Optimal Fire Departments:

Evaluating Public Policy in the Face of Externalities

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Introduction

Fires -- and the fire brigades that fight them -- have evoked our interests and served as insightful parables for many generations. The purpose of this paper is therefore to explore the optimal design of a fire department when individuals are allowed to choose the design of their own homes, and when fires can spread from one house to another. A key finding is that the analytical prism used to evaluate an issue is critical: not surprisingly, using an inappropriate prism will often yield an inappropriate answer. *Distorted models produce distorted decision-making*. And on numerous issues involving the fire department, conventional paradigms offer a misleading perspective on the problem.

The basic intuition of the model below is that individuals do not take into account the benefits of building safer houses in reducing fire damages to their neighbors. Their level of care is therefore below the level that would produce the minimum social cost *at any given fire department size.* In other words, the presence of the externality – that homeowner 1 does not take into account the effects of his house design on the welfare of homeowner 2, and vice versa – implies that the individuals do not build safe enough homes relative to the social optimum for any given fire department.

The presence of the externalities also affects the optimal size of the fire department, however. The lower level of individual care makes it desirable to have a larger fire department, since the marginal return to a larger fire department is higher the less safe the homes are. The complication arises because expanding the fire department would encourage homeowners to build even less safe homes, thus exacerbating the externality. The insight is applicable to a wide number of real-world examples, including the use of unobservable security measures to thwart crime or terrorism and the policies adopted by international policy-makers to address financial crises.

The model

To begin with, imagine a town with a fire department and two houses. The town planner's objective function is to minimize the total costs of building the houses and protecting them against fire damage. The two homeowners choose what types of houses to build, and can choose from a variety of models – some of which are more fire-retardant than others. To begin with, we assume that the homeowners are risk neutral.

The optimization problem faced by the town planner is solved in two steps. First, we allow the individuals to choose the types of the houses they build conditional on each other's choices and on the quality of the fire department. Then we allow the town planner to choose the quality of the fire department, given the homeowners' (already derived) reactions to any change in the quality of the fire department.

Given the choices of homeowner 2 and the quality of the fire department, homeowner 1's objective is to minimize the *total* expected costs of building her house and protecting it against fire:

$$\underset{\{\alpha_1|\alpha_2,\ \beta\}}{\text{Min}}\ \text{p}(\alpha_1,\alpha_2)\text{L}(\alpha_1,\alpha_2,\ \beta)+\text{c}(\alpha_1)+\text{f}(\beta) \qquad \{1\}$$

where α_1 is a vector of characteristics of the first house, α_2 is a vector of characteristics of the second house, β is a metric of the quality of the fire department, p is the probability of a fire in homeowner 1's house, L is the loss function conditional on a fire occurring, c is the cost of building the house, and f is the cost of maintaining the fire department (per home). Note that p is a function of both α_1 and α_2 because of contagion effects: since fires can spread from one home to another, the probability of a fire in the first house is affected by the structure of the second house.

The vector α includes such attributes of the house as whether it is constructed of brick or wood; whether it has fire escapes; whether it has fire extinguishers; whether it has fire alarms; and other physical characteristics that affect both the probability of fire and the losses from a fire. Not all of these characteristics are observable to outsiders. We assume that the characteristics are defined such that increases in any element of α result in decreases in the probability of a fire and in the losses resulting from a fire, but increase the cost of building the house:

$$p_{\alpha_1}(\alpha_1,\alpha_2)<0 \qquad \{2\}$$

$$L_{\alpha_1}(\alpha_1, \alpha_2) < 0 \qquad \{3\}$$

$$c_{\alpha_1}(\alpha_1) > 0 \qquad \{4\}$$

Externalities are present because an improvement in the second house decreases the probability of a fire and the losses from a fire in the first house. The explanation for these externalities could be several-fold. For example, the probability of a fire in the first home could depend on the structure of the second home because it is easy for fires to spread from one home to another – and therefore the probability of a fire in the second home (which depends on its structure) affects the probability of a fire in the first home. The *losses* from a fire in the first home could depend on the structure of the second home either because a more severe fire in the second home is more likely to cause more severe damage in the first home, or because of the limited resources of the fire department. In particular, if the fire department is less effective at fighting two fires at once, then the structure of the second home will affect the damage from a fire in the first home because of the possibility that the fire department will have to fight two fires at once (which is a function of both $\alpha_1$and α_2). Hence:[2]

$$p_{\alpha_2}(\alpha_1, \alpha_2) < 0 \qquad \{5\}$$

$$L_{\alpha_2}(\alpha_1, \alpha_2) < 0 \qquad \{6\}$$

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[2] These conditions are natural, but not inevitable. For example, it is conceivable that there are some improvements that benefit my house but worsen my neighbor's risk. One example is putting fire doors within my house, and offsetting the additional cost by using wood shingles on the roof. That combination could improve the fire safety of my house, but expose the neighbors to more risk. (Similarly, in the context

We further assume that improvements in the quality of the fire department, as measured by β, reduce the losses from a fire but increase the costs of maintaining the fire department. β reflects the quality and number of the department's firefighters, as well as its fire trucks, radio equipment, and other life-saving equipment. References below to a "larger" or "smaller" fire department refer to increases or decreases in β respectively. These assumptions imply:

$$L_{\beta}(\alpha_1, \alpha_2, \beta)<0 \qquad \{7\}$$

$$f'(\beta) > 0 \qquad \{8\}$$

We also assume that there are diminishing marginal returns to improvements in the house and the quality of the fire department, non-decreasing marginal costs of improvements to the house and the fire department, and positive cross-partial derivatives (so that the marginal reduction in the probability of fire or the extent of loss from building a safer home is smaller the larger the fire department, or the safer the other home):

$$p_{\alpha_1\alpha_1}(\alpha_1, \alpha_2) > 0 \qquad \{9\}$$

$$L_{\alpha_1\alpha_1}(\alpha_1, \alpha_2, \beta) > 0 \qquad \{10\}$$

$$L_{\beta\beta}(\alpha_1, \alpha_2, \beta) > 0 \qquad \{11\}$$

$$c_{\alpha_1\alpha_1}(\alpha_1) \geq 0 \qquad \{12\}$$

$$f''(\beta) \geq 0 \qquad \{13\}$$

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of crime or terrorism, it is possible that observable security measures that I undertake merely displace more crime or terrorism onto you.)

$$p_{\alpha_1\alpha_2}(\alpha_1, \alpha_2) > 0,\ L_{\alpha_1\alpha_2}(\alpha_1, \alpha_2,\ \beta) > 0,\ L_{\alpha_1\beta}(\alpha_1, \alpha_2,\ \beta) > 0 \qquad \{14\}$$

Then the first-order conditions for the minimization problem defined by {1} are given by:[3]

$$p_{\alpha_1}(\alpha_1, \alpha_2)L(\alpha_1, \alpha_2, \beta) + p(\alpha_1, \alpha_2)L_{\alpha_1}(\alpha_1, \alpha_2, \beta) + c_{\alpha_1}(\alpha_1) = 0 \qquad \{15\}$$

The second homeowner will face a parallel minimization problem. We assume that homeowners 1 and 2 are symmetrical, so that in equilibrium, $\alpha_1=\alpha_2$. Then {15} and its equivalent for the second homeowner will implicitly define an $\alpha*$, where:

$$\alpha_1*=\alpha_2*=\Omega(\beta) \qquad \{16\}$$

where $\Omega'(\beta) < 0$, since totally differentiating {15} yields:

$$\frac{d\alpha}{d\beta} = -\frac{p_{\alpha_2}L_\beta + L_{\alpha_2\beta}}{p_{\alpha_1\alpha_1}L + p_{\alpha_1\alpha_2}L + 2p_{\alpha_1}L_{\alpha_1} + pL_{\alpha_1\alpha_1} + pL_{\alpha_1\alpha_2} + p_{\alpha_2}L_{\alpha_1} + p_{\alpha_1}L_{\alpha_2} + c_{\alpha_1\alpha_1}} \qquad \{17\}$$

which is unambiguously negative given {2}, {3}, {7}, {9), {12}, and {14}. In other words, the better the fire department, the less homeowners will invest in making their homes fire-proof. We assume $\Omega''(\beta)$<0, which imposes restrictions on the direct and cross-partial derivatives, to ensure appropriate second-order conditions. Given $\Omega(\beta)$, the town planner optimizes the size of the fire department by choosing β to minimize the total costs to society:

$$\text{Min } \{\beta\} \qquad p(\Omega(\beta), \Omega(\beta))L(\Omega(\beta), \Omega(\beta), \beta)+c(\Omega(\beta))+f(\beta) \qquad \{18\}$$

The first-order conditions for this minimization problem are:

$$p_{\alpha_1} L\Omega'(\beta) + p_{\alpha_2} L\Omega'(\beta) + L_{\alpha_1} p\Omega'(\beta) + L_{\alpha_2} p\Omega'(\beta) + L_{\beta} p + c_{\alpha_1} \Omega'(\beta) + f'(\beta) = 0 \qquad \{19\}$$

{19} implicitly defines a β* that minimizes the total costs to society of building houses and protecting against fires (given that the individuals build their own houses and decide what features to include).

Theorem 1 (rescue risk in the presence of externalities):

At β*, $L_\beta p + f'(\beta) < 0$**. In the presence of externalities, it will always appear at the equilibrium that a higher-quality fire department is a good social investment. But the town planner purposefully does not expand the fire department, since doing so would encourage individuals to build less safe homes and thus exacerbate the social costs of the externality.**

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[3] The second-order conditions for a minimum obtain from {9}, {10}, and {12}, along with {2} and {3}.

Proof: From {19}, we know that at $\beta*$ (determined in the presence of externalities),

$$L_{\beta}p + f'(\beta) < 0 \text{ if } p_{\alpha_1}L\Omega'(\beta) + p_{\alpha_2}L\Omega'(\beta) + L_{\alpha_1}p\Omega'(\beta) + L_{\alpha_2}p\Omega'(\beta) + c_{\alpha}\Omega'(\beta) > 0.$$

Dividing through by $\Omega'(\beta)$, which is negative, we see that the result obtains if:

$$p_{\alpha_1}L + p_{\alpha_2}L + L_{\alpha_1}p + L_{\alpha_2}p + c_{\alpha_1} < 0 \qquad \{20\}$$

From {15}, we know that $p_{\alpha_1}L + pL_{\alpha_1} + c_{\alpha_1} = 0$ at $\alpha*$. Therefore, since $p_{\alpha_2} < 0,\ L_{\alpha_2} < 0$, $p_{\alpha_1}L + p_{\alpha_2}L + L_{\alpha_1}p + L_{\alpha_2}p + c_{\alpha_1}$ must be negative. Thus Theorem 1 is true.

Without the externalities, however, the theorem would not obtain. This point can be seen in two ways. First, note that without the externalities, $p_{\alpha_2} = 0,\ L_{\alpha_2} = 0$. Thus {15}, {19}, and {20} would imply that $L_{\beta}p + f'(\beta) = 0$. Alternatively (and equivalently), note that instead of eliminating the existence of the externality, its adverse social effects could be addressed by having the town planner build the houses rather than the homeowners. If instead of allowing each homeowner to minimize {1}, the town planner undertook the minimization over both α_1 and α_2, and imposing symmetry, the first-order condition would be:[4]

[4] The Hessian of {1} in α_1 and α_2 is assumed to be positive definite, because the cross-partial derivatives between α_1 and α_2 are assumed to be small relative to the direct second derivatives. In other words, the effects of the second home on marginal improvements in the safety of the first home are assumed to be small relative to the direct effects of the safety of the first home on those same marginal improvements.

$$p_{\alpha_1} L + p_{\alpha_2} L + L_{\alpha_1} p + L_{\alpha_2} p + c_{\alpha 1} = 0 \qquad \{21\}$$

The town planner would then, as above, solve for β given α. By comparing {21} to {19}, one can see that at the equilibrium that would obtain in the absence of externalities, $L_\beta p + f'(\beta) = 0$.

The key insight is that given the externalities, it will always appear at the equilibrium of the system that the fire department "should" be larger. But a larger fire department would reduce social welfare, because it would induce people to build homes that were not as safe. The wrong paradigm thus yields a misleading answer.

This insight is a particular manifestation of the phenomenon that has come to be known as moral hazard.[5] Important applications include:

- Terrorism. Individuals and firms can affect the probability and severity of terrorist attacks by undertaking additional security measures.[6] In the presence of such externalities, it will always *appear* that the government should undertake more

[5] We are using the term "moral hazard" in the generic sense of situations in which an incentive distortion -- that is, a discrepancy between the individual's private returns (costs) and those facing society -- exists. In some uses, the term "moral hazard" is limited to incentive problems that arise in insurance markets, in which the incentive distortion arises from the fact that part of the social costs of the accident-inducing activity is borne by the insurer rather than the individual. We discuss that issue later in the paper.
[6] Here we are implicitly assuming that such security measures *reduce* the probability of attack, or the severity of loss from an attack, on others. Such should be the case for unobservable security measures. As noted in a footnote above, it is entirely possible that observable security measures raise the probability of attack on others, by displacing the focus of terrorist attacks toward those with lower observable security precautions.

security against terrorists – but only because the analysis ignores the impact of the government action on private anti-terrorism effort.

- Contagious diseases. Individuals can take actions that affect the likelihood that they are infected with a contagious disease. The incentive to do so is attenuated by the provision of public health facilities and insurance, which serve to reduce the costs borne by the individual should she come down with the disease. The social benefit of taking preventive actions, however, includes the benefits to others who might catch the disease. The optimal provision of public health facilities and insurance needs to take the impact on preventive actions into account. Given the level of care taken, it will always appear as if there should be more public provision of health facilities and insurance.

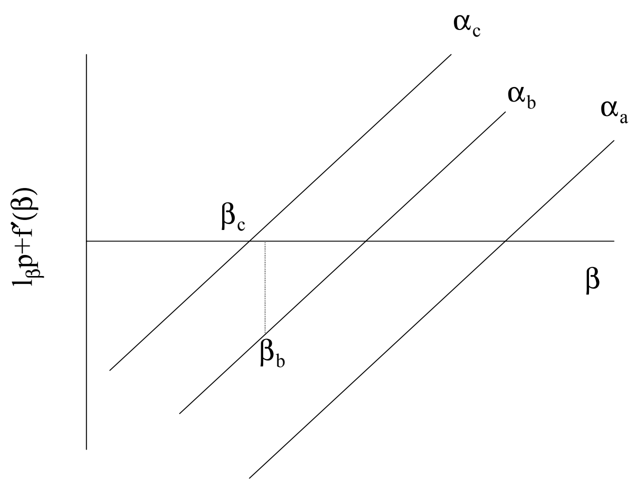
- Financial contagion. Financial crises in one country could have adverse effects on others. While the magnitude and significance of these contagion effects remain the subject of significant controversy, it is clear that if such externalities exist, the incentive of country to take actions to prevent crises at home will be sub-optimal. The implications for international rescue packages are then readily discernible.

Interactions between the externality and the public good

The previous section has shown that it may appear as if the level of provision of fire protection is suboptimal if one ignores the induced effect of a better fire department on the level of safety in construction. We now ask the somewhat more subtle question: When these externalities exist, should the fact that individuals take too little care result in a larger or smaller fire department than in the first best situation?

The figure below illustrates the situation in the context of the fire department model. The curves show the marginal change in social welfare from a larger fire department as a function of the size of the fire department, conditional on the safety of the homes. (For expositional simplicity, the curves are presented as linear even though they would not be under the assumptions above.) Safer homes shift the curve upward ($\alpha_c>\alpha_b>\alpha_a$), since $L_{\beta\alpha} > 0$. Assume that α_c represents the safety of the homes that would obtain with full internalization of the externalities (e.g., through a town planner) and that β_c is the corresponding optimal level of the fire department in the absence of externalities.

Then consider the impact of the externalities. At β_c, we know that the safety of the homes must be less than α_c, say α_b. And given the line associated with α_b, the town planner has an incentive to improve the fire department, since $L_\beta p + f'(\beta) < 0$ for α_b at β_c. Suppose therefore that the town planner decides to improve the fire department to β_b. That would appear to be welfare-improving if $\Omega'(\beta)$=0. But since $\Omega'(\beta)$<0, the increase in β would induce a further decline in α, say to α_a. And at {β_b, α_a}, the marginal incentive to expand the fire department (disregarding $\Omega'(\beta)$) is even larger than at {β_c, α_b}. The town planner must thus carefully balance the gains from building a fire department more appropriate to the level of care, and the losses from the lower level of care. In other words, given the externality-induced distortion in α, the town planner must trade off movements up a given line in the graph versus shifts in that line. The size of the shifts is determined by $L_{\beta\alpha}$ and $\Omega'(\beta)$.



One can see from the graph that depending on the slope of $L_\beta p + f'(\beta)$ and the magnitudes of $L_{\beta\alpha}$ and $\Omega'(\beta)$, β and α could be either higher or lower with the externality than without it. For large $L_{\beta\beta} p + f''(\beta)$ and small $\Omega'(\beta)$ and $L_{\beta\alpha}$, β is likely higher with the externality than without it, and α is likely lower. In that case, the externality-induced decline in α creates a substantial incentive to expand the fire department, and the homeowners' reaction to a larger β is relatively weak – inducing the town planner to expand β relative to β_c. If $\Omega'(\beta)$ and $L_{\beta\alpha}$ are large in absolute value, however, the town planner would have an incentive to *reduce* β relative to β_c, since reducing β would engender a significant increase in α. Indeed, it is not impossible that the equilibrium with the externality involves $\alpha>\alpha_c$ and $\beta<\beta_c$.

Example

A simplified example may be helpful as an illustration. Assume for simplicity that p is a constant (although this assumption violates {2} and {5} above, it simplifies the algebra and, when combined with an externality in the damage function, still preserves the fundamental results above). We also assume that the loss function is given by:

$$L = \alpha_1^{\ a} \alpha_2^{\ b} \beta^c \qquad \{22\}$$

where a<0, b<0, and c<0, and that $f(\beta)=\beta$, $c(\alpha)=\alpha$, where α is assumed to be a scalar. (Note that in this context, the parameters a, b, and c are not the same nor related in any way to the subscripts denoting different scenarios in the graph above.) Let z=a+b-1, where z<0. Then:

$$\alpha^{**} = \left\{ \frac{-1}{ap} \right\}^{\frac{1}{z}} \beta^{-\frac{c}{z}} \qquad \{23\}$$

Note that $\alpha^{**}>0$ since a<0. Further note that $\frac{d\alpha^{**}}{d\beta} < 0$. The town planner then optimizes over β:

$$\beta ** = \frac{-1}{ap}^{\frac{z+1}{z}} \frac{pc}{z}^{\frac{z}{c+z}} \qquad \{24\}$$

Theorem 1 holds at α**, β**, since:

$$L_{\beta} p + f'(\beta **) = pc\alpha_1{}^{a}\alpha_2{}^{b}\beta^{c-1} = pc\alpha **^{a+b}\beta **^{c-1} + 1 < 0 \qquad \{25\}$$

Now compare α** and β** to the social optimum, which would obtain if the town planner optimized over the characteristics of both homes, instead of allowing the individual homeowners to choose their own levels of care. Then:

$$\alpha_{soc} ** = \frac{-1}{(a+b)p}^{\frac{1}{z}} \beta^{\frac{-c}{z}} \qquad \{26\}$$

Note that $\alpha_{soc} ** > \alpha **$ *at any given* β, since b<0. The intuition is that *ceteris paribus*, taking account of the externalities when the houses are built results in safer houses than under the decentralized equilibrium. But what about β**?

$$\beta_{soc} ** = \frac{-1}{(a+b)p}^{\frac{z+1}{z}} \frac{pc}{z}^{\frac{z}{c+z}} \qquad \{27\}$$

Note that $\beta_{soc}{**} < \beta{**}$ since b<0. Thus, given decentralized control over house quality, the fire department is larger than under the social optimum. The larger fire department, in turn, reduces the quality of homes even further below $\alpha_{soc}{**}$ than the level that would obtain with the externalities and a fire department of $\beta_{soc}{**}$.

Regulation

Theorem 1 shows that the town planner keeps the fire department "small" to encourage homeowners to build safer homes. The only alternative to a small fire department is regulation: the fire department could mandate safer homes. Indeed, given the presence of externalities, the town planner could achieve the first-best solution if α were fully observable and the planner could regulate levels of α. In particular, the town planner could achieve the first best solution by simply mandating that α be set at the optimal level (where "optimal" takes the externality into account). In particular, the regulatory α would be set such that:

$$\begin{aligned} &\mathrm{p}_{\alpha_1}(\alpha_1,\alpha_2)\mathrm{L}(\alpha_1,\alpha_2,\beta) + \mathrm{p}_{\alpha_2}(\alpha_1,\alpha_2)\mathrm{L}(\alpha_1,\alpha_2,\beta) + \mathrm{p}(\alpha_1,\alpha_2)\mathrm{L}_{\alpha_1}(\alpha_1,\alpha_2,\beta) + \\ &\mathrm{p}(\alpha_1,\alpha_2)\mathrm{L}_{\alpha_2}(\alpha_1,\alpha_2,\beta) + \mathrm{c}_{\alpha_1}(\alpha_1) = 0 \end{aligned} \qquad \{28\}$$

where $\alpha_1 = \alpha_2$.

Theorem 2 (optimal regulation with observable characteristics): The planner could achieve the first-best solution if all elements of α could be regulated.

The proof follows from the ability of the town planner to choose α according to {21}, which defines the socially optimal level of α. The problem is that by assumption, not all elements of α are observable.

An alternative condition under which regulation can attain a first-best optimum obtains when some observable variable s (for safety) exists such that the probability of a fire and the losses incurred in a fire (p and L) depend *only* on s in the two houses (and the fire department). In other words, s is a sufficient statistic for the vectors α. In that case, the social planner would simply set s=s*, and each household would attain s in a way that minimizes costs.

If the probability of a fire and the losses resulting from such a fire are a function of the "s safety" of the house and the neighbor's house, as well as a summary statistic m of the contagion across the two houses, then optimal regulation can be attained (assuming s and m are observable) by imposing m* and s* from the town planner's optimization problem. Individual households can then minimize the cost of attaining m* and s*. If m is not observable, however, then regulation can not obtain a first best optimum. Even if the town planner mandates s*, *how* s* is attained by each household would then matter --

some methods of attaining s may produce larger m than others. In minimizing the cost of attaining s, individuals will have no incentive to take the impact on m into account.

User fees with a single fire-fighting technology

Typically, if the government can attain a first best optimum through regulation, it can also attain it through taxes and fees. This condition clearly holds when s and m are observable -- a charge can be imposed for any deviations from s* and m*. At high enough charges, individuals will be induced to set s=s* and m=m*. But the possibility of fees opens up a more subtle question: If the only thing that can be observed is that a fire has occurred, can the government induce the appropriate level of care simply by charging for use of the fire department? We show that the town planner *can* achieve the optimal solution with a user fee if there is a single element to α, even if that element is not observable. (α is therefore a scalar, either because we are interested in only one of the attributes of the house, or because a single sufficient statistic exists for all the various housing attributes in terms of their fire effects.[7])

In particular, assume that homeowners must pay the fire department a charge τ for fighting a fire. The cost minimization problem facing homeowner 1 is then:

$$\underset{\{\alpha_1|\alpha_2,\,\beta\}}{\text{Min}}\ p(\alpha_1,\alpha_2)L(\alpha_1,\alpha_2,\,\beta)+p(\alpha_1,\alpha_2)\tau+c(\alpha_1)+f(\beta) \qquad \{29\}$$

The first-order conditions for this minimization problem are given by:

$$p_{\alpha_1}(\alpha_1,\alpha_2)L(\alpha_1,\alpha_2,\ \beta)+p(\alpha_1,\alpha_2)L_{\alpha_1}(\alpha_1,\alpha_2,\ \beta)+p_{\alpha_1}(\alpha_1,\alpha_2)\tau+c_{\alpha_1}(\alpha_1)=0 \qquad \{30\}$$

Note that for τ>0, the first-order conditions defined by {30} will produce α***, where α***>α*. The proof follows from $p_{\alpha_1}(\alpha_1,\alpha_2)<0$ and {15}, combined with {9}, {10}, and {12}. The intuition is clear: by providing an additional disincentive against fires, the user fee induces homeowners to build safer houses.

Theorem 3 (optimal user fees):

If α is a scalar, the town planner can achieve the first-best solution, even if α is not observable, by setting $\tau=\frac{(p_{\alpha_2}L+L_{\alpha_2}p)}{p_{\alpha_1}}$. If α is a vector, then a user fee can not achieve the first best as long as more than one element of α is not observable (unless a sufficient statistic, s, captures the impact of all such non-observable elements).

Proof for scalar: Given $\tau=\frac{(p_{\alpha_2}L+L_{\alpha_2}p)}{p_{\alpha_1}}$, the individual homeowner's first-order conditions become:

$$p_{\alpha_1}L+pL_{\alpha_1}+p_{\alpha_2}L+L_{\alpha_2}p+c_{\alpha_1}=0 \qquad \{31\}$$

[7] As above, a sufficient statistic s exists if p and L are functions of s alone.

which is identical to the social optimum given by {21} above for scalar α. From the monotonicity of the p, L, and c functions in α, the user fee thus achieves the social optimum. Note further that no user fee is necessary to achieve the social optimum in the absence of externalities. In particular, the absence of externalities would imply that τ=0.

Proof for vector: If all elements of α are observable, then a non-linear vector tax h(α) can always achieve the social optimum by aligning private incentives for each element of α with the social cost of the externality. In particular, by setting each element of h(α) such that its derivative is equal to each element of $p_{\alpha_2} L + L_{\alpha_2} p$, the town planner can align social and private incentives. If only one element of α is not observable, then the town planner can combine a user fee for the fire department with the rest of the h(α) schedule to achieve the first best outcome. If more than one element of α is not observable, however, the first best is not achievable because the town planner has no way of providing appropriate incentives for the unobservable elements of α.

Note that if the loss from a fire is observable, then the number of unobservable elements of α consistent with achieving the social optimum could be expanded to two. In particular, by imposing a user fee on calling the fire department (related to p) and another user fee on the loss from the fire (related to L), the town planner achieves two degrees of freedom with which to align private and social incentives for two elements of α.

Finally, assume some elements of α are observable, and others not. Then the fire department could impose conditions on a house to receive fire assistance (assuming that the commitment was credible), and the fees charged would be adjusted appropriately to provide the correct incentives. On the other hand, assume that the fire department could observe α after the fire (for example, it could determine the structural soundness of the house by examining the remains of the house and the manner in which it burned down). It could then charge fees dependent on the observed (ex post) characteristics of the house. It is easy to show that in general, a fee structure can then be devised to induce the optimal α. Given the risk neutrality of our agents, such a fee structure could be devised and the first-best obtained even if the characteristics of the house are observed *ex post* with noise.

User fees with uncertainty over the fire

The model can be extended by assuming that once a fire starts, there is some uncertainty over the extent of the loss from a fire – some fires spread over the entire house and are difficult to control, and other fires remain contained in a small part of the house. The result we obtain is similar to that in the real options literature: the combination of the uncertainty over the future course of the fire and the fixed cost of calling the fire department creates an option for the homeowner. Exercising that option by calling the fire department is an additional cost to the homeowner, and thus the homeowner will delay calling the fire department beyond the point at which a simple cost-benefit analysis (excluding the option) would suggest such a call. *Again, viewing the problem from a* *traditional policy perspective -- using conventional cost-benefit analysis -- would produce distorted decisions.*

To model the uncertainty surrounding the loss function, we will assume that l follows a geometric Brownian motion with drift:

$$dl = \mu l dt + \sigma l dz \qquad \{32\}$$

where μ is the drift rate, which may be a function of α and of efforts to fight the fire with fire extinguishers, and where $dz = \varepsilon\sqrt{dt}$ is the increment in Wiener process and ε is identically and independently distributed as a standard normal variable.

Assuming that the fire department can immediately extinguish any fire, the benefit of calling the fire department immediately is then the losses avoided in the future by stopping the fire now:

$$B(s) = E_s\left\{\int_{t=s}^{t=\infty} L(s)e^{-\rho(t-s)}dt\right\} \qquad \{33\}$$

where ρ is the discount rate, and we assume ρ>μ. This implies:

$$B(s) = \frac{L(s)}{\rho - \mu} \qquad \{34\}$$

Given {34} and the fixed cost of calling in the fire department, τ, the homeowner must decide when to dial 911 or when not to. For well-behaved problems, there exists a L* such that for L>L*, the homeowner will call the fire department and for L<L*, the homeowner will not call the fire department.[8] Following standard approaches to such problems, the solution for L* is:

$$L^* = (\rho - \mu)\frac{r}{r-1}\tau \qquad \{35\}$$

where:

$$r = \frac{\frac{1}{2}\sigma^2 - \mu + \sqrt{(\frac{1}{2}\sigma^2 - \mu)^2 + 2\rho\sigma^2}}{\sigma^2} \qquad \{36\}$$

and r>1 since $\rho > \mu$.

Note that if there were no uncertainty over the future course of the fire and therefore no option value to calling the fire department, the homeowner would dial 911 whenever $\frac{L}{\rho - \mu} > \tau$, or whenever the capitalized value of stopping the fire exceeded the fixed cost of doing so. But given the uncertainty over how quickly the fire will spread and the amount of damage it will do, the homeowner would not be willing to call the fire

[8] Sufficient conditions for being "well behaved" are that B(L) is increasing in L, and that the distribution function of L conditional on a higher current L exhibits first order stochastic dominance over the distribution function conditional on a lower current L. See A. Dixit and R. Pindyck, *Investment under Uncertainty* (Princeton University Press: Princeton, 1994), pp. 128-130.

department at that point – he or she would wait until the damage had grown further, since $\frac{r}{r-1} > 1$ for $r > 1$.

Theorem 4: Given uncertainty over the severity of the fire, and a fixed fee for calling the fire department, homeowners will wait to call the fire department beyond the point a simple cost-benefit analysis would suggest.

Proof: Follows from $\frac{r}{r-1} > 1$ for $r > 1$.

This result is common in parts of the investment under uncertainty literature: the homeowner is hesitant to undertake the fixed cost of calling the fire department without being sure that the call is absolutely necessary. In effect, the homeowner waits to see what happens, in the hope that the fire will not turn out to be particularly harmful and that using a fire extinguisher may be sufficient to fight it.

If the fixed fee is set optimally to reflect the underlying uncertainty and irreversibility of calling the fire department, the delays are socially optimal -- and yet the fire department will always complain that homeowners *should* have called earlier. And empirical studies using traditional tools will generally show that the delays were costly, wasteful, and unnecessary. The situation is even worse if the fee is not set optimally. For example, assume the fixed fee is higher than socially optimal. Then the delays caused by the arbitrary fixed fees imposed by the fire department result in more fire damage with no offsetting social benefit. The fire department will continue to blame the homeowner for the delays, pointing out how costly they are, *even though the delays are induced by the fire department's own fee structure.*

CONCLUSION

The paper has examined a simple problem, in which fires can spread from home to home and homeowners do not take that effect into account when building their homes. The conclusions that can be drawn from the models above include:

- In many situations, an apparently reasonable set of analytical tools produces very misleading insights into the nature of the problem and the best policy response. An appropriate degree of modesty is thus warranted for both economists and policy-makers examining even these relatively simple problems, let alone the much more complicated ones existing in reality. It is all too easy to fall prey to the tyranny of conventional models.

- Externalities complicate the analysis of whether a larger fire department is beneficial. It is easy to be misled by the apparent effectiveness of the fire department in reducing losses from fires.

- The appropriate role of regulations and user fees depends on many factors, including how easy it is to observe the actions of homeowners. In general, both regulations and user fees have a role to play in the most effective overall policy stance.

- Irreversibility and uncertainty may make homeowners hesitant to call the fire department. Despite appearances to the contrary, delays in calling the fire department are not necessarily socially costly. But if the irreversibility is created solely by the fire department's fee policies, and does not reflect any underlying social calculus, the delays *are* costly. In that case, however, the socially costly delays are caused by the fire department itself, and the fire department should change its fee structure.

Extensions to a more realistic model would include situations in which homeowners are risk averse, in which the fire department has its own political objectives, and in which the fire department is unsure of the best way of fighting the fire. The appendix briefly explores the impact of risk aversion.

Appendix: Risk aversion, insurance, and moral hazard

In the main text, we assume that homeowners are risk neutral. We can expand the model to allow for risk aversion, and then examine the effects of an insurance market on homeowners' choices. The insurance market does not provide perfect insurance in equilibrium, because of the moral hazard created by not being able to fully observe α.

To begin with, assume that α is fully observable. Then the homeowner will maximize:

$$\underset{\{\alpha_1 . y|\alpha_2 , \beta\}}{\text{Max}} \ \text{p}(\alpha_1, \alpha_2)\text{U}\{\text{W - L}(\alpha_1, \alpha_2,\ \beta) + y\} + (1 - \text{p}(\alpha_1, \alpha_2))\{\text{U}(\text{W - z})\} - \text{c}(\alpha_1) - \text{f}(\beta)$$

{A1}

where y is the amount of coverage (net of the premium) taken out by the homeowner, so that if a fire occurs, the individual is paid y; U is the utility function, where $U' > 0$ and $U'' < 0$; W is initial wealth; and z is the premium paid for insurance.

The first order conditions with respect to y are:

$$\frac{\partial}{\partial y} = \text{p}(\alpha_1, \alpha_2)\text{U}'\{\text{W - L}(\alpha_1, \alpha_2,\ \beta) + y\} - (1 - \text{p}(\alpha_1, \alpha_2))\{\text{U}'(\text{W - z})\}\frac{\text{dz}}{\text{dy}} = 0 \qquad \{A2\}$$

In a competitive market with α fully observable, free entry and exit into the insurance market will ensure that:

$$(1-p)z = py \qquad \{A3\}$$

Thus, $\frac{dz}{dy} = \frac{p}{1-p}$. Substituting into {A2}, we see that the first-order conditions imply:

$$U'\{W - L(\alpha_1, \alpha_2, \beta) + y\} = U'\{W - z\} \qquad \{A4\}$$

The monotonocity of U therefore implies that

$$L(\alpha_1, \alpha_2, \beta) - y = z \qquad \{A5\}$$

and that the insurance is perfect, in the sense that the homeowner's utility does not depend on whether a fire occurs. Furthermore, z (the premium) is a function of α. Therefore, the premium reflects the level of care taken, and no "moral hazard" problem arises.

By assumption, however, some elements of α are not observable. For simplicity, assume that none of the elements of α is observable. Then the insurance premium, z, will be an increasing function of y, the amount of protection afforded. In particular, assume that $z = yq(y)$, where $q'(y) > 0$. Then $\frac{dz}{dy} = q + yq'(y)$. Plugging this into the first-order conditions, we see that:

$$pU'\{W - l(\alpha_1, \alpha_2, \beta) + y\} = pU'\{W - z)\} + (1-p)U'\{W - z)\}yq'(y) \qquad \{A6\}$$

Thus, $U'\{W - l(\alpha_1, \alpha_2\ \beta) + y\} > U'\{W - z\}$ and from the first and second derivatives of U, we can conclude that $W - l(\alpha_1, \alpha_2\ \beta) + y < W - z$.

This result is the classic implication of moral hazard: since the insurance agents can not observe α, individuals cannot obtain perfect insurance – for if the insurance agents offered such insurance, homeowners would build very unsafe homes. The moral hazard arises in this case because α is not observable, and therefore z can not be a function of α.

Theorem 5 (moral hazard):

Moral hazard not only results in a lack of perfect insurance, but also reduces care relative to the social optimum. In particular, two distortions in the housing market both point in the same direction: the externality itself, which reduces care below the optimal level, and the existence of the insurance market, which reduces care even in the absence of the externality. Increasing the quality of the fire department exacerbates these distortions by inducing even less care. Thus moral hazard provides an additional incentive to the town planner not to expand the fire department.

The consumer's choices of α and y are both functions of β. In particular, assume that α=N(β) and that y=Q(β). The town planner can therefore maximize over β, taking into account the reactions of consumers and the insurance market:

$$\underset{\{\beta\}}{\text{Max}}\ \Psi(\beta) - \text{f}(\beta) \qquad \{A7\}$$

where:

$$\Psi(\beta) = \text{p}(\text{N}(\beta))\text{U}\{\text{W - L}(\text{N}(\beta),\beta) + Q(\beta)\} + (1 - \text{p}(N(\beta)))\{\text{U}(\text{W - z})\} - \text{c}(\text{N}(\beta)) = V(N(\beta),Q(\beta),\beta,z)$$

The first-order condition is given by $\Psi'(\beta) = \text{f}'(\beta)$. The externality introduces an additional term into that first-order condition, which then affects the size of the fire department.[9]

There is a certain parallel between the risk-moral hazard effects examined here and the externality effects highlighted in the main body of the text. In both cases, individuals undertake less than the optimal level of care. This by itself would suggest a larger fire department than if care were at the optimal level. But increasing the size of the fire department reduces the level of care from the second best level, worsening the distortion. The key issue is what happens to the magnitude of the marginal return to a larger fire department. The two effects are offsetting, and the result is ambiguous.

[9] The algebra is available upon request to the authors.