Balassa–Samuelson in the Long Run: Qualitative Success, Quantitative Limits

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Motivation: Real exchange rate in the long-run

- **Determinants of Real Exchange Rate (RER) in the long-run**
  - Textbook: Balassa-Samuelson (BS)
    - If relative productivity of tradable to non-tradable ↑, then RER ↑

Refresher: Textbook Balassa-Samuelson

Country i, j , Sector $s = T, N$, fixed labor endowment: $\overline{L_i} = \sum \ell_{i,s}$

Production $Y_{i,s} = A_{i,s} \cdot \ell_{i,s}$

Sectoral price $P_{i,s} = w_i / A_{i,s}$

$$\text{LOOP: } P_{i,T} = P_{j,T} \rightarrow \text{wages } w_i/w_j = A_{i,T}/A_{j,T} \rightarrow \frac{P_{i,N}}{P_{j,N}} = \frac{A_{i,T}/A_{j,T}}{A_{i,N}/A_{j,N}}$$

Cobb-Douglas across sectors $\rightarrow$ CPI: $P_i = P_{i,T}^{\alpha_{i,T}} \cdot P_{i,N}^{\alpha_{i,N}}$

Real exchange rate: relative productivity

$$RER_{i,j} = \frac{P_i}{P_j} = \frac{\left(A_{i,T}/A_{i,N}\right)^{\alpha_{i,N}}}{\left(A_{j,T}/A_{j,N}\right)^{\alpha_{j,N}}}$$

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  - Textbook: Balassa-Samuelson (BS)
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What we know (?)

- Empirical literature: Mixed
  - Different periods / countries / specifications → different results
  - Controlling for wages is a ``bad control", testing "Residualized?" BS
- Quantitative literature: Mixed
  - All are short-run business-cycle models—not for long-run effects

Research Questions and My Answers

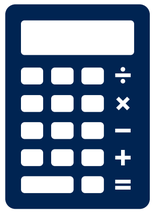
- Can BS **qualitatively** explain the long-run RER? - **Yes**
  - Show that traditional specifications are not robust and are ``inconsistent''
  - Adding time FEs resolves the inconsistency and leads to robust results
  - Qualitatively, BS is right *on average*

- Can BS **quantitatively** explain long-run RER - **No**
  - Set up multiple versions of quantitative trade models
    - Costly trade, N-country, IO linkages, S-sector
  - Quantitatively, BS cannot explain RER (effects are close to zero)

Today's plan

1. Qualitative validity

2. Quantitative (un)importance



3. Reasons for the failure of the model?

Traditional empirical design

- Traditional empirical design

$$\ln \text{RER}_{i,j,t} = \beta \ln \left(\frac{A_{i,T,t}/A_{i,NT,t}}{A_{U,T,t}/A_{U,NT,t}} \right) + \mu_i + \varepsilon_{i,t}$$

  - U : reference country (typically the US); μ_i : country FEs

- Mixed Findings from previous papers (different year/country/data):
  - ✓ : Officer (1976), Hsieh (1982), Lee and Tang (2007), Cardi and Restout (2015)
  - × : Canzoneri et al (1999), Berka-Devereux-Engel (2018), Berka and Steenkamp (2018), Devereux-Fujiwara-Granados (2025),…

Data

- RER: Penn World Table
  - CPI (``pl_c") and Nominal exchange rate

- Labor productivity: real value added per hour from KLEMS
  - EU KLEMS (Europe, US, Japan) — baseline, balanced panel sample
  - Asia KLEMS (Korea, Taiwan, India)
  - CIP from RIETI (China)

- Years: 1970-2021 (or 1995-2021)

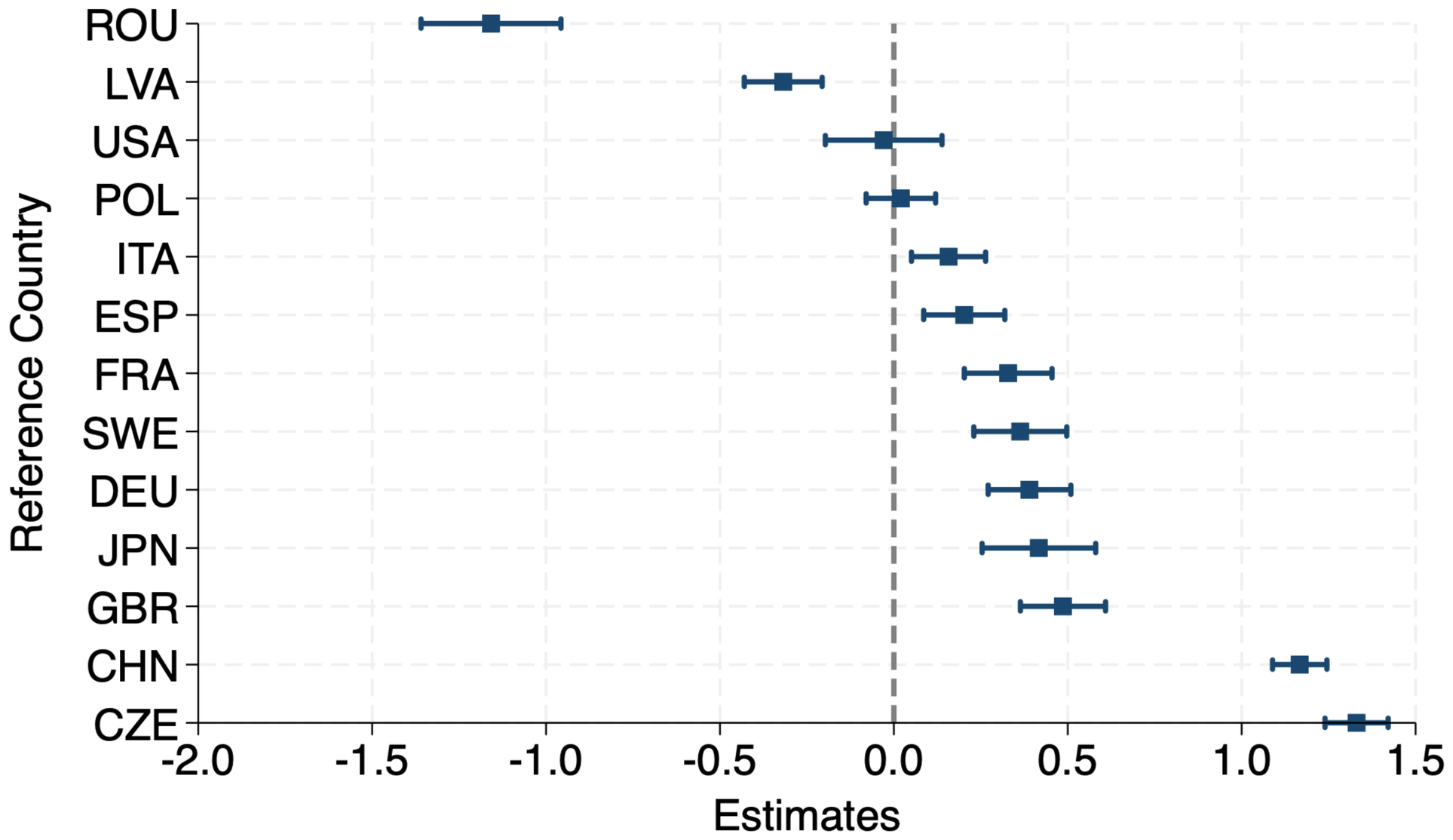
- HP-filtered ($\lambda = 6.25$, Ravn and Uhlig (2002))

Real exchange rate and relative productivity

Table 1: Real Exchange Rate and Relative Productivity

	(1)	(2)	(3)	(4)
Log Rel. ALP				
Observations	1,307	849	849	849
Num of Countries	33	33	33	33
Num of Years	52	27	27	27
Sample Years	1970-2021	1995-2021	1995-2021	1995-2021
Ref. Country	U.S.	U.S.	Germany	U.K.

Results


ROU
LVA
USA
POL
ITA
ESP
FRA
SWE
DEU
JPN
GBR
CHN
CZE
Reference Country
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Estimates

Econometric issues of traditional approach

- Traditional approach requires $T \to \infty$ ($N \to \infty$ alone is not enough)
  - $N \to \infty$ is a typical consistency assumption in panel-data analyses

- **Reference-country-specific errors** do not wash out as $N \to \infty$
  - ``US enters the regression NT times, while others enter T times"

$$\ln \text{RER}_{i,j,t} = \beta \ln \left(\frac{A_{i,T,t}/A_{i,NT,t}}{A_{U,T,t}/A_{U,NT,t}} \right) + \mu_i + \varepsilon_{i,t}$$

Easy Fix: Including Time Fixed Effects

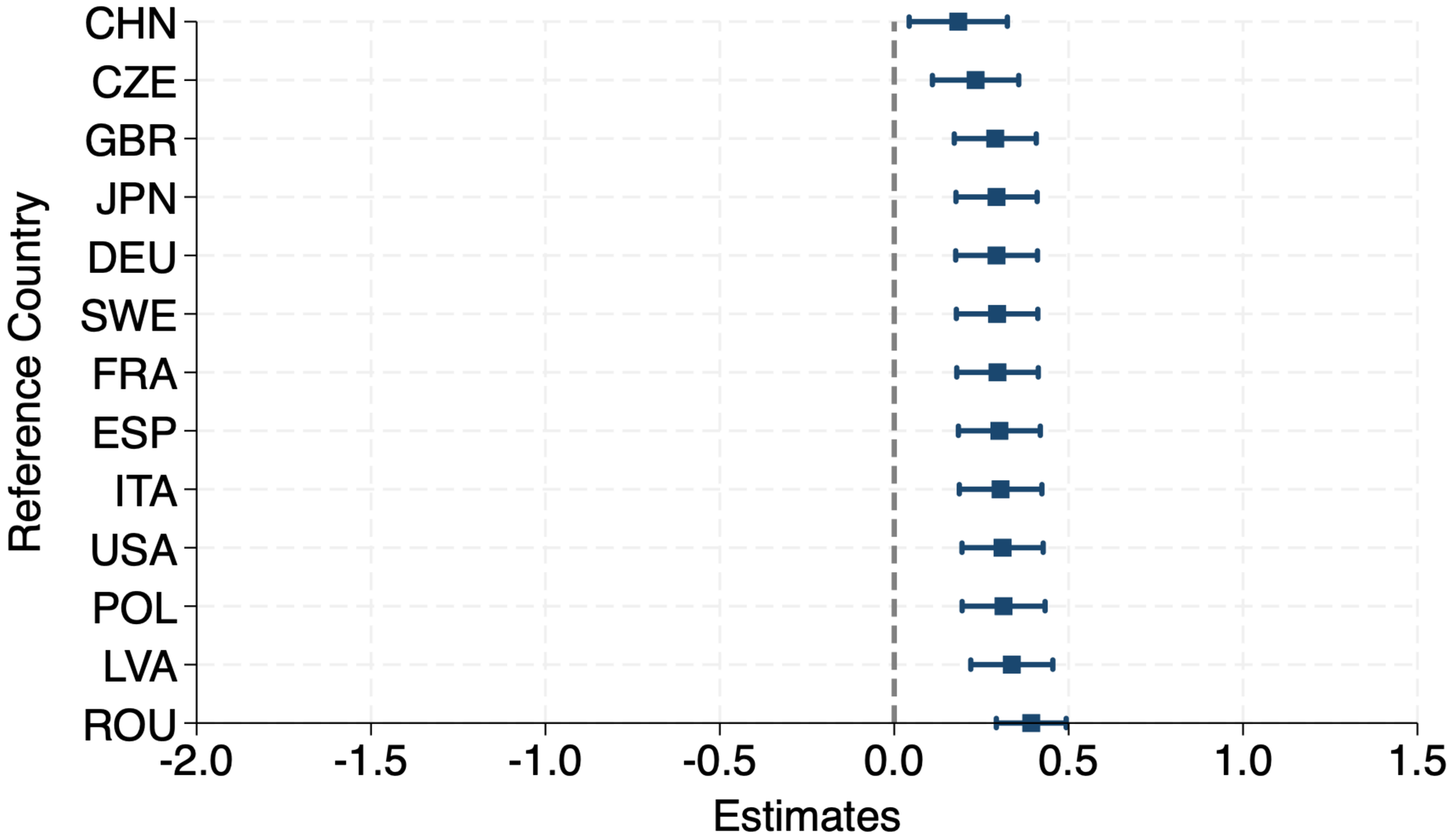
- Easy fix: include Time FEs

$$\ln \text{RER}_{i,j} = \beta \ln \left(\frac{A_{i,T,t}/A_{i,NT,t}}{A_{U,T,t}/A_{U,NT,t}} \right) + \mu_i + \eta_t + \varepsilon_{i,t}$$

- Variations are **across non-reference** countries

- Do not mix up two different sources of variations
  - US-UK, US-Japan, US-Germany → give up! (Or run pair-level)
  - UK-Japan, Japan-Germany, Germany-UK → focus on these

Robust results across reference countries



Robust results across different settings

- Previous papers: Mixed results across different settings

- My specification is robust across
  - Different years (1970-2021, 1995-2021, etc)
  - Different frequencies (1, 5, 10, 20 years)
  - Different country sets (Only EU, Adv, Balanced, Dev. countries)
  - Different data (10 sector DB, instead of KLEMS)
  - Different productivity measure (VA per workers, Composition adjusted)
  - Different covariates (GDP per workers = Penn Effect)
  - Different RER measures (BIS's REER)
  - Different filtering (Christiano–Fitzgerald, no filtering,..)
  - Different NT shares (by pairwise design)

Summary of empirical validity

- Traditional Specifications are not robust
  - The issue comes from single-reference without time FEs
  - Not consistent even with $N \to \infty$

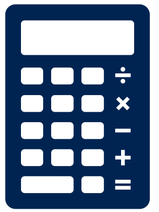
- Including **time fixed effects** leads to robust results

- Balassa-Samuelson Effects are qualitatively (empirically) present

Today's plan

1. Qualitative validity

2. Quantitative (un)importance



3. Reasons for the failure of the model?

Quantitative accounting

- Goal: Quantitatively evaluate how much productivity can explain RER

- Variants of textbook trade models
  - 2x2 Free Trade, 2x2, Nx2, NxS, NxSxS (IO linkages)
    - Break LOOP (= Costly Trade)
  - Similar spirit to Davis-Weinstein (2001 AER) for Heckscher-Ohlin-Vanek

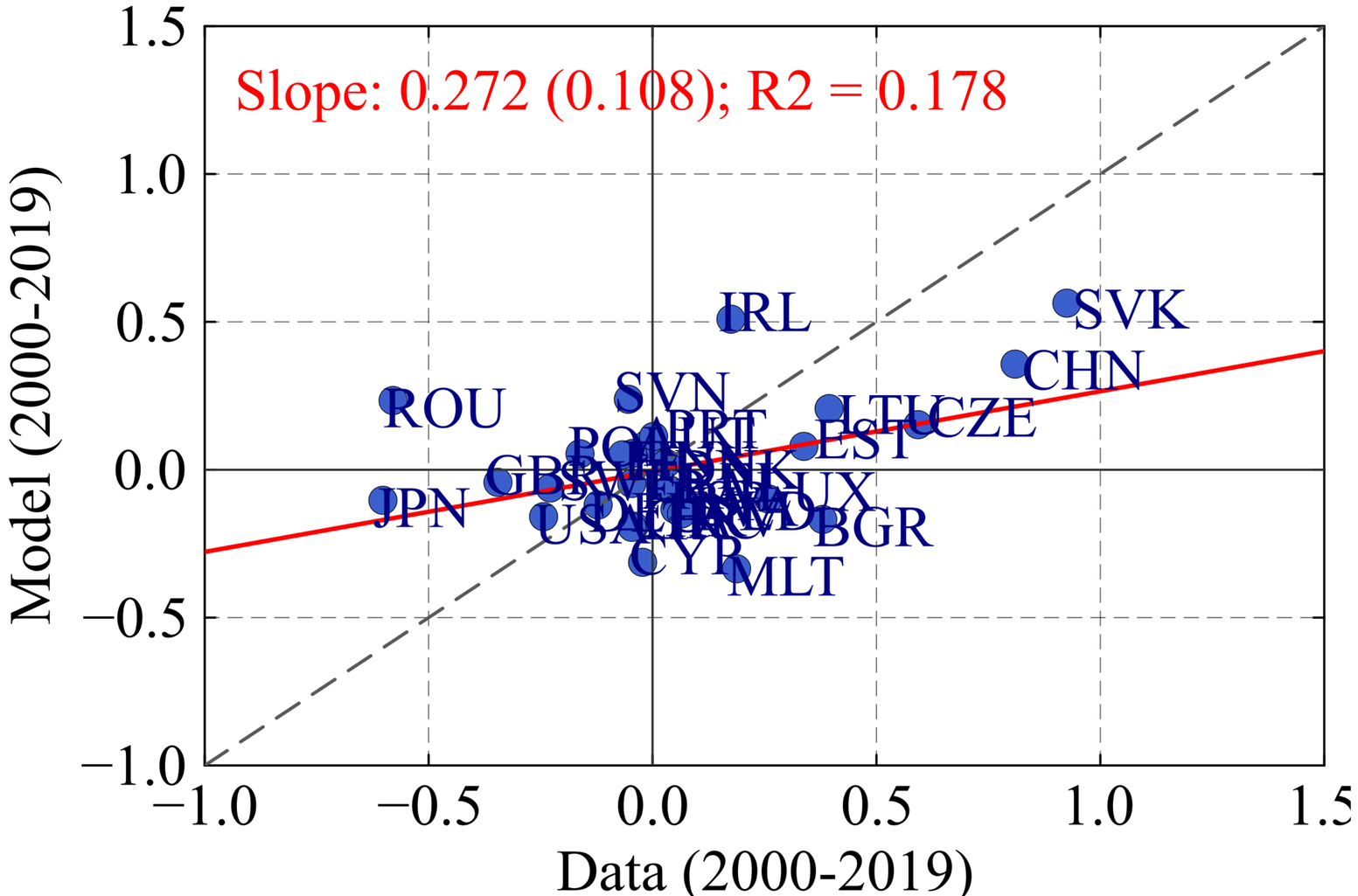
- Calibrate the model to 2000 (Exact Hat Algebra)
  - Shock the economy by feeding sectoral productivity $\{A_{i,s,t}\}$
    - Later add shocks to trade costs, total labor, and deficits as well
  - Focus on 31 countries with non-missing data (= Europe, US, Japan, China)

- Evaluate changes of R**E**ER between 2000-2019 against data

2x2 free trade model cannot explain long-run RER

Change in REER in Data and 2x2 Free Trade Model

2000=1, HP-filtered ($\lambda = 6.25$)

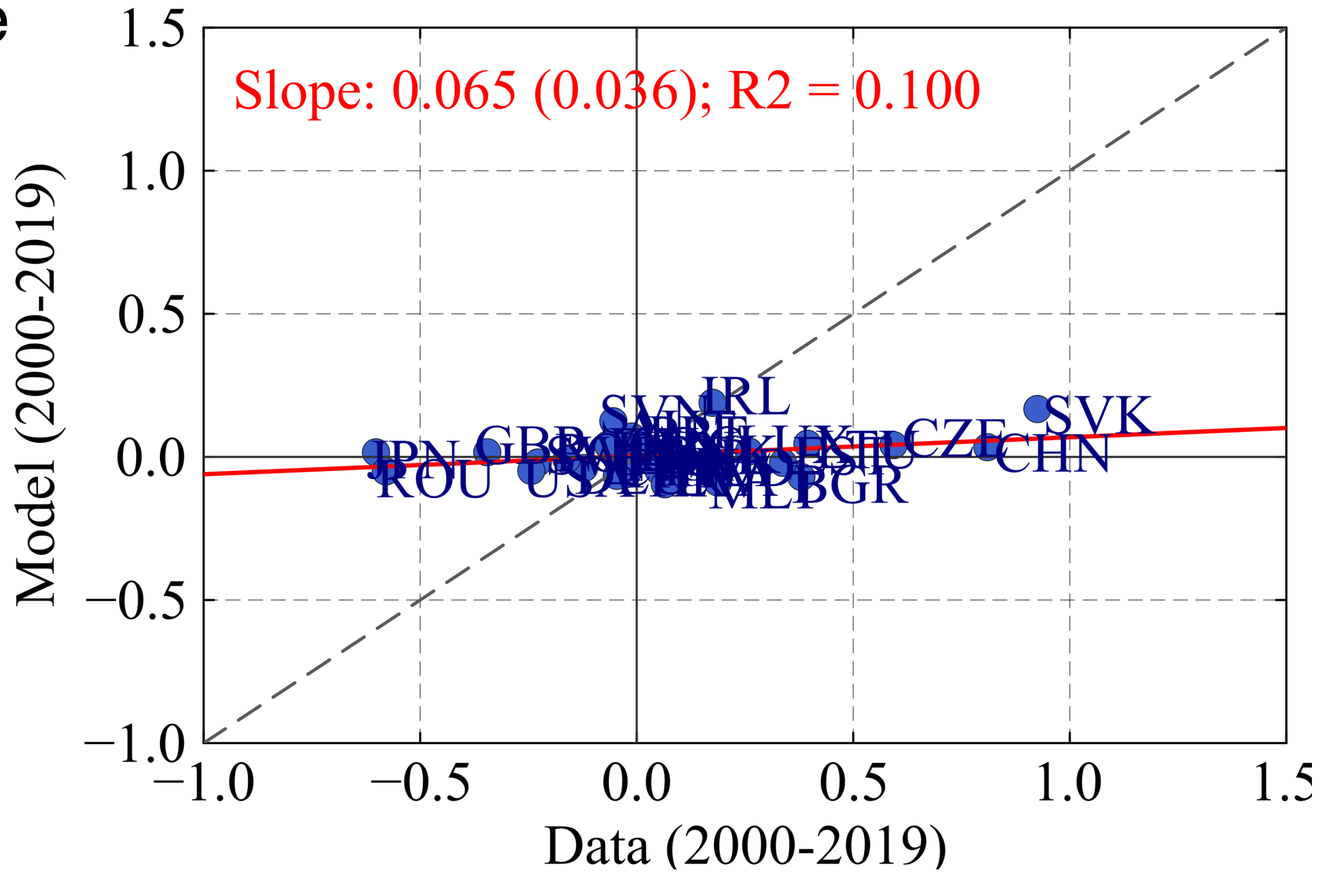


Costly trade

- Textbook model
  - Free trade in Tradables/MFG
  - No trade in Non-tradables/Service
- Reality
  - LOOP does not hold in MFG
  - Rises of Service trade
- 2x2 Costly Trade
  - Allow both sectors to costly trade

Changes in REER: 2x2 Costly Trade Model

2000 = 1, HP-filtered ($\lambda = 6.25$)



Extending to N countries

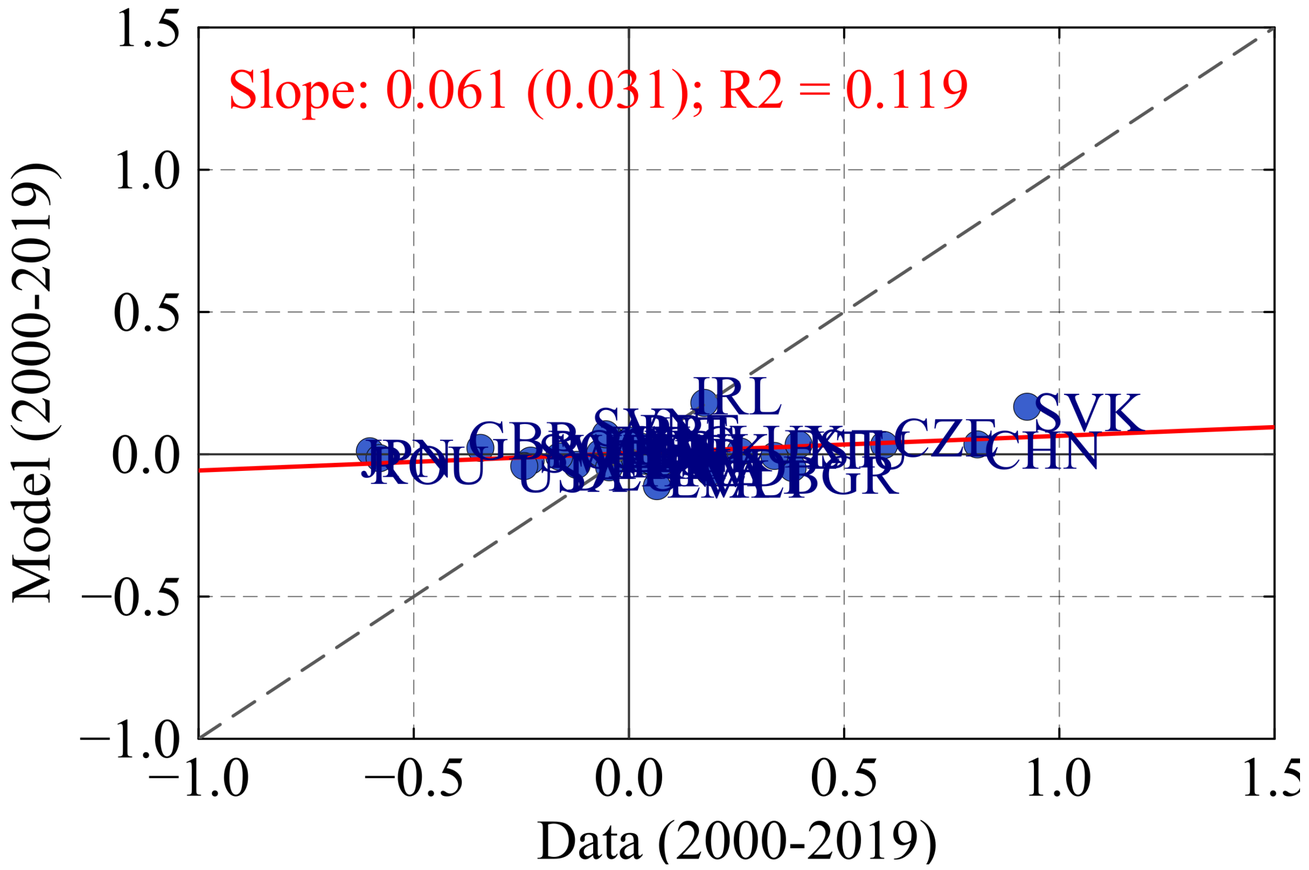
- 2x2 (Costly/Free) Trade Model
  - Solve 2x2 models (Nx(N-1))2 times
  - Asm. no China effect on JPY-USD

- Reality
  - We have N countries
    - Matsuyama (2009, JEEA)

- N (31) x2 Costly Trade Model
  - KLEMS countries + ROW

Changes in REER: Nx2 Costly Trade Model

2000 = 1, HP-filtered ($\lambda = 6.25$)



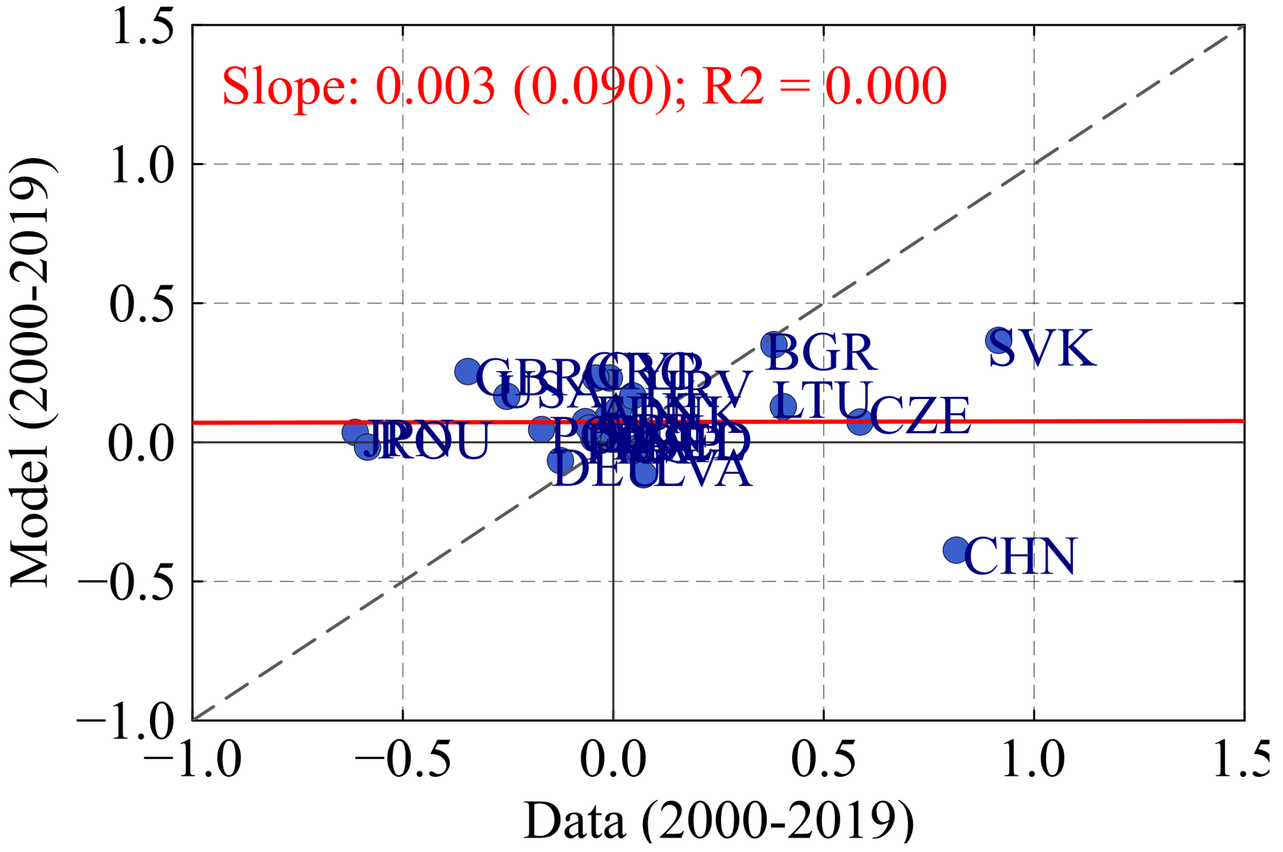
Richer models with more shocks

- Nx2 Trade Model
  - No Input-Output linkages
  - 2 sector (distinctions matter)

- NxS (31x23) with IO linkages

Changes in REER: NxS Costly Trade Model w/ IO

2000 = 1, HP-filtered ($\lambda = 6.25$)



Summary of REER results

- Textbook 2x2 Free Trade model is the least problematic

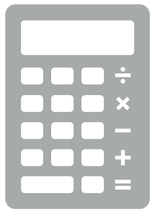
- N country, S sector, IO linkages (+ other bells and whistles) worsens the fit

- Seemingly, standard quantitative trade models cannot account for
  - Changes in real exchange rates (= prices) across countries

Today's plan

1. Qualitative validity

2. Quantitative (un)importance

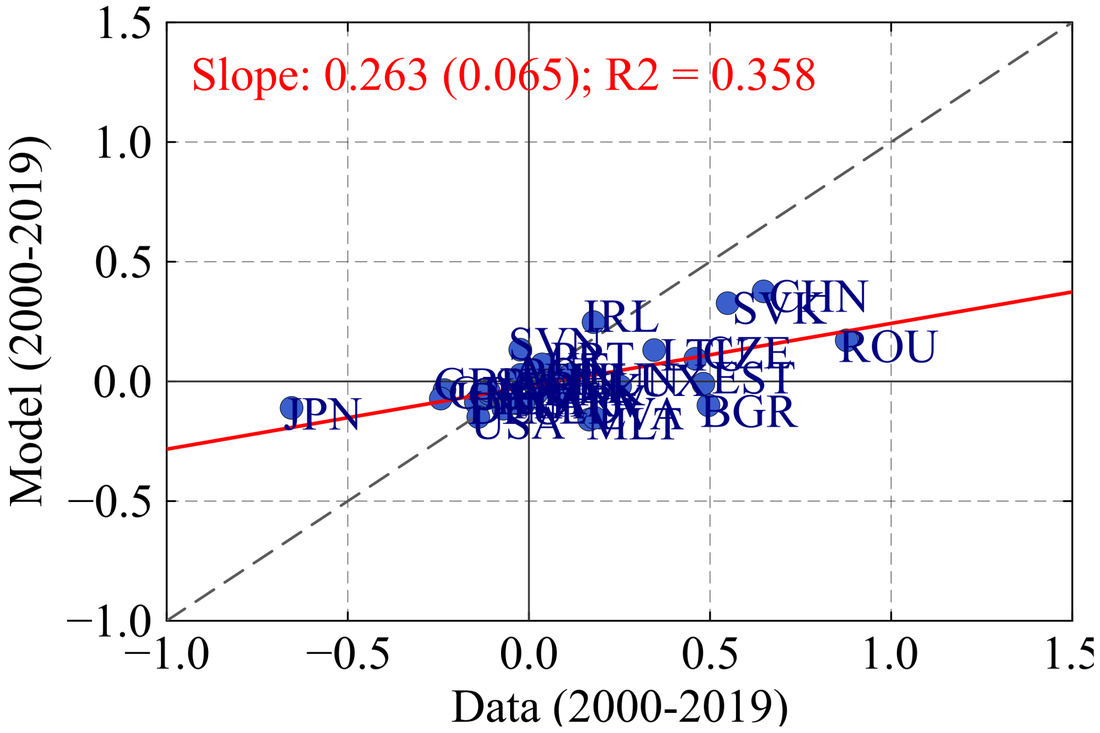


3. Reasons for the failure of the model?

Where do we fail: Goods REER are off in Costly Trade

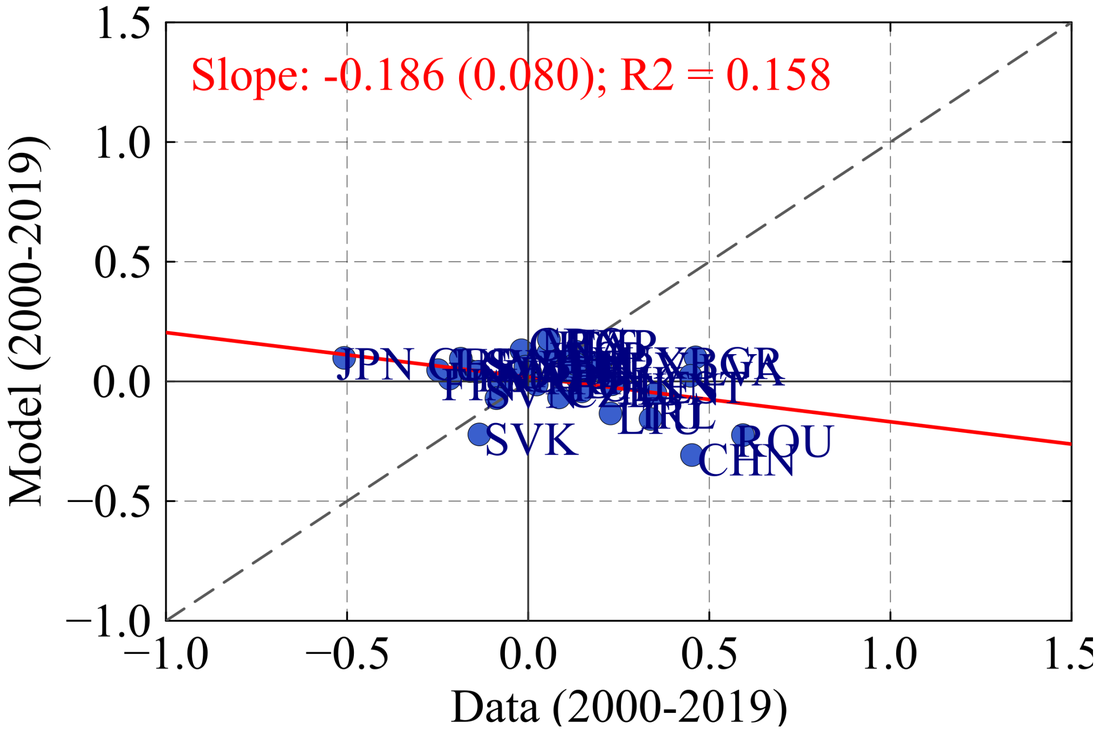
Change in Service REER

2000 = 1, Nx2 Costly Trade



Change in Goods REER

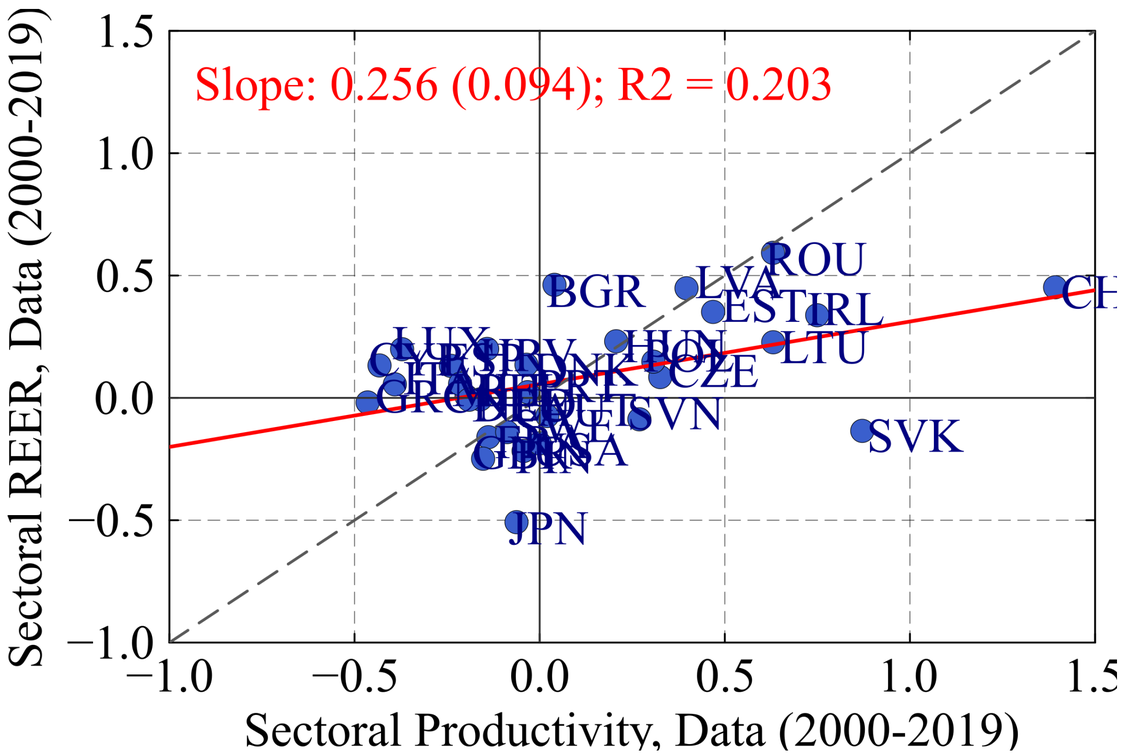
2000 = 1, Nx2 Costly Trade



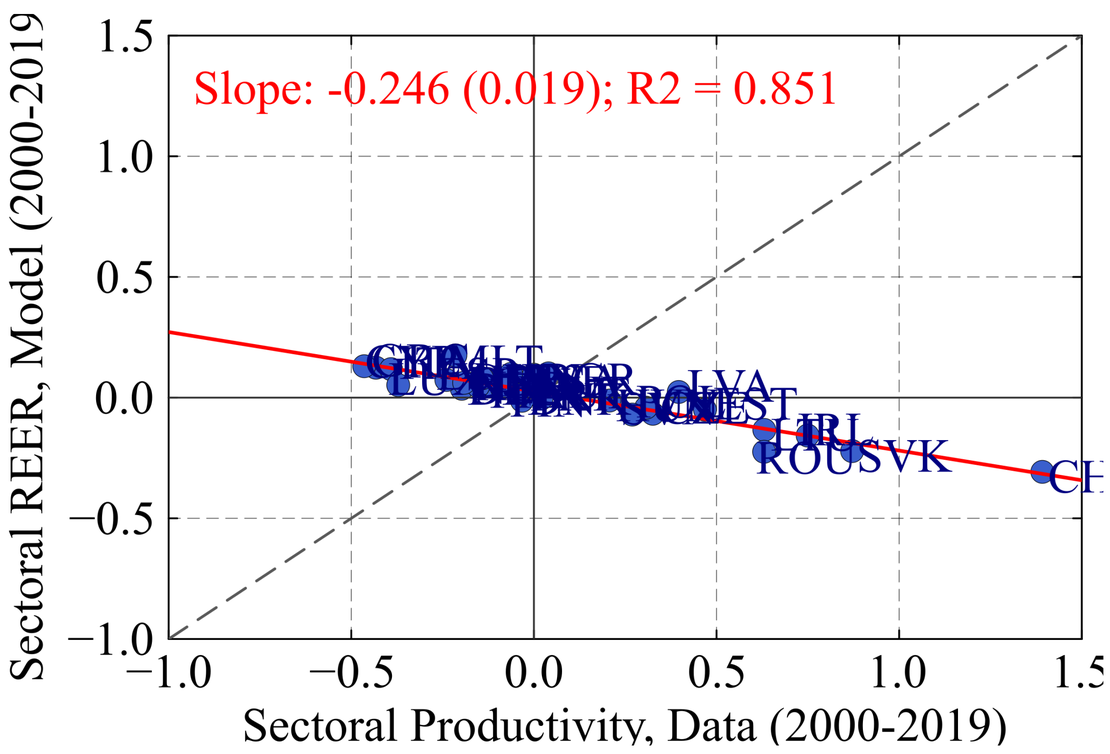
High Productivity, High Price (Data), Low Price (Model)

Change in Goods REER against Productivity; 2000 = 1

Observed Data



Nx2 Costly Trade Model



Discussion: Sources of failures

Measuring Shocks

- Productivity
- Trade costs
- Total labor endow.
- Trade deficits

Models

- Input: Shocks
- Output: **Wages**

Price Aggregator

- Input: **Wages** (+ data π)
- Outputs:
  - Sectoral PPI/CPI
  - CPI
  - **RER**

Engel (1999): pS - RER

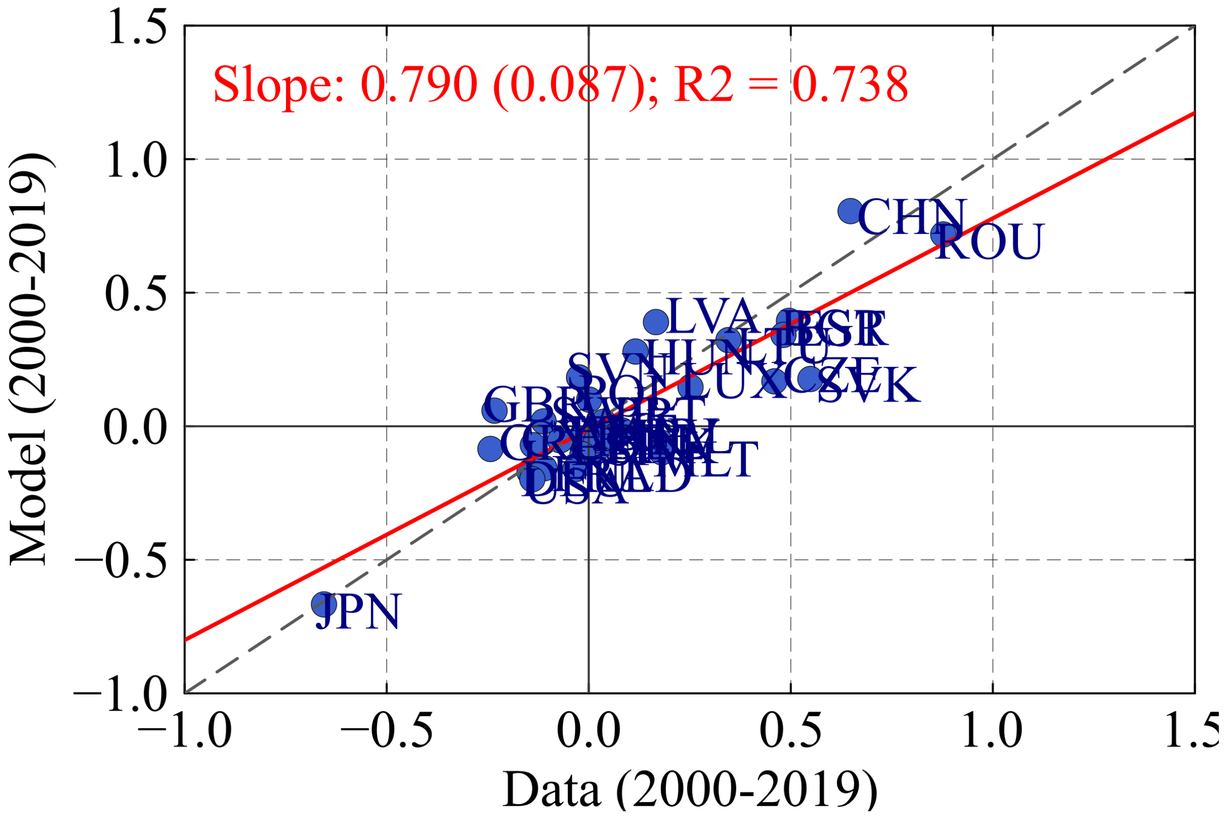
Next:

1. **Feeding observed changes in wages (in national currency)**
2. **Check sectoral RER (using Nx2 model) and aggregate RER**

Feeding wages replicate sectoral RERs

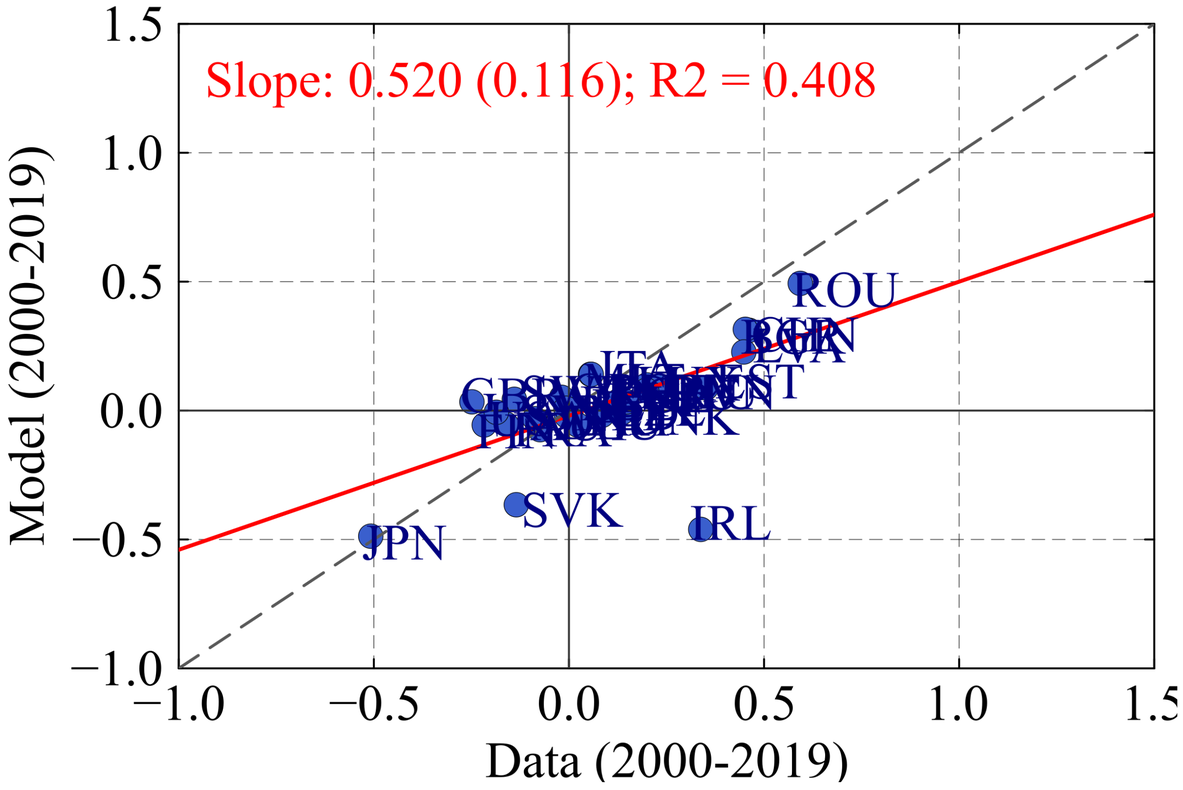
Change in Service REER

2000 = 1, Nx2 Costly Trade + Wage



Change in Goods REER

2000 = 1, Nx2 Costly Trade + Wage



Discussion: Sources of the quantitative failure of BS

- Goods REER are particularly off
  - (Unobserved) quality upgrading within sector?
- Given observed wages, REER fits well
  - Wage responses (to productivity) are too small?

- Traditional candidates as a source of the failure of Balassa-Samuelson
  - ~~Breakdown of LOOP~~ — maybe not b/c Relaxing LOOP worsens!
  - ~~Changing tradability~~ — maybe not
    - ~~Mechanical weight changes~~— b/c aggregator worked with fixed shares
    - ~~Structural reasons~~ — b/c changing trade costs did not help
  - **Inability of macro/trade model to replicate wage changes** — maybe.

Conclusion

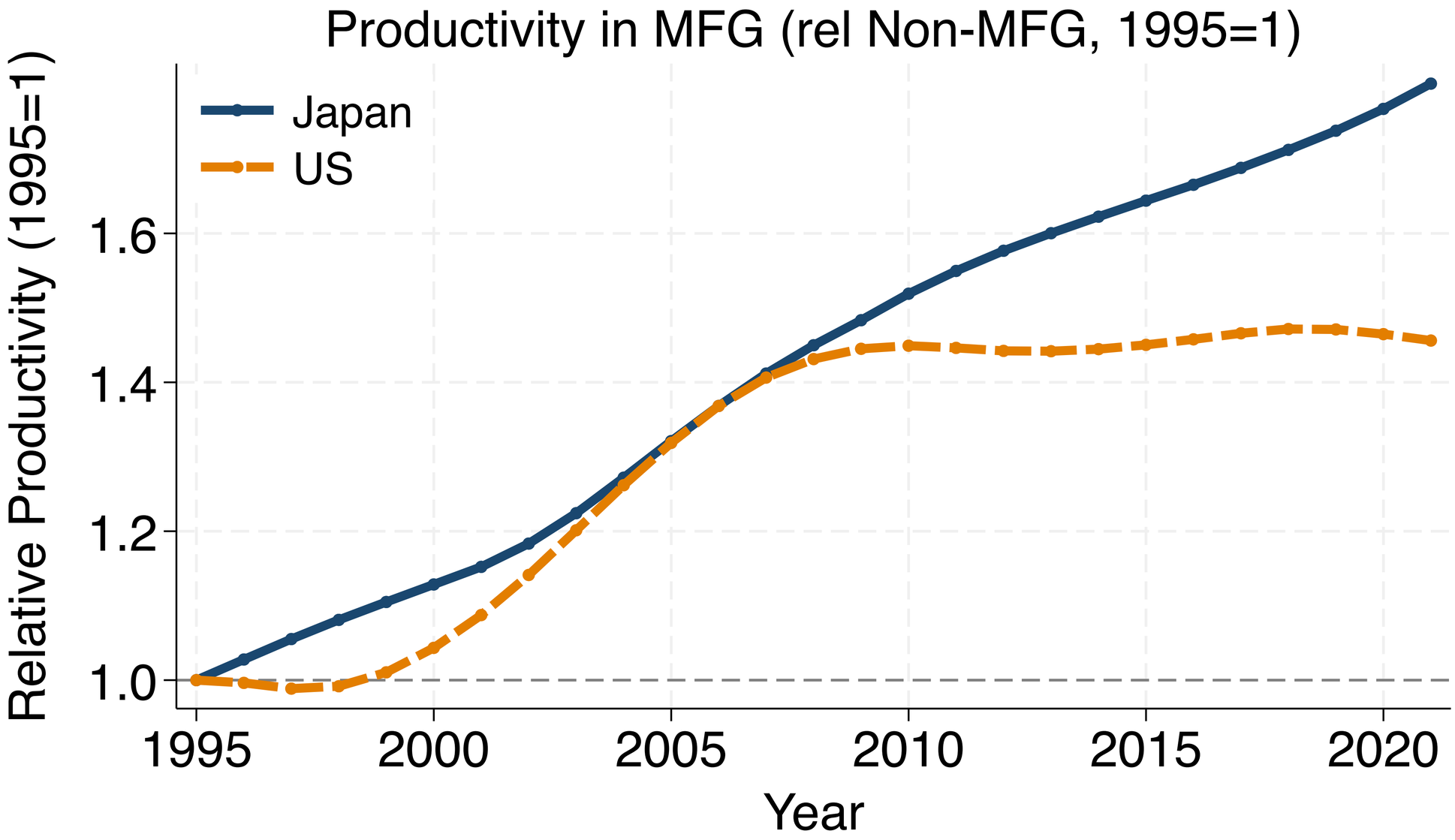
- Testing Balassa-Samuelson qualitatively and quantitatively

- Qualitatively, BS is supported
  - Traditional approach, yielding mixed results, had econometrics issues
  - Time fixed effects resolve the issue

- Quantitatively, BS performs poorly
  - Richer models do not help
  - Failures might come from the inability of the models to explain wages
    - Productivity measurement? Price measurement? — quality, “appeal”
    - Markdown?

Appendix

Japanese MFG's Relative Productivity Grow Faster



Econometric issues of traditional approach 1/2

DGP for CPI: $\ln P_{i,t} = \beta\, a_{i,t} + \alpha_i + g_t + u_{i,t}, \qquad a_{i,t} \equiv \ln\left(\frac{A_{i,T,t}}{A_{i,NT,t}}\right)$

In ``relative'' to reference country U,

$$r_{i,t} \equiv \ln RER_{i,U,t} = \ln\left(\frac{P_{i,t}}{P_{U,t}}\right) = \ln P_{i,t} - \ln P_{U,t}, \qquad i \neq U$$

Then, $r_{i,t} = \beta\left(a_{i,t} - a_{U,t}\right) + (\alpha_i - \alpha_U) + \left(u_{i,t} - u_{U,t}\right),$

Leading to the traditional specification

$$r_{i,t} = \beta x_{i,t} + \mu_i + \varepsilon_{i,t}$$

Within-estimator: $\tilde{r}_{i,t} = \beta\, \tilde{x}_{i,t} + \tilde{\varepsilon}_{i,t}$ and

$$\hat{\beta} = \beta + \frac{S_{x\varepsilon}}{S_{xx}}, \qquad S_{x\varepsilon} \equiv \sum_{i,t} \tilde{x}_{i,t}\tilde{\varepsilon}_{i,t}, \quad S_{xx} \equiv \sum_{i,t} \tilde{x}_{i,t}^2 .$$

Econometric issues of traditional approach 2/2

Error terms:

$$S_{x\varepsilon} = \sum_{i,t} (a_{i,t} - \bar{a}_i)(u_{i,t} - \bar{u}_i) - \sum_{i,t} (a_{i,t} - \bar{a}_i)(u_{U,t} - \bar{u}_U)$$

$$- \sum_{i,t} (a_{U,t} - \bar{a}_U)(u_{i,t} - \bar{u}_i) + N \sum_{t} (a_{U,t} - \bar{a}_U)(u_{U,t} - \bar{u}_U)$$

$$S_{xx} = \sum_{i,t} (a_{i,t} - \bar{a}_i)^2 + N \sum_{t} (a_{U,t} - \bar{a}_U)^2 + \text{cross terms}\,.$$

However, even under standard exogeneity,

$$\hat{\beta} - \beta = \frac{S_{x\varepsilon}}{S_{xx}} = \frac{O_p(N\sqrt{T})}{O_p(NT)} + o_p(1) = O_p\left(\frac{1}{\sqrt{T}}\right),$$

Robust results after controlling Penn effects

Table 3: Single-reference time-FE regressions with Penn effect control

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.31		0.26	0.25
	(0.06)		(0.07)	(0.05)
Log Rel. GDP per Workers		0.22	0.11	-0.33
		(0.08)	(0.09)	(0.11)
Observations	849	849	849	1,307
Sample Countries	All	All	All	All
Num of Countries	33	33	33	33
Sample Years	1995-2021	1995-2021	1995-2021	1970-2021
Num of Years	27	27	27	52
Country & Year FE	✓	✓	✓	✓

Robust results with different data and variables

Table 4: Single-reference time-FE regressions: GGDC 10-sector data

	(1)	(2)	(3)	(4)
Log Rel. ALP	0.84	0.45	0.65	0.38
	(0.07)	(0.06)	(0.07)	(0.06)
Log Rel. GDP per Workers			1.42	-0.36
			(0.13)	(0.06)
Observations	2,106	741	2,106	741
Sample Countries	All	All	All	All
Num of Countries	39	39	39	39
Sample Years	1960-2013	1995-2013	1960-2013	1995-2013
Num of Years	54	19	54	19
Country & Year FE	✓	✓	✓	✓

Overview of quantitative model

- Goal: Quantitatively evaluate how much productivity can explain RER
- Vehicles: Variants of textbook trade models
  - 2x2 Free Trade, 2x2 Costly Trade, **Nx2 Costly Trade** (+ IO linkages)

- Country: $i, j = 1,..,N$; Sector: $s = 1,2$
- All prices are expressed in USD (nominal)

- Textbook Armington Trade Model
  - Preferences: Cobb-Douglas across sectors, CES within sectors
  - Technology: linear in labor
  - Market clearing: exogenous trade surplus

Preference: (Armington Trade)

Cobb-Douglas across sectors

$$U_j = \prod_{s=1}^{S} \left(\frac{C_{j,s}}{\alpha_{j,s}} \right)^{\alpha_{j,s}}$$

CES within sectors

$$C_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s}^{1/\theta} C_{i,j,s}^{(\theta-1)/\theta} \right)^{\theta/(\theta-1)}$$

Aggregate CPI

$$P_j = \prod_{s=1}^{S} P_{j,s}^{\alpha_{j,s}}$$

Sectoral CPI and Trade Share

$$P_{j,s} = \left(\sum_{i=1}^{N} \mu_{i,j,s} P_{i,j,s}^{1-\theta} \right)^{1/(1-\theta)}$$

$$\pi_{i,j,s} \equiv \frac{P_{i,j,s} C_{i,j,s}}{\sum_{\ell=1}^{N} P_{\ell,j,s} C_{\ell,j,s}} = \frac{\mu_{i,j,s} P_{i,j,s}^{1-\theta}}{\sum_{\ell=1}^{N} \mu_{\ell,j,s} P_{\ell,j,s}^{1-\theta}}$$

Technology and trade

Linear Production in Labor

$$Y_{i,s} = A_{i,s} \cdot \ell_{i,s}, \qquad \overline{L_i} = \sum_s \ell_{i,s}$$

Unit production cost

$$c_{i,s} = \frac{w_i}{A_{i,s}}$$

Costly trade

$$P_{i,j,s} = \tau_{i,j,s} \cdot c_{i,s} = \frac{w_i \cdot \tau_{i,j,s}}{A_{i,s}}$$

Equilibrium

An equilibrium is $\{w_i\}_{i \in \mathcal{N}}$ s.t.

Goods markets clear ($\sum_i \beta_i = 0$)

$$\sum_{j=1}^{N} \sum_{s=1}^{S} \pi_{i,j,s} \cdot \alpha_{j,s} \cdot w_j L_j = w_i L_i + \beta_i \cdot \sum_{\ell=1}^{N} w_\ell L_\ell$$

where trade share is given by

$$\pi_{i,j,s} = \frac{\mu_{i,j,s} \left(\frac{w_i \tau_{i,j,s}}{A_{i,s}} \right)^{1-\theta}}{\sum_{\ell=1}^{N} \mu_{\ell,j,s} \left(\frac{w_\ell \tau_{\ell,j,s}}{A_{\ell,s}} \right)^{1-\theta}}$$

Wage to Price

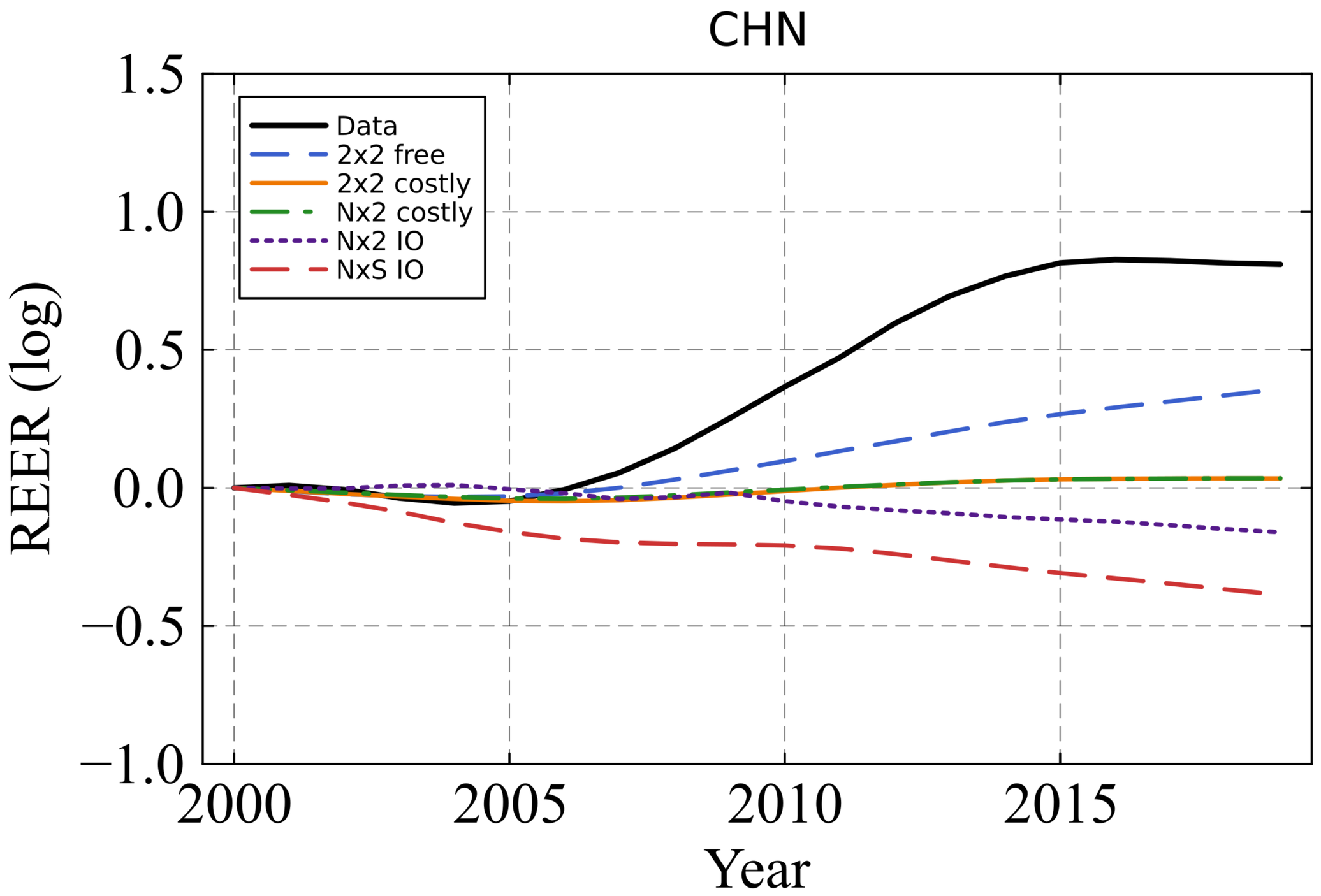
$$P_{i,j,s} = \frac{w_i \cdot \tau_{i,j,s}}{A_{i,s}}$$

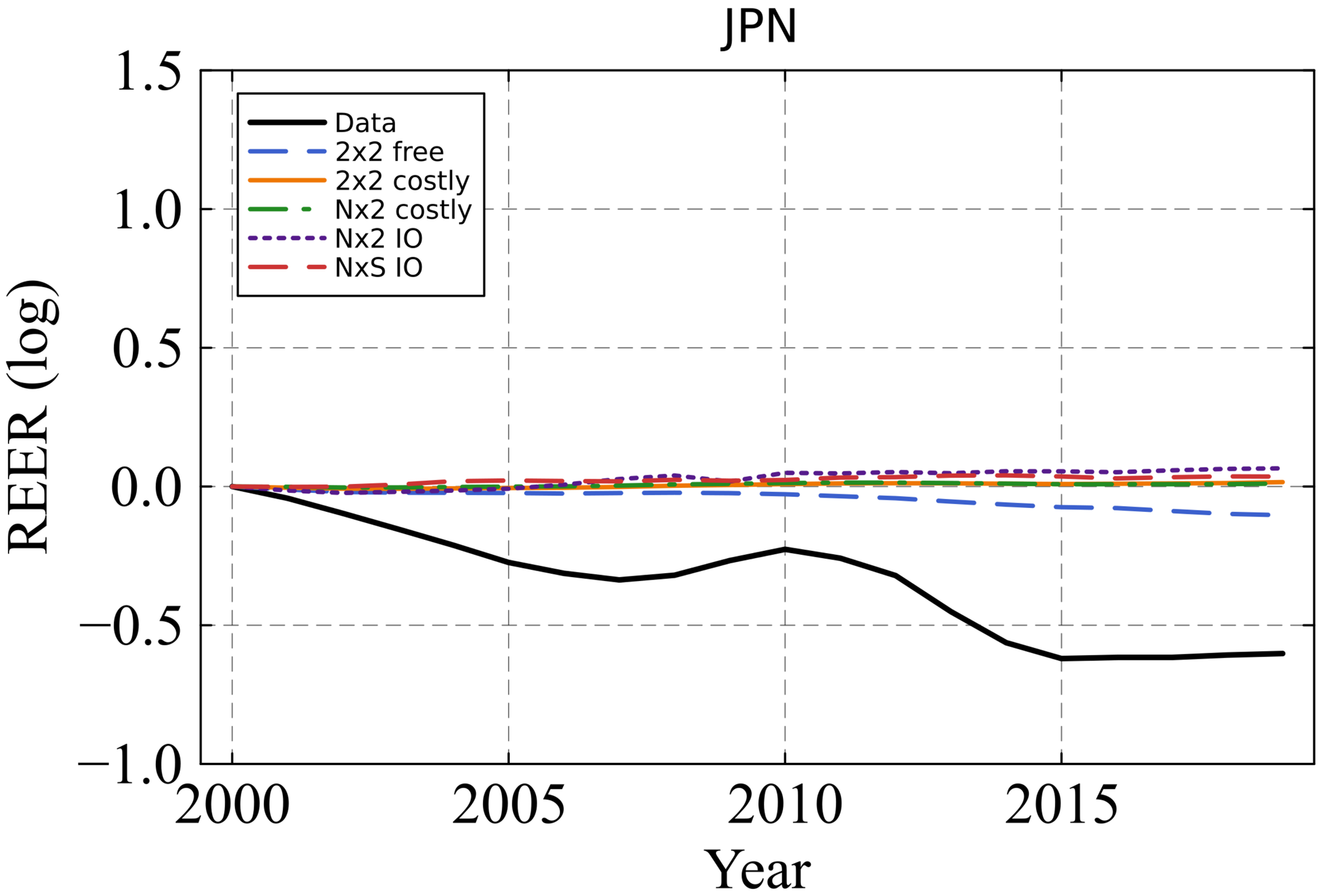
Price to Aggregate CPI

CES + Cobb-Douglas

Real Exchange Rate

$$\text{RER}_{i,j} = \frac{P_i}{P_j}$$


CHN
Data
2x2 free
2x2 costly
Nx2 costly
Nx2 IO
NxS IO
REER (log)
1.5
1.0
0.5
0.0
−0.5
−1.0
2000
2005
2010
2015
Year


JPN
Data
2x2 free
2x2 costly
Nx2 costly
Nx2 IO
NxS IO
REER (log)
1.5
1.0
0.5
0.0
−0.5
−1.0
2000
2005
2010
2015
Year