One-Sided and Two-Sided Commitments

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1986

Research Working Paper Series, Center for Telecommunications and Information Studies, Columbia University Graduate School of Business, 809 Uris, New York, NY 10027. (212) 280-4222.

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Revised September 1986

Abstract

We analyse through a series of examples the effects of precommitment in a strategic variable (quality). We show that while precommitment increases the profits of the precommitting firm it may harm or benefit its passive opponent depending on the marginal cost of the precommitting variable. The benefit to a firm of precommitment increases when the opponent precommits. The option to increase the quality in the second stage never helps the precommitting firm and in fact it harms it when marginal costs of quality are high.

Financial support from the National Science Foundation is gratefully acknowledged.

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One-sided and Two-sided Commitments

1. Introduction

In this paper we analyse with the help of examples the significance of credible commitment in a strategic variable. There are numerous situations where firms have the option to commit themselves. For example, firms can commit themselves by acting earlier rather than later or by writing binding contracts about their actions in later periods.

That a firm may be better off by committing in some strategic variable is known. Here we want to discuss the effects of precommitment on the opponent firm which may also have the possibility to commit itself. In particular we are interested to compare market structures where both firms can commit themselves to ones where only one firm can commit itself and to market structures where no firm can make a credible commitment. We discuss a number of games in the context of a simple example of quality and price duopoly. Firms may commit themselves in quality according to rules specified below.[1] Quality level here is an "aggressive" strategic variable in the sense that increases in quality of a firm has direct positive effects on its own demand, as well as direct negative effects on the demand of the opponent.

In the first game to be analysed, quality levels and prices are chosen simultaneously. This we name the one-stage game. In the second game we allow firms to choose simultaneously their quality levels in a first stage while price choices are made simultaneously in the second stage. To assure credibility we seek subgame-perfect equilibria. We refer to this as the <u>two-stage simultaneous precommitment</u> game. In the third game we give firm 1 the opportunity to choose its quality level in stage one, while firm 2 has to choose its quality in stage two, together with the price levels choices of both firms. Again we seek subgame-perfect equilibria. This we name the <u>one-sided precommitment</u> game.

In these three games the ability of each firm to choose quality in a particular stage of the game is specified in advance. Subsequently we allow firms to choose to commit or not, i.e. to choose the game structure in which they will compete. Depending on their choice the firms will find themselves in one of the above three games or in the mirror image of the third, where only firm 2 precommits. Viewing the choice of game structures as a game in itself we analyse it and find its non-cooperative equilibria. Next, we analyse a fifth game where firm 2 has the same moves as in the one-sided precommitment game, while firm 1 has the added option to increase quality in the second stage. The sixth game is like the simultaneous precommitment game with the added option to one firm to increase its quality level in the second stage. Finally, the seventh game views the choice of game structures as a game in itself; firms are given the option to choose quality in any stage as well as increase the level of the precommittted quality in the second stage.

2. The One-stage, No Precommitment Game

We first outline the basic market conditions. Two firms $j = 1, 2$, choose quality levels a_j and price levels p_j. They face demand $D_j = b + p_i - p_j + a_j - a_i$.[2,3] The costs of quality are $C(a_j) = ca_j^2/2$. There are no costs of production. However, a fixed cost plus constant marginal cost technology leaves the results virtually unaffected.[4] The profit function of firm j is

$$\Pi_j = p_j(b + p_i - p_j - a_i + a_j) - ca_j^2/2.$$

In the one-stage game quantities and quality levels are set simultaneously by both firms. First order conditions are[5]

$$(1) \qquad b - 2p_1 + p_2 + a_1 - a_2 = 0,$$

$$(2) \qquad b - 2p_2 + p_1 + a_2 - a_1 = 0,$$

$$(3) \qquad p_1 = ca_1$$

$$(4) \qquad p_2 = ca_2,$$

which result in equilibrium levels and profits

$$(5) \quad p_1^{1s} = p_2^{1s} = b, \quad a_1^{1s} = a_2^{1s} = b/c, \quad \Pi_1^{1s} = \Pi_2^{1s} = b^2(1 - 1/(2c)).$$

The superscript (1s) is used to denote that all choices happen in one stage.[6]

3. The Two-stage Simultaneous Precommitment Game

In this game firms choose simultaneously quality levels in the first stage, and price levels in the second stage. Subgame perfection assures that precommitments are credible. Imposing subgame perfection restricts the choices of firms in each stage of the game. In the last stage quality cannot be varied, while in the first stage quality has to be chosen taking into account the effects of such choice in the prices chosen subsequently.

The extent to which firms profit or lose from this strategic restriction in comparison to the simultaneous game is a priori unclear.

Consider the derivative of profits of firm 1 with respect to a_1:

$$d\Pi_1^{2s}/da_1 = \partial\Pi_1/\partial a_1 + \partial\Pi_1/\partial p_1 \cdot dp_1^*/da_1 + \partial\Pi_1/\partial p_2 \cdot dp_2^*/da_1,$$

where $p_j^*(a_1, a_2)$, $j = 1, 2$, are the equilibrium strategies in the subgame. When evaluated at the equilibrium of the simultaneous choice game (1s) the first two terms are zero and therefore

$$d\Pi_1^{2s}/da_1 = \partial\Pi_1/\partial p_2 \cdot dp_2^*/da_1.$$

Since the products of firms 1 and 2 are substitutes, $\partial\Pi_1/\partial p_2 > 0$. The sign of the second factor dp_2^*/da_1 is ambiguous. It measures the price response by a firm to changes in the quality that the opponent played in the earlier stage. If the result is adverse, $dp_2^*/da_1 < 0$, we call a_1 and p_2 "conflict strategies." In this particular game $dp_2^*/da_1 < 0$, as seen below. This implies $d\Pi_1^{2s}/da_1 < 0$, and therefore $a_1^{2s} < a_1^{1s}$ -- firms advertise less in the simultaneous precommitment game than in the simultaneous play game.

In the price subgame first order conditions on prices, (1), (2) are solved in

(6) $$p_1^* = (3b + a_1 - a_2)/3, \quad p_2^* = (3b + a_2 - a_1)/3$$

Equilibrium profits in the subgame, which are also the payoff functions of the first-stage game, are $\Pi_j^* = (3b + a_j - a_i)^2/9 - ca_j^2/2$. A non-cooperative equilibrium in the first-stage game is defined by $\partial\Pi_j^*/\partial a_j = 0$, $j = 1, 2$, which are solved to define the equilibrium quality levels, the implied prices, and equilibrium profits:[7]

$$(7) \qquad a_1^{2s} = a_2^{2s} = 2b/(3c), \quad p_1^{2s} = p_2^{2s} = b,$$
$$\Pi_1^{2s} = \Pi_2^{2s} = b^2(1 - 2/(9c)).^8$$

We note that the availability of precommitment to both sides in this two-stage game increased their profits over the no-precommitment, one-stage game, i.e. $\Pi_j^{2s}(c) > \Pi_j^{1s}(c)$. In the precommitment game firms support the same equilibrium prices as in the one-stage game with lower expenditure on quality: $p_j^{1s} = p_j^{2s}$ but $a_j^{1s} > a_j^{2s}$ from the comparison of equations (5) and (7). Thus their quality costs are lower and their profits higher when they precommit.

<u>Proposition 1: The two-stage simultaneous precommitment game results in lower levels of quality and higher profits than the one-stage simultaneous move game.</u>

This result may lead one to believe that quality precommitment is profitable in general. One may even jump to the conjecture that had firm 1 precommitted alone it would have reaped even higher profits. The next section shows the fallacy of this conjecture.

4. Precommitment by One Firm Only

In the following game only firm 1 is allowed to precommit in quality in the first stage. Firm 2 chooses quality in the second stage, where also prices are chosen by both firms. We denote the equilibrium outcomes with superscript (1c) because firm 1 commits itself.

In the second stage game, first order conditions in prices of both firms and in quality choice of firm 2 are (1), (2) and (4). Solving (1) and (2) yields (6), which together with (4) implies

$$a_2 = (3b - a_1)/(3c - 1), \quad p_2 = c(3b - a_1)/(3c - 1),$$
$$p_1 = [ca_1 + (3c - 2)b]/(3c - 1).$$

Equilibrium profits of firm 1 in the subgame are

$$\Pi_1 = [ca_1 + (3c - 2)b]^2/(3c - 1)^2 - ca_1^2/2.$$

Firm 1 maximizes these with respect to a_1 in the first stage. Maximization yields

(8a) $$a_1^{1c} = 2(3c - 2)b/[(3c - 1)^2 - 2c].$$

The full equilibrium includes

(8b) $$a_2^{1c} = b(9c - 7)/[(3c - 1)^2 - 2c],$$

(8c) $$p_2^{1c} = cb(9c - 7)/[(3c - 1)^2 - 2c],$$

(8d) $$p_1^{1c} = b(3c - 1)(3c - 2)/[(3c - 1)^2 - 2c].$$

Equilibrium profits are[9]

(8e) $$\Pi_1^{1c} = b^2(3c - 2)^2/[(3c - 1)^2 - 2c],$$

(8f) $$\Pi_2^{1c} = cb^2(c - 1/2)(9c - 7)^2/[(3c - 1)^2 - 2c]^2.$$ [10]

It is interesting to compare these values with those of the equilibria of the two previously analysed games. For low costs of quality, $c < 1$, firm 1 charges a higher price and firm 2 charges a lower price than the ones they charged in the simultaneous one-stage game: $p_1^{1c} > p_1^{1s} = p_2^{1s} > p_2^{1c}$. In the same range of costs, quality and profits levels compare similarly: $a_1^{1c} > a_1^{1s} = a_2^{1s} > a_2^{1c}$, $\Pi_1^{1c} > \Pi_1^{1s} = \Pi_2^{1s} > \Pi_2^{1c}$.[11] See figures 1 and 2. Further, the profits of the precommitting firm are never higher than its profits in the simultaneous precommitment game:

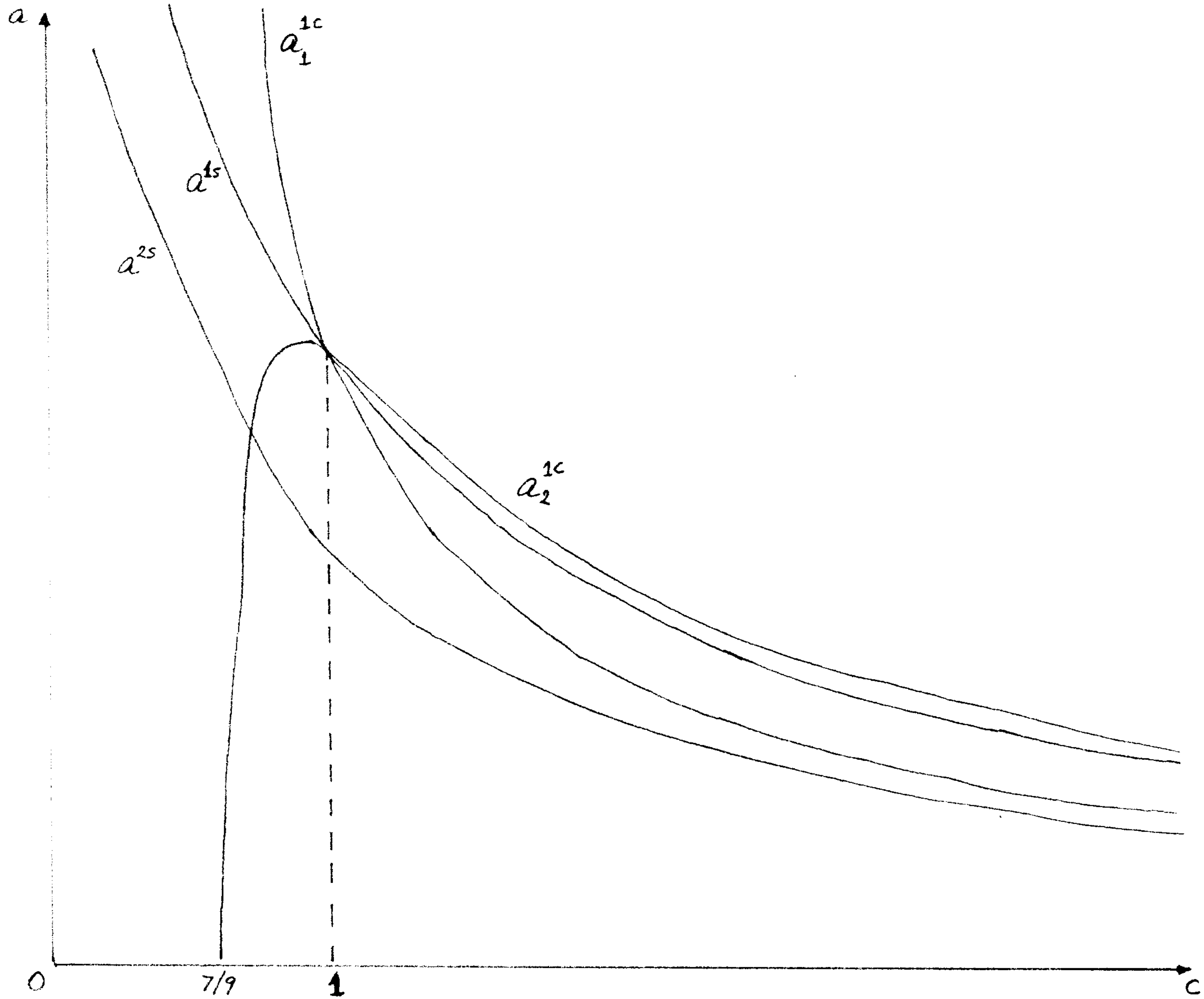

Figure 1: Equilibrium advertising levels.

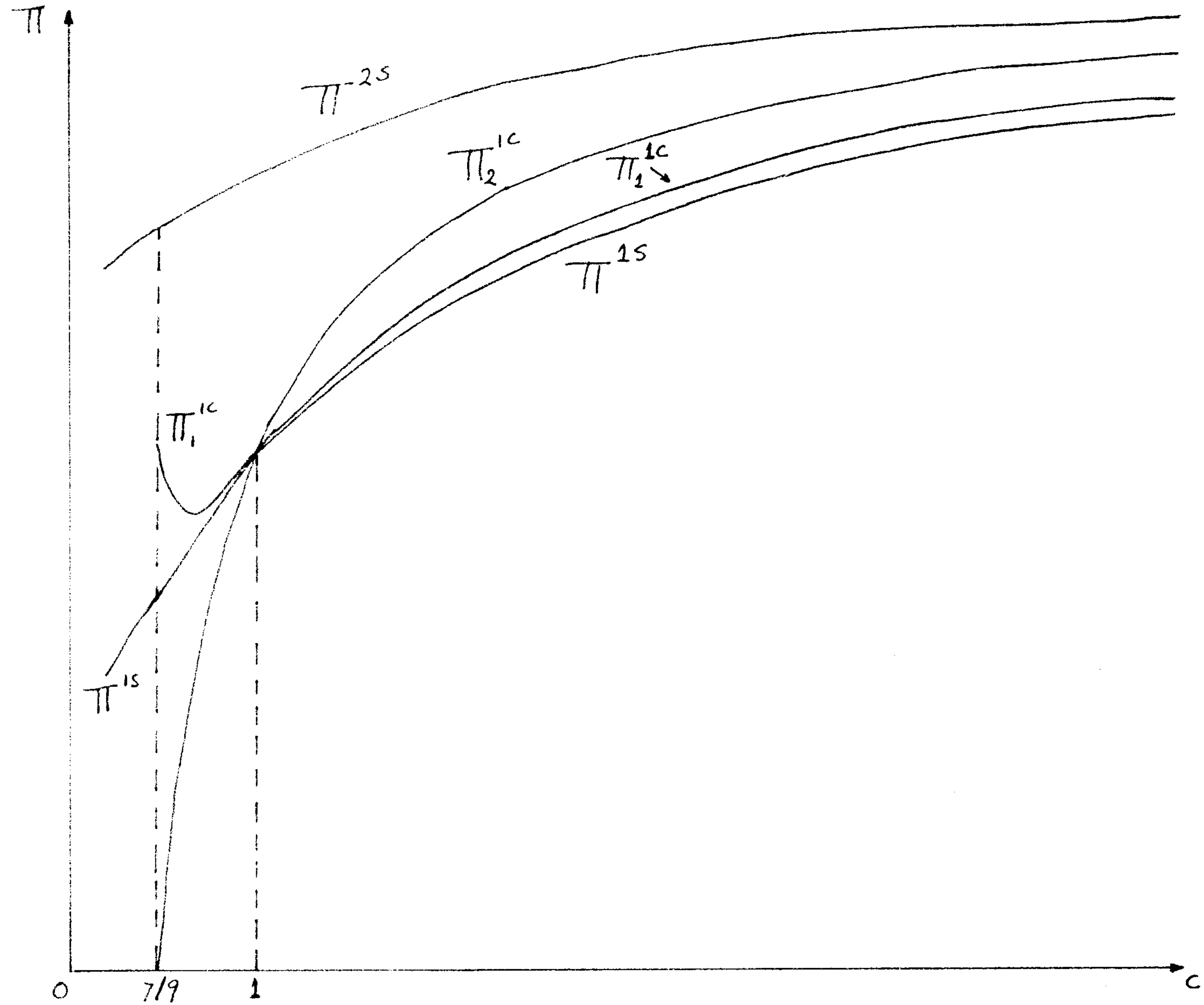

Figure 2: Equilibrium profit levels.

$\Pi_1^{1c} < \Pi_1^{2s}$.[12] This low cost case exhibits equilibria that confirm most of our intuition on the effects of commitment. The strategic power of the precommitting firm is utilized to ensure a higher price and quality level in comparison with its rival in this game as well as in comparison to itself in the no-precommitment game. However the precommitting firm is worse off than in the game where both firms simultaneously precommit.

When costs of quality are high, c > 1, the precommitting firm charges a lower price and sets a lower quality level while firm 2 charges a higher price and sets a higher quality level than in the simultaneous game: $p_1^{1c} < p_1^{1s} = p_2^{1s} < p_2^{1c}$, $a_1^{c} < a_1^{1s} = a_2^{1s} < a_2^{1c}$. See figure 1. Profits for the precommitting firm are higher than those of the simultaneous game, but lower than the ones of the firm which does not precommit, which are themselves lower than the profits of the simultaneous precommitment game: $\Pi_1^{1s} < \Pi_1^{1c} < \Pi_2^{1c} < \Pi_j^{2s}$. See figure 2. These results run counter to our intuition. The availability of precommitment increases the profits of the precommitting firm over the simultaneous game, but it increases even more the profits of the "passive" opponent. This is reminiscent of the equilibrium of the game of a cartel acting as a price leader with a fringe of competitive followers. There both sides gain from an increase of the size of the cartel, but the "passive" side gains much more than the cartel members.[13]

<u>Proposition 2: When only firm 1 precommits in quality in stage 1, its profits are higher than in the no-precommitment game, but lower than in the game of simultaneous precommitment by</u>

both sides. The precommitting firm reaps lower (higher) profits than the passive opponent when costs are high (low).

It is important to see how and why our results are sensitive on quality costs. Firm 1 is restricted to announce its quality level, a_1^1, in advance. This can be of help if it is in the interest of firm 1 to announce a large a_1^1, so that firm 2 is forced to respond with a very small a_2. This is the case when quality costs are small, $c < 1$. However, when quality costs are high, $c > 1$, firm 1 does not want to commit a large a_1^1. Since it has to commit in advance, firm 1 is in a weak strategic position. Firm 2 is able to take full advantage of this and have higher quality, prices and profits.

5. Choices when the Option to Precommit is Available

Up to this point we have considered precommitment as not costly. In some cases using a precommitting strategy may be no more costly than using a no-commitment strategy. In most cases, however, we expect that a precommitting firm will have to incur some costs F_j to make the commitment credible.[14] Now we are in position to analyse the behavior of firms that are given the opportunity to precommit, but do not have to take it. Consider the two-person game where each side has two strategies, C for commitment and NC for no commitment. When both firms play NC the simultaneous play one-stage game (1s) results. When both play C the two-stage game (2s) results. When firm 1 plays C while firm 2 plays NC the result is the one-sided precommitment game (1c). Similarly the play (NC, C) results in game (2c) where only firm 2 precommits. The payoff matrix of this game is:

		Player 2	
		C	NC
Player 1	C	$(\Pi_1^{2s}-F_1,\ \Pi_2^{2s}-F_2)$	$(\Pi_1^{1c}-F_1,\ \Pi_2^{1c})$
	NC	$(\Pi_1^{2c},\ \Pi_2^{2c}-F_2)$	$(\Pi_1^{1s},\ \Pi_2^{1s})$

The equilibria of this game are as follows. A precommitment equilibrium (C, C) exists when $\Pi_1^{2s} - F_1 > \Pi_1^{2c}$ and $\Pi_2^{2s} - F_2 > \Pi_2^{1c}$ are true. A no-commitment equilibrium (NC, NC) exists when $\Pi_1^{1c} - F_1 < \Pi_1^{1s}$ and $\Pi_2^{2c} - F_2 < \Pi_2^{1s}$. Since $\Pi_1^{1c} - \Pi_1^{1s} < \Pi_2^{2s} - \Pi_2^{1c}$, (C, C) is the unique equilibrium for $F_1, F_2 < \Pi_1^{1c} - \Pi_1^{1s}$; (NC, NC) is the unique equilibrium for $F_1, F_2 > \Pi_2^{2s} - \Pi_2^{1c}$; and both (C, C) and (NC, NC) are equilibria for $\Pi_1^{1c} - \Pi_1^{1s} < F_1, F_2 < \Pi_2^{2s} - \Pi_2^{1c}$. The gain of a firm through precommitment is larger under the assumption that the opponent has precommitted: $\Pi_2^{2s} - F_2 - \Pi_2^{1c} > \Pi_2^{2c} - F_2 - \Pi_2^{1s} = \Pi_1^{1c} - F_2 - \Pi_1^{1s}$. Therefore when firms have the same precommitment technology, $F_1 = F_2$, there are no equilibria where only one side precommits. When the precommitment costs differ across firms, asymmetric equilibria can arise. Equilibrium (C, NC) exists if $F_2 > \Pi_2^{2s} - \Pi_2^{1c}$ and $F_1 < \Pi_1^{1c} - \Pi_1^{1s}$. Since $\Pi_2^{2s} - \Pi_2^{1c}$ lies always above $\Pi_1^{1c} - \Pi_1^{1s}$ the existence of an asymmetric equilibrium requires that precommiment costs are substantially different across firms. Because the incentive to commit is strengthened when the opponent precommits ($\Pi_1^{2s} - \Pi_1^{2c} - F_1 > \Pi_1^{1c} - \Pi_1^{1s} - F_1$), when (C, NC) is an equilibrium then the "C" strategy is dominant and therefore no other equilibria exist. The equilibrium regions are shown in figure 3.

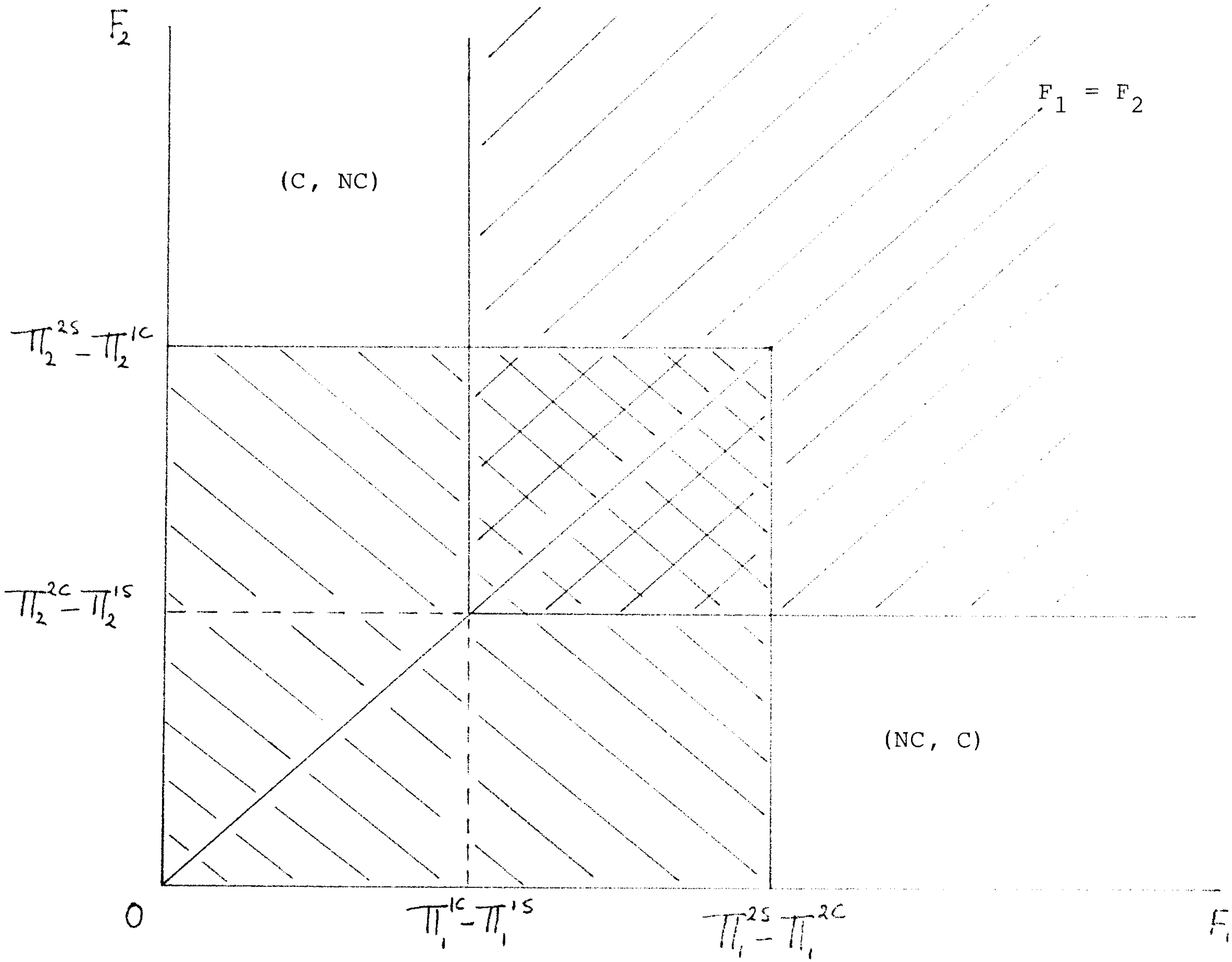

Figure 3: Regions of existence of equilibrium. (NC, NC) ///, (C, C) \\\\\\.

Proposition 3: When firms faced with the same precommitment technology choose non-cooperatively on precommitment, there is a unique two-sided commitment equilibrium for low commitment costs, a unique no-commitment equilibrium for high commitment costs, and both equilibria for intermediate commitment costs.

Proposition 4: For sufficiently different commitment costs $F_1 < \Pi_1^{1c} - \Pi_1^{1s} < \Pi_2^{2s} - \Pi_2^{1c} < F_2$ there exists a unique asymmetric equilibrium where only firm 1 commits.

When both (NC, NC) and (C, C) are equilibria and costs are high, c > 1, (C, C) is Pareto superior. Profits for firm 1 at (C, C) are $\Pi_1^{2s} - F$, larger than Π_1^{1s}, which are the profits at (NC, NC): $\Pi_1^{2s} - \Pi_1^{1s} - F = \Pi_1^{2s} - \Pi_1^{1s} - (\Pi_1^{2s} - \Pi_1^{2c}) + (\Pi_1^{2s} - \Pi_1^{2c}) - F > \Pi_1^{2s} - \Pi_1^{1s} - (\Pi_1^{2s} - \Pi_1^{2c}) = \Pi_1^{2c} - \Pi_1^{1s} = \Pi_2^{1c} - \Pi^{1s} > 0$, where the first inequality holds because (C, C) is an equilibrium, and the second inequality holds for c > 1. For low costs of quality, c < 1, it is not clear which non-cooperative equilibrium Pareto dominates.

6. The Option to Increase the Level of Quality in the Second Stage

We now analyse a game which is like the one-sided precommitment game analysed earlier except for the added possibility for firm 1 to increase its quality level in stage 2. Firm 2 is restricted to choose quality in stage 2, and both firms choose prices in stage 2. Let a_1^1 (respectively a_1^2) be the quality choice of firm 1 in stage 1 (2). In the stage 2 subgame the payoff functions are

$$\Pi_1 = p_1(b + p_2 - p_1 - a_2 + a_1^1 + a_1^2) - C(a_1^1 + a_1^2)$$

$$\Pi_2 = p_2(b + p_1 - p_2 + a_2 - a_1^1 - a_1^2) - C(a_2)$$

It is important that the costs of firm 1 are <u>not</u> represented by $C(a_1^1) + C(a_1^2)$. Such representation would give firm 1 a direct cost advantage because $C(a_1^1) + C(a_1^2) < C(a_1^1 + a_1^2)$ since the function C(.) is convex. Of course, in the subgame it is required that $a_1^2 \geq 0$ so that the first stage commitment is credible.

We first solve for the equilibrium of the second stage subgame. If the constraint $a_1^2 \geq 0$ is not binding, the subgame is equivalent to the one-stage no-precommitment game with $a_1^1 + a_1^2$ substituted for a_1 in conditions (1)-(4). Their solution is $p_1 = p_2 = b$, $a_1^1 + a_1^2 = a_2 = b/c$, $\Pi_1 = \Pi_2 = \Pi^{1s} = b^2[1 - 1/(2c)]$. This is the equilibrium of the subgame, provided that $a_1^1 < a_1^{1s}$. If firm 1 commits $a_1^1 \geq a_1^{1s}$ in stage 1, in stage 2 it has no incentive to increase quality; hence $a_1^2 = 0$. Then the subgame has the same equilibrium as the preceding game where the option of increasing quality by firm 1 in stage 2 was not available -- where the superscript "1c" was used. Thus, the equilibrium is

$$a_1^1 = a_1^{1c},\quad a_1^2 = 0,\quad a_2 = a_2^{1c},\quad p_1 = p_1^{1c},\quad p_2 = p_2^{1c},\quad \Pi_j = \Pi_j^{1c}$$
$$\text{if} \quad a_1^{1c} > a_1^{1s},$$

$$a_1^1 + a_1^2 = a_2 = a_1^{1s},\quad 0 \leq a_1^1 \leq a_1^{1s},\quad p_1 = p_2 = b,\quad \Pi_1 = \Pi_2 = \Pi^{1s}$$
$$\text{if} \quad a_1^{1c} < a_1^{1s}.$$

It can be calculated that $a_1^{1c} > a_1^{1s} = b/c \iff 1/3 < c < 1$.[15] Therefore, when costs are low, $c \leq 1$, the equilibrium of this game coincides with the one of the similar game without the option of adding quality in the second stage. The payoff remains the same despite the added option in the second stage. This is not surprising. When it is best for it to set a high quality, as in the present case of low quality costs, firm 1 should do all spending on quality in period 1 where it has the added effect of diminishing the quality of the opponent. When costs are high, $c > 1$, the added option is detrimental. It decreases the payoff of the precommitting firm to the level of the one-stage no-precommitment game. In this case of high quality costs, firm 1 would have liked to set a low quality level (a_1^{1c}) in stage 1, and not increase quality in stage 2. The added option "forces" the precommitting firm to increase quality in the second stage -- an act it would avoid if it could.

Proposition 5: The option of firm 1 to increase its stage 1 quality precommitment in stage 2 never increases profits. When costs of quality are low, $c < 1$, firm 1 is as well off as when the option was not available. When quality costs are high, $c > 1$, the profits of firm 1 are lower than when the option was not available.

This proposition underlines the fact that the availability of a strategic option can be detrimental. In the high cost case firm 1 is willing to pay to write a contract that will prohibit it from increasing quality in the second stage. Note further, that in the high cost case the profits of firm 2 are decreased as well when firm 1 has the added option. Thus it is in the common interest of firms to make sure that firm 1 never has the option to increase quality in the second stage.

To complete the picture we now analyse the game where both firms set quality levels $a_1^1 > 0$, $a_2 = a_2^1 > 0$, in the first stage and only firm 1 has the option to increase its quality level in the second stage, $a_1^2 \geq 0$, $a_2^2 = 0$. Prices are also chosen in the second stage. This game is similar to the simultaneous precommitment game (2s) except for the added option for firm 1.

Clearly there are two possibilities. If firm 1 decides not to increase its quality level in the second stage, then the game is equivalent to the (2s) game of simultaneous precommitment by both firms. If firm 1 decides to increase its quality level in the second stage, then the game is equivalent to the one-sided commitment game where firm 2 precommits (2c). Thus, if $a_1^{2c} > a_1^{2s}$ firm 1 chooses $a_1^2 > 0$ and game (2c) results; if $a_1^{2c} < a_1^{2s}$ firm 1 chooses $a_1^2 = 0$ and game (2s) results. Now $a_1^{2c} > a_1^{2s} \iff c > \bar{c} \simeq .825$.[16] Since the function Π_1^{2c} lies always below Π^{2s} (see figure 2), for high costs, $c > \bar{c}$, the addition of the strategic possibility of increasing quality in the second stage has detrimental effects for firm 1. Further, since Π_2^{2c} lies below Π^{2s}, firm 2 loses from the additional possibility of firm 1 when costs are high.

Proposition 6: The addition to the simultaneous precommitment game of the possibility to increase quality in the second stage of the game never increases profits. When costs of quality are low, $c < \bar{c}$, profits remain the same as when the option was not available. When costs of quality are high, $c > \bar{c}$, profits are lower for both firms than when the option was not available.

7. Choices when the option to precommit in the first stage as well as to increase quality in the second stage are available.

In this section we give firms the opportunity to choose the strategic options of the game they will play. The first option of a firm is to precommit in the first stage and not increase its quality level in the second stage -- the commitment strategy (C). The second option is not to precommit in the first stage and set just set quality in the second stage -- the no-commitment strategy (NC). The third option is to precommit in the first stage and increase its quality level in the second stage of the game -- commitment and subsequent increase (CA).

Consider the situation where both firms play (CA) strategies and when it arises. Given $a_2^1 > 0$, $a_2^2 > 0$, firm 1 can choose $a_1^2 = 0$ thereby reducing the game to (1c), the game of one-sided precommitment by firm 1. It can alternatively choose $a_1^2 > 0$ thereby reducing the game to (1s), the game of no precommitment. Firm 1 will choose $a_1^2 = 0$ if $a_1^{1c} \geq a_1^{1s} \iff c \leq 1$. Firm 1 will choose $a_1^2 > 0$ if $a_1^{1c} < a_1^{1s} \iff c > 1$. The outcome (CA, CA) can arise for $c > 1$ and it results in payoffs $(\Pi_1^{1s} - F_1, \Pi_2^{1s} - F_2)$. Thus we have two alternative payoff matrices depending on the value of c. For $c > 1$ the payoff matrix is:

Player 2

		CA	C	NC
Pl. 1	CA	$(\Pi_1^{1s}-F_1, \Pi_2^{1s}-F_2)$	NA	$(\Pi_1^{1s}-F_1, \Pi_2^{1c})$
	C	NA	$(\Pi_1^{2s}-F_1, \Pi_2^{2s}-F_2)$	$(\Pi_1^{1c}-F_1, \Pi_2^{1c})$
	NC	$(\Pi_1^{1s}, \Pi_2^{1s}-F_2)$	$(\Pi_1^{2c}, \Pi_2^{2c}-F_2)$	(Π_1^{1s}, Π_2^{1s})

,

where NA signifies that this event cannot arise for the specified parameters.

Strategy (CA) will never be played by player 1 because (NC, CA) is preferred by firm 1 to (CA, CA), and (C, NC) is simlarly preferred to (CA, NC). A similar argument can be made for player 2 never using (CA). Thus, the possibility of increasing quality in the second stage is not utilized.

For $c \leq 1$ the payoff matrix is:

Player 2

		CA	C	NC
	CA	NA	$(\Pi_1^{2c}-F_1,\ \Pi_2^{2c}-F_2)$	NA
Pl. 1	C	$(\Pi_1^{1c}-F_1,\ \Pi_2^{1c}-F_2)$	$(\Pi_1^{2s}-F_1,\ \Pi_2^{2s}-F_2)$	$(\Pi_1^{1c}-F_1,\ \Pi_2^{1c})$
	NC	NA	$(\Pi_1^{2c},\ \Pi_2^{2c}-F_2)$	$(\Pi_1^{1s},\ \Pi_2^{1s})$

Strategy (CA) will never be played by player 1 because he prefers (NC, C) to (CA, C). Similarly, player 2 prefers (C, NC) to (C, CA). Again the added possibility of increasing quality in the second stage is not utilized.

<u>Proposition 7: When firms are faced with the option to precommit in the first stage as well as increase their quality in the second stage they will never increase quality in the second stage. Resulting equilibria will be as described in propositions 3 and 4.</u>

8. Concluding Remarks

We have analysed through simple examples the effects of precommitment in quality (or advertising). We found that, while precommitment increases the profits of the precommitting firm, it may harm or benefit its opponent depending on the marginal cost of quality. When quality is very costly, the passive opponent is better off at equilibrium than the precommitting firm. The profits of the passive player when the opponent precommits are never larger than when both precommit provided that precommitment is not costly. With costly precommitment, and when firms have access to the same precommitment technology, the non-cooperative game where firms have the option but not the obligation to precommit can lead to a unique symmetric commitment equilibrium, a unique symmetric no-commitment equilibrium, or both. Sufficiently asymmetric precommitment costs lead to asymmetric non-cooperative equilibria. The option to increase quality in the second stage never helps the precommitting firm, and does in fact harm it when the marginal cost of quality is high. Thus, when firms have the option to avoid increasing their qualities in the second stage they will avoid doing so.

Footnotes

1. There is no particular significance to the name of the precommitment strategic variable. For example, the quality variable can also be thought of as denoting advertising. Note also that the demand can be derived from a model of locationally differentiated products. See Economides (1986).

2. Note that the specification of the demand defines a very competitive environment where total (industry) demand is independent of the levels of prices and quality. Decreases in the price of a firm (or an increase in its quality) hurt the opponent directly.

3. Quality is normalized so that "a" is measured in the units of price. A unit of "a" is the effectiveness of quality required so that consumers are willing to pay $1 more for the same quantity.

4. If a fixed cost plus constant marginal cost technology is used, the resulting prices $\hat{p}_j$ are related to the prices p_j of the zero cost technology by $\hat{p}_j = p_j + m$, where m is the marginal cost. Profits are reduced by the fixed cost G, $\hat{\Pi}_j = \Pi_j - G$.

5. Second order conditions are also satisfied. Equilibrium exists for cost functions specified by $c \geq 1/2$.

6. Equilibria are stable for $c > 2/3$.

7. Equilibrium exists for $c \geq 2/9$.

8. Equilibria are stable for $c > 4/6$.

9. From the maximization of Π_1 with respect to a_1 we have $ca_1^{1c} + 3bc - 2b = a_1^{1c}(3c - 1)^2/2$. Therefore,

$$\Pi_1^{1c} = [ca_1^{1c} + (3c - 2)b]^2/(3c - 1)^2 - c(a_1^{1c})^2/2 =$$
$$(3c - 1)^2(a_1^{1c})^2/4 - c(a_1^{1c})^2/2 =$$
$$\{[(3c - 1)^2 - 2c]/4\}\cdot\{4(3c - 2)^2b^2/[(3c - 1)^2 - 2c]^2 =$$
$$b^2(3c - 2)^2/[(3c - 1)^2 - 2c].$$

10. The equilibrium in this game exists for values of "c" which make both the numerators and the denominators of Π_1^{1c} and Π_2^{1c} non-negative. For the numerators we require $c \geq \max(2/3, 1/2,$

7/9) = 7/9. For the denominators we require, $(3c - 1)^2 - 2c > 0$ $\Leftrightarrow$ $9c^2 - 8c + 1 > 0$ $\Leftrightarrow$ $c < (4 - \sqrt{7})/9$ or $c > (4 + \sqrt{7})/9 < 7/9$. Therefore the equilibrium exists for $c \geq 7/9$. Stability is guaranteed for $c > 3/5$. Thus all equilibria are stable.

11. $\Pi_1^{1s}(c) \leq \Pi_1^{1c}(c)$ $[1 - 1/(2c)] \leq (3c - 2)^2/[(3c - 1)^2 - 2c]$ $\Leftrightarrow$ $(9c^2 - 8c + 1)(2c - 1) \leq 2c(9c^2 - 12c + 4)$ $\Leftrightarrow$ $0 \leq (c - 1)^2$.

12. $\Pi_1^{1c}(c) < \Pi_1^{2s}(c)$ $\Leftrightarrow$ $b^2[1 - 2/(9c)] > b^2(3c - 2)^2/[(3c - 1)^2 - 2c]$ $\Leftrightarrow$ $(9c - 2)(9c^2 - 8c + 1) > 9c(9c^2 - 12c + 4)$ $\Leftrightarrow$ $18c^2 - 11c - 2 > 0$. The positive root is $(11 + \sqrt{265})/36 \cong 0.757 < 7/9$. Therefore, in the region of the existence of the one-sided precommitment equilibrium, $[7/9, \infty)$, the above inequality is true.

13. See Donsimoni, Economides and Polemarchakis (1986).

14. F could represent costs of making a contract with a third party which makes the commitment credible, or costs of shifting the activity of quality to an earlier stage.

15. $a_1^{1c} > b/c$ $\Leftrightarrow$ $2(3c - 2)/[(3c - 1)^2 - 2c] > 1/c$ $\Leftrightarrow$ $0 > 3c^2 - 4c + 1$ $\Leftrightarrow$ $0 > (3c - 1)(c - 1)$ $\Leftrightarrow$ $1/3 < c < 1$.

16. $a_2^{1c} = a^{2s}$ $\Leftrightarrow$ $9c^2 - 5c - 2 = 0$. The root between 0 and 1 is $\bar{c}$ $(5+\sqrt{97})/18 \simeq .825 > 7/9$.

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