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OPTIMAL PROVISION OF LOANS AND INSURANCE AGAINST UNEMPLOYMENT FROM A LIFETIME PERSPECTIVE

Joseph Stiglitz
Jungyoll Yun

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ABSTRACT

In an earlier paper, we showed that integrated individual accounts, allowing individuals to borrow against future pensions when they are unemployed, can be welfare increasing, because it allows increased inter-temporal consumption smoothing without attenuating incentives to search. Here, we examine from a lifetime perspective how the optimal mix between publicly provided unemployment insurance (UI) and loans against pension accounts changes over time in a model where unemployment may occur in any period. Even loans can have an adverse effect on search, because they attenuate the consequences of unemployment; and even more so when there is a chance that the loan will not be repaid. As we present the optimal mix of loans and UI as the one that balances the adverse incentive costs with the benefits of inter-state and inter-temporal smoothing while taking into consideration the interactions between loans and UI benefits, we provide general conditions under which loans should still be a part of the unemployment package for the young unemployed. We also show that, if the incidence of long-term unemployment is relatively low, the optimal mix entails more loans and a smaller UI benefit for the young than for the old, while the amount of consumption for the unemployed young is greater than for the unemployed old. We demonstrate that there will be incentives to save excessively in good states as well as to borrow excessively from the market when unemployed. Individuals and markets do not take into account the externalities of such actions: they affect search, and thus the magnitude of UI payments and loan defaults in subsequent periods. Finally, we show how non-market groups can improve welfare through loan-cosigning, which may be voluntarily provided within the group, as it allows income smoothing with lower incentive costs, and while the income sharing is less effective than market pooling, the incentive benefits of co-signing dominate.

Joseph Stiglitz
Uris Hall, Columbia University
3022 Broadway, Room 212
New York, NY 10027
and NBER
jes322@columbia.edu

Jungyoll Yun
Department of Economics,
Ewha University,
Seoul
Korea
jyyun@ewha.ac.kr

I. Introduction

Unemployment insurance has been criticized because of its adverse incentive effect.[1] For most individuals, the fraction of life time income that is lost as a result of episodic unemployment is small, so that individuals are close to risk neutral with respect to such losses. Hence, individuals should, with perfect capital markets, smooth their consumption, and using loans to do so, rather than insurance, would avoid the adverse incentives. The problem is that, with imperfections in capital markets, temporarily unemployed individuals are forced to cut consumption. Several studies (Chatty (2008)) have shown that the liquidity constraint is one of the most serious difficulties facing unemployed individuals. But allowing borrowing against one's retirement savings can effectively "resolve" the market imperfection created by capital market constraints. Self-insurance has the advantage that there are not associated moral hazard (adverse incentive) effects.

With capital market imperfections, while a loan provides inter-temporal consumption smoothing with little incentive costs, if there are large unemployment shocks (e.g. extending over a significant fraction of an individual's working life), it does not provide any inter-state (across states of the world where there is and there is not episodes of unemployment) consumption smoothing (insurance).[2] Unemployment insurance does this, but with some incentive costs. This suggests that a desirable form of income support for unemployed individuals may involve a combination of loans and UI benefit. In a model where the unemployment shock is small (so that there is no risk of default associated with loans), Stiglitz-Yun(2005) showed that under seemingly fairly weak conditions, provided that the duration of unemployment is limited, self-insurance through borrowing e.g. against future retirement benefits, could enhance welfare of workers by providing them with intertemporal income smoothing without attenuating incentives.[3] The optimal mix between loans and insurance always entails a positive amount of loans, collateralized by pension savings, suggesting that (contrary to current practice) individuals should be allowed to borrow at least a limited amount against their future retirement benefits when they are unemployed. There should not be complete reliance on unemployment insurance (UI). Indeed, when

[1] See Flemming (1978), Hopenhayen and Nicolini (1991).
[2] As the discussion below will make clear, this is not quite accurate, because if there is a risk of default, there is some inter-state smoothing.
[3] See also Altman and Feldstein (1998), Costain (1999)

unemployment risk is negligible compared to lifetime income, there should be no reliance on UI. Since there is negligible risk to lifetime income, the only market failure is related to the inability to borrow, and government loan programs (effectively collateralized by retirement savings) should be relied upon.

This paper analyzes the optimal combination of loans and UI in a model where unemployment may occur at any point in time in an individual's life, or alternatively, when an individual may experience long-term unemployment or may (again) get unemployed later in his lifetime. The precise mix depends upon the relative benefits and costs of the one compared to the other, which vary with one's employment history and with the point an individual is in his lifetime. It also depends on the importance of incentive effects (e.g. the elasticity of search). Since the amount of loans at any point is based upon one's lifetime income expected at the time of unemployment, the introduction of loans necessitates designing social insurance against unemployment and retirement from a lifetime perspective.[4] The package of unemployment benefit that is optimal from a lifetime perspective, on the other hand, has to be one that addresses the possible interactions between UI and loans across time and the possible response (in terms of savings and loans) from the private sector. Specifically, we will examine in this paper how the possibility of long-term (or extended) unemployment affects the optimal mix for the young unemployed and how the timing of being unemployed changes the optimal package.

While our earlier analysis suggested that when the risks of loss of income from temporary unemployment were very small, loans were preferable to UI (income was smoothed, but there was no attenuation of incentives), in a life-time model, there is a risk of extended unemployment. When that is the case, we show the benefits of loans may be limited, while an incentive costs associated with loans may arise. With the chance of extended unemployment, there is the risk that the requisite borrowing against retirement savings results in individuals depleting their pension accounts. This has two effects. First, it means that the ability to smooth consumption through loans is limited. And secondly, it means that there is a risk that individuals will not be able to repay what they have borrowed. The "bailout" that then results can be thought of as a form of UI for extended bouts of unemployment, but that, in turn, means that there are incentive effects associated with loans

[4] The analysis below will make it clear that even without a formal government program of lending against pensions (as is considered here), the fact that individuals borrow and save affects the optimal design of UI programs, and necessitates taking a life-time perspective.

as well as with UI; loans, like UI, attenuate incentives to get reemployed. Furthermore, if UI benefits are given to those with extended periods of unemployment, there may be interactions between the adverse incentive effects associated with the UI benefit and those associated with loans, implying a reduction in the scale of loans (from what they would be in the absence of a UI benefit). For instance, a larger UI benefit *in the future,* when they are unemployed again, exerts a negative externality upon the loans. It induces less search, and thus a higher likelihood of unemployment, and hence a higher likelihood of non-repayment.

The government program we characterize in this paper not only takes into account the externality among different unemployment benefits and between UI and loans, it also balances out adverse incentive effects with inter-temporal and inter-state consumption smoothing. The optimal program of this sort, which has not been dealt with in the literature, has some important features about the optimal mix between loans and UI, the optimal consumption for the unemployed and about the desirability of introducing loans for the unemployed, among others.

This paper shows that a loan should be a part of unemployment benefit package even when extended unemployment is highly probable, if the search elasticity is also high. This is because a high search elasticity leads to small UI, which increases the need for loans. This is true even if the probability of extended unemployment is high--so there is a high chance of non-repayment (a bailout). Despite the high incentive costs for the long-term unemployed, the optimal mix entails a positive amount of loans in earlier episodes of unemployment--in a sense loans become *more* desirable under the stipulated conditions, for under these conditions, the UI benefit (in the later period) becomes zero, when the search elasticity is high because of the high incentive costs, and thus the cross period/cross market externality associated with loans (normally, higher loans reduce incentives for search in later periods, and thus induce greater expenditures on unemployment insurance) is diminished.

This paper also allows us to understand how the optimal package of unemployment benefit changes with timing of being unemployed. Not surprisingly, the optimal mix of loans and UI changes over time and does not achieve perfect consumption smoothing: Unless the probability of being unemployed long-term is high, it should entail more loans and a smaller UI benefit when unemployed young than when old, while the amount of consumption for those unemployed young should be greater than for those unemployed old. After all, those unemployed when they are young anticipate that the losses are likely to be made up over the

rest of their lives; while those who are unemployed when they old know that that cannot be the case. This in turn means that there is greater need for insurance when an unemployment episode occurs later in life: the UI benefit should be smaller in the earlier periods than in the later periods. But this in turn means that those that are unemployed when young especially need intertemporal consumption smoothing—that is, there is a role for loans. But the possibility of long-term unemployment reduces the amount of loans for the young unemployed (from what it would be if there were not this risk), and the amount of consumption for those unemployed is decreasing in the probability of extended unemployment. Unless the probability of extended unemployment is fairly high, however, the loan provision may lead to a level of consumption for the young unemployed greater than that for the long-term unemployed (since there is a good chance that their lifetime income will not be too adversely affected by this bout of unemployment), while the opposite may be true in the absence of the provision of loans.

One interesting implication of our analysis is that, given the optimal package by the government, individuals may save excessively, and the private market may have excessive incentives for offering loans, as private individuals ignore the externality exerted upon the government program by savings and loans. This suggests that in implementing the optimal package the government needs to take into account the adverse incentive on the part of private markets. First, the excessive precautionary savings, while increasing the ability of individuals to smooth out consumption on their own, aggravates incentive costs (when individuals have a large "nest egg," they search less intensively). The excessive private savings implies smaller UI benefits (as well as a greater reliance on self-insurance) in the future optimal package of benefits than would be the case if the level of savings could be controlled by the government.

Similarly, unfettered markets may offer excessive loans, since lenders will not take into account the adverse externalities to either the public (retirement) loans or UI programs. With more private loans, individuals incentives to search are reduced, and the risk of a “bail-out” on the retirement-lending program is increased. This paper thus uncovers a new market failure—the risk that the market provides too much income smoothing.[5] To discourage the

[5] In a sense, this market failure is related to that analyzed by Arnott and Stiglitz (1991), who point out that the provision of insurance against one risk may affect risk taking affecting other insurance contracts. This, in turn, is related to the fundamental non-decentralizatibility theorem of Greenwald and Stiglitz [1986).

market from offering excessive loans for the unemployed, the government has to offer more UI (than it would in the absence of private loan markets). In short, unrestrained loan markets are socially dysfunctional: It leads to too little efforts at job search. It is markets, not government, that, in some sense, is responsible for *excessively* high unemployment. Of course, with government programs, unemployment may be higher than it would be without government programs, but optimally designed government programs balance out carefully the benefits of risk reduction and the costs of any induced unemployment.[6] Alleged improvements in capital markets--increasing the availability of private loans for the unemployed--can be welfare reducing.

On the other hand, we show that, unlike the market, a *non-market* group (such as family, village, etc.) that has a superior monitoring ability (to that of government or markets) and a sense of peer pressure among its members, can be used to improve welfare as they supplement publicly provided UI benefits through loan-cosigning.[7] The informational advantage and the peer pressure associated with a non-market group can interact with each other, leading to an equilibrium where loans for one member are voluntarily cosigned by another member. This equilibrium can be Pareto superior to one without co-signing. In these situations, there is a positive externality between individual actions and government programs.

There is, of course, an extensive literature on optimal unemployment insurance (Hopenhayn-Nicolini (1997), Kocherlakota(2004), Shimer-Wernings (2005)). The existing literature focuses, however, on single episodes of unemployment, e.g. how consumption changes during the unemployment tenure of an individual, while this paper analyzes *multiple episodes of unemployment,* how the optimal consumption for the unemployed changes with the timing of unemployment in one's lifetime career, and how that affects the design of public programs.

The rest of this paper is organized as follows. The next section describes the basic model

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Chetty and Saez(2010) discuss, in a general framework of insurance provision, how the presence of private insurance market affects the optimal social insurance.

[6] Of course, some governments may have provided UI benefits in excess of the optimum. Our analysis shows that to obtain the optimal level of unemployment, restrictions have to be placed both on the amount of insurance that government provides and the amount of borrowing that individuals can undertake.

[7] This can be compared to Arnott-Stiglitz(1991), who argues that the presence of a non-market group may not be welfare-increasing in the provision of insurance unless it has perfect control of the actions taken by its members.

that characterizes the optimal mix of UI benefits and loans from the lifetime perspective and analyzes how it varies with changes in the probability of unemployment in later periods and the possibility of the extended unemployment. Section III explores how the externality associated with private loans and savings affect the optimal program for the unemployed by the government, while Section IV addresses the welfare implications of loan-cosigning. Section V collects the main results of this paper with some concluding remarks.

II. The Model and Baseline Optimum

Consider a 3-period model in which an individual may work for period 1 and 2 at the wage w per period, and then retires in period 3 (Fig.1). For simplicity, we assume w is fixed and there is no discounting (the safe rate of interest is zero). The worker may be confronted with an unemployment shock in each of the two periods. The probability of an unemployment shock occurring to an individual in period 1 is q, while that in period 2 depends upon whether or not he is unemployed in period 1. The probability of a shock in period 2 for a worker who was previously employed is p_N, while that for a worker who was unemployed is p_U.[8]

There are thus three different unemployment shocks in the model: unemployment shock in period 1 (called unemployment shock 1), unemployment shock in period 2 for those who have not been unemployed (unemployment shock 2), and unemployment shock in period 2 for those who have previously been unemployed (unemployment shock 3). Each unemployment shock occurs at the beginning of the relevant period (that is, the individual at any point of time does not know whether he will experience future unemployment shocks, though we assume he knows all the relevant probabilities). After each shock, a worker may choose to search or not to search for a job. If he expends sufficient search effort e, then he finds a job; if he doesn't search, he is unemployed that period.[9] [10] Search costs may differ

[8] A couple of different interpretations of the parameter p_U should be noted. It could be interpreted as the correlation coefficient between the two unemployment shocks in period 1 and 2, i.e. $p_U > p_N$ implies that an individual who is unemployed today has a higher probability of facing an "unemployment shock" next period than an individual who is employed. $p_U = 1$ means that an individual who is unemployed today will be unemployed next period (in the absence of search) . In a slightly different formulation, with analogous results, p_U p_U could also refer to the duration of unemployment relative to one's lifetime income.

[9] An individual who is unemployed in period 1 and does not experience an unemployment shock in period 2 can be thought of as having been laid off for one period.

across the three shocks: we denote e_1, e_2, e_3, for the amount of search required to find a job, given the unemployment shock 1, 2, 3, respectively. The search cost $\{e_1, e_2, e_3\}$ are independent random variables with distribution functions F_1, F_2, F_3, respectively. The individual finds out his search costs before committing to search.

It is easy to show that there exists a threshold level $\bar{e}_i$ $(i = 1,2,3)$, such that when the realized search cost for an individual in a period is lower (higher) than a threshold level, he will choose to search (not to search). Hence, if the threshold for the unemployment shock 1 is $\bar{e}_1$, then the first period unemployment rate *among those who have faced the shock is* $1 - F_1(\bar{e}_1)$. We denote by $h_i \equiv \frac{f_i(e)}{1-F_i(e)}$ $(i = 1,2,3)$ the search elasticity of unemployment, i.e., the degree of sensitiveness of unemployment with respect to search activity, for a shock i, and assume that h_i is constant over e.

An unemployed worker (under shock i) receives money from the government consisting of two components: an unemployment insurance (UI) benefit r_i $(i = 1,2,3)$, which does not have to be repaid), and a loan in the amount of R_i $(i = 1,2,3)$. The loan for a worker is to be repaid out of his retirement income, which consists of the savings he has made out of his wage income during his working career. If an individual gets unemployed again in period 2 as well as in period 1, his retirement income will become zero so that he may be unable to repay what he borrowed in period 1 when he was unemployed. In this case the government bails him out. But at the time the government provides the "loan" in period 1, it does not know whether the individual will have to be bailed out.[11]

The UI benefit provided to the unemployed workers is financed by an (unemployment) tax imposed upon employed workers. For analytic simplicity, we assume that there are three separate UI insurance programs, each insuring against each of the risks (i.e. against the three

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[10] The search activity in the model takes no time and guarantees a job for the worker with probability 1. Thus, a worker choosing to search will not be unemployed in the period, like the one with no unemployment shock. Here we also assume away the possible externality that can be caused by search by assuming that the number of job openings is so large that an individual search activity may not affect the return to search of the others.

[11] We put the word "loan" in quotations, to remind the reader that it is a "contingent" contract, to be paid back only if the individual is employed in the second period. But, of course, all loans are, in this sense, contingent, to be paid back if and only if the individual can, e.g. if he does not declare bankruptcy.

The government would not provide a loan in period 2 to those who were unemployed in period 1 as the government knows that they would not have any income left for retirement. This point will be made clear later in this section.

different shocks.) In particular, the UI benefits for shock 1, 2 and 3, - r_1, r_2 and r_3 - are financed by the tax T_1, T_2 and T_3 that are imposed upon those who get employed under shock 1, 2 and 3, respectively. (See Figure 1) Also, we assume that the *expected* cost of the bail-out for defaulted loans is, in effect, borne by borrowers at the time of borrowing as part of the price for the loan. Heuristically, we can think of the loan a facilitating life time consumption smoothing while the UI benefit focuses on inter-state smoothing. Note, however, that a loan with the possibility of bail-out provides insurance against an unemployment shock in period 2. Intertemporal smoothing and interstate smoothing are interlinked.

In this section, we assume that the government is the only provider of unemployment insurance and loans for the unemployed workers[12] and that individuals are homogeneous.[13]

Let $V_i(i = N, U)$ denote the lifetime expected utility for a worker who is employed or unemployed in period 1. The lifetime expected utility V for an individual at the beginning of period 1 will be

$$V = \mathrm{Max}_{\bar{e}_1, s_1}\ (1 - \bar{q})V_N + \bar{q}V_U - q\int_0^{\bar{e}_1} e dF_1 \qquad (1)$$

where q is the probability that the shock 1 occurs, and

$$\bar{q} \equiv q(1 - F_1(\bar{e}_1)),$$

giving the probability of being unemployed under shock 1. $\bar{e}_1$ is the threshold search cost in period 1, above which individuals do not search; it is determined as follows:

$$\bar{e}_1 = V_N - V_U \qquad (2)$$

where V_N and V_U are the life-time expected utility of an individual who is employed (not

[12] Or, we can assume that the government can perfectly control provisions of benefits and loans from private sectors so that it may implement optimal package of unemployment compensations.

[13] When individuals are heterogenous and they are privately informed of their types, i.e., when they differ in the probability of getting re-unemployed in period 2 after being unemployed in period 1, for example, the government provision of loans may be desired compared to the market provision.

employed) in period 1. The individual searches if the lifetime benefits from search are worth the costs.

We will now calculate these lifetime values. Let V_{ij} $(i, j = N, U)$ indicate the lifetime utility expected at the beginning of period 2 for a worker who is employed or unemployed in period 2 after having been employed or unemployed in period 1. Normalizing the constant wage to 1 for simplicity, we then have the individual maximizing with respect to e_i, given s_i, and the government maximizing with respect to s_i:

$$V_N = \mathrm{Max}_{s_1}\{U(1 - s_1 - T_1)$$
$$+\mathrm{Max}_{\bar{e}_2}\{[1 - p_N(1 - F_2(1 - \bar{e}_2))]V_{NN} + p_N(1 - F_2(1 - \bar{e}_2))V_{NU} - \int^{\bar{e}_2} e dF\}\}$$
$$V_{NN} = \mathrm{Max}_{s_2}\{U(1 - s_2 - T_2) + U(s_1 + s_2)\} = 2U(\tfrac{1+s_1-T_2}{2})$$
$$V_{NU} = \mathrm{Max}_{R_1}\{U(r_2 + R_2) + U(s_1 - R_2)\} = 2U(\tfrac{r_2+s_1}{2})$$
$$V_U = U(r_1 + R_1(1 - P))$$
$$+\mathrm{Max}_{\bar{e}_3}\{(1 - p_U(1 - F_3(\bar{e}_3)))\, V_{UN} + p_U(1 - F_3(\bar{e}_3))V_{UU} - \int^{\bar{e}_3} e dF\}$$
$$V_{UN} = \mathrm{Max}_{s_3}\{U(1 - s_3 - T_3) + U(s_3 - R_1)\} = 2U(\tfrac{1-R_1-T_3}{2})$$
$$V_{UU} = 2U(\tfrac{r_3}{2})$$

where $\{ s_1, s_2 , s_3 \}$ denote savings for an individual in the three states of nature and $\bar{e}_2, \bar{e}_3$ indicate the threshold search cost upon unemployment shock 2 and 3, respectively. (In this formulation, the government gets to set the savings rate s_i; later, we will deal with the more general case.) We can then see that

$$\bar{e}_2 = V_{NN} - V_{NU} \qquad (3)$$
$$\bar{e}_3 = V_{UN} - V_{UU} \qquad (3')$$

and

$$T_1 = \frac{\bar{q}}{(1-\bar{q})} r_1$$
$$T_2 = \frac{\bar{p}_N}{(1-\bar{p}_N)} r_2$$

$$T_3 = \frac{\bar{p}_U}{1-\bar{p}_U} r_3$$

where the probability of a person who is employed or unemployed the first period being unemployed the second is given by $\bar{p}_N$ or $\bar{p}_U$, respectively, i.e.,

$$\bar{p}_N \equiv p_N\left(1 - F_2(1 - \bar{e}_2)\right)$$
$$\bar{p}_U \equiv p_U\left(1 - F_3(1 - \bar{e}_3)\right)$$

The price of loan, P, refers to the price of public loans provided to those unemployed in period 1, which is equal to the probability of default, $\bar{p}_U$. An individual who gets a loan that obligates him to repay R_1 receives only $R_1(1 - , \bar{p}_U)$ and has consumption of $r_1 + R_1(1 - , \bar{p}_U)$. Note that the only "bail out" occurs in the case of an individual who is unemployed in the first period andf remains unemployed in the second. Note also that individual savings s_2, s_3 are determined so as to equalize consumption across periods.

The optimal savings s_1 in period 1 is determined so as to balance inter-temporal consumption smoothing against the disincentive associated with savings:

$$-U'(1 - s_1^* - T) + (1 - \bar{p}_N)U'\left(\frac{1+s_1^*-T}{2}\right) + \bar{p}_N U'\left(\frac{r_2+s_1^*}{2}\right)$$
$$-h_2 \frac{\bar{p}_N}{1-\bar{p}_N} r_2\{U'\left(\frac{r_2+s_1^*}{2}\right) - U'\left(\frac{1+s_1^*-T}{2}\right)\}U'\left(\frac{1+s_1^*-T}{2}\right) = 0. \qquad (5)$$

In this paper we treat the savings in period 1 as the mandatory ("social security') savings for retirement. This can be justified by the possible moral hazard behavior in the choice of savings that can be caused by the government subsidy r_3 to those with no income for retirement, which is a part of the government program in this paper.[14]

On the other hand, the private savings s_1^o that individuals would like to make, which is different from s_1^*, is determined as

$$-U'(1 - s_1^o - T) + (1 - \bar{p}_N)U'\left(\frac{1+s_1^o-T}{2}\right) + \bar{p}_N U'\left(\frac{r_2+s_1^o}{2}\right) = 0. \qquad (5')$$

[14] Without the mandatory retirement savings in period 1, the loan for the unemployed in period 2 may not be necessary, because individuals can consume a part of their savings when unemployed.

We can then prove the following Proposition.

Proposition 1

$s_1^* < s_1^o$.

This is clear from the comparison of (5) with (5'). That is, because private individuals do not take into account the incentive costs caused by their savings, *the level that individuals would save on their own account (private savings)* s_1^o *is greater than the socially optimal level* s_1^*. This implies that individuals will make saving beyond the optimal level s_1^* mandated by the government. We will assume for the remaining part of this paper that, whenever applicable, the government can implement the optimal savings s_1^* on the part of individuals by imposing taxes upon the extra (private) savings individuals make in period 1. We will check later, however, how, if this is not the case, the uncontrolled private savings may affect the government program for unemployed individuals.[15]

The interior solution for the optimum unemployment insurance/loan program $\{r_i^*, R_i^*\}(i = 1,2,3)$, which maximizes the lifetime expected utility V will then satisfy the following conditions (by the envelope theorem on the savings s_1):

r_1^* :

$$U'\left(r_1^* + R_1^*(1-\bar{p}_U)\right) - M_1(T_1) - \frac{h_1}{1-\bar{q}} r_1^* \, U'\left(r_1^* + R_1^*(1-\bar{p}_U)\right) \cdot M_1(T_1) = 0 \qquad (6)$$

R_1^*:

$$U'\left(r_1^* + R_1^*(1-\bar{p}_U)\right) - M_3(T_3) - \frac{h_3\bar{p}_U}{1-\bar{p}_U}\{R_1^* U'\left(r_1^* + R_1^*(1-\bar{p}_U)\right) + \frac{1}{1-\bar{P}_U} r_3^* M_3(T_3)\} M_3(T_3) = 0 \qquad (7)$$

r_2^* :

[15] Our analysis is based on full individual rationality. In practice, individuals may be excessively myopic, so that they do not fully anticipate future risks and needs; in that case, private savings will be less than the desired mandatory level. Our welfare criterion focused on maximizing social welfare, given the actual probabilities of unemployment, not ex ante expected utility, given individual's failure to take fully into account future risks of unemployment.

$$U'(r_2^* + R_2^*) - M_2(T_2) - h_2 \frac{1}{1-\bar{p}_N} r_2^* U'(r_2^* + R_2^*) \cdot M_2(T_2) = 0 \qquad (8)$$

R_2^*:

$$R_2^* = \frac{s_1^* - r_2^*}{2} \qquad (9)$$

r_3^* :

$$U'\left(\frac{r_3^*}{2}\right) - M_3(T_3) - h_3\left\{R_1^* U'\left(r_1^* + R_1^*(1-\bar{p}_U)\right) + \frac{1}{1-\bar{p}_U} r_3^* M_3(T_3)\right\} U'\left(\frac{r_3^*}{2}\right) = 0 \qquad (10)$$

R_3^*:

$$R_3^* = 0 \qquad (11)$$

where $M_1(T_1) \equiv U'(1 - s_1^* - T_1)$, $M_2(T_2) \equiv U'\left(\frac{1+s_1^*-T_2}{2}\right)$ and $M_3(T_3) \equiv U'\left(\frac{1-R_1^*-T_3}{2}\right)$, is the marginal disutility of tax T_1, T_2 (for an individual employed either in period 1 or in both of the two periods) and of tax T_3 (for an individual employed in period 2 only), respectively. The above conditions imply that the optimal UI benefit and loan in each of the optimal government programs is determined so as to balance its benefit of consumption smoothing across states or periods with the moral hazard (adverse incentive) costs associated with them.[16]

The first order conditions make clear that there are some interesting interactions between unemployment insurance and loans that can affect the optimal mix in an important way. The first period loan R_1 and the second period UI benefit r_3 for the long-term unemployed, for example, exert an externality upon each other as one affects the probability of reemployment which in turn affects the incentive costs of the other. This interaction highlights an important aspect of the optimal package of unemployment benefits from a lifetime perspective, which has not been dealt with in the optimal UI literature.

Using the necessary conditions presented above, we will analyze the desirability of introducing loans as a part of unemployment benefits in the presence of its default possibility and how the optimal package of UI benefit and loans changes over lifetime depending upon timing of unemployment.

Desirability of Loan Provision

[16] Equations (6) –(10) highlight the different incentive effects and how these are affected by changes in various of the policy variables. In (6), for instance, the incentive effect of UI in shock 1 is affect both by UI and by loans. Interestingly, in our formulation, r_1 only affects first period search, R_1 only second period search (in shock 3), and r_2 only second period search (in shock 2).

We will first explore whether or not it is desirable to introduce loans when there is some chance that an individual remains unemployed for a long time and thus may not repay what he has borrowed,[17] and how the loan provision affects optimal consumption of the young unemployed relative to that of the long-term unemployed. In particular, we will address this issue by analyzing loans R_1 and consumption $(r_1 + R_1(1 - \bar{p}_U))$ for the unemployed under shock 1.

We will first consider as a benchmark a set of parameters (q, h_1)'s, Ω, under which loan provision is desirable. For that purpose we substitute $R_1 = 0$ into (6) and (10) to get the following two conditions:

$$U'(r_1^*) = \frac{M_1}{1 - h_1 \frac{r_1}{1-\bar{q}} M_1} \equiv G_1(q, h_1) \qquad \text{(A1)}$$

$$h_3 \frac{1}{1-\bar{p}_U} r_3^* M_3 = \frac{U'\left(\frac{r_3^*}{2}\right) - M_3}{U'\left(\frac{r_3^*}{2}\right)} \equiv G_2(p_U, h_3) \qquad \text{(A2)}$$

We can then prove the following Lemma on $G_1(q, h_1)$ and $G_2(p_U, h_3)$:

Lemma

1) There exists a unique r_1^* or r_3^* that satisfies (A1) or (A2), respectively.

2) $\frac{\partial G_1}{\partial q} > 0$, $\frac{\partial G_1}{\partial h_1} > 0$ and $\frac{\partial G_2}{\partial p_U} > 0$, $\frac{\partial G_2}{\partial h_3} > 0$.

The proof can be found in the Appendix.

Let

$$\varphi(q, h_1; p_U, h_3) \equiv G_1(q, h_1) - M_3\left(1 + \frac{\bar{p}_U}{1-\bar{p}_U} G_2(p_U, h_3)\right),$$

to define

$$\Omega(p_U, h_3) \equiv \{(q, h_1) | \varphi(q, h_1; p_U, h_3) > 0\}.$$

[17] Recall that for this part of the paper, in analyzing the optimal government program $\{r_i^*, R_i^*\}$, we will assume that the government can control private savings so that $s_1 = s_1^*$.

That is, $\Omega(p_U, h_3)$ is a set of (q, h_1)'s under which the optimal benefit package involves a positive amount of loans in the presence of a possibility of extended unemployment. We can then establish the following proposition.

Proposition 2

i) *Condition for positive* R_1^*:

(a) If $(q, h_1) \in \Omega(p_U, h_3)$, $(q', h_1) \in \Omega(p_U, h_3)$ and $(q, h_1') \in \Omega(p_U, h_3)$ for $q' > q$ and $h_1' > h_1$.

(b) There exists $K(p_U, h_3)(> 0)$ for any given p_U, h_3 such that, if $U'(0) > K(p_U, h_3)$, $\Omega(p_U, h_3) \neq \emptyset$.

(c) Suppose $(q, h_1) \in \Omega(0,0)$. There then exists $p_U'(> 0)$ for *any given* h_3 such that $(q, h_1) \in \Omega(p_U, h_3)$ for $p_U \leq p_U'$. .

ii)) *Relative Consumption of Long-term Unemployed*:

There exists $k(> \frac{1}{2})$ such that, for $p_U \leq k$, $r_1^* + R_1^*(1 - \bar{p}_U) > \frac{r_3^*}{2}$.

Proposition 2 (i) suggests that the desirability of having positive loans as part of an unemployment benefit program is not only affected by the probability (p_U) of unemployment being extended or search elasticity (h_3) associated with future reemployment, but also has to be viewed in its relationship with the current UI (r_1) and the future UI (r_3) for the long-term unemployed. The loan is to some extent a substitute for the current UI (r_1), suggesting that higher q and/or higher h_1, which reduces r_1, increases the desirability of positive loans R_1, as is shown in Proposition 2 i)(a). While R_1 may improve search incentives in earlier periods, on the other hand, it exerts a negative externality upon the future UI for the long-term unemployed, as it affects the probability of reemployment in the future. Based upon these relationships between R_1 and r_1 and between R_1 and r_3, Proposition 2 i) demonstrates two results.

First, for the unemployed whose marginal utility of income is high at zero income, loans become a part of the optimal benefit package even when the probability of extended unemployment (p_U) is high if the first period incentive costs (h_1) or probability (q) of unemployment shock is high enough to yield a small UI benefit for them. In particular, *if search elasticity (h_1) associated current unemployment is very high, loans would always be a*

part of the optimal unemployment benefit under shock 1, no matter how high the probability of extended unemployment (p_U) or search elasticity (h_3) associated with future unemployment may be.

Second, high search elasticity (h_3) may not prevent loans from being a part of optimal benefit package, as is implied by Proposition i) (c). To see this we need to understand that the search elasticity (h_3) and the probability (p_U) of extended unemployment affect the UI r_3 for the future unemployment and the loan R_1 differently: higher search elasticity (h_3) reduces r_3 more than it reduces R_1, whereas higher p_U reduces R_1 more than it reduces r_3. The externality relationship between r_3 and R_1 would then suggest that *when search elasticity (h_3) associated with the future unemployment is high, unless the probability of long-term unemployment (and the resulting default) is high, it is optimal to introduce some loans as a part of the benefit package for unemployed individuals.*[18]

The intuition for this result is the following: Loans and UI benefit r_3 adversely affect search, exerting in effect negative externalities on each other and implying that, the presence of loans decreases the optimal level of UI and vice versa. Now, as the sensitivity of search later in life to effort increases (h_3 increases), overall benefits when the individual is unemployed later in life decrease. The question is, as h_3 increases, which of the two – loans and r_3 - becomes zero earlier than the other. Proposition 2 shows that r_3 does, unless the probability p_U of extended unemployment is high. The main reason is the following. Although the consumption-smoothing (across states) effect is larger for r_3 than for loans, the incentive effect is also larger. The incentive effect for loans is limited by the probability p_U of extended unemployment; that is not the case for r_3, so that a lower p_U increases the difference in the incentive effect between the two. Note, on the other hand, that when UI benefit r_3 for long-term unemployment is zero because of high incentive costs (indicated by a high value of h_3), the loans R_1 would not exert any externality upon any other unemployment benefit so that it will be a part of the optimal benefit package .

Proposition 2 (ii) says that there is imperfect consumption smoothing: Those who are unemployed the first period have a higher level of consumption the first period than they do in the second. This is perhaps not surprising: The first period, unless the probability of long-term unemployment is high, their expected income is still high because of the

[18] Note that it is the externality loans exert upon r_3 that leads to zero loans in the optimal package. If r_3 is set to be zero exogenously, the optimal amount of loans R_1 would always be positive despite a high probability of extended unemployment.

expectation of a second period job, and they wish to smooth consumption over their lifetime. Insurance does not fully replace lost income, and so when the adverse shock occurs in the second period, consumption has to decrease. Furthermore, the loan provision for the young would reduce UI benefit he could have in the future because of the externality between the two.

Loans for the long-term unemployed are zero in this model, as they have no future income to borrow against. If we solve, in an unrestricted way, for the optimal value of R_2, loans for those unemployed in period 2 only, which does not involve any possibility of loan-default (since it is effectively backed by first period savings), it may be positive or negative, depending upon the relative size of UI r_2 and savings s_1 made in period 1. Negative loans implies that the unemployed would like to save (in period 2) out of UI benefit r_2 for retirement. We will leave discussion of loans for those unemployed in period 2 to Proposition 6, which deals with the optimal mix in the presence of private savings.

Now let us turn to the optimality of introducing UI benefit in the presence of loan provision. While the substitutability between UI benefit and loans may lead to the optimal mix involving no loans, it does not imply that the optimal mix may involve no UI benefit. We can prove the following Proposition on this issue, with the proof being delegated to the Appendix.

Proposition 3

It is always the case that $r_i^* > 0$ for $i = 1,2$, while it may be true that $r_3^* = 0$.

Note that the amount of expected lifetime consumption per period for an unemployed individual with no UI benefit is smaller than that for an employed individual. This implies that the optimal mix always entails positive amount of UI benefit unless it exerts some externality upon some other unemployment benefits (including the implicit benefit associated with a loan bailout). The optimal mix for the young unemployed always entails positive UI r_1 or r_2 as it does not exert any externality upon other unemployment benefits, while it may involve zero UI r_3 for the long-time unemployed as it exerts an externality upon the loans R_1 for the young unemployed through its price $\bar{p}_U$.

Now let us examine $\{r_i^*, R_i^*\}$ (i=1,2), analyzing how the government loan program and the resulting unemployment benefit will be affected by the timing of unemployment. In this analysis we will mostly presume that the probability of long-term unemployment is small in order to focus on the effect of timing of unemployment upon optimal mix, i.e., the optimal mix for those unemployed only in period 1 in comparison with that for those unemployed only in period 2. We can state the following proposition.

Proposition 4

Suppose that $F_1(x) = F_2(x)$ for any x and that $p_N = q$. There then exists $p'_U(> 0)$ such that, for $p_U \leq p'_U$, the followings are true: a) $r_1^* < r_2^*$ and $R_1^* > R_2^*$, b) there exists $p(> 0)$ such that for $r_1^* + R_1^*(1 - \bar{p}_U) > r_2^* + R_2^*$ for $p_N = q < p$, while $r_1^* < r_2^*$ in the absence of loan provision.

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Proposition 4 compares the optimal mix for the young unemployed with that for the old unemployed. It looks at the central case where the search cost distributions in the first period, and in the second period, for those who have been employed in the first, are identical and where the probability (P_N) of an unemployment shock the second period to someone who was not unemployed the first is equal to the probability (q) of an unemployment shock the first period**.** It makes two assertions. First, if the probability of extended unemployment is low enough, then the first period UI benefit is smaller than the second period UI benefit, but the first period loan (to those unemployed) is greater than the second period loan. Second, if the incidence of unemployment is not high, the increase in the first period loan outweighs the loss in the UI benefit in the same period, implying that the loan provision enables the consumption for the unemployed in the first period to be greater than that for those unemployed in the second period only, while the opposite is true in the absence of loan provision.[19]

The intuitions for these results are as follows. In comparing the optimal mix for the

[19] The intuition for the result is the following. When the probability of unemployment being extended is small, the first period unemployed would suffer from smaller reduction in the expected utility compared to those unemployed in the second period. Those unemployed in the first period expect to be employed in the second, and hence can effectively smooth their income, while those unemployed in the second period have just savings they made (in the first period) for retirement. Note also that this result should hold in the case of a corner solution for loans, i.e., when no provision of loans is optimal as in the case described by Proposition 2 (when q is very low, for example).

young unemployed with that of the old, we need to take into consideration the two points: (a) an unemployed young person has a longer time over which to smooth consumption; (b) one can only smooth going forward, not going backward. First of all, the larger UI benefit r_2^* for the old unemployed compared to r_1^* for the young ones is mainly because, when p_U is so small that an individual could be unemployed either young or old (not both), those unemployed young have more periods over which they smooth out their lifetime income *after being unemployed* than do those unemployed when old.[20] By the concavity of utility function this leads to the smaller difference in the lifetime expected payoff between unemployment and employment under shock 1 compared to that under shock 2.[21]. Secondly, the larger amount of loans R_1^* for the young unemployed compared to that R_2^* for the old ones is not only due to the substitutability between UI benefit and loans but also to the fact that the young unemployed have larger amount of future income to borrow against than the old ones. Specifically, in the absence of insurance, the expected lifetime resources for those unemployed in period 1(at the beginning of that period) is close to 1 (or w) while that for those unemployed in period 2 (at the beginning of that period) will only be the savings s_1 they made in the previous period. So long as the incidence of unemployment is small, consumption for the young unemployed will be greater than that of the old, because precautionary savings will be low, R_1^* is greater than R_2^* (by an amount that is larger than s_1), while the difference in UI benefit between the young and the old becomes smaller due to the smaller difference in incentive costs associated with UI. When there are no loans, however, consumption of the young unemployed would be lower, because UI benefit is still subject to a more serious incentive problem under shock 1 than under shock 2 when $p_U \approx 0$.

Comparative statics: effects of changes in the incidence of unemployment and its expected duration

We now examine how the optimal mix of unemployment benefits changes as the incidence of unemployment or its (expected) duration varies. In doing this we will focus upon its impact on $\{r_1^*, R_1^*\}$.[22] In particular, we assume for simplicity that $P_N = 0$ and that the UI

[20] Note that, prior to unemployment shock, individuals would not be able to effectively smooth out their consumption over time.

[21] This will be true even for small p_N or q. When p_N or q gets greater, the case for smaller r_1 (relative to r_2) becomes stronger as the expected payoff from being employed under shock 1 is decreasing in p_N.

[22] Note that $R_3^* = 0$ while R_2^* is uniquely determined by r_2^* through inter-temporal smoothing (as R_2^* does not entail any bailout costs).

benefit r_3 for shock 3 is exogenously given as a socially acceptable minimum level $\bar{r}$ of consumption.[23] In this case individuals choose the socially optimal level of savings s_1 in period 1[24]:

$$s_1^* = \frac{1-T}{3} = s_1^o. \qquad (12)$$

Confining ourselves to the case when the optimal mix of UI benefit and loans involves a positive amount of each type of benefit, we can establish the following Proposition:

Proposition 5 (Comparative statics of unemployment insurance)

As the two unemployment shocks in period 1 and 2 get more highly correlated (as p_u increases), the optimal mix of unemployment benefits involves a greater UI benefit, r_1^*, and smaller loan-based self-insurance, $R_1^*(1-\bar{p}_U)$. The converse will be true, however, when the probability q of the period 1 unemployment shock 1 gets higher. The total unemployment benefits $(r_1^* + R_1^*(1-\bar{p}_U))$ is decreasing in q or in p_U.

The results of Proposition 5 are intuitive. First, as an unemployed worker is more likely to experience a longer duration of unemployment, i.e., as p_u gets higher, the welfare benefit from loan-based self-insurance decreases, because loans provide no interstate smoothing and limited inter-temporal smoothing but still suffer from incentive costs. This leads to a partial substitution of UI benefits for loans, while the total amount of unemployment benefits (UI plus loans) decreases, not surprisingly, given the decrease in total expected lifetime income. Second, when the probability q of the period 1 unemployment shock gets higher, the burden of financing UI benefit gets larger, leading to an optimal mix involving a smaller total amount of unemployment compensation, with a smaller fraction of the benefits in the form of UI benefits.

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[23] Those who get unemployed for both of the two periods should be the ones who need to be assisted by the government through the various social assistance programs. These expenditures represent the "bail-out" described earlier. But earlier, we chose that level optimally based on our "expected utility" model. In many cases the level of assistance for these individuals tend to be determined by social and political factors, as well as economic ones.

[24] So the first problem on which we focus in the next section does not arise.

III. Presence of Private Loan Market and Excess Savings

We have thus far assumed that the government can ensure that individuals' savings s_1 in period 1 are at the optimal level s_1^*. If the government cannot control individual savings, however, it has to take into account the change in private savings in response to the government unemployment insurance program. The fundamental problem is suggested by Proposition 1: individuals wish to save more than the government would like them to. The reason is simple: if individuals have more savings, if they become unemployed, their incentive to search is lessened, and accordingly there are externalities to government loan and UI programs, which individuals will fail to take into account.[25]

So too, so long as the optimal provision of loans entails incomplete inter-temporal smoothing due to the moral hazard, private lenders would have an incentive to provide additional loans, as they would not take into consideration the effect of their loan provision upon search and losses under the government loan and UI programs.[26]

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Excess savings

We can prove the following Proposition on the government program (r_2^0, R_2^0) in the presence of uncontrolled private savings.

Proposition 6

$r_2^0 < r_2^*$ if $U''' \approx 0$.

The proof can be found in the Appendix. The proposition says that, if the government cannot intervene to cure the distortion in savings caused by the externality, private savings

[25] If individuals had wanted to save less than the government wanted them, then all the government would have had to worry about is individual borrowing against their retirement earnings privately, undoing, in effect, the "forced" government savings. Restrictions on the set of admissible enforceable contracts and/or requiring individuals to register loan contracts to be enforceable, and imposing taxes (to compensate for the externality generated) could be used to restrict such excessive lending. (These comments are relevant to the discussion in the second subsection below. It is obviously much more difficult to restrict excessive savings, simply because savings can take so many different forms, some of which are hard for government to detect.)

[26] While this paper should be viewed as mainly a contribution to the literature on the design of unemployment insurance schemes, it can also been seen as a contribution to the literature on search, which has, for the most part, ignored the impact of the variables to which we have called attention (savings and loans) on search intensity. See, e.g. Mortensen [1986, 2010], Pissarides [2000] and Diamond [1981].

induces government to provide smaller UI benefits while increasing retirement income.[27] In response to the larger amount of savings the government offers a smaller amount of UI benefit for those unemployed in the later periods, as its incentive costs becomes higher due to the enhanced probability of unemployment for them and as the resulting reduction in the probability of unemployment has the further benefit of reducing incentives to save[28]. The amount of self-insurance for the unemployed under shock 2, on the other hand, which would be sum of private savings plus loans R_2 would be larger than would be the case when individual savings can be controlled by the government.

The policy implications of this are not clear, mainly because this form of precautionary savings is inseparable from other forms of savings. Many governments have taken the stance that they wish to encourage savings. In that policy discourse, little note is given to the adverse effect that any induced savings (stimulated, for instance, by favorable tax treatment) might have in discouraging search.

Excess borrowing in periods of unemployment

Because under shock 2, government loans achieve complete inter-temporal consumption smoothing, there is no incentive for private markets to provide loans then. But the presence of the market would affect the loan provision under shock 1. Obviously, if the government can observe and control the provision of loans by private lenders, the private provision of loans would not matter. Suppose, however, that the government cannot observe the private provision of loans by the market, but, of course, it can *infer* what the market will do. In this case the presence of the market would affect the government program in important ways which we discuss in this section.[29] We will suppose that the government delegates the provision of loans to the private market, which, in any case, will exceed the amounts it would have provided i.e., that, once the government offers UI r_1^0, the private market responds to it

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[27] This result can be compared to the one reported by the existing literature, which analyzes how unobservable savings affects the inter-temporal consumption structure for the unemployed during the unemployment duration (Kocherakota(2004), Shimer-Wernings(2005)), who found the optimal UI benefit to be non-decreasing in unemployment tenure. This should also be contrasted with to Hopenhyen-Nicolini (1998) who focuses on the depletion of savings as unemployment persists. In this paper we focus upon how the unobservable savings affects the optimal mix of benefits in the government program for the *next* unemployment bout in one's future career.

[28] The possible effect of lower UI benefit upon increased savings would be small if $U''' \approx 0$.

[29] This is the case, even if, after the government adjusts its loan and UI program, the market chooses not to make any loans.

by offering loans R_1^o.[30] To keep the analysis simple, we will assume and that $r_3 = \bar{r}$, a socially acceptable minimum level of consumption that may be determined by the government based upon social and political factors as well as economic ones.

We will first analyze the set of sustainable loan contracts in the presence of a competitive loan market and then analyze the optimal response on the part of the government. Private lenders always have incentives to provide loans to individuals whenever additional loan can improve intertemporal consumption smoothing. That is, for given r_1, R_1 and P (price of loans), additional loans will be offered whenever

$$U'\big(r_1 + R_1(1-P)\big) - U'\left(\frac{1-R_1-T_3}{2}\right) > 0,$$

because the price P of additional loan in a competitive loan market is equal to the (expected) probability that an individual is unemployed in period 2[31], that is, $P = p_U(1 - F(\bar{e}_3))$, where $\bar{e}_3 = U\left(\frac{1-R_1-T_3}{2}\right) - U(\frac{\bar{r}}{2})$. The amount of loan $R_1(P, r_1)$ for an individual that is sustainable in the market should then satisfy

$$r_1 + R_1(P, r_1)(1-P) = \frac{1-R_1(P,r_1)-T_3}{2}. \qquad (12)$$

That is, the "sustainable" loan contract for the individual has to entail complete inter-temporal smoothing between period 1 and 2 in the presence of a private loan market. Note, however, that it does not imply that the contract entails complete insurance against shock 3, and thus not full consumption smoothing. In other words, an individual choosing the contract would have some (albeit insufficient) incentive to search under shock 3.[32]

We can see from (12) that the package (r_1^o, R_1^o), which is provided by the government

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[30] Under certain circumstances, however, the government may choose not to leave loan provision to the market but to provide loans by itself. When individuals are heterogenous in the probability of extended unemployment and are privately informed of their types, for example, the utilitarian optimum, which might entail some subsidy from one type to the others, can be implemented only by a compulsory provision of loans by the government.

[31] These individuals will not repay their loans. Our analysis assumes that those who can repay do, that lenders are risk neutral, and charge the actuarial fair interest rate to compensate for the risk of non-payment.

[32] In a more general model, if the private sector could observe the amount of loans outstanding, it would take into account the effect of lending on search in the event of shock 3. In the absence of such information, though, there is an additional market failure.

and the market, has to satisfy

$$R_1^o = \frac{1-2r_1^o-T_3}{3-2\bar{p}_U}, \qquad (13)$$

where $\bar{p}_U = p_U(1 - F(\bar{e}_3^o))$ and $\bar{e}_3^o = U\left(\frac{1-R_1^o-T_3}{2}\right) - U(\frac{\bar{r}}{2})$.

The UI benefit r_1^o provided by the government should then be the one that satisfies the following condition:

$$U'\left(r_1^o + R_1^o(1-\bar{p}_U)\right) - U'\left(\frac{2-T_1}{3}\right) - \frac{h_1}{1-\bar{q}} r_1^o U'\left(r_1^o + R_1^o(1-\bar{p}_U)\right)U'\left(\frac{2-T_1}{3}\right)$$
$$- \frac{\partial R_1}{\partial r_1}\frac{\bar{p}_U}{1-\bar{p}_U} h_3\{R_1^o U'\left(r_1^o + R_1^o(1-\bar{p}_U)\right) + \bar{r}U'(\frac{1-R_1^o-T_3}{2})\}U'(\frac{1-R_1^o-T_3}{2}) = 0 \qquad (14)$$

where, from (13),

$$\frac{\partial R_1}{\partial r_1} = \frac{-2}{1+2(1-\bar{p}_U-\bar{p}_U h_3\{R_1^o U'\left(r_1^o+R_1^o(1-\bar{p}_U)\right)+\bar{r}U'(\frac{1-R_1^o-T_3}{2})\}U'(\frac{1-R_1^o-T_3}{2}))} < 0 \text{ (by (7)).}$$

In the presence of the market, loan provision is increased; and the government must take this into account in setting the UI benefit. (14) shows what that entails.[33] We can establish the following proposition on the optimal provision of UI benefit in response to the market provision of loans.

Proposition 7

In the presence of a private market for loans that cannot be controlled by the government, the amount of unemployment benefits (including loans provided by the market) under shock 1 is greater than that in the absence of the private market, i.e., $r_1^o + R_1^o(1-\bar{p}_U) > r_1^* + R_1^*(1-\bar{p}_U)$. Welfare is lower than in the case without private loans.

The proof is delegated to the Appendix. The last result is almost obvious: the problem

[33] As the full inter-temporal smoothing is maintained in the presence of private loan market, more provision of UI would always reduce the need for loans and thereby reduce the scope for private provision of loans.

considered here is “second best” to that considered in the previous section, so that necessarily, welfare is lower. The amount of reduction in welfare depends upon the seriousness of moral hazard associated with loans.

A priori, there might seem to be two effects on UI from excessive borrowing. As the market provides loans so as to secure complete inter-temporal smoothing, which is costly in terms of search incentives, to encourage more search, the government might want to cut back on insurance—to increase the private cost of not finding a job. But more UI makes borrowing from the market less necessary.[34] As a result, it may be that the government offers a larger amount of UI to reduce the amount of loans that the market offers.[35] We have not been able to derive a general result depicting when one effect dominates the other. But what we can show is that total unemployment benefits increase. Private borrowing obviously supplants government borrowing, but if there is a reduction in UI benefits, it only partially offsets the “excess” provision of markets.

IV. Welfare Effects of Loan-Cosigning in the Provision of Loans

Faced with possibility of default associated with the loans on the part of the unemployed, the government may want the loans to be cosigned by other employed workers. A worker who has a job who has co-signed a loan with a worker who is unemployed makes (partially) good on the loan. This provides him with an incentive to monitor—to ensure that the person for whom he co-signs searches for a job. In this section we ask whether the introduction of a loan-cosigning program would increase welfare and whether such a program could be made to work, i.e. would a potential cosigner have an incentive to cosign the loans.

We first note that an individual who is close to the cosignee, such as member of the same informal group (like the same family, close friends, etc.), is in a superior position for loan-

[34] A similar argument can be applied to the UI benefit $\bar{r}$ offered for those unemployed in period 1 and 2. The presence of private provision of loans may lead the government to increase $\bar{r}$ to reduce private incentive for lendings as higher $\bar{r}$ will yield higher probability $\bar{p}_U$ of unemployment under shock 3 and thereby lower loans R_1.

[35] This can be compared to the arguments of the literature (Bailey(1978), Crossley-Low(2011)) that the presence of a borrowing constraint increases optimal UI. Here the presence of uncontrolled market for loans may reduce optimal UI. It is the possibility of default associated with loans and of excessive borrowing in the private market that plays a role in the model, whereas it is not considered in the literature.

cosigning for a couple of reasons. First, he may be in a position to monitor the actions taken by the cosignees more effectively than others. Second, the cosignee may be subject to so-called peer pressure from an individual within the same informal group.[36] How much an individual cares about the peer pressure within a group and how much one can effectively control actions taken by the others would depend upon many other factors (cultural ones, for example) exogenous to this model.[37] The peer pressure may affect the behavior of both cosigner and cosignee, as will be discussed below.

In the model below examining the welfare effect of loan-cosigning we formulate a precise specification of both the informational advantage and the effect of peer pressure. When the search cost for an unemployed individual is realized, the cosignee can notice it with some probability γ (<0). The probability γ thus indicates the degree of informational advantage that a cosigner has over the government, who cannot identify the realized search cost at all. We will next suppose that the co-signing group determines the target threshold cost of search, $\bar{e}_3'$, which is the one that is optimal from the group's point of view, as will be specified below. We will also suppose that whenever the search cost is found to be lower than the target level $\bar{e}_3'$, the cosignee chooses to search, because of the peer pressure.[38] Then, the probability that an unemployed individual chooses to search under the loan-cosigning, $\bar{F}_3$, would be

$$\bar{F}_3(\bar{e}_3, \bar{e}_3') = F_3(\bar{e}_3) + \gamma\{F_3(\bar{e}_3') - F_3(\bar{e}_3)\} = (1-\gamma)F_3(\bar{e}_3) + \gamma F_3(\bar{e}_3') \quad , \quad (15)$$

,

where $\bar{e}_3$ is the threshold search cost that is chosen by an unemployed individual in the absence of loan-cosigning.[39]

Let us consider a "group" of two individuals, within which an employed individual is supposed to cosign the loans provided to the unemployed individual.[40] The government demands that all individuals who participate in the government loan program have a cosignee,

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[36] The incentive effects created by the peer pressure in a group have been discussed by Lazear (1990).

[37] Here we suppose that an individual does not care about the well-being of the others within a group, implying that there is no need for intra-group transfers, *ex post.*

[38] We assume here that the amount of peer pressure for a cosignee choosing not to search is greater than his benefit associated with pursuing a no-search strategy.

[39] In the assumed information structure, an unemployed individual knows whether the peer group has observed his search costs before he makes a decision on whether to search. In a slightly different formulation with similar results, the peer group announces $\bar{e}_3$'e and a punishment should they observe a deviation from the peer level.

[40] The results can be easily generalized.

who, if the borrower defaults, must repay a fraction c of the amount owed . The government sets c, as well as UI benefit and loan for the unemployed, to maximize the expected utility of an individual.

Suppose that an employed individual cosigns the loans R_1 for his colleague (within the same group) who is unemployed in period 1. If both of the two individuals in a group are employed or unemployed, there would be no loan-cosigning.[41] We will let the price of the loan vary with rate of loan-cosigning. For the sake of expositional simplicity we assume in this section that $p_N = 0$ and $p_U = 1$ and that r_3 is exogenously given as a socially acceptable minimum level of consumption, $\bar{r}$.

Using this formulation we can write the expected payoff of an individual in the group as follows:

$$V = (1 - \bar{q})V_N + \bar{q}V_U - q\int^{\bar{e}_1} edF_1$$

where

$$V_N = U(1 - s - T_1) + (1 - \bar{q}(1 - \bar{F}_3)2U\left(\frac{1 + s}{2}\right) + \bar{q}(1 - \bar{F}_3)2U\left(\frac{1 + s - cR_1}{2}\right)$$

$$V_U = \bar{q}\left\{U\left(r_1 + R_1\left(1 - P(0)\right)\right) + F_3(\bar{e}_3)2U\left(\frac{1-R_1-T_3}{2}\right) + \left(1 - F_3(\bar{e}_3)\right)2U\left(\frac{\bar{r}}{2}\right) -$$

e3edF3+1−q[Ur1+R11−Pc+F32U1−R1−T32+1−F3{2Ur2}−e3′edF3].

,

Here s is the savings in period 1, and T_1, T_3 are taxes for UI benefit r_1, $\bar{r}$, respectively, i.e.,

$$T_1 = \frac{\bar{q}}{1-\bar{q}}r_1$$

$$T_3 = \frac{1-\bar{F}_3}{\bar{F}_3}\bar{r} \qquad (16)$$

[41] In general, an individual who is unemployed in period 1 may have different packages of UI benefit and loans, depending upon whether his colleague is employed or not. In this section, however, for expositional convenience we will assume that the same amount of UI benefit and loans are provided to unemployed individuals in period 1 regardless of whether or not the loan can be cosigned. This assumption does not matter for the analysis below, because we will be examining whether or not the *introduction* of loan-cosigning is welfare-increasing. .

Also, $P(c)$ is the price of loan that is cosigned at the "rate" c,[42] where

$$P(c) = (1 - \bar{F}_3)(1 - c) \quad \text{for } c \geq 0.$$

This suggests that with the loan-cosigning cR_1 the price of loan decreases by $c(1 - \bar{F}_3)$ as the expected default cost gets lowered. In that respect the loan-cosigning provides an inter-personal (and, most importantly from our perspective, inter-state) income transfer within the group. In particular, we can notice that it plays a similar role to UI benefit, in that the cost is to be borne by an employed individual and that the total resources available to the unemployed increases. The loan-cosigning is different from the UI benefit, however, in that the risk for an individual is effectively pooled (among the entire population) in the latter case whereas it is not in the former. On the other hand, loan-cosigning can improve individual search incentives (especially in period 2) due to the informational advantage on the part of cosigners.

The threshold search costs $\bar{e}_1$, $\bar{e}_3$ and $\bar{e}_3'$ will be determined as follows:

$$\bar{e}_1 = V_N - V_U$$
$$\bar{e}_3 = 2U\left(\frac{1-R_1-T_3}{2}\right) - 2U(\bar{r})$$
$$\bar{e}_3' = 2U\left(\frac{1-R_1-T_3}{2}\right) - 2U(\bar{r}) + 2\{U\left(\frac{1+s}{2}\right) - U\left(\frac{1+s-cR_1}{2}\right)\}, \qquad (17)$$

The target threshold search cost $\bar{e}_3'$ set by the group maximizes the welfare of the group, i.e., the sum of utilities of cosigner and cosignee.[43] We can also notice from (15) (16) and (17) that the incentive effect of loan-cosigning hinges upon cR_1, a measure of the extent of loan cosigning, as well as the degree of informational advantage indicated by γ. Using the above conditions for the threshold search costs, we can express the incentive effects of the loan-cosigning as follows:

$$\frac{\partial \bar{e}_1}{\partial c} = -(1 - \bar{F}_3)R_1\{U'\left(r_1 + R_1(1 - P)\right) + \frac{\bar{q}}{1-\bar{q}}U'\left(\frac{1+s-cR_1}{2}\right)\} 0$$
$$\frac{\partial \bar{e}_3'}{\partial c} = R_1U'\left(\frac{1+s-cR_1}{2}\right) > 0$$

[42] Note that $\bar{F}_3 = F_3$ when $c = 0$.
[43] We can think of the group as agreeing on this ex ante, before they know which (if any) of the members of the group will be unemployed.

That is, the loan-cosigning has a positive incentive effect under shock 3, while it has a negative incentive effect under shock 1 (since individuals know that if they do not get employed, some of the costs are borne by their cosignees.[44])

On the other hand, the savings by an employed individual will be determined by inter-temporal smoothing[45]:

$$-\mathrm{U}'(1-\mathrm{s}-\mathrm{T}_1)+\bar{\mathrm{F}}_3\mathrm{U}'\left(\frac{1+s}{2}\right)+(1-\bar{\mathrm{F}}_3)\mathrm{U}'\left(\frac{1+s-cR_1}{2}\right)=0 \qquad (18)$$

Given this behavior of individuals in a group, the government sets the amounts of UI benefit r_1 and loan R_1, and "c" to maximize the expected payoff V subject to the government constraints (16). We can then establish the following Proposition.

Proposition 8

The introduction of loan-cosigning in the government provision of loans for the unemployed increases welfare so long as an individual cosigner is better informed of the realized search cost for the cosignee than the government.

The proof can be found in the Appendix. Although loan-cosigning is similar to UI benefit in its risk sharing properties, the former would not be able to fully replace the latter, as the risk is not perfectly pooled. Because at $\mathrm{c}=0$, however, the risk pooling effect is trivial by the envelope theorem, the benefits of co-signing arise solely from the improved incentives. Co-signing is only advantageous because of the informational advantage compared to the government. The mix of UI and loan-cosigning balances the imperfect risk-pooling and the

[44] This negative incentive effect is due to the risk sharing of cosigning, which lowers the difference in expected utility between getting a job and getting unemployed under shock 1. Risk itself has a positive incentive effect. (Arnott-Stiglitz, 1988, Stiglitz, 1982). It is still the case that incentives with co-signing are better than without co-signing.

[45] Notice that we have allowed peer pressure to effect only search costs the second period for an individual who is unemployed the first period. The group could exercise peer pressure to effect search in other contingencies and savings. If so, the welfare benefits of group co-signing would be enhanced.

informational advantage associated with loan-cosigning.[46]

Finally, there is the question of whether individuals will agree to co-sign. Note that the loan cosigned by an employed member of a group is priced lower than it would otherwise be because the loan cosigned is subject to a lower default probability, and all members of the group benefit in expected utility. Thus, ex ante, it makes sense for them to agree to co-sign, Even if it weren't, one could argue that peer pressure might suffice. Moreover, it is reasonable to assume that peer pressure is sufficiently great (at least for small c) that they would not renege from such an agreement.[47]

V. Conclusion

There is little doubt that under current arrangements, those who face a bout of unemployment suffer a great deal: there is imperfect consumption smoothing over time and imperfect risk-sharing across states.

Markets on their own did not provide unemployment insurance. Government had to step in to fill the lacuna. But government programs have been criticized for their adverse effects on search. In the U.S., for instance, normally benefits are cut off after six months because of the concern that extended benefits would greatly attenuate search and job acceptance.

One of the reasons that unemployment extracts such high costs--even when the loss of lifetime income is relatively small, as a result of a short term bout of unemployment-- is that

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[46] The question arises over the possibility of private incentives for loan-cosigning beyond the rate mandated by government. As long as the group does not take into account the effect of its choice upon the UI tax or the price of loan, it is easy to imagine circumstances under which the private group might have an incentive to go beyond the level required by the government. But unless the cosigners fully control the action of unemployed cosignee, this may decrease welfare, because the additional risk sharing attenuates first period search effort (The argument is analogous to Arnott-Stiglitz(1991).) In this paper, moreover, there is no incentive for additional loan-cosigning on the part of a group once unemployment risk is realized in period 1, because there is no interdependence of utility functions by members of the group.

[47] More formally, we can assume that the marginal peer pressure associated with the initial cosigning (i.e., at $c = 0$) is greater than the marginal cost (for the cosigner) of cosigning at $c = 0$, so that the cosigner would voluntarily accept some level of cosigning for his (unemployed) colleague.

individuals are unable to smooth consumption over their lifetime. Capital markets are imperfect. That is why models assuming perfect capital markets ascribe so little costs to economic fluctuations (Lucas(1987)). It was that insight that motivated our earlier paper, which showed that, indeed, if episodes of unemployment are short enough, one could make extensive use of loans: a well-designed government loan program could ensure income smoothing without the attenuation of incentives, while only a limited UI program might be needed.

Previous analyses within the search literature, which have focused on the adverse effects on search of government UI programs, have not attempted to balance the adverse incentive costs with the benefits of cross-state and cross-time income smoothing. In fact, most advanced industrial countries do not make use of loans, whereas Singapore, with its Provident Fund, in effect relied heavily on loans.

But some individuals do experience episode(s) of unemployment that represent a significant fraction of their potential life-time income, and in that case, one cannot rely simply on loans. In particular, there is a risk that (in a utilitarian-optimal unemployment scheme), loans may not be repaid, if individuals have (unexpected) repeated episodes of unemployment. In such circumstances, there needs to be greater reliance on UI: from a life-time utility maximization perspective, those with extended unemployment have greater need for insurance and even less of an ability to smooth income over time. This paper analyzes the optimal combination of UI benefit and loans for unemployed individuals from a lifetime perspective: taking into consideration the possible interactions between UI's and loans and the interactions between the government program and private savings (and loans), it examines how the level and composition of benefits (say the proportion of benefits provided in the form of loans) changes over time in a model where unemployment may occur in any period.

Methodological contributions. This paper is a contribution both to theory and policy. At the center of this paper are three fundamental ideas, which are relevant more generally in situations where moral hazard (incentive) effects may arise. First, in the context of an intertemporal model, a shock (an "accident" or, as here, the loss of a job) has an effect on the individual's current position that is normally greater than the impact on his lifetime prospects, which means that the adverse effects can be diminished by lifetime smoothing. Simply enhancing opportunities for lifetime smoothing has the advantage that there are not the

adverse incentive effects typically associated with insurance.[48] But with uncertainties associated with future income, lifetime budget constraints are uncertain. This poses limits on the extent of lifetime smoothing that is possible in the absence of perfect cross-state insurance (i.e. insurance that would eliminate uncertainty in the lifetime budget constraint.) Because of the adverse incentive effect of such insurance, there will be only limited cross-state insurance; but that in turn implies a limit to the extent of lifetime smoothing. In short, we need to simultaneously solve for the optimal lifetime smoothing and the optimal cross-state smoothing. This paper shows how this can be done, in the context of a particular model of some policy relevance.[49]

Secondly, this paper explores the implications of a number of externalities which, to date, have received insufficient attention, and which result in market inefficiency with and without government programs (Greenwald-Stiglitz, 1986, 1988). While earlier literature had called attention to the externalities within and across insurance markets (that is, the provision of insurance by one firm affected the risk-taking behavior of individuals, and therefore the losses experiences by other insurance firms (Arnott and Stiglitz, 1990), here we show that such externalities are more pervasive, e.g. between savings, credit markets, and insurance. We show, for instance, that such effects also arise from private savings and borrowing decisions--indeed, there is a presumption that a private unemployment insurance system that carefully balanced the benefits of cross-state and cross-time smoothing with the costs of adverse incentives would *not* be Pareto efficient. Well-designed government programs take these into account, thereby limiting the effects.

Thirdly, this paper examines the interactions between government programs, markets and non-market institutions and their implications. The latter is particularly important, given the limitations in the ability of government to monitor and control externality generating

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[48] There can still be effects on incentives, because, for instance, the expected marginal utility associated with additional work (or search) will change as a result of the increased opportunities for lifetime consumption smoothing.

[49] Income contingent loans represent a particular way of combining interstate and intertemporal smoothing. (See Chapmen and Hunter, 2009, Chapman and Tan, 2009, and Chapman, 2010.) Because in our model, repayment depends on whether individuals get a job in subsequent period, the loans are, in a sense, "income contingent," but, of course, they would be more so, if the wages individuals received when employed were variable, and repayment depended on those wages. If the extent of search affected the wages individuals received, then the design of the income contingent loans would affect the extent of search. See Stiglitz and Yun, 2013.

activities. In particular, we show how non-market institution (such as family, etc.) may be used to enhance welfare performance of government programs: a government program of co-signing can be welfare enhancing, and can be implemented voluntary by members of a group, if members of the co-signing group have an informational advantage over the government and can exert peer pressure to induce "better" behavior (in this context, ensure that individuals with low search costs search for a job.)

The externalities/markets failures that we have analyzed here arise whenever there are insurance markets (public or private), and whenever individuals take actions at one date that may affect their willingness to take actions at a subsequent date that will affect the magnitude or likelihood of a loss. These are examples of *diffuse* externalities—hard for any insurance firm to control. A health insurance company might naturally try to restrict smoking, which is directly related to a number of risks insured by the company. But individuals save and borrow for many reasons, and savings and debts affect behavior in many ways—including individual's risks taking behavior, and their incentives and ability to repay loans to others. Our paper illustrated the risk of excessive lending (borrowing) in one particular context, but the problem is more pervasive. A loan by one lender may reduce the likelihood of another lender being repaid. Excesses in this market played a big role in the financial crisis of 2008. Those excesses may partially be attributable to miscalculations of risk (hardly consistent with assumptions of rational expectations); but even with rational expectations such problems could arise, as we have noted, simply because individual market participants do not take into account the externalities that their actions impose on others.

Redesigning unemployment insurance programs The analysis of this paper suggests that there is considerable room for improvement in most UI programs. Given the heterogeneity of the population (e.g. with respect to utility functions, search cost functions, etc), a precise model of the kind formulated here can only be suggestive[50]. Several of the results of our analysis we think are likely to prove robust.

[50] In principle, one could design unemployment programs (mixes of UI and loans) contingent on observed characteristics, and offer choices, using principles of self-selection, to differentiate among individuals with unobservable but relevant characteristics; but practically, it is unlikely that governments would adopt programs of such complexity.

Most important, our analysis suggests that optimal policy maximizing lifetime utility involves greater reliance on loan programs, since with a loan program there can be (especially for the young) greater consumption smoothing with less incentive-attenuation. The loan programs, like the insurance program, have to be sensitive to (i) the impact that they have on search; (ii) the risk of not being repaid; and (iii) private incentives for savings and lending which, too, may attenuate search incentives. We noted, for instance, that there will be excessive precautionary savings,[51] which will lead to too little search.[52]

Another robust result concerns the relationship between unemployment benefits and past unemployment experience. In our model, the optimal policy involves a larger UI benefit for those with past episodes of unemployment. This stands in contrast to current US programs, which pays no attention to longer term employment experience, but gives a smaller benefit to those who have been unemployed (at least in the recent past).

On the other hand, we are not convinced that our result on how the relative reliance on UI benefits vs. loans should change with age is robust. In our idealized model, those who are unemployed when older have less ability to compensate for an adverse shock by smoothing over time, and that suggests the UI benefit for the young should be smaller. But in a more general model, the relationship between UI and age is not so clear: because the lifetime benefits of search are greater when young, adverse effects on search may be weaker, suggesting a higher UI benefit for the young.

There is one more qualification to the direct application of our results: We have conducted our analysis within the usual rational individuals with rational expectations framework, modified only by a recognition of certain market failures. If young people excessively discount the consequences to their retirement consumption[53], then switching from UI benefits to loans may not have the positive incentive effects predicted by this paper; instead, it may simply result in greater poverty among the elderly and/or a greater need to "bail out" those who have had the misfortune of experiencing several unemployment shocks over their lives.

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[51] In some countries, such as the United States, there seems to be insufficient precautionary savings, but this probably has to do more with "behavioral economics" considerations, e.g. individual's underestimation of the risk of their facing spells of unemployment. In this model, we have assumed rational expectations.

[52] In our simplified model, in which there are no problems of adverse selection, there is excessive private lending. In more general models, this may not be the case.

[53] As asserted by much of the recent literature in behavioral economics (Kahneman-Tversky(1991), Benartzi-Thaler(2004))

The paper has focused on how governments, with limited information, and limited ability to correct these market failures, can intervene to improve welfare. It assumes that when individuals search for a job, there is a job to be found; as we noted earlier (in footnote 13), if the number of jobs is less than the number of job seekers, more search simply increases the length of queues for the jobs that are available, and can worsen welfare. The implication is clear: UI systems should be made contingent on the state of the economy.

We have focused too on how market forces affect individuals willingness to search. There are other market externalities that might affect his ability to search, some manifest in the current crisis. Excessive indebtedness may, for instance, restrict his ability to invest in search over longer distances and may inhibit his ability to purchase a home in another locale (particularly important in countries with limited rental markets.[54])

Given the suffering associated especially with extended periods of unemployment, and given the significance of potentially adverse incentive effects for job search posed by poorly designed unemployment systems, there is need for thinking carefully about how one can structure systems which improve the well-being of the unemployed while limiting societal costs from adverse incentives. It is hoped that this paper will have made a contribution to that understanding.

[54]. Delli Gatti *et al* [2011] have raised this possibility in the context of the current recession. The role of capital constraints in affecting migration was long noted in the development literature (Stiglitz, 1969).

APPENDIX <Proofs of Propositions>

<Lemma>

We can see from (A1) that LHS is decreasing in r_1^* and RHS is increasing in r_1^* for any given q and h_1, while LHS is greater (or smaller) than RHS as $r_1^* \to 0$ (or as $r_1^* \to 1$). Also (A2) implies that LHS is increasing in r_3^* for any given p_U and h_3 and RHS is decreasing in r_3^*, while LHS is smaller (or greater) than RHS as $r_3^* \to 0$ (or as $r_3^* \to 1$). These prove Lemma 1). As RHS of (A1) is increasing in q and h_1 while LHS of (A2) is increasing in p_U and h_3, Lemma 2) is proved.

<Proposition 2>

i) (a) is proved by Lemma.

i) (b) Since $r_3 < 1 - \bar{p}_U$ and $G_2(\ ) < 1$, $M_3\left(1 + \frac{\bar{p}_U}{1-\bar{p}_U} G_2\right) < \frac{U'(\frac{1-\bar{p}_U}{2})}{1-\bar{p}_U} < \frac{U'(\frac{1-p_U}{2})}{1-p_U} \equiv K(p_U) < \infty$. As h_1 and/or q becomes large, $U'(r_1^*) \to U'(0)$. If $U'(0) > K(p_U)$, therefore, there will exist $(q, h_1) \in \Omega(p_U, h_3)$, proving i) (b).

i) (c) Consider a (q, h_1) such that $(q, h_1) \in \Omega(0,0)$, i.e., that $G_1(q, h_1) > U'(\frac{1}{2})$. This implies that there exists $p_U'(> 0)$ for any h_3 such that $G_1(q, h_1) > M_3(1 + \frac{\bar{p}_U}{1-\bar{p}_U} G_2(p_U, h_3))$ for $p_U \le p_U'$, suggesting that $(q, h_1) \in \Omega(p_U, h_3)$ for $p_U \le p_U'$. This proves (i) (c).

ii) Since $U'\left(\frac{r_3}{2}\right) > U'(\frac{1-R_1-T_3}{2})$ (from (10)), there exists $k(> \frac{1}{2})$ such that, for $p_U < k$, $U'\left(\frac{r_3}{2}\right) > \frac{\bar{p}_U}{1-\bar{p}_U} U'(\frac{1-R_1-T_3}{2})$. This implies that $U'(r_1 + R_1(1-\bar{p}_U)) < U'(\frac{r_3}{2})$ by (7) and (10), suggesting that $r_1^* + R_1^*(1-\bar{p}_U) > \frac{r_3^*}{2}$.

Proposition 3

Suppose that $r_1^* = 0$. Note that $R_1^*(1-\bar{p}_U) < \frac{1-T_3}{3}$, while (6) implies that $R_1^*(1-\bar{p}_U) = M_1(T_1)$, where $M_1(T_1) \equiv U'(1 - s_1^* - T_1)$. Since $1 - s_1^* - T_1 > \frac{1-T_1}{3} = \frac{1}{3}$ (because $T_1 = 0$), $U'(R_1^*(1-\bar{p}_U)) > M_1(T_1)$, and thus that $r_1^* > 0$ by (6). Suppose also that $r_2^* = 0$. (9) implies that $R_2^* = \frac{s_1^*}{2}$, which is smaller than $\frac{1+s_1^*}{2} = M_2$, whereas $R_2^* = M_2$ (by (8)), suggesting that $r_2^* > 0$. These will prove the first part of Proposition. Finally, when $p_U < p_U(q; h_1, h_3)$ so that $R_1^* > 0$, we can see from (10) that a higher h_3 can lead to zero r_3^*.

Proposition 4

i) Suppose that $r_1^* = r_2^*$. It then implies that $R_1 > R_2$ because $R_1 - R_2 = \frac{1+r_1}{3} - \frac{s_1+r_2}{2} = \frac{1}{6}[2\{(1-s_1) - \frac{s_1+r_1}{2}\}] > 0$ by (5) (when $p_U \cong 0$). This suggests that $\bar{e}_1 = V_N - V_U < V_{NN} - V_{NU} = \bar{e}_2$ because $(V_{NN} - V_{NU}) - (V_N - V_U) = 2\bar{p}_N\left\{U\left(\frac{1+s_1-T}{2}\right) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} + 3\left\{U(r_1 + R_1(1-\bar{p}_U)) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} - \left\{U(1-s_1-T) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} > 0$(because $(r_1 + R_1) - \frac{s_1^*+r_2}{2} = \frac{1}{3}\{(1-s_1) - \frac{s_1^*+r_2}{2}\}$) when $p_U \approx 0$, implying that $\bar{q} > \bar{p}_N$ because $q = p_N$. Since $\bar{q} > \bar{p}_N$, (6) and (8) imply that $r_1^* < r_2^*$ because $M_2(T_2) < M_1(T_1)$ (by (5)). Since $R_2 - R_1 = \frac{1}{6}[3(r_2 - r_1) + 2\{(1-s_1) - \frac{s_1+r_1}{2}\}]$ by (6) and (8) (when $p_U \cong 0$), $R_1^* > R_2^*$ by (5)

and because $r_1^* < r_2^*$.

Also, suppose that $r_1 + R_1(1-\bar{p}_U) < r_2 + R_2$. Then, by (6) and (8), $(V_{NN} - V_{NU}) - (V_N - V_U) = 2\bar{p}_N\left\{U\left(\frac{1+s_1-T}{2}\right) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} + 3\left\{U\left(r_1 + R_1(1-\bar{p}_U)\right) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} - \left\{U(1-s_1-T) - U\left(\frac{s_1^*+r_2}{2}\right)\right\}$
$< 2\bar{p}_N\left\{U\left(\frac{1+s_1-T}{2}\right) - U\left(\frac{s_1^*+r_2}{2}\right)\right\} - \left\{U(1-s_1-T) - U\left(\frac{s_1^*+r_2}{2}\right)\right\}$
< 0 by (5) if $p_N \approx 0$. Thus, $\bar{e}_1 = V_N - V_U > V_{NN} - V_{NU} = \bar{e}_2$, suggesting that $\bar{q} < \bar{p}_N$, implying that $r_1 + R_1(1-\bar{p}_U) > r_2 + R_2$ by (6) and (8), which is contradictory. Finally, suppose that no loan is provided to the unemployed. When $r_1 = r_2 = r$, $(V_{NN} - V_{NU}) - (V_N - V_U) =$

$$\left[2U\left(\frac{1+s_1-T_2}{2}\right) - \{U(r) + U(s_1)\}\right] - [U(1-s_1-T_1) + \{(1-\bar{p}_N)2U\left(\frac{1+s_1-T_2}{2}\right) + \bar{p}_N\{U(r) + U(s_1)\}\} - \left\{U(r) + 2U\left(\frac{1}{2}\right)\right\}]$$

$= \bar{p}_N\left[2U\left(\frac{1+s_1-T_2}{2}\right) - \{U(r) + U(s_1)\}\right] + [2U\left(\frac{1}{2}\right) - \left(U(1-s_1-T_1) + U(s_1)\right)]$

> 0 (because $2U\left(\frac{1}{2}\right) > \left(U(1-s_1-T_1) + U(s_1)\right)$ by the concavity of U(.)). That is, $\bar{e}_1 = V_N - V_U < V_{NN} - V_{NU} = \bar{e}_2$, suggesting that $\bar{q} > \bar{p}_N$, implying that $r_1^* < r_2^*$ in the absence of loan provision.

<Proposition 5>

If $p_N = 0$, $1 - s_1 - T_1 = \frac{1+s_1-T_1}{2} = \frac{2(1-T_1)}{3}$, so that the condition (6) can be rewritten as

$$U'(r_1 + X) - U'\left(\frac{2(1-T_1)}{3}\right) - \frac{h_1}{1-\bar{q}} r_1 U'(r_1 + X) U'\left(\frac{2(1-T_1)}{3}\right) = 0 \qquad (6')$$

where $X \equiv R_1(1-\bar{p}_U)$. We can then rewrite (7) as

$$U'(r_1 + X) - U'\left(\frac{1-\frac{X}{1-\bar{p}_U}-T_1}{2}\right) - \frac{h_3\bar{p}_U}{1-\bar{p}_U} U'\left(\frac{1-\frac{X}{1-\bar{p}_U}-T_1}{2}\right)\left\{\frac{X}{1-\bar{p}_U} U'(r_1 + X) + \frac{1}{1-\bar{p}_U} r_3 U'\left(\frac{1-\frac{X}{1-\bar{p}_U}-T_1}{2}\right)\right\} = 0 \qquad (7')$$

Let $r_1^* = r(X; q, h_1)$ or $X = {r_1}^{-1}(r_1^*; q, h_1)$ from (6'), and let $X^* = X(r_1;\ p_U, h_3, r_3)$ or $r_1 = X^{-1}(X^*;\ p_U, h_3, r_3)$ from (7'). See Figure 2. We can see from (6') and (7') that $\frac{\partial {r^{-1}}}{\partial r_1} < \frac{\partial X}{\partial r_1} < 0$, because $\frac{\partial \bar{q}}{\partial r_1} > 0$, $\frac{\partial \bar{p}_U}{\partial r_1} = 0$ and $\frac{\partial \bar{q}}{\partial X} = 0$, $\frac{\partial \bar{p}_U}{\partial R_1} > 0$. Since $\frac{\partial X}{\partial R_1} > 0$ (from (7')), $\frac{\partial \bar{p}_U}{\partial X} > 0$. Also, from (6') and (7'), ${r_1}^{-1}(0; q, h_1) > X(0;\ p_U, h_3, r_3)$ because, for $p_N = 0$, ${r_1}^{-1}(0; q, h_1) = 1 - s_1 = \frac{2}{3} > \frac{1-T_1}{3} > X(0;\ p_U, h_3, r_3)$.

Suppose that $r_1(0; q, h_1) < X^{-1}(0;\ p_U, h_3, r_3)$ so that an interior solution (r_1^*, X^*) for (6') and (7') exists. Since $\frac{\partial {r_1}^{-1}}{\partial r_1} < \frac{\partial X}{\partial r_1} < 0$ and $r_1(0; q, h_1) < X^{-1}(0;\ p_U, h_3, r_3)$, the solution, which is unique, can be illustrated by the intersection A of $r_1(R_1; q, h_1)$ and $X(r_1;\ p_U, h_3, r_3)$ curves as in Figure 2. As q increases, $r_1(R_1; q, h_1)$ curve shifts leftward, so that r_1^* decreases while X^* increases. On the other hand, as p_U increases, $X(r_1;\ p_U, h_3, r_3)$ curve shifts leftward, so that r_1^* increases while X^* decreases. Note also from (6') and (7') that $\frac{\partial {r_1}^{-1}}{\partial r_1} < -1 < \frac{\partial X}{\partial r_1} < 0$, suggesting that $\frac{\partial (r_1^* + X^*)}{\partial q}$, $\frac{\partial (r_1^* + X^*)}{\partial p_U} < 0$.

<Proposition 6>

Differentiating (5') with respect to s_1 and r_2, we have $\frac{\partial s_1}{\partial r_2} > 0$, because

$$\frac{\partial s_1}{\partial r_2} = -\frac{\partial SS/\partial r_2}{\partial SS/\partial s_1},$$

where $SS \equiv -U'(1 - s_1^o - T_1) + (1-\bar{p}_N)U'\left(\frac{1+s_1^o-T_2}{2}\right) + \bar{p}_N U'\left(\frac{r_2^o+s_1^o}{2}\right)$.

Since $\frac{\partial SS}{\partial s_1} < 0$ (by the second-order condition) and

$\frac{\partial SS}{\partial r_2} = \bar{p}_N\{U''(\frac{s_1+r_2}{2})\text{-}U''\left(\frac{1+s_1-T_2}{2}\right)\} + \frac{\partial \bar{p}_N}{\partial r_2}\{U'(\frac{s_1+r_2}{2}) - U'\left(\frac{1+s_1-T_2}{2}\right)\} - \frac{1}{1-\bar{p}_N}\frac{\partial \bar{p}_N}{\partial r_2}U''\left(\frac{1+s_1-T_2}{2}\right) > 0$

(as $U''' \approx 0$ and $\frac{\partial \bar{p}_N}{\partial r_2} > 0$), $\frac{\partial s_1}{\partial r_2} > 0$.

Differentiating the lifetime utility V with respect to r_2 to find the optimal r_2, r_2^o,

$\frac{dV}{dr_2} = \frac{\partial V}{\partial r_2} + \frac{\partial V}{\partial s_1}\frac{\partial s_1}{\partial r_2} < 0$ at $r_2 = r_2^*$,

because $\frac{\partial V}{\partial s_1} < 0$ at $s_1 = s_1^o$ by Proposition 1. This implies that $r_2^o < r_2^*$.

<Proposition 7>

Figure 3 depicts (by A) how the original optimal mix $(r_1^*, R_1^*(1-\bar{P}_U))$ is determined by (6)$(r_1 = r_1(X,;q,h_3))$ and (7) $(X = X(r_1;p_U,h_3,r_3))$. Let (12) and (14) be represented by the curve $X = \tilde{X}(r_1;p_U,h_3,r_3)$, and by the curve $r_1 = \tilde{r}_1(X,;q,h_3)$ in Figure 3, respectively. Comparing (14) and (12) with (6) and (7), respectively, we can see that the curve $r_1 = \tilde{r}_1(X,;q,h_3)$ and the curve $X = \tilde{X}(r_1;p_U,h_3,r_3)$ are on the right hand side of the curve $r_1 = r_1(X,;q,h_3)$ and of the curve $X = X(r_1;p_U,h_3,r_3)$, respectively. Since $\frac{\partial r_1^{-1}}{\partial r_1} < -1 < \frac{\partial X}{\partial r_1} < 0$, we can see (from A and B in Figure 3) that $R_1^o(1-\bar{P}_U) + r_1^o > R_1^*(1-\bar{P}_U) + r_1^*$ as B is to the right of the 45° line $(r_1 + X = r_1^* + X^*)$.

<Proposition 8>

To see whether or not the loan-cosigning enhances welfare we will evaluate the expected payoff V of an individual at $c = 0$ and differentiating V with respect to c at $c = 0$, to obtain

$$\frac{\partial V}{\partial c} = (1-\bar{q})\bar{q}(1-F_3)R_1[U'(r_1 + R_1(1-P)) - U'\left(\frac{1+s-cR_1}{2}\right)]$$
$$-h_1\frac{r_1}{1-\bar{q}}\{U'(r_1 + R_1(1-P)) + \frac{\bar{q}}{1-\bar{q}}U'\left(\frac{1+s-cR_1}{2}\right)\}U'(1-s-T_1)]$$
$$+h_3\frac{1}{1-\bar{q}}\{R_1U'(r_1 + R_1(1-P)) + \bar{r}U'(\frac{\bar{r}}{2})\}\gamma U'\left(\frac{1+s-cR_1}{2}\right)]$$

Since $1-s-T_1 = \frac{1+s-cR_1}{2}$ when $c = 0$ by (18), and since the necessary condition for r_2 is

$U'(r_1 + R_1(1-P)) - U'\left(\frac{1+s-T_1}{2}\right)] - h_1\frac{r_1}{1-\bar{q}}\{U'(r_1 + R_1(1-P)) + \frac{\bar{q}}{1-\bar{q}}U'\left(\frac{1+s-cR_1}{2}\right)\}U'(1-s-T_1) = 0,$

we can see that, if $\gamma > 0$,

$\frac{\partial V}{\partial c} > 0$ at c = 0.

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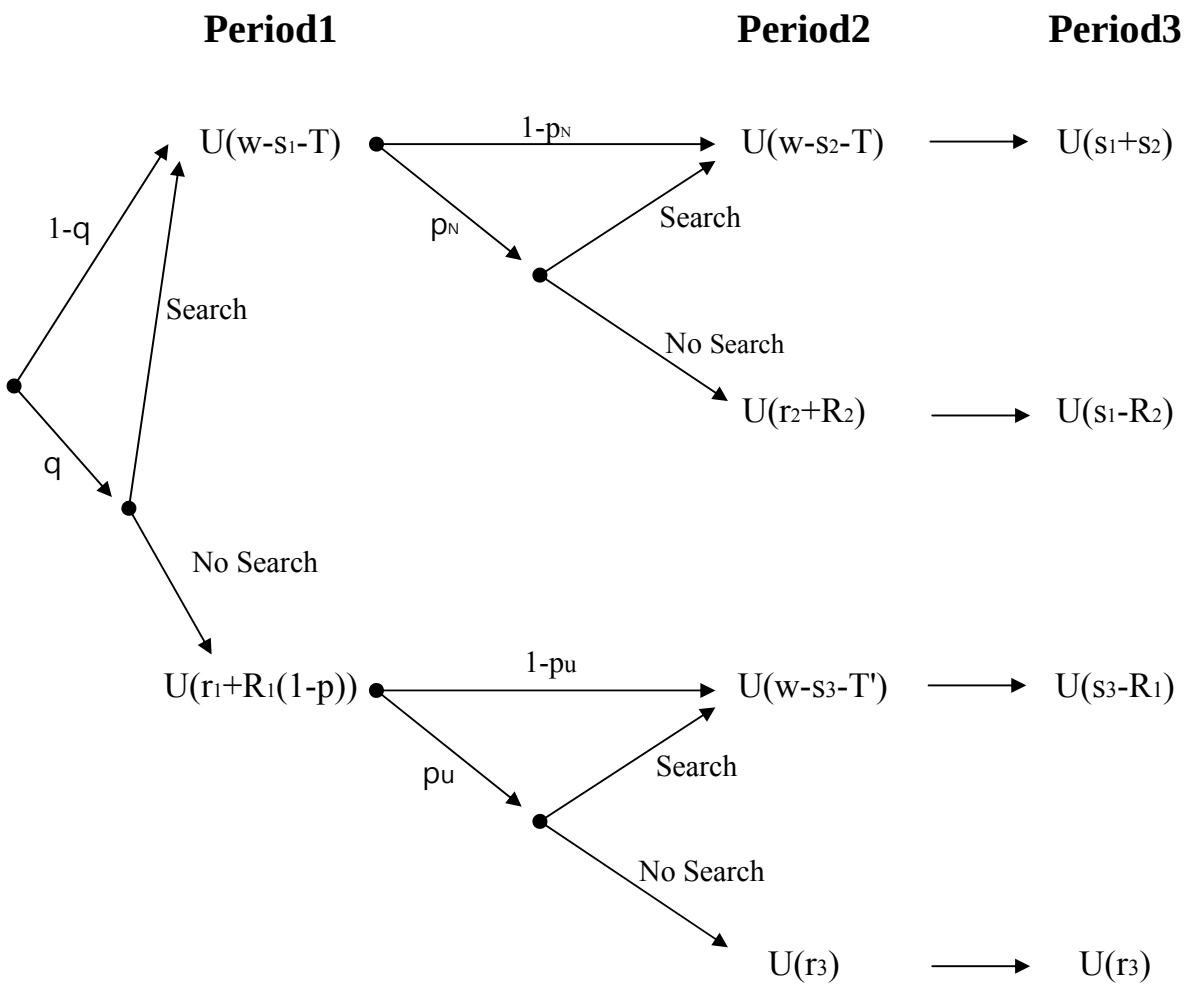
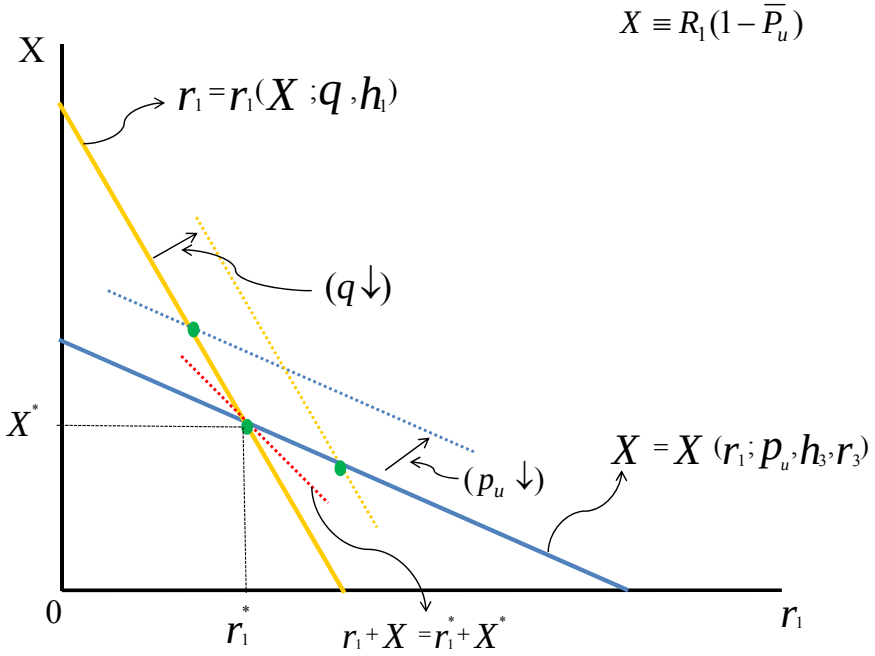


$T = \frac{1}{2-\bar{p}_N}\left(\frac{\bar{q}}{1-\bar{q}}\, r_1 + \bar{p}_N\, r_2\right)$ $\qquad T' = \frac{\bar{p}_u}{1-\bar{p}_u}\, r_3$ $\qquad$ p: price of loan

< Figure 1 > Model

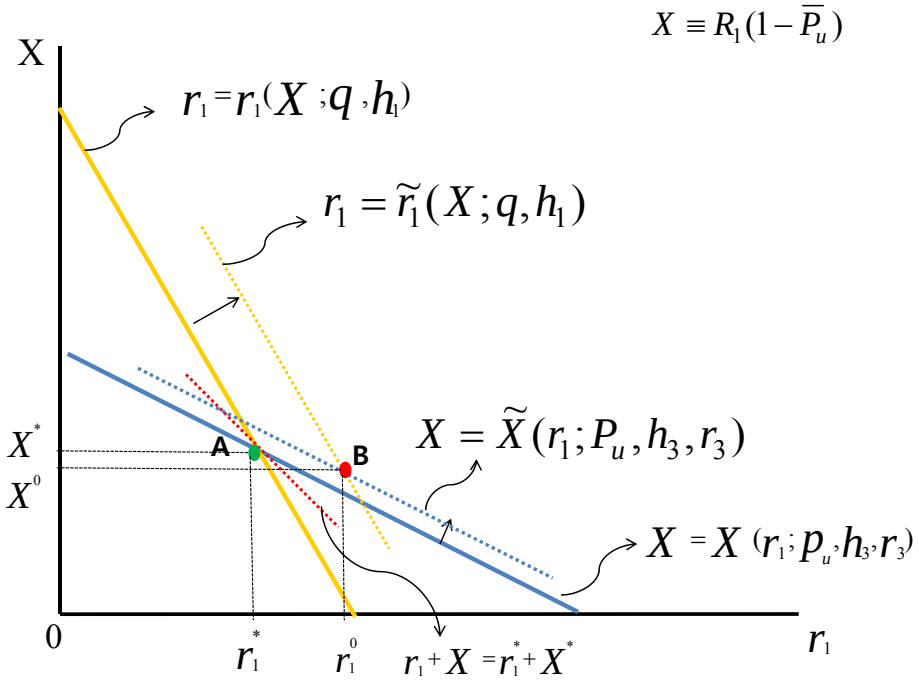
<Figure 2> Optimal Mix (r_1^*, R_1^*)



$X \equiv R_1 (1 - \overline{p_u})$

< Figure 2 > Optimal Mix (r_1^*, R_1^*)

<Figure 3> Optimal Mix (r_1^0, R_1^0)



$X \equiv R_1 (1 - \overline{p_u})$

< Figure 3 > Optimal Mix (r_1^0, R_1^0)