
The Impact of Central Bank Stock Purchases: Evidence from Discontinuities in Policy Rules

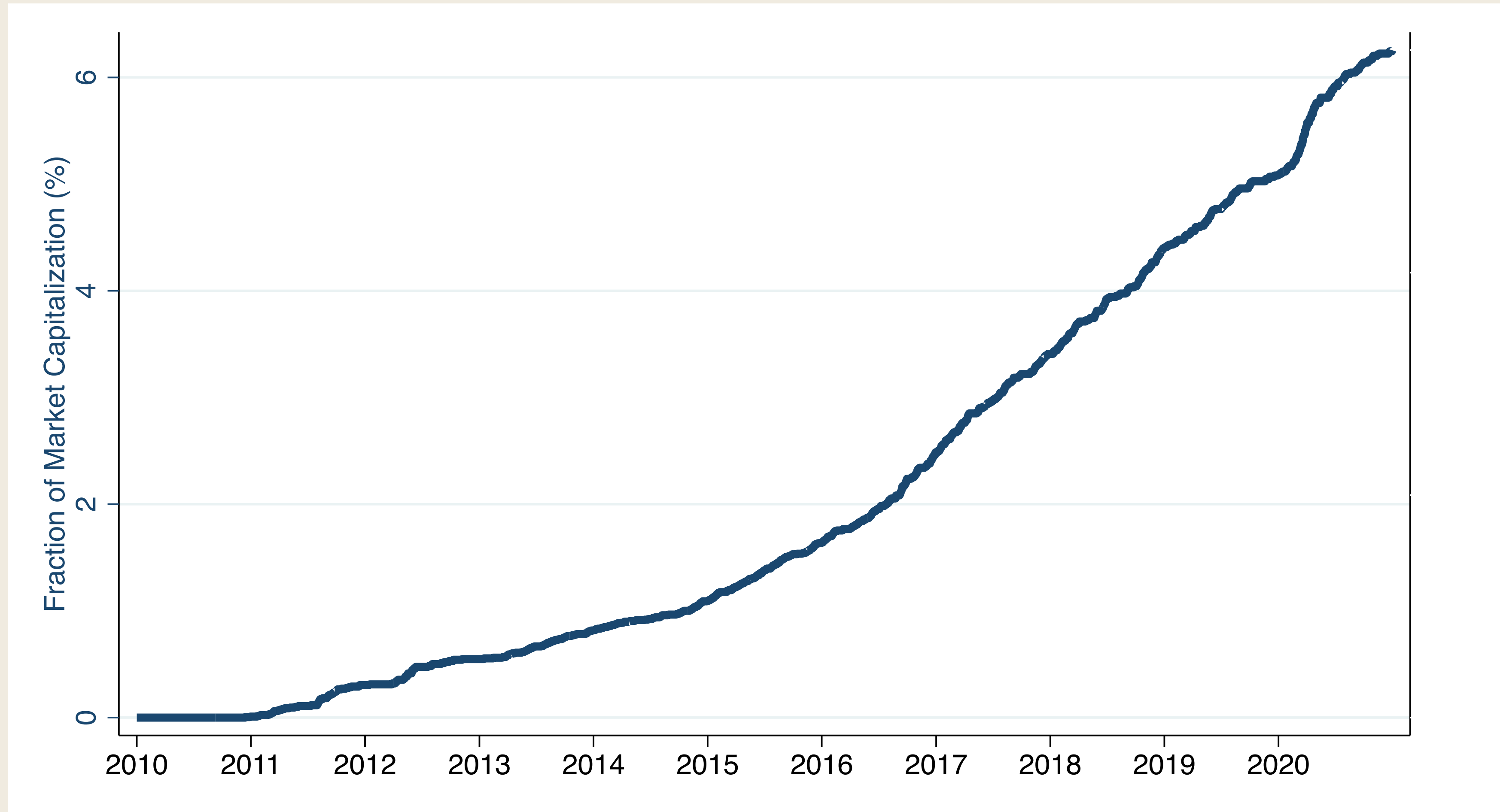
Masao Fukui
Boston University

Masayuki Yagasaki
ESRI & University of Tokyo

Japan Economic Seminar

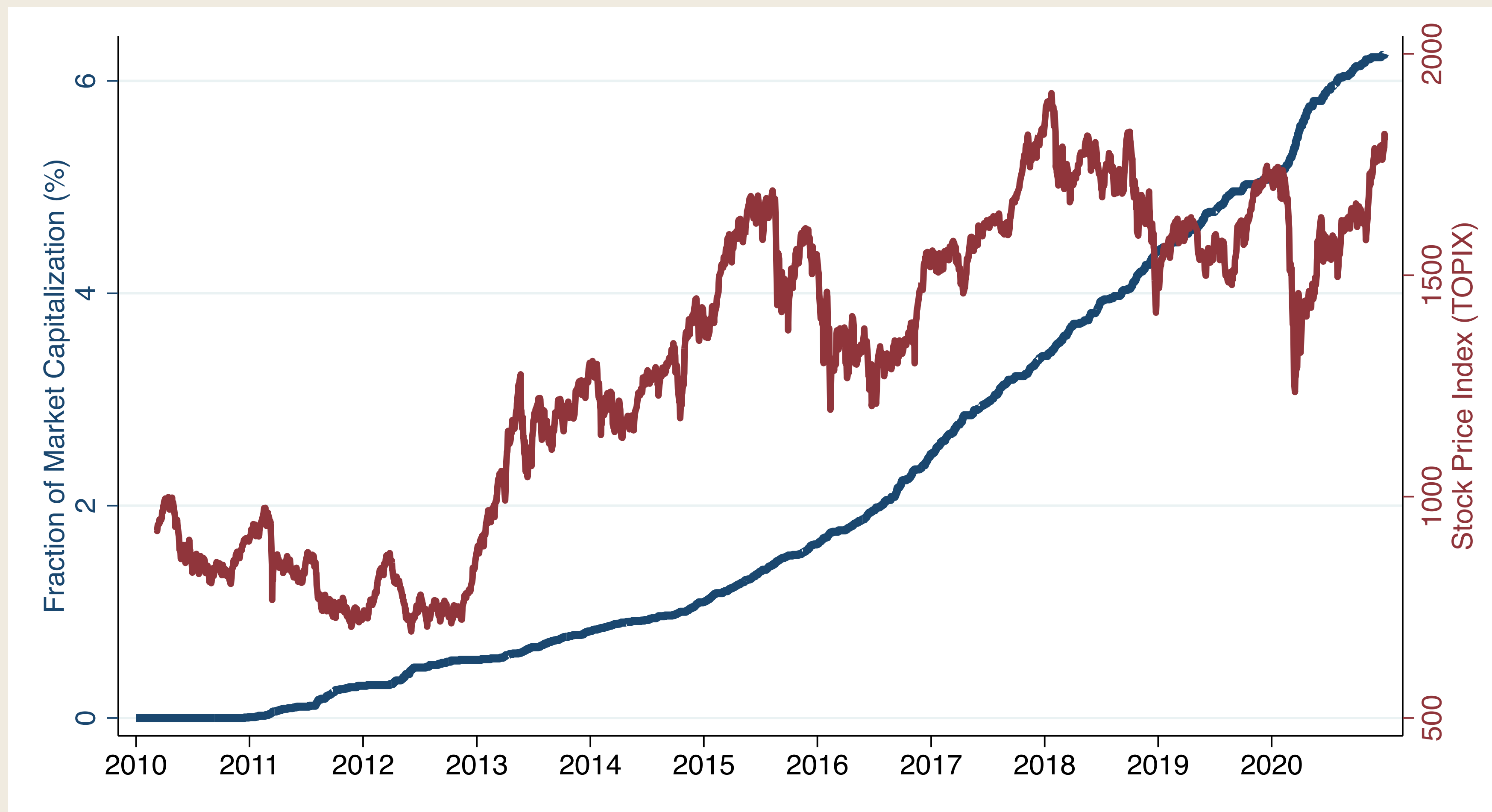
February 2023

Motivation



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Question

- What is the impact of central bank stock purchases?
- Why do we care?
 1. Frontier of “quantitative easing”
 2. Ideal laboratory to test new theories of stock market fluctuations

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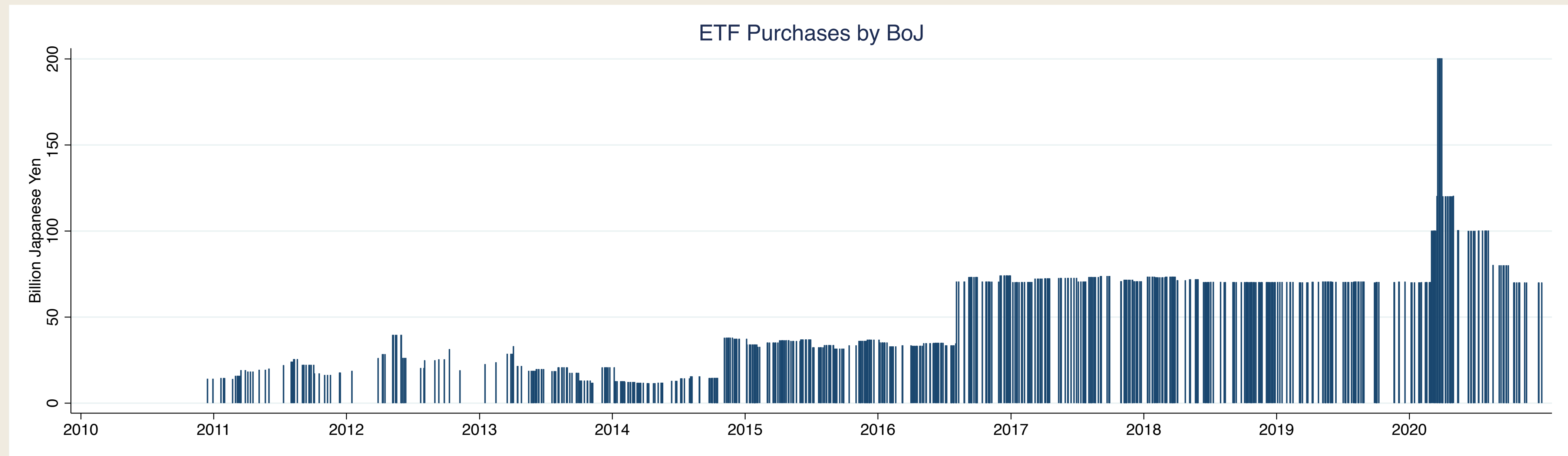
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 - ✓ Inelastic stock & bond market model → Yes

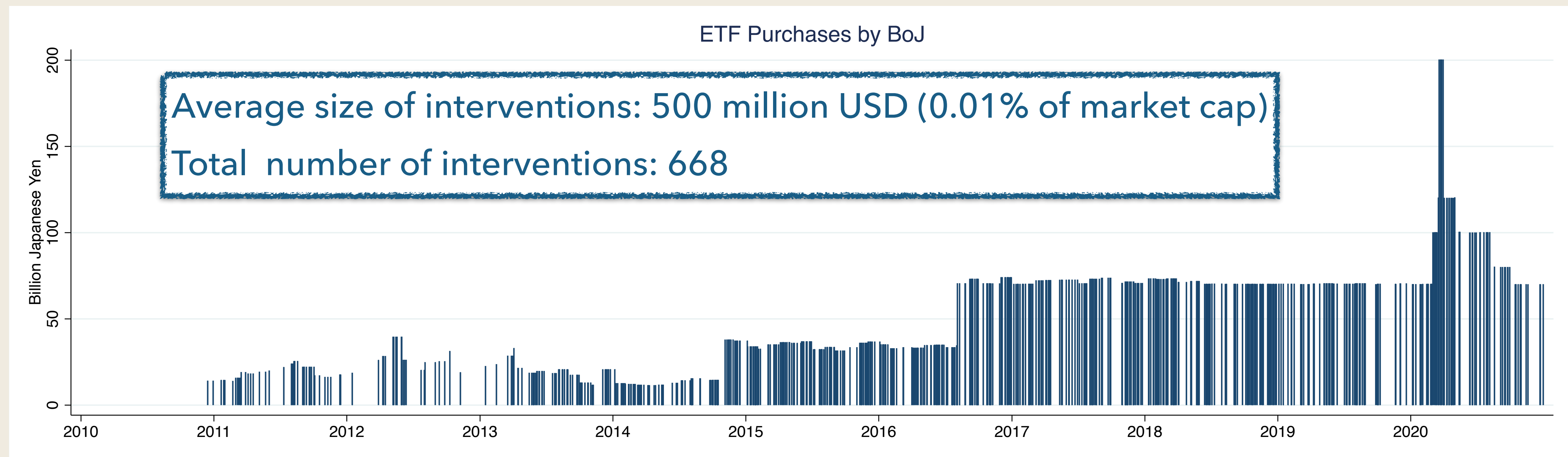
Empirical Results

Goal



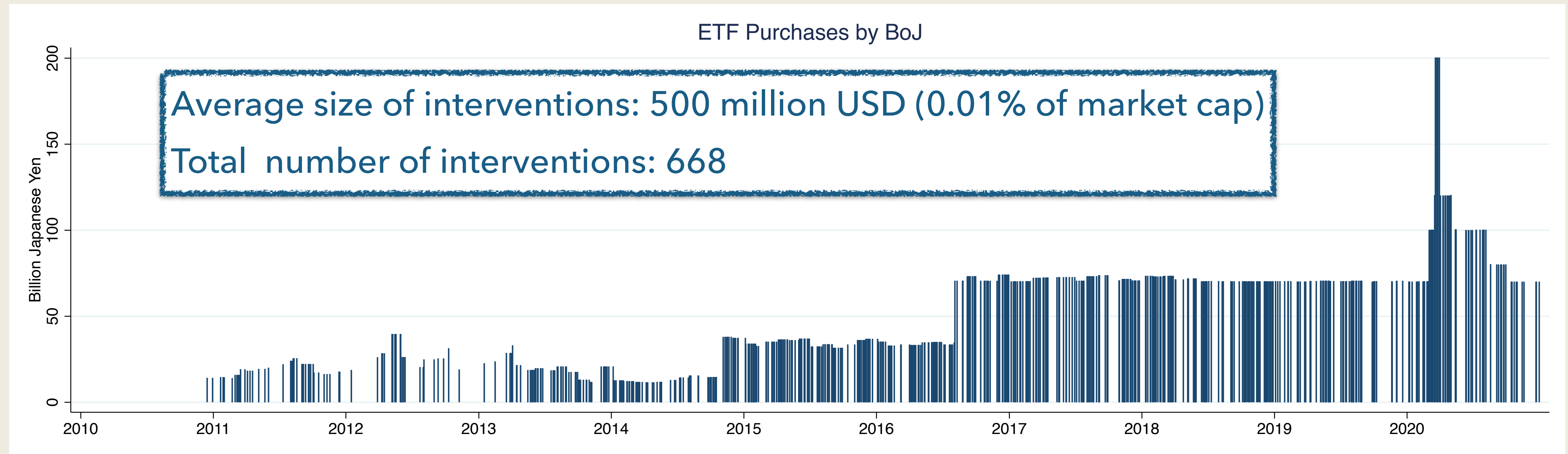
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- The goal is to estimate

$$y_{t,h} - y_{t,0} = \beta_h E_t / S_t + \gamma_h' \mathbf{X}_t + \epsilon_{t,h}$$

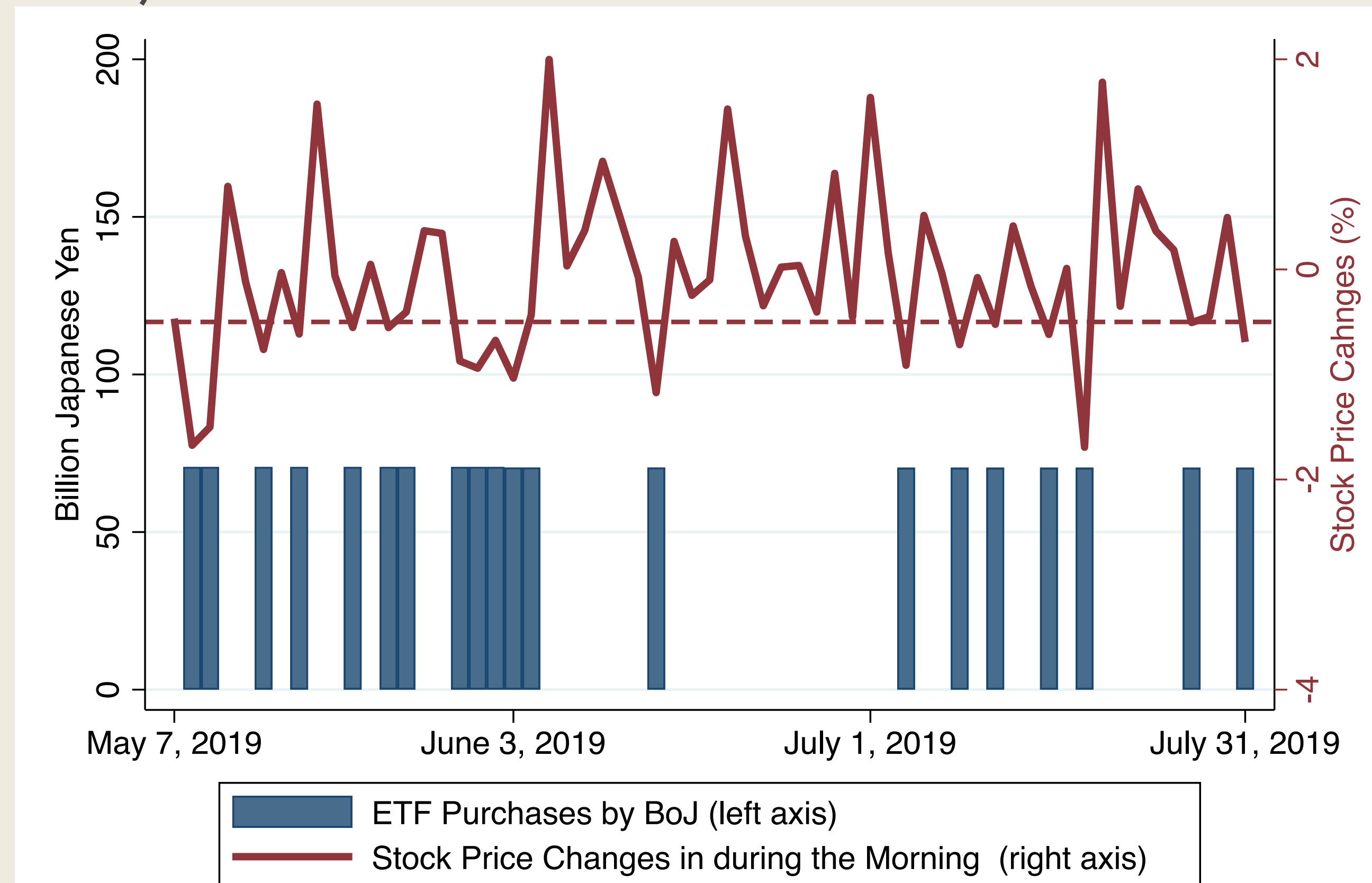
$y_{t,h}$: outcome at horizon h , E_t : BoJ purchases, S_t : stock market capitalization

Identification Strategy

- *“While the central bank has never made those conditions explicit, a decline in the Topix of 0.5% during the morning session was at one point seen to trigger purchases”*
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Selecting a Cutoff

- In general, the cut-off seems to be time-varying and therefore unknown
- We estimate the cut-off so as to maximize the discontinuity (Porter and Yu, 2015)
- In each sample split, for a given guess of the cut-off, we estimate

$$\Pr(E_t > 0 \mid \Delta p_t) = \Pr_{-,t}(E_t > 0 \mid \Delta p_t) \mathbb{I}(\Delta p_t < c_t) + \Pr_{+,t}(E_t > 0 \mid \Delta p_t) \mathbb{I}(\Delta p_t \geq c_t)$$

- Then we select the cut-off, c_t^* , so as to maximize the discontinuity

$$c_t^* \in \arg \max_{\bar{c} \in \mathbb{C}} \left[\lim_{\Delta p \uparrow \bar{c}} \widehat{\Pr}_{-,t}(E_t > 0 \mid \Delta p) - \lim_{\Delta p \downarrow \bar{c}} \widehat{\Pr}_{+,t}(E_t > 0 \mid \Delta p) \right]$$

- Sample split is based on (i) six major announcements by BoJ; and (ii) the stock market performance in the past two days

Identification

$$\beta_h = \frac{\lim_{c \uparrow c^*} \mathbb{E}[y_{t,h} - y_{t,0} \mid \Delta p_t = c, \mathbf{X}_t] - \lim_{c \downarrow c^*} \mathbb{E}[y_{t,h} - y_{t,0} \mid \Delta p_t = c, \mathbf{X}_t]}{\lim_{c \uparrow c^*} \mathbb{E}[E_t/S_t \mid \Delta p_t = c, \mathbf{X}_t] - \lim_{c \downarrow c^*} \mathbb{E}[E_t/S_t \mid \Delta p_t = c, \mathbf{X}_t]}$$

- Identification assumptions for fuzzy RDD:
 1. There is a jump in the size of intervention around the cutoff
 2. Structural error $\epsilon_{t,h}$ is continuous in Topix changes in the morning session
- ⇒ Substantially weaker assumptions than any of the existing works

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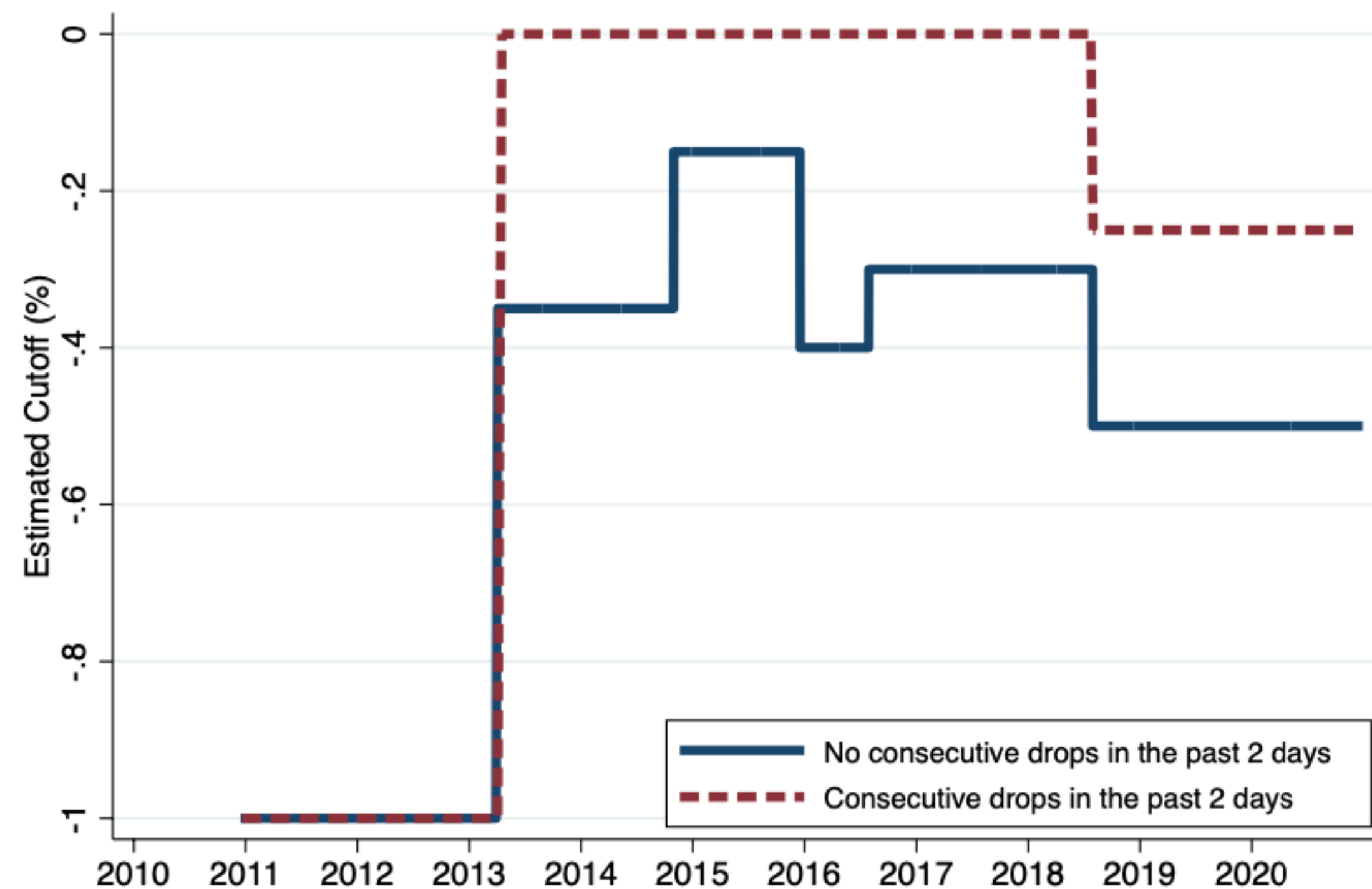
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■ What are we identifying?

- ✓ The impact of a shock to flow from bonds to stocks (**liquidity channel**)
- ✓ Unlikely to be **signaling channel**
 - BoJ announces the target amount of purchases in advance
- ✓ We cannot identify the effect of policy rules

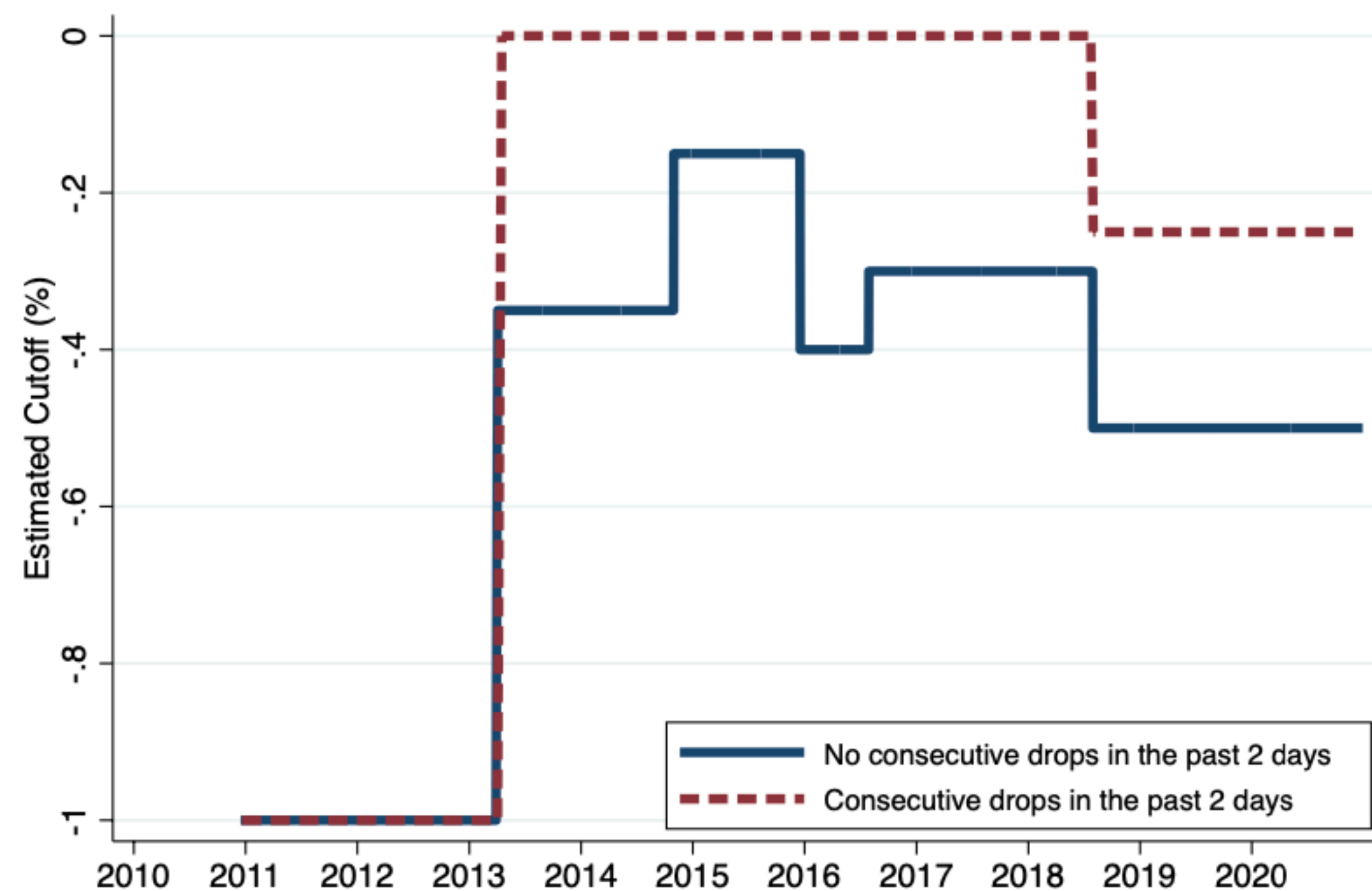
Cutoffs Estimation

Estimated Cutoffs over Time

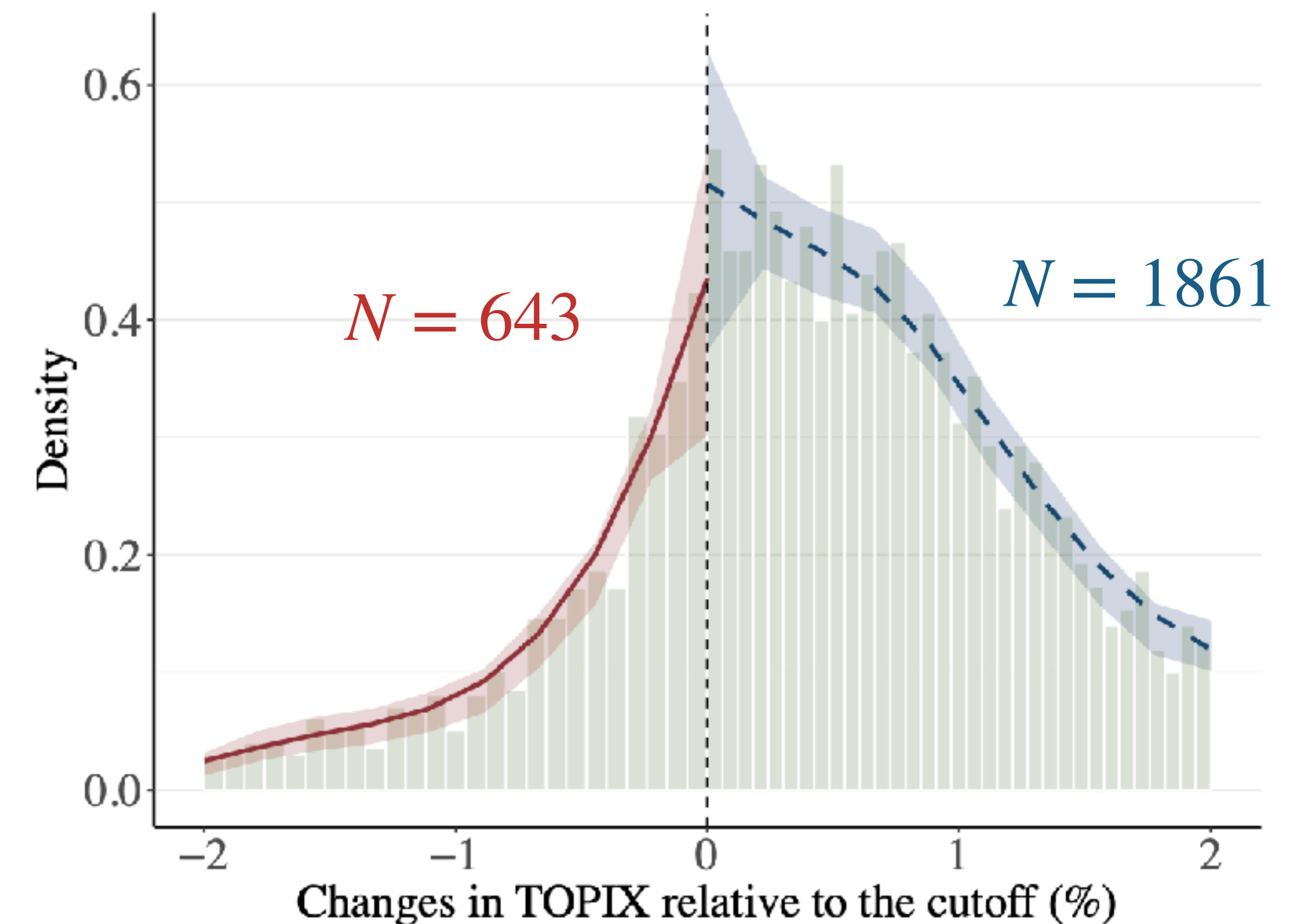


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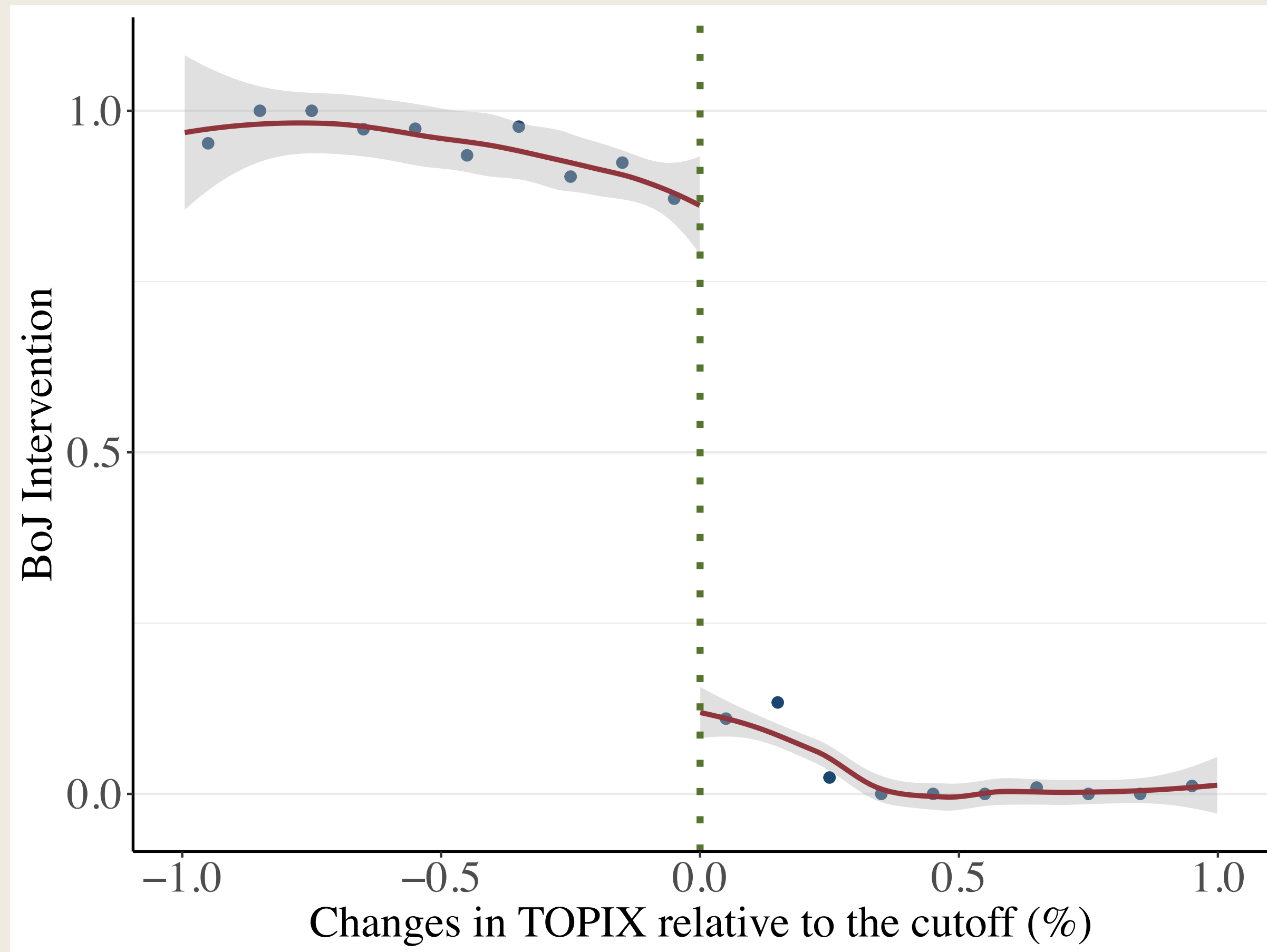


Density around the cutoff

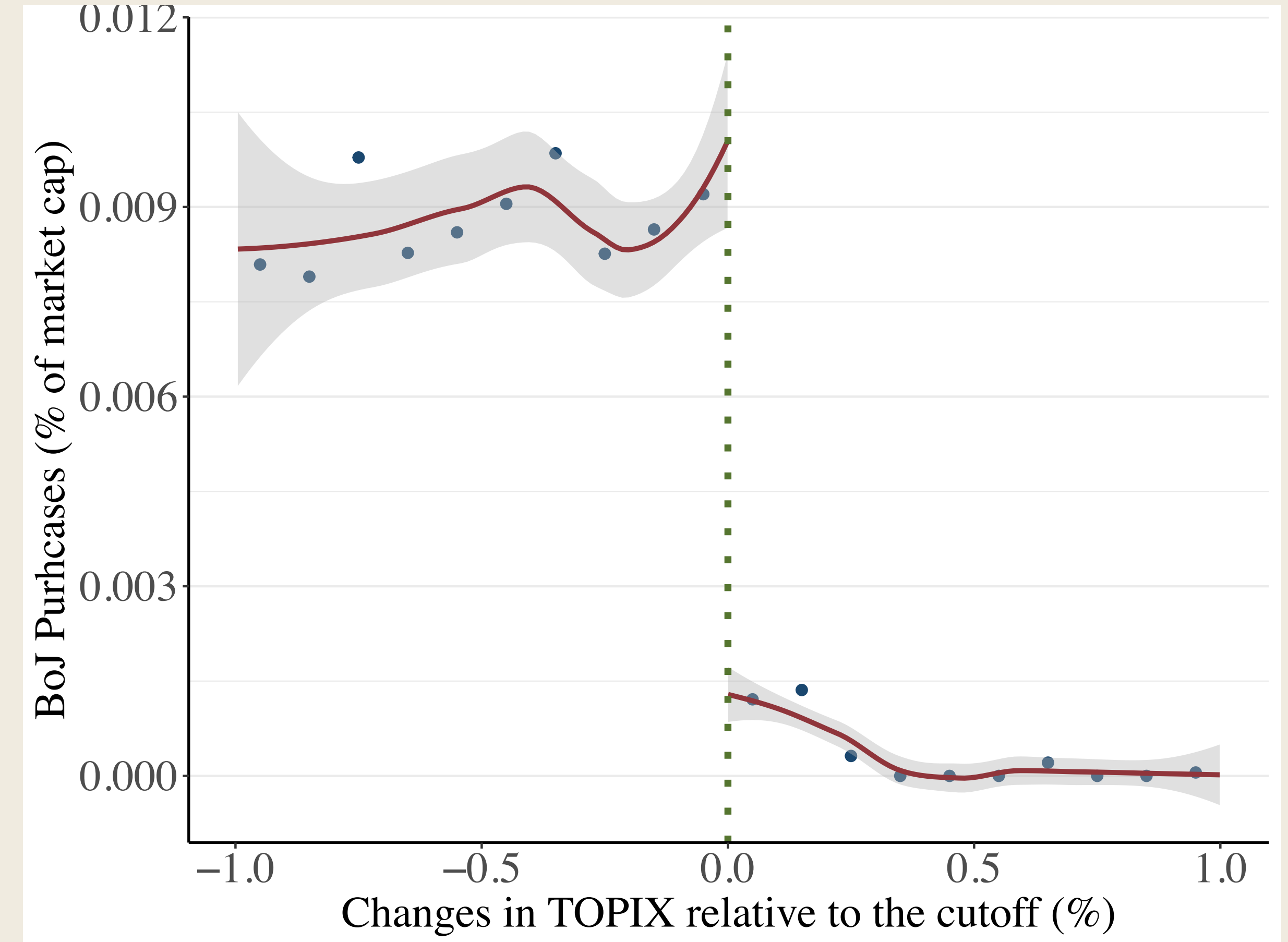


Discontinuity in Policy

Intervention Indicator

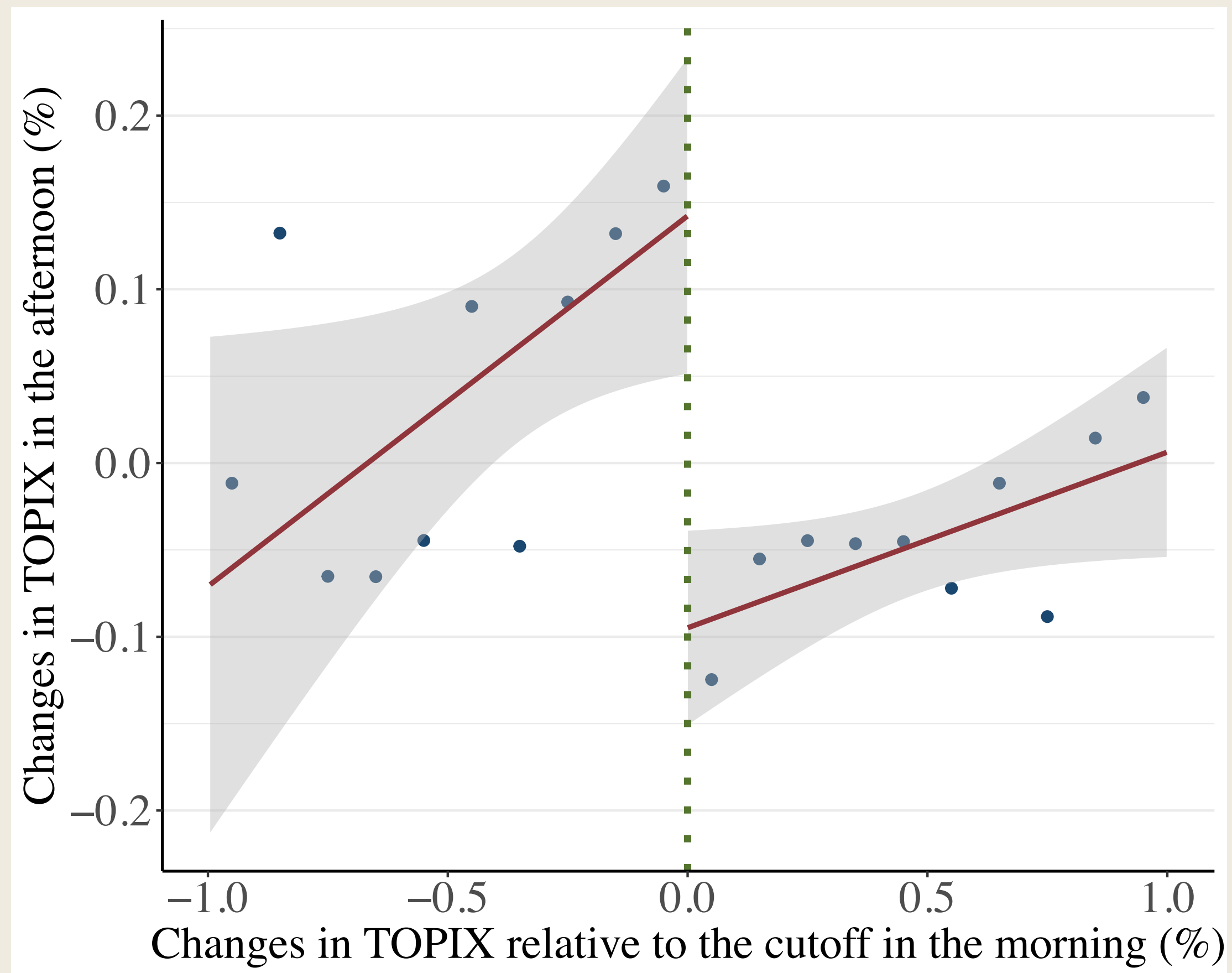


BoJ Purchases Amount



Discontinuity in Stock and Bond Prices

Stock (TOPIX) Price

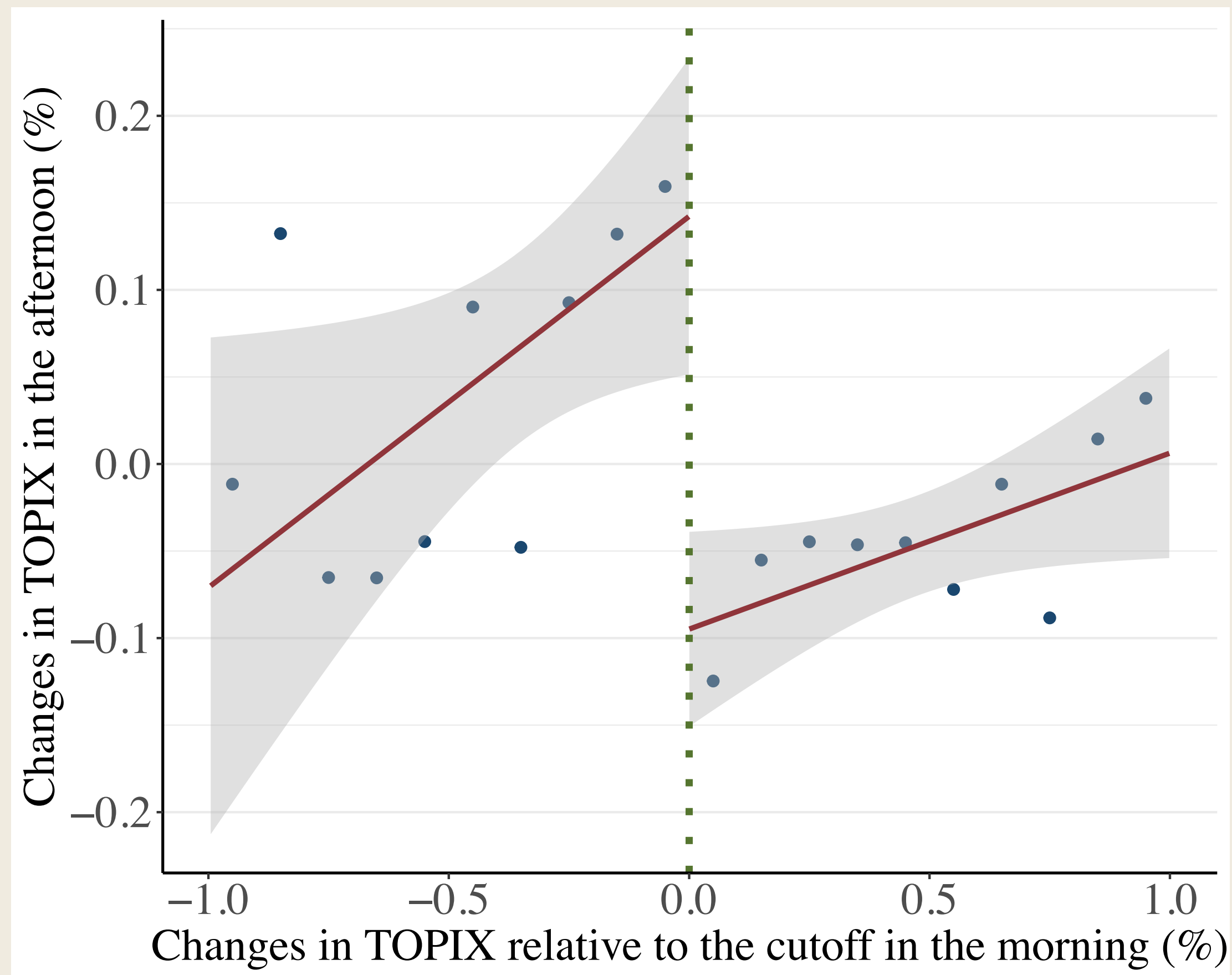


Note: binned-scatter plot with bin size 0.1%.

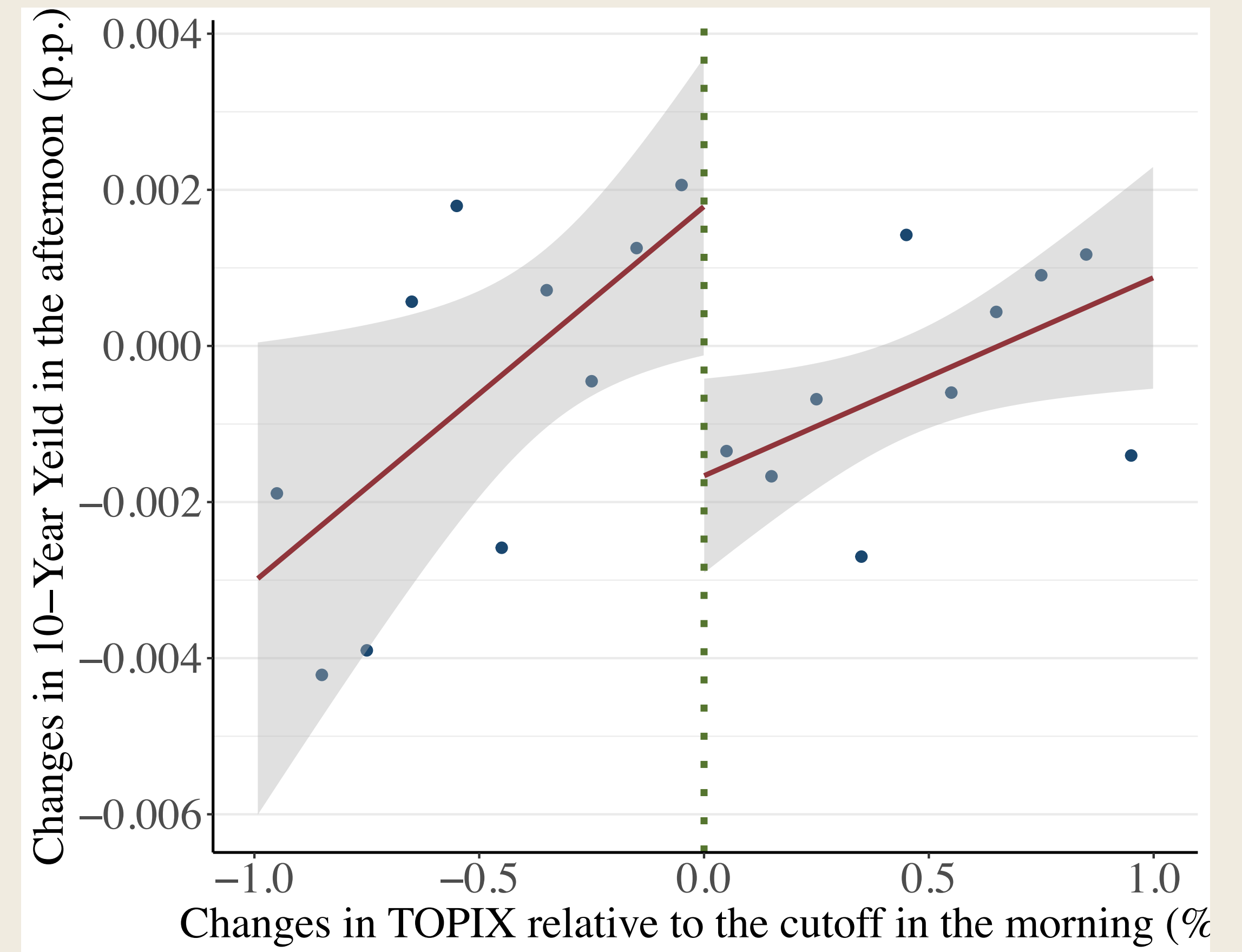
- Discontinuity in policy leads to discontinuities in stock return **and** JGB yield

Discontinuity in Stock and Bond Prices

Stock (TOPIX) Price



10-year JGB Yield



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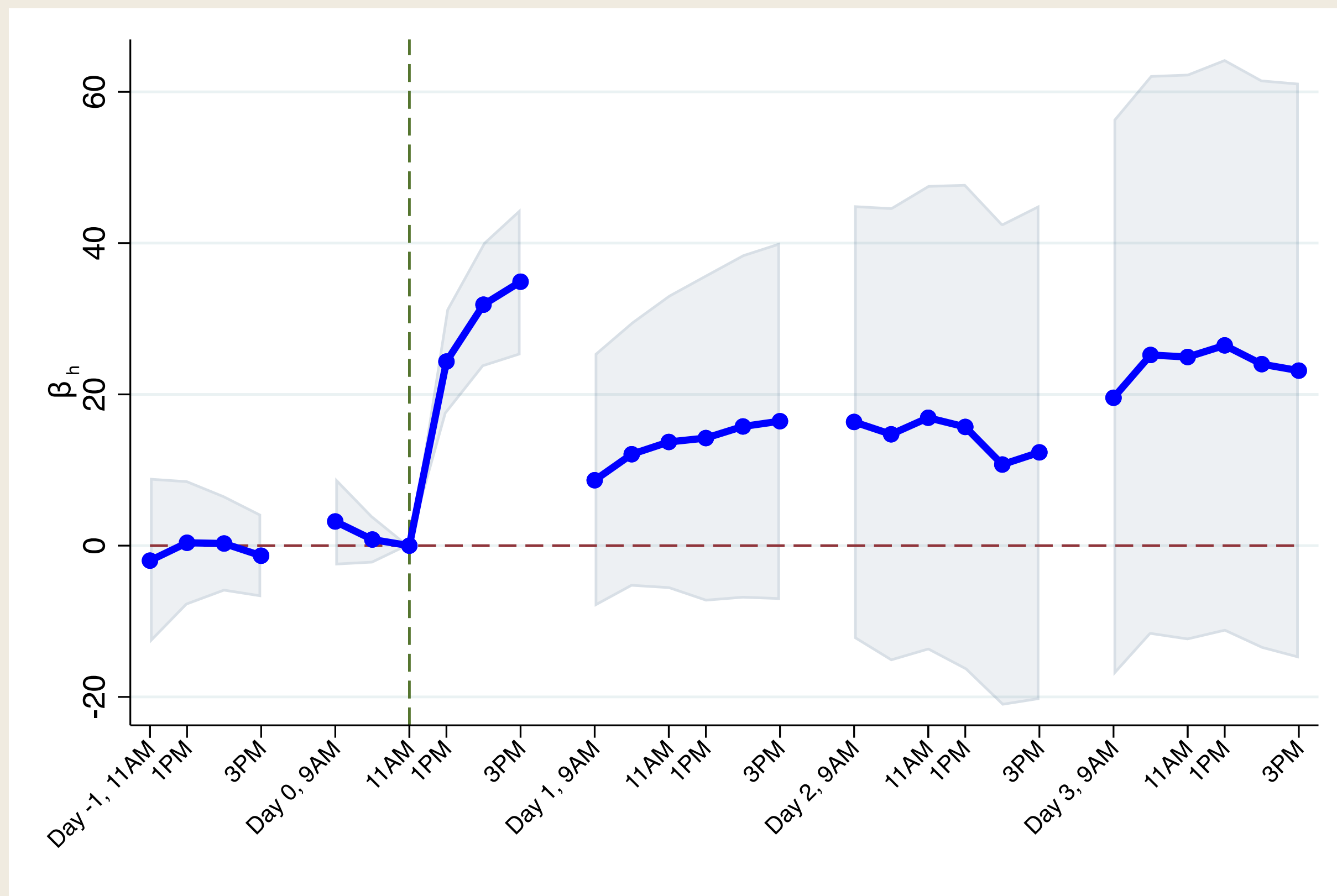
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Homogenous Responses

■ Fuzzy RDD:

$$y_{t,h} - y_{t,0} = \beta_h E_t / S_t + \gamma_h' \mathbf{X}_t + \epsilon_{t,h}$$

Stock (TOPIX) Price



Note: The shaded area represents 90% confidence interval, which accounts for heteroskedasticity and autocorrelation.

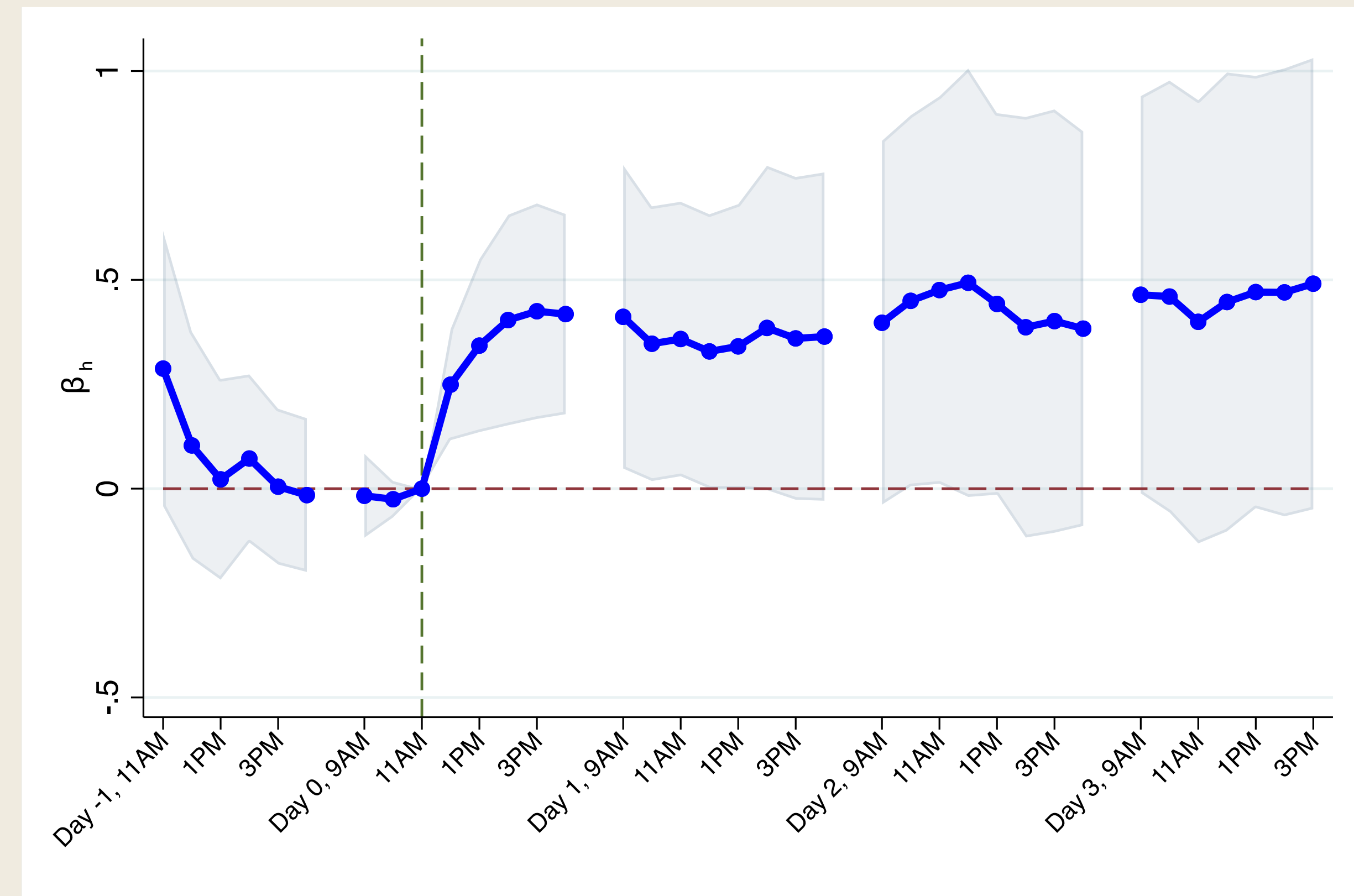
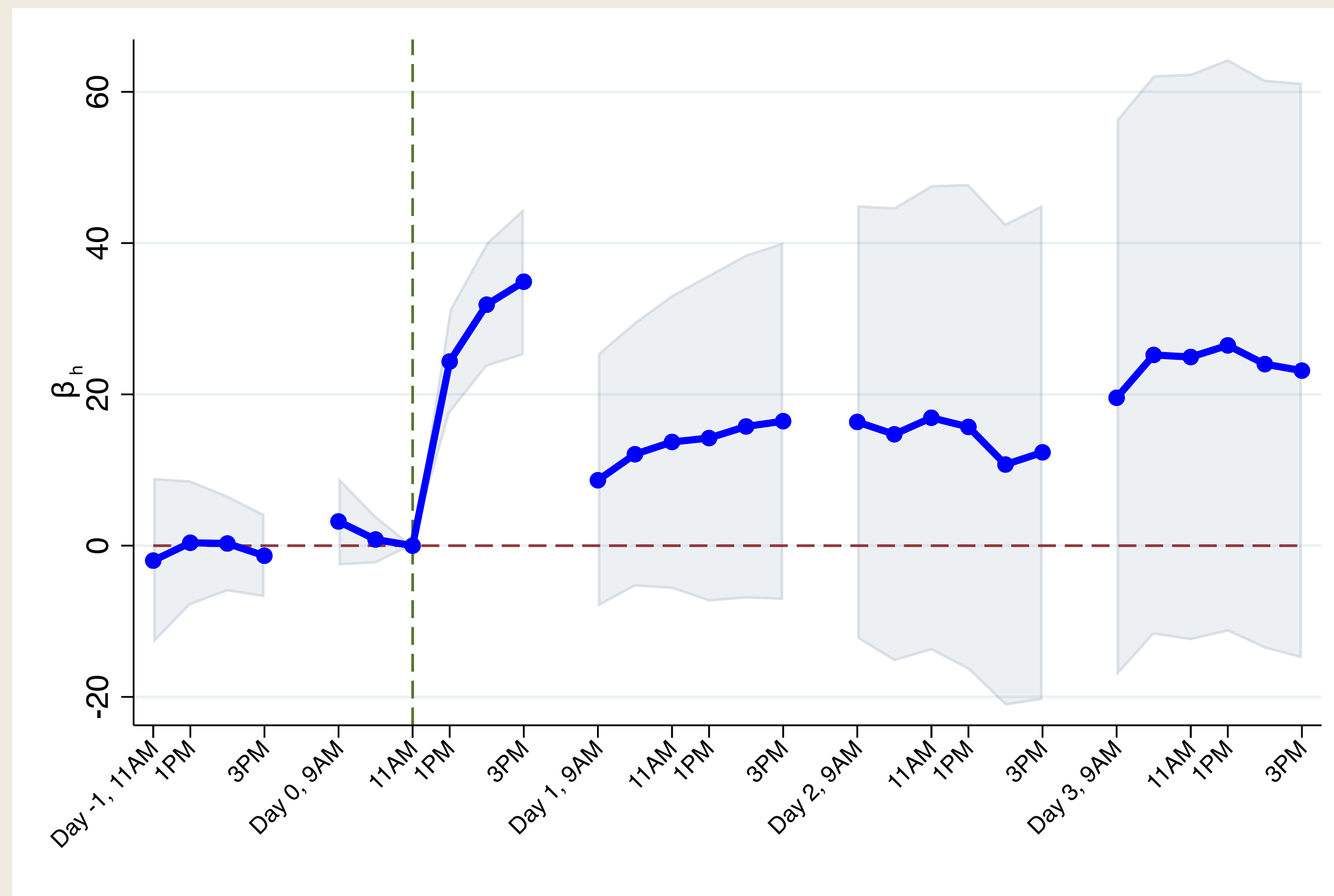
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Yield Curve Control (YCC)

- On Sep 2016, BoJ introduced “yield curve control” (YCC)
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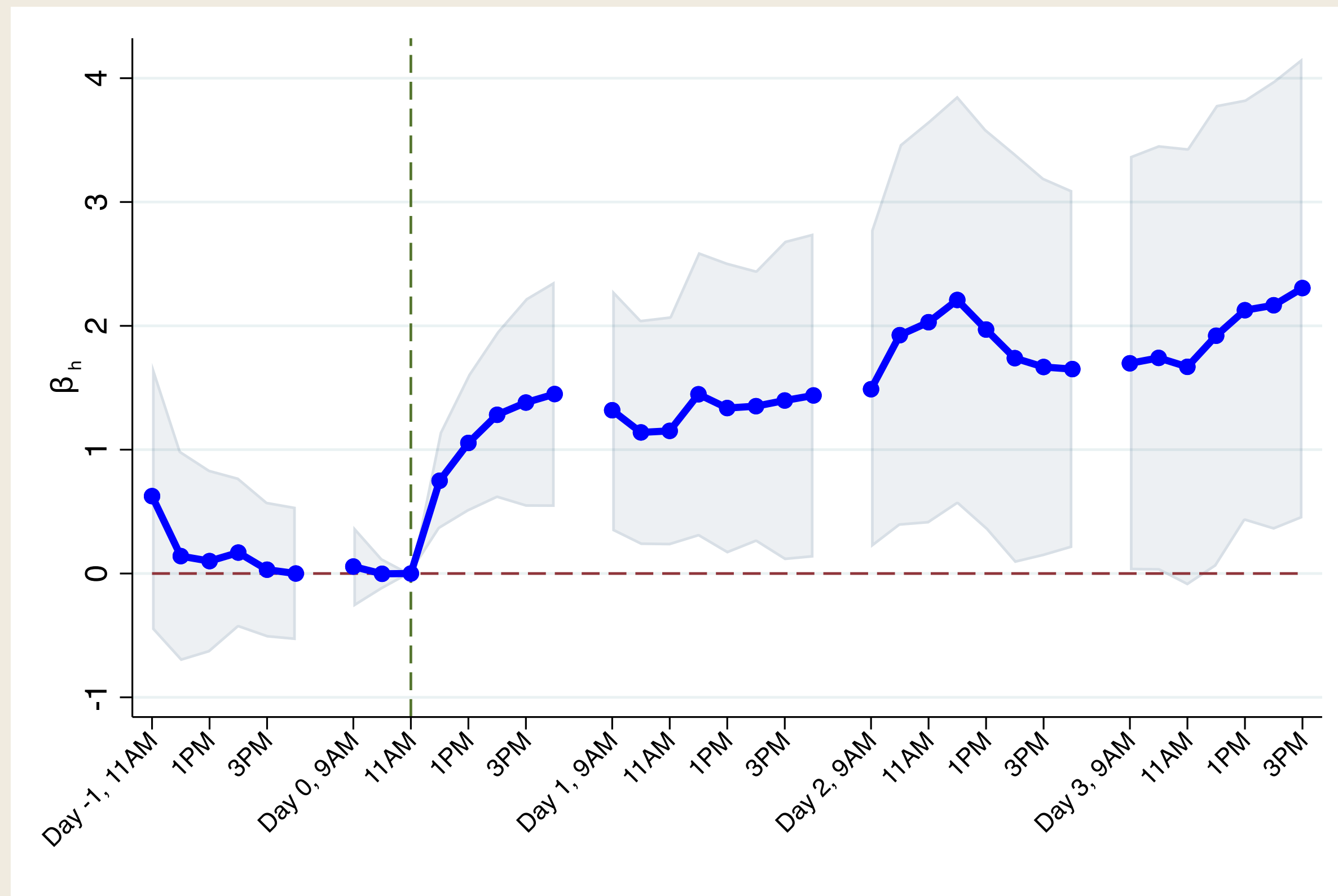
10-year Government Bond Yield



Heterogenous Interest Rate Responses

10-year JGB Yield

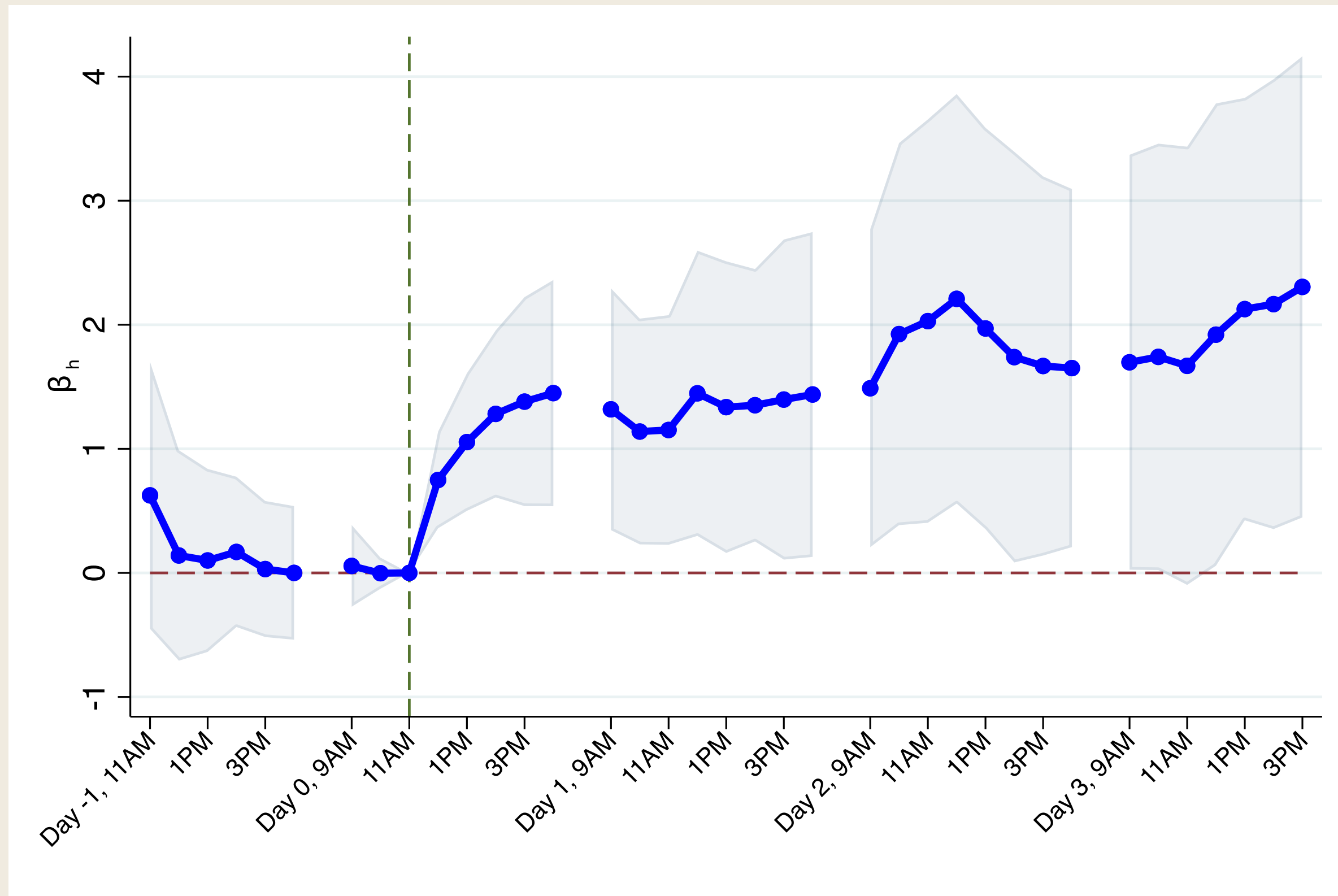
Before YCC



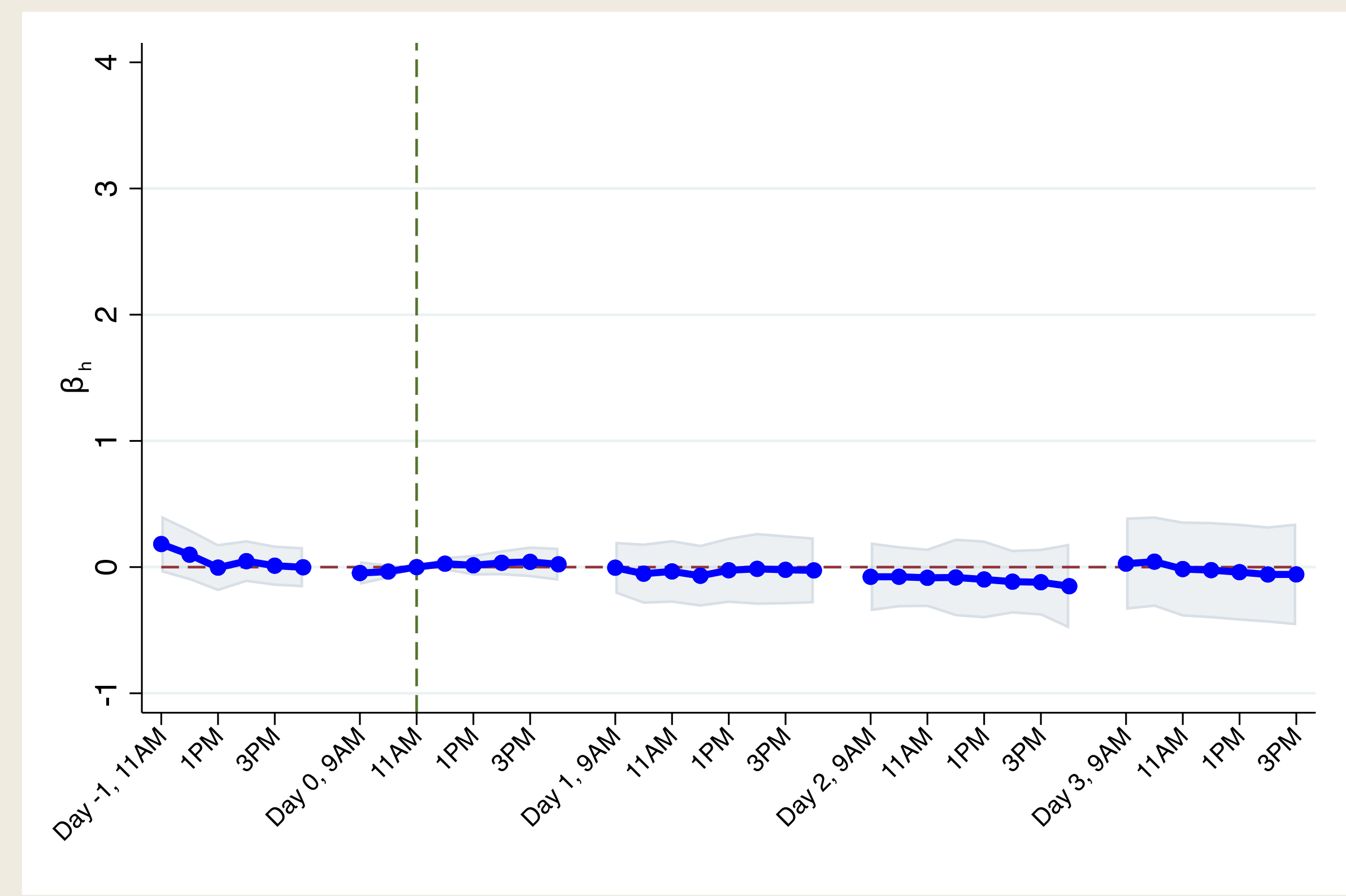
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10-year JGB Yield

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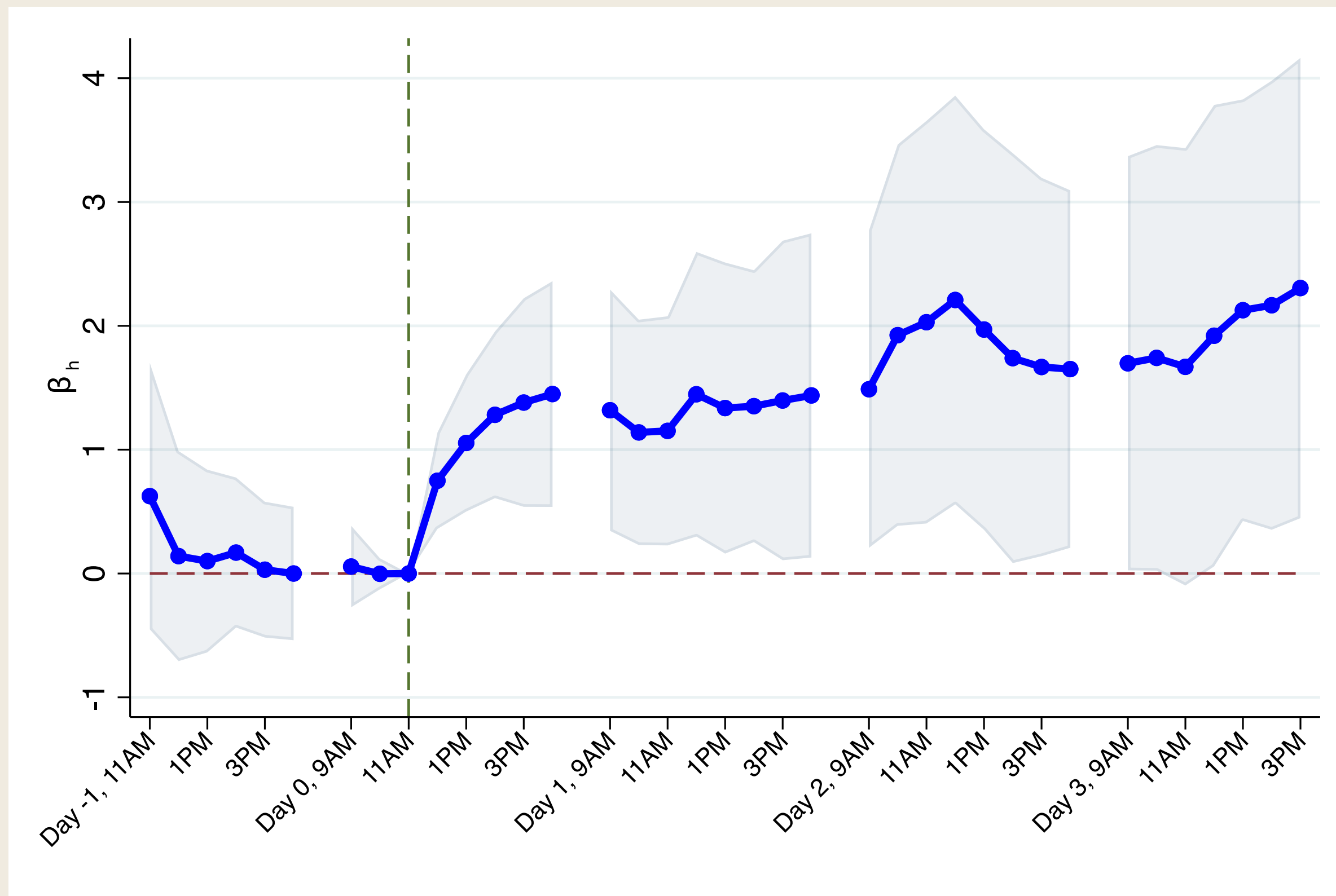
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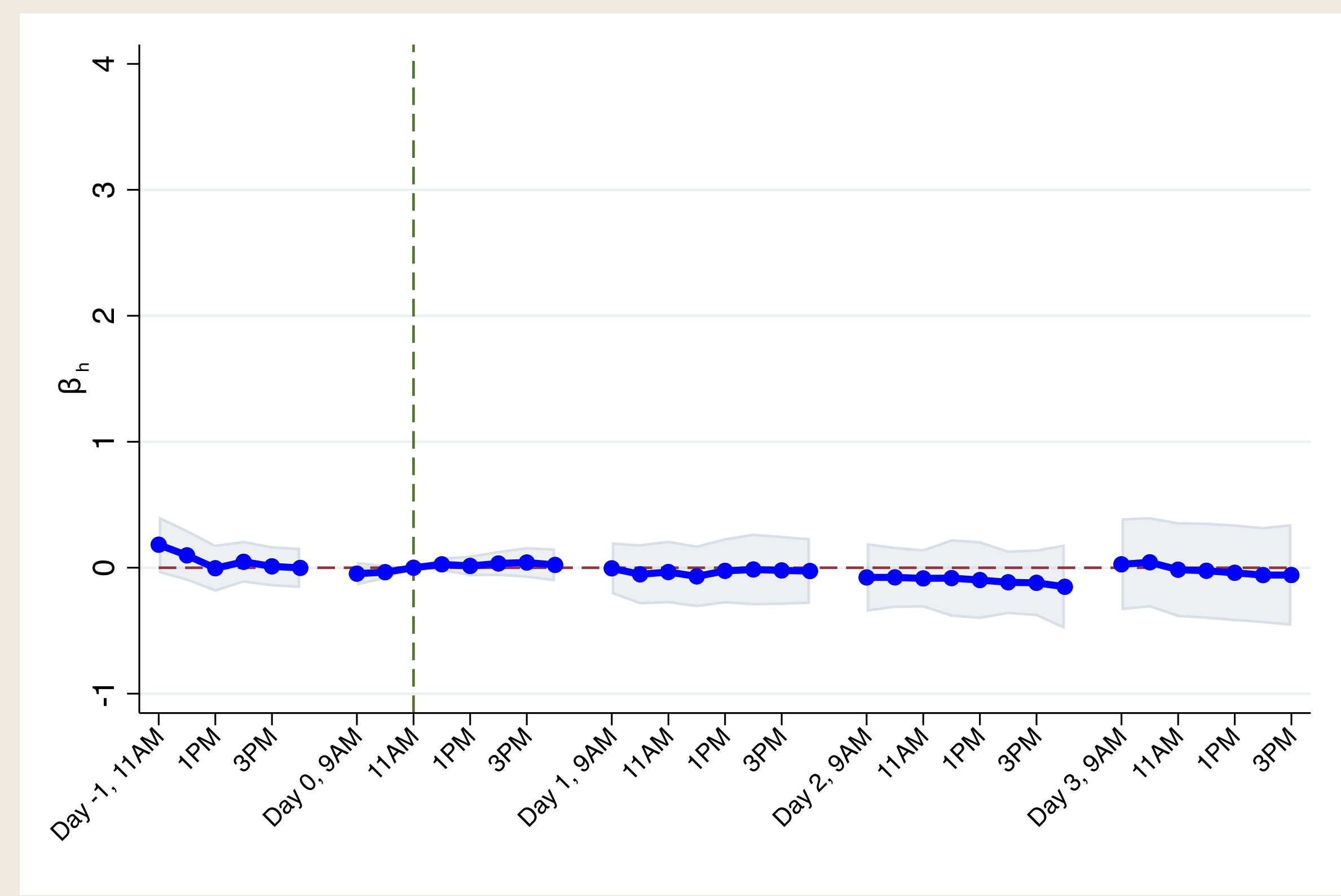
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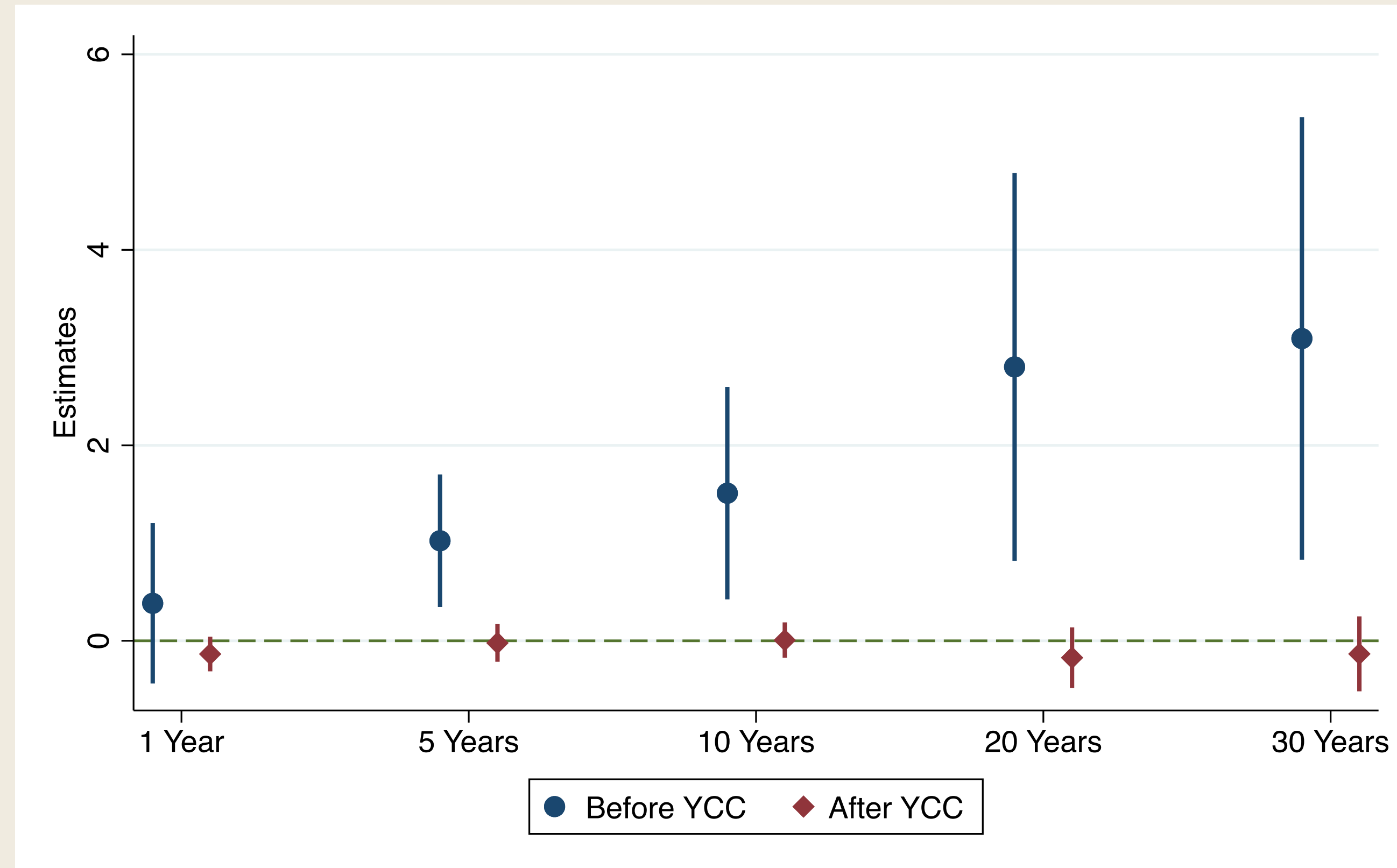
After YCC



- In response to a 0.01% purchase of stocks by BoJ, long-term rate (i) rose by 1.4 b.p. before YCC; (ii) stopped responding after YCC

Response of Yield Curve

Next-day Response of Yields across Different Maturities

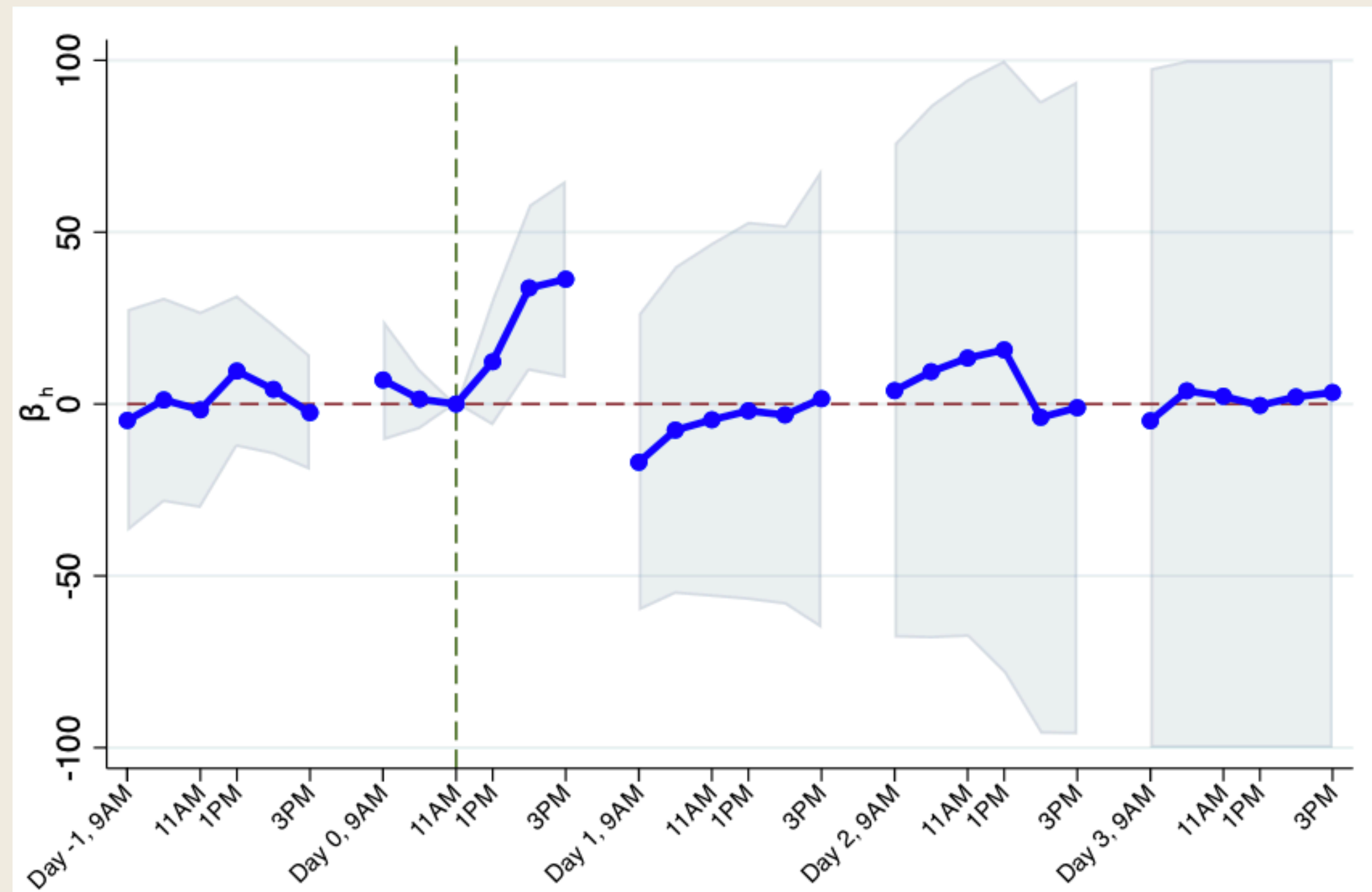


- Smaller responses for shorter maturity before YCC
⇒ Our interpretation is that ZLB prevented the response of shorter maturity

Heterogenous Stock Price Responses

Stock (TOPIX) Price Changes

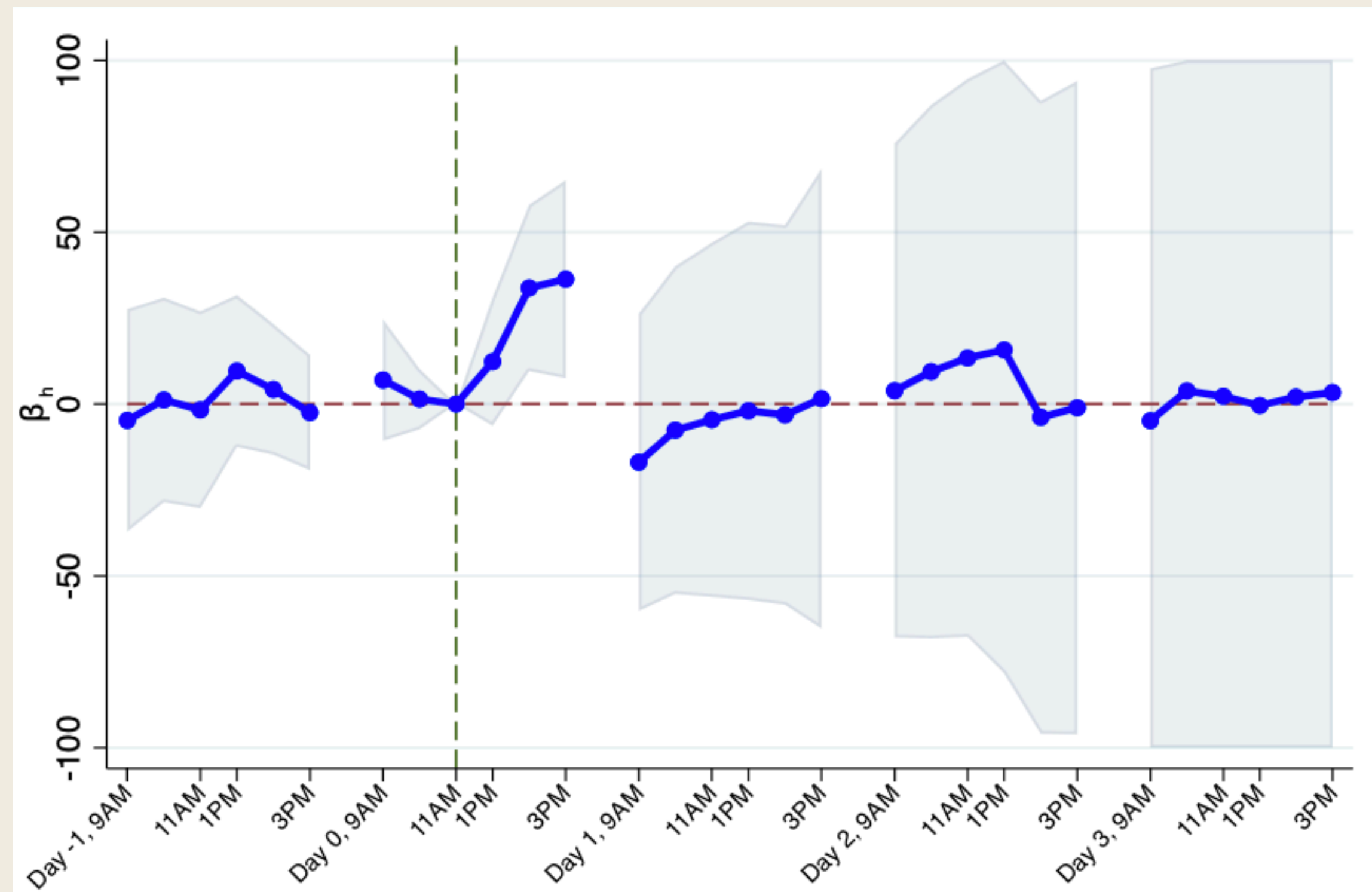
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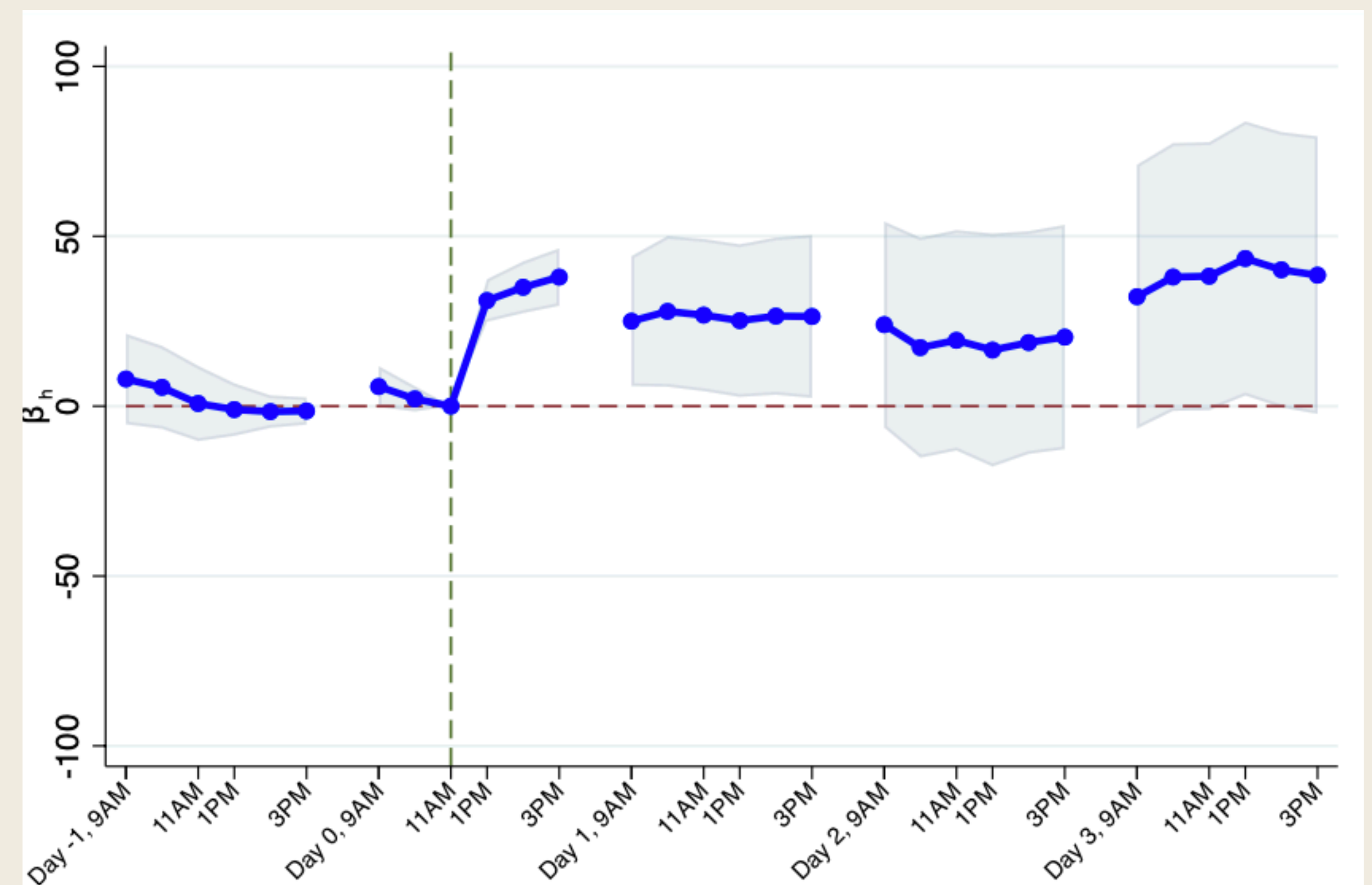
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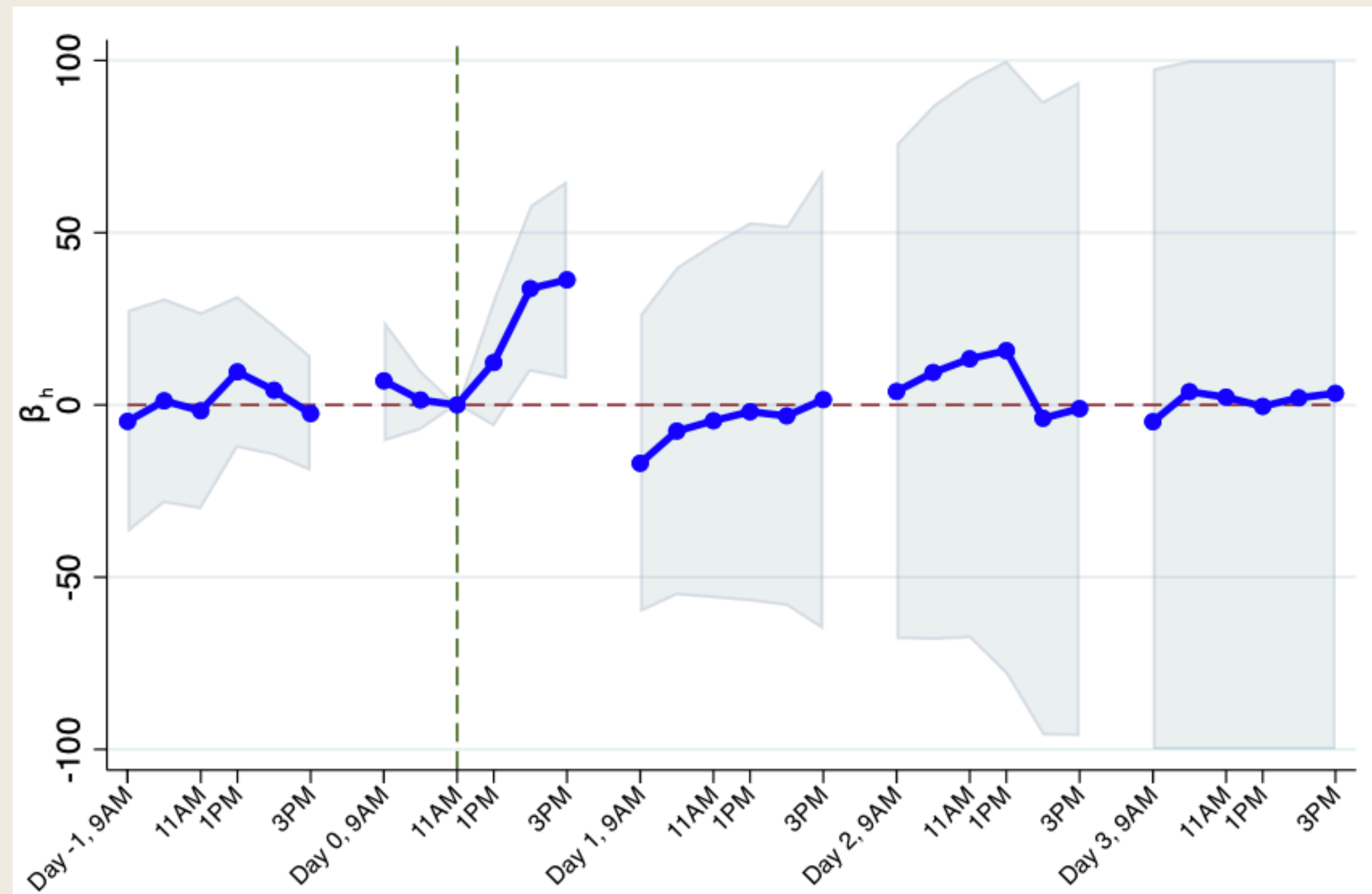
After YCC



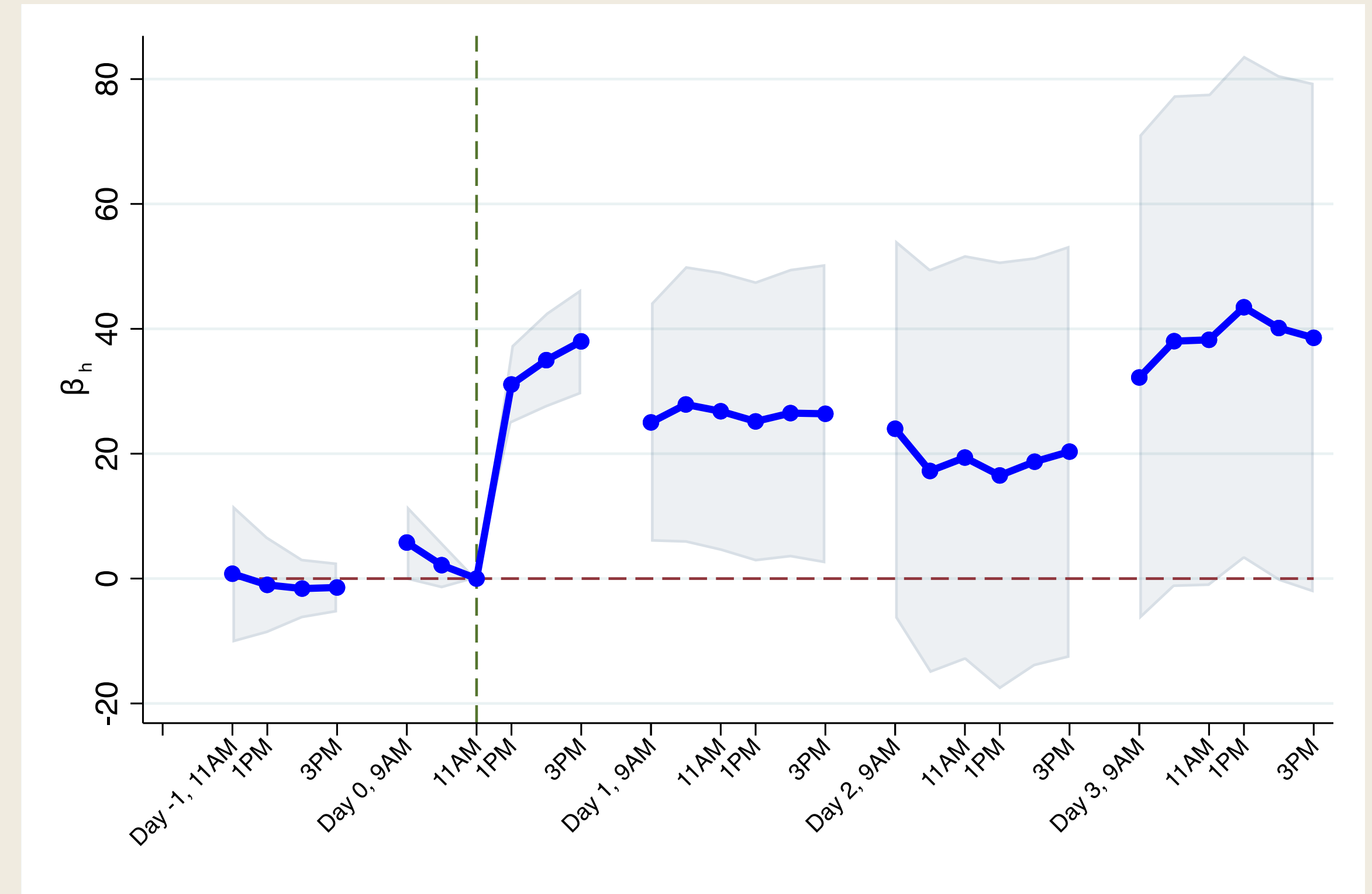
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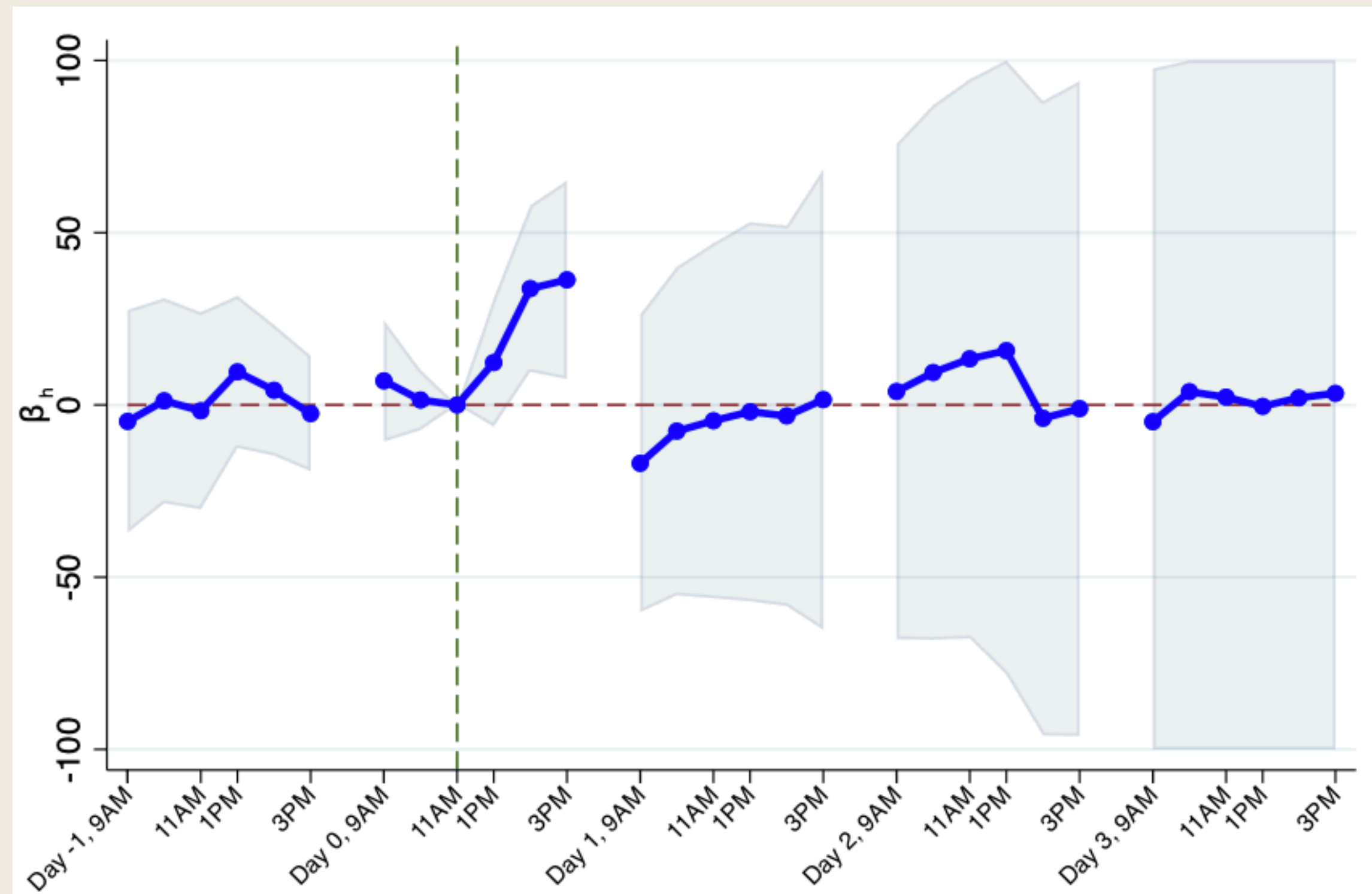
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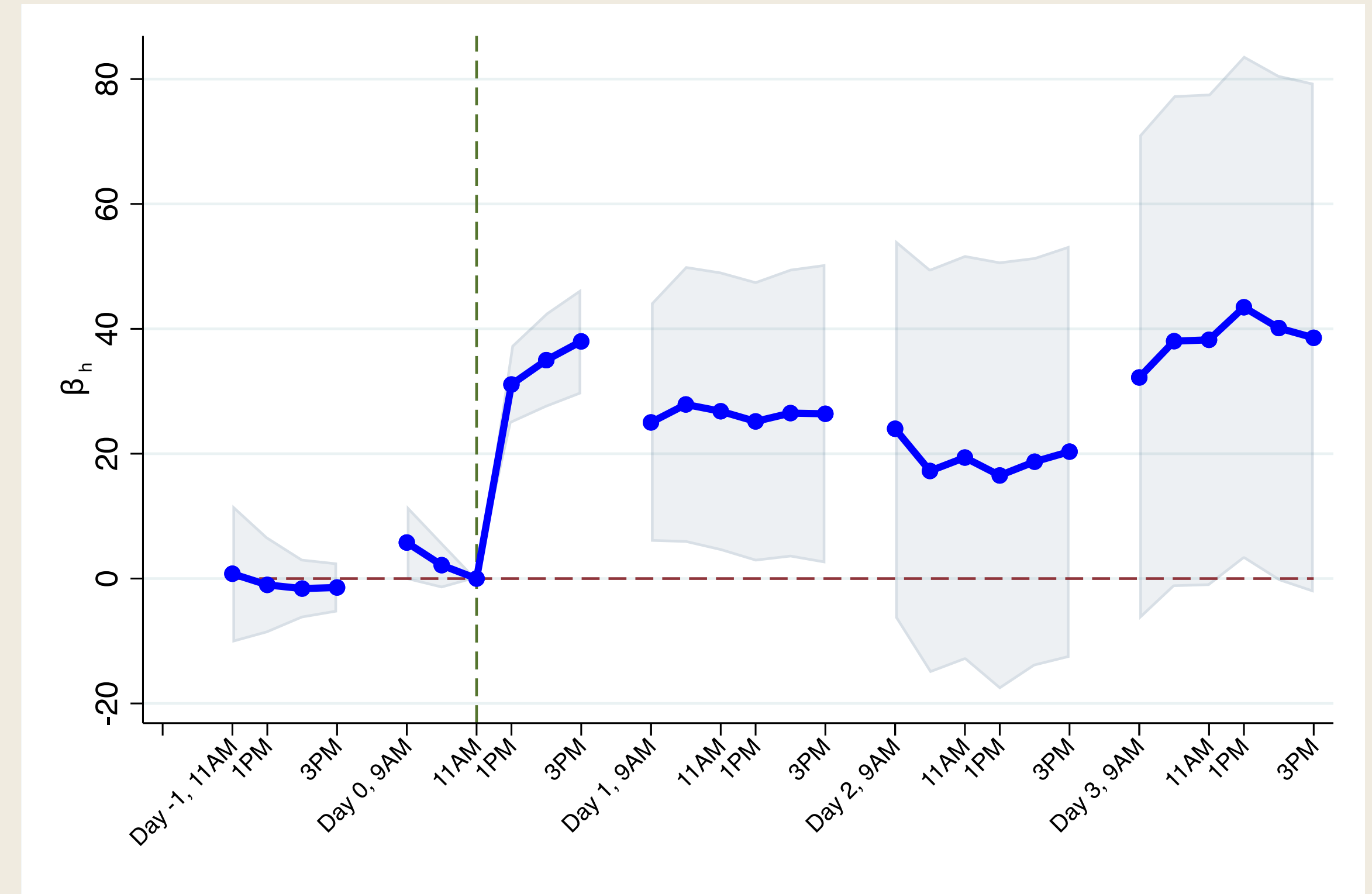
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After YCC

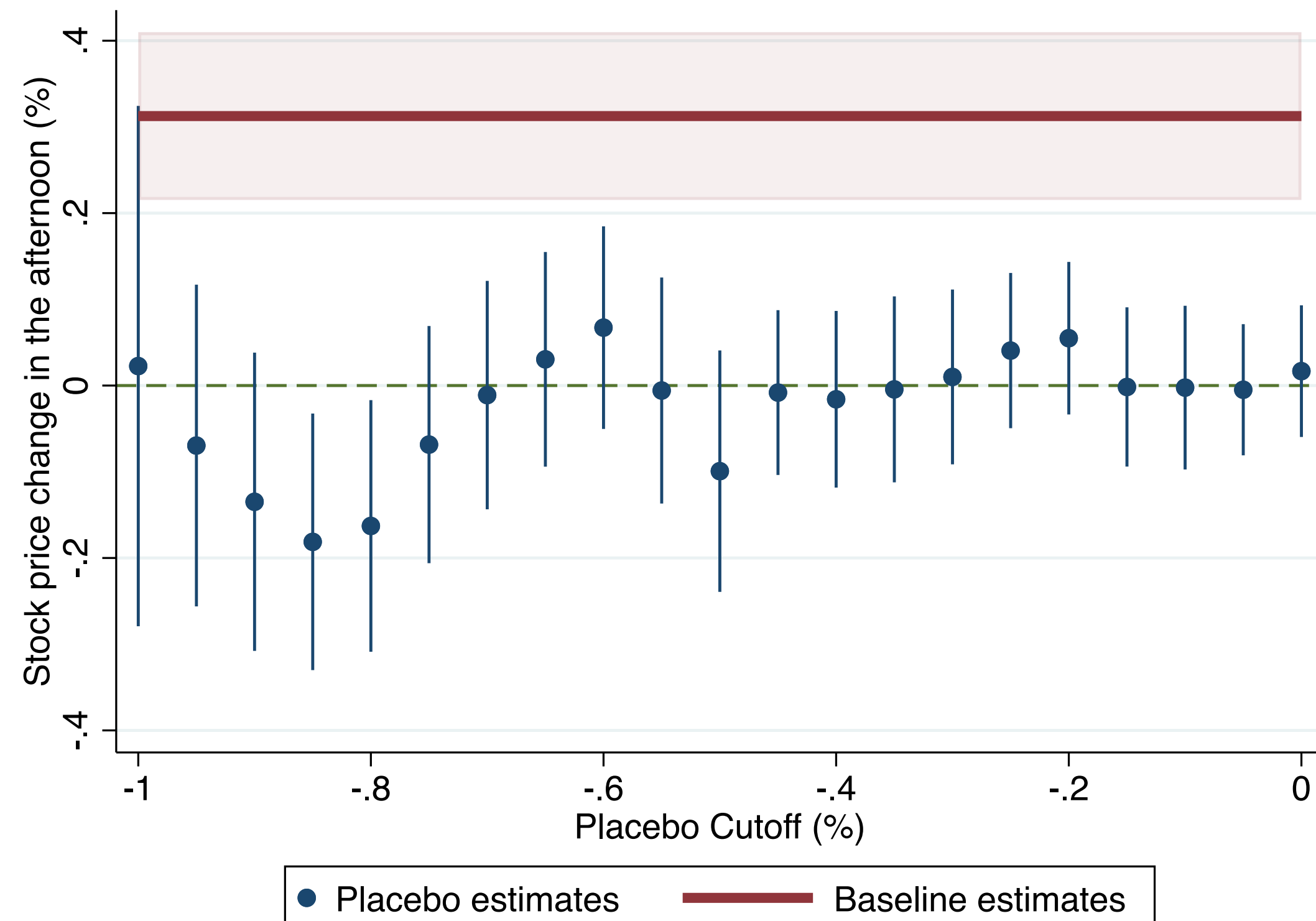


- In response to a 0.01% purchase of stocks by BoJ,
(i) noisy zero effect before YCC; (ii) 0.22% increase in stock prices after YCC

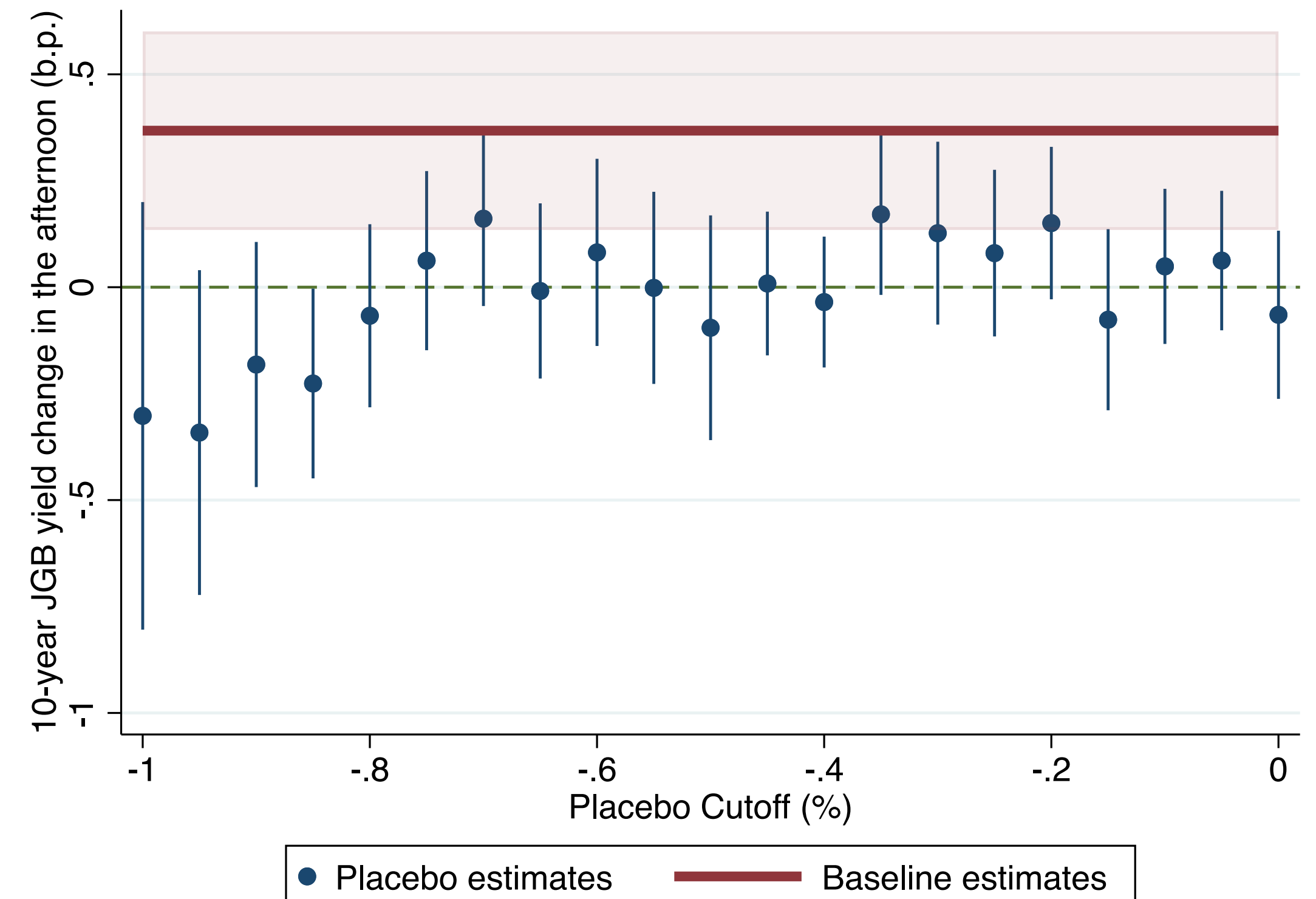
Placebo Tests

- Run the same regression with arbitrary chosen cutoffs (excluding true ones)
 - no significant effect around the cutoff in which there is no policy discontinuity

Stock (TOPIX) Price



10-year JGB Yield



Taking Stock

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- Summary of empirical results
 1. Before YCC, BoJ stock purchases mostly ended up increasing interest rates
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 1. Before YCC, BoJ stock purchases mostly ended up increasing interest rates
 2. After YCC, interest rates stopped responding and stock prices robustly rise
- Results robust to ([Table](#))
 - ✓ alternative bandwidths & polynomial orders
 - ✓ controls for past outcomes, policies
 - ✓ dropping the periods around cut-off changes

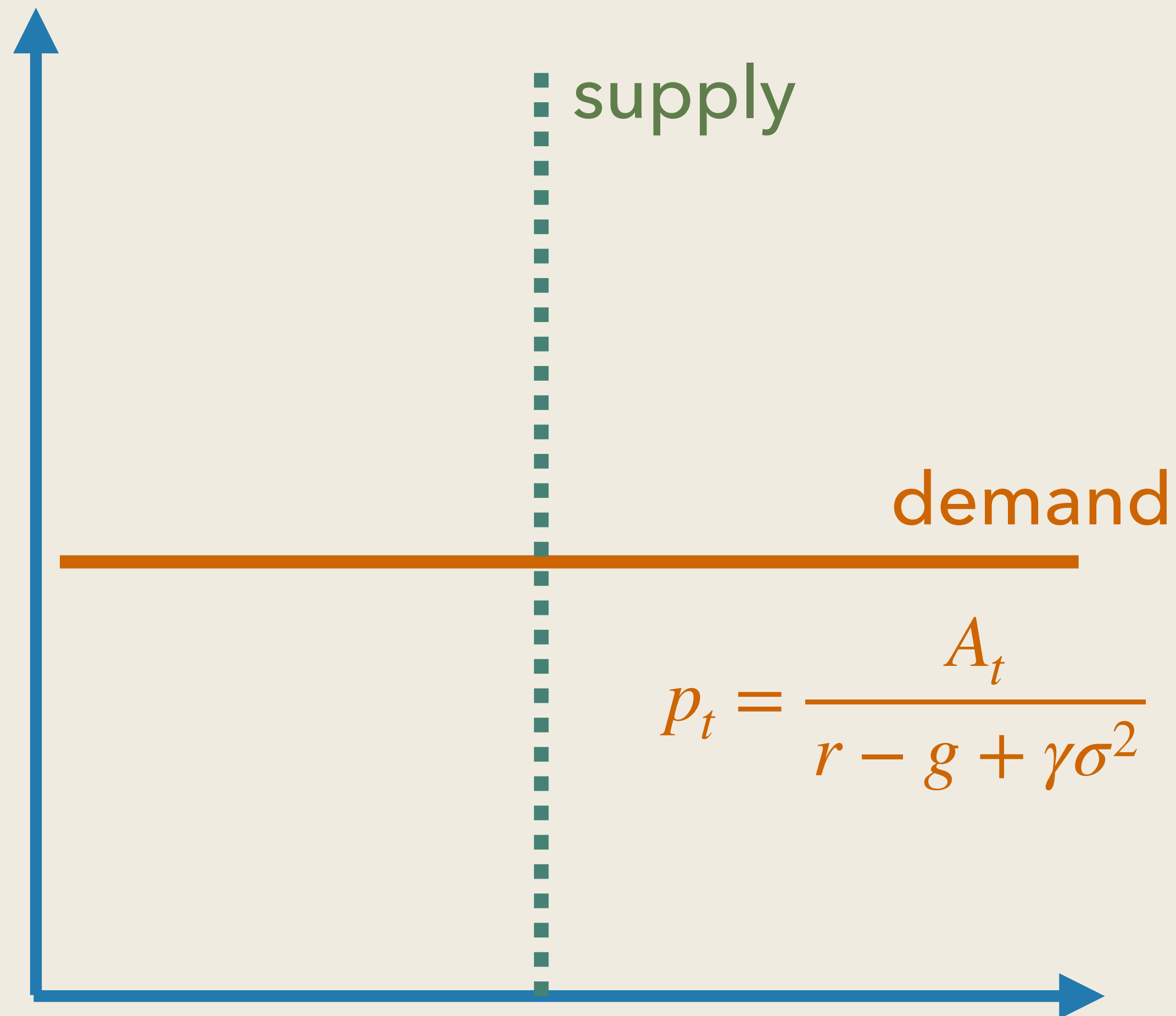
Theoretical Framework

Model Summary

- Single fixed factor (capital) with $Y_t = A_t K$ and $\log A_{t+1}/A_t \sim N(g - \sigma^2/2, \sigma^2)$
- Households decide saving with bonds in utility function, $\sum_t \beta^t [u(C_t) + v_b(B/Y)]$
- Mutual funds invest in stocks with portfolio adjustment cost, $v_s(s)$
- Central bank chooses the holdings of bonds and stocks as well as lump-sum taxes

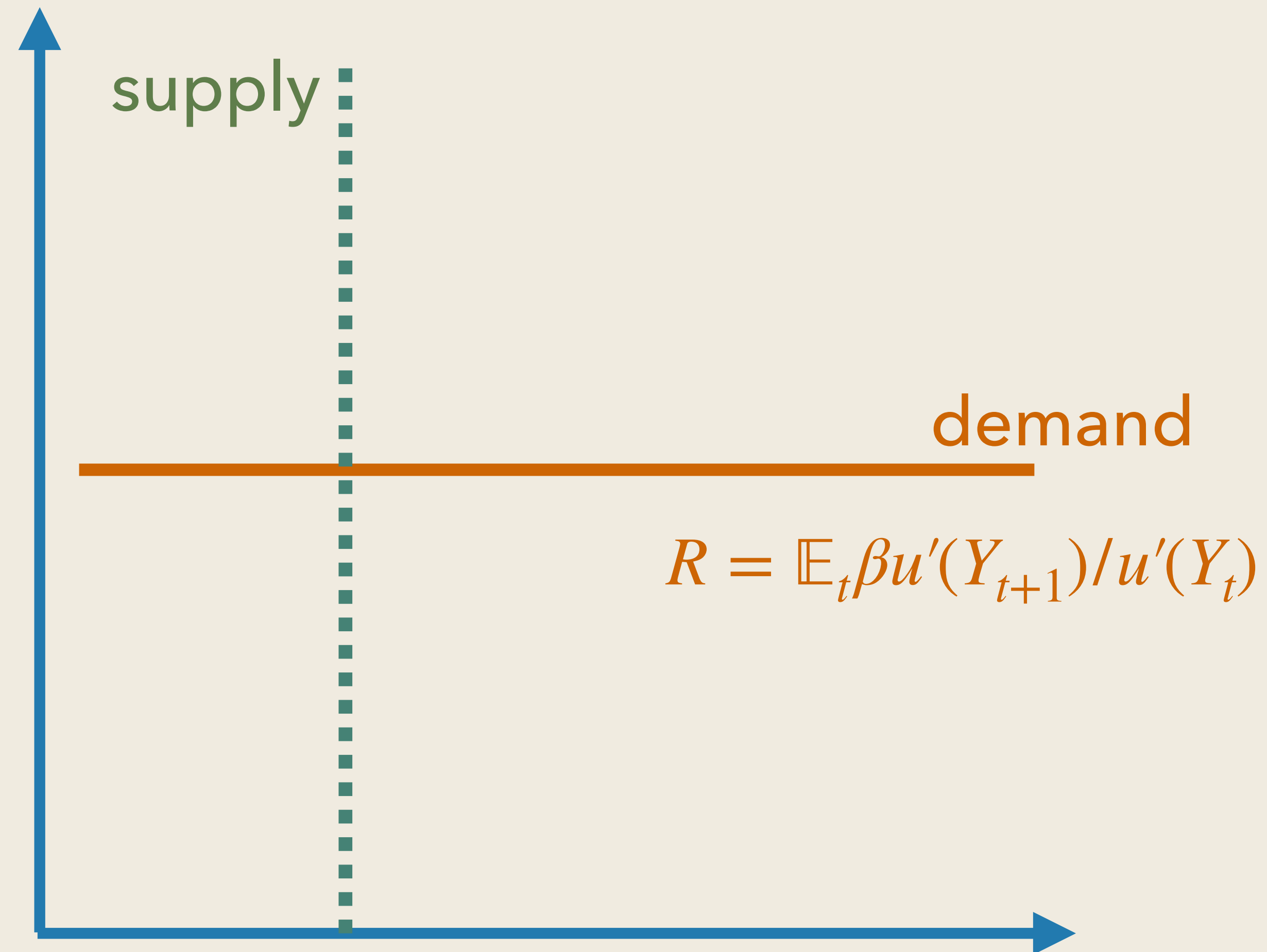
Frictionless Economy ($v_b'' = v_s'' = 0$)

stock price, p



stock quantity in the market, S

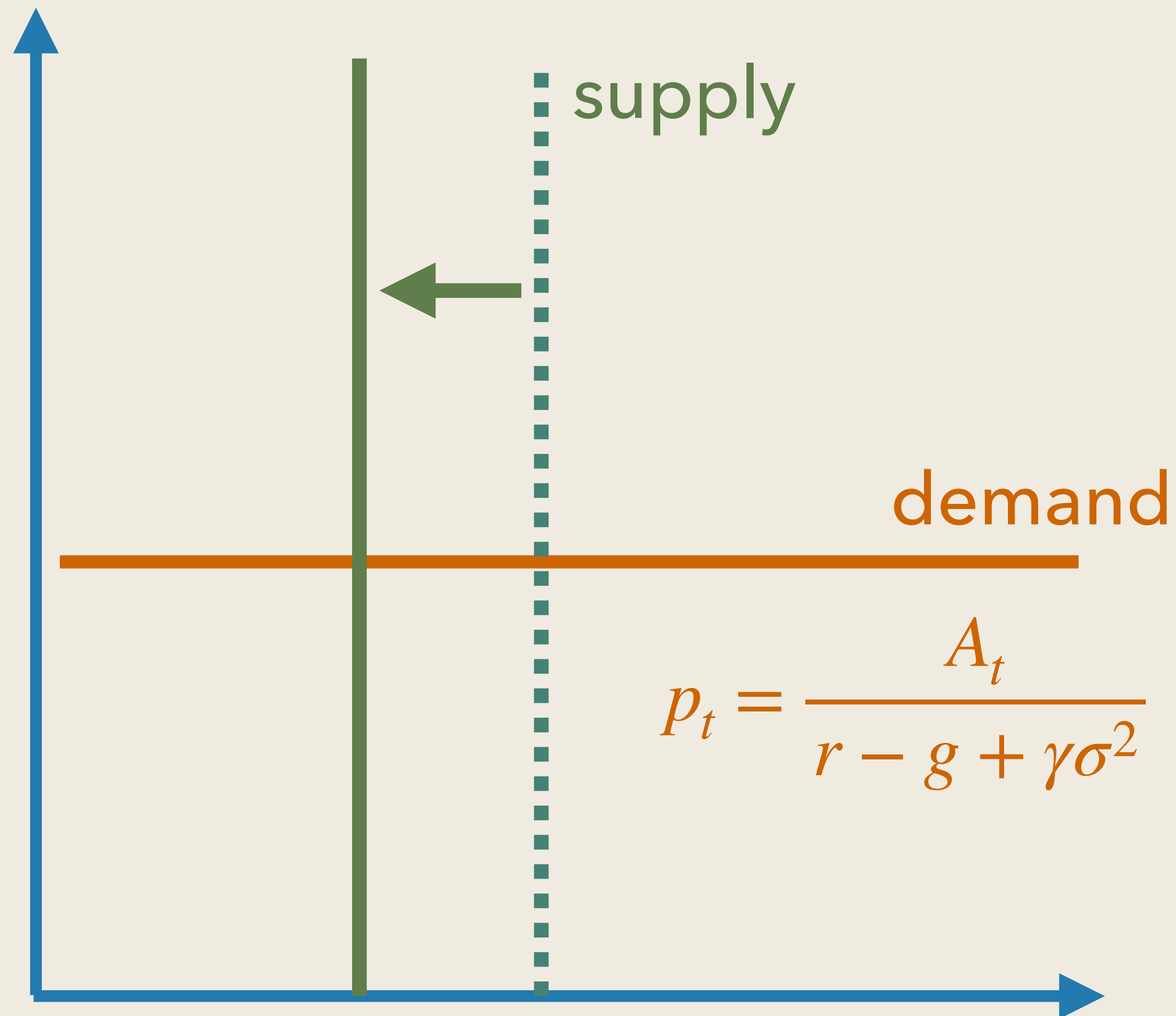
interest rate, R



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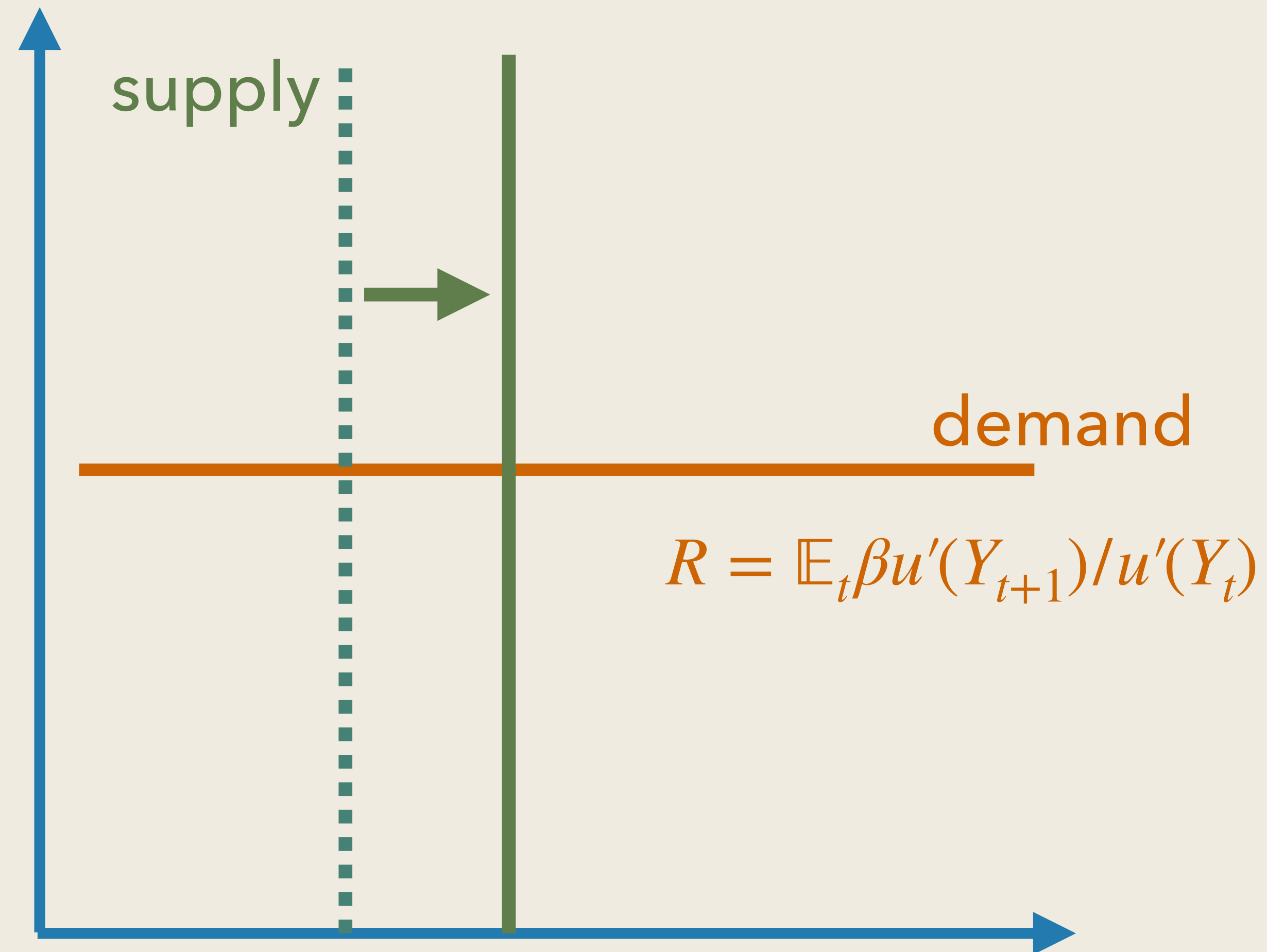
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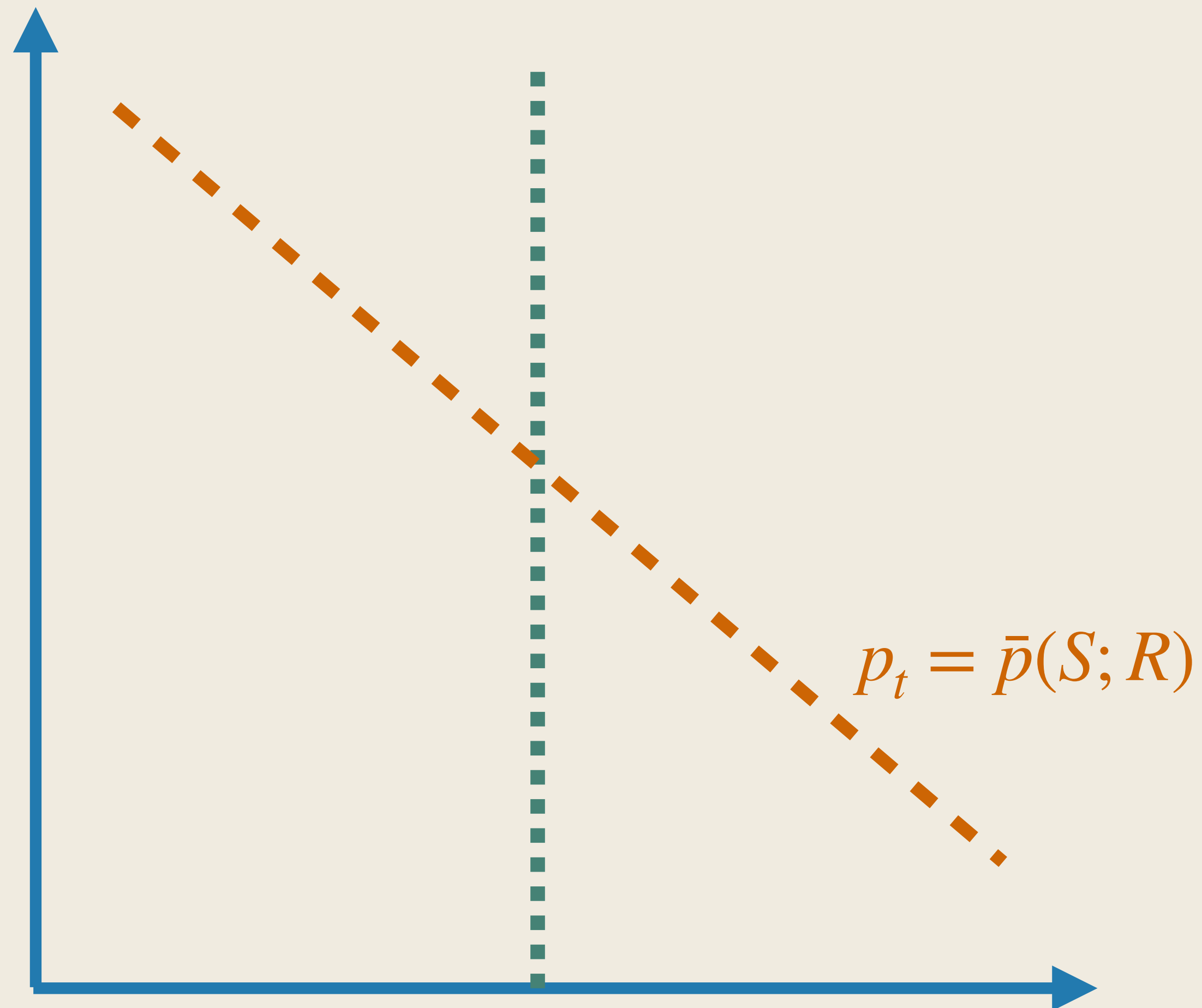
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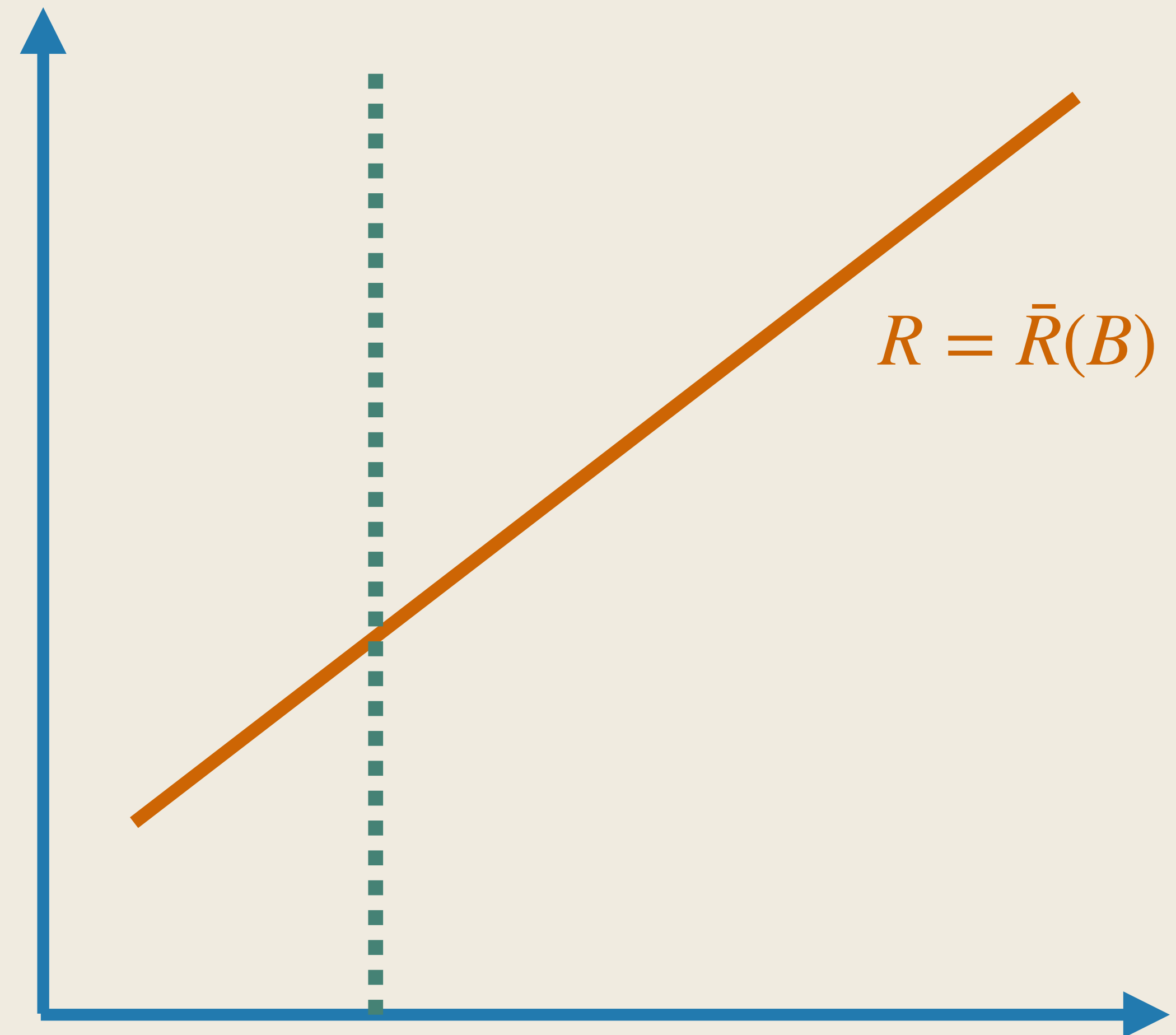
Inelastic Stock and Bond Market ($v_s'' > 0, v_b'' < 0$)

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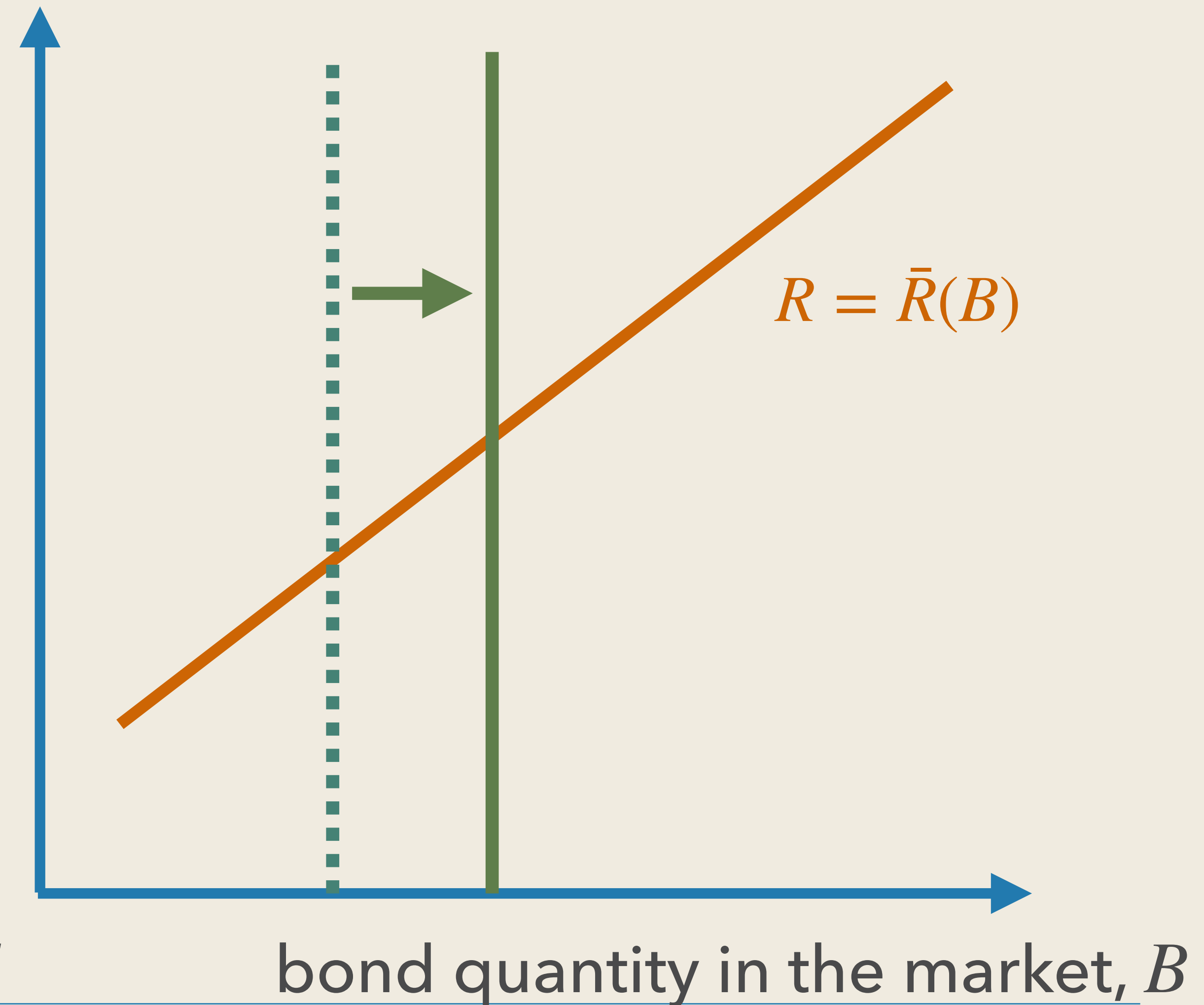
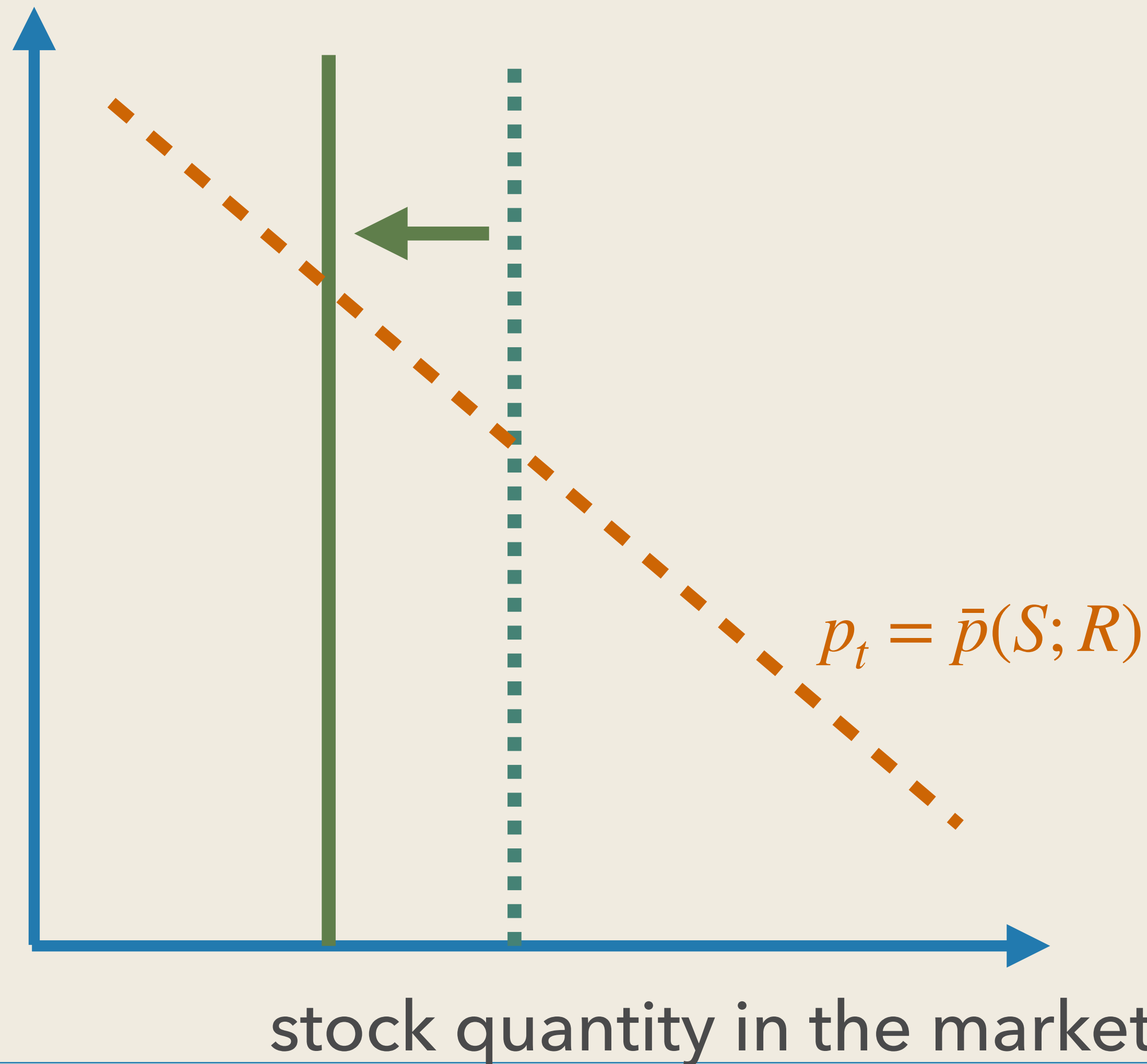


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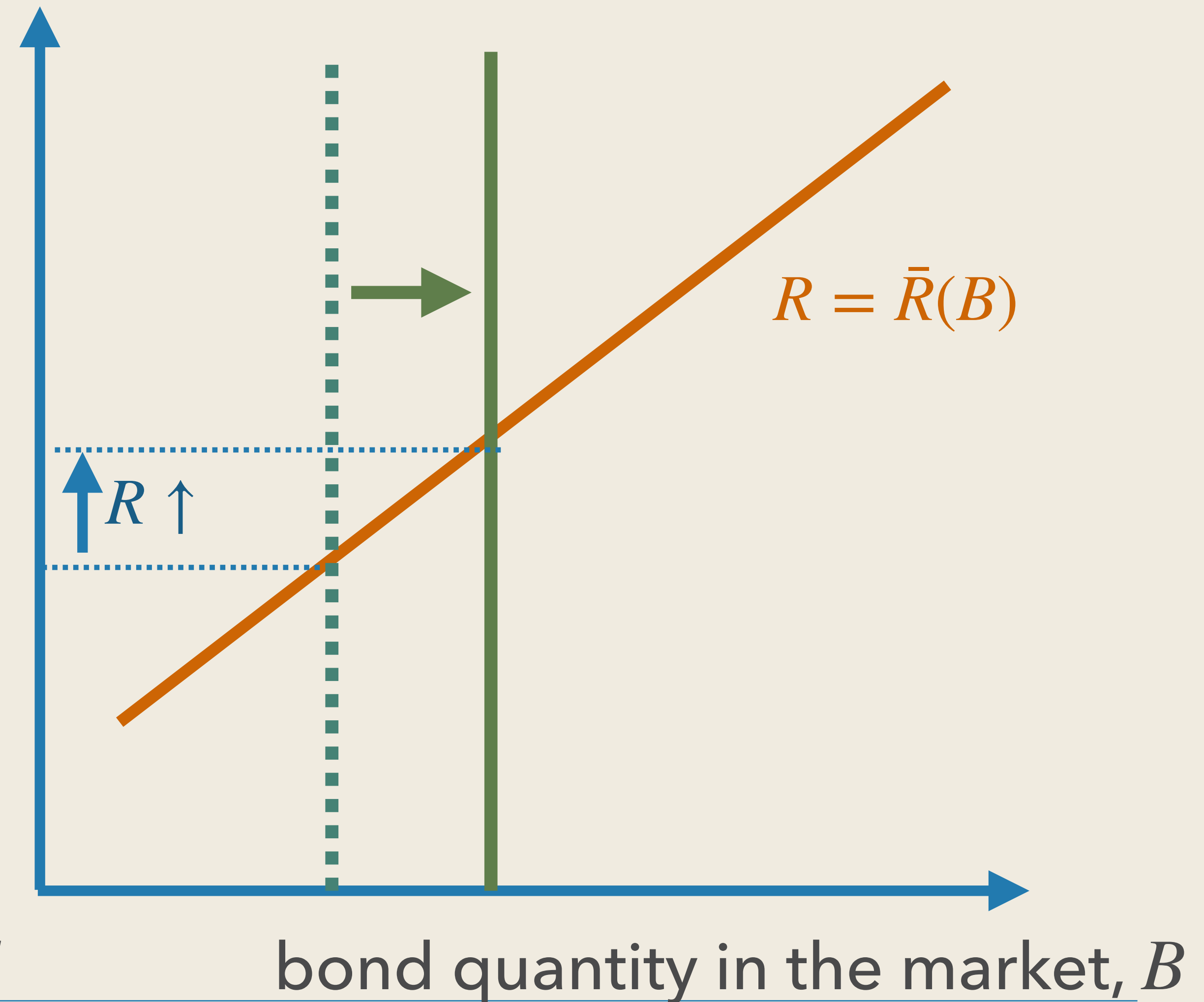
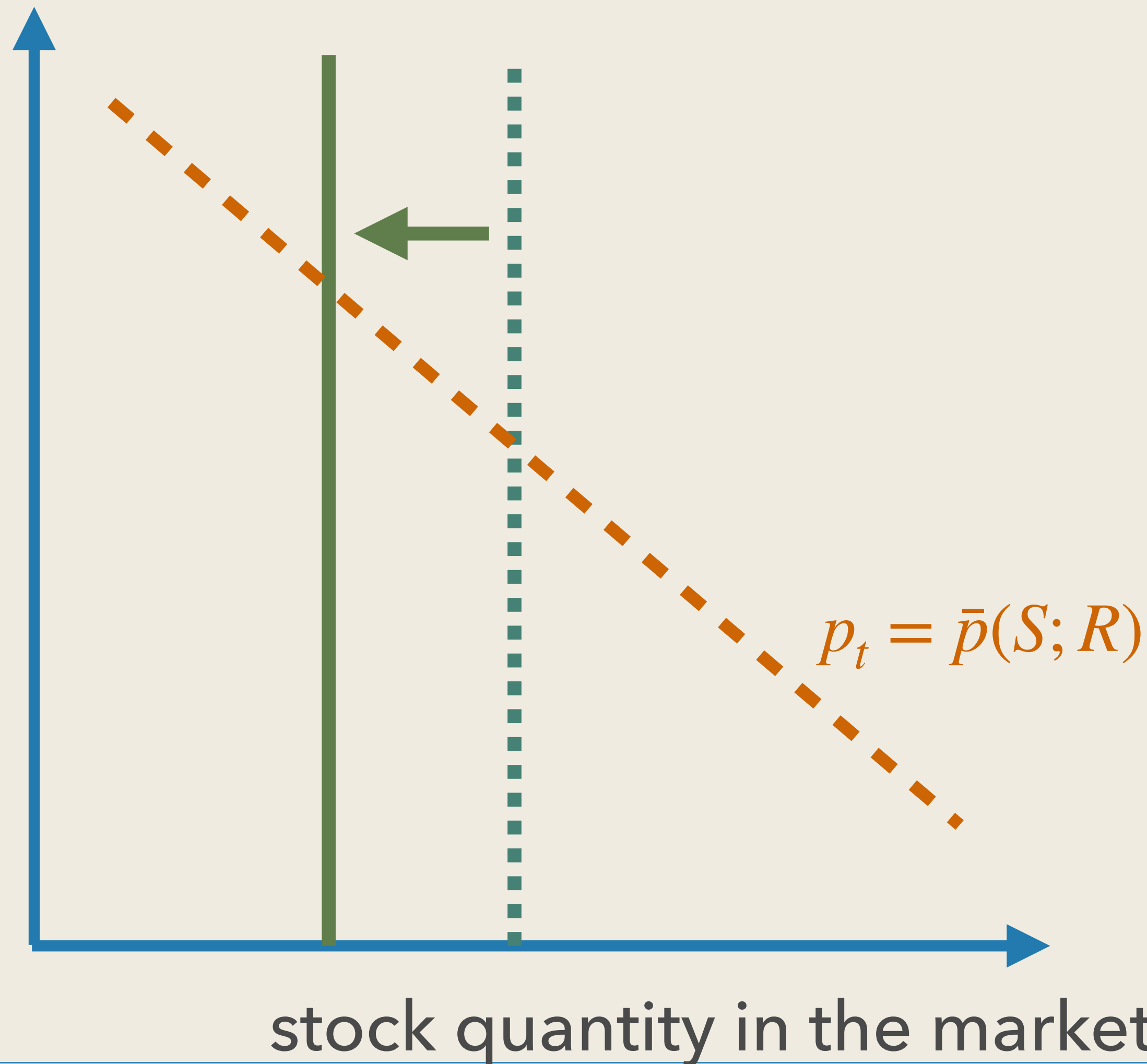
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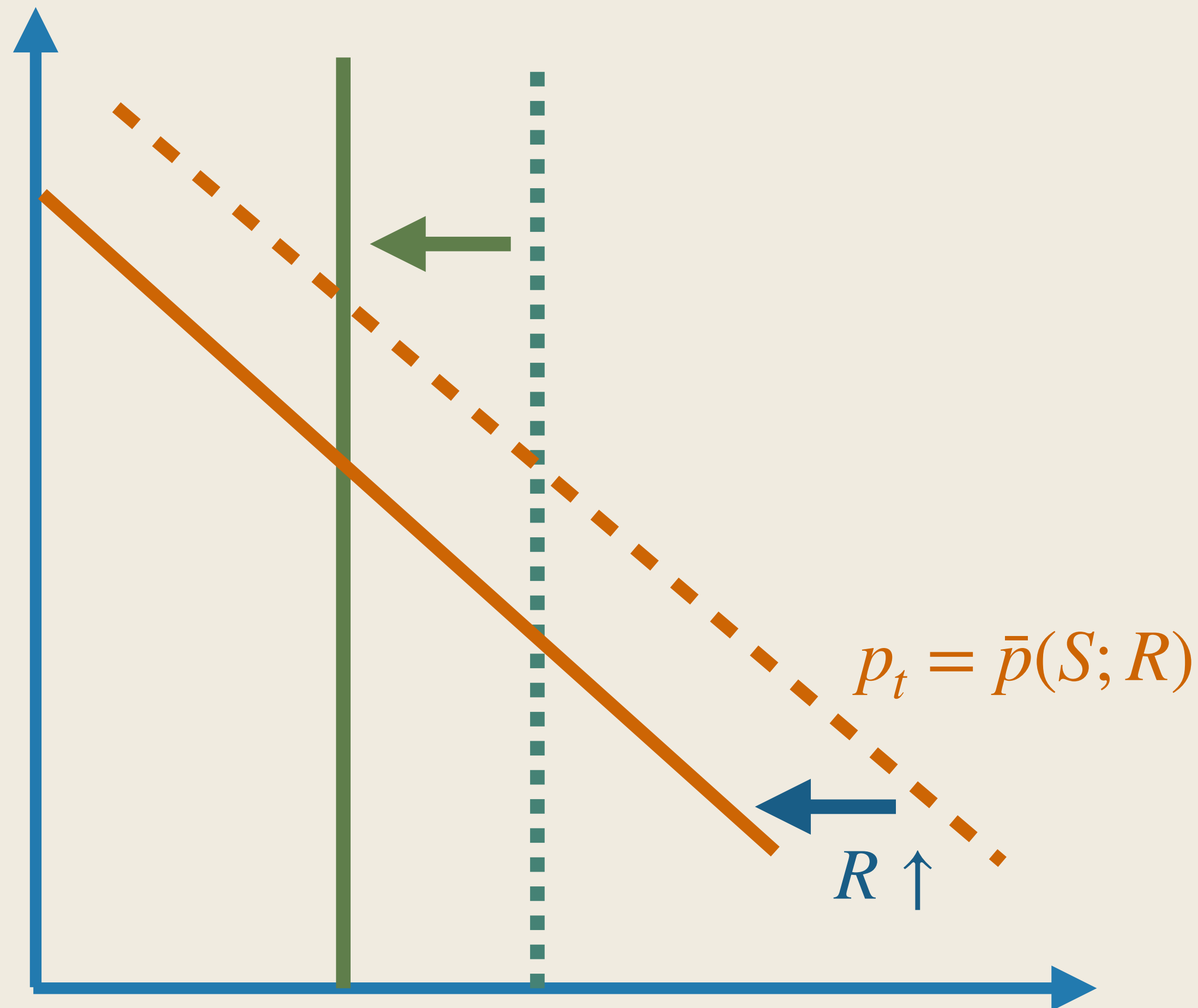
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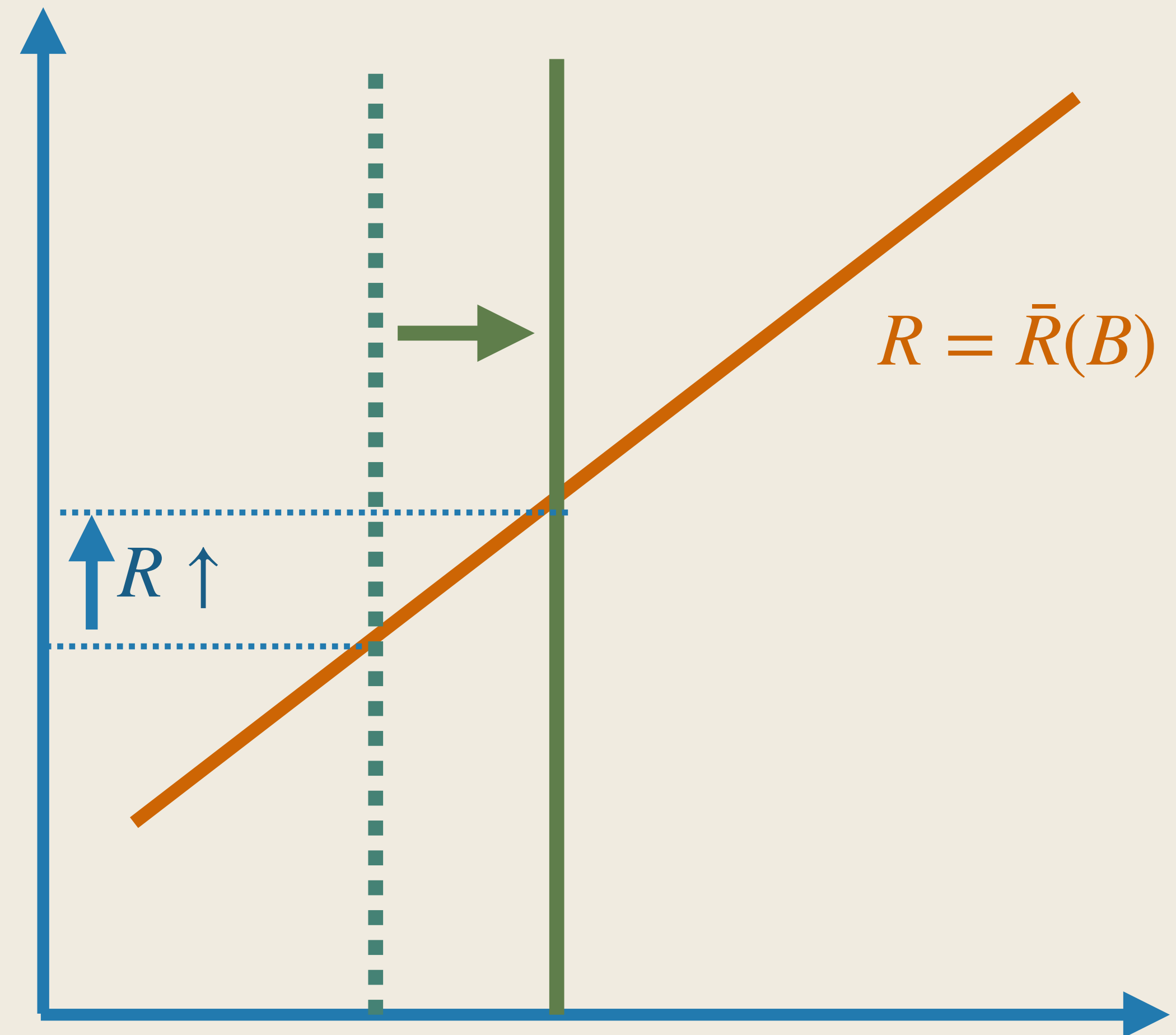
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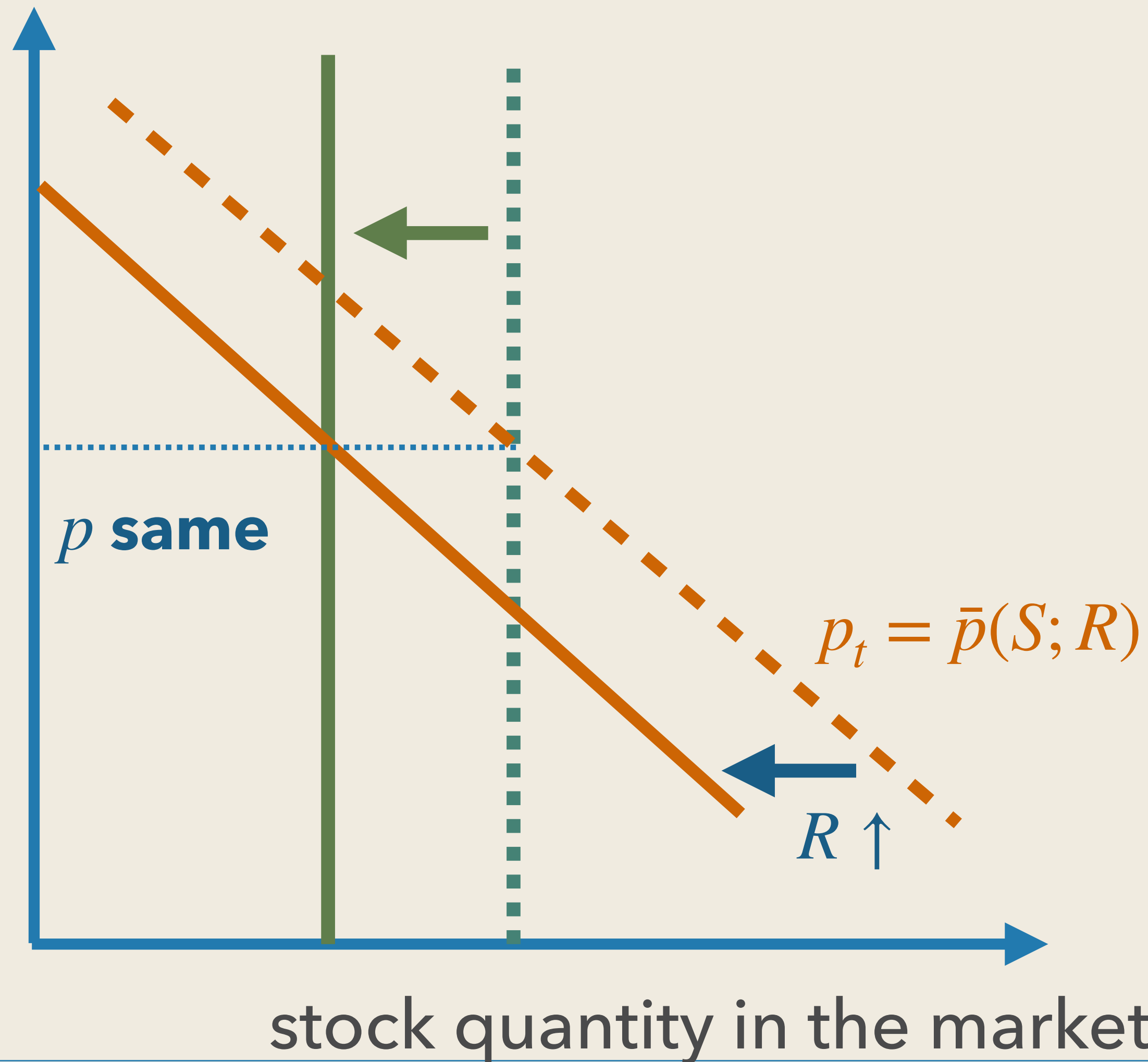
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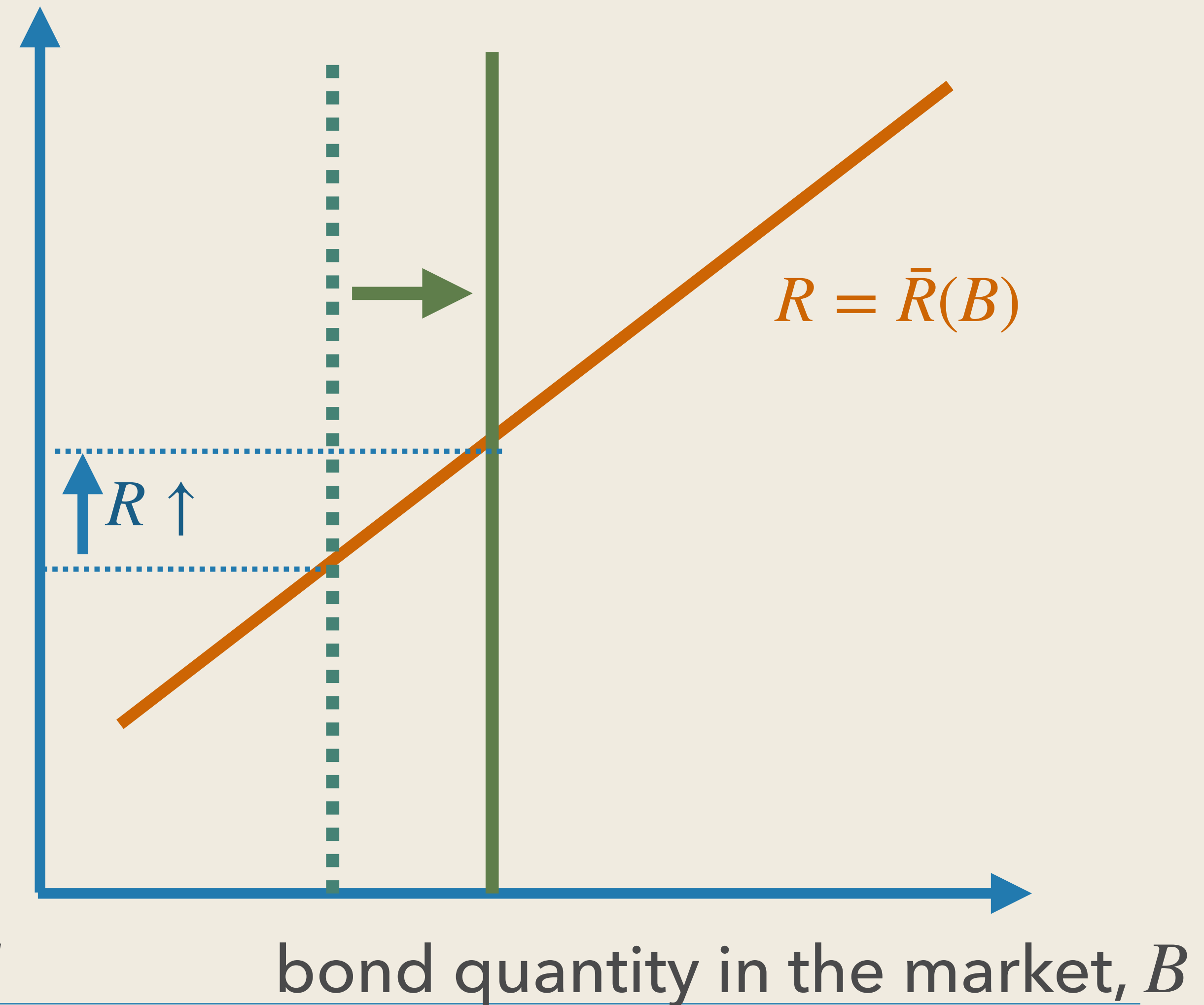
bond quantity in the market, B

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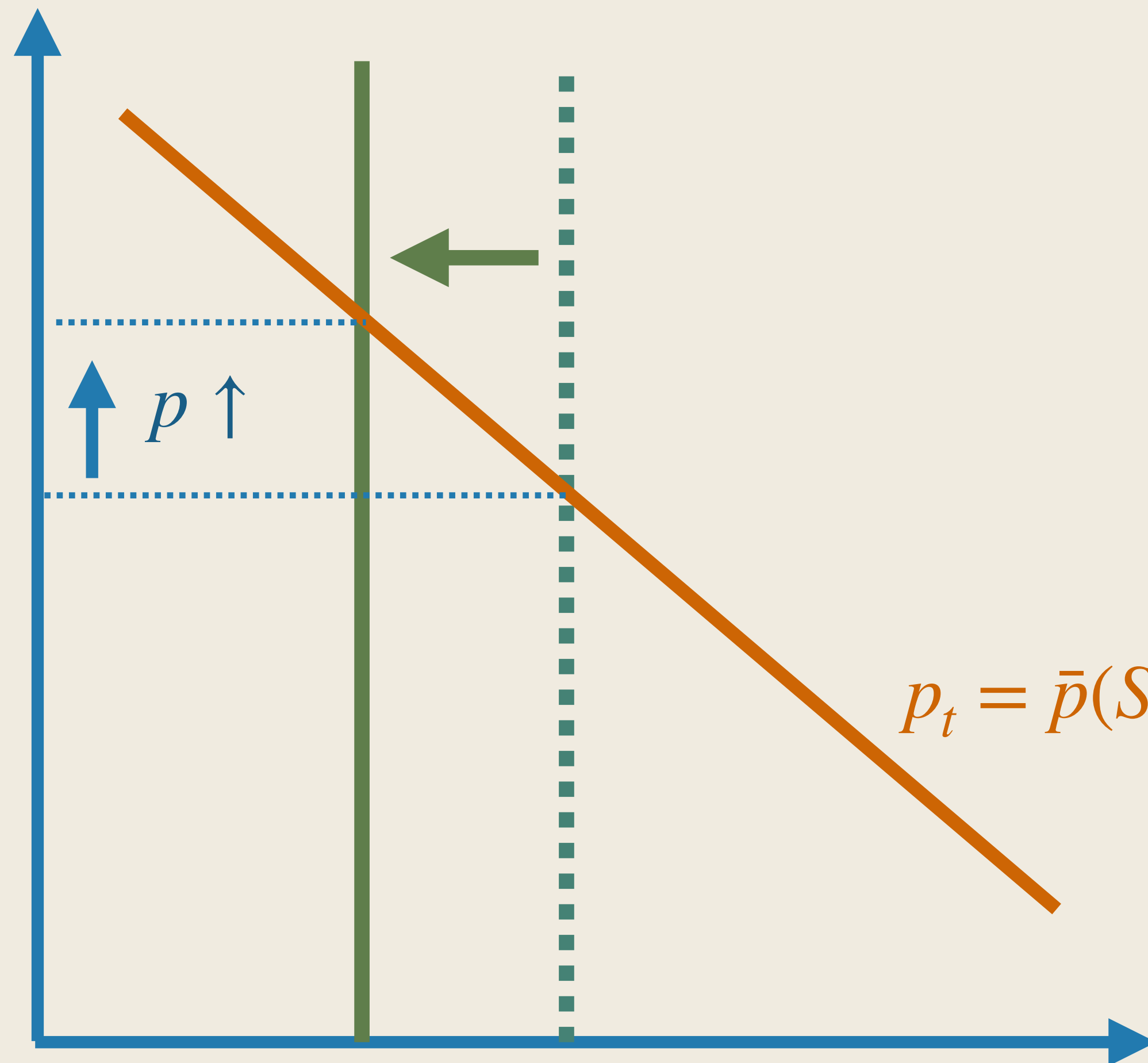


interest rate, R



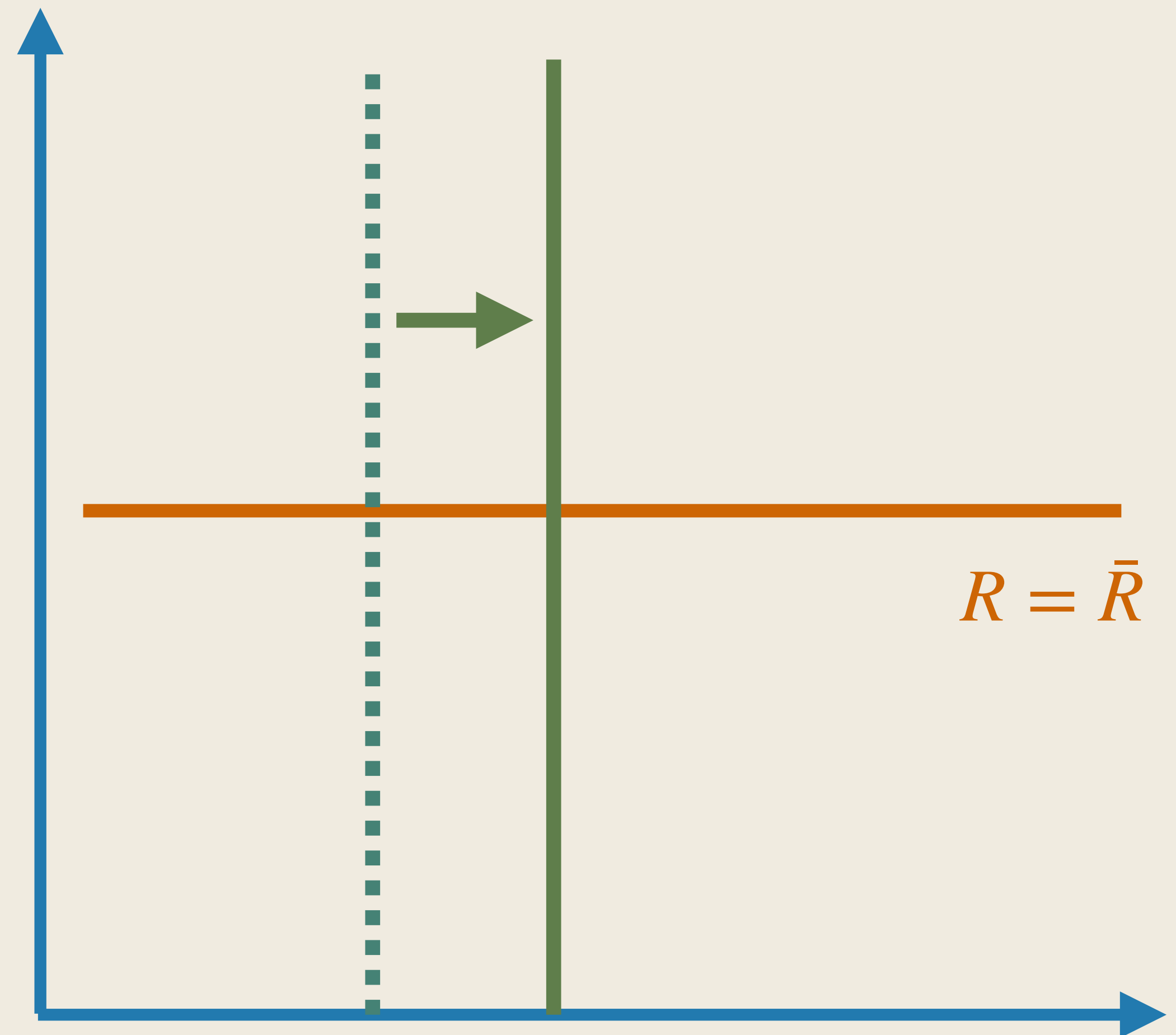
Yield Curve Control

stock price, p



stock quantity in the market, S

interest rate, R



bond quantity in the market, B

Identification

- Three sufficient stats:

Identification

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$$-\frac{\partial \ln \bar{p}(S, R)}{\partial \ln S}$$

stock market inelasticity
holding R fixed

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bond market
inelasticity

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bond market
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Interest rate
sensitivity

Identification

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stock market inelasticity
holding R fixed

≈ 22

$$\frac{\partial \ln \bar{R}(B)}{\partial \ln(B/S)}$$

bond market
inelasticity

≈ 1.4

$$\frac{\partial \ln \bar{p}(S, R)}{\partial \ln R}$$

Interest rate
sensitivity

≈ -15

Identification

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inelasticity

≈ 1.4

$$\frac{\partial \ln \bar{p}(S, R)}{\partial \ln R}$$

Interest rate
sensitivity

≈ -15

- Gabaix and Koijen's (2020) estimates: $-\frac{d \ln \bar{p}(S, R(S))}{d \ln S} \approx 5$

Conclusion

- Two strands of literature:
 1. bond market is inelastic (Krishnamurthy & Vissing-Jorgensen, 2012, Vayanos & Vila, 2021)
 2. stock market is inelastic (Gabaix & Koijen, 2021)
- How a flow from bond to stock drives assets prices depends **jointly** on two elasticities
- Using a RDD design, we show
 - ✓ such a flow mostly ended up moving bond prices in a normal time
 - ✓ once bond market becomes elastic, stock prices start to respond substantially
⇒ evidence that *both* bond and stock market are substantially inelastic
- Central bank stock purchases “works” only when combined with YCC

Question

- What is the impact of central bank stock purchases?
- Why do we care?
 1. Frontier of “quantitative easing”
 2. Ideal laboratory to test new theories of stock market fluctuations

Literature

■ Empirical studies on QE:

- Krishnamurthy & Vissing-Jorgensen (2011,2013), Baba et al. (2006), Gagnon et al. (2010, 2011), Sarkar and Shrader (2010), Ashcraft et al. (2011), Hancock and Passmore (2011), Joyce et al. (2011), Swanson (2011, 2015), Stroebel and Taylor (2012), D'Amico and King (2013), Kandrak and Schlusche (2013), Koijen et al. (2018), Di Maggio et al (2020), Beraja et al (2020), Droste, Gordnichenko, and Ray (2021)

■ Studies on BoJ stock purchases:

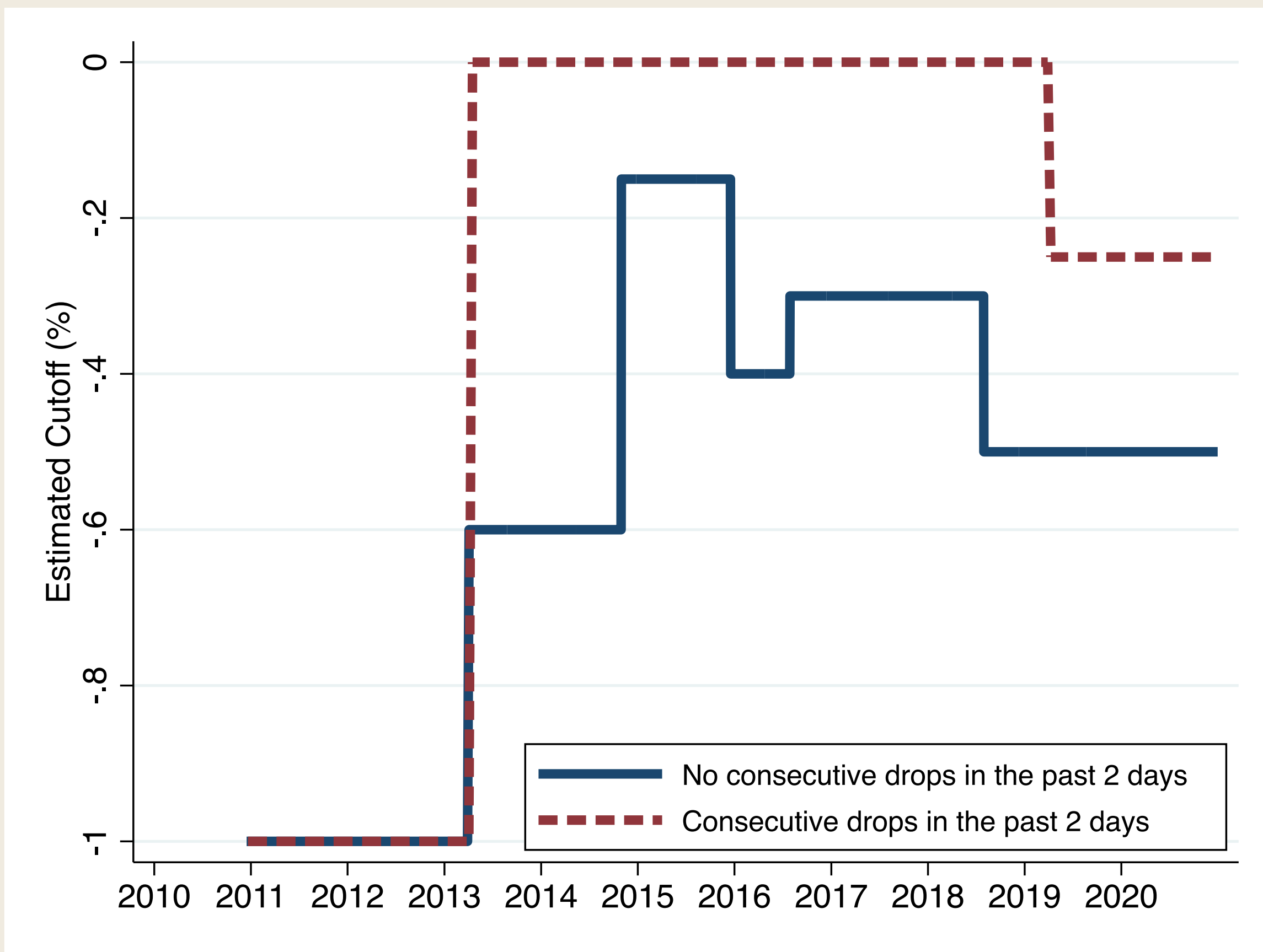
- Charoenwong et al. (2019), Fukuda and Tanaka (2019), Shirota (2019), etc

■ Market inelasticity:

- Bonds market: Vayanos-Vila (2009), Krishnamurthy-Vissing-Jorgensen (2012)
- Stock market: Koijen and Yogo (2019), Gabaix and Koijen (2020)

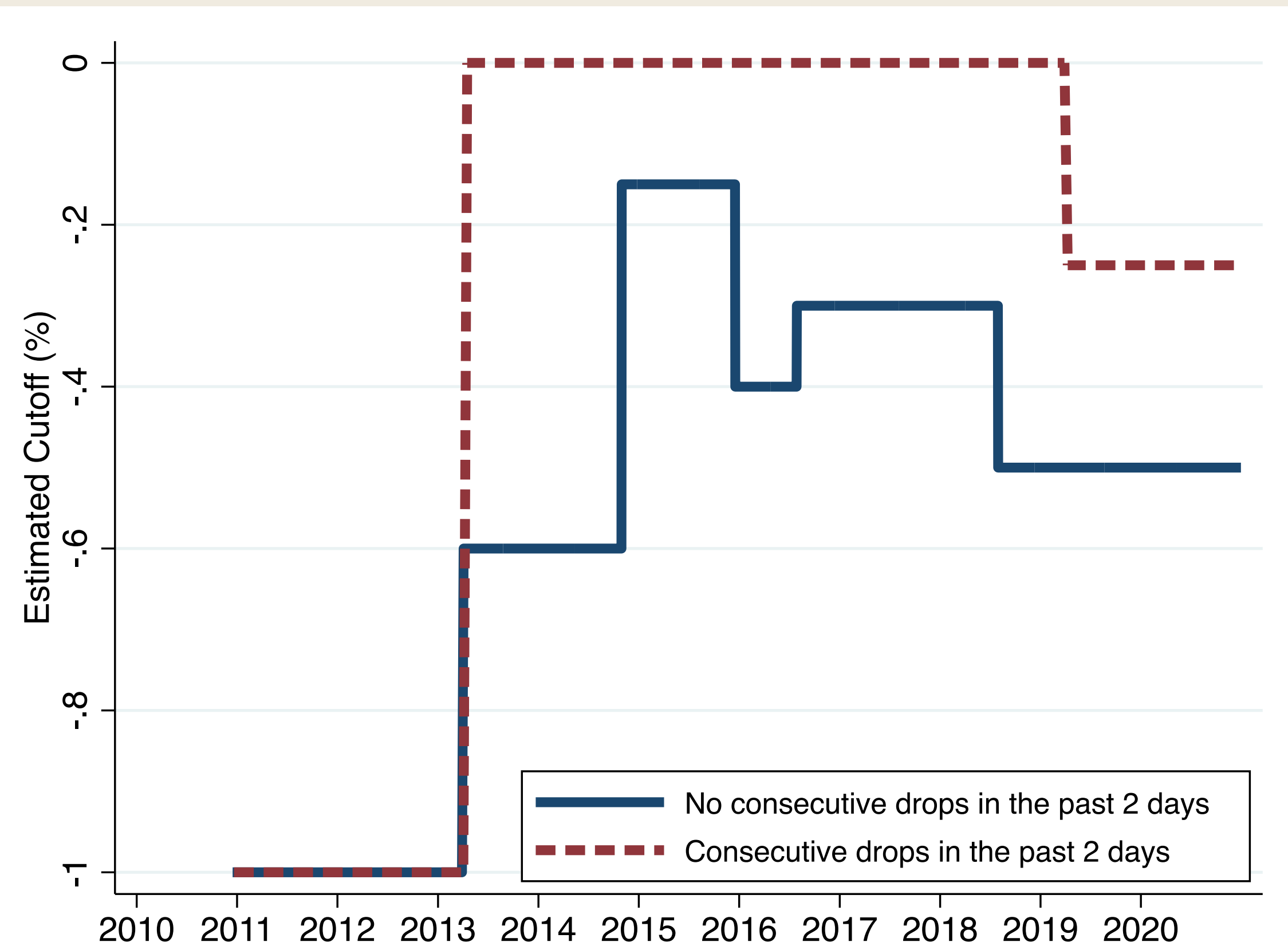
Cutoffs Estimation

Estimated Cutoffs over Time

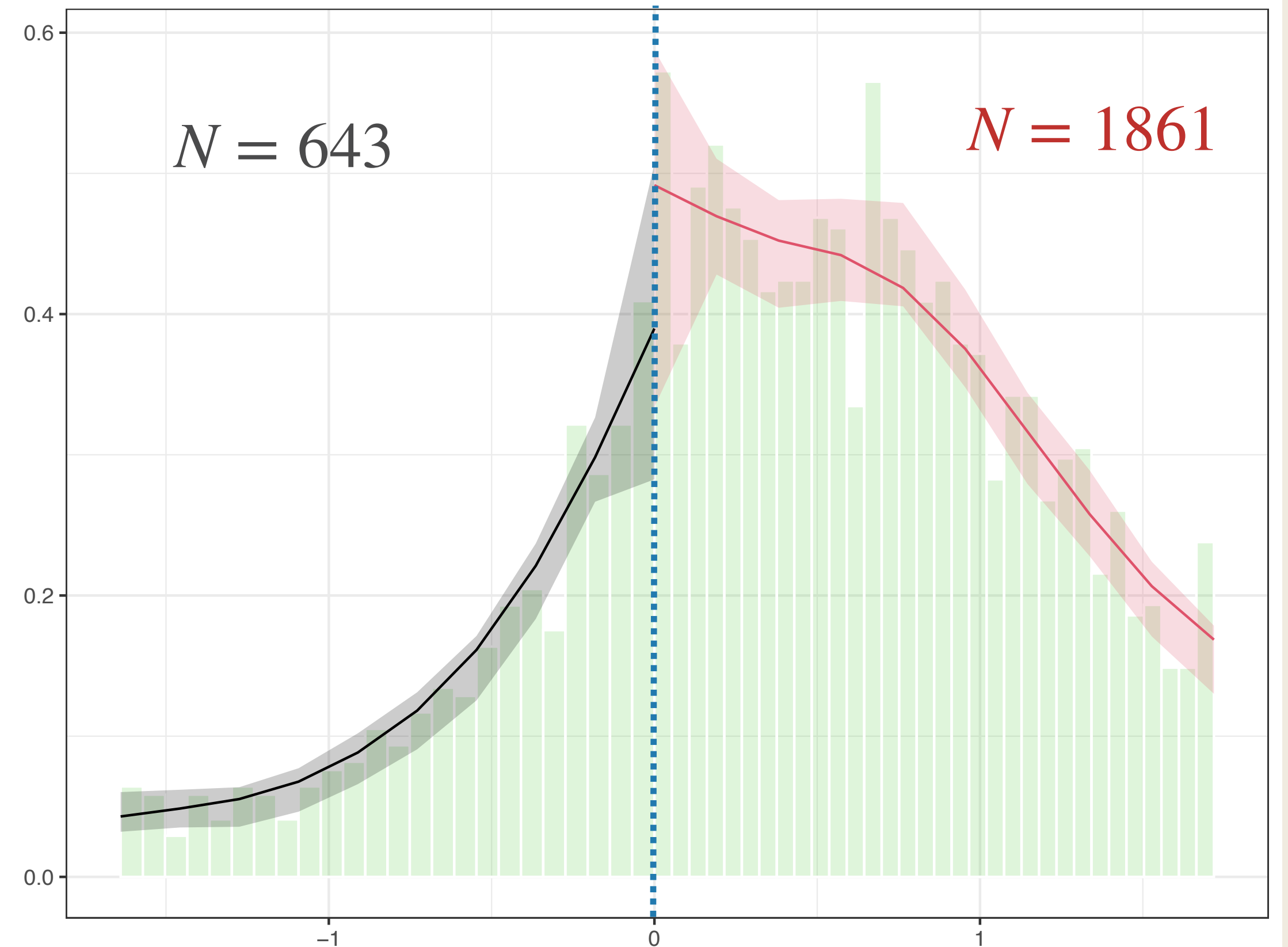


Cutoffs Estimation

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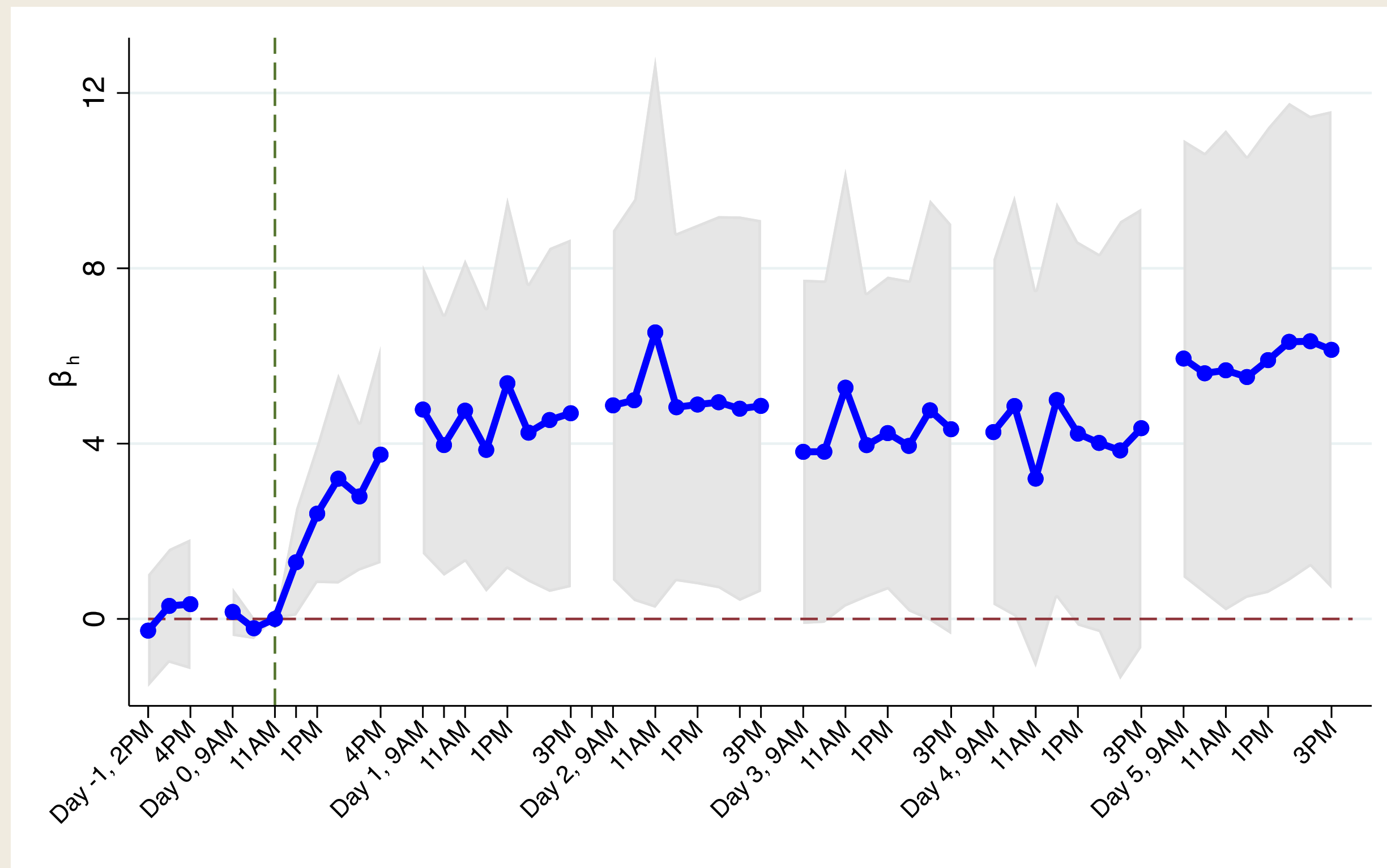
Density around the cutoff



Heterogenous Interest Rate Responses

10-year JGB Yield

Before YCC

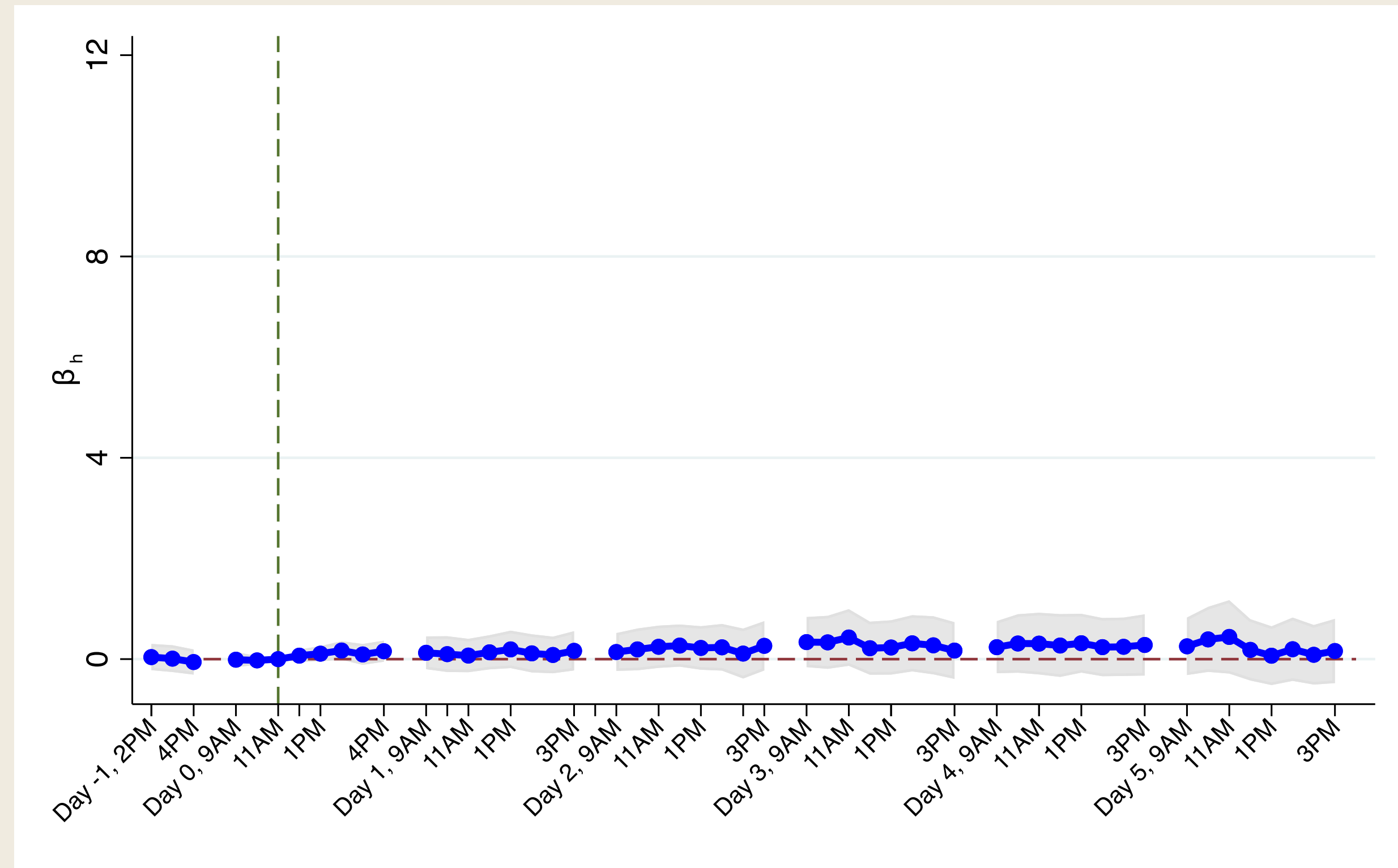
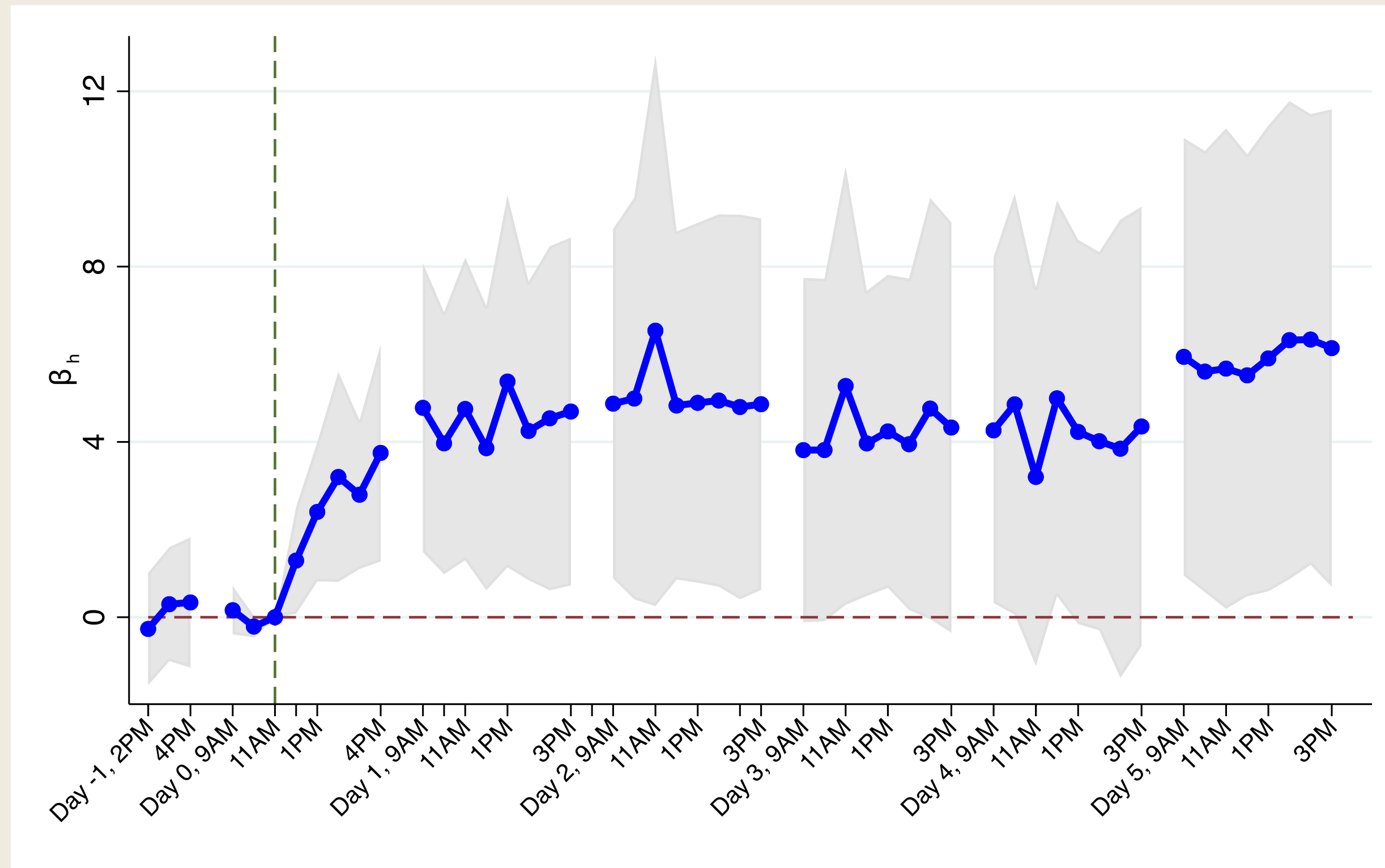


Heterogenous Interest Rate Responses

10-year JGB Yield

Before YCC

After YCC

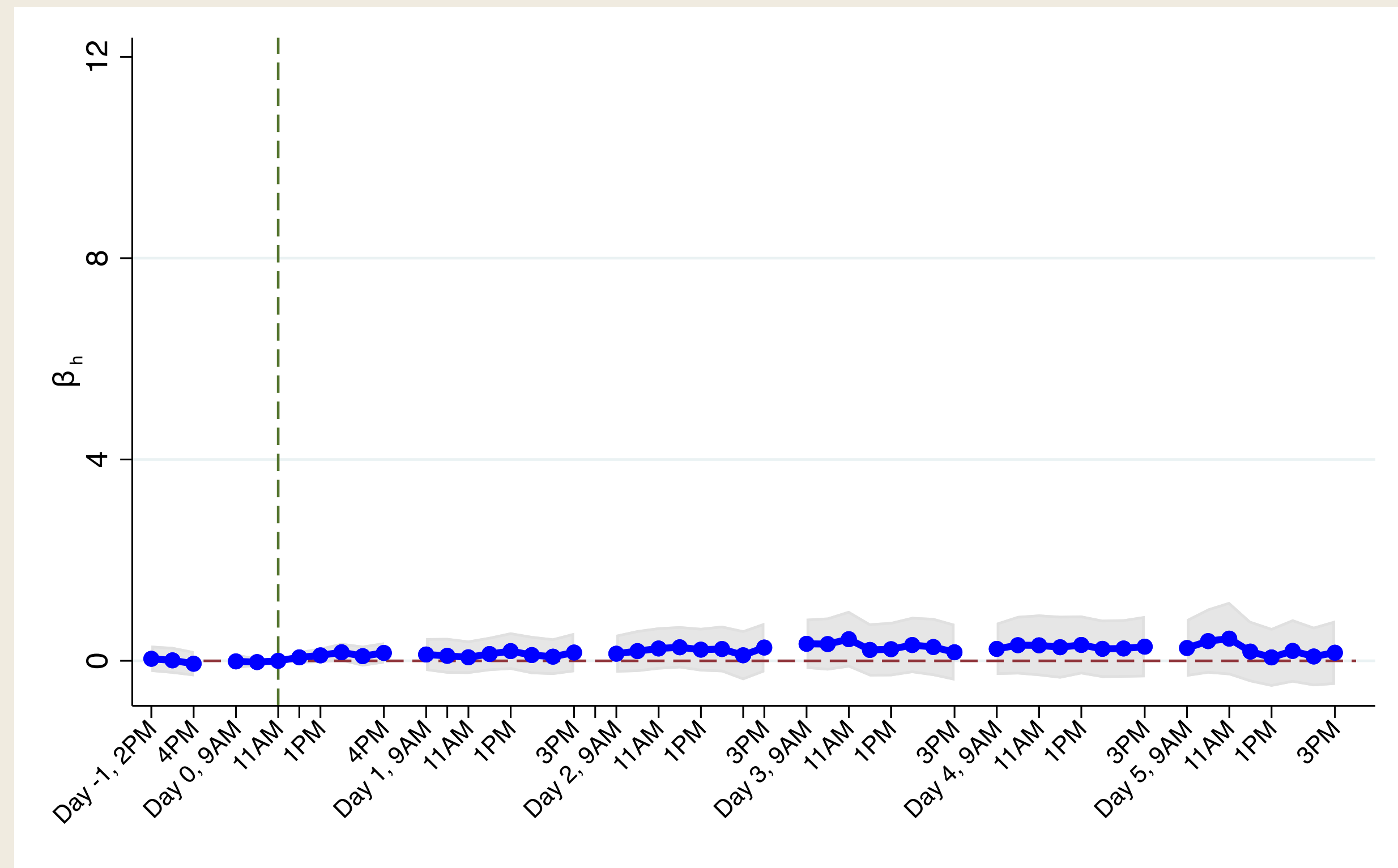
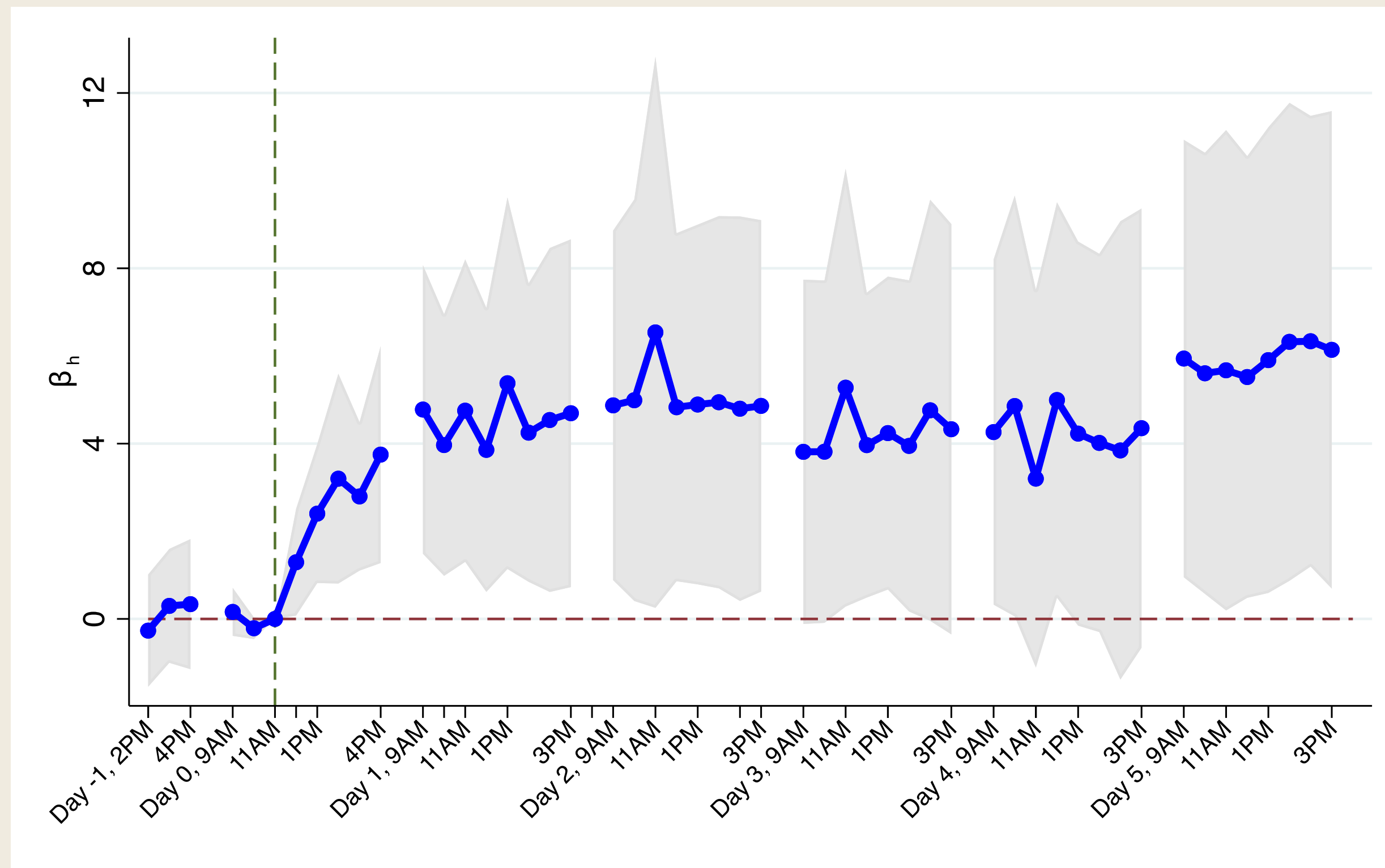


Heterogenous Interest Rate Responses

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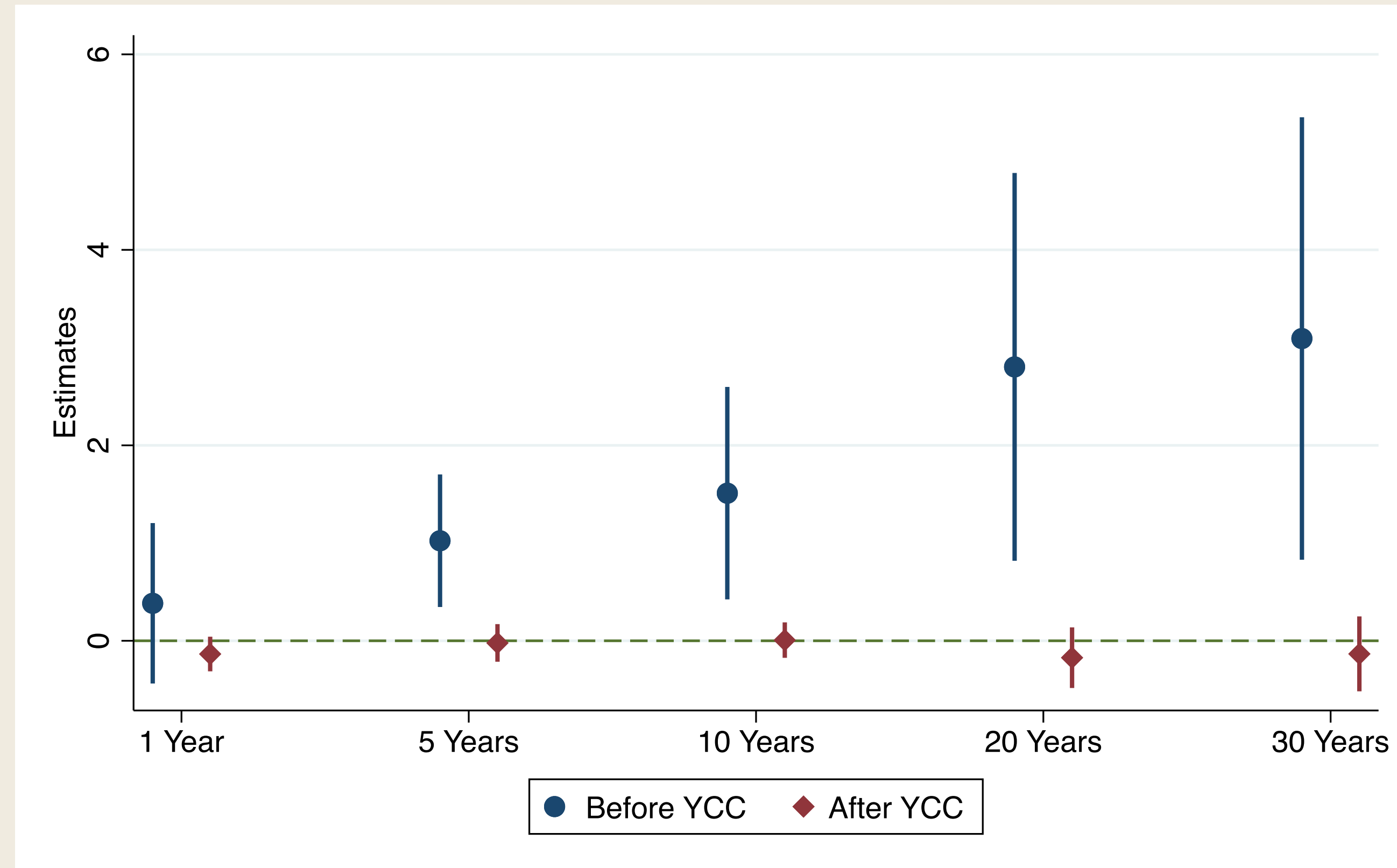
After YCC



- In response to a 1% purchase of stocks by BoJ, long-term interest rate (i) rose by 4% before YCC; (ii) stopped responding after YCC

Response of Yield Curve

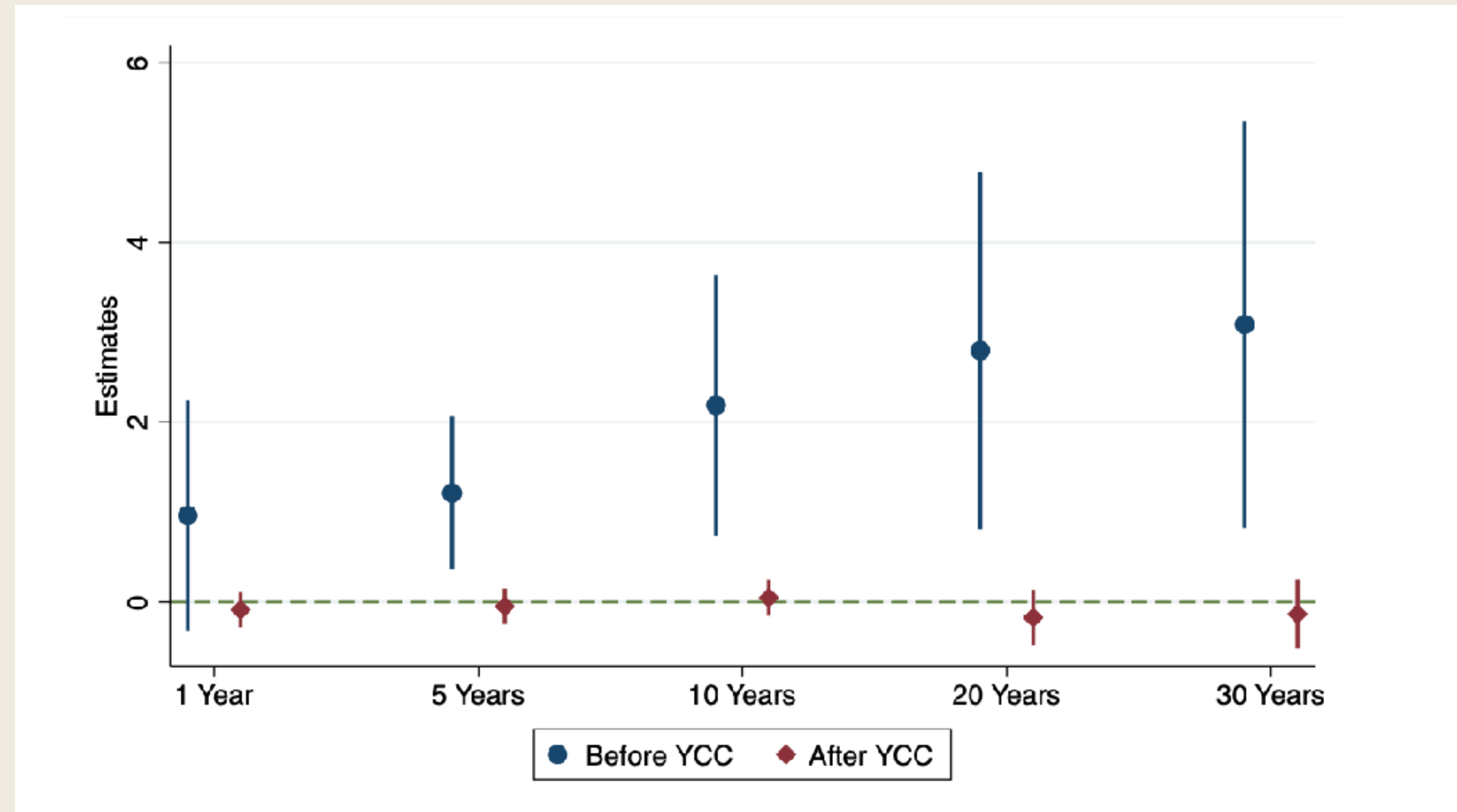
Next-day Response of Yields across Different Maturities



- Smaller responses for shorter maturity before YCC
⇒ Our interpretation is that ZLB prevented the response of shorter maturity

Response of Yield Curve (Yagasaki added)

Intra-day Response of Yields across Different Maturities

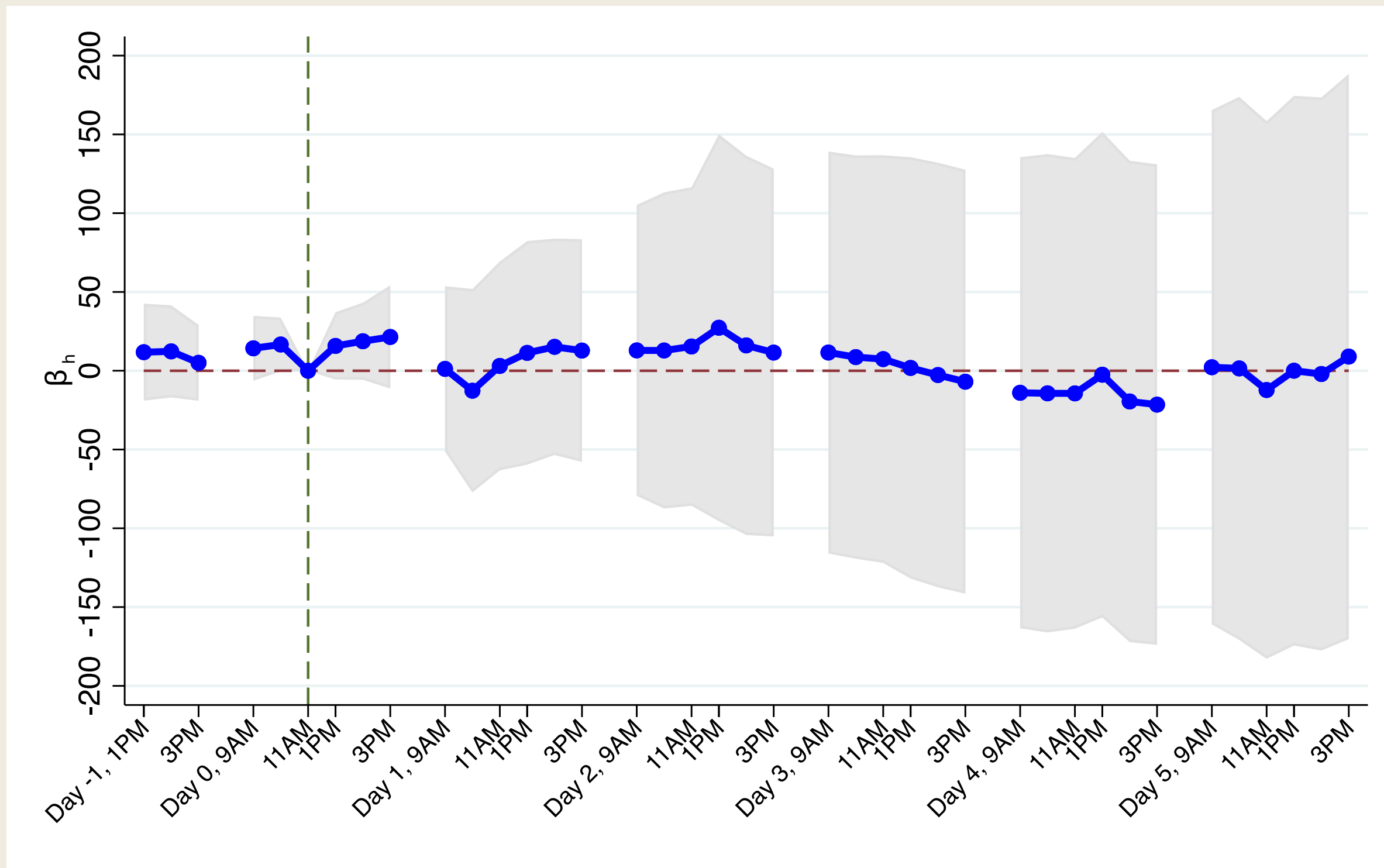


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Heterogenous Stock Price Responses

Stock (TOPIX) Price Changes

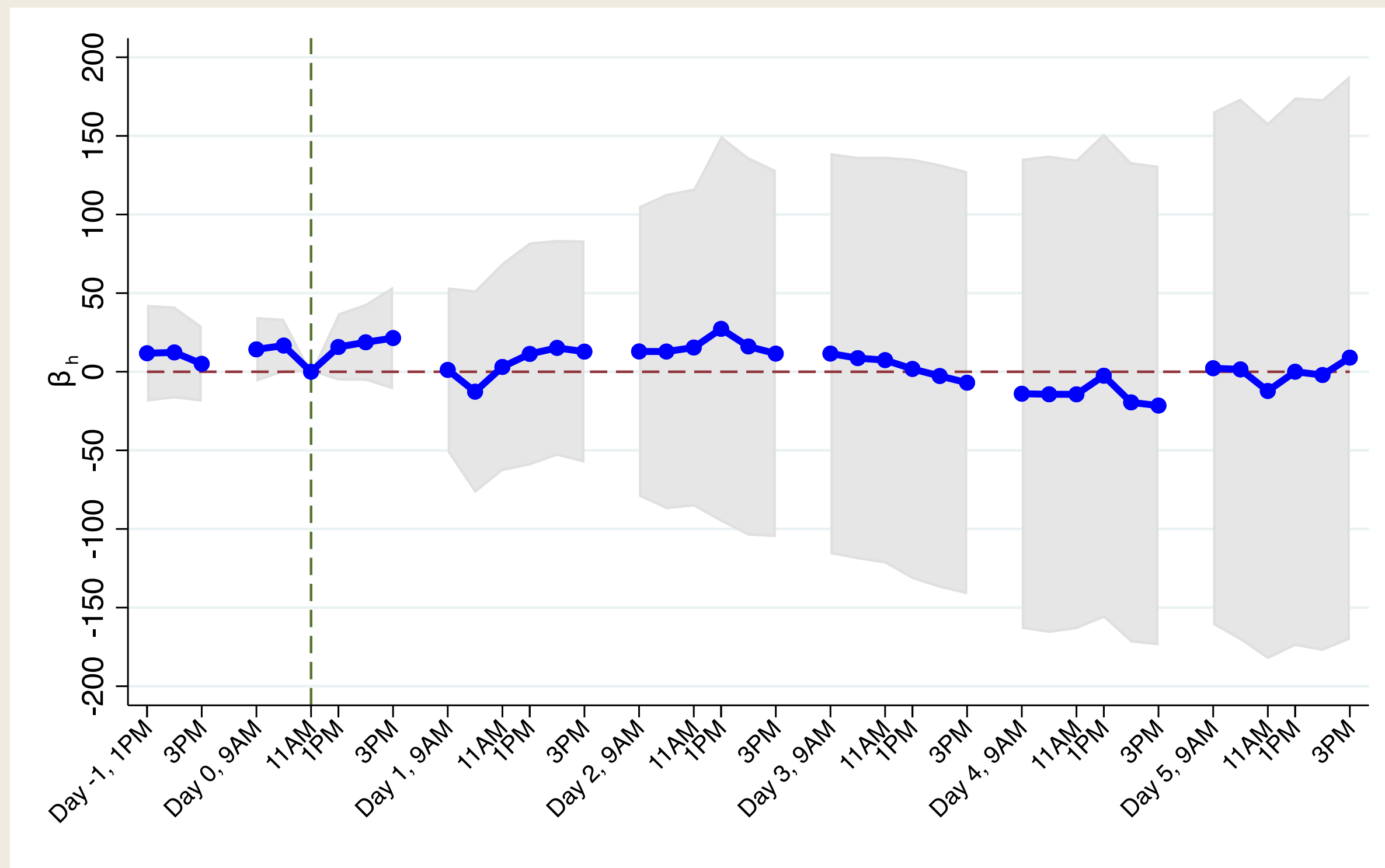
Before YCC



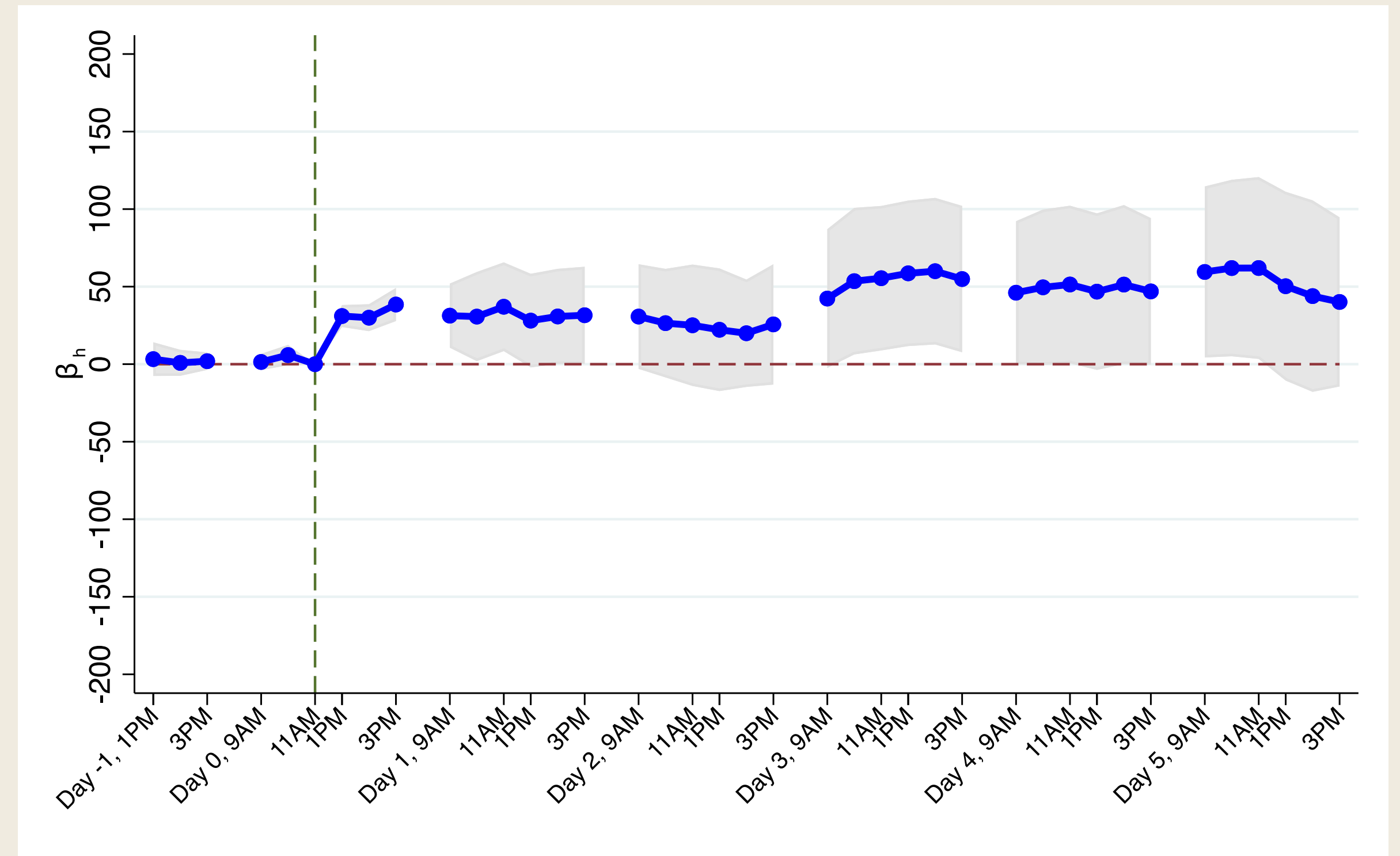
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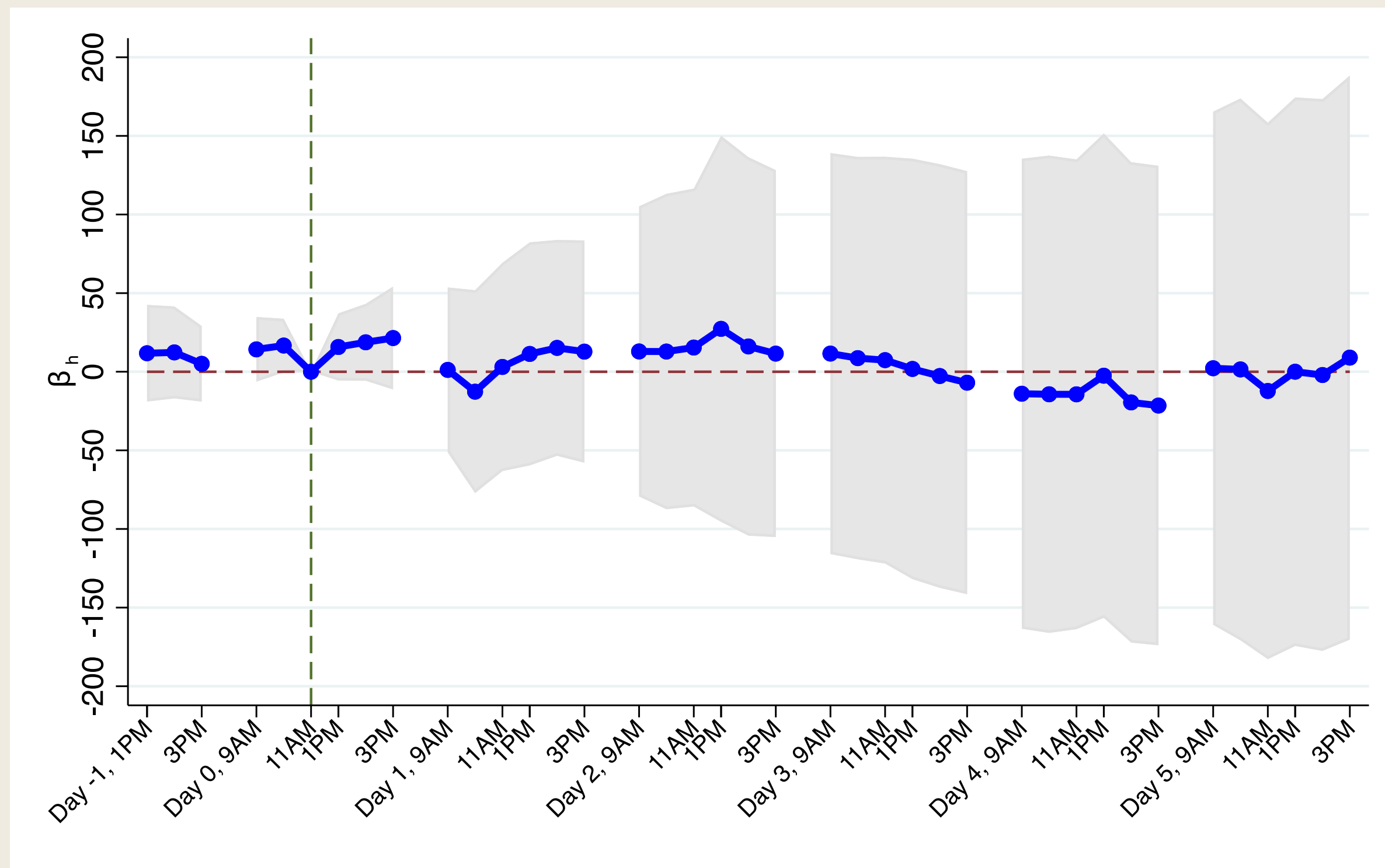
After YCC



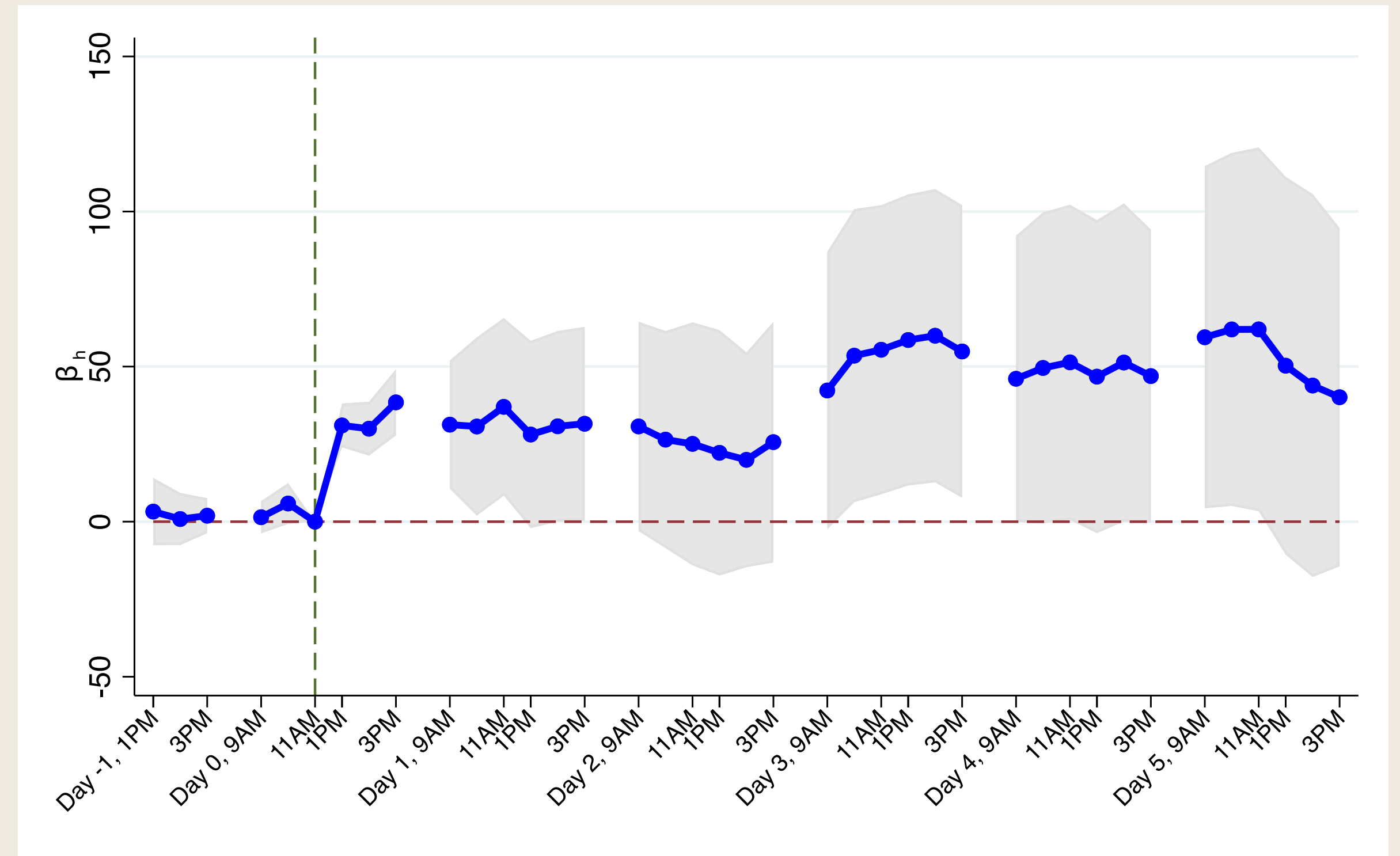
Heterogenous Stock Price Responses

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Before YCC



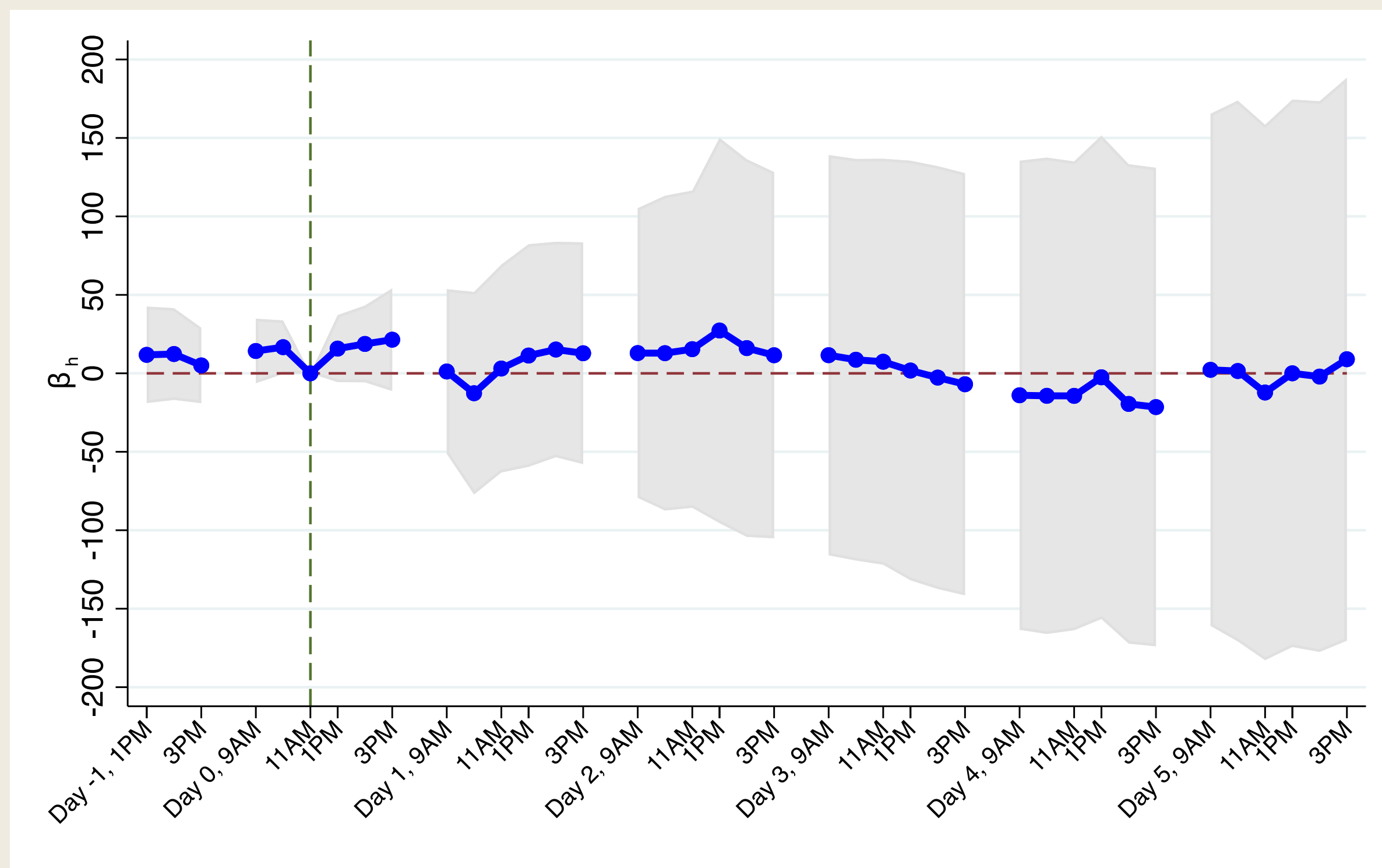
After YCC



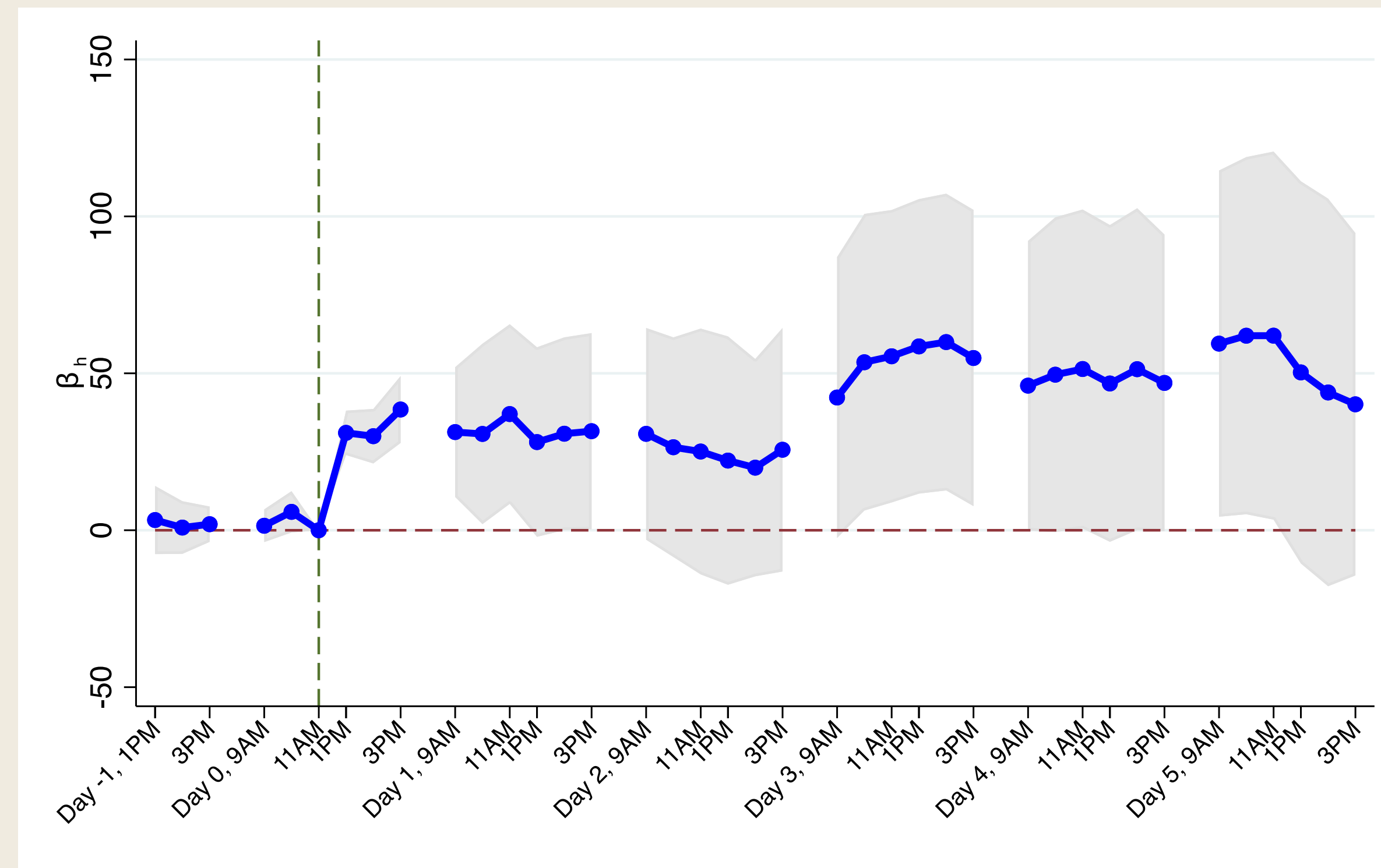
Heterogenous Stock Price Responses

Stock (TOPIX) Price Changes

Before YCC



After YCC

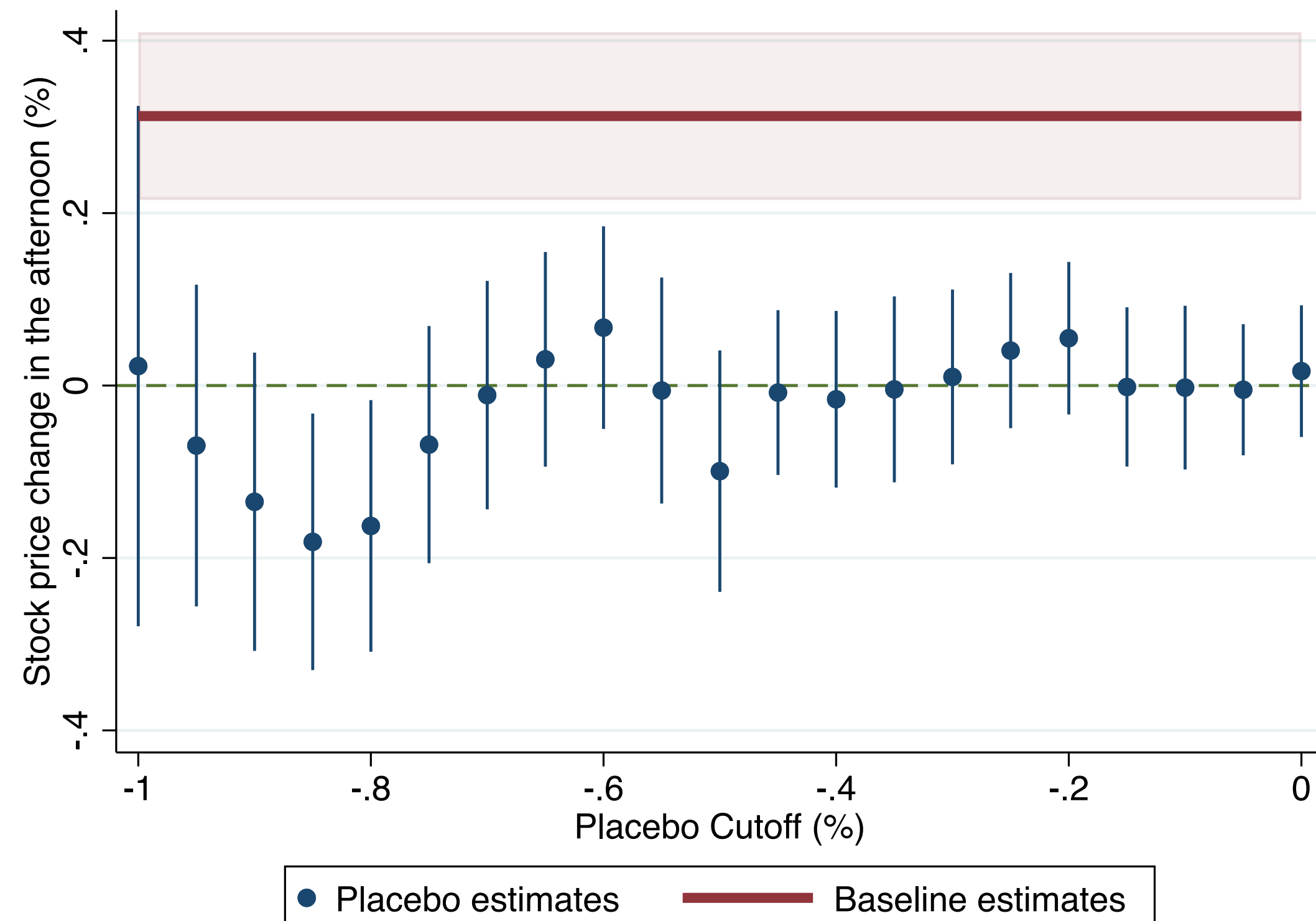


- In response to a 1% purchase of stocks by BoJ,
 - (i) noisy zero effect before YCC; (ii) 40-50% increase in stock prices after YCC

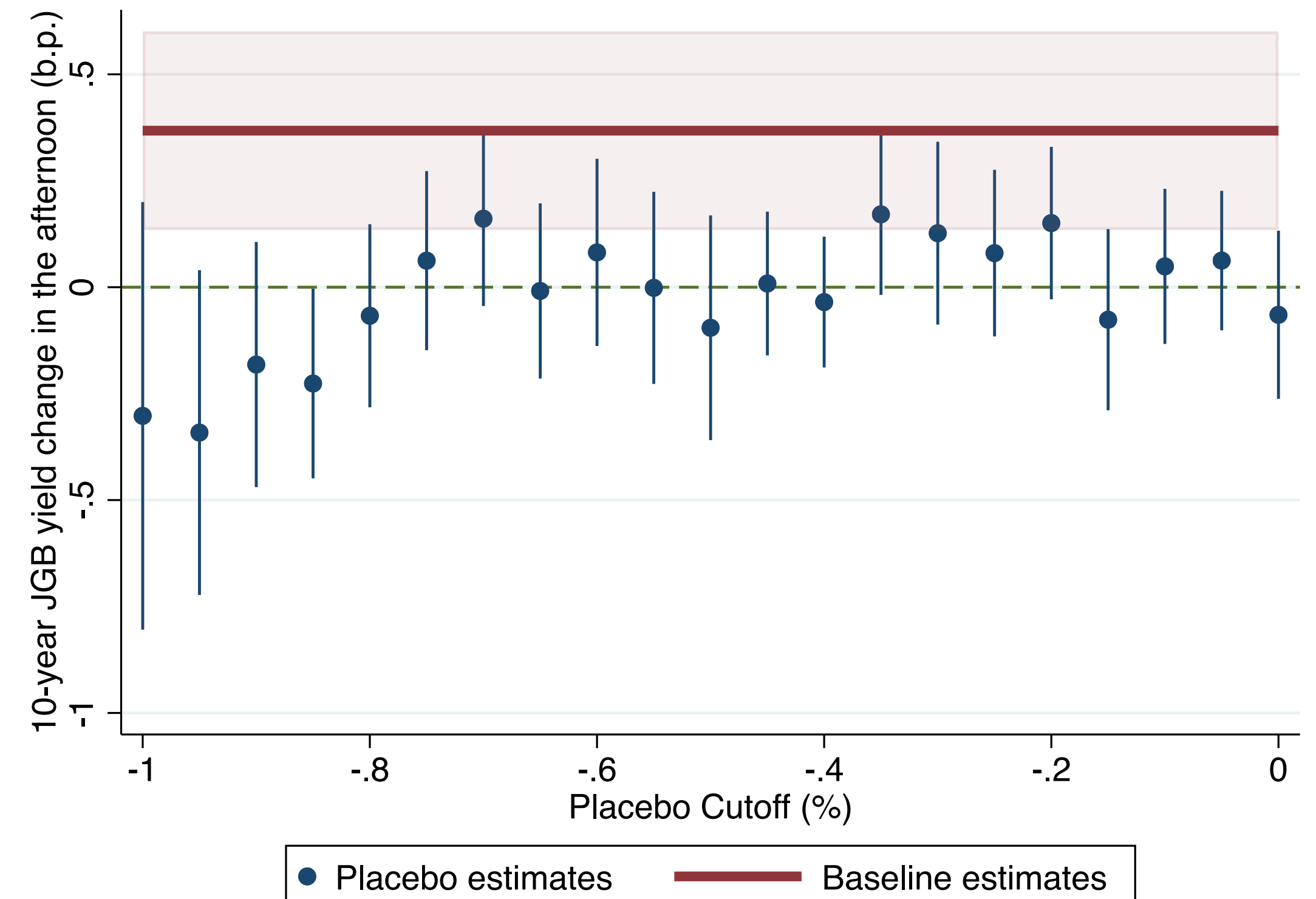
Placebo Tests

- Run the same regression with arbitrary chosen cutoffs
 - no significant effect around the cutoff in which there is no policy discontinuity

Stock (TOPIX) Price



10-year JGB Yield



Taking Stocks

- Summary of empirical results
 1. Before YCC, BoJ stock purchases mostly ended up increasing interest rates
 2. After YCC, interest rates stopped responding and stock prices robustly rise

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 - ✓ controls for past outcomes, policies
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- Results robust to ([Table](#))
 - ✓ alternative bandwidths & polynomial orders
 - ✓ controls for past outcomes, policies
 - ✓ dropping the periods around cut-off changes
- What do bond price responses reflect?
 - ✓ Inflation? – no significant response of inflation swap ([Figure](#))
 - ✓ Default risk? – no significant response of credit default swap ([Figure](#))

Preferences

- A household with preferences

$$\sum_t \beta^t [u(c_t \exp(v_b(b_t/y_t)))],$$

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$$c_t + a_t = R_{t-1}a_{t-1} + D_t, \quad b_t \equiv a_t - S_t$$

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- Assets, a_t , are managed by a mutual fund:

$$\max_{S_t, w_{t+1}} \frac{(w_{t+1} \exp(-v_s(s_t - \bar{s})))^{1-\gamma}}{1-\gamma}$$

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$$s_t \equiv S_t/a_t, \quad D_{t+1} \equiv w_{t+1} - R_t a_t$$

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Mandate, inattention, etc

$$v'_s(0) = 0, v''_s > 0$$

$$\text{s.t.} \quad w_{t+1} = R_t^s S_t + R_t(a_t - S_t) - T_t$$

$$s_t \equiv S_t/a_t, \quad D_{t+1} \equiv w_{t+1} - R_t a_t$$

Closing the Model

- A fixed supply of capital, k :

$$y_t = A_t k, \quad \log(A_{t+1}/A_t) \sim N(g - \sigma^2/2, \sigma^2)$$

- Return on stock is

$$R_t^s = \frac{A_{t+1} + p_{t+1}}{p_t}$$

- Central bank's budget constraint

$$S_t^{CB} + B_t^{CB} = R_{t-1}^s S_{t-1}^{CB} + R_{t-1} B_{t-1}^{CB} + T_t$$

- Market clearing

$$c_t = y_t$$

$$s_t a_t + S_t^{CB} = p_t k$$

$$b_t + B_t^{CB} = 0$$

Steady State and Shock

- Along the BGP without the Central Bank, $\{R, p_t\}$ solve

$$1 - v'_b(0) = \beta R \mathbb{E} \frac{u'(y_{t+1} \exp v_b(0))}{u'(y_t \exp v_b(0))}$$
$$p_t/A_t \approx \frac{1}{r + \gamma\sigma^2 - g} \equiv \bar{p}. \quad r \equiv R - 1$$

- Consider the impact of permanent Central Bank stock purchases around the BGP:

$$ds_t^{CB} \equiv \frac{dS_t^{CB}}{p_t k} = - \frac{dB_t^{CB}}{p_t k} > 0, \quad \text{for all } t \geq 0$$

Identification

$$d \ln p_t \approx \underbrace{\frac{1}{r + \gamma \sigma^2 - g} v_s''(0)}_{\kappa_p: \text{ Stock market inelasticity holding } R \text{ fixed}} ds^{CB} - \underbrace{\frac{1}{r + \gamma \sigma^2 - g}}_{\gamma_r: \text{ Interest rate sensitivity of stock price}} d \ln R$$

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κ_p : Stock market inelasticity *holding R fixed*

$$d \ln R_t \approx \frac{-v_b''(0)}{1 - v_b'(0)} \bar{p} ds^{CB}$$

κ_b : Bond market inelasticity

γ_r : Interest rate sensitivity of stock price

The diagram illustrates the identification of parameters in the log-price equations. It features three boxed coefficients: a light blue box for the first term in the first equation, an orange box for the second term in the first equation, and a light blue box for the coefficient in the second equation. Arrows point from these boxes to their respective parameter labels: a blue arrow from the first box to κ_p , an orange arrow from the second box to γ_r , and a blue arrow from the third box to κ_b .

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$$d \ln R_t \approx \underbrace{\frac{-v_b''(0)}{1 - v_b'(0)} \bar{p}}_{\kappa_b: \text{Bond market inelasticity}} ds^{CB}$$

- Frictionless model: $d \ln p_t = d \ln R_t = 0$

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- Frictionless model: $d \ln p_t = d \ln R_t = 0$
- Inelastic stock market model (Gabaix & Koijen, 2020): $d \ln p_t > 0, d \ln R_t = 0$
- Identification using the estimates before YCC ($d \ln p_t \approx 0, d \ln R_t = 1.4$)
 $\Rightarrow \kappa_p - \gamma_r \kappa_b \approx 0 \text{ \& } \kappa_b = 1.4$

Over-identification Test

- With YCC, the Central Bank finances stock purchases with lump-sum tax

$$ds_t^{CB} \equiv \frac{dS_t^{CB}}{p_t k} > 0, \quad \frac{dB_t^{CB}}{p_t k} = 0 \quad \Rightarrow \quad d \ln R_t = 0$$

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- The impact on stock prices will be

$$d \ln p_t = \kappa_s ds_t^{CB}$$

- ✓ We found $d \ln p_t \approx 22$. This implies $\kappa_s \approx 22$
- ✓ These moments will exactly identify $(\kappa_s, \kappa_b) \approx (22, 1.4)$
- ✓ As a byproduct, we recover $\gamma_r = -15$.
Existing estimates of γ_r range from -10 to -16 (Kubota & Shintani 2021)

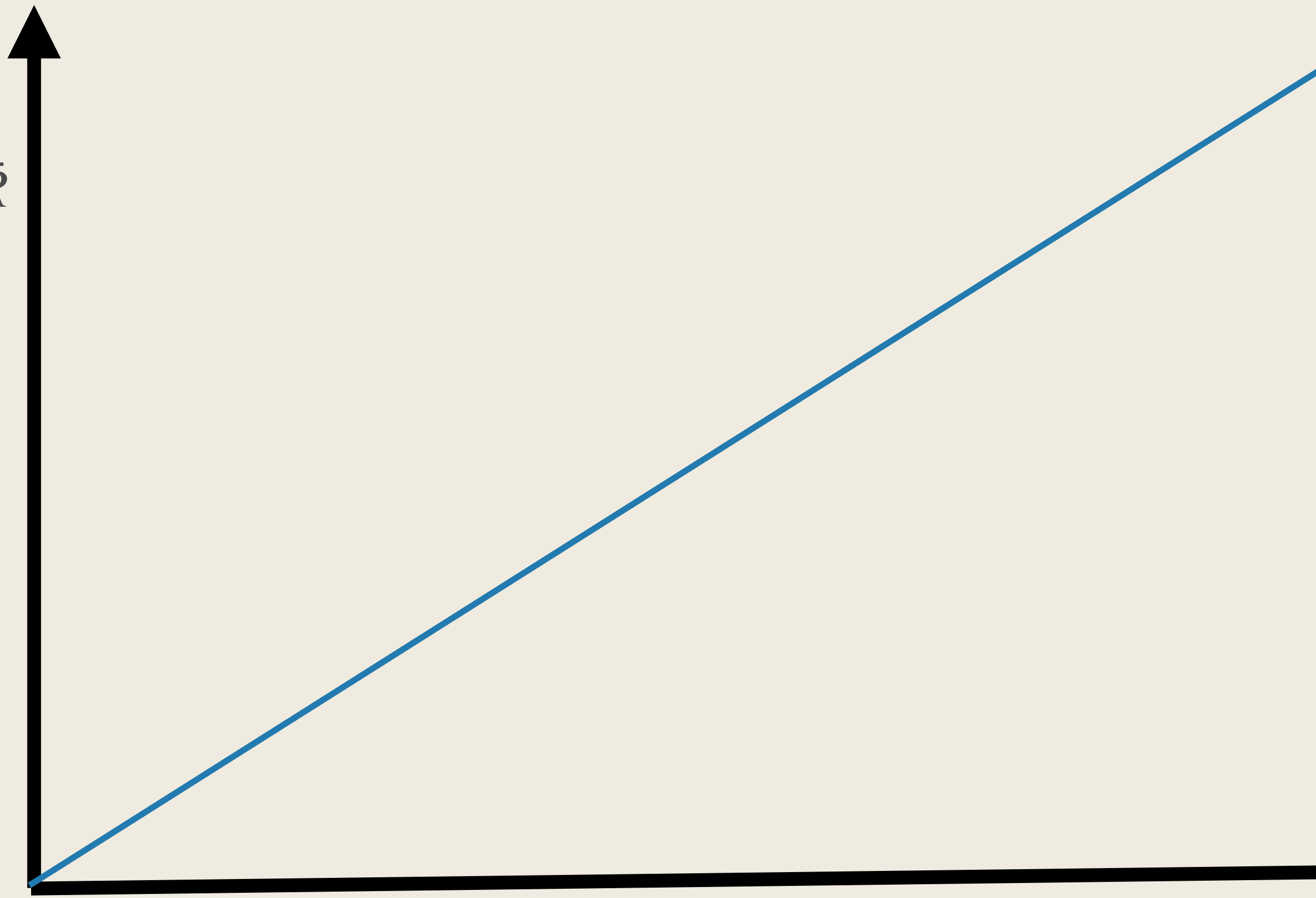
Taking Stock

Stock inelasticity

$$\kappa_s \equiv \left. \frac{\partial \ln p_t}{\partial \ln s^{CB}} \right|_{R=\bar{R}}$$



0



Bond inelasticity

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Stock price goes up
without YCC

Stock price goes down
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Gabaix & Koijen
(0, 5)



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Our estimates
(1.4, 22)

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- Flows may not explain stock price volatility, without constraints on interest rates

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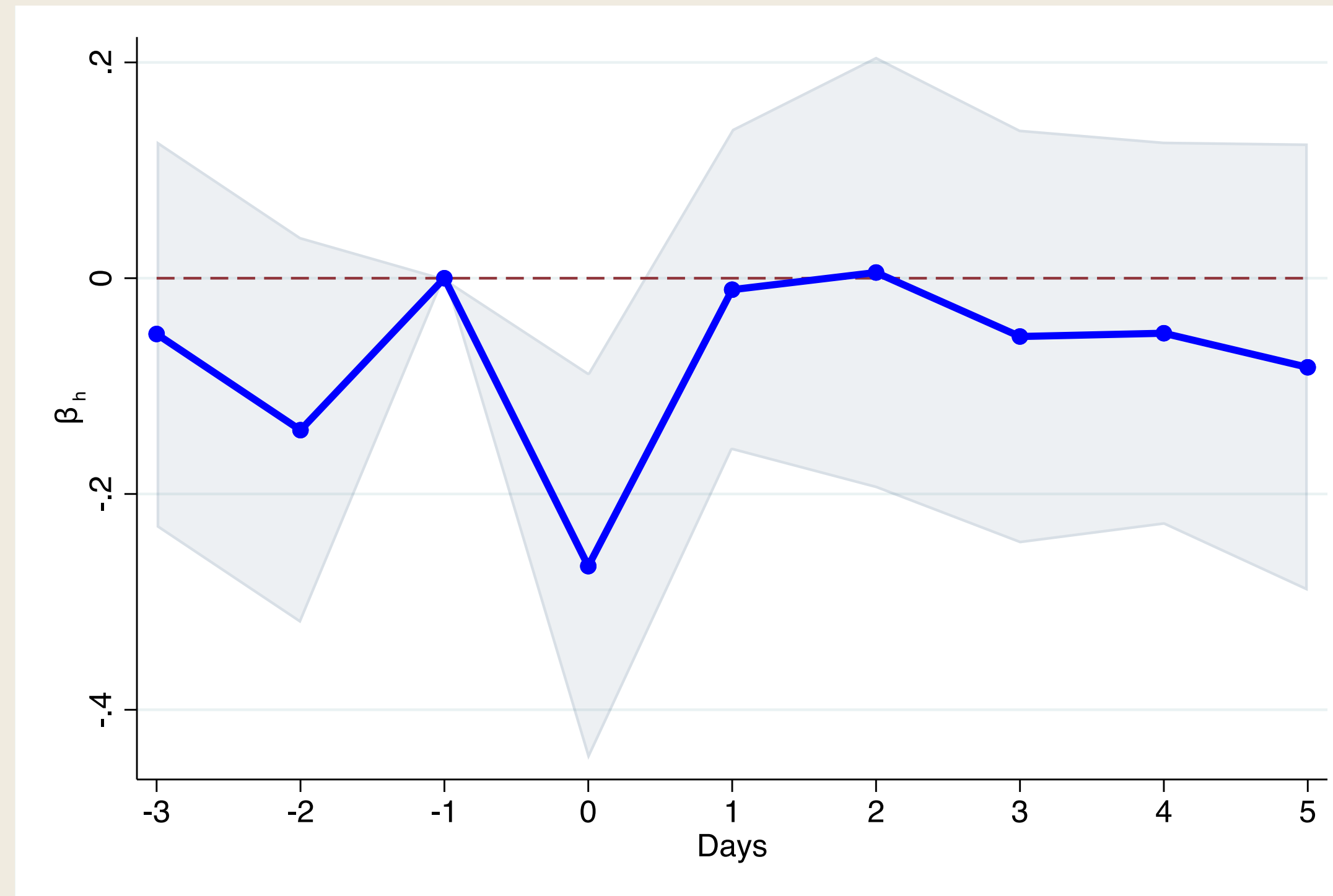
$$\kappa_b \equiv \frac{\partial \ln R_t}{\partial \ln b^{CB}}$$

- Flows may not explain stock price volatility, without constraints on interest rates
- But it may help explain why interest rate in the data is ten times more volatile than RBC (Winberry, 2020)

Exploration of Real Effect

Response of Newspaper Sentiments

- Newspaper sentiments negatively react to interventions:

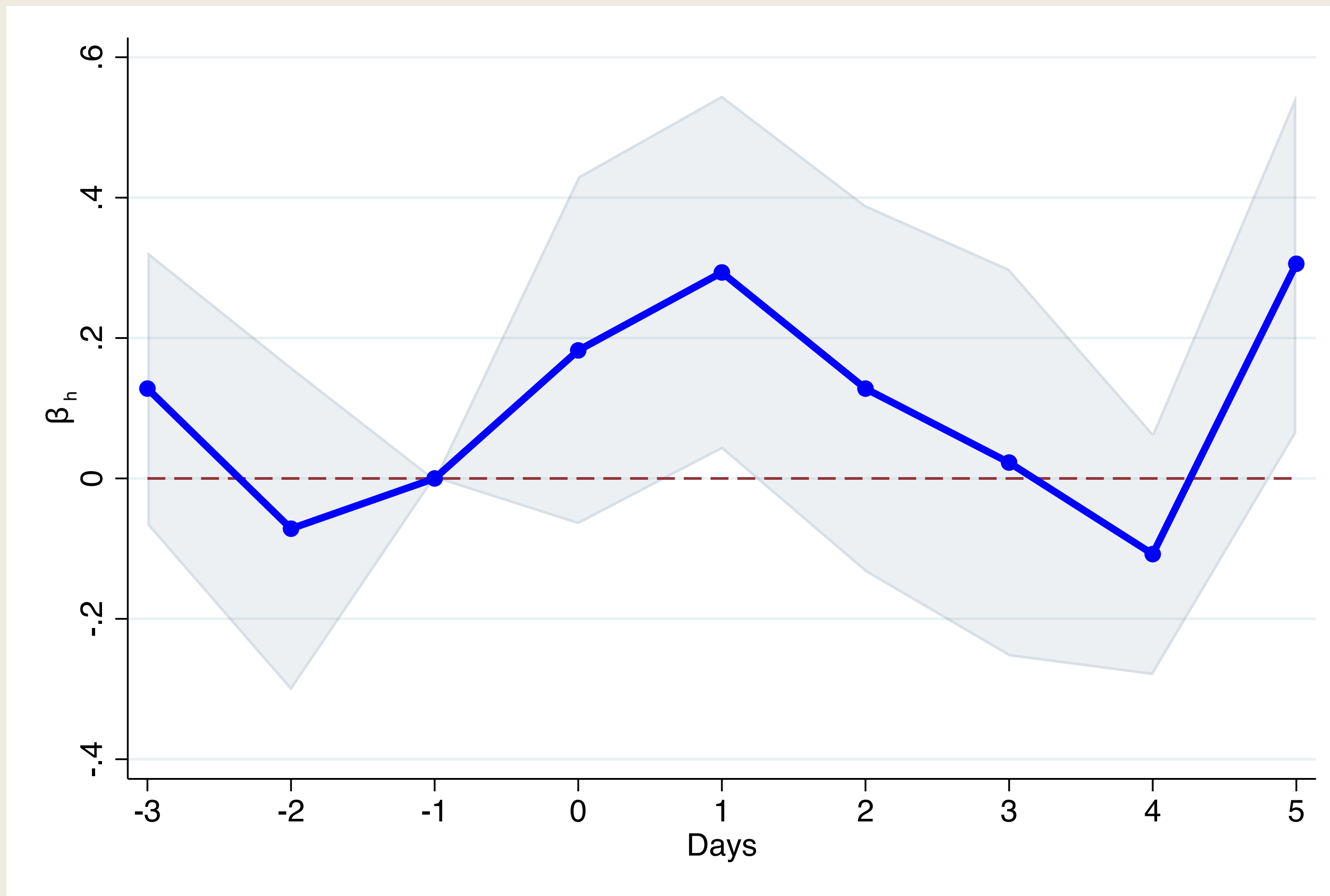


Two possibilities:

1. Growing concern for “distortion” that policy might cause
2. Stock price decline more likely to appear in the newspaper

Response of Retail Sales

- Some positive response of retail sales:



Robustness: Stocks

Panel A. Stock Price Response

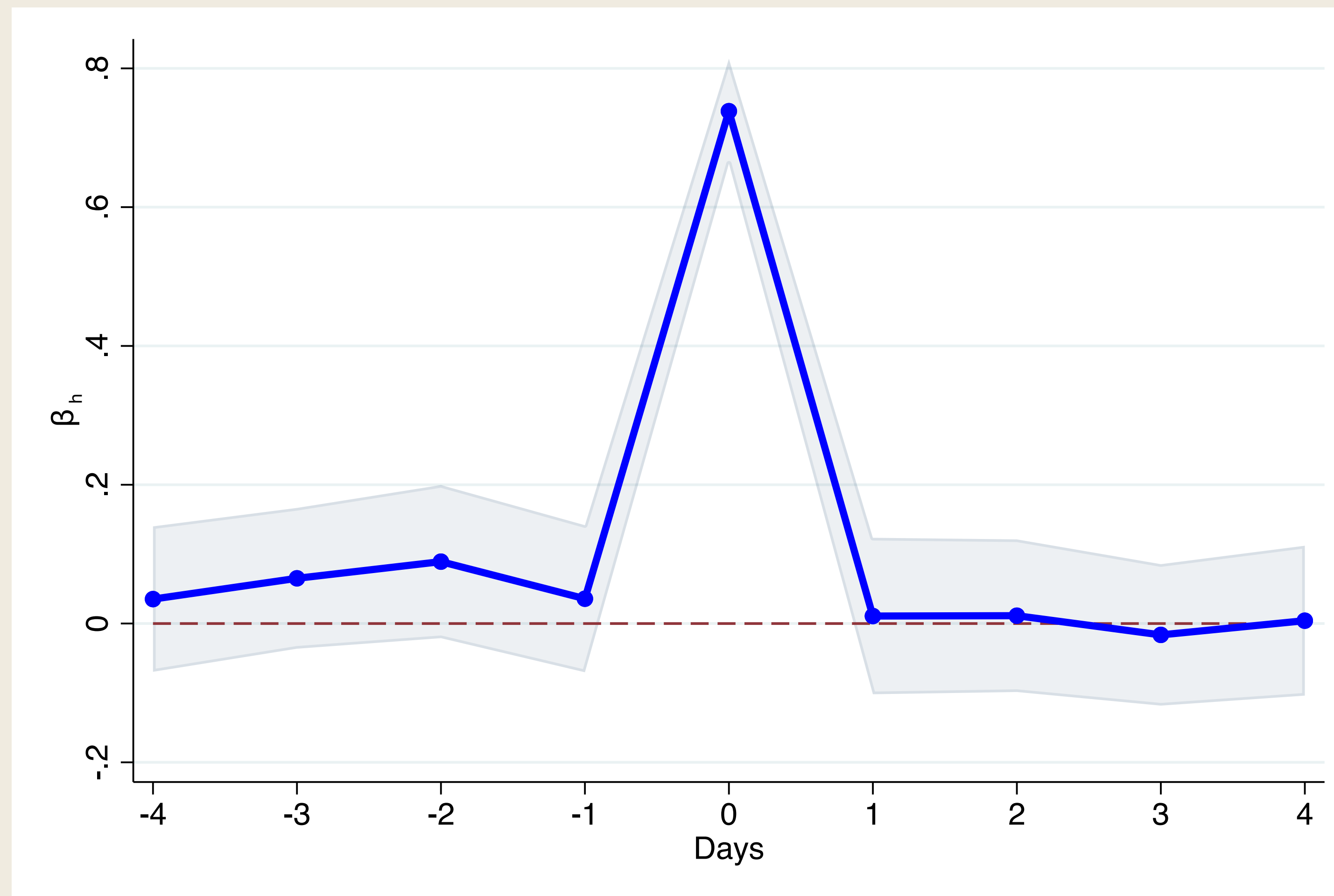
	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	34.90 (5.89)	8.65 (10.18)	36.31 (17.50)	-16.95 (26.27)	35.24 (4.68)	22.23 (11.13)
1. Narrower Bandwidth	41.65 (7.16)	16.22 (16.09)	30.14 (22.18)	-21.25 (39.57)	47.26 (7.15)	33.08 (16.25)
2. Wider Bandwidth	29.49 (4.84)	6.07 (9.36)	30.63 (13.46)	-2.44 (20.74)	29.41 (4.69)	15.66 (9.14)
3. Polynomial Order 2	43.40 (7.47)	16.98 (15.64)	41.13 (21.64)	-22.70 (42.42)	47.24 (7.14)	30.63 (14.39)
4. Control Past Interventions	39.50 (8.09)	12.56 (15.10)	54.57 (25.67)	-16.44 (34.04)	37.32 (6.61)	29.65 (14.48)
5. Control Past Stock Returns	34.91 (5.89)	8.62 (10.20)	36.15 (17.70)	-17.33 (26.21)	35.18 (4.77)	20.51 (11.76)
6. Control Past 10-Year Yield	37.86 (6.80)	10.35 (12.53)	49.85 (25.39)	-32.66 (36.34)	34.71 (4.97)	27.41 (13.66)
7. Drop Around the Cutoff Changes	34.41 (6.15)	7.64 (11.12)	28.76 (16.28)	-22.46 (28.58)	35.17 (5.49)	26.23 (14.97)

Robustness: Bonds

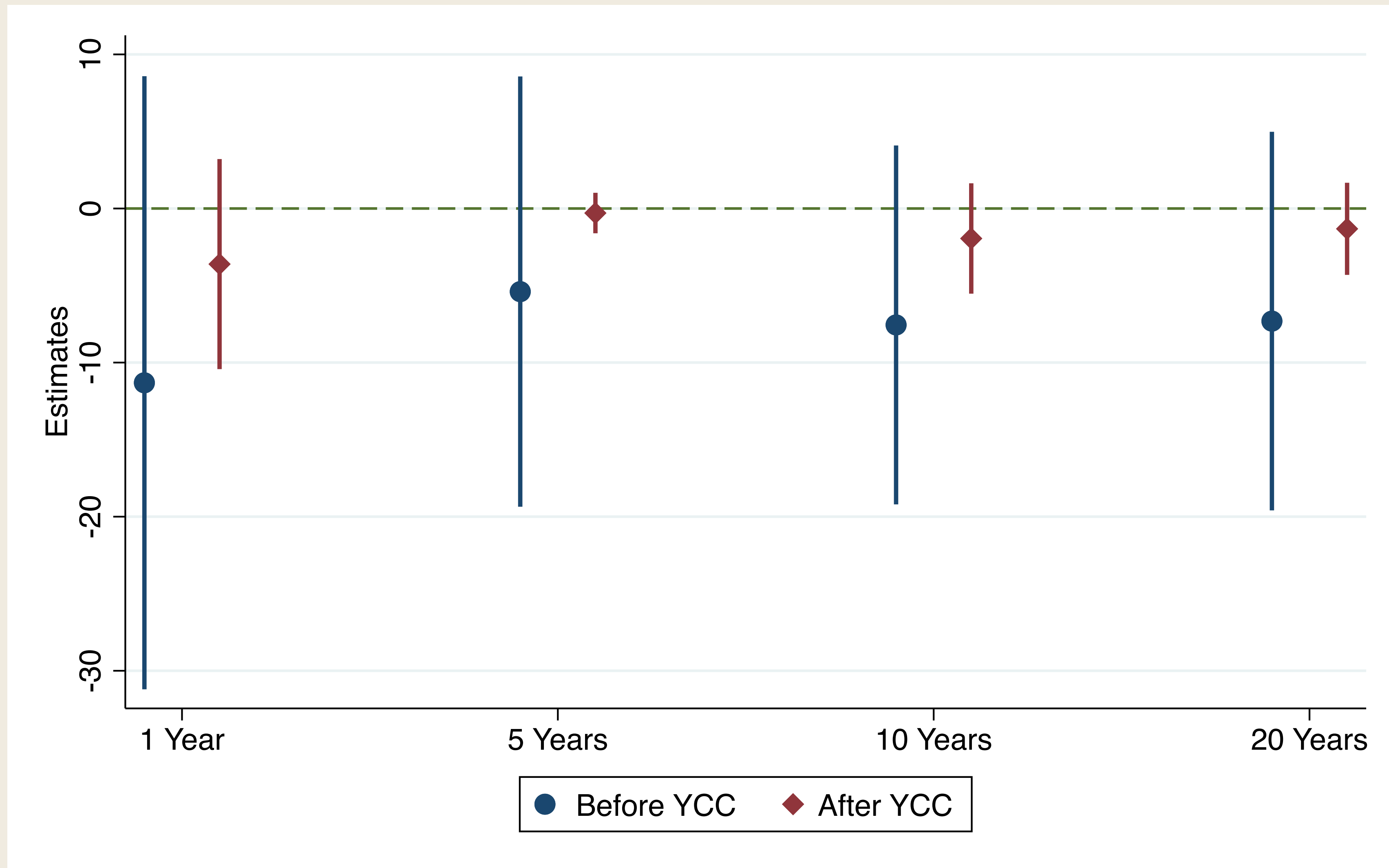
Panel B. JGB 10-Year Yield Response

	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	0.60 (0.22)	0.54 (0.26)	2.03 (0.69)	1.80 (0.76)	0.06 (0.08)	0.03 (0.13)
1. Half Bandwidth	0.64 (0.25)	0.59 (0.32)	2.51 (0.98)	2.43 (1.02)	0.02 (0.09)	-0.06 (0.20)
2. Wider Bandwidth	0.48 (0.18)	0.44 (0.23)	1.52 (0.56)	1.11 (0.53)	0.02 (0.06)	-0.04 (0.10)
3. Polynomial Order 2	0.69 (0.26)	0.69 (0.31)	2.51 (0.93)	2.69 (1.15)	0.10 (0.10)	0.08 (0.19)
4. Control Past Interventions	0.56 (0.27)	0.41 (0.34)	2.12 (0.91)	1.96 (0.97)	-0.04 (0.11)	-0.15 (0.23)
5. Control Past Stock Returns	0.59 (0.22)	0.53 (0.26)	2.04 (0.69)	1.82 (0.76)	0.05 (0.08)	0.01 (0.13)
6. Control Past 10-Year Yield	0.57 (0.22)	0.48 (0.27)	2.14 (0.78)	1.83 (0.83)	0.04 (0.08)	0.04 (0.14)
7. Drop Around the Cutoff Changes	0.48 (0.20)	0.42 (0.25)	1.51 (0.64)	1.60 (0.79)	0.02 (0.08)	0.02 (0.17)

ETF Shock



Inflation Responses



CDS Response

