Variable Compatibility without Network Externalities

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by

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Abstract

I analyze a model where systems are composed of two components. Hybrid systems, composed of components produced by different firms, require an adapter or interface to function. Through design manipulations, component-producing firms control the price of the adapter which is produced by a competitive sector. I show that, for symmetric demand, when firms choose non-cooperatively design specifications and prices, they produce fully compatible components, both when the choices are simultaneous and when they are taken in sequence, with the specification choice preceding the price choice. However, if the demand for hybrid systems is very small, at equilibrium firms choose to maximize the degree of incompatibility of their components. If the demand for one single-producer system is very large, then only the small-demand firm wants compatibility, and a regime of limited incompatibility results.

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Many complex products are composed of complementary components. For example, a personal computer is composed of a central processing unit and a monitor. The good "use of a spreadsheet" requires three components, a spreadsheet program such as Lotus 123, a computer operating system, such as PC-DOS and a personal computer, such as the IBM-AT. Some components are immediately and freely combinable to produce a system in which they function together. For example, IBM-AT's central processing unit can connect to a NEC monitor, and a NEC computer can connect to an IBM monitor. However, some manufacturers make it difficult for their components to connect with components made by other firms. For example, the original Apple Macintosh was built as a single unit containing the computer and the monitor. Attachment of a different monitor was very difficult and costly.

In general, I assume that the combination of two components produced by different manufacturers requires an investment in an interface, adapter or translator to create a functioning hybrid system.[1] In most cases adapters are produced by a competitive sector. However, the component-producing firms can determine the cost of an adapter through their choice of the design of the components. The cost of the adapter defines the degree of compatibility of components produced by competing firms. Full compatibility means that no adapter (or an adapter of zero cost) is required for operation of a hybrid system.

I explicitly assume away any positive consumption externalities, commonly called network externalities. In the presence of a network externality, the value of the n_ith unit sold by firm i is increasing with the total number of units sold in the "network" of compatible products, $\sum_i n_i$. This creates a natural tendency for firms to gravitate towards full compatibility with the products of their competitors.[2] I show that, because of the complementarity between components, <u>there is a natural tendency towards full compatibility, even in the absence of network externalities</u>.

At first glance, it seems that firms will try to choose component specifications that would make it difficult and expensive to combine them with components made by the opponent firms. It seems that in this way they would ensure that more of their own production is sold. However, this intuition is incorrect. We establish that firms acting non-cooperatively will try to make their components directly and freely combinable with components made by an opponent. Because components are complementary with the adapter, the elasticity of demand for components that a firm faces is increasing in the price of the adapter. When the price of the adapter is zero, a firm faces the most inelastic demand and realizes the highest price and profits. Thus, each firm decides to design its components so the price of the adapter is minimized. As a result, full compatibility arises in this model as a <u>non-cooperative</u> equilibrium and not as a coordinated decision of cooperating firms adhering to an agreement. Firms have the possibility to establish two different standards but they decide non-cooperatively to adhere to the same standard.

In the context of locationally differentiated components, Carmen Matutes and Pierre Regibeau (1988) and Nicholas Economides (1989) have shown that, faced with the choice between a regime of full compatibility and a regime of full incompatibility, firms prefer full compatibility. The present paper improves on these results by allowing firms to continuously vary the degree of compatibility of their components with those of the competitor. Therefore, the choice is not between two extreme regimes, but from a continuum of possible regimes. In this paper the choice of regime is explicit and non-cooperative. Thus, full compatibility and standardization arise as a non-cooperative equilibrium of the game. The model of this paper is specified for a general demand function. Thus, I avoid the restrictions on demand and computational limitations imposed by a locational structure.

Two exceptions to the full compatibility results are analyzed, both arising from demand asymmetries. In the first case, I assume that the demand for hybrid systems is smaller than the demand for simple-producer systems. When the demand for hybrids is low enough, it is not a high enough reward to balance the cost of the extra competition that compatibility implies. Then, each firm chooses a high cost of the adapter and there are two different "standards" in the industry. The second exception arises when only the demand for one single-producer system is high, and the demand for the second firm's single-producer system as well as the demand for hybrids are low. In this setup, the firm with the low demand wants compatibility to get access to the markets for hybrids. However, the high-demand firm will prefer compatibility only when the markets for the other three systems are large enough. When the demands of the other three systems are low enough, compatibility does not reward enough the high-demand firm for the added competition. It will then opt for a high adapter cost, and this will result in an adapter with an intermediate cost, since the small-demand firm always opts for compatibility. These exceptions point to the significance of the relative scale of the demand in the non-cooperative creation of standards.

The rest of the paper is organized as follows. Section I sets-up the model. Section II analyzes the game of simultaneous choice of adapter's cost and prices under symmetrical demand conditions. Section III analyzes the game of adapter's cost in the first stage and price cost in the second stage. Section IV analyzes the first asymmetric case when hybrids' demand is small relative to the demand for single-producer systems. Section V analyzes the second asymmetric case when only the demand for one single-producer system is large. Section VI contains extensions and concluding remarks.

I. Model Set-up

In the basic model of this paper there are two firms, each producing a component of type 1 and a component of type 2. Components of different types can be combined to produce systems demanded by consumers. Components produced by the same firm are readily combinable. Components produced by different firms require an adapter to allow them to function together as a working system.

Let p_1 and p_2 be the increments of the prices above their constant marginal costs of components of type 1 and type 2 produced by firm 1. Similarly, let q_1 and q_2 be the increments of prices above their constant marginal costs of the components of type 1 and 2 produced by firm 2.[3] Let ij denote the identity of a system composed of component 1 produced by firm i and of component 2 produced by firm j. System 11 is available to the consumers at price $p_1 + p_2$, and system 22 is available to the consumers at price $q_1 + q_2$. Hybrid systems 12 and 21 require an adapter.

Interfaces are produced by a competitive sector and sold at marginal cost. However, the duopolists have the ability to determine how costly the adapter will be through decisions concerning the design of the components they produce. Let $x \in [0, \bar{x}]$ be the cost of the adapter attributed to the design choice of firm 1. Similarly, let $y \in [0, \bar{y}]$ be the cost of the adapter that is the result of the design choice of firm 2. Thus, system 12 is available to the consumer at price $p_1 + q_2 + x + y$. Similarly, system 21 is available at price $q_1 + p_2 + x + y$. The cost of the adapter, $x + y$, defines the degree of compatibility of components made by opponent firms. When $x + y = 0$, there is full compatibility. The higher the cost of the adapter, the less compatible the components of the hybrid systems.

The assumption that the influences of each firm on the cost of the adapter are additive plays no important role in the derivation of the basic results. A more general cost function for the adapter can be used, $\varphi(x,y)$, where x and y are the decision variables of firms 1 and 2 respectively. The requirements on such a function would be that it is symmetric, $\varphi(x, y) = \varphi(y, x)$, increasing, $\partial\varphi/\partial x > 0$, $\partial\varphi/\partial y > 0$, and passes through the origin, $\varphi(0, 0) = 0$.[4]

Two game structures are analyzed. In the first game structure, the decisions of the degree of compatibility are taken simultaneously with the pricing decisions. The second game structure has two stages. Design decisions (choices of x and y) are taken at the first stage. Pricing decisions are taken in the second stage. I seek subgame-perfect equilibria. The two-stage game models many situations where it is more difficult to vary the design rather than the price in the short run.

In both game structures, I show that, for a symmetric demand system, firms decide non-cooperatively to be in an environment of full compatibility by choosing their products' designs so that the price of an adapter is zero. Thus, an adapter will not be needed at equilibrium.

II. The Game of Simultaneous Choice

I first analyze the simultaneous' choices model. Firm 1 chooses x, p_1 and p_2, while firm 2 chooses y, q_1 and q_2. There are four products, product 11 available at price $p_1 + p_2$, product 22 available at price $q_1 + q_2$, product 12 available at price $p_1 + q_2 + x + y$, and product 21 available at price $q_1 + p_2 + x + y$. I use superscripts to denote the demand and profit functions.

The profits of firm 1 come from sales of component 1, $D^{11} + D^{12}$, and from sales of component 2, $D^{11} + D^{21}$. The profit function of firm 1 is,

$$\Pi^1(p_1, p_2, q_1, q_2, x, y) = p_1(D^{11} + D^{12}) + p_2(D^{11} + D^{21}). \tag{1a}$$

Similarly, the profit function of firm 2 is,

$$\Pi^2(p_1, p_2, q_1, q_2, x, y) = q_1(D^{21} + D^{22}) + q_2(D^{12} + D^{22}) \tag{1b}$$

where D^{ij} is the demand for product ij. The demand for product 11 can be written as a function of four variables; first, of its own price; second, of the price of the product that differs from 11 in the second component, i.e. of product 12; third, as a function of the price of the product that differs from 11 in the first component, i.e. of product 21; and fourth, as a function of the price of the product that differs from 11 in both components, i.e. of product 22. Thus D^{11} is written as

$$D^{11}(p_1 + p_2, \; p_1 + q_2 + x + y, \; q_1 + p_2 + x + y, \; q_1 + q_2).$$

We can write the demand function for any other product in a similar manner, as a function of its own price, the price of the product that differs from it in the second component, the price of the product that differs from it in the first component and the price of the system that differs from it in both components. For example, the demand for product 12 can be written as

$$D^{12}(p_1 + q_2 + x + y, \; p_1 + p_2, \; q_1 + q_2, \; q_1 + p_2 + x + y).$$

I assume that the demand system is symmetric, so that, when written in the above manner, one function represents the demand for any product. Thus, I assume,

<u>A1</u>: <u>The demand system is symmetric so that</u>

$$D^{11}(a, b, c, d) \equiv D^{12}(b, a, d, c) \equiv D^{21}(c, d, a, b) \equiv D^{22}(d, c, b, a). \quad (2)$$

<u>where</u> a, b, c, d <u>are the prices of systems 11, 12, 21 and 22 respectively</u>.

Note, however, that no restrictions have been placed on the substitutability between systems. Thus, system 11 is not necessarily equally substitutable with systems 12, 21 and 22. Function $D^{11}(a, b, c, d)$ is not assumed to be symmetric with respect to b, c, and d.

By assumption <u>A1</u>, the substitutability between products 11 and 12 is the same in the demands of products 11 and 12. Further, the price of product 21 has the same effect on the demand for product 11 as the price of product 22 has on the demand of product 12, since in both cases the comparable products differ in the first component. Finally, the price of product 22 is seen in the demand for product 11 as the price of product 21 is seen in the demand for product 12, since in both cases the comparable products differ in both components.

The demand for any product is decreasing in its own price and increasing in the price of any of its three substitutes. As a regularity condition, I assume that the own price effects on demand outweigh the effects of the prices of substitutes.

A2: An equal increase in the price of all four goods decreases the demand of any particular good, that is, for all i and j,

$$\sum_{k=1}^{4} D_k^{ij}(a,\ b,\ c,\ d) < 0,$$

where subscripts denote partial derivatives.

I also make a technical weak concavity assumption on the demand.

A3: $D_{11}^{11} \leq D_{14}^{11} \leq D_{13}^{11} \leq D_{12}^{11}$ and $D_{41}^{11} \leq D_{43}^{11} \leq D_{44}^{11} \leq D_{42}^{11}$.

Consider the effect on the profits of firm 1 of an increase in the price of the adapter, caused by a change in the design specification of firm 1. It is

$$\partial\Pi^1/\partial x = p_1\partial(D^{11} + D^{12})/\partial x + p_2\partial(D^{11} + D^{21})/\partial x. \tag{3}$$

Consider first the effect on the sales of component 1 produced by firm 1. Sales of firm 1 are composed of D^{11} and D^{12}. Remember the definition of $D^{11} = D^{11}(a,\ b,\ c,\ d)$ and $D^{12} = D^{12}(b,\ a,\ d,\ c)$, where $a = p_1 + p_2$, $b = q_1 + p_2 + x + y$, $c = p_1 + q_2 + x + y$, $d = q_1 + q_2$ are the prices of the four available systems. The effect of an increase in the price of the adapter on sales of firm 1's component 1 is

$$\begin{aligned}\partial(D^{11} + D^{12})/\partial x &= D_2^{11}(a,\ b,\ c\ ,d) + D_3^{11}(a,\ b,\ c\ ,d)\\ &\quad + D_1^{12}(b,\ a,\ d,\ c) + D_4^{12}(b,\ a,\ d,\ c).\end{aligned} \tag{4}$$

By the symmetry of the demand system assumed in A1,

$$D_1^{12}(b, a, d, c) = D_1^{11}(b, a, d, c), \quad D_4^{12}(b, a, d, c) = D_4^{11}(b, a, d, c). \tag{5}$$

Using the weak concavity assumption A3, I show in the Appendix that

$$D_1^{11}(b, a, d, c) \leq D_1^{11}(a, b, c, d), \tag{6}$$

and

$$D_4^{11}(b, a, d, c) \leq D_4^{11}(a, b, c, d). \tag{7}$$

Substituting (5), (6) and (7) in (4) it follows that

$$\partial(D^{11} + D^{12})/\partial x = D_2^{11}(a, b, c, d) + D_3^{11}(a, b, c, d) + D_1^{12}(b, a, d, c) + D_4^{12}(b, a, d, c) \leq \sum_{k=1}^{4} D_k^{11}(a, b, c, d) < 0. \tag{8}$$

Thus, the effect of an increase in the price of the adapter on the sales of component 1 produced by firm 1 is smaller than or equal to the effect of an equal increase in all four prices. By assumption A2, the demand falls when the prices of all four goods rise equally. It follows that increasing the price of the adapter decreases the demand for component 1 of firm 1. Similarly, it is shown in the Appendix that an increase in the adapter's price decreases the demand for component 2 of firm 1, i.e.,

$$\partial(D^{11} + D^{21})/\partial x \leq \sum_{k=1}^{4} D_k^{11}(a, b, c, d) < 0. \tag{9}$$

By substitution of (8) and (9) in (3) it follows that profits for firm 1 fall in the price of the adapter,

$$\partial \Pi^1/\partial x \leq (p_1 + p_2) \cdot \sum_{k=1}^{4} D_k^{11} < 0. \tag{10}$$

Lemma 1: An increase in the price of the adapter results in a decrease in profits.[5]

Proof: See the Appendix.

It follows that firm 1 will choose $x^* = 0$. Similarly, firm 2 will choose $y^* = 0$. Therefore, the price of an adapter or interface is zero, $x^* + y^* = 0$. In other words, no adapter or interface is needed at equilibrium, and the products are fully compatible.

Proposition 1: *At the non-cooperative equilibrium of the game of simultaneous choice of prices and design specifications, firms choose to produce fully compatible products*.

Note that full compatibility was established as a non-cooperative equilibrium of independently acting firms, and no cooperation, agreements, or enforcement of thereof was required. A single "standard" evolves from non-cooperative behavior.

III. The Two-stage Game

In the two-stage game structure, firms choose the degree of compatibility of their components in the first stage, while they set prices in the second stage. This game structure describes many situations where prices are more flexible in the short run than are design specifications. I seek a subgame-perfect equilibrium.

I first analyze the equilibrium of the last stage. Each firm chooses prices to maximize its profits so that,

$$\partial \Pi^1/\partial p_1 = \partial \Pi^1/\partial p_2 = \partial \Pi^2/\partial q_1 = \partial \Pi^2/\partial q_2 = 0. \tag{11}$$

Equilibrium prices that solve (11) can be written parametrically as $p_1^*(x, y)$, $p_2^*(x, y)$, $q_1^*(x, y)$, $q_2^*(x, y)$. By equilibrium perfection, firms predict correctly in the first stage the equilibrium prices of the last stage. Thus, the objective function of firm 1 in the first stage is,

$$\Pi^{1d}(x, y) \equiv \Pi^1(p_1^*(x, y), p_2^*(x, y), q_1^*(x, y), q_2^*(x, y), x, y),$$

where the superscript "d" is used for the stage of design specifications. Similarly, the objective function of firm 2 in the first stage is

$$\Pi^{2d}(x, y) \equiv \Pi^2(p_1^*(x, y), p_2^*(x, y), q_1^*(x, y), q_2^*(x, y), x, y).$$

In the stage of design specifications, firms choose how compatible their components are going to be with the ones of the opponent. This means that they decide on the price (cost) of the required adapter or adapter, $x+y$, where x is a choice of firm 1, and y is a choice of firm 2.

In the previous section I have established that, ceteris paribus, profits fall in the cost of the adapter, $\partial \Pi^1/dx < 0$. I will now show that the incentive to reduce the cost of the adapter is higher in the two-stage game than in the game of simultaneous choice, i.e. that $d\Pi^{1d}/dx < \partial \Pi^1/dx$. This will imply that firms will choose zero as the adapter's price at the perfect equilibrium of the two-stage game. Thus, at the perfect equilibrium there will be again full compatibility.

Consider the variation of profits with the price of the adapter in the first stage. It is

$$\begin{aligned} d\Pi^{1d}/dx &= \partial\Pi^1/\partial x + \partial\Pi^1/\partial p_1 \cdot dp_1^*/dx + \partial\Pi^1/\partial p_2 \cdot dp_2^*/dx + \partial\Pi^1/\partial q_1 \cdot dq_1^*/dx + \partial\Pi^1/\partial q_2 \cdot dq_2^*/dx \\ &= \partial\Pi^1/\partial x + \partial\Pi^1/\partial q_1 \cdot dq_1^*/dx + \partial\Pi^1/\partial q_2 \cdot dq_2^*/dx, \qquad (12) \end{aligned}$$

since $\partial\Pi^1/\partial p_1 = \partial\Pi^1/\partial p_2 = 0$ at the subgame equilibrium $(p_1^*(x, y), p_2^*(x, y), q_1^*(x, y), q_2^*(x, y))$. The contribution of firm 1 to the cost of the adapter, x, influences profits directly, and indirectly through the prices of the components produced by the opponent. By symmetry,

$$q_1^*(x, y) = q_2^*(x, y),$$

and (12) simplifies to

$$d\Pi^{1d}/dx = \partial\Pi^1/\partial x + 2\cdot\partial\Pi^1/\partial q_1 \cdot dq_1^*/dx. \qquad (13)$$

Thus, the difference between the one-stage and two-stage games in the individual firm's incentive to reduce the price of the adapter is proportional to the cross price effect on profits, $\partial\Pi^1/\partial q_1$, and on the influence of the price of the adapter on the opponent's price, dq_1^*/dx. Since components made by different manufacturers are substitutes, the cross price effect on profits is positive, so that an increase in the price of a component produced by firm 2 increases the profits of firm 1,

$$\partial \Pi^1/\partial q_1 > 0. \tag{14}$$

In the appendix I show that an increase in the price of the adapter implies a downward shift in the demands for components and this implies more elastic demands and lower prices for components, and, in particular,

$$dq_1^*/dx < 0. \tag{15}$$

The crucial assumption to guarantee that a firm faced with reduced demand for its components responds by cutting their prices is that the adapter and the component are strategic substitutes. Formally we assume

A4: An adapter and a component are strategic substitutes so that $\partial^2\Pi^1/\partial p_i\partial x = \partial^2\Pi^1/\partial p_i\partial y < 0$, $\partial^2\Pi^2/\partial q_i\partial x = \partial^2\Pi^2/\partial q_i\partial y < 0$, $i = 1, 2$.

Substituting in (14) and (15) in (13) it follows that

$$d\Pi^{1d}/dx < \partial\Pi^1/\partial x < 0.$$

Using the same method, I can show that

$$d\Pi^{2d}/dy < \partial\Pi^2/\partial y < 0.$$

Below I provide an intuitive illustration of how profits decrease in response to increases in the price of the adapter. Figure 1 was constructed under the assumption that $p_1 = p_2 = p$ and $q_1 = q_2 = q$. Referring to Figure 1, I compare profits at the original equilibrium at A and at the new equilibrium at B after x has increased. From A3 we know that an increase of x shifts the best reply function of firm 1 to the left to R1' and the best reply function of firm 2 to the right to R2'. Let Π_A denote firm 1's iso-profit curve through A and let $\Pi(A)$ be level of profits at A. Define point C on the new best reply curve of firm 1, R1', at firm 2's price level of the original equilibrium, q_A. Before the increase of x, profits are lower at C than at A because of the natural order of the iso-profit curves. Thus $\Pi(A) > \Pi(C)$, where $\Pi(C)$ denotes the level of profits at C before x increased. Let $\Pi'(C)$ denote the level of profits at C after x increased. From equation (10) and the results of the simultaneous game

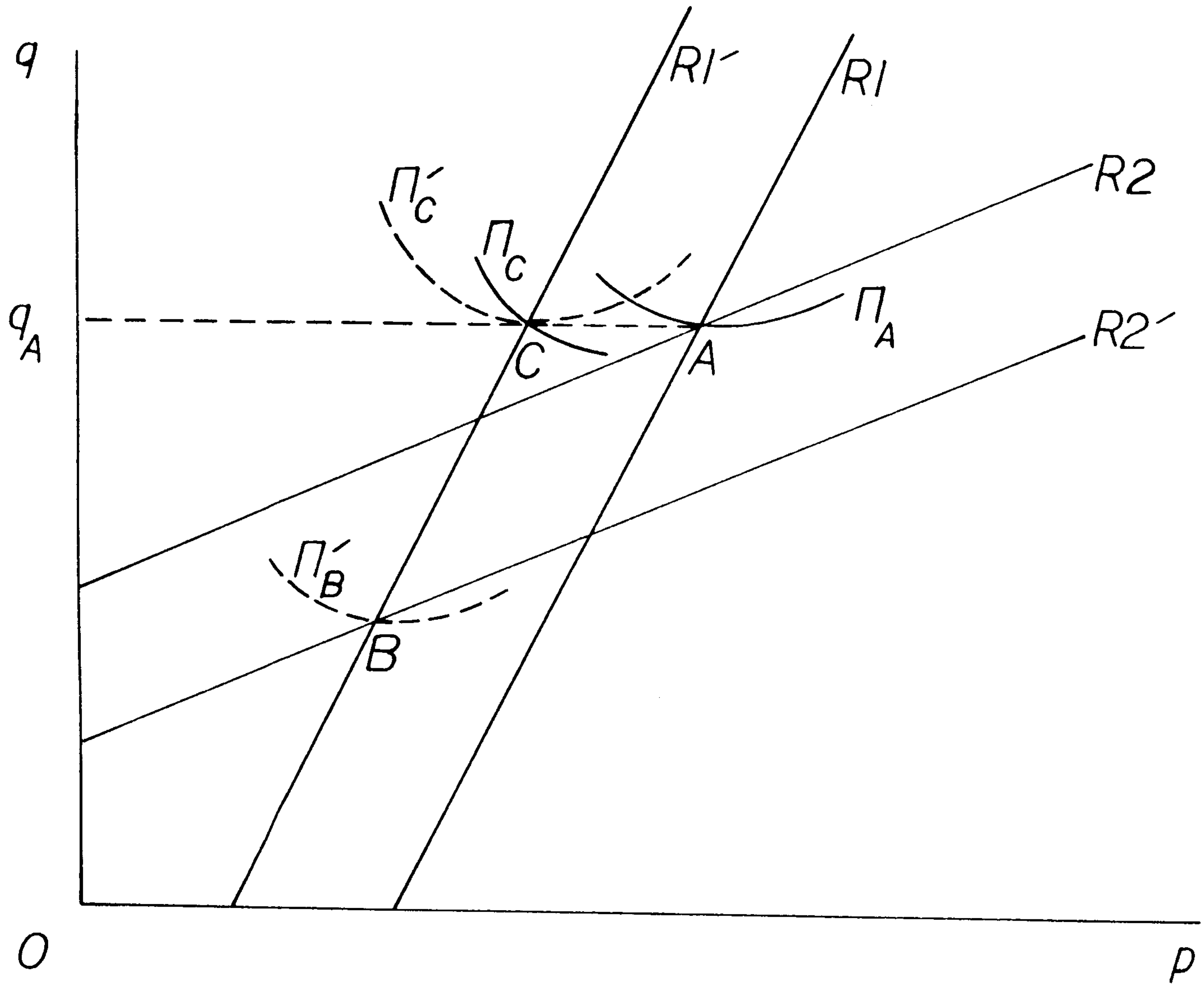

Figure 1: Comparison of equilibrium profits for firm 1 at A and B.

we know that increases in x decrease profits <u>ceteris paribus</u>. Thus, $\Pi(C) > \Pi'(C)$. In Figure 1, the new iso-profit curve through C, Π_C' (broken line), is tangent to the horizontal, while the old iso-profit through C, Π_C (solid line) is not, since C is on the new best reply but not on the old best reply. Now, because of the natural order of the iso-profit curves after the increase of x, $\Pi'(C) > \Pi'(B)$. Combining the inequalities, it follows that

$$\Pi(A) > \Pi(C) > \Pi'(C) > \Pi'(B).$$

Thus, equilibrium profits fall as x increases.

Therefore, in the first stage, given any y, firm 1 will choose $x^* = 0$. Similarly, given any x, firm 2 will choose $y^* = 0$. Thus, there exists a unique subgame-perfect equilibrium at $x^* = y^* = 0$. At equilibrium, no adapter is needed and there is full compatibility between components produced by different firms.

<u>Proposition 2</u>: <u>At the perfect equilibrium of the two-stage game of design specifications choices in the first stage and prices choices in the second stage, competing firms produce fully compatible components</u>.

<u>Proof</u>: See the Appendix.

Although these results were established for a world of two firms, they can easily be extended to a world of three or more firms, each producing two components. In the three-firm situation, product 33 needs no adapter, while products 13 and 31 require an adapter of cost x'+z', and products 23 and 32 require an adapter of cost y'+z. Firm 1 chooses x and x', firm 2 chooses y and y', and firm 3 chooses z and z'. It is not difficult to work through the arguments made for two firms problem and see that at equilibrium all adapters cost zero, i.e. x = x' = y = y' = z = z' = 0, and there is full compatibility.

The results of this paper can also be extended to a world of systems composed of as many components as producers, with each firm producing each type of component. For example, in a world of three firms in this framework there are twenty seven (i.e. 3^3) systems available, starting with 111, 121, 131, 211, 221, 231, 311, 321, 331, 112, 122, 132, etc. I assume that a different adapter is needed for each hybrid system, and that each firm controls a part of the adapter's cost through its decisions on the design of its components. It is easy to see that the problem is very similar to the one discussed above. The proof that full compatibility arises at equilibrium is a straightforward repetition of the proofs of this and the previous sections.

IV. Sensitivity to Symmetry I: Small Demand for Hybrids

The results of the previous two sections have been established for symmetric demand systems, where the demand for hybrid systems is equal to the demand for single-producer systems, under assumption A1. If the demand for hybrid systems is small compared to the demand for single-producer systems, increases in the cost of the adapter have a small negative effect on profits generated from sales of hybrid systems, but have a significant positive effect on profits generated from sales of single-producer systems. Thus, when the demand for hybrid systems is relatively small, increases in the price of the adapter tend to increase prices and profits. Below I establish this result for the two-stage game structure. The proof for the one-stage game structure is similar. Demand for hybrid systems can be small if such systems are always perceived as inferior to the single-producer systems or when manufacturers promote their complete systems.

Consider the case of a linear demand system where the demand for a hybrid system is k times the demand of a single-producer system, i.e.

$$D^{11}(a, b, c, d) = D^{12}(b, a, d, c)/k = D^{21}(c, d, a, b)/k = D^{22}(d, c, b, a),$$

where $k \in (0, 1]$. The demand system can be written as

$$\begin{bmatrix} D^{11} \\ D^{12} \\ D^{21} \\ D^{22} \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ k \\ k \\ 1 \end{bmatrix} + \begin{bmatrix} -\beta & \delta & \gamma & \epsilon \\ \delta k & -\beta k & \epsilon k & \gamma k \\ \gamma k & \epsilon k & -\beta k & \delta k \\ \epsilon & \gamma & \delta & -\beta \end{bmatrix} \cdot \begin{bmatrix} p_1 + p_2 \\ p_1 + q_2 + x + y \\ q_1 + p_2 + x + y \\ q_1 + q_2 \end{bmatrix},$$

or equivalently,

$$\begin{bmatrix} D^{11} \\ D^{12} \\ D^{21} \\ D^{22} \end{bmatrix} = \begin{bmatrix} \delta-\beta & \gamma-\beta & \gamma+\epsilon & \delta+\epsilon & \gamma+\delta & 1 \\ k(\delta-\beta) & k(\delta+\epsilon) & k(\gamma+\epsilon) & k(\gamma-\beta) & k(\epsilon-\beta) & k \\ k(\gamma+\epsilon) & k(\gamma-\beta) & k(\delta-\beta) & k(\delta+\epsilon) & k(\epsilon-\beta) & k \\ \gamma+\epsilon & \delta+\epsilon & \delta-\beta & \gamma-\beta & \gamma+\delta & 1 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ x + y \\ \alpha \end{bmatrix},$$

where $\alpha, \beta, \gamma, \delta, \epsilon, \kappa > 0$.

Profit maximization by firm 1 implies $\partial\Pi^1/\partial p_1 = 0$, <=>

$$(2(1+k)(\delta-\beta) \quad \gamma+\delta-2\beta+k(\gamma+\delta+2\epsilon) \quad (1+k)(\gamma+\epsilon) \quad \delta+\epsilon+k(\gamma-\beta) \quad \gamma+\delta+k(\epsilon-\beta) \quad 1+k)\cdot$$

$$(p_1 \quad p_2 \quad q_1 \quad q_2 \quad x+y \quad \alpha)^t = 0, \tag{16}$$

where superscript "t" denotes a transposed vector. By symmetry, $p_1^* = p_2^* = p^*$, $q_1^* = q_2^* = q^*$ and equation (13) simplifies to

$$[\gamma+3\delta-4\beta+k(\gamma+3\delta+2\epsilon-2\beta)]p^* + [\gamma+\delta+2\epsilon+k(2\gamma+\epsilon-\beta)]q^* + [\gamma+\delta+k(\epsilon-\beta)](x+y) + \alpha(1+k) = 0.$$

Similarly, from profits maximization of firm 2,

$$[\gamma+\delta+2\epsilon+k(2\gamma+\epsilon-\beta)]p^* + [3\delta+\gamma-4\beta+k(\gamma+3\delta+2\epsilon-2\beta)]q^* + [\gamma+\delta+k(\epsilon-\beta)](x+y) + \alpha(1+k) = 0.$$

Their common solution is

$$p^*(x,y) = q^*(x,y) = \{\alpha(1+k)+[\gamma+\delta+k(\epsilon-\beta)](x+y)\}/[2(2\beta-2\gamma-\delta-\epsilon)+3k(\beta-\gamma-\delta-\epsilon)]. \tag{17}$$

By assumption A2, increases in the price of all four goods decrease the demand of any good. Thus,

$$\beta > \gamma + \delta + \epsilon \tag{18}$$

and the denominator of (17) is positive. For the numerator to be positive, I require that

$$k < [\alpha + (\gamma + \delta)(x + y)]/(\beta - \epsilon)(x + y) - \alpha]. \tag{19}$$

The realized profits for firm 1 at the equilibrium of the subgame are

$$\Pi^{1d}(x, y) = [2(2\beta - \gamma - \delta) + 2k(\beta - \gamma - \delta - \epsilon)](p^*(x, y))^2. \tag{20}$$

In the first stage, firms choose x and y non-cooperatively. From (20) it is evident that the effect of changes in the price of the adapter on profits come entirely through the equilibrium prices,

$$\partial\Pi^{1d}/\partial x = 2[2(\beta - \gamma - \delta) + 2k(\beta - \gamma - \delta - \epsilon)]p^*\partial p^*/\partial x.$$

Prices vary with the adapter cost as

$$\partial p^*/\partial x = [\gamma + \delta + k(\epsilon - \beta)]/[2(2\beta - 2\delta - \gamma - \epsilon) + 3k(\beta - \gamma - \delta - \epsilon)]. \tag{21}$$

The denominator is always positive, as noted above in (18). However, the sign of the numerator depends on the scale of the hybrid demand, k. Let

$$k_1 = (\gamma+\delta)/(\beta-\epsilon)$$

be the value of k that makes the numerator of $\partial p^*/\partial x$ zero. For small k, $k < k_1$, the numerator, $\gamma+\delta+k(\epsilon-\beta)$, is positive and therefore $\partial\Pi^{1d}/\partial x > 0$, i.e. profits increase with the cost of the adapter. See Figure 2. Then firm 1 chooses the maximum x possible, i.e. $x = \bar{x}$. Similarly, firm 2 chooses the maximum y possible, $y = \bar{y}$, and we observe maximum incompatibility with the adapter's cost at $\bar{x} + \bar{y}$. Conversely, for k large, $k > k_1$, the numerator of (21) is negative and firm 1 will choose $x = 0$. Similarly, firm 2 will choose $y = 0$ and there will be full compatibility. The range of k for this event includes $k = 1$, the symmetric case where the hybrid-system demand equals the single-producer-system demand, that was discussed in generality in the previous sections. I have shown that a single standard results when the demand for hybrids is comparable to the demand for single-producer systems (k close to 1), while two different standards result when the demand for hybrid systems is small (k small).

Proposition 3: For a linear demand system, at the perfect equilibrium firms choose full compatibility if and only if the relative scale of the demand for hybrid systems is sufficiently large.[6]

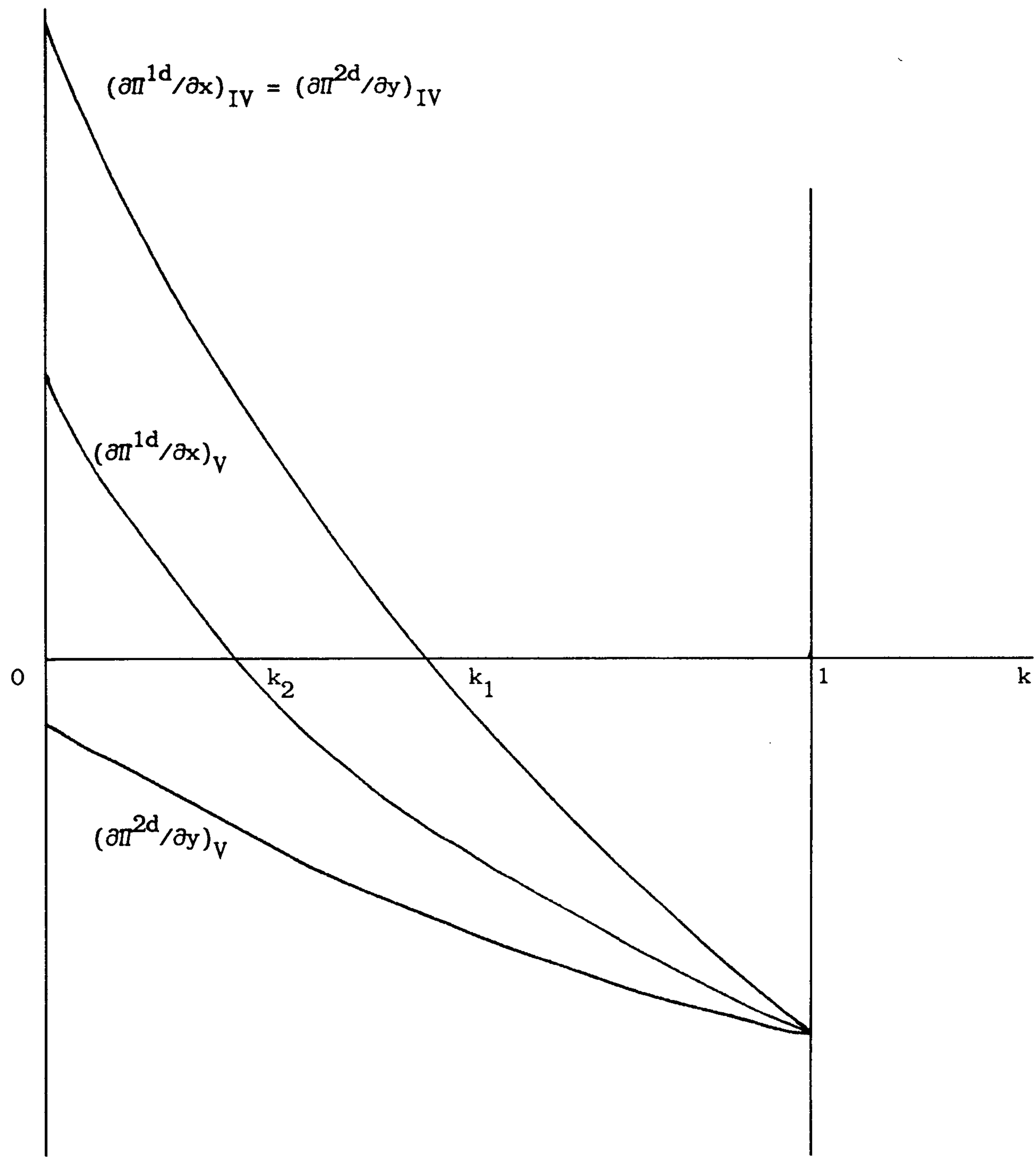

Figure 2: The rate of change of profits with the cost of the adapter for the demand structures of Sections IV and V.

V. Sensitivity to Symmetry II: Demand for one Single-producer System is Large

This section analyzes the case when the demand for one of the single-producer systems is large, while demands for all other systems are small. This case can arise naturally in a setting where one firm enjoys good reputation from past achievements in related fields, while the second firm is relatively unknown. Then the demand for any systems that embody a component produced by the second firm will be small.

Formally, consider a linear demand system where the demand for for systems 12, 21 and 22 is k times the demand for system 11,

$$D^{11}(a, b, c, d) = D^{12}(b, a, d, c)/k = D^{21}(c, d, a, b)/k = D^{22}(d, c, b, a)/k,$$

where k is in (0, 1]. This demand system can be written as

$$\begin{bmatrix} D^{11} \\ D^{12} \\ D^{21} \\ D^{22} \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ k \\ k \\ k \end{bmatrix} + \begin{bmatrix} -\beta & \delta & \gamma & \epsilon \\ \delta k & -\beta k & \epsilon k & \gamma k \\ \gamma k & \epsilon k & -\beta k & \delta k \\ \epsilon k & \gamma k & \delta k & -\beta k \end{bmatrix} \cdot \begin{bmatrix} p_1 + p_2 \\ p_1 + q_2 + x + y \\ q_1 + p_2 + x + y \\ q_1 + q_2 \end{bmatrix},$$

or equivalently as,

$$\begin{bmatrix} D^{11} \\ D^{12} \\ D^{21} \\ D^{22} \end{bmatrix} = \begin{bmatrix} \delta-\beta & \gamma-\beta & \gamma+\epsilon & \delta+\epsilon & \gamma+\delta & 1 \\ k(\delta-\beta) & k(\delta+\epsilon) & k(\gamma+\epsilon) & k(\gamma-\beta) & k(\epsilon-\beta) & k \\ k(\gamma+\epsilon) & k(\gamma-\beta) & k(\delta-\beta) & k(\delta+\epsilon) & k(\epsilon-\beta) & k \\ k(\gamma+\epsilon) & k(\delta+\epsilon) & k(\delta-\beta) & k(\gamma-\beta) & k(\gamma+\delta) & k \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ q_1 \\ q_2 \\ x + y \\ \alpha \end{bmatrix},$$

where $\alpha, \beta, \gamma, \delta, \epsilon, \kappa > 0$.

Although the degree of compatibility is variable, to understand the intuition let us consider two extreme situations, full compatibility where $x = y = 0$, and total incompatibility when the cost of the adapter is so high that the demand for hybrids is zero. The incentive for compatibility of the (small) second firm is significant. By making its components compatible with those of firm 1, firm 2 attempts to approximately triple its demand from D^{22} to $D^{22} + D^{12} + D^{21} \cong 3D^{22}$. The incentive for compatibility for the large firm is much smaller. By making its components compatible with those of firm 2, firm 1 will increase its demand from D^{11} to $D^{11} + D^{12} + D^{21} \simeq D^{11} + 2kD^{11}$. Thus, the incentive for compatibility clearly depends on k. For small k, the demand reward for compatibility to firm 1 is small and does not compensate it for the increase in competition. Hence, for small k, firm 1 will attempt to maximize incompatibilities by choosing the maximal x available. However, for k large, the demand reward is sufficient to compensate firm 1 for the increase in competition, and firm 1 will choose full compatibility. In particular for $k = 1$ this case reduces to the general symmetric case analyzed in section III, where compatibility always results.

The profit maximization by firm 1 is identical to the one of the previous section. It implies $\partial\Pi^1/\partial p_1 = 0 \iff$

$$(2(1+k)(\delta-\beta) \quad \gamma+\delta-2\beta+k(\gamma+\delta+2\epsilon) \quad (1+k)(\gamma+\epsilon) \quad \delta+\epsilon+k(\gamma-\beta) \quad \gamma+\delta+k(\epsilon-\beta) \quad 1+k)\cdot$$
$$(p_1 \quad p_2 \quad q_1 \quad q_2 \quad x+y \quad \alpha)^t = 0. \tag{16}$$

Maximization by firm 2 implies $\partial\Pi^2/\partial p_2 = 0 \iff$

$$(2k(\gamma+\epsilon) \quad k(\gamma+\delta+\epsilon-\beta) \quad 4k(\delta-\beta) \quad 2k(\gamma+\delta+\epsilon-\beta) \quad k(\gamma+\delta+\epsilon-\beta \quad 2k)\cdot$$
$$(p_1 \quad p_2 \quad q_1 \quad q_2 \quad x+y \quad \alpha)^t = 0. \tag{22}$$

Given symmetry across components, $p_1 = p_2 = p^*$, $q_1 = q_2 = q^*$, and (16) and (22) reduce to

$$p^*(\gamma+3\delta-4\beta+k(\gamma+3\delta+2\epsilon-2\beta)) + q^*(\gamma+\delta+2\epsilon+k(2\gamma+\epsilon-\beta)) + (x+y)(\gamma+\delta+k(\epsilon-\beta)) + \alpha(1+k) = 0$$

and

$$p^*(3\gamma+3\epsilon+\delta-\beta) + q^*(2\gamma+6\delta+2\epsilon-6\beta) + (x+y)(\gamma+\delta+\epsilon-\beta) + 2\alpha = 0.$$

I solve these to derive $p^*(x, y)$, $q^*(x,y)$ and dp^*/dx and dq^*/dx,

$$dp^*/dx = [(\beta-\gamma-\delta-\epsilon)(\gamma+\delta+2\epsilon+k(2\gamma+\epsilon-\beta)) - (k(\beta-\epsilon)-\gamma-\delta)(2\gamma+6\delta+2\epsilon-6\beta)]/D \tag{23}$$

$$dq^*/dx = [(k(\beta-\epsilon)-\gamma-\delta)(3\gamma+3\epsilon+\delta-\beta) - (\beta-\gamma-\delta-\epsilon)(\gamma+3\delta-4\beta+k(\gamma+3\delta+2\epsilon-2\beta))]/D \tag{24}$$

where

$$D = (3\gamma+3\epsilon+\delta-\beta)(\gamma+\delta+2\epsilon+k(2\gamma+\epsilon-\beta)) - (2\gamma+6\delta+2\epsilon-6\beta)(\gamma+3\delta-4\beta+k(\gamma+3\delta+2\epsilon-2\beta)) \quad (25)$$

The realized profits for each firm at the equilibrium of the subgame are proportional to the firm's price,

$$\Pi^{1d} = 2(p^*)^2[2\beta - \gamma - \delta + k(\beta - \gamma - \delta - \epsilon)] \quad (26a)$$

$$\Pi^{2d} = 2k(q^*)^2(3\beta - 2\gamma - 2\delta - \epsilon). \quad (26b)$$

In the first stage, firms choose x and y non-cooperatively, anticipating the equilibrium prices and profits of the last stage. From (26a,b) it is evident that the direction of change of profits with a change in the price of the adapter is the same as the direction of change of the equilibrium price. We show in the Appendix that the (small) second firm's price decreases with the price of the adapter, while the first firm's price increases with the price of the adapter for small k and decreases with the price of the adapter for large k.

<u>Lemma 2</u>: $dp^*/dx > 0$ and $\partial\Pi^{1d}/\partial x > 0$ <u>for</u> $0 \leq k < k_2$, $dp^*/dx < 0$ and $\partial\Pi^{1d}/\partial x < 0$ <u>for</u> $1 \geq k > k_2$, <u>while</u> $dq^*/dy < 0$ and $\partial\Pi^{2d}/dy < 0$ <u>for all</u> k. <u>Further</u>, $k_2 < k_1 < 1$.

Functions $\partial\Pi^{1d}/\partial x$ and $\partial\Pi^{2d}/dy$ are pictured in Figure 2. From Lemma 2 it follows that firm 2 chooses always compatibility by setting $y = 0$. However, firm 1 chooses compatibility and sets $x = 0$ only if $k > k_2$. Otherwise, firm 1 chooses incompatibility and sets $x = \bar{x}$. Therefore there can be two qualitatively different equilibria. For $k > k_2$, there is full compatibility and the cost of the adapter is zero. For $k < k_2$, there is partial compatibility and the cost of the adapter is $\bar{x}$.

<u>Proposition 4</u>: <u>When the demand for one single-producer system is high, the opponent always chooses compatibility. The high demand firm chooses incompatibility if its demand is large; it chooses compatibility if its demand is close to the demand of the other three systems.</u>

Thus, for $k > k_2$ a single "standard" results. For $k < k_2$ two "standards" result, but their difference is smaller than in the case of the previous section, as signified by the fact that here the required adapter costs $\bar{x}$ and there it costed $\bar{x} + \bar{y}$. the dominance of the market by one firm results in total compatibility or only partial incompatibility.

Note that, since $k_2 < k_1 < 1$, the range of values of k such that compatibility prevails, $(k_2, 1]$, is larger than the similar range $(k_1, 1]$ of the case of the previous section where only hybrids demand was small. This shows clearly the workings of competition among single-producer and hybrid systems. Small demand for hybrid systems tends to drive the market towards incompatibility. However, this drive is <u>weakened</u> when there is disparity in the single-producer systems demands as well. Thus, compatibility tends to arise more often, and incompatibilities when they arise are smaller, when only one single-producer demand is large. Table 1 summarizes the compatibility/incompatibility regimes and the corresponding price for the adapter for the two asymmetric demand structures as well as for the symmetric structure.

<u>Table 1</u>

<u>Market Demand Specification</u>	<u>Range of k</u> $(0, k_2)$	(k_2, k_1)	$(k_1, 1]$
Small Demand for Hybrids	Total Incompatibility $x^* + y^* = \bar{x} + \bar{y}$	Total Incompatibility $x^* + y^* = \bar{x} + \bar{y}$	Total Compatibility $x^* + y^* = 0$
One Large Single-Producer Demand	Partial Incompatibility $x^* + y^* = \bar{x}$	Total Compatibility $x^* + y^* = 0$	Total Compatibility $x^* + y^* = 0$
All Symmetric	Total Compatibility $x^* + y^* = 0$		

V. Extensions and Concluding Remarks

So far I have assumed that the markets for adapters were perfectly competitive so that increases in marginal costs were directly reflected in their prices. But, for the arguments of this paper to hold, all that is required is that the price of an adapter is increasing in its marginal cost, and this can come about from many market structures of the adapter industry.

When the players in the adapter market are strategically active, I postulate a three-stage game structure. In the first stage, the component-producing firms choose the design specifications of their components, and thereby the cost of production $x+y$ of the adapters for hybrid systems is determined. In the second stage, the producers of adapters choose their prices. In the third stage, the component-producing firms choose prices. As before, in each stage firms consider the decisions of previous stages as final, and correctly anticipate the equilibrium of the subsequent subgame.

Let z be the equilibrium price for the adapters. Then z takes the position of $x+y$ in the formulas for the prices of the components in the subgame that follows the choice of z. Provided that x and y have a positive influence on z, the arguments of all the previous sections go through. Note that $x+y$ is the marginal cost of an adapter and z is its equilibrium price. Thus, the results of this article generalize to market structures for the production of adapters such that the equilibrium adapter price is increasing with marginal cost. This holds for monopoly and symmetric oligopoly in the adapter market where the component-producing firms do not participate.

If a component-producing firm participates in the production of adapters, the results are ambiguous. A firm still has the incentive to lower the cost of an adapter and its price so that the profits from sales of components are maximized. However, the firm also has an incentive to increase the adapter's price so that its profits from the sale of adapters increase. Of course, the balance of these incentives depends on the degree of competition in the market for adapters. This problem, as well as the problem of the incentive of a component-producing firm to enter the market for adapters are still open for further research.

This paper has shown that, in the absence of network externalities and for symmetric demand systems, full compatibility arises as an equilibrium in a game where firms choose non-cooperatively the degree of compatibility of their components and the prices at which they sell them, even in the absence of network externalities. Full compatibility arises again as the equilibrium of a two-stage game where design specifications are chosen in the first stage and prices are chosen in the second stage. However, when demand for hybrid systems is relatively low, the reverse result is true: firms choose to maximize the incompatibility of their components.

When the demand for only one single-producer system is relatively high, the small-demand firm chooses always full compatibility while the high-demand firm chooses full compatibility only if its demand is of comparable size to the demands of the other three systems. When the demand for one single-producer system is very high, the firm that produces it chooses to maximize incompatibility as much as possible, thus resulting in partial incompatibility since the small-demand firm chooses not to add to the incompatibility.

These results point to the significance of the relative scale of the demand for single-producer systems and hybrid systems in the determination of standards. When the scale of single-producer and hybrid systems is the same, a single standard always results. When hybrid demand is low, two quite different standards evolve. When only the demand for one single producer system is very large, then two standards result but they do not differ (as measured by the cost of the required adapter) as much as the opposing standards of the case of low demand for hybrids only. Finally, when the demand for one single-producer system is large but comparable to the demands for the other three systems, a single standard evolves. The worst scenario for compatibility arises when the demand for single-producer systems is large while the demand for hybrids is small, while the best scenario for compatibility arises when the demand is of equal scale for all systems.

Appendix

Proof of Lemma 1:

To complete the proof of the validity of equation (8),

$$\partial(D^{11} + D^{12})/\partial x \leq \sum_{k=1}^{4} D_k^{11}(a, b, c, d) < 0 \tag{8}$$

we need to show equations (6) and (7),

$$D_1^{11}(b, a, d, c) \leq D_1^{11}(a, b, c, d), \tag{6}$$

$$D_4^{11}(b, a, d, c) \leq D_4^{11}(a, b, c, d). \tag{7}$$

To show equation (6), note that

$$D_1^{11}(b, a, d, c) \leq D_1^{11}(a, b, d, c) \tag{A1}$$

provided that $(b - a) \geq 0$ and $D_{11}^{11} \leq D_{12}^{11}$. Without loss of generality we assume $b - a = q_1 + x + y - p_1 \geq 0$. The condition $D_{11}^{11} \leq D_{12}^{11}$ is satisfied by assumption A3. Further,

$$D_1^{11}(a, b, d, c) \leq D_1^{11}(a, b, c, d) \tag{A2}$$

provided that $(c - d) \geq 0$ and $D_{14}^{11} \leq D_{13}^{11}$. Without loss of generality we assume $c - d = p_1 + x + y - q_1 \geq 0$. The condition $D_{14}^{11} \leq D_{13}^{11}$ is satisfied by assumption A3. Combining conditions (A1) and (A2) results in (6).

To show equation (7), note that

$$D_4^{11}(b, a, d, c) \leq D_4^{11}(a, b, d, c) \tag{A3}$$

provided that $(b - a) \geq 0$ and $D_{41}^{11} \leq D_{42}^{11}$, as assumed in assumption A3. Further,

$$D_4^{11}(a, b, d, c) \leq D_4^{11}(a, b, c, d) \tag{A4}$$

provided that $c - d \geq 0$, and $D_{43}^{11} \leq D_{44}^{11}$, as assumed in A3. Combining conditions (A3) and (A4) results in (7).

To prove equation (9),

$$\partial(D^{11} + D^{21})/\partial x \leq \sum_{k=1}^{4} D_k^{11}(a, b, c, d) < 0, \tag{9}$$

note that

$$\partial D^{21}/\partial x = D_1^{21}(c, d, a, b) + D_4^{21}(c, d, a, b),$$

while as before,

$$\partial D^{11}/\partial x = D_2^{11}(a, b, c, d) + D_3^{11}(a, b, c, d).$$

Using the same technique as above, it can be shown that

$$D_1^{21}(c, d, a, b) = D_1^{11}(c, d, a, b) \leq D_1^{11}(a, d, c, b) \leq D_1^{11}(a, b, c, d) \quad \text{(A5)}$$

provided that $D_{11}^{11} \leq D_{13}^{11}$ and $D_{14}^{11} \leq D_{12}^{11}$, and these are satisfied by assumption A3. Further,

$$D_4^{21}(c, d, a, b) = D_4^{11}(c, d, a, b) \leq D_4^{11}(a, d, c, b) \leq D_4^{11}(a, b, c, d) \quad \text{(A6)}$$

provided that $D_{41}^{11} \leq D_{43}^{11}$ and $D_{44}^{11} \leq D_{42}^{11}$, and these are satisfied by assumption A3. Equation (9) follows immediately, and, together with (8), it implies

$$\partial \Pi^1/\partial x = p_1 \partial(D^{11} + D^{12})/\partial x + p_2 \partial(D^{11} + D^{21})/\partial x < 0.$$

Q.E.D.

Proof of Proposition 2:

Differentiating $\partial \Pi^1/\partial p_1 = 0$ with respect to x results in

$$\partial^2\Pi^1/\partial p_1 \partial x + \partial^2\Pi^1/\partial p_1^2 \cdot dp_1^*/dx + \partial^2\Pi^1/\partial p_1 \partial p_2 \cdot dp_2^*/dx$$
$$+ \partial^2\Pi^1/\partial p_1 \partial q_1 \cdot dq_1^*/dx + \partial^2\Pi^1/\partial p_1 \partial q_2 \cdot dq_2^*/dx = 0$$

By symmetry, the expression simplifies to

$$\partial^2\Pi^1/\partial p_1 \partial x + 2[\partial^2\Pi^1/\partial p_1^2 \cdot dp_1^*/dx + \partial^2\Pi^1/\partial p_1 \partial q_1 \cdot dq_1^*/dx] = 0. \quad \text{(A7)}$$

Similarly, differentiating $\partial \Pi^2/\partial q_1 = 0$ with respect to x results in

$$\partial^2\Pi^2/\partial q_1 \partial x + 2[\partial^2\Pi^2/\partial q_1 \partial p_1 \cdot dp_1^*/dx + \partial^2\Pi^2/\partial q_1^2 \cdot dq_1^*/dx] = 0. \quad \text{(A8)}$$

Solving the system of (A7, A8) with respect to dp_1^*/dx and dq_1^*/dx we derive

$$dq_1^*/dx = [-\partial^2\Pi^1/\partial p_1^2 \cdot \partial^2\Pi^2/\partial q_1 \partial x + \partial^2\Pi^2/\partial q_1 \partial p_1 \cdot \partial^2\Pi^1/\partial p_1 \partial x]/D, \quad \text{(A9)}$$

where the denominator

$$D = \partial^2\Pi^1/\partial p_1^2 \cdot \partial^2\Pi^2/\partial q_1^2 - \partial^2\Pi^1/\partial p_1 \partial q_1 \cdot \partial^2\Pi^2/\partial q_1 \partial p_1 > 0$$

is positive from second order conditions. Further, $-\partial^2\Pi^1/\partial p_1^2 > 0$ and

$\partial^2\Pi^2/\partial q_1\partial p_1 > 0$, while from A4 (strategic substitutability) $\partial^2\Pi^1/\partial p_1\partial x < 0$ and $\partial^2\Pi^2/\partial q_1\partial x = \partial^2\Pi^2/\partial q_1\partial y < 0$. Substituting in (A9) it follows that $dq_1^*/dx < 0$. Q.E.D.

Proof of Lemma 2:

dp^*/dx can be written as

$$dp^*/dx = (A + Bk)/D$$

where

$$A = (\beta - \gamma - \delta - \epsilon)(-5\gamma - 5\delta + 2\epsilon) - (\gamma + \delta)(4\gamma + 4\epsilon),$$

$$B = (\beta - \epsilon)(5\beta - 5\delta - \gamma - \epsilon) + 2\gamma(\beta - \gamma - \delta - \epsilon),$$

$$D = R + Sk,$$

$$R = (2\gamma + 2\epsilon)(3\gamma + 7\delta + 2\epsilon - 8\beta),$$

$$S = -11(\beta - \gamma - \delta - \epsilon)^2 - (\beta - \gamma - \delta - \epsilon)(17\gamma - 5\delta + 9\epsilon) + 2(\delta - \gamma)(\gamma + \epsilon).$$

Clearly, $R < 0$. Since

$$R + S = -35(\beta - \gamma - \delta - \epsilon)^2 - (\beta - \gamma - \delta - \epsilon)(52\gamma + 2\delta + 51\epsilon)$$
$$- 6(2\gamma + \epsilon)(\gamma + \epsilon) - \epsilon(5\gamma + \delta + 6\epsilon) < 0,$$

it follows that $D < 0$ for all $k \in [0, 1]$.

For $A < 0$, it is sufficient that β is sufficiently large,

$$\beta > \gamma + \delta + \epsilon + 4(\gamma + \delta)(\gamma + \epsilon)/(2\epsilon - 5\gamma - 5\delta).$$

Thus $dp^*/dx > 0$ at $k = 0$. B is immediately positive, $B > 0$. Since

$$A + B = (\beta - \gamma - \delta - \epsilon)(5\beta - 5\delta + \gamma + \epsilon) > 0,$$

it follows that $dp^*/dx < 0$ at $k = 1$. There exists a unique $k = k_2 = -A/B$ such that, for all $k < k_2$, $dp^*/dx > 0$, and, for all $k > k_2$, $dp^*/dx < 0$.

dq^*/dx can be written as

$$dq^*/dx = (C + Ek)/D,$$

where

$$C = 4(\beta - \gamma - \delta - \epsilon)^2 + (\beta - \gamma - \delta - \epsilon)(2\delta + 4\gamma + 4\epsilon) - 2(\gamma + \delta)(\gamma + \epsilon)$$

and

$$E = (\beta - \gamma - \delta - \epsilon)(\beta + \gamma - 3\delta + \epsilon) + 2(\gamma + \delta)(\gamma + \epsilon).$$

Both C and E are positive for β sufficiently large. Thus dq^*/dx for all $k \in (0, 1]$.

Finally, $k_2 < k_1 \iff$

$$[(\beta-\gamma-\delta-\epsilon)(5\gamma+5\delta-2\epsilon)+4(\gamma+\delta)(\gamma+\epsilon)]/[(\beta-\epsilon)(5\beta-5\delta-\gamma-\epsilon)+2\gamma(\beta-\gamma-\delta-\epsilon)] < (\gamma+\delta)/(\beta-\epsilon)$$

which is equivalent (after few steps) to

$$-2\epsilon(\beta-\gamma-\delta-\epsilon)^2-2(\gamma+\epsilon)(\gamma+\delta)(\beta-\gamma-\delta-\epsilon) < 0,$$

which is immediately true. Q.E.D.

Footnotes

1. Sometimes this adapter should be interpreted as the translation or conversion of a program to work with a different operating system then the one it was originally written for. For the purposes of this paper, an adapter or interface can also be interpreted as any device, such as translator, converter or gateway, that allows two components to function together in a hybrid system.

2. A positive consumption externality of good X typically arises because of the existence of a complementary good Y with some public good features. Higher consumption of X implies a lower effective price for Y. Therefore the effective price of the last unit of X sold is lower than the effective price of the first unit. Under full compatibility, the total sales of all the compatible goods have a negative effect on the price of Y. Thus, a firm that joins a large "network" of compatible goods enjoys a higher willingness-to-pay for its good. For example, a firm adhering to the VHS standard for video cassette recorders enjoys a higher demand than if it chose the Beta standard, because the libraries for VHS tapes are typically larger than the libraries for Beta tapes. See Michael Katz and Carl Shapiro (1985) among others for an analysis of network externalities.

3. For the rest of this exposition, p_1, p_2, q_1, and q_2 are called prices as if marginal costs were zero, but the equivalence with the constant marginal cost case is obvious.

4. This specification excludes functional forms such as $\varphi(x, y) = xy$ where each firm has the opportunity to unilaterally impose compatibility on the

industry. This is because, given all the dimensions on which design specifications can be varied, it seems unlikely that all incompatibilities introduced by the opponent can be anticipated and neutralized. Also, such functional forms would create multiplicity of equilibria such as $x^* = 0$, $y^* = y$, $y \geq 0$, that are all essentially equivalent compatibility equilibria.

5. This result can be interpreted as showing that adapters and components are substitutes.

6. Note that for a linear demand system, strategic substitutability of adapters and components is equivalent to substitutability between components and adapters. This is because $\partial\Pi^1/\partial x$ and $\partial^2\Pi^1/\partial p_1\partial x$ are of the same sign,

$$\partial\Pi^1/\partial x = p_1(\gamma+\delta+k(\epsilon-\beta)), \quad \partial^2\Pi^1/\partial p_1\partial x = (p_1 + p_2)(\gamma+\delta+k(\epsilon-\beta)).$$

Note further that for the symmetric linear demand system ($k = 1$) substitutability and strategic substitutability are equivalent to condition A1, i.e. that an increase in all four prices decreases demand for each system, since

$$\partial\Pi^1/\partial x = p_1(\gamma+\delta+\epsilon-\beta) = p_1 \cdot \sum_{k=1}^{4} D_k^{11} < 0.$$

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