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Sequential Leadership and Social Inefficiency

by Nicholas Economides

Columbia Institute for Tele-Information
Graduate School of Business
Columbia University
809 Uris Hall
New York, NY 10027
(212)854-4222

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Abstract

We analyze a game of simultaneous free entry and sequential output choices. At its perfect equilibrium, the production level of a firm is decreasing with the order of the firm in the decision making. The firm which is the last to choose output produces the same amount as a typical firm in the symmetric Cournot game. Moreover, industry output is identical in the sequential and the Cournot games. It follows that in the sequential game there are <u>fewer</u> active firms and <u>higher</u> total surplus than in the symmetric Cournot game. Strategic asymmetry results in higher concentration and higher total surplus without an increase in price.

Key words: sequential leadership, Cournot oligopoly, social inefficiency.

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**Columbia University, U.C. Berkeley and Stanford University.

Sequential Leadership and Social Inefficiency

In the context of quantity-setting oligopoly, entry deterrence has been recently discussed by Dixit [1979], Nti and Shubik [1981], Gilbert [1986], and Gilbert and Vives [1987] among others. These authors focus on the effect of quantity and investment decisions of incumbents on the entry decision of a potential entrant. Traditionally, however, (von Stackelberg [1934]) the ability of a "leader" to act first has been taken to have an effect on output decisions of "followers", but not on their presence in the market. In this paper we discuss a model of sequential leadership where firms that act earlier in the sequence of decisions are able to influence the output of those who act later, but are not able to throw them out of competition. In our framework, at the time of output decisions the entry phase has been completed and the number and identity of competitors cannot be altered. Thus, entry deterrence as in Dixit [1979] or Gilbert and Vives [1987] cannot occur.

Our model has two phases. The first phase has one stage in which firms enter simultaneously incurring a fixed cost F. The n entrants are assigned an order and they are labelled $i = 1, \ldots, n$. We do not discuss in detail the mechanism which assigns order to the firms.[1] In the second phase firms choose quantities sequentially. The second phase has n stages. In stage i a quantity choice, q_i, is made by the ith firm. In making quantity choices, each firm considers all decisions of prior stages as given and beyond its control. Thus, firm k acts as a (quantity) follower to all firms of index $j < k$ and considers their output choices as given. However, each firm k realizes the influence of its output choice on the subsequent decisions of firms of higher index $j > k$ which are made in later stages. Thus, each firm is a (quantity) leader with respect to firms of higher index. We seek subgame perfect equilibria in this two-phase game.

It is important to understand that this model of sequential leadership does not describe sequential entry. Here entry is simultaneous. At the juncture of output choice (second phase) the number of active firms and their order is already known and given. Therefore output choices cannot be used to prevent a potential entrant from entering. It follows that our results are expected to be quite different from the ones of models of sequential entry.

Quite surprisingly, we show that the industry output of our sequential leadership model is equal to the industry output of the Cournot model of simultaneous output choices. Therefore the two game structures result in the same price and consumers' surplus. On the producers side, there are significant differences between the resulting market structures. The sequential leadership game results in a market structure with significant inequality in the production levels of active firms. An earlier-acting firm has a strategic advantage which it exploits by producing more than a later-acting firm. Thus, outputs in sequential leadership are inversely related to their order of action. We show that the last-acting firm in sequential leadership produces as much as a typical firm in the Cournot game. Since in sequential leadership earlier-acting firms produce higher outputs while total output is the same as in Cournot, it follows that the number of active firms in sequential leadership is smaller than in Cournot. Thus, in sequential leadership there are fewer active firms than in Cournot with significant inequalities in their outputs. While price and consumers surplus are the same in both market structures, sequential leadership results in positive total profits and therefore higher total surplus than the Cournot model. Thus, we have the surprising result that a market structure of significant inequality and a smaller number of firms achieves the same price and consumers' surplus but higher total surplus than a market structure of total equality.

The rest of the paper is organized as follows. Section I presents the general results. Section II presents an important special case of linear demand and constant marginal cost, noting the significant differences in Herfindahl indices and total surpluses across the two market structures. In Section III we summarize our results and discuss extensions and possibilities of further research.

I. General Results

We make the following standard assumptions on the demand and cost functions:

A1. All firms have the same technology represented by cost function $F + C(q)$, where $C(0) = 0$, $C'(q) \geq 0$, $C''(q) \geq 0$ and $F > 0$.

A2. Industry demand is downward sloping and weakly concave, $P'(Q) < 0$, $P''(Q) \leq 0$.

Let a subgame perfect equilibrium of the game of sequential leadership (S.L.E.) described above be a number of active firms n_s and their corresponding outputs $(q_1^s, \ldots, q_{n_s}^s)$. We also define the standard Cournot game to be able to compare its outcome with the one of the sequential game. The quantity-setting game of Cournot has two stages. In stage 1 firms enter simultaneously incurring a fixed cost F. The n entrants of the first stage choose quantities simultaneously in the second stage. A Cournot Equilibrium (C.E.) consisting of n_c active firms and $(q_1^c, \ldots, q_{n_c}^c)$ corresponding output levels is a subgame-perfect equilibrium of this game structure. We will confine our attention to symmetric Cournot equilibria (C.E).[2]

We will characterize the equilibrium of the sequential leadership game and compare it with the Cournot equilibrium. We first characterize the industry output and the output of the last firm in the sequential game. Consider the output decision of the firm that acts last. Let Q denote the industry output, and let y be the output of all firms other than the last firm n_s, so that $Q = y + q_{n_s}$. The profit function of the last firm,

$$\Pi(q_{n_s}, y, F) = q_{n_s} P(q_{n_s} + y) - C(q_n) - F, \tag{1}$$

is maximized with respect to q_{n_s} at $q^*(y)$, the solution of

$$P(q^* + y) + q^* P'(q^* + y) - C'(q^*) = 0. \tag{2}$$

Note that q^* is monotonically decreasing in y.[3] Last firm n_s responds to an increase of output of previously-acting firms through a cut in its own output. Total output, $Q(y) = q^*(y) + y$, is monotonically increasing in y,[4] as an increase of production by previously-acting firms is not totally absorbed by the decrease of the output of the last firm.

Ignoring integer constraints, the realized profits of the last firm are zero,

$$\Pi(q^*(y), y, F) = 0. \tag{3}$$

It is easily checked that the realized profits of the last firm are monotonically decreasing in y and in F. It follows that, for each level of fixed cost F, there exists a unique output of other firms, $y = y(F)$, such that the last firm realizes zero profits when it optimally responds to this output. Thus, at a subgame perfect equilibrium, for a given F, we have a unique output of other firms y(F), a unique output of the last firm $q_{n_s} = q^*(y(F))$, and a unique total output $Q(F) = y(F) + q^*(y(F))$ consistent with the last firm optimizing and realizing zero profits.

Looking now at the Cournot game we note that the decision problem of a <u>typical</u> firm is <u>identical</u> to the decision problem of the <u>last</u> firm of the sequential game. Firm i in the Cournot game maximizes

$$\Pi(q_i, y, F) = q_i P(q_i + y) - C(q_i) - F \tag{1'}$$

which is a re-writing of (1) with the index n_s substituted by i. Profits are maximized at $q_i^c = q^*(y)$, the solution of (2). As before, total output is $Q(y) = q^*(y) + y$. At the free entry Cournot equilibrium, profits of a typical firm are zero, i.e.,

$$\Pi(q^*(y), y, F) = 0, \tag{3'}$$

which is identical to (3). The same F implies the same y in the two games. Therefore, the output of the typical firm at the Cournot equilibrium (C.E.) is equal to the output of the last firm at the sequential leadership equilibrium (S.L.E.),

$$q_i^c = q^*(y(F)) = q_{n_s}^s . \tag{4}$$

It also follows that, for the same fixed cost F, the sequential leadership and the Cournot games have the same equilibrium industry output,

$$Q(y(F)) = q^*(y(F)) + y(F).$$

<u>Theorem 1</u>: **Equilibrium industry output is the same in the game of sequential output decisions as in the symmetric Cournot game. The output of the last firm of the sequential game is equal to the output of a typical firm of the symmetric Cournot game.**

Since industry output is the same in the market of sequential leadership and in the Cournot market, concentration indices such as the Lerner index that are based on total output or price will not reveal any difference between the two markets. This is despite the fact that these markets exhibit significant differences in concentration and profits as we show next.

Consider now the output decision of the penultimate firm n_s-1 in the sequential leadership game. This firm is aware of the influence of its output choice on the last firm's output. This influence is precisely

$$dq_{n_s}/dq_{n_s-1} = dq_{n_s}/dy = -(P' + q^*P")/(2P' + q^*P" - C") < 0. \tag{5}$$

An increase of the output of firm n_s-1 precipitates a decrease in the output of firm n_s. Taking into account its strategic advantage, firm n_s-1 produces more output than firm n_s. This is seen through an analysis of its first order condition,

$$\partial\Pi_{n_s-1}(q_{n_s-1},y)/\partial q_{n_s-1}=P(q_{n_s-1}+y) + q_{n_s-1}P'(q_{n_s-1}+y)(1+dq_{n_s}/dq_{n_s-1})-C'(q_{n_s-1})=0. \tag{6}$$

Evaluating $\partial\Pi_{n_s-1}/\partial q_{n_s-1}$ at $q_{n_s-1} = q^*$, and using (2) yields,

$$\partial\Pi_{n_s-1}(q^*, y)/\partial q_{n_s-1} = q^*P'(q^* + y)[dq_{n_s}/dq_{n_s-1}] > 0,$$

since $dq_{n_s}/dq_{n_s-1} < 0$ from (5) and $P' < 0$. Since the profit function of firm n_s-1 is concave in its own production, it immediately follows that $q^s_{n_s-1} > q^* = q^s_{n_s}$, i.e. the penultimate firm produces more than the last firm.

Using the same technique, we show in the Appendix that the output of firm n_s-2 is higher than the output of firm n_s-1, $q^s_{n_s-2} > q^s_{n_s-1}$. Similarly, it can be shown that output of firms that act earlier is even larger,

$$q^s_1 > q^s_2 > \ldots > q^s_{n_s-2} > q^s_{n_s-1} > q^s_{n_s}.$$

<u>Theorem 2</u>: **A firm's output level varies inversely with its order in the chain of decisions.**

Since industry output is the same in the Cournot and in the sequential leadership game while all firms except the last one produce higher outputs at the sequential leadership game, it follows that there are fewer active firms at the sequential leadership equilibrium.

<u>Corollary 1</u>: **There are fewer active firms at the sequential leadership equilibrium than at the Cournot equilibrium.**

We now compare equilibrium profits of firms j and j-1 for any $j \in \{2, \ldots, n_s\}$. They can be written as

$$\Pi_{j-1}^{s} = \Pi(q_{j-1}^{s}, y_{j-1}^{s}, F), \quad \Pi_{j}^{s} = \Pi(q_{j}^{s}, y_{j}^{s}, F),$$

where

$$y_{j-1}^{s} = \sum_{i=1}^{n_s} q_i^s - q_{j-1}^{s}, \quad y_j^s = \sum_{i=1}^{n_s} q_i^s - q_j^s.$$

Now, $q_{j-1}^{s} > q_j^s$ implies $y_j^s > y_{j-1}^{s}$ by their definition, and, since $\partial\Pi/\partial y = qP' < 0$, it follows that

$$\Pi(q_j^s, y_{j-1}^{s}, F) > \Pi(q_j^s, y_j^s, F) = \Pi_j^s. \tag{7}$$

Since q_{j-1}^{s} maximizes $\Pi(q, y_{j-1}^{s}, F)$ with respect to q, we have

$$\Pi_{j-1}^{s} = \Pi(q_{j-1}^{s}, y_{j-1}^{s}, F) \geq \Pi(q_j^s, y_{j-1}^{s}, F). \tag{8}$$

Combining (7) and (8) it follows that firm j realizes lower profits than the one preceding it, $\Pi_{j-1}^{s} > \Pi_j^s$. This is true for any j, $2 \leq j \leq n_s$, so that,

$$\Pi_1^s > \Pi_2^s > \ldots > \Pi_{n_s}^s = 0.$$

We have shown,

<u>**Theorem 3**</u>**: Equilibrium profits decrease with a firm's position in the chain of decisions. Except for the last firm that makes zero profits, all firms make positive profits.**

Next we compare social welfare at the equilibrium of the sequential leadership game with that of the symmetric Cournot game. Each consumer is equally well-off in both games because they result in the same equilibrium price. All active producers except the last one make positive profits at the equilibrium of the sequential leadership game and thus are better off than at the Cournot equilibrium where all had zero profits. The last active producer of the equilibrium of the sequential game and all the inactive producers at that equilibrium realize zero profits and are therefore equally well-off as at the Cournot equilibrium. Therefore the outcome of the sequential game is Pareto superior to the Cournot outcome.

<u>**Theorem 4**</u>**: The equilibrium of the sequential leadership game is Pareto superior to the symmetric equilibrium of the Cournot game with free entry.**

Total consumers' surplus is equal in the two games since they result in equal industry outputs and prices. At the sequential leadership equilibrium, as noted above, total industry profits are positive, while they are zero at the Cournot equilibrium. It follows that,

<u>Corollary 2</u>: **Total surplus is higher at the sequential leadership game than at the symmetric Cournot game.**

As expected, market concentration is positively correlated with positive profits. But, in this framework, <u>market concentration and profits are also positively correlated with social welfare</u>. This is not surprising because the increase in profits is achieved without a decrease of output that would create a "dead weight loss". The welfare increase arises from the more efficient utilization of the production technology. Social welfare increases because of savings of fixed costs of firms that were active at the Cournot equilibrium but are inactive at the sequential equilibrium. These savings may be tempered by variable cost inefficiencies that may arise from the asymmetric distribution of production at the sequential equilibrium. Of course, the sequential game cannot achieve the "first best" because its equilibrium industry output, being equal to the Cournot output, differs from the competitive one.

In the next section we present an example of an industry of linear demand and constant marginal cost. We calculate the sequential output decisions and the Cournot equilibria and compare them.

<u>II. An example</u>

In the following example, we assume linear demand and constant marginal cost c. The units can be normalized so that the industry demand function is $P = A - Q$. We can substitute $p = P - c$, $a = A - c$, so that $p = a - Q$. We first characterize the n-firm equilibrium of the second phase. In the Appendix, we prove the following Theorem.

Theorem 5: For linear demand and constant marginal cost, in the sequential leadership game outputs form a geometric sequence, with the first firm producing as much as a monopolist would have produced!

Firm i produces

$$q_i^m = a/2^i \tag{9}$$

Total output and price are

$$Q^s = \sum_{i=1}^{n} q_i^m = a \cdot \sum_{i=1}^{n} 2^{-i} = a(1 - 2^{-n}), \quad p^s = a/2^n, \; P^s = c + a/2^n. \tag{10}$$

It follows that equilibrium profits for firm i and industry profits of phase 2 are

$$\Pi_i^s = a^2/2^{n+i} - F, \quad \sum_{i=1}^{n} \Pi_i^s = a^2(1 - 2^{-n+1})2^{-n-1} - nF \tag{11}$$

respectively. Since the last firm at an n_s-firm equilibrium makes zero profits, $a^2/2^{2n_s} = F$, or equivalently,

$$2^{n_s} = a/\surd F \;\text{ or }\; n^s = [\log a - (\log F)/2]/\log 2. \tag{12}$$

This equilibrium exists if $n^s > 1 \iff a > 2\surd F$. Substituting in (9)-(11) we derive the full equilibrium of the sequential choice game

$$q_i^s = a/2^i, \quad Q^s = a - \surd F, \quad P^s = c + \surd F, \quad \Pi_i^s = a\surd F/2^i - F. \tag{13}$$

Profits can also be written as $\Pi_i^s = F(2^{n_s - i} - 1)$. From this formula it is immediate that $\Pi_{n_s}^s = 0$, $\Pi_{n_s-1}^s = F$, $\Pi_{n_s-2}^s = 3F$, $\Pi_{n_s-3}^s = 5F$, etc. Total profits are $\sum_{i=1}^{n_s} \Pi_i^s = 2a\surd F - (n_s + 1)F > 0$.

In contrast, the n-firm symmetric Cournot equilibrium is

$$q_i^c = a/(n + 1), \quad Q^c = na/(n + 1), \quad p^c = a/(n + 1), \quad P^c = c + a/(n + 1). \tag{14}$$

Individual and industry profits are

$$\Pi_i^c(n) = a^2/(n+1)^2 - F, \quad \sum_{i=1}^{n} \Pi_i^c = na^2/(n+1)^2 - nF. \tag{15}$$

Solving $\Pi_i^c(n_c) = 0$ yields n_c active firms, where

$$n_c + 1 = a/\sqrt{F} \quad \text{or} \quad n_c = a/\sqrt{F} - 1. \tag{16}$$

Substituting in (14) we derive the full equilibrium of the symmetric Cournot game,

$$q_i^c = \sqrt{F}, \quad Q^c = a - \sqrt{F}, \quad P^c = c + \sqrt{F}, \quad \Pi_i^c = 0. \tag{17}$$

We now compare the production levels, prices, and profits of the two game structures given by (13) and (17). Of course, as proposition 1 assures us, $Q^s = Q^c$, $q_i^c = q_{n_s}^s$. Comparing n_c and n_s we see that they are equal ($n_c = n_s = 1$) for $a/\sqrt{F} = 2$. For all $a/\sqrt{F} > 2$, $n_c - n_s$ is positive and increasing in $a/\sqrt{F}$. The Herfindahl indices for the sequential game and the Cournot games are,

$$H_s(n_s) = (a/\sqrt{F} + 1)/[3(a/\sqrt{F} - 1)], \quad H_c(n_c) = 1/(a/\sqrt{F} - 1)$$

respectively. Their ratio, $H_s(n_s)/H_c(n_c) = (a/\sqrt{F} + 1)/3$, is increasing in $a/\sqrt{F}$, the ratio of the willingness to pay over the root of the fixed cost. Two factors contribute to this result. First, $H_s(n)/H_c(n)$ increases in n. The degree of concentration in a sequential decisions industry increases with the number of active competitors. Second, the equilibrium number of active firms increases in the willingness to pay much more quickly in the symmetric Cournot game than in the sequential game.

The difference in total surplus between the sequential and Cournot equilibria is the sequential industry profits, $\sum_i \Pi_i^s = 2a\sqrt{F} - (n_s + 1)F$. These profits are increasing in $a/\sqrt{F}$. Thus, profits and social welfare are positively correlated with the relative degree of concentration as expressed by the ratio of the Herfindahl indices, $H_s(n_s)/H_c(n_c)$. The conclusion does not change if we use other concentration indices, such as the Gini coefficient or a k-firm concentration ratio.

Table 1 shows the numbers of active firms, the corresponding deviations from maximum social surplus S_o and the Herfindahl indices for the Cournot and sequential equilibria for varying values of the demand and cost parameters.[5]

Table 1

$a/\sqrt{F}$	n_c	n_s	$(S_o-S_c)/F$	$(S_o-S_s)/F$	$(S_o-S_c)/(S_o-S_s)$	H_c	H_s
2	1	1	1/2	1/2	1	1	1
4	3	2	5/2	3/2	5/3	1/3	5/9
8	7	3	13/2	5/2	13/5	1/8	3/7
16	15	4	19/2	7/2	19/7	1/16	17/42
32	31	5	61/2	9/2	61/9	1/32	11/31
64	63	6	125/2	11/2	125/11	1/64	65/189

Note that there are significant differences in the number of active firms in the two market structures (columns two and three). As seen in column six of Table 1, at high levels of the demand intercept, "a", or relatively low fixed cost, "F", <u>the welfare loss in Cournot compared to optimality $S_o - S_c$ is a large multiple of the welfare loss of the sequential game</u> $S_o - S_s$, this multiple being over eleven for $a/\sqrt{F} = 64$. For the same value of the parameters (last line in Table 1), the concentration index in Cournot is 1/64 while for the sequential game it is 65/189, approximately 21 times larger!

III. Extensions and Discussion

We have shown that sequential decisions in a model of leaders and followers improved total surplus without causing a price increase over an equilibrium of simultaneous output (Cournot) decisions. Strategic asymmetry resulted in inequality in production levels, fewer active firms and increased social welfare. This came as a result of the more efficient utilization of the production technology in the model of sequential leadership. It was essential that firms did not have the ability to stop potential entrants from entering. This was guaranteed by a game structure in which the subgame of output decisions followed the choice of entry/exit.

Our results for a homogeneous market can easily be extended to markets of symmetrically (not locationally) differentiated products where output is the strategic variable. In markets where prices are the strategic variables, we cannot directly apply the methodology of this paper. In such markets it is the last firm that has the advantage, and its profits cannot be compared to the profits of a typical firm in a symmetric game. An analysis of sequential price-setting with free entry remains an open question.

Footnotes

1. For example firms may bid among themselves for positions. Other mechanisms may also be used. For example, an auctioneer can collect all the rents the firms would realize by auctioning the positions. For the validity of the results it is essential that the mechanism does not levy a cost on the last firm.

2. See Novshek [1984] for a discussion of the existence and algorithms of calculation of asymmetric Cournot equilibria.

3. $dq^*/dy = -[\partial^2\Pi_{n_s}/\partial q_{n_s}\partial y]/[\partial^2\Pi_{n_s}/\partial q^2_{n_s}] = -[q^*P''(q^* + y) + P'(q^* + y)]/[q^*P''(q^* + y) + 2P'(q^* + y) - C''(q^*)] < 0$ because both numerator and denominator are negative.

4. $dQ/dy = 1 + dq^*/dy = [P'(q^* + y) - C''(q^*)]/[q^*P''(q^* + y) + 2P'(q^* + y) - C''(q^*)] > 0$ since both numerator and denominator are negative.

5. Maximized social surplus is $S_o = a^2/2 - F$. Social surplus at the Cournot and sequential equilibria are $S_c = a^2/2 - 3F/2 - a\sqrt{F}$ and $S_s = a^2/2 - F(1 + 2n_s)/2 = a^2/2 - F(1/2 + [\log a - (\log F)/2]/\log 2)$ respectively.

Appendix

Consider the output decision of firm n_s-2. It is aware of its influence on the outputs of firms n_s-1 and n_s. Its first order condition is

$$\partial\Pi_{n_s-2}(q_{n_s-2},y)/\partial q_{n_s-2} = P(q_{n_s-2}+y)$$

$$+ q_{n_s-2}P'(q_{n_s-2}+y)(1+dq_{n_s}/dq_{n_s-2}+dq_{n_s-1}/dq_{n_s-2}(1+dq_{n_s}/dq_{n_s-1})) - C'(q_{n_s-2})=0. \tag{A1}$$

Evaluating $\partial\Pi_{n_s-2}/\partial q_{n_s-2}$ at $q_{n_s-2} = q^s_{n_s-1}$ and using (6) we have,

$$\partial\Pi_{n_s-2}(q^s_{n_s-1}, y)/\partial q_{n_s-2} = q^s_{n_s-1}P'(q^s_{n_s-1} + y)dq_{n_s-1}/dq_{n_s-2}(1 + dq_{n_s}/dq_{n_s-1}). \tag{A2}$$

In this expression,

$$dq_{n_s-1}/dq_{n_s-2} = dq_{n_s-1}/dy = -[\partial^2\Pi_{n_s-1}/\partial q_{n_s-1}\partial y]/[\partial^2\Pi_{n_s-1}/\partial q^2_{n_s-1}] =$$

$$-[P' + q^s_{n_s-1}P''(1 + dq_{n_s}/dy) + q^s_{n_s-1}P'd^2q_{n_s}/dy^2]/$$

$$[P'(2 + dq_{n_s}/dy) + q^s_{n_s-1}P''(1 + dq_{n_s}/dy) + q_{n_s-1}P'd^2q_{n_s}/dy^2 - C''].\tag{A3}$$

Since $dq_{n_s}/dy < 0$ and $d^2q_{n_s}/dy^2 > 0$, both terms in brackets in the numerator and the denominator of (A3) are negative, so that $dq_{n_s-1}/dq_{n_s-2} < 0$. The last term of (A2) is positive,

$$(1 + dq_{n_s}/dq_{n_s-1}) = (P' - C'')/(2P' + q_{n_s-1}P'' - C'') > 0,$$

because both numerator and denominator are negative. Therefore $\partial\Pi_{n_s-2}(q^s_{n_s-1}, y)/\partial q_{n_s-2} > 0$. Since Π_{n_s-2} is concave, it follows that $q^s_{n_s-2} > q^s_{n_s-1}$. Q.E.D.

<u>Proof of Theorem 5</u>:

The last (nth) firm maximizes

$$\Pi_n = q_n(a - \sum_{i=1}^{n-1} q_i - q_n) - F$$

at

$$q_n^m = (a - \sum_{i=1}^{n-1} q_i)/2. \tag{A4}$$

The n-1th firm maximizes

$$\Pi_{n-1} = q_{n-1}(a - \sum_{i=1}^{n-2} q_i - q_{n-1} - q_n^m) - F$$
$$= q_{n-1}(a - \sum_{i=1}^{n-2} q_i - q_{n-1})/2 - F$$

after substituting from (A4). Π_{n-1} is maximized at

$$q_{n-1}^m = (a - \sum_{i=1}^{n-2} q_i)/2.$$

In general, the n-jth firm maximizes

$$\Pi_{n-j} = q_{n-j}(a - \sum_{i=1}^{n-j-1} q_i - q_{n-j} - \sum_{k=1}^{j} q_{n-j+k}^m) - F. \tag{A5}$$

Firm $n-j$ recognizes the influence of its quantity choice q_{n-j} on quantities chosen later. On these latter ones we have put the superscript m.

<u>Claim:</u> <u>For all</u> $j = 1, \ldots, n-1$,

$$\sum_{k=1}^{j} q_{n-j+k}^m = (a - \sum_{i=1}^{n-j} q_i)(1 - 2^{-j}). \tag{A6}$$

Proof is by mathematical induction. As equation (A4) shows, (A6) holds for $j = 1$. We show that if (A6) holds for $j = t$ then it also holds for $j = t+1$. Assuming that (A6) holds for $j = t$, substituting (A6) in (A5) yields

$$\Pi_{n-t} = q_{n-t}(a - \sum_{i=1}^{n-t-1} q_i - q_{n-t})/2^t. \tag{A7}$$

Firm $n-t$ maximizes Π_{n-t} at

$$q_{n-t}^m = (a - \sum_{i=1}^{n-t-1} q_i)/2. \tag{A8}$$

Therefore

$$\sum_{k=1}^{t+1} q_{n-t-1+k}^m = \sum_{k=0}^{t} q_{n-t+k}^m = \sum_{k=1}^{t} q_{n-t+k}^m + q_{n-t}^m.$$

Substituting the first term from (A6) and the second from (A8) yields

$$\sum_{k=1}^{t+1} q^m_{n-t-1+k} = (1 - 2^{-j})(a - \sum_{i=1}^{n-t-1} q_i)(1 - 1/2) + (a - \sum_{i=1}^{n-t-1} q_i)/2$$

$$= (a - \sum_{i=1}^{n-t-1} q_i)(1 - 2^{-t-1}),$$

which is equation (A6) for $j = t+1$. Thus, the claim is proved.

Therefore, it is clear that the profit function of firm n-t takes the form of (A7), and is maximized at q^m_{n-t} given by (A8). For $t = n - 1$, we have from (A8) that $q^m_1 = a/2$. It follows that $q^m_2 = a/4$, $q^m_3 = a/8$, and in general

$$q^m_i = a/2^i, \tag{A9}$$

i.e. outputs form a geometric sequence. Q.E.D.

References

Cournot, A. A. [1960], Researches into the Mathematical Principles of the Theory of Wealth, Kelly, NY (English translation by N.T. Bacon).

Dixit, A. [1979], "A Model of Duopoly Suggesting a Theory of Entry Barriers," Bell Journal of Economics, vol. 10, pp. 20-32.

Gilbert, R. [1986], "Pre-emptive Competition" in G.F. Mathewson and J. Stiglitz (eds.) New Developments in the Analysis of Market Structure, MIT Press, Cambridge, Massachusetts.

Gilbert, R. and X. Vives [1986], "Entry Deterrence and the Free Rider Problem", Review of Economic Studies, vol. 53 (1), pp. 71-83.

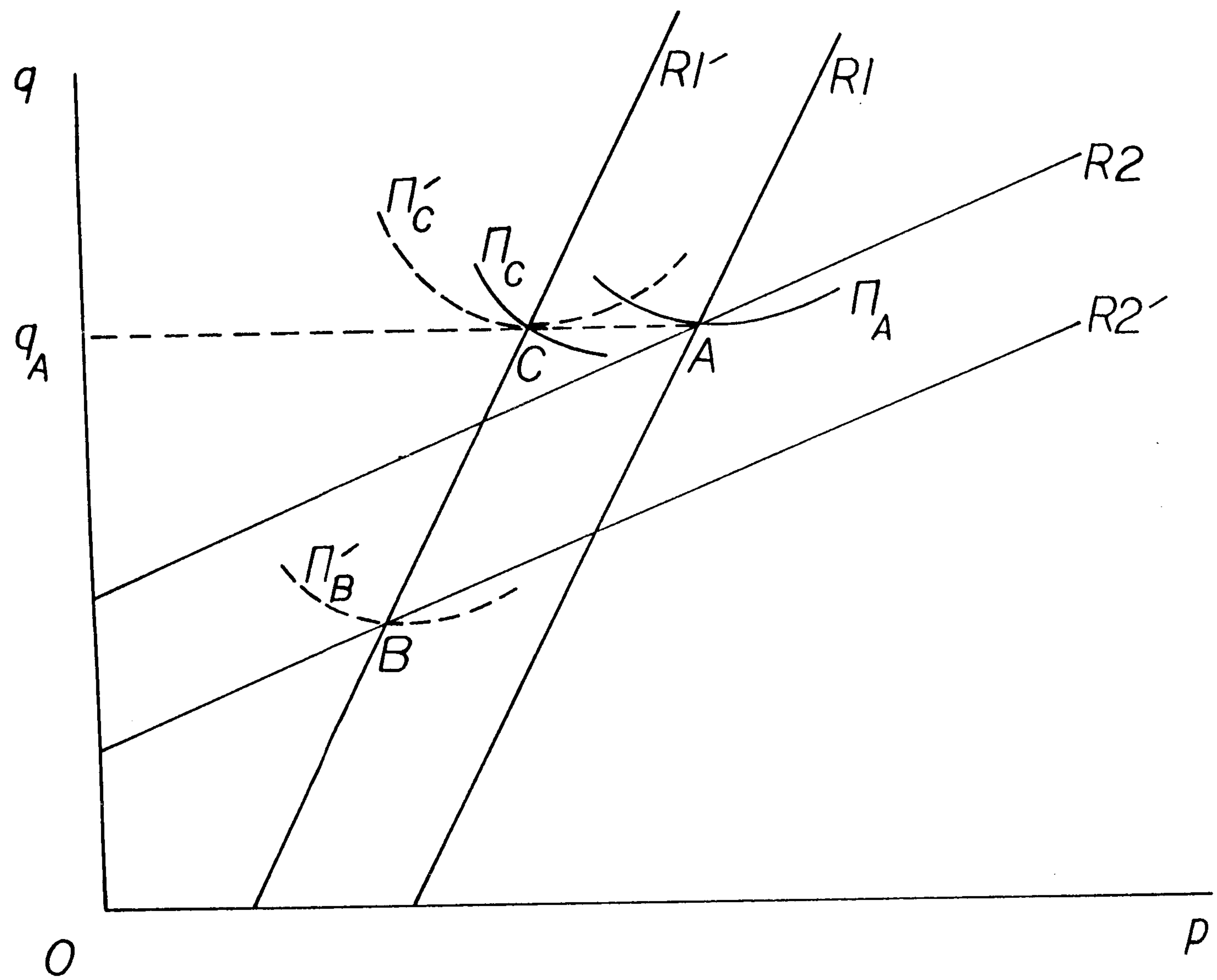
Novshek, W. [1984], "Finding All n-Firm Equilibria," International Economic Review, vol. 25, no. 1, pp. 61-70.

Nti, K. and M. Shubik [1981], "Non-cooperative Oligopoly with Entry," Journal of Economic Theory, vol. 24, pp. 187-204.

Seade, J. [1980], "On the Effects of Entry," Econometrica, vol. 48, no. 2, pp. 479-490.

Stackelberg, H. von [1934], Marktform und Gleichgewicht, Julius Springer, Vienna.

Figure 1: Comparison of equilibrium profits for firm 1 at A and B.



q
R1′
R1
R2
Π′c
Πc
ΠA
R2′
qA
C
A
Π′B
B
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p