Optimal Strategic Pricing of Reproducible Consumer Products

by Wilfried R. Vanhonacker
and Fernando Nascimento

Columbia Institute for Tele-Information
Graduate School of Business
Columbia University
809 Uris Hall
New York, NY 10027
(212)854-4222

OPTIMAL STRATEGIC PRICING OF REPRODUCIBLE CONSUMER PRODUCTS

Wilfried R. Vanhonacker*

and

Fernando Nascimento*

Revised January 1987

*Associate Professor and doctoral student

Graduate School of Business

Center for Telecommunications and Information Studies

809 Uris Hall, Columbia University, New York, NY 10027

OPTIMAL STRATEGIC PRICING OF

REPRODUCIBLE CONSUMER PRODUCTS

ABSTRACT

This paper investigates the strategic pricing of consumer durable products which can be acquired through either purchase or reproduction (e.g., computer software). As copy piracy results in an opportunity loss, its adverse effect on profits needs to be incorporated in strategic decisions such as pricing. Using a dual diffusion model which parsimoniously describes sales and copying, and employing control theory methodology, optimal price trajectories are derived for the period of monopoly. The results indicate that (a) in absence of any protection, skimming pricing strategies are generally optimal, and (b) copy protection is warranted only when sales diffuse much faster than copying and the protection technology does not significantly raise the marginal production cost.

Key Words: DYNAMIC PRICING; NEW PRODUCT DIFFUSION; REPRODUCIBLE PRODUCTS.

1. Introduction

A number of durable consumer products which are easily copied or reproduced by consumers recently have become available in the market.[1] Computer software, for example, can be acquired either through purchase or simply by copying an available version of the program. The extent of copying or reproduction is not well documented but is generally believed to be widespread among individual consumers. As copying represents an opportunity loss, some efforts have been made to discourage or prevent it.[2] These efforts range from the physical protection of the product to legal protection using copyright laws. Despite these efforts, this kind of piracy is likely to persist. Copying technology advances at the same pace as copy protection technology, and legal prosecution of individuals is hard to enforce. As such, this activity needs to be incorporated and dealt with in a number of strategic decisions.

An important strategic decision for a company that tries to recoup its investment in a reproducible product is the pricing decision. A high initial price, or skimming price strategy (Dean 1969), might provide healthy margins but could depress sales volume and encourage copying. Some managers suggest, however, that copies will always be made and that one simply should not worry about them and should try to make profits on the actual (but limited) sales base. A low initial price, or penetration price strategy (Dean 1969), might create less incentive to copy but the low margins might not be financially attractive, particularly as the life cycle for many of these products is relatively short. Nevertheless, some managers favor this strategy since it is likely to result in substantial sales potential through quicker and broader acceptance and use. No guidelines exist in the current pricing literature which could help managers make this decision. This paper is an initial attempt to clarify the issues involved and provide some normative guidelines for this pricing decision.[3]

The pricing decision is framed within a parsimonious model of the adoption environment of reproducible consumer products. The model consists of two interacting diffusion processes. One describes the diffusion of sales whereas the other describes the diffusion of copying. Both processes interact.

Each individual in the population at each point in time is classified as either a potential buyer, a potential copier, or an uninterested consumer of the product. The classification, which depends on the relative magnitude of the product's market price, the individual's reservation price, and the individual's cost of copying, is similar to the one used by Johnson (1985). Both the reservation price and the cost of copying follow distributions across the population. Thus, the potential buyer population and the potential copier population vary over time as a function of the market price.

Both sales and copying are modeled explicitly as Bass diffusion processes (Bass 1969). Accordingly, sales and copying are assumed to diffuse in a contagion fashion.[4] The more units of the product there are around (i.e., both sold units <u>and</u> copies), the higher the individual's probability to "adopt" (either buy or copy) the product. Hence, the number of copies and the number of units sold together affect the hazard rate in both processes.

The diffusion rate of copying is also a function of the copy cost. This cost is modeled as a search cost incurred as the individual tries to locate a unit suitable for copying. The search cost depends on whether or not copy protection exists. In the case of no copy protection ("naive"-copier model), any available unit can be copied as soon as it has been located. Here, the search cost (and, hence, the diffusion rate of copying) is a function of the number of units available (i.e., units sold plus units copied). If any kind of copy protection exists ("expert"-copier model), the search cost contains the costs of locating a unit suitable for copying and the copying technology. In this instance, it is assumed that the search cost (and, hence, the diffusion rate of copying) is a function of the number of copies available. Accordingly, an inherent assumption is made that the owner of a copy has knowledge of the copy technology.

In this framework, profit-maximizing price trajectories are derived numerically for the period of the firm's monopoly. A summary of these results provides insights into (a) the shape of the price trajectory under "expert" and "naive" copying relative to the optimal trajectory if copying were ignored (or complete protection existed), (b) the critical parameters that impact on the cumulative profits over a finite time horizon, and (c) the instances in which some investment in copy protection is warranted.

2. The Model

1. The Bass Diffusion Model

The dynamic demand function found valid for a number of new durable products models the probability of an individual adoption at time t given no previous purchase as a linear function of the number of people who have adopted the product by t. In other words, many potential adopters are influenced by their social environment in making a purchase decision and, hence, will buy the product when a number of people around them possess it. This epidemic-type diffusion process was proposed by Bass (1969) and has been widely used in normative pricing studies (see, e.g., Robinson and Lakhani 1975, Bass 1980, Dolan and Jeuland 1981, Jeuland 1981).

Consistent with the basic demand process, the adoption rate can be expressed in a differential equation form as

$$\dot{Q}B(t) = [PB(t) - QB(t)][b_0 + b_1 QB(t)] \quad (1)$$

where PB(t) denotes the market potential at time t, QB(t) denotes the cumulative number of units sold by time t, and b_0 and b_1 are the parameters of the diffusion process. b_0 and b_1 are commonly referred to as the <u>innovation</u> parameter and the <u>imitation</u> parameter, respectively.

In the initial work by Bass (1969), the market potential was assumed to be fixed over time. More recently, Jeuland (1981) introduced the notion of reservation price into the modeling framework to capture the non-stationarity in the size of the adopter population. The reservation price is the maximum price an individual is willing to pay to acquire the product given a budget constraint and preferences for other products. Thus, if an individual's reservation price is larger than the market price, he will be a potential buyer of the product. The reservation price is distributed across the population, essentially capturing heterogeneity in preferences, tastes, etc. Accordingly, as the market price falls below the reservation price of more people, the potential market expands. This link between market price and market potential has been incorporated in recent normative pricing studies (Jeuland and Dolan 1981, Dolan and Jeuland 1981, Feichtinger 1982, Kalish 1983, 1985, Nascimento and Vanhonacker 1986).

The adoption of a reproducible consumer product can be modeled in a similar fashion. Both sales and copying can be captured by the Bass diffusion model. Trial sales are essentially a straightforward application of the model discussed elsewhere for durable products (e.g., Jeuland 1981, Kalish 1983). Copying, on the other hand, is an activity of which not much is known. Nevertheless, it seems intuitively appropriate to assume that copies are made and adopted in much the same way that sales units are acquired.

A unique aspect of this environment is the relation between the potential markets of the two diffusion processes. Specifically, the cumulative sales volume by time t affects product awareness which stimulates adoption not only through sales but also through copying. Similarly, the number of copies made by time t affects product awareness which stimulates the diffusion rates of both processes. Accordingly, the sales rate $\dot{Q}B(t)$ is modeled as

$$\dot{Q}B(t) = [PB(t) - QB(t)][b_0 + b_1[QB(t) + QC(t)]] \qquad (2)$$

where QB(t) denotes the cumulative number of <u>buyers</u> of the product by time t, PB(t) denotes the number of <u>potential buyers</u> at time t, and QC(t) denotes the cumulative number of <u>copiers</u> by time t. Without loss of generality, we will assume that buyers only purchase one unit and copiers only copy and retain one unit. In a similar fashion, the copying rate $\dot{Q}C(t)$ is modeled as

$$\dot{Q}C(t) = [PC(t) - QC(t)][c_0 + c_1[QB(t) + QC(t)]] \qquad (3)$$

where PC(t) denotes the number of potential copiers at time t. PB(t) and PC(t) are derived shortly.

2. Potential Buyers and Potential Copiers

The numbers of potential buyers and potential copiers are a function of the reservation price distribution, the copy cost distribution, and the market price of the product. An individual whose reservation price is larger than the market price will be a potential buyer. If his cost of copying is below his reservation price, he will be a potential copier. If both the market price and the cost of copying are below his reservation price, he could be either a potential buyer or potential copier. In this case, it is assumed that the individual will select the cheapest option. In other words, he will be a potential copier if his cost of copying is below the market price and a potential buyer when the reverse is true. Figure 1 illustrates graphically the assumed breakdown of the population at one point in time.

The population heterogeneity in both the reservation price and the cost of copying is fixed over time and incorporated as follows. The reservation price is assumed to be uniformly distributed across the population between zero and p_{Max}. The uniform assumption is restrictive but it simplifies the analytic derivations considerably as will be seen shortly. The distribution of the cost of copying is assumed to be independent of the reservation price distribution and is derived from an assumption that the copy cost involves a random search cost as discussed next.

3. Cost of Copying

The cost of copying is assumed to consist of two components: a fixed cost and a variable search cost. The fixed cost captures all the costs an individual would incur in trying to obtain a copy of the product once a suitable unit has been located. These costs include material costs as well as costs associated with time spent in the copying process. Intangible costs associated with possible quality differences between an original and a copy, moral issues, and fear of legal prosecution can also be considered as part of these costs. The search cost involves all expenses an individual would incur in attempting to locate a unit of the product.

The first step in reproducing the product is the location of a unit of the product suitable for copying. Two situations can occur. On one hand, the individual might already know where to find such a unit, in which case his search cost is zero. On the other hand, the individual might have to search to locate such a unit. The assumption made here is that the search process is affected by the number of units suitable for copying which are available throughout the population. In other words, if many units have been sold and/or many copies have been made already, the individual will have little trouble locating a unit and, hence, his search cost will be minimal. However, if only a few units have been sold and little or no copying has been done, his search cost will be much higher. The search is modeled here as a pure random process, and the distribution of the cost of copying is derived accordingly.

The probability of finding a unit of the product suitable for copying in k trials at time t equals

$$p(k,t) = \theta(t)[1 - \theta(t)]^{k-1} \qquad (4)$$

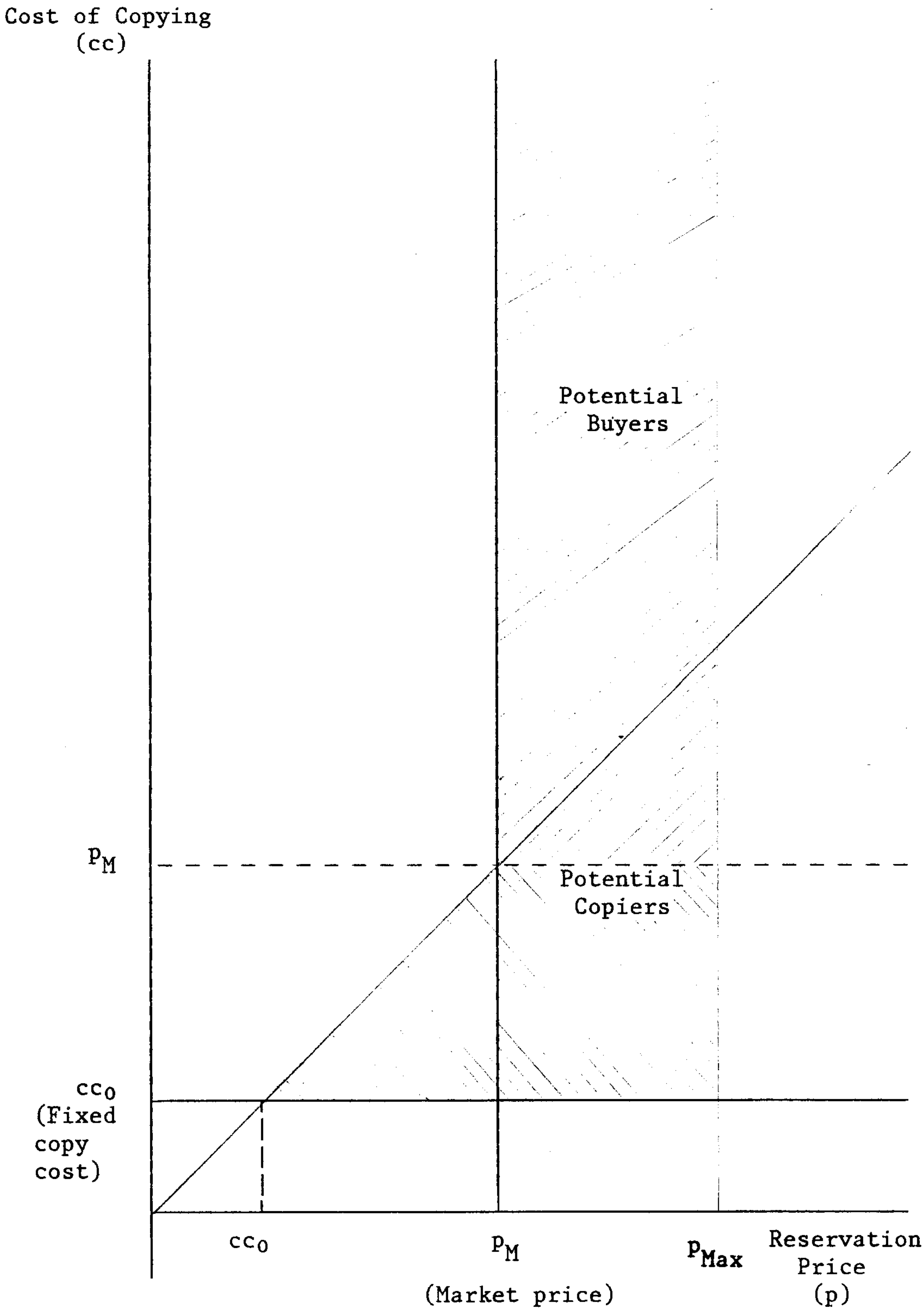


Figure 1

Breakdown of Population of Adopters at One Point in Time where $\theta(t)$ is the probability of locating a unit suitable for copying at time t. Accordingly, search is modeled as a geometric process. $\theta(t)$ is defined in two different ways in order to capture the possibility that, due to copy protection, some expertise might be required for successful copying.

In one instance, no expertise is needed and any unit of the product (sold or copied) is suitable for copying. This is called the "naive-copier" model. Here, $\theta(t) = [QB(t) + QC(t)]/N$ where N denotes the size of the population. Thus, the probability of locating a unit suitable for copying equals the relative proportion of units available in the population.

When expertise is required, the individual has to obtain both a unit suitable for copying and the copying technology. This process is more constrained than the naive-copier process as not every original of the product is suitable for reproduction. In this "expert-copier" model, it is assumed that the individual obtains a suitable unit only when he locates a person who has a copy. Therefore, the probability of locating a unit suitable for copying will be smaller on each trial in the expert-copier model than in the naive-copier model. It is assumed here that $\theta(t) = [QC(t)/N]$ with $QC(0) = QC_0 > 0$ where QC_0 denotes the "seed" of the copying process.[5] This represents the very small segment of the population which is quickly able to break any copy protection (e.g., the so-called "hackers" in computer software). The model assumes that nobody in this segment will commercialize the copy technology, as is frequently done with computer software (e.g., UNLOCK and COPY II software). If the copy technology is widely made available in this fashion, one essentially falls back into the naive-copier environment discussed above.

Using an exponential approximation (Johnson and Kotz 1969, p. 123), $p(k,t)$ in (4) can be expressed as

$$p(k,t) = \theta(t)\exp[-k\theta(t)]. \tag{5}$$

An additional assumption is made that the marginal cost of copying is homogeneous across the population and constant over time. Thus, total cost is linear in the number of trials, or $cc = cc_0 + cc_1 k$, where cc_1 denotes the fixed marginal cost of an additional search. Expressed alternatively, $k = (cc - cc_0)/cc_1$, and, since $dk/dcc = 1/cc_1$, we transform (5) into the distribution of the cost of copying at time t:

$$g(cc,t) = \frac{\theta(t)}{cc_1}\exp[-(\frac{cc-cc_0}{cc_1})\theta(t)]. \tag{6}$$

It is important to note that in the model framework adopted here there is no assumption about an a priori distribution of the fixed cost of copying. The material costs are more or less fixed and constant across the population. Realistically, one would expect the adoption decision (whether it be a buy or copy decision) to be influenced by the expected copy cost prior to the decision. There is likely to be heterogeneity in that cost as different individuals might have conflicting expectations about the degree of difficulty in (a) locating a unit suitable for copying, (b) obtaining that unit, and (c) copying that unit once it has been obtained. Although it is theoretically more sound to relate the adoption decision to those cost expectations, the actual total cost incurred in copying and its distribution across the population is relied upon here. Accordingly, the assumption of perfect foresight is made to the extent that each individual correctly anticipates the costs associated with the search process he is engaging in.[6]

Given the distribution of the cost of copying in (6), the uniform distribution of reservation prices, and the assumed independence between them, the number of <u>potential copiers</u> at time t is described by the following double integral (see Figure 1)

$$PC(t) = N \int_{cc_0}^{p_M(t)} \int_{cc}^{p_{Max}} \frac{1}{p_{Max}} \frac{\theta(t)}{cc_1} \exp[-(\frac{cc-cc_0}{cc_1})\theta(t)]dpdcc$$

where $p_M(t)$ denotes the market price of the product at time t and p denotes the reservation price. Evaluating this double integral, it can be shown that

$$PC(t) = N[(\frac{p_M(t)-p_{Max}{}^{-1}}{p_{Max}})\exp[\frac{\theta(t)}{cc_1}(cc_0-p_M(t)]-(\frac{cc_0-p_{Max}{}^{-1}}{p_{Max}})] \quad (7)$$

where $\theta(t)$ is replaced with the formula derived above for either the expert-copier or the naive-copier model.

The number of potential buyers at time t can be expressed in a similar fashion (see Figure 1), or

$$PB(t) = N \int_{p_M(t)}^{p_{Max}} \int_{p_M(t)}^{\infty} \frac{1}{p_{Max}} \frac{\theta(t)}{cc_1} \exp[-\frac{\theta(t)}{cc_1}(cc-cc_0)]dccdp.$$

Evaluating this double integral, it can be shown that

$$PB(t) = N(\frac{p_{Max}-p_M(t)}{p_{Max}})\exp[\frac{\theta(t)}{cc_1}(cc_0-p_M(t))] \quad (8)$$

where $\theta(t)$ is replaced with the appropriate formula derived above. Moreover, the diffusion equations in (2) and (3) together with the potentials in (7) and (8) capture the diffusion environment of a reproducible consumer product as modeled in this research.

4. Analytic Statement of the Problem

If the objective of the company is to maximize discounted profit over a finite time horizon T, the optimal trajectory of price over that horizon can be obtained from

$$\max_{p_M(t)} \int_0^T [\exp(-rt)](p_M(t)-c)QB(t)dt \tag{9}$$

subject to expressions (2) and (3) and with $QB(0) = QB_0$ and $QC(0) = QC_0$. In this finite time horizon problem, r denotes the discount factor, and c denotes the marginal production cost of the product. Using Pontryagin's Maximum Principle (Kamien and Schwarz 1981), the optimal trajectory $p_M(t)$ can be shown to be characterized by a system of four differential equations (see Appendix 1). These differential equations are highly nonlinear which prevents the derivation of analytic results. Some numerical results were derived using the approach discussed in Appendix 1. The uniqueness of the optimal price trajectories derived is addressed in Appendix 2.

3. Numerical Results

1. Introduction

Some numerical results were derived using the model developed above. The variables manipulated were: marginal production cost (c), the sales and copy diffusion parameters (b_0,b_1,c_0, and c_1), the number of initial buyers and copiers (QB_0 and QC_0) and the finite time horizon (T). The marginal production cost was varied to assess its effect on the price trajectories. Furthermore, there was an interest in its interaction with the minimum cost of copying. As the new product will want to "compete" effectively against copies, prices are expected to fall to either the marginal production cost c (the minimum level from a profit perspective) or the minimum copy cost cc_0 (the minimum level from a competitive perspective) whichever is higher. Accordingly, the marginal production cost was set at levels above and below the minimum copy cost. The parameters of both diffusion processes were varied. An attempt was made to use realistic parameter values. For sales diffusion, the consumer durables reported in Bass (1969) have an averabe b_0 value of 0.016 and an average b_1 value of 0.00015. In contrast, little is known about copy diffusion and no previous results can be relied upon. In the numerical analysis, parameter values of both diffusion processes were set around the average sales diffusion numbers as the impact of the relative rate of diffusion on the price trajectories and cumulative profits was particularly of interest.

The finite time horizon T was changed to assess the effect of the size of the window of opportunity on the trajectories and profits. As stated at the outset of this study, many of the reproducible consumer products currently available in the market have very short life cycles. Accordingly, the time period during which the company has a monopoly is short and there is little time to recoup (often rather hefty) investments in the product. All other variables were kept constant. The specific values used in the numerical analyses are shown in Table 1.

First, the results were obtained for the "monopoly" case. Here, the focus is entirely on sales without considering copy piracy. Accordingly, price trajectories are derived for the basic Bass diffusion model and the numerical results will be consistent with the theoretical results provided in Kalish (1983). Note that the "monopoly" case can be interpreted here in a number of different ways. If copying or reproduction exists, the results of the "monopoly" model would indicate the non-optimal price trajectories found if that activity and its associated opportunity loss were ignored. In a broader framework, the price trajectories could be considered optimal if copying is either impossible because of complete protection or not attempted because the copy costs are very high (i.e., $cc \to \infty$). The latter instance essentially describes the situation for products other than the reproducible products which are investigated here. Mainly, the "monopoly" model results serve as a valuable benchmark against which other model results can be compared.

Second, results for the "naive"-copier model were derived. This model is the exact opposite of the monopoly case. Copying or reproduction is easy, as no form of protection exists. Within the search process described above, once a unit of the product has been located and obtained, a copy can be made. Third, the "expert"-copier model is considered. This model describes an environment between the monopoly model and the naive-copier model. Copying is possible here but some expertise is needed in order to successfully reproduce a unit of the product. As discussed above, expertise is not modeled explicitly but is incorporated as a constraint on the search process in which potential copiers engage.

Two sets of numerical results are reported and discussed hereafter. First, a detailed analysis of four scenarios is provided. Second, the

Table 1

Parameter Values of Numerical Investigation

Variable Notation	Variable Description	Values
c	marginal unit production cost	3, 10
p_{Max}	maximum reservation price	40
N	size of population	2,000
b_0 } b_1	sales diffusion parameters	{ 0.02, 0.002 0.0003, 0.0001
c_0 } c_1	copy diffusion parameters	{ 0.02, 0.002 0.0003, 0.0001
cc_0	fixed copy cost	5
cc_1	marginal search cost per trial	1
QB_0	initial buyers	5, 50
QC_0	initial copiers	5, 50
T	finite time horizon	2, 4, 7, 10

insights obtained from these four scenarios are corroborated against a summary of 10 period horizon results for all scenarios.

2. A Detailed Look at Four Scenarios

2.A Four Scenarios

For illustrative purposes, four scenarios were singled out for a detailed investigation. The four scenarios investigated differ only in the magnitude of the parameters of the sales diffusion process and the copy diffusion process. They can be described as follows:

Scenario 1: Both diffusion processes have identical parameters with relatively large coefficients of innovation. Specifically, $b_0 = c_0 = 0.02$ and $b_1 = c_1 = 0.0003$;

Scenario 2: Both diffusion processes have identical parameters with relatively small coefficients of innovation. Specifically, $b_0 = c_0 = 0.002$ and $b_1 = c_1 = 0.0003$;

Scenario 3: Both diffusion processes have different parameters with copying having a faster rate of diffusion than sales. Specifically, $b_0 = c_0 = 0.02$, $b_1 = 0.0001$, and $c_1 = 0.0003$;

Scenario 4: Both diffusion processes have different parameters with sales having a faster rate of diffusion than copying. Specifically, $b_0 = c_0 = 0.02$, $b_1 = 0.0003$, and $c_1 = 0.0001$.

All other parameters were kept constant across the four scenarios and equalled: $c = 3$, $QB_0 = QC_0 = 5$, $cc_0 = 5$, $cc_1 = 1$, $p_{Max} = 40$, and $N = 2000$.

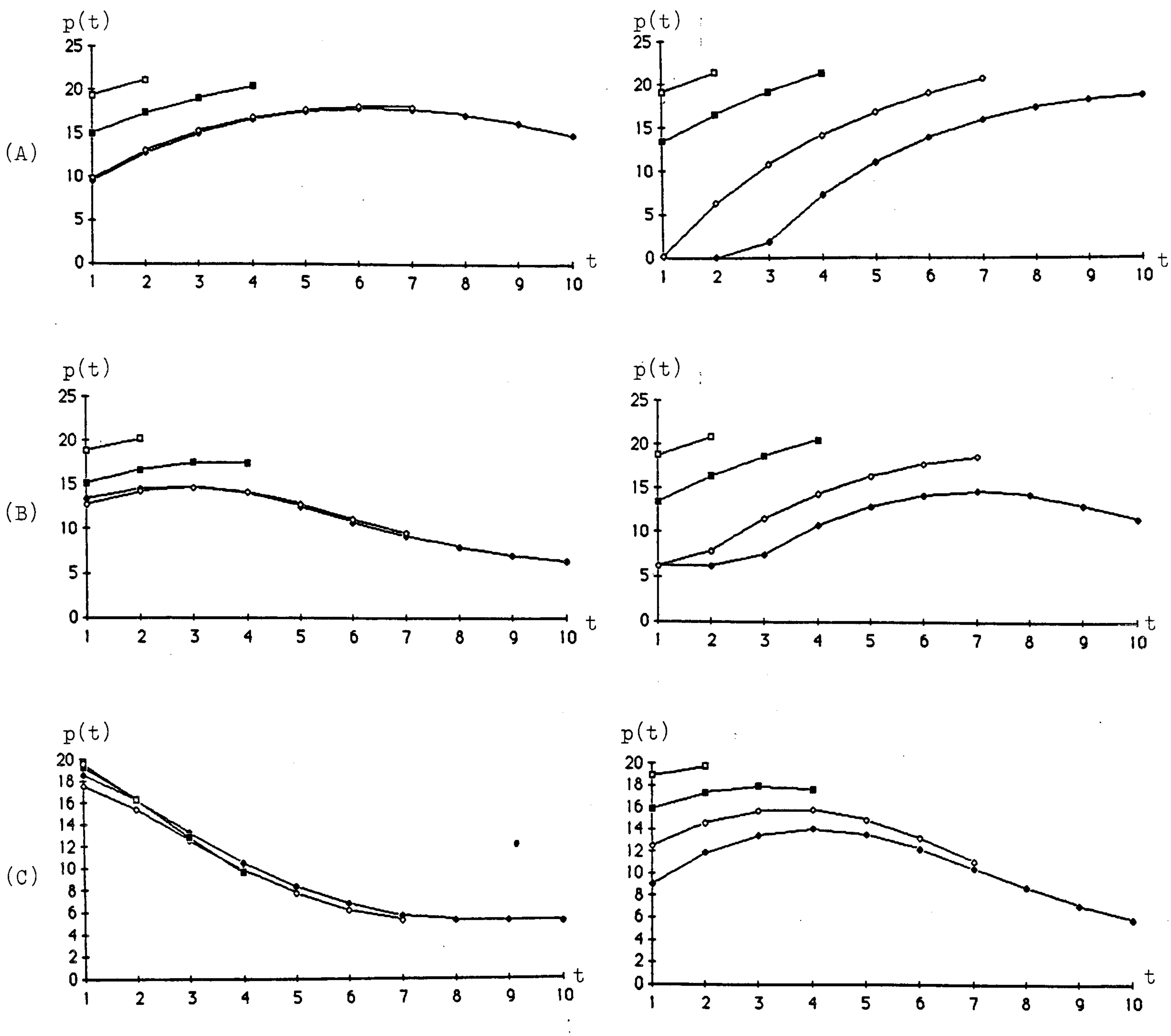
2.B Optimal Price Trajectories

The optimal price trajectories are summarized in Figure 2. For scenario 1, the shape of the optimal price trajectories seem to be quite different for the three models. The starting prices are, however, below the myopic optimal price of 22.45 in all instances.[7] For the monopoly model, prices increase monotonically over time for short time horizons. For larger time horizons ($T \geqq 7$), prices start out lower, reach a maximum and decline subsequently.

For the expert-copier model, the optimal price trajectories for short time horizons are quite similar to the ones derived for the monopoly case. This result is intuitive as there is little time for the copying process to diffuse from its small base, and the opportunity loss of copying is negligible. For larger time horizons, the trajectories are similar in shape to the ones of the monopoly case except that initial prices are higher and prices peak earlier in the expert-copier model. Here, prices come down sooner to make copying less attractive and to keep the opportunity loss associated with it to a minimum.

For the naive-copier model, the trajectories are similar across the various time horizons, but different from those of the previous two models. Here, the optimal pricing strategy is one of skimming: start out at a high level and let prices fall over time until they asymptote with the minimum copy cost cc_0. Apparently, both diffusion processes fuel each other to the extent that the rate of diffusion of sales does not

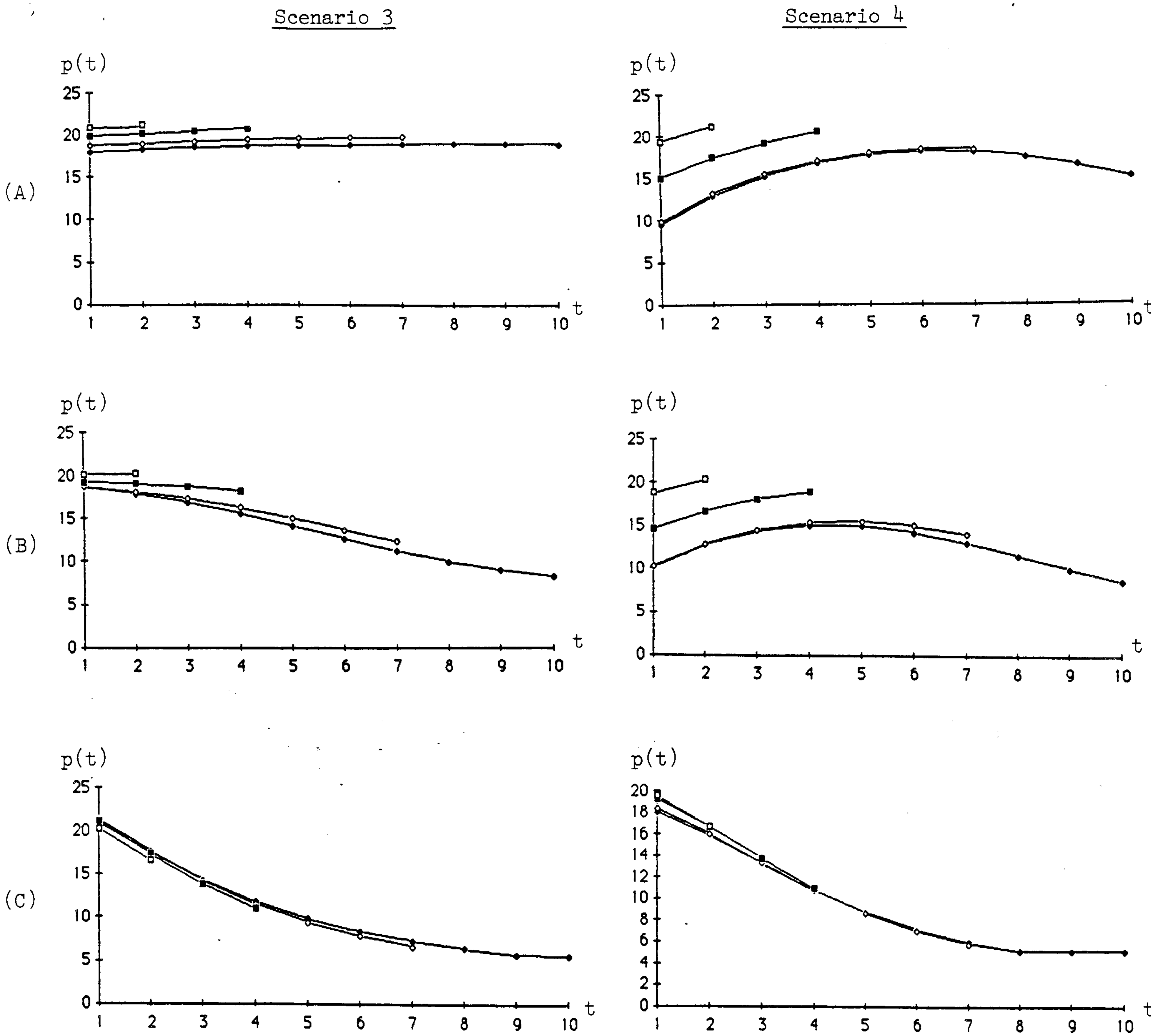
Scenario 1 | Scenario 2



(A) Monopoly Model
(B) Expert-Copier Model
(C) Naive-Copier Model

Figure 2

Optimal Price Trajectories
for Various Time Horizons



(A) Monopoly Model
(B) Expert-Copier Model
(C) Naive-Copier Model

Figure 2 (cont'd)

Optimal Price Trajectories
for Various Time Horizons

have to be enhanced by "subsidizing" early buyers as is needed in the monopoly and expert-copier cases.

The shapes of the trajectories across the three models for scenario 2 are similar to the results for scenario 1. However, prices are generally lower here particularly for larger time horizons. This is directly a result of the small innovation parameters of both sales and copy diffusion in scenario 2. Accordingly, penetration will be necessary to stimulate the diffusion rate of sales as the number of initial purchasers (i.e., the "innovators") will be rather small. These penetration prices also will limit the opportunity loss associated with copying, as lower prices make copying less attractive. For the monopoly model, aggressive penetration pricing is found to be optimal for larger time horizons with prices starting out at a level which is even below the marginal production cost c. For the expert-copier model, similar results are obtained, but penetration pricing is not that aggressive.

In the naive-copier model, prices increase monotonically, reach a maximum, and decline subsequently. This pattern is drastically different from the skimming strategy found optimal in scenario 1. Apparently, some subsidizing of early adopters is needed to stimulate the diffusion of sales, but prices peak early and decline rapidly to discourage copying.

The optimal price trajectories for scenario 3 are quite different from the trajectories obtained for the previous scenarios. As the copy process diffuses faster than the sales process in this case, one would expect no penetration pricing in the naive-copier and expert-copier models. Any kind of subsidizing will further stimulate the copying process and erode potential profits. The results do indeed show skimming strategies to be optimal here irrespective of the length of the time horizon. Note that for the expert-copier model prices drop at a slower rate than for the naive-copier model. This is a direct result of the slower diffusion of copies in the former relative to the latter because the potential number of copiers is limited. In the monopoly case, prices only increase slightly. No aggressive penetration is found to be optimal despite the small coefficient of imitation in the sales diffusion model.

The fourth scenario is the reverse of the third scenario. Here, the sales process diffuses faster than the copy process. The results for the monopoly are exactly identical to the monopoly results of scenario 1 as the parameters of the sales diffusion process are exactly identical. For the expert-copier model, the results are very close to the monopoly model results. This is rather intuitive as copying will not be all that extensive given the expert-copier environment and the fact that sales diffuse a lot faster than copies. In the naive-copier model, a skimming pricing strategy is again identified as being optimal. Despite the difference in diffusion rates between sales and copies, the piracy has to be reckoned with from the beginning.

Across the four scenarios, one interesting observation can be made. The shapes of the optimal price trajectories are quite similar across the three models except that the time at which the price is highest is reached first in the naive-copier model, then in the expert-copier model, and finally in the monopoly model. Actually, the maximum price for the naive-copier model is often the initial price of the optimal skimming strategy. Hence, this seems to suggest that the length of the time period over which prices increase monotonically is inversely related to the degree of protection. In other words, the penetration phase will last much longer for a protected (or difficult to copy) product than for an unprotected product.

2.C Cumulative Profits

The cumulative profits for the four scenarios are shown in Figure 3. For scenario 1, the monopoly model provides consistently higher cumulative profits than either the expert-copier model or the naive-copier model. For small time horizons ($T \leqq 4$), however, monopoly profits and expert-copier profits are close. This is because the price trajectories are very similar here given the minimal adverse effect of copying on sales revenue and profits. In the naive-copier model, profits are lower than for the other two models. For larger time horizons ($T \geqq 7$), the cumulative profits for this model are less than half the cumulative profits for the monopoly model. Clearly, the rapid diffusion of copies cuts into the sales potential and drastically affects profits over time.

The difference in cumulative profits between the models can be interpreted as the maximum amount (i.e., upperbound) one would be willing to invest in any form of protection to prevent copy piracy. For example, the difference between the cumulative profits in the monopoly model and the naive-copier model gives an upperbound on the investment for complete protection. The cumulative profit results for scenario 1 indicated that over larger time horizons, the marginal profit obtained for each additional dollar spent on protection seems to be substantial. In other words, if the product in the monopoly stage of its life cycle has a wide strategic window to recoup its investment, it seems that spending money upfront on protection in order to secure profitability in the long run is warranted. The difference in cumulative profits between the expert-

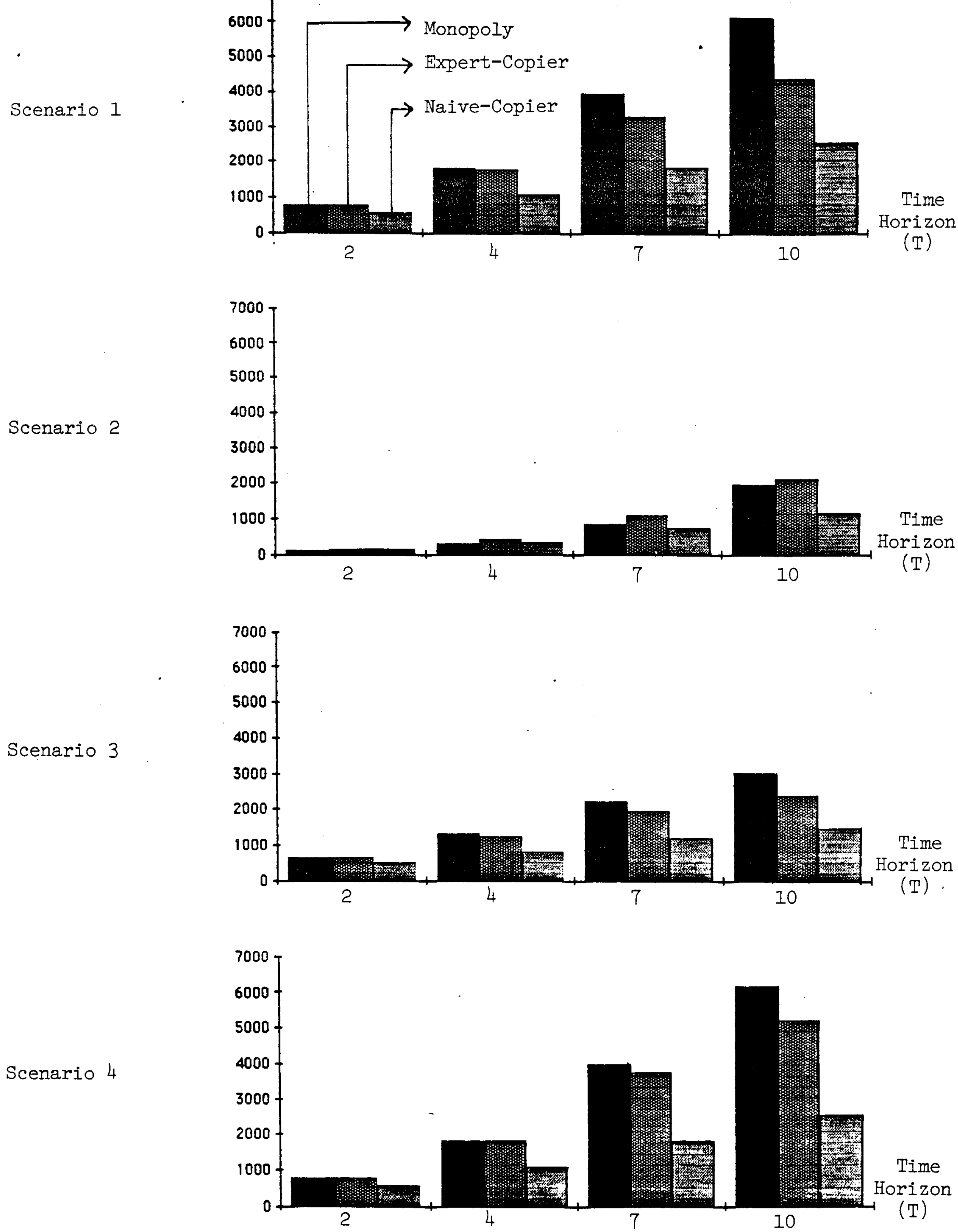


Figure 3

Cumulative Profits for Monopoly, Expert-Copier, and Naive-Copier Models copier model and the naive-copier model suggest that even a limited form of protection is desirable in this scenario.

For scenario 2, the cumulative profits are much lower than for scenario 1. This is clearly a direct result of the aggressive penetration pricing which has to be pursued to stimulate the diffusion rate of sales. As the diffusion of copies also stimulates sales diffusion, there seems to be less of an incentive to spend money upfront on protection as is evidenced by the small difference in cumulative profits. One important observation here is that the expert-copier model provides larger cumulative profits than the monopoly model but both provide larger cumulative profits than the naive-copier model. Accordingly, some protection seems to be desirable, but not complete protection. This scenario seems to describe a situation where awareness created through limited copying has a positive effect on the bottom line.

The cumulative profit results for scenario 3 shown in Figure 3 are in absolute numbers between the results for scenarios 1 and 2. Note that despite the rapid diffusion rate of copies throughout the population, the results seem to imply that it pays little to go for complete protection. This is evident in the small difference between cumulative profits in the monopoly model and the expert-copier model (particularly relative to scenario 1). This is primarily a result from the relatively slow diffusion of sales in this case.

The results for the fourth scenario are similar to the ones for scenario 1 except for the expert-copier model where the cumulative profits are higher as a direct result of the slower diffusion of copies relative to sales. Accordingly, and despite the slower diffusion of copies in this fourth scenario, the adverse effects of copying on profits remains significant. The relative magnitude of cumulative profits suggest that in this instance, some protection might be warranted. Particularly for larger time horizons, the difference in cumulative profits between the naive-copier model and the expert-copier model is larger than in any of the other three scenarios. Hence, it appears that it does pay to invest upfront in copy protection even when copies diffuse slower than sales.

Before general conclusions can be drawn, however, the results from all scenarios have to be investigated. A summary of these results follows.

3. Summary of Numerical Results on Cumulative Profits

The numerical results derived for a 10 period finite time horizon were summarized using a set of regression equations. Focus was on the effect of various model parameters on cumulative profits, differences in cumulative profits between the three models, the initial price level, the point in time at which the maximum was reached, and the average price. The predictor variables in each of the equations were the manipulated parameters listed in Table 1 and their pairwise interactions (i.e., 6 main effects for c, b_0, b_1, c_0, c_1, and $QB_0 = QC_0$, and 15 pairwise interaction effects evaluated over 64 scenarios). Higher-order interactions were not considered as to assure an adequate number of degrees-of-freedom for estimation and statistical testing. Standardized regression coefficients significant at the 0.05 level are reported and discussed. The results related to the initial number of buyers and copiers are not reported. Consistent with the fact that b_0 and QB_0 play a very similar role in the diffusion process, the estimates for QB_0 =

QC_0 were very similar in magnitude, direction, and significance to the estimates for b_0. This also holds for c_0 and QC_0.

Note that the regression framework is only adopted as a convenient way to summarize results. As is evident from the model development, some of the criterion variables are nonlinear functions of the selected predictors and linearity might be a crude approximation.[8] The results should be evaluated with that caveat in mind.

The results for cumulative profits are summarized in Table 2. For the level of cumulative profits, the sales diffusion parameters b_0 and b_1 have strong positive effects across the three models. Accordingly, a stronger diffusion of sales leads to higher profits. The positive interaction of the sales diffusion parameters in the monopoly model further enhances profits.[9]

For all three models, the marginal production cost has a negative interaction effect on cumulative profits with both sales diffusion parameters (b_0 and b_1). Accordingly, when sales diffuse quickly, profits are enhanced in all instances as long as marginal production costs are kept down. These results are in accordance with intuition but it is interesting to note that for the monopoly model the negative interaction is out of synergy with the main effects of its components. Specifically, higher marginal costs push up profits slightly.[10] However, in combination with rapidly diffusing sales (i.e., large values for b_0 and b_1), it pays to keep costs down. Note that the interaction is quite large relative to the main effect of marginal production cost. In protecting a product from reproduction or piracy, it is likely that the marginal production cost increases. Accordingly, the above results indicate that <u>in terms of cumulative profits, protection might be justified only to</u>

Table 2

Summary of Numerical Results on Cumulative Profits Over 10 Period Time Horizon[a]

	Cumulative Profits			Difference in Cumulative Profits		
	Monopoly	Expert	Naive	Monopoly-Expert	Monopoly-Naive	Expert-Naive
MAIN EFFECTS						
(1) Marginal Production Cost (c)	0.31				0.29	
Sales Diffusion Parameters:						
(2) Innovation Parameter (b_0)	0.88	1.21	1.06		0.67	1.28
(3) Imitation Parameter (b_1)	0.60	1.00	0.73		0.45	1.23
Copy Diffusion Parameters:						
(4) Innovation Parameter (c_0)						
(5) Imitation Parameter (c_1)						
PAIRWISE INTERACTIONS						
(1) × (2)	-0.44	-0.59	-0.63		-0.28	-0.51
(1) × (3)	-0.51	-0.67	-0.64		-0.38	-0.64
(1) × (4)						0.20
(2) × (3)	0.26				0.33	0.27
(2) × (5)						-0.36
(3) × (4)						-0.24
(3) × (5)						-0.36
R^2	0.97	0.92	0.88		0.95	0.95

[a]Apart from the R^2 values, table entries are the standardized estimates (beta coefficients) significant at 0.05.

the extent that it fails to increase marginal production costs, particularly when sales diffuse rapidly over time.

The regression results for the pairwise differences in cumulative profits are also contained in Table 2. These results essentially identify the key parameters affecting the upfront investment justified for piracy prevention. For the monopoly versus expert-copier case, no particular pattern emerges from the data and it is not clear when one would go from partial to complete protection. Note, however, that this does not imply that complete protection would not be warranted in some specific instances (see, e.g., scenario 1 in Figure 3).

For the monopoly versus naive-copier case, the regression results are identical to the ones obtained on cumulative profits for the monopoly model (shown in column one of Table 2). Accordingly, it pays to protect the product, and profits will be enhanced, when sales diffuse quickly and marginal production costs can be kept low. The latter is true despite a positive main effect of the marginal production cost because of the relatively large interactions with the parameters of the sales diffusion process. Moreover, one should not protect a product when marginal production costs are high and sales diffusion is slow.

The results in the expert-copier versus naive-copier case suggest that even some partial protection is justified to enhance profits in certain situations. Specifically, and consistently with previous results, protection is warranted when sales diffuse quickly and marginal production costs are low. This is particularly the case when copies diffuse at a relatively slow rate given the strong negative interactions of the copy diffusion parameters with the sales diffusion parameters.

The cumulative profit results can be summarized as follows. In general, cumulative profits will be large for all three models considered when sales diffuse rapidly and marginal production costs are kept down. Copy protection is warranted when sales diffuse much faster than copies and only so when the marginal production cost (which includes the protection cost) is kept to a minimum. Given the high cost of protection and the rapid diffusion of copies in computer software, for instance, these results indicate that no protection should be done. This is consistent with the current move away from protected products in that industry (see, e.g., the Wall Street Journal 1986).

4. Summary of Numerical Results on Optimal Price Trajectories

The regression results for the optimal price trajectory variables are summarized in Table 3. The results for the initial price of the trajectory indicate that for the monopoly model the initial price will be low when the imitation parameter of the sales diffusion process is large, particularly when the innovation parameter is small. This is intuitively reasonable as these parameter values identify instances where subsidizing early adopters is justified in maximizing discounted profits. A high marginal production cost reduces the magnitude of the subsidy, however, as a positive main effect pushes up the initial price. Similar results were obtained for the expert-copier model. The marginal production cost is no longer significant but subsidizing is enhanced when initial copiers are numerous.

For the naive-copier model, the regression results exhibit some inconsistent synergies. For example, the innovation parameter of the copy diffusion process has negative interaction effects with the marginal production cost and the innovation parameter of the sales diffusion process. As both of these parameters have positive main effects, one would expect a small value for c_0 to drive up the skimming price. However, a positive interaction with b_1, which itself has a negative main effect, would indicate otherwise. The standardized regression coefficients indicate that the main effects are the dominant forces driving the initial price level. Accordingly, skimming will be enhanced when marginal production costs are high and when the sales diffusion process has a large number of innovators but diffuses slowly over time. These results are consistent with the insights obtained from the four scenarios investigated above.

Table 3 also contains regression results on the time period in which the price reached its maximum level. As discussed above in relation to the four scenarios, the price trajectories across the models are similar in shape except that the maximum occurs later as one goes from the monopoly model to the expert-copier model to the naive-copier model. This result is reflected in the intercept estimates for the regressions shown in Table 3. For the monopoly model, the maximum of the trajectory will occur earlier when sales develop rapidly and the marginal production costs are high. These variables exhibit a positive interaction but the estimates indicate that the parameters of the sales diffusion process are dominant in this case. These results are also consistent with the "ceiling" effect which is inherent in Bass model results (see, e.g., Kalish 1983). Specifically, as sales diffuse rapidly, the market will become saturated earlier and prices will decline sooner.

For the expert-copier model, the dominant effects on the time at which prices will peak were the innovation parameters of both diffusion

Table 3

Summary of Numerical Results on Prices Over 10 Period Time Horizon[a]

	Initial Price			Time Period of of Maximum Price			Mean Price Level		
	Monopoly	Expert	Naive	Monopoly	Expert	Naive	Monopoly	Expert	Naive
MAIN EFFECTS									
(1) Marginal Production Cost (c)	0.55		0.55	-0.40			0.92	0.64	0.81
Sales Diffusion Parameters:									
(2) Innovation Parameter (b_0)			0.59	-1.35	-1.05	-1.48			-0.40
(3) Imitation Parameter (b_1)	-1.24	-1.11	-0.60	-0.56	0.49		-0.97	-0.73	-0.47
Copy Diffusion Parameters:									
(4) Innovation Parameter (c_0)					-1.03	-0.96			
(5) Imitation Parameter (c_1)									
PAIRWISE INTERACTIONS									
1 × 2									0.27
1 × 3			0.31	0.52					
1 × 4			-0.23						-0.30
1 × 5									
2 × 3	0.51	0.36		0.55			0.41		
2 × 4		-0.30	-0.34		0.37	0.61		-0.21	
2 × 5									
3 × 4			0.30		0.39				0.24
3 × 5					-0.35				
INTERCEPT				13.86	9.80	4.92	17.42	19.52	16.72
R^2	0.92	0.87	0.93	0.90	0.89	0.89	0.90	0.93	0.94

[a] Apart from the intercept and R^2 values, the table entries are the standardized estimates (beta coefficients) significant at 0.05.

processes and the imitation parameter of the sales diffusion process. Accordingly, prices will peak early when both diffusion processes have large innovation parameters but sales diffuse slowly over time. For the naive-copier model, the results are similar and consistent with the skimming strategies found to be optimal in many instances.

The mean price level results are shown next in Table 3. The intercept estimates seem to indicate that on average prices are highest for the expert-copier model and lowest for the naive-copier model. The less aggressive penetration pricing often optimal in the expert-copier model relative to the monopoly model explains why the former dominates the latter on this dimension. The skimming strategies in the naive-copier model provide high prices initially but they drop rapidly as to remain competitive with copies, explaining the overall low price level here. As expected across all three models, the main effects indicate prices to be higher when the marginal production cost is higher and sales diffuse relatively slowly. This pattern is enhanced in the naive-copier case by a negative main effect of the innovation parameter of the sales diffusion process.

For the monopoly model, a positive interaction was obtained between the sales diffusion parameters. This interaction is much smaller, however, than the main effect of the imitation parameter. In the expert-copier model, the innovation parameters of both diffusion processes interact negatively. Accordingly, the pattern of the main effects is enhanced when both processes have a large number of innovators. These results are consistent with disincentives to subsidize aggressively early in the sales development process in this case.

For the naive-copier model, the three significant interactions (marginal production cost with the innovation parameters of both diffusion processes and the sales imitation parameter with the copy innovation parameter) are small relative to the main effects. Accordingly, the average price level will be high here when marginal production costs are high and sales diffuse slowly. This is particularly the case when the initial number of copiers is small. In essence, this result is a synthesis of the monopoly and expert-copier findings.

In summary, the major findings from the extensive numerical results on the optimal price trajectories over a 10 period time horizon can be stated as follows. For the expert-copier model, pricing will be aggressive initially to stimulate sales if the sales diffusion process develops slowly by itself. The aggressiveness will be less than in the monopoly model, however, so as to protect profits. As sales diffuse over time, prices eventually will come down to aggressively compete with copies. For the naive-copier model, skimming strategies are optimal in general. Initial skimming is enhanced when sales diffusion is slow, but eventually prices will come down to stimulate the rate of diffusion and, as such, protect profits (or minimize opportunity loss). It seems that in the naive-copier case, it is less desirable to actively compete with copies and emphasis is more on maximizing and protecting profits on sales. This could be a result from the fact that in contrast to the expert-copier model, there is little to be gained from aggressive competition with copies.

4. Discussion of Model Assumptions and Limitations

Despite the fact that the model developed contains the main ingredients of the pricing environment investigated, it is based on a set of simplifying assumptions. As discussed above already, the model was developed for a monopolistic environment. Furthermore, sales and copying are modeled as contagion processes. Price trajectories were derived maximizing discounted profits assuming no learning effects on the cost side. Consumer heterogeneity in preferences was confined to a reservation price distribution postulated a priori.

Consumer heterogeneity in copy costs was modeled in a posterior sense to reflect the actual cost of copying if each consumer engaged in a random search process. The search process is probably an oversimplification in itself. However, it allows for a parsimonious representation which is descriptively quite appealing. Note that within the search process framework, it was assumed that the marginal cost per search and the minimum copy cost are constant and homogeneous across the population. It is probably more realistic to assume that the marginal search cost increases with the number of searches. Furthermore, the minimum copy cost is likely to have some distribution across the population. Even when material costs are likely to be the same, intangible costs (i.e., costs related to, e.g., moral issues, fear of legal prosecution, etc.) are likely to vary from consumer to consumer. Obviously, additional work is possible and warranted on this aspect of the model.

Furthermore, the model assumed that consumers act myopically. Accordingly, they have neither price nor copy cost expectations. Consumer do not make their acquisition decisions (buy or copy) in anticipation of future price movements. Furthermore, the cost of copying was modeled simply as a search cost and was incorporated in the decision as the actual cost known to the consumer at the time of acquisition. These assumptions are certainly restrictive as consumers are likely to have expectations which will influence their decisions. These expectations are difficult to incorporate into a diffusion framework, particularly if one wants to maintain parsimony and a certain level of generalizability. They provide an avenue for future research.

Another assumption underlying the numerical work is that the reservation price is independent of whether or not the product is copy protected. Copy protection can reduce the value of a product. First, a protected product is more difficult to use in certain instances (e.g., using hard disks with copy protected computer software). Second, the product is not easily shared with others. Shareability creates additional value as established in the economics literature on indirect appropriability (Liebowitz 1985, Novos and Waldman 1984). The model framework developed here is, however, still applicable as value differences can be captured in the reservation price distributions underlying the various models. For example, comparing the results of the naive-copier model and the expert-copier model with results for the monopoly model based on a reservation price distribution with a lower mean provides insights into the pricing problem recognizing value differences. Nevertheless, additional work is warranted here and the issue will have to be addressed directly in empirical work.

5. Conclusion and Directions for Future Research

This paper investigated the strategic pricing of consumer durable products which can be acquired through either purchase or reproduction (copying). Using a dual diffusion model which parsimoniously describes sales and copy diffusion over time, and employing control theory methodology profit maximizing price trajectories were derived for the period of the company's monopoly in the market. The results indicate that when the product is not protected against copying, skimming pricing strategies tend to be optimal. The initial skimming prices are high when sales diffuse slowly over time. These prices will eventually come down to stimulate the rate of sales diffusion and, hence, protect profits on units sold.

When the product is somewhat protected, pricing will be aggressive initially, particularly when sales diffuse slowly over time. The aggressiveness will be less, however, than in the case of complete protection so as to maintain profitability. As sales develop, prices will peak and eventually decline monotonically to a level where sales can compete aggressively with copies.

Cumulative profits are generally the highest when the product is fully protected against copy piracy. In some specific cases, however, a partially protected product provided higher cumulative profits than a fully protected product. In general, copy protection is warranted when sales diffuse much faster than copies and only so when the marginal production cost (which includes the cost of protection) can be kept to a minimum.

These results were obtained in a simplified modeling framework of the pricing problem of interest. Despite the fact that the main characteristics of the environment were captured, this paper still provides only initial insights into the complex pricing problem. Future research is needed to enhance the validity and practicality of the various results derived. One obvious avenue of future research is the relaxation of the many assumptions underlying the modeling framework.

Another fruitful avenue is empirical work. As the results in this paper point out the significant impact of the diffusion characteristics of copies on the optimal pricing of the product, research and data are needed on the diffusion of copies. Once data are available, the model developed in this research can be estimated and tested empirically, and optimal pricing trajectories for a specific product can be derived. At that point, the practical value and managerial usefulness of the model will have been established.

FOOTNOTES

1. Examples are pre-recorded music tapes, pre-recorded video tapes, computer software, etc.

2. The economics literature has been focusing on the short term social welfare gain versus long term discouragement of creative activity in relation to patent and copyright protection (see, e.g., Nordhaus 1969, Scherer 1972, Novos and Waldman 1984, Braunstein et al. 1979). Price discrimination issues in relation to photocopying have been researched by Liebowitz (1981, 1985).

3. Vanhonacker (1984) provides myopic optimal results for the pricing of reproducible products.

4. Johnson's (1985) analysis of the economics of copying takes into account neither demand dynamics nor the effect of copies on demand for originals.

5. QC_0 and QB_0 are starting values of, respectively, copying (or $QC(0) = QC_0$) and sales (or $QB(0) = QB_0$). Both were set at non-zero values for numerical analysis purposes. Note that conceptually and numerically, they play the same role as the innovation parameters in the corresponding diffusion processes.

6. This is a restrictive assumption but it should not affect the derived results qualitatively to the extent that the model adequately describes actual adoption.

7. This value is derived from the myopic optimal price $\tilde{p}(t)$ which equals $\tilde{p}(t) = [(p_{Max}+c)/2]-[(QB(t)\cdot p_{Max})/2N]$. For $t = 0$, $QB(0) = QB_0 = 5$ and, hence, $\tilde{p}(0) = 22.45$. Note that as $QB(t)$ increases, the myopic optimal price decreases monotonically (consistent with Bass and Bultez (1982)) which is quite different from the optimal trajectories shown in Figure 2 for this scenario. Given that only positive diffusion effects are operating here, these results are entirely consistent with the analytic derivations in Kalish (1983).

8. The high R^2 values which will be reported shortly do, however, provide some support for the adequacy of the linear approximation.

9. It is interesting to note that the copy diffusion parameters do not have a significant effect on profits levels, not even a negative one.

10. Investigation of individual scenarios indicates that in instances where only the marginal production costs differ, price trajectories are very similar in shape but they manifest themselves at somewhat higher price levels when marginal production costs are higher. Even myopic prices are higher in these instances as is evident from the expression in footnote 7.

REFERENCES

Bass, F. M. (1969), "A New Product Growth Model for Consumer Durables," Management Science, 15, 215-227.

Bass, F. M. and A. V. Bultez (1982), "A Note on Optimal Strategic Pricing of Technological Innovations," Marketing Science, 1, 371-378.

Braunstein, Y., D. Fisher, J. A. Ordover, and W. J. Baumol (1979), "Economics of Property Rights as Applied to Computer Software and Data Bases," in Technology and Copyright: Sources and Materials, G. P. Bush and R. H. Dreyfuss (eds.). Mount Airy, MD: Lomond Publications.

Dean, J. (1969), "Pricing Pioneering Products," Journal of Industrial Economics, 17, 165-179.

Dolan, R. J. and A. P. Jeuland (1981), "Experience Curves and Dynamic Demand Models: Implications for Optimal Pricing Strategies," Journal of Marketing, 45, 52-62.

Feichtinger, G. (1982), "Optimal Pricing in a Diffusion Model with Concave Price-Dependent Market Potential," Operations Research Letters, 1, 236-240.

Jeuland, A. P. and R. J. Dolan (1982), "An Aspect of New Product Planning: Dynamic Pricing," TIMS Studies in the Management Sciences, Special Issue on Marketing Planning Models, A. Zoltners (ed.), 1-21.

Johnson, N. L. and S. Kotz (1969), Discrete Distributions. Boston, Mass.: Houghton Mifflin Co.

Johnson, W. R. (1985), "The Economics of Copying," Journal of Political Economy, 93, 158-174.

Kalish, S. (1983), "Monopolistic Pricing with Dynamic Demand and Production Costs," Marketing Science, 2, 135-159.

Kalish, S. (1985), "A New Product Adoption Model with Price, Advertising, and Uncertainty," Management Science, 31, 1569-1585.

Kamien, M. I. and N. L. Schwartz (1981), Dynamic Optimization. New York, NY: North-Holland.

Kendrick, D. and A. Meeraus (1985), GAMS. Washington, D.C.: The World Bank.

Liebowitz, S. J. (1981), The Impact of Reprography on the Copyright System. Ottawa, Canada: Dept. of Consumer and Corporate Affairs.

Liebowitz, S. J. (1985), "Copying and Indirect Appropriability: Photocopying of Journals," Journal of Political Economy, 93, 945-956.

Murtagh, B. A. and M. A. Saunders (1978), "Large-Scale Linearity Constrained Optimization," Mathematical Programming, 14, 41-72.

Murtagh, B. A. and M. A. Saunders (1982), "A Projected Lagrangian Algorithm and Its Implementation for Sparse Nonlinear Constraints," Mathematical Programming Study, 16, 84-117.

Nascimento, F. and W. R. Vanhonacker (1986), "Optimal Strategic Value Pricing for Non-Durables in Multibrand Markets," Working Paper, Columbia University.

Nordhaus, W. D. (1969). Invention, Growth, and Welfare: A Theoretical Treatment of Technological Change. Cambridge, MA: MIT Press.

Novos, I. E. and M. Waldman (1984), "The Effects of Increased Copyright Protection: An Analytic Approach," Journal of Political Economy, 92, 236-246.

Pindyck, R. S. (1978), "Gains to Producers from the Cartelization of Exhaustible Resources," Review of Economics and Statistics, 60, 238-251.

Robinson, B. and C. Lakhani (1975), "Dynamic Price Models for New Product Planning," Management Science, 10, 1113-1122.

Scherer, F. M. (1972), "Nordhaus' Theory of Optimal Patent Life: A Geometric Reinterpretation," American Economic Review, 62, 422-427.

The Wall Street Journal (1986), "Business Bulletin: A Special Background Report on Trends in Industry and Finance," March 27, 1.

Vanhonacker, W. R. (1984), "Optimal Pricing of Reproducible Products," Working Paper, Columbia University.

Appendix 1. Analytic Statement and Numerical Solution Approach

The Hamiltonian corresponding to the optimization problem characterized in (9) equals

$$H = [[\exp(-rt)][p_M(t)-c]+\lambda_B(t)][N(\frac{p_{Max}-p_M(t)}{p_{Max}})$$

$$\exp[\frac{\Theta(t)}{cc_1}(cc_0-p_M(t))]-QB(t)][b_0+b_1(QB(t)+QC(t))]]$$

$$+\ \lambda_c(t)[N(\frac{p_M(t)-p_{Max}^{-1}}{p_{Max}})\exp[\frac{\Theta(t)}{cc_1}(cc_0-p_M(t))]$$

$$-\ (\frac{cc_0-p_{Max}^{-1}}{p_{Max}})][c_0+c_1(QB(t)+QC(t))]$$

where $\lambda_B(t)$ and $\lambda_C(t)$ are costate factors associated with the differential equations (2) and (3) respectively. The solution implies $\partial H/\partial p_M(t) = 0$ everywhere on the trajectory and is characterized by the following differential equations (dropping the time varying indicator for simplicity)

$$\dot{\lambda}_B = \frac{\partial H}{\partial QB} = -\{[\exp(-rt)](p_M-c)+\lambda_B[(\frac{\partial PB}{\partial QB}-1)h_B+(PB-QB)b_1]+\lambda_C[\frac{\partial PC}{\partial QB}h_C+(PC-QC)c_1]\}$$

$$\dot{\lambda}_C = \frac{\partial H}{\partial QC} = -\{[[\exp(-rt)](p_M-c)+\lambda_B][\frac{\partial PB}{\partial QC}h_B+(PB-QB)b_1]+\lambda_C[(\frac{\partial PC}{\partial QC}-1)h_C+(PC-QC)c_1)]\}$$

$$\dot{QB} = (PB-QB)h_B$$

and

$$\dot{QC} = (PC-QC)h_C$$

where

$$h_B = b_0 + b_1(QB+QC)$$

$$h_C = c_0 + c_1(QB+QC)$$

$$PB = N[(p_{Max}-p_M)/p_{Max}]\ \exp[(\Theta/cc_1)(cc_0/p_M)]$$

and

$$PC = N\{[(p_M-p_{Max}-1)/p_{Max}]\exp[(\Theta/cc_1)(cc_0-p_M)]-[(cc_0-p_{Max}-1)/p_{Max}]\}.$$

Since $\Theta = (QC+QB)/N$ in the naive-copier model and $\Theta = (QC/N)$ in the expert copier model,

$$\frac{\partial PB}{\partial QB} = N[(p_{Max}-p_M)/p_{Max}][(cc_0-p_M)/Ncc_1]\ \exp[(\Theta/cc_1)(cc_0-p_M)]$$

in the naive-copier model

$= 0$ in the expert-copier model,

$$\frac{\partial PB}{\partial QC} = N[(p_{Max}-p_M)/p_{Max}][(cc_0-p_M)/Ncc_1]\ \exp[(\Theta/cc_1)(cc_0-p_M)]$$

in both models,

$$\frac{\partial PC}{\partial QB} = N[(p_M-p_{Max}-1)/p_{Max}][(cc_0-p_M)/Ncc_1]\ \exp[(\Theta/cc_1)(cc_0-p_M)]$$

in the naive-copier model

$= 0$ in the expert-copier model, and

$$\frac{\partial PC}{\partial QC} = N[(p_M-p_{Max}-1)/p_{Max}][(cc_0-p_M)/Ncc_1]\ \exp[(\Theta/cc_1)(cc_0-p_M)]$$

in both models.

Using an approach similar to the one taken in Pindyck (1978), the problem was solved numerically using the corresponding Lagrangian (i.e., discrete analog in each discrete time period within time horizon T) subject to

$$PB(t) - QB(t) \geqq 0 \quad \text{for all } t,$$

$$PC(t) - QC(t) \geqq 0 \quad \text{for all } t,$$

and

$$p_M(t) \geqq 0 \quad \text{for all } t.$$

The results were obtained using GAMS (Kendrick and Meeraus 1985), a powerful optimization program for large and complex problems. The algorithm used is described in Murtagh and Saunders (1978, 1982).

Appendix 2. Uniqueness of Optimal Price Trajectories

In order to prove uniqueness, the concavity of the Hamiltonian has to be established. Accordingly, it has to be shown that the second-order derivative of the current Hamiltonian, $\hat{H}$, with respect to price is negative over the optimal trajectory.

Using the simplified notation of Appendix 1,

$$\hat{H} = [(p_M-c)+y_B](PB-QB)h_B+y_C(PC-QC)h_C$$

where $y_B = \lambda_B \exp(rt)$ and $y_C = \lambda_C \exp(rt)$.

It can be shown that

$$\frac{\partial^2 \hat{H}}{\partial p_M^2} = A_1[A_2(p_M-c+y_B)+A_3+A_4 y_C] \tag{2-1}$$

where $A_1 = (N/p_{Max}) \exp[(\Theta/cc_1)(cc_0-p_M)]$,

$A_2 = h_B[2+(p_{Max}-p_M)(\Theta/cc_1)](\Theta/cc_1)$,

$A_3 = -2h_B[1+(p_{Max}-p_M)(\Theta/cc_1)]$,

and

$A_4 = -h_C(\Theta/cc_1)[2+(p_{Max}-p_M+1)(\Theta/cc_1)]$.

Accordingly, it is evident that $A_1 > 0$, $A_2 > 0$, $A_3 < 0$, and $A_4 < 0$. If it can be shown that A_3 with its negative sign dominates the second-term in (2-1), concavity will have been established. In order to do this, the signs of (p_M-c+y_B) (and, hence, y_B) and y_C have to be known. Both y_B and y_C are, however, defined by the system of differential equations shown in Appendix 1. As the highly non-linear system is difficult to solve, the signs of y_B and y_C are not easily determined. Nevertheless, some conjecture can be made.

By definition, the co-state factors λ_B and λ_C measure the total impact on profits caused by variations in QB and QC. Three forces are operating here:

(1) a "diffusion" effect: $\nearrow$ QB,QC => $\nearrow$ h_B, h_C => $\nearrow$ $\frac{\partial QB}{\partial t}$, $\frac{\partial QC}{\partial t}$;

(2) a "facilitate copying" effect: $\nearrow$ QB,QC => $\nearrow$ PC, $\searrow$ PB => $\nearrow$ $\frac{\partial QC}{\partial t}$, $\searrow$ $\frac{\partial QB}{\partial t}$;

(3) a "direct" effect: $\nearrow$ QB,QC => $\searrow$ (PB-QB),(PC-QC) => $\searrow$ $\frac{\partial QB}{\partial t}$, $\frac{\partial QC}{\partial t}$.

In early periods, the first effect will be dominant. In late periods, the third effect will be dominant. Therefore, λ_B and λ_C (and, hence, y_B and y_C) should be positive in early periods, negative in later periods, and equal to zero for $t = T$ because of the transversality conditions.

Accordingly, in early periods, (p_M-c+y_B) and y_C are positive and could be large. At the same time, however, Θ is very small and, hence, A_2 and A_4 are small (in absolute value) relative to A_3. Hence, A_3 will dominate the second-order derivative as $A_2(p_M-c+y_B)$ and $A_4 y_C$ will tend to cancel each other out. As time goes on, Θ increases and y_B and y_C decrease. Even though (p_M-c) could be large, $\Theta < 1$ and A_3 will continue to dominate the second-order derivative. In late periods, y_B and y_C approach zero from the negative side and $p_M \to c$ as $QB \to PB$. Again, A_3 will dominate the second-order derivative. Moreover, this conjecture establishes the negativity of the second-order derivative of the Hamiltonian over the optimal price trajectory and, hence, indicates its uniqueness.