# The Optimal Tax on Capital is Greater than Zero 

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## Early work

- Concerned that Ramsey tax seemed to imply that there should be high taxes on necessities like food
- Low demand elasticities
- Very regressive
- Ramsey model had a single individual
- Optimal to have lump sum tax
- Atkinson-Stiglitz 1972 analyzed optimal taxes taking into account distributional effects
- Derived Ramsey-like formula, with distributional weights
- Generated expected results—reversed Ramsey's presumption


## Optimal income tax and redistribution

- But when individuals differ in a systematic observable way, we may have better ways of addressing issues of distribution
- Mirrlees (1971) had solved for optimal income tax, trade-offs between benefits of redistribution and losses of efficiency from disincentive
- Individuals differed in productivities; wages fixed
- Most of results could be extended to the analysis of an Pareto efficient tax structure (Stiglitz, 1982, 1987)
- Obvious question: if we already have an optimal income tax, do we need any commodity taxation at all


## Atkinson-Stiglitz (1976)

- Combined optimal commodity taxation with an optimal income tax (no restrictions on forms of either tax)
- If there is separability between consumption of goods and leisure, then there should be no commodity taxation if there is an optimal income tax
- In the absence of separability, taxation of commodities had nothing to do with Ramsey taxation
- Similar results hold even if the income tax is not optimal (e.g. a linear income tax)(Stiglitz, 2009)
- Mathematics: key question is how commodity taxation affects the self-selection constraints
- Insight has important implications for key generalization-where commodity taxation can affect the distribution of income (e.g. between skilled and unskilled workers)
- Desirable to impose commodity taxes that improve the market distribution of income
- Less burden on redistribution
- Real reason: loosens self-selection constraints


## Wrong interpretation

- Treating consumption at different dates as different goods, AtkinsonStiglitz 1976 implies that there should be no tax on interest (no capital taxation)
- Separability assumption not plausible
- In absence of separability, may be capital tax or subsidy
- Depends on complementarity/substitutability between leisure and retirement consumption
- Wrong model to study the issue of capital taxation
- Individuals differ in other respects
- Inherited human and financial capital
- Abilities to obtain returns out of financial assets
- Capital taxation is directed at addressing these inequalities


## Two class model

- Simplification of a more general model with individuals inheriting different amounts of financial capital, and some individuals only saving for life-cycle purposes, others leaving bequests to children
- Workers: overlapping generations
- All identical
- Maximize two period utility, working only first period
- Capitalists: so much wealth that we can ignore wage income
- Dynastic utility function


## Capital tax used to fund public education

- Fixed labor supply, normalized at unity
- Productivity of labor depends on public expenditure on education

$$
\begin{equation*}
Q=F\left(K, \phi\left(K_{g}\right)\right) \tag{1}
\end{equation*}
$$

where $\phi\left(K_{g}\right)$ describes the increased productivity from public education $K_{g}$ on labor, i.e. $\phi\left(K_{g}\right)$ is the effective labor supply, with $\phi^{\prime}\left(K_{g}\right) \geq 0$ and $\phi^{\prime \prime}\left(K_{g}\right) \leq 0$

## Tax rate determines effective capital labor ratio

$$
\begin{equation*}
Q=\phi f(K / \phi)=\phi f(k) \tag{2}
\end{equation*}
$$

where $k=\frac{K}{\phi}$, the capital-"effective labor" ratio.

Capital tax on wealth of capitalists
$L R$ equilibrium $k$ depends only on capitalists, so long as they exist

$$
f^{\prime}-\eta-\tau=\delta
$$

can be solved for $k$ as a function of $\tau$

$$
k=\psi(\tau)
$$

## Solving for w

$$
\begin{equation*}
w=\phi\left(K_{g}\right) g(k)=\phi\left(K_{g}\right)\left(f-k f^{\prime}\right) \tag{3}
\end{equation*}
$$

Where

$$
\begin{equation*}
K_{g}=\tau\left(K-K_{w}\right)=\tau\left(\psi(\tau) \phi\left(K_{g}\right)-K_{w}\right) \tag{4}
\end{equation*}
$$

And

$$
\begin{equation*}
K_{w}=s_{w}(r) \phi\left(K_{g}\right) g(k)=s_{w}(\delta+\tau) \phi\left(K_{g}\right) g(k) \tag{5}
\end{equation*}
$$

(4) and (5) can be solved for $\mathrm{K}_{\mathrm{g}}$ and $\mathrm{K}_{\mathrm{w}}$ as function of $\tau$ and hence $w$ as a function of $\tau$, the solution to
(6) $\quad K_{g}=\tau\left[\psi(\tau) \phi\left(K_{g}\right)-s_{w}(\delta+\tau) \phi\left(K_{g}\right) g(\psi(\tau))\right]=H(\tau)$

## Solving for $w$ and maximizing workers' welfare

(7) $\quad w=\phi(H(\tau)) g(\psi(\tau))$

If there is enough inequality, and enough inequality aversion in social welfare function, maximizing social welfare maximizes welfare of workers, represented by indirect utility function

$$
(8) \mathrm{V}^{\mathrm{w}}=V^{w}(w, r)=V(\phi(H(\tau)) g(\psi(\tau)), \tau+\delta)
$$

If $\phi^{\prime}$ is large enough, then $\frac{d V}{d \tau}>0$

## Optimal tax rate

Can also use tax revenue for public capital goods

- Public investments can reduce adverse incidence of capital tax-or reverse, if (a) public capital goods are productive enough, and (b) are complementary to labor

Pure transfers may be welfare reducing

- Adverse incidence effect

Can easily solve for optimal tax rate

## Upper bound

- There is an upper bound to the tax rate
- Upper bound can be established by looking at tax rate at which capitalists get extinguished, i.e. $\mathrm{K}_{\mathrm{w}}=\mathrm{K}$, the solution to
(9) $s_{w}(\delta+\tau) \phi(0) g(\psi(\tau))=\psi(\tau) \phi(0)$

Can solve numerically

- In case where workers have logarithmic utility function, additively separable, with discount rate $\delta_{w}$ (fixed savings rate)
- And capitalists have fixed savings rate



## Conclusions

- Wrong to conclude for Atkinson-Stiglitz 1976 that there should be no taxation of capital: there should be
- If we could, we would tax transfers of wealth across generations
- In practice, we don't
- Large fraction of wealth inherited
- This model takes into account the incidence of the tax on wages
- Main factor limiting taxation
- Always desirable to impose tax on capital provided we can avoid adverse incidence effect, which we can through investments in education and public goods
- I believe Tony would have agreed with, and liked, this result

