Aging, Housing, and Macroeconomic Inefficiency*

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JEL Classifications: E22, D15, J11, R21
Keywords: aging, the natural rate of interest, overlapping generations model, bequest motives, intergenerational transfer of housing, Japan

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Abstract

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I. Introduction

Population aging affects a wide range of markets and macroeconomy (for example, see Wise, 1989, and subsequent publications from the NBER Program on the Economics of Aging). Our study focuses on the ownership of large housing capacities by elderly households, part of which is underutilized and may eventually become vacant after being transferred to the younger generation (Seko et al., 2023). Because housing consumption increases with age in many countries (e.g., Yang, 2009; Cocco and Lopes, 2020), a large stock of accumulated housing will become a common feature of an aging society.

Despite the accumulation of housing stock, the younger generation may still face housing affordability problems if the housing stock is not readily available to them due to excess housing capacity held by the elderly and intergenerational mismatches in preferences. Thus, a natural prediction is that an aging society will allocate more capital to the construction industry, which has low productivity in many countries (e.g., Goolsbee and Syverson, 2023; Garcia and Molloy, 2023). Thus, one consequence of population aging may be low overall productivity growth and low interest rates.

Insights into the macroeconomic impact of aging can be gained by studying Japan, which is at the forefront of aging societies. Its potential support ratio (the number of working-age people (15-64) per elderly person (65+)) is the lowest at 2.0 in 2021, compared with 3.9 for the US, 5.3 for China, and 9.9 for India. In addition, Japan, like other countries, shows increasing housing consumption with age. The homeownership rate increases monotonically with the age of the household head from 3% for households under 25 to 83% for those 85 and older (2018 Housing and Land Survey). A similar upward-sloping age profile is also observed for housing wealth in Japan's 2019 National Survey of Family Income, Consumption and Wealth. Moreover, total factor productivity (TFP) in the construction and housing sector also declined in the four decades leading up to the 2000s (figure 1), in stark contrast to the growing productivity of other sectors. The Japanese economy is thus an ideal case for our study of how the ownership of large housing capacities by elderly households affects the macroeconomy.

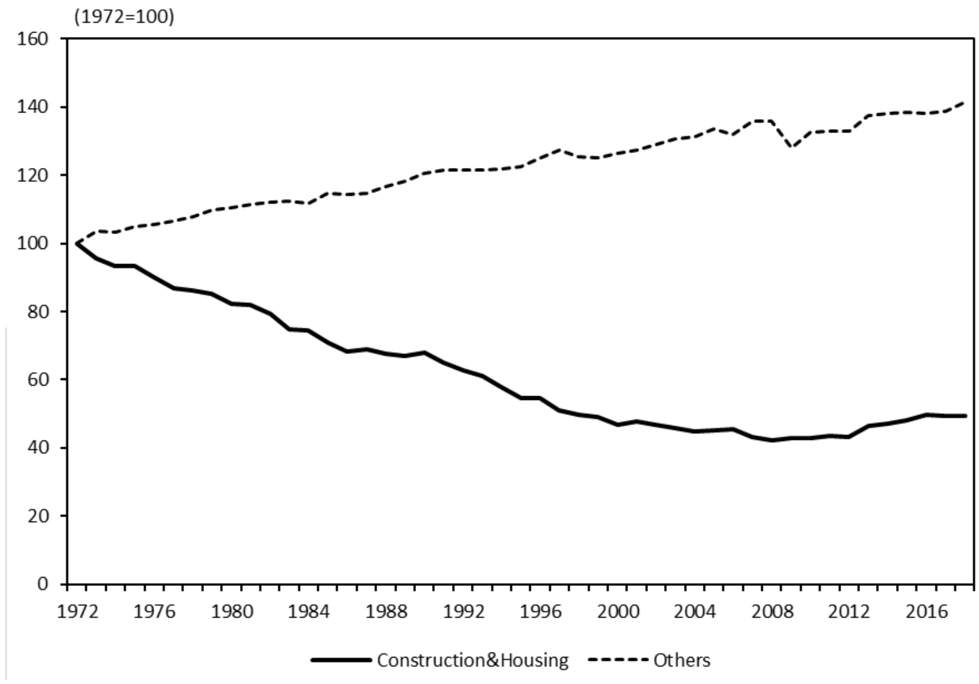


Figure 1: Total factor productivity level for construction and housing sector in Japan. Source: Japan Industrial Productivity Database.

We quantify the effect of elderly households' accumulated housing by calibrating a quantitative, perfect-foresight, overlapping generations (OLG) model to the Japanese economy. The model replicates well the historical GDP growth rate, the real interest rate, the housing-to-consumption ratio, and the housing vacancy rate. It also replicates the age profile of consumption, housing assets, and savings relatively well. These replications and calibrations are based on the exogenously specified trajectory of fertility rates, mortality rates, labor-augmenting technology for each sector, government spending, and government debt, in addition to constant parameters for age-specific labor productivity, the sectoral share of labor supply, and the pension replacement rate.

We analyze the natural (real) rate of interest, gross domestic product, total factor productivity, capital allocation between two production sectors, and house prices between 1980 and 2050. We first decompose the baseline prediction of the equilibrium outcome into the six exogenous factors: mortality, fertility, government debt, government spending, technology growth, and the technology gap between the two sectors. To identify the contribution of each factor, we "knock out" each factor at a time by keeping its 1980 value throughout the remaining years as Bielecki et al. (2020) do.[1] This decomposition analysis shows that the natural rate of interest will decline by 1.5-2.0 percentage points after 1980, mainly driven by stagnant technology growth and aging. The GDP growth rate will also decline by 4 percentage points for similar reasons. Housing value will increase from 2020, driven by the growing demand of the elderly.

The model also identifies the general-equilibrium effect of bequest driven financial and housing asset holdings, the balance between financial and housing bequests, the efficiency of intergenerational housing transfers, and inheritance tax rates. First, bequest motives increase the level of GDP but decrease the growth of GDP and TFP. Households consume less and allocate more capital to housing. The interest rate is lower because both productivity and the marginal product of capital are lower. Houses are less affordable because prices are inflated by both the extra demand and the less efficient production of housing. Second, the above effects are exacerbated when housing bequests are preferred to financial bequests because these effects are primarily driven by the extra demand for housing driven by bequest motives. Third, the same effects will be further exacerbated if more inherited houses become vacant due to inefficient intergenerational transfers. Finally, changing the inheritance taxe rate makes only a small impact on the outcome because the government will substitute the inheritance tax for the income tax. Overall, the Japanese economy can improve its efficiency by reducing the bequest-motivated housing demand of the elderly and improving the efficiency of house matching.

Our model has several special features. First, we introduce age-specific mortality rates. One challenge with a life-cycle model with a fixed life span is that the model predicts a flat age profile of consumption, whereas data show a hump-shaped age profile. By incorporating actual mortality rates into our model, we generate a hump-shaped age profile of consump-

[1] For the fertility rate, we use the average rate for 1980-1989 because the growth rate of the 20-year-old population is negative between 1970 and 1980 due to WWII.

tion. However, the actual mortality rates lead to a consumption peak much later (after age 80) than in the data (around age 50). Considering that households take health-adjusted life expectancy (HALE) into account when making consumption decisions, we adjust the empirical mortality rates for healthy life expectancy. Our calibrated model shifts the consumption peak earlier to around 70 yeas of age. Second, we introduce motives for bequeathing housing. In the OLG model that include housing consumption, households' first-order conditions require that the age profile of housing consumption be parallel to that of general consumption. However, the data show that housing consumption increases monotonically with age, unlike non-housing consumption. Housing bequest motives create an upward age profile of housing consumption (and wealth) in our model.

Third, we also introduce motives for bequeathing financial assets. In an OLG model, savings usually peak just before retirement age and decline monotonically to zero as a household ages. However, the data show that households do not dissave after retirement age (French et al., 2023). Our model introduces the financial bequest motive, which generates an increasing amount of saving that offsets the decreasing amount of saving in the standard life-cycle profile.

However, we note that several factors are not included in our model. First, our perfect foresight model does not allow for precautionary saving, which could help explain the persistently high level of savings after retirement. Second, our rational model does not allow for behavioral biases, such as myopia and suboptimal housing choices. Behavioral biases could explain an early peak in consumption and large housing capacities, but we intend to study the rational benchmark. Third, our model does not account for age-specific consumption goods, such as children's education and health services. While middle-aged parents may spend a significant amount on education, this is not reflected in our model. In addition, our model does not account for health care costs or the public health care system. Fourth, our model does not include mortgage loan terms, such as required down payments. The data suggest that households borrow and increase housing wealth significantly when their heads are in their 30s and 40s, which are likely driven by mortgage-financed first-time home purchases. Last, our model does not include housing transaction costs beyond inefficient intergenerational house transfers. One might think that the absence of downsizing is due to transaction costs, but they cannot explain excess housing capacity because rational households maintain undercapacity in their middle years to avoid costly downsizing.

Our study contributes to the literature in four ways. First, we show that homeownership is a new channel through which population aging affects the economy. This channel is based on the empirical regularity that the elderly are likely to own large housing capacities. Our model features the bequest motive of rational households to generate additional housing demand from elderly households, which leads to the allocation of capital to a less productive sector. Although other factors, such as a behavioral bias, could generate a similar result, we emphasize that downsizing constraints alone cannot generate an upward-sloping age profile of housing consumption if households are rational.

Second, our study provides quantitative insights into the impact of population aging on the Japanese economy, such as the real interest rate, GDP growth, tax rate, productivity growth, and house prices. By decomposing the changes in these variables between 1980 and 2050, we quantify the effect of the expected changes in mortality and fertility rates. These factors lead to lower interest rates and higher taxes, but are almost neutral for GDP and TFP growth.

Third, our study shows the general equilibrium effect of age-specific housing demand on macroeconomic variables, whereas existing studies typically show partial equilibrium effects and age-independent housing choices. The increased housing capacity of the elderly due to bequest motives significantly affects consumption, output, productivity growth, and the natural rate of interest. Specifically, the additional demand for housing suppresses consumption and real interest rates, while it expands the level of output.

Fourth, our model demonstrates that an OLG model can replicate a hump-shaped age profile for consumption and an upward-sloping age profile for housing and savings by incorporating both age-specific mortality rates and bequest motives for financial and housing assets. It provides a benchmark modeling choice for future studies.

The remainder of this paper is organized as follows. Section II reviews the related literature, Section III describes our model, and Section IV explains the model calibration. Sections V and VI presents the results, followed by discussions in Section VII. Section VIII concludes the paper.

II. Literature Review

A. Population aging

Population aging can have a profound effect on the economy. A series of conference proceedings from the NBER Project on the Economics of Aging include a large number of contributions since 1989 (Wise, 1989, 1990, 1992, 1994, 1996, 1998b,a, 2001, 2004, 2005, 2009, 2011, 2012, 2014, 2017). These studies show that aging has not only an obvious impact on health care and the social security system but also a wide range of indirect effects on savings, consumption, and investment. Furthermore, Weil (1997) and Lee (2016) review the effect of aging on the macro economy. The literature generally predicts a tendency toward increased capital intensity, higher wages, and lower returns on capital. In an open economy setting, flows of capital and labor into the aging country are also expected to increase. The effect on output is generally expected to be negative (e.g., Sheiner et al., 2007; Sheiner, 2014; Börsch-Supan et al., 2014; Gagnon et al., 2021), but the literature does not completely agree on this prediction because capital deepening can overturn the effect (e.g., Cutler et al., 1990). In a recent study, Maestas et al. (2023) further point out that population aging slows economic growth by diminishing labor productivity growth in addition to employment growth. These results critically depend on conditions such as labor supply at older ages, per capita consumption of the elderly relative to younger ages, strength of public pension and health care systems, and health and vitality of the elderly.

B. Large-scale overlapping generations models

Following the seminal work by Auerbach and Kotlikoff (1987), overlapping generations (OLG) models have been a leading tool to evaluate the macroeconomic effect of demographic change because they can incorporate empirical age distributions to replicate realistic life-cycle behavior. Recent OLG models predict a downward trend of the real interest rate due to population aging (e.g., Bielecki et al., 2020; Papetti, 2021). Longevity can result in more precautionary savings. In particular, Bielecki et al. (2020) is most closely related to the present study. They develop a quantitative OLG model calibrated to match the life-cycle profiles of the European economy. They particularly investigate the impact of demographics on the natural rate of interest (NRI) in the euro area, with a particular focus on the role played by economic openness, migrations and pension system design. They show that population aging explains approximately two thirds of the secular decline in the NRI between 1985 and 2030. They also find that an increase in the retirement age can revert the downward trend on the interest rate, but the replacement rate exacerbate the trend. Papetti (2021) also confirms that demographics account for a decrease in the natural real interest rate in the euro area due to scarce labor input and more savings. However, the results depend on substitutability between labor and capital, intertemporal elasticity of substitution, higher productivity, and labor participation by older individuals. Gagnon et al. (2021) predicts that GDP growth and interest rates will remain low in the U.S. Carvalho et al. (2016) show that the equilibrium real interest rate will be lower through potentially competing three channels: larger savings driven by longevity, larger capital per-worker, and a lower savings rate. Eggertsson et al. (2019) quantify the secular stagnation hypothesis and predict a natural rate ranging from -1.5% to -2% in the U.S. Sudo and Takizuka (2018) estimate that demographic changes account for approximately 270 basis points out of the 640 basis-point decline in real interest rates during the last 50 years in Japan. Braun and Ikeda (2022) provide a quantitative theory of deflation and secular stagnation using Japanese data.

C. Macroeconomic models with bequests and housing

A bequest is a type of intergenerational transfers in addition to public pensions and education and plays a central role in investment decisions. Thus, it critically affects the consequences of population aging. Artle and Varaiya (1978) is an early study that incorporates homeownership and bequest motives in the life cycle consumption model. However, they generally predict a monotonic age profile of the optimal consumption, either sloping downward or upward, depending on financial constraints. Whether a bequest motive is a critical factor for the economy has been debated. Hurd (1989) estimates a life cycle model augmented by a bequest motive and finds that the bequest motive is statistically significant but economically trivial. Hurd (1987) argues that bequests are merely accidental resulting from precautionary responses to imperfect insurance markets. However, Bernheim (1991) finds that the typical household chooses to maintain a positive fraction of its resources in bequeathable forms even if insurance markets were perfect. Kopczuk and Lupton (2007) find that roughly three quarters of the elderly single population has a bequest motive and spends less on consumption expenditure, doubling the amount of bequeathed net wealth. Intentional bequests can emerge either from altruism (Tomes, 1981) or self-interested exchange with one's heirs (Bernheim et al., 1985). Bequests are also incorporated in quantitative macro models (e.g., Hurd, 1989; Hviding and Mérette, 1998; Fougère and Mérette, 1999; Kraft and Munk, 2011).

A model that features bequest motives in the utility function can also be seen as a version of models featuring wealth in the utility function (Michaillat and Saez, 2021; Nakajima, 2003; Barberis et al., 2001; Carroll, 1998; Dynan et al., 2002, 2004). Wealth in the utility function may resolve macroeconomic puzzles related to savings rate and consumption around zero lower bound. Wealth in the utility function is often incorporated in partial-equilibrium consumer models and DSGE models, but there are not many OLG models that feature wealth in the utility function.

Housing plays a critical role in consumption choices and investment decisions because housing plays a dual role as a consumption goods and investment and collateral assets. The role of real estate as collateral is a key insight provided by Kiyotaki and Moore (1997), Bernanke et al. (1999), and subsequent studies on the effect of financial constraints on the financial crisis and monetary policy transmissions (e.g., Wong, 2021). Housing also affects household portfolio choices and equilibrium asset prices (e.g., Flavin and Yamashita, 2002; Piazzesi et al., 2007). Iacoviello (2005) and Iacoviello and Neri (2010) incorporate housing in dynamic stochastic general equilibrium (DSGE) models. They show that homeownership with mortgage financing significantly affects how demand and supply shocks propagate. Also, they find that slow technological progress in the housing sector is a major cause of house price appreciation. Furthermore, they show that housing is increasingly important in explaining consumption. More recent DSGE models also incorporate housing (Liu et al., 2013; Garriga et al., 2021; Adam and Woodford, 2021). Housing is also incorporated in multi-sector OLG models (Galor, 1992; Farmer and Wendner, 2003; Yang, 2009; Kraft and Munk, 2011; Waters, 2020; Nakajima, 2020). In particular, as in the present study, Nakajima (2020) focuses on the substitution between housing and non-housing capital and finds that preferential tax treatment for owner-occupied housing significantly decreases the optimal capital income tax rate. A low capital income tax rate improves welfare by narrowing a tax wedge between housing and non-housing capital. Nakajima and Telyukova (2020) study why retired homeowners dissave slowly. They show that the accumulated home equity during a housing boom cannot be easily cashed out. Cocco and Lopes (2020) study the role of housing wealth in retirement consumption and saving decisions by the elderly. They find that reverse mortgages combined with insurance against a forced home sale is Pareto improving.

III. Model

A. The Model Setup

Consider a model for a closed-economy populated by overlapping generations (OLG) of households that solve a standard life-cycle problem. There are four agents in this model economy: households, two-sector firms (general goods and housing sectors) and fiscal authority. The model is mainly based on Bielecki et al. (2020) and Papetti (2021).

B. Households

Each household consists of a single agent, who appears in our model at the age of 20, indexed as $j = 1$. Agents can live up to 104 years ($j = J = 85$) but may die at year t according to an age- and time-dependent mortality rate $\omega_{j,t}$. Hence, at each point in time, the model economy is populated by 85 cohorts of overlapping generations. The size of the age-j cohort is denoted by $N_{j,t}$.

The representative age-j household at time t maximizes its expected remaining lifetime utility until the maximum age J:

$$U_{j,t} = \sum_{i=0}^{J-j} \beta^i \left[\frac{N_{j+i,t+i}}{N_{j,t}} \left(\ln c_{j+i,t+i} + \chi \ln h_{j+i,t+i} \right) + \left(\frac{N_{j+i,t+i} - N_{j+i+1,t+i+1}}{N_{j,t}} \right) \psi \{ \nu \ln bi_{j+i,t+i} + (1-\nu) \ln h_{j+i,t+i} \} \right]. \tag{1}$$

where β denotes the subjective discount factor. The per period utility function has two components. The first term represent felicity as a function of the consumption stream $c_{j,t}$ and the housing services $h_{j,t}$, conditional on survival; $N_{j+i,t+i}/N_{j,t}$ denotes the probability of surviving for at least i more years. Parameter χ determines the importance of housing consumption. The second term represents bequest motives conditional on death; the probability of dying in year $t+i$ equals $\left(N_{j+i,t+i} - N_{j+i+1,t+i+1} \right) / N_{j,t}$, with $N_{J+1,t+J+1-j} = 0$. Parameter ψ determines the importance of bequest motives relative to felicity. Bequest motives are specified as the weighted average of the log utilities from financial bequest contributions $bi_{j,t}$ and housing bequests $h_{j,t}$. Parameter ν denotes the utility weight for financial bequests.

An age-j household has the following budget constraint:

$$c_{j,t} + a_{j,t} + bi_{j,t} + P_t^h hi_{j,t} = (1 + r_t)\, a_{j-1,t-1} + \mathbf{1}_{j \geq JR} pen_t$$
$$+ (1 - \mathbf{1}_{j \geq JR}) \left[(1 - T_t) \left\{\sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h\right\} + (1 - T_t^a)\, i_t - T_t^h P_t^h i_t^h\right]. \quad (2)$$

Households inelastically supply labor until reaching the age of 64 ($j = JR = 45$), either in the general goods production sector g or housing production sector h. Labor income in sector $s = \{g, h\}$ depends on the sector-specific gross wage rate w_t^s and the exogenously determined age-specific labor productivity ξ_j^s. Working-age households also inherit financial assets i_t and housing assets i_t^h evaluated at the market price of housing P_t^h after paying inheritance taxes at rates T^a and T^h, respectively. Retirees, as indicated by $\mathbf{1}_{j \geq JR}$, receive pension benefits pen_t that are independent of their age. Participation in the pension system is mandatory, so that all workers have to make a social security contribution that is levied as part of personal income tax at the rate of T_t. All households can smooth their consumption by adjusting liquid assets $a_{j,t}$, which are claims on productive capital and government debt. We allow households to borrow across different cohorts, so that asset holdings $a_{j,t}$ can be negative at any age j. Households can also adjust housing, either positively or negatively, with housing investment $hi_{j,t}$ at the market price of housing. Finally, households invest $bi_{j,t}$ in the financial bequest account $b_{j,t}$. At death, agents leave the accumulated financial bequest, the outstanding liquid asset, and the outstanding housing asset as bequests.

In Equation (2), a household smoothly adjusts housing by the amount $hi_{j,t}$ by spending $P_t^h hi_{j,t}$. The sign of $hi_{j,t}$ can be negative if a household disposes part of its housing. This smooth adjustment of housing can also be understood as a household selling and buying houses each period if Equation (4) is substituted into Equation (2). The direct interpretation of this budget constraint is that a household in our model is a homeowner who spends on housing assets at the time of purchase but does not pay periodic user costs (or housing rents). However, this specification is equivalent to an alternative one that includes periodic user costs but not purchase costs. An advantage of our specification is that house prices are directly specified. Because a household enjoys housing services without paying periodic monetary expenses, this implicit service benefit is regarded as the income return from housing investment. Thus, we can derive the user cost rate by taking the difference between the total return to housing assets $1 + r_t$ and house price appreciation rate P^h_{t+1} / P^h_t.

Financial assets for bequest ($b_{j,t}$) and occupied and vacant houses ($h_{j,t}$ and $v_{j,t}$) are accumulated according to the following equations, respectively.

$$b_{j,t} = (1 + r_t)\, b_{j-1,t-1} + bi_{j,t}, \tag{3}$$

$$h_{j,t} = (1 - \delta_h)\, h_{j-1,t-1} + hi_{j,t} + \phi\, (1 - \mathbf{1}_{j \geq JR})\, i^h_t, \tag{4}$$

$$v_{j,t} = (1 - \delta_h)\, v_{j-1,t-1} + (1 - \phi)\, (1 - \mathbf{1}_{j \geq JR})\, i^h_t. \tag{5}$$

Financial bequest assets are invested in productive capital and earn real interest rate r_t, whereas housing assets depreciate at the rate of δ_h. Parameter ϕ denotes the intergenerational house matching efficiency, as defined by the proportion of inherited housing i^h_t used by younger generations. The initial and terminal value conditions are:

$$b_{0,t} = a_{0,t} = h_{0,t} = v_{0,t} = 0, \tag{6}$$

$$a_{J,t} = 0. \tag{7}$$

Inherited financial assets and housing are defined by:

$$i_t = (1 + r_t)\, \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t})\, (a_{j,t-1} + b_{j,t-1})}{\sum_{j=1}^{JR} N_{j,t}}, \tag{8}$$

$$i^h_t = (1 - \delta_h)\, \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t})\, h_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}}, \tag{9}$$

where i_t and i_t^h denote inherited financial assets and housing, respectively. Bequeathed financial assets and housing are equally allocated to all working-age households.

In our model, demographic processes are governed by exogenous changes in the size of the youngest cohort $n_{1,t} = N_{1,t}/N_{1,t-1} - 1$ and the mortality rate $\omega_{j,t}$. Then, the total number of living agents N_t and the population growth rate n_t are given by:

$$N_t = \sum_{j=1}^{J} N_{j,t}, \tag{10}$$

$$n_t = \frac{N_t}{N_{t-1}} - 1. \tag{11}$$

The number of agents in each cohort is given by:

$$N_{j,t} = \left(1 - \omega_{j-1,t-1}\right) N_{j-1,t-1}. \tag{12}$$

Table I summarizes the timing convention for each variable.

C. Firms

Firms consist of general goods and housing sectors: $s = \{g, h\}$. A representative firm in sector s hires capital and labor and produces non-differentiated output Y_t^s according to the following Cobb-Douglas production function:

$$Y_t^s = \left(K_t^s\right)^{\alpha_s} \left(\zeta_t^s L_t^s\right)^{1-\alpha_s}, \tag{13}$$

where ζ_t^s denotes labor-augmenting technology growing at the rate of g_t^s:

$$\zeta_t^s = \left(1 + g_t^s\right) \zeta_{t-1}^s. \tag{14}$$

Period		t	
Date	$t-1$ (Beginning of period t)		t (End of period t)
Asset	$A_{t-1}, a_{j-1,t-1}$		$A_t, a_{j,t}$
Housing	$H_{t-1}, h_{j-1,t-1}$		$H_t, h_{j,t}$
Accumulated bequest	$B_{t-1}, b_{j-1,t-1}$		$B_t, b_{j,t}$
Capital	K_{t-1}		K_t
Government debt	D_{t-1}		D_t
Population/Size of corhort		$N_t, N_{j,t}$	
Consumption		$C_t, c_{j,t}$	
Bequest		$bi_{j,t}$	
Housing investment		$hi_{j,t}$	
Mortality rate		$\omega_{j,t}$	
Goods production		Y_t^g	
Housing production		Y_t^h	
Capital for goods sector		K_t^g	
Capital for housing sector		K_t^h	
Government spending		G_t	
Inheritance		i_t	
Housing Inheritance		i_t^h	
Interest rate		r_t	

Table I: Timing Convention for Each Variable

Let θ_t denote the technology gap between the housing and general goods sectors:

$$\theta_t \equiv 1 - \frac{\zeta_t^h}{\zeta_t^g}. \tag{15}$$

Then, the housing-technology growth rate equals the general-technology growth rate adjusted for the change in the technology gap:

$$1 + g_t^h = (1 + g_t^g)\,\frac{1 - \theta_t}{1 - \theta_{t-1}}. \tag{16}$$

The periodic profit flow is:

$$\pi_t^s = P_t^s Y_t^s - w_t^s L_t^s - (r_t + \delta_k)\, K_t^s, \tag{17}$$

where P_t^g equals unity.

D. Government

The government purchases general goods G_t in the market and spends on pensions pen_t. It rolls over public debt by paying off the outstanding debt $(1 + r_t)\, D_{t-1}$ and issuing a new debt D_t. It also levies taxes from ordinary income at a rate T_t, inherited housing at a rate T_t^h, and inherited financial assets at a rate T_t^a. The government's budget constraint at time t is:

$$\sum_{j=1}^{JR-1} T_t N_{j,t}\left[\sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h\right] + \left(T_t^h P_t^h i_t^h + T_t^a i_t\right) \sum_{j=1}^{JR-1} N_{j,t} + D_t$$
$$= G_t + pen_t \sum_{j=JR}^{J} N_{j,t} + (1 + r_t)\, D_{t-1} \tag{18}$$

Pensions per retired household are determined as the product of the replacement rate ρ_t

and the economy-wide average wage:

$$pen_t = \rho_t \frac{w_t^g \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g + w_t^h \sum_{j=1}^{JR-1} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h}{\sum_{j=1}^{JR-1} N_{j,t}}, \tag{19}$$

where σ_j^g is the proportion of j-aged household choosing to work for the general goods sector. The paths of public debt, government purchases, the replacement rate, and inheritance tax rates are exogenously determined, whereas the income tax rate T_t adjusts so that the government budget constraint (18) is satisfied. Thus, for the government, three types of taxes are perfect substitutes.

E. Market clearing and other aggregate conditions

The financial market clears with

$$D_t + K_t = A_t + B_t \tag{20}$$

$$K_{t-1} = K_t^g + K_t^h \tag{21}$$

Labor market clears with:

$$L_t^g = \sum_{j=1}^{J} \sigma_j^g N_{j,t} \xi_j^g, \tag{22}$$

$$L_t^h = \sum_{j=1}^{J} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h. \tag{23}$$

New housing market clears with:

$$Y_t^h = \sum_{j=1}^{J} N_{j,t} hi_{j,t}. \tag{24}$$

The good market clears as:

$$Y_t^g = \sum_{j=1}^{J} N_{j,t} c_{j,t} + K_t - (1 - \delta_k) K_{t-1} + G_t. \tag{25}$$

The aggregate consumption, assets and housing are:

$$C_t = \sum_{j=1}^{J} N_{j,t} c_{j,t}, \tag{26}$$

$$A_t = \sum_{j=1}^{J} N_{j,t} a_{j,t}, \tag{27}$$

$$B_t = \sum_{j=1}^{J} N_{j,t} B_{j,t}, \tag{28}$$

$$H_t = \sum_{j=1}^{J} N_{j,t} h_{j,t}, \tag{29}$$

$$V_t = \sum_{j=1}^{J} N_{j,t} v_{j,t}. \tag{30}$$

F. *Exogenous process*

The model economy is driven by several exogenous forces. Demographic processes are characterized by the growth rate of the 20-year-old cohort $n_{1,t}$ and the age-specific mortality risk $\omega_{j,t}(j = 1, ..., J)$. The fiscal authority exogenously determines its purchases of general goods G_t, the replacement rate in the pensions system ρ_t, and the path of public debt D_t. The total factor productivity is also exogenous for the general goods and housing sectors. The age-specific mortality rate $\omega_{j,t}(j = 1, ..., J)$ is adjusted for healthy life expectancy.

Table II shows the list of the exogenous variables.

G. *Equilibrium*

For all periods $t = 0, 1, ..., \infty$, given exogenously given demographics $\{N_{j,t}\}_{j=1}^{J}$, N_t, n_t and variables $n_{1,t}$, ρ_t, ζ_t^g, ζ_t^h, G_t, $\{\omega_{j,t}\}_{j=1}^{J}$, $\left\{\xi_j^g\right\}_{j=1}^{J}$, $\left\{\xi_j^h\right\}_{j=1}^{J}$, and $\left\{\sigma_j^g\right\}_{j=1}^{JR-1}$, the equilibrium

Name	Description	Type
$n_{1,t}$	Fertility rate (the growth rate of 20-year-old cohort)	periods x 1
$\omega_{j,t}$	Age-specific modified mortality rate	periods x cohorts
ζ_t^g	Labor-augumenting technology in general production sector	periods x 1
θ_t	Technology gap between the housing and general production sectors	periods x 1
$\mathbf{1}_{j \geq JR}$	Retirement indicator	periods x cohorts
ρ_t	Replacement rate	periods x 1
σ_j^g	Age-specific proportion for working in general production sector	cohorts x 1
ξ_j^g	Age-specific labor productivity in general production sector	cohorts x 1
ξ_j^h	Age-specific labor productivity in housing sector	cohorts x 1
$\overline{G/Y}$	Government spending ratio	periods x 1
$\overline{D/Y}$	Government debt ratio	periods x 1

Table II: List of Exogenous Variables

of this economy is a sequence of aggregates C_t, A_t, B_t, H_t, V_t, prices w_t^g, w_t^h, P_t^h, r_t, transfers T_t, pen_t, and quantities $\{c_{j,t}, a_{j,t}, b_{j,t}, bi_{j,t}, h_{j,t}, hi_{j,t}, v_{j,t}\}_{j=1}^J$, i_t, i_t^h, K_t, K_t^g, K_t^h, L_t^g, L_t^h, Y_t^g, Y_t^h, such that:

1. Demographics develops as (10)-(12) and the aggregation satisfies (26)-(30);
2. Households maximize the life-time utility (1) subject to constraints (2)-(9);
3. Firms maximize profits (17) given the production technology (13)-(16);
4. The government balances budget (18) and (19) by setting the income tax rate;
5. Factor markets (20)-(23), the housing market (24) and the general goods market (25) clear.

IV. Baseline Calibration

A. *Exogenous variables and structural parameters*

We calibrate the model to the Japanese economy using the data from 1925 to 2095. We exogenously specify a demographic scenario, technological progress, and the fiscal conditions of the government. Appendix G contains the graphs of each exogenous variables.

For our demographic scenario, we use past data and projections on mortality rates and the size of the 20-year-old cohort from the Population Projection for 2020 through 2050 from the National Institute of Population and Social Security Research. We smooth the mortality rate and the cohort size by using the stochastic trend component from the Hodrick-Prescott (HP) filter. We further adjust the mortality rate $\omega_{j,t}$ for the health-adjusted life expectancy (HALE) by using the Life Tables for 2019 and the Comprehensive Survey of Living Conditions from the Ministry of Health, Labour and the Welfare. In Japan, HALE is shorter than the actual life expectancy by 8 to 12 years (figure 3). We assume that mortality rates are constant for 1925-1970 at the rate in 1970. Regarding the growth rate of the 20-year-old cohort $n_{1,t}$, we use the Population Estimates between 1925 and 2014 and the Population Projection between 2015 and 2050 from the Statistics Bureau. After 2050, we assume that both the mortality rate and the size of the 20-year-old cohort stabilize at the 2050 level. Figure 2 contrasts the actual age distribution of the population with the model-generated age profile for 1995, 2019, and 2050. The model replicates the actual demographics quite well.

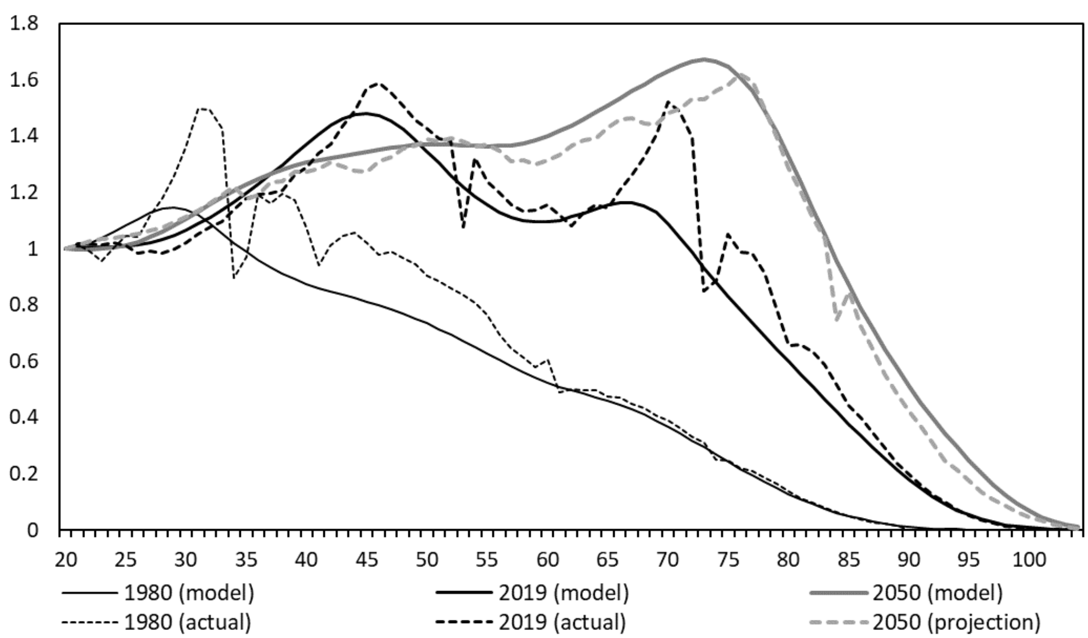


Figure 2: Demographics in 1980, 2019 & 2050

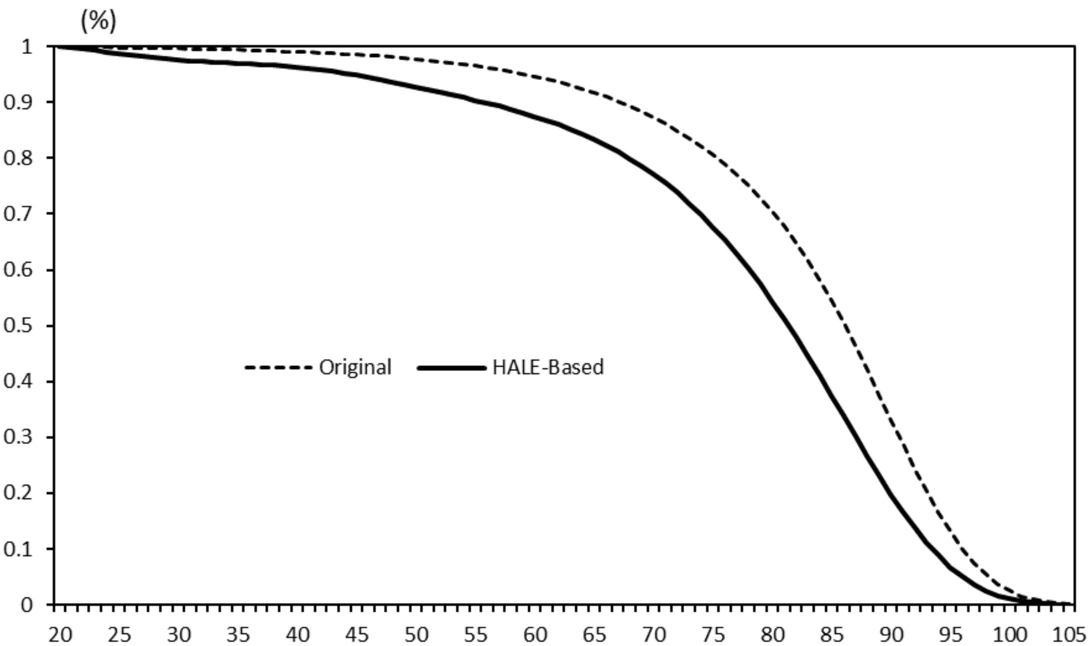


Figure 3: Original and HALE-Based Survival Rates

We also estimate the age-specific labor productivity for the general goods sector ξ_j^g and housing sector ξ_j^h and the age-specific proportion of workers in the general goods sector ρ_j^g, using the 2020 Basic Survey on Wage Structure by the Ministry of Health, Labour and Welfare. Our empirical measure of labor productivity is the estimated hourly labor income based on a regression on the constant, age, and age squared. The retirement age JR is 65 for all periods.

For technological progress, we estimate the labor-augmenting technology for the general goods sector ζ_t^g and housing sector ζ_t^h using the JIP Database between 1972 and 2018 from the Research Institute of Economy. We use the stochastic trend component out of the Hodrick-Prescott filter. fter 2019, we assume that the TFPs grow at the same rates as 2018.

For the fiscal condition of the government, we use the government spending ratio (of GDP) and government debt ratio (of GDP) from the Japanese System of National Accounts (JSNA) data for 1980-2021. We assume that those are constant for 1925-1980 at the rate in 1980 and for 2021-2095 at the rate in 2021. We also fix the replacement rate at 45%, referring to the literature.

Table III summarizes the parameters. The discount factor β, the capital depreciation rate δ_g, the housing depreciation rate δ_h are based on the literature. The inheritance tax rates for financial assets (T^a) and housing (T^h) capture the preferential tax treatment for housing. Inherited financial assets are heavily taxed up to the maximum tax rate of 55%, although there are some basic exemptions. In contrast, the assessed value of housing is reduced by 80% up to a 330 m^2 lot. Thus, many heirs who inherit only a house often pay no inheritance taxes at all. Instead of specifying the complex inheritance tax codes, we simply use a 40% effective tax rate for inherited financial assets and no effective tax for inherited housing. Other countries also have various forms of preferential treatment of housing in estate taxes, such as a step-up in basis. Thus, this difference in the effective tax rate is relevant for many countries.

We determined the importance of housing consumption χ, the relative importance of bequest motives ψ in the lifetime utility function, and the weight on financial bequests ν relative to housing bequests by matching the following statistics with the 2019 data: 1. the ratio of housing consumption to non-housing consumption ($p^h h/c$) for ages 20-64 and 65-104 and 2. the average saving-to-consumption ratio ($(a+b)/c$) for 65-104.

The efficiency parameter for intergenerational housing transfer ϕ is set to 0.2 to match the inverse vacancy rate (20% of the inherited houses are used by the younger generation). The capital shares for general goods sector α_g and for the housing sector α_h are calibrated to the JSNA data for 1994-2018.

Table IV shows the parameter calibration results. The ratio of housing consumption to non-housing consumption in the model ($P^h h/c$) is identical to the data for both the 20-64 group (5.3) and the 65-104 group (8.4). However, these perfect matches and the realistic age profile of housing consumption come at the expense of a high saving-to-consumption ratio ($(a+b)/c = 17.6$) compared to the acutual average ratio (6.4) for 65-104. A lower value of ψ can decrease the saving-to-consumption ratio but worsens the housing-to-non-housing consumption ratio and the age profile of saving and housing consumption. Because the

Name	Description	Value	How to decide
β	Discount factor	0.996	Litereture
χ	Importance of housing	0.45	Match the average $P^h h/c$ ratio among age 20-64
ψ	Importance of bequest	24	Match the average $P^h h/c$ ratio among age 65-104
ν	Relative importance of financial bequest	0.56	Match the average $(a+b)/c$ ratio among age 65-104
δ_g	Capital depreciation rate	0.15	Litereture
δ_h	Housing depreciation rate	0.075	Litereture
ϕ	Resusable rate of inherit housing	0.2	Match the average h/v ratio
T^a	Tax rate for the financial bequest	0.4	Litereture
T^h	Tax rate for the housing bequest	0	Litereture
α_g	Capital share in general production sector	0.37	Calibrate with JSNA
α_h	Capital share in housing sector	0.31	Calibrate with JSNA

Table III: List of Structural Parameters

main focus of this study is on the effect of housing consumption, we accept a high saving-to-consumption ratio for the baseline. However, we perform the counterfactual analysis by changing the value of ψ and ν. The inverse vacancy rate from the model ($c/v = 6.1$) is reasonably similar to the data (5.0). In fact, if we remove vacant rental units from the data, the empirical inverse vacancy rate increases to 7.14. Thus, the efficiency parameter $\phi = 0.2$ is reasonable.

Variable	Target Value	Model Value
Average $P^h h/c$ ratio among age 20-64	5.3	5.3
Average $P^h h/c$ ratio among age 65-104	8.4	8.4
Average $(a+b)/c$ ratio among age 65-104	6.4	17.6
Average h/v ratio	5.0	6.1

Table IV: Calibration Results

B. Baseline Calibration

Figure 4 illustrates the age profile of consumption, housing, and savings in 2019. In contrast to the basic model for the life-cycle and permanent income hypotheses (LCH/PIH), our model generates a hump-shaped age profile of per capita consumption due to variable mortality rates (panel 4a). The consumption growth for a household of age j between t and

$t+1$ provides an insight into this cross-sectional age profile. Our model's Euler equation is as follows:

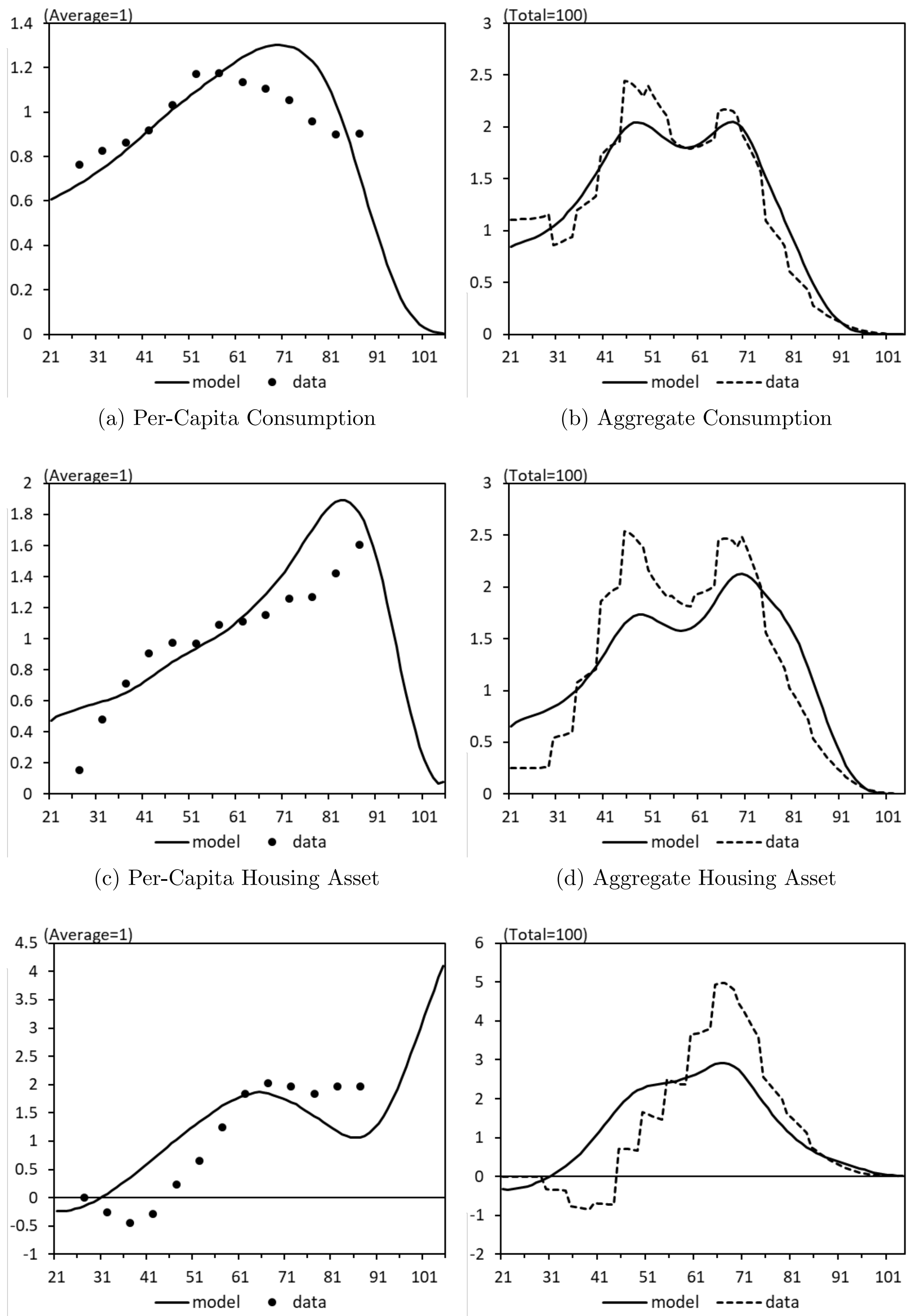
$$\frac{c_{j+1,t+1}}{c_{j,t}} = \beta\left(1+r_{t+1}\right)\left(1-\omega_{j,t}\right). \tag{31}$$

This equation shows that the consumption growth rate is determined not only by the standard factors (the subjective discount factor and the interest rate) but also the survival rate $(1-\omega_{j,t})$. Younger generations increase consumption each year based on the standard factors because changes in survival rates are negligible. However, the survival rate decreases rapidly towards the end of the lifespan. As a result, consumption growth becomes less than one when the survival rate is significantly low. Panel 4a shows a hump-shaped age profile of per capita consumption for both the model and data. A difference is that the peak occurs at around 50 years old in the data, whereas it occurs at around 70 years old in the model. One possible explanation for the discrepancy is the significant amount of educational expenses for children when the household head is around 50 years old. Our model does not account for age-specific consumption goods and services, which prevents it from replicating this early consumption peak. However, when consumption is weighted by the cohort size, the age profile of aggregate consumption is reasonably similar between the model and data (see panel 4b).

Housing consumption, unlike per capita consumption, increases with age, as shown in panel 4c. This upward-sloping age profile of housing consumption is also observed in many countries, including the U.S. (Yang, 2009). In the standard two-good model without bequest motives, the ratio of housing to non-housing consumption should be independent of age because both consumption and housing consumption have parallel age profiles (Yang, 2009). Our model replicates the realistic age profile of housing by incorporating bequest motives and age-dependent mortality rate.

Housing consumption is determined by the following first-order condition:

$$\frac{h_{j,t}}{c_{j,t}} = \left[\chi+\psi\left(1-\nu\right)\omega_{j,t}\right]\left[P_t^h - \frac{\left(1-\delta_h\right)}{\left(1+r_{t+1}\right)}P_{t+1}^h\right]^{-1}. \tag{32}$$



(a) Per-Capita Consumption

(b) Aggregate Consumption

(c) Per-Capita Housing Asset

(d) Aggregate Housing Asset

(e) Per-Capita Liquidity Asset

(f) Aggregate Liquidity Asset

Figure 4: Age Profile of Consumption, Housing and Liquidity Asset: Data and Model

As in the standard model, the ratio of housing to non-housing consumption increases with the importance of housing χ in the felicity function and decreases with housing costs. However, unlike the standard model, equation (32) includes the product of the mortality rate $\omega_{j,t}$, the bequest motive parameter ψ, and the housing bequest weight $(1-\nu)$ for retirees only ($\mathbf{1}_{j\geq JR}$). Elderly households increase their housing consumption relative to non-housing consumption as mortality increases due to their bequest motives. Note that housing consumption is a linear transformation of housing assets in the model. Thus, the age profile shown in panel 4d is based on housing assets due to data availability. The model agrees well with the data, except for a sharp increase for households in their 30s and 40s in the data. This sharp increase is likely due to first-time home purchases around child births, which the model does not account for. However, cohort-weighted aggregate age profile is quite similar between the model and the data (panel 4d).

Savings exhibit a hump-shaped age profile in the standard life-cycle model because households save during a high-income period and dissave during a low-income period to smooth consumption. In our model, liquid asset savings $a_{j,t}$ has that age profile. However, as French et al. (2023) discuss, retired households decumulate their assets very slowly in the US and other countries. Our data confirm this tendency even for liquid assets (panel 4e); savings peak at the retirement age and remain at the same level throughout the remaining life. Our model address this issue by incorporating bequest motives. The second component of savings in the form of financial bequests $b_{j,t}$ sharply increase with mortality rates. Thus, the sum of liquid assets $a_{j,t}$ and financial bequests $b_{j,t}$ exhibit a high level of savings for the elderly. However, the data show negative savings for ages between 30 and 50. This borrowing corresponds to a sharp increase in housing assets. Thus, this dip in savings for middle-aged household heads likely corresponds to mortgages for first-time home purchases driven by the birth of children. Because our model does not account for such mortgage financing or a rapid change in the household size, it deviates from the data for these age groups.

Figure 5 contrasts the model results with the data for the dynamics of key aggregate variables. For annual real GDP growth rates (panel 5a), we use JSNA. We estimate the stochastic trend by using the HP filter by setting parameter λ equal 100. The model shows a decreasing trend in GDP growth as in the data, but the predicted change is smaller than the actual change. However, both the model and the data show no growth around 2010. Our model predicts negative real GDP growth rates after the 2000s.

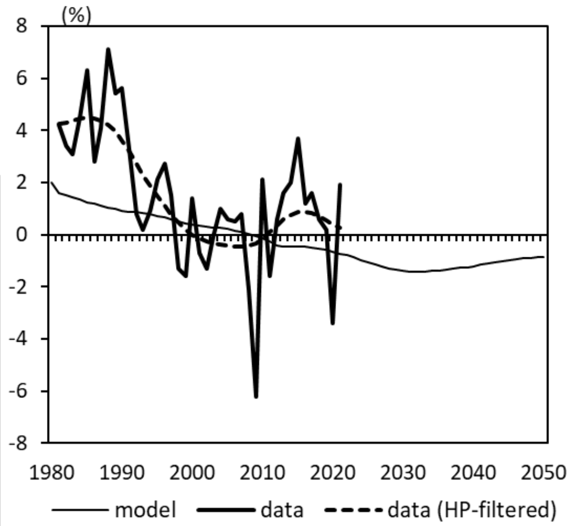
For the real interest rate (panel 5b), we use the prime lending interest rate adjusted for the GDP deflator, provided by the World Bank. The real prime lending rate is used as the average rate for safe government notes and risky borrowing by businesses and households, because the model has the single interest rate for both public and private sectors. The model predicts decreasing real interest rates until the 2010s, which are consistent with the data, except for the period of the Bank of Japan's quantitative and qualitative easing regime since 2013. From the 2010s, the model interest rate is stabilized around 2%.

The capital stock ratio (panel 5c) decreases between 1980 and 2020 in both the model and the data. The model predicts continued decline in the ratio until 2050. For household assets, we include land but exclude pension assets to make the data consistent with the model.

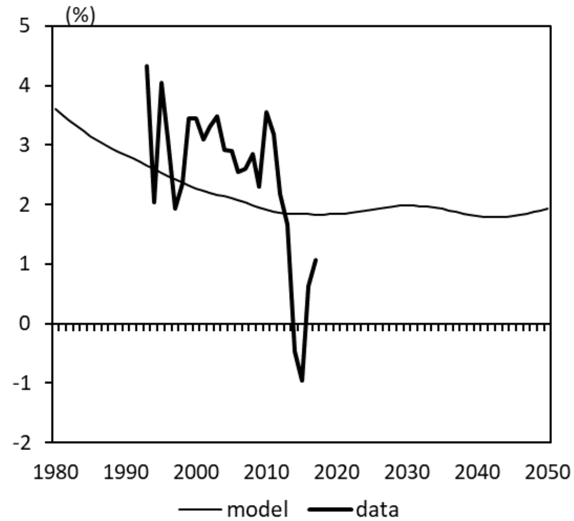
The real interest rate can be characterized by both household's Euler equation (equation (31)) and each production sector's first-order condition with respect to capital:

$$r_t + \delta_k = \alpha_g \left(\frac{K_t^g}{\zeta_t^g L_t^g} \right)^{\alpha_g - 1} = \alpha_h \left(\frac{K_t^h}{\zeta_t^h L_t^h} \right)^{\alpha_h - 1} P_t^h. \tag{33}$$

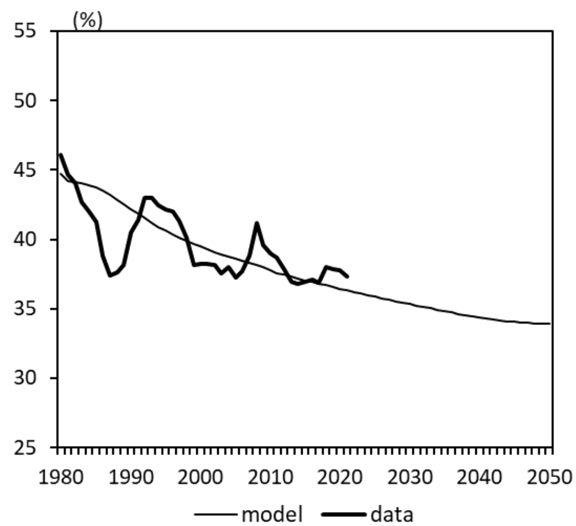
The secular decrease in the interest rate between the 1980s and 2010s inversely reflects the dynamics of capital K_t^g, which peaks in 2011. The interest rate also reflects the secular decrease in productivity ζ_t^g.



(a) Real GDP Growth



(b) Real Interest Rate



(c) Ratio of Capital Stock to Household Asset

Figure 5: Dynamics of Baseline Estimation: Data and Model

V. Factor decomposition

We specify our exogenous variables including demographic ratios between 1925 and 2095 and obtain the model outcome between 1980 and 2050. The detailed solution method is outlined in Appendix D.

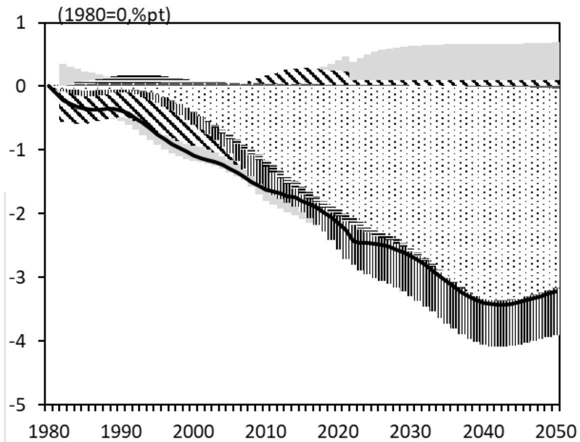
We obtain the model outcome for all equilibrium variables: aggregates $\{C_t, A_t, B_t, H_t, V_t\}_{t=0}^{\infty}$, prices $\{w_t^g, w_t^h, P_t^h, r_t\}_{t=0}^{\infty}$, transfers $\{T_t, pen_t\}_{t=0}^{\infty}$, and quantities $\Big\{\{c_{j,t}, a_{j,t}, b_{j,t}, bi_{j,t}, h_{j,t}, hi_{j,t}, v_{j,t}\}_{j=1}^{J}, i_t, i_t^h, K_t^g, K_t^h, L_t^g, L_t^h, Y_t^g, Y_t^h\Big\}_{t=0}^{\infty}$. However, for brevity, we discuss real interest rate, GDP growth rate, total house value, and the ratio of liquid assets to total assets.

To gain insights into the dynamics of the equilibrium outcome, we decompose each variable into the effects of six exogenous factors: mortality rates, fertility rates, government debt, government spending, technology growth rates for the general goods sector, and technology gap between the housing and general goods sectors. We "knock out" each factor at a time by fixing the exogenous factor value at the 1980 level and measuring the difference between the baseline outcome and the outcome in the knock-out case. The residual that cannot be attributed to any single factor is considered interactions.

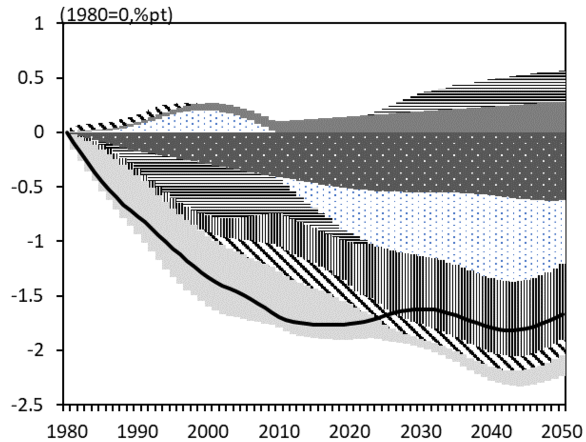
Figure 6 shows the results. Panel 6a shows that the model's GDP growth rate decreases by more than three percentage points between 1980 and 2050, mainly because low fertility rates reduce the working-age population. In addition, low technology growth rates in the general goods sector also reduce GDP growth.

Panel 6b shows that after the 2000s, the natural rate of interest is approximately 1.7 percentage points below its 1980 level. The main factors driving the decline in interest rates are low technological growth and low mortality and fertility rates. These effects are consistent with those found in the literature (e.g., Kitao and Mikoshiba, 2020). After 2010, however, rising government spending financed by public debt will create demand for capital and raise interest rates.

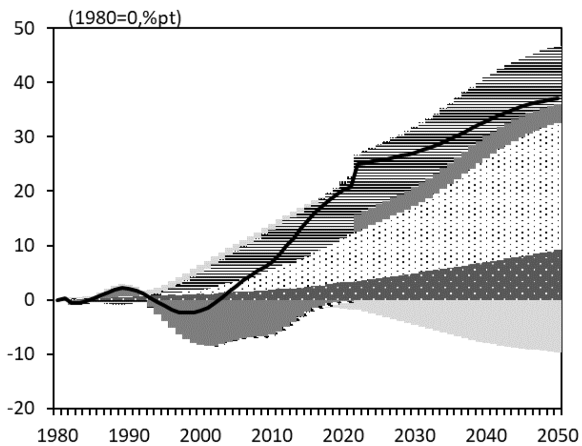
In panel 6c, income tax rates increase by approximately 37 percentage points by 2050, mainly because of declining fertility rates and increasing government spending. Note that



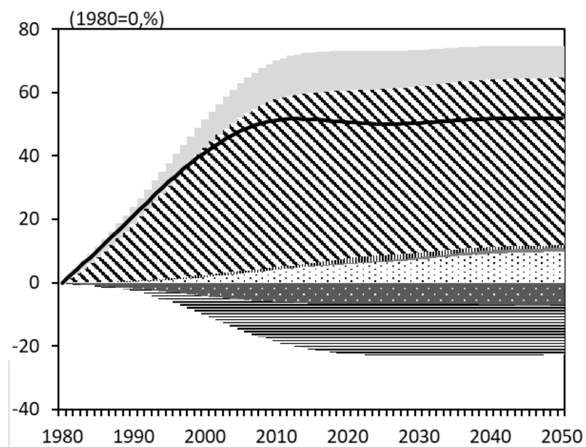
(a) GDP Growth Rate



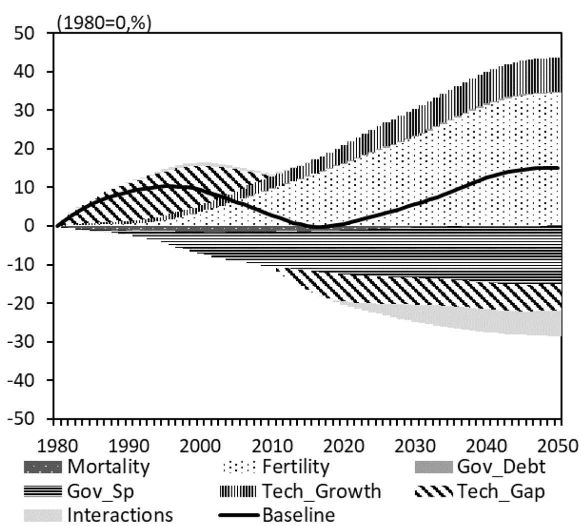
(b) Natural Rate of Interest



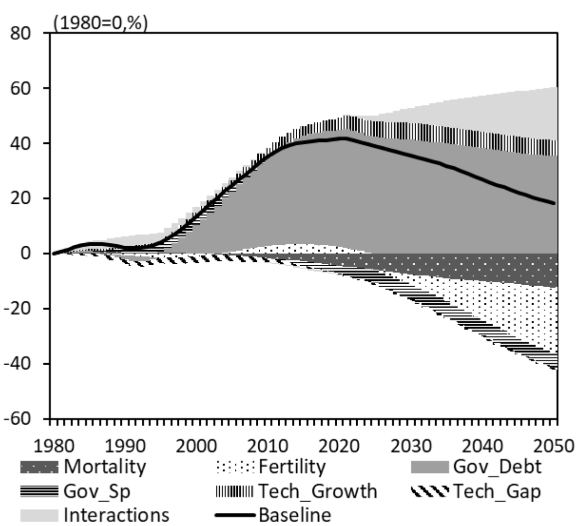
(c) Tax Rate



(d) Housing Price



(e) Per Capita Housing Value



(f) Per Capita Financial Assets

Figure 6: Decomposition of Changes

we do not calibrate the model tax rate to the actual income tax rate because the model does not include other sources of government revenue. Thus, this large increase in tax rates should not be taken as a prediction, but as the necessary government revenue to finance the exogenously specified government spending, pension replacement rate, and government debt. If a 37 percentage point increase in the tax rate is unacceptable to voters, the Japanese government would naturally reduce spending, debt, and the pension replacement rate. To put this rate in perspective, the tax wedge for Japan in 2023 is 32.6%, which is significantly lower than Belgium's 53.0%.[2] Also, the national burden rate in 2020 is 47.9% for Japan compared to the rates for the Scandinavian countries (65.9% for Denmark, 59.7% for Finland, 54.5% for Sweden, and 53.4% for Norway).

Panel 6d shows that the model housing price increases by 51% between 1980 and 2010, but then levels off. The main source of the increase in housing prices is the gap in TFP growth, a supply-side factor. As figure 1 shows, housing production became increasingly less efficient than the other sector until 2010, when TFP growth rates converged in the 2010s. As a result, the relative price of housing rose steadily for three decades. Another factor contributing to price increases is declining fertility rates, which create housing demand through population aging. However, our model does not replicate the rapid rise and fall of housing prices during the so-called "bubble economy" following the Plaza Accord in 1985. The actual national residential land price index showed a 266% premium over the 1980 value at its peak in 1991, and then declined steadily to a 33% premium in 2010. Thus, our model replicates the long-term real price changes relatively well, although it does not incorporate overly optimistic economic forecasts or monetary easing and tightening during this period.

Panel 6e shows that per capita housing value decreases by more than 10% between the 1990s and 2010s, but increases more by 2050.[3] There are competing effects on house values, which are the product of price and quantity. On the one hand, government spending has a

[2] See OECD data for tax wedge statistics: `https://www.oecd.org/tax/tax-policy/taxing-wages-japan.pdf`

[3] The per capita values of housing and financial assets in this figure do not include the effect of trend growth, but the trend adjustment does not change the dynamics significantly.

negative effect on house values by suppressing the demand for housing. On the other hand, declining fertility rates, which cause population aging, have positive effects by increasing housing demand.

In panel 6f, the per capita value of financial assets (the sum of liquid assets A_t and financial bequests B_t) increases by approximately 41.7% by 2020, mainly due to government debt. In equilibrium, household savings are used to finance government debt. However, as the population declines due to low fertility and high mortality rates, total assets begin to decline after 2015.

VI. Counterfactual Analysis

We conduct four types of counterfactual simulations. Table V summarizes the variations we analyze. We obtain the counterfactual values for all of the endogenous variables but discuss only the GDP level, the consumption level, the capital level, GDP growth rates, Total TFP growth rates, real interest rate, house prices and quantities, and the capital share of the general goods sector.

Name	Description	Baseline	CF1	CF2	CF3	CF4	CF5	CF6	CF7	CF8
ψ	Importance of bequest	24	0	36	-	-	-	-	-	-
ν	Relative importance of financial bequest	0.56	-	-	0.2	0.8	-	-	-	-
ϕ	Resusable rate of inherit housing	0.2	-	-	-	-	0	1	-	-
T^a	Tax rate for the financial bequest	0.4	-	-	-	-	-	-	0	0.4
T^h	Tax rate for the housing bequest	0	-	-	-	-	-	-	0	0.4
β	Discount factor	0.996	-	-	-	-	-	-	-	-
χ	Importance of housing	0.45	-	-	-	-	-	-	-	-
δ_g	Capital depreciation rate	0.15	-	-	-	-	-	-	-	-
δ_h	Housing depreciation rate	0.075	-	-	-	-	-	-	-	-
α_g	Capital share in general production sector	0.37	-	-	-	-	-	-	-	-
α_h	Capital share in housing sector	0.31	-	-	-	-	-	-	-	-

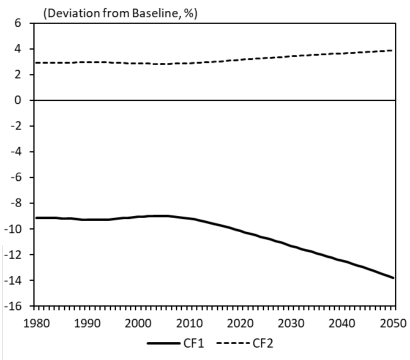
Table V: List of Parameters' Value

First, we change the parameter ψ, which represents the importance of bequest motives. Starting with the baseline value of $\psi = 24$, we examine 0 (no motives) and 36 (stronger motives). Second, we change the parameter ν, the weight on the financial bequest motive within the total bequest motives. We change the parameter from the baseline value of 0.55 to 0.2 and 0.8. Third, we change the parameter ϕ, which represents the matching efficiency for intergenerational housing transfers, from the baseline value of 0.2 to 0 and 1. When $\phi = 0$, all bequeathed houses become vacant because the younger generation's housing demand does not match the inherited houses due to geographical and taste gaps. In contrast, when $\phi = 1$, all inherited houses are fully utilized by younger generations; there is no vacant housing in the economy. Matching efficiency can be improved by helping the elderly remodel their houses to match young households' tastes. Last, we change inheritance tax rates. The baseline rates are 0 for housing and 0.4 for financial assets. We change them to $(0, 0)$ and $(0.4, 0.4)$ by removing preferential treatment of housing assets in the inheritance tax code.

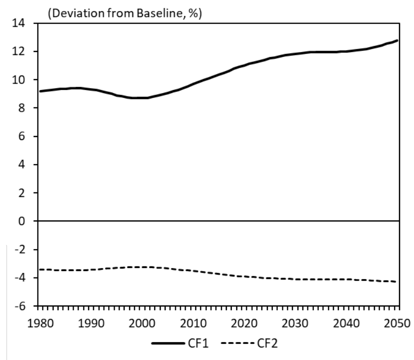
A. Bequest Motives

Figure 7 shows the results when we change the importance of bequest motives. The bold line (CF1) corresponds to the case with no bequest motives and the dotted line (CF2) corresponds to the case with greater bequest motives. The removal of bequest motives for both financial and housing assets reduces GDP by 9% to 14% (panel 7a) and the capital stock by 16% to 25% (panel 7c) because the demand for the less productive housing production is reduced. However, consumption will increase by 9% to 13% (panel 7b) because a larger share of household income will be spent on general consumption after each household optimizes housing consumption only for its own flow utility.

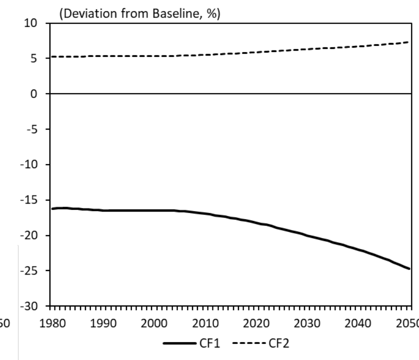
The growth rate of GDP (panel 7d) and average TFP (panel 7e) would have been $6 - 11$ percentage points higher until the 1990s if households did not have bequest motives. During this period, technological growth in the construction sector consistently lagged behind that of the general production sector. A lower demand for housing would have reduced the share of the housing production sector in the economy and thus increased the overall productivity of the economy. However, because we assume that technological progress in the two sectors



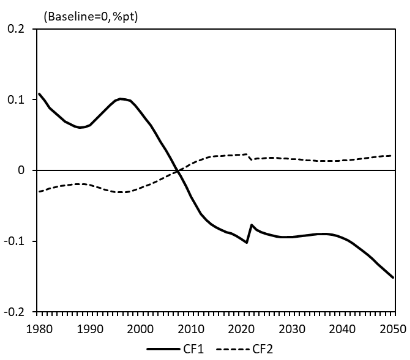
(a) GDP Level



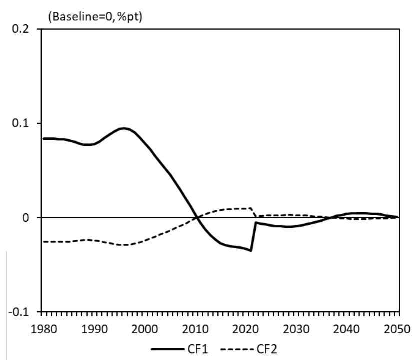
(b) Consumption Level



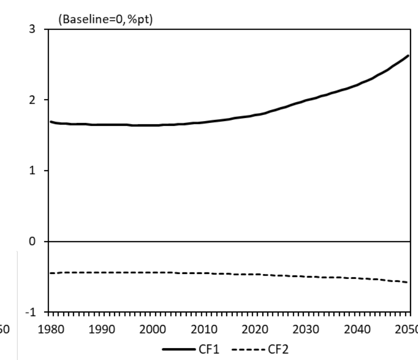
(c) Capital Level



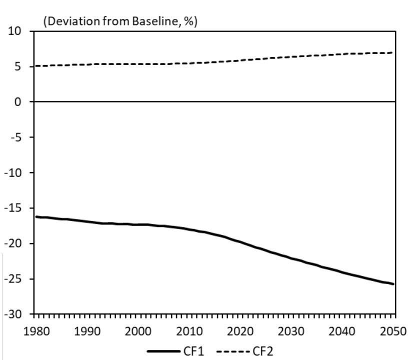
(d) GDP Growth



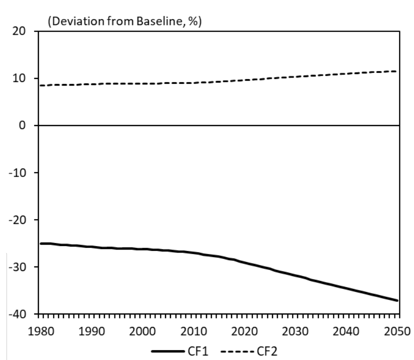
(e) Total TFP Growth



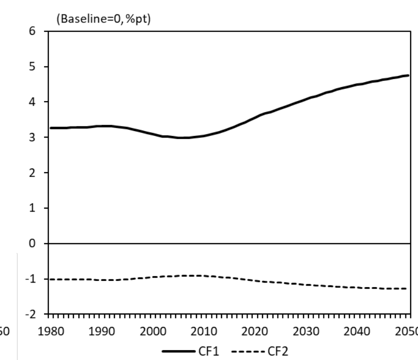
(f) Real Interest Rate



(g) House Prices



(h) Per Capita Housing Value



(i) Goods Sector Capital Share

Figure 7: The Effect of Bequest Motives

will converge in the future, the effect of reduced housing production has a negligible impact on overall TFP growth after 2020. In contrast, GDP growth would be depressed by the reduced demand for housing in the absence of bequest motives.

Interest rates would be 2.6 percentage points higher than the benchmark rate in 2050 in the absence of bequest motives (panel 7f). Because the benchmark rate is projected to decrease from 3.6% to 1.9% over the same period, this upward and increasing adjustment will keep the natural rate of interest relatively constant. This upward shift corresponds to a higher marginal product of capital (MPK) due to a smallerscale of production and higher aggregate TFP.

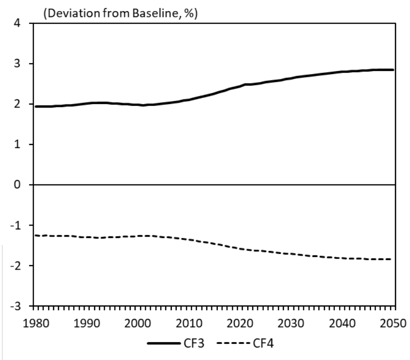
House prices would be 16% to 26% lower without bequest motives (panel 7g); that is, housing becomes more affordable as bequest driven housing demand is removed. In other words, the current equilibrium house prices are inflated by the additional demand for bequest motives. Similarly, the equilibrium level of the housing stock will also be 11% to 15% smaller (panel 7h). The reduction in both price and quantity also confirms that this change is caused by a negative shift in demand.

Furthermore, without bequest motives, 3% to 5% more capital will be allocated to the general goods production sector (panel 7i). A reduction in the capital share of the housing sector will make the economy less exposed to the technological gap between the two sectors.

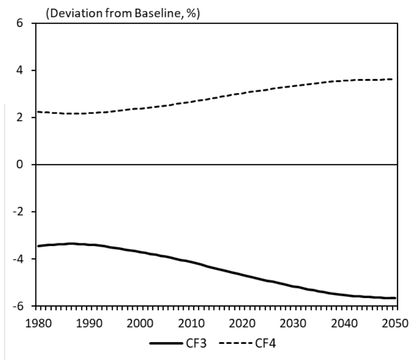
B. Utility Weight on Financial Bequests

Figure 8 shows the results when we change the utility weight for financial bequests as opposed to housing bequests. The bold line (CF3) corresponds to a smaller weight on financial bequests, and the dotted line (CF4) corresponds a larger weight.

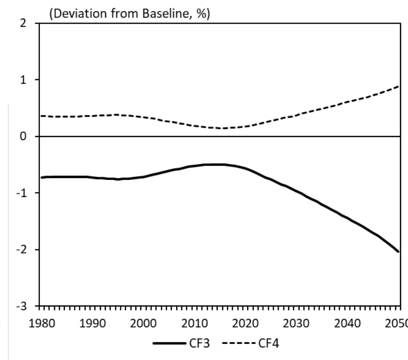
The results are qualitatively similar to those for a change in the importance of the total bequest motives. Thus, increasing the relative importance of financial bequests has an analogous effect to the case of decreasing a bequest motive itself. However, the effect on real interest rates and capital stock is opposite from that of changing bequest motives.



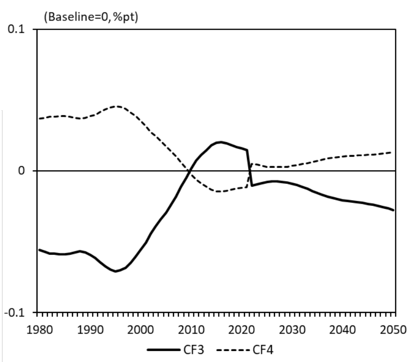
(a) GDP Level



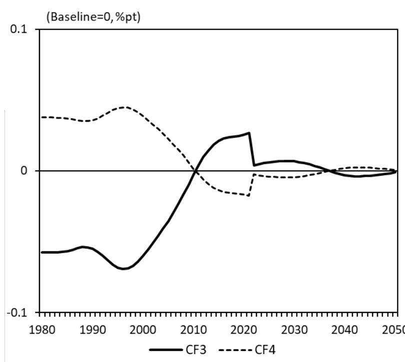
(b) Consumption Level



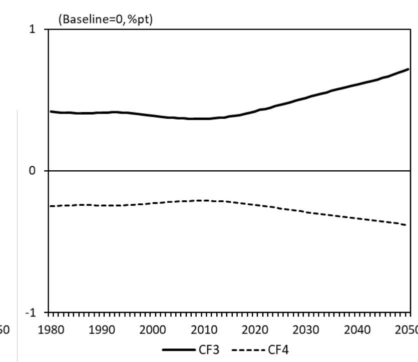
(c) Capital Level



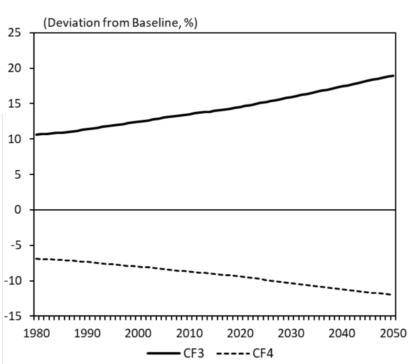
(d) GDP Growth



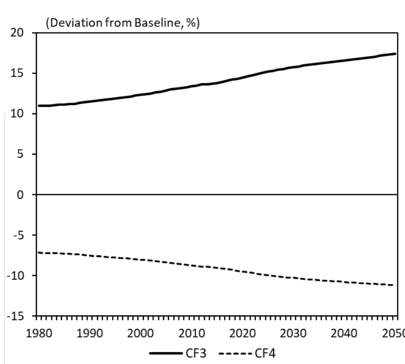
(e) Total TFP Growth



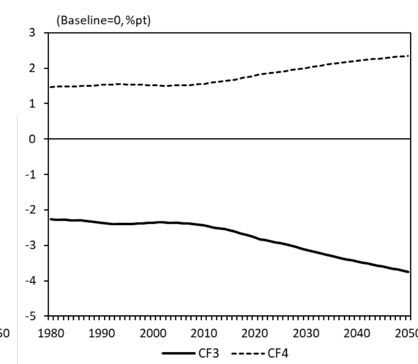
(f) Real Interest Rate



(g) House Prices



(h) Per Capita Housing Value



(i) Goods Sector Capital Share

Figure 8: The Effect of Relative Bequest Motives

Specifically, a higher weight on financial bequests will increase the total capital stock (panel 8c) and decrease the real interest rate (panel 8f). However, the magnitude of effects is generally small because housing bequests and financial bequests are substitutes in generating capital demand. Thus, a weight change will create offsetting effects on capital intensity.

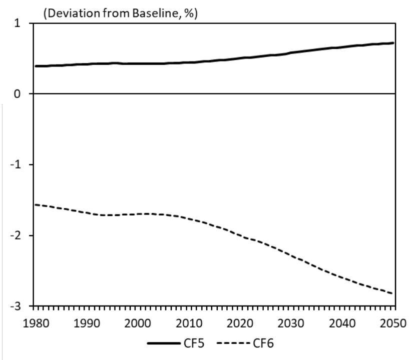
C. *Matching Efficiency of Intergenerational Housing Transfers*

Table 9 shows the effect of changing the house matching efficiency–the rate of successful transfers of bequeathed houses to younger generations. The bold line (CF5) corresponds to a completely inefficient transfer–all transferred houses become vacant, and the dotted line (CF6) corresponds to a completely efficient transfer–no vacancies. The efficient house matching can also be interpreted as an environmentally sustainable society, in which all of the existing stock is reused. In such a sustainable economy, production activities are reduced.

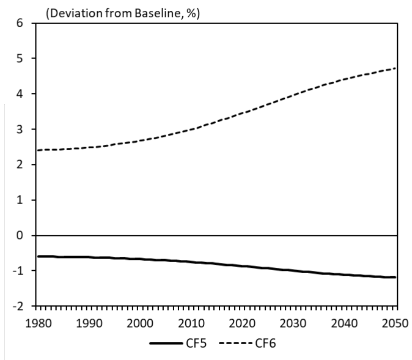
Efficient house matching will reduce GDP by $1.6-2.9\%$ (panel 9a) because less construction is needed. It will also reduce the total capital stock employed in the production sectors (panel 9c). Thus, in a sustainable society, households can enjoy $2.4-4.7\%$ more consumption (panel 9b) without expanding the size of the economy or increasing capital intensity. Most importantly, efficient house matching makes housing affordable: house prices would be $7-14\%$ lower, while more housing units are available (panels 9g and 9h). The opposite directions of price and quantity changes suggest that an efficient house matching acts as a positive supply shift in the housing market; i.e., housing is supplied without using additional labor and capital. Also, a smaller presence of the less productive housing production sector would have increased the GDP growth (panel 9d) and the average TFP growth (panel 9e) during the period of the stagnant construction productivity.

D. *Inheritance Tax Rates*

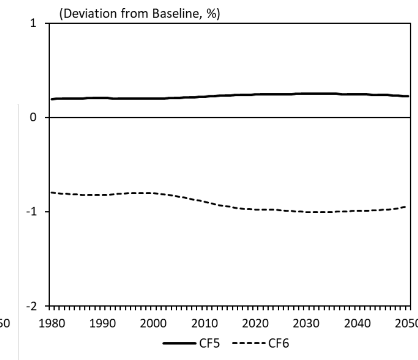
Table 10 shows the effect of changing inheritance tax rates. We explore two possibilities of removing the preferential treatment for housing. The bold line (CF7) corresponds to no



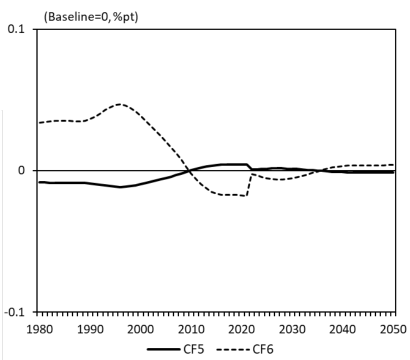
(a) GDP Level



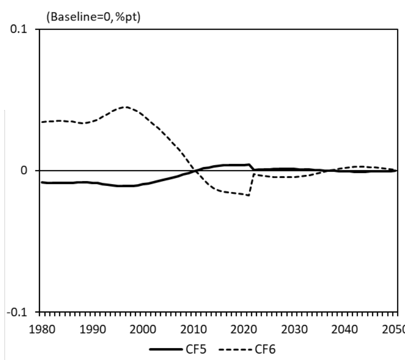
(b) Consumption Level



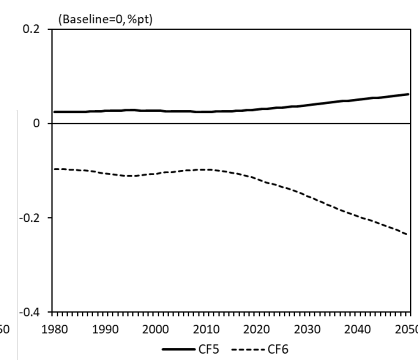
(c) Capital Level



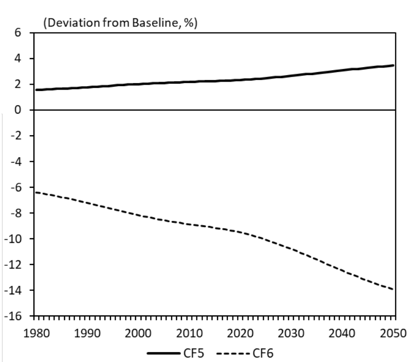
(d) GDP Growth



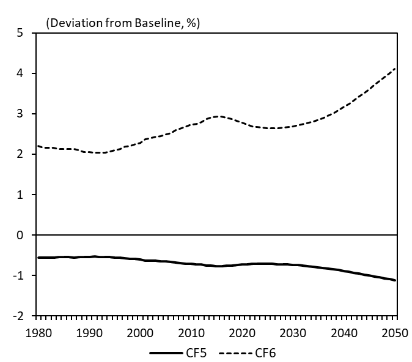
(e) Total TFP Growth



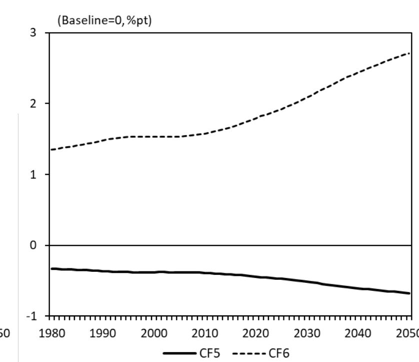
(f) Real Interest Rate



(g) House Prices



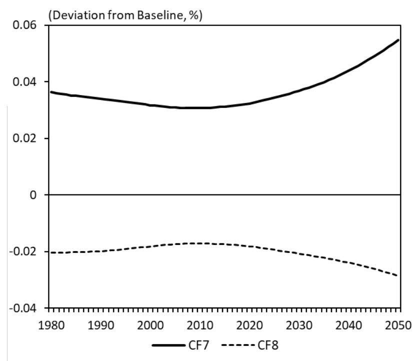
(h) Per Capita Housing Value



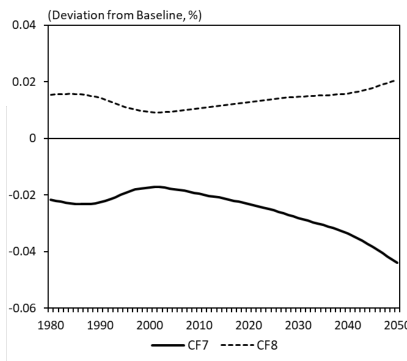
(i) Goods Sector Capital Share

Figure 9: The Effect of Matching Efficiency

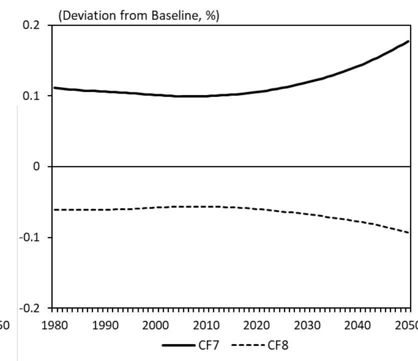
effective inheritance taxes for both assets, and the dotted line (CF8) corresponds to the same 40% effective tax rate for both assets, possibly by removing the assessment reduction for housing.



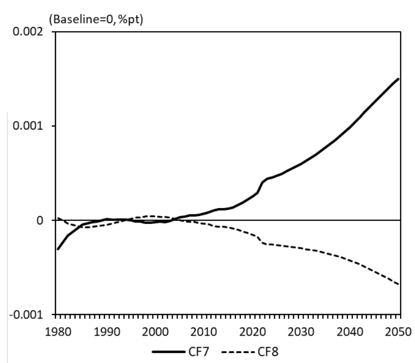
(a) GDP Level



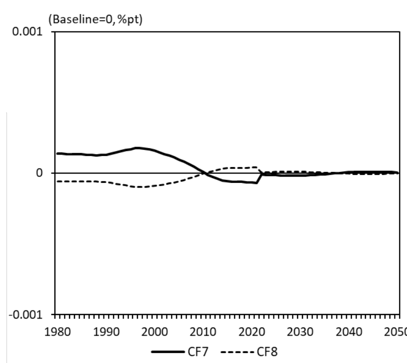
(b) Consumption Level



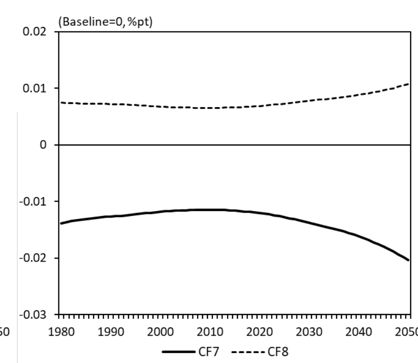
(c) Capital Level



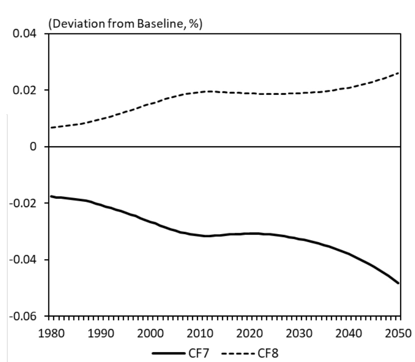
(d) GDP Growth



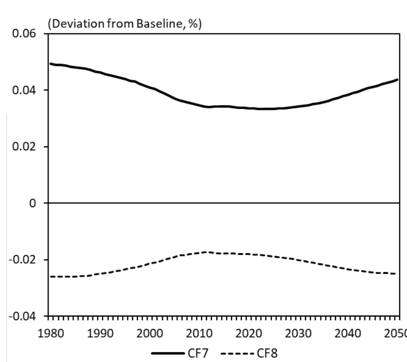
(e) Total TFP Growth



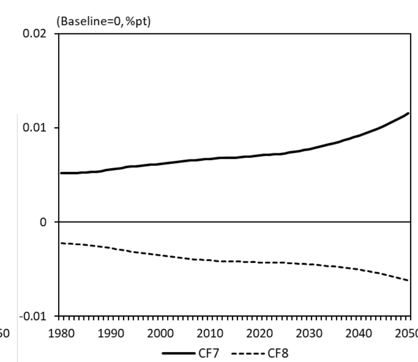
(f) Real Interest Rate



(g) House Prices



(h) Per Capita Housing Value



(i) Goods Sector Capital Share

Figure 10: The Effect of Inheritance Tax

The results suggest that the effects are negligible; for each variable, the effect is less than one percentage point. The reason is that these tax changes do not affect the household's decision. In our model, households cannot decide to forgo inheritance when tax rates are high. Thus, a change in inheritance tax rates could affect the macroeconomy only through its effect on households' after-tax wealth. For the government, however, inheritance taxes are a substitute for income taxes. As a result, the effect of a change in inheritance taxes is offset by the opposite effect of a change in income taxes on household wealth. Thus, if our model allows for households to choose whether to accept inheritances, the effect will be larger. Nevertheless, our analysis suggests that a inheritance tax reform may have only a small effect compared to changes in household preferences.

VII. Discussions

Bequest motives for housing create additional housing demand and increase GDP. Furthermore, capital demanded by the housing sector additionally increases general goods production. Thus, housing bequest motives increase aggregate demand. The effect of bequest motives continues to increase as society ages due to low fertility and mortality rates.

However, housing bequest motives have negative effects on the efficiency of capital usage. First, the growth rate of GDP and TFP is lower until the 2000s than in the counterfactual case of no bequest motives because the housing sector's productivity growth lagged the general goods sector's. The negative effect of bequest motives on TFP growth largely disappears after 2020 because we assume that technology growth rates converge between the two sectors. Thus, future aggregate TFP growth rates depend more on the technological progress of the increasingly important housing sector. The effect of bequest motives on GDP growth is greater than the effect on TFP growth because housing demand increases due to the growing elderly population.

Second, with housing bequest motives, household assets are used less for production capital than in the case of no-bequest motives, while housing services provide only small marginal utility. Without bequest motives, households hold more liquid assets, which are eventually invested in production capital.

Third, greater housing production suggests a lower marginal product of capital or real interest rates in two ways; the marginal product is diminishing with more capital employed, and the marginal product is lower in the housing sector because of lower technology. Thus, a reduction of housing bequest motives will increase the marginal product of capital or interest rates.

Bequest motives for housing also suppress household consumption because households spend more on housing. Still, the elderly are satisfied by leaving housing for their heirs despite frugal consumption. This satisfaction is partly derived from the inheritance-tax benefit of housing assets. If this tax benefit is removed, households would enjoy more consumption.

Bequest motives have an obvious direct effect on the housing market. They create additional demand and thus increase both price and quantity of housing. In other words, housing is less affordable for new buyers because old generations occupy their housing underutilized.

However, as long as bequeathed houses are efficiently transferred to the young generation, the crowding-out issue is minimal. A problem of housing bequest is that bequeathed houses do not perfectly match the young generation's housing demand. The young generation typically has different tastes for housing and lives in different cities from the old generation. In the calibrated baseline model, we incorporate such inefficiencies by assuming that 40% of bequeathed houses become vacant.

With such vacancies due to generational mismatch, housing stock does not accumulate as quickly as in the case of no wasted housing. Thus, the reduction of wasted housing works as the additional housing supply without the need for new housing production. Thus, a more efficient intergenerational transfer of housing will decrease house prices while increasing the aggregate housing quantity. As long as transfers are efficient, housing bequest motives have a small effect on aggregate housing affordability. In reality, there will be inequality between those who inherited housing and those who do not. We do not model household heterogeneity in housing assets.

Improving the matching efficiency can be achieved by improving house remodeling. Inefficiencies arise from taste and geographical gaps between generations. Thus, a better physical alteration of housing at the time of intergenerational transfer will solve taste gaps. Such policies include providing remodeling prototypes. However, a geographical mismatch cannot be resolved by remodeling. To address geographical mismatch, it would be effective to facilitate multi-party exchanges beyond within-family inheritance. For example, if taxes are exempted for the transaction of an inherited house to a local buyer, housing assets will be utilized better.

Our present study provides valuable insights but omits several aspects of the economy. We will explore these omitted aspects in future extensions. First, we introduce only age heterogeneity and omit other heterogeneity in income, wealth, and household size. As Mian et al. (2021) emphasize, demand for saving by wealthy households has important implications, especially because age and wealth are positively correlated. Second, we omit geographical heterogeneity. Even when intergenerational transfers look efficient at the aggregate level, geographical concentrations of particular generations and housing assets may make transfers infeasible in reality. For example, housing shortages for young generations in Tokyo will not be resolved by houses in rural areas bequeathed by old generations. Third, we do not introduce environmental externalities. Housing construction and usage account for a large proportion of carbon emissions. More carbon is emitted from housing construction if old generations' underutilized housing is further wasted as vacant units. Such externalities should be taken into account in the social planner's problem. Fourth, we specify a log-linear utility function, which suggests a restrictive assumption on intra and inter-temporal consumption substitutions. More realistic utility functions, such as the CES-CRRA utility and the Epstein-Zin recursive utility function, can suggest a more significant role of housing in the economy. Fifth, ours is a closed-economy model, but flows of capital and labor will increase in an open economy model and is expected to mitigate some of the effects.

VIII. Conclusion

Japan is at the forefront of aging societies. Studying the macroeconomic effects of aging in Japan will provide valuable insights into the future of the global economy. This study focuses on the ownership of large houses by elderly households, some of which are wasted as vacant units after being transferred to the younger generation. Ownership of large houses by the elderly is a common phenomenon in many countries, and the housing sector tends to be less productive than other sectors. A natural prediction is that an aging society will allocate more capital to housing assets and a less productive housing sector. We quantify the impact of this prediction by calibrating a quantitative OLG model to the Japanese economy. The model with housing bequest motives explains the age profile of housing assets well.

The model shows that bequest driven demand for housing increases GDP, but decreases GDP and TFP growth when technological growth is slow in the housing sector. Households consume less and allocate less resources to productive capital. The interest rate is lower because a larger scale of production reduces the marginal product of capital. Houses are less affordable because prices are inflated by the additional demand. These effects are exacerbated when inherited houses are more likely to become vacant and when the proportion of elderly people is higher. The efficiency of the Japanese economy can be increased by reducing bequest demand for housing and improving the efficiency of housing allocation.

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Appendix A List of Model Equations

This section presents the full set of model equilibrium conditions that jointly determine the evolution of real allocations and prices. Endogenous variables are $\{N_{j,t},\ c_{j,t},\ a_{j,t},\ b_{j,t},\ bi_{j,t},\ h_{j,t},\ hi_{j,t},\ v_{j,t}\}_{j=1}^{J}$, $a_{0,t}$, $b_{0,t}$, $h_{0,t}$, N_t, n_t, C_t, A_t, B_t, H_t, V_t, w_t^g, w_t^h, P_t^h, r_t, T_t, pen_t, i_t, i_t^h, K_t, K_t^g, K_t^h, L_t^g, L_t^h, Y_t^g, Y_t^h, which are $533 (= 6 \times 85 + 23)$ variables.

- Household

Budget Constraint

$$\begin{aligned} c_{j,t} + a_{j,t} + bi_{j,t} + P_t^h hi_{j,t} &= (1 + r_t)\, a_{j-1,t-1} + \mathbf{1}_{j \geq JR} pen_t \\ &+ (1 - \mathbf{1}_{j \geq JR}) \left[(1 - T_t) \left\{ \sigma_j^g w_t^g \xi_j^g + \left(1 - \sigma_j^g\right) w_t^h \xi_j^h \right\} + (1 - T_t^a)\, i_t - T_t^h P_t^h i_t^h \right] \\ & \qquad for\ j = 1, ..., J \quad \text{(A.1)} \end{aligned}$$

Bequeathed Assets Accumulation

$$b_{j,t} = (1 + r_t)\, b_{j-1,t-1} + bi_{j,t} \quad for\ j = 1, ..., J \qquad \text{(A.2)}$$

Housing Accumulation

$$h_{j,t} = (1 - \delta_h)\, h_{j-1,t-1} + hi_{j,t} + \phi\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h \quad for\ j = 1, ..., J \qquad \text{(A.3)}$$

Underutilized Housing Accumulation

$$v_{j,t} = (1 - \delta_h)\, v_{j-1,t-1} + (1 - \phi)\, (1 - \mathbf{1}_{j \geq JR})\, i_t^h \quad for\ j = 1, ..., J \qquad \text{(A.4)}$$

Initial Holdings of Asset and Housing

$$b_{0,t} = a_{0,t} = h_{0,t} = v_{0,t} = 0 \qquad \text{(A.5)}$$

Terminal Asset Holdings

$$a_{J,t} = 0 \tag{A.6}$$

Consumption FOC

$$c_{j+1,t+1} = \beta\left(1 + r_{t+1}\right)\left(1 - \omega_{j,t}\right) c_{j,t} \quad for\ j = 1, ..., J - 1 \tag{A.7}$$

Housing FOC

$$\left[\omega_{j,t}\psi\left(1 - \nu\right) + \chi\right] c_{j,t} = \left[P_t^h - \frac{\left(1 - \delta_h\right)}{\left(1 + r_{t+1}\right)} P_{t+1}^h\right] h_{j,t} \quad for\ j = 1, ..., J - 1 \tag{A.8}$$

$$\left[\psi\left(1 - \nu\right) + \chi\right] c_{J,t} = \left[P_t^h - \frac{\left(1 - \delta_h\right)}{\left(1 + r_{t+1}\right) P_{t+1}^h}\right] h_{J,t} \tag{A.9}$$

Bequest FOC

$$bi_{j,t} = \omega_{j,t}\psi\nu c_{j,t} \quad for\ j = 1, ..., J - 1 \tag{A.10}$$

$$bi_{J,t} = \psi\nu c_{J,t} \tag{A.11}$$

- Demography

Survival Rate

$$\frac{N_{j,t}}{N_{j-1,t-1}} = \left(1 - \omega_{j-1,t-1}\right) \quad for\ j = 2, ..., J \tag{A.12}$$

Growth Rate of Initial Young

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1 \tag{A.13}$$

Total Population

$$N_t = \sum_{j=1}^{J} N_{j,t} \tag{A.14}$$

Population Growth

$$n_t = \frac{N_t}{N_{t-1}} - 1 \tag{A.15}$$

- Aggregation over households

Aggregate Consumption

$$C_t = \sum_{j=1}^{J} N_{j,t} c_{j,t} \tag{A.16}$$

Financial Asset

$$A_t = \sum_{j=1}^{J} N_{j,t} a_{j,t} \tag{A.17}$$

Bequest Asset

$$B_t = \sum_{j=1}^{J} N_{j,t} b_{j,t} \tag{A.18}$$

Housing Stock

$$H_t = \sum_{j=1}^{J} N_{j,t} h_{j,t} \tag{A.19}$$

Underutilized Housing Stock

$$V_t = \sum_{j=1}^{J} N_{j,t} v_{j,t} \tag{A.20}$$

Financial Bequest

$$i_t = (1 + r_t) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) (a_{j,t-1} + b_{j,t-1})}{\sum_{j=1}^{JR} N_{j,t}} \tag{A.21}$$

Housing Bequest

$$i_t^h = (1 - \delta_h) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) h_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}} \tag{A.22}$$

- Firms

Labor Demand-Goods

$$w_t^g = (1 - \alpha_g) \zeta_t^g \left(\frac{K_t^g}{\zeta_t^g L_t^g} \right)^{\alpha_g} \tag{A.23}$$

Labor Demand-Housing

$$w_t^h = (1-\alpha_h)\,\zeta_t^h \left(\frac{K_t^h}{\zeta_t^h L_t^h}\right)^{\alpha_h} P_t^h \tag{A.24}$$

Capital Demand

$$r_t + \delta_k = \alpha_g \left(\frac{K_t^g}{\zeta_t^g L_t^g}\right)^{\alpha_g - 1} = \alpha_h \left(\frac{K_t^h}{\zeta_t^h L_t^h}\right)^{\alpha_h - 1} P_t^h \tag{A.25}$$

General Goods Production

$$Y_t^g = (K_t^g)^{\alpha_g} (\zeta_t^g L_t^g)^{1-\alpha_g} \tag{A.26}$$

Housing Production

$$Y_t^h = \left(K_t^h\right)^{\alpha_h} \left(\zeta_t^h L_t^h\right)^{1-\alpha_h} \tag{A.27}$$

- Government

Government Budget Constraint

$$\sum_{j=1}^{JR-1} T_t N_{j,t} \left[\sigma_j^g w_t^g \xi_j^g + \left(1-\sigma_j^g\right) w_t^h \xi_j^h\right] + \left(T_t^h P_t^h i_t^h + T_t^a i_t\right) \sum_{j=1}^{JR-1} N_{j,t} + D_t$$
$$= G_t + pen_t \sum_{j=JR}^{J} N_{j,t} + (1+r_t)\,D_{t-1} \tag{A.28}$$

Pension determination

$$pen_t = \rho_t \frac{w_t^g \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g + w_t^h \sum_{j=1}^{JR-1} \left(1-\sigma_j^g\right) N_{j,t} \xi_j^h}{\sum_{j=1}^{JR-1} N_{j,t}} \tag{A.29}$$

- Market clearing

Capital Market

$$D_t + K_t = A_t + B_t, K_{t-1} = K_t^g + K_t^h \tag{A.30}$$

Labor Market—General Goods

$$L_t^g = \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g, \tag{A.31}$$

Labor Market—Housing

$$L_t^h = \sum_{j=1}^{JR-1} \left(1 - \sigma_j^g\right) N_{j,t} \xi_j^h. \tag{A.32}$$

New Housing

$$\sum_{j=1}^{J} N_{j,t} hi_{j,t} = Y_t^h. \tag{A.33}$$

General Goods Market

$$Y_t^g = C_t + K_t - (1 - \delta_k) K_{t-1} + G_t. \tag{A.34}$$

Appendix B Detrending

We detrend from the equations with ζ_t^g. We define that $\hat{c}_{j,t} \equiv c_{j,t}/\zeta_t^g$, $\hat{a}_{j,t} \equiv a_{j,t}/\zeta_{t+1}^g$, $\hat{bi}_{j,t} \equiv bi_{j,t}/\zeta_t^g$, $\hat{b}_{j,t} \equiv b_{j,t}/\zeta_{t+1}^g$, $\hat{h}_{j,t} \equiv h_{j,t}/\zeta_{t+1}^g$, $\hat{hi}_{j,t} \equiv hi_{j,t}/\zeta_t^g$, $\hat{v}_{j,t} \equiv v_{j,t}/\zeta_{t+1}^g$, $\hat{C}_t \equiv C_t/\zeta_t^g$, $\hat{A}_t \equiv A_t/\zeta_{t+1}^g$, $\hat{B}_t \equiv B_t/\zeta_{t+1}^g$, $\hat{H}_t \equiv H_t/\zeta_{t+1}^g$, $\hat{V}_t \equiv V_t/\zeta_{t+1}^g$, $\hat{w}_t^g \equiv w_t^g/\zeta_t^g$, $\hat{w}_t^h \equiv w_t^h/\zeta_t^g$, $\hat{pen}_t \equiv pen_t/\zeta_t^g$, $\hat{i}_t \equiv i_t/\zeta_t^g$, $\hat{i}_t^h \equiv i_t^h/\zeta_t^g$, $\hat{K}_t \equiv K_t/\zeta_{t+1}^g$, $\hat{K}_t^g \equiv K_t^g/\zeta_t^g$, $\hat{K}_t^h \equiv K_t^h/\zeta_t^g$, $\hat{Y}_t^g \equiv Y_t^g/\zeta_t^g$, $\hat{Y}_t^h \equiv Y_t^h/\zeta_t^g$.

$$\hat{c}_{j,t} + \left(1 + g_{t+1}^g\right)\hat{a}_{j,t} + \hat{bi}_{j,t} + P_t^h \hat{hi}_{j,t} = (1 + r_t)\,\hat{a}_{j-1,t-1} + \mathbf{1}_{j \geq JR}\hat{pen}_t + (1 - \mathbf{1}_{j \geq JR})\left[(1 - T_t)\left\{\sigma_j^g \hat{w}_t^g \xi_j^g + \left(1 - \sigma_j^g\right)\hat{w}_t^h \xi_j^h\right\} + (1 - T_t^a)\,\hat{i}_t - T_t^h P_t^h \hat{i}_t^h\right] \quad for\ j = 1, ..., J \tag{B.1}$$

$$\left(1 + g_{t+1}^g\right)\hat{b}_{j,t} = (1 + r_t)\,\hat{b}_{j-1,t-1} + \hat{bi}_{j,t} \quad for\ j = 1, ..., J \tag{B.2}$$

$$\left(1 + g_{t+1}^g\right)\hat{h}_{j,t} = (1 - \delta_h)\,\hat{h}_{j-1,t-1} + \hat{hi}_{j,t} + \phi\,(1 - \mathbf{1}_{j \geq JR})\,\hat{i}_t^h \quad for\ j = 1, ..., J \tag{B.3}$$

$$\left(1 + g_{t+1}^g\right)\hat{v}_{j,t} = (1 - \delta_h)\,\hat{v}_{j-1,t-1} + (1 - \phi)\,(1 - \mathbf{1}_{j \geq JR})\,\hat{i}_t^h \quad for\ j = 1, ..., J \tag{B.4}$$

$$\hat{b}_{0,t} = \hat{a}_{0,t} = \hat{h}_{0,t} = \hat{v}_{0,t} = 0 \tag{B.5}$$

$$\hat{a}_{J,t} = 0 \tag{B.6}$$

$$\left(1 + g_{t+1}^g\right)\hat{c}_{j+1,t+1} = \beta\,(1 + r_{t+1})\,(1 - \omega_{j,t})\,\hat{c}_{j,t} \quad for\ j = 1, ..., J - 1 \tag{B.7}$$

$$\left[\omega_{j,t}\psi\,(1 - \nu) + \chi\right]\hat{c}_{j,t} = \left[P_t^h - \frac{(1 - \delta_h)}{(1 + r_{t+1})}P_{t+1}^h\right]\left(1 + g_{t+1}^g\right)\hat{h}_{j,t} \quad for\ j = 1, ..., J - 1 \tag{B.8}$$

$$\left[\psi\,(1 - \nu) + \chi\right]\hat{c}_{J,t} = \left[P_t^h - \frac{(1 - \delta_h)}{(1 + r_{t+1})}P_{t+1}^h\right]\left(1 + g_{t+1}^g\right)\hat{h}_{J,t} \tag{B.9}$$

$$\hat{bi}_{j,t} = \omega_{j,t}\psi\nu\hat{c}_{j,t} \quad for\ j = 1, ..., J - 1 \tag{B.10}$$

$$\hat{bi}_{J,t} = \psi\nu\hat{c}_{J,t} \tag{B.11}$$

$$\frac{N_{j,t}}{N_{j-1,t-1}} = (1 - \omega_{j-1,t-1}) \quad for\ j = 2, ..., J \tag{B.12}$$

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1 \tag{B.13}$$

$$N_t = \sum_{j=1}^{J} N_{j,t} \tag{B.14}$$

$$n_t = \frac{N_t}{N_{t-1}} - 1 \tag{B.15}$$

$$\hat{C}_t = \sum_{j=1}^{J} N_{j,t}\hat{c}_{j,t} \tag{B.16}$$

$$\hat{A}_t = \sum_{j=1}^{J} N_{j,t}\hat{a}_{j,t} \tag{B.17}$$

$$\hat{B}_t = \sum_{j=1}^{J} N_{j,t}\hat{b}_{j,t} \tag{B.18}$$

$$\hat{H}_t = \sum_{j=1}^{J} N_{j,t}\hat{h}_{j,t} \tag{B.19}$$

$$\hat{V}_t = \sum_{j=1}^{J} N_{j,t}\hat{v}_{j,t} \tag{B.20}$$

$$\hat{i}_t = (1 + r_t) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) \left(\hat{a}_{j,t-1} + \hat{b}_{j,t-1}\right)}{\sum_{j=1}^{JR} N_{j,t}} \tag{B.21}$$

$$\hat{i}_t^h = (1 - \delta_h) \frac{\sum_{j=1}^{J} (N_{j,t-1} - N_{j+1,t}) \hat{h}_{j,t-1}}{\sum_{j=1}^{JR} N_{j,t}} \tag{B.22}$$

$$\hat{w}_t^g = (1 - \alpha_g) \left(\frac{\hat{K}_t^g}{L_t^g}\right)^{\alpha_g} \tag{B.23}$$

$$\hat{w}_t^h = (1 - \alpha_h) (1 - \theta_t) \left(\frac{\hat{K}_t^h}{(1 - \theta_t) L_t^h}\right)^{\alpha_h} P_t^h \tag{B.24}$$

$$r_t + \delta_k = \alpha_g \left(\frac{\hat{K}_t^g}{L_t^g}\right)^{\alpha_g - 1} = \alpha_h \left(\frac{\hat{K}_t^h}{(1 - \theta_t) L_t^h}\right)^{\alpha_h - 1} P_t^h \tag{B.25}$$

$$\hat{Y}_t^g = \left(\hat{K}_t^g\right)^{\alpha_g} (L_t^g)^{1-\alpha_g} \tag{B.26}$$

$$\hat{Y}_t^h = \left(\hat{K}_t^h\right)^{\alpha_h} \left(\left(1-\theta_t\right) L_t^h\right)^{1-\alpha_h} \tag{B.27}$$

$$\sum_{j=1}^{JR-1} T_t N_{j,t} \left[\sigma_j^g \hat{w}_t^g \xi_j^g + \left(1-\sigma_j^g\right) \hat{w}_t^h \xi_j^h\right] + \left(T_t^h P_t^h \hat{i}_t^h + T_t^a \hat{i}_t\right) \sum_{j=1}^{JR-1} N_{j,t} + \left(1+g_{t+1}^g\right) D_t$$
$$= G_t + p\hat{e}n_t \sum_{j=JR}^{J} N_{j,t} + \left(1+r_t\right) D_{t-1} \tag{B.28}$$

$$p\hat{e}n_t = \rho_t \frac{\sum_{j=1}^{JR-1} N_{j,t} \left[\sigma_j^g \hat{w}_t^g \xi_j^g + \left(1-\sigma_j^g\right) \hat{w}_t^h \xi_j^h\right]}{\sum_{j=1}^{JR-1} N_{j,t}} \tag{B.29}$$

$$D_t + \hat{K}_t = \hat{A}_t + \hat{B}_t, \hat{K}_{t-1} = \hat{K}_t^g + \hat{K}_t^h \tag{B.30}$$

$$L_t^g = \sum_{j=1}^{JR-1} \sigma_j^g N_{j,t} \xi_j^g, \tag{B.31}$$

$$L_t^h = \sum_{j=1}^{JR-1} \left(1-\sigma_j^g\right) N_{j,t} \xi_j^h. \tag{B.32}$$

$$\sum_{j=1}^{J} N_{j,t} \hat{hi}_{j,t} = \hat{Y}_t^h. \tag{B.33}$$

$$\hat{Y}_t^g = \hat{C}_t + \left(1+g_{t+1}^g\right) \hat{K}_t - \left(1-\delta_k\right) \hat{K}_{t-1} + G_t. \tag{B.34}$$

Appendix C FOCs of Households

A representative j-aged household at time t maximize their utility with respect to $c_{j,t}$, $a_{j,t}$, $bi_{j,t}$, $h_{j,t}$ and $hi_{j,t}$. The Lagrangian is

$$\begin{aligned}
\mathcal{L}_{j,t} = & \sum_{i=0}^{J-j} \beta^i \left(\frac{N_{j+i,t+i} - N_{j+i+1,t+i+1}}{N_{j,t}} \right) \psi \left\{ \nu \ln bi_{j+i,t+i} + (1-\nu) \ln h_{j+i,t+i} \right\} \\
& + \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \{ (\ln c_{j+i,t+i} + \chi \ln h_{j+i,t+i}) + \lambda_{j+i,t+i} [c_{j+i,t+i} + a_{j+i,t+i} + bi_{j+i,t+i} \\
& + P^h_{t+i} \{ hi_{j+i,t+i} + (1 - \mathbf{1}_{j+i \geq JR}) T^h_{t+i} i^h_{t+i} \} - (1 + r_{t+i}) a_{j+i-1,t+i-1} - \mathbf{1}_{j+i \geq JR} pen_{t+i} \\
& - (1 - \mathbf{1}_{j+i \geq JR}) \left[(1 - T_{t+i}) \{ \sigma^g_{j+i} w^g_{t+i} z^g_{j+i} + \left(1 - \sigma^g_{j+i}\right) w^h_{t+i} z^h_{j+i} \} + \left(1 - T^a_{t+i}\right) i_{t+i} \right]] \\
& + \lambda^h_{j+i,t+i} \left[h_{j+i,t+i} - (1 - \delta_h) h_{j+i-1,t+i-1} - hi_{j+i,t+i} - \phi (1 - \mathbf{1}_{j+i \geq JR}) i^h_{t+i} \right] \} \\
& for\ j = 1, ..., J
\end{aligned}$$

FOCs with respect to $c_{j+i,t+i}$, $a_{j+i,t+i}$, $bi_{j+i,t+i}$, $h_{j+i,t+i}$ and $hi_{j+i,t+i}$ are respectively

$$\beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \left(\frac{1}{c_{j+i,t+i}} + \lambda_{j+i,t+i} \right) = 0$$

$$\iff \quad \lambda_{j+i,t+i} = -\frac{1}{c_{j+i,t+i}} \tag{C.1}$$

$$\beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \lambda_{j+i,t+i} - \beta^{i+1} \frac{N_{j+i+1,t+i+1}}{N_{j,t}} \lambda_{j+i,t+i} (1 + r_{t+i+1}) = 0$$

$$\iff \quad \lambda_{j+i,t+i} = \beta \frac{N_{j+i+1,t+i+1}}{N_{j+i,t+i}} \lambda_{j+i+1,t+i+1} (1 + r_{t+i+1}) \tag{C.2}$$

$$\beta^i \left(\frac{N_{j+i,t+i} - N_{j+i+1,t+i+1}}{N_{j,t}} \right) \frac{\psi \nu}{bi_{j+i,t+i}} + \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \lambda_{j+i,t+i} = 0$$

$$\iff \quad \left(1 - \frac{N_{j+i+1,t+i+1}}{N_{j+i,t+i}} \right) \frac{\psi \nu}{bi_{j+i,t+i}} = -\lambda_{j+i,t+i} \tag{C.3}$$

$$\beta^{i}\left(\frac{N_{j+i,t+i}-N_{j+i+1,t+i+1}}{N_{j,t}}\right)\frac{\psi\left(1-\nu\right)}{h_{j+i,t+i}}+\beta^{i}\frac{N_{j+i,t+i}}{N_{j,t}}\left(\frac{\chi}{h_{j+i,t+i}}+\lambda^{h}_{j+i,t+i}\right)$$
$$-\beta^{i+1}\frac{N_{j+i+1,t+i+1}}{N_{j,t}}\lambda^{h}_{j+i+1,t+i+1}\left(1-\delta_{h}\right)=0$$

$$\iff \left[\left(1-\frac{N_{j+i+1,t+i+1}}{N_{j+i,t+i}}\right)\psi\left(1-\nu\right)+\chi\right]\frac{1}{h_{j+i,t+i}}$$
$$+\lambda^{h}_{j+i,t+i}-\beta\frac{N_{j+i+1,t+i+1}}{N_{j+i,t+i}}\lambda^{h}_{j+i+1,t+i+1}\left(1-\delta_{h}\right)=0 \quad \text{(C.4)}$$

$$\beta^{i}\frac{N_{j+i,t+i}}{N_{j,t}}\left(\lambda_{j+i,t+i}P^{h}_{t+i}-\lambda^{h}_{j+i,t+i}\right)=0$$

$$\iff \lambda_{j+i,t+i}P^{h}_{t+i}=\lambda^{h}_{j+i,t+i} \quad \text{(C.5)}$$

From C.1 and C.2, we have

$$c_{j+1,t+1}=\beta\left(1+r_{t+1}\right)\left(1-\omega_{j,t}\right)c_{j,t} \quad for\ j=1,...,J-1 \quad \text{(C.6)}$$

From C.1, C.2, C.4 and C.5, we have

$$\left[\omega_{j,t}\psi\left(1-\nu\right)+\chi\right]c_{j,t}=\left[P^{h}_{t}-\frac{\left(1-\delta_{h}\right)}{\left(1+r_{t+1}\right)}P^{h}_{t+1}\right]h_{j,t}$$
$$for\ j=1,...,J-1 \quad \text{(C.7)}$$

$$\left[\psi\left(1-\nu\right)+\chi\right]c_{J,t}=\left[P^{h}_{t}-\frac{\left(1-\delta_{h}\right)}{\left(1+r_{t+1}\right)}P^{h}_{t+1}\right]h_{J,t} \quad \text{(C.8)}$$

From C.1 and C.3, we have

$$bi_{j,t}=\omega_{j,t}\psi\nu c_{j,t} \quad for\ j=1,...,J-1 \quad \text{(C.9)}$$

$$bi_{J,t} = \psi \nu c_{J,t} \tag{C.10}$$

Appendix D Solution Method

Our solution method is the same as Bielecki et al. (2020).

Although we use the scenarios of our demographic and other exogenous forces from 1925 to 2095 (consisting of artificial data, past data and projections), our focus is on the development from XXX1990 to 2050. We first estimate the steady state value for initial state (=1925) and end state (=2095). A steady state is defined as a stationary equilibrium where the exogenous demographic variables assume a fixed value forever. Then, we do our deterministic simulation (with perfect foresight) over the same periods. The deterministic simulation is calculated using Matlab and dynare.

A steady state value is calculated by the following algorithm. Variables for convergence calculation are $\frac{K^g}{L^g}$, $\frac{K^h}{L^h}$, i, i^h and pen.

1. We set initial values for each variable.
2. Calculate factor prices w^g_t, w^h_t, P^h_t, r_t and T_t.
3. Calculate the sum of lifetime discount factors and calculate the NPV of lifetime labor income and pension.
4. Calculate initial consumption and other household's variables $\{N_{j,t}, c_{j,t}, a_{j,t}, bi_{j,t}, h_{j,t}, hi_{j,t}\}_{j=1}^{J}$ as in Appendix E.
5. Calculate the variables for convergence calculation and guess the next loop values.
6. Judge the convergence.

	$\frac{K^g}{L^g}$	$\frac{K^h}{L^h}$	i	i^h	pen
Initial values	3	1.5	2.0	4.0	0.2

Table A1: Details of the Algorithm

Appendix E Initial Value Calculation

Set $w_j = (1-\mathbf{1}_{j\geq JR})\left[(1-T)\left\{\sigma_j^g w^g z_j^g + \left(1-\sigma_j^g\right) w^h z_j^h\right\} + (1-T^a)\, i - P^h T^h i^h\right]$ $+\mathbf{1}_{j\geq JR} pen$ and $1+rg = \frac{1+r}{1+g^g}$,

$$c_J + (1+g^g)\, a_J + bi_J + P^h hi_J = w_J + (1+r)\, a_{J-1}$$

$$\begin{aligned}\iff\quad & c_J + beq_J + P^h\left\{(1+g^g)\, h_J - (1-\delta_h)\, h_{J-1} - \phi\,(1-\mathbf{1}_{J\geq JR})\, inh^h\right\}\\ =& w_J + (1+rg)\left\{-c_{J-1} - bi_{J-1} - P^h hi_{J-1} + w_{J-1} + (1+r)\, a_{J-2}\right\}\end{aligned}$$

$$\begin{aligned}\iff\quad & c_J + (1+rg)\, c_{J-1} + bi_J + (1+rg)\, bi_{J-1} + P^h\,(1+g^g)\, h_J - P^h\,(1-\delta_h)\, h_{J-1}\\ & + (1+rg)\, P^h\left\{(1+g^g)\, h_{J-1} - (1-\delta_h)\, h_{J-2} - \phi\,(1-\mathbf{1}_{J-1\geq JR})\, i^h\right\}\\ =& w_J + (1+rg)\, w_{J-1} + P^h\phi i^h\,(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1+r)\, a_{J-2}\end{aligned}$$

$$\begin{aligned}\iff\quad & c_J + (1+rg)\, c_{J-1} + (1+rg)^2\, c_{J-2} + bi_J + (1+rg)\, bi_{J-1} + (1+rg)^2\, bi_{J-2}\\ & + P^h\,(1+g^g)\left\{h_J + (1+rg)\, h_{J-1} + (1+rg)^2\, h_{J-2}\right\}\\ & - P^h\,(1-\delta_h)\left\{h_{J-1} + (1+rg)\, h_{J-2} + (1+rg)^2\, h_{J-3}\right\}\\ =& w_J + (1+rg)\, w_{J-1} + (1+rg)^2\, w_{J-2}\\ & + P^h\phi i^h\left\{(1-\mathbf{1}_{J\geq JR}) + (1+rg)\,(1-\mathbf{1}_{J-1\geq JR}) + (1+rg)^2\,(1-\mathbf{1}_{J-2\geq JR})\right\}\\ & + (1+rg)^2\,(1+r)\, a_{J-3}\end{aligned}$$

$$
\begin{aligned}
\iff\quad & c_J + (1+rg)\, c_{J-1} + \cdots + (1+rg)^{J-1} c_1 \\
& + bi_J + (1+rg)\, bi_{J-1} + \cdots + (1+rg)^{J-1} bi_1 \\
& + P^h (1+g^g) \left\{ h_J + (1+rg)\, h_{J-1} + \cdots + (1+rg)^{J-1} h_1 \right\} \\
& - P^h (1-\delta_h) \left\{ h_{J-1} + (1+rg)\, h_{J-2} + \cdots + (1+rg)^{J-1} h_0 \right\} \\
= & w_J + (1+rg)\, w_{J-1} + \cdots + (1+rg)^{J-1} w_1 \\
& + P^h \phi i^h \left\{ (1-\mathbf{1}_{J\geq JR}) + (1+rg)(1-\mathbf{1}_{J-1\geq JR}) + \cdots + (1+rg)^{J-1}(1-\mathbf{1}_{1\geq JR}) \right\} \\
& + (1+rg)^{J-1} (1+r)\, a_0
\end{aligned}
$$

Recall that $h_j = \frac{(1+rg)\{\omega_j \psi(1-\nu)+\chi\}}{(r+\delta_h)P^h} c_j$ and $bi_j = \omega_j \psi \nu c_j \quad for\ j = 1, ..., J-1$, we have

$$
\begin{aligned}
& c_J + (1+rg)\, c_{J-1} + \cdots + (1+rg)^{J-1} c_1 \\
& + \psi\nu \left\{ c_J + (1+rg)\, \omega_{J-1} c_{J-1} + \cdots + (1+rg)^{J-1} \omega_1 c_1 \right\} \\
& + \frac{1+r}{r+\delta_h} \left[\{\psi(1-\nu)+\chi\} c_J + (1+rg)\{\omega_{J-1}\psi(1-\nu)+\chi\} c_{J-1} \right. \\
& \left. + \cdots + (1+rg)^{J-1} \{\omega_1 \psi(1-\nu)+\chi\} c_1 \right] \\
& - \frac{1-\delta_h}{r+\delta_h} \left[(1+rg)\{\omega_{J-1}\psi(1-\nu)+\chi\} c_{J-1} + \cdots + (1+rg)^{J-1} (\omega_1 \psi(1-\nu)+\chi\} c_1 \right] \\
= & w_J + (1+rg)\, w_{J-1} + \cdots + (1+rg)^{J-1} w_1 \\
& + P^h \phi i^h \left\{ (1-\mathbf{1}_{J\geq JR}) + (1+rg)(1-\mathbf{1}_{J-1\geq JR}) + \cdots + (1+rg)^{J-1}(1-\mathbf{1}_{1\geq JR}) \right\}
\end{aligned}
$$

$$\iff \left[1+\left\{\nu+\frac{1+r}{r+\delta_h}(1-\nu)\right\}\psi+\frac{1+r}{r+\delta_h}\chi\right]c_J$$

$$+(1+rg)\{1+\omega_{J-1}\psi+\chi\}c_{J-1}+\cdots+(1+rg)^{J-1}\{1+\omega_1\psi+\chi\}c_1$$

$$=w_J+(1+rg)w_{J-1}+\cdots+(1+rg)^{J-1}w_1$$

$$+P^h\phi i^h\left\{(1-\mathbf{1}_{J\geq JR})+(1+rg)(1-\mathbf{1}_{J-1\geq JR})+\cdots+(1+rg)^{J-1}(1-\mathbf{1}_{1\geq JR})\right\}$$

Recall that $c_{j+1}=\beta(1+rg)(1-\omega_j)c_j \quad for\ j=1,...,J-1$, we have

$$(1+rg)^{J-1}\left[\beta^{J-1}(1-\omega_{J-1})(1-\omega_{J-2})\cdots(1-\omega_1)\left\{1+\psi\nu+\frac{1+r}{r+\delta_h}\psi(1-\nu)+\frac{1+r}{r+\delta_h}\chi\right\}\right.$$

$$\left.+\beta^{J-2}(1-\omega_{J-2})\cdots(1-\omega_1)\{1+\omega_{J-1}\psi+\chi\}+\cdots+\beta(1-\omega_1)\{1+\omega_1\psi+\chi\}\right]c_1$$

$$=w_J+(1+rg)w_{J-1}+\cdots+(1+rg)^{J-1}w_1$$

$$+P^h\phi i^h\left\{(1-\mathbf{1}_{J\geq JR})+(1+rg)(1-\mathbf{1}_{J-1\geq JR})+\cdots+(1+rg)^{J-1}(1-\mathbf{1}_{1\geq JR})\right\}$$

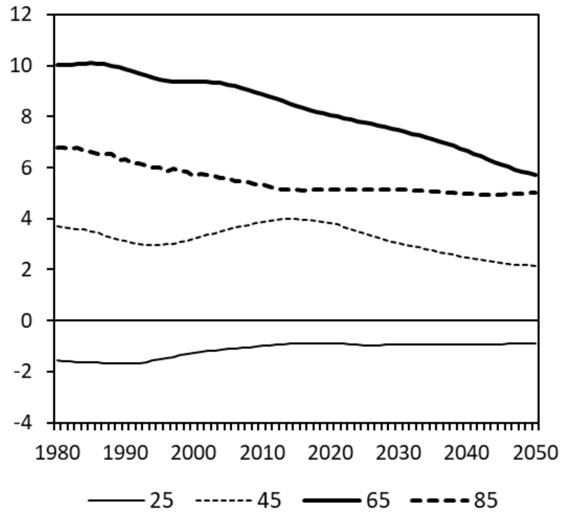
$$\iff (1+rg)^{J-1}\left[\beta^{J-1}\left(\prod_{k=1}^{J-1}(1-\omega_k)\right)\left\{1+\psi\nu+\frac{1+r}{r+\delta_h}\psi(1-\nu)+\frac{1+r}{r+\delta_h}\chi\right\}\right.$$

$$\left.+\sum_{j=1}^{J-1}\left\{\beta^{j-1}\left(\prod_{k=1}^{j-1}(1-\omega_k)\right)(1+\omega_j\psi+\chi)\right\}\right]c_1$$

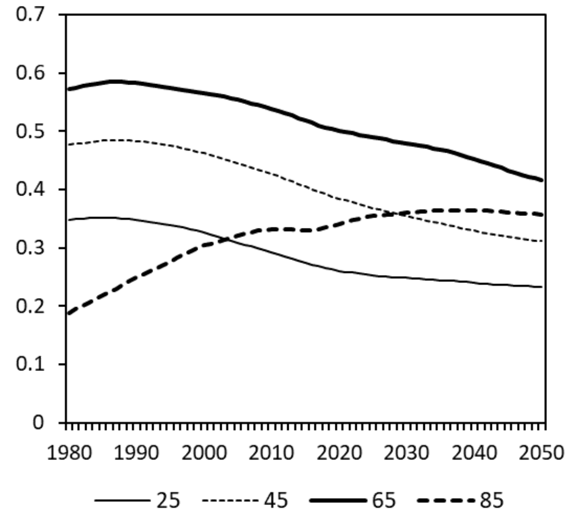
$$=\sum_{j=1}^{J}(1+rg)^{J-j}\left\{w_j+P^h\phi i^h(1-\mathbf{1}_{j\geq JR})\right\}$$

Appendix F Dynamics by Age Groups

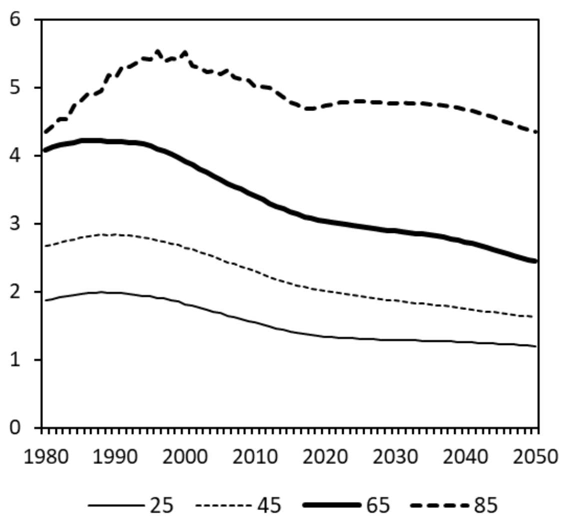
The households with 85-year-old increase asset and consumption whereas the other households do not, while they decreases housing sharper.



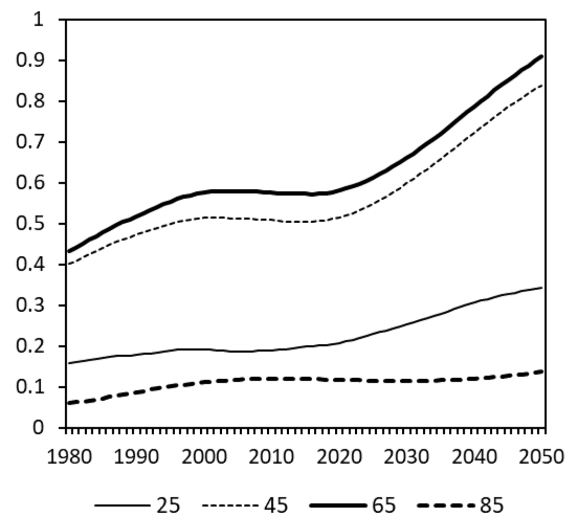
(a) Financial Asset Per HH



(b) Consumption Per HH



(c) Housing Per HH

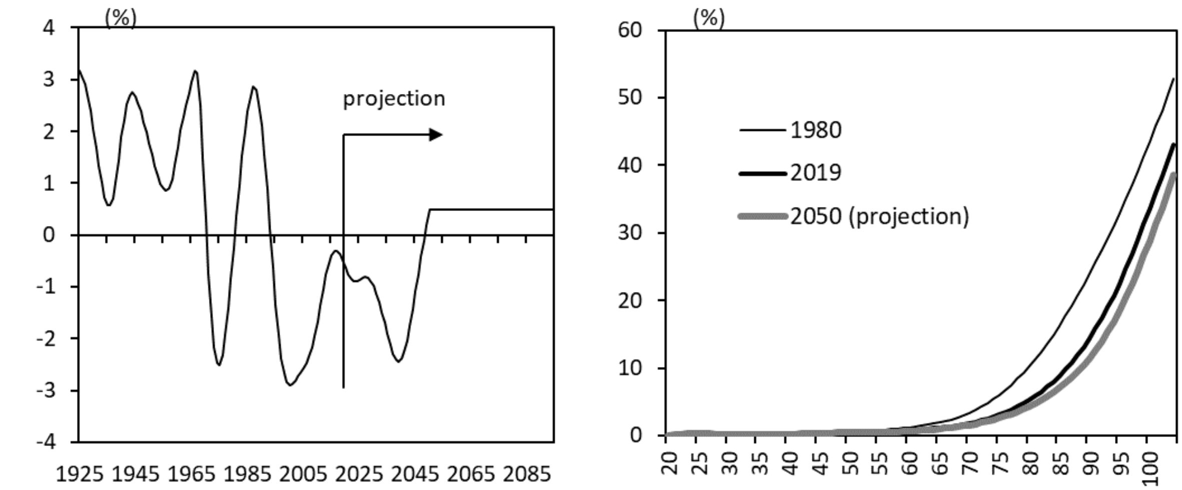


(d) Underutilized House Per HH

Figure A1: The Effect of Bequest Motives: GDP, Consumption, and TFP Growth

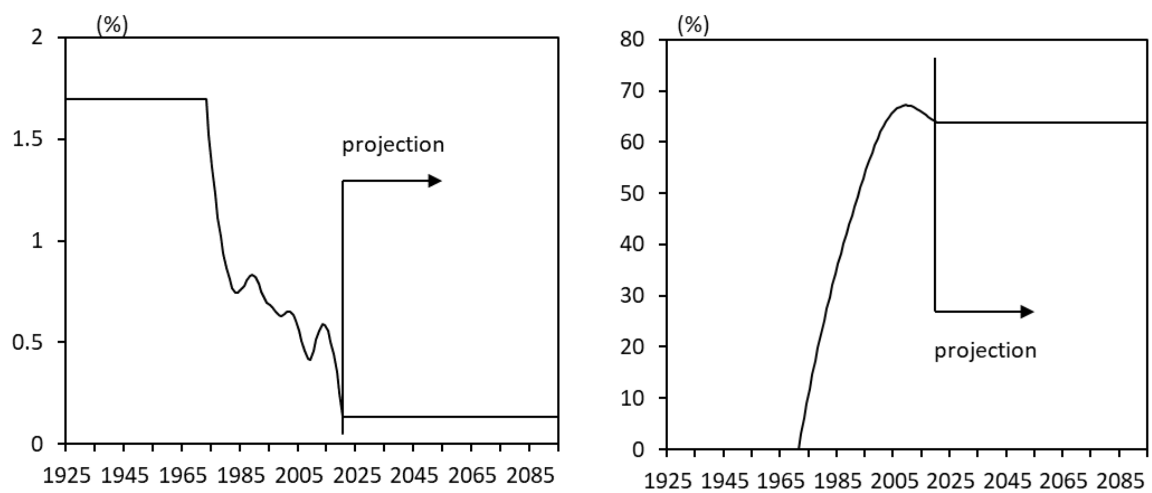
Appendix G Exogenous Variables

This section contains the graphs of each exogenous variables. The specification methodology is elaborated in IV.A.



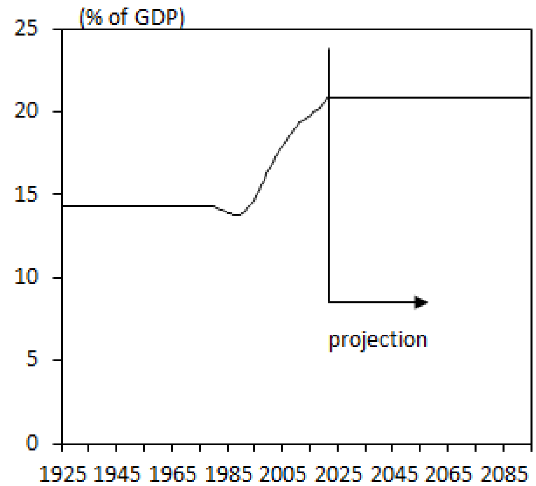
(a) Fertility Rate (Growth Rate of 20-year-olds) (b) Mortality Rate (over the Next 1-Year)

Figure A2: Demographics

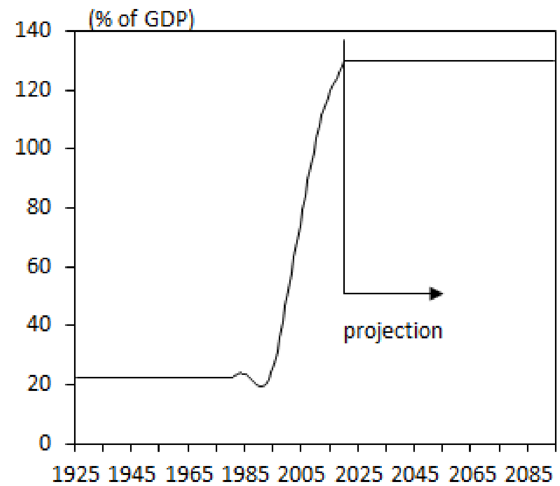


(a) Technology Growth in General Goods Sector (b) Technology Gap Between the Housing and General Goods Sectors

Figure A3: Technology

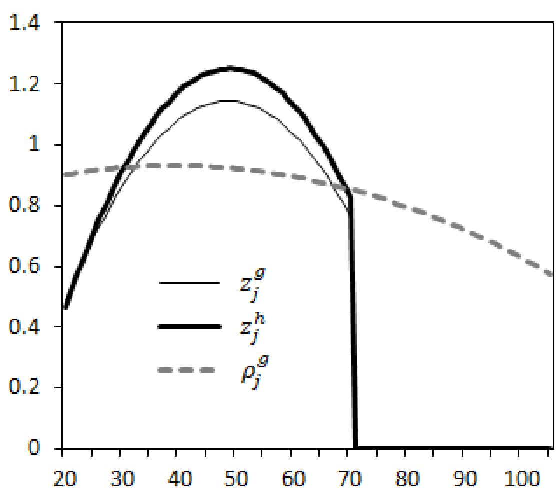


(a) Government Spending

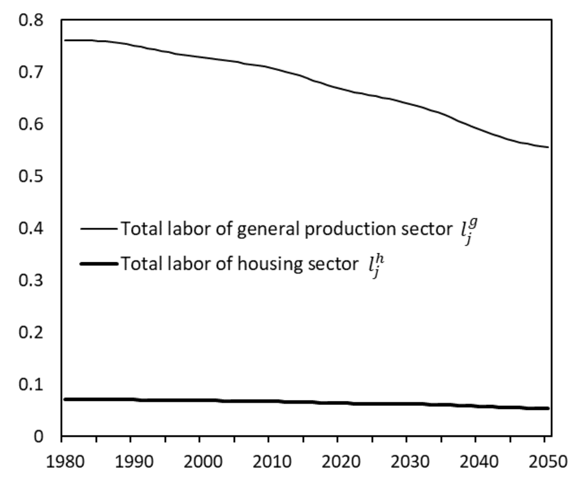


(b) Government Debt

Figure A4: Fiscal Condition



(a) Labor Properties



(b) Total Labor

Figure A5: Labor

Appendix H Return of Housing

Figure A6a compares the dynamics of real interest rate and the return of housing price in the model. They move parallel and the gap between them is considered as “user cost”.

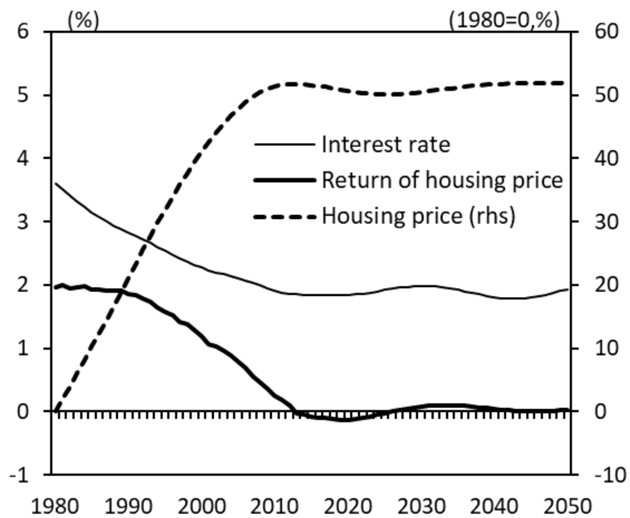


Figure A6: Interest Rate and Return of Housing Price