The Incentive for Vertical Integration

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Abstract

We evaluate the incentive to integrate vertically in a simple 2X2 Bertrand model of two substitutes that are each comprised of two complementary components. We confirm that all prices fall as a result of a vertical merger. Further, we find that, when the composite goods are poor substitutes, producers of complementary components are better off after integration. Thus at equilibrium, each pair of complementary goods is produced by a single firm (parallel vertical integration). In contrast, when the composite goods are close substitutes, vertical integration reduces profits of the merging firms and is therefore undesirable. Thus, at equilibrium, all firms are independent (independent ownership). The reason for the change in the incentive to merge is that, as the composite goods become closer substitutes, competition between them reduces prices (in comparison to full monopoly) thereby eliminating the usefulness of a vertical merger in accomplishing the same price effect. We also find that, for intermediate levels of substitution, firms producing complementary components prefer to merge only if the substitute good is produced by an integrated firm. Thus, for intermediate levels of substitution, both parallel vertical integration and independent ownership are equilibria.

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The Incentive for Vertical Integration

1. Introduction

This paper evaluates the incentive of firms to vertically integrate in a simple setting. Cournot (1838) considered the case of two firms that produce complementary components, and each is a price-setting monopolist in its product line. He showed that each of them has an incentive to integrate vertically and become a single monopolist and that the single monopolist has a lower price than the sum of the prices of the independent firms, while also earning higher profits.

How do these results fare in the presence of imperfect competition? The reduction of price resulting from vertical integration has been the subject of much discussion, especially in the context of the possibility or lack of substitution in the technology that combines the components.[1] The incentive for vertical integration has received relatively less attention. This paper will focus on the incentives for vertical mergers in a market where two composite goods are substitutes to each other.

The formulation of this paper is conceptually simple. We start with the model of Cournot where two complementary components are combined in fixed proportions to produce a composite good. We introduce a second composite good comprised of two new complementary components. We will vary the degree of substitution between the two composite goods and assess the incentive for vertical mergers. As long as the two composite goods are distant substitutes, by continuity, the result of Cournot must continue to hold: vertical integration of the components of one of the composite goods results in a lower price of that good and in higher profits.[2] As the composite goods become closer substitutes, we show that prices still fall as a result of a vertical merger. However, we also show that, ***when the composite goods are***

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[1] See, for example, Schmalensee (1973), Greenhut and Ohta (1979), and Salinger (1988, 1989), among others.

[2] Of course this result has to be slightly qualified to make sure that the increase in price of composite good 1 is sustained in the presence of a substitute. Further, it is of interest to observe the effects of vertical integration on the price of the substitute composite good.

relatively close substitutes, a vertical merger results in a reduction of the total profits of the merged entities, and therefore it is undesirable to the merging firms.

Why does the incentive for merger change as the degree of substitution changes, and in particular why are vertical mergers unprofitable in the presence of close substitutes? To give an intuitive answer to this question, we consider four ownership structures, ***independent ownership***, where each of the four components is produced by a separate firm; ***partial vertical integration***, where a single firm produces the two components that comprise the first composite good, while the two components of the second composite good are each produced by separate firms; ***parallel vertical integration***, where for each composite good, the pair of components comprising it are produced by the same firm; and ***joint ownership***, where all components are produced by the same firm. A merger between one of the pairs of firms that produce complementary components changes the market structure from independent ownership to partial vertical integration. A further merger between the two firms that produce the other two complementary components changes the market structure from partial vertical integration to parallel vertical integration. From the point of view of the vertically integrated firm, Cournot's world of a single composite good is identical to our setting with zero substitution among the two composite goods, since, for zero substitution, the outcomes of partial vertical integration, parallel vertical integration, and joint ownership coincide.

The most important effect of a vertical merger is a reduction of all prices because of the elimination of double marginalization. When there is zero (or very weak) substitution between the composite goods, the reduction in price is desirable to the merged firms because it leads them away from the distortion of double marginalization and into the full (or almost full) monopoly profits. However, when the composite goods are closer substitutes, the price reduction from the equilibrium prices of independent ownership is not necessarily desirable to the merging firms. This is because, when the composite goods are close substitutes, competition in the composite goods market drives prices under independent ownership ***below*** the prices of joint ownership. A merger that changes market structure from independent ownership to partial vertical integration reduces the prices even further, again ***below*** the prices of joint ownership. Thus, when the composite goods are close substitutes, the major effect of the merger in reducing the prices of the merging firms (because of the elimination of double marginalization) is detrimental to the merging firm, which instead would like to find a way to increase prices. It follows that when the composite goods are close enough substitutes, independent firms producing complements prefer not to merge. Thus, this argument shows that ***as the composite goods become closer substitutes, competition between them reduces prices (in comparison to joint ownership) thereby eliminating the usefulness of a vertical merger in accomplishing the same price effect.*** A similar argument shows that the remaining independent firms under partial vertical integration would like to stay independent if the composite goods are sufficiently close substitutes.[3]

It is clear by now that our analysis does not utilize the traditional setting of upstream and downstream industries, where every good of type 1 can be combined with every good of type 2. Further, the complements that comprise composite good 1 do not have to be of the same types as the complements that comprise composite good 2. Thus, our discussion does not immediately apply to issues of compatibility among components in mix and match settings.[4] Also in this paper we do not consider ***horizontal*** mergers.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 presents the equilibria in the various ownership structures. Section 4 discusses the

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[3] Our setting has firms choosing their prices simultaneously. Our general results also hold when prices are chosen sequentially, as for example when one of the firms chooses its price after the price for the complementary good has been disclosed.

[4] See, among others, Economides (1989, 1993), Economides and Salop (1992), Matutes and Regibeau (1988, 1992). Economides (1994) discusses the issue of vertical mergers and compatibility in a similar model which also allows for the hybrid combinations.

individual incentive of firms to integrate. This section contains the essential comparisons of prices and profits across ownership structures. Section 5 defines the equilibrium ownership structures in the full game. Section 6 presents concluding remarks.

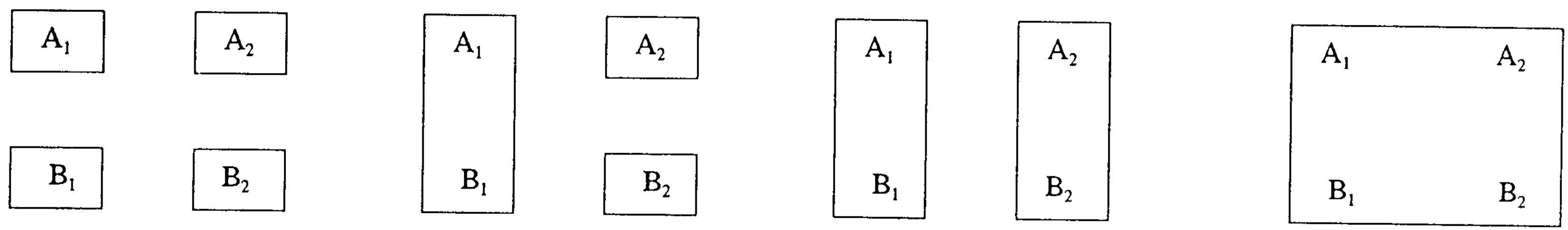
2. The Model

Let there be two types of goods, A and B, which are complementary to each other. Let there be two varieties of each type of good, A_1, A_2, B_1 and B_2. Suppose that A_i is only combinable with B_i to form composite good A_iB_i. Thus, consumers demand composite goods A_1B_1 and A_2B_2 that are assumed to be substitutes.

We consider four possible market structures. In the first market structure (***independent ownership***, "i"), there are four independent firms, each producing one of the four components. In the second market structure (***partial vertical integration***, "pvi"), components A_i and B_i are produced by the same firm (i = 1 or 2), while components A_j and B_j (j = 1 or 2; $j \neq i$) are each produced by an independent firm. In the third market structure (***parallel vertical integration***, "plvi"), components A_1 and B_1 are produced by the same firm (firm 1), and components A_2 and B_2 are produced by the same firm (firm 2). In the fourth (reference) market structure (***joint ownership***, "j"), all components are produced by the same firm. Figure shows all of these market structures. Products contained in the same box are sold by the same firm. Since we want to focus on vertical mergers, we do not consider other ownership structures, where a single firm controls one component of each composite good.

We model competition as a two-stage game. Firms choose the degree of integration in stage 1, while prices are chosen in stage 2. We seek subgame-perfect equilibria.

Ownership Structures



Independent Ownership "i"

Partial Vertical Integration "pvi"

Parallel Vertical Integration, "plvi"

Joint Ownership "j"

Figure 1

3. Equilibria in the Price Game

3.1 Independent Ownership

We first find all non-cooperative equilibria in prices for every ownership structure. In the regime of ***independent ownership*** (i), each of the four components is provided by a different firm. Firm A_i, i = 1, 2, sells component A_i at price p_i, and firm B_j, j = 1, 2, sells component B_j at price q_j. Thus, composite good A_1B_1 is sold at $p_1 + q_1$, and composite good A_2B_2 is sold at $p_2 + q_2$. Let a single consumer have a quadratic utility function in A_1B_1 and A_2B_2 which is separable in the outside good; i.e,

$$U(D_0, D_1, D_2) = D_0 + \alpha_1 D_1 + \alpha_2 D_2 - [\beta_1 D_1^2 + \beta_2 D_2^2 + 2\gamma D_1 D_2]/2. \tag{1}$$

with $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma > 0$. Maximization of utility $U(D_0, D_1, D_2)$ subject to the budget constraint $D_0 + (p_1 + q_1)D_1 + (p_2 + q_2)D_2 = I$ yields linear inverse demands

$$p_1 + q_1 = \alpha_1 - \beta_1 D_1 - \gamma D_2, \quad p_2 + q_2 = \alpha_2 - \gamma D_1 - \beta_2 D_2. \tag{2}$$

As long as $\beta_1\beta_2 \neq \gamma^2$, this system can be inverted to give the demand equations

$$D_1 = a_1 - b_1(p_1 + q_1) + c(p_2 + q_2), \quad D_2 = a_2 + c(p_1 + q_1) - b_2(p_2 + q_2). \tag{3}$$

with $a_1 = (\alpha_1\beta_2 - \alpha_2\gamma)/(\beta_1\beta_2 - \gamma^2)$, $a_2 = (\alpha_2\beta_1 - \alpha_1\gamma)/(\beta_1\beta_2 - \gamma^2)$, $b_1 = \beta_2/(\beta_1\beta_2 - \gamma^2)$, $b_2 = \beta_1/(\beta_1\beta_2 - \gamma^2)$, $c = \gamma/(\beta_1\beta_2 - \gamma^2)$. The expression $c^2/(b_1b_2) = \gamma^2/(\beta_1\beta_2)$ measures ***the degree of substitution*** between the two composite goods. It can be shown that $\gamma^2/(\beta_1\beta_2) = \epsilon_{12}\epsilon_{21}/(\epsilon_{11}\epsilon_{22})$, where ϵ_{ii} is the own elasticity of demand of composite good i and ϵ_{ij} is the cross elasticity of demand of good i with respect to changes in the price of good j.[5] Thus, the expression $\gamma^2/(\beta_1\beta_2)$ measures the relative size of cross elasticities in comparison with the own

[5] Since $\epsilon_{11} = (\partial D_1/\partial p_1)/(D_1/(p_1 + q_1))$, $\epsilon_{12} = (\partial D_1/\partial p_2)/(D_1/(p_2 + q_2))$, we have $-\epsilon_{12}/\epsilon_{11} = b_1/c$. Similarly, $-\epsilon_{21}/\epsilon_{22} = b_2/c$, so that $\epsilon_{12}\epsilon_{21}/(\epsilon_{11}\epsilon_{22}) = c^2/(b_1b_2) = \gamma^2/(\beta_1\beta_2)$.

elasticity of demand.[6] A zero value for $\gamma^2/(\beta_1\beta_2)$ means that the two composite goods are independent. Restricting b_1, b_2, and c to positive and finite values implies $\gamma^2/(\beta_1\beta_2) < 1$.[7] Thus we restrict the degree of substitution $\gamma^2/(\beta_1\beta_2)$ to lie in the range [0, 1).

At the consumer's optimal choice, the realized consumers' surplus is[8]

$$CS = U(D_0^*, D_1^*, D_2^*) = I + (\beta_1 D_1^{*2} + \beta_2 D_2^{*2} + 2\gamma D_1^* D_2^*)/2. \tag{4}$$

Under the assumption of zero costs, the profit functions are[9]

$$\begin{aligned}
\Pi_{A_1} &= p_1 D_1 = p_1[a_1 - b_1(p_1 + q_1) + c(p_2 + q_2)],\\
\Pi_{B_1} &= q_1 D_1 = q_1[a_1 - b_1(p_1 + q_1) + c(p_2 + q_2)],\\
\Pi_{A_2} &= p_2 D_2 = p_2[a_2 + c(p_1 + q_1) - b_2(p_2 + q_2)],\\
\Pi_{B_2} &= q_2 D_2 = q_2[a_2 + c(p_1 + q_1) - b_2(p_2 + q_2)].
\end{aligned}$$

The solution of first order conditions,[10] $\partial\Pi_{A_1}/\partial p_1 = \partial\Pi_{B_1}/\partial q_1 = \partial\Pi_{A_1}/\partial p_1 = \partial\Pi_{B_1}/\partial q_1 = 0$, defines the equilibrium prices and profits:[11]

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[6] Of course these relations hold in general for linear demand, and are not dependent on our focus on vertical relations.

[7] $b_1, b_2, c > 0$ requires $\beta_1\beta_2 - \gamma^2 > 0$, i.e., $\gamma^2/(\beta_1\beta_2) < 1$. A slightly stronger restriction, $b_1 > c$, $b_2 > c$, (equivalent to $\beta_1 > \gamma$, $\beta_2 > \gamma$) can be interpreted as "an increase in the price of all differentiated goods reduces the demand for each good." This restriction is commonly used in oligopoly models. To have $a_1, a_2 > 0$, we require $\alpha_1\beta_2 > \alpha_2\gamma$ and $\alpha_2\beta_1 > \alpha_1\gamma$.

[8] From the budget constraint, $D_0 = I - (p_1 + q_1)D_1 - (p_2 + q_2)D_2 = I - \alpha_1 D_1 - \alpha_2 D_2 + \beta 1 D_1^2 + \beta_2 D_2^2 + 2\gamma D_1 D_2$. Substituting in the definition of $U(D_0, D_1, D_2)$ yields $CS = U(D_0^*, D_1^*, D_2^*) = I + (\beta_1 D_1^{*2} + \beta_2 D_2^{*2} + 2\gamma D_1^* D_2^*)/2$.

[9] Clearly the same results hold for non-zero but constant marginal costs.

[10] Second order conditions also hold.

[11] In terms of the parameters of the utility function, the equilibrium prices, demand and profits are

$$p_1^i = q_1^i = (3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)/(9\beta_1\beta_2 - 4\gamma^2),$$

$$p_1^i = q_1^i = (3a_1b_2 + 2a_2c)/(9b_1b_2 - 4c^2), \quad p_2^i = q_2^i = (3a_2b_1 + 2a_1c)/(9b_1b_2 - 4c^2). \quad (4a,b)$$

$$\Pi_{A_1}^i = \Pi_{B_1}^i = b_1(3a_1b_2 + 2a_2c)^2/(9b_1b_2 - 4c^2)^2, \quad \Pi_{A_2}^i = \Pi_{B_2}^i = b_2(3a_2b_1 + 2a_1c)^2/(9b_1b_2 - 4c^2)^2. \quad (5a,b)$$

3.2 Partial Vertical Integration

In ***partial vertical integration*** (pvi), components A_1 and B_1 are sold by the same firm, while A_2 and B_2 are each provided independently. Let the price of good A_1B_1 (provided by integrated firm 1) be s_1. The prices of the two components A_2 and B_2 of the second good are p_2 and q_2 as before. The demand and profit functions are now

$$D_1 = a_1 - b_1s_1 + c(p_2 + q_2), \quad D_2 = a_2 + cs_1 - b_2(p_2 + q_2),$$
$$\Pi_1 = s_1D_1 = s_1(a_1 - b_1s_1 + c(p_2 + q_2)),$$
$$\Pi_{A_2} = p_2D_2 = p_2(a_2 + cs_1 - b_2(p_2 + q_2)),$$
$$\Pi_{B_2} = q_2D_2 = q_2(a_2 + cs_1 - b_2(p_2 + q_2)).$$

The solution of first order conditions,[12] $\partial\Pi_1/\partial s_1 = \partial\Pi_{A_2}/\partial p_2 = \partial\Pi_{B_2}/\partial q_2 = 0$, defines the equilibrium prices

$$s_1^{pvi} = (3a_1b_2 + 2a_2c)/(6b_1b_2 - 2c^2), \quad (6a)$$
$$p_2^{pvi} = q_2^{pvi} = (2a_2b_1 + a_1c)/(6b_1b_2 - 2c^2). \quad (6b)$$

$$p_2^i = q_2^i = (3\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - 2\alpha_2\gamma^2)/(9\beta_1\beta_2 - 4\gamma^2),$$
$$D_1^i = \beta_2(3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)/((\beta_1\beta_2 - \gamma^2)(9\beta_1\beta_2 - 4\gamma^2)),$$
$$D_2^i = \beta_1(3\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - 2\alpha_2\gamma^2)/((\beta_1\beta_2 - \gamma^2)(9\beta_1\beta_2 - 4\gamma^2)),$$
$$\Pi_{A_1}^i = \Pi_{B_1}^i = \beta_2(3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)^2/[(9\beta_1\beta_2 - 4\gamma^2)^2(\beta_1\beta_2 - \gamma^2)],$$
$$\Pi_{A_2}^i = \Pi_{B_2}^i = \beta_1(3\alpha_2\beta_1\beta_2 - \alpha_1\beta_1\gamma - 2\alpha_2\gamma^2)^2/[(9\beta_1\beta_2 - 4\gamma^2)^2(\beta_1\beta_2 - \gamma^2)].$$

[12] $\partial\Pi_1/\partial s_1 = a_1 - 2b_1s_1 + c(p_2 + q_2) = 0$, $\partial\Pi_{A_2}/\partial p_2 = a_2 + cs_1 - 2b_2p_2 - b_2q_2 = 0$, $\partial\Pi_{B_2}/\partial q_2 = a_2 + cs_1 - 2b_2q_2 - b_2p_2 = 0$.

Note that, if the demands for the two composite goods are equal at equal prices (i.e., if $a_1 = a_2$ and $b_1 = b_2$), the composite good of the unintegrated firms is sold at a higher price, $p_2^{pvi} + q_2^{pvi} > s_1^{pvi}$. This is because the unintegrated firms faces double marginalization.

Equilibrium profits are[13]

$$\Pi_1^{pvi} = b_1(3a_1b_2 + 2a_2c)^2/[4(3b_1b_2 - c^2)^2], \quad (7a)$$

$$\Pi_{A_2}^{pvi} = \Pi_{B_2}^{pvi} = b_2(2a_2b_1 + a_1c)^2/[4(3b_1b_2 - c^2)^2]. \quad (7b)$$

3.3 Parallel Vertical Integration

We now consider ***parallel vertical integration*** (plvi), where each substitute composite good is provided by an integrated firm. Let the price of composite good A_iB_i (sold by integrated firm i) be s_i, $i = 1, 2$. Demand and profit functions are now

$$D_1 = a_1 - b_1s_1 + cs_2, \ \ D_2 = a_2 + cs_1 - b_2s_2, \ \ \Pi_1 = s_1D_1, \ \ \Pi_2 = s_2D_2.$$

The solution of first order conditions,[14] $\partial\Pi_1/\partial s_1 = \partial\Pi_2/\partial s_2 = 0$, defines the equilibrium prices and profits as[15]

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[13] In terms of the parameters of the utility function, the equilibrium prices, demand, and profits are

$$s_1^{pvi} = (3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)/[2(3\beta_1\beta_2 - \gamma^2)],$$
$$p_2^{pvi} = q_2^{pvi} = (2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)/[2(3\beta_1\beta_2 - \gamma^2)],$$
$$D_1^{pvi} = \beta_2(3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)/(2(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2))$$
$$D_2^{pvi} = \beta_1(2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)/(2(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2))$$
$$\Pi_1^{pvi} = \beta_2(3\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - 2\alpha_1\gamma^2)^2/[4(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2)^2],$$
$$\Pi_{A_2}^{pvi} = \Pi_{B_2}^{pvi} = \beta_1(2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)^2/[4(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2)^2].$$

[14] They are $\partial\Pi_1/\partial s_1 = a_1 - 2b_1s_1 + cs_2 = 0$ (as in parallel vertical integration), and $\partial\Pi_2/\partial s_2 = a_2 + cs_1 - 2b_2s_2 = 0$.

[15] In terms of the parameters of the utility function, the equilibrium prices, demand, and profits are

$$s_1^{plvi} = (2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - \alpha_1\gamma^2)/(4\beta_1\beta_2 - \gamma^2),$$
$$s_2^{plvi} = (2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)/(4\beta_1\beta_2 - \gamma^2),$$

$$s_1^{plvi} = (2a_1b_2 + a_2c)/(4b_1b_2 - c^2), \quad s_2^{plvi} = (2a_2b_1 + a_1c)/(4b_1b_2 - c^2), \qquad (8a,b)$$

$$\Pi_1^{plvi} = b_1(2a_1b_2 + a_2c)^2/(4b_1b_2 - c^2)^2, \quad \Pi_2^{plvi} = b_2(2a_2b_1 + a_1c)^2/(4b_1b_2 - c^2)^2. \qquad (9a,b)$$

3.4 Joint Ownership

In ***joint ownership*** (j), all products are sold by the same firm. Its profit function is

$$\Pi = \Pi_1 + \Pi_2 = s_1D_1 + s_2D_2 = s_1(a_1 - b_1s_1 + cs_2) + s_2(a_2 + cs_1 - b_2s_2),$$

The solution of first order conditions, $\partial\Pi/\partial s_1 = \partial\Pi/\partial s_2 = 0$, defines the equilibrium prices

$$s_1^j = (a_1b_2 + a_2c)/[2(b_1b_2 - c^2)], \quad s_2^j = (a_2b_1 + a_1c)/[2(b_1b_2 - c^2)]. \qquad (10)$$

Equilibrium profits from A_1B_1 and A_2B_2 are

$$\Pi_1^j = [a_1(a_1b_2 + a_2c)]/[4(b_1b_2 - c^2)], \quad \Pi_2^j = [a_2(a_2b_1 + a_1c)]/[4(b_1b_2 - c^2)]. \qquad (11)$$

so that the total profits of the monopolist are[16]

$$\Pi^j = \Pi_1^j + \Pi_2^j = (a_1^2b_2 + a_2^2b_1 + 2a_2a_1c)]/[4(b_1b_2 - c^2)].$$

4. The Choice to Integrate Vertically

4.1 Mergers that Lead from Independent Ownership to Partial Vertical Integration

We are interested in the comparison of prices and profits across regimes. We are most interested in the comparisons of profits or losses from mergers that lead to vertical integration.

$$D_1^{plvi} = \beta_2(2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - \alpha_1\gamma^2)/((\beta_1\beta_2 - \gamma^2)(4\beta_1\beta_2 - \gamma^2)),$$
$$D_2^{plvi} = \beta_1(2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)/((\beta_1\beta_2 - \gamma_2)(4\beta_1\beta_2 - \gamma^2)),$$
$$\Pi_1^{plvi} = \beta_2(2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\gamma - \alpha_1\gamma^2)^2/[(\beta_1\beta_2 - \gamma^2)(4\beta_1\beta_2 - \gamma^2)^2],$$
$$\Pi_2^{plvi} = \beta_1(2\alpha_2\beta_1\beta_2 - \alpha_1\beta_2\gamma - \alpha_2\gamma^2)^2/[(\beta_1\beta_2 - \gamma^2)(4\beta_1\beta_2 - \gamma^2)^2].$$

[16] In terms of the parameters of the utility function, the equilibrium prices and profits are $s_1^j = \alpha_1/2$, $s_2^j = \alpha_2/2$, $\Pi^j = (\alpha_2^2\beta_1 + \alpha_1^2\beta_2 - 2\alpha_1\alpha_2\gamma)/[4(\beta_1\beta_2 - \gamma^2)]$.

Starting with "independent ownership," we consider a merger of firms A_1 and B_1. Such a merger will result in "partial vertical integration". We first compare prices. All prices fall as a result of the merger,[17]

$$s_1^{pvi} < p_1^i + q_1^i, \;\; p_2^{pvi} < p_2^i.$$

The reduction in price is a direct effect of the elimination of double marginalization and is true under weak restrictions on general demand functions. Keeping for the moment the prices of firms A_2 and B_2 constant, we note that $\partial\Pi_1^{pvi}/\partial s_1 = \partial\Pi_{A_1}^i/\partial p_1 + \partial\Pi_{B_1}^i/\partial p_1 = \partial\Pi_{B_1}^i/\partial p_1 < 0$, when evaluated at the equilibrium prices of independent ownership, as long as A_1 and B_1 are complements. Given the concavity of Π_1^{pvi}, the integrated firm chooses a smaller price for A_1B_1 (as a response to $p_2 + q_2$) than the sum of the prices of its components in independent ownership. Since the composite goods are strategic complements, the competitors reduce prices in response.[18] In this model, hybrid composite goods A_iB_j, $i \neq j$, do not exist, and therefore the mergers considered have only vertical effects. This is in contrast with Economides and Salop (1992) where such "vertical" mergers have both horizontal and vertical effects.

<u>Proposition 1</u>: *Starting with independent ownership, a vertical merger of firms A_1 and B_1 (leading to partial vertical integration) reduces all prices.*

We now compare the profits of the firms that merge as we move from independent ownership to partial vertical integration. The incentive for firms A_1 and B_1 to merge is measured by $\Pi_1^{pvi} - (\Pi_{A_1}^i + \Pi_{B_1}^i)$. We find that this is negative when the composite goods are

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[17] $p_1^i + q_1^i - s_1^{pvi} = 3b_1b_2(3a_1b_2 + 2a_2c)/[2(9b_1b_2 - 4c^2)(3b_1b_2 - c^2)] > 0$, $p_2^i - p_2^{pvi} = b_1c(3a_1b_2 + 2a_2c)/[2(9b_1b_2 - 4c^2)(3b_1b_2 - c^2)] > 0$.

[18] The reduction in the prices of A_2 and B_2 has an extra dampening effect on s_1.

close substitutes: for $x < 0.44$,[19] where $x = c^2/b_1b_2 = \gamma^2/\beta_1\beta_2$ measures the degree of substitution between the composite goods. Thus, when the composite goods are not close substitutes, it pays for firms A_1 and B_1 to merge (given that A_2 and B_2 are independent). The firms that did not participate in the merger are made worse off as a result of the merger, $\Pi_{A_2}^{pvi} < \Pi_{A_2}^{i}$, $\Pi_{B_2}^{pvi} < \Pi_{B_2}^{i}$.[20]

__Theorem 1__: Starting with independent ownership, firms A_1 and B_1 will find a merger with each other to be profitable if and only if the composite goods A_1B_1 and A_2B_2 are not close substitutes: for $c^2/b_1b_2 = \gamma^2/\beta_1\beta_2 < 0.44$. Firms A_2 and B_2 are made worse off as a result of the above merger.

To understand these results, we need to elaborate on the effects of the merger on prices. Figure 2 shows the equilibrium price for composite good 1 as a function of the degree of substitution $\gamma^2/\beta_1\beta_2$ for independent ownership, partial vertical integration, parallel vertical integration, and joint ownership. The merger causes the composite price to jump from the "i" line to the "pvi" line. For $\gamma = 0$, $s_1^j = s_1^{pvi} = s_1^{plvi}$. Therefore at $\gamma = 0$ (and for small γ), $p_1^i + q_1^i > s_1^j$. As γ increases, competition depresses the composite prices in independent ownership in comparison with joint ownership. Thus, for high enough γ, $p_1^i + q_1^i < s_1^j$. Thus, the reduction in the composite price resulting from the merger, although beneficial for small γ because it brings the composite price closer to the composite price of joint ownership, is

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[19] $(\Pi_{A_1}^{i} + \Pi_{B_1}^{i}) - \Pi_1^{pvi} = b_1(24b_1b_2c^2 - 8c^4 - 9b_1^2b_2^2)(3a_1b_2 + 2a_2c)^2/[4(9b_1b_2 - 4c^2)^2(3b_1b_2 - c^2)^2]$. In this expression, all terms are positive except for the term in the first set of parentheses. This term can be written as $24b_1b_2c^2 - 8c^4 - 9b_1^2b_2^2 = -(b_1b_2)^2(8x^2 - 24x + 9) = 8(b_1b_2)^2(2.56 - x)(x - 0.44)$, where $x = c^2/b_1b_2 = \gamma^2/\beta_1\beta_2$. Since $0 \leq x < 1$, it follows that $\Pi_{A1}^{i} + \Pi_{B1}^{i} < \Pi_{A1}^{pvi}$ for relatively small x, $x < 0.44$.

[20] $\Pi_{A_2}^{i} - \Pi_{A_2}^{pvi} = \{b_1b_2c(3a_1b_2 + 2a_2c)[a_2b_1(36b_1b_2 - 14c^2) + a_1c(21b_1b_2 - 8c^2)]\}/[4(9b_1b_2-4c^2)^2(3b_1b_2 - c^2)^2] > 0$, since $b_1b_2 > c^2$, and similarly for firm B_2.

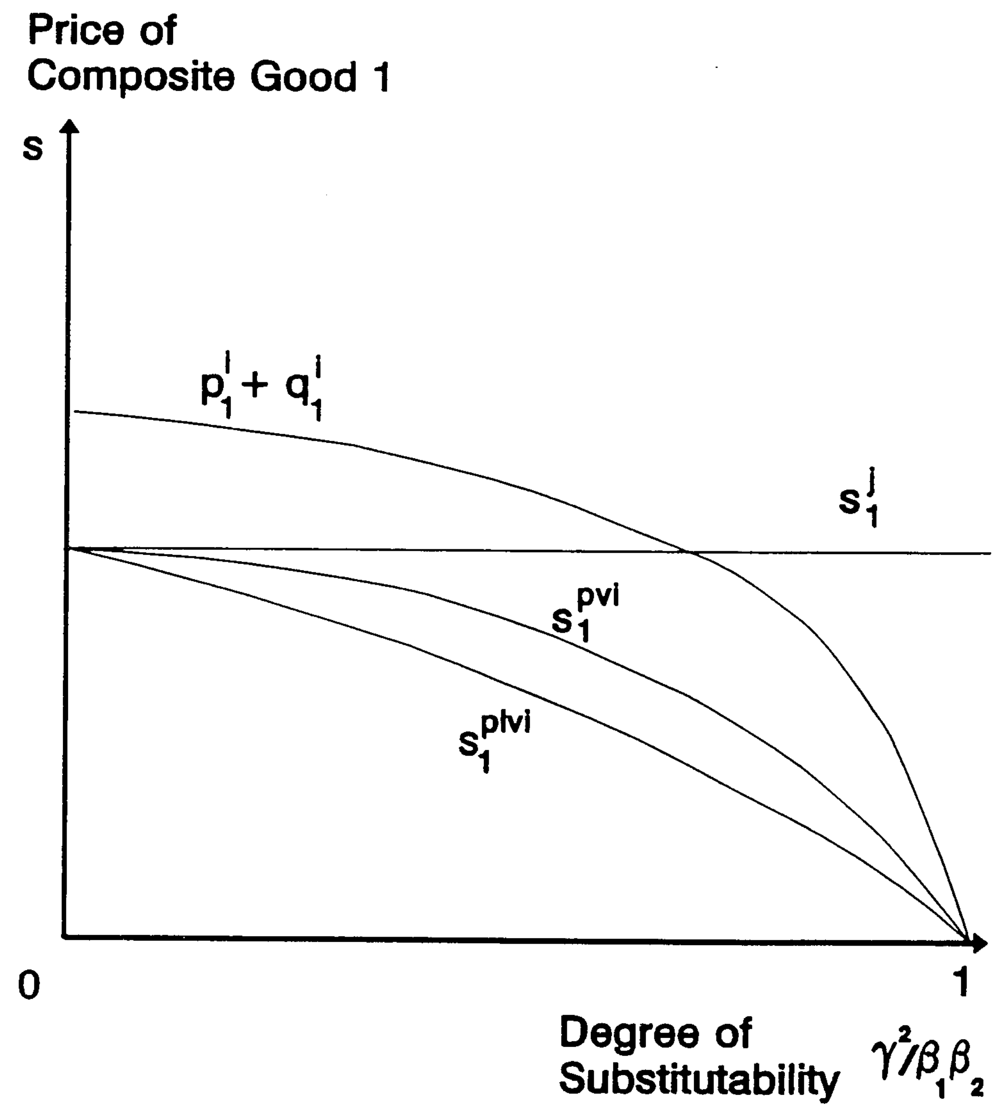

Figure 2

detrimental for large γ because the composite price in independent ownership is already close to or lower than the composite price in joint ownership. Firms A_2 and B_2 are significantly disadvantaged as a result of the merger. Not only do their prices fall, but also the quantity they produce falls as seen below.

Consumers' surplus will rise as long as both quantities increase. Because the composite goods are substitutes, there is no guarantee that a reduction in the price of both will reduce the quantities sold of both. It can be shown that the equilibrium demand changes in opposite directions as the market structure changes from independent ownership to partial vertical integration. Despite the reduction in both prices as a result of the merger, the quantity of the firms that remain independent falls, while the quantity of the merging firms increases.[21] Therefore, we cannot use a general rule for surplus comparisons, and we have to rely on specific calculations. In the case of symmetric demand, defined by $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, consumers surplus rises as we switch from independent ownership to partial vertical integration.[22] Total welfare, $TS = CS + \Sigma_i \Pi_i$, also increases for a very wide range of values of the parameters.[23]

4.2 Mergers that Lead from Partial Vertical Integration to Parallel Vertical Integration

Starting from "partial vertical integration" (i.e., integration of A_1 and B_1) consider a merger between A_2 and B_2 that leads to parallel vertical integration. All prices of composite

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[21] To see this, note that the demand differences are

$$D_1^{pvi} - D_1^i = \Phi\beta_2(3\beta_1\beta_2 - 2\gamma^2)/[2(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2)(9\beta_1\beta_2 - 4\gamma^2)],$$
$$D_2^{pvi} - D_2^i = -\Phi\beta_1\beta_2\gamma/[2(\beta_1\beta_2 - \gamma^2)(3\beta_1\beta_2 - \gamma^2)(9\beta_1\beta_2 - 4\gamma^2)],$$

where $\Phi = \beta_1(\alpha_1\beta_2 - \alpha_2\gamma) + 2(\beta_1\beta_2 - \gamma^2)$. All terms in parenthesis are positive in the relevant range because $\gamma^2/\beta_1\beta_2 < 1$ and $\alpha_1\beta_2 - \alpha_2\gamma = a_1$ is the intercept of the demand for firm 1. Therefore, $\Phi > 0$, and $D_1^{pvi} > D_1^i$, $D_2^{pvi} < D_2^i$

[22] $CS^{pvi} - CS^i = \alpha^2(1.59 - y)(1 - y)(1.23 + y)(2.86 + y)/[8(1 + y)(3 - 2y)^2(3 - y^2)^2\beta]$, where $y = \sqrt{x} = \gamma/\beta$. Therefore $CS^{pvi} > CS^i$ for all $y \in [0, 1)$.

[23] $TS^{pvi} - TS^i = 2\alpha(1 - y)(1.62 - y)(0.98 - y)(2.47 + 3.1y + y^2)/[(1 + y)(3 - 2y)^2(3 - y^2)^2\beta]$, where $y = \sqrt{x} = \gamma/\beta$. Therefore $TS^{pvi} > TS^i$ for $0 < \gamma/\beta < 0.98$ and $TS^{pvi} < TS^i$ for $0.98 < \gamma/\beta < 1$.

goods decrease as a result of the integration of firms A_2 and B_2, $s_1^{pvi} > s_1^{plvi}$, $p_2^{pvi} + q_2^{pvi} > s_2^{plvi}$.[24]

__Proposition 2__: Starting with partial vertical integration of firms A_1 and B_1, a vertical merger of firms A_2 and B_2 leading to parallel vertical integration reduces all prices.

Total profits of the firms that integrate are higher after integration when the composite goods are not close substitutes, $\Pi_2^{plvi} > \Pi_{A_2}^{pvi} + \Pi_{B_2}^{pvi}$ if and only if $x < 0.59$, where $x = c^2/b_1b_2 = \gamma^2/\beta_1\beta_2$ measures the degree of substitution between the composite goods.[25] Thus, when the composite goods are not close substitutes, it pays for firms A_2 and B_2 to merge (given that A_1 and B_1 are already merged). Profits of the firm that remains integrated fall as a result of the integration of the competitors, $\Pi_1^{plvi} < \Pi_1^{pvi}$.[26]

__Theorem 2__: Starting with partial vertical integration of firms A_1 and B_1, firms A_2 and B_2 find a merger with each other profitable if and only if the composite goods A_1B_1 and A_2B_2 are not close substitutes: for $c^2/b_1b_2 = \gamma^2/\beta_1\beta_2 < 0.59$. The firm that is already integrated is made worse off as a result of the above merger.

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[24] $s_1^{pvi} - s_1^{plvi} = b_2c(2a_2b_1 + a_1c)/[2(3b_1b_2 - c^2)(4b_1b_2 - c^2)] > 0$, $p_2^{pvi} + q_2^{pvi} - s_2^{plvi} = b_1b_2(2a_2b_1 + a_1c)/[(3b_1b_2 - c^2)(4b_1b_2 - c^2)] > 0$.

[25] $\Pi_{A_2}^{pvi} + \Pi_{B_2}^{pvi} - \Pi_2^{plvi} = b_2(4b_1b_2c^2 - 2b_1^2b_2^2 - c^4)(2a_2b_1 + a_1c)^2/[2(3b_1b_2 - c^2)^2(4b_1b_2 - c^2)^2]$. In this expression, all terms are positive except for the term in the first set of parentheses. This term can be written as $4b_1b_2c^2 - 2b_1^2b_2^2 - c^4 = -(b_1b_2)^2(x^2 - 4x + 2) = (b_1b_2)^2(3.41 - x)(x - 0.59)$, where $x = c^2/b_1b_2 = \gamma^2/\beta_1\beta_2$. Since $0 \le x < 1$, it follows that $\Pi_{A_2}^{pvi} + \Pi_{B_2}^{pvi} < \Pi_2^{plvi}$ for relatively small x, $x < 0.59$.

[26] $\Pi_1^{pvi} - \Pi_1^{plvi} = b_1b_2c(2a_2b_1 + a_1c)(24a_1b_1b_2^2 + 14a_2b_1b_2c - 7a_1b_2c^2 - 4a_2c^3)/[4(3b_1b_2 - c^2)^2(4b_1b_2 - c^2)^2] = b_1b_2c(2a_2b_1 + a_1c)[a_1b_2(24b_1b_2 - 7c^2) + 2a_2c(7b_1b_2 - 2c^2)]/[4(3b_1b_2 - c^2)^2(4b_1b_2 - c^2)^2] > 0$.

Again, to understand the incentives for the vertical merger we need to look carefully at the price relationships. Figure 3 shows the equilibrium prices for composite good 2 as functions of the degree of substitution $\gamma^2/\beta_1\beta_2$ for independent ownership, partial vertical integration, parallel vertical integration, and joint ownership. For $\gamma = 0$, s_2^j coincides with s_2^{plvi}, and $p_2^i + q_2^i$ coincides with $p_2^{pvi} + q_2^{pvi}$. Under partial vertical integration (of firms A_1 and B_1), the second composite good is sold at $p_2^{pvi} + q_2^{pvi}$, which lies always below $p_2^i + q_2^i$ and above s_2^{plvi}. For small γ, $p_2^{pvi} + q_2^{pvi}$ is above s_2^j. As γ increases, competition drives prices down under partial vertical integration, and eventually $p_2^{pvi} + q_2^{pvi}$ falls below s_2^j. Thus, the reduction in prices resulting from the merger of A_2 and B_2 to parallel vertical integration, although beneficial for small γ because it brings the composite price closer to the composite price of joint ownership, is detrimental for large γ because the composite price in partial vertical integration is already lower than in joint ownership.

Seen from a different angle, the vertical integration of A_2 and B_2 puts them on equal footing with the integrated firm 1, and as a result, the quantity of composite good 2 sold increases.[27] However, this is done at the expense of the increased competition that accompanies vertical integration. When the market environment is already competitive, for large γ, the decrease in the composite price (due to the increase in competition) is costly to the merging firms and is not sufficiently offset by the increase in its sales. Thus, a vertical merger of the second set of firms is undesirable to them when the composite goods are close substitutes. The already vertically integrated firm loses as a result of the merger, since both its price and

[27] The quantities sold of composite good 2 under partial vertical integration and under vertical integration are $d_2^{pvi} = b_2(2a_2b_1 + a_1c)/(6b_1b_2 - 2c^2)$, $d_2^{plvi} = b_2(2a_2b_1 + a_1c)/(4b_1b_2 - c^2)$. Their difference is $d_2^{plvi} - d_2^{pvi} = b_2(2a_2b_1 + a_1c)(2b_1b_2 - c^2)/[(6b_1b_2 - 2c^2)(4b_1b_2 - c^2)] > 0$.

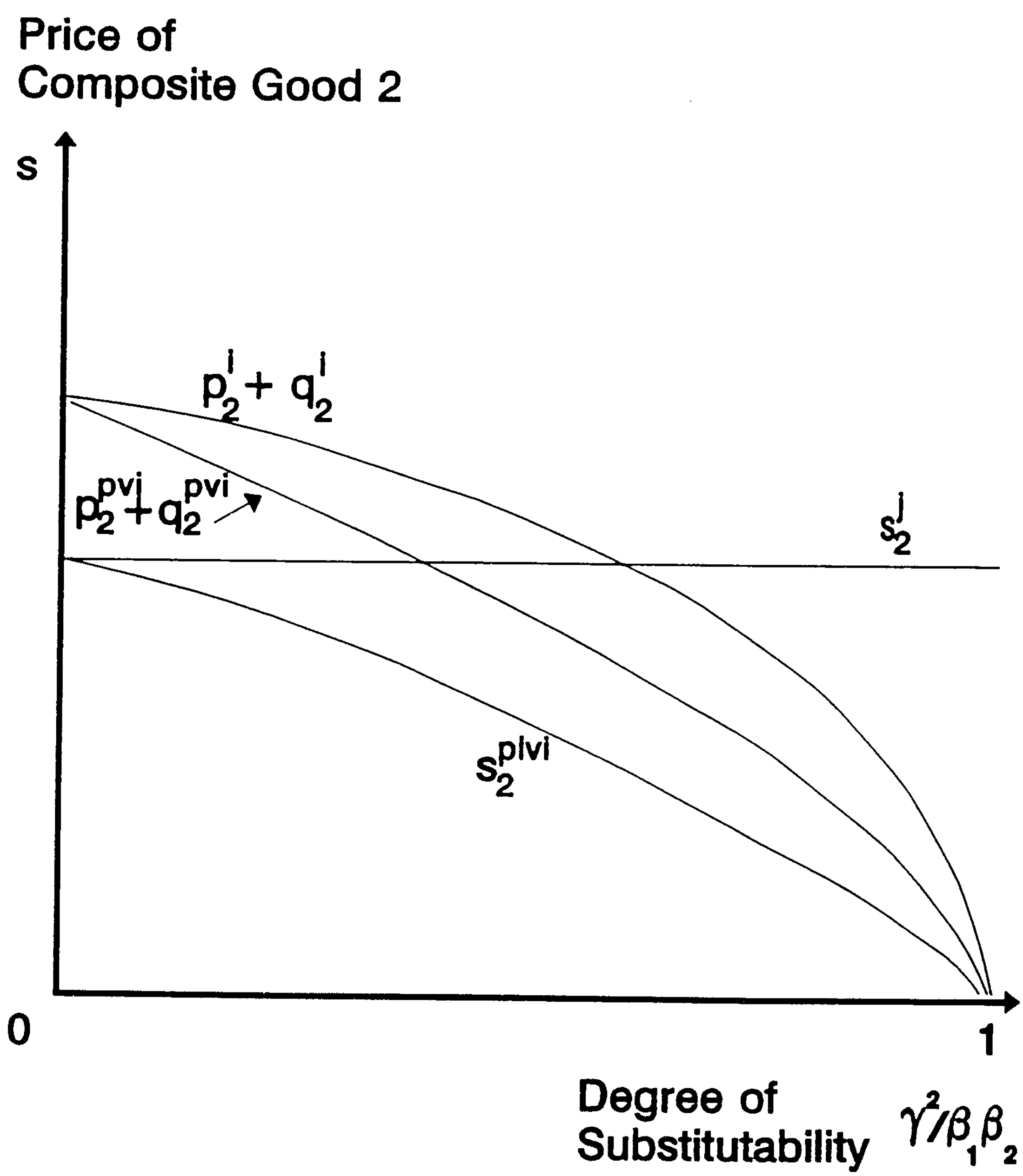
Price of
Composite Good 2
s
p2i + q2i
p2pvi + q2pvi
s2j
s2plvi
0
1
Degree of
Substitutability
γ2/β1β2

Figure 3

its quantity fall.[28]

In terms of consumers' surplus, we are again unable to make general comparisons based solely on quantities, since the quantities of the different composite goods change in opposite directions. In the case of symmetric demand, we find that consumers' surplus increases as we switch from partial vertical integration to parallel vertical integration.[29] Total welfare, TS = CS + $\Sigma_i \Pi_i$, also increases for all symmetric demands.[30]

5. Equilibrium Ownership Structures

Suppose that firms A_1, B_1, A_2, and B_2 play a merger game where two firms merge if their total post-merger profits are higher. Assume that firms move simultaneously, and each pair of merging firms assumes that the rest of the ownership structure of the rest of the industry does not change. Finally, assume that horizontal mergers are ruled out.

Clearly, different levels of substitution between the composite goods imply different equilibria. There are three relevant ranges of parameter values. First, for far substitutes, $\gamma^2/\beta_1\beta_2 < 0.44$, a pair of firms producing complementary components prefers to merge irrespective of the ownership structure. Therefore, for far substitutes, we will observe parallel vertical integration. Second, for an intermediate range of closer substitutes, $0.44 < \gamma^2/\beta_1\beta_2 < 0.59$, a pair of independent firms producing complementary components prefers to merge if the substitute composite good is produced by an integrated firm but not otherwise. For every market configuration that falls in this case, there are two equilibria: one at parallel vertical

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28 Its sales under partial vertical integration and under vertical integration are $d_1^{pvi} = b_1(3a_1b_2 + 2a_2c)/(6b_1b_2 - 2c^2)$, $d_1^{plvi} = b_1(2a_1b_2 + a_2c)/(4b_1b_2 - c^2)$. Their difference is $d_1^{pvi} - d_1^{plvi} = b_1b_2c(2a_2b_1 + a_1c)/[(4b_1b_2 - c^2)(6b_1b_2 - 2c^2)] > 0$.

29 $CS^{plvi} - CS^{pvi} = \alpha^2(1 - y)(20 + 12y - 5y^2 - 4y^3)/[8\beta(y - 2)^2(1 + y)(y^2 - 3)^2] > 0$, where $y = \sqrt{x} = \gamma/\beta$.

30 $TS^{plvi} - TS^{pvi} = \alpha^2(1 - y)(28 - 12y - 23y^2 + 4y^3 + 4y^4)/[8\beta(y - 2)^2(1 + y)(y^2 - 3)^2)] = \alpha^2(1 - y)(y - 1.87)(y - 1.03)(1.53 + y)(2.37 + y))/[2\beta(y - 2)^2(1 + y)(y^2 - 3)^2)] > 0$, where $y = \sqrt{x} = \gamma/\beta$.

integration, and one at independent ownership. Finally, for very close substitutes, $0.59 < \gamma^2/\beta_1\beta_2$, independent firms producing complementary components will not merge no matter what is the ownership structure in the rest of the industry. Thus, in this case the outcome is independent ownership. These results are summarized in the following table.

Table 1

Substitution Between the Composite Goods	Parameter Values	Equilibrium Outcome
Far	$0 \leq \gamma^2/\beta_1\beta_2 < 0.44$	**Parallel Vertical Integration**
Intermediate	$0.44 < \gamma^2/\beta_1\beta_2 < 0.59$	**Parallel Vertical Integration or Independent Ownership**
Close	$0.59 < \gamma^2/\beta_1\beta_2 < 1$	**Independent Ownership**

Theorem 3: The subgame-perfect ownership structure is parallel vertical integration when the composite goods are close substitutes; it is independent ownership when the composite goods are far substitutes; and it can be either parallel vertical integration or independent ownership when the degree of substitution is intermediate.

It is interesting to compare profits, consumers' and total surplus in the region of parameters where two equilibria (parallel vertical integration or independent ownership) exist. General comparisons across equilibria are inconclusive since they depend on the relative sizes of the demand. For symmetric demand, profits are lower under parallel vertical integration than under independent ownership for the whole ranges of intermediate and close substitution between the composite goods. Therefore in the intermediate range, where both parallel vertical integration and independent ownership can be equilibria, profits are higher under independent ownership.[31] However, for symmetric demand, the quantities of both goods increase as we switch from independent ownership to vertical integration; therefore parallel vertical integration has higher consumers' surplus.[32] Finally, for symmetric demand, total surplus is higher under parallel vertical integration.[33]

6. Concluding Remarks

Extending the results of Cournot (1838), we showed that the producers of complementary components of a composite good that does not have a close substitute have an incentive to merge. This incentive is diminished and eventually it is reversed when the composite good faces competition from a close substitute. Thus, ***vertically-related firms that face competition from a close substitute prefer to stay unintegrated.*** In general, the welfare consequences of these vertical mergers are ambiguous. For the special case of a symmetric demand system, consumers' and total surplus increase in both mergers from independent ownership to partial vertical integration and from partial vertical integration to parallel vertical integration. The results are expected to generalize to competition in the presence of more than one substitutes.

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[31] $\Pi_1^{plvi} - (\Pi_{A_1}^i + \Pi_{B_1}^i) = \beta_2(36\alpha_1^2\beta_1^4\beta_2^4 - 132\alpha_1\alpha_2\beta_1^4\beta_2^3\gamma + 49\alpha_2^2\beta_1^4\beta_2^2\gamma^2 - 84\alpha_1^2\beta_1^3\beta_2^3\gamma^2 + 226\alpha_1\alpha_2\beta_1^3\beta_2^2\gamma^3 - 56\alpha_2^2\beta_1^3\beta_2\gamma^4 + 95\alpha_1^2\beta_1^2\beta_2^2\gamma^4 - 132\alpha_1\alpha_2\beta_1^2\beta_2\gamma^5 + 14\alpha_2^2\beta_1^2\gamma^6 - 48\alpha_1^2\beta_1\beta_2\gamma^6 + 24\alpha_1\alpha_2\beta_1\gamma^7 + 8\alpha_1^2\gamma^8)/[(9\beta_1\beta_2 - 4\gamma^2)^2(\beta_1\beta_2 - \gamma^2)(4\beta_1\beta_2 - \gamma^2)^2)]$. For a symmetric demand system, $\Pi_1^{plvi} - (\Pi_{A_1}^i + \Pi_{B_1}^i) = \alpha^2(1 - y)(y - 1.70)(y - 0.29)/[2\beta(y - 2)^2(y - 1.5)^2 (1 + y)]$, where $y = \sqrt{x} = \gamma/\beta$. This is negative for $0.29 < y < 1$.

[32] In general, $D_1^{plvi} - D_1^i = \beta_2(6\alpha_1\beta_1^2\beta_2^2 - 5\alpha_2\beta_1^2\beta_2\gamma - 6\alpha_1\beta_1\beta_2\gamma^2 + 3\alpha_2\beta_1\gamma^3 + 2\alpha_1\gamma^4)/[(\beta_1\beta_2 - \gamma^2)(4\beta_1\beta_2 - \gamma^2)(9\beta_1\beta_2 - 4\gamma^2)]$. For a symmetric demand system, $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, $D_1^{vi} - D_1^i = \alpha(1 - y)/[\beta(y - 2)(1 + y)(2y - 3)] > 0$, where $y = \sqrt{x} = \gamma/\beta$.

[33] $TS^{plvi} - TS^i = \alpha^2(7 - 4y)(y - 1)^2/[\beta(y - 2)^2(1 + y)(2y - 3)^2] > 0$.

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