COLUMBIA UNIVERSITY
Center for Pricing and Revenue Management

WORKING PAPER SERIES: NO. 2013-1

Optimizing Prices for Consumer Credit

Robert Phillips
Columbia University, Nomis Solutions

2013

http://www.cprm.columbia.edu

Optimizing Prices for Consumer Credit

Robert Phillips
Columbia University Business School
Nomis Solutions

Uris Hall 408
3022 Broadway
New York, NY 10027
e-mail: rp2051@columbia.edu

March 22, 2013

Optimizing Prices for Consumer Credit

Abstract

Pricing of consumer credit was long considered to be straightforward. However, the recent financial crisis has shown that mispricing and misallocating consumer credit can have severe consequences for the global economy. For many years, risk-based pricing has been the state-of-the art in credit pricing. In the past few years, lenders have begun to adopt pricing optimization approaches that consider customer willingness-to-pay as well as risk in setting prices for credit. In this paper, we describe the pricing problem faced by providers of consumer credit. We focus on aspects of the problem that differ from other pricing settings – in particular, the non-linearity of incremental contribution in rate, the uncertainty of the return on a loan, the role of information, and the interaction between pricing and risk. We describe how these aspects can be incorporated into the determination of optimal rates for consumer credit. Finally we discuss current challenges and open areas for research.

Keywords: Pricing, consumer credit, financial services, banking, price optimization

Author: Robert Phillips is Professor of Professional Practice at the Columbia University Graduate School of Business. He is also chief science officer of Nomis Solutions, a company specializing in the application of price optimization to financial services, which he founded in 2003. Prior to founding Nomis, Dr. Phillips served as chief technology officer of Manugistics and as founder and chief executive officer of Talus Solutions, a pricing and revenue optimization company. Robert Phillips is author of the book *Pricing and Revenue Optimization* and is co-editor of the *Oxford Handbook of Pricing Management*.

Optimizing Prices for Consumer Credit

Pricing of consumer credit was once considered simple. A commonly cited maxim was the *3 - 6 - 3 rule*: "Borrow at 3%, lend at 6%, on the golf course by 3:00." However, in the wake of the financial crisis of 2008 - 2009, it has become apparent that such simple rules are no longer adequate (if they ever were). Furthermore, "getting it right" is more important than ever: the globalization of banking means that bad pricing decisions by a bank in Florida can lead to the collapse of pension funds in Norway. Add to this the fact that, in the pursuit of market share and profit, banks have segmented their customer base more and more finely and introduced a multitude of new products into the marketplace. It is fair to say that the pricing capabilities at most financial services institutions have not kept pace with this vastly increased complexity. The purpose of this paper is to motivate the problem of pricing consumer credit, describe an analytical approach to determining optimal credit prices, and discuss current challenges and open issues.

The question of how lenders should price consumer credit has received little attention in the academic literature. Chapter 3 of Thomas (2009) summarizes the mathematical foundations of risk-based pricing and some extensions. Phillips and Raffard (2011) consider pricing in a model in which low-risk customers are more price-sensitive than high-risk customers. Sundarajan et al. (2011) consider pricing in the context of determining the optimal product offering for customers at a retail bank. Oliver and Oliver (2012) consider the problem of determining the optimal price for a loan in the face of price-dependent risk when the goal of the lender is to maximize return-on-equity. Caufield (2012) provides an overview of pricing issues in different global consumer credit markets and discusses the application of pricing optimization in different markets.

In the next section, we briefly outline the market for consumer credit with a focus on the different forms of consumer credit and the channels through which it is sold. In Section 2 we describe the processes through which consumer credit is sold and priced. In Section 3, we formulate credit pricing as an optimization problem and give conditions under which a unique set of optimal prices is guaranteed to exist. In Section 4, we discuss the current state of consumer credit pricing optimization and a number of open issues

1 The Consumer Credit Market

Consumer credit refers to loans and lines extended to individual consumers as opposed to those extended to businesses or government entities. Consumer credit in advanced economies such as North America, Western Europe, and Eastern Asia is provided by a large number of lenders in a wide variety of forms through a number of different channels. Sources of credit include both banks and non-bank lenders, where a bank can be loosely defined as an institution that accepts deposits. Non-bank lenders include credit card issuers such as American Express and specialized lenders such as the Ford Motor Credit Corporation. Forms of credit include mortgages, other secured loans such as auto and boat loans, unsecured loans, credit cards, and loans against pensions and insurance. In North America, mortgages represent the largest total balance for consumer loans, however, credit cards have a large lead in the number of total transactions.

At the most basic level, loans can vary by the amount lent (the *starting principal*), the repayment period (the *term*) and the interest rate. In addition, lending products may vary by other dimensions such as:

- *Collateralization.* A loan may be secured (collateralized) or unsecured. The most common forms of security or collateral are real-estate and automobiles. Credit cards, student loans, and "unsecured loans" are usually not backed by collateral. In general, rates for uncollateralized loans are higher due to the fact that a non-performing collateralized loan could be partially or entirely repaid by seizing and selling its collateral.

- *Revolving versus non-revolving.* A loan may be for a fixed amount with a fixed repayment schedule or it may be *revolving* with a minimum payment specified. Typically, a revolving loan has a "cap" representing the maximum amount that can be borrowed. In each period, the borrower can borrow more up to the cap or pay some portion of the remaining principal. Credit cards and "lines of credit" are the most popular forms of revolving credit.

- *Fixed versus variable rate.* The APR on a loan may be fixed for the life of the loan or it may change as a function of some external rate such as LIBOR or the prime rate. As an example, a variable rate mortgage might have an APR equal to the prime rate plus 2% with the APR being updated each month based on the value of the prime rate on the last day of the month. Combinations of variable and fixed rates are common. A popular mortgage in the United States combines a low fixed rate for some initial period of time converting to a variable rate after that.

The "typical" loan varies from country to country. The most popular mortgage in the United States is the 35-year fixed-rate mortgage in which the APR for the mortgage is determined up-front and does not change for the duration of the mortgage. By contrast, in Canada, the most popular mortgage is a "five-year renewable" mortgage. This type of mortgage is usually amortized over 25 years, however, the remaining balance is due at the end of five years. When the mortgage is due, the borrower will renew the loan with the same lender – possibly at a different rate – or choose to borrow from a different lender. With the 35-year fixed-mortgage, the risk of future interest changes lies with the lender, while the five-year renewable rate means that the borrower assumes some of this risk.

One other difference among lending markets is the extent to which loans are sold on the secondary market. The process of bundling and selling loans is known as *securitization* and it has grown in popularity since its introduction in the United States in the 1970's. The increasing prevalence of securitization combined with the use of ever more "creative" ways to package loans together and an underestimation by purchasers of the systematic risk inherent in those packages has been claimed as a major contributor to the global financial crisis of 2008/2009 – see, for example, Lewis (2010). Despite this, securitization remains extremely common for non-revolving loans such as mortgages and auto loans in the United States: about 85% of the mortgages issued in the United States in 2010 were securitized within a year (Bureau of Labor Statistics, 2011). While common in the United States, the sale of loans on the secondary market is still rare elsewhere in the world.

One of the salient characteristics of consumer lending markets is that they demonstrate a surprising amount of *price dispersion* – different lenders offer the same loan to the same customer at different – sometimes drastically different – rates. Table 1 shows some of the APR's advertised on-line in the UK for an unsecured personal loan of £5,000 with a 36 month term as advertised through the web-site moneyfacts.com in October, 2012. The rates shown were for a 55-year old customer[1] with "good credit". Given that the product and the customer information are identical, there is a remarkable range of rates on offer: the highest APR on-offer is more than 3 times the lowest – as is the total amount of interest that would be paid over the price of the loan.

Table 1 About Here

The amount of price dispersion shown in Table 1 is not unusual. Similarly high levels of price dispersion in the British unsecured lending marketing were documented in 2004 (Phillips, 2005) and in 2007 (Thomas, 2009). High levels of price dispersion can be found in the American car-lending and mortgage markets, with different lenders offering very different rates for the same loan.

The reasons for the high levels of price dispersion found in consumer credit markets have not been fully explained. However, it is notable that many consumer credit markets are not highly concentrated: there tend to be many players, each with a relatively small share of the market. In 2006, the two largest sources of motor finance in the United States were General Motors Acceptance Corporation (GMAC) and Ford Motor Credit Company with market shares of 12.6% and 7.8% respectively. No other company had a market share greater than 5%. This means that there is typically no dominant player or pricing leader.

One implication of the wide range of price-dispersion found in most consumer lending markets is that there is considerable room for individual lenders to move prices without creating massive swings in demand. This supports the need of analysis to determine the "right price" for each particular loan.

[1]It is common in the UK to offer different rates to customers of different age – a practice that is illegal in the US.

2 How Credit is Sold and Priced

Consumer credit is typically sold through a *customized pricing* process as defined by Phillips (2005, 2012A) in which the lender obtains information about a customer and her desired loan prior to quoting a price and the lender has information about customers who accept a loan offer as well as those who reject it. A typical consumer credit sales and pricing process is shown in Figure 1. Customers enter the process by submitting an application that discloses information both about their current financial position customer (e.g. income, assets, existing loans, etc.) and the characteristics of the desired loan such as amount and term. The lender then determines whether or not they wish to offer the loan to the customer – the so-called *underwriting* decision. The underwriting decision is based on the lender's estimate of the probability that the customer would default on the loan and the loss if she did default. This estimate is typically based on the "credit history" of the customer (i.e., their track record in paying back previous obligations) as well as their current financial state. Most lenders in the United States subscribe to one or more commercial credit-scoring services that provide credit scores for potential customers that reflect their past behavior and their current obligations. The first and best known of these scores is the FICO score developed by the Fair Isaac Company (see Poon [2007] for a history of the FICO score). The calculation of credit scores and the use of these scores and other information by a bank to determine which loan applicants are creditworthy is a substantial field in itself – see Thomas et al. (2002) and Siddiqi (2005) for overviews.

Figure 1 About Here

The fraction of customers declined for credit varies by institution and market segment, but around 50% would be typical for a typical "prime" lender such as a major bank. For those customers who are accepted, the bank needs to determine the price (APR) of the loan. This may be based on such aspects as the size of the loan, the term, the channel through which the customer entered, the riskiness of the customer, and other aspects of the product and customer. In some markets the price is "take it or leave it", in other markets there may be the possibility of negotiation. At any rate, the final decision is on the part of the customer whether or not to accept the loan at the price offered. The fraction of customers accepting the final offer depends again on market: the percentages shown in Figure 1 were derived from an unsecured lender in the UK.

There are many variations on the process shown in Figure 1. The underwriting process may not be yes/no but may involve adjustments to the loan – a customer may be approved for a loan of $10,000 but not one of $20,000. There may also be an intermediary in the process. For example, a mortgage broker may send a customer's application to several different lenders, accepting the best offer from among those extended by the lenders. However, the general structure of an initial application, followed by a decision on the part of the lender whether or not to accept the application and, if so, at what price, followed by a final customer decision to take or not take the loan, is almost universal.

2.1 Segmentation

Most lenders do not offer the same price of credit for all their products to all customers through all channels. Rather, they differentiate prices according to product characteristics, customer characteristics, and channel:

Product differentiation. Most lenders offer a range of different products. Perhaps the most prolific in this regard is the American mortgage market in which a lender will often offer a wide range of fixed-rate products, variable-rate products, and combinations of the two (e.g. an introductory fixed-rate with conversion to variable-rate after a specified period). While mortgage lenders probably offer the widest range of products, even a relatively simple product such as an auto loan has many different variations including:

Term: e.g. 60 month versus 72 months,

Size of loan: e.g. $10,000 versus $15,000,

Loan-to-Value ratio[2] *(LTV)*: e.g. a loan for 85% of the purchase price of a car versus one for 95%.

Type of transaction: Finance vs. re-finance.

Lien Position

Nature of collateral: New car vs. 1-year old, 2-year old, etc.

Customer characteristics. Lenders have far more opportunity to differentiate based on customer characteristics than most consumer markets. Typical dimensions used for segmentation include:

Credit Score

Geography,

Relationship: New vs. existing customer. For existing customers, number of accounts with the lender, tenure, etc.

Channel. Lenders often offer their products through a variety of channels. A bank offering unsecured personal loans in the UK may offer loans through its retail branches, on-line through the Internet, and through a call center. In addition it might offer "white label" loans through a number of different affiliates such as grocery or retail stores.

[2] Loan-to-Value ratio denotes the size of a loan as a fraction of the underlying collateral value. Thus, a $90,000 mortgage secured with property worth $100,000 has a 90% LTV.

We use the term *pricing segment* to refer to any combination of product characteristics, customer characteristics and channel for which a lender can set a different price. As an example, assume a mortgage lender ranks applicants on a scale from 0 to 100 based on credit quality. Then, the lender must decide what rate to offer a customer with a credit score of 90 who is applying for a 60 month, £140,000 mortgage at an 80% LTV through a branch. The rate offered to this customer could be different than the rate offered to a customer with a credit score of 80 who is applying for a £180,000 mortgage at 90% LTV through a broker. The number of pricing segments used by a lender can range to a few hundred to a few hundred thousand or even millions. The primary challenge of pricing optimization is to find the rate for each segment that best meets the lender's goals.

2.2 Risk Based Pricing

The most common approach to pricing consumer credit in the United States is *risk-based pricing* (Edelberg, 2006). Under risk-based pricing, accepted applicants are classified into two or more *risk bands* based on the lender's judgment of their riskiness. The rate in each band is computed by $r_i = r_c + m + \ell_i$ where r_i is the rate offered to a customer in risk band i, r_c is the cost of capital, m is a standard margin and ℓ_i is a factor that varies by risk band. Typically, ℓ_i is chosen to compensate for the expected *loss rate* associated with customers in that risk band. While, in theory, a lender could offer a different risk-based price to each customer, due to operational and practical considerations, most lenders group customers into a relatively small number of bands – often five to ten.

Different lenders use different approaches to evaluating risk so that a customer judged risky by one bank may be judged much less risky by another. In addition, different lenders use different risk-bands with different break points. This differential treatment of risk is one reason why customers may be charged different rates for the same loan by different lender.

The rationale for risk-based pricing is clear: a riskier customer should pay a higher price in order to compensate for the higher probability of default and the associated cost to the lender. However, there are other factors at play as well. Empirical evidence has shown that riskier customers tend to be less price-sensitive than low risk customers which provides an additional rationale for charging them a higher rate. However, the riskiness of a loan is not independent of the rate. As discussed in Section 3.3, charging a higher rate for a particular type of loan to a particular group of customers will result in a higher rate of default. The interplay of price and risk leads to much of the complexity in setting optimal rates for credit.

2.3 Decentralized Pricing and Discretion

Almost all consumer lenders generate a price list that specifies rates for all pricing segments. However, in many cases, the rate on the price list is not necessarily the rate that is quoted to the customer. Rather, the rate quoted may be determined by a local agent of the lender during a process of negotiation with the customer. For example in Canada, the local lending agent in a branch typically has the authority to offer a lower rate to a customer for a mortgage if he believes it is necessary to acquire or retain the customer. A customer purchasing a car through a dealership in the United States can negotiate a rate lower than the "list rate".

Local pricing discretion is the norm in many lending markets. An Oliver Wyman study (2012) noted that local discretion was allowed in more than 50% of the unsecured loans and more than 70% of secured loans offered by major European banks. In Canada, more than 80% of mortgages and somewhere between 30 and 40 percent of loans and lines are originated or renewed at discretionary prices that differ from the corresponding list prices (Nomis, 2012).

This combination of a centralized price list with local adjustments within specified limits is similar to the process used in many business-to-business settings. In both business-to-business pricing and consumer lending, three rationales for allowing local adjustments are often given:

1. *Negotiation and price-adjustment are customary in the market.* Sales staff argue that attempting to go to a "take it or leave it" price without possibility of negotiation would alienate customers and drive them to the competition.

2. *Customers perceive value in receiving a discount.* A customer might be more willing to accept a loan at 3.5% if he believes that he is getting a 100 basis point discount from a loan that would normally cost 4.5%.

3. *Sales agents have information about individual customers that can be used to better target individual prices.* The sales agent at an auto dealership may judge that a particular customer is extremely eager to buy a car and may therefore be willing to accept a higher rate while another is more reluctant and may require a lower rate to close the deal.

Regarding the first rationale, Phillips (2012a) discusses the role of custom and history in determining the process by which prices are set in a market and Gelber (2008) argues that the process of price negotiation in automobile sales has survived due to custom. Regarding the second, the idea that "perceived discount" stimulates sales has received considerable empirical and experimental support – see Özer and Zheng (2012) for a survey. The third rationale has received less attention in the financial-services literature. Phillips et al. (2012) present empirical evidence that local lenders in auto dealerships do, on the average, negotiate rates that come closer to customer willingness-to-pay than the list rates. This would indicate that, in this case at least, a decentralized element of pricing adds value to the lender.

A decentralized component to the pricing process may introduce *endogeneity* into price-response estimation – that is, the willingness-to-pay of a customer may influence the price offered. The typical approach to estimating price-response in a customized pricing setting is to apply Bernoulli regression to the win-loss history based on some assumed underlying price-response function such as probit or logit (Agrawal and Ferguson, 2007; Phillips, 2012). This approach assumes that the prices charged are a function of the product, segment, and customer characteristics recorded in the data plus a random residual. However, when customized pricing has a decentralized component, it is likely that the residual will be correlated with take-up. This can be due to the lender's observation of individual characteristics or of the local competitive environment. Correlation of the residual with the target variable is known as *endogeneity*. Phillips et al. (2012) found that such endogeneity was strongly present in a data set of approved loans from a nationwide indirect lender (i.e., one that delivered loans though dealerships) but absent in a data set of approved loans from an on-line lender. They provide evidence that failure to adjust for such endogeneity can lead to significant underestimation of price-sensitivity in consumer lending.

3 Optimizing Consumer Credit Prices

The primary pricing optimization problem facing a lender is to determine what rate (or APR) to charge for each pricing segment at each time. As noted above, a lender may need to set prices for thousands, hundreds of thousands, or even millions of pricing segments. These prices needed to be updated periodically in response to market changes, changes in the cost of funds, or competitive actions. For mortgage lenders, such updates can occur daily or even twice daily. For other forms of consumer credit, prices are routinely updated on a weekly or monthly basis with ad hoc updates in response to changes in the cost of funds or competitive actions. The number of prices that need to be maintained and updated, the complexity of the problem, and the frequency of changes makes consumer lending a prime candidate for automated pricing optimization software and, indeed, such software is offered by companies such as Earnix and Nomis Solutions.

Optimizing the price for a consumer loan involves trading off the probability that a loan will fund with the incremental profitability to the lender if the loan does fund. Increasing the APR for a prospective loan reduces the probability that the customer will accept the loan but increases profitability if the customer does accept. In lending, the incremental profit from a loan is not a linear function of price as it is in most industries but a more complex function of rate. In this section, we discuss the calculation of incremental loan profitability and specify the pricing optimization problem faced by a lender.

3.1 Incremental Loan Profitability

In most consumer markets, the incremental profit from a sale is simply the price minus the unit cost. Furthermore, the incremental profitability can be calculated at the time of sale. However, calculating incremental profitability for consumer credit is more complicated. The primary "price" of a loan is its *Annual Percentage Rate* (APR). However, the profitability of a loan is a non-linear function of its APR. In addition, the profitability of a loan will depend on the post-acquisition behavior of the borrower: whether or not she defaults, whether or not she repays early, and the amount of fees that she pays. In the case of revolving accounts such as a line-of-credit or a credit card, profitability will also depend on the extent to which the line is utilized and the speed with which it is repaid. This means that, at the time a loan is funded, its actual profitability is unknown and must be considered as a random variable. Furthermore, loan payments occur take place periodically over the future. This means that the lender needs to consider the present value of a future uncertain stream of payments.

Several different measures of loan profitability are used by lenders including *Net Interest Income* (NII), *Net Income After Capital Charge*, *Return on Equity* (ROE), and *Return on Risk Adjusted Capital* (RAROC). We use *Present Value of Net Income* (PVNI) as the appropriate measure of incremental profitability for use in pricing optimization. Specifically, PVNI measures the present value of the expected after-tax *incremental* contribution of a loan or line to the overall profitability of the lender, consistent with the basic idea of pricing optimization. PVNI can be broken into following components:

- The *Present Value of Net Interest Income.* This is the present value of the *net* interest received during the duration of the loan net of interest (or opportunity cost) that the lender pays on the principal.
- The *Present Value of Expected Non-Interest Income* including various fees.
- The *Present Value of Operating Expenses* including the cost of mailing statements, processing payments, etc.

In this paper, we focus on the influence of rate on the Present Value of Net Interest Income. In general, Net Interest Income is the key component of loan profitability. In addition, while fees can be a significant source of revenue in some settings (particularly credit cards), it is a reasonable assumption that they do not depend on the rate. In addition, operating expense for most loans are also not dependent on the rate. In addition, they are quite small relative to the level of net interest income.

Consider a *simple loan* with a fixed APR. In this case, the loan is repaid in equal periodic payments with each payment equal to:

$$p(P, r, n) = Pr\left[\frac{(1+r)^n}{(1+r)^n - 1}\right], \tag{1}$$

where P is the starting principal (the amount borrowed), r is the rate, and n is the term. We assume that loans have a monthly payment schedule. Thus, for a five year loan, $n = 60$ and $r \approx APR/12$ would be the monthly interest rate of the loan.

Assume that the lender pays a monthly rate r_c for capital and has an internal discount rate r_d. r_c can be interpreted as the rate at which the lender borrows money (say from the Federal Reserve) or, alternatively, the rate of return on a risk-free investment. In this case, the present value of net interest income from a riskless loan is:

$$\begin{aligned} PVNII_R(P, r, n) &= P\left[\sum_{i=1}^{n}\left(\frac{1}{(1+r_d)^i}\right)\left(\frac{r(1+r)^n}{(1+r)^n - 1} - \frac{r_c(1+r_c)^n}{(1+r_c)^n - 1}\right)\right] \\ &= P\left(\frac{r(1+r)^n}{(1+r)^n - 1} - \frac{r_c(1+r_c)^n}{(1+r_c)^n - 1}\right)\left(\frac{(1+r_d)^{n+1} - (1+r_d)^n}{(1+r_d)^n - 1}\right). \end{aligned} \tag{2}$$

The subscript R in $PVNII_R(P, r, n)$ indicates that this is the present value of net interest income for a it riskless loan – i.e., one that is certain to go to term. However, at the time of funding, the lender faces the possibility that the borrower may default – that is, stop making payments – at some point during the life of the loan. Let p_i be the probability that the lender defaults in period i. Then, the probability that the borrower will make payment i is given by:

$$s_i = \prod_{j=1}^{i}(1 - p_i) \quad \text{for} \quad i = 1, 2, \ldots n.$$

In this case, the expected value of future net interest income is:

$$PVNII(P, r, n) = P\left[\sum_{i=1}^{n}\left(\frac{1}{(1+r_d)^i}\right)\left(\frac{s_i r(1+r)^n}{(1+r)^n - 1} - \frac{r_c(1+r_c)^n}{(1+r_c)^n - 1}\right)\right] \tag{3}$$

The lender is obliged to repay the principal even if the borrower defaults. Because of this, a high-risk loan can have a negative expected present value. In this case, an expected-profit maximizing lender should either raise the rate it charges for the loan or, if this is not possible, not extend credit to this customer. In fact, as discussed in Section 3.3, for a sufficiently risky group of borrowers, it may be the case that there is no rate at which a loan can be made profitable and a profit-maximizing lender would not lend to those borrowers.

Consider the case in which $r_d \approx 0$. This is a reasonable assumption because the discount rate r_d is being applied over and above the risk-free cost-of-capital, r_c. Also, note that $r(1+r)^n/[(1+r)^n - 1] \approx$

1 for large values of n. This motivates the approximation:

$$\begin{aligned} PVNII(P,r,n) &\approx P\Big[r\sum_{i=1}^{n} s_i - nr_c\Big] \\ &= Pn(r-r_c) - Pr\big[\sum_{i=1}^{n}(1-s_i)\big] \\ &= Pn(r-r_c) - (1-s_n)\times Pr\big(\sum_{i=1}^{n}(1-s_i)/(1-s_n)\big) \\ &= Pn(r-r_c) - PRD \times LGD \end{aligned} \tag{4}$$

where $Pn(r-r_c)$ is an approximation of the profitability of the risk-free loan, PRD stands for *probability of default* and LGD stands for *loss given default*. The approximation in Equation (4) has the obvious advantage of being linear in rate. In addition, it does not require the full vector of default probabilities $(p_1, p_2, \ldots, p_n)$ but only the probability that the borrower will default and the loss given she defaults. These quantities may be more easily observable (and stable) given past observations than the full default probability vector. For this reason, Equation (4) is a commonly used approximation for the Present Value of Net Interest Income.

Equation 3 gives the present value of net interest income in the case in which a lender either repays the loan on schedule or defaults with the remaining balance being lost. In actuality, there are a number of additional possibilities. For many loans, the borrower may repay the principal early – either with or without a penalty, depending upon the loan agreement. Additionally, the lender may be able to recover some portion of the balance from a loan in default either through its own collection actions or by selling the defaulted loan to a third party at "pennies on the dollar".

Incorporating all of these possibilities adds considerable complexity into the calculation of the expected PVNII. However, from the point of view of optimizing the interest rate, the most important fact is that the expected PVNII is, under most reasonable assumptions, a continuous, increasing, log-concave function of the rate. As discussed in the next section, this guarantees the existence and uniqueness of an expected-profit maximizing rate.

The computation of expected net interest income in this section has been based on the assumption that the lender will retain the loan on her balance sheet – that is, she will not sell the loan in the secondary market. As noted in Section 1, in some markets a majority of loans are not held by the original lender but are sold – often several times over – in the secondary market. If the secondary market for loans has perfect information, all participants are expected profit maximizers, and there are complete arbitrage opportunities, then the fact that a borrower plans to sell a loan on the secondary market should make no difference to the price (or to the lender's underwriting policy). In practice, these conditions do not always hold. There is evidence that increasing competition for market share led lenders to relax underwriting policies for loans prior to the 2008/2009 crisis and that this increase in risk was not incorporated in the secondary market (Simkovic, 2011). To the extent that risk perceptions vary between buyers and sellers, a lender who plans to securitize and sell his loans may charge a different rate and use a different underwriting policy than one who plans to keep the loans on the book.

3.2 Optimization

As noted in Section 2.1, lenders typically define pricing segments based on combinations of product, channel, and customer type and offer different rates to each combination. There are two reasons to charge different prices to different segments. First of all, incremental cost and risk vary by pricing segment: customers with lower FICO scores are more risky than those with higher FICO scores. Loans with a high LTV are riskier than loans with a lower LTV. In addition, borrower price sensitivity is different for different segments: borrowers applying for larger loans are more price-sensitive than those applying for smaller loans and higher risk customers tend to be less price sensitive than lower risk customers. The price optimization problem facing the lender is to determine which price to charge to each pricing segment in order to maximize expected total profitability. For a lender offering only simple loans, this problem can be written:

$$\begin{aligned} \max_r \quad TR(r) \quad &= \sum_i D_i \bar{F}_i(r_i)[PVNII(P_i, r_i, n_i) + v_i] \\ \text{s.t.} \quad\quad &\quad r_i \geq 0 \end{aligned} \tag{5}$$

where:

- $N \geq 1$ is the number of pricing segments.
- $r = r_1, r_2, \ldots, r_N$ is the vector of rates offered to each pricing segment.
- $D_i > 0$ is the total demand (in number of loans) in pricing segment i.
- $P_i > 0$ is the average loan size in pricing segment i.
- $n_i \geq 1$ is the typical term in pricing segment i.
- $PVNII(P, r, n)$ is the present value of net interest income calculated according to Equation 3.
- v_i is the present value of expected non-interest items (such as fees and operating cost) in segment i.
- $\bar{F}_i(r_i)$ is the fraction of successful applicants who will accept the loan as a function of the rate, r_i.

In this formulation, we have expressed the demand for loans in segment i as $D_i\bar{F}(r_i)$ where D_i is the demand in pricing segment i from applicants who would be accepted by the lender and $\bar{F}_i$

is a function such as the probit or logit that can be interpreted as the c.c.d.f. of a probability distribution.

Note that the optimization problem (5) is valid only if there is a different pricing cell for each term, and size of loan. Otherwise the expected PVNII for a cell may be different from the PVNII based on the expected values of term and principal size due to Jensen's inequality. This would seemingly impose the requirement for a high level of disaggregation in pricing segments. In practice, the problem can be avoided by adding artificial pricing segments or modifying the objective function to incorporate the mean PVNII for each cell as a function of r.

The optimization problem (5) is an example of the so-called *customized pricing* problem discussed by Phillips (2005, 2012a). The problem as written is separable in r_i and it is well-known that, when the incremental profit term is linear and $\bar{F}$ corresponds to an *increasing failure rate* (IFR) distribution that a unique solution to (5) is guaranteed to exist. However, in the case of consumer credit, $PVNII(P, r, n)$ is not linear in r. However, $PVNII(P, r, n)$ is log-concave in r and we show that this is sufficient for existence and uniqueness of a solution. Log-concavity is a weaker condition than concavity – all concave functions are log-concave, but the converse is not true. More discussion of log-concavity can be found in Chapter 3.5 of Boyd and Vanderberghe (2010).

Proposition 1 *Let $\rho(x) = D\bar{F}(x)$ be a continuous decreasing price-response function whose associated density function $f(x)$ satisfies the increasing failure rate property. Let $g(x)$ be a continuous increasing strictly log-concave incremental profit function with $g(x_0) = 0$ for some $x_0 \geq 0$ and $\lim_{x\to\infty} g(x) = \infty$. Then, $\pi(x) = \rho(x)g(x)$ has a unique maximizer $x^* \geq 0$*

The proof is in the appendix. Proposition 1 is a generalization of the well-known result that $D\bar{F}(p)(p - c)$ has a unique maximizer $p^* \geq c$ if $\bar{F}(p)$ is IFR (Ziya, et al., 2004).

Proposition 2 *The optimization problem (5) is guaranteed to have a unique solution when every $\bar{F}_i$ is IFR and $PVNII(P, r, n)$ is calculated according to Equation (3).*

The proof is in the appendix. The general idea is that profitability is proportional to $\bar{F}_i(r_i)PVNII(P_i, r_i, n_i)$ where $PVNII(P_i, r_i, n_i)$ is log-concave. This, along with the IFR condition on $\bar{F}_i(r_i)$ is sufficient for quasiconcavity of each $\pi_i(r)$.

Except for the requirement that rates be non-negative, Problem (5) is unconstrained. In practice, lenders impose a number of constraints on their rates. Some of these constraints are driven by regulations: for example, usury laws specify a maximum APR. Other constraints may be due to marketing or operational considerations or the desire to pursue different goals in different pricing segments – e.g., to pursue increased market share in some segments versus increased profitability in others. Typical constraints include:

- *Bounds.* Most lenders apply bounds such that $p_i^+ \geq p_i^* \geq p_i^-$ for some specified $p_i^+ \geq p_i^- > 0$. Bounds may used to guarantee that a new price does not deviate too greatly from a previous one under the belief that some price stability from period to period is desirable. Additionally, the price-response curve may only be predictive in some range representing historical prices. Finally, regulations and business practices may define a cap on the maximum lending rate that can be quoted.

- *Monotonicity Constraints.* In many cases, lenders want prices to be monotonic along certain dimensions. Most lenders want to charge less risky borrowers lower rates for the same loan than more risky borrowers. In the same vein, most lenders want to charge lower rates for larger rates, all else being equal.

- *Rate Relationships.* Lenders typically require other relationships among rates. For example, a lender might want on-line rates to be no greater than the rates offered through a branch in order to support the on-line channel.

- *Business Performance Constraints.* It is quite common that a lender may wish to maximize expected contribution as long as volume does not drop below a certain level – either for all new lending as a whole for certain segments or combinations of segments. This would imply a constraint of the form:

$$\sum_{i \in I} D_i \bar{F}_i(r_i) \geq B$$

  where B is the minimum total origination volume required from the set of segments I.

- *Risk Constraints*: It is also common that a lender may not want to take on a portfolio that includes too much risk. In this case, the lender might impose a constraint of the form:

$$\sum_i D_i \bar{F}_i(r_i) \ell_i \leq L \sum_i D_i \bar{F}_i(r_i)$$

  where ℓ_i is the expected loss rate for segment i and L is the accepted loss rate for the portfolio.

The constraints described above are straightforward to implement and generally result in a convex feasible region for r. In this case, standard non-linear optimization approaches work well and the property of a unique maximum to the objective function (5) is preserved. However, there are plausible business constraints that can result in non-convex feasible regions. For one example, it may that the lender wishes to impose a "value at risk" (VaR) constraint on the portfolio. Such a constraint would take the form of "the probability of a loss rate higher than 10% for the portfolio cannot exceed .05." This would we require imposition of a complex non-linear (and generally non-convex) constraint on the portfolio, particularly if losses are correlated among segments.

We note that the imposition of business performance constraints usually reflects conflicting objectives on the part of a lender. When asked for what her goal for the loan portfolio, a bank executive is likely to say something like "I want to maximize originations and net income while minimizing risk." Of course, these goals conflict with each other. For this reason, many pricing optimization software packages enable users to generate efficient frontiers that illustrate the tradeoffs between origination volume and net income or net income and risk. Figure 2 shows an efficient frontier between balances originated and the present value of net interest income (PVNII). Points on the efficient frontier represent combinations of the two measures that the lender could attain by feasible combinations of prices for different segments. Points outside the efficient frontier are unobtainable. The expected outcomes from prices currently in place are shown as Point A. Point B represents the outcome of a set of prices that maximizes PVNII while maintaining the current level of originations. Point C is the maximum balances that could be originated while maintaining the current PVNII. Points between B and C on the frontier represent the outcomes of policies that generate both more income and higher balances than the current prices in place. Displaying the efficient frontier to management enables them to understand the tradeoff between balances and net interest income and enables them to choose a set of prices that achieves their objectives considering these tradeoffs. A discussion of the use of efficient frontiers in pricing and revenue management can be found in Phillips (2012b).

Figure 2 About Here

3.3 Price-Dependent Risk

It is well-known among lenders that the riskiness of a loan can depend upon its price (or rate) – all else equal, higher rates or fees will lead to higher loss rates. Both Phillips and Raffard (2011) and Oliver and Thaker (2012) present statistical evidence of the phenomenon. There are four reasons commonly given for the phenomenon of *price-dependent risk*:

1. *Adverse Selection.* Adverse selection was first identified by Akerlof (1970) as a phenomenon in markets where either the buyer or the seller possesses *adverse private information* that is not known to the other party. A lending example would be a customer who just received a bad job review and, as a result, believes that she is likely to be laid off. This might prompt her to accept a high-priced loan because of the concern that she might not be able to qualify for a loan if she loses her job.

2. *Financial Sophistication.* Even if borrowers do not possess adverse private information, it is possible that borrowers who appear *ex ante* identical differ in their levels of financial sophistication. If this is the case, than among two borrowers with the same credit history, the one that is the less sophisticated would probably be more likely to accept a higher APR loan without shopping for alternatives. However, this same lack of financial sophistication would

mean that this borrower is more likely to mismanage his finances in the future and therefore default on the loan.

3. *Affordability.* The ability of a borrower with limited resources to repay a loan might depend upon the monthly payment. Such a borrower might be able to make payments on a loan with 7% APR but might find that they cannot make the monthly payment on a loan with a 21% APR. To a lender, this would manifest itself as price-dependent risk.

4. *Winner's Curse.* Lenders differ in the information and methodologies that they use to estimate the riskiness of a prospective loan. Some lenders are more sophisticated and more capable of distinguishing the riskiness of various groups of prospective borrowers than others. This can lead to a "winner's curse" situation in which the lender who is least sophisticated wins the highest proportion of risky loans. If the less-sophisticated lender raises his rates relative to the competition, he will continue to retain the riskier customers while losing the less risky ones. His loss rate would therefore increase.

Phillips and Raffard (2011) showed that a sufficient condition for price-dependent risk to exist in a population is for high-risk customers to be less price sensitive than low-risk customers. Such a difference in price-sensitivity could be a consequence of adverse selection, differential financial sophistication, or the winner's curse. To see how this would lead to price-dependent risk, consider a population that consists of two types of customer – "goods" who do not default and "bads" who default with certainty. For any given price, r, the demand from goods is $d_g(r)$, the demand from bads is $d_b(r)$ and the total demand is $D(r) = d_b(r) + d_g(r)$. Then, the loss rate at any price r is:

$$LR(r) = \frac{d_b(r)}{D(r)}$$

and

$$LR'(r) = (e_g(r) - e_b(r))\left[\frac{d_b(r)d_g(r)}{D(r)^2 r}\right] \tag{6}$$

Where $e_i(r)$ is the own-price response elasticity for customers of type $i = b, g$ defined as:

$$e_i(r) = \left|\frac{d_i'(r)p}{d_i(r)}\right|.$$

Since the bracketed term in equation 6 is always positive, the sign of $LR'(r)$ will depend upon the relative values of $e_g(r)$ and $e_b(r)$. For the reasons noted above, we anticipate that $e_g(r) > e_b(r)$: low-risk (good) customers are more price elastic than high-risk customers, so $LR'(r) > 0$ and risk will increase with price. Furthermore, according to this model, we would anticipate the risk to be strongest among sub-prime customers. Rewrite equation 6 as:

$$LR'(r) = (e_g(r) - e_b(r))LR(r)(1 - LR(r))/r. \tag{7}$$

From 7, it is clear that if a population consists entirely of “goods” ($LR(r) = 0$) or entirely of “bads” ($LR(r) = 1$), the loss rate will not change with rate. For any value of r, $LR'(r)$ is maximized when $LR(r) = .5$. Since the expected loss rate is typically well below 50% even for highly sub-prime populations, we would expect the change in risk as a function of the change in price to be greatest in sub-prime populations. This is, indeed what is seen: the effect of price on risk appears to be negligible in low risk “super-prime” populations, but can be quite significant in “sub-prime populations”.

If price-dependent risk is expected to be significant in a population, then the objective function of the pricing optimization problem in (5) needs to be modified to incorporate the effect. Phillips and Raffard (2011) show that ignoring price-dependent risk will result in rates that are too high relative to the optimal rates incorporating price-dependent risk. Oliver and Oliver (2012) present an approach using differential equations to set prices and cutoff limits in the presence of price-dependent risk when the goal of the lender is to maximize Return on Equity.

4 Current Challenges

The pricing of consumer credit has evolved considerably over the last two decades. Prior to the widespread adoption of credit scores, loans were usually priced on a “one size fits all” basis. Once credit scores had become widely available, most lenders moved to some form of risk-based pricing, often supplemented with ad hoc adjustments to meet competition in different markets. More recently, many lenders have begun to adopt systems that explicitly optimize their pricing – either developing the capabilities in-house or licensing systems from third-party developers such as Earnix or Nomis Solutions. As in other industries, companies that have adopted pricing optimization systems have reported substantial improvements in revenue and profitability (Britting, 2006). However, adoption of pricing optimization systems among consumer lenders is much lower than among their counterparts in the passenger airline industry and the issues associated with setting optimal rates for consumer loans has received less attention from researchers. In this section, we discuss some of the important current issues.

4.1 Acquisition versus Retention Pricing

This paper has focused on *acquisition pricing* – determining the price for a customer who is applying for a loan. However, for many loans, the lender will have the opportunity to change the APR (or other components of the price such as fee rates) after acquisition. This flexibility is particularly common in revolving credit products such as credit cards. In this case, the lender also faces the problem of *retention pricing* – how to change the rate over time in order to maximize return.

Loan rates are often changed due to changes in funding cost or borrower behavior. A lender is likely to increase the APR of a customer who has a number of late payments. Experience has also shown that customer price sensitivity decreases with customer tenure. Customers are generally more price-sensitivity at acquisition than after acquisition. And customers with longer tenure are less price sensitive as a group than customers with shorter tenure[3]. The natural consequence is that, all else equal, it is optimal to charge an existing customer more than a new customer – and to charge customers with longer tenure more than newer customers. However, this conflicts with the widespread idea that "loyal" customers should not get higher rates than new customers. Many lenders offer introductory discounts or so-called "teaser rates" as a way to entice new price-sensitive customers with lower rates without antagonizing existing customers. The problem of how to manage both acquisition and retention pricing simultaneously over the borrower life-cycle in a manner that both maximizes return and is acceptable to customers is still very much open.

4.2 Household Decision Making

This paper has primarily considered setting the APR a simple loan which can be fully characterized by its rate, term, and amount. However, lending products are often far more complicated and multi-dimensional. For example, a mortgage may have a fixed rate or a variable rate or some combination of the two. In addition, it will have a number of associated fees associated with origination as well as a schedule of late fees and a possible pre-payment penalty. Lenders often offer borrowers the option of a reduced APR if they pay "points" – essentially a fee determined as a percentage of the balance borrowed. What combinations of prices and loan product features a company should offer in general and which ones they should offer to a particular customer is a decision faced by most lenders. For example, is an applicant for a credit line more likely to accept a \$25,000 line at 5.9% or a \$50,000 line at 6.9%? Which offer is likely to be more profitable for the lender considering acquisition and attrition probabilities and risk?

These questions are similar to those faced by other industries with non-linear pricing and bundling opportunities (Oren, 2012) and those with simultaneous service design and product design issues (Gallego and Stefanescu, 2012). In these situations, the usual approach is to present a menu of options at different prices in a way that encourages customers with different preferences to choose products in a way that maximizes profitability to the seller. The design of such approaches assumes that customers choose the product/price combination that maximizes their expected utility – in other words, that customer choice is rational.

The complexity of financial decisions means that many customers do not make the "optimal" choice among alternatives. That is, they do not choose the loan that minimizes the net present value of their expected future payments after tax. The determination of what type of mortgage

[3]This phenomenon is not unique to consumer lending. A similar behavior pattern has been seen in newspaper subscriptions (Lewis, 2005).

(fixed rate with different terms versus different variable rate options) in the United States will result in the lowest expected total future discounted payments to a borrower is a complex problem which depends, among other things, on the household's current and expected future tax bracket, expected length of time before selling the house, and expected future inflation (Campbell and Cocco, 2003). Households that do not include a PhD economist are unlikely to make the "optimal choice" among the 10 or 20 or more different mortgage options that may be available to them. Anecdotal evidence suggests that consumers use simple heuristics to reduce the number of alternatives. For example, mortgage lenders often hear prospective borrowers say things such as "I will not consider any mortgage with points" – even those this may *ex ante* exclude the best alternative. This is consistent with the finding that customers faced with complex alternatives do not typically choose the "best" one among them and the propensity to choose poorly increases with the number and complexity of alternatives presented (Iyengar, 2011).

Non-rational behavior by consumers presents lenders with a dilemma – to what extent should they take advantage of "deviations from rationality" in designing, selling, and pricing their products? Or, is it the lenders' responsibility to help each borrower find the best product for herself – even if this is may not be the most profitable loan for the lender? Which types of pricing constitute accepted business practice and which types are "predatory" or "abusive"? Regulators are wrestling with these questions across North America and Europe. In most cases there are no widely accepted answers. Better understanding of household financial decision making could lead to both better regulation and better business practice.

4.3 Competition

As with most industries, initial implementations of pricing optimization systems in financial services have not explicitly considered competitive response in generating their recommendations. This apparent omission can be justified in two ways. First of all, it can be argued that past data on pricing and loan demand already contains the effects of competitive response. If competitors respond to our price changes in the future in a way similar to the way that they have done in the past, it could be argued that price-sensitivity estimated from historical data already includes competitive response. A second argument is that, in many consumer lending markets, individual pricing is opaque – the rates that are offered to customers for loans are not visible to competitors since they require an application before a rate is quoted. The level of price opacity varies widely from market to market. In the U.K., unsecured consumer loan rates are widely advertised and comparison shopping is relatively easy through various web-sites. As a result, lenders see high levels of price-sensitivity among customers. In Canada, unsecured loan rates are much more opaque and lenders see much lower levels of customer price-sensitivity.

A unique aspect of consumer loan pricing is the potential for *risk shifting*. Consider two banks, A and B competing for two customer segments – a high-risk segment and a low-risk segment. Under simple risk-based pricing, the two banks will calculate the risk for each segment, and add a margin. Assuming that their estimates of the risk associated with each segment are the same as are their margins, they will charge the same amount to each segment and will split the market 50/50. Assume now, that bank A adopts pricing optimization. As discussed in Section 3.3, low-risk customers are more price-sensitive than high-risk customers. Thus, the likely outcome is that bank A will lower his price for low-risk customers and raise his price for high-risk customers. Assuming bank B does not change his prices, his customer mix will have a higher proportion of high-risk customers and, consequently, higher overall risk.

A related question is the effect on rates and industry profitability when multiple competing lenders adopt pricing optimization. If every lender adopts pricing optimization, will it be "good for the industry" in the sense that lenders will profit? Or is pricing optimization a "zero-sum game" in which lenders invest millions in pricing systems and capabilities only to find that industry profit has not increased?[4] Kuckuk and Phillips (2009) performed a high-level analysis based on data from the UK Unsecured Lending Market. They considered a duopoly in which each competitor either set prices according to one of four methods:

1. Optimal single price.
2. Risk-based pricing with a single optimized margin for all risk bands.
3. Risk-based pricing with optimized prices for all risk bands.
4. Full Pricing Optimization.

They found that, in this simple setting, the strategy in which both competitors choose Full Pricing Optimization is the unique Nash equilibrium ignoring the costs of development and implementation. Furthermore, the total profit earned by both lenders increases monotonically with the sophistication of their pricing approach. This suggests that there is motivation for lenders to improve the sophistication of their pricing and that pricing optimization would be "good for the industry", but far more work needs to be done for the effects of more sophisticated pricing by all of the lenders in a market to be understood.

4.4 Final Word

The use of explicit optimization approaches on the part of lenders to determine the prices to offer for their products is quite new relative to the use of similar approaches in industries such as retail and passenger airlines. While a number of lenders have adopted such approaches, there

[4] Belobaba and Wilson (1997) used simulation to address the question of whether or not increased revenue management capabilities would increase overall profitability for the airline industry.

are still a number of important open issues that have not yet been fully addressed. These include the interaction of risk and pricing, the effects of front-line discretion in pricing, the implications of "non-rational" household financial decision making, and the incorporation of competition into pricing. These represent important opportunities for further research.

5 Appendix: Proofs

5.1 Proof of Proposition 1

We note that a differentiable function $g(x)$ is log-concave if $\ln[g(x)]$ is concave, or equivalently, the ratio $g'(x)/g(x)$ is decreasing (Boyd and Vanderberghe, 2010). The first order necessary condition for any x^* that maximizes $\pi(x) = D\bar{F}(x)g(x)$ can be written:

$$\frac{g'(x^*)}{g(x^*)} = \frac{f(x^*)}{\bar{F}(x*)} \tag{8}$$

The left-hand side of Equation (8) is continuous and, by the log-concavity of $g(x)$, strictly decreasing. Also, $g'(0)/g(0) = \infty$ and $\lim_{x\to\infty}[g'(x)/g(x)] = 0$ as a direct consequence of the assumptions. Because $f(x)$ is IFR, the right-hand side of (8) is increasing and continuous. Therefore, there will be exactly one crossing point satisfying Equation (8). It must be a maximum, because $\pi'(0) > 0$.

5.2 Proof of Proposition 2

We first show that the net present value of net interest income, $PVNII(P, r, n)$ as defined in Equation (3) is a log-concave function of the rate, r for $r \geq 0$ and term $n \geq 1$. Since $PVNII(P, r, N)$ is a linear function of the monthly payment $p(P, r, n)$ as defined in Equation (1), it is sufficient to show that $p(P, r, n)$ is log-concave. Define $x = 1 + r$ and let $g(x) = \ln[p(P, x-1, n)]$. Then,

$$\begin{aligned} g(x) &= \ln(P) + \ln(x-1) + n\ln(x) - \ln(x^n - 1), \\ g'(x) &= \frac{1}{x-1} - \frac{n}{x(x^n-1),} \\ g''(x) &= -\frac{1}{(x-1)^2} + \frac{n[(n+1)x^n - 1]}{x^2(x^n-1)^2}. \end{aligned}$$

For log-concavity, we must have $g''(x) \leq 0$ which will be the case iff:

$$\frac{1}{(x-1)^2} \geq \frac{n[(n+1)x^n - 1]}{x^2(x^n-1)^2}$$

or, equivalently,

$$x^2\Psi_n^2(x) \geq n(n+1)x^n - n \tag{4}$$

where we define

$$\Psi_n(x) = \sum_{i=0}^{n-1} x^i.$$

For $n \geq 1$ and $x \geq 1$, $\Psi_n(x)$ is a convex function of x. Therefore, $\Psi_n(x) \geq (n+1)x^{n/2}$ and

$$x^2\Psi_n^2(x) \geq (n+1)^2x^{n+2} \geq n(n+1)x^n - n,$$

for $x \geq 1, n \geq 1$, which shows that condition 4 holds.

References

[1] Agrawal, V. and Ferguson, M. 2007. "Bid-Response Models for Customized Pricing", *Journal of Revenue and Pricing Management*, **6** 3, 212-228.

[2] Akerlof, G. A. 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* **84** 3. 212-218.

[3] Belobaba, P. P. and Wilson, J. L. 1997. "Impacts of Yield Management in Competitive Airline Environment," *Journal of Air Transport Management*, **3** (1) 3-9.

[4] Boyd, S. and Vandenberghe, L. 2004. *Convex Optimization*. Cambridge, UK: Cambridge University Press.

[5] Britting, B. 2006. *Profit-Based Pricing: Time to Stop Leaving Money on the Table*. Tower Group Report V48:13C. Tower Group. Needham, MA.

[6] Bureau of Labor Statistics. 2011. *2011 Mortgage Market Statistical Annual Vol. 2: Inside MBS and ABS*

[7] Campbell, J. Y. and Cocco, J. F. 2003. *Household Risk Management and Optimal Mortgage Choice The Quarterly Journal of Economics* **118** (4) 1449-1494.

[8] Caufield, S. 2012. "Consumer Credit Pricing," in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[9] Edelberg, W. 2006. "Risk-based Pricing of Interest Rates for Consumer Loans," *Journal of Monetary Economics*. **53**, 2283-2298.

[10] Gallego, G. and Stefanescu, C. 2012. "Service Engineering: Design and Pricing of Service Features," in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[11] Gelber, S. 2008. *Horse Trading in the Age of Cars: Men in the Marketplace*. Johns Hopkins Press. Baltimore, MD.

[12] Iyengar, S. 2010. *The Art of Choosing*. Twelve. New York, NY.

[13] Kuckuk, M. and Phillips, R. L., 2009. "Pricing Optimization and Segmentation for Sustainable Competitive Advantage," Nomis White paper available at www.nomissolutions.com.

[14] Lewis, M. 2005. "A Dynamic Programming Approach to Customer Relationship Pricing," *Management Science* **51** 6, 986-994.

[15] Lewis, M. 2010. *The Big Short: Inside the Doomsday Machine*. W. W. Norton. New York, NY.

[16] Nomis Solutions. 2012. *Discretion Management – Leveraging Pricing Excellence at the Point of Sale*. Executive White paper. Available at www.nomissolutions.com.

[17] Oren, S. 2012. "Nonlinear Pricing," in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[18] Oliver, B. V. and Oliver, R. M. 2012. "Optimal ROE Loan Pricing With or Without Adverse Selection," *Journal of the Operational Research Society* Advance Online Publication doi:10.1057/jors2012.87.

[19] Oliver, R. M. and Thaker, A. M. 2012. "Adverse Selection and Non-Take Inference with Coherent Risk and Response Scoring," *Journal of the Operational Research Society* Advance Online Publication doi:10.1057/jors.2012.3.

[20] Özer, Ö. and Zheng, Y. 2012. "Behavioral Issues in Pricing Management," in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[21] Phillips, R.L. 2005. *Pricing and Revenue Optimization*. Stanford University Press. Stanford, CA.

[22] Phillips, R.L. 2012a. "Customized Pricing," in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[23] Phillips, R. L. 2012b. "Efficient Frontiers in Revenue Management," *Journal of Revenue and Pricing Management* **11** 371-385.

[24] Phillips, R.L. 2012c. "Why are Prices Set the Way they Are?" in Özer, Ö. and Phillips, R. eds, *Oxford Handbook of Pricing Management*. Oxford University Press. Oxford, UK.

[25] Phillips, R., Simsek, A. S., VanRyzin, G. 2012. "Endogeneity and Price Sensitivity in Customized Pricing," Columbia University Center for Pricing and Revenue Management Working Paper 2012-004, http://www7.gsb.columbia.edu/cprm/sites/ default/files/files/ Working_Paper_ 2012_4%281%29.pdf.

[26] Poon, M. 2007. "Scorecards as Devices for Consumer Credit: The Case of Fair, Isaac & Company Incorporated" in *Market Devices.* eds. Callon, M., Muniesa, F. and Millio, Y., Blackwell. London.

[27] Siddiqi, N., 2005. *Credit Risk Scorecards, Developing and Implementing Intelligent Credit Scoring*, Wiley. New York, NY.

[28] Simkovic, M. 2011. "Competition and Crisis in Mortgage Securitization," *Indiana Law Journal* **88**, 2013. Available at SSRN: http://ssrn.com/abstract=1924831.

[29] Sundarajan, R., Bhaskar, T., Sarkar, A., Bal, D., Marasanapalle, J. K., Zmudzka, B. and Bak, K. 2011 "Marketing Optimization in Retail Banking," *Interfaces* **41** 5. 485-505.

[30] Thomas, L. C., 2009. *Consumer Credit Models: Pricing, Profit, and Portfolios* Oxford University Press. Oxford, UK.

[31] Thomas, L.C., Edelman, D.B., and Crook, J. N., 2004. *Readings in Credit Scoring.* Oxford University Press. Oxford, UK.

[32] Ziya, S., H. Aryhan, R. D. Foley. 2004. "Relationships among three assumptions in revenue management." *Operations Research* **52** (5) 804-809.

Provider	APR	Monthly Payment	Total Cost (Interest and Fees)
Zopa	6.8%	£153.46	£584.56*
Sainsbury	7.1%	154.11	547.96
Derbyshire	7.2%	154.32	555.52
Santander	7.6%	155.18	586.48
Clydesdale	7.8%	155.61	601.96
Tesco	7.8%	155.61	601.96
Post Office	8.4%	156.90	648.40
First Direct	8.9%	157.97	686.92
AA	13.9%	168.68	1,072.48
Barclays	15.9%	172.95	1,226.20
HSBC	16.9%	175.09	1,303.24
NatWest	17.9%	177.22	1,379.92
RBS	17.9%	177.22	1,379.92
Bank of Scotland	20.9%	217.70	1,822.36

Table 1: Advertised interest rates (APR) and monthly payments for a 3-year unsecured personal loan of £5,000 for a 55 year old borrower with "good credit" as published by various UK lenders on-line in October, 2012. * Includes a £60.00 application fee. Source: moneyfacts.co.uk/compare/loans, accessed October 2, 2012.

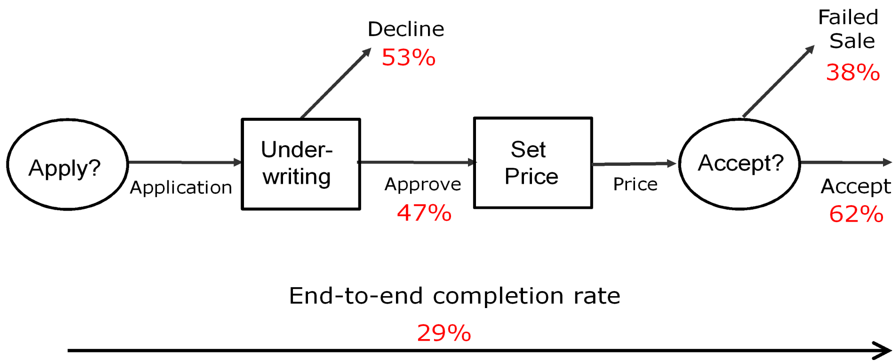

Figure 1: Steps in a typical consumer credit pricing and sales process. Percentages are taken from a UK unsecured personal loan lender.

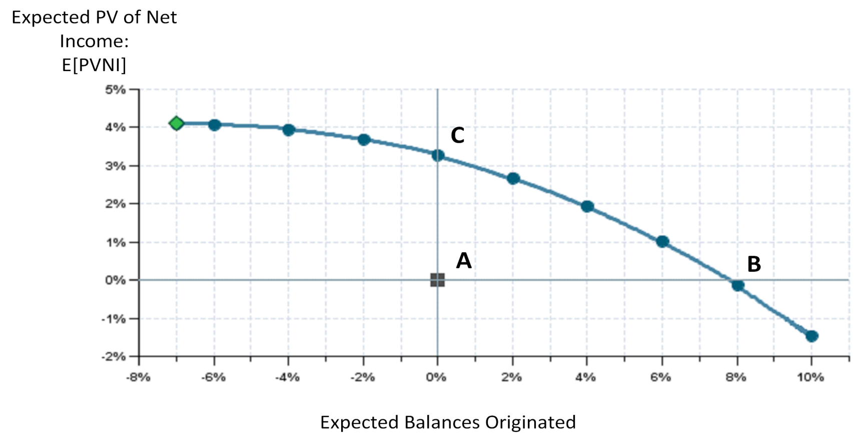

Figure 2: An example efficient frontier between balances and net income. Point C is the current operating point. Points between A and B on the efficient frontier represent combinations of price that result in both higher balances and higher net income.