Optimality and Sustainability: Regulation and Intermodal Competition in Telecommunications

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REGULATION AND INTERMODAL COMPETITION IN TELECOMMUNICATIONS

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1. Introduction

Over the past decade, federal and state regulators have permitted increasing amounts of competition in markets that had been restricted exclusively to public utility monopolies. Consequently, a pressing issue in today's policy-making arena involves whether residences and businesses should have the right to forego public utility service in favor of alternative technologies that existing competitors or potential entrants may offer; Brautigam (1979) has termed this competitive fringe intermodal competition. The issue is most relevant in the case of large users of utility's services, who may be able to install their own private technologies and leave the public utility system altogether; e.g, large manufacturers frequently construct their own power plants and large companies may install their own telephone facilities. As the future of competition unfolds, intermodal competitors will probably focus their attention primarily on the utility's largest users.

Utility prices at some point must incorporate the substantial fixed costs of the company's plant and equipment; consequently, they must exceed, on average, their associated marginal costs. These uneconomic price signals may induce large

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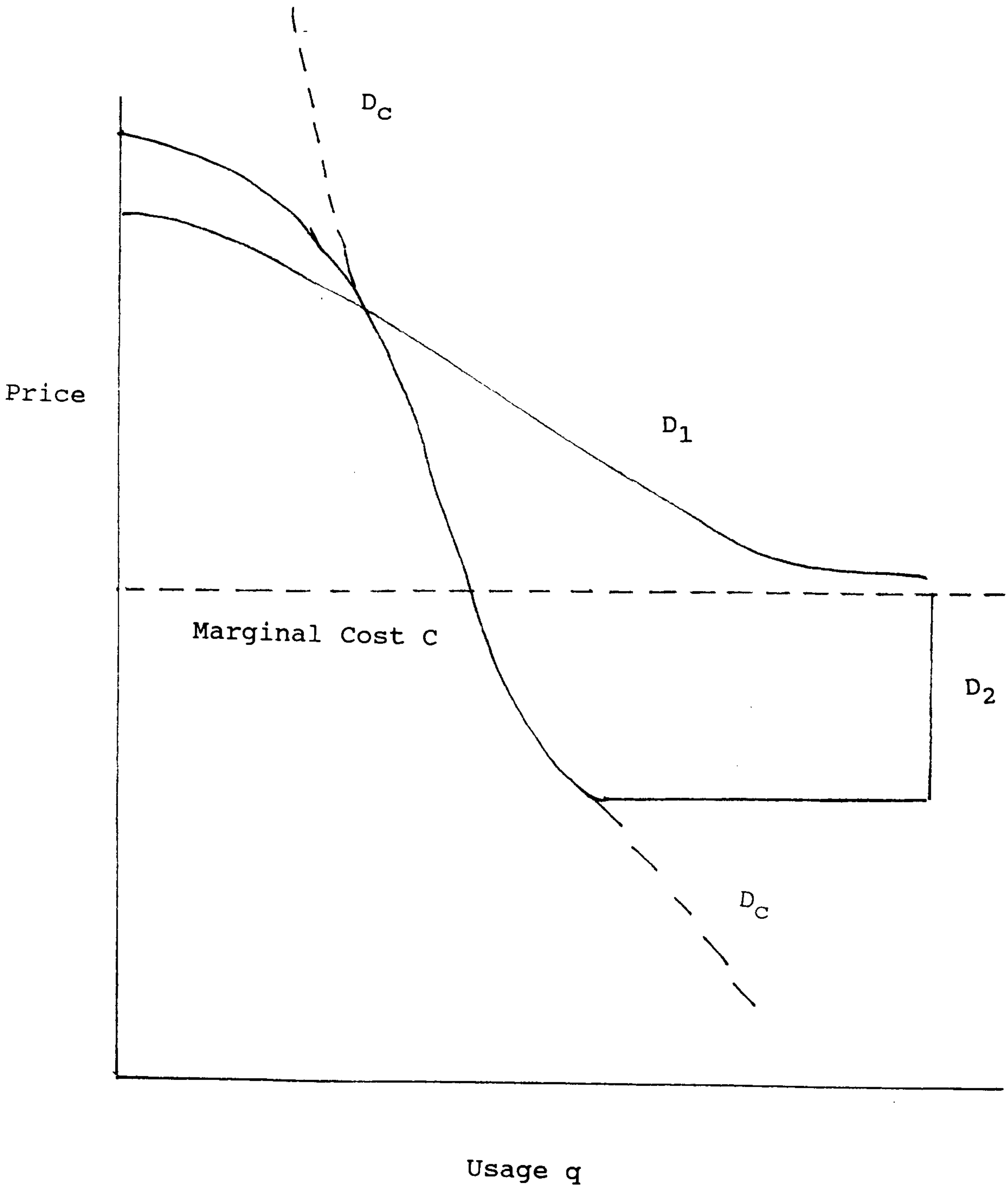
customers to forego the utility's service altogether. Once these large customers exit, they would install substantial amounts of private plant and equipment for alternative service; though profitable to the exiting customers, these investments can be economically inefficient from a social perspective nonetheless. Consequently, social welfare could be reduced if intermodal competition is permitted without allowing the utilities the opportunity to design appropriate pricing strategies.

Perhaps the most prominent (but certainly not the only) contemporary example of large customer exit for competitive alternatives arises in telecommunications. Under current policy, a long-distance user may install and use alternative facilities to reach his long-distance carrier; when customers do this, they are said to bypass the local phone company which would ordinarily provide long-distance access. Consequently, regulators and local companies are caught in a bind. If long-distance access is priced too high, the largest long-distance users will bypass the local company. If regulators price long-distance access too low, the local company cannot cover its fixed costs. In the case of telecommunications, these fixed costs include both the local company's fixed costs of plant and equipment as well as a contribution that toll usage must generate to subsidize single-line customer access to the system; single-line access is now priced -- largely for political reasons -- well below its associated marginal cost.

Contemporary theoretical research on optimal nonuniform pricing (Spence (1977), Roberts (1979), Mirman and Sibley (MS, 1980), and Goldman, Leland, and Sibley (GLS, 1984)) has not considered the problem that intermodal competition for large customers poses to sustainability. Each of these papers designs a nonuniform price schedule to maximize either social welfare or consumer surplus subject to a binding profit constraint on the utility's net earnings; the number of customers is fixed, but usage per customer depends upon price. Each paper then prescribes a nonuniform price schedule which prices some customer usage above its associated marginal cost. The usage price eventually falls to, but is never lower than, the associated marginal cost of usage; the price schedule need not be a monotonically decreasing function of usage. (Curve D_1 in Figure 1 illustrates one possibility.) Based on a regulatory environment of a decade past, these papers do not recognize the possibility that some customers, especially large ones, may leave the utility for an intermodal supplier. Once the possibility of large customer exit is recognized, we shall show that the optimal price schedule can eventually fall below the associated marginal cost of usage. Curve D_2 in Figure 1 illustrates one possibility; the curve marked D_c will be explained later in the paper.

This paper designs an optimal nonuniform price schedule for a profit-maximizing or profit-constrained welfare-maximizing utility that is faced with possible customer exit, especially by large users. Respective utility and bypass costs are assumed to be such that one cannot cost-dominate the other; i.e., there exist usage levels at which each technology is the low-cost technology. The principal conclusions are as follows. First, the utility can and should retain some of these large users by

Figure 1: Optimal Nonuniform Price Schedules



pricing some high-level customer usage below its associated marginal cost; prices for low-level customer usage still should exceed the marginal cost of production. Second, under an optimally designed nonuniform price schedule, the total payments from each customer would exceed the total costs that the customer imposes; though some usage by large customers is subsidized, these customers generate a positive net contribution nonetheless. Third, the utility's price of providing access should be equal to its associated marginal cost; this contrasts with optimal two-part tariff pricing schemes (see Oi (1971), Ng and Weisser (1974), Schmalensee (1981)) which sometimes charge customers access prices which differ from the marginal costs of providing it. Fourth, an optimal nonuniform price schedule would provide economically correct signals to large customers regarding exit; i.e., a customer will leave the utility if and only if the competitor's costs of providing service are lower than the utility's.

This paper is organized as follows. Section 2 provides a basic theoretical and institutional discussion of the telephone pricing problem and provides an intuitive justification of the notion that some usage prices must be below marginal cost when alternatives are available for large customers. Section 3 develops a mathematical model that is consistent with the problem outlined in Section 2; necessary first-order conditions are obtained. Section 4 mathematically derives several properties of the optimal schedule, including below-marginal cost pricing and the fact that no customer is subsidized. Section 5 extends the analysis to some different kinds of bypass possibilities. Section 6 investigates whether large customers make economically efficient decisions (from a social welfare perspective) when deciding whether to leave the utility. Section 7 introduces positive access costs. Section 8 concludes the paper with some implications for policy-making in the contemporary regulatory arena.

2. A Simple Problem Illustrated

The Problem and Its Relevance

This section will discuss the cost-recovery problem, its relevance to contemporary telecommunications regulation, and an economically optimal solution. In specific, we shall show that a profit-maximizing or profit-constrained welfare-maximizing utility, when faced with intermodal competition for large users, should price some high-level usage below its associated marginal cost.

Assume that consumers may purchase products from a utility which is either profit-maximizing or profit-constrained welfare-maximizing. It has positive fixed costs K and constant marginal cost C per unit of usage. Each utility customer pays a marginal price per unit of usage of P(q); P may vary with a customer's level of usage q. Each utility customer faces the same price schedule; resale is not possible. The customer also pays an initial access fee of A; the cost to the utility of providing access to each customer is Z.

Any large consumer can entirely avoid purchasing from the utility by installing an alternative technology. These technologies would involve high flat-rate costs (Z^*) and lower costs per unit of production (C^*) compared with the utility's respective costs. Assuming that these alternative technologies cannot be dominated at high usage levels, these technologies represent the economically preferable choice for the largest customers. Also assume that the intermodal market is competitive (or contestable); i.e., flat-rate prices and usage prices A^* and P^* of all non-utility alternatives equal their respective costs Z^* and C^*. Consumer i's total payments for alternative services are $A^* + C^*q_{i*}$ where q_{i*} is the usage of consumer i on the alternative technology.

This framework represents aspects of the contemporary market for telecommunications, where intermodal competitors may slice the largest users away from the local companies. In particular, most long-distance calls that are routed over local company facilities are switched, meaning that the call is processed and held through the local company's switching facilities and transmitted to long-distance carriers over common transmission lines. As a result, the marginal cost of a minute of usage includes the associated operation and capacity costs of the switching equipment which processes and holds the call. Local companies now bill each long-distance carrier for its share of switched access minutes that the utility handles; carriers pass these charges back to their customers. Switched access charges include an allocated portion of the local company's fixed costs.

Alternative bypass technologies (including the utility's own special access service which competes against other companies as well as its own switched access) directly connect the customer to the long-distance carrier, thereby avoiding some of the costs that switched access usage entails. As a result, the marginal cost of using a bypass technology can be lower than switched access; furthermore, bypass includes no allocated fixed cost subsidy. Because bypass installation entails substantial initial costs, long-distance carriers have an incentive to induce their largest customers to circumvent local company switched access facilities with bypass circuits; because fixed utility costs are allocated to switched access but not bypass minutes, there is an uneconomic incentive favoring bypass.

Reliable estimates of the relevant costs of switched access and bypass are hard to come by. New York Telephone (1986, vol. 1, page 1-29, chart 2) estimates that its average "traffic-sensitive" cost per access minute (as determined by existing accounting procedures) is 3.67 cents; however, the company also claims that the true marginal cost of a switched access minute is 1.28 cents (vol 3, page 5-37, tab 1). The monthly levelized cost of installing and maintaining a switched access line is somewhere between $25 and $30 per month. The cost of a bypass channel varies with the specific technology and the size of the user; there are economies of scale in bypass installation. While present costs of bypass may be in the vicinity of $100 per circuit per month, these costs may fall substantially by the end of the decade. $50 per circuit or lower seems very possible in some locations. (For estimates of bypass and local company costs, see Bell Communications Research (1984), Jackson and Rohlfs (1985))

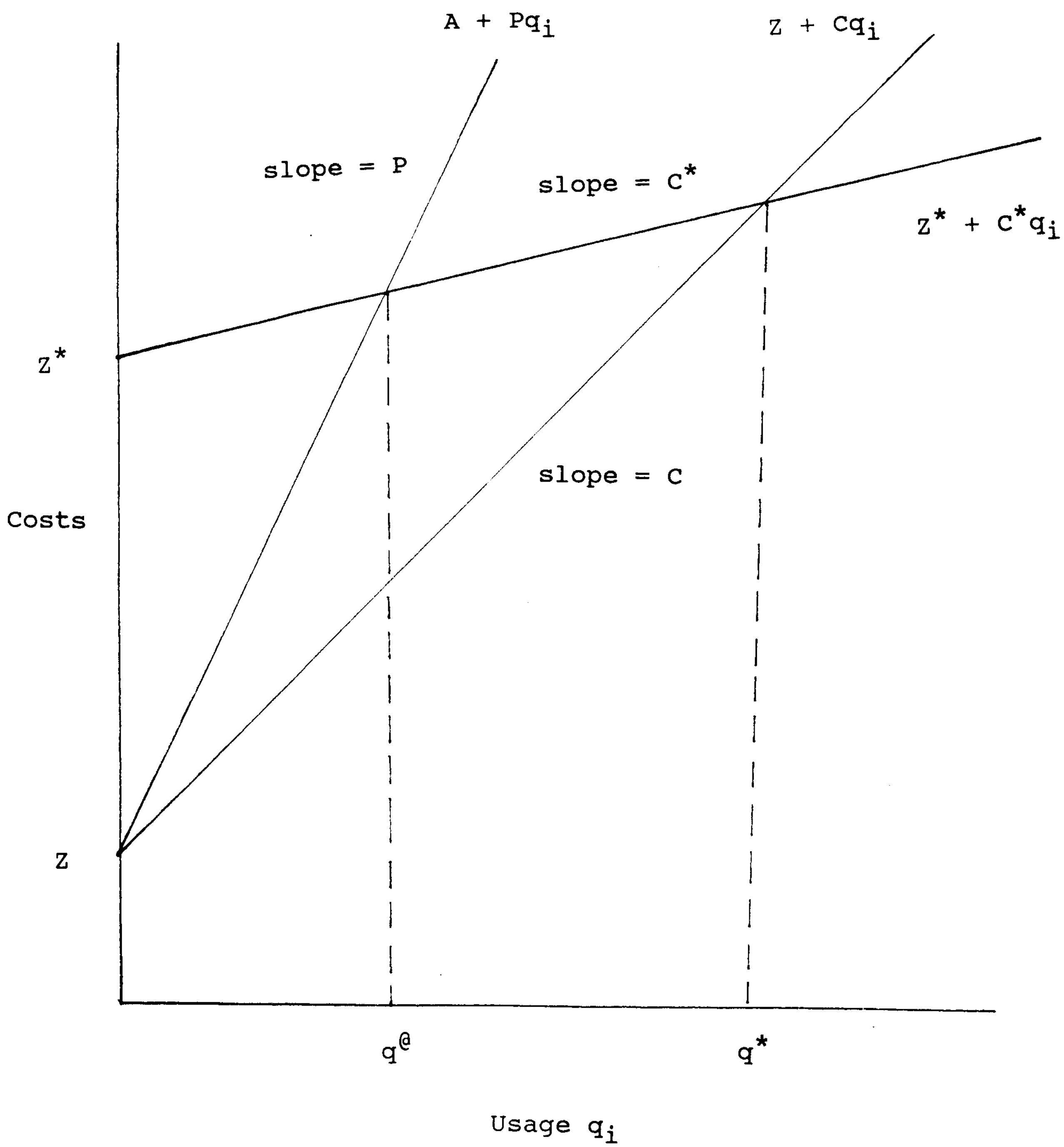
Setting $C = 1.0$ cents per minute, $Z = \$25$ per line per month, and $Z^* = \$75$ per circuit per month, bypass would be economically efficient if monthly circuit usage q_i exceeded $(Z^* - Z)/C = 5000$ switched access minutes per month. At this level of usage, bypass cost-dominates switched access. Since the average large WATS caller uses 5500 access minutes per line per month (Griffin and Egan (1985)), economic bypass by large long-distance users may soon be, if it is not already, a realistic threat to the local company.

Bypass Illustrated

In the remainder of this section, assume that customer usage is inelastic with respect to its own price; Section 3 will relax this assumption. If customer i were to remain with the utility, he would impose costs of $Z + Cq_i$, where q_i represents his usage. If the customer were to choose an alternative supplier, he would impose costs of $Z^* + C^*q_i$. Figure 2 represents the respective costs of each. In Figure 2, customers who use less (more) than q^* would minimize social costs by choosing the utility (bypass technology); q^* is the intersection of the two cost schedules and equals $(Z^* - Z)/(C - C^*)$. From a standpoint of economic efficiency, utility (bypass) technology is the dominant service when usage is less (greater) than q^*.

For _first-best_ pricing, the utility would set A and P(q) at marginal costs Z and C respectively. Under these circumstances, payments to the utility $A + Pq_i = Z + Cq_i$ would be less (greater)

Figure 2: Comparative Costs of Utility and Intermodal Competitor



than payments to competitors $A^* + P^*q_i = Z^* + C^*q_i$ if $q_i < (>) q^*$. Bypass would then occur at q^* and above. However, under first-best marginal cost pricing, the utility would not be able to cover its fixed costs (and necessary single-line subsidies) K. As a result, this first-best solution could not be satisfactory for any welfare-maximizing utility that faces a positive fixed costs requirement. Furthermore, a profit-maximizing utility certainly would not price at marginal cost. For a payment schedule to generate positive profits, payments at some usage levels q_i must exceed costs; i.e., $A + R(q_i) > Z + Cq_i$.

In order to cover their variable and fixed costs, local companies now bill long-distance carriers for their switched access usage with a per minute traffic-sensitive component (which is a flawed overestimate of actual marginal cost) plus a per minute additional charge to cover the fixed costs K of the local company. Even if the per minute traffic-sensitive costs accurately reflected their associated marginal costs, the effective price P of a switched access minute would still exceed marginal cost C. Given the present pricing structure, a long-distance carrier would be able to induce customer i to bypass if $A + Pq_i > Z^* + C^*q_i$. Assuming that A = Z (which is approximately true for multiline telephone customers), bypass would occur if $q_i > q^@ = (Z^* - A)/(P - C^*) < q^*$; see Figure 2. From a social welfare-maximizing perspective, bypass by consumers who use between $q^@$ and q^* should not occur; because local company service is actually cost-dominant, bypass is said to be uneconomic. Bypass by users who use more than q^* is (socially) cost-justified and is termed economic.

Optimal Pricing

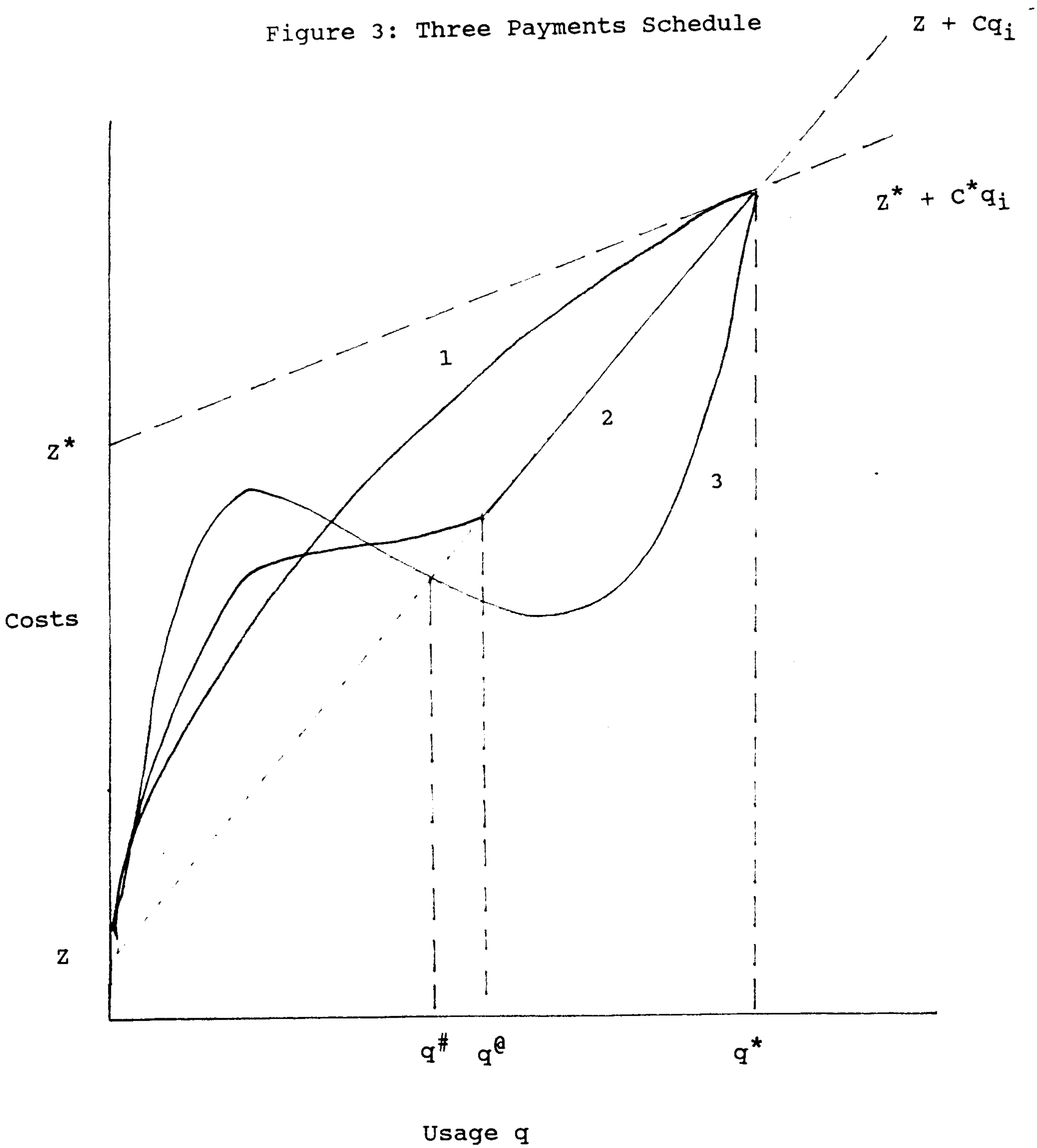
If permitted to implement a nonuniform price schedule, a profit-maximizing or a profit-constrained welfare-maximizing utility should be able to attract any customer that uses less than q^* units. To see this, note that the customer will forego utility service only if payments to the utility $A + R(q_i)$ exceed payments under an alternative technology $A^* + P^*q_i$, which have been assumed to be equal to costs $Z^* + C^*q_i$. If exit should occur, the utility would make no profit from this customer; furthermore, social welfare would be reduced since utility service provides the less expensive choice when $q_i < q^*$. Consequently, the utility may simultaneously increase its profits and improve social welfare by changing its price schedule so that $Z + Cq_i < A + R(q_i) < A^* + P^*q_i$ for all $q_i < q_i^*$. The permissible region for the payment schedule lies between the curves $Z + Cq_i$ and $Z^* + Cq_{i*}$ in Figure 2.

By contrast, there is no reason why a profit-maximizing or profit-constrained welfare-maximizing utility should ever have a customer who uses above q^*. To see this, note again that since alternative suppliers are competitive with one another, payments $A^* + P^*q_i$ must equal costs $Z^* + C^*q_i$. To attract the customer, the utility would have to offer a lower-priced service, which means that it must price service below $Z^* + C^*q_i$; this is below the utility's own costs of $Z + Cq_i$ (since $q_i > q^*$; see Figure 2). Consequently, the utility could attract the customer only by losing money on it. Furthermore, the social cost of the alternative service is below the utility's for customer usage more than q^*. Therefore, customer i's choosing the utility is irrational from both a welfare-maximizing and a profit-maximizing perspective.

If permitted to price with a nonuniform price schedule, a profit-maximizing or a profit-constrained welfare-maximizing utility will then retain (lose) all customers that use less (more) than q^* units; in this case, bypass will result if and only if it is economic. For either a profit-maximizing or a profit-constrained welfare-maximizing firm, $A + R(q^*) = Z + Cq^*$ must hold at the bypass point q^*. Figure 3 illustrates three possible payments schedules; note that each permits economic bypass but disallows uneconomic bypass. Consequently, first-best marginal cost pricing (i.e., $A = Z$, $P = C$) is sufficient, <u>but not necessary</u>, for providing the economically correct signals for bypass in the case, assumed here, when usage is perfectly price-inelastic.

In payment schedule 1, each customer pays to the utility an amount which always exceeds the respective costs that it imposes; i.e., $A + R(q_i) > Z + Cq_i$ for all $q_i < q^*$. Consequently, each customer makes a positive contribution to the utility's profits and no customer is subsidized. In payment schedule 2, each customer who uses more than $q^@$ pays just enough to cover its imposed costs; each customer who uses less than $q^@$ makes a positive contribution to the utility's profits. No customer is subsidized in schedule 2. In payment schedule 3, each customer

Figure 3: Three Payments Schedule



who uses more than $q^{\#}$ pays less than its imposed costs; each of these customers must be subsidized by customers who use less than $q^{\#}$.

We now shall demonstrate that regardless of which payments schedule is implemented, the price of usage must at some point be below its associated marginal cost. First, note that $A + R(q_i) > Z + Cq_i$ (for some $q_i < q^*$) must hold somewhere if non-negative profits are to be secured. Let $R_{i,*}$ represent the revenue that the utility secures from a customer's usage between q_i and q^*; $C(q^* - q_i)$ represents the associated cost of this additional usage. At usage q^*, payments $A + R(q^*) = A + R(q_i) + R_{i,*}$ and costs $Z + Cq^* = Z + Cq_i + C(q^* - q_i)$. Since $A + R(q^*) = Z + Cq^*$ and $A + R(q_i) > Z + Cq_i$ at some $q_i < q^*$, $R_{i*} < C(q^* - q_i)$ must hold. The incremental price of usage $P(q)$ must then be below its associated marginal cost C somewhere between q_i and q^*.

Consequently, the utility will subsidize some high-level usage by large customers to keep them from uneconomically bypassing. This does not mean that the utility necessarily subsidizes any of these customers in total; e.g., see payment schedule 1. In payment schedule 1, prices for low-level usage (near 0) are above marginal usage cost and each customer, large or small, generates in these low-level blocks a positive contribution toward the utility's profits. For larger customers (nearer to but still less than q^*), this "profit-cushion" is returned via some subsidized usage at the high-end. Prior to q^*, each customer, in total, pays more to the utility than it costs. Once the "profit-cushion" is burned off entirely (at q^*, where $A + R(q^*) = Z + Cq^*$), the utility is powerless to prevent bypass;

customers that use more than q^* cannot be profitably restrained from leaving. However, from an economic perspective, these customers should leave because the alternative supplier can serve them at lower cost.

In payment schedule 1, some usage prices are below their associated marginal costs. However, no customer is subsidized "in the whole". The next section will demonstrate that payment schedule 1 presents the basic paradigm for the optimal nonuniform price schedule.

3. A Mathematical Model: Optimizing Conditions

This section will develop a mathematical model of a utility and its maximizing strategies, its customer subscriptions, and resulting usage patterns. We shall state useful assumptions and definitions, define an objective function and an optimal control problem, and derive necessary first-order optimizing conditions.

Variables, Definitions, and Assumptions

Customers of a utility vary among one another depending upon their intensities of usage. Assume that consumer demand-for-product curves do not cross (see Faulhaber and Panzar (1977); Spence; MS; GLS); i.e., $q_m(P_A) > q_n(P_A)$ if and only if $q_m(P_B) > q_n(P_B)$ for all customers m and n and all usage prices P_A and P_B. The same is true for q_{m*}, q_{n*}, P_{A*}, and P_{B*} where the asterisk represents usage and prices for an alternative technology. Given the non-crossing assumption, we may index each consumer by an ordinal parameter i which represents the intensity of his preference for the product. i increases with intensity and lies between 0 and 1; i is continuously distributed with frequency f(i). Let a (e) designate the infimum (supremum) of intensities i of customers who are served by the utility; $(a, e) \in [0, 1]$.

Assume that customer intensity 0 is such that $q_0 = 0$ at $p \geq 0$. In this section, assume that the utility cannot provide any additional benefit to its customers beyond paid usage; consequently, people subscribe to the utility only in order to purchase service. Section 6 will relax this assumption. In making these assumptions, we have implicitly disallowed any price design which could cover K without distorting customer usage or subscription in some manner; while this possibility would be fortuitous from a welfare-maximizing standpoint, the resulting optimization problem would be trivial.

Represent usage by customer i as q_i; his marginal price is $P_i = P(q_i) = dR/dq_i$. The net welfare W_i of consumer i on each system is written:

3.1a) $W_i(q_i) = U_i(q_i) - R(q_i) - A$

3.1b) $W_{i*}(q_{i*}) = U_i(q_{i*}) - C^* q_{i*} - A^*$

where:

$U_i(q_i)$ = consumer i's willingness to pay for usage q_i

For an individual to maximize personal utility, $dU_i/dq_i = dR/dq_i = P_i$ or $dU_{i*}/dq_{i*} = P^* = C^*$.

Society's consumers can be divided into two groups. If utility services were unavailable, <u>small consumers</u> would rather forego service altogether rather than install an alternative technology; i.e., $0 > W_{i*}$. By contrast, <u>large consumers</u> would install an alternative system; i.e., $W_{i*} > 0$. Let b designate the borderline customer between the small and the large group.

Assumption 1: $a < b < e$. In words, the utility serves both small and large customers.

If they exist, utility customers i with $W_i = 0$ will be indifferent between having utility service and having no service at all. While a small utility customer need not be indifferent, utility customers with $W_i = 0$ must be small consumers. To prove this, recall that a large consumer i has $W_{i*} > 0$. If $0 = W_i$, a large consumer would then drop off the utility system altogether (since $W_i = 0 < W_{i*}$). By contrast, a small consumer has $W_{i*} < 0$; consequently, he would prefer being an indifferent customer of the utility rather than move to an alternative technology. Consequently, we shall call the state of indifference where $W_i = 0$ as <u>small-indifference</u>.

Utility customers for whom $W_{i*} = W_i$ will be indifferent between purchasing utility service and using an alternative technology. By reasoning similar to that in the above paragraph, we may show that large consumers are the only utility customers for whom $W_{i*} = W_i$ is possible; however, not all large utility customers need be indifferent. The state of indifference where $W_i = W_{i*}$ will be termed <u>large-indifference</u>.

Given the definitions of infimal and supremal intensities a and e and Assumption 1, a consumer with $i < a$ $(i > e)$ will prefer having no service at all (an alternative technology) even if utility services are available. That is, $W_i < 0$ for $i < a$; $W_i <$

W_{i*} for $i > e$.

The set of utility customers S is <u>complete</u> if $S = (a, e)$ except for a countable number of points. The set S is <u>incomplete</u> if $\exists$ $(a', e') \in [a, e]$ such that $(a', e') \notin S$.

The remainder of this section will employ the following assumptions:

Assumption 2: S is complete

Assumption 3: The price schedule is continuous (though not necessarily smooth) over (a, e).

Assumption 2 means that if consumers i and k ($k > i$) are utility customers, all consumers j where $i < j < k$ will be utility customers as well; no gaps are permitted in the spectrum of customer intensities. As a result, we may integrate over customer intensities between endpoints a and e. Assumption 3 disallows downward or upward jumps in the price-schedule. A downward jump in the price schedule produces a nonconvex budget set (see Burtless and Hausman (1978)). As a result, no consumer may consume within a finite neighborhood of the kink; consequently, gaps arise in customer usage levels (even if intensities are continuously distributed). By contrast, an upward jumping price schedule produces a kinked convex budget set; in this case, customers tend to bunch up at the kink. (for more on gapping and bunching, see Burtless and Hausman, GLS.)

The appendix proves that a profit-maximizing or a profit-constrained welfare-maximizing firm would meet Assumption 2;

furthermore, no upward discontinuities would ever arise in an optimal nonuniform price schedule. While downward discontinuities do seem possible, the basic results of this paper hold if these discontinuities are present.

We now shall define a very important property of a nonuniform price schedule:

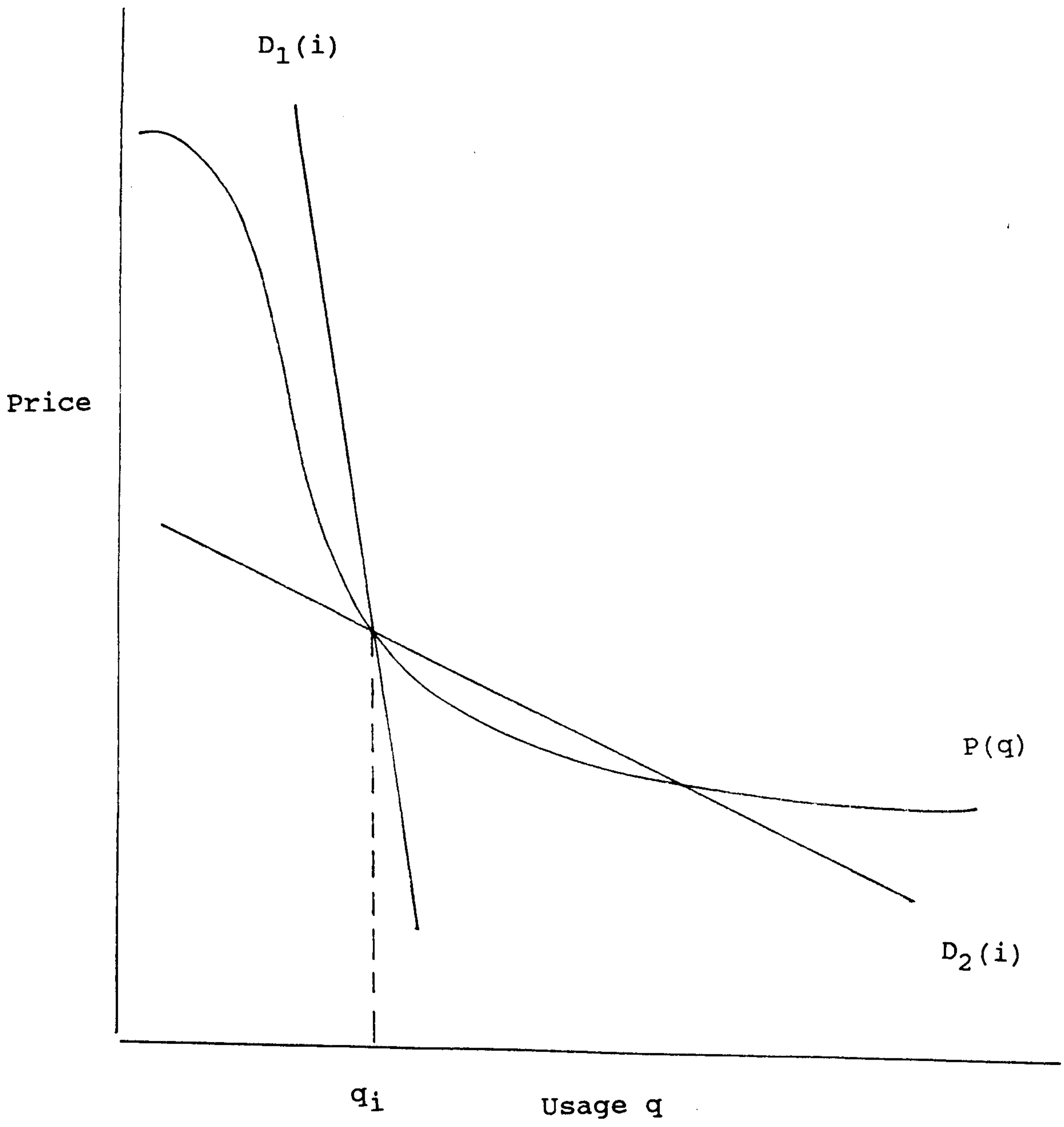
Definition: A price schedule $P(q)$ is <u>single-crossing</u> at q if, for any i such that $P(q) \leq \partial U_i(q)/\partial q_i$, $P(q') \leq \partial U_i(q')/\partial q'$ for $q' \leq q$ (GLS).

Assumption 4: The price schedule is single-crossing

Assumption 4 ensures that second-order conditions for a profit-maximum or a profit-constrained welfare maximum are always met; it is equivalent to assuming that customer usage increases with customer intensity at all prices P_i. This is known as the <u>monotonicity constraint</u> (Roberts, GLS). Note that $u_i = \partial U_i(q)/\partial q_i$ is consumer i's marginal willingness to pay for an incremental change in usage at q_i. Consequently, u_i represents the price on consumer i's demand function that corresponds to q_i. The single-crossing assumption then insures that customer i's demand curve intersects the price schedule from above at q_i; $D_1(i)$ in Figure 4 is single-crossing. If single-crossing did not hold at a particular q_i, then q_i would not be a point where utility is maximized; since $D_2(i)$ in Figure 4 is double-crossing, utility is minimized at q_i (since $u_i(q) < P(q)$ for $q < q_i$). Note that $\partial q_i/\partial i < 0$ at q_i for $D_2(i)$ as well.

Some authors (Roberts, GLS) have considered what constraints

Figure 4: Single-Crossing and Double-Crossing Price Schedules



on the price schedule are necessary if the monotonicity constraint fails to hold on an unconstrained price schedule which meets necessary, but not sufficient, first-order conditions. The monotonicity constraint must either be assumed to hold (Spence, MS) or imposed as a constraint (Mirrlees 1976); Roberts; GLS).

First-Order Maximizing Conditions

Turning to the formal optimization problem, we define aggregate consumer welfare W:

$$3.2) \quad W = \int_a^e W_i dF(i) + \int_e^1 W_{i*} dF(i)$$

Utility profits are defined:

$$3.3) \quad X = \int_a^e (R_i - Cq_i) dF(i) - K$$

A profit-maximizing utility will attempt to maximize profits X (eq. 3.3). A profit-constrained welfare-maximizing utility will attempt to maximize consumer welfare (eq. 3.2) subject to the constraint that profits are non-negative ($X \geq 0$).

For each customer i served by the utility, there are two additional constraints that face either type of strategist. First, for i to remain a utility customer instead of foregoing service altogether, $W_i \geq 0$; customers for whom equality holds are small-indifferent. Second, for i to remain a utility customer instead of jumping to an alternative supplier, $W_i \geq W_{i*}$; customers for whom equality holds are large-indifferent.

Given the appropriate maximands, profit constraints, and indifference constraints, we may construct the objective function:

$$3.4) \quad L = (1 - g)W + gX + \int_a^e h_i(W_i - 0) + \int_a^e j_i(W_i - W_{i*})$$

The last two terms on the right hand side of eq. 3.4 represent the constraints that are needed to ensure that $W_i \geq 0$ and $W_i \geq W_{i*}$ for all customers i. If h_i (j_i) > 0, customer i is small- (large-) indifferent. If h_i (j_i) = 0, customer i is not small- (large-) indifferent. It is impossible for either h_i or j_i to be negative.

Eq. 3.4 can be converted to a straightforward profit-maximizing problem by setting g = 1 and to an unconstrained welfare-maximizing problem by setting g = 1/2. A profit-constrained welfare-maximizing problem must weight profits X more heavily than consumer surplus W; consequently, 1/2 < g < 1 for a profit-constrained welfare-maximizing problem. (For further explanation of this Lagrangean formulation, see Schmalensee, MS.)

Since $W_i = U_i - R_i$ for all $i \in (a, e)$:

$$3.5) \quad dW/di = \partial U/\partial i + (\partial U_i/\partial q_i - \partial R/\partial q_i)(dq_i/di)$$

$$= \partial U/\partial i$$

The second equality follows since consumers are utility-maximizers and, therefore, $dU_i/dq_i = dR/dq_i = P_i$.

Write W_i:

$$3.6) \quad W_i = W_a + \int_a^i (\partial U/\partial i)dj$$

Substituting 3.6 into 3.4:

$$3.7) \quad L = (1 - g) \int_a^e W_i dF(i) + (1 - g) \int_e^1 W_{i*} dF(i)$$

$$+ \; g \int_a^e (U_i - W_i - Cq_i)\,dF(i)$$

$$+ \int_a^e h_i W_i\,di + \int_a^e j_i (W_i - W_{i*})\,di$$

$$= \int_a^e [g(U_i - Cq_i) + (1 - 2g + h_i + j_i)W_a + (1 - 2g + h_i + j_i) \int_a^i (\partial U/\partial i)\,dj]\,dF(i)$$

$$+ (1 - g) \int_e^1 W_{i*}\,dF(i) - \int_a^e j_i W_{i*}\,dF(i)$$

$$= \int_a^e [g(U_i - Cq_i)f(i) + (1 - 2g + h_i + j_i)W_a f(i) + (1 - 2g + h_i + j_i)(\partial U/\partial i) \int_i^e f(j)\,dj]\,di$$

$$+ (1 - g) \int_e^1 W_{i*}\,dF(i) - \int_a^e j_i W_{i*}\,dF(i) .$$

The third equality in equation 3.7 involves reversing the order of integration in the last term. The basic derivation is due to Spence.

We shall displace eq. 3.7 with respect to q_i, a, and e. Herein lies a crucial distinction between this model and the earlier optimal nonuniform pricing models of Spence, Roberts, MS[1], and GLS; while these authors displace welfare with q_i or its equivalent, they assume that the endpoint intensities a and e are fixed. However, profound differences will arise if the endpoint e is a variable of choice. Note that if a is an interior solution ($a > 0$), $W_a = 0$ must hold; if e is an interior solution ($e < 1$), $W_e = W_{e*}$ must hold.

The optimizing conditions are then:

3.8a) $g(dU_i/dq_i - C)f(i) + (1 - 2g + h_i + j_i)(\partial^2U/\partial i\partial q)[F(e) - F(i)] = 0$

3.8b) $(g - h_a - 1)W_af(a) - g(R_a - Cq_a) = -g(R_a - Cq_a) \leq 0$

$a \geq 0;\ ag(R_a - Cq_a) = 0$

3.8c) $(1 + j_e - g)(W_e - W_{e*}) + g(R_e - Cq_e) = g(R_e - Cq_e) \geq 0$

$e \leq 1;\ (e - 1)g(R_e - Cq_e) = 0$

The first equalities in 3.8b and 3.8c result from the fact that $W_a = 0$ and $W_e = W_{e*}$ at interior solutions for a and e.

The relevant Kuhn-Tucker conditions are:

3.8d) $X \geq 0;\ g \geq 0;\ gX = 0$

3.8e) $W_i \geq 0;\ h_i \geq 0;\ h_iW_i = 0$

3.8f) $W_i \geq W_i^*;\ j_i \geq 0;\ j_i(W_i - W_i^*) = 0$

For all i, $dU_i/dq = dR/dq = P(q_i)$. Equation 3.8a can then be reexpressed:

3.8a') $P_i = P(q_i) = C + (2g - 1 - h_i - j_i)t(i)(d^2U/didq)/g$

where:

$t(i) = [F(e) - F(i)]/f(i) \geq 0$

4. An Optimal Nonuniform Price Schedule

This section will demonstrate that the curve D_2 in Figure 1 is an accurate prototype of an optimal nonuniform payments schedule once large customer exit is possible. We shall derive the shape of the price schedule through a series of lemmas and theorems; if short, proofs of some lemmas and theorems have been relegated to endnotes. In deriving the shape of the optimal price schedule, we shall prove in the process that usage price $P(q)$ must be below its associated marginal cost C at some point. We shall then demonstrate that no customer is subsidized.

Prices for Low-Level Usage

The first subsection derives results that are relevant to the small-usage end ($q < q_b$) of the price schedule. Most importantly, we shall show that consumer a is the only small customer who may be indifferent, usage prices for non-indifferent customers (including all small customers $i > a$) must exceed marginal cost, and the smallest utility customer (a) will have a usage q_a equal to zero.

Lemma 1.1: If $a = \inf(S)$, $W_i > 0$ for all $i > a$.[2]

By the definition of small-indifferent, Lemma 1.1 proves that no customer $i > a$ can be small-indifferent. This means $h_i = 0$ for $i > a$.

Lemma 1.2: If customer i is not small- or large-indifferent, $P_i > C$.[3]

Lemma 1.3: If $P_i \leq C$, then customer i is indifferent.[4]

Note that the converses of both lemmas are not necessarily true.

Lemma 1.4: For all $i \in (a, b]$, $P_i > C$.[5]

Theorem 1: $q_a = 0$.

Proof: Two cases are possible: $a = 0$ (a corner solution) or $a > 0$ (an interior optimal point).

If $a = 0$, for any $p \geq 0$, $q_a = 0$. This follows from the definition of customer intensity 0 (i.e, for $a = 0$, $q = 0$ when $P \geq C$).

Let $a > 0$. We shall prove that $q_a = 0$ by contradiction. Suppose that $q_a > 0$. For $i > a$, customer i is not indifferent; therefore, $P_i > C$ (Lemma 1.2). Because jumps are not permitted, $P_a > C$. At a , $W_a = 0$; $U_a = R_a$. Because we have assumed single-crossing (see definition), $W_a = 0$ requires $u_a = dU_a/dq = P(q)$ for all $q \leq q_a$ (see proof of Lemma 1.1). This means that the price schedule and the demand curve for customer a must coincide between 0 and q_a; i.e., the price schedule is monotonically downward-sloping to some point $P_a > C$. At an optimum $a > 0$, $R_a = Cq_a$ must hold (see eq. 3.8b); this implies for some prices at $q < q_a$ that $P(q) \leq C$. But this contradicts the fact that the price schedule is monotonically downward-sloping. It is then impossible for $q_a < 0$. E.O.P.

<u>Prices for High-Level Usage</u>

We now turn to properties of the schedule at $q > q_b$. We shall show that some large customers must be indifferent (if $e < 1$); furthermore, usage price $P(q)$ for these customers must be at C^* (and therefore below C). Although some high-level usage by large users is subsidized, no customer is subsidized "in the whole".

Theorem 2: If $e < 1$, then some customers $i \in (b, e)$ must be large-indifferent between having utility services or not; i.e., $W_i = W_{i*}$ for some $i \in (b, e)$.

Proof: Proof by contradiction; assume that no customer is large-indifferent. Therefore, $j_i = 0$. At infimal a, $q_a = 0$ (see Theorem 1); therefore, $R(q_a) = Cq_a = 0$. Lemma 1.1 demonstrated that for $i > a$, $h_i = 0$. Since demand curves do not cross one another, $d^2U/dqdi > 0$; furthermore, $g > 1/2$ in a profit-maximizing or a profit-constrained welfare-maximizing optimization problem. From eq. 3.8a' and the above facts, $P(q_i) > C$ for all $i \in (a, e)$. If e is a customer, $t(e) = [F(e) - F(e)]/f(e) = 0$ (see definition of t(i) following eq. 3.8a'); therefore, $P(q_e) = C$ (see eq. 3.8a').

Eq. 3.8c requires $R_e = Cq_e$ for $e < 1$; this clearly cannot occur if $q_a = 0$, $P_i > C$ for all $q_i \in (q_a, q_e)$, and $P_e = C$. Therefore, some customers must be large-indifferent. E.O.P.

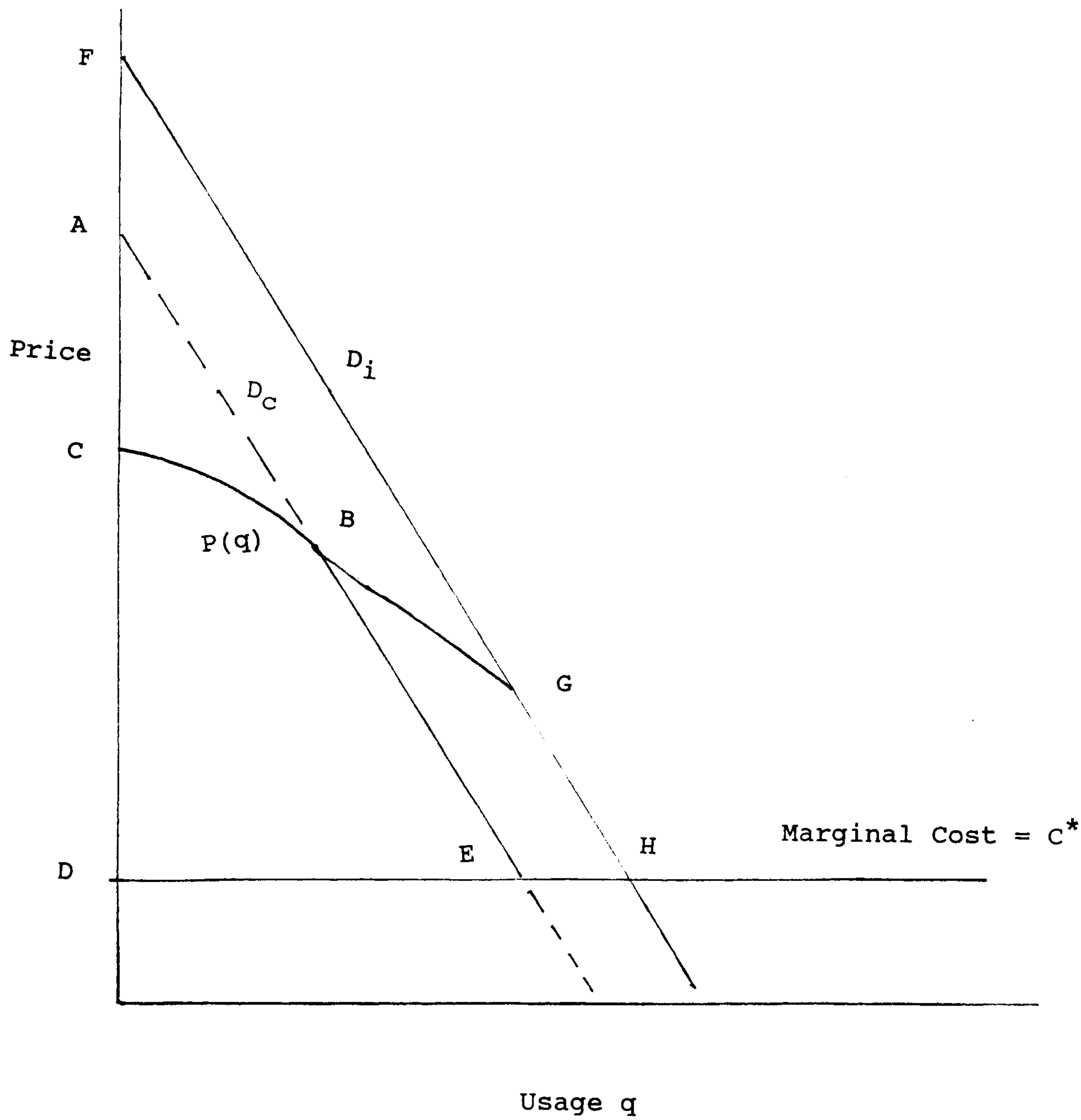
Corollary 2.1: If $e < 1$, $P_i < C$ at some point on the utility's price schedule.[6]

We now shall begin to derive the shape of the price schedule after intensity b. A preliminary lemma will be useful.

Lemma 3.1: Let c represent the intensity of the smallest large-indifferent customer. Suppose for some i ($i > c$), $P_i > C^*$. Then for some $j \in (c, i)$, $P_j < C^*$.

Proof: Proof by contradiction. Suppose that $P(q_j) \geq C^*$ for all $j \in (c, i)$ and that $P(q_i) \geq C^*$. Figure 5 displays the respective demand curves for customers c and i D_c and D_i, one possible price schedule P(q), and the bypasser's usage price $P^* = C^*$. Area ABC (FGC) represents the net consumer surplus that customer c (i) enjoys with utility service; with an alternative technology, customer c (i) would enjoy a net consumer surplus of area ADE (FDH) minus the flat-rate fee A^* (or Z^*). Since c is large-indifferent, area ADE - A^* = area ABC; therefore, A^* = area BCDE. Since i is a utility customer, area FDH - $A^* \leq$ Area FGC; therefore, $A^* \geq$ area GCDH. But area GCDH = area BCDE + area GBEH.

Figure 5: Two Demand Curves for Customers c and i



Unless area GBEH = 0, A^* cannot possibly equal both area BCDE and area GCDH. Furthermore, area GBEH cannot equal zero if $P_j \geq C^*$ for j ε (c, i) and $P_i > C^*$. E.O.P.

Given Lemma 3.1 and Assumption 4, the utility's price schedule P(q) must correspond with customer c's demand curve $u_c(q)$ from point B to point E; i.e., the price schedule falls along customer c's demand schedule from some price P_i ($\geq$ C) down to C^*. Immediately beyond E, $P(q) \leq C^*$. Theorem 4 will demonstrate that only equality may hold.

Theorem 3: Let q_{C*} represent the usage level of customer c at marginal price c^* (i.e., point E in Figure 5). Then $R(q_{C*}) = Z^* + C^*q_{C*}$.

Proof: Given Lemma 3.1, usage price P_c for consumer c might fall to C^* or lower; the utility's price schedule P(q) and consumer c's demand curve must coincide between points B and I, which might be the same as E. At point E, consumer c can purchase usage at marginal usage price C^*, which is below marginal cost C; therefore, user c must be large-indifferent between utility and bypass services at point E. (see Lemma 1.3) By extension, the consumer must be indifferent between E and any price between B and I. Since the marginal usage price at E (C^*) is also the marginal usage price under bypass, it follows (from the idea of indifference) that customer payments for usage prior to q_{C*} must be equal under utility and bypass payment schedules; this means $R(q_{C*}) = Z^* + C^*q_{C*}$. E.O.P.

Theorem 4: C^* is the minimum price for usage.

Proof: Proof by contradiction. Let (i, j) represent the first interval of prices P_i, P_j that are below C^*; $i \geq c$ and $j \leq e$. This interval must precede any $P > C^*$ for intensities above c; see Lemma 3.1. $i = c$ is possible if the price schedule falls below E to I in Figure 5.

Let P_j represent the marginal price of usage for consumer j; his usage is $q_j = q_j(P_j)$. Given the nature of the interval (i, j) and Assumption 3, $P_j \leq C^* < C$. Since $P_j < C$, consumer j is indifferent between utility and bypass service; see Lemma 1.3. Therefore,

$$4.1) \quad U_j(q_j) - R(q_j) = U_j(q_{j*}) - Z^* - C^*q_{j*}$$

where:

q_{j*} = optimal usage by consumer j on a bypass system

q_j represents consumer j's optimal usage on the utility's price schedule; consequently, q_{j*} cannot be preferred to q_j if P(q) is the relevant price schedule. Therefore,

$$4.2) \quad U_j(q_j) - R(q_j) \geq U_j(q_{j*}) - R(q_{j*})$$

Revenue $R(q_{j*})$ is the sum $R(q_{c*}) + R_{c*,j*}$, where the latter term represents utility revenue between q_{c*} and q_{j*}. Theorem 3 proved that $R(q_{c*}) = Z^* + C^*q_{c*}$; substituting, $R(q_{j*}) = Z^* + C^*q_{c*} + R_{c*,j*}$. Given that the interval (i, j) must precede any $P > C^*$ after consumer c, $R_{c*,j*} < C^*(q_{j*} - q_{c*})$. Therefore,

$$\begin{aligned} 4.3) \quad U_j(q_{j*}) - R(q_{j*}) &= U_j(q_{j*}) - Z^* - C^*q_{c*} - R_{c*,j*} \\ &> U_j(q_{j*}) - Z^* - C^*q_{c*} - C^*(q_{j*} - q_{c*}) \\ &= U_j(q_{j*}) - Z^* - C^*q_{j*} \end{aligned}$$

Combining eqs. 4.2 and 4.3, $U_j(q_j) - R(q_j) > U_j(q_{j*}) - Z^* - C^*q_{j*}$. The inequality in 4.3 contradicts eq. 4.1, which defines the indifference condition. Therefore, we have just contradicted our initial assumption that customer j was indifferent. E.O.P.

Corollary 4.1: $P_i = C^*$ for all customers $i \in (c, e)$[7].

In words, Theorem 4 and Corollary 4.1 mean that if $P_i = C^*$ at usage q_{c*}, the price schedule is flat until usage q_e. In Figure 5, price P(q) falls along consumer c demand curve from point B to point E and then stays fixed at C^*. Area GBEH then equals 0. Consequently, both consumer c and $i > c$ can remain utility customers, although both are large-indifferent. (see proof of Lemma 3.1.)

Corollary 4.2: If $e < 1$, e is not a customer.[8]

We now prove that although some high-level usage is subsidized, no customer is subsidized "in the whole".

Theorem 5: $R_i > Cq_i$ for all customers $i \in (a, e)$.

Proof: Since c is the first indifferent customer, $P(q_i) > C$ for $i < c$ (see Lemma 1.2); the theorem is then obviously true for $i \in (a, c)$. For $i \in [c, e)$, R_e can be expressed:

4.4) $R_e = R_i + R_{i,e}$

where:

R_i = revenue from customer with intensity $i \in [c, e)$

$R_{i,e}$ = additional revenue gain from usage between

q_i and q_e

Similarly, Cq_e can be expressed:

4.5) $Cq_e = Cq_i + Cq_{i,e}$

If $e < 1$, $R_e = Cq_e$ is a necessary first-order optimizing condition (see eq. 3.8c). Subtracting eq. 4.5 from 4.4 and rearranging terms, $R_i - Cq_i = Cq_{i,e} - R_{i,e}$. Since usage above q_{c*} is priced above C^*, $R_{i,e} = C^*(q_e - q_i) < C(q_e - q_i) = Cq_{i,e}$. Therefore, $R_i - Cq_i > 0$ or, equivalently, $R_i > Cq_i$. E.O.P.

Theorem 5 then proves that each customer makes some positive contribution toward the utility's revenue requirement; payment schedule 1 in Figure 3 is an accurate representation of this. Each unit of customer usage before q_c is priced above marginal cost C. Consequently, a customer's high-level usage can be subsidized -- for an interval -- through the excess revenue that the utility secured from his own low-level usage prior to q_{c*}.

To summarize this section, the utility sells each unit of output $q < q_c$ above marginal cost C. At customer c, the schedule falls from a price that is above C down to C^*, which is below C. For $q > q_c$, $P_i = C^* < C$; the utility actually subsidizes the last units that it sells to its largest customers. The minimum price for usage is C^*. No customer that uses q_e or more would remain a customer. Finally, no customer is subsidized in the whole.

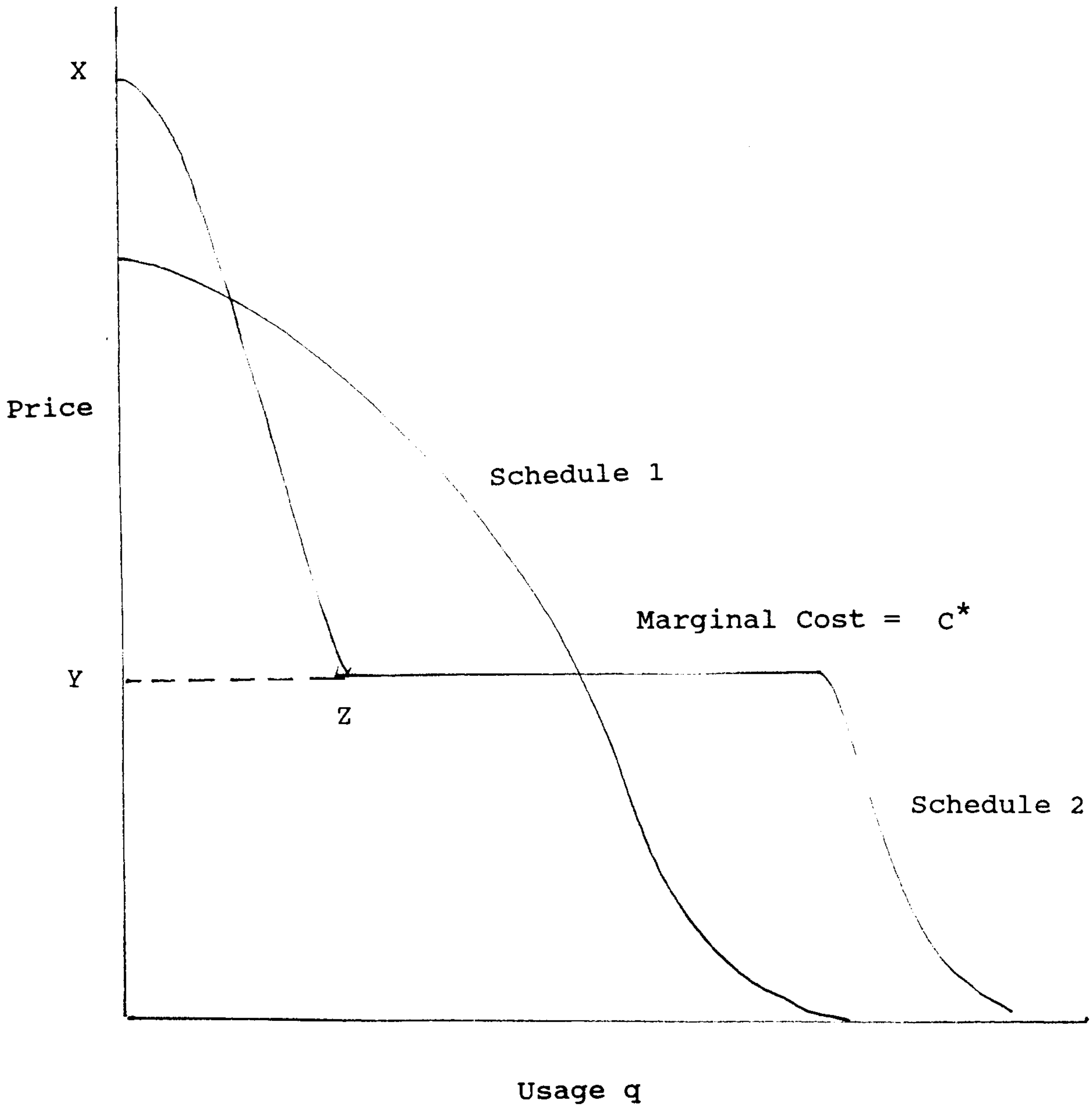
Figure 1 then depicts the basic shape of an optimal nonuniform price schedule when a utility faces a bypass technology which cannot be dominated at large levels of usage; the price schedule need not be monotonically decreasing prior to q_c. The curve marked D_c -- until now unidentified -- represents the demand curve of the smallest large-indifferent customer c.

5. Creamskimming

Up until now, we have assumed that the bypass alternative is the more efficient technology at a sufficiently large level of usage; i.e, $C^* < C$ and $Z^* + C^*q < Z + Cq$. Another possibility which may occur in some markets is that bypass alternatives may be less efficient than utility technologies at all levels of usage (i.e, marginal cost is C^* where $C < C^* < P$). If the utility's usage price is above its marginal cost, inefficient technologies still may attract some large customers; this is known as creamskimming. From an economic perspective, creamskimming should never occur; under earlier nonuniform price schedules (Spence, Roberts, MS, GLS) represented by Schedule 1 in Figure 6), it can occur. However, a utility can eliminate creamskimming entirely by implementing a constrained nonuniform price schedule like Schedule 2; $P = C^*$ is a plateau which is now above C. Area XYZ equals $Z^* + C^*q_{C*}$, where c represents the first large-indifferent customer. I believe that the shape of this curve is intuitively obvious given my previous discussion; a mathematical derivation is available (Einhorn (1986)).

6. Efficient Bypass

Figure 6: Optimal Nonuniform Price Schedule vs. Creamskimming



Section 2 demonstrated that if usage is completely price-inelastic, customers will bypass under a nonuniform price schedule if and only if it is socially beneficial to do so. We now shall introduce a non-zero price elasticity and formally confirm the validity of Section 2's results.

Customer i will stay with the utility (choose bypass) if his net consumer surplus is greater (less) under the utility; i.e.,

$$6.1) \quad U_i(q_i) - R(q_i) > (<) \quad U_i(q_{i*}) - Z^* - C^*q_{i*}$$

From a social perspective, customer i should stay with the utility (choose bypass) if social surplus is greater (less) under the utility; i.e.,

$$6.2) \quad U_i(q_i) - Cq_i > (<) \; U_i(q_{i*}) - Z^* - C^*q_{i*}$$

We first consider a customer who elects to stay with the utility. From Theorem 5, $R(q_i) > Cq_i$ for all customers $i < e$. Therefore, $U_i(q_i) - Cq_i > U_i(q_i) - R(q_i)$ for these customers. Since eq. 6.1 must hold with a > sign for a customer who stays with the utility, it immediately follows that $U_i(q_i) - Cq_i > U_i(q_{i*}) - Z^* - C^*q_{i*}$. From eq. 6.2, this is socially optimal.

Now consider whether it would be efficient for the utility to attract bypassers with $i > e$. In order for the utility to attract these users, it must continue to price usage beyond q_e at C^*, which is below its own marginal cost C. These customers will be large-indifferent; therefore,

$$6.3) \quad U_i(q_i) - R(q_i) = U_i(q_{i*}) - Z^* - C^*q_{i*}$$

Furthermore, $q_i = q_{i*}$ for $i > c$. Since $P_i = C^*$ for $i > e$, $R(q_i)$

$= R(q_e) + C^*(q_i - q_e) = Cq_e + C^*(q_i - q_e) < Cq_i$; the second equality follows since $R(q_e) = Cq_e$ at $e < 1$ (see eq. 3.8c). Therefore, $U_i(q_i) - Cq_i < U_i(q_i) - R(q_i)$. It follows from eq. 6.3 that $U_i(q_i) - Cq_i < U_i(q_{i*}) - Z^* - C^*q_{i*}$. From eq. 6.2, consumers $i > e$ should choose bypass.

7. Positive Access Costs

This section will consider two complications which apply to telephone service. First, customer flat-rate access costs Z are positive. Second, telephone customers can enjoy some free benefits without having to purchase additional usage; e.g., simply by installing a phone, a customer may make free local calls, receive calls, and have the security of being able to reach emergency numbers whenever necessary. The need to purchase toll usage q is consequently not the only motivation for a customer selecting service; i.e., $U_i(q_i) > 0$ even if $q_i = 0$. $U_i(0)$ differs between consumers; assume that $U_i(0) > U_j(0)$ if and only if $i > j$.

Allowing for both non-zero A and Z and $U_i(0) > 0$, eq. 3.1a defines the welfare W_i of customer i. Eq. 3.2 still represents aggregate consumer welfare. Utility profits are now expressed:

$$3.3') \quad X = \int_a^e (A + R_i - Z - Cq_i)dF(i) - K$$

The objective function in eq. 3.4 is still relevant, *mutatis mutandis*. Optimizing condition eq. 3.8a does not change, but eq. 3.8b and 3.8c do:

$$3.8b') \quad -g(A + R_a - Z - Cq_a) \leq 0; \; a \geq 0;$$

$$ag(A + R_a - Z - Cq_a) = 0$$

3.8c') $$g(A + R_e - Z - Cq_e) \geq 0; \quad e \leq 1;$$
$$(e - 1)g(A + R_e - Z - Cq_e) = 0$$

Regarding eq. 3.8b', intensity a represents a marginal customer who is willing to pay a flat-rate fee of A in order to access a utility's product; $q_a \geq 0$ and $a \geq 0$. We shall assume, quite realistically, that $a > 0$ (since $a > 0$, not every small consumer joins the utility system) but $q_a = 0$ (some people who do subscribe to the utility do not purchase any toll usage). Consequently, a necessary first-order condition in eq. 3.8b' is that $A + R_a - Z - Cq_a = 0$. Clearly, it makes no sense -- in a nonuniform price schedule -- for either a profit-maximizing or a profit-constrained welfare-maximizing utility to keep off any potential customer as long as the customer is willing to cover the incremental costs that he imposes upon the utility; furthermore, all potential customers who are unwilling to cover their incremental costs should be kept off. Since $q = 0$ for some customers, this would require $A = Z$. If $q = 0$, then $q_a = 0$; therefore, $R_a = 0 = Cq_a$ must hold as well; the necessary first-order condition of eq. 3.8b is then met.

Note that $A = Z$ is quite different from the usual result regarding access or entry prices in the two-part tariff literature (Oi, Ng and Weisser, Schmalensee). In this literature, $A \gtrless Z$ is possible; if $q_a = 0$ (where a is the smallest customer), $A > Z$ results.

All of the theorems, lemmas, and corollaries of Section 4 and 5 carry forward to the case when A and Z are positive. The proofs of these are straightforward.

8. Conclusion

Current pricing policy in telecommunications is a fair distance away from economically efficient pricing. As a result, regulators and local telephone companies now ponder the dangers of uneconomic bypass which may result when usage charges are increased to cover the company's fixed costs. Politicians and consumer advocates continue to argue that low cost flat-rate fees are necessary. Finally, regulators and economists are left to debate whether divestiture was desirable and whether further extensions of competition can prove worthwhile.

The price strategy developed in this paper can go some way to resolve several of these conflicting concerns. Under optimal nonuniform price schedule, large long-distance users would enjoy substantial volume discounts for using local company switched access facilities to reach their long-distance carriers. Consequently, these customers would face "more correct" (though not perfect) signals when deciding whether to bypass or not. While small customers should optimally pay flat-rate fees A to cover their associated flat-rate costs Z, it is possible to subsidize these customers as well; that is, the monthly flat rate fee could be constrained below its associated marginal cost. The resulting shortfall in company revenues must be recovered by modifying usage charges elsewhere.

ENDNOTES

[1]Mirman and Sibley incorporate some notion of bypass alternatives in their paper (p. 664); they implicitly assume that bypass systems are <u>in place</u> and that customers may route calls over these circuits if utility prices get too high. However, this really begs the question; unless bypass technologies can be installed cost-free, the nonuniform price schedule of Mirman and Sibley does not incorporate the costs of installing the bypass alternatives. That is, a discrete decision must first be made whether or not to install a bypass technology and a comprehensive welfare-maximizing analysis should incorporate the costs of that decision.

[2]Proof of Lemma 1.1: Since $a = \inf(S)$, $W_a(q_a) = 0$ and $U_a(q_a) = R(q_a)$ for this user (see eq. 3.1a). Let $u_a(q) = \partial U_a/\partial q$ represent the marginal value of unit q ($\leq q_a$) to customer a. Given the single-crossing assumption (Assumption 4), $u_a(q) = P(q)$ for $q \leq q_a$. Since demand curves do not cross one another, $(d^2U/dqdi) > 0$; therefore, $u_i(q) > u_a(q) = P(q)$ for all q if $i > a$. It immediately follows that $U_i(q_a) > U_a(q_a) = R(q_a)$ at usage q_a. therefore, $W_i(q_a) = U_i(q_a) - R(q_a) > 0$. Since W_i is maximized at q_i, $W_i(q_i) > W_i(q_a) > 0$. Therefore, $W_i(q_i) > 0$.

[3]Proof of Lemma 1.2: If customer i is not indifferent, $h_i = j_i = 0$. $P_i > C$ follows from eq. 3.8a'.

[4]Proof of Lemma 1.3: Contrapositive of Lemma 1.2.

[5]Proof of Lemma 1.4: If $P_i \leq C$, the customer would be indifferent (Lemma 1.3); since $i \leq b$, this would mean that $W_i = 0$. This contradicts Lemma 1.1.

[6]Proof of Corollary 2.1: One of two states must hold -- all customers are indifferent or some customers are not indifferent. Given Lemma 1.1, the first state is impossible. For customers who are not indifferent, $P_i > C$ (Lemma 1.2). For eq. 3.8c to hold when $e < 1$, $R_e = Cq_e$; in order for this to hold, $P_j < C$ must hold for some other customers $j > b$.

[7]Proof of Corollary 4.1: From Theorem 4, $P_i < C^*$ is impossible. From Lemma 3.1, $P_i > C^*$ requires $P_j < C^*$ for j e (c, i); therefore, $P_i > C^*$ is impossible as well. Therefore, $P_i = C^*$ is the only possibility for prices after point E in Figure 5.

[8]Proof of Corollary 4.2: If $e < 1$, $R_e = Cq_e$ is a necessary first-order optimizing condition (see eq. 3.8c); since $t(e) = [F(e) - F(e)]/f(e) = 0$ (see immediately after eq. 3.8a'), $P_e = C$ if consumer e were a customer. From the proof of Theorem 4, it is obvious that consumer e will leave the utility since its marginal price is above C^*.

APPENDIX:ADDITIONAL THEOREMS

This appendix will prove two assumptions that were made in Section 3.

Theorem A1: S cannot be incomplete if the utility maximizes profits or constrained social welfare.

Proof: With no loss of generality, suppose that there is one gap in S. S then consists of (a, e), (a', e') and maybe any endpoint; $a' > e$.

The new objective function is written:

$$\text{A.1)} \quad L = (1 - g)\left[\int_a^e W_i dF(i) + \int_e^{a'} W_{i'} dF(i) + \int_{a'}^{e'} W_i dF(i) + \int_{e'}^{1} W_{i*} dF(i) \right] + g\left[\int_a^e (R_i - Cq_i) dF(i) + \int_{a'}^{e'} (R_i - Cq_i) dF(i) \right] + \int_S h_i(W_i - 0)\, di + \int_S j_i(W_i - W_{i*})\, di$$

where:

$W_{i'} = 0$ if $i \in (e, a') \leq b$

$W_{i'} = W_{i*}$ if $i \in (e, a') > b$

$P(q)$, a, e, a', and e' are under the utility's control.

Assuming that $0 < a < e < 1$ and that the profit constraint is binding, the optimizing conditions are:

A.2a) $P_i = C + (2g - 1 - h_i - j_i)t(i)(d^2U/didq)/g$ for $i \in (a, e)$

A.2b) $P_i = C + (2g - 1 - h_i - j_i)t'(i)(d^2U/didq)/g$ for $i \in (a', e')$

A.2c) $R_i = Cq_i$ for $i = a, e, a', e'$

A.2d) $\int_a^e (R_i - Cq_i)dF(i) + \int_{a'}^{e'} (R_i - Cq_i)dF(i) = K$

A.2e) $W_i \geq 0$ for all $i \in S$

A.2f) $W_i \geq W_i^*$ for all $i \in S$

Either $b \geq e$ or $b < e$. If $b \geq e$, $W_i > 0$ for all $i \in (a, e)$ (see Lemma 1.1); therefore, $P_i > C$ (Lemma 1.2) for $i \in (a, e)$. At a, Theorem 1 would still hold; $q_a = 0$. At e, $P_e = C$; see eq. A.2a. Under these circumstances, eq. A.2c cannot hold for $i = e$; i.e., $R_e = Cq_e$ is not possible.

If $b < e$, then $R_e = Cq_e$ can hold. The results of Section 4 still carry forward, <u>mutatis</u> <u>mutandis</u>. Therefore, for some customer $c \in (a, e)$, $P_c = C^*$. From Theorem 4 for all $i \in (c, e)$, $P_i = C^*$ and $W_i = W_{i*}$; therefore, $R_{c,i} = C^*(q_i - q_c)$. For consumer $j > e$,

A.3) $R_j = R_e + R_{e,j}$

A necessary condition for consumer j to choose to be a customer is that:

A.4) $R_{c,j} \leq C^*(q_j - q_c)$

Suppose that $j > a'$. From eq. A.2c, $R_e = Cq_e$ and $R_{a'} = Cq_{a'}$; therefore, $R_{e,a'} = C(q_{a'} - q_e)$. Then $R_{c,j} = R_{c,e} + R_{e,a'} + R_{a',j} = C^*(q_e - q_c) + C(q_{a'} - q_e) + R_{a',j}$. Eq. 4 can be satisfied only when $R_{a',j} < C^*(q_j - q_{a'}) < C(q_j - q_{a'})$ for all $j \in (a', e')$. But this condition is impossible if two other optimizing conditions in eq. A.2c hold; $R_{a'} = Cq_{a'}$ and $R_{e'} = Cq_{e'}$, implying $R_{a'e'} = R_{e'} - R_{a'} = C(q_{e'} - q_{a'})$. Since the presence of a gap leads to impossible conclusions for a profit-maximizing or social welfare-maximizing firm, no gap in S will occur.

Theorem A2: The price schedule can have no upward discontinuities.

Proof: In eq. 3.8a', i can be scaled, without loss of generality, so that $d^2U/dqdi$ is constant; i will be continuously distributed. Since $h_i = 0$ (Lemma 1.1), upward discontinuities at P_i can only result from downward discontinuities in j at P_i. Since $j_i \geq 0$, $\lim_{h \to i} j_h > 0$ is necessary if downward discontinuities are to arise in j; since $j_h > 0$, $W_h^* = W_h$ for consumers h immediately before i. By Theorem 3 and 4, $P_i = C^*$ for all $i > h$; since C^* is a minimum price, no upward discontinuities are possible.

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