NBER WORKING PAPER SERIES

OVERCOMING WEALTH INEQUALITY BY CAPITAL TAXES THAT FINANCE PUBLIC INVESTMENT

Linus Mattauch
David Klenert
Joseph E. Stiglitz
Ottmar Edenhofer

Working Paper 25126
http://www.nber.org/papers/w25126

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2018

We thank Max Franks, Cameron Hepburn, Olga Heismann, Katharina Jochemko, Ulrike Kornek, Marc Morgan, Salvatore Morelli, Jacquelyn Pless and Anselm Schultes for helpful comments and Fiona Spuler and Simona Sulikova for excellent research assistance. We further thank seminar audiences in Berlin and Oxford, at ZEW as well as participants of meetings of the European Economic Association, the Journees Louis-Andre Gerard Varet, the Royal Economic Society, the International Economic Association, the Society for the Study of Economic Inequality and the Verein fuer Socialpolitik and for useful comments. Linus Mattauch's research was supported by a postdoctoral fellowship of the German Academic Exchange Service (DAAD). The article is also part of a project that has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 681228. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Overcoming Wealth Inequality by Capital Taxes that Finance Public Investment
Linus Mattauch, David Klenert, Joseph E. Stiglitz, and Ottmar Edenhofer
NBER Working Paper No. 25126
October 2018
JEL No. D31,E21,H31,H41,H54

ABSTRACT

Wealth inequality is rising in rich countries. Capital taxation used simply to finance redistribution may not be able to counteract this trend, but can increased public investment financed by higher capital taxes? We examine how such a policy affects the distribution of wealth in a setting with distinct wealth groups: dynastic savers and life-cycle savers. Our main finding is that public investment financed through capital taxes always decreases wealth inequality when the elasticity of substitution between capital and labor is moderately high. Indeed, for all elasticities of substitution greater than a threshold value, at high enough capital tax rates, dynastic savers disappear in the long run. Below these rates, both types of households co-exist in equilibrium with life-cycle savers gaining from the higher capital tax rates. These results are robust with respect to the different roles of public investment in production. We calibrate our model to OECD economies and find the threshold elasticity to be 0.82.

Linus Mattauch
Institute for New Economic Thinking
at the Oxford Martin School
University of Oxford
Oxford Ox2 6Ed
United Kingdom
linus.mattauch@inet.ox.ac.uk

David Klenert
EUREF Campus 19
Torgauer Straße 12-15
Berlin 10829
Germany
klenert@mcc-berlin.net

Joseph E. Stiglitz
Uris Hall, Columbia University
3022 Broadway, Room 212
New York, NY 10027
and NBER
jes322@columbia.edu

Ottmar Edenhofer
Potsdam Institute for Climate
P.O. 60 12 03
D-14412 Potsdam
Ottmar.Edenhofer@pik-potsdam.de

1 Introduction

Wealth inequality is rising in rich countries (Piketty and Saez, 2014). At the same time, it is widely recognized that public capital, notably in education and infrastructure, is underfunded even though it increases efficiency in the long term (Bom and Ligthart, 2014; Calderón and Servén, 2014). There is no consensus, however, about how additional public investment should be financed. One prominent proposal is to tax capital when wealth inequality is seen as too high, using the proceeds to increase public investment (Diamond and Saez, 2011; Piketty, 2014; Piketty and Zucman, 2014; Stiglitz, 2012, 2016a).

In this article, we examine whether taxing capital income and investing the proceeds into public capital decreases wealth inequality. Our main contribution is to show how the success of this policy depends critically on the elasticity of substitution between capital and labor. We prove that wealth inequality decreases in the long term with a moderately high elasticity of substitution, and we completely characterize how the possible distributional outcomes depend on the tax rate, forms of public investment, and substitutability.

Stiglitz (2015b) argues that there is a new set of stylized facts regarding growth and distribution to be explained by macroeconomic modelling. These stylized facts include an increase in the wealth-income ratio; growing wealth disparity; a wealth distribution more skewed than the labor income distribution; and rising top income and wealth shares in nearly all countries in recent decades (Alvaredo et al., 2017; Novokmet et al., 2017; Wolff, 2017). Taken together, these stylized facts seem to replace the Kaldor facts that were integral in the development of the neoclassical growth model. Growing empirical evidence further suggests that individuals at the top of the wealth distribution display saving behavior that is markedly different from the remainder of the population: Rich individuals have higher saving rates, which is consistent with lower time preference rates, obtain a greater share of their income from capital and, when compared to the rest of society, save more for posterity rather than for retirement (Attanasio, 1994; Dynan et al., 2004; Lawrance, 1991; Saez and Zucman, 2016). Models should thus account for heterogeneous preferences (Krusell and Smith, 1998; Foley and Michl, 1999), especially with respect to saving behavior (Stiglitz, 2015b, 2016b).

To address these realities, in a simple framework that allows for inequality between households even in the long run, we consider how increased public investment that is financed by capital taxes affects wealth inequality. We combine the two standard approaches to saving behavior—the dynastic model and the life-cycle model—in a single framework. This simplification captures some of the stylized facts by representing the saving behavior of most citizens as occurring within their “life cycle”, while representing a second group of citizens with such high levels of inherited wealth as to make their labor income irrelevant and their saving behavior dynastic. Stiglitz (2015d) shows this simplification is a limiting case of a model in which households with highly non-linear savings functions can transition between (endogenous) wealth groups.[1]

Our model distinguishes the two groups by income source, time preference rate and saving behavior: "Workers" receive income from labor and capital and save for life-cycle purposes. "Capitalists", the top wealth owners, receive only capital income[2] and have a dynastic saving motive. This type of model was originally introduced by Pasinetti (1962) and has been taken up by Samuelson and Modigliani (1966), Stiglitz (1967, 1969) and Judd (1985). More recently Baranzini (1991), Klenert et al. (2018), Mattauch et al. (2016), Michl (2009) and Stiglitz (2015b, 2016b, 2018a) have analyzed related models in which workers also save in a life-cycle fashion, thus accounting for the importance of retirement savings. In particular, Mattauch et al. (2016) and Stiglitz (2017) proved that public investment financed through capital taxes is Pareto-improving for low tax rates and that workers prefer higher capital tax rates than capitalists.[3]

The present article characterizes all possible distributional outcomes that can result from financing public investment through (proportional) capital taxes in an otherwise unregulated closed economy. We prove that, depending on the value of the elasticity of substitution between capital and labor σ, and on the level of the capital tax τ, three cases can result in the long term. Both classes can co-exist ("Pasinetti-regime"), but at the margins of the model the capitalists disappear as their absolute income becomes zero ("Anti-Pasinetti-regime") or workers' savings becomes zero ("Anti-Anti-Pasinetti-regime"). We thus generalize and unify claims about the distributional outcome of two-class models with public capital made in Klenert et al. (2018), Mattauch

[1] The literature analyzing the effect of capital taxation on wealth inequality can be subdivided into four strands: first, determining the optimal level of capital taxes under standard weak assumptions about heterogeneity (Atkinson and Stiglitz, 1976); second, determining capital tax levels in models with idiosyncratic shocks to productivity or earnings (Aiyagari, 1994; Bewley, 1977); third, studying the effect of capital taxes when productivities and endowments are stochastically determined (Champernowne, 1953; Jones, 2015; Piketty and Saez, 2013); fourth, studying capital tax increases where the distribution itself is endogenous with varying heterogeneity assumptions and shocks, giving rise to endogenous stochastic processes (Becker, 1980; Bevan, 1979; Bevan and Stiglitz, 1979; Chatterjee and Turnovsky, 2012; Stiglitz, 1976, 1978a,b). Our contribution is related to the fourth strand, though here we do not introduce shocks.

[2] Individuals at the top of the wealth distribution are more likely to be self-employed entrepreneurs and to receive a higher share of capital income (Wolff, 1998; Diaz-Gimenez et al., 2011; Wolff and Zacharias, 2013), but see Appendix B.1 for an analysis with "capitalists" who also work.

[3] Klenert et al. (2018) analyze the cases of financing public investment with (linear) capital-, labor- and consumption taxes in a calibrated numerical framework with endogenous labor supply, also considering the distribution of income and utility explicitly. They find that, while capital taxes reduce inequality in income, wealth and utility, consumption and labor taxes do not.

et al. (2016) and Stiglitz (2015c, 2016b, 2018a).

Specifically, we establish that public investment financed through capital taxes always decreases wealth inequality for a general production function as long as the factor share of labor does not approach zero for very high tax rates. For constant elasticity of substitution (CES) production functions, the main result is that for any σ there exists a capital tax rate τ_{lim} such that either capitalists disappear or workers' savings become zero, as long as steady states exist. In particular, we prove that there exists a threshold elasticity σ_1 such that for any elasticity $\sigma \geq \sigma_1$, there exists a capital tax rate τ_{lim} at which the economy switches from a Pasinetti to an Anti-Pasinetti state, so that capitalists disappear. If $\sigma < \sigma_1$ there exists a capital tax rate τ_{lim} at which the economy switches from a Pasinetti to an Anti-Anti-Pasinetti state, so that workers savings (and income), relative to output, become zero. The Anti-Anti-Pasinetti case results as a two-class steady state is no longer sustained; the Anti-Pasinetti case is unrelated to steady-state existence, but due to high savings of workers. The relationship between the elasticity of substitution and τ_{lim} is monotone in all cases. We also establish by numerical simulations, however, that workers still gain in relative terms from moderate capital tax rates, even for very low elasticities. The suggested policy only harms workers for implausible parameter choices.[4]

The main intuition behind our results is simple: For high elasticities, capital taxes increase the share of labor, both directly (from the scarcity of capital) and indirectly (from the increased productivity resulting from public investment). So workers, who provide a fixed amount of labor, earn more and can afford to save more as the value of their labor increases. For low elasticities, as capital becomes relatively scarce, its share increases. The division of the population into two distinct groups permits us to represent these effects in general equilibrium and study the implications for wealth inequality.

There has been a recent debate over the value of the elasticity of substitution. For example, Chirinko (2008) show that 26 out of 31 studies find an elasticity of substitution between capital and labor (significantly) below 1. By contrast, Piketty and Saez (2014) and Piketty and Zucman (2015) argue that the elasticity must be higher than 1.[5] Our contribution is the first

[4]The only conditions under which the Anti-Anti-Pasinetti regime arises are those under which capitalist asymptotically disappear. In practice, a government would finance public capital by other taxes long before this particular case could arise, it is relevant for understanding the behavior of the economy as a limiting case only.

[5]In particular, Piketty and Saez (2014) argue that "it makes sense to assume that σ tends to rise over the development process, as there are more diverse uses and forms for capital and more possibilities to substitute capital for labor." (p. 841). But this argument is not fully persuasive, because advances in technology can result in dominating technologies, leading to a lower elasticity of substitution (Atkinson and Stiglitz, 1969; Korinek and Stiglitz, 2017). In any case, the discussion of the aggregate elasticity of substitution entails delicate issues of capital aggregation (Stiglitz, 2015b), with much of

to analyze systematically how the success of capital taxes financing public investment in terms of addressing inequality depends on σ. For that purpose we calibrate our model to OECD economies and solve it numerically, including the top 5 % of the population in the group of dynastic savers. We find that the threshold elasticity σ_1 is approximately 0.82, indicating that both the Anti-Pasinetti and the Anti-Anti-Pasinetti case are conceivable as limiting cases. The simulation also indicates, however, that workers are still better off in absolute terms for even lower elasticities as long as capital tax rates are moderate.

The remainder of this article is structured as follows: In the next section, we set out the model. In Section 3, we analyze the steady-state properties for a general production function and characterize wealth inequality. In Section 4, we derive results with a CES production function in which public capital is labor-enhancing. In Section 5, we consider an alternative specification with capital-enhancing public capital. Section 6 presents our numerical application. Section 7 concludes and outlines how the impact of automation reinforces the policy implications of our results.

2 Model

We model an economy with a single consumption good in which the government can finance productivity-enhancing public investment by a capital tax. There are two types of households in the population, "capitalists" and "workers". The workers are modeled as representative overlapping generation agents that live for two periods each. They provide labor in the first period and save for their own retirement in the second period, but they leave no bequests to future generations. The capitalists are modeled as a representative infinitely-lived agent that saves dynastically. Their source of income are interest payments on their capital holdings and, in some cases, firms' profits. Factor markets clear and on the capital market, the supply consists of both agents' capital holdings. We examine the distribution of wealth, but since consumption is linear in wealth in our basic model for both groups and utility only depends on consumption, results qualitatively carry over to the distribution of consumption and utility.

Capitalists The capitalist owns a capital stock K_t^c and maximizes intertemporal utility given by

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho_c)^t} \ln(C_t^c), \tag{1}$$

the increase in the value of capital associated not with changes in the amount of capital but in relative prices.

with consumption C_t^c and time preference rate ρ_c. Its budget constraint is

$$K_{t+1}^c - K_t^c = (1-\tau)r_t K_t^c - C_t^c + \Pi_t, \tag{2}$$

where r_t is the before-tax interest rate. A capital income tax τ is imposed on all capital.[6] Firms' profits Π_t may be zero, depending on the production structure. The initial capital stock is given as K_0^c. The capitalist respects a transversality condition: $\lim_{t\to\infty}\left(K_t^c \prod_{s=1}^{t-1} \frac{1}{1+r_s}\right) \geq 0$.

Solving the maximization problem yields an Euler equation for this household:

$$\frac{C_{t+1}^c}{C_t^c} = \frac{1+(1-\tau)r_{t+1}}{1+\rho_c}. \tag{3}$$

Workers The worker lives for two periods, a "young" (y) and an "old" (o) stage. It maximizes its lifetime utility, with utility from consumption in the second period being discounted by the time preference rate ρ_w:

$$\ln(C_t^y) + \frac{1}{1+\rho_w}\ln(C_{t+1}^o). \tag{4}$$

In the first period, the agent sells its fixed labor L to the producing firm, which in turn pays a wage rate w_t. Labor income can either be consumed or saved for the old age:

$$w_t L = S_t + C_t^y. \tag{5}$$

In the second period the agent consumes its savings and the interest on them:

$$C_{t+1}^o = (1+(1-\tau)r_{t+1})S_t. \tag{6}$$

Solving the optimization problem subject to the budget constraints leads to an Euler equation for this household:

$$\frac{C_{t+1}^o}{C_t^y} = \frac{1+(1-\tau)r_{t+1}}{1+\rho_w}. \tag{7}$$

From Equations (5-7) an explicit expression for saving can be derived:

$$S_t = \frac{1}{2+\rho_w} w_t L. \tag{8}$$

This implies a constant savings rate of $1/(2+\rho_w)$, as is standard in discrete OLG models when the utility function is logarithmic.[7]

[6] Slightly different results are obtained if the tax is imposed only on capitalists' capital. See Stiglitz (2015d).

[7] Thus, standard Keynesian style models that begin with a constant savings rate can easily be provided micro-foundations. Note however that modern behavioral economics suggests that it may be misguided to demand such micro-foundations, see Stiglitz (2018b). Note too that savings rates will depend on the real interest rate when the utility function is not logarithmic, but the equilibrium can still be analyzed with techniques similar to those presented here, see Stiglitz (2018a).

Production Consider a production sector given by the neoclassical production function $F(P_t, K_t, L)$, with P_t public capital. K_t denotes the sum of the individual capital stocks

$$K_t = K_t^c + S_{t-1}. \tag{9}$$

Throughout we assume constant returns to scale in all three factors:
$F(P_t, K_t, L) = F_K K + F_L L + F_P P$. This implies that constant returns in the *accumulable* factors K and P are impossible and thus there is no endogenous growth.

Government The sole function of the government in this model is the provision of public capital. It finances its investments using the capital income tax revenue, thus influencing the interest rate. Hence the government's activity is summarized as the change in the stock of public capital (with δ_P denoting its depreciation):

$$P_{t+1} = (1 - \delta_P)P_t + \tau r_t K_t. \tag{10}$$

This means that increased investment into public capital in one period will yield increased production in the next period.

Return to public investment There are a number of ways to close the model by specifying how the return to public investment is distributed to the agents and whether it modifies returns to the other production factors, with different economic interpretations (see also Section 5). For the basic model, we focus on the case of public capital as investment into education, enhancing the productivity of labor. For this case, assume that workers' enhanced labor is a constant-returns-to-scale sub-production function J

$$J(P_t, L) = LJ(P_t/L, 1) \tag{11}$$

a function of the labor supply and education expenditures. Total production is then given by $F(K, J)$ and is constant-returns to scale in K and J. With this specification, the functions F and J, in combination with the parameters ρ_w and ρ_c determine the equilibrium—including the equilibrium distribution of wealth. It is natural to define w as the wage per efficiency unit of labor, J :

$$w_t = \frac{\partial F(K_t, J_t)}{\partial J}. \tag{12}$$

and total wage income is $w_t J_t$. Workers appropriate all the return to labor, so the budget constraint of workers becomes

$$w_t J_t = S_t + C_t^y, \tag{5a}$$

so that

$$S_t = \frac{1}{2+\rho_w} w_t J_t. \tag{8a}$$

Profit maximization of the firm yields the standard rates of return to capital (with δ_K denoting depreciation of private capital):

$$r_t + \delta_K = \frac{\partial F(P_t, K_t, L)}{\partial K_t}. \tag{13}$$

Profits in Equation (2) are set to zero as a consequence. We employ this version of the model in the following unless stated otherwise.

3 Results for general production functions

In this section, we determine the basic properties of the model for general production functions, before turning to specific parametrizations in the later sections. First, we discuss the difference between absolute improvements for workers—including partial "burden-shifting" of the tax—and changes in the distribution of wealth, for different uses of the tax revenue. Second, we demonstrate the existence of an optimal tax rate for the purpose of redistribution. Third, we derive the main result of this section: with an asymptotically positive factor share of augmented labor (or elasticity of substitution between capital and augmented labor asymptotically greater than 1), there is always a capital tax rate sufficiently high that capitalists vanish, that is a switch from a Pasinetti to an Anti-Pasinetti regime. Fourth, we characterize the boundary between the Pasinetti and the Anti-Pasinetti regime.[8]

With respect to the difference between absolute improvements for workers and changes in the wealth distribution, we exclusively focus on cases in which the government uses the capital-tax revenue for public investment. Prior work established that this policy, under certain conditions, constitutes a Pareto improvement, i.e. it makes all classes better off in absolute terms (see Klenert et al. 2018, Mattauch et al. 2016 and Stiglitz 2017).[9] If the capital tax revenue would instead be used to finance lump-sum transfers to the workers, the burden of the capital tax would be shifted to them. This would make workers worse off than before in absolute terms, as shown by

[8]Meade (1966) and Samuelson and Modigliani (1966) were the first to describe the conditions under which an Anti-Pasinetti outcome can occur in a neoclassical framework—however, they did not relate their finding to capital tax-financed public investment and elasticities of substitution.

[9]To clarify, we focus in this article only on comparisons of steady states: there are steady states in which both workers and capitalists are worse off than in some other steady state, but to make the transition from the inferior steady state to the better would require some of those in the intervening generations to be worse off.

Stiglitz (2015d), Stiglitz (2016b) and Stiglitz (2018a). This result can also be shown in our framework, but is not the focus of our article, see Appendix D. Instead, we characterize relative changes in the wealth distribution induced by different types of capital tax-financed public investments. Further we focus on steady-state distributions, that is, long term outcomes. However, see Klenert et al. (2018) for a related analysis of transitional dynamics.

The model is solved for steady states for general production functions F, J (which we assume in this section to be such that stable steady states exists, see below). Steady-state values of variables are denoted by a tilde.

If capitalist initial wealth is zero, then capitalists remain in such a steady state with. The model becomes a variant of the standard overlapping generations model. Corresponding to any value of τ, there is a steady state equilibrium given by the solution to

$$\widetilde{K} = \frac{1}{2+\rho_w}\widetilde{w}(\widetilde{K})\widetilde{J}(\tau\widetilde{K}). \tag{14}$$

We focus, however, on the steady-state equilibria which emerge if K_c initially is greater than zero. There are three possible patterns: a steady state in which both classes exist; one in which only workers exist; and one in which the entire economy collapses, with both wages and capitalists' capital go to zero.

It follows from the capitalist's Euler Equation (3), in any equilibrium with a steady-state capitalist, that the steady-state interest rate $\widetilde{r}$ is given by

$$\widetilde{r} = \frac{\rho_c}{(1-\tau)}. \tag{15}$$

This means that the steady-state interest rate is solely determined by the capitalists' time preference rate and if there is a steady state with capitalists, then there is full shifting of capital taxation (see Pasinetti (1962) and Appendix D).[10]

The share of workers' wealth for a general production function can be determined by dividing Equation (8) by total capital:

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{1}{2+\rho_w}\frac{\widetilde{w}\widetilde{J}}{\widetilde{K}}. \tag{16}$$

Workers' saving propensity determines the distribution of capital between both classes. The above equation only holds, of course, if $\widetilde{S} < \widetilde{K}$. For a wealth ratio of $\widetilde{S}/\widetilde{K} = 1$, only workers would exist (the Passinetti regime), which would yield a standard discrete overlapping generations model with

[10] It is no surprise that we obtain consequences similar to Pasinetti (1962), because under our assumptions, savings rates are effectively fixed. More precisely, if profits are zero, Equations (2), (6) and (7) imply that all consumption variables are linear in wealth for the two groups, but with different factors. We have simply provided the obvious micro-foundations.

public capital, see, for example, Heijdra (2009), [Ch. 17]. Much of our analysis concerns examining behavior towards this boundary.

The steady-state level of public capital is given by:

$$\widetilde{P} = \frac{1}{\delta_P} \tau \widetilde{r} \widetilde{K}. \tag{17}$$

Equations (15) and (17) determine the allocation of total private and public capital in the economy. They thus define two equations in only two variables and can be solved for $\widetilde{K}$ and $\widetilde{P}$, fixing the tax rate τ. By the Extreme Value Theorem, a solution in $\widetilde{K}$ and $\widetilde{P}$ will always yield a maximum (and minimum) value for $\widetilde{S}/\widetilde{K}$ as a function of τ. This remains true as long as $\tau \in [0 + \epsilon, 1 - \epsilon]$ for $\epsilon > 0$ small, because of the continuity of the respective functions. The result establishes that a specific capital tax rate will be optimal from the point of view of redistributing as large a share of wealth as possible to workers. However, it does not determine whether the maximum and minimum values of the wealth ratio are such that both classes actually co-exist or whether solutions are unique. For a fixed tax rate τ, and no restrictions on production functions, there may be multiple steady states, there may exist a (unique) equilibrium with both classes, or there may be only a single class equilibrium.[11] There may be multiple equilibria in the two-class case because Equations (15) and (17) determine a unique value of $k \equiv K/J$, but this may not result in unique values for K and J for arbitrary production functions.

By contrast, it is possible to characterize the limiting behavior of the model by making assumptions about the elasticity of substitution between capital and (augmented) labor and the factor shares. Define factor shares Ω_J and Ω_K as usual, but recalling that labor is augmented by public investment to yield the composite J:

$$\Omega_J = \frac{F_J J}{Y} \qquad \Omega_K = \frac{F_K K}{Y}. \tag{18}$$

From Equation (16) one obtains

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau) = \frac{1}{2 + \rho_w} \widetilde{\Omega}_J(\tau) / \widetilde{\Omega}_K(\tau) \Big(\frac{\rho_c}{1 - \tau} + \delta_k \Big) \tag{19}$$

(see also Stiglitz, 2015d, 2016b).

It is immediate that since $\rho_c/(1 - \tau)$ tends to infinity as $\tau \to 1$, if the factor share accruing to enhanced labor is strictly positive as $\tau \to 1$, then $\frac{\widetilde{S}}{\widetilde{K}}(\tau)$ exceeds 1 (and in fact diverges). If for some tax rate both classes exist, then there will be a tax rate at which capitalists vanish, by the Intermediate

[11] In such a one class equilibrium, there can be multiple steady-states for a single value of τ, too, see Heijdra (2009) [Ch. 17].

Value Theorem. Further, $f(k) = F(K,J)/J = F(K/J,1)$, as usual and let $\sigma(k)$ be the elasticity of substitution between K and J defined via f. One can then establish that more generally:

Proposition 1. *Assume $0 < \frac{\widetilde{S}}{K}(\epsilon) < 1$, i.e. for small taxes rates $\epsilon > 0$ both classes coexist. Further assume $\lim_{\tau\to 1} k(\tau) = 0$. If $\sigma(0) > 1$, with $\sigma(0)$ the limiting elasticity of substitution as $\tau \to 1$, then there always exists a capital tax rate $\tau_{\lim}$ such that capitalists vanish (Anti-Pasinetti case).*

Proof. It is known that $\sigma(0) > 1$ implies $\lim_{k\to 0} \widetilde{\Omega}_K(\tau) = 0$ (see for instance Barelli and de Abreu Pessôa, 2003). Thus Equation (19) diverges to infinity as $\tau \to 1$. Therefore, the conclusion follows from the Intermediate Value Theorem. □

Later, we examine specific production functions for which the condition is satisfied. When $\sigma(0) < 1$ instead, the above argument does not work because it does not lead to convergence of Equation (19) to a finite value less than 1. Further, the weaker premise $\lim_{\tau\to 1} K(\tau) = 0$, might not suffice, because in our setting it is then still possible that $\lim_{\tau\to 1} k(\tau) = K(\tau)/J(\tau) > 0$, so that the first step in the proof does not necessarily follow.[12]

Next we characterize the boundary of the regime in which both classes co-exist for the case of public investment as education. Proposition 1 merely states that a tax rate exists at which capitalists eventually vanish: the number of switches between regimes cannot be concluded from it. Let $\Phi(\tau) = \frac{\Omega_J(\tau)}{\Omega_K(\tau)}$. At the boundary

$$1 = \frac{1}{2+\rho_w}\Phi(\tau)\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big). \tag{20}$$

Any value of τ for which the above equation is satisfied is a switch-line. If $\Phi'(\tau) > 0$ there is a unique solution in τ. But for there to be a definite number of solutions requires wealth inequality to be a strictly concave function in the tax rate, not merely Φ to be concave.[13] More precisely, we can prove Proposition 2, see Figure 1 for an illustration:

Proposition 2 (General characterization of Pasinetti-regime boundary)**.** *Suppose $0 < \frac{\widetilde{S}}{K}(\epsilon) < 1$, i.e. for small taxes rates $\epsilon > 0$ both classes coexist. Suppose that $\Phi(\tau)$ is a monotone, continuous function on $(0,1)$.*

1. *Suppose Φ is increasing. Then only one switch from the Pasinetti regime to the Anti-Pasinetti regime occurs.*

[12]By the assumption $0 < \frac{\widetilde{S}}{K}(\epsilon) < 1$, we exclude the case of $\lim_{\tau\to 0} k(\tau) = +\infty$.

[13]It implies the below condition on Φ, which holds as long as its "curvature" is not too big and so long as the elasticity of substitution is enough below 1. This is because a tax increase for a small tax would lead to a decrease in share of labor if elasticity of substitution is less than unity.

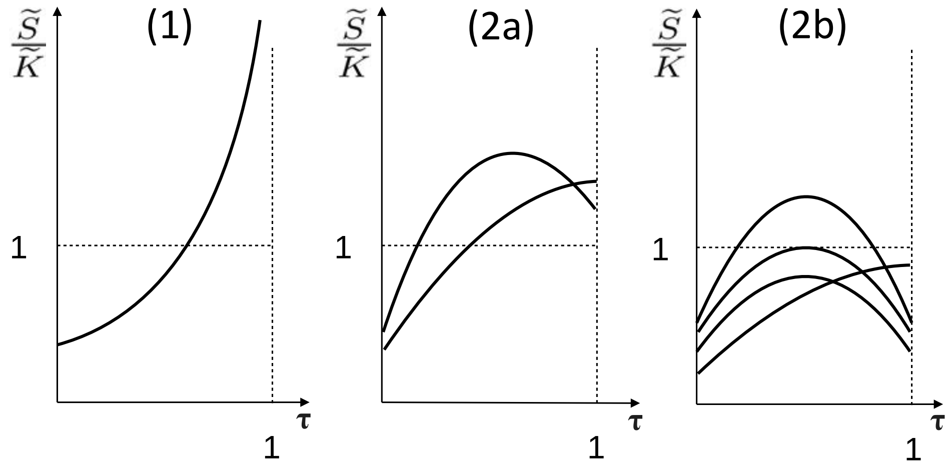

Figure 1: Schematic illustration of the possibilities for the Pasinetti-regime boundary occuring in the cases of Proposition 2

2. *Suppose instead that Φ is decreasing. Suppose further that Φ satisfies the following inequality:*

$$\frac{1}{2+\rho_w}\Phi(\tau)''\left(\frac{\rho_c}{1-\tau}+\delta_k\right)+2\Phi(\tau)'\left(\frac{\rho_c}{(1-\tau)^2}\right)+\Phi(\tau)\frac{2\rho_c}{(1-\tau)^3}<0 \tag{21}$$

Then

   (a) *if $lim_{\tau\to 1}\frac{\widetilde{S}}{\widetilde{K}} > 1$, only one switch from the Pasinetti to the Anti-Pasinetti regime occurs.*

   (b) *if $lim_{\tau\to 1}\frac{\widetilde{S}}{\widetilde{K}} \leq 1$, either the Pasinetti regime persists for all tax rates, or there may be a switch from the Pasinetti to the Anti-Pasinetti regime for some tax rate followed by a switch back for a higher tax rate (which can coincide).*

Proof. Part 1 follows from monotonicity and the Intermediate Value Theorem.

For Part 2, the second-order differential inequality (Equation 21) on $\Phi(\tau)$ ensures that $\partial^2\frac{\widetilde{S}}{\widetilde{K}}/\partial\tau^2 < 0$, i.e. wealth inequality is a strictly concave (and continuous) function of the tax rate. If $\lim_{\tau\to 1}\widetilde{S}/\widetilde{K} > 1$, the conclusion follows again by the Intermediate Value Theorem and the strict concavity implies there can be no more than one switch.

If instead $\lim_{\tau\to 1}\frac{\widetilde{S}}{\widetilde{K}} \leq 1$, note that wealth inequality has a unique maximum at $\Phi'(\tau^*)(\frac{\rho_c}{(1-\tau^*)}+\delta_k) = \Phi(\tau^*)(\frac{\rho_c}{(1-\tau^*)^2})$. Suppose this maximum occurs in the relevant range of $(0,1)$. If the value of this maximum is smaller than 1, no switch to an Anti-Pasinetti regime occurs. If the value is greater or equal to 1, there is a switch for some tax rate and a switch back for a higher tax rate, applying the Intermediate Value Theorem again.[14] The reason is that by definition of strict concavity of a function, it can take a single value at most two times. If the maximum is not in the interval $(0, 1)$ there will be no switch to the Anti-Pasinetti regime. □

In Section 4 we prove that for the specific case of CES production functions with labor-enhancing public capital, the cases (1) and (2b) of Proposition 2 apply for the relevant part of the capital tax range. We also characterize the limiting behavior more fully than is possible by Proposition 1.

Proposition 1 and 2 include the case where increasing labor-enhancing public capital (financed by the tax) increases the factor share of labor monotonically. However, if the factor share is instead decreasing, then there is the possibility that wealth is distributed more unequally with more public investment. For instance, if the increase of labor productivity is low but capitalists reduce their investment a lot in response to the tax, and if the elasticity of substitution is small enough, then the adverse distributional effect of capitalists saving less exceeds the positive benefit of public investment. So the policy of investing capital tax revenue into labor-enhancing public investment cannot always be assumed to reduce inequality. In cases in which public investment enhancing labor could harm workers in relative terms, one may wonder whether the government could instead invest into capital-enhancing public investment. The latter might raise the factor share accruing to labor in those cases in which labor-enhancing public investment lowers it. In Section 5 below we show that this is not generally the case. The simplest representation of capital-enhancing public investment is in fact entirely symmetrical, so that, in relative terms, it does not make a difference which factor public investment augments (Subsection 5.1). Even when public investment enters production as an entirely separate production factor that yields rents (Subsection B.2) workers could still be harmed in relative terms by a capital tax. However, we are able to show that for a decreasing factor share of labor, workers are still better off in absolute terms, for moderate capital tax rates (see Figure 4 below).

An implication of Equation (15) is that as the tax rate increases, the before tax return to capital has to increase, so that K/J has to decrease. For general production functions there are two cases. Consider the factor price frontier, the dual to the production function, which gives the maximum interest rate corresponding to any wage. Either, there is no upper bound to the interest rate compatible with a steady state, so that regardless of the tax imposed, there is a positive wage. Or there is a maximum before-tax interest rate. If there is a maximum before tax interest rate, then there is

[14]There is a singular case where the maximum value is just unity.

a maximum tax rate at which Equation (15) can be satisfied. At tax rates above that critical level, the Euler equation implies that capitalists converge to zero. Note that at that maximum interest rate, the wage is zero, so that workers' capital also is zero.

Further, for a marginal change in the tax rate, it is also possible to characterise the effect of an increase in the tax rate on relative capital holdings in terms of the elasticity of substitution $\sigma(k)$ for general production functions, in a two class equilibrium:

Proposition 3. *$sgn(\mathrm{d}\ln(S/K)/\mathrm{d}\ln\tau) = sgn(1-\Omega_K/\sigma(k))$. That is, wealth inequality decreases if the elasticity is greater than 1, holding factor shares constant.*

Proof.

$$\begin{aligned}\operatorname{sgn}(\mathrm{d}\ln(S/K)/\mathrm{d}\ln\tau) &= -\operatorname{sgn}(\mathrm{d}\ln(S/K)/\mathrm{d}\ln k)\\ &= \operatorname{sgn}(1-\mathrm{d}\ln w/\mathrm{d}\ln k) = \operatorname{sgn}(1-\Omega_K/\sigma(k))\end{aligned}$$

□

Hence, if the elasticity of substitution is greater than 1, wealth inequality is always decreased, *ceteris paribus*. If instead the elasticity is less than one, the effect of a marginal capital tax increase additionally depends on the factor share of capital.

It is worth noting that the properties of J do not enter at all in the above analysis, though they are central to the analysis of the welfare effects on workers themselves. The above analysis can only hold, of course, for $0 \leq S/K \leq 1$, and so it is important to understand the domain over which that is true, and what happens when it is not. To investigate this more closely, we narrow our focus in the next section on constant elasticity of substitution functions.

4 Labor-enhancing public investment

This section investigates the role of the elasticity of substitution between capital and labor in assessing whether capital taxation leads to a reduction in wealth inequality. In particular, public investment is assumed (as in the previous section) to be labor-enhancing (as is education). We use the model of Section 2, but parametrize the production function by a CES function between capital and labor. We classify all possible long-term outcomes, that is we analyze whether one or both classes exist.[15]

[15] It is well-known that for CES functions, a steady state might not exist for all values of the elasticity in a neoclassical growth model because for each value of the elasticity, one of the Inada conditions is violated. In this section, existence would also depend on

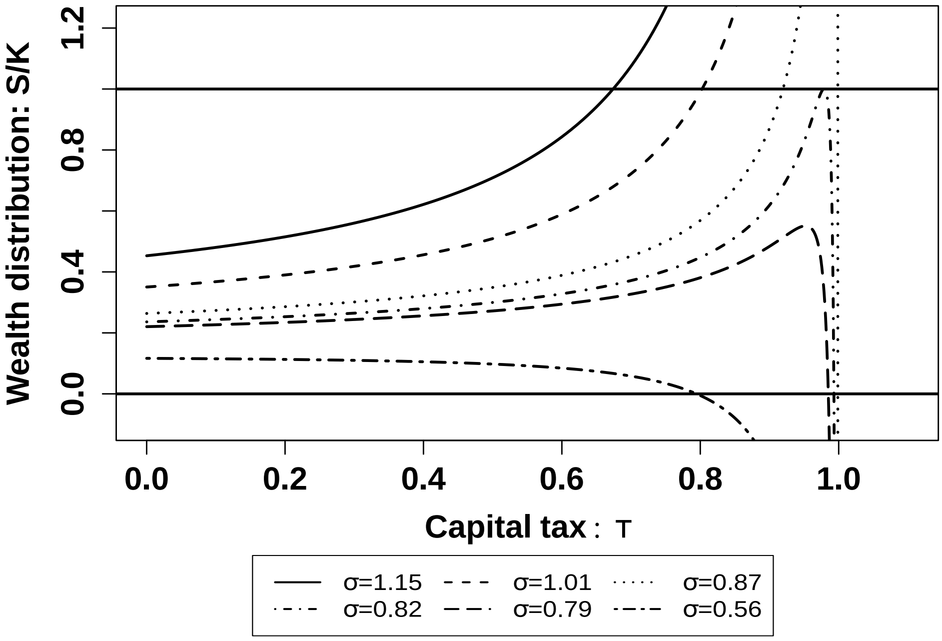

Figure 2: Wealth inequality as a function of the capital tax rate for various elasticities as an illustration of Proposition 4, 5 and Corollary 6. (Values above 1 and below 0 are not economically meaningful, for model calibration see Section 6).

We proceed as follows: We first prove that for a given elasticity of substitution between capital and labor σ with $\sigma \geq 1$, there exists a capital tax rate τ_{lim} at which the economy switches from a Pasinetti to an Anti-Pasinetti state so that capitalists disappear (Proposition 4). We then show that, if $\sigma < 1$, there exists a capital tax rate τ_{lim} at which the economy either switches from a Pasinetti to an Anti-Pasinetti or to an Anti-Anti-Pasinetti state, in which the capital share of workers goes to zero (Proposition 5). Further, there exists a threshold value $\sigma_1 < 1$ such that for $\sigma \geq \sigma_1$ the switch is to an Anti-Pasinetti state, while for $\sigma \leq \sigma_1$, the switch is to an Anti-Anti-Pasinetti state (Corollary 6). In all cases the relationship between the elasticity of substitution and τ_{lim} is monotone. The results are illustrated by Figure 2.

Assume the production is given by the following function, in which public

the function J, so we assume it is such that a steady state exists for relevant ranges of the Pasinetti regime. In Appendix C we show that there are plausible specifications of J for which this is true. Further, whenever a unique steady state exists, the model converges to it because it inherits the dynamics of a neoclassical growth model with public capital, see Appendix C.

capital P is labor-enhancing:

$$F(P,K,L) = \big(\alpha K^{\gamma} + (1-\alpha)(J)^{\gamma}\big)^{\frac{1}{\gamma}} \tag{22}$$

with $0 < \alpha < 1$, $\gamma < 1, \gamma \neq 0$.[16] The elasticity of substitution between capital and labor σ is given by $\sigma = 1/(1-\gamma)$. The relative capital intensity at any given wage-interest-ratio is reflected by α. Throughout, we assume that the function $J(P,L)$ is such that a steady state in relevant ranges of the Pasinetti regime exists (and we verify this below for J specified as Cobb-Douglas).

We derive in Appendix A.1 that

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau;\gamma) = \frac{1}{\alpha(2+\rho_w)}\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big)\Big(\big(\frac{1}{\alpha}(\frac{\rho_c}{1-\tau} + \delta_k)\big)^{\frac{\gamma}{1-\gamma}} - \alpha\Big). \tag{23}$$

Note that one can show by straightforward computation that this expression is monotonically decreasing in α. This means that wealth inequality increases with higher capital intensity, as is to be expected (capitalists derive all their income from capital; workers only a fraction of their income). We hence focus on substitution elasticities and so compare different values of production parameters (different economies) on long-run outcomes.[17]

We assume throughout this section that there is an economically meaningful state in which both classes co-exist for a capital tax of zero (Pasinetti-case).

Assumption 1. *(a) For a capital tax of nearly zero both agents co-exist. This implies that* $0 < \widetilde{S}/\widetilde{K}(\epsilon,\gamma) < 1$ *for* $\epsilon > 0$ *small.*

(b) $\delta_K > \alpha$.

Both of these assumptions hold for the economically relevant range of the parameters used in our model by a very large margin.[18]

First, consider the case that $\gamma > 0$, that is, the substitution elasticity between capital and labor is greater than 1.

[16] To avoid a confusion in units, as K is measured in capital goods while J in equivalent labor units, one should, strictly speaking, account these different goods in "aK" and "b," where a and b are such as to ensure equivalency of services provided, i.e. so that if K is the only factor of production, $Q/K = a$, and similarly for J. We choose our units so that a = b = 1.

[17] Changing only one of the two parameters of a CES function, as we explore in this section, changes the distribution of income at any given ratio of capital and (effective) labor. The net effect on the equilibrium if we change both parameters simultaneously so as to preserve the distribution of income in the initial situation would be different from that when we perturb only one parameter.

[18] For the content of Propositions 4 and 5, the weaker claim $\rho_c + \delta_K > \alpha$ would suffice for Part (b) of the assumption. See Section 6 for calibration of the model, which includes employing time steps of 30 years.

Proposition 4. *Let production be specified as above and assume $\gamma \geq 0$.*

(a) For every γ, there exists a capital tax rate τ_{lim}, such that capitalists vanish, that is, the solution to Equation (23) *entails $\widetilde{S}/\widetilde{K} = 1$ (Anti-Pasinetti case). For tax rates above τ_{lim}, only equilibria with solely workers exist.*

(b) This relationship is monotone: the higher the value of γ, the lower the tax rate at which capitalists vanish.

Proof. It is straightforward to show that $\widetilde{S}/\widetilde{K}(\tau;\gamma)$ is monotonically increasing in τ and γ for $\tau, \gamma \in (0,1)$, keeping the other parameters fixed. Moreover, it can be established, using Equation (23) that

$$\lim_{\tau \to 1} \frac{\widetilde{S}}{\widetilde{K}}(\tau, \gamma) = \infty.$$

So as $\widetilde{S}/\widetilde{K}(\tau;\gamma)$ is continuous in $\tau \in (0,1)$, the proposition follows from the Intermediate Value Theorem.

□

Now consider the case $\gamma < 0$, that is, the substitution elasticity between capital and labor is smaller than 1.

Proposition 5. *Let production be specified as above. Assume $\gamma < 0$ and that assumption 1 still holds.*

(a) For every γ, there exists a capital tax rate, such that either capitalists vanish (Anti-Pasinetti case) or workers vanish (Anti-Anti-Pasinetti case).

(b) In both cases, the relationship is monotone: For the Anti-Pasinetti case, the higher the elasticity, the lower the tax rate at which capitalists vanish. For the Anti-Anti-Pasinetti case, the lower the elasticity, the lower the tax rate at which workers vanish.

Proof. The idea of the proof is to realize that for $\gamma < 0$ with $|\gamma|$ small, the function $\widetilde{S}/\widetilde{K}(\tau;\gamma)$ has a unique maximum that may or may not be greater than 1 depending on parameter choices.

To prove part (a), first note that

$$\lim_{\tau \to 1} \frac{\widetilde{S}}{\widetilde{K}}(\tau;\gamma) = -\infty.$$

This again follows from the algebra of limits, as

$$\lim_{\tau \to 1} \left(\left(\frac{1}{\alpha} \left(\frac{\rho_c}{1-\tau} + \delta_k \right) \right)^{\frac{\gamma}{1-\gamma}} - \alpha \right) = -\alpha.$$

By straightforward computation, it can be shown that $\widetilde{S}/\widetilde{K}(\tau)$ has a unique maximum in $\tau \in (0, 1)$ for

$$\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} > \delta_K. \tag{24}$$

Else $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically decreasing in (0,1) (see Appendix A.2).

First consider the case that a unique maximum exists. If the value of this maximum is below 1 (or outside of the range (0,1)), the Anti-Anti-Pasinetti case occurs, by the Intermediate Value Theorem, as $\widetilde{S}/\widetilde{K}(\tau)$ is continuous in $\tau \in (0, 1)$. If instead the value of this maximum is above 1 and it is in the range $(0, 1)$, the Anti-Pasinetti case occurs. However, if condition (24) is not fulfilled, the Anti-Anti-Pasinetti case occurs.

To prove part (b), note that the proof of monotonicity of $\widetilde{S}/\widetilde{K}(\tau;\gamma)$ in γ in the proof of Proposition 4 does not depend on γ being positive. $\gamma/(1-\gamma)$ is still a monotonically increasing function for $\gamma < 0$, given Assumption 1. □

The following corollary characterizes exactly under which condition the Anti- and the Anti-Anti-Pasinetti case occur for $\gamma < 0$.

Corollary 6. *Let production be specified as above and assume $\gamma < 0$. Assumption 1 is still given.*

(a) There exists $\gamma_1 < 0$, such that: If $\gamma > \gamma_1$, for every γ, there exists a capital tax rate, such that capitalists vanish (Anti-Pasinetti case). If $\gamma < \gamma_1$, for every γ, there exists a tax rate such that workers vanish (Anti-Anti-Pasinetti case).

(b) In both cases, the relationship is monotone: For the Anti-Pasinetti case, the higher the elasticity, the lower the tax rate extinguishing capitalists. For the Anti-Anti-Pasinetti case, the lower the elasticity, the lower the tax that extinguishes workers.[19]

Proof. See Appendix A.3. □

The previous results imply that, given our specification of the production function, for *fixed* τ, the workers' wealth share increases in γ. This is a consequence of the fact that, so long as there are capitalists, the interest rate remains fixed by the capitalists' time preference rate even if the elasticity between capital and labor is changed. In our formulation, workers' fixed supply of labor is worth more the higher the elasticity, so that they save more.[20]

[19] As we noted earlier, however, there may (and in general will) exist an Anti-Pasinetti equilibrium. By contrast, the Anti-Anti-Pasinetti case is a limiting case: the only conditions under which it arises are such that capitalists aymptocially disappear, too, because there does not exist a steady state any more.

[20] This is not a general property, but is a consequence of the specification of our CES

5 Robustness: Public investment that enhances capital or generates rents

In this section we analyze the robustness of the findings from Section 4. We consider alternative ways in which public investment might act on the economy: in particular, public capital can be an imperfect substitute for private capital, as in the case of state-owned companies. This can happen when public investment augments the interest rate (Section 5.1). Alternatively both labor- and capital-enhancing public investment can generate rents, implying that firms make profits (Section 5.2). The final subsection sketches a more general formulation with classes holding different capital goods and distinct forms of public investment (Subsection 5.3).

5.1 Capital-enhancing public investment

As an alternative to labor-enhancing public investment such as education, one can study capital-enhancing public investment. Core infrastructure investments may be plausibly represented as predominantly capital- , not labor-enhancing. In this subsection, we consider a variant of our model, introducing capital-enhancing public investment in a way entirely symmetric to Section 4.[21] For this case, assume that there is a constant-returns-to-scale sub-production function H of both types of capital

$$H(K_t, P_t) = P_t H(K_t/P_t, 1). \tag{25}$$

Total production is then given by $F(H, L)$ and is constant-returns to scale in H and L. With this modification, one defines the wage as:

$$w_t = \frac{\partial F(H_t, L)}{\partial L}. \tag{12a}$$

Here we assume that public investment modifies the return to capital, with capitalists appropriating the full return to H (just as workers appropriated the full return to education-augmented-labor) so that profit maxi-

production function (which is standard). Reducing the elasticity of substitution simply means increasing the curvature. If this is done around the initial equilibrium point, the marginal rate of substitution (and hence the wage) remains unchanged at the point. The focus of the article is on understanding in which situations the policy proposal of capital tax-financed public investment is effective. The effect of a given change in the before tax return on capital on the capital-effective labor ratio depends, in turn, on the elasticity of substitution.

[21]Note that because we consider a *public capital stock*, not technological change and our model *converges to a steady state in which private and public capital are constant*, Uzawa's theorem does not apply to our setting. "Factor-enhancing" public investment has very different properties from standard "factor-enhancing" technological change in growth theory.

mization yields the following rate of return:

$$r_t + \delta_K = \frac{\partial F(H_t, L)}{\partial H_t}. \tag{13a}$$

Further, Equations (1) to (10) of the original model are assumed to hold, but with K, K^c and S replaced by H, H^c and H^S, that is H^c is capitalists' aggregate capital holding and H^S is workers aggregate capital.

Assuming the steady state exists, as above, for capital-enhancing public investment, one can then conclude that

$$\frac{\widetilde{H}^S}{\widetilde{H}}(\tau) = \frac{1}{2+\rho_w}\widetilde{\Omega}_L(\tau)/\widetilde{\Omega}_H(\tau)\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big) \tag{26}$$

(with Ω_L and Ω_H the respective factor shares) and hence prove

Proposition 7. *Assume* $0 < \frac{\widetilde{H^S}}{\widetilde{H}}(\epsilon) < 1$, *i.e. for small taxes rates* $\tau > 0$ *both classes coexist. If the factor share accruing to capital is non-increasing as* $\tau \to 1$, *there always exists a capital tax rate* $\tau_{\lim}$ *such that capitalists vanish (Anti-Pasinetti case).*

Proof. Equation (26) can be derived by analogy to the case of Equation (19), so the result follows by the Intermediate Value Theorem. □

However, as mentioned in Section 3, this raises the question whether labor-enhancing public investment decreasing the share of labor happens if and only if capital-enhancing public investment increases the share of labor. We prove next that this is not in general the case, by considering a specific production function.

Assume now again an explicit CES function between H and L in production, with the same parameters as in Equation (23), but H replacing K and L replacing J there. One finds that, entirely symmetrical to Equation (23) and by the method given in Appendix A.1 that wealth inequality given by H_S/H ratio is given by:

$$\frac{\widetilde{H_S}}{\widetilde{H}} = \frac{(\widetilde{r} + \delta_K)(1-\alpha)}{\alpha(2+\rho_w)}(L/\widetilde{H})^{\gamma}. \tag{27}$$

This can be transformed to an expression with parameters only, as in Equation (23) and similarly to the method given in Appendix A.1:

$$\frac{\widetilde{H_S}}{\widetilde{H}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\Big(\frac{\rho_c}{1-\tau} + \delta_k\Big)\Big(\frac{1}{(1-\alpha)}\big((\frac{1}{\alpha}(\frac{\rho_c}{1-\tau} + \delta_k))^{\frac{\gamma}{1-\gamma}} - \alpha\big)\Big). \tag{28}$$

So we have established:

Proposition 8. *For the case of public investment that augments a production factor as given by Equations (11-13) or Equations (12a-13a) and (25), it is irrelevant* for distributional outcomes *which factor is augmented.*

Proof. The right-hand side of Equation (28) is identical to that of Equation (23). □

Note the emphasis on distributional outcome in the proposition. In absolute terms, it of course matters which production factor is augmented by public investment. It is only through the set-up with two classes that there is no difference in the distributional outcomes regardless of which factor is enhanced. However, this variant of the model might not be convincing if one objects to the idea that public capital adds to the stock of private capital holdings. We next explore the alternative that public capital is a fully separate production factor that yields rents.

5.2 Public investment that creates rents appropriated by firms

Consider a version of the model in which public investment generates a return, which firms obtain as profits:

$$\Pi_t = \frac{\partial F(P_t, K_t, L)}{\partial P} P_t. \tag{29}$$

In Equation (2) we assumed that capitalists appropriate profits, for example as shareholders of the firms. Alternatively, one may think of the government as appropriating the rent and redistributing the returns to the capitalists. In this version, factor returns are given by

$$w_t = \frac{\partial F(K_t, L, P_t)}{\partial L}. \tag{12b}$$

and

$$r_t + \delta_K = \frac{\partial F(K_t, L, P_t)}{\partial K_t}. \tag{13b}$$

This formulation is plausible if the capitalist (who is also a shareholder of the firm) does not optimize for the rents he may receive as the government provides public investment. This would be so if there are many firms. If we think of the rents on public capital being appropriated in proportion to K, then an individual who invests more appropriates more of the public capital, so to him, the observed return to capital is the marginal return to private capital plus his increased share of the rents of public capital. For this reason, we study the details of this approach in Appendix B only, but note the following results here:

Findings from Section 4 are robust up to a multiplicative constant representing the productivity of public capital for the case of labor-enhancing public investment that generates rents (Subsection B.1). We further show that for capital-enhancing public investment that generates rents, the Anti-Anti-Pasinetti case, workers disappearing, can still occur for poor substitution possibilities between aggregate capital and labor when the different forms of capital are highly substitutable (Subsection B.2). Finally, for the special case of perfect substitutability between private and public capital, the tax rate at which a switch from the Pasinetti to the Anti-Pasinetti regime occurs is determined explicitly (Subsection B.3). Importantly, in a formulation with rents obtained as profits, one can treat analytically the case in which capitalists also receive labor income. We find that, by comparison to a case in which only workers provide labor, they are relatively worse off, as expected (Subsection B.1).

5.3 A more general formulation of public investment

There are two possible criticisms of the above analysis. The first is that it ignores differences in the kinds of capital goods in which life-cycle savers and capitalists save. The former have less wealth and are accordingly naturally more risk averse. If there are costs associated with portfolio diversification and obtaining information concerning the relative merits of different assets, it is natural that (at any level of wealth holding) they are less diversified and that they spend less on information acquisition. Data bear out that there are large differences in compositions of assets and liabilities, which can have important implications for the distributive consequences of different policies (Franks et al., 2018; Stiglitz, 2016b). For instance, since equities are disproportionately owned by the wealthy (capitalists, in our model), monetary policies like quantitative easing which disproportionately benefits equity owners contributed to an increase in wealth inequality (Galbraith, 2012; Stiglitz, 2010, 2015a; Turner, 2017).

Secondly, public investment in physical capital can take on a number of forms: it can increase or decrease workers wages, or increase or decrease the returns to private investment in capital. In the simple specifications explored so far, public capital is complementary to both labor and private capital, but that is not true more generally. (In a constant returns to scale production function with two factors, the two factors are necessarily complements, but in a production function with three or more factors, all that one can say is that each factor must be complementary with at least one other factor, i.e. $F_{ij} > 0$ for some j for every i.) Thus, we can formulate a more general production function

$$Y = F(L, K_w, K_c, P_1, P_2) \tag{30}$$

in which workers' and capitalists' capital may not be perfect substitutes for each other, and P_1 and P_2 are different forms of public goods, with, say

$$F_{LP_1} > 0, F_{LP_2} < 0, F_{K_wK_c} < 0 \tag{31}$$

and

$$F_{K_c P_1} < 0 \qquad \text{and} \qquad F_{K_c P_2} > 0. \tag{32}$$

A tax on the return to only (particularly) capitalists' capital with the proceeds invested in P_1 could (a) lead to an increase in wages and thereby K_w; but (b) leads to a decrease in capitalists' capital, so that wealth inequality would decrease. On the other hand, if the proceeds of the tax were invested in P_2, and $F_{K_c P_2}$ is large enough, then K_c might have to increase $F_{K_c P_2}$ to drive down the marginal return to private investment to the long-run equilibrium value, while wages and therefore K_w actually decrease. In this case, wealth inequality has increased. Government investment in research to create human-replacing robots is an example of public investment decreasing the return to labor, and government investment in roads may be an example of public investment which increased the private returns to a particular kind of private investment, railroads. In short, once one takes into account the variety of forms of public investment, it is clear that simplistic models suggesting that the incidence of a capital tax are adverse to the interests of workers have to be re-examined.

6 Application: Quantitative implications of labor-enhancing public investment

In this section we calibrate the version of the model with labor-enhancing public investment (Section 4) to quantify the theoretical properties and more closely examine the possibility of coming close to the Pasinetti-regime boundaries. We calculate a set of threshold values for the substitution elasticity between capital and labor ($\sigma = 1/(1-\gamma)$) and the capital tax rate (τ), as they completely determine which class benefits from increasing capital taxes to finance public investment. Exploring this space numerically matters in particular since there is empirical disagreement about the value of the substitution elasticity (Chirinko, 2008; Piketty and Saez, 2014; Rognlie, 2014).

The main results are illustrated in Figures 2 and 3. Figure 2 shows the wealth ratio changes with increasing capital taxes, for different elasticities. Figure 3 shows all conceivable model outcomes and the prevailing regimes in the form of a "phase diagram". To the left of the gray line, capitalists are better off in relative terms from an increase in capital taxes, to its right, workers are relatively better off. Figure 4 translates this to *absolute terms*, addressing the question when workers are actually better off in absolute terms (see Mattauch et al. (2016) and Stiglitz (2018a) for analytical results). Here we find that the higher the elasticity, the higher can be the tax rate up to which workers are still better off in absolute terms.[22]

[22]We calculated the maximum capital stock of workers as a function of the tax rate

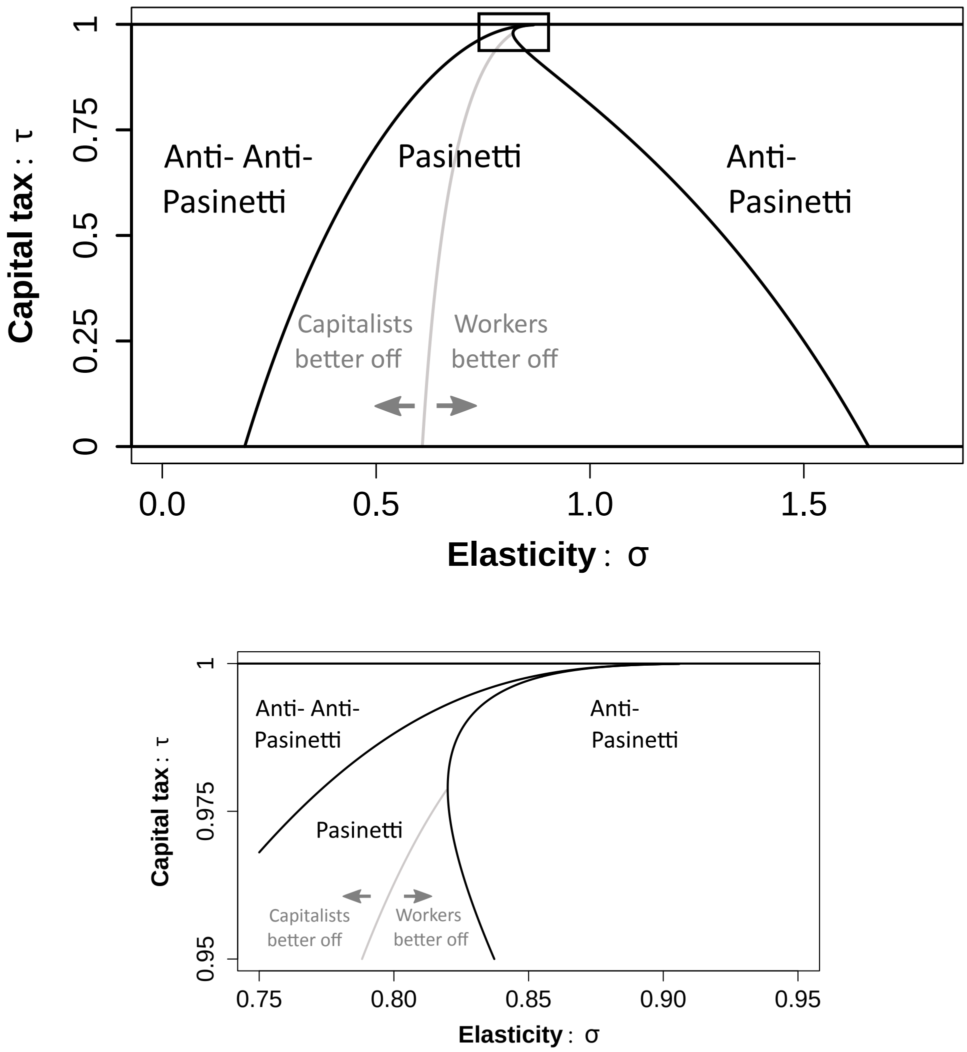

Figure 3: Equilibrium outcomes of the model with labor-enhancing public investment as a function of the elasticity of substitution between capital and labor σ and the capital tax rate τ. The lower half is an enlarged representation of the rectangle in the upper part. The gray line represents the frontier from which on capital tax-financed public investment harms or benefits either capitalists or workers *in relative terms*. Cases: 'Pasinetti': both classes exist; 'Anti-Pasinetti': only workers exist; 'Anti-Anti-Pasinetti': only capitalists exist and no steady-state exists.

numerically. The model is such that workers' consumption and welfare is also maximized at the maximal capital stock, see Mattauch et al. (2016). For verification that the model converges to the steady state for almost the entire Pasinetti regime depicted in the figures of this section, see Appendix C.

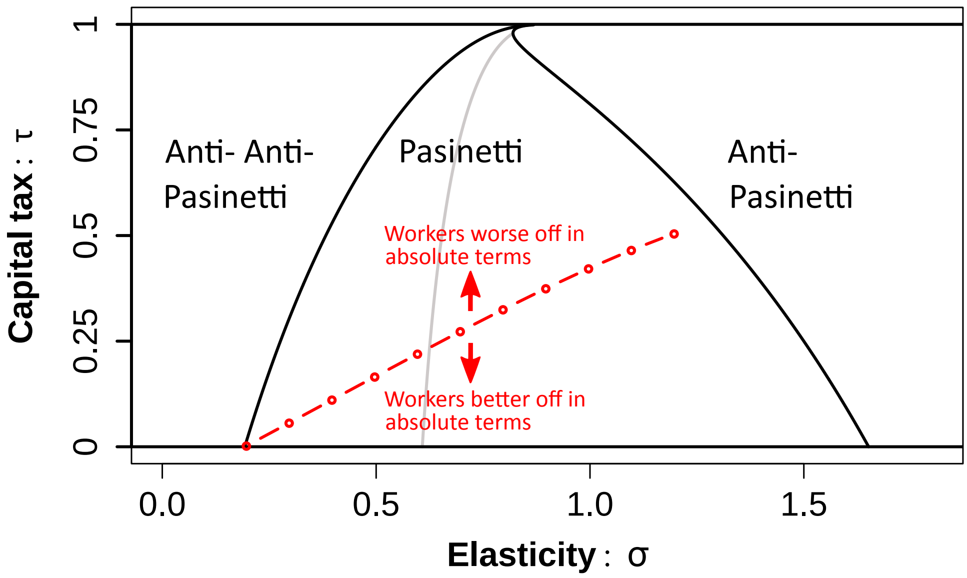

Figure 4: Equilibrium outcomes of the model with public investment as a function of the elasticity of substitution between capital and labor σ and the capital tax rate τ. In addition to Figure 3, this graph shows up to which tax rates workers are better off in *absolute terms*, benefitting from the effect of public investment on their wages. Cases: 'Pasinetti': both classes exist; 'Anti-Pasinetti': only workers exist; 'Anti-Anti-Pasinetti': only capitalists exist and no steady-state exists.

The calibration of the model, which is summarized in Table 1, is justified as follows: The capital share of income α is set at 0.38, in accordance with observations that in OECD countries the labor share of income was dropping from 66.1 % to 61.7 % from 1990 to 2009 (OECD, 2012). The productivity of public capital β, is chosen at 0.2 in accordance with estimates that it is under-provided (Aschauer, 1989; Bom and Ligthart, 2014; Gramlich, 1994). Labor is fixed; however, since we only display results about the distribution of wealth, its value is irrelevant. As we focus on the case of labor-enhancing public investment as specified in Section 4, we parametrize the general sub-function $J(L, P)$ assumed in that section. We use $J_t = P_t^{\beta} L^{(1-\beta)}$, with $0 < \beta < 1$, and $\alpha + \beta < 1$, so that there is no long-run or explosive growth. Time is measured in steps of 30 years because workers are assumed to live for two periods; therefore we choose corresponding values for time preference and depreciation rates.

The wealth distribution for the OECD is not known (Alvaredo et al., 2018). Therefore, we calibrate the model to the U.S. wealth distribution (Wolff, 2010) and check robustness below. In 2007, 62 % of net worth were held by the top 5 % of the population and almost 38 % of net worth by the remaining 95 %. We thus set the time preference rates ρ_c and ρ_w such that for a capital tax of 21 %, (the average capital tax rate in OECD countries between the years 1970 and 2000, Carey and Rabesona, 2002) and an elasticity of substitution between capital and labor of 1, this wealth distribution results. In accordance with evidence that richer households are more patient (Lawrance, 1991; Green et al., 1996), time preference rates of capitalists are chosen significantly lower than that of workers.

Parameter	Standard value	Corresponding annual value
ρ_c	0.56	1.5%
ρ_w	3.98	5.5%
δ_k	0.7	4%
δ_P	0.7	4%
α	0.38	–
β	0.2	–
L	100	–

Table 1: Model calibration

Finally, we conduct a sensitivity analysis of the threshold value of the elasticity σ_1. For elasticities greater than this value, high tax rates lead to an Anti-Pasinetti regime (capitalists disappearing). The threshold is given by setting Equation (A.17) to 1 and thus depends only on the parameters ρ_w and α. Table 2 shows that the threshold is hardly dependent on economically plausible values for the capital share and workers' time preference rate. The reason is that the peaks of the curves $\widetilde{S}/\widetilde{K}(\tau)$ are very steep as they approach $\tau = 1$ and lead to Anti-Pasinetti outcomes.

ρ_w	σ_1	α	σ_1
3	0.812	0.33	0.793
3.2	0.814	0.34	0.799
3.4	0.816	0.35	0.804
3.6	0.817	0.36	0.81
3.8	0.819	0.37	0.815
3.98	**0.82**	**0.38**	**0.82**
4.2	0.822	0.39	0.825
4.4	0.823	0.4	0.83
4.6	0.824	0.41	0.835
4.8	0.825	0.42	0.839
5	0.826	0.43	0.844

Table 2: Dependency of the threshold elasticity on capital share and workers' time preference rate. Standard values in bold.

7 Conclusion

This article examines whether taxing capital at higher rates in order to finance underfunded public capital helps to mitigate wealth inequality. Importantly, we consider disparities in saving behavior in a novel way. Wealth inequality continues to rise in rich countries, which is at least partially due to heterogeneous saving behavior across the wealth distribution. Rich individuals have lower time preference rates, obtain a greater share of their income from capital and save for posterity, not for retirement, when compared to the rest of society. Our study develops the simplest possible framework representing these disparities by combining a standard life-cycle saving working class with dynastically saving top earners.[23]

We prove that the success of the proposed policy in reducing wealth inequality depends on the substitutability of capital and labor. When these factors are highly substitutable, there exists a positive capital tax rate at which dynastic savers disappear in the limit. Life-cycle savers will therefore always obtain a higher share of aggregate capital in this scenario and wealth inequality will be reduced. For low values of the elasticity of substitution between capital and labor, there exists a positive capital tax rate at which life-cycle savers disappear. However, this is a boundary case with little practical relevance, as our numerical results indicate.

Our study thus confirms that capital tax-financed public investment—the major policy recommendation resulting from Piketty (2014)—reduces inequality in wealth in the long term, if capital and labor are close substitutes. Conversely, our results can be seen as a note of caution against this policy recommendation if capital and labor tend to be complements. While empirically there is disagreement about the value of the substitution elasticity (Chirinko, 2008; Karabarbounis and Neiman, 2014; Piketty and Saez, 2014; Knoblach et al., 2016), we find that for standard parameter values the critical threshold between the different limiting cases is around 0.82. This indicates that both cases are conceivable. Future research needs to clarify the value of the elasticity in different macroeconomic settings but, nonetheless, we demonstrate numerically that even for lower elasticities, wealth inequality decreases and the middle class is better off in absolute terms, if capital taxes are only moderately high. Further research could also examine welfare-optimal policies, not merely Pareto-improvements or inequality, if welfare weights were assigned to the different groups in the wealth distribu-

[23] Franks et al. (2018) use a model with a more fine-grained heterogeneity structure, in which there are several intermediate classes that display a mixture between bequest and life-cycle motivated saving in order to compare the distributional incidence of different wealth-based taxes. Their analysis complements this study in indicating that it is more effective to tax certain components of aggregate wealth, namely land rents or bequests than aggregate capital. This is a feature from which the present analysis abstracts, as does most of the literature on capital taxation.

tion. Yet this is an open question in the literature on class models (but see Calvo and Obstfeld, 1988).

Our results can be linked to recent economic studies of ever more intelligent machines (Autor, 2015; Acemoglu and Restrepo, 2017, 2018b; Guerreiro et al., 2017; Korinek and Stiglitz, 2017)—given the critical dependence of our findings on the elasticity of substitution between capital and labor. Advances in automation amplify concerns about growing wealth inequality due to the decreased importance of labor as a production factor and the concentration of capital in the hands of a relatively small share of the population (Autor, 2015; Piketty and Saez, 2014). Yet, the implications of the development of intelligent machines for wealth inequality are presently unclear.

At least three different approaches to the macroeconomics of automation are generally explored: First, if machines become more intelligent this could be represented as technological progress which enhances capital (Acemoglu and Restrepo, 2018a).[24] Second, intelligent machines may be more usefully represented as a different form of capital that becomes a perfect substitute for labor over time (Acemoglu and Restrepo, 2018a). Third, one may think of automation as a rise in the capital intensity of the economy as well as an enhanced substitutability between capital and labor.

In this last case, the results of this study can be interpreted as being about increasing automation. Capital (or robot) taxation would be a useful means for reducing wealth inequality if automation implies a high substitutability between capital and labor.[25] If the development of intelligent machines raises the elasticity close to or above 1, our contribution shows that capital-tax financed public investment emerges as the adequate recipe to keep wealth inequality in check and labor income sufficiently high.

Acknowledgements We thank Max Franks, Cameron Hepburn, Olga Heismann, Katharina Jochemko, Ulrike Kornek, Marc Morgan, Salvatore Morelli, Jacquelyn Pless and Anselm Schultes for helpful comments and Fiona Spuler and Simona Sulikova for excellent research assistance. We further thank seminar audiences in Berlin and Oxford, at ZEW as well as participants of meetings of the European Economic Association, the Journées Louis-André Gérard Varet, the Royal Economic Society, the International Economic Association, the Society for the Study of Economic Inequality and the Verein fuer Socialpolitik and for useful comments. Linus Mattauch's research was supported by a postdoctoral fellowship of the German

[24]This case is related to a large body of literature on factor-biased technological change, which has elaborated on the role of innovation possibility curves, implying that there is a trade-off between labor- and capital-enhancing technological progress (Kennedy, 1964; Samuelson, 1965; Acemoglu, 2010; Stiglitz, 2006, 2014). This literature, however, has to date not been linked to analyses of wealth inequality by class models.

[25]The Anti-Anti-Pasinetti case, by contrast, seems relevant for understanding dystopian visions about intelligent machines. In a world with very little substitution possibilities between increasingly intelligent machines and workers, taxing machines and financing labor-enhancing investment would suppress wages and thus decrease the workers' share of wealth.

Academic Exchange Service (DAAD). The article is also part of a project that has received funding from the European Unions Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 681228.

References

Acemoglu, D. (2010). When does labor scarcity encourage innovation? *Journal of Political Economy*, 118(6):1037–1078.

Acemoglu, D. and Restrepo, P. (2017). Robots and jobs: Evidence from US labor markets. NBER Working Paper 23285.

Acemoglu, D. and Restrepo, P. (2018a). Modeling automation. NBER Working Paper 24321.

Acemoglu, D. and Restrepo, P. (2018b). The race between man and machine: Implications of technology for growth, factor shares and employment. *American Economic Review, forthcoming.*

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Alvaredo, F., Chancel, L., Piketty, T., Saez, E., and Zucman, G. (2017). Global inequality dynamics: New findings from wid. world. *American Economic Review*, 107(5):404–09.

Alvaredo, F., Chancel, L., Piketty, T., Saez, E., and Zucman, G. (2018). WID – World Wealth and Income Database. wid.world, retrieved 2018-02-07.

Aschauer, D. A. (1989). Is public expenditure productive? *Journal of Monetary Economics*, 23(2):177–200.

Atkinson, A. B. and Stiglitz, J. E. (1969). A new view of technological change. *The Economic Journal*, 79(315):573–578.

Atkinson, A. B. and Stiglitz, J. E. (1976). The design of tax structure: direct versus indirect taxation. *Journal of Public Economics*, 6(1-2):55–75.

Attanasio, O. P. (1994). Personal saving in the United States. In Poterba, J. M., editor, *International Comparisons of Household Saving*, pages 57–124. The University of Chicago Press, Chicago.

Autor, D. H. (2015). Why are there still so many jobs? The history and future of workplace automation. *The Journal of Economic Perspectives*, 29(3):3–30.

Baranzini, M. (1991). *A theory of wealth distribution and accumulation.* Oxford University Press.

Barelli, P. and de Abreu Pessôa, S. (2003). Inada conditions imply that production function must be asymptotically Cobb–Douglas. *Economics Letters*, 81(3):361–363.

Becker, R. A. (1980). On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households. *The Quarterly Journal of Economics*, 95(2):375–382.

Bevan, D. L. (1979). Inheritance and the distribution of wealth. *Economica*, 46(184):381–402.

Bevan, D. L. and Stiglitz, J. E. (1979). Intergenerational transfers and inequality. *The Greek Economic Review*, 1(1):8–26.

Bewley, T. (1977). The permanent income hypothesis: A theoretical formulation. *Journal of Economic Theory*, 16(2):252–292.

Bom, P. R. D. and Ligthart, J. E. (2014). What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys*, 28(5):889–961.

Calderón, C. and Servén, L. (2014). Infrastructure and growth. In Durlauf, S. N. and Blume, L. E., editors, *The New Palgrave Dictionary of Economics. Online Edition.* Palgrave Macmillan, Basingstoke. Accessed: 1 December 2014.

Calvo, G. A. and Obstfeld, M. (1988). Optimal time-consistent fiscal policy with finite lifetimes. *Econometrica*, 56(2):411–432.

Carey, D. and Rabesona, J. (2002). Tax ratios on labour and capital income and on consumption. *OECD Economic Studies*, 35(2):129–174.

Champernowne, D. G. (1953). A model of income distribution. *The Economic Journal*, 63(250):318–351.

Chatterjee, S. and Turnovsky, S. J. (2012). Infrastructure and inequality. *European Economic Review*, 56(8):1730–1745.

Chirinko, R. S. (2008). σ: The long and short of it. *Journal of Macroeconomics*, 30(2):671–686.

Diamond, P. and Saez, E. (2011). The case for a progressive tax: from basic research to policy recommendations. *The Journal of Economic Perspectives*, 25(4):165–190.

Diaz-Gimenez, J., Glover, A., and Rios-Rull, J.-V. (2011). Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update. *Federal Reserve Bank of Minneapolis Quarterly Review*, 34(1).

Dynan, K. E., Skinner, J., and Zeldes, S. P. (2004). Do the rich save more? *Journal of Political Economy*, 112(2):397–444.

Foley, D. K. and Michl, T. R. (1999). *Growth and distribution.* Harvard University Press.

Franks, M., Klenert, D., Schultes, A., Edenhofer, O., and Lessmann, K. (2018). Is capital back? the role of land ownership and savings behavior. *International Tax and Public Finance*, 25(5):1252–1276.

Galbraith, J. K. (2012). *Inequality and instability: A study of the world economy just before the great crisis.* Oxford University Press.

Gramlich, E. M. (1994). Infrastructure investment: a review essay. *Journal of Economic Literature*, 32(3):1176–1196.

Green, L., Myerson, J., Lichtman, D., Rosen, S., and Fry, A. (1996). Temporal Discounting in Choice Between Delayed Rewards : The Role of Age and Income. *Psychology and Aging*, I:79–84.

Guerreiro, J., Rebelo, S., and Teles, P. (2017). Should robots be taxed? NBER Working Paper 23806.

Heijdra, B. J. (2009). *Foundations of Modern Macroeconomics*. Oxford University Press, Oxford.

Jones, C. I. (2015). Pareto and Piketty: The macroeconomics of top income and wealth inequality. *The Journal of Economic Perspectives*, 29(1):29–46.

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics*, 28(1):59–83.

Karabarbounis, L. and Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, 129(1):61–103.

Kennedy, C. (1964). Induced bias in innovation and the theory of distribution. *The Economic Journal*, 74(295):541–547.

Klenert, D., Mattauch, L., Edenhofer, O., and Lessmann, K. (2018). Infrastructure and inequality: Insights from incorporating key economic facts about household heterogeneity. *Macroeconomic Dynamics*, 22:864–895.

Knoblach, M., Rößler, M., and Zwerschke, P. (2016). The elasticity of factor substitution between capital and labor in the US economy: a meta-regression analysis. CEPIE Working Paper 03/16.

Korinek, A. and Stiglitz, J. E. (2017). Artificial intelligence and its implications for income distribution and unemployment. NBER Working Paper 24174.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Lawrance, E. C. (1991). Poverty and the rate of time preference: evidence from panel data. *Journal of Political Economy*, 99(1):54–77.

Mattauch, L., Edenhofer, O., Klenert, D., and Bénard, S. (2016). Distributional Effects of Public Investment when Wealth and Classes are Back. *Metroeconomica*, 67(3):603–629.

Meade, J. E. (1966). The outcome of the pasinetti-process: a note. *The Economic Journal*, 76(301):161–165.

Michl, T. R. (2009). *Capitalists, workers, and fiscal policy: a classical model of growth and distribution*. Harvard University Press, Cambridge, MA.

Novokmet, F., Piketty, T., and Zucman, G. (2017). From Soviets to Oligarchs: inequality and property in Russia, 1905-2016. NBER Working Paper 23712.

OECD (2012). *Labour Losing to Capital: What Explains the Declining Labour Share?*, chapter 3, pages 109–161. OECD.

Pasinetti, L. L. (1962). Rate of profit and income distribution in relation to the rate of economic growth. *Review of Economic Studies*, 29(4):267–279.

Piketty, T. (2014). *Capital in the Twenty-First Century*. Harvard University Press.

Piketty, T. and Saez, E. (2013). A theory of optimal inheritance taxation. *Econometrica*, 81(5):1851–1886.

Piketty, T. and Saez, E. (2014). Inequality in the long run. *Science*, 344(6186):838–843.

Piketty, T. and Zucman, G. (2014). Capital is back: Wealth-income ratios in rich countries 1700-2010. *Quarterly Journal of Economics*, 129(3):1255–1310.

Piketty, T. and Zucman, G. (2015). Wealth and inheritance in the long run. In Atkinson, A. B. and Bourguignon, F., editors, *Handbook of income distribution. Vol. 2*, pages 1303–1368. Elsevier.

Rognlie, M. (2014). A note on Piketty and diminishing returns to capital. http://gabriel-zucman.eu/files/teaching/Rognlie14.pdf. retrieved 15-10-07.

Saez, E. and Zucman, G. (2016). Wealth Inequality in the United States Since 1913: Evidence from Capitalized Income Tax Data. *Quarterly Journal of Economics*, 131(2):519–578.

Samuelson, P. A. (1965). A theory of induced innovation along Kennedy-Weisäcker lines. *Review of Economics and Statistics*, 47(4):343–356.

Samuelson, P. A. and Modigliani, F. (1966). The Pasinetti paradox in neoclassical and more general models. *Review of Economic Studies*, 33(4):269–301.

Stiglitz, J. E. (1967). A Two-Sector Two Class Model of Economic Growth. *The Review of Economic Studies*, 34(2):227–238.

Stiglitz, J. E. (1969). Distribution of Income and Wealth Among Individuals. *Econometrica*, 37(3):382–397.

Stiglitz, J. E. (1976). Estate taxes, growth and redistribution. In Grieson, R., editor, *Essays in Honor of W. Vickrey*, pages 225–232. Lexington Publishing Company.

Stiglitz, J. E. (1978a). Equality, taxation, and inheritance. In Krelle, W. and Shorrocks, A., editors, *Personal Income Distribution*, pages 271–303. North-Holland Publishing Company.

Stiglitz, J. E. (1978b). Notes on estate taxes, redistribution, and the concept of balanced growth path incidence. *Journal of Political Economy*, 86(2, Part 2):S137–S150.

Stiglitz, J. E. (2006). Samuelson and the factor bias of technological change: Toward a unified theory of growth and unemployment. In Szenberg, M., Ramrattan, L., and Gottesman, A., editors, *Samuelsonian economics and the twenty-first century*, pages 235–251. Oxford University Press.

Stiglitz, J. E. (2010). *Freefall: America, free markets, and the sinking of the world economy*. WW Norton & Company.

Stiglitz, J. E. (2012). *The price of inequality: How today's divided society endangers our future*. WW Norton & Company.

Stiglitz, J. E. (2014). Unemployment and innovation. NBER Working Paper 20670.

Stiglitz, J. E. (2015a). Fed policy, inequality, and equality of opportunity. Roosevelt Institute Report.

Stiglitz, J. E. (2015b). New theoretical perspectives on the distribution of income and wealth among individuals: Part i. the wealth residual.

Stiglitz, J. E. (2015c). New theoretical perspectives on the distribution of income and wealth among individuals: Part ii: Equilibrium wealth distributions. NBER Working Paper 21190.

Stiglitz, J. E. (2015d). New theoretical perspectives on the distribution of income and wealth among individuals: Part iii: Life cycle savings vs. inherited savings. NBER Working Paper 21191.

Stiglitz, J. E. (2016a). How to restore equitable and sustainable economic growth in the united states. *American Economic Review*, 106(5):43–47.

Stiglitz, J. E. (2016b). New Theoretical Perspectives on the Distribution of Income and Wealth Among Individuals. In Basu, K. and Stiglitz, J. E., editors, *Inequality and Growth: Patterns and Policy, Volume I: Concepts and Analysis*. IEA Conference Volume No. 156-I, Houndmills, UK and New York: Palgrave Macmillan.

Stiglitz, J. E. (2017). Pareto efficient taxation and expenditures: Pre-and redistribution. NBER Working Paper 23892.

Stiglitz, J. E. (2018a). Pareto efficient taxation and expenditures: Pre-and redistribution. *Journal of Public Economics*, 62:101–119.

Stiglitz, J. E. (2018b). Where modern macroeconomics went wrong. *Oxford Review of Economic Policy*, 34(1-2):70–106.

Turner, A. (2017). *Between debt and the devil: money, credit, and fixing global finance*. Princeton University Press.

Wolff, E. (2017). Household wealth trends in the united states, 1962 to 2016: Has middle class wealth recovered? NBER Working Paper 24085.

Wolff, E. N. (1998). Recent trends in the size distribution of household wealth. *Journal of Economic Perspectives*, 12(3):131–150.

Wolff, E. N. (2010). Recent trends in household wealth in the United States: rising debt and the middle-class squeeze - an update to 2007. *The Levy Economics Institute Working Paper Collection 589*.

Wolff, E. N. and Zacharias, A. (2013). Class structure and economic inequality. *Cambridge Journal of Economics*, 37:1381–1406.

Zamparelli, L. (2017). Wealth distribution, elasticity of substitution and Piketty: an anti-dual pasinetti economy. *Metroeconomica*, 68(4):927–946.

Appendices

A Derivations when public investment is labor-augmenting

A.1 Derivation of $\widetilde{S}/\widetilde{K}$

In this section we derive an explicit formula for the capital share of the workers $\widetilde{S}/\widetilde{K}$ (Equation 23).

We divide the expression for workers' saving (Equation 8) by total capital and then insert the firm's first-order conditions (Equations 13 and 12):

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{\widetilde{J}\widetilde{w}}{(2+\rho_w)\widetilde{K}} = \frac{(1-\alpha)\widetilde{J}^{\gamma}}{(2+\rho_w)\widetilde{K}\widetilde{Y}^{-(1-\gamma)}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\Big(\frac{\rho_c}{1-\tau}+\delta_k\Big)\Big(\frac{\widetilde{J}}{\widetilde{K}}\Big)^{\gamma}. \tag{A.1}$$

Here we used that

$$w_t = \frac{\partial F(K_t, J_t)}{\partial J}. \tag{A.2}$$

To find an explicit solution for expression (A.1), solve the model for $\widetilde{K}/\widetilde{J}$. For this purpose, let $k = K/J$, and let $y = Y/J$. Then

$$y = \big(\alpha(k)^{\gamma} + (1-\alpha)1^{\gamma}\big)^{\frac{1}{\gamma}}. \tag{A.3}$$

From standard growth theory, we know that for any constant-returns-to-scale function

$r_t + \delta_k = \widetilde{Y}_K = \widetilde{y}'(k)$,

so that

$$\widetilde{y}'(k) = \frac{\rho_c}{1-\tau} + \delta_k. \tag{A.4}$$

To solve this, use that

$$\widetilde{y}'(\widetilde{k}) = \alpha\widetilde{k}^{\gamma-1}\big(\alpha\widetilde{k}^{\gamma} + (1-\alpha)\big)^{\frac{1-\gamma}{\gamma}}. \tag{A.5}$$

Substituting this into Equation (A.4) gives

$$\Big(\frac{1}{\alpha}\Big(\frac{\rho_c}{1-\tau}+\delta_k\Big)\Big)^{\frac{\gamma}{1-\gamma}} = \widetilde{k}^{-\gamma}(\alpha\widetilde{k}^{\gamma} + (1-\alpha)) = \alpha + (1-\alpha)\widetilde{k}^{-\gamma}. \tag{A.6}$$

This is an equation that can be solved for $\widetilde{k}$,[26] as it is equivalent to

[26] Evidently solutions to Equation (A.7) could be complex if the term inside the exponent is negative. This reflects that outside of the Pasinetti regime, the model would not converge to a steady state given by the above equations as one class disappears. For further economic analysis, only the term's appearance in Equation (A.8) is used, which has an exponent equal to one. For $\sigma > 1$, our parametrized version of $F(J, K)$ thus fulfills the condition $\lim_{\tau\to 1} K/J = 0$ assumed in Proposition 1.

$$\frac{\widetilde{K}}{\widetilde{J}} = \widetilde{k} = \Big(\frac{1}{(1-\alpha)}\big((\frac{1}{\alpha}(\frac{\rho_c}{1-\tau}+\delta_k))^{\frac{\gamma}{1-\gamma}}-\alpha\big)\Big)^{\frac{-1}{\gamma}} \tag{A.7}$$

This expression can be substituted into Equation (A.1) to obtain an explicit solution for the capital ratio:

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{(1-\alpha)}{\alpha(2+\rho_w)}\big(\frac{\rho_c}{1-\tau}+\delta_k\big)\Big(\frac{1}{(1-\alpha)}\big((\frac{1}{\alpha}(\frac{\rho_c}{1-\tau}+\delta_k))^{\frac{\gamma}{1-\gamma}}-\alpha\big)\Big). \tag{A.8}$$

A.2 Properties of $\widetilde{S}/\widetilde{K}$

We determine the sign and zero of the derivative of $\widetilde{S}/\widetilde{K}(\tau)$. For this purpose, let $x(\tau) = (\rho_c/(1-\tau)+\delta_K)$, and note that $x'(\tau) = \rho_c(1-\tau)^{-2}$.

Then:

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau) = \frac{1}{\alpha(2+\rho_w)}\big((\frac{1}{\alpha})^{\frac{\gamma}{1-\gamma}}(x(\tau))^{\frac{1}{1-\gamma}}-\alpha x(\tau)\big). \tag{A.9}$$

Thus:

$$\begin{aligned}(\frac{\widetilde{S}}{\widetilde{K}})'(\tau) &= \frac{1}{\alpha(2+\rho_w)(1-\gamma)}(\frac{x(\tau)}{\alpha})^{\frac{\gamma}{1-\gamma}}x'(\tau)-\frac{x'(\tau)}{(2+\rho_w)}\\ &= (\frac{\rho_c}{(2+\rho_w)(1-\tau)^2})\Big(\frac{1}{\alpha(1-\gamma)}(\frac{1}{\alpha}(\frac{\rho_c}{(1-\tau)}+\delta_K))^{\frac{\gamma}{1-\gamma}}-1\Big)\end{aligned} \tag{A.10}$$

We now compute the zero of the derivative by setting the second term of the product to 0:

$$\frac{1}{\alpha(1-\gamma)}(\frac{1}{\alpha}(\frac{\rho_c}{(1-\tau)}+\delta_K))^{\frac{\gamma}{1-\gamma}} = 1 \tag{A.11}$$

This is equivalent to

$$(\frac{1}{\alpha}(\frac{\rho_c}{(1-\tau)}+\delta_K)) = (\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} \tag{A.12}$$

and further equivalent to

$$\frac{\rho_c}{(1-\tau)} = \alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}}-\delta_K. \tag{A.13}$$

Therefore,

$$\tau_z = 1-\frac{\rho_c}{\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}}-\delta_K}. \tag{A.14}$$

Further, replacing the equalities by inequalities, one can determine the sign of the derivative. This is, in general, dependent on the value of all relevant parameters. However, for non-restrictive parameter conditions, its sign can be determined for the economically relevant cases as follows.

Consider the above four equations as inequalities: First, note that for values $\gamma < 0$ the direction of the inequality changes from Equation (A.11) to (A.12). Second, noting that $\tau \in (0,1)$, there is also a change in the direction of the inequality from Equation (A.13) to (A.14) if

$$\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} > \delta_K. \tag{A.15}$$

For $|\gamma|$ small, it can be verified that this inequality holds for $\gamma < 0$, but not for $\gamma > 0$, for a wide parameter range for α and δ_k around their standard values of 0.38 and 0.7, respectively. For part of this parameter range, it also holds for large values of $|\gamma|$. Taken together, this means that the derivative is positive for $\tau < \tau_z$ and negative for $\tau > \tau_z$. Thus τ_z is a local maximum. The only economically relevant case that differs is for $\gamma < 0$ and $|\gamma|$ large ($\gamma < -0.95$ for the standard parametrization): in this case τ_z is a local minimum. However, for this case, $\tau_z > 1$ and thus $\widetilde{S}/\widetilde{K}$ is decreasing on $\tau \in (0,1)$.

Further, it can be verified, by inserting τ_z into the function, that the value of the maximum is given by

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau_z) = -\frac{\alpha\gamma}{(2+\rho_w)}\big(\alpha(1-\gamma)\big)^{\frac{1-\gamma}{\gamma}}. \tag{A.16}$$

A.3 Proof of Corollary 6

It is established in Appendix A.2 that $\widetilde{S}/\widetilde{K}(\tau)$ has a unique maximum if $\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} > \delta_K$. (If this condition is not fulfilled, which is the case for $|\gamma|$ large, $\widetilde{S}/\widetilde{K}(\tau)$ has a minimum. The minimum, however, occurs for $\tau > 1$, and $\widetilde{S}/\widetilde{K}(\tau)$ can be shown to be decreasing within $\tau \in (0,1)$. See Appendix A.2 for details.) The value of this maximum is given by

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau_z) = -\frac{\alpha\gamma}{(2+\rho_w)}\big(\alpha(1-\gamma)\big)^{\frac{1-\gamma}{\gamma}}. \tag{A.17}$$

Consider this value as a function of γ :

$$f(\gamma) = -\frac{\alpha\gamma}{(2+\rho_w)}\big(\alpha(1-\gamma)\big)^{\frac{1-\gamma}{\gamma}}. \tag{A.18}$$

The corollary is shown by proving the following properties, which are derived in Appendix A.2:

1. $f(\gamma)$ has a unique minimum with respect to γ at $\gamma = \ln(\alpha)$. It is monotonically increasing with respect to γ for $\gamma > \ln(\alpha)$.

2. $\lim_{\gamma\to 0+} f(\gamma) = +\infty$.

Assumption 1 implies that $f(\ln(\alpha)) < 1$ because one can deduce that there exists a γ with $\alpha(\alpha(1-\gamma))^{\frac{1-\gamma}{\gamma}} > \delta_K$. such that $\tau_z = 0$.[27]

The corollary is then deduced from the two properties in the following way: by the Intermediate Value Theorem, a value γ_1 exists, such that $f(\gamma_1) = 1$, since $f(\gamma)$ is continuous. This implies that for $\gamma < \gamma_1$, $f(\gamma_1) < 1$ and hence the Anti-Anti-Pasinetti case occurs. If $\gamma > \gamma_1$, then $f(\gamma_1) > 1$ and the economy is in an Anti-Pasinetti state.

Finally, note that Part (b) would not follow if it were the case that $\ln(\alpha) > \gamma_{\text{crit}}$ with γ_{crit} given by $\alpha(\alpha(1-\gamma_{\text{crit}}))^{\frac{1-\gamma_{\text{crit}}}{\gamma_{\text{crit}}}} = \delta_K$. In fact, it would violate the monotonicity of $\widetilde{S}/\widetilde{K}(\tau)$ throughout. Below we show why this cannot occur.

We now complete the proof of Part (a) of Corollary 6 by showing the two properties that

1. $f(\gamma)$ has a unique minimum with respect to γ at $\gamma = \ln(\alpha)$. It is monotonically increasing with respect to γ for $\gamma > \ln(\alpha)$.

2. $\lim_{\gamma\to 0+} f(\gamma) = +\infty$.

Regarding the first property, note that $f(\gamma)$ can be rewritten as

$$f(\gamma) = -\gamma\alpha^{1/\gamma}\Big(\frac{1}{(2+\rho_w)}\big(1-\gamma\big)^{\frac{1-\gamma}{\gamma}}\Big). \tag{A.19}$$

Let $g(\gamma) = -\gamma\alpha^{1/\gamma}$ and $h(\gamma) = 1/(2+\rho_w)\big((1-\gamma)\big)^{\frac{1-\gamma}{\gamma}}$. $h(\gamma)$ is monotonically increasing for all $\gamma > 0$, as is obtained from the fact that the function x^x is monotonically increasing. Further, it can be calculated that

$$\frac{\mathrm{d}g}{\mathrm{d}\gamma} = \alpha^{1/\gamma}\left(\frac{1}{\gamma}\ln(\alpha)\right). \tag{A.20}$$

This derivative equals zero for $\gamma = \ln(\alpha)$ and is positive for $\gamma > \ln(\alpha)$ and negative for $\gamma < \ln(\alpha)$. Since $f(\gamma)$ is the product of function g, which has a minimum at $\gamma = \ln(\alpha)$ and the monotonically increasing function h, it also has a minimum at $\gamma = \ln(\alpha)$. From this also follows that $f(\gamma)$ is monotonically increasing for $\gamma > \ln(\alpha)$.

Regarding the second property, factor $f(\gamma)$ into

$$f(\gamma) = \big(-\gamma\alpha^{1/\gamma}\big)\cdot\frac{1}{(1-\gamma)(2+\rho_w)}\cdot\big(1-\gamma\big)^{\frac{1}{\gamma}}. \tag{A.21}$$

Taking limits with respect to $\gamma \to 0$ from below, the second factor of this product tends to $1/(2+\rho_w)$. Note the third factor is equivalent to

[27] Condition $f(\ln(\alpha)) < 1$ is true if $-\frac{\alpha\ln(\alpha)}{(2+\rho_w)}\big(\alpha(1-\ln(\alpha))\big)^{\frac{1-\ln(\alpha)}{\ln(\alpha)}} < 1$, a condition that is satisfied by our standard parametrization (see Section 6) by a large margin.

$\exp(1/x \ln(1-x))$. Applying L'Hôpital's rule to its exponent yields that this factor tends to e^{-1}.

It remains to consider the first term, $-\gamma\alpha^{1/\gamma}$. Substituting $\gamma = -1/y$ and applying L'Hôpital's rule to $(1/\alpha)^y/y$ as $y \to +\infty$ shows that this term tends to $+\infty$. This establishes the behavior at $\gamma = 0$ from below and completes the proof of Part (a).

We finally explain that given Assumption 1 it is always the case that $\ln(\alpha) < \gamma_{\text{crit}}$ as mentioned in the proof of Part (b) of Corollary 6. Recall that γ_{crit} is given by

$$\alpha(\alpha(1-\gamma_{\text{crit}}))^{\frac{1-\gamma_{\text{crit}}}{\gamma_{\text{crit}}}} = \delta_K. \tag{A.22}$$

Suppose for contradiction that $\ln(\alpha) > \gamma_{\text{crit}}$. Then by definition

$$\alpha(\alpha(1-\ln(\alpha)))^{\frac{1-\ln(\alpha)}{\ln(\alpha)}} > \delta_K. \tag{A.23}$$

Rearranging gives:

$$(1-\ln(\alpha))^{\frac{1}{\ln(\alpha)}} > \delta_K(1-\ln(\alpha)). \tag{A.24}$$

Noting that for $0 < \alpha < 1$, the right-hand side is bigger than δ_K and the left-hand side is smaller than α, one establishes a contradiction to Assumption 1 (b) in Section 4 of the main text.

B Labor- and capital-enhancing public investment generating rents

B.1 Labor-enhancing public investment with rents

We first consider the case in which public capital is labor-enhancing in the sense of Equation (11). From Equation (16), one immediately obtains a modified expression with factor shares Ω_J, Ω_K :

$$\frac{\widetilde{S}}{\widetilde{K}}(\tau) = \frac{1}{2+\rho_w}\frac{\partial \widetilde{J}}{\partial L}\widetilde{\Omega}_J(\tau)/\widetilde{\Omega}_K(\tau)\Big(\frac{\rho_c}{1-\tau}+\delta_k\Big). \tag{B.1}$$

In comparison with Equation (19), the marginal product of the composite factor with respect to labor enters as an additional multiplicative term.

Assume an explicit parametrization for J :

$$J_t = P_t^{\beta} L^{(1-\beta)}, \tag{B.2}$$

with $\alpha + \beta < 1$, to exclude the case of long-run or explosive growth.

Then $\partial J/\partial L = (1-\beta)$. For a parametrized CES function as in Section 4, one thus finds that all results in Section 4 hold, but are modified by the multiplicative constant $(1-\beta)$.

In this formulation, one can also characterize a regime in which capitalists also work. Assume total labor is divided between workers' labor L_c and capitalists' labor L_w. Then

$$\widetilde{S}/\widetilde{K} = 1/(2+\rho_w)(\widetilde{w}L_w)/\widetilde{K}, \tag{B.3}$$

so by comparison to a case in which only workers provide labor, they are relatively worse off, as expected. However, by analogy to Proposition 1, one can still show that the elasticity of substitution between capital and labor determines whether there exists a tax rate at which capitalists disappear. So the results of this manuscript do not change qualitatively when it is assumed that capitalists also provide labor.

B.2 Capital-enhancing public investment with rents

Here we treat the case in which capital-enhancing public investment generates a rent, instead of augmenting the marginal product of private capital. Firms generate profits and we assume again that these are appropriated by the capitalists, so that households hold stocks of private capital, not augmented capital.

For a general production function with capital-enhancing public investment in the sense of Equation (25), one obtains Equation (26) again. Next, we analyze when the Anti-Pasinetti and Anti-Anti-Pasinetti may occur depending on substitution possibilities.

Therefore, in this subsection we analyze a production function of the nested CES type (instead of a single CES function and an unspecified subfunction, as with labor-enhancing public capital in Section 4). Assuming a general subfunction H will not help in this case because the interest rate is now given by $\partial F/\partial K$. Let production thus be given by:

$$Y_t = F_t(H_t, L) = (\theta H_t^\mu + (1-\theta)L^\mu)^{(1/\mu)} \tag{B.4}$$

Public and private capital G and K are combined into generic capital H_t by means of a CES function:

$$H_t(K_t, P_t) = (\zeta K_t^\eta + (1-\zeta)P_t^\eta)^{(1/\eta)}, \tag{B.5}$$

with $0 < \zeta < 1$ being the share parameter of private capital and $s = 1/(1-\eta)$ being the elasticity of substitution between private and public capital with $-\infty < \eta \leq 1$.

The ratio for wealth inequality in the steady state is then given by:

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{(\frac{\rho_c}{(1-\tau)} + \delta_K)(1-\theta)}{\theta\zeta(2+\rho_w)} \left(\frac{L}{H}\right)^\mu \left(\zeta + (1-\zeta)\left(\frac{\tau\rho_c}{(1-\tau)\delta_P}\right)^\eta\right). \tag{B.6}$$

An explicit expression for $(\frac{L}{H})^{\mu}$ can be determined by using the intensive form of the production function:

$$\left(\frac{L}{H}\right)^{\mu} = \frac{\left(\left(\frac{\frac{\rho_c}{(1-\tau)}+\delta_k}{\zeta\theta}\right)\left(\zeta+(1-\zeta)\left(\frac{\tau\rho_c}{\delta_p(1-\tau)}\right)^{\eta}\right)^{\left(\frac{\eta-1}{\eta}\right)}\right)^{\left(\frac{\mu}{1-\mu}\right)} - \theta}{(1-\theta)}. \quad (B.7)$$

To obtain Equation (B.6), insert Equations (12*b*) and (13*b*) into Equation (26) for the specified production structure, noting that

$$\left(\frac{H}{K}\right)^{\eta} = \left(\zeta + (1-\zeta)\frac{\widetilde{r}\tau}{\delta_P}\right)^{\mu}. \quad (B.8)$$

To further obtain the variable-free expression (B.7), proceed analogously to Appendix A.1 and let $h = H/L$, $y = Y/L$, etc. Computing the marginal product of capital in the intensive form, one finds that

$$\frac{\widetilde{r}+\delta_K}{k^{\eta-1}h^{1-\eta}} = \theta\zeta h^{\mu-1}\left(\theta h^{\mu}+(1-\theta)\right)^{\frac{1}{\mu}-1}. \quad (B.9)$$

Using Equation B.8 for the denominator on the left-hand side and solving for h, similar as in Appendix A.1, yields Equation (B.7).

In the derivations, it is additionally used that from Equation (10) in the steady state it follows that

$$\widetilde{P} = \frac{\tau\widetilde{r}\widetilde{K}}{\delta_P}. \quad (B.10)$$

We next examine whether the Anti-Pasinetti and Anti-Anti-Pasinetti regimes still can occur.

Proposition 9. *Let $\eta > 0$. For capital-enhancing public investment that does not augment factor prices, with the explicit production structure given by Equations (B.4-B.5) the Anti-Anti-Pasinetti case can occur for $\mu < 0$. For $\mu > 0$, only the Anti-Pasinetti regime can occur.*

The limiting behavior as $\tau \to 1$ of wealth inequality is now a more complicated combination in the space of the two elasticity parameters. We limit the treatment here to a partial result with high substitution possibilities between the two forms of capital. Proposition 9 is illustrated by Figure 5.

Proof. Let $\eta > 0$. Taking limits as $\tau \to 1$ in Equation (B.6) yields terms straightforwardly tending to positive infinity except those implicit in $\left(\frac{L}{H}\right)^{\mu}$ and given by Equation (B.7). It can be shown, by applying L'Hôpital's rule, that

$$\lim_{\tau\to 1}\left(\frac{L}{H}\right)^{\mu} = \begin{cases} +\infty & \text{if } \mu > 0, \\ -\theta/(1-\theta) & \text{if } \mu < 0 \end{cases} \quad (B.11)$$

and therefore

$$\lim_{\tau\to 1}\frac{\widetilde{S}}{\widetilde{K}}=\begin{cases}+\infty & \text{if } \mu>0,\\ -\infty & \text{if } \mu<0\end{cases} \tag{B.12}$$

□

Setting $\mu = 0$ in Equation (B.6), one finds the steady-state wealth distribution for the special case in which the upper level of the CES-nest is Cobb-Douglas:

$$\frac{\widetilde{S}}{\widetilde{K}}=\frac{(\widetilde{r}+\delta_K)(1-\alpha)}{\alpha\zeta(2+\rho_w)}\left(\zeta+(1-\zeta)\left(\frac{\tau\widetilde{r}}{\delta_P}\right)^{\eta}\right). \tag{B.13}$$

From this expression, one can readily deduce the following special case:

Proposition 10. *With a nested CES production structure as assumed in Equations* (B.4) *and* (B.5) *and* $\mu = 0$

1. *the Anti-Anti-Pasinetti case cannot occur.*
2. *for every* $1 \geq \eta > 0$ *there exists a capital tax rate* τ_{lim} *from which on the economy switches from a Pasinetti to an Anti-Pasinetti state.*

In proving this proposition, we assume that Assumption 1 still holds, i.e. that $0 < \widetilde{S}/\widetilde{K}(\epsilon) < 1$ for ϵ small, which is the case for the meaningful parameter range and the second part is only meaningful if steady states exist (see discussion in Footnote 15).

Proof. Part 1 can be inferred directly from Equation (B.6): since $\widetilde{r}$, δ_K, δ_P, α, ζ, ρ_w are greater than zero, and $0 \leq \tau \leq 1$, the expression for $\widetilde{S}/\widetilde{K}$ is always strictly positive and has a strictly positive limit.

For Part 2, the idea of the proof is to show that $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically increasing in τ, starting from a value lower than one and converging to infinity for $\tau \to 1$. The proof proceeds in two steps:

1. we show that $\lim_{\tau\to 1}\widetilde{S}/\widetilde{K}(\tau) = \infty$.
2. we show that $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically increasing in $0 \leq \tau < 1$.

Regarding the first step, we insert the explicit expression for $\widetilde{r} = \rho_c/(1-\tau)$ and expand the products in Equation (B.6). This yields the following expression:

$$\begin{aligned}\frac{\widetilde{S}}{\widetilde{K}}&=\frac{(1-\alpha)}{\alpha\zeta(2+\rho_w)}\left[\left(\frac{\rho_c}{1-\tau}+\delta_K\right)\left(\zeta+(1-\zeta)\left(\frac{\tau\rho_c}{(1-\tau)\delta_P}\right)^{\eta}\right)\right]\\&=\frac{(1-\alpha)}{\alpha\zeta(2+\rho_w)}\left[\lambda\zeta+(1-\zeta)\left(\left(\frac{\rho_c^{1+\eta}}{\delta_P^{\eta}}\mu\right)+\frac{\delta_K}{\delta_P^{\eta}}\nu\right)\right],\end{aligned}$$

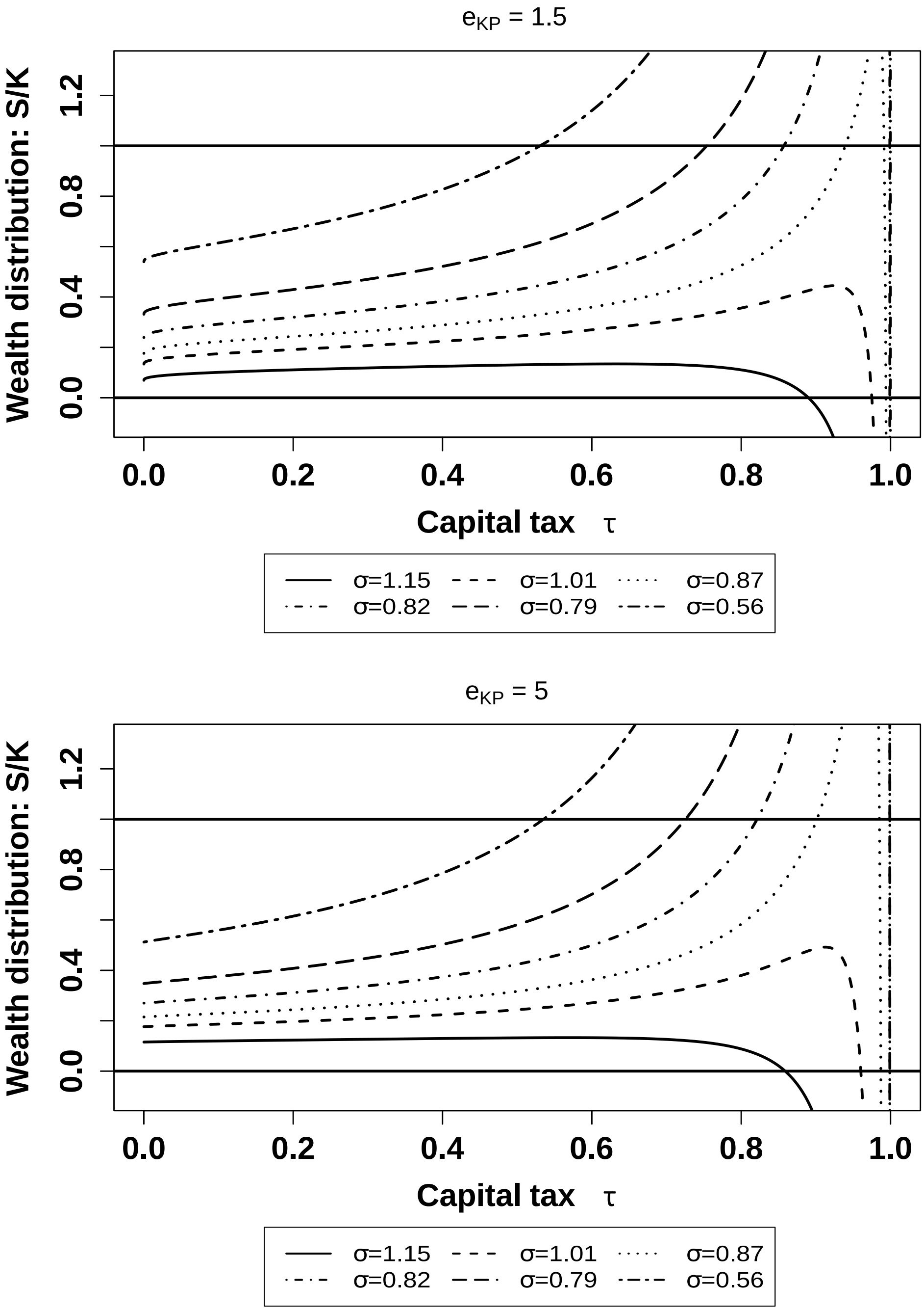

Figure 5: Wealth inequality as a function of the capital tax rate for various elasticities of substitution between aggregate capital and labor as an illustration of Proposition 8. The upper panel illustrates these for a value of the elasticity of substitution between private and public capital of 1.5. This value is 5 in the lower panel. Values above 1 and below 0 are not economically meaningful, but illustrate the ideas of the proofs.

with

$$\lambda(\tau) = \left(\frac{\rho_c}{1-\tau} + \delta_K\right),$$

$$\mu(\tau) = \frac{\tau^\eta}{(1-\tau)^{(1+\eta)}}$$

and

$$\nu(\tau) = \left(\frac{\tau\rho_c}{(1-\tau)}\right)^\eta.$$

It can be inferred from these equations directly that for $\tau \in (0,1)$

$$\lim_{\tau\to 1} \lambda(\tau) = \lim_{\tau\to 1} \mu(\tau) = \lim_{\tau\to 1} \nu(\tau) = \infty,$$

which implies that $\lim_{\tau\to 1} \widetilde{S}/\widetilde{K}(\tau) = \infty$.

Regarding the second step, it remains to show that $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically increasing for all $\tau \in (0,1)$.

Since we only consider $\eta > 0$, that is, the case of elasticities between public and private capital greater than or equal to one, this is straightforward to show: $\widetilde{S}/\widetilde{K}(\tau)$ is the sum of the monotonically increasing functions $1/(1-\tau)$, $\tau^\eta/(1-\tau)^{(1+\eta)}$ and $(\tau/(1-\tau))^\eta$, multiplied by positive constants. All these functions are monotonically increasing for $\eta > 0$. This implies that the function $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically increasing.

Since we assume that $0 < \widetilde{S}/\widetilde{K}(0,\gamma) < 1$ and we showed that $\widetilde{S}/\widetilde{K}(\tau)$ is monotonically increasing in τ and $\lim_{\tau\to 1} \widetilde{S}/\widetilde{K}(\tau) = \infty$, it follows from the Intermediate Value Theorem that for a given $0 < \eta < 1$, there exists a $\tau_{lim} \in (0,1)$, with $\widetilde{S}/\widetilde{K}(\tau_{lim}) = 1$. For this τ_{lim} the Pasinetti regime changes into an Anti-Pasinetti regime.

□

Note that the case $\eta < 0$, that is substitution elasticity $s < 1$, is not treated in Propositions 9 and 10. The reason is that one can show that for small tax rates and $\mu = 0$, capitalists vanish because the limit of S/K tends to infinity as the tax rate approaches 0. This is not a surprising finding: The assumption that private and public capital are highly complementary implies that, for low taxes, the value of private capital is strongly diminished and capitalists' income is decreased. However, as this setting only considers good substitutability between capital and *labor,* this increases wages and explains how for low tax rates the Anti-Pasinetti case can reappear.

B.3 The case of perfect substitutability between private and public capital

Finally consider the special case of Proposition 10 of a perfect elasticity of substitution between public and private capital, for which the value of the Anti-Pasinetti tax rate can be calculated explicitly. Set $\zeta = 0.5$ and $\eta = 1$ in Equation (B.13) and assume $\delta_K = \delta_P = \delta$.

Proposition 11. *For the case of a perfect elasticity of substitution between public and private capital, there exists a capital tax rate τ_{lim} at which the Pasinetti regime changes to the Anti-Pasinetti regime. For equal depreciation across capital stocks, this tax rate is given by the $\tau_{lim}^{1,2}$ which is in the economically meaningful range of $(0,1)$:*

$$\tau_{lim}^{1,2} = \frac{\frac{\rho_c^2}{\delta} - 2(\delta - \frac{1}{x}) \pm \frac{\rho_c}{\delta}\sqrt{1 + 4\frac{\delta}{x}}}{2\frac{1}{x} - \delta + \rho_c}. \tag{B.14}$$

Proof. For the case at hand, wealth inequality is given by:

$$\frac{\widetilde{S}}{\widetilde{K}} = \frac{(\widetilde{r} + \delta_K)(1-\alpha)}{\alpha(2+\rho_w)}\left(1 + \left(\frac{\tau\widetilde{r}}{\delta_P}\right)\right). \tag{B.15}$$

To determine the capital tax rate τ_{lim} at which the regime changes from a Pasinetti to an Anti-Pasinetti state we set $S/K = 1$ in Equation (B.15). Let $x = \frac{(1-\alpha)}{\alpha(2+\rho_w)}$, then:

$$1 = x\left(\frac{\rho_c}{(1-\tau_{\mathrm{lim}})} + \delta_K\right)\left(1 + \left(\frac{\tau_{\mathrm{lim}}}{\delta_P}\frac{\rho_c}{(1-\tau_{\mathrm{lim}})}\right)\right). \tag{B.16}$$

Solving for τ yields the following quadratic equation:

$$(\tau_{\mathrm{lim}})^2 \underbrace{\left[\frac{1}{x} - \delta_k + \rho_c\frac{\delta_K}{\delta_P}\right]}_{a} + \tau_{\mathrm{lim}} \underbrace{\left[\rho_c\left(1 - \frac{\delta_K}{\delta_P} - \frac{\rho_c}{\delta_P}\right) + 2\left(\delta_k - \frac{1}{x}\right)\right]}_{b} + \underbrace{\left[\frac{1}{x} - \delta_k - \rho_c\right]}_{c} = 0. \tag{B.17}$$

Therefore,

$$\tau_{\mathrm{lim}}^{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{B.18}$$

Set $\delta = \delta_k = \delta_P$, to obtain the expression in Proposition 11.

□

Equation (B.14) permits to study the dependency of the critical tax rate on parameters. For example, one finds that it increases monotonically in the pure time preference rate of the capitalists, while it decreases monotonically in the workers' time preference rate (details available upon request).[28]

[28] For the standard parametrization (see Section 6) the economically meaningful tax rate at which the economy switches from a Pasinetti to an Anti-Pasinetti state is 54 %. The sensitivity to changes in the capitalists' time preference rate is much stronger than the sensitivity to changes in the workers' time preference rate.

C Convergence to steady state

For the version of the model used in Section 4, it can be shown that a (unique non-trivial) steady state exists whenever

$$\frac{\partial F}{\partial K}(K, J) = \frac{\rho_c}{(1-\tau)} + \delta_k. \tag{C.1}$$

It is well-known that the CES function does not fulfill this condition for all values of the elasticity. Here we use the parametrized version of our model from Section 6 to verify that a steady state exists for almost the entire range of the Pasinetti regime. Let again $J(P, L) = P_t^{\beta} L^{(1-\beta)}$. Figure 6 shows that for various values of β around its empirically plausible value of 0.2, the steady state to which the model converges exists for almost the entire Pasinetti range. Here we additionally simulated the parametrized version of Equation C.1. We checked that this finding holds for extensive variation of the further parameters (details available upon request).

Note that the steady state is saddle-point stable whenever it exists. This is because the dynamical system given by Equations (2), (3), (9) and (10) inherits the dynamics of the neoclassical growth model with public captial (Heijdra, 2009). To see this, note that Equation (9) only adds to the standard dynamics that in Equations (2) and (3) the interest rate is lower than if K^c was the only private capital input. This implies that there are no qualitative differences in the dynamics, only the steady-state value of K^c is smaller than the Keynes-Ramsey level of capital K (further details upon request).

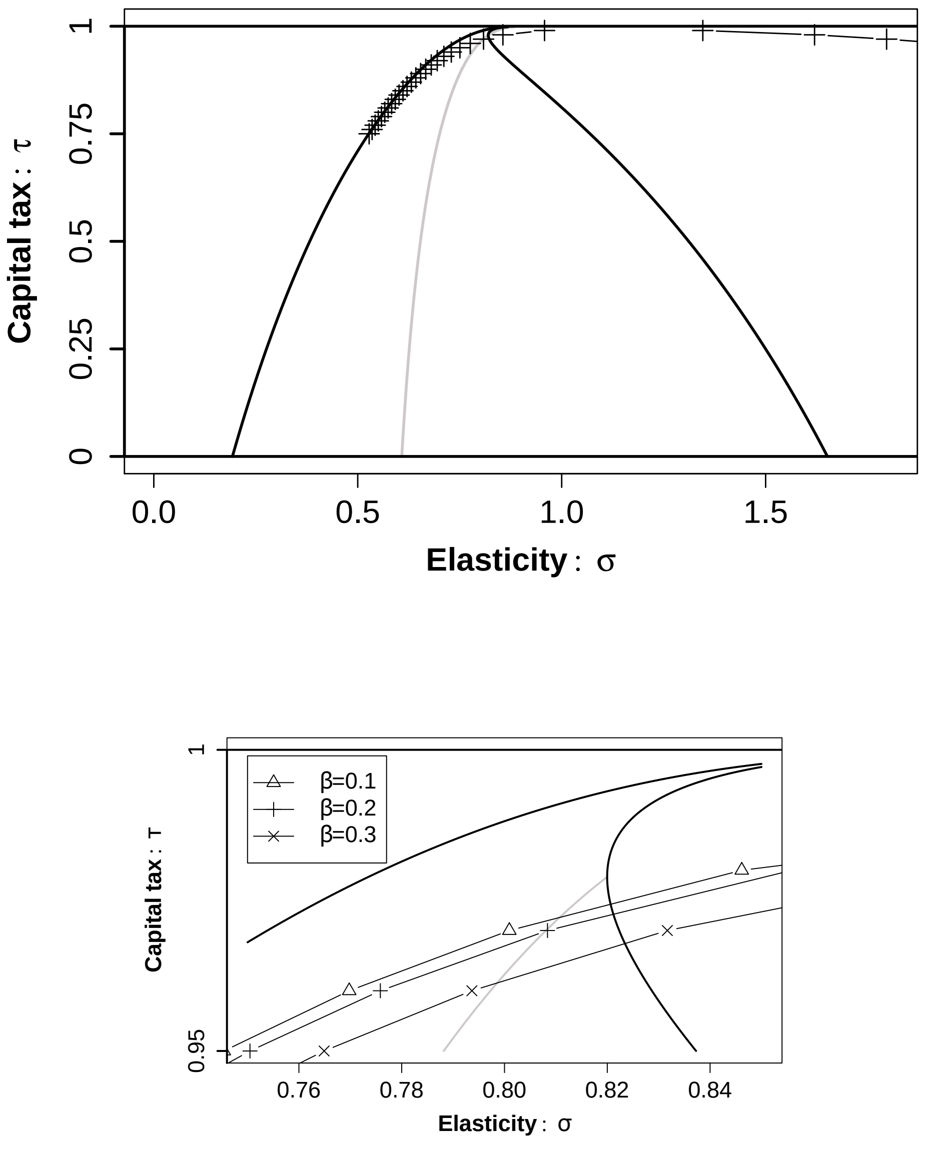

Figure 6: Equilibrium outcomes of the model with labor-enhancing public investment as a function of the elasticity of substitution between capital and labor σ and the capital tax rate τ as in Section 6, Figure 3. In the upper panel, above the line consisting of crosses no steady state exists for $\beta = 0.2$. The lower panel visualizes differences in the existence of steady states for different values of β : No steady state exists above these lines.

D Burden shifting in two class models

In this manuscript we focus on cases in which the government uses the capital tax revenue for public investment, since this policy, under certain conditions, constitutes a Pareto improvement. If the capital tax revenue was instead used to finance lump-sum transfers to the workers, the burden of the capital tax would be fully shifted to the workers (Stiglitz, 2016b). A capital tax in a two-class model hence only redistributes without making one class worse off if its proceeds are invested in public capital. In some more detail, in prior work we have already established the following:

- In a model with dynastically saving capitalists and life-cycle saving workers and tax revenue spent as transfers to workers, the burden of the capital tax is fully shifted to workers (Stiglitz, 2015d, 2017).
- If in such a model the tax is instead a tax on capitalists return to capital only, i.e. it is an inheritance tax, and tax revenue is spent as transfers to workers, workers can benefit in absolute terms (Stiglitz, 2015d).
- If in such a model the capital tax on the return to capitalists only is invested in public capital, workers benefit in absolute terms (Stiglitz, 2017). Mattauch et al. (2016) proves this is the case even if the tax on capital is also levied on workers' returns, but only for specific production functions.

These results mostly hold for small tax rates only and they do not consider that the capitalists might also benefit from public investment. Therefore, it is still unclear – and the rationale for this manuscript – how wealth inequality evolves for all tax rates (up to the boundary) with public investment. It was also not discussed in relative terms in most of these studies. Zamparelli (2017) shows the related result that in a two-agent Solow model with endogenous growth dynamics, a tax on capital income benefits workers in relative terms even without public investment.

In this Appendix, we translate the formal arguments on burden shifting that lead to the results just highlighted to the model used in this manuscript (with endogenous savings). We briefly sketch the proof of full burden shifting under a capital tax recycled as a lump-sum transfer and then outline why it is not conclusive for public investment.

Proposition 12. *If the capital tax revenue is redistributed lump-sum to the workers, the burden of the capital tax is shifted to the workers.*

In the following proof we work with per capita variables only. Let $k_t = K_t/L$, $f(k_t) = F(K_t, L)/L$ etc. as usual.

Proof. If the capital tax revenue were redistributed to the workers through a lump-sum transfer λ_t, the workers young-period budget equation would be given by:

$$w_t + \lambda_t = s_t + c_t^y. \tag{D.1}$$

The lump-sum transfer would be given by

$$\lambda_t = \tau r_t k_t. \tag{D.2}$$

The workers' per capita saving would be given by

$$s_t = \frac{1}{2+\rho_w}(w_t + \lambda_t). \tag{D.3}$$

In the following we show that $(\partial \tilde{s})/(\partial \tau) < 0$. We use Equation (D.2) and the fact that without public investment and in per capita variables $w_t = f(k_t) - f_k(k_t)k_t$. We use $f_k(k)$ as a shorthand for the first derivative of $f(k)$ with respect to k. Also, we only consider the change in steady-state values, so the steady-state interest rate is still determined by Equation (15).

$$\begin{aligned}
\frac{\partial \tilde{s}}{\partial \tau} &= \frac{\partial}{\partial \tau}\left(\frac{1}{2+\rho_w}\left(\left(f(\tilde{k}) - f_k(\tilde{k})k\right) + \tau r \tilde{k}\right)\right) \\
&= \frac{1}{2+\rho_w}\Big(f_k(\tilde{k})\frac{\partial \tilde{k}}{\partial \tau} - f_k(\tilde{k})\frac{\partial \tilde{k}}{\partial \tau} - f_{kk}(\tilde{k})\tilde{k}\frac{\partial \tilde{k}}{\partial \tau} \\
&+ \tilde{k}(f_k(\tilde{k}) - \delta_k) + \tau f_{kk}(\tilde{k})\tilde{k}\frac{\partial \tilde{k}}{\partial \tau} + \tau\frac{\partial \tilde{k}}{\partial \tau}(f_k(\tilde{k}) - \delta_k)\Big) \\
&= \frac{1}{2+\rho_w}\Big(\Big(-f_{kk}(\tilde{k})\tilde{k}(1-\tau) + \tau(f_k(\tilde{k}) - \delta_k)\Big)\frac{\partial \tilde{k}}{\partial \tau} + (f_k(\tilde{k}) - \delta_k)\tilde{k}\Big)
\end{aligned} \tag{D.4}$$

Applying the Implicit Function Theorem to Equation (15) yields: $\rho_c/(1-\tau)^2 1/f_{kk}(\tilde{k}) = \partial\tilde{k}/\partial\tau$. Hence, Equation (D.4) becomes:

$$\begin{aligned}
\frac{\partial \tilde{s}}{\partial \tau} &= \frac{1}{2+\rho_w}\Big(-\frac{\rho_c \tilde{k}}{(1-\tau)} + \frac{\rho_c \tau}{(1-\tau)^2}\frac{(f_k(\tilde{k}) - \delta_k)}{f_{kk}(\tilde{k})} + (f_k(\tilde{k}) - \delta_k)\tilde{k}\Big) \\
&= \frac{1}{2+\rho_w}\Big(\frac{\rho_c^2 \tau}{(1-\tau)^3}\frac{1}{f_{kk}(\tilde{k})}\Big) < 0, \qquad \text{since} \qquad f_{kk}(\tilde{k}) < 0.
\end{aligned} \tag{D.5}$$

The last equality follows because the first and third summand are equal, applying several times Equations (13) and (15).

□

The last expression in this derivation is analogous to Equation (1.11) in (Stiglitz, 2015d).

Furthermore, note that if the capital tax revenue is invested in public capital, it is generally unclear if full burden shifting will occur, since the sign of $(\partial\tilde{S})/(\partial\tau)$ is ambiguous.[29] A non-technical way of thinking about this case would be to argue that public investment P follows a "Laffer curve". Therefore sign in Equation (D.7) below will be ambiguous. If the effect of a tax increase on augmented labor-income is very high, as can be expected for low tax rates, workers likely benefit from public investment in absolute terms. This corroborates the messages of Figure 4 in the main part.

To see this formally, note that for the case of labor-enhancing public investment, the change in workers' savings in the intensive form as in the proof of Proposition 12 is no longer a meaningful indicator of the distributional impact of the policy. We hence have to look at the change in workers' aggregate savings $(\partial\tilde{S})/(\partial\tau)$.

Aggregate savings are given by $S_t = \frac{1}{2+\rho_w} J_t w_t$. So the change in workers' aggregate savings induced by capital tax-financed public investment in the steady state is given by:

$$\begin{aligned}
\frac{\partial(\tilde{S})}{\partial\tau} &= \frac{1}{2+\rho_w}\frac{\partial}{\partial\tau}\left(\tilde{J}\left(f(\tilde{k}) - f_k(\tilde{k})k\right)\right) \\
&= \frac{1}{2+\rho_w}\left(\left(\frac{\partial\tilde{J}}{\partial\tau}\right)\left(f(\tilde{k}) - f_k(\tilde{k})k\right) + \tilde{J}\frac{\partial}{\partial\tau}\left(f(\tilde{k}) - f_k(\tilde{k})k\right)\right) \\
&= \frac{1}{2+\rho_w}\left(\left(\frac{\partial\tilde{J}}{\partial\tau}\right)\left(f(\tilde{k}) - f_k(\tilde{k})k\right) - \tilde{J}f_{kk}(\tilde{k})\tilde{k}\frac{\partial\tilde{k}}{\partial\tau}\right) \\
&= \frac{1}{2+\rho_w}\left(\left(\frac{\partial\tilde{J}}{\partial\tau}\right)\left(f(\tilde{k}) - f_k(\tilde{k})k\right) - \tilde{J}\tilde{k}\frac{\rho_c}{(1-\tau)^2}\right).
\end{aligned} \tag{D.6}$$

The second summand within the brackets is unambiguously negative, but the first summand is positive in case $\left(\frac{\partial\tilde{J}}{\partial\tau}\right)$ is, in which case the sign may or may not be positive. It thus remains to determine the sign of $\left(\frac{\partial\tilde{J}}{\partial\tau}\right)$:

$$\left(\frac{\partial\tilde{J}}{\partial\tau}\right) = \left(\frac{\partial}{\partial\tau}\right)\tilde{P}^{\beta}L^{(1-\beta)}. \tag{D.7}$$

Steady-state public investment levels are given by $\delta_P\tilde{P} = \tau\tilde{r}\tilde{K}$, with $\tilde{r} =$

[29]This statement is in line with Footnote 32 in Stiglitz (2015c), although for a differing production structure.

$\rho_c/(1-\tau)$, so the above equation becomes:

$$\begin{aligned}\left(\frac{\partial \tilde{J}}{\partial \tau}\right) &= \left(\frac{\partial}{\partial \tau}\right)\left(\frac{\tau}{(1-\tau)}\frac{\rho_c}{\delta_P}\tilde{K}\right)^{\beta} L^{(1-\beta)} \\ &= L^{(1-\beta)}\left(\frac{\rho_c}{\delta_P}\right)^{\beta}\beta\left(\frac{\tau}{(1-\tau)}\tilde{K}\right)^{\beta-1}\left(\frac{\tilde{K}}{(1-\tau)^2}+\frac{\tau}{(1-\tau)}\frac{\partial \tilde{K}}{\partial \tau}\right).\end{aligned} \tag{D.8}$$

Using the implicit function theorem (for the non-intensive version of the production function) on Equation (15) yields:

$$\frac{\partial \tilde{K}}{\partial \tau} = \left[\frac{\rho_c}{(1-\tau)} - F_{KP}\frac{\rho_c}{\delta_P(1-\tau)^2}\tilde{K}\right]\left(F_{KK}+F_{KP}\frac{\tau\rho_c}{(\delta_P(1-\tau))}\right)^{(-1)}. \tag{D.9}$$

The expression in Equation (D.9) can be bigger, smaller or equal to zero. Therefore the sign of $\frac{\partial(\tilde{J})}{\partial \tau}$ and hence $\frac{\partial(\tilde{S})}{\partial \tau}$ is ambiguous.