The Division of Markets is Limited by the Extent of Liquidity (Spatial Competition with Externalities)

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Abstract

This paper studies the role of lack of liquidity as an endogenous trading friction in limiting the number of markets in a competitive economy. Each agent in the economy, faced with uncertain endowments, prefers to trade in a market with high liquidity rather than less. Liquidity at a market can only be increased by increasing the number of traders at that market. The traders are spatially separated so that as more traders go to a particular market, they are incurring increasing transportation cost. Given this tradeoff between liquidity and transportation cost, not all agents in the economy will go to the same market.

This paper considers three kinds of market structures. The first assumes that traders may participate in markets without charge (as in standard Walrasian markets). The second structure assumes that "autioneers" run individual markets and compete among themselves in the fees they charge for providing market services. The third market structure assumes that a monopolist auctioneer runs all available markets.

A basic positive result is that even with free market services, liquidity considerations will limit the number of markets in a competitive economy. The welfare implications of the competitive division of markets in this economy are ambigious. Since liquidity is an externality, there may be too little liquidity in equilibrium because each agent acts only in his own self interest. Then there are too many markets to be efficient. On the other hand, liquidity is self reinforcing. Given an existing equilibrium, new markets may find it impossible to open because nobody wants to use a new market with low liquidity. There may be too few markets to be efficient and new markets will not open.

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THE DIVISION OF MARKETS IS LIMITED BY THE EXTENT OF LIQUIDITY

I. Introduction

How many markets are there in a competitive economy? The standard general equilibrium model [e.g. Debreu (1959)] assumes that there are as many markets as there are commodities. This assumption may be justified when there is no trading friction, that is no cost to setting up a market. When there are trading frictions due to technological or demographic contraints, the number of markets may be less than the number of commodities [e.g. Cass and Shell (1983), Diamond (1982), Townsend (1983)].[1]

This paper studies the role of lack of liquidity as an endogenous trading friction in limiting the number of markets in a competitive economy. In many markets, the variance of (competitive) price fluctuations is negatively correlated with the volume of trade in that market. A market exhibits high liquidity when the volume of trade is high and the corresponding variance of prices is low.[2] The problem of liquidity is most apparent in financial markets. For example, liquidity is a particularly important factor in determining the success of futures contracts. The futures market for any asset has only a small number of maturity dates. In principle, many more maturity dates may be admitted. However, if there were many maturity dates, the market at each maturity date would be thin. Market participants may face large competitive price fluctuations arising only from the thinness of the markets. Traders may prefer fewer maturity dates so that liquidity is enhanced in the remaining markets, even though they will have less maturity dates to choose from. Thus, there is a fundamental tradeoff between liquidity and the number of markets.[3] While the question of liquidity is appreciated in financial markets, it is also an issue in other sectors of the economy. For example, the historical development of spatially separated towns may be attributed to the tradeoff that farmers faced between liquidity in the trading place and costs of transporting the commodities to the market.

This paper was motivated by Diamond's search models [e.g. Diamond (1982, 1984), Mortensen (1976), Siow (1982)]. Many issues addressed in this paper were raised by him and our analysis complements his work. Our formal model uses a spatial location framework which is related to work by Townsend (1983, 1984). Townsend's models address some of the same concerns as ours. The main differences between our work and his are that our model is analytically more tractable and we use the Nash Equilibrium as our main equilibrium concept. We briefly consider the core as an alternative equilibrium concept in the final section of the paper.

In Section II we show that each agent in the economy, faced with uncertain endowments, prefers to trade in a spot market with high liquidity rather than less. Liquidity at a market can only be increased by increasing the number of traders at that market. The traders in our economy are spatially separated so that as more traders go to a particular market, they incur increasing trasportation costs. Given this tradeoff between liquidity and transportation cost, not all agents in the economy will go to the same market.

This paper considers two kinds of competitive market structures. The first assumes that traders may participate in markets without charge (as in standard Walrasian markets). The second structure assumes that it is costly to operate a market which means that the "auctioneer" must be paid for providing market services. In Section III we establish and characterize the non-cooperative symmetric market equilibria for the economy with free market services. A basic positive result from this section is that even without fixed cost, liquidity considerations will limit the number of markets in a competitive economy. The welfare implications of the competitive division of markets in this economy are ambigious. Since liquidity is an externality, there may be too little liquidity in equilibrium because each agent acts only in his own self interest. In this case there are too many markets to be efficient. On the other hand, liquidity is self-reinforcing. Given an existing market structure, new markets may find it impossible to open because nobody wants to use a new market with low liquidity. There may be fewer markets than is necessary for efficiency and yet new markets will not open. We conclude the section with comparative static results.

In Section IV we make it costly to operate a market. Each market must be operated by a market maker whose opportunity cost is the expected utility that he can get as a trader. Arbitrage between being a market maker or a trader reduces the number of equilibria (relative to the economy in Section III). However, competition between market makers for customers is not sufficient to internalize the externality caused by liquidity. This result runs counter to that of Knight (1924) who suggested that profit maximizing ownership of a congested facility leads to efficient pricing.[4]

Our model is applicable to financial markets. Many such markets in the United States are organized by a few financial exchanges. In section V we study the market structure when all markets are organized and run by a monopoly exchange. We show that the monopolist will overcrowd the space with small markets because he is able to appropriate a larger percentage of the surplus generated in smaller markets.

Liquidity in our model is not tied to the spot market specification in the economy. In Section VI we show that the same issues arise when the spot markets are replaced by state contingent claims between agents at a specific market location. State contingent claims markets cannot reduce the intrinsic uncertainty of any specific market location. Only the addition of traders at a specific market location may reduce the uncertainty in that location. Since contingent claims markets cannot reduce transportation costs, the same issues remain. We also briefly consider the concept of the core as an alternative equilibrium concept. Final remarks are also in this Section.

II. The Model

Consider a two-goods, x, y, economy with consumers (traders) located at equal distances (d) on the real line. See figure 1. All consumers have identical preferences but their endowments are stochastic. Specifically a consumer is endowed with commodity vector (1, 0) with probability $1 - \theta$ and with (0, 1) with probability θ. Consumers, who are expected utility maximizers, meet to exchange goods at specific locations which we call markets. The decision whether or not to participate in the market is made before the endowment vector is realized. If the consumer does not participate, he consumes his endowment and receives a utility of zero. If he participates in a market, his endowment is realised after he arrives at the market. After the endowments are realised, all consumers in the same market may trade with each other in a competitive spot market for the two goods. Since he has to make his participation decision before he learns of his endowment, an expected utility maximizing consumer prefers to be in a market of high liquidity (with many traders) because in such a market the variance of price is lower. Note that the expectation of price is independent of the size of participation. A consumer incurs a cost of travelling to the market, so that, ceteris paribus, he prefers a market closer to his location on the line. Large participation requires that some consumers travel from afar. Thus, there is a tradeoff between market liquidity and the distance between markets in this economy. This tradeoff determines the equilibrium distribution of markets in the economy.

When the endowment (1, 0) (respectively (0, 1)) is realized for a specific trader, we call him of type 1 (respectively type 2). Let a market of N participants consist of X traders of type 1 and Y traders of type 2. Let $k = Y/N$ be the realized proportion of type 2 traders. Thus, a realized market can be described by a pair (X, Y) or alternatively by a pair (k, N). X and Y are random variables distributed binomially $(N, 1 - \theta)$ and (N, θ) respectively. Then $E(k) = E(Y/N) = \theta$, and $E(1 - k) = E(X/N) = 1 - \theta$.

We assume that exchange in every realized market is Walrasian. Aggregate supply and demand for each commodity are proportional to N, so that the equilibrium price is independent of N. Calling $x_i^d(e_i, P)$ the demand of a type i trader, where e_i is the respective endowment vector and P is the price of y relative to x, market clearing is defined by $Xx_1^d(e_1, P) + Yx_2^d(e_2, P) = X$ $\iff (1 - k)x_1^d(e_1, P) + kx_2^d(e_2, P) = 1 - k$, which defines a price P(k) independent of N.[5]

Let $V_1(k)$ (respectively $V_2(k)$) denote the indirect utility of a trader of type 1 (respectively of type 2) in a market (k, N). For any N, the expected utility of a trader who does not know his type conditional on realization k is

(1) $W(k) \equiv (1 - k)V_1(k) + kV_2(k)$,

i.e. the probability of being type 1 times the utility of type 1 plus the probability of being type 2 times the utility of being type 2. The (unconditional) expected utility of a trader in a market of N traders $(N > 1)$ is

$U(N) = E_k W(k)$.

For large N, W(K) can be approximated by a Taylor expansion up to second order around $k = \theta$:

$$(2) \quad U(N) = E_k W(k) \simeq E_k [W(\theta) + (k - \theta)W'(\theta) + (k - \theta)^2 W''(\theta)/2]$$
$$= W(\theta) + W''(\theta)\theta(1 - \theta)/(2N).$$

since $E_k k = \theta$, $E_k (k - \theta)^2 = \text{var}(k) = \theta(1 - \theta)/N$.[6] W(k) is assumed to be concave, so that $W''(\theta) < 0$. We show in appendix A that this is true for Cobb-Douglas and CES utility functions.[7] Concavity of W means that a trader, facing uncertainty about his type, prefers to be in a market where the sizes of two groups of traders facing each other are expected to be equal rather than unequal. Further he loses at an increasing rate as the groups become more and more unequal (in expectation).

U(N), the benefit of a trader from participating in a market of N traders, is an increasing and concave function of N.[8] A trader always prefers to be in a larger market, but the marginal advantage drops as the market becomes larger. Traders prefer larger markets because they provide superior liquidity through lower price variance. On the cost side, a trader has to travel from his location to the market to be able to participate in it. This cost must be subtracted from U(N) to determine the benefits of participation. We assume that costs are linear in the distance traveled with cost coefficient c.[9] Thus, a trader participating in a market of N traders at distance α from his original location has net benefit

$$(2') \quad Z(N, \alpha) = U(N) - c\alpha.$$

III. Non-cooperative Equilibria With Free Market Services

We are now in position to look for an equilibrium with free market services. Let agents j = 1, ... be located on a real line at consecutive positions at distance d apart. Each agent decides either to stay home and consume his endowment (earning zero utility) or to travel to a location on the real line. Each agent has expectations on the decisions of all other agents. When he travels a distance α to a market of N traders his objective function is $Z(N, \alpha)$ given by (2´), above. We define a market structure to be a (non-cooperative) equilibrium if the participation of an agent in a particular market maximizes his expected utility under the expectation that all other agents will not change their decisions with respect to their market affiliation.

There are many equilibria in this game, including quite unreasonable ones. For example, there is an equilibrium where everybody stays home because everybody expects others to stay home. We restrict our attention to symmetric equilibria Nd apart, where the market size, N, will be defined by the equilibrium conditions. With reference to figure 1, let the marginal consumer of market m_1, at distance $(N/2 - 1/2)d$ from m_1, weakly prefer to participate in m_1 rather than at neighboring m_2, which is located at distance $(N/2 + 1/2)d$ from him, i.e

$$U(N) - dc(N - 1)/2 \geq U(N + 1) - cd(N + 1)/2,$$ [10]

or equivalently

$$U(N + 1) - U(N) \leq cd,$$

which is approximated by

$$(3) \qquad U'(N) \leq cd.$$

Let N_1 be the minimum N obeying this inequality.[11] (3) is fulfilled for all $N \geq N_1$. <u>When liquidity is important, thin markets will not survive.</u> In thin markets, the influence of a single agent on the variance of price is large. Thus, in a thin market, $N < N_1$, an agent located in the "natural" area of market 1 prefers to participate in the distant market 2, thereby upsetting the existence of a symmetric non-cooperative equilibrium.

For the equilibrium to exist, it is also required that the marginal trader, at distance $(N - 1)d/2$ from m_1, be better off by participating in the market rather than staying home, i.e.

(5) $U(N) - cd(N - 1)/2 \geq 0.$

Let N_2 be the solution of (5) as equality. Then all $N \leq N_2$ satisfy inequality (5). Markets cannot be too large because traders from afar do not wish to participate. (5) together with (3) are necessary and sufficient for the existence of a symmetric equilibrium. Thus, all markets of sizes N in $[N_1, N_2]$ are symmetric non-cooperative equilibria.[12]

Both N_1 and N_2 are decreasing in the cost coefficient c and in the distance d separating the original positions of consecutive traders.[13,14]. It can be shown that the distance $N_2 - N_1$ is decreasing in c and d. For high transportation costs we have that $N_2 < N_1$, so that there are no equilibria.[15]

Proposition 1: Symmetric non-cooperative equilibrium market structures exist for non-prohibitive transportation cost, c. Typically there are many such equilibria characterized by the number of traders per market, N, which lies in an interval $[N_1, N_2]$.

We now consider social welfare in this economy. Let the planner's problem be to set up markets so as to maximize welfare. When N consumers participate in a market, average utility is

$$S(N) = [NU(N) - 2cd(1/2 + 3/2 + \ldots + (N - 1)/2)]/N = U(N) - cdN/4.$$

S(N) is approximately maximized (ignoring integer constraints) at

(6) $U'(N) = cd/4,$

the solution of which we call N^*.[16] The concavity of U(N) implies $N^* > N_1$. The result that N^* is larger than N_1 is a consequence of liquidity being a positive externality. When the transportation costs coefficient, c, is low,

N^* is lower than N_2 and therefore the optimum can be achieved as a non-cooperative equilibrium. This is in contrast with the usual result in price-location models as in Economides (1984a, 1984b), Lancaster (1979).

Since N^* falls in (N_1, N_2) there can also exist equilibria with more or less traders per market than is efficient. It is striking that the non-cooperative equilibrium in this economy may have more traders per market than the social optimum. This possibility arises because liquidity is self-reinforcing. Given an existing equilibrium, new markets may find it impossible to open because nobody wants to use a new market with low liquidity. There may be too few markets to be efficient, $N_2 > N > N^*$, and yet new markets do not open. So even though liquidity is a positive externality, too much liquidity can result from non-cooperative behavior!

When c is large, the social optimum lies beyond N_2 traders per market. To achieve the optimal market structure traders have to be subsidized to participate in larger and fewer markets. This case is close to the received wisdom in Diamond (1982), (1984).

<u>Proposition 2:</u> **The surplus maximizing market structure is a non-cooperative equilibrium when transportation cost, c, is low. Non-cooperative equilibrium market size could also be larger or smaller than is optimal.**

Next we consider the effects of addition of traders to the economy. At first sight it may seem that the addition of traders (say through replication) should decrease the variance of price in every market, increase liquidity and result in higher expected utility for all traders. In fact the addition of agents may not reduce the variance of price if the uncertainty is location-specific, so that agents at the same location get identical draws. To see this, suppose we double all agents at the old positions. First, let us consider the case when uncertainty is not location specific. Let any two agents at the same location get independent draws from the distribution of endowments. At every market, the distribution of types is preserved. Given k, the equilibrium price is unaffected and so is the indirect utility functions $V_1(k)$, $V_2(k)$ of each trader and their weighted sum $W(k)$. The function $U(N)$ is the same as before, but now it has to be evaluated at 2N. Let markets be $\hat{N}_1 d$ apart at the lower bound of the equilibrium existence region (where the corresponding number before doubling was $N_1 d$). There are now $2\hat{N}_1$ traders per market. Equation (3) for this equilibrium is

$$U'(2\hat{N}_1) = cd,$$

which implies $2\hat{N}_1 = N_1$, or $\hat{N}_1 = N_1/2$, since N_1 solves $U'(N_1) = cd$. Thus, when agents are doubled at the old locations and receive independent draws, the number of agents per market remains unaffected ($2\hat{N}_1 = N_1$)[17] while the markets markets are twice as dense ($\hat{N}_1 = N_1 d/2$) compared with $N_1 d$). The expected utility of agents is now higher because they have to travel half the distance to find a market of the same liquidity as before doubling.

Alternatively, suppose that all uncertainty is location-specific, such as uncertainty associated with the local weather. After replication, let agents receive identical endowments. Again $V_1(k)$, $V_2(k)$ and $W(k)$ are unaffected. Further, the variance of k is the same as before replication, although there are now twice as many agents per market. The original expected utility function

(2) $$U(N) = W(\theta) + W''(\theta)\theta(1 - \theta)/(2N)$$

has been replaced by a new utility function for the replicated model

$$U_1(2N) = W(\theta) + W''(\theta)\theta(1 - \theta)/(2N),$$

and therefore

$$U_1(N) \equiv U(N/2).$$

Let markets be $\hat{N}_1'd$ apart at the lower bound of the equilibrium existence region. There are now $2\hat{N}_1'$ traders per market. Equation (3) is now

$$U_1'(2\hat{N}_1') = cd \iff U'(\hat{N}_1') = cd \iff \hat{N}_1' = N_1.$$

The resulting markets are at the same distance as before replication, with twice as many traders per market, but all traders receive the same utility as before, $Z_1(2N, \alpha) = U_1(2N) - c\alpha = U(N) - c\alpha = Z(N, \alpha)$. We have shown that

Proposition 3: Replicating the number of agents when uncertainty is location-specific leaves liquidity per market constant. Thus traders' utility is also unaffected. On the other hand, replication of the number of agents when uncertainty is not location-specific results in increased liquidity *ceteris paribus*. The resulting equilibrium will have denser markets with increased utility for all agents.

IV. Competitive Equilibrium With Costly Market Services

In many situations market services are not free. We now allow explicit competition in the provision of market services. Instead of having market services provided free of charge, let each market be operated by a real auctioneer (market maker).[18] Suppose that agents can choose between being market makers or traders. When making this choice they are ignorant of the location they will receive on the line if they decide to be traders. After the choice of occupation, the market makers choose their positions on the line to set up their markets. There is no cost to setting up a market except for the opportunity cost of being a trader. The market maker charges each trader a fee, F, for using his market.[19] After the markets are set up, traders learn their position on the line. Assume that traders are restricted to choose from the set of markets which is offered. Then the traders' problem is similar to the one we have discussed before except for the possible difference in fees across markets.

Consider the problem of the market maker located at m_1. He can lower the fee and hope to attract traders from his competitors, assuming that his competitors will not respond to his price cutting. When he gets additional customers just due to the price cut, his closest competitor to his right, m_2, will lose customers to him. But m_2 will also lose customers to the next market to the right, m_3, because m_2's market is now less liquid. This in turn will drive even more customers away from m_2 toward m_1. Therefore the number of new customers that m_1 gets from cutting his price depends on the difference in fees and on how liquidity is affected in all other markets. Formally, the new distribution of customers across markets due to m_1 cutting his fee is described by the solution to a second order difference equation.[20] m_1 will choose the price that will maximize his profits. But his competitors are doing the same thing, resulting in a Nash equilibrium in fees. Consider the case of symmetrically spaced markets so that all markets charge the same fee at equilibrium attracting N traders each. Let the equilibrium fee with N traders per market be $F^*(N)$. The wage of a market maker is then $N \cdot F^*(N)$. Since any agent can choose to be a market maker or trader, the wage of the market maker must be equal to the expected utility of a trader from participating in a market of size N. This closes the model and determines the unique market structure in our economy.[21]

We first analyse the game in fees among market makers. After the equilibria of this game are computed, we will return to the (earlier) stage of occupation choice. Formally, let market maker j (operating market m_j) charge a fee F_j for the participation of a trader in his market. For a symmetric positioning of the markets Nd distance apart, with N traders between every adjacent markets, we seek a symmetric non-cooperative equilibrium in fees, $F^*(N)$. We now calculate the demand facing market maker j+1. Let the marginal trader who is indifferent between going to market m_{j+1} and m_{j+2} be located at $(N_{j+1} - 1/2)d$ to the right of m_{j+1} (and this implies that he will be $(N - N_{j+1} + 1/2)d$ away from m_{j+2}). If he goes to market m_{j+1} there will be $N - N_j + N_{j+1}$ traders in that market and he has to travel $(N_{j+1} - 1/2)d$ and pay fee F_{j+1}. Therefore his utility will be $U(N - N_j + N_{j+1}) - (N_{j+1} - 1/2)cd - F_{j+1}$. Similarly, if he goes to market m_{j+2} there will be $N - N_{j+1} + N_{j+2} + 1$ traders at m_{j+2} and his utility will be $U(N - N_{j+1} + N_{j+2} + 1) - (N - N_{j+1} + 1/2)cd - F_{j+2}$. Since he is marginal he has to fulfill[22] $U(N - N_j + N_{j+1}) - (N_{j+1} - 1/2)cd - F_{j+1} = U(N - N_{j+1} + N_{j+2} + 1) - (N - N_{j+1} + 1/2)cd - F_{j+2}$. The system of these equations (j integer) determines the marginal consumers and the demand faced by all market makers as functions of the fees charged.

Let all market makers charge the same fee $F_j = F$ except for one market, say F_1, and let $\Delta F = F_1 - F$. Then the positions of the marginal consumers are the solution of the system of equations:

(7) $U(N - N_j + N_{j+1}) - (N_{j+1} - 1)cd = U(N - N_{j+1} + N_{j+2} + 1) - (N - N_{j+1})cd,$
$j \neq 0, -1,$

(8) $U(N - N_0 + N_1) - (N_1 - 1)cd - \Delta F = U(N - N_1 + N_2 + 1) - (N - N_1)cd,$
$j = 0,$

(9) $U(N - N_{-1} + N_0) - (N_0 - 1)cd + \Delta F = U(N - N_0 + N_1 + 1) - (N - N_0)cd,$
$j = -1.$

Defining $\Delta N_j = N_j - N/2$ and linearizing $U(.)$ around $U(N)$ results in

(10) $\Delta N_{j+2} + \gamma \Delta N_{j+1} + \Delta N_j = \gamma/2$, $j \neq 0, -1,$

(11) $\Delta N_2 + \gamma \Delta N_1 + \Delta N_0 = \gamma/2 - \Delta F/U'$, $(j = 0),$

(12) $\Delta N_1 + \gamma \Delta N_0 + \Delta N_{-1} = \gamma/2 + \Delta F/U'$, $(j = -1),$

where $\gamma = 2(cd - U'(N))/U'(N)$.

In Appendix B we solve the system of (10) - (12). Imposing the condition that perturbations at market 1 should have minimal effect at markets far away from m_1, we show that the general solution of (10) with a convergent path is

(13a) $\Delta N_j = \gamma/2(2 + \gamma) + A\rho_1{}^j, \quad j = 1, 2, \ldots$

(13b) $\Delta N_j = \gamma/2(2 + \gamma) + B\rho_2{}^j, \quad j = 0, -1, \ldots$

where $\rho_2 = \rho_1{}^{-1}$ are the roots of the characteristic equation of equation (10) and $\rho_2 < -1 < \rho_1 < 0$ for $U'(N) < cd/2$.[23]

Imposing (11) and (12) determines A and B (see appendix B) as $A = \Delta F/[\rho_1(1 - \rho_1 - \gamma)U']$ and $B = -\Delta F/[(1 - \rho_1 - \gamma)U']$. Thus, demand for firm 1 when it deviates $\Delta F = F_1 - F$ from the fees of all others is $N_1 - N_0 + N = \Delta N_1 - \Delta N_0 + N = N + 2\Delta F/[(1 - \rho_1 - \gamma)U']$. The profit function of firm 1 is $\Pi_1(F_1) = NF_1 + 2(F_1 - F)F_1/[(1 - \rho_1 - \gamma)U']$ is concave in F_1, and is maximized at $4F_1/[(1 - \rho_1 - \gamma)U'] - 2F/[(1 - \rho_1 - \gamma)U'] + N = 0$. At the symmetric equilibrium $F_1 = F$ which implies that the equilibrium fee is

(13´) $F^*(N) = N(\rho_1 + \gamma - 1)U'/2$.[24]

Symmetric equilibrium profits of a market maker when markets are Nd apart are

(14) $\Pi_E(N) = N^2(\rho_1 + \gamma - 1)U'/2 = N^2\{cd - 2U' + [(cd)^2 - 2cdU']^{1/2}\}/2$.

This is an increasing function of N.[25]

__Proposition 4:__ **The symmetric fee structure $F_1 = F^*(N)$ given by (13´) is a non-cooperative equilibrium of the game among market makers when markets are set Nd apart. Equilibrium profits are given by (14).**

Now we return to the choice of occupations. The overall equilibrium market structure is determined by the condition that equalizes the expected profits of a market maker with the expected utility of a trader. The expected utility of a trader when he does not know his position on the line (following the analysis of Section III) is $S(N) = U(N) - cdN/4 - F^*(N)$, a concave function of N passing through the origin. The equilibrium market structure is determined by the intersection of $\Pi_E(N)$ and S(N). Since $\Pi_E(N)$ is defined for $N > N_0$, where $U'(N_0) = cd/2$, the equilibrium N_E exists if $S(N_0) - \Pi_E(N_0) \geq 0$. Equilibrium fees can be calculated by sustitution of N_E in (13′).

In general, the equilibrium N_E is not the same as the optimal market structure N^*, which is analogous to the one of Section III adjusted for the wages of the market makers. In fact, results for the Cobb Douglas utility function show that N_E can be smaller or larger than N^*. So even when market makers are aware of the gains from liquidity and there is competition in fees between market makers and free entry of market makers, the externality caused by liquidity is still not completely internalized. As noted in the introduction, this result is counter to Knight′s (1924), where he argued that profit maximizing ownership of a congested facility leads to efficient pricing.

Proposition 5: **There exists a unique symmetric equilibrium market structure with N_E traders per market, where the ex-ante expected utility of an agent is the same in either occupation. Liquidity in this equilibrium may exceed or fall short of the level needed for surplus maximization.**

Note that function S(N) is decreasing in c (and d) while $\Pi_E(N)$ is increasing in c (and d).[26] Therefore the intersection N_E of S(N) and $\Pi_E(N)$ decreases in c (and d). A decrease in the cost of travel results in larger markets. An increase in the density of the distribution of consumers (1/d) results in larger markets but the distance between markets $N_E \cdot d$ may decrease or increase.

V. The Monopolist's Solution

So far this paper has only considered competitive market structures. However, many financial markets in the United States, where liquidity considerations are important, are organized by a few financial exchanges. A study of competition between financial exchanges using oligopolistic models of product differentiation is beyond the scope of this paper. As a benchmark, we discuss the market configuration chosen by a monopolist exchange that acts as a market maker in all markets. For example, the monopolist's problem is particularly relevant for a futures exchange that has to determine the maturity dates of a futures contract in a commodity. The maturity dates compete among themselves for liquidity. Therefore it is a non-trival choice problem for the monopolist to determine the optimal number of maturity dates to maximize its profits.

Within the context of our model, the objective of the monopolist is to maximize total revenue collected from all markets. Once the locations of markets and the fee structure are anounced by the monopolist, the traders decide whether to participate in a market and in which market to do so. The monopolist will serve all agents on the line. This is because, for any fee structure which leaves some agents at home, the monopolist can bring the markets closer together, close the gap, and increase revenue by establishing a market in the freed space. Thus the monopolist wants to make the marginal agent, at distance $cd(N-1)/2$, indifferent between participating and staying home. From each market the monopolist collects $F(N) \cdot N$. Since the frequency of the markets is $1/(Nd)$, total revenues are proportional to $F(N)$. Thus, the monopolist's problem is:

$$\underset{N}{\text{Maximize}} \quad F(N) \quad \text{subject to}$$

$$U(N) - cd(N-1)/2 - F(N) \geq 0,$$

i.e. that traders come to the market rather than stay home, and

(3) $U'(N) \leq cd$,

i.e. that the symmetric equilibrium fee structure of the monopolist is a non-cooperative equilibrium for traders. It is equivalent to

(15) $\underset{N}{\text{Maximize}} \quad U(N) - cd(N-1)/2 \quad \text{subject to} \quad U'(N) \leq cd.$

Its solution is at N_M defined by

(16) $U'(N) = cd/2.$

Clearly N_M is in (N_1, N_2). In comparison with the surplus maximizing outcome the monopolist will operate a larger number of smaller markets.[27] This is because the surplus maximizing outcome is defined by $U'(N^*) = cd/4$ and U(.) is concave. See figure 2.

Comparing the monopolist's market structure with the equilibrium of independent market makers of section IV, we see that $U'(N_M) = cd/2 > U'(N_E)$. By the concavity of U(.), $N_M < N_E$. The monopolist will have smaller and more numerous markets than independent market makers. Starting from the market structure of independent market makers, the monopolist, facing no opponents, will increase fees in all markets until the least well-off consumers, located in the middle of the distance between markets, are indifferent between staying home or participating in a market. Then he will increase fees further while spacing markets closer together (keeping the middle consumers indifferent between participating or staying home) until equation (16) is satisfied. Thus the monopolist's market structure will have a larger number of smaller markets than the equilibrium of independent market makers.

<u>Proposition 6:</u> **A monopolist will operate smaller and more numerous markets than independent operators. Further, his markets are always smaller and more numerous than is optimal.**

This result shows that the monopolist's incentive to reduce output and increase price is dominant. Lacking the ability to price discriminate and appropriate the whole surplus, the monopolist avoids creating large markets with high surplus. Instead he institutes a large number of smaller markets where he can appropriate a larger percentage of the surplus. We note that this overcrowding of the space with markets happens despite the fact that there is no threat of entry. Overcrowding of the product space to deter potential entrants has been noted by Schmalensee (1978) among others.

VI. Discussion

Consider replacing each spot market in this economy with state-contingent claims markets [e.g. Debreu, Chapter 7] for agents that go to the same market location. Agents will still go to a specific market location. But each agent now trades in state contingent claims with other agents at the same location before he knows the realization of endowments. Every agent at the same market location is ex-ante identical. So every agent will have the same excess demand functions for these contingent claims commodities. The equilibrium prices must be such that all agents will have the same ex-post consumption bundle. Therefore, when k is realised, each agent at the same market will consume (1 - k, k). Assuming that the representative agent has a utility function that is concave in the two goods, his indirect utility function will also be concave in k.[28] Substituting this indirect utility function in equation (2) in the place of W(k), one can derive an expected benefit function with the same properties as U(N). The results of the rest of the paper follow. Therefore the qualitative features of the symmetric non-cooperative equilibria are the same whether agents are faced with spot markets or state contingent claims markets.

The difference between the state contingent claims markets in this economy and in the standard general equilibrium model is that agents in this economy must go to a specific market location before they can participate in the state contingent claims markets. There is risk sharing within a market location but not across market locations. In the standard model, agents can participate in a complete market structure without first having to go to any specific location. The standard model allows risk sharing between all individuals in the economy, whereas our model only allows risk sharing among endogenous subsets of individuals.

The alternative market setup considered above shows that it is not the spot market setup of our problem that is important. The normative results on liquidity is due to the Nash Equilibrium concept that we employ. Some readers have questioned whether alternative equilibrium concepts, such as the core (as used in studies on financial intermediation by Boyd and Prescott (1986) and Townsend (1983)), may give different normative results. In our problem, the core is efficient because it allows a redistribution of income from agents close to a market to those who are far away. The redistribution means that all agents that go to the same market will have the same ex-ante utility, making them indifferent to their distance from the market. The efficient market structure allows for the largest surplus to be redistributed which means no other coalition can be formed that will satisfy all agents in this other coalition. Therefore the core will provide the efficient level of liquidity. The main difficulty with using the concept of the core in our problem is that the redistribution that is necessary seems difficult to enact for many relevant problems. In particular, the identity of every agent (i.e. his location) has to be common knowledge to all agents in order to implement the core allocation. The identity of agents are not necessary for constructing the Nash Equilibria. If the identities of agents were somehow known, it is possible that price discrimination in fees by market makers within the Nash equilibrium construct may mimic the core solution.

An area where the model with free market services may apply is in the choice of standards when there is a variety of products. For example, the personal computers industry has a large variety of potential and actual products. In this industry a few standards have already arisen. Many consumers buy an IBM or IBM-compatible machine although it is not the "best" for what they currently want to do. Other products may be able to do what they want better and at a cheaper price. However, most consumers know that they may use the computer to solve other problems in the future. By buying "the standard", they are buying insurance that accessories (software and hardware) will be available for solving those problems. On the other side of the market, firms may not produce products which accomplish a task most efficiently, but will rather produce IBM-compatible products. Since firms do not have to pay a fee in choosing the "standard" there will be a range of indeterminacy for the equilibrium standards, as predicted by our model. The standards that obtain in an actual industry can often be predicted by participants in that industry from their knowledge of initial conditions. For example most informed observers expected IBM to become a standard in the personal computer industry. This observation is not inconsistent with the fact that at a point in time, "the standard" may look arbitrary given the available knowledge and technology in that society. Recent work on network externalities consider closely related issues (e.g. Carlton and Klamer (1983), Farrell and Saloner (1985), Katz and Shapiro (1985)).

Notes

1. There are also models where the incomplete market structure is assumed even though there is no trading friction [e.g. Hart (1975), Newbery and Stiglitz (1984)]. In these models, results often depend on which markets are assumed to be missing.

2. Our definition ignores the speed at which sales can be consumated. For example, Lippman and McCall (1986) defines an asset as liquid "if it can be sold quickly and at a predictable price". Their paper contains a comprehensive discussion of the attributes of liquidity that are important to a seller of an asset.

3. See Carlton (1984), Garbade and Silber (1979), (1983), Telser (1981) and the references therein for discussions of liquidity and the success of futures markets. Black (1985) contains empirical evidence on the same subject.

4. Exceptions to Knight's result have been noted elsewhere. For references and a study of duopoly pricing of congested facilities, see Braid (1986).

5. In an extension one may assume that the equilibrium price may depend on N as well, to reflect the inefficiency of thin markets caused by bargaining among a small number of participants.

6. Since Y is binomial (N, θ), $var(k) = var(Y/N) = (1/N^2)var(Y) = \theta(1 - \theta)/N$. The approximation of equation 2 is good for large N.

7. The concavity of W(k) can also be derived by allowing agents to participate in location specific state contingent markets, where we only assume agents have concave utility functions. See Section VI.

8. $dU(N)/dN = -W''(\theta)\theta(1 - \theta)/(2N^2) > 0$, $d^2U/dN^2 = W''(\theta)\theta(1 - \theta)/N^3 < 0$.

9. Linear transportation cost is assumed for expositional purposes. None of the results hinge on the linear specification.

10. Given that the marginal consumer N_m prefers to go to market m_1 rather than m_2 (under the expectation that all consumers between him and m_1 go to m_1) any other consumer i closer to m_1 also prefers m_1 over m_2 (under the expectation that all other consumers between m_1 and N_m go to m_1). This comes directly from the concavity of U(N) in N.

11. Using the definition of U(N) in equation (2), N_1 can be calculated as $N_1 = [-\theta(1 - \theta)W''(\theta)/2cd]^{1/2}$.

12. Clearly the existence of equilibrium does not depend on the linearity of the travelling cost. Since U(N) is concave, any weakly convex travelling cost function is sufficient for the existence of equilibrium.

13. Totally differentiating $U'(N) = cd$ we have $dN_1/d(cd) = 1/U'' < 0$. Similarly, from $U(N) - cd(N - 1)/2 = 0$ we deduce $dN_2/d(cd) = (N - 1)/(2U' - cd) < 0$ since $U' - cd/2 < 0$ at N_2.

14. In the determination of the equilibrium market structure the distance between consecutive agents and the marginal cost of distance enter together as cd. Therefore a proportional widening of the spacing of agents together with a proportional decrease of marginal cost of distance leaves the equilibrium number of consumers per market unaffected, although markets are now more widely spaced.

15. Let the left hand side of (5) be defined as $F(N) \equiv U(N) - cd(N - 1)/2$. $F(N_1) \gtrless 0 \iff N_1 \lessgtr N_2$. Using (2), $F(N_1) = W(\theta) + cd(1 - 3N_1)/2$. Since $N_1 > 1/3$, the term in parenthesis is negative. Therefore $F(N_1)$ will be negative for large c, implying $N_1 > N_2$, and thus there will be no N where both (3) and (5) are satisfied simultaneously.

16. Using the definition of U(N) in equation (2), it is straightforward to show that $N^* = [-2p(1 - p)W''(p)/cd]^{1/2} = 2N_1$.

17. More precisely we are referring to comparisons of the lower bounds of the equilibrium sizes of the market.

18. Siow (1982) and Townsend (1983) also studied similar costly financial intermediation.

19. The fee is in utility units.

20. We are grateful to Mike Woodford for his help with the difference equations.

21. Because the number of agents is finite, the demand and profit functions are discontinuous. We thus establish ε-equilibria, where market-makers optimize up to ε.

22. This equation approximates the position of the marginal consumer when the N's are treated as integers. Because we are establishing an ε-equilibrium we can treat this relation (and equations (7) - (9) below) as equalities rather than inequalities.

23. We could also have considered agent $N_{j+1} + 1$ (at distance $d(N_{j+1} + 1/2)$ from m_{j+1}) being indifferent between going to market m_{j+1} or m_{j+2}. Then the resulting system of equations is similar to (10)-(12) except that $\gamma/2$ is replaced by $-\gamma/2$. As is seen next in the text, this change only shifts equally the boundaries of the market for any market maker. Therefore the demand for market maker j+1, being

$$N_{j+1} - N_j + N = \Delta N_{j+1} - \Delta N_j + N,$$

remains unchanged and there will be no effect on the equilibrium.

24. This analysis was done under the assumption that all consumers participate in the market, i.e. that $N(\gamma + \rho_1 - 1)U'/2 = F^*(N) \leq U(N) - cd(N - 1)/2$. $F^*(N)$ is positive since it is proportional and of the same sign as $\rho_1 + \gamma - 1 = [cd - 2U' + (cd)^{1/2}(cd - 2U')^{1/2}]/U' > 0$ because $U' < cd/2$.

25. For convexity it is sufficient that $U''' < 0$.

26. $d\Pi_E(N)/dc = d - 2U' + d(dc - U')/[(cd)^2 - 2cdU']^{1/2} > 0$ for $0 \leq c \leq 1$ because the second term is always positive since $U' < cd/2 < cd$, and the first term is positive if $U' < cd/2 \leq d/2 \; <= \; c \leq 1$.

27. Using the definition of U(N) in equation (2) it is easy to show that $N^*/N_M = \sqrt{2}$ so that the monopolist operates approximately 40% more markets than is optimal.

28. In this setup $W(k) = w(1-k, k)$ where $w(\, , \,)$ is concave. Then under regularity $W''(k) = w_{11} + w_{22} - 2w_{12}$. For concavity of W(.) we need to show $|w_{11}| + |w_{22}| + 2w_{12} > 0$. This is obviously true for $w_{12} > 0$. For $w_{12} < 0$, it is sufficient to show that $|w_{11}| + |w_{22}| - 2|w_{12}| > 0$. By concavity of w, we know that $(|w_{11}w_{22}|)^{1/2} > |w_{12}|$. Thus $|w_{11}| + |w_{22}| - 2|w_{12}| > |w_{11}| + |w_{22}| - 2(|w_{11}w_{22}|)^{1/2} = (|w_{11}|^{1/2} - |w_{22}|^{1/2})^2 \geq 0$.

Appendix A

A solution of the problem when agents have Cobb-Douglas utility functions follows. Let P be the relative price of good y with respect to x. A consumer with utility function $U(x, y) = x^{\alpha}y^{\beta}$ when endowed with A units of x he has budget constraint $x + Py = A$ and (gross) demands $x_1^d = \alpha A/(\alpha + \beta)$, $y_1^d = \beta A/(\alpha + \beta)P$. When endowed with A units of y he has budget constraint $x + Py = AP$ and demands $x_2^d = \alpha AP/(\alpha + \beta)$, $y_2^d = \beta A/(\alpha + \beta)$. Market clearing implies $x_1^d X + x_2^d(N - X) = XA \iff P = \beta X/\alpha(N - X) = \beta(1 - k)/\alpha k$. The equilibrium indirect utility function of a consumer endowed with A units of x participating in a market (N, k) is $V_1(k) = (\alpha A/(\alpha + \beta))^{\alpha+\beta}(k/(1 - k))^{\beta}$. For a consumer endowed with A units of y the corrersponding indirect utility is $V_2(k) = (\beta A/(\alpha + \beta))^{\alpha+\beta}((1 - k)/k)^{\alpha}$. Let $W(k) = (1-k)V_1(k) + kV_2(k)$ as in equation (1) in the text. A straightforward calculation will show that W(k) is concave as long as α, β are in the open interval (0, 1).

When the utility functions are CES, $U(x, y) = (x^{\rho} + y^{\rho})^{1/\rho}$, $\rho < 1$, a type 1 consumer, facing constraint $x + Py = A$, will demand $x_1^d = A/(1 + P^r)$, $y_1^d = AP^{r-1}/(1 + P^r)$, where $r = \rho/(\rho - 1)$. Similarly a type 2 consumer, facing constraint $x + Py = AP$, will demand $x_2^d = AP/(1 + P^r)$, $y_2^d = AP^r/(1 + P^r)$. Market clearing implies $x_1^d X + x_2^d(N - X) = XA \iff$

(A.1) $\quad 1 - k + kP = (1 - k)(1 + P^r) \iff P = (k/(1 - k))^{1/(r-1)}$.

The indirect utility functions are $V_1 = A(1 + P^r)^{-1/r}$, $V_2 = AP(1 + P^r)^{-1/r}$. Thus, $W(k) \equiv (1 - k)V_1 + kV_2 = (1 - k + kP)V_1 = (1 - k)A(1 + P^r)^{1-1/r}$, using (A.1). Substituting the clearing price we have

$$W(k) = A(1 - k)\ [1 + (k/(1 - k))^{r/(r-1)}]^{(r-1)/r}.$$

Direct computation reveals

$$W'(k) = A[1 + (k/(1 - k))^{r/(r-1)}]^{-1/r}[-1 + (k/(1 - k))^{1/(r-1)}],$$

and

$W''(k) = A(k/(1-k))^{1/(r-1)}[1+(k/(1-k))^{r/(r-1)}]^{-1-1/r}/[(r-1)k(1-k)^2] < 0$, where all terms except the denominator are positive. $r - 1$ is negative for all $\rho < 1$, i.e. for the whole range of definition of the CES.

Appendix B

The homogeneous version of equation (10), $\Delta N_{j+2} + \gamma\Delta N_{j+1} + \Delta N_j = 0$, has has characteristic equation $\rho^2 + \gamma\rho + 1 = 0$ with solutions $\rho_1 = (-\gamma + (\gamma^2 - 4)^{1/2})/2$, $\rho_2 = 1/\rho_1$, or equivalently, since $\gamma = 2(cd - U')/U'$, $\rho_1 = (U' - cd + [(cd)^2 - 2cdU']^{1/2})/U'$, $\rho_2 = (U' - cd - [(cd)^2 - 2cdU']^{1/2})/U'$. The roots are real and distinct for $U' < cd/2$. For $U' = cd/2$ they are real and coinciding $\rho_1 = \rho_2 = -1$. Then the solution of the homogeneous equation $\Delta N_j = A(-1)^j + Bj(-1)^j$ diverges as j goes to infinity. For $U' > cd/2$ the roots are complex and the solution is $\Delta N_j = A\cos(\theta j) + B\sin(\theta j)$, where $\cos\theta = -\gamma/2 = (U' - cd)/U'$, an exact oscillation. In the last two cases a disturbance at $j = 1$ has large effects at markets far away, an event we rule out.

In the case of distinct roots arising for $U' < cd/2$ it is easy to see that $\rho_2 < -1 < \rho_1 < 0$. The general solution of the homogeneous equation for markets to the right of m_1, $\Delta N_j = A_1\rho_1{}^j + A_2\rho_2{}^j$, converges as $j \to \infty$ if and only if A_2 is zero. Similarly the general solution for markets to the left of m_1, $\Delta N_j = B_1\rho_1{}^j + B_2\rho_2{}^j$, converges as $j \to -\infty$ if and only if B_1 is zero. Hence the convergent solution is $\Delta N_j = A\rho_1{}^j$, $j = 1, 2, \ldots$, and $\Delta N_j = B\rho_2{}^j$, $j = 0, -1, \ldots$ The inhomogeneous equation (10) has a particular solution $\Delta N_j = \gamma/2(2 + \gamma)$. Thus the solution of (10) is

$$\Delta N_j = \gamma/2(2+\gamma) + A\rho_1{}^j, \quad j = 1, 2, \ldots,$$

$$\Delta N_j = \gamma/2(2+\gamma) + B\rho_2{}^j, \quad j = 0, -1, \ldots$$

Conditions (11) and (12) can thus be written as

$$A\rho_1^{\;2} + A\gamma\rho_1 + B = -\Delta F/U'$$

$$A\rho_1 + B\gamma + B\rho_1 = \Delta F/U'$$

which are solved by $A = \Delta F/[(1 - \rho_1 - \gamma)U']$, $B = -\Delta F/[(1 - \rho_1 - \gamma)U']$, so that the solution of the system is

$$\Delta N_j = \gamma/2(2 + \gamma) + \Delta F\rho_1^{\,j-1}/[(1 - \rho_1 - \gamma)U'], \quad j = 1, 2, \ldots$$

and

$$\Delta N_j = \gamma/2(2 + \gamma) - \Delta F\rho_2^{\,j}/[(1 - \rho_1 - \gamma)U'], \quad j = 0, -1, \ldots$$

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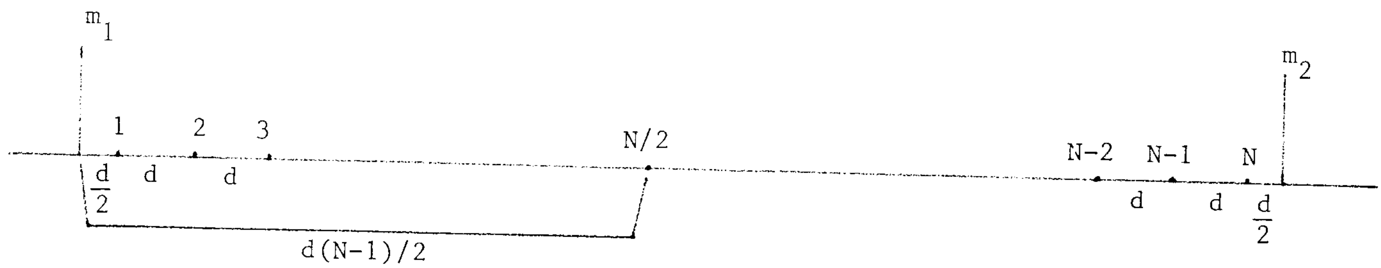

Figure 1: Locations of agents and markets.

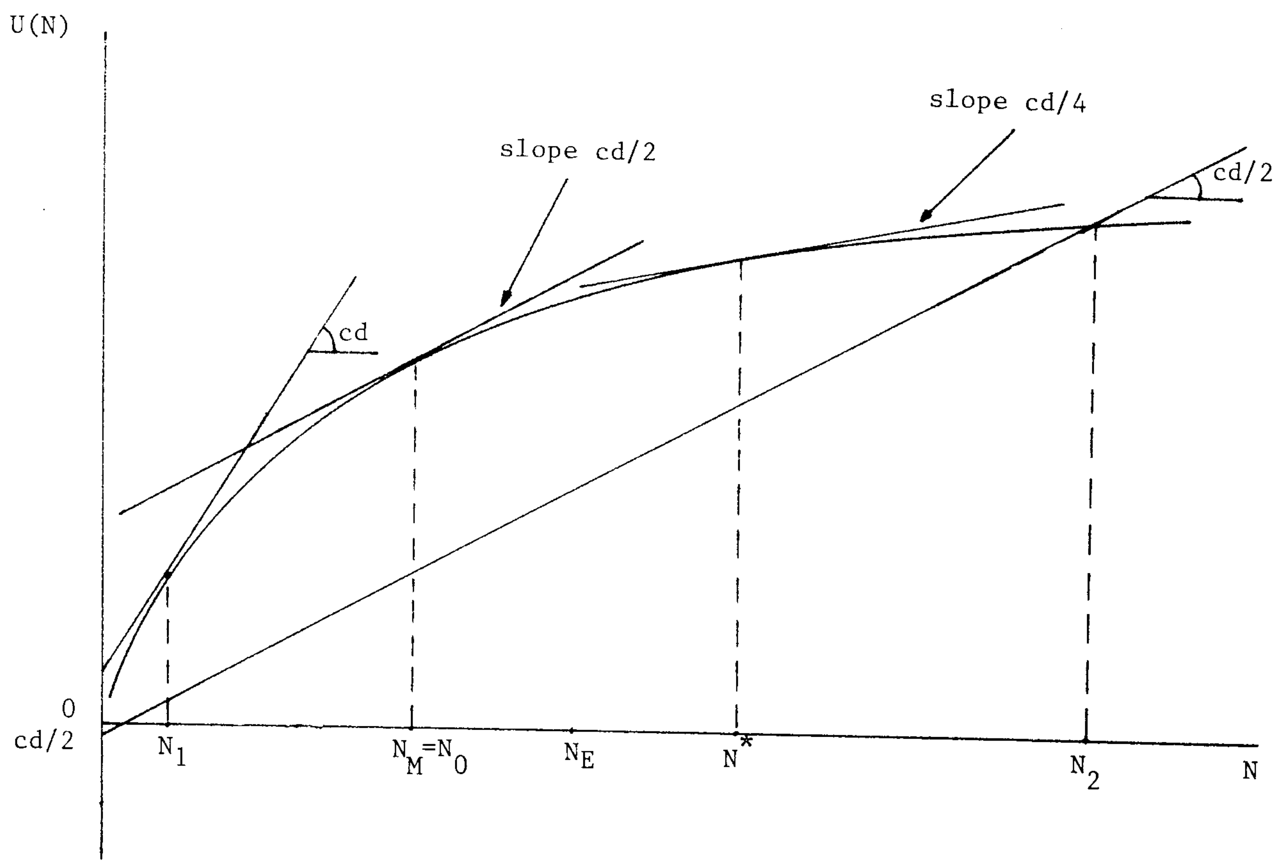

Figure 2: Markets of sizes N in (N_1, N_2) are non-cooperative equilibria. N^* is the optimal market size. N_M is the monopoly market size. N_E is the equilibrium market size with independent market makers.