A Theory of Communication and Coordination in a Polarized Society

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Abstract

This paper examines individuals' use of private information and expression of polarized sentiments in the presence of coordination motives. Agents have polarized sentiments in the sense that their beliefs about the state can be biased in opposite directions. Before the coordination game is played, agents may publicly communicate their polar types. In equilibrium, truthful communication generally takes place only when the expected population composition is relatively more balanced; otherwise, the minority group of agents have an incentive to mimic their majority counterparts. In addition, when the uncertainty about sentiments is sufficiently high, the minority can even be induced to direct their sentiments in the opposite way so that their actions are aligned with the majority.

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1. Introduction

Social media has become increasingly relevant in modern life, and so has individuals' expression of their sentiments on social media. Financial market regulators conclude that "social media is landscape-shifting," converting the traditional two-party communication into an interactive, multi-party dialogue where users actively create content (SEC, 2012, p. 1). On one hand, individual investors regularly express their polarized opinions (e.g., bullish or bearish) about stocks on social media (Cookson and Niessner, 2020; Cookson, Engelberg and Mullins, 2021). On the other hand, trading strategies based on the sentiment extracted from social media have become increasingly popular.[1] In the more recent GameStop mania, retail traders not only spread their investment ideas, but also spurred mass coordination that caused substantial disruption in the stock price (The Wall Street Journal, 2021).[2] The aforementioned expression of polarized sentiments seems particularly puzzling as it may happen even before investors take trading positions.[3] If the sentiments are value-relevant, why do these investors spread their investment ideas on social media, but not trade and profit from the valuable information they possess instead? If the sentiments are value-irrelevant, why do money managers, the most sophisticated in-

[1]As of 2012, one third of affluent investors in the U.S. are reported to have directly relied on investment advice transmitted via social media outlets (Cogent Research, 2012). Hedge funds and high-frequency-trading firms also use sentiment indicators for market-making strategies (The Wall Street Journal, 2015).

[2]Consistent with this anecdotal incident, there is abundant empirical evidence indicating that social interactions affect investment decisions by individuals and money managers (see, e.g., Kelly and O Grada, 2000; Duflo and Saez, 2003; Hong, Kubik and Stein, 2004; Bursztyn, Ederer, Ferman and Yuchtman, 2014; Campbell, DeAngelis and Moon, 2019). Relatedly, media sentiment is also found to affect investors' trading activity and stock prices. For example, Goldman, Gupta and Israelsen (2021) observe that investors respond to news about a stock published in politically polarized newspapers by trading in the same direction as other investors who read the same newspaper, and Tetlock (2007) finds that media pessimism predicts lower market prices followed by a reversion to fundamentals.

[3]For instance, individual investors on StockTwits publicly identify themselves as "bulls" or "bears" by attaching a corresponding sentiment indicator to their account names, and such labeling of identities seems to be quite persistent over time.

stitutional investors, base their trading strategies on the sentiment extracted from social media? Either way, what are the incentives for such expression of sentiments?

Some common intuition holds that insiders may share privileged information to manipulate markets (Benabou and Laroque, 1992) or accelerate price discovery (Ljungqvist and Qian, 2016). However, neither vindicates the observed variations in expression of sentiments across time and individuals (see, e.g., Cookson and Niessner, 2020). In this paper, I propose that it is the coordination motive that drives symmetrically informed investors to express their polarized sentiments. Such communication changes the market's perceived population composition between agents of two polar types and hence affects the aggregate sentiment incorporated into the aggregate investment. Investors communicate their sentiments in a way that increases the value of their private information in coordinating their investment with the others. To put it differently, sentiments are payoff-relevant to the extent of investment complementarities, so even the most sophisticated institutional investors like money managers will base their trading strategies on the sentiment extracted from social media. Nevertheless, they are value-irrelevant to the extent of being independent from the intrinsic value of investment, so investors do not lose any information advantage by spreading their investment ideas. This novel theory complements the existing explanations for investors' information-sharing behavior and provides a single consistent explanation for the expression of sentiments.

The analysis presented here departs from the typical assumption that the unknown state (e.g., the intrinsic value of investment or some nonstraightforward matter of fact) is the only payoff-relevant uncertainty. Specifically, this paper examines agents' decision-making with multidimensional uncertainty by injecting uncertainty about the population composition in addition to uncertainty about the state. This additional dimension of uncertainty is non-fundamental yet affects agents' payoffs.

For example, in stock markets populated with short-horizon investors, stock prices can deviate from the underlying fundamental by the aggregate sentiment. Hence, even a bearish investor who believes the stock is overvalued may still take long positions when he expects the bullish investors to be the dominating majority driving the short-run price up. Similarly, in social interactions where individuals have a preference for conformity, an authoritarian may instead advocate liberty when he believes the majority population are libertarians. In a word, the *perception* about the population composition plays a vital role in agents' decision-making process, which gives rise to individuals' incentive to communicate their polarized sentiments in social interactions.

To study investors' communication incentives of their polarized beliefs in an economy with social interactions, I abstract from a capital market setting but instead adopt a canonical model of beauty contests to capture the coordination motive.[4] There is a finite set of agents making investment decisions. As in Angeletos and Pavan (2004), the return of the investment depends not only on a common state of nature but also on how much other agents invest. Agents thus would like to match their own actions with both the state (the *fundamental motive*) and the average action (the *coordination motive*). Each agent observes a private signal about the state. Belief polarization is operationalized as opposite sentiments across the two groups of agents, e.g., bulls and bears, while the agents are symmetrically informed about the fundamental in a statistical sense.[5] More specifically, I decompose the noise of each

[4]The beauty-contest metaphor for stock markets was originally proposed by Keynes (1936) and was formalized by Allen, Morris and Shin (2006).

[5]Prior literature shows that a unidimensional belief polarization can arise endogenously in the long-run with persuasion bias (see, e.g., DeMarzo, Vayanos and Zwiebel, 2003). More recently, Nimark and Sundaresan (2019) demonstrate that rational inattention can also lead the beliefs of ex-ante identical agents to cluster in two distinct groups at opposite ends of the belief space. Experimentally, Plous (1991) shows that people process information in a biased manner to support their initial beliefs, whereas Benoît and Dubra (2019) find a population of people to be more prone to polarization if their initial opinions have largely been based on very similar evidence.

private signal into a common component and an idiosyncratic component.[6] The division of agents into two sentiment groups is based on the direction of their exposure to the common noise. Thus, the common noise represents the polarized sentiment across groups, and the idiosyncratic noise captures the within-group disagreement. Conditional on the realization of the state, the private signals of any two agents are positively correlated within the same group, but negatively correlated across different groups. Consequently, the private signal of one agent is asymmetrically informative about that of another agent across the two groups. I first characterize agents' equilibrium actions given their perceived population composition, and then apply the main model to study agents' expression of their sentiments by introducing the option for agents to publicly communicate their types before the coordination game is played.[7]

The central finding of the paper pertains to the agents' strategic communication of their polarized sentiments. I show that agents communicate their types in a way that increases the value of their private information. In particular, communication of types affects the informativeness of an agent's private signal about the average investment by changing others' perception about the population composition and, hence, their investment decisions. Put differently, it is the coordination motive that gives rise to agents' communication incentives. Without the coordination motive, the population composition has no bearing on agents' investment decisions and hence the average investment.[8]

In the presence of the coordination motive, whether truthful communication takes

[6]Myatt and Wallace (2012) provide a micro-foundation of such information structure with the notion of "rational-inattention"; an investor's understanding of a public signal is costly and thus depends on the investor's information-acquisition choice. See also Corona and Wu (2021) for a similar information structure to operationalize correlations in private signals.

[7]Restricting communication to be only about the type but not the private signal may be justified as a consequence of the communication bandwidth constraints such as character limits imposed by social media platforms and time constraints in conversation (see, e.g., Hirshleifer, 2020).

[8]This result continues to hold even if action complementarity is replaced by action substitutability.

place depends not only on the degree of belief polarization, but also on the expected population composition. When forming investment decisions, agents' reliance on their private signals is jointly determined by the signal informativeness about the fundamental and the average investment. I refer to the former as the *fundamental value* and the latter as the *strategic value* of private signals. While the fundamental value of information is independent of the population composition, the strategic value can be non-monotonic in the population composition. To see this, consider two extreme cases where (i) the population composition is perfectly balanced between the two types, and (ii) the population composition is extremely unbalanced and concentrated on one type. In the first case, the two types are symmetric and the aggregate investment contains no sentiment. The strategic value of private signals is low as the individual sentiment is purely noise in predicting the average investment. In the second case, the individual sentiment is perfectly correlated with the aggregate sentiment and, thus, results in a higher strategic value of private signals. Moreover, strong coordination motive will lead the extreme minority type to "follow the crowd" and use their private signals in the opposite way when there is more uncertainty about the sentiment than about the fundamental.

The non-monotonicity of the strategic value of private information with respect to the population composition may lead the minority to lie about their sentiment types. Agents communicate their types in a way that increases the value of their private information through the others' perception about the population composition. Truthful communication from the minority type can induce the majority type to use less of their private signals due to the perceived more neutral aggregate sentiment. This decreases the strategic value of private information of the minority type, leading them to "hide in the shadows" by mimicking the majority type. As a result, a separating equilibrium exists only when the expected population composition is relatively balanced.

The rest of the paper is organized as follows. Section 2 reviews the related literature and outlines the contribution of the paper. Section 3 develops the model and characterizes agents' equilibrium investment. Section 4 introduces an option for agents to publicly communicate their types before investing and derives their equilibrium communication strategies. Section 5 elaborates on the implications of belief polarization in various social interactions. Section 6 concludes.

2. Related Literature

The idea that sentiments can drive financial decision-making in uncertain environments has been at the core of capital market dynamics described by Keynes (1936), which he referred to as "animal spirits." Meanwhile, individual sentiments have been increasingly polarized in various social and political contexts since the early 1990s (see, e.g., Baldassarri and Gelman, 2008; Fiorina and Abrams, 2008). Applying these insights to a beauty contest setting, this paper makes several contributions by examining the implications of polarized sentiments on individuals' communication and coordination decisions.

First, it contributes to the recent thread of theoretical literature that studies disclosure incentives of self-interested economic agents in settings of investment beauty contests.[9] Closely connected to the general beauty contest model (see, e.g., Morris and Shin, 2002), investment beauty contests are first examined in Angeletos and Pavan (2004, 2007). Arya and Mittendorf (2016) examine the incentives of firms to take preemptive action and publicly disclose their investments in beauty contests. This paper complements their findings in the sense that an investor discloses his sentiment not to

[9]For thorough summaries on the disclosure literature, see Verrecchia (2001), Dye (2001), and Beyer, Cohen, Lys and Walther (2010).

establish norms, but to influence how the others use their private information. The strategic communication incentives may also explain the mixed empirical evidence on the ability of crowdsourced information to predict financial market movements (see, e.g., Tumarkin and Whitelaw, 2001; Antweiler and Frank, 2004; Das and Chen, 2007; Garcia, 2013; Chen, De, Hu and Hwang, 2014). Indeed, the optimistic conclusion of Condorcet's jury theorem on efficient aggregation of information is preserved only for a relatively balanced population composition.[10]

More generally, it complements the thread of literature on information transmission among traders in financial markets. I focus on communication of sentiments among symmetrically informed agents driven by coordination motives. Different from Benabou and Laroque (1992) and Goldstein, Xiong and Yang (2021), agents express their sentiments to improve their own prediction ability of the average action rather than to manipulate the market. Although several existing papers have rationalized the information-sharing behavior of short-horizon investors by arguing that information revelation can be used to accelerate price correction (Liu, 2018; Kovbasyuk and Pagano, 2020; Schmidt, 2020), to my knowledge, the current paper is the first one to focus on disclosure of non-fundamental private information from a coordination perspective.

Second, this paper is also related to the literature of social learning. One of the most closely related papers in this thread of literature is Hagenbach and Koessler (2010), who study endogenous communication networks formed by agents with publicly observable preference heterogeneity.[11] With social media reducing the cost of

[10] Condorcet's jury theorem states that sincere reporting of their information by each individual is sufficient for efficient aggregation of information (Condorcet, 1785). In contrast to this background, a number of papers argue that the selfish behavior of individuals in game theoretic situations may prevent efficient aggregation of dispersed information (see, e.g., Acemoglu, Ozdaglar and Parande-hGheibi, 2010; Acemoglu and Ozdaglar, 2011; Acemoglu, Dahleh, Lobel and Ozdaglar, 2011).

[11] See Mobius and Rosenblat (2014) for a complete review of the existing empirical and theoretical literatures on social learning.

communication to almost zero, I instead focus on the information content of the equilibrium communication strategy in a public communication setting. Yet some of the main results in this paper seem analogous. First, there is asymmetric information transmission between the majority and the minority groups. Second, the minority's tendency to truthfully communicate in a public network decreases as the population becomes more unbalanced. Empirically, Bursztyn et al. (2014) conclude that both *social learning* and *social utility* channels have statistically and economically significant effects on investment decisions. While the social utility effect is captured by the coordination motive, the social learning effect differs in my model in the sense that an investor's expression of his sentiment does not directly inform the others about his investment decision, but rather affects the others' *belief* about his prospective investment.

Finally, this paper examines decision-making with multidimensional uncertainty. Following Avery and Zemsky (1998), Goldstein and Yang (2015) confirm that strategic complementarities in trading and information acquisition can arise in a setting where the asset value is affected by different fundamentals. Subsequently, they examine how public disclosures affect market efficiency when the market and the firm possess superior information along different dimensions (Goldstein and Yang, 2019). This paper instead studies how fundamental uncertainty affects investors' disclosure of their private information regarding strategic uncertainty.

3. Main Model

3.1. Setup

There are a *finitely* large number of agents in the economy. Each agent $i \in \{1, 2, \ldots, N\}$ is privately informed about his type $t_i \in \{-1, 1\}$, where $t_i = 1$ with probability $\lambda \in (0, 1)$. The type affects the information structure of the agent's private signal about a common state of nature such that

$$\tilde{s}_i = \tilde{v} + t_i \tilde{\eta} + \tilde{\epsilon}_i, \tag{1}$$

where $\tilde{v} \sim \mathcal{N}(\mu, \sigma_v^2)$ is the state, $\tilde{\eta} \sim \mathcal{N}(0, \sigma_\eta^2)$ is the within-type common noise, and $\tilde{\epsilon}_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is the idiosyncratic noise independent and identically distributed across all agents. The opposite signs in t_i thus capture the idea of belief polarization in the sense that conditional on the realization of the state, the private signals of any two agents from different groups are negatively correlated.[12] Examples of such polarized beliefs include bulls and bears, authoritarians and libertarians, and democrats and republicans, etc. For the following analysis, we refer to type-1 and type-2 agents as those with $t_i = 1$ and $t_i = -1$, respectively, and use them in subscripts, e.g., s_{ki} denotes the signal of agent i of type-k for $k \in \{1, 2\}$. Regarding information quality, σ_v^2 measures the prior uncertainty about the state which can be based on some public information, σ_η^2 measures the across-type disagreement which serves as a proxy for the degree of polarization across groups, and σ_ϵ^2 captures the within-type disagreement. I assume information quality is well defined, i.e., σ_v^2, σ_η^2, and σ_ϵ^2 are positive and finite,

[12] One interpretation of polarized sentiments is that it arises from individuals' diverging interpretations even when they are presented with the same evidence. For example, some investors may interpret an increase in inventory on a firm's balance sheet as a good sign, believing it indicates the firm's confidence in the future sales; others may interpret it as bad sign, thinking it shows the firm's declining ability in selling the inventory in time.

and that all random variables are independent of each other.

Following Angeletos and Pavan (2004) and Arya and Mittendorf (2016), agent i's payoff function is given by

$$u_i(a, \tilde{v}) = ((1-\omega)\tilde{v} + \omega\bar{a})\, a_i - \frac{1}{2}a_i^2, \tag{2}$$

where $\bar{a} \equiv \frac{1}{N}\sum_{j=1}^{N} a_j$ is the average action, and $\omega \in (0,1)$ parameterizes agents' coordination motive arising from the strategic complementarity in their actions. We may also interpret such a payoff function as one with investment complementarity, where the marginal return of investment is a weighted average of the fundamental and the average investment, with ω capturing the investment complementarity, and the quadratic cost may be interpreted as an investment adjustment cost from the status quo.

3.2. Equilibrium Analysis

In this section I first characterize agents' equilibrium actions fixing exogenously their perception about the population composition which is denoted as $\hat{\lambda}$. Then I proceed to analyze how the equilibrium actions and, more specifically, agents' reliance on their private information, are affected by such perception about the population composition. In the next section I endogenize the perceived population composition and characterize agents' equilibrium communication strategies about their sentiment types. In particular, in the absence of any communication or in a babbling equilibrium, $\hat{\lambda}$ converges to the prior probability λ with a large number of agents; in a separating equilibrium, $\hat{\lambda}$ will depend on the number and the type of the messages sent by each agent.

Proposition 1. *There exists a unique linear equilibrium where the action of agent i of type-k is characterized by*

$$a^*_{ki} = (1-\phi_k)\mu + \phi_k s_{ki}, \tag{3}$$

where

$$\phi_1 = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega(1-\hat{\lambda}))\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2}, \tag{4}$$

$$\phi_2 = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega\hat{\lambda})\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2}. \tag{5}$$

An agent's equilibrium action is a weighted average of his expectation about the state and the average action. Consequently, his reliance on the private signal is a weighted average of the signal informativeness about the underlying fundamental (the *fundamental value* of information) and the average action (the *strategic value* of information). As in the prior literature (see, e.g., Morris and Shin, 2002), the coordination motive induces agents to overweight public information and underweight private information. However, the two types' polarized sentiments makes the strategic value depend not only on the information structure of private signals, but also on the population composition. More precisely,

$$\begin{aligned}\phi_1 &= (1-\omega)\rho_v + \omega\left(\hat{\lambda}\rho_s\phi_1 + (1-\hat{\lambda})\rho_d\phi_2\right),\\ \phi_2 &= (1-\omega)\rho_v + \omega\left(\hat{\lambda}\rho_d\phi_1 + (1-\hat{\lambda})\rho_s\phi_2\right),\end{aligned} \tag{6}$$

where $\rho_v \equiv \frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ measures the signal informativeness about the state, and $\rho_s \equiv \frac{\sigma_v^2+\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ and $\rho_d \equiv \frac{\sigma_v^2-\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$ respectively represent within-type and cross-type signal correlations. While being symmetrically informed about the fundamental, agents are asymmetrically informed about the average action due to the existence of the common noise.

The extent of such asymmetry is determined by the population composition. In the special case where the population composition is perceived to be perfectly balanced, i.e., $\hat{\lambda} = \frac{1}{2}$, neither of the two types of agents expects to play a dominating role in determining the average action. All agents expect to be equally well informed about the average action, and hence place the same weight on their private signals, i.e., $\phi_1 = \phi_2 = \frac{(1-\omega)\sigma_v^2}{(1-\omega)\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}$. Indeed, the common noise is cancelled out in the average action, and the average action is proportional to the fundamental. In the more general case where $\hat{\lambda} \neq \frac{1}{2}$, the majority type of agents expect to influence the average action to a larger extent than their minority counterparts, hence being better informed about the average action. Thus, they place a higher weight on their private signals than their minority counterparts.[13] We summarize these observations in the following corollary.

Corollary 1. *The private information is always underweighted in the presence of the coordination motive, i.e.,* $\phi_k < \rho_v$. *Moreover, the perceived majority always place a higher weight on their private signals than the perceived minority, i.e.,* $\phi_1 > \phi_2$ *if and only if* $\hat{\lambda} > \frac{1}{2}$.

Nevertheless, agents' reliance on their private signals does not always increase in the expected proportion of their own types. The following proposition summarizes how beliefs about the population composition affect agents' subsequent actions.

Proposition 2. *Agents' reliance on their private information,* ϕ_k, *can be non-monotonic in* $\hat{\lambda}$. *In particular, when* $\omega > \omega^T$ *and* $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$, $\frac{\mathrm{d}\phi_k}{\mathrm{d}\hat{\lambda}} < 0$ *for* $\hat{\lambda} \in (0, \lambda_k^*)$,

[13] In fact, since the weight on the private signal ϕ_k can be negative, this claim holds true in absolute values as well. Figure 1(c) provides a graphical illustration for such a case.

and $\frac{\mathrm{d}\phi_k}{\mathrm{d}\hat{\lambda}} > 0$ *for* $\hat{\lambda} \in (\lambda_k^*, 1)$*, where* $\omega^T \in (\frac{1}{3}, 1)$ *and* $0 < \lambda_1^* < \frac{1}{2} < \lambda_2^* < 1$ *are defined in the appendix; otherwise,* ϕ_1 *monotonically increases in* $\hat{\lambda}$ *and* ϕ_2 *monotonically decreases in* $\hat{\lambda}$*, i.e.,* $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} > 0$ *and* $\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} < 0$ *for any* $\hat{\lambda} \in (0, 1)$*.*

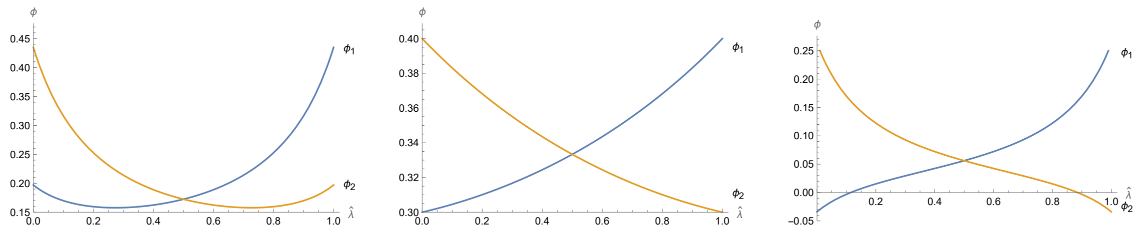
Moreover, when $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*, there exist* $0 < \lambda_1^\dagger \leq \lambda_1^* < \lambda_2^* \leq \lambda_2^\dagger < 1$ *such that* $\phi_1 < 0$ *for* $\hat{\lambda} \in (0, \lambda_1^\dagger)$ *and* $\phi_2 < 0$ *for* $\hat{\lambda} \in (\lambda_2^\dagger, 1)$*.*

As discussed earlier, the belief about the population composition affects agents' reliance on their private signals only through the strategic value of information. More precisely, equation (7) shows that the perceived population composition affects the strategic value of information not only directly by itself, but also indirectly through the other agents' use of their information. Taking type-1 agents' reliance on their private signals as expressed in equation (6) as an example, its first-order derivative with respect to the expected proportion of their types is

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} = \omega\Bigg(\underbrace{\rho_s\phi_1 - \rho_d\phi_2}_{\text{direct effect}} + \underbrace{\hat{\lambda}\rho_s\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} + (1-\hat{\lambda})\rho_d\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}}}_{\text{indirect effect}}\Bigg). \tag{7}$$

The direct effect is always positive. Intuitively, more individuals of the same type leads an agent to use more of his private signal by making the average action more contingent on the information that is strongly correlated with his information.

On the other hand, the indirect effect can be negative for an agent who belongs to the minority type. When the population composition is relatively unbalanced, the indirect effect is dominated by the majority type's response in their reliance on private signals. In the extreme case where $\hat{\lambda} = 0$, the indirect effect reduces to $\rho_d\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}}$. Since an increase in the proportion of the minority type neutralizes the aggregate sentiment, the private signals of the majority type become less informative about the average action. As a result, the majority type of agents rely less on their



(a) (b) (c)

Figure 1: The Weight of the Private Signals in Equilibrium Actions

Note: The above figures demonstrate how the weight of the private signals in equilibrium actions change in $\hat{\lambda}$. Fixing $\sigma_\eta^2 = 1$, the other parameter values are (a) $\omega = \frac{7}{8}, \sigma_v^2 = 2, \sigma_\epsilon^2 = \frac{1}{5}$, (b) $\omega = \frac{1}{2}, \sigma_v^2 = 2, \sigma_\epsilon^2 = 1$, and (c) $\omega = \frac{7}{8}, \sigma_v^2 = \frac{1}{2}, \sigma_\epsilon^2 = \frac{1}{20}$, respectively.

private signals, i.e., $\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} < 0$. The negative indirect effect dominates the positive direct effect only when $\omega > \omega^T$ and $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$, leading the minority type to rely less on their private signals, i.e., $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} < 0$ for $\hat{\lambda} < \lambda_1^*$ (see Figure 1(a)). The former condition prescribes a strong coordination motive so that agents' reliance on their private signals is sufficiently sensitive to the population composition. The latter condition on relatively low uncertainty about the sentiment results in a lower ρ_s and a higher ρ_d, which make the direct effect weaker but the indirect effect stronger. Under these two conditions, as the supermajority type use less of their private signals when their population proportion decreases, the extreme minority type respond by also decreasing their reliance on private signals even though their population proportion increases. Otherwise, the conventional wisdom would prevail: agents always rely more on their private signals when they anticipate a higher proportion of their own types (see Figure 1(b)).

It is worth noting that high uncertainty about the sentiment, σ_η^2, can even lead the extreme minority type to "follow the crowd" by using their signals in the opposite way, i.e., $\phi_i < 0$, given the sufficiently negative cross-type signal correlation ρ_d. In this case, an increase in ϕ_i then translates to decreased reliance on private information in absolute terms (see Figure 1(c)). As the proportion of the extreme minority type increases, at some point the minority may even completely ignore their private information and solely rely on the public information.

4. Expression of Sentiments

In this section, we consider the situation where agents have the option to publicly communicate their types before their private signals are realized and the coordination game is played. Formally, agents may send a message indicating their types before taking the action, i.e., $m_i \in \{-1, 1\}$. Thus, agent i's information set when he is about to choose an action is $\mathcal{I}_i \equiv \{t_i, s_i, \langle n_1, n_2 \rangle\}$, where $n_1 \equiv \sum_{i|m_i=1} m_i$ and $n_2 \equiv \left|\sum_{i|m_i=-1} m_i\right|$ are the total number of type-1 and type-2 messages (up to relabelling of messages), respectively. I restrict attention to pure strategy equilibria, so each agent's communication strategy may take one of only two forms: the truthful one, $m_i = t_i$, and the babbling one, $m_{1i} = m_{2i}$.[14] For any perfect Bayesian equilibrium, agents' belief about the population composition is given by $\hat{\lambda} = \frac{n_1}{n_1+n_2}$ in a separating equilibrium, and $\hat{\lambda} = \lambda$ in a babbling equilibrium.

Before characterizing agents' equilibrium communication strategies, I illustrate with the following corollary that it is the combination of both the coordination motive and the fundamental motive that gives rise to agents' incentive to (truthfully) communicate their types.

Corollary 2. *Without the coordination motive or the fundamental motive, agents have no incentive to communicate their sentiments.*

[14] In fact, a mixed strategy equilibrium exists with a probability measure zero. In addition, in such an equilibrium, only one type of agents play a mixed communication strategy. For more details on the characterization of such mixed strategy equilibria, please refer to footnote 17.

When there is no coordination motive, i.e., $\omega = 0$, agents have no communication incentive because the average action is irrelevant to their payoffs and so is the aggregate sentiment. Since agents of both types are equally well informed about the fundamental, they rely on their private signals to the same extent when choosing their actions; that is, $\phi_k = \rho_v$ for $k \in \{1, 2\}$ and is independent of the posterior belief about the population composition $\hat{\lambda}$. When there is no fundamental motive, i.e., $\omega = 1$, agents have no communication incentive either because the coordination motive is so strong that they coordinate solely on the public information and ignore their private information; that is, $\phi_k = 0$ for $k \in \{1, 2\}$ and is again independent of the expected population composition $\hat{\lambda}$.

When the coordination motive and the fundamental motive coexist, i.e., $0 < \omega < 1$, agents may be incentivized to truthfully communicate their types so as to boost the others' belief about their population proportion. One exception is that, as in every cheap-talk game (see, e.g., Crawford and Sobel, 1982), babbling equilibria always exist. For the following analysis, I therefore focus on the separating equilibrium, if it exists, where agents of different types adopt different communication strategies.

4.1. Value of Private Information

Consider an agent's expected payoff given the perceived population composition:

$$\mathbb{E}[u_{ki}] = \mathbb{E}\left[\frac{1}{2}a_{ki}^2\right] = \frac{1}{2}\Big(\mu^2 + (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\phi_k^2\Big). \tag{8}$$

The magnitude of ϕ_k captures the value of private information and directly translates into type-k agents' payoff level. An agent's optimal communication strategy thus maximizes the value of his private information by affecting the others' perception about the population composition. Moreover, in any separating equilibria, the realized population composition is fully revealed and converges to the prior probability λ with sufficiently many agents by the central limit theorem.[15] By deviating from his equilibrium communication strategy, the agent will induce a small change of the other agents' posterior belief about the population composition. He then adjusts his own action accordingly in response to the change in the other agents' actions. More formally, suppose the other agents' posterior belief becomes λ' following such a deviation, then the deviating agent's best response in terms of his reliance on the private signal is

$$\begin{aligned}
\phi_1' &= \phi_1'(\lambda, \lambda') = (1-\omega)\rho_v + \omega\Big(\lambda\rho_s\phi_1(\lambda') + (1-\lambda)\rho_d\phi_2(\lambda')\Big), \\
\phi_2' &= \phi_2'(\lambda, \lambda') = (1-\omega)\rho_v + \omega\Big(\lambda\rho_d\phi_1(\lambda') + (1-\lambda)\rho_s\phi_2(\lambda')\Big),
\end{aligned} \tag{9}$$

respectively, where $\phi_k(\lambda')$'s represent the other agents' actions as given by *Proposition 1*.

The assumption of finitely many agents simplifies the analysis of agents' communication strategies by rendering the effect of each agent's message on the other agents' posterior belief marginal. Consequently, agents' communication incentive becomes independent of the size of the population. The incentive compatibility constraints for truthful communication reduce to

$$\begin{aligned}
\left.\frac{\mathrm{d}\mathbb{E}[u_{1i}']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\,\phi_1 \left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} > 0, \\
\left.\frac{\mathrm{d}\mathbb{E}[u_{2i}']}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\,\phi_2 \left.\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} < 0,
\end{aligned} \tag{IC}$$

where u_{ki}' differs from u_{ki} in equation (8) by replacing ϕ_k with ϕ_k' in agent i's action. The truthful communication strategy can be sustained in equilibrium if it increases

[15] There is a subtle difference between a large but finite number of agents and infinitely many agents. In the latter case, the realized population composition *equals* the prior probability λ. This eliminates the incentive to communicate sentiments.

the value of the communicating agent's private information. Hence, a separating equilibrium exists only when the above IC constraints hold for both types.[16] The following lemma establishes how an agent's use of his private signal is affected by his deviation from truthful communication through its marginal effect on the other agents' posterior belief.

Lemma 1. *When $\sigma_\eta^2 < \sigma_v^2$, there exist $0 < \lambda_1^* < \lambda_1^\ddagger < \frac{1}{2} < \lambda_2^\ddagger < \lambda_2^* < 1$ such that $\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_k^\ddagger)$, and $\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_k^\ddagger, 1)$. Otherwise, $\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for any $\lambda \in (0,1)$.*

Similar to all agents' common belief, the other agents' belief about the population composition affects the focal agent's action through the strategic value of his private information, but only indirectly through the others' use of their information; that is,

$$\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} = \omega\left(\lambda\rho_s\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda} + (1-\lambda)\rho_d\frac{\mathrm{d}\phi_2}{\mathrm{d}\lambda}\right). \tag{10}$$

As discussed earlier, the indirect effect can be negative for the minority type, implying that they will rely less on their private signals following an increase in the others' belief about their population proportion.

To take a closer look at the indirect effect, we may decompose it into a weighted average effect on the response of each type. When the population composition is perfectly balanced, the two types change their reliance on private signals to the same

[16] If we assume λ is a random variable *ex-ante* such that $\tilde{\lambda} \sim Beta(\alpha, \beta)$, then the left-hand-side of the IC constraints is

$$\frac{\mathrm{d}\mathbb{E}[u_{ki}']}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} = (\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\mathbb{E}_\lambda\left[\phi_k\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}\right],$$

which admits no closed-form solutions due to the complexity in the nonlinear moment function. Alternatively, we may view the conditions we derive in *Propositions 3* as one class of distribution whose probability density is sufficiently concentrated around λ among all potential distributions that allow for the existence of a separating equilibrium.

extent but in opposite directions. Nevertheless, since within-type signal correlation ρ_s is always higher than cross-type signal correlation ρ_d, the indirect effect is positive. When the population composition is relatively unbalanced, the indirect effect is dominated by the majority type's response. In particular, the majority type rely less on their private signals drastically following an increase in the minority population. Recall that in the extreme case where $\lambda = 0$, the indirect effect reduces to $\rho_d \frac{\mathrm{d}\phi_2}{\mathrm{d}\lambda}$ which is negative as long as ρ_d is positive, that is, $\sigma_\eta^2 < \sigma_v^2$. As a result, the negative effect through the majority type's actions translates into the minority type's decreased reliance on their private signals.

Combined with the result from *Proposition 2*, the following proposition characterizes the conditions under which there exists a separating equilibrium.

Proposition 3. *Truthful communication from both types can be sustained in equilibrium only for (i)* $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$ *if* $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*; (ii)* $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$ *if* $\sigma_\eta^2 < \sigma_v^2$*; and (iii)* $0 < \lambda < 1$ *otherwise, where* $\lambda_1^\dagger, \lambda_1^\ddagger \in (0, \frac{1}{2})$ *and* $\lambda_2^\dagger, \lambda_2^\ddagger \in (\frac{1}{2}, 1)$ *are defined in the appendix.*

Recall that agents prefer truthful communication whenever it increases the value of their private information. In the first case, strong coordination motives and relatively high uncertainty about the sentiment induce agents of the minority type to use their private signals in the opposite way (see *Proposition 2*). Hence, the minority's increased reliance on their private signals following a perceived increase in their population proportion manifests decreased value of their private information. In the second case, agents' reliance on their private signals is always positive, but as shown in *Lemma 1*, a perceived increase in the proportion of the minority induces them to rely less on their private signals. This also results in decreased value of the minority's private information. In both cases, agents of the minority type are reluctant to truthfully communicate their sentiments. Instead, they are incentivized to mimic the majority type in order to increase the value of their private information. Therefore, truthful communication can only be sustained as equilibrium strategies when the population composition is relatively balanced; otherwise, the babbling equilibrium is the only equilibrium.[17]

To illustrate the validity of the assumption of a large number of agents, an example of two agents is provided below.

Example $(N = 2)$**.** When there are only two agents in the economy, we modify the payoff function to $u_i(a, \tilde{v}) = \big((1-\omega)\tilde{v} + \omega a_j\big)a_i - \frac{1}{2}a_i^2$, so that the agent's own action does not affect his marginal return.

In a separating equilibrium, we have

$$\phi_i = \begin{cases} \phi_s = \frac{(1-\omega)\sigma_v^2}{(1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2} & \text{if } t_i = t_j, \\ \phi_d = \frac{(1-\omega)\sigma_v^2}{(1-\omega)\sigma_v^2+(1+\omega)\sigma_\eta^2+\sigma_\epsilon^2} & \text{if } t_i \neq t_j. \end{cases}$$

Recall from equation (8) that an agent's expected payoff increases in his (expected) squared reliance on the private signal. Hence, a separating equilibrium can be sustained if and only if

$$\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2 \quad \text{and} \quad \lambda\phi_d^2 + (1-\lambda)\phi_s^2 > \phi_2^2,$$

which is always satisfied around $\lambda = \frac{1}{2}$.

Suppose $t_i = 1$, note that $\phi_s = \phi_1(1)$. By the convexity of ϕ_1^2 (see *Proposition*

[17] There also exist mixed strategy equilibria at $\lambda = \lambda_i^\dagger$ and $\lambda = \lambda_i^\ddagger$ where only the minority type randomize between truthful communication with probability q and lying with probability $1-q$, while the majority type always tell the truth. Without loss of generality, suppose type-1 is the minority type, then agents' posterior belief about the population composition is given by $\hat{\lambda} = \frac{\frac{n_1}{\hat{q}}}{n_1+n_2}$, where $\hat{q}$ is the agents' conjecture of type-1's truth-telling probability, and in equilibrium, $\hat{q} = q$. Indeed, any $q \in (0, 1)$ can be sustained as an equilibrium strategy of the minority type when $\lambda = \lambda_i^\dagger$ or $\lambda_i^\ddagger$.

2), the relative magnitude of $\lambda\phi_s^2 + (1-\lambda)\phi_d^2$ to ϕ_1^2 then depends on that of ϕ_d^2 to $(\phi_1(0))^2$.[18] More specifically, if $\phi_d^2 \geq (\phi_1(0))^2$, then $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for any $\lambda \in (0,1)$; otherwise, there exists $0 < \lambda_{1,N=2}^T < \frac{1}{2}$ such that $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 < \phi_1^2$ for $\lambda < \lambda_{1,N=2}^T$, and $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for $\lambda > \lambda_{1,N=2}^T$. The case of $t_i = -1$ is symmetric to $t_i = 1$ about $\lambda = \frac{1}{2}$ and thus follows the similar analysis.

To conclude, babbling equilibrium is the only equilibrium when $\lambda \leq \lambda_{1,N=2}^T$ or $\lambda \geq \lambda_{2,N=2}^T$, where $0 < \lambda_{1,N=2}^T < \frac{1}{2} < \lambda_{2,N=2}^T < 1$, if (i) $\sigma_\eta^2 > \frac{\sigma_v^2+\sigma_\epsilon^2+\sqrt{(\sigma_v^2+\sigma_\epsilon^2)(\sigma_v^2+5\sigma_\epsilon^2)}}{2}$ and $\omega > \omega_{N=2}^T$ where $\omega_{N=2}^T = \left(\sqrt{\sigma_v^4 - 2\sigma_v^2\sigma_\eta^2 + 5\sigma_\eta^4} - \sigma_v^2 - \sigma_\eta^2\right)\frac{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}{2\sigma_\eta^2(\sigma_\eta^2-\sigma_v^2)} > \frac{1}{2}$ or (ii) $\sigma_\eta^2 < \sigma_v^2$. Otherwise, a separating equilibrium is also an equilibrium.

Comparing with *Proposition 3*, the analysis with a large number of agents provides a sufficient but not necessary condition for the existence of a separating equilibrium in general. Indeed, whenever there exists a separating equilibrium with a large number of agents, the value of private information is monotonically increasing in the other agents' perceived population composition.

4.2. Truthful Communication Region

By Jensen's inequality, agents' expected payoffs in a separating equilibrium is always higher than that in a babbling equilibrium. In other words, truthful communication is Pareto superior to babbling. The intuition is that information about the exact realization of population composition facilitates coordination among all agents. Accordingly, we examine how the truthful communication region is affected by the information structure and the coordination motive.

[18] To be more precise, it is possible that $\phi_d^2 > (\phi_1(0))^2$ yet ϕ_1^2 is concave for small values of λ. Nevertheless, $\lambda\phi_s^2 + (1-\lambda)\phi_d^2 > \phi_1^2$ for any $\lambda \in (0,1)$ whenever that is the case. Therefore, the concavity of ϕ_1^2 does not affect our following analysis.

Corollary 3. *The truthful communication region always expands with the within-type disagreement and shrinks with the coordination motive, but may expand or shrink with uncertainty about the fundamental and the sentiment. More specifically,*

(i) if $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*, then* $\Delta^\dagger \equiv \lambda_2^\dagger - \lambda_1^\dagger$*, and*

  (a) $\frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_v^2} > 0, \frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_\eta^2} < 0, \frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\sigma_\epsilon^2} > 0$*;*

  (b) $\frac{\mathrm{d}\Delta^\dagger}{\mathrm{d}\omega} < 0$*;*

(ii) if $\sigma_\eta^2 < \sigma_v^2$*, then* $\Delta^\ddagger \equiv \lambda_2^\ddagger - \lambda_1^\ddagger$*, and*

  (a) $\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_v^2} < 0, \frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_\eta^2} > 0, \frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\sigma_\epsilon^2} > 0$*;*

  (b) $\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}\omega} < 0$*.*

The information structure affects the truthful communication region through agents' reliance on their private signals. When $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, an agent prefers to disclose his type only when he uses his signal in a positive way. Thus, whether the truthful communication region expands or shrinks following a change in the information structure depends on whether such a change increases or decreases agents' reliance on their private signals. Since higher prior uncertainty increases both the fundamental value and the strategic value of private signals, it makes agents rely more on their private signals and hence willing to truthfully communication their types for a broader range of underlying population composition.

Although both cross-type and within-type heterogeneities render a private signal noisier about the fundamental and hence reduces its fundamental value, they affect the truthful communication region in opposite directions due to their different impacts on the strategic value. More cross-type heterogeneity decreases cross-type signal correlation and makes it more negative. Consequently, when the agent is of the minority type, it decreases the strategic value of his private signal and results in a smaller truthful communication region. Conversely, more within-type heterogeneity increases the strategic value of his private signal by attenuating the negative cross-type signal correlation. Moreover, this positive effect on the strategic value of private signals dominates the negative effect on the fundamental value of private signals. The overall positive effect makes agents more inclined to disclose their types and thus results in a larger truthful communication region.

When $\sigma_\eta^2 < \sigma_v^2$, the effect of a change in the information structure on the truthful communication region depends on how it changes the importance of the perceived population composition in agents' actions. As prior uncertainty about the fundamental increases, private information is directed more towards forecasting the fundamental. This diminishes the strategic value of private information, and makes the perceived population composition influence agents' actions to a lesser extent. Conversely, across-type and within-type heterogeneity have an opposite effect to that of prior uncertainty about the fundamental. Noisier private information about the fundamental elevates the importance of its strategic value. Agents' perception about the population composition thus becomes more important in forming their actions, which encourages more truthful communication. This result implies that there can still be heated discussions on some social topics even as more information about the fundamental becomes publicly available.

Note that although the effects of within-type disagreement are qualitatively the same in part (i) and part (ii) of *Corollary 3*, their intuitions are quite different. Nevertheless, a stronger coordination motive leads to a tighter truthful communication region in both cases. Intuitively, as the coordination motive becomes stronger, agents are inclined to rely more on the common prior rather than their private signals, which makes the aggregate sentiment and hence the population composition less important.

In the extreme case where there is no fundamental motive, recall from *Corollary 2* that agents rely solely on the common prior belief and completely ignore their private information.

5. Discussion

The Role of Public Information To the extent that communication of sentiments facilitates coordination, the welfare effect of enhanced dissemination of public information related to how it affects individuals' incentives to truthfully communicate their sentiments remains underexplored in the literature. As shown in *Corollary 3*, whether more public information makes truthful communication of sentiments more likely or less likely depends on the specific case. When there is limited prior knowledge about the fundamental, provision of public information encourages more communication of sentiments by making the strategic value of private information more prominent. This effect goes in the same direction as the informational effect of improving transparency in the prior literature (see, e.g., Morris and Shin, 2002; Gao, 2008), indicating an overall positive social value of public information.

However, the effect is reversed when the coordination motive is relatively strong and the degree of polarization is high. In this case, more public information about the fundamental makes private information less relevant. Hence, more transparency suppresses individuals' incentive to express themselves and can thus be detrimental in a setting like Angeletos and Pavan (2007) where coordination is socially valuable. This implies that more public disclosure is not necessarily always desirable, especially when speculative investors are severely polarized. Thus, caution should be exercised before enhancing dissemination of public information when the public already possess significant knowledge about the underlying fundamental.

To conclude, from the perspective of facilitating coordination, provision of more public disclosure is beneficial when market participants have limited common prior knowledge about the fundamental, but can be detrimental when the market is relatively divided in sentiments yet the coordination motive is strong.

Diversity and Bias in Aggregate Decision-Making Conventional wisdom predicts that diversity enables better decision-making (Rock, Grant and Grey, 2016; Reynolds and Lewis, 2017). However, the following analysis on the aggregate action suggests that it may not always be the case when individuals are subject to great pressure for conformity or reputation concerns. Note that the joint decision is biased by the aggregate sentiment:

$$\bar{a} = \mu + \big(\lambda\phi_1 + (1-\lambda)\phi_2\big)\tilde{v} + \big(\lambda\phi_1 - (1-\lambda)\phi_2\big)\tilde{\eta},$$

and its relative exposure to sentiment is $R \equiv \frac{\lambda\phi_1-(1-\lambda)\phi_2}{\lambda\phi_1+(1-\lambda)\phi_2}$. While the joint decision's absolute exposure to sentiment monotonically increases in λ and vanishes to zero with a perfectly balanced population, its relative exposure to sentiment may not be monotonically decreasing in diversity. In particular, recall from *Proposition 2* that when the degree of polarization is high, strong coordination motives can induce the extreme minority type to direct their sentiments in the opposite way so that their actions are aligned with the majority. Such "herding" of the minority not only exacerbates, instead of neutralizing, the aggregate sentiment, but also countervails the majority's reliance on their private information and reduces the loading on the fundamental component in the joint decision. As a result, the magnitude of R may increase as the population becomes more diverse from an extremely homogenous one.

Corollary 4. *The magnitude of the joint decision's relative exposure to sentiment* *is symmetric about and minimized at* $\lambda = \frac{1}{2}$*. In particular, when* $(2\omega - 1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$*,* $\frac{\mathrm{d}|R|}{\mathrm{d}\lambda} > 0$ *for* $\lambda \in (0, \frac{1}{2}\left(1 - \sqrt{\frac{\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right))$*, and* $\frac{\mathrm{d}|R|}{\mathrm{d}\lambda} < 0$ *for* $\lambda \in (\frac{1}{2}\left(1 - \sqrt{\frac{\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2}{\omega\sigma_\eta^2}}\right), \frac{1}{2})$*; otherwise,* $\frac{\mathrm{d}|R|}{\mathrm{d}\lambda} < 0$ *for* $\lambda \in (0, \frac{1}{2})$*.*

In a nutshell, while it conforms to conventional wisdom that a perfectly diverse population minimizes the magnitude of bias in the joint decision-making process, marginally improving diversity of an originally homogenous decision-making body does not always lead to a less biased joint decision, especially when the the coordination motive is strong and the degree of polarization is high. Conversely, diversity enhancement efforts would be beneficial when the decision-making body is already relatively diverse. In such a case, further improving diversity not only neutralizes the aggregate sentiment by increasing the presence of the minority, but also encourages the minority to integrate more of their opinions into their voice without discouraging the majority too much.

6. Conclusion

Polarized beliefs have been pervasive in various social and political contexts (see, e.g., Baldassarri and Gelman, 2008; Fiorina and Abrams, 2008). On one hand, media and political interpreters of American politics have been promulgating a polarization narrative since the early 1990s. On the other hand, prior literature shows that belief polarization can arise endogenously in the long-run with persuasion bias (DeMarzo et al., 2003) or rational inattention (Nimark and Sundaresan, 2019). Yet there is little work examining the implications of belief polarization. This paper thus attempts to fill the gap by examining individuals' use of private information and strategic expression of polarized sentiments in a setting with strategic complementarities.

Although sentiments are value-irrelevant in the sense of being uninformative about the common state of nature, they are payoff-relevant in the presence of strategic complementarities. Individuals communicate their polarized sentiments, not aiming at manipulating others' beliefs about the fundamental, but instead to influence subsequent actions of the others in a way that maximizes the value of their private information in forecasting the others' actions. The model is general in that it applies to other economic situations where agents with polarized beliefs seek not only to adapt to an unknown state of the world but also to coordinate behavior with others. The analysis on individuals' aggregate action sheds some light on settings of collective decision-making such as corporate boards or state legislatures. The result implies that improving diversity of an originally homogenous decision-making body does not always lead to a less biased joint decision, especially when the the coordination motive is strong and the degree of polarization is high.

The framework set up in this paper also opens up many new opportunities for future research. One direction is to endogenize the evolution of belief polarization in the presence of coordination motives, perhaps as a consequence of private communication with endogenous communication networks. Moreover, embedding endogenously determined prices into a model with trade, expression of polarized beliefs may have substantial implications on trading volume and price efficiency. Another interesting direction is to allow for communication about both fundamental uncertainty, i.e., private signals, and strategic uncertainty, i.e., polarized beliefs, which gives rise to interaction between the market's inferences along the two dimensions of private information.

Appendix

Proof of Proposition 1. Taking the FOC yields $a_i^* = (1-\omega)\mathbb{E}_i[\tilde{v}] + \omega\mathbb{E}_i[\bar{a}^*]$. To solve for the linear equilibrium, we thus assume $a_i^* = \mu + \phi_i(s_i - \mu)$. Hence,

$$\bar{a} = \mu + \frac{N_1}{N_1+N_2}\phi_1(\bar{s}_1 - \mu) + \frac{N_2}{N_1+N_2}\phi_2(\bar{s}_2 - \mu). \tag{A1}$$

Then

$$\begin{aligned} \mathbb{E}_{1i}[\bar{a}^*] &= \mu + \left(\hat{\lambda}\rho_s\phi_1 + (1-\hat{\lambda})\rho_d\phi_2\right)(s_{1i} - \mu), \\ \mathbb{E}_{2i}[\bar{a}^*] &= \mu + \left(\hat{\lambda}\rho_d\phi_1 + (1-\hat{\lambda})\rho_s\phi_2\right)(s_{2i} - \mu), \end{aligned} \tag{A2}$$

where $\hat{\lambda} \equiv \mathbb{E}[\frac{N_1}{N_1+N_2}]$. Plugging $\mathbb{E}_i[\tilde{v}] = \mu + \rho_v(s_i - \mu)$ and equation (A2) in the FOC, the coefficient array (ϕ_1, ϕ_2) is determined by the system of simultaneous equations (4) – (5) by matching coefficients. □

Proof of Corollary 1. Note that

$$\phi_1, \phi_2 \le \phi_1(1) = \phi_2(0) = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\eta^2 + \frac{\sigma_\epsilon^2}{1-\omega}} < \phi|_{\omega=0} = \rho_v, \text{ and}$$

$$\phi_1 - \phi_2 = \frac{2\omega(1-\omega)(2\hat{\lambda}-1)\sigma_v^2\sigma_\eta^2}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2} > 0 \iff \hat{\lambda} > \frac{1}{2}.$$

□

Proof of Proposition 2.

$$\phi_1(0) = \phi_2(1) = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2)} > 0 \text{ iff } \sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2 > 0,$$

$$\phi_1(1) = \phi_2(0) = \frac{(1-\omega)\sigma_v^2\big(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2\big)}{(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2)} = \frac{(1-\omega)\sigma_v^2}{(1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2} > 0,$$

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} = \frac{8\omega^3(1-\omega)(\sigma_v^2\sigma_\eta^2)^2}{\big((\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2\big)^2} F_1(\hat{\lambda}) \text{ where}$$

$$F_1(\hat{\lambda}) \equiv \left(\hat{\lambda} + \frac{\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2}\right)^2 - \frac{(\sigma_v^2 - \sigma_\eta^2)(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}{4\omega^2\sigma_v^2\sigma_\eta^4}.$$

If $\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2 < 0$, then $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} > 0$. Moreover, $\phi_1 < 0$ for $\hat{\lambda} \in (0, \lambda_1^\dagger)$, and $\phi_1 > 0$ for $\hat{\lambda} \in (\lambda_1^\dagger, 1)$ where

$$\lambda_1^\dagger \equiv -\frac{\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2}. \tag{A3}$$

Otherwise, if $\sigma_v^2+(1-2\omega)\sigma_\eta^2+\sigma_\epsilon^2 \geq 0$, i.e., $\omega \leq \frac{1}{2}$ or $\omega > \frac{1}{2}$ and $(2\omega-1)\sigma_\eta^2 \leq \sigma_v^2+\sigma_\epsilon^2$, then $\phi_1(\hat{\lambda}) \geq 0$ for all $\hat{\lambda} \in [0,1]$, and

(i) if $(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big((1-\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2 - 2\omega\sigma_v^2\big) + 4\omega^2\sigma_v^2\sigma_\eta^2 \geq 0$, then $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} > 0$; and

(ii) otherwise, $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} < 0$ for $\hat{\lambda} \in (0, \lambda_1^*)$, and $\frac{\mathrm{d}\phi_1}{\mathrm{d}\hat{\lambda}} > 0$ for $\hat{\lambda} \in (\lambda_1^*, 1)$ where

$$\lambda_1^* \equiv \frac{\sqrt{(\sigma_v^2 - \sigma_\eta^2)(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)} - \sigma_v\big(\sigma_v^2 + (1-2\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}{2\omega\sigma_v\sigma_\eta^2}. \tag{A4}$$

The condition for this case further reduces to $\sigma_\eta^2 < \frac{(\sigma_v^2+\sigma_\epsilon^2)(2\sigma_v^2-\sigma_\epsilon^2)}{2\sigma_v^2+\sigma_\epsilon^2}$ and $\omega > \omega^T$ where

$$\omega^T \equiv \left(3\sigma_v^2 + \sigma_\eta^2 - \sqrt{(\sigma_v^2 - \sigma_\eta^2)(9\sigma_v^2 - \sigma_\eta^2)}\right)\frac{\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2}{8\sigma_v^2\sigma_\eta^2} \in \left(\frac{1}{3}, 1\right). \tag{A5}$$

Since $\phi_2(\hat{\lambda})$ is symmetric to $\phi_1(\hat{\lambda})$ about $\hat{\lambda} = \frac{1}{2}$, the proof for ϕ_2 is omitted.[19]

[19] More formally,

$$\frac{\mathrm{d}\phi_2}{\mathrm{d}\hat{\lambda}} = \frac{8\omega^3(1-\omega)(\sigma_v^2\sigma_\eta^2)^2}{\big((\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)((1-\omega)(\sigma_v^2 + \sigma_\eta^2) + \sigma_\epsilon^2) + 4\omega^2\hat{\lambda}(1-\hat{\lambda})\sigma_v^2\sigma_\eta^2\big)^2} F_2(\hat{\lambda}) \quad \text{where}$$

$$F_2(\hat{\lambda}) \equiv -\left(\hat{\lambda} - \frac{\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2}{2\omega\sigma_\eta^2}\right)^2 + \frac{(\sigma_v^2 - \sigma_\eta^2)(\sigma_v^2 + \sigma_\eta^2 + \sigma_\epsilon^2)\big(\sigma_v^2 + (1-\omega)\sigma_\eta^2 + \sigma_\epsilon^2\big)}{4\omega^2\sigma_v^2\sigma_\eta^4}.$$

Instead, the expressions for $\lambda_2^\dagger$ and λ_2^* are specified as follows:

$$\lambda_2^\dagger \equiv \frac{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}{2\omega\sigma_\eta^2}, \tag{A6}$$

$$\lambda_2^* \equiv \frac{\sigma_v\big(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2\big)-\sqrt{(\sigma_v^2-\sigma_\eta^2)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)}}{2\omega\sigma_v\sigma_\eta^2}. \tag{A7}$$

□

Proof of Corollary 2. The proof follows immediately by plugging in $\omega=0$ and $\omega=1$, respectively, into equations (4) – (5), and observing that ϕ_i's are independent of $\hat{\lambda}$ in both cases. □

Proof of Lemma 1. Taking the first-order derivative of ϕ_1' expressed in equation (9) with respect to λ' yields

$$\begin{aligned}
\left.\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\right|_{\lambda'=\lambda} &= \omega\frac{\lambda\frac{\mathrm{d}\phi_1}{\mathrm{d}\lambda}(\sigma_v^2+\sigma_\eta^2)+(1-\lambda)\frac{\mathrm{d}\phi_2}{\mathrm{d}\lambda}(\sigma_v^2-\sigma_\eta^2)}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}\\
&= \frac{2\omega(1-\omega)\sigma_v^2\sigma_\eta^2}{\big((\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2)+4\omega^2\lambda(1-\lambda)\sigma_v^2\sigma_\eta^2\big)^2}G(\lambda) \quad \text{where}\\
G(\lambda) \equiv 4\omega^2\sigma_v^2\sigma_\eta^2F_1 &- \left((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2+4\omega^2\lambda(1-\lambda)\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2}\right)\\
&\quad \big((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2+2\omega\lambda\sigma_v^2\big).
\end{aligned}$$

For function G, we have

$$\begin{aligned}
G(0) &= \omega(\sigma_\eta^2-\sigma_v^2)\big((1+\omega)\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big)\\
&\implies \operatorname{sgn}(G(0)) = \operatorname{sgn}\left(\sigma_\eta^2-\sigma_v^2\right),\\
G(1) &= \omega(\sigma_\eta^2+\sigma_v^2)\big((1+\omega)\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\big) > 0,
\end{aligned}$$

$$\frac{\mathrm{d}G}{\mathrm{d}\lambda} = \frac{2\omega\sigma_v^2}{\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2} G^1(\lambda) \text{ where}$$
$$G^1(\lambda) \equiv (\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2-4\omega(1-\lambda)\sigma_\eta^2\big)$$
$$-2\omega(1-2\lambda)\sigma_\eta^2\big((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2\big)-4\omega^2\lambda(2-3\lambda)\sigma_v^2\sigma_\eta^2.$$

For function G^1, we have

$$G^1(0) = (\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2-4\omega\lambda\sigma_\eta^2\big)-2\omega\sigma_\eta^2\big((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2\big),$$
$$G^1(1) = (\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)\big((1+\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2\big)+2\omega\sigma_\eta^2\big((1-\omega)(\sigma_v^2+\sigma_\eta^2)+\sigma_\epsilon^2\big)+4\omega^2\sigma_v^2\sigma_\eta^2>0,$$
$$\frac{\mathrm{d}G^1}{\mathrm{d}\lambda} = 4\omega\sigma_\eta^2 G^2(\lambda) \text{ where } G_2(\lambda)=\big(3\omega(2\lambda-1)+2\big)\sigma_v^2+(2-\omega)\sigma_\eta^2+2\sigma_\epsilon^2, \text{ and}$$
$$G^2(0) = (2-3\omega)\sigma_v^2+(2-\omega)\sigma_\eta^2+2\sigma_\epsilon^2,$$
$$G^2(1) = (2+3\omega)\sigma_v^2+(2-\omega)\sigma_\eta^2+2\sigma_\epsilon^2>0.$$

1. If $G^2(0)\geq 0$, i.e., $\sigma_\eta^2 \geq \frac{(3\omega-2)\sigma_v^2-2\sigma_\epsilon^2}{2-\omega}$, then $\frac{\mathrm{d}G^1}{\mathrm{d}\lambda}\geq 0$ and
   (a) if $G^1(0)<0$, i.e., $\sigma_\eta^2 > \frac{\sqrt{9\sigma_\epsilon^4+8(\sigma_v^2+\sigma_\epsilon^2)(17\sigma_v^2-\sigma_\epsilon^2)}-3\sigma_\epsilon^2}{4}$ and $\omega > \frac{\big(5\sigma_\eta^2-\sigma_v^2-\sqrt{(\sigma_\eta^2-\sigma_v^2)(17\sigma_\eta^2-\sigma_v^2)}\big)(\sigma_v^2+\sigma_\eta^2+\sigma_\epsilon^2)}{4\sigma_\eta^2(\sigma_v^2+\sigma_\eta^2)}$, then $G(\lambda)>0$ for all $\lambda\in(0,1)$;
   (b) otherwise, $G^1(\lambda)\geq 0$ for all $\lambda\in(0,1)$, and
      i. if $G(0)<0$, i.e., $\sigma_\eta^2<\sigma_v^2$, then $G(\lambda)<0$ for $\lambda\in(0,\lambda_1^\ddagger)$, and $G(\lambda)>0$ for $\lambda\in(\lambda_1^\ddagger,1)$, where $\lambda_1^\ddagger$ is the unique solution to $G(\lambda_1^\ddagger)=0$ on $(0,1)$.
      ii. otherwise, $G(\lambda)>0$ for all $\lambda\in(0,1)$.
2. Otherwise, if $G^2(0)<0$, i.e., $\sigma_\eta^2 < \frac{(3\omega-2)\sigma_v^2-2\sigma_\epsilon^2}{2-\omega}$, then $G^1(\lambda)>0$ for all $\lambda\in(0,1)$, and we go back to case 1b.

To conclude, if $\sigma_\eta^2<\sigma_v^2$, then $\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}<0$ for $\lambda\in(0,\lambda_1^\ddagger)$, and $\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}>0$ for $\lambda\in(\lambda_1^\ddagger,1)$; otherwise, $\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}>0$ for any $\lambda\in(0,1)$. Moreover, since $G(\lambda_1^*)<0$ and $G(\frac{1}{2})>0$, $\lambda_1^*<\lambda_1^\ddagger<\frac{1}{2}$.

The analysis for $\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}$ is similar and thus omitted. $\square$

Proof of Proposition 3. I first show that the incentive compatibility constraints can indeed be determined by the sign of $\frac{\mathrm{d}\mathbb{E}[u_i']}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda}$ where λ and λ' are the other agents' equilibrium belief and off-equilibrium belief following a deviation, respectively.

Consider agent i of type-1 and adopts truthful communication $m_i^* = t_i = 1$. By deviating to $m_i = -t_i = -1$, $\hat{\lambda}' = \frac{n_1-1}{(n_1-1)+(n_2+1)} = \frac{n_1-1}{n_1+n_2}$. Since $\hat{\lambda} = \frac{n_1}{n_1+n_2} > \hat{\lambda}'$ and $\lim\limits_{N\to\infty} \hat{\lambda} = \lim\limits_{N\to\infty} \hat{\lambda}' = \lambda$, no-deviation of agent i from truthful communication requires

$$\lim_{N\to\infty} \mathbb{E}[u_{1i}] - \mathbb{E}[u_{1i}'] > 0 \iff \lim_{\hat{\lambda}'\to\hat{\lambda}^-} \frac{\mathbb{E}[u_{1i}] - \mathbb{E}[u_{1i}']}{\hat{\lambda} - \hat{\lambda}'} > 0 \iff \frac{\mathrm{d}\mathbb{E}[u_{1i}']}{\mathrm{d}\hat{\lambda}'}\bigg|_{\hat{\lambda}'=\hat{\lambda}=\lambda} > 0.$$

Similarly, for agent i of type-2 and adopts truthful communication $m_i^* = t_i = -1$, no deviation to $m_i = 1$ requires

$$\lim_{N\to\infty} \mathbb{E}[u_{2i}] - \mathbb{E}[u_{2i}'] > 0 \iff \lim_{\hat{\lambda}'\to\hat{\lambda}^+} \frac{\mathbb{E}[u_{2i}] - \mathbb{E}[u_{2i}']}{\hat{\lambda} - \hat{\lambda}'} < 0 \iff \frac{\mathrm{d}\mathbb{E}[u_{2i}']}{\mathrm{d}\hat{\lambda}'}\bigg|_{\hat{\lambda}'=\hat{\lambda}=\lambda} < 0.$$

Therefore, any separating equilibrium requires $\phi_1\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\phi_2\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ (see IC constraints). Combining the results from *Proposition 2* and *Lemma 1*, we have

(i) if $(2\omega-1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$, then $\phi_k\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_k^\dagger)$, $\phi_k\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_k^\dagger, 1)$, and hence there exists a separating equilibrium iff $\lambda_1^\dagger < \lambda < \lambda_2^\dagger$;

(ii) if $\sigma_\eta^2 < \sigma_v^2$, then $\phi_k\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$ for $\lambda \in (0, \lambda_k^\ddagger)$, $\phi_k\frac{\mathrm{d}\phi_k'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ for $\lambda \in (\lambda_k^\ddagger, 1)$, and hence there exists a separating equilibrium iff $\lambda_1^\ddagger < \lambda < \lambda_2^\ddagger$;

(iii) otherwise, $\phi_1\frac{\mathrm{d}\phi_1'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} > 0$ and $\phi_2\frac{\mathrm{d}\phi_2'}{\mathrm{d}\lambda'}\Big|_{\lambda'=\lambda} < 0$, and hence there always exists a separating equilibrium. $\square$

Proof of Corollary 3. For the first part where $\Delta^\dagger = \lambda_2^\dagger - \lambda_1^\dagger$, the proof follows immediately by inspecting equations (A3) and (A6). For the second part where $\Delta^\ddagger = \lambda_2^\ddagger - \lambda_1^\ddagger$, note that $\lambda_1^\ddagger + \lambda_2^\ddagger = \frac{1}{2}$, and hence

$$\operatorname{sgn}\left(\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}x}\right) = -\operatorname{sgn}\left(\frac{\mathrm{d}\lambda_1^\ddagger}{\mathrm{d}x}\right) \text{ for } x \in \{\sigma_v^2, \sigma_\eta^2, \sigma_\epsilon^2, \omega\}.$$

Recall from the proof of *Lemma 1* that $\frac{\partial G}{\partial \lambda} > 0$. Therefore, by implicit function theorem,

$$\begin{aligned} \operatorname{sgn}\left(\frac{\mathrm{d}\lambda_1^\ddagger}{\mathrm{d}x}\right) &= \operatorname{sgn}\left(-\left.\frac{\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial \lambda}}\right|_{\lambda=\lambda_1^\ddagger}\right) \\ \Longrightarrow \operatorname{sgn}\left(\frac{\mathrm{d}\Delta^\ddagger}{\mathrm{d}x}\right) &= \operatorname{sgn}\left(\left.\frac{\partial G}{\partial x}\right|_{\lambda=\lambda_1^\ddagger}\right). \end{aligned}$$

In particular, since $\sigma_\eta^2 < \sigma_v^2$ and $\lambda_1^\ddagger \in (0, \frac{1}{2})$, we have

(i) $\left.\frac{\partial G}{\partial \sigma_v^2}\right|_{\lambda=\lambda_1^\ddagger} < 0$: Note that $\frac{\partial^2 G}{\partial (\sigma_v^2)^2} < 0$, and hence,

$$\left.\frac{\partial G}{\partial \sigma_v^2}\right|_{\lambda=\lambda_1^\ddagger} < \left.\frac{\partial G}{\partial \sigma_v^2}\right|_{\sigma_v^2=\sigma_\eta^2,\ \lambda=\lambda_1^\ddagger} = -\omega(2\sigma_\eta^2 + \sigma_\epsilon^2) < 0.$$

(ii) $\left.\frac{\partial G}{\partial \sigma_\eta^2}\right|_{\lambda=\lambda_1^\ddagger} > 0$: Note that $\frac{\partial^2 G}{\partial (\sigma_\eta^2)^2} > 0$ and hence,

$$\left.\frac{\partial G}{\partial \sigma_\eta^2}\right|_{\lambda=\lambda_1^\ddagger} > \left.\frac{\partial G}{\partial \sigma_\eta^2}\right|_{\sigma_\eta^2=0,\ \lambda=\lambda_1^\ddagger} = \omega\left((1-\omega)\sigma_v^2 + \sigma_\epsilon^2\right) > 0.$$

(iii) $\left.\frac{\partial G}{\partial \sigma_\epsilon^2}\right|_{\lambda=\lambda_1^\ddagger} > 0$: Note that $\frac{\partial^2 G}{\partial (\sigma_\epsilon^2)^2} > 0$ and hence,

$$\left.\frac{\partial G}{\partial \sigma_\epsilon^2}\right|_{\lambda=\lambda_1^\ddagger} > \left.\frac{\partial G}{\partial \sigma_\epsilon^2}\right|_{\sigma_\epsilon^2=0,\ \lambda=\lambda_1^\ddagger} = \omega \frac{\begin{array}{c}(\sigma_v^2+\sigma_\eta^2)^2(\sigma_\eta^2-(1-2\lambda_1^\ddagger)\sigma_v^2) \\ -4\omega^2\lambda_1^\ddagger(1-\lambda_1^\ddagger)\sigma_v^2\sigma_\eta^2((1-2\lambda_1^\ddagger)\sigma_v^2+\sigma_\eta^2)\end{array}}{(\sigma_v^2+\sigma_\eta^2)^2} > 0.$$

In addition, $\lambda_1^\ddagger > \lim_{\sigma_\epsilon^2 \to \infty} \lambda_1^\ddagger = \frac{1}{2}\left(1 - \frac{\sigma_\eta^2}{\sigma_v^2}\right)$.

(iv) $\left.\frac{\partial G}{\partial \omega}\right|_{\lambda=\lambda_1^\ddagger} < 0$: Note that $\frac{\partial^3 G}{\partial \omega^3} > 0$ and hence,

$$\left.\frac{\partial G}{\partial \omega}\right|_{\lambda=\lambda_1^\ddagger} < \max\left\{ \left.\frac{\partial G}{\partial \omega}\right|_{\omega=0,\ \lambda=\lambda_1^\ddagger}, \left.\frac{\partial G}{\partial \omega}\right|_{\omega=1,\ \lambda=\lambda_1^\ddagger} \right\} = 0. \qquad \square$$

Proof of Corollary 4. Note that

$$\mathrm{sgn}\frac{\mathrm{d}|R|}{\mathrm{d}\lambda} = \mathrm{sgn}\frac{\mathrm{d}R^2}{\mathrm{d}\lambda} = \mathrm{sgn}(2\lambda-1)\left(\sigma_v^2 + (1-2\omega(1-2\lambda(1-\lambda)))\sigma_\eta^2 + \sigma_\epsilon^2\right).$$

Solving for $\sigma_v^2 + (1-2\omega(1-2\lambda(1-\lambda)))\sigma_\eta^2 + \sigma_\epsilon^2 = 0$ yields $\lambda = \frac{\omega\sigma_\eta^2 \pm \sqrt{\omega\sigma_\eta 2\left(\sigma_v^2+(1-\omega)\sigma_\eta^2+\sigma_\epsilon^2\right)}}{2\omega\sigma_\eta^2}$, which is in $(0,1)$ if and only if $(2\omega-1)\sigma_\eta^2 > \sigma_v^2 + \sigma_\epsilon^2$. $\square$

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