's profits decrease as a result of a switch to bundling by the opponent, both when the firm is bundling and when it is not. For example, firm 2, following a non-bundling strategy, is hurt by the switch of firm 1 to bundling,

$$\Pi_2^{B,N} < \Pi^{N,N},$$

and firm 1, following a bundling strategy is hurt by the switch of firm 2 to bundling,

$$\Pi^{B,B} < \Pi_1^{B,N}.$$

These two comparisons, are of course not important for the equilibrium of the game, since they compare profits of firm i across situations that differ only in the strategy of firm $j \neq i$.

Comparing equations (6) and (12), we find that when composite goods are not very close substitutes, i.e., for $b/c > 3.24$, firms realize lower profits when they both bundle than when they both do not bundle,[10]

$$\Pi^{B,B} < \Pi^{N,N}.$$

Conversely, when the composite goods are very close substitutes, firms are better off when they both bundle.

<u>Theorem 2</u>: **When the composite goods are not very close substitutes, firms are better off when they both commit *not* to use mixed bundling than when they both use such strategies.**

Thus, when goods are not close substitutes, firms end up bundling when they would have been better off not bundling. This is a typical prisoners dilemma situation. Firms would be better off if they did not have the mixed bundling strategy available, for example, if bundling were illegal.[11]

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[10] $\Pi^{B,B} - \Pi^{N,N} = -a^2(b + c)(31b^2 - 177bc + 248c^2)/[9(7b - 17c)^2(2b - 5c)^2]$. The roots of the profit difference in b/c are -1, 3.242, and 2.46. Therefore it is positive for $3 < b/c < 3.242$ and negative for $3.242 < b/c$.

[11] These equilibria can also be compared to an equilibrium of "independent ownership" (described in Economides and Salop (1992)) where each of the four components is produced by a different firm. Then the equilibrium prices are

$$p^I = q^I = a/(3b - 7c).$$

These prices are above the equilibrium prices in the (No Bundling, No Bundling) regime, $p^{N,N} + q^{N,N}$, and below the prices for separately sold components in the (Bundling, Bundling) regime, $p^{B,B} + q^{B,B}$,

$$p^{N,N} + q^{N,N} < p^I + q^I < p^{B,B} + q^{B,B}.$$

7. Concluding Remarks

In the traditional literature, mixed bundling by monopolists is a profitable strategy. In duopoly, we showed two results. First, mixed bundling is used at equilibrium by both firms; in fact, the use of mixed bundling is a dominant strategy for both firms. Second, when goods are not very close substitutes, using mixed bundling leads to lower profits for both firms in comparison to the profits achieved when both firms avoid using mixed bundling. Thus, this is a typical Prisoners' Dilemma situation: a profit-enhancing strategy under monopoly is still used by all firms in duopoly, although all players would be better off if this "profit-enhancing" strategy were not available.

In many areas of regulation, including, for example, telephone utility regulation, there are strict restrictions against bundling. Although they were primarily meant to be against pure bundling, they restrict mixed bundling as well. The theoretical rationale of these restrictions comes from the analysis of monopoly bundling. Our results show that equilibrium in duopoly competition is qualitatively different, and therefore these restrictions are misguided. Firms would compete more vigorously and would realize lower profits if restrictions on *mixed* bundling were relaxed.

An important implication of this paper is that the application to duopoly of results derived from the analysis of monopoly is at least suspect. In fact, intuitively it seems likely that the use of other price discriminating strategies may produce similarly contrasting results in monopoly and duopoly settings. The analysis of this conjecture is an interesting open question.

In fact for $3 < b/c < 6$ the prices under independent ownership also lie below the price for a hybrid system in the (B, N) regime,

$$p^I + q^I < p_1^{B,N} + q_2^{B,N} < p^{B,B} + q^{B,B}.$$

Profits in independent ownership are in general smaller that in both the (Bundling, Bundling) and the (No bundling, No bundling) regimes. $\Pi^I < \Pi^{B,B}$ for $b/c > 3.11$, and $\Pi^I < \Pi^{N,N}$ for $b/c > 5.61$.

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