Two-Part Tariff Pricing in the Presence of Bypass Technologies

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TWO-PART TARIFF PRICING IN THE PRESENCE OF BYPASS TECHNOLOGIES

1. Introduction

Previous economic research on two-part tariffs has considered social welfare-maximizing tariffs for companies subject to a profit constraint (see Ng and Weisser, N/W; Faulhaber and Panzar, F/P). This work is immediately applicable to price regulation for the utility industry. In these papers, customers pay both for initial access to the system and for usage of the system's services. Oi has likened this to Disneyland pricing, as we pay an entrance fee to get into the park and also pay for the rides themselves. (Like Oi, F/P and N/W assumed that the marginal cost per unit of service is positive but the marginal cost per unit of access, i.e. customer, is zero. In a forthcoming paper, I have attempted to extend the theory of two-part tariffs to incorporate positive costs of entry.)

All of the articles referenced above assumed that customers have no choice but the utility's access/service system. That is, go to Disneyland or stay home. For some services which the utility can provide, customers indeed have alternatives. For example, in the telephone industry, long-distance callers might elect to purchase access to any long-distance carrier through the utility's local loop; alternatively, these customers can access a long-distance carrier with a private line.

how do we extend the theory of two-part tariffs to handle utility pricing when bypass systems are available?

In the papers that we have cited, some customers stay off the system because the value of the consumer surplus that they derive from usage is less than the access fee. Except for some pathological instances, this means that the smallest customers stay off. Why go to Disneyland if you intend to go on one or two rides? However, many bypass systems (e.g., private lines) best appeal to the largest customers. That is, imagine a Disneyland 2, where entrance fees are higher than at Disneyland 1 but where ride fees are lower. The largest ride-users go off to Disneyland 2.

In this paper, I shall consider appropriate social-welfare maximizing prices for utility services subject to a profit constraint. Both access and service have positive per unit marginal costs. In addition, the largest users of the service can now elect to bypass the utility altogether by accessing an alternative system. This alternative system is assumed to be a profit-maximizing enterprise.

Section 2 of this paper develops a simple model where the utility offers only one service to its customers; large users can bypass the utility to purchase the service. Section 3 allows only small customers to bypass the system. Section 4 allows both small and large customers to bypass the system while Section 5 extends Section 4 to include separate tariff structures for the small and large customers. Section 6 extends the analysis to the telephone industry, where callers may access their long-distance carrier either via the operating company's loop or through their own private lines.

2. A Simple Model

We shall now consider the simplest case of customer bypass. In the model below, the customer can choose between two systems that each provide one service. The nature and the quality of the service are the same in the two systems. One of the systems is regulated, the other not. The unregulated system is the bypass system and is a profit-maximizing enterprise. The regulated system is the utility; it is subject to a binding profit constraint. As a result, first-best marginal cost pricing is not possible for the utility's services.

Access prices per customer and usage prices per unit of service can differ among the systems. Respective access and usage costs can differ as well. We assume that the access price for the bypass system is greater than for the utility; the per unit usage cost of the bypass system is less than its utility counterpart. The bypass system will then appeal to larger users of the service.

Users vary among one another depending upon their respective amounts of service demanded. Each user h purchases his quantity q_h subject to his income constraint. If he chooses the utility system, his access price is A_1 and his per unit usage price is P_1. For the bypass system, these respective prices are A_2 and P_2. The respective costs for access and usage are Z_1 and C_1 for the utility and Z_2 and C_2 for the bypass system.

In selecting a system, each customer must compare the consumer surplus of using each system. For system i (i=1,2), the relevant consumer surplus is:

$$(2.1) \quad CS_i = \int_0^{q_i} (P(X)-P_i)dX - A_i$$

where:

$$q_i = q(P_i)$$

In comparing alternative technologies, some customers will be completely indifferent between the two. Unfortunately, without additional assumptions, there is no one unique level of marginal consumption q^* where all and only customers using q^* are indifferent. We must therefore assume that customer demand curves have the following relationship. If $q_m(P_A) > q_n(P_A)$, then $q_m(P_B) > q_n(P_B)$ for all prices P_A and P_B and all customers m and n. With this assumption, it is possible to define one unique set of points q_1^* and q_2^*; these are the respective usages of the marginal user on each system. For this marginal customer, the consumer surplus of using either technology is the same:

$$(2.2) \quad \int_0^{q_1^*} (P(X)-P_1)dX - A_1 = \int_0^{q_2^*} (P(X)-P_2)dX - A_2$$

Because $P_2 < P_1$, $q_1^* < q_2^*$.

If any customer h selects the utility service, he imposes total costs upon the utility of C_{1h}:

$$(2.3) \quad C_{1h} = Z_1 + C_1 q_h(P_1)$$

His total revenue payments are:

$$(2.4) \quad R_{1h} = A_1 + P_1 q_h(P_1)$$

For the bypass system, the respective costs and revenues are:

(2.5) $C_{2h} = Z_2 + C_2 q_h(P_2)$

(2.6) $R_{2h} = A_2 + P_2 q_h(P_2)$

For the marginal customer on the bypass system, marginal revenues and marginal costs must be equal:

(2.7) $A_2 + P_2 q_2^* = Z_2 + C_2 q_2^*$

This condition arises because the bypass system is a profit-maximizing enterprise; the analog for the utility system will not be true.

Utility profits are the difference between total revenue and total costs:

(2.8) $\Pi = (A_1 - Z_1)N_1 + (P_1 - C_1)Q_1 - K$

where:

Π = profits

N_1 = total number of utility customers

Q_1 = total quantity of service demanded

K = fixed cost (e.g., return on common capital)

In order for the utility to remain in business, Π must be non-negative. If there were no binding profit constraint, the socially optimal prices A_1 and P_1 would be, respectively, Z_1 and C_1. If the resulting profit stream is inadequate, second-best pricing rules permit distortions which maximize social welfare while simultaneously meeting the profit constraint (see Ramsey, Baumol and Bradford). If the profit constraint does hold, there is no reason to assume $A_1 = Z_1$, $P_1 = C_1$, or $A_1 + P_1 q_1^* = Z_1 + C_1 q_1^*$

The general theory behind Ramsey rules is as follows. Utility profit Π is a "good" that we must purchase at the expense of total social welfare W, which is measured as the sum of aggregate consumers' plus producers' surplus. We then attempt to maximize $W(Q_1,N_1)$ subject to the constraint that $\pi(Q_1,N_1)$ is non-negative. Therefore, our optimizing conditions are $(\partial W/\partial Q_1)/(\partial\pi/\partial Q_1) = (\partial W/\partial N_1)/(\partial\pi/\partial N_1) = \lambda$, where λ is a Lagrangean multiplier.

Marginal social welfare for service i equals marginal social benefit (where total benefit is defined as the sum of consumer surplus CS_i plus total producer revenue -- i.e, utility plus bypass system) minus marginal social cost. Marginal profit for service i equals marginal revenue to the utility minus its marginal cost. Therefore, our generalized Ramsey rule is:

(2.9) $(MSB_i-MSC_i)/(MUR_i-MUC_i) = (MSB_j-MSC_j)/(MUR_j-MUC_j)$

where:

MSB_i = marginal social benefit of an additional unit of i

MSC_i = marginal social cost of an additional unit of i

MUR_i = marginal utility revenue of an additional unit of i

MUC_i = marginal utility cost of an additional unit of i

Equation 2.9 must hold for a one service/one access system as well. Subscript i represents the marginal benefits, costs, and revenues of an additional customer on the utility system. Subscript j represents the marginal benefits, costs, and revenues of an additional unit of service.

For service, the marginal social benefit MSB_j of an additional unit is P_1 while the marginal social cost MSC_j

is C_1. Marginal utility revenue is:

(2.10) $MUR_j = dTUR/dQ_j|_{\bar{P}} + dTUR/dQ_j|_{\Delta P}$

where:

TUR = total utility revenue = $P_1Q_1 + A_1N_1$

We can reexpress equation 2.10:

(2.11) $MUR_j = P_1 + (\partial P_1/\partial Q_1)Q_1 + (\partial A_1/\partial Q_1)N_1$

$P_1 + P_1(e_{11} + e_{A1}r_{A1})$

where:

$e_{11} = (\partial P_1/\partial Q_1)(Q_1/P_1)$

$e_{A1} = (\partial A_1/\partial Q_1)(Q_1/A_1)$

$r_{A1} = A_1N_1/P_1Q_1$

For access, the marginal customer is indifferent between the utility service and the bypass service; therefore, the marginal consumer surplus of converting him to the utility system is zero. The utility gains producer revenue of $A_1 + P_1q_1^*$ while the bypass system loses $A_2 + P_2q_2^*$. The marginal cost to the utility of the new customer is $Z_1 + C_1q_1^*$ while the marginal (saved) cost to the bypass system is $Z_2 + C_2q_2^*$. Therefore, the marginal social benefit of an additional customer on the utility system is:

(2.12) $MSB_i = A_1 + P_1q_1^* - A_2 - P_2q_2^*$

The marginal social cost is then:

(2.13) $MSC_i = Z_1 + C_1q_1^* - Z_2 - C_2q_2^*$

The marginal revenue to the utility MUR_i is then:

(2.14) $MUR_i = \partial TUR/\partial N$

$= P_1q_1^* + A_1 + (\partial P_1/\partial N_1)Q_1 + (\partial A_1/\partial N_1)N_1$

$$= P_1 q_1^* + A_1 + P_1 \bar{q}_1 (e_{1A} + r_{A1} e_{AA})$$

where:

$$e_{1A} = (\partial P_1/\partial N_1)(N_1/P_1)$$

$$e_{AA} = (\partial A_1/\partial N_1)(N_1/A_1)$$

$$r_{A1} = A_1 N_1/P_1 Q_1$$

Substituting equations 2.10-2.14 into 2.9, imposing 2.7, and inverting yields:

(2.15) $$[(A_1-Z_1)+(P_1-C_1)q_1^* + P\bar{q}u]/[A_1-Z_1+(P_1-C_1)q_1^*] = (P_1-C_1+P_1 v)/(P_1-C_1)$$

where:

$$u = e_{1A} + e_{AA} r_{A1}$$

$$v = e_{11} + e_{A1} r_{A1}$$

With some simple algebra, we can simplify equation 2.15:

(2.16) $$[(A_1-Z_1)+(P_1-C_1)q_1^*]/u = (P_1-C_1)\bar{q}_1/v$$

Constraining our profits in 2.8 to be non-negative:

(2.17) $$(A_1-Z_1) \geq K/N_1-(P_1-C_1)\bar{q}_1$$

Assuming equality in 2.17 (if $\pi > 0$, we either do not have a binding profit constraint or we are not at a second best pptimum) and substituting 2.17 into 2.16:

(2.18) $$[K/N_1-(P_1-C_1)\bar{q}_1+(P_1-C_1)q_1^*]/u = (P_1-C_1)\bar{q}_1/v$$

We can then solve equation 2.18

(2.19) $$(P_1-C_1)Q_1/K = 1/(x+1-q_1^*/\bar{q}_1)$$

where:

$$x = u/v$$

Combining 2.19 and 2.17, we solve for $(A_1-Z_1)N_1/K$

(2.20) $$(A_1-Z_1)N_1/K = (x-q_1^*/\bar{q}_1)/(x+1-q_1^*/\bar{q}_1)$$

Equations 2.19 and 2.20 define the optimal shares of $(P_1-C_1)Q_1$ and $(A_1-Z_1)N_1$ in K. These respective shares can be analysed further. We note that each depends only upon x and $q_1^*/\bar{q}_1$. In addition, x and $q_1^*/\bar{q}_1$ are related to one another. In a mathematical appendix, I demonstrate the following results concerning x and $q_1^*/\bar{q}_1$.

1. When $q^* = 0$, the cross-price elasticity between access and service is zero also. Both e_{A1} and e_{1A} equal zero. As a result, $x = e_{AA}r_{A1}/e_{11} > 0$.

2. $\lim_{q_1^* \to \bar{q}_1^-} x = \infty \qquad \lim_{q_1^* \to \bar{q}_1^+} x = -\infty$
3. If $q_1^* \gtrless \bar{q}_1$, $x \lesseqgtr 0$.
4. $dx/dq_1^* > 0$. x is an increasing function of q_1^*.
5. $\lim_{q^* \to \pm\infty} x = 0$

Figure 1 shows the relationship between x and q^*.

From these five results, we can go on to prove:

6. $A > 0$.
7. If $q_1^* = \bar{q}_1$, $P_1 = C_1$ and $(A_1 - Z_1)N_1 = K$.
8. If $q_1^* = 0$, $P_1 > C_1$ and $A_1 > Z_1$.
9. If $0 < q_1^* < \bar{q}_1$, $P_1 > C_1$ and $A_1 \gtrless Z_1$

Each of these results is proved in the appendix.

For our applied concerns, the interesting case involves $q_1^* > \bar{q}_1$; consumers who consider switching to the bypass technology -- with its presumed low per unit usage charge and high access fee -- will most likely be the largest customers on the utility system. As stated in result 3 above and proved in the appendix, $x < 0$ when $q_1^* > \bar{q}_1$. Consequently, the denominator of equation 2.19 is negative; therefore, $P_1 < C_1$. The numerator of equation 2.20 is negative as well; consequently, $A_1 > Z_1$.

We have reached a useful conclusion. If regulators are worried about a bypass technology attracting the largest customers away from a utility that faces a binding profit constraint, the appropriate social welfare maximizing rules for pricing services and access are given by equations 2.19 and 2.20. Interestingly, the price of the service should be below its marginal cost of production. In this way, the utility to some extent competes with the bypass system which offers cheap service. The access fee is higher than the marginal cost of customer entry. This would seem to be particularly harsh on small users; since our model has assumed that they can't leave the system, they bear the large share that anyone with a low elasticity bears under Ramsey pricing. The remaining sections of this paper allow small customers to drop off or bypass.

3. Small Customer Bypass

Suppose now that bypass technologies could attract only the smallest customers. That is, the bypass technology offers a low access price but a high usage charge. Presuming that the bypass system is still profit-maximizing, the only difference between this case and the large customer case in Section 2 is that $q_1^* < \overline{q_1}$. As a result, x is positive; see our result 3 in Section 2. From our result 9 in Section 2, it follows that $P_1 > C_1$; the service is priced above marginal cost. Furthermore, since $A_1 \lesseqgtr Z_1$, it may be appropriate to subsidize customer access. The appendix derives the conditions needed for a subsidy.

This makes sense. Because small customers can be pulled away from the utility by the bypass system, we allow P_1 to rise above C_1; these people are less interested in cheap usage than they are in cheap access. On the other hand, we may actually subsidize access to make the utility competitive with the bypass system.

Now suppose that small customers have no bypass alternative but can drop off the system. That is, some prefer having no access and service to any system at all. In this case, $Z_2 = A_2 = C_2 = P_2 = 0$. As a result, eq. 2.7 still holds. Equations 2.12 and 2.13 simplify to:

(2.12') $MSB_i = A_1 + P_1 q_1^*$
(2.13') $MSC_i = Z_1 + C_1 q_1^*$

Then 2.15 is valid as before except now $q_1^* > \bar{q}_1$. The rest of the discussion is precisely the same as the case of small customer bypass. From the standpoint of designing suitable second-best pricing rules for access and service, small customer bypass and small customer dropoff are equivalent.

4. Simultaneous Bypass by Large and Small Customers

Section 2 developed a model which sets access and usage prices for a utility that offers only one service and which can be bypassed only by large users. Section 3 reverses this model and applies it only to small users. In each of these sections, we assumed that bypass occurred only at one end and that customers at the other end were locked in. how should we set A_1 and P_1 when bypass and/or dropoffs can occur at both ends?

Since small customer bypass has been shown to be equivalent to dropoff, we lose no generality by assuming that small customers can choose either the utility's service or nothing at all. Large customers do have a formal bypass option. They of course can go with the utility as well.

We now distinguish two kinds of customers. Given existing bypass prices A_2 and P_2, a large customer is one who would rather bypass than go without service altogether. There are N_L of these customers on the utility system and they use Q_L units of service. By contrast, a small customer is one who would rather go without any service than use the available bypass technology. There are N_S of these customers on the utility system and they use Q_S units of service. (This is easily generalized to the small bypass case as well.)

N_L, N_S, Q_S, and Q_L each depend upon P_1, A_1, P_2, and A_2. The regulators can set only the first two prices. Total utility revenue $TUR = P_1(Q_S+Q_L)+A_1(N_S+N_L)$. Total utility cost $TUC = C_1(Q_S+Q_L)+A_1(N_S+N_L)$

In order to determine the second-best optimal P_1 and A_1, we can set up our problem precisely as we did in Section 2. That is, we will compare the marginal costs, benefits, and revenues of adding one more large customer N_L and one more unit of "large usage"

Q_L. However, unlike Section 2, we now make a crucial allowance. Whenever we bring on an additional customer or unit of usage (both large), we must have changed P_1 and/or A_1. These price changes will both affect N_S and Q_S as well. We now shall include these additional effects.

Table 1 displays the marginal social benefits (MSB), marginal social costs (MSC), and marginal utility revenue (MUR) of adding an additional large customer and large unit of usage. The customer-relevant terms are subscripted with an i; the usage j. Presuming once again that bypass systems are profit-maximizing, equation 2.7 holds; the marginal revenue of adding a customer to the bypass system equals its marginal costs. That is, $Z_2+C_2q_L^* = A_2+P_2q_L^*$. We therefore can reconstruct the Ramsey rule of equation 2.9:

$$(2.9') \quad \frac{P_1\bar{q}u+(A_1-Z_1)(1+t_1)+(P_1-C_1)(q_L^*-t_1q_S^*)+(P_1-C_1)t_2}{(A_1-Z_1)(1+t_1)+(P_1-C_1)(q_L^*-t_1q_S^*)+(P_1-C_1)t_2}$$

$$= \frac{s_1(A_1-Z_1+(P_1-C_1)q_S^*)+(P_1-C_1)(1+s_2)+P_1v}{s_1(A_1-Z_1+(P_1-C_1)q_S^*)+(P_1-C_1)(1+s_2)}$$

where:

$u = (e_{AA}r_{A1}+e_{1A}w)(\bar{q}_L/\bar{q})$

$v = e_{A1}r_{A1}+e_{11}w$

$e_{AA} = (\partial A_1/\partial N_L)(N_L/A_1)$

$e_{1A} = (\partial P_1/\partial N_L)(N_L/P_1)$

$e_{A1} = (\partial A_1/\partial Q_L)(Q_L/A_1)$

$e_{11} = (\partial P_1/\partial Q_L)(Q_L/P_1)$

$r_{A1} = AN/PQ_L$

TABLE 1

Marginal Benefits, Costs, and Revenues of One Additional Customer and One Additional Unit of Usage

$$MSB_i = A_1 + P_1 q_L^{*} - A_2 - P_2 q_L^{**} + t_1(A_1 + P_1 q_S^{*}) + t_2 P_1$$

$$MSC_i = Z_1 + C_1 q_L^{*} - Z_2 - C_2 q_L^{**} + t_1(A_1 + P_1 q_S^{*}) + t_2 P_1$$

$$MUR_i = A_1 + P_1 q_L^{*} + t_1(A_1 + P_1 q_S^{*}) + t_2 P_1 + (\partial A_1/\partial N_L)N + (\partial P_1/\partial N_L)Q$$

$$MUC_i = Z_1 + C_1 q_L^{*} + t_1(Z_1 + C_1 q_S^{*}) + t_2 C_1$$

$$MSB_j = P_1 + s_1(A_1 + P_1 q_S^{*}) + s_2 P_1$$

$$MSC_j = C_1 + s_1(Z_1 + C_1 q_S^{*}) + s_2 C_1$$

$$MUR_j = P_1 + s_1(A_1 + P_1 q_S^{*}) + s_2 P_1 + (\partial A_1/\partial Q_L)N + (\partial P_1/\partial Q_L)Q$$

$$MUC_j = C_1 + s_1(Z_1 + C_1 q_S^{*}) + s_2 C_1$$

$$t_1 = (\partial N_S/\partial A_1)(\partial A_1/\partial N_L) + (\partial N_S/\partial P_1)(\partial P_1/\partial N_L)$$

$$t_2 = (\partial Q_S/\partial A_1)(\partial A_1/\partial N_L) + (\partial Q_S/\partial P_1)(\partial P_1/\partial N_L)$$

$$s_1 = (\partial N_S/\partial A_1)(\partial A_1/\partial Q_L) + (\partial N_S/\partial P_1)(\partial P_1/\partial Q_L)$$

$$s_2 = (\partial Q_S/\partial A_1)(\partial A_1/\partial Q_L) + (\partial Q_S/\partial P_1)(\partial P_1/\partial Q_L)$$

q_L^{*} = usage by marginal large customer on utility system

q_S^{*} = usage by marginal small customer on utility system

q_L^{**} = usage by marginal large customer on bypass system

q_S^{**} = usage by marginal small customer on bypass system

$w = 1 + Q_S/Q_L$

Imposing our profit constraint (2.17) and using some algebraic reasoning similar to that in Section 2 (re equations 2.15--2.19), we can obtain expressions for $(P_1-C_1)Q/K$ and $(A_1-Z_1)N/K$:

(4.1a) $(P_1-C_1)Q/K = (1 + t_1 - xs_1\bar{q})/D$

(4.1b) $(A_1-Z_1)N/K = (xs_2 + xs_1q_S^* - q_L^*/\bar{q} - t_2/\bar{q} + t_1q_S^*/\bar{q} + x)/D$

where:

$D = D_0 + t_1 - xs_1\bar{q} + xs_2 + xs_1q_S^* - t_2/\bar{q} + t_1q_S^*/\bar{q}$

$D_0 = 1 + x - q_L^*/\bar{q} < 0$

In our mathematical appendix, we prove that u is negative and v is positive; consequently, x is negative. Furthermore, our appendix proves that t_1 is non-negative and s_1 is non-positive; both t_2 and s_2 prove to be able to be positive, negative, or zero.

It is no longer the case that P_1 is less than C_1 all the time. Whether this is true depends upon the signs of both the numerator and the denominator in 4.1a. It is possible for the numerator and the denominator to have the same sign, thereby implying that P_1 is greater than C_1.

It is also no longer true that A_1 exceeds Z_1 all the time. From equation 4.1b, we see that A_1 is greater than Z_1 when both the numerator and the denominator have the same sign. This is no longer guaranteed as it was in Section 2.

It of course makes sense to see the signs become indefinite. In Section 2, we ignored the "feedback" effects upon Q_S and N_S of adding an additional customer N_L and unit of service Q_L. We found that the utility should compete most aggressively with the bypass system that threatened to siphon off its largest customers. The utility did this by dropping the price of service P_1 well below C_1. A_1 had to exceed Z_1 to meet the profit constraint. And in Sectinn 3, we found that when bypass is only possible on the small customer side, the utility should again meet the competition by lowering A and increasing P_1 above C_1. It should be no surprise that when we allow for simultaneous bypass by both small and large customers, the sign of P_1-C_1 is ambiguous; the two groups countervail one another and which one dominates is a matter of relative magnitudes.

5. Bypass with Two Sets of Tariffs

We now can consider an interesting extension of the above problem. Why doesn't the utility offer two sets of tariffs to its customers? The first set would include an access charge of A_L and a service price of P_L, the second A_S and P_S. $A_L > A_S$ and $P_L < P_S$. Any customer can select either set. The first set of tariffs would appeal more to larger users while the second would appeal to smaller. We represent the number of customers that select the (A_L, P_L) and (A_S, P_S) sets respectively as N_L and N_S: their units of service demanded are Q_L and Q_S.

Given a double tariff, the utility can use A_L and P_L to compete against large user bypass and can use A_S and P_S to compete against small user bypass/attrition. Furthermore, setting two sets of tariffs can result in greater social welfare than setting one set of tariffs; after all, the "one-set" problem is merely a constrained version of the "two-set" problem. Regulators can therefore secure a greater level of social welfare subject to the utility's necessary profit constraint if they allow a second set of tariffs.

To fix second-best prices A_L, A_S, P_L, and P_S, regulators must consider the marginal social benefits, social and utility costs, and utility revenues that would result from an additional large or small customer as well as from an additional unit of service from each of these customer classes. To add a customer (e.g. N_L) or unit of service (e.g. ω_L), we can change the relevant prices A (e.g. A_L) and P (e.g. P_L). Any time any price changes, tariff-switches can result; i.e., utility customers will change their selected set of tariffs. For the sake of simplicity, however, we shall assume that no tariff-switching is present; i.e., the distribution of per customer usage is distinctly bimodel so that a clear and wide "buffer zone" exists between the two classes of customers.

Table 2 presents the relevant marginal revenues, costs, and benefits associated with adding one more customer or one more unit of service. The subscript S refers to small customer/usage while L refers to large customer/usage. The subscript i refers to the addition of a customer while j refers to the addition of a unit of service. In this case, the relevant Ramsey conditions are:

TABLE 2

Marginal Benefits, Costs, and Revenues of One Additional Customer and One Additional Unit of Usage

$$MSB_{Ji} = A_1 + P_1 q_J^{*} - D(A_2 - P_2 q_J^{**})$$

$$MSC_{Ji} = Z_1 + C_1 q_J^{*} - D(Z_2 - C_2 q_J^{**})$$

$$MUR_{Ji} = A_1 + P_1 q_J^{*} + (\partial A_1/\partial N_J)N + (\partial P_1/\partial N_J)Q$$

$$MUC_{Ji} = Z_1 + C_1 q_J^{*}$$

$$MSB_{Jj} = P_1$$

$$MSC_{Jj} = C_1$$

$$MUR_{Jj} = P_1 + (\partial A_1/\partial Q_J)N + (\partial P_1/\partial Q_J)Q$$

$$MUC_{Jj} = C_1$$

$J = S,L$

$D = 1$ if marginal customer chooses between utility and bypass system

$D = 0$ if marginal customer chooses between utility and no service at all.

$$(5.1)\quad (MSB_{Si}-MSC_{Si})/(MUR_{Si}-MUC_{Si}) = (MSB_{Sj}-MSC_{Sj})/(MUR_{Sj}-MUC_{Sj})$$
$$= (MSB_{Lj}-MSC_{Lj})/(MUR_{Lj}-MUC_{Lj}) = (MSB_{Li}-MSC_{Li})/(MUR_{Li}-MUC_{Li})$$

Substituting from Table 2 to equation 5.1 produces three independent equations:

$$(5.2a)\quad (A_S+P_Sq_S^*-Z-Cq_S^*)/(A_S+P_Sq_S^*-Z-Cq_S^*+P\bar{q}_Su_S) = (P_S-C)/(P_S-C+P_Sv_S)$$

$$(5.2b)\quad (P_S-C)/(P_S-C+P_Sv_S) = (P_L-C)/(P_L-C+P_Lv_L)$$

$$(5.2c)\quad (P_L-C)/(P_L-C+P_Lv_L) = (A_L+P_Lq_L^*-Z-Cq_L^*)/(A_L+P_Lq_L^*-Z-Cq_L^*+P\bar{q}_Lu_L)$$

where:

$$u_J = e^J_{AA}r^J_{A1} + e^J_{1A} \qquad J = S,L$$
$$v_J = e^J_{A1}r^J_{A1} + e^J_{11}$$
$$e^J_{AA} = (\partial A_J/\partial N_J)(N_J/A_J)$$
$$e^J_{1A} = (\partial P_J/\partial N_J)(N_J/P_J)$$
$$e^J_{A1} = (\partial A_J/\partial Q_J)(Q_J/A_J)$$
$$e^J_{11} = (\partial P_J/\partial Q_J)(Q_J/P_J)$$
$$r^J_{A1} = A_JN_J/P_JQ_J$$

Using some simple algebra, we can re-express equations 5.2 as:

$$(5.3a)\quad (A_S-Z) = (P_S-C)(\bar{q}_Sx_S-q_S^*)$$
$$(5.3b)\quad (P_S-C) = (P_L-C)P_Sv_S/(P_Lv_L)$$
$$(5.3c)\quad (A_L-Z) = (P_L-C)(\bar{q}_Lx_L-q_L^*)$$

where

$$x_J = u_J/v_J \qquad J = S,L$$

Our profit constraint is now:

$$(5.4)\quad (A_S-Z)N_S+(A_L-Z)N_L+(P_S-C)Q_S+(P_L-C)Q_L = K$$

Equations 5.3a, 5.3b, 5.3c, and 5.4 provide four equations which can be solved for four unknowns: $(P_S-C)Q_S/K$, $(P_L-C)Q_L/K$, $(A_S-Z)N_S/K$, and $(A_L-Z)N_L/K$. Our mathematical appendix derives these four formulae. The results are:

(5.5a) $(P_S-C)Q_S/K = 1/D_S$

(5.5b) $(P_L-C)Q_L/K = (P_S-C)Q_S y_{LS}/K = y_{LS}/D_S = 1/D_L$

(5.5c) $(A_S-Z)N_S/K = \left[(P_S-C)Q_S/K\right](\bar{q}_S x_S - q_S^*)(N_S/Q_S) = (x_S - q_S^*/\bar{q}_S)/D_S$

(5.5d) $(A_L-Z)N_L/K = \left[(P_L-C)Q_L/K\right](q_L^* x_L - \bar{q}_L)(N_L/Q_L) = (x_L - q_L^*/\bar{q}_L)/D_L$

where:

$$D_S = 1 - q_S^*/\bar{q}_S + x_S + y_{LS}(1 - q_L^*/\bar{q}_L + x_L)$$
$$D_L = 1 - q_L^*/\bar{q}_L + x_L + y_{SL}(1 - q_S^*/\bar{q}_S + x_S)$$
$$y_{LS} = v_L P_L Q_L/(v_S P_S Q_S)$$
$$y_{SL} = v_S P_S Q_S/(v_L P_L Q_L)$$

If we make the assumption that the ranking of small customer usages does not change for all prices P_S, it must follow that the marginal small customer uses less service than the average small customer (see Section 2); $q_S^* < \bar{q}_S$. A similar assumption regarding large customers implies that $q_L^* > \bar{q}_L$. Because we have assumed that no tariff-switching is present, u_S, v_S, and u_L must be negative while v_L must be positive (see appendix). Consequently, x_S is positive and x_L is negative; y_{LS} and y_{SL} are negative.

From equations 5.5a, b, c, and d, we see that the denominator D_S must be positive while D_L must be negative. Consequently, P_S must exceed C while P_L is below C. The price of service to small customers exceeds marginal cost; the price of service to large customers is below marginal cost.

Because x_L is negative, the numerator in 5.5d is negative. Consequently, A_L-Z is positive. This means that large customers are charged an access charge above marginal cost.

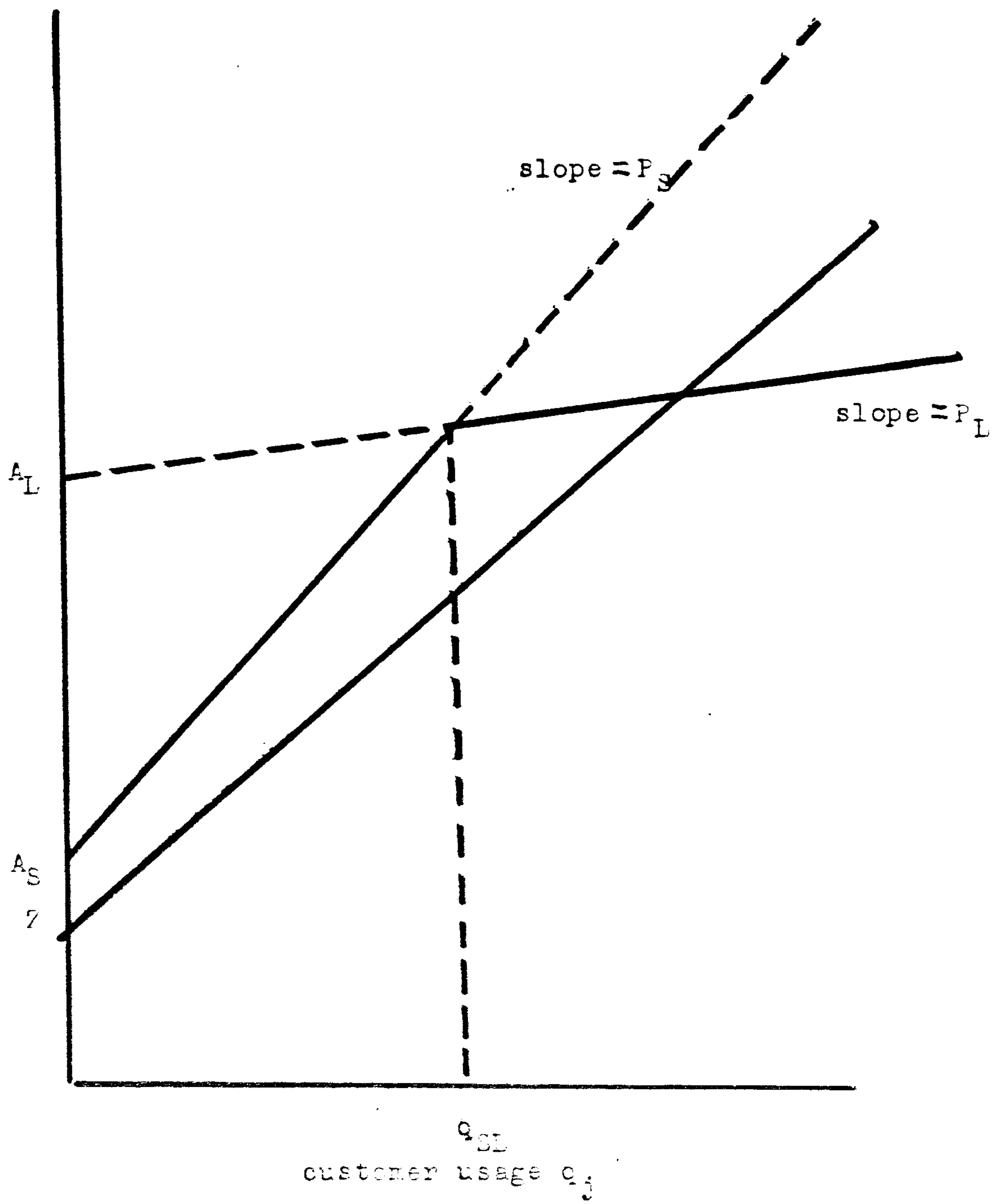
The numerator in 5.5c can have either sign. A_S-Z is positive (negative, zero) when $x > q_S^*/\bar{q}_S$ $(<, =)$. Our appendix proves that there always is a level of q_S^* where A_S-Z is positive; specifically, if q_S^* is zero, $A_S > Z$. Having A_S-Z be negative or zero is possible but not guaranteed; if it does occur, it occurs for levels of marginal usage q_S^* in a closed compact interval $[a,b]$ which is a proper subset of $[0,\bar{q}_S]$. Figure 2 illustrates. Equality is assured at the endpoints a and b while inequality is assured in the interior. Thus it is possible for $A_S \leq Z$.

The above analysis assumed that no customers switch tariffs. If tariff-switching is possible, then changes in any price can cause customer addition and attrition at "both ends" of the tariff's usage spectrum; i.e., some customers move between the bypass system and the utility while others switch tariffs but remain utility customers. As had been the case in Section 4, these two effects countervail one another and together influence whether prices are above or below marginal costs. Very little can be said in general about the final outcome of the utility's prices once we allow for tariff-switching.

Willig has pointed out that offering customers a choice of two two-part tariffs is equivalent to a two-block declining block rate structure. Figure 3 illustrates for A_S, P_S and A_L, P_L. $A_L > A_S > Z$, $P_S > C > P_L$. Furthermore, it is not necessary that we stop at two blocks; we can construct additional blocks to allow the utility more profit and each consumer more utility (see Willig for a detailed proof.) In future work, I shall consider in greater detail how declining block rates can be structured to do this. At this point, it is beyond the scope of this paper to explore in any detail the declining block topic; I shall presume for the duration of this paper that the utility implements a simple $A_S, P_S / A_L, P_L$ two-block declining block structure.

Willig also points out that any two-part tariff might seem somewhat "unfair" to small customers; this is because each must pay hookup fees above marginal cost even if its usage [illegible] is minimal. If this is a problem, regulators must set the rate for low-end customers at "fair" levels instead of optimal ones. Willig suggests using the average cost of usage that would prevail if no two-part tariff structure were adopted. Another idea would be prices sufficiently low to allow universal service. If "fair" prices are implemented at the low end, equations 5.5a--d are obviously no longer suitable. However, prices for large customers can still be optimized; we must then modify equations 2.19 and 2.20 to obtain optimal pricing rules for large customers. In this modification, K must be replaced by K', where K' equals K minus all profits obtained from all usage which is antecedent to the final block.

Figure 3: Declining Block Rates



q_{SL} $(A_L - A_S)/(P_S - P_L)$

6. Extension to the Telephone Industry

To extend our models to the telephone industry, we need to acknowledge that local telephone companies provide two major services. First, they currently have a legal monopoly to provide local and intraLATA toll message service; customers are free to go without this service but they cannot bypass it. Second, any utility customer may use the utility's loop and central office to access any available long-distance carrier. however, any of these customers may instead access a long-distance carrier directly by private line, consequently bypassing the local phone company altogether. With these remarks in mind, we must develop a model with two different kinds of service.

In the models which follow, a customer can use utility facilities for local/intraLATA toll service only or for both local/intraLATA service and access to the long-distance carriers. however, no customer can use utility facilities for long-distance access unless it is a local customer as well.

As in Section 2, we shall assume that the size ranking of each customer's usage is the same regardless of usage price; this is true for either local/intraLATA or interLATA usage. But there is no necessary correspondence between a user's local/intraLATA usage and his interLATA usage. however, we shall further assume that any customer that has both local/intraLATA and utility-provided interLATA access service would continue its local/intraLATA service alone if the latter were discontinued. That is, the availability of utility-provided long-distance access or service does not influence anyone's decision to have local/intraLATA service. We assume that all inter-service cross price elasticities = 0

In our model below, the utility then offers two services. Any customer, large or small, is free to bypass/drop off using the utility loop to access the long-distance carriers. Furthermore, any customer may drop off the utility system altogether, thereby foregoing both services. Customers that drop off the system altogether undoubtedly will have been small users of the local service. We shall allow two sets of tariffs for long-distance users and a third set for local users.

In the model below, the access charge for local/intraLATA users is A_R while the marginal cost of access is Z_R. The price of a local call is P_R while the marginal cost of a local call is C_R. There are two sets of tariffs for long-distance access; we assume that the customer onee again has a choice between either. Tariffs in the first set are A_S, P_S and tariffs in the second set are A_L, P_L. As in Section 5, we assume that $A_L > A_S$ and $P_L < P_S$. The respective marginal costs per long distance customer and call are Z_E and C_E. We assume that these marginal costs are the same regardless of which tariff the customer chooses.

All local prices are billed directly to the customer as is the long-distance access charge. The price per call for long-distance access is passed on to the long-distance carrier, which passes it on to the ultimate caller. We may note here that I have not explicitly incorporated any non-traffic sensitive access charge for long-distance carriers that hook into the loop. If implemented, these carrier access charges would be passed along to callers just like usage charges, i.e., on a per call basis. We might then want to think of P_S and P_L containing a component which represents a share of the non-traffic sensitive carrier access charge. Assuming both P_S and P_L constant simplifies the exposition considerably.

There are N_R local/intraLATA customers who place Q_R calls. On the long-distance side, there are N_S customers with tariffs A_S, P_S; they place Q_S calls. There are N_L customers with tariffs A_L, P_L; they place Q_L calls.

We then must set six prices: A_R, A_S, A_L, P_R, P_S, and P_L. To set second-best optimal prices, we must use the following Ramsey rule:

$$(6.1)\qquad \begin{aligned} (MSB_{Si}-MSC_{Si})/(MUR_{Si}-MUC_{Si}) &= (MSB_{Sj}-MSC_{Sj})/(MUR_{Sj}-MUC_{Sj}) \\ (MSB_{Li}-MSC_{Li})/(MUR_{Li}-MUC_{Li}) &= (MSB_{Lj}-MSC_{Lj})/(MUR_{Lj}-MUC_{Lj}) \\ (MSB_{Ri}-MSC_{Ri})/(MUR_{Ri}-MUC_{Ri}) &= (MSB_{Rj}-MSC_{Rj})/(MUR_{Rj}-MUC_{Rj}) \end{aligned}$$

Subscripts R, S, and L of course represent local, A_S/P_S long-distance, and A_L/P_L long-distance services. The subscript i represents an additional customer while the subscript j represents an additional phone call.

The first four terms of equation 6.1 have turned up in Section 5; they represent precisely the same concepts now as then. We have added two additional terms for local/intraLATA usage/customers; these are analogous to the first four terms. Each customer term J (J = R,S,L) can therefore be represented:

$$(6.2) \qquad (A_J+P_J q_J^{*}-Z_J-C_J q_J^{*})/(A_J+P_J-Z_J-C_J q_J^{*}+P_J \bar{q}_J u_J)$$

where:

$\bar{q}_J$ = average number of calls per customer, class J

q_J^{*} = number of calls per marginal customer, class J

u_J is the same as was defined in equation 5.2.

Each "phone call" term can be represented:

$$(6.3) \qquad (P_J - C_J)/(P_J - C_J + P_J v_J)$$

v_J is the same as was defined in equation 5.2

Using reasoning similar to that in Section 5, we can reexpress 6.1 with five independent equations:

$$(6.4a) \qquad (A_R - Z_R) = (P_R - C_R)(\bar{q}_R x_R - q_R^{*})$$

$$(6.4b) \qquad (A_S - Z_E) = (P_S - C_E)(\bar{q}_S x_S - q_S^{*})$$

$$(6.4c) \qquad (A_L - Z_E) = (P_L - C_E)(\bar{q}_L x_L - q_L^{*})$$

$$(6.4d) \qquad (P_S - C_E) = (P_R - C_R)P_S C_S/(P_R C_R)$$

$$(6.4e) \qquad (P_L - C_E) = (P_R - C_R)P_L C_L/(P_R C_R)$$

where:

$x_J = u_J/v_J \qquad\qquad J = R,S,L$

Our profit constraint is now:

$$(6.5) \qquad (A_R - Z_R)N_R + (A_S - Z_E)N_S + (A_L - Z_E)N_L + (P_R - C_R)Q_R + (P_S - C_E)Q_S + (P_L - C_E)Q_L = K$$

Equations 6.4(a)-(e) and 6.5 can be solved for six dependent variables in precisely the same manner as we solved for four dependent variables in Section 5; Section 5 of the appendix illustrates:

(6.6a) $(P_J - C_J)Q_J/K = 1/D_J$

(6.6b) $(A_J - Z_J)N_J/K = (x_J - q_J^*/\bar{q}_J)/D_J$

where:

$$D_J = 1 - q_J^*/\bar{q}_J + x_J + \sum_{K \neq J} y_{KJ}(1 - q_K^*/\bar{q}_K + x_K)$$

$$y_{KJ} = v_K P_K Q_K / v_J P_J Q_J$$

Because we have assumed that the size rankings of both local/intraLATA and interLATA per customer usage do not change with price per phone call, $\bar{q}_R > q_R^*$, $\bar{q}_S > q_S^*$, and $\bar{q}_L < q_L^*$. As a result, u_J, v_R, and v_S are negative; v_L is positive (see appendix). Therefore, x_R, x_S, y_{RS}, and y_{SR} are all positive; x_L, y_{RL}, y_{LR}, y_{SL}, and y_{LS} are all negative. From equations 6.6, we can see that D_R and D_S are positive and D_L is negative. Therefore, $P_R > C_R$, $P_S > C_S$, and $P_L < C_L$; this is immediately obvious from 6.6a. In addition, $A_L > Z_L$; this results because the numerator in 6.6b is negative when $J = L$. The sign of $A_R - Z_R$ (and $A_S - Z_S$) depends on the relationship between x_R and $q_R^*/\bar{q}_R$ (and x_S and $q_S^*/\bar{q}_S$). We have seen in Section 5 that these signs can be positive or negative.

Therefore, usage by large interLATA customers should be priced below marginal cost while usage by small interLATA customers and local/intraLATA customers should be priced above marginal cost. Access for large interLATA customers should be priced above marginal cost. Access for small interLATA customers and local/intraLATA customers can be priced either above or below marginal cost.

7. Conclusion

One of the most important issues currently facing the telephone industry involves the design of spitable prices to charge a customer for use of the local loop to access his or her long distance carrier. A grave fear of local telephone company personnel is that high access charges will lead large customers to connect directly with the long-distance carrier via a private line. Someone must pay for the common costs of the utility's loop and central office equipment; what large customers do not pay, small customers must. This presents a greater possibility of small customer attrition and places univeral telephone service that much farther away.

This paper provides the framework for an economic solution for the above problem. The utility should offer (at least) two sets of tariffs for usage of the loop for long-distance access. In this way, the utility can more readily compete against bypass systems which appeal to larger users and also can more readily safeguard against small user attrition. Furthermore, not only will the utility's competitive edge be better secured, consumers will be better off than they would be with only one set of long-distance tariffs (unless, of course, the two sets of optimal tariffs happen to coincide.)

In general, we cannot say anything about whether access or usage will be priced above or below marginal costs without making some restrictive assumptions. however, assuming that the cross-price elasticities between local/intraLATA prices and interLATA access/usage (and vice versa) and that there is little or no tariff switching among long-distance users, we have seen that very definite recommendations are indeed possible. In specific, long-distance phone calls made by large long-distance users which are routed through the loop should be priced below marginal cost. However, the access charge for these users should be priced above marginal cost. In addition, all local/intraLATA phone calls and long-distance calls made by small long-distance users should be priced above marginal cost. Access for local/intraLATA customers and for small long-distance customers may be priced above or below marginal cost.

APPENDIX

This appendix proves several propositions claimed in the text. It is divided into sections which correspond to sections in the text. however, the material buiids sequentially from the begininning of the appendix; material in Section 5 uses propositions proved in Section 2.

2. A Simple Model

Much of the material in this section was originally proved in Einhorn. Parts of the mathematical appendix of that paper are reprinted here verbatim.

For the sake of simplicity, we shall refer to the number of customers as N and the number of units of usage as Q. (The text referred to them as N_1 and Q_1 respectively.) The prices of access and service are A and P respectively.

The number of customers N that have access to service depends upon the price of access A and the price of service P. The demand for service Q depends upon both A and P as well. Therefore:

(A.1) $Q = Q(P, A)$

(A.2) $N = N(P, A)$

The total differential of A.1 and A.2 is

(A.3) $$\begin{bmatrix} dQ \\ dN \end{bmatrix} = \begin{bmatrix} Q_P & Q_A \\ N_P & N_A \end{bmatrix} \begin{bmatrix} dP \\ dA \end{bmatrix}$$

Let us call the 2 x 2 matrix on the right hand side M.

The M matrix normally combines both substitution and income effects; we shall assume that the income effects are negligible. As a result, $Q_P < 0$, $N_A < 0$, and det $M > 0$ must be true. We can solve A.3 for (dP dA)':

$$\text{(A.4)} \quad \begin{bmatrix} dP \\ dA \end{bmatrix} = \begin{bmatrix} Q_p & Q_A \\ N_p & N_A \end{bmatrix}^{-1} \begin{bmatrix} dQ \\ dN \end{bmatrix}$$

N_A is the derivative of the demand for access N with respect to its own price A. This can be written:

$$\text{(A.5)} \quad N_A = (\partial N/\partial CS^*)(\partial CS^*/\partial A)$$

where $CS^* = CS^*(A,P)$ represents the per customer consumer surplus of the marginal customer. Clearly, $\partial CS^*/\partial A = -1$. Furthermore, we write $\partial N/\partial CS^* \equiv N^*(A,P)$ where N^* is the number of marginal customers when the respective prices for access and service are A and P. Therefore, $N_A = -N^*$.

N_p is the derivative of N with respect to the price of service P. This can be written:

$$\text{(A.6)} \quad N_p = (\partial N/\partial CS^*)(\partial CS^*/\partial P)$$

If P rises by one unit, CS^* declines by q^* units: $\partial CS^*/\partial P = -q^*$.

Therefore,

$$\text{(A.7)} \quad N_p = -N^* q^*$$

Q_A can be written:

$$\text{(A.8)} \quad Q_A = (\partial Q/\partial N)(\partial N/\partial A)$$

$\partial Q/\partial N$ is simply the demand for Q by the marginal customers: $\partial Q/\partial N = q^*$. Therefore,

$$\text{(A.9)} \quad Q_A = -q^* N^*$$

Note that $Q_A = N_p$, a symmetry which makes sense and which should hold.

Q_p is the derivative of Q with respect to its own price P. Since $Q \equiv \bar{q}N$ ($\bar{q}$ = demand for Q by the average customer),

$$\text{(A.10)} \quad \partial Q/\partial P = (d\bar{q}/dP)N + (\partial N/\partial P)\bar{q}$$
$$= (d\bar{q}/dP)N - N^* q^* \bar{q}$$

Substituting A.5, A.7, A.9, and A.10 into A.4 and using Cramer's Rule, we obtain:

$$(A.11) \quad \partial P/\partial Q = -N^*/D < 0$$

$$\partial P/\partial N = N^* q^*/D > 0$$

$$\partial A/\partial Q = N^* q^*/D > 0$$

$$\partial A/\partial N = \left[(d\bar{q}/dP)N - q^* N^* \bar{q}\right]/D < 0$$

where:

$$D = \det M = \left[(d\bar{q}/dP)N - N^* q^* \bar{q}\right](-N^*) - (N^* q^*)^2 > 0$$

In the text, we derive equations 2.19 and 2.20 which allocate to service and access their respective shares of costs K:

$$(2.19) \quad (P - C)Q/K = 1/(x + 1 - q^*/\bar{q}) \qquad (A.12)$$

$$(2.20) \quad (A - Z)N/K = (x - q^*/\bar{q})/(x + 1 - q^*/\bar{q}) \qquad (A.13)$$

where

$$x = (e_{1A} + e_{AA} r_{A1})/(e_{11} + e_{A1} r_{A1})$$

$$e_{1A} = (\partial P/\partial N)(N/P) = N^* q^* N/(PD)$$

$$e_{AA} = (\partial A/\partial N)(N/A) = \left[(d\bar{q}/dP)N - q^* N^* \bar{q}\right]N/(AD)$$

$$e_{11} = (\partial P/\partial Q)(Q/P) = -N^* Q/(PD)$$

$$e_{A1} = (\partial A/\partial Q)(Q/A) = N^* q^* Q/(AD)$$

$$r_{A1} = AN/PQ$$

We can then substitute A.11 into A.12 and A.13 to derive the respective shares of service and access, using x:

$$(A.14) \quad x = (d\bar{q}/dP)(P/\bar{q})/\left[(N^*/N)P(q^* - \bar{q})\right]$$

In the text, we expressed x as the ratio of u to v; x u/v. $u = e_{1A} + e_{AA} r_{A1}$ and $v = e_{11} + e_{A1} r_{A1}$.

Therefore, $u = (d\bar{q}/dP)N^2/(\bar{q}PD) < 0$; $v = N^* Q(q^*/\bar{q} - 1)/(PD)$.

The sign of v depends on whether $q^* \gtreqless \bar{q}$. The same is true for the sign of x.

From here, we can proceed to prove the nine propositions claimed in the text after equation 2.20.

1. When $q^* = 0$, then the cross-price elasticities are zero. (see A.11). Consequently, $x = e_{AA} r_{A1} / e_{11} > 0$. From A.12 and A.13, we can see that $P > C$ and $A > Z$ when $x > 0$ and $q^* = 0$.

2. In equation A.14, the denominator is negative if $q^* < \bar{q}$, zero if $q^* = \bar{q}$, and positive if $q^* > \bar{q}$. The numerator is always negative. Therefore, $\lim_{q^* \to \bar{q}^-} x = +\infty$ and $\lim_{q^* \to \bar{q}^+} x = -\infty$.

3. From the preceding proof, it is obvious that if $q^* < \bar{q}$, $x > 0$. If $q^* > \bar{q}$, $x < 0$.

4. $dx/dq^* = -\text{numerator}/\text{denominator}^2$. See eq. A.14. The numerator is negative; dx/dq^* is positive for all values of q^*

5. As q^* approaches positive or negative infinity, the denominator in A.14 approaches positive or negative infinity as well. Consequently, x approaches zero.

Having proved these five statements, we can then draw the relation between x and q^* as shown in Figure 1.

6. If A were negative or zero, the marginal consumer's usage would be $q^* = 0$, since anybody who can derive any benefit from consuming any amount will come on the system. Substituting into A.13 (2.20), $-ZN/K = x/(x+1)$. Since $x > 0$ and $-ZN/K \leq 0$, this equality is not possible. Therefore, the access price A is positive.

7. In Proposition 2, if $q^* = \bar{q}$, $x = \pm\infty$. Therefore, we see from A.12 and A.13 that $P = C$ and $(A - Z)N = \pm\infty$.

8. If $q^* = 0$, we know from Proposition 1 that x is positive. From A.12 and A.13, we can immediately see that $P > C$ and $A > Z$.

9. If $0 < q^* < \bar{q}$, x is positive; see Proposition 3. $P > C$ and $A \gtreqless Z$.

3. Small Customer Bypass

Under what conditions will access price A be less than the cost of access Z?

We proved in Section 2 of this appendix that if $q^* < \bar{q}$, x is positive. Then it is immediately obvious from equation A.13 that $A \gtreqless Z$ iff $x \gtreqless q^*/\bar{q}$. Given result 4 and our Figure 1, we can see that the subsidy range is between a and b in Figure 2.

The subsidy range is a compact subset of $[0,\bar{q}]$. Equality of A and Z occurs at the endpoints a and b; a positive subsidy occurs in the interior. Although this range can occur, it need not occur; this depends on the relationship between the $q^*/\bar{q}$ line and $x(q^*)$. It cannot include either 0 or $\bar{q}$.

4. Simultaneous Bypass by Large and Small Customers

This section will prove that the u and v defined after equation 2.9' in Section 4 are respectively negative and positive. We also prove that $t_1 \geq 0$, $s_1 \leq 0$, $s_2 \gtreqless 0$, and $t_2 \gtreqless 0$.

From our text, $u = (e_{AA}r_{A1} + e_{1A}w)(\bar{q}_L/\bar{q})$ where $r_{A1} = AN/PQ_L$ and $w = 1 + Q_S/Q_L$. The sign of u is obviously the same as the sign of $e_{AA}r_{A1} + e_{1A}w$. From the definitions of e_{AA} and e_{1A} which follow equations A.12 and A.13 in Section 2:

$$\text{(B.1a)} \quad e_{1A} = N_L^* q_L^* N_L / (PD)$$

$$\text{(B.1b)} \quad e_{AA} = (d\bar{q}_L/dP - N_L^* q_L^* \bar{q}_L)/(AD)$$

Substituting our definitions of e_{1A}, e_{AA}, r_{A1}, and w into the formula for u:

$$\text{(B.2)} \quad u = \left[(d\bar{q}_L/dP)N_L N/\bar{q}_L - N_L^* q_L^* N + N_L^* q_L^* N_L + N_L^* q_L^* N_L Q_S/Q_L\right]/(PD)$$

Since $N_S + N_L = N$, $N_L^* q_L^* N_L - N_L^* q_L^* N = -N_L^* q_L^* N_S < 0$. Furthermore, since $\bar{q}_S < \bar{q}_L$ (or equivalently, $Q_S/N_S < Q_L/N_L$), $N_S > Q_S N_L/Q_L$. Consequently, we can reexpress B.2:

$$\text{(B.3)} \quad u = \left[(d\bar{q}_L/dP)(N_L N)/\bar{q}_L + N_L^* q_L^* (N_L Q_S/Q_L - N_S)\right]/(PD)$$

Both terms in the numerator of B.3 are negative. In Section 2, we prove that D is positive. Consequently, u is negative.

Our text defined $v = e_{A1}r_{A1} + e_{11}w$. r_{A1} and w are the same as were defined above. e_{A1} and e_{11} were defined in Section 2 of this appendix (see the discussion following equations A.12 and A.13):

$$\text{(B.4a)} \quad e_{11} = -N_L^* Q_L/(PD)$$

$$\text{(B.4b)} \quad e_{A1} = N_L^* q_L^* Q_L/(AD)$$

Substituting our definitions of e_{11}, e_{A1}, r_{A1}, and w into the definition of v:

(B.5) $v = N_L^* q_L^* N/(PD) - N_L^* Q_L (1 + Q_S/Q_L)/(PD)$

$= N_L^* q_L^* N/(PD) - N_L^* Q/(PD)$

where:

$Q = Q_S + Q_L$

Equation B.5 can be reexpressed:

(B.6) $v = (q_L^* - \bar{q}) N N_L^*/(PD)$

Since $q_L^* > \bar{q}$, v is positive.

The text defined t_1, t_2, s_1, and s_2 (see Table 1):

(B.7a) $t_1 = (\partial N_S/\partial A)(\partial A/\partial N_L) + (\partial N_S/\partial P)(\partial P/\partial N_L)$

(B.7b) $t_2 = (\partial Q_S/\partial A)(\partial A/\partial N_L) + (\partial Q_S/\partial P)(\partial P/\partial N_L)$

(B.7c) $s_1 = (\partial N_S/\partial A)(\partial A/\partial Q_L) + (\partial N_S/\partial P)(\partial P/\partial Q_L)$

(B.7d) $s_2 = (\partial Q_S/\partial A)(\partial A/\partial Q_L) + (\partial Q_S/\partial P)(\partial P/\partial Q_L)$

From Section 2 of our appendix, we know that

(B.8a) $\partial N_S/\partial A = -N_S^*$

(B.8b) $\partial N_S/\partial P = -N_S^* q_S^*$

(B.8c) $\partial Q_S/\partial A = -N_S^* q_S^*$

(B.8d) $\partial Q_S/\partial P = (d\bar{q}_S/dP) N_S - N_S^* q_S^* \bar{q}_S$

We also know from equations A.11 that

(B.9a) $\partial P/\partial Q_L = -N_L^*/D$

(B.9b) $\partial P/\partial N_L = N_L^* q_L^*/D$

(B.9c) $\partial A/\partial Q_L = N_L^* q_L^*/D$

(B.9d) $\partial A/\partial N_L = [(d\bar{q}_L/dP) N_L - N_L^* q_L^* \bar{q}_L]/D$

Substituting B.8a--d and B.9a--d into B.7a--d:

(B.10a) $t_1 = -N_S^*[(d\bar{q}_L/dP)N_L - N_L^*q_L^*\bar{q}_L]/D - N_S^*N_L^*q_S^*q_L^*/D$

(B.10b) $t_2 = -N_S^*q_S^*[(d\bar{q}_L/dP)N_L - N_L^*q_L^*\bar{q}_L]/D$
$+ N_L^*q_L^*[(d\bar{q}_S/dP)N_S - N_S^*q_S^*\bar{q}_S]/D$

(B.10c) $s_1 = [-N_L^*N_S^*q_L^* + N_L^*N_S^*q_S^*]/D$

(B.10d) $s_2 = -N_L^*N_S^*q_L^*q_S^*/D - N_L^*[(d\bar{q}_S/dP)N_S - N_S^*q_S^*\bar{q}_S]/D$

Equation B.10a can be expressed:

(B.11) $t_1 = -(d\bar{q}_L/dP)N_LN_S^*/D + N_S^*N_L^*q_L^*(\bar{q}_L - q_S^*)/D$

Since $d\bar{q}_L/dP$ is negative and $\bar{q}_L > q_S^*$, t_1 is positive.

Equation B.10b can be expressed:

(B.12) $t_2 = -N_S^*N_Lq_S^*(d\bar{q}_L/dP)D + N_L^*N_S^*q_L^*(d\bar{q}_S/dP)/D$
$+ N_S^*N_L^*q_S^*q_L^*(\bar{q}_L - \bar{q}_S)/D$

Since $\bar{q}_L > \bar{q}_S$, the third term in B.12 is positive; since $d\bar{q}_L/dP$ is negative, the first term is positive. However, $d\bar{q}_S/dP$ is negative; the second term is negative. As a result, t_2 could be either positive or negative; it seems that positive would be more likely.

Equation B.9c can be expressed:

(B.13) $s_1 = N_L^*N_S^*(q_S^* - q_L^*)/D$

Since $q_S^* < q_L^*$, s_1 is negative.

Equation B.10d can be expressed:

(B.14) $s_2 = -(d\bar{q}_S/dP)N_SN_L^*/D + N_L^*N_S^*q_S^*(\bar{q}_S - q_L^*)/D$

Since $\bar{q}_S < q_L^*$, the second term is negative. Since $d\bar{q}_S/dP$ is negative, the first term is positive. Therefore, s_2 can be positive, negative, or zero.

5. Bypass with Two Sets of Tariffs

We shall derive equations 5.5a--d from 5.3a--c and 5.4. We first substitute 5.3a and 5.3c into 5.4:

(C.1) $(P_S-C)(\bar{q}_S x_S - q_S^*)N_S + (P_L-C)(\bar{q}_L x_L - q_L^*)N_L$

$+ (P_S-C)Q_S + (P_L-C)Q_L = K$

Solving 5.3b for (P_L-C) and substituting into C.1:

(C.2) $(P_S-C)(\bar{q}_L x_L - q_L^*)N_L P_L v_L/(P_S v_S) + (P_S-C)(\bar{q}_S x_S - q_S^*)N_S$

$+ (P_S-C)Q_L P_L v_L/(P_S v_S) + (P_S-C)Q_S = K$

Factoring out $(P_S-C)Q_S$ and rearranging terms:

(C.3) $(P_S-C)Q_S[1+x_S-q_S^*/\bar{q}_S+y_{LS}(1+x_L-q_L/\bar{q}_L)] = (P_S-C)Q_S D_S = K$

where:

$y_{LS} = v_L P_L Q_L/(v_S P_S Q_S)$

$D_S = 1+x_S-q_S^*/\bar{q}_S+y_{LS}(1+x_L-q_L^*/\bar{q}_L)$

Multiplying both sides of C.3 by $1/(D_S K)$ gives:

(C.4) $(P_S-C)Q_S/K = 1/D_S$

This is identical to equation 5.5a.

We know from equation 5.3b that:

(C.5) $(P_S-C)Q_S = (P_L-C)Q_L v_S P_S Q_S/(v_L P_L Q_L)$

Substituting C.5 into C.3,

(C.6) $(P_L-C)Q_L = y_{LS}/D_S$

We can reexpress y_{LS}/D_S:

(C.7) $y_{LS}/D_S = 1/[1+x_L-q_L^*/\bar{q}_L+y_{SL}(1+x_S-q_S^*/\bar{q}_S)] = 1/D_L$

We term the denominator in C.7 D_L. Equations C.6 and C.7 are identical to equation 5.5b.

From 5.3a and 5.3c:

(C.8a) $(A_S-Z)N_S = (P_S-C)Q_S(x_S-q_S^*/\bar{q}_S)$

(C.8b) $(A_L-Z)N_L = (P_L-C)Q_L(x_L-q_L^*/\bar{q}_L)$

We can combine C.8a with C.4 and C.8b with C.6 or C.7 to obtain:

(C.9a) $(A_S-Z)N_S/K = (x_S-q_S^*/\bar{q}_S)/D_S$

(C.9b) $(A_L-Z)N_L/K = (x_L-q_L^*/\bar{q}_L)/D_L$

Equations C.9a and C.9b are respectively 5.5c and 5.5d.

6. Extension to the Telephone Industry

We define u_J and v_J as after equation 5.2:

(D.1a) $u_J = e^J_{AA} r_{A1} + e^J_{1A}$

(D.1b) $v_J = e^J_{A1} r^J_{A1} + e^J_{11}$

where:

$$e^J_{AA} = (\partial A_J/\partial N_J)(N_J/A_J)$$

$$e^J_{1A} = (\partial P_J/\partial N_J)(N_J/P_J)$$

$$e^J_{A1} = (\partial A_J/\partial Q_J)(Q_J/A_J)$$

$$e^J_{11} = (\partial P_J/\partial Q_J)(Q_J/P_J)$$

$$r^J_{A1} = A_J N_J/(P_J Q_J)$$

In these formulations of u and v, we have implicitly assumed that any price change in one of the three sets of tariffs J (local/intraLATA, small interLATA, and large interLATA) does not affect either usage Q_K or access N_K for $K \neq J$. Each N_J, Q_J, P_J, and A_J is then a self-contained system.

We know from Section 2 of this appendix:

(D.2a) $\partial N_J/\partial A_J = -N^*_J$

(D.2b) $\partial N_J/\partial P_J = q^*_J N^*_J$

(D.2c) $\partial Q_J/\partial A_J = q^*_J N^*_J$

(D.2d) $\partial Q_J/\partial P_J = \left[(d\bar{q}_J/dP_J)N_J - q^*_J N^*_J \bar{q}_J\right]$

We then showed in A.11:

(D.3a) $\partial P_J/\partial Q_J = -N^*_J/D_J < 0$

(D.3b) $\partial P_J/\partial N_J = q^*_J N^*_J/D_J > 0$

(D.3c) $\partial A_J/\partial Q_J = q^*_J N^*_J/D_J > 0$

(D.3d) $\partial A_J/\partial N_J = \left[(d\bar{q}_J/dP_J)N_J - q^*_J N_J \bar{q}_J\right]/D_J < 0$

where D_J is positive.

Substituting D.3(a)--(d) into D.1(a)--(b)

(D.4a) $u_J = (d\bar{q}_J/dP_J)N_J^2/(P_J D_J \bar{q}_J)$

(D.4b) $v_J = N_J^* Q_J(-1+q_J^*/\bar{q}_J)/(P_J D_J)$

u_J is negative as long as $d\bar{q}_J/dP_J$ is negative. v_J is positive (negative, zero) if q_J^* is less than (exceeds, equals) $\bar{q}_J$.

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