

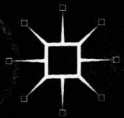
COMPLEXITY AND INSTITUTIONS: MARKETS, NORMS AND CORPORATIONS

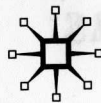
Edited by

Masahiko Aoki, Kenneth Binmore,
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3

Toward an Analytical Solution for Agent-Based Models: An Application to a Credit Network Economy*

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3.1 Introduction

The mainstream approach, as it is formalized in the DSGE (dynamic stochastic general equilibrium) models, is based on the process of intertemporal maximization of utility in the market-clearing context of the standard competitive equilibrium theory. It is built upon the representative agent (RA) framework, which rules out direct interactions among agents by assumption. Its fundamental hypotheses trivially lead to conclusions that there can be no inefficiencies and any pathology in general.¹

In short, the RA framework of the DSGE models adopts the most extreme form of conceptual reductionism in which macroeconomics and the financial network are reduced to the behavior of an individual agent.² The deep understanding of the interplay between the micro and macro levels is ruled out, as well as any 'pathological' problems, such as coordination failure. The RA in economics is tantamount to stating that 'macroeconomics equals microeconomics.'

We believe that a change of focus is necessary: an appropriate micro-foundation should consider the interaction at the agent-based level. This is feasible if the economy is represented by a framework with heterogeneous agents. The development of sound micro-founded models should also involve the links among agents in a networked economy. This approach can provide

better insights on how a crisis emerges from the microeconomic interaction and how it propagates in the economic system.

The literature provided, up to now, two different tools for dealing with interaction and heterogeneity: game theory and computational economics (CE). In the game theory framework, each agent takes into account the behavior of every other, which is supposed to be known. But a complete network populated by rational and fully informed agents would require infinite computational ability, which is beyond any plausible assumption. On the other hand, there has been a strong development of CE models, populated by interacting and boundedly rational agents. In these models, macro outcomes are very sensitive to different configurations of the parameters. Furthermore, the relationship between results and initial conditions is undefined, not to mention the issue of the trade-off between tractability and realism.

The idea of this paper is neither to ignore interactions between agents, nor to get hopelessly mired in complicated details by trying to model those interactions in their completeness, but rather to strike for some middle ground in which the consequences of interconnectedness can be at least crudely assessed. To achieve this, we start with a 'minimal' model of the economy as a credit network, in which firms interact directly with banks. Rather than simulating these interactions, we represent them by means of probabilities, and derive equations describing the evolution of the network and how its structure changes with time. The equations we derive provide some qualitative insights into the systemic fragility of the credit network.

The results of the present paper are based on the solution algorithms proposed by Di Guilmi et al. (2011) and are here presented without the full detail of the derivation. The complete demonstrations are available upon request. The remainder of the paper is structured as follows: section 3.2 presents the general hypotheses of the model, introducing firms and banks; section 3.3 illustrates the solution of the model dynamics; section 3.4 interprets the results and proposes some policy implications; section 3.5 offers some final remarks.

3.2 The model

This section introduces the main hypotheses about the structure of the model, concerning the firms, the banks and their match on the credit market.

3.2.1 The firms sector

The productive sector is modeled along the lines of Delli Gatti et al. (2010). Each firm sets the optimal quantity of output on the basis of its financial condition, according to the following rule

$$Q_{f,t} = \alpha A_{f,t}^\beta, \quad (1)$$

where $\alpha > 0$ and $\beta \in (0, 1)$ are constant parameters and A is the net worth of firm f . As shown in Greenwald and Stiglitz (1993), equation (1) comes from a profit maximization problem, when bankruptcy is costly and the probability of default is inversely related to the net worth. For the sake of simplicity, all firms are assumed to have the same linear production function, defined as

$$Q_{f,t} = (1/\varpi)N_{f,t}, \quad (2)$$

where Q is the physical output, N is the quantity of labor and $\varpi > 0$ is the inverse of labor productivity, assumed to be a constant parameter. From (1) and (2) it follows that $N_{f,t} = \varpi \alpha A_{f,t}^\beta$. The wage bill W is

$$W_{f,t} = wN_{f,t}, \quad (3)$$

where the nominal wage w is constant and uniform across firms.

Firms do not know in advance the quantity of goods that will be demanded and this can produce uncertainty about the final price. For this reason, following Greenwald and Stiglitz (1993), we model the selling price for each firm u_f as a stochastic variable coming from a uniform distribution, defined in the interval $[u_{min}, u_{max}]$.

Because of imperfect information there is a hierarchy in the sources of firms financing, from internal to external finance. The firms that can finance their whole wage bill with internal resources are defined as *self-financing* (SF), while the others are *non-self-financing* (NSF). The latter ones resort to the credit market as, by assumption, they are completely rationed on the equity market. The demand for credit for a NSF firm is given by

$$D_{f,t} = W_{f,t} - A_{f,t}, \quad (4)$$

while $D_{f,t} = 0$ for SF firms. Firms' profits $\pi_{f,t}$ are defined as

$$\pi_{f,t} = u_{f,t}Q_{f,t} - W_{f,t} - r_{f,t}D_{f,t}, \quad (5)$$

where r is the rate of interest. The net worth of firms is the sum of past profits:

$$A_{f,t+1} = A_{f,t} + \pi_{f,t} \quad (6)$$

If $A < 0$ the firm goes bankrupt and it is replaced by a new one, such that the total number of firms is constant in time.

3.2.2 The banking sector

A bank can lend to different firms and sets unilaterally the interest rate for each of them. In particular, the bank b asks to the firm f an interest rate which is inversely related to the financial soundness of borrower and lender, according to the formula

$$r_{b,t}^f = a \left[A_{b,t}^{-a} + \left(\frac{D_{f,t}}{A_{f,t}} \right)^a \right] = a \left[A_{b,t}^{-a} + v_{f,t}^a \right] \quad (7)$$

where $a > 0$, $A_{b,t}$ is the bank's net worth and $v_{f,t}$ is the firm leverage ratio.

The dependence on $A_{b,t}$ represents the fact that, for a bank, the lower its equity, the higher is its probability of default. Consequently, the owners of the bank's capital will demand a premium for risk which is inversely proportional to its internal financial resources. As a consequence, a higher net worth means a lower premium paid by the bank to investors and allows the bank to charge a lower interest rate to customers (Gambacorta, 2008). The second factor in the interest rate formula, $(v_{f,t})^a$, quantifies the higher risk premium requested by the bank to highly-leveraged firms.

This formula analytically devices the mechanism described by Stiglitz and Greenwald, (2003, p. 145): 'The high rate of bankruptcy is a cause of the high interest rate as much as a consequence of it.' This is because the demise of one or more firms generates bad debt for the lender and, thus, brings about a deterioration in its financial conditions (a lower $A_{b,t}$). As a consequence, the bank will raise the interest rates, worsening the positions of its customers and, possibly, leading to the bankruptcy of some of them. The new bad debt has further negative impact on the financial soundness of the lender in a downward spiral.

The profits of the bank b are given by

$$\pi_{b,t} = \sum_f (1 + r_{b,t}^f) D_{b,t}^f \quad (8)$$

where $D_{b,t}^f$ is the credit supplied by the bank b to the firm f . The bank's net worth is computed as

$$A_{b,t+1} = A_{b,t} + \pi_{b,t} - BD_{b,t} \quad (9)$$

where BD is the bad debt, which is the debt that cannot be paid back due to the bankruptcy of borrowers. If $A_{b,t} < 0$, the bank goes bankrupt, and it is replaced by a new one. Thus, also the number of banks is constant.

3.2.3 Partner selection and network evolution

A NSF firm signs a one-period credit contract with one bank. In order to capture the fact that firms have limited information about the credit market, at each point in time a NSF firm can demand for credit only to a subset of banks. If we indicate with B the total number of banks in the economy, the number of banks that are 'visible' to a firm is given by mB , where $m \in (0, 1]$ is a parameter constant across firms and in time. The subset of banks for each firm is randomly selected at each period. Hence every firm possibly surveys a different pool of banks every period.

The firm sorts the banks in its randomly selected pool according to the proposed interest rates and then sends a signal to the bank which offers the lowest interest rate, demanding for credit.

Credit rationing can arise as the banks must comply with a regulatory framework. In particular, the monetary authority determines an adequacy ratio along the lines of the Basel II accord. Consequently, the total lending of the bank b , denoted by $L_{b,t}$, must be lower than $\bar{\Theta} A_{b,t}$ where $\bar{\Theta} \geq 1$ is a constant parameter. Let us define

$$\bar{L}_{b,t} = A_{b,t} \bar{\Theta} \quad (10)$$

as the limit for lending that the bank b can supply.

In order to select how many and which requests for credit will be satisfied, the bank adopts a prudential criterion. It first sorts the potential customers in ascending order of leverage ratios v . Then it considers the demand for credit starting from the firm with the lowest v and accepts their demand as long as condition (10) is satisfied.

When a firm is refused credit from a bank, it sends a signal to the bank which follows in its list, that is a bank who offers a higher interest rate. If all the banks in the firm's pool decline to supply credit, the firm must reduce its production in order to meet its financial constraint. It can pay salaries only with its internal resources, and, accordingly, the quantity produced by a fully credit rationed firm is equal to $q_{f,t} = A_{f,t} / (w\varpi)$.

The structure of the network is represented by a series of islands, or cliques: each island is composed by the lending bank and its borrowing firms. The composition and the number of cliques vary from a period to another. The composition changes as a NSF firms can become SF (therefore not connected to any bank) or change its lender. The number of cliques is variable as a bank can have no customers at a given time (and so not be included in the network) and been chosen by one or more firms in the following period (forming a clique).

3.3 Solution of the model

This section introduces and applies the analytical techniques for the solution of the model. We have already partitioned the population of firms into two groups: SF and NSF. The evolution of the density for the number of agents belonging to a particular cluster can be modeled by means of the master equation (ME). The ME is a differential equation which describes the variation of the probability of observing a certain number of agents in a given group; it can be solved under asymptotic conditions. In this treatment, the final outcome of the solution is a system of equations which describes the evolution of the network and the degree distribution. To this aim we first need to quantify the number of NSF firms, as they compose the network, and then use this result to study the evolution of the network. We set up two master equations:

- the first one (NSF-ME) to describe the evolution of the number of NSF firms; the solution of this ME will be plugged in
- the second one (K-ME) which models the dynamics of the degree and, through further passages, makes possible the identification of the degree distribution.

One of the novelties is in that the first ME is *nested* into the second to model the dependence between the two.

In the first subsection the NSF-ME is introduced and solved. It is worth noticing that the transition rates for this ME are endogenous and dependent upon the financial conditions of firms. The second subsection presents the K-ME and its solution.

3.3.1 Stochastic evolution of firms

The evolution of two different groups of firms is modeled along the lines of Di Guilmi et al. (2010), who study an analogous problem using a ME to model the dynamics of the densities of different groups of firms. The ME is a function of the transition probabilities for the agents to move between the two groups. In this model, each firm has a different transition probability, which is dependent on its financial condition (the equity A) and on the price shock u . In order to make the problem analytically tractable we need to quantify an average probability for each of the two transitions. Therefore, we identify two *representative* firms, one SF and one NSF by taking the average net worth, indicated respectively by A_0 for SF and A_1 for NSF firms. This reduction in the degrees of freedom of the problem is defined as *mean-field approximation* in statistical mechanics. From these two values we can compute the targeted productions, the costs and the financial needs for the two average firms by using equations (1)–(4).

3.3.1.1 The transition rates and the NSF-ME

The probability for a NSF firm f to become SF depends on the capacity of the firm of having at time $t - 1$ a profit large enough to pay the salary bill at time t . This condition can be expressed as

$$A_{f,t-1} + \pi_{f,t-1} \geq W_{f,t},$$

which, using equation (5), becomes

$$A_{f,t-1} + u_{f,t-1} Q_{f,t-1} - W_{f,t-1} - r_{f,t-1} D_{f,t-1} \geq W_{f,t}. \quad (11)$$

The only exogenous variable in (11) is the price u_f and thus it is convenient to specify the probability of switching as a function of it. In particular, since the distribution of u is known, it is possible to quantify the minimum price threshold above which the NSF firm can obtain a profit sufficient to become

SF. We denote this threshold with \bar{u} . Rearranging equation (11) and using the subscript 1 for the mean-field variables of the representative NSF firm, we can write

$$u_{1,t-1} \geq \frac{W_{1,t} + W_{1,t-1} + r_1 D_{1,t-1} - A_{1,t-1}}{Q_{1,t-1}} = \bar{u}_t. \quad (12)$$

Using the uniform probability function of u , we can write the probability of becoming SF ι as

$$\iota_t = 1 - F(\bar{u}_t) = 1 - \frac{\bar{u}_t - u_{\min}}{u_{\max} - u_{\min}}. \quad (13)$$

In the same fashion, we can specify the condition for the mean-field SF firm to become NSF as

$$A_{0,t-1} + u_{0,t-1} Q_{0,t-1} - W_{0,t-1} < W_{0,t}, \quad (14)$$

which can be written as follows

$$u_{0,t-1} < \frac{W_{0,t} + W_{0,t-1} - A_{0,t-1}}{Q_{0,t-1}} = \underline{u}_t, \quad (15)$$

where \underline{u} is the upper threshold of the price shock. Denoting with ζ the probability of becoming NSF and making use of the known uniform probability function of u , we can write

$$\zeta_t = F(\underline{u}_t) = \frac{\underline{u}_t - u_{\min}}{u_{\max} - u_{\min}}. \quad (16)$$

In order to obtain the transition rates, the transition probabilities need to be conditioned on the probability of being NSF or SF. In particular, the probability to find a firm belonging to one particular group is higher the larger is the size of that group. This fact can be modelled by means of the two following environmental externality functions

$$\begin{aligned} \psi_{1,t} &= \psi_1 \left(\frac{N_{1,t} - \vartheta}{N} \right) = \frac{b_1 + b(N_{1,t} - \vartheta)}{N}, \\ \psi_{0,t} &= \psi_0 \left(\frac{N_{1,t}}{N} \right) = \frac{b_0 + b[N - (N_{1,t} - \vartheta)]}{N}, \end{aligned} \quad (17)$$

where $b > 0$, $b_1 > -bN_1$, $b_0 > -b(N - N_1)$, $\vartheta = \{0, 1\}$ is the observed variation in N_1 and ψ_1 and ψ_0 are constants. Accordingly the transition rates β and δ are given by the following homogeneous functions

$$\begin{aligned} N\beta_t &= \left(\frac{N_{1,t} - \vartheta}{N} \right) = N\zeta_t \left[\frac{b_1 + b(N_{1,t} - \vartheta)}{N} \right] \left[\frac{N - (N_{1,t} - \vartheta)}{N} \right], \\ N\delta_t &= \left(\frac{N_{1,t} + \vartheta}{N} \right) = N\iota_t \left\{ \frac{b_0 + b[N - (N_{1,t} + \vartheta)]}{N} \right\} \left(\frac{N_{1,t} + \vartheta}{N} \right). \end{aligned} \quad (18)$$

The NSF-ME is a differential equation which quantifies the variation in the probability to observe a given number of NSF firms $N_{1,t}$. It is given by the probability of observing $N_{1,t} + 1$ or $N_{1,t} - 1$ and having a transition of one firm, respectively, out from or in the NSF condition, less the probability of already having a number $N_{1,t}$ of NSF firms and observing any transition. Consequently, we have

$$\frac{dP_t(N_{1,t})}{dt} = \frac{[\beta_t(N_{1,t} - 1)P_t(N_{1,t} - 1) + \delta_t(N_{1,t} + 1)P_t(N_{1,t} + 1)]}{\text{in flow probabilities}} - \frac{[(\beta_t(N_{1,t}) + \delta_t(N_{1,t}))P_t(N_{1,t})]}{\text{out flow probabilities}} \quad (19)$$

3.3.1.2 ME solution and dynamics of the proportions of firms

The solution algorithm involves three main steps:

1. split the state variable N_1 in two components:
 - the *drift* (ϕ), which is the expected value of $n_1 = N_1/N$;
 - the *spread* (ϵ), which quantifies the aggregate fluctuations around the drift.

Accordingly, the state variable is re-formulated in the following way (Aoki, 1996):

$$N_{1,t} = N\phi_t + \sqrt{N}\epsilon_t; \quad (20)$$

2. expand in Taylor's series the modified master equation;
3. equate the terms with the same order of power for N .

This process yields an ordinary differential equation, known as macroscopic equation, which describes the dynamics of the trend, and a stochastic partial differential equation, known as Fokker-Planck equation, which describes the dynamics of the density R of the fluctuations (see Aoki, 1996; van Kampen, 1992). Hence, the final solution is given by the mean-field system of coupled equations

$$\begin{cases} \dot{\phi} - \Delta_t(\phi) = \beta_t(\phi) - \delta_t(\phi) = \rho_t\phi - \rho_t\phi^2 \\ \partial_t R_t(\epsilon) = -\partial_\phi \Delta_t(\phi) \partial_\epsilon (\epsilon R_t(\epsilon)) + \frac{1}{2} \sum_t(\phi) \partial_\epsilon^2 R_t(\epsilon) \end{cases} \quad (21)$$

where $\rho_t = b(\zeta_t - \iota_t)$ and $\sum_t(\phi) = \beta_t(\phi) + \delta_t(\phi)$.

The first of the (21) is a logistic equation with two equilibrium points: $\phi^* = \{0, 1\}$. The system approaches the two boundaries without hitting them. Indeed, the quadratic term $-\rho_t\phi^2$ acts as a break with increasing intensity as ϕ_t approaches one of the equilibrium points. Both equations depend on the transition rates and, therefore, on the average financial conditions of the firms.

Following van Kampen (1992), we substitute the formulation of the transition rates (18) into (21) and integrate the resulting expression. The final solution, with the trend dynamics ϕ and the distribution probability for the fluctuation component ϵ , can be written as

$$\begin{cases} \phi(t) = \left[1 + \left(\frac{1}{\phi_0} - 1\right) \exp(-\rho_t t)\right]^{-1}, \\ \epsilon(t) \overset{i.i.d}{\rightarrow} \mathcal{N}(\mu_{\epsilon,t}, \sigma_{\epsilon,1}^2, t) \end{cases} \quad \text{s.t.} \quad (22)$$

$$\begin{aligned} \mu_{\epsilon,t} &= \langle \epsilon_0 \rangle e^{\Delta'_t(\phi)} \\ \sigma_{\epsilon,1}^2 &= \langle \epsilon_0^2 \rangle (1 - e^{2\Delta'_t(\phi)t}), \end{aligned}$$

with $\phi_0 \in [0, 1)$ and $\langle \epsilon_0^2 \rangle = -\sum_t(\phi)/(2\Delta'_t(\phi))$.

By using the mean-field values of the average production of the firms in the two groups Q_1 and Q_0 , it is possible to obtain the trend and the fluctuations of the aggregate output, as for equation (20). The total output can be expressed as

$$\begin{aligned} Q_t &= N_{1,t}Q_{1,t} + (N - N_{1,t})Q_{0,t} \\ &= N\{\phi_t + N^{-1/2}\epsilon_t\}Q_{1,t} + [1 - \phi_t - N^{-1/2}\epsilon_t]Q_{0,t} \\ &= N\{Q_{0,t} - [\phi_t + N^{-1/2}\epsilon_t][Q_{0,t} - Q_{1,t}]\}. \end{aligned} \quad (23)$$

It is possible to show that $Q_1 < Q_0$, therefore the dynamics of trend and fluctuations of aggregate production are dependent upon ϕ and ϵ in system (22).

3.3.2 Stochastic evolution of the network: K-ME

In this section we develop the ME for the network degree. The NSF-ME is nested into the K-ME; to the best of our knowledge, this method has never been developed before. The problem is analysed by studying the evolution of the probability for two firms to be financed by the same bank. This (indirect) connection between two firms defines an (indirect) link in the network. The solution algorithm makes use of the concept of *giant component*, which is the node with the highest number of connection. In this model, it represents the bank which has the largest number of customers.³

As done for the NSF-ME, in the same fashion we split the state variable in the trend and fluctuations components. The volume of NSF firms N_l that are borrowers from a bank with degree K_l is assumed to be given by

$$N_{l,t} = N_{1,t}[K_l\phi_{l,t} + \sqrt{K_l}\epsilon_{l,t}], \quad (24)$$

where ϕ_l is the expected value and ϵ_l is the fluctuations component. Equation (24) shows a direct correlation between N_l and the expected number of borrowers for a bank. It can be rearranged and written in intensive form as

$$n_{l,t} = \frac{N_{l,t}}{N_{1,t}} = K_l\phi_{l,t} + \sqrt{K_l}\epsilon_{l,t}. \quad (25)$$

3.3.2.1 The transition rates and the K-ME

The transition probabilities in this setting concern the creation or destruction of a link between two firms. We introduce the variable ω and set $\omega_{ij} = 1$, if there is a link between the two firms i and j (they share the same bank), and $\omega_{ij} = 0$ otherwise. Accordingly, the creation and destruction rates are equal to, respectively,

$$\begin{aligned}\mathbb{P}(\omega_{t+1,i,j} = 1 | \omega_{t,i,j} = 0) &= \zeta, \\ \mathbb{P}(\omega_{t+1,i,j} = 0 | \omega_{t,i,j} = 1) &= \iota.\end{aligned}\quad (26)$$

Analogously to the NSF-ME case, two externality functions need to be defined in order to quantify the transition rates. These functions are assumed to be dependent on the size of the giant component. In particular, the market share of the largest bank (the giant component) is given by $\gamma_t = S_t/N_{1,t}$, where S_t is the number of its customers. Due to the interest rate formula in equation (7), the size of the giant component impacts on the morphology of the network by exerting a *gravitational* effect. The larger is a bank, the higher is its capacity to attract new borrowers by offering a lower interest rate. Thus, the greater is the giant component, the more it attracts firms and, consequently, the smaller are the chances of an inflow of firms into another component of the network. Accordingly, the externality function $\psi_{1,l,t}$ for the inflows into a generic component (bank) is assumed to be an inverse function of γ_t and $k_l = K_l^{-1}$. Symmetrically, the outflow externality function $\psi_{0,l,t}$ is a direct function of the two quantities. The externality functions can be specified as follows

$$\psi_{1,l,t} = \exp(p_1^2(1 - \gamma_t)(\phi_l^0/k_l)), \quad (27)$$

$$\psi_{0,l,t} = \exp(p_1^2\gamma_t(\phi_l^0/k_l)), \quad (28)$$

where p_1 is the firm-bank matching probability.⁴ Consequently the formulations of the transition rates are the following

$$\begin{aligned}\beta_t(n_{l,t} - \vartheta\rho_{l,t}) &= \lambda_{l,t}[1 - (n_{l,t} - \vartheta\rho_{l,t})], \\ \delta_t(n_{l,t} + \vartheta\rho_{l,t}) &= \mu_{l,t}[n_{l,t} + \vartheta\rho_{l,t}],\end{aligned}\quad (29)$$

where $\lambda_{l,t} = \zeta\psi_{1,l,t}$ and $\mu_{l,t} = \iota\psi_{0,l,t}$. The term $\pm\vartheta\rho_{l,t}$ introduces a correction to take into account that the number of firms in this case is variable, being represented by the NSF firms. In section 3.1, the total population of firms is constant and equal to N ; for the network dynamics we need to consider only the firms who enter the credit market, whose number comes from the solution of the NSF-ME. In particular, we indicate with ϑ the observed variation in $n_{l,t}$ and $\rho_{l,t} = \rho(K_{l,t}, N_{1,t})$.

The K-ME describes the evolution of the probability distribution for the degree in each level. It can be expressed as

$$\begin{aligned}\frac{dP_t(n_{l,t})}{dt} &= \underbrace{[\beta_t(n_{l,t} - 1)P_t(n_{l,t} - 1) + \delta_t(n_{l,t} + 1)P_t(n_{l,t} + 1)]}_{\text{inflow probability}} \\ &\quad - \underbrace{(\beta_t(n_{l,t}) + \delta_t(n_{l,t}))P_t(n_{l,t})}_{\text{outflow probability}}.\end{aligned}\quad (30)$$

3.3.2.2 ME solution: dynamics of the degree and degree distribution

In order to solve (30), we adopt the same methodology used for the NSF-ME. Also in this case, the final solution is a system analogous to (22). Splitting the state variable as in (24) and following the steps of the solution algorithm defined in subsection 3.3.1.2, we obtain the following equation for the trend

$$\phi_{l,t} = (\phi_l^0 - \phi_l^*) \exp\{-\rho_{l,t}[\lambda_{l,t} + \mu_{l,t}]t\} + \phi_l^* \quad (31)$$

where

$$\phi_l^* = [1 + \mu_{l,t}/\lambda_{l,t}]^{-1} \quad (32)$$

is the steady-state value of the degree.

Finally, the Fokker-Planck equation provides a Gaussian law for the fluctuations about the expected l -th degree level with mean $\mu_{\epsilon_{l,t}} = \langle \epsilon_l^0 \rangle \exp(-\rho_{l,t}\Delta'_t(\phi_l)t)$ and variance $\sigma_{\epsilon_{l,t}}^2 = \langle \epsilon_l^{*2} \rangle [1 - \exp(2\rho_{l,t}\Delta'_t(\phi_l)t)]$, where ϵ_l^* is the stationary value of fluctuations in the l -th degree level and ϵ_l^0 is the initial condition.

The trend equation and the distribution of fluctuations can be used to compute the degree distribution and its evolution in time. If we define a vector of possible initial starting points ϕ_0 for the average degree, there will be a different trajectory for each starting point according to the dynamics described by equation (32). Consequently, at each point in time, the empirical distribution of the degree is obtained by the different values of ϕ generated by the different trajectories.

3.4 Results

The systems of equations that compose the solution illustrate the role that the levels of indebtedness and concentration play in shaping the dynamics of the network evolution. The description of the network by means of the ME makes possible an analytical representation of the concept of *too big to fail* (proxied by the giant component) and *too interconnected to fail* (the degree). The model is also able to endogenously generate a dynamics for the proportion of NSF and SF firms, and, consequently, of aggregate production. These dynamics affect the size and the evolution of the network.

3.4.1 Interpretation

The analytical solution of the model describes the effects of the interaction among agents in the system. Indeed, the dynamics of the degree is modeled as dependent on the interaction among firms through the banking system. In particular, the solution shows how the interaction can cause coordination failures; as a consequence, the system oscillates between different steady states. The transition from an equilibrium point to another can be originated by avalanches of bankruptcies of firms and banks when the level of concentration becomes critical.

In order to identify the effects of the level of concentration on the market structure, we need to study the effect of the size of the giant component on the equilibrium solution. We can substitute the definitions of λ and μ into (32) and take the derivative with respect to γ . It can be demonstrated that this derivative is always positive. As a consequence, both the value of the degree during the adjustment and its steady value positively depend on the size of the giant component. The variance of the fluctuations of the degree is directly proportional to its steady-state value and, therefore, to the giant component. Consequently, a market with a relatively high level of concentration (large giant component and large degree) will display a higher volatility, due to the expected larger fluctuations in the average degree. Therefore, the degree distribution will appear as platycurtik. Since in the model the degree distribution is a proxy of the size distribution of banks, this implies larger instability, due to the possibility of rapid and deep modifications in the market structure. This analytical result is confirmed by the numerical analysis of the model⁵ as shown by Figure 3.1.

In respect of the aggregate output of the economy, the solution shows that instability in the credit market structure brings about higher volatility in output. An increase in the number of NSF firms has a negative effect on aggregate output, as shown by equation (23), and brings about an increase in the average degree, as for equation (24). Moreover, due to the solution equations, a larger N_1 causes a larger volatility in output due to equation (23); in the same way, the bigger is the average degree, the larger is the variance of its fluctuations. Accordingly, there is a positive correlation between variance of the degree and variance of output.

With regard to the dynamics of the network, the increasing concentration follows the growth of the economy: as firms and banks profit, interest rates decrease due to the accumulation of internal finance. Sounder banks are able to attract customers and grow faster. This virtuous cycle can lead to the emergence of a *big bank* which controls the biggest share of the credit market (represented by the giant component). This builds the set-up for the subsequent crisis, according to a pattern analogous to the one envisaged by Minsky (1982): during a boom credit becomes cheaper and firms are led to increase their production, as they

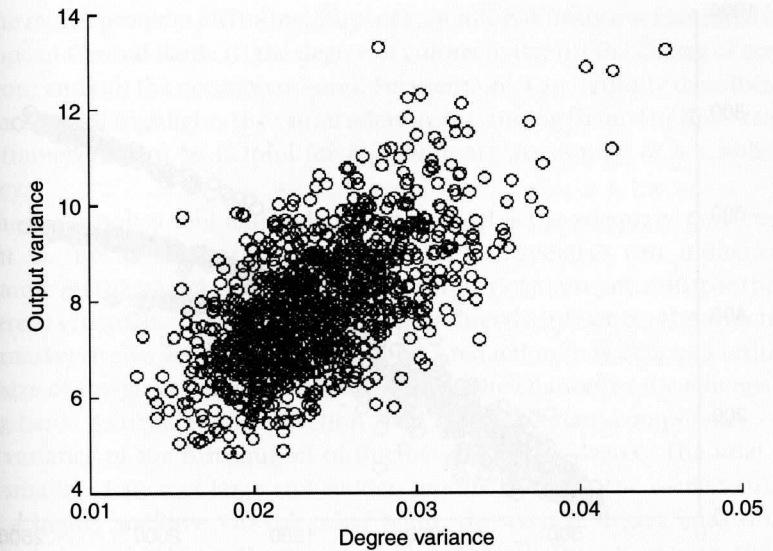


Figure 3.1 Variance of the degree and of the output. Monte Carlo simulation (correlation: 0.62)

accumulate profits. This growth increases the concentration in the credit market and makes firms and banks more vulnerable to negative shocks. Indeed, the presence of a big bank can have a destabilizing effect on the system through the variance of the distribution of the fluctuations of the degree. Every bank is potentially subject to large shocks, which consequently impact on the conditions of credit for firms through equation (7); as a consequence, borrowers can experience substantial variation in the cost of debt and, in the case of a significant negative shock, become insolvent, worsening the conditions of other banks and eventually spiraling down the whole system. The network economy displays an endogenous cyclical behavior, in which the tendency to concentration heightens the probability for the system to be hit by systemic financial distress (Figures 3.2 and 3.3).

3.4.2 Policy implications

Despite the fact that the representation of the credit market is simplified, the model captures the basic features of a credit network economy, such as the emergence of nodes with systemic relevance and the possibility of crises through propagation effects, when these nodes are in financial distress (De Masi et al., 2010).

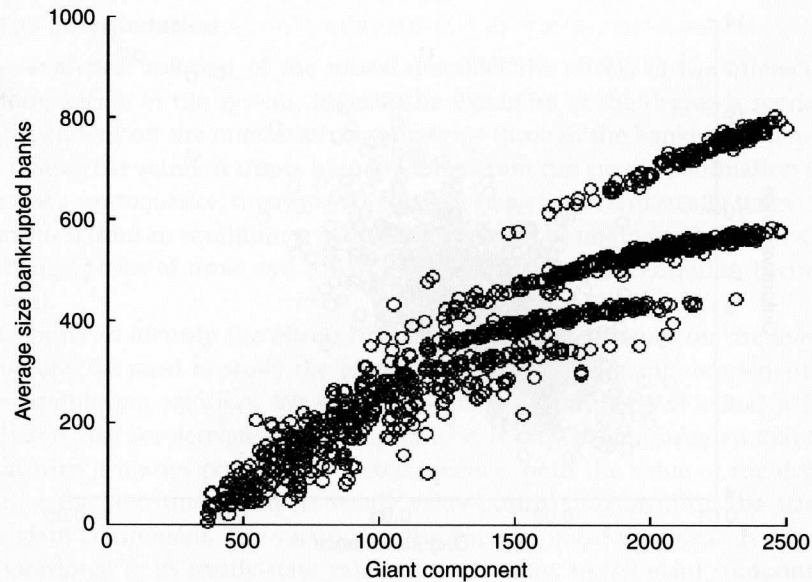


Figure 3.2 Giant component and average lending of bankrupted banks. Single simulation (correlation in Monte Carlo simulations: 0.71)

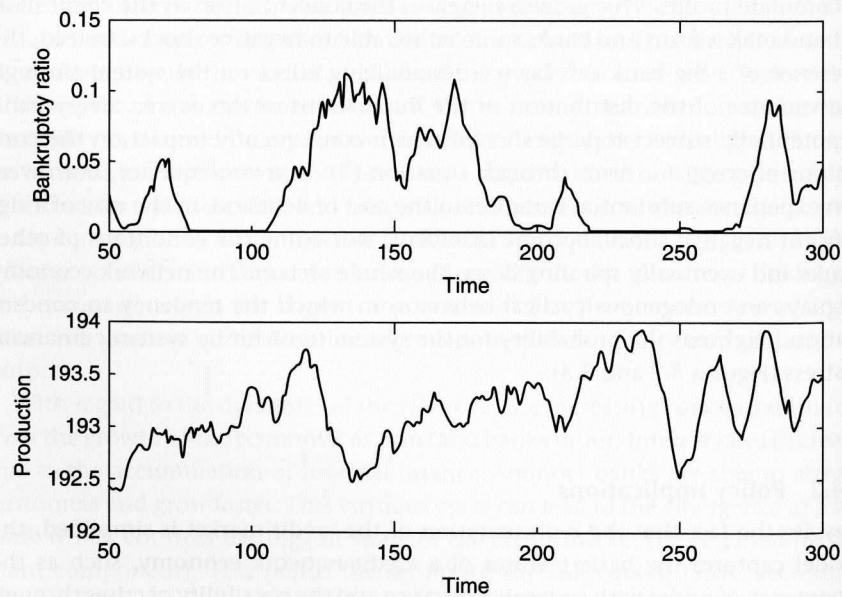


Figure 3.3 Banks' bankruptcy ratio and aggregate output

The model provides all the measures of systemic risk measures identified by the European Central Bank: (i) the degree of connectivity; (ii) the degree of concentration; and (iii) the size of exposures. Furthermore it analytically describes their dynamics and highlights the causal relationship among them. For these reasons, the framework can be helpful for a preliminary assessment of a stabilization policy.

The main policy tool embedded in the model is the adequacy ratio requirement $\bar{\Theta}$. By handling this parameter, the policymaker can influence the dynamics of the model and reduce the probability of a systemic collapse through different channels. The capital requirement directly influences the structure of the market in two ways. On the one hand, a reduction in $\bar{\Theta}$ defines a ceiling for the size of the giant component, diminishing the chances of the emergence of a big bank. As detailed in subsection 3.4.1, a smaller giant component reduces the variance of the distribution of fluctuations of the degree. The final effect is a smaller chance of large and sudden modifications in the market structure and a higher stability. On the other hand, the average degree (and thus the average size of banks) is directly proportional⁶ to $\bar{\Theta}$. This effect is amplified by the fact that a smaller $\bar{\Theta}$ is also likely to increase the competition in the credit market, lowering the interest rate spread among banks. The ultimate outcome is therefore a higher dispersion in the market and no big banks. Hence, the model illustrates that the size limit is also a limit on the interconnectivity, linking the concepts of *too big to fail* and *too interconnected to fail*. The introduction of a capital requirement allows the policymaker to shape the network topology and the market structure.

The parameter $\bar{\Theta}$ indirectly affects the probability of bankruptcy for firms. In fact, with stricter lending limits for banks, a firm with high leverage ratio is likely to be credit rationed, due to the banks' selection process, and thus to reduce its production and financial needs, consequently lowering its chances of bankruptcy.

At the micro level, $\bar{\Theta}$ indirectly influences the transition rates. A lower lending limit will bring about, on average, smaller interest rates, through equation (7), as only the NSF firms with the lowest debt ratio will be financed. The threshold \bar{u} in (12) will be lower, increasing the probability for a NSF firm to become SF. This chain of effects is particularly relevant as it allows the policymaker to influence the path of evolution of the system between different equilibriums. Indeed the economic policy, by influencing the transition rates, can impact on the dynamics described by the first of the (22). In this way, it can drive the proportion of NSF between the two limits 0 and 1 in order to set the economy on the preferred equilibrium path. The impact of the variations in $\bar{\Theta}$ in the numerical simulations are illustrated by Figure 3.4.

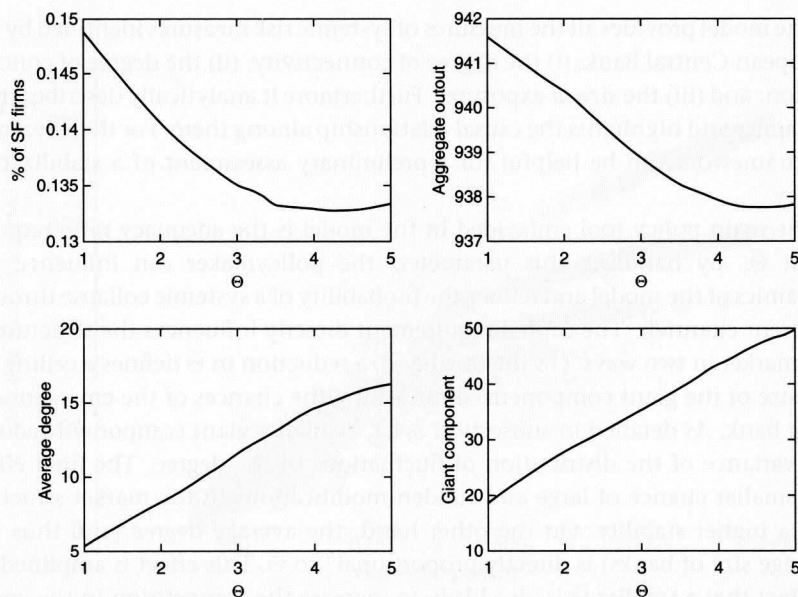


Figure 3.4 Average proportion of SF firms, output, degree and giant component for different values of q . Monte Carlo simulation

Summarizing, with reference to the three measures of systemic risk, the model shows that the capital requirement threshold influences: (i) the degree of connectivity, as it impacts on the average degree of network; (ii) the degree of concentration, as it directly determines the maximum size of banks and, indirectly, their average size; (iii) the level of exposures, by imposing a limit to lending. The solution of the model makes possible a quantitative and qualitative analysis of the impact of such a policy on the average degree, on the shape of the degree distribution and on the resilience of the credit market structure.

Through the stabilization of the credit market, policymakers can also influence the dynamics of production. An increase in the probability for NSF firms to become SF brings about a lower number of NSF firms in steady state and, consequently, a lower variance of the fluctuation component ϵ in equation (22). Equation (23) demonstrates that it causes a higher level and a smoother dynamics for aggregate output. Indeed, as the dynamics of output is dependent on the density of NSF firms, a lower variance of this density is accompanied by a smaller variance of output.

3.5 Concluding remarks

In this paper we propose a technique for the analytical solution for models with heterogeneous and interacting agents and apply it to a credit network model. In

particular, we describe the dynamics of the behavior of the agents by means of two MEs, one nested into the other. Their asymptotic solutions yield the trend and fluctuations of the two state variables: the proportion of NSF firms and the network degree.

The solution identifies some emerging properties of a credit network. We find that rising economic output, and the consequent increase in the overall wealth of firms, turns out to be proportional to how much the loans in the system come to be concentrated among a few banks. In network terms, this concentration can be measured by the average degree for banks. There is a natural tendency for this quantity to rise as the economy expands and banks and firms profit. This rise in concentration is potentially destabilizing for the system: the failure of a single bank can bring trouble to a large number of firms, which pass it on to other banks, leading to further failures, in a downward spiral. Cascades of failures put financial pressure on all firms, raising the costs of borrowing and slowing down the economy.

The model is able to endogenously generate feedback between economic growth and rising interconnectedness which leads to cycles of booms and busts. The solution of the model highlights the causal links among micro-, meso- and macro-variables. In this perspective, the present work provides a starting point for the development of more refined models of the credit network in order to test possible stabilization policies.

Notes

- * With the usual caveats we thank the participants to the Eastern Economic Association meeting 2011 in New York, the International Economic Association meeting in Beijing, the DGES in Paris, the CIES in Wien and the Econophys Colloquium 2011 for very helpful comments.
- 1. There are attempts of reconciling the empirical evidence with the mainstream model by introducing various imperfections. Unfortunately, small departures from the perfect information hypothesis have been shown to undermine most of the key propositions of the standard competitive model (Greenwald and Stiglitz, 1986).
- 2. In natural sciences, the notion of reductionism is much more limited since it amounts to represent the nature of macro-phenomena by analysing the constitutive elements, whose interaction allows for emergent phenomena, i.e. characteristics that are not present in the single element.
- 3. In the solution algorithm, without loss of generality, we consider it as an exogenous stochastic random variable.
- 4. This probability can be quantified as the product of the probability for a bank to be in a firm's pool, m , and the probability of being chosen by the firm. The latter should be different for each bank, as it is dependent on $\bar{\Theta}$ and A_b . Since the probability of matching needs to be defined at the mean-field level, we express it as a function only of $\bar{\Theta}$ and write it as: $1 - \exp(-c\bar{\Theta})$, with $c > 0$. Accordingly, p_1 is given by

$$p_1(\bar{\Theta}) = m[1 - \exp(-c\bar{\Theta})]. \quad (33)$$

As a consequence, the probability for two firms to be connected to the same bank is p_1^2 .

5. Computer simulations of the model with full degree of heterogeneity for banks and firms have been performed in order to provide some insights on the dynamics generated by the behavioral rules of agents. It is worth stressing that, in our study, numerical simulations are used just as a test of the analytical outcomes.
6. See equation (29).

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