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# Inflation and the stock market: Understanding the "Fed Model"☆

Geert Bekaert [a,*], Eric Engstrom [b,1]

[a] Columbia Business School and NBER, USA
[b] Federal Reserve Board of Governors, USA

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ABSTRACT

The so-called Fed model postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated. In US data there is indeed a strikingly high time series correlation between the yield on nominal bonds and the dividend yield on equities. This positive correlation is often attributed to the fact that both bond and equity yields comove strongly and positively with expected inflation. Contrary to some of the extant literature, we show that this effect is consistent with modern asset pricing theory incorporating uncertainty about real growth prospects and habit-based risk aversion. In the US, high expected inflation has tended to coincide with periods of heightened uncertainty about real economic growth and unusually high risk aversion, both of which rationally raise equity yields.



## 1. Introduction

The so-called Fed model postulates that the dividend or earnings yield on stocks should equal the yield on nominal Treasury bonds, or at least that the two should be highly correlated.[2] Both investment professionals (see for instance

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* Corresponding author.
*E-mail address:* gb241@columbia.edu (G. Bekaert).

[1] The views expressed in this article do not necessarily represent those of the Federal Reserve System, its Board of Governors, or staff. Supplemental materials including data, MATLAB code, and appendices describing data construction and methodology for this paper are available online at doi:10.1016/j.jmoneco.2010.02.004.

[2] The Fed Model may have gained its moniker from Prudential Securities strategist Ed Yardeni (Yardeni, 1999) who noted that in the Federal Reserve Humphrey Hawkins Report for July 1997, a chart plotted the time series for the earnings–price ratio of the S&P 500 against the 10-year constant-maturity nominal treasury yield.

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Asness, 2003) and academics (see for instance Thomas and Zhang, 2008) have long been struck by the strength of the empirical regularity. Fig. 1 shows a graph of the yield on a 10-year nominal bond and the equity yield (using dividends) for the US aggregate stock market. The correlation between the two yields is 0.77! While some investment professionals are using the Fed model as a model of equity valuation (see the references in Estrada (2005)), both practitioners and academics have concluded that the model is inconsistent with a rational valuation of the stock market (see for instance, Asness, 2003; Feinman, 2005; Campbell and Vuolteenaho, 2004; Cohen et al., 2005; Ritter and Warr, 2002; Sharpe, 2002).

The difficulty in squaring the model with rational valuation can be illustrated using a simple decomposition of the dividend yield and the nominal bond yield. In the Gordon model, the equity cash yield, *EY*, on the aggregate stock market consists of three components:

$$EY = -EDIV + RRF + ERP, \tag{1}$$

where *EDIV* is the expected growth rate of real equity dividends, *RRF* is the real risk free rate of interest and *ERP* is the equity risk premium. Similarly, the yield on a nominal bond is:

$$BY = EINF + RRF + IRP, \tag{2}$$

where *EINF* is expected inflation, *RRF* is again the real interest rate, and *IRP* is the inflation risk premium. The high correlation between dividend yields and nominal bond yields is difficult to reconcile with rational models because expected inflation is a dominant source of variation in nominal yields and the extant literature seems to have concluded that it is impossible for expected inflation to have a large (rational) effect on any of the *real* components that drive the equity cash yield. In fact, the aforementioned authors all resort to the simple behavioral model proposed by Modigliani and Cohn in 1979 to explain the empirical regularity: inflation (or money) illusion. Inflation illusion suggests that when expected inflation increases, bond yields duly increase, but because equity investors incorrectly discount real cash flows using nominal rates, the increase in nominal yields leads to equity underpricing (the equity yield rises with bond yields to levels that are too high) and vice versa. Alternatively, one can view equity investors as correctly discounting nominal cash flows and using nominal discount rates, but failing to increase expected nominal cash payouts in response to increases in expected inflation.

The importance of this conclusion extends beyond the narrow confines of testing the Fed model. If behavioral biases induced by inflation cause misvaluation in the equity market, then the potential exists for informed practitioners to devise trading strategies to take advantage of the mispricing. For policy makers, if money illusion causes undue variation in equity prices during periods of inflation uncertainty, this suggests another motive for inflation stabilization policies, as Campbell and Vuolteenaho (2004) point out.

In this article, we carefully re-examine the evidence by constructing dynamic versions of Eqs. (1) and (2) in a vector autoregressive (VAR) framework, building on Campbell and Shiller's (1988) seminal work. The benchmark VAR includes

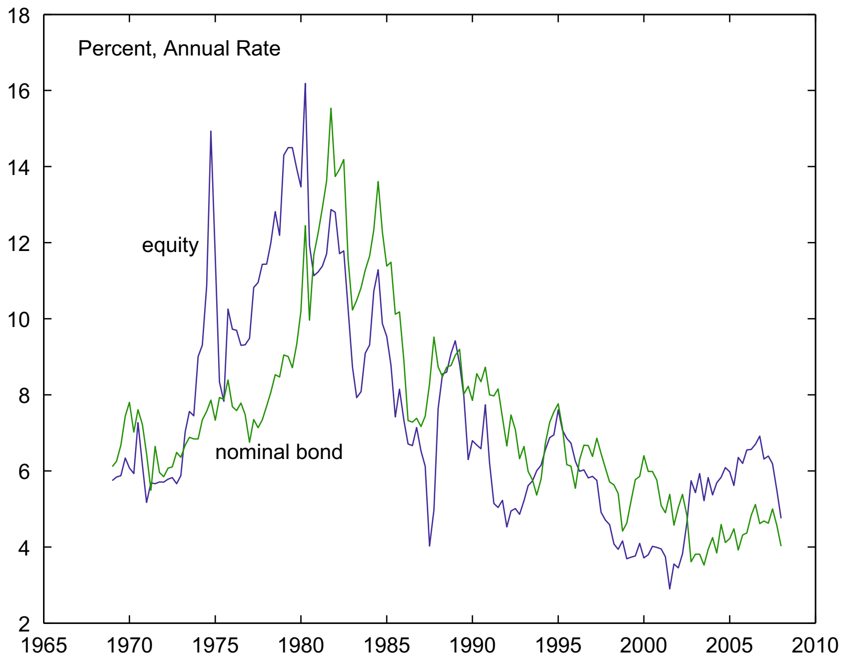


**Fig. 1.** Equity and bond yield time series for the US. This figure plots time series for the equity yield, $ey_t$ (left scale), and the bond yield, $by_t$ (right scale). We measure the equity yield, $ey_t$ as the dividend yield for the S&P500, and the nominal bond yield, $by_t$, as that of the 10-year constant-maturity Treasury. For illustration, both yields have been plotted in levels (that is, the $ey_t$ series has been exponentiated), and in units of percentage points at annual rates.

earnings growth and survey expectations of earnings to help predict cash flow growth and uses empirical proxies for real rates and expected inflation. However, we construct the risk premium components of yields as residuals since they are not directly measurable. We find that expected inflation is indeed the primary bond yield component responsible for the high stock–bond yield correlation. This is a remarkable stylized fact that any macro-economic model of the stock market must seek to explain. In the context of a rational model, expected inflation must be positively correlated with the dividend yield through some combination of positive correlation with the real rate and the equity risk premium, or a negative correlation with expected cash flow growth. However, only a relatively small portion of the overall comovement between expected inflation and the dividend yield can be ascribed to the correlation between expected inflation and real rates or expected cash flow growth.[3] The bulk of the positive covariance between the dividend yield and expected inflation comes from positive comovement between expected inflation and the equity risk premium. Importantly, because the equity premium is measured as a residual, these initial results do not identify whether money illusion-induced misvaluation or rational equity risk premiums are responsible for the high correlation with expected inflation.

However, our subsequent analysis strongly supports the latter explanation. The high correlation between expected inflation and the dividend yield is almost entirely due to the positive correlation between expected inflation and two plausible proxies for rational time-varying risk premiums: a measure of economic uncertainty (the uncertainty among professional forecasters regarding real GDP growth) and a consumption-based measure of risk aversion. These measures of rationally time-varying risk premiums feature prominently in recent asset pricing articles showing that they help to explain a number of salient asset return features. Bansal and Yaron's (2004) model features a small persistent expected growth component in consumption and dividend growth and fluctuations in economic uncertainty. While "long-run risk" plays an important role in their model, the fluctuations in risk premiums are entirely driven by fluctuations in economic uncertainty, which is modeled as an autoregressive volatility process for the fundamentals. Campbell and Cochrane (1999, CC henceforth) built a model of external habit in which negative (positive) shocks to consumption move actual consumption closer to (away from) a habit level, and consequently drive up (decrease) risk aversion. Using this model, we construct a measure of time-varying risk aversion using only current and past consumption data, which is counter-cyclical. Bekaert et al. (2009) formulate a model in which the agent has external habit preferences and consumption growth features time-varying economic uncertainty. Both aspects of the model are important in driving equity risk premiums, but stochastic risk aversion dominates. While the Bekaert et al. (2009) model fits the data, including term structure data, rather well, it is fair to say that all three models are too tightly parameterized to fit the comovements between equity and bond yields and their components. We therefore do not use a structural model to analyze the link between bond and equity yields, but rather use empirical proxies for the structural determinants of risk premiums suggested by these models. With these proxies we show that a rational channel explains why the Fed model "works": high expected inflation coincides with periods of high risk aversion and/or economic uncertainty. Therefore our explanation is very different from the prevailing explanations based on money illusion.

Our work is related to but distinct from another "old" hypothesis regarding the relationship between inflation and the stock market: Fama's (1981) proxy hypothesis. Fama argues that the strong negative relationship between stock returns and inflation is due to stock returns anticipating future economic activity and inflation acting as a proxy for expected real activity; hence, the hypothesis also relies on stagflation being an important part of US data. Using our VAR's implicit cash flow expectations to capture expected real activity suggests that the proxy hypothesis is part of the explanation, but that a risk-based story dominates. An out-of-sample test also supports our interpretation of the US data. Specifically, our results suggest that the correlation between equity and bond yields ought to be higher in countries with a higher average incidence of stagflation. It is. Our US results also prove robust in a wide variety of alternative VAR specifications. The concluding section summarizes our results and discusses how they hold up outside our particular sample period.

## 2. Empirical methodology

In this section, the first sub-section presents a dynamic version of the Gordon model alluded to in the introduction. In the second sub-section, we decompose the different yields using a VAR methodology. The third sub-section describes how our framework generates estimates of equity–bond yield correlations and their components. The fourth sub-section shows how to identify a rational component in the equity yield to test the main hypothesis. In the fifth sub-section, alternative hypotheses involving cash flow expectations are formulated.

### 2.1. Yield decompositions

Our goal is to construct dynamic versions of Eqs. (1) and (2). Beginning with the latter task, we simply assume the nominal yield decomposition relationship holds at each point in time using continuously compounded rates, denoted by

[3] This confirms Modigliani and Cohn's careful work that the effect is not due to expected real cash flow growth rates being adversely affected by expected inflation.

lower case letters. In particular, $by_t$, the continuously compounded bond yield, is given by,

$$by_t = einf_t + rrf_t + irp_t, \tag{3}$$

where $rrf_t$ is a real risk free rate assumed to have maturity equal to that of the nominal bond, $einf_t$ is the average (annualized) expected inflation over the life of the bond, and $irp_t$ is the inflation risk premium associated with the bond. In principle, all three components are unobserved. We achieve identification by finding observable proxies for the real rate and expected inflation, and use Eq. (3) to infer the inflation risk premium.[4] The next section contains empirical variable definitions and data sources.

To decompose the equity yield into its components, we use the Campbell and Shiller (1988, CS henceforth) decomposition. CS arrive at the following formula for the logarithmic equity yield, $ey_t$:

$$ey_t = -\frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j (r_{t+j+1} - \Delta d_{t+j+1})\right], \tag{4}$$

where $k$ and $\rho$ are linearization constants, $r_t$ is the one-period real return to holding equity, and $\Delta d_t$ is logarithmic one-period real dividend growth. Without loss of generality, the expected rate of return on equity can be decomposed into a risk-free component and a risk premium component,

$$E_t[r_{t+1}] = rrf_t + erp_t, \tag{5}$$

where $erp_t$ is the continuously compounded one-period equity risk premium. Given the implicit definition of $rrf_t$ in Eq. (3), the equity premium is defined relative to a long-term real risk free rate. Substituting,

$$ey_t = -\frac{k}{1-\rho} - E_t\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j+1} + E_t\sum_{j=0}^{\infty} \rho^j rrf_{t+j} + E_t\sum_{j=0}^{\infty} \rho^j erp_{t+j} \tag{6}$$

which is the dynamic version of Eq. (1). Here too, we treat the risk premium component as the residual, with the two other components constructed empirically using our assumed data generating process, described next.

### 2.2. Empirical model: VAR

To model the joint dynamics of stock and nominal bond yields and their components, we stack the following variables into a vector, $Y_t$,

$$Y_t = [einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x_t']', \tag{7}$$

with $x_t$ denoting a vector of time—$t$ observable information variables that will be useful in interpreting the results:

$$x_t = [ra_t, vr_t, \Delta ern_t, gern_t^{su}]'. \tag{8}$$

Hence, there are a total of nine variables in the VAR. The first two elements of the information vector, $x_t$, are designed to capture rational components of the equity risk premium, $erp_t$. First, $ra_t$, is a measure of rational risk aversion based on the specification of external habit persistence in CC. Second, $vr_t$ is a measure of uncertainty about real economic growth. BY use uncertainty in the context of a data generating process for dividend and consumption growth and demonstrate that a modest amount of time-varying uncertainty about real growth can, under some assumptions about investor preferences, generate nontrivial variation in the equity risk premium. The other two variables in $x_t$ represent contemporaneous realized real earnings growth, $\Delta ern_t$, and a subjective measure of expected earnings growth, $gern_t^{su}$. These variables help predict future dividends and help us test some alternative hypotheses. Further details are provided in Section 2.5.

We proceed by assuming a simple data generating process for $Y_t$, and using the fully observable vector,

$$W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t']', \tag{9}$$

to identify the dynamics of $Y_t$. Specifically, assume $Y_t$ follows a first-order VAR:

$$Y_t = AY_{t-1} + u_t, \tag{10}$$

where drift terms are suppressed since we are only interested in variance decompositions. The matrix $A$ is comprised of parameters governing the conditional mean of $Y_t$, and $u_t$ is a vector of i.i.d shocks with covariance matrix $\Omega$. Once the $Y_t$ dynamics are specified to take this form, Eqs. (3) and (4) imply that $W_t$ is a linear combination of concurrent values of $Y_t$ as well as expectations of future values of $Y_t$ (with constant terms suppressed):

$$W_t = M_1 Y_t + M_2 E_t \sum_{j=0}^{\infty} \rho^j Y_{t+j+1}, \tag{11}$$

[4] In a robustness exercise, we also conduct our main analysis using a different identification scheme for real rates that assumes the inflation risk premium is a function of inflation uncertainty.

where the matrices $M_1$ $(9\times 9)$ and $M_2$ $(9\times 9)$ are comprised of known parameters. Under the VAR(1) structure for $Y_t$, this has the implication that $Y_t$ and $W_t$ are related by a linear transformation,

$$Y_t=\Theta W_t, \tag{12}$$

and we must solve for $\Theta$. Consequently, $W_t$ also follows a linear VAR:

$$W_t=A^w W_{t-1}+u_t^w,$$

where $u_t^w$ has covariance matrix $\Omega^w$. Under the mapping in Eq. (12), $A$ and $\Omega$ satisfy:

$$A=\Theta A^w \Theta^{-1}$$

$$\Omega=\Theta\Omega^W\Theta^{'}. \tag{13}$$

To solve for $\Theta$, combine Eqs. (11) and (12) to obtain,

$$W_t=M_1\Theta W_t+M_2E_t\sum_{j=0}^{\infty}\rho^j\Theta W_{t+j+1} \tag{14}$$

Defining for notational convenience $\Phi_1=A^w(I-\rho A^w)^{-1}$ and solving the expectations terms yields

$$W_t=M_1\Theta W_t+M_2\Theta\Phi_1 W_t. \tag{15}$$

Equating $W_t$ coefficients on both sides of the equations yields a solution for $\Theta$:

$$vec(\Theta)=(I^{'}\otimes M_1+\Phi_1^{'}\otimes M_2)^{-1}vec(I). \tag{16}$$

Using Eqs. (13) and (16), we can completely specify the dynamics of $Y_t$ in terms of parameters estimated from the data. That is, $\{\hat{A},\hat{\Omega}\}=F\{\hat{A}^w,\hat{\Omega}^w\}$.

### 2.3. Decomposing yields under the VAR

As stated above, the nominal bond yield is affine in components of $Y_t$, as the right hand side terms of Eq. (3) are direct elements of $Y_t$. We can also now more explicitly describe our decomposition of the equity yield into three components,

$$ey_t=const+ey_t^{\Delta d}+ey_t^{rrf}+ey_t^{erp}, \tag{17}$$

where $ey_t^{\Delta d}=-E_t\sum_{j=0}^{\infty}\rho^j\Delta d_{t+j+1}$ represents the total effect of cash flow expectations, $ey_t^{rrf}=E_t\sum_{j=0}^{\infty}\rho^j rrf_{t+j}$, represents the total effect of real interest rates, and $ey_t^{erp}=E_t\sum_{j=0}^{\infty}\rho^j erp_{t+j}$ represents the total effect of equity risk premiums. We use objective conditional expectations under the VAR to calculate each of these quantities, and because of the simple VAR structure, the three equity yield components are affine in $Y_t$. For example, ignoring constant terms, and defining $e_{\Delta d}^{'}$ such that $\Delta d_t=e_{\Delta d}^{'}Y_t$,

$$ey_t^{\Delta d}=-e_{\Delta d}^{'}E_t\sum_{j=0}^{\infty}\rho^j Y_{t+j+1}=-e_{\Delta d}^{'}\rho A(I-\rho A)^{-1}Y_t$$

which is indeed a linear function of $Y_t$. For our baseline specification then, $M_1$ is an identity matrix and $M_2$ is the zero matrix, except for the rows pertaining to $ey_t$ and $by_t$:

$$M_1^{ey}=e_{rrf}^{'}+e_{erp}^{'},\quad M_2^{ey}=-e_{\Delta d}^{'}+\rho e_{rrf}^{'}+\rho e_{erp}^{'}$$

$$M_1^{by}=e_{einf}^{'}+e_{rrf}^{'}+e_{irp}^{'},\quad M_2^{by}=0, \tag{18}$$

where $M_1^{ey}$ denotes the relevant row of $M_1$ for the equity yield, and similarly for the other superscripts.

A decomposition of the covariance between stock and bond yields into its nine components allows us to trace the source of the high value of the overall covariance:

$$\begin{aligned}COV(ey_t,by_t)=&COV(ey_t^{\Delta d},einf_t)+COV(ey_t^{\Delta d},rrf_t)+COV(ey_t^{\Delta d},irp_t)+COV(ey_t^{rrf},einf_t)+COV(ey_t^{rrf},rrf_t)\\&+COV(ey_t^{rrf},irp_t)+COV(ey_t^{erp},einf_t)+COV(ey_t^{erp},rrf_t)+COV(ey_t^{erp},irp_t)\end{aligned} \tag{19}$$

Each of these covariances is readily calculated using VAR arithmetic. For instance,

$$COV(ey_t^{\Delta d},einf_t)=-e_{\Delta d}^{'}\rho A(I-\rho A)^{-1}COV(Y_t)e_{einf}^{'}, \tag{20}$$

where $vec[COV(Y_t)]=(I-A\otimes A)^{-1}vec(\Omega)$. Note that every element of $COV(ey_t,by_t)$ is ultimately a function of the parameters of the observable VAR, $\{\hat{A}^w,\hat{\Omega}^w\}$.

### 2.4. Orthogonalizing the equity risk premium

The equity risk premium component of equity yields in our decompositions above, $ey_t^{erp}$, is essentially a residual, the difference between the observed equity yield and the summed presented values, calculated under the VAR, of future cash flows and real risk free rates. A disadvantage of this approach is that model misspecification could contaminate the equity risk premium estimates. To try to isolate the component of the equity risk premium that is consistent with rational pricing, we draw on recent theoretical advances in the empirical asset pricing literature. CC and BY suggest that $erp_t$ is approximately linear in risk aversion, $ra_t$, or real uncertainty, $vr_t$ respectively.

Let us start with describing our fundamental measure of risk aversion; more details can be found in a self-contained supplemental data appendix. In CC's external habit model, (logarithmic) risk aversion is a negative affine function of the log "consumption surplus ratio," which in turn is aggregate consumption minus the "habit stock" divided by consumption. As aggregate consumption moves closer to the habit stock (as would happen in recessions), aggregate risk aversion increases. CC model the surplus ratio as a heteroskedastic autoregressive process, with its shocks perfectly correlated with consumption shocks. We use data on nondurables and services consumption growth and CC's parameters and model to create an empirical proxy for risk aversion. The resulting measure is clearly counter-cyclical.

In BY, it is the heteroskedasticity in consumption growth itself that leads to time-variation in risk premiums. BY introduce two latent variables, a time-varying mean for consumption growth, and time-varying volatility for consumption (and dividend) shocks. The volatility process follows an AR(1) process. In one robustness exercise, we literally use BY 's model, parameters and US consumption data to filter out an economic uncertainty process. However, there are more direct measures of economic uncertainty available using the Survey of Professional Forecasters that do not rely on consumption data or a specific ARIMA model. As detailed in the supplemental data appendix, for our benchmark specification, we combine information from a survey about the probability of a recession the next quarter and from the dispersion across respondents about next year's real GDP growth. The potential usefulness of real uncertainty is foreshadowed in the work of Hasbrouk (1984). He shows that the negative relationship between inflation and stock returns cannot be explained by survey forecasts of economic activity (casting doubt on Fama's proxy hypothesis), but is explained by measures of forecaster dispersion. He then surmises such result is due to the role of uncertainty in capturing discount rate variation.

In a recent article by Bekaert et al. (2009), both economic uncertainty and risk aversion drive equity risk premiums. However, in their model, risk aversion is imperfectly correlated with fundamentals. For our exercise here, it is important to keep the rational part of the equity premium tied to fundamentals. Therefore, we parse $ey_t^{erp}$ into two components: one spanned-by and one orthogonal-to the vector $[ra_t, vr_t]$. Fig. 2 plots $ra_t$ and $vr_t$. Because this vector is a subset of the information variable vector in the VAR, $x_t$, $ey_t^{erp}$ can be decomposed into the two orthogonal pieces without any further estimation. Conceptually, the process is analogous to running a regression of $ey_t^{erp}$ on $ra_t$ and $vr_t$ and interpreting the regression residual as the orthogonal component, denoted $ey_t^{erp-re}$; that is,

$$ey_t^{erp-sp} = \beta^{erp\prime}[1, ra_t, vr_t]$$

$$ey_t^{erp-re} = ey_t - ey_t^{erp-sp}, \tag{21}$$

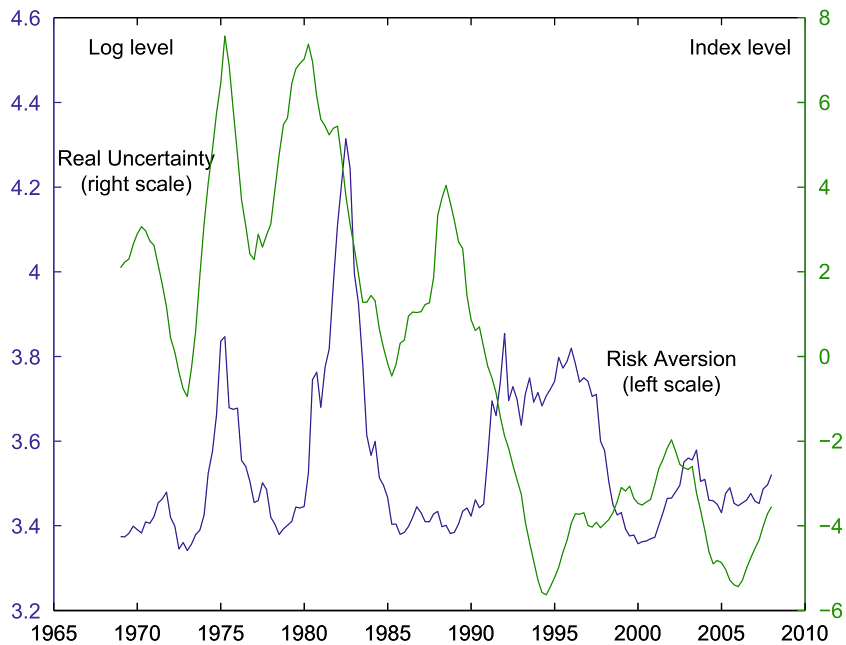


**Fig. 2.** Risk aversion and real uncertainty. This figure plots time series for risk aversion, $ra_t$ (left scale), and real uncertainty, $vr_t$ (right scale). Data construction is described in the supplemental appendix.

where the coefficients, $\beta^{erp}$ are given under OLS as,

$$\beta^{erp} = E([1,ra_t,vr_t][1,ra_t,vr_t]')^{-1}E(ey^{erp}[1,ra_t,vr_t]')$$

and the two unconditional expectations that comprise the coefficients are readily calculated from the VAR. With this additional decomposition, there are now six potential components to the covariance between the equity premium component of stock yields and bond yields,

$$COV(ey_t^{erp},by_t) = COV(ey_t^{erp-sp},einf_t)+COV(ey_t^{erp-re},einf_t)+COV(ey_t^{erp-sp},rrf_t)+COV(ey_t^{erp-re},rrf_t) \\ +COV(ey_t^{erp-sp},irp_t)+COV(ey_t^{erp-re}, irp_t) \tag{22}$$

If money illusion were present in the data, the covariance between the equity yield, measured as a residual, and expected inflation, $COV(ey_t^{erp-re},einf_t)$ would be positive because all the other covariances with expected inflation are constructed in a manner consistent with rational pricing.

### 2.5. Cash flow expectations

Our model for cash flow expectations is much richer than the models featured in CC and BY. All the variables in the VAR, including realized and expected earnings growth, can affect expected future dividends. We allow these effects for several reasons. First, in our decomposition we measure cash flow expectations directly and must make sure we have predictive power for future dividends. Both realized and expected earnings growth are helpful in this respect. In an supplemental appendix table, we report regressions of one quarter and one year dividend growth on these variables, finding significant coefficients for at least one of the variables in each regression and joint significance at about the 10 percent level. Having a reasonable model for cash flow expectations is also helpful in distinguishing Fama's proxy hypothesis from our interpretation of the data. If Fama is correct, inflation may be negatively correlated with real future activity when stagflations dominate the data and the correlation between equity yields and inflation really reflects a link between equities and future real activity. In our decomposition, the proxy hypothesis effect can be measured using the covariance between expected inflation, $einf_t$ and, $ey_t^{\Delta d}$.

Second, we can use our framework and the difference between "subjective" and "objective" cash flow forecasts to cast some direct doubt on "money illusion" as an alternative interpretation of the data. We compute the equity premium residual assuming that agents use "correct" objective cash flow forecasts from the VAR. However, some descriptions of money illusion suggest that the effect comes through incorrect subjective cash flow predictions by market participants which are correlated with inflation expectations. Of course, in our VAR system, subjective errors in cash flow forecasts would end up in the "residual," the equity premium, and if not related to $ra_t$ and $vr_t$, they will still be attributed to the residual component of the equity premium, $ey_t^{erp-re}$. To shed light on whether a subjective bias in cash flow expectations is related to the variation in equity yields and expected inflation, we use our VAR to estimate the bias and then check for comovement of the bias with inflation and equity yields. Specifically, the subjective bias in profit expectations is the difference between the subjective measure of real profit expectations, $gern_t^{su}$ and an objective growth estimate under the VAR, $gern_t^{ob}$, at the same horizon (four quarters). The latter is readily calculated using VAR mathematics because we include realized real earnings growth, $\Delta ern_t$, as an element of the information vector in the VAR, $x_t$. Because the subjective earnings expectations measure predicts four-quarter earnings growth, and because the VAR uses quarterly data, the relevant expectations are (ignoring constant terms):

$$gern_t^{ob} = e'_{\Delta ern}(A+A^2+A^3+A^4)Y_t. \tag{23}$$

The subjective bias is then,

$$bias_t = gern_t^{su}-gern_t^{ob} \tag{24}$$

which is clearly affine in $Y_t$ given that $gern_t^{su}$ is also in the information vector, $x_t$. If this bias is not significantly related to either equity yields or expected inflation, it is hard for money illusion to play a major role in explaining equity–bond yield correlations.

## 3. VAR results

This section first briefly discusses the data and the estimation methodology. We then move to the main results regarding the equity–bond yield correlations.

### 3.1. Data and empirical methods

The VAR uses quarterly data extending from the fourth quarter of 1968 through the end of 2007. The data are described in detail in the supplemental appendix. Here we give a short overview. The bond yield is the yield to maturity on a nominal

**Table 1**
VAR statistics.

Panel A: VAR feedback parameters

| | $einf_{t-1}$ | $rrf_{t-1}$ | $\Delta d_{t-1}$ | $ra_{t-1}$ | $vr_{t-1}$ | $\Delta ern_{t-1}$ | $gern_{t-1}$ | $ey_{t-1}$ | $by_{t-1}$ |
|---|---|---|---|---|---|---|---|---|---|
| $einf_t$ | 0.9628 | −0.3155 | −0.0026 | −0.0013 | −0.0009 | 0.0019 | 0.0011 | 0.0007 | 0.0538 |
| | (20.33) | (2.94) | (1.43) | (3.61) | (0.44) | (2.59) | (1.06) | (3.76) | (1.52) |
| $rrf_t$ | 0.0413 | 0.8403 | −0.0020 | 0.0002 | −0.0006 | −0.0003 | −0.0004 | −0.0002 | 0.0247 |
| | (1.25) | (11.23) | (1.63) | (0.80) | (0.40) | (0.58) | (0.61) | (1.28) | (1.00) |
| $\Delta d_t$ | 2.1129 | 0.3139 | −0.3921 | −0.0288 | −0.1419 | 0.0542 | 0.0110 | 0.0052 | −0.4203 |
| | (1.15) | (0.07) | (5.36) | (2.11) | (1.72) | (1.80) | (0.26) | (0.73) | (0.29) |
| $ra_t$ | 6.5293 | −17.978 | −0.0087 | 0.8890 | −0.3999 | 0.0372 | −0.0516 | 0.0091 | 6.0400 |
| | (1.77) | (2.12) | (0.06) | (31.77) | (2.40) | (0.65) | (0.63) | (0.60) | (2.18) |
| $vr_t$ | −0.1903 | −3.4005 | −0.0143 | −0.0093 | 0.9711 | 0.0065 | −0.0413 | 0.0057 | 0.6250 |
| | (0.31) | (2.38) | (0.59) | (2.07) | (35.2) | (0.69) | (2.98) | (2.39) | (1.33) |
| $\Delta ern_t$ | −1.0339 | 9.8396 | −0.1741 | −0.0247 | 0.0796 | 0.0233 | 0.2574 | 0.0116 | −4.0855 |
| | (0.19) | (0.78) | (0.83) | (0.63) | (0.34) | (0.29) | (2.12) | (0.58) | (0.99) |
| $gern_t$ | 0.6332 | 21.008 | 0.0720 | 0.0907 | 0.2400 | −0.1112 | 0.6473 | −0.0274 | −7.1636 |
| | (0.39) | (5.29) | (1.05) | (7.35) | (3.24) | (4.11) | (16.83) | (4.33) | (5.47) |
| $ey_t$ | 8.2109 | 2.2792 | −0.2389 | −0.1096 | 0.0172 | 0.1769 | 0.1893 | 0.9192 | 0.3415 |
| | (1.47) | (0.18) | (1.09) | (2.63) | (0.07) | (2.01) | (1.49) | (41.71) | (0.08) |
| $by_t$ | 0.3561 | 0.7577 | −0.0026 | −0.0005 | −0.0008 | 0.0004 | −0.0040 | −0.0006 | 0.6408 |
| | (3.30) | (3.06) | (0.61) | (0.61) | (0.15) | (0.23) | (1.63) | (1.44) | (7.84) |

Panel B: VAR lag length tests

| | $VAR(1)$ | $VAR(2)$ | $VAR(3)$ | $VAR(4)$ |
|---|---|---|---|---|
| BIC | −73.5 | −72.3 | −70.7 | −70.0 |
| AIC | −75.2 | −75.7 | −75.7 | −75.6 |

Panel C: Cumby–Huizinga tests ($p$-values)

| | $VAR(1)$ | $VAR(2)$ |
|---|---|---|
| $einf_t$, | 0.44 | 0.42 |
| $rrf_t$ | 0.01 | 0.28 |
| $\Delta d_t$ | 0.03 | 0.04 |
| $ra_t$ | 0.00 | 0.05 |
| $vr_t$ | 0.00 | 0.77 |
| $\Delta ern_t$ | 0.38 | 0.36 |
| $gern_t^s$ | 0.03 | 0.00 |
| $ey_t$ | 0.79 | 0.85 |
| $by_t$ | 0.69 | 0.46 |

Results in this table are based on the observable VAR, $W_t=\mu^w+A^wW_{t-1}+\Sigma^w\varepsilon_t$, where $W_t=[einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$ and $x_t=[ra_t, vr_t, \Delta ern_t, gern_t^s]'$. The endogenous variables are: $einf_t$ is survey-expected inflation, $rrf_t$ is the real short rate, $\Delta d_t$ is dividend growth, $ey_t$ is the equity yield, $by_t$ is the bond yield, $ra_t$ is risk aversion, $vr_t$ is survey macroeconomic uncertainty, $\Delta ern_t$ is earnings growth, $gern_t^s$ is survey expected growth. Panel A reports the estimated parameters, $\hat{A},^w$ with $t$-ratios based on bootstrapped standard errors below in parentheses. Panel B presents information criteria for optimal VAR lag length. The row labeled BIC contains standard Schwartz test results and the row labeled AIC reports results for the Akaike test. In Panel C, $p$-values for Cumby–Huizinga (1992) tests for residual autocorrelation are presented. Each VAR equation is tested separately. We test for autocorrelation at up to four lags.

10 year US Treasury bond.[5] As a proxy for the real rate, we use the estimate for the 5 year zero coupon real rate provided in Ang et al. (2008). As is well known, real term structures are relatively flat at longer maturities so that this maturity is a reasonable proxy for a coupon bond with duration significantly lower than 10 years. There is a voluminous literature on inflation forecasting, but recent work by Ang et al. (2007) strongly suggests that professional surveys provide the best out-of-sample forecasts of inflation. Therefore, we use a proxy for inflation expectations from the Survey of Professional Forecasters (SPF), the four-quarter ahead, four quarter expected change in the GDP deflator. The availability of the SPF data determines the starting point for our sample; Section 5 considers several robustness checks to the measurement of real rates and inflation expectations.

Our analysis uses standard equity data, which represent information on the S&P500 Index. In our base results, cash flows (dividends) do not include repurchases, but results with an adjusted measure are included as a robustness check in Section 5. Consequently, real earnings, dividend growth and the equity yield all refer to the S&P500 Index. Subjective expectations regarding earnings growth are also extracted from the SPF.

[5] While the coupon bonds on which these yields are based have a roughly stable maturity, their duration naturally varies over time. We can roughly gauge the degree of this variation under some simplifying assumptions: If (1), the bonds pay semi-annual coupons, and (2) trade at par, then the bonds' duration is function of yield alone. These calculations yield a Macaulay duration series for the bonds that has a mean of around 7.5 years and a standard deviation of about 0.8 years.

Finally, the empirical proxies for ''fundamental risk aversion'' and for economic uncertainty, described earlier, also use standard data sources. We use CC's risk aversion specification together with nondurables and services consumption data from the NIPA tables, starting the process in 1947. Consequently, the effect of initial conditions has died out by the time our main sample starts. We filter the uncertainty measure from the dispersion in SPF forecasts for real GDP growth over the next year and a real volatility measure inferred from the assessments of recession probability by SPF respondents.[6] The procedure is detailed in the supplemental appendix.

The VAR on $W_t$ is estimated by OLS. Table 1 reports some of the VAR parameter estimates and a few specification tests on the VAR residuals (some summary statistics of the 9 endogenous variables are reported in the supplemental Appendix). Panel A reports the estimated feedback parameters. These parameters show, not surprisingly, that many of our variables are highly persistent. Nevertheless a number of strong cross relations emerge as well. For example, the real rate is not only a significant predictor of bond yields, but also of expected inflation, uncertainty and earnings expectations.

Panel B reports the standard Schwarz (BIC) and Akaike (AIC) criteria. The BIC criterion clearly selects a first-order VAR whereas the AIC criterion selects a second-order VAR. Panel C reports Cumby-Huizinga (1992) tests on the residuals of a first and second-order VAR for each variable separately, using the first four autocorrelations for the test. While the selection criteria in Panel B suggest that a VAR(1) adequately describes the dynamics of the data, the Cumby-Huizinga tests in Panel C suggest some serial correlation remains with a first-order VAR and that a second-order VAR may be more appropriate. Nevertheless, given the length of our sample, we use a first-order VAR as the benchmark specification and consider a second order VAR only as a robustness check.

Our data sample is comprised of 157 quarterly observations of a nine-variable vector. In addition to the 9 unconditional means, the first-order VAR feedback matrix, $A^w$, has 81 elements and the innovation covariance matrix, $\Omega^w$, has 45 distinct elements. The ''saturation ratio,'' or the ratio of the number of the total number of data points to the number of estimated parameters, is thus $(157\cdot 9)/(9+81+45)=10.5$. This is satisfactory but suggests many VAR coefficients may not be statistically significant. To make sure our results are not due to over-fitting, a robustness check in Section 5 considers VARs with insignificant coefficients zeroed out and smaller VARs.

Because all of the statistics of interest for this exercise are functions of the VAR parameters, it is possible to derive standard errors for them using the parameter standard errors and the delta method. However, there are many reasons to suspect asymptotic theory may not work well in this context: some of the variables are very persistent, the saturation ratio is not exceedingly large and the residuals are likely to have fat-tailed distributions. Therefore, standard errors are derived from a standard bootstrap procedure, which is further described in the supplemental appendix. The bootstrap procedure yields 90 percent confidence intervals for all our statistics.

### 3.2. Main results

Table 2 contains the main results. In Panel A, the top line simply reports the variance of the bond and equity yields, their covariance and their correlation. The heart of the puzzle is that the correlation between $ey_t$ and $by_t$ is 77 percent. Under the VAR point estimates, a (bootstrapped) 90 percent confidence interval for this correlation ranges from 64 to 86 percent. This illustrates the Fed Model puzzle because, as shown under the variance decompositions for the two yields, 54 percent of the variance of the bond yield is driven by expected inflation, whereas 76 percent of the variation of the equity yield is driven by the equity risk premium.[7]

Let us first comment on the realism of the variance decompositions. That discount rate variation is the dominant source of equity yield variation is by now well accepted (see Cochrane, 1992). Nevertheless, different theoretical models imply starkly different predictions. The CC model has no predictable cash flow variation, so that dividend yield variation is *entirely* driven by discount rate variation. The persistent time-varying mean for consumption (and dividend) growth in BY naturally implies that cash flows constitute a more important fraction of equity yields variation, with the BY article claiming a roughly 50–50 split. Models that fit the data more closely such as Bekaert et al. (2009) imply that discount rate variation dominates. For bonds, it is generally accepted that expected inflation is a dominant source of bond yield variation, although concrete estimates are actually hard to find. Ang et al. (2008) report that 71 percent is accounted for by expected inflation, and indirect estimates by Mishkin (1990) and many others also suggest expected inflation is the dominant source of bond yield variation, especially at longer horizons. Again, our estimates are consistent with the extant literature.

With the equity premium the main driver of equity yields and expected inflation the main driver of bond yields, for the yields to comove so strongly, expected inflation, a nominal concept, must correlate highly with the equity premium, a real concept. This fact is confirmed in the covariance decomposition on the right side of Panel A. More than half of the comovement comes from the positive correlation between expected inflation and the equity premium. The other two relatively large contributors are the covariance between the real rate and the equity premium, which is positive and contributes 17 percent to the $ey_t-by_t$ covariance, and the covariance between expected inflation and the cash flow component of the equity yield, which contributes 16 percent. The latter effect implies that expected inflation is on average

[6] Giordani and Soderlind (2003) show that such disagreement measures are good proxies for uncertainty.

[7] Note that when we use the concept of ''equity premium'' here, we refer to the summation of current and (expected) future equity premiums, as defined in Eq. (17).

**Table 2**
US VAR results.

Panel A: Decomposing yield (co-)variation

| | $STD(by_t)^*$ | $STD(ey_t)$ | $COV(by_t, ey_t)^*$ | $CORR(by_t, ey_t)$ |
|---|---|---|---|---|
| | 0.61 | 0.44 | 0.21 | 0.77 |
| | (0.44,0.70) | (0.30,0.53) | (0.09,0.30) | (0.64,0.86) |

Fractional contributions

| | $VAR(by_t)$ | | $VAR(ey_t)$ | | $COV(by_t,ey_t)$ | | |
|---|---|---|---|---|---|---|---|
| | | | | | $einf_t$ | $rrf_t$ | $irp_t$ |
| $einf_t$ | 0.54 | $ey_t^{\Delta d}$ | 0.18 | $ey_t^{\Delta d}$ | 0.16 | 0.02 | $-0.01$ |
| | (0.43,0.64) | | (0.07,0.28) | | (0.09,0.25) | $(-0.01,0.05)$ | $(-0.07,0.03)$ |
| $rrf_t$ | 0.23 | $ey_t^{rrf}$ | 0.08 | $ey_t^{rrf}$ | 0.07 | 0.02 | 0.00 |
| | (0.21,0.25) | | (0.06,0.10) | | (0.06,0.09) | (0.01,0.03) | $(-0.01,0.01)$ |
| $irp_t$ | 0.23 | $ey^{erp}$ | 0.76 | $ey^{erp}$ | 0.55 | 0.17 | 0.04 |
| | (0.15,0.34) | | (0.63,0.88) | | (0.40,0.66) | (0.13,0.21) | $(-0.03,0.16)$ |

Panel B: Decomposing $ey_t^{erp}$ into $ey_t^{erp-sp}$ and $ey_t^{erp-re}$
Fractional contributions

| | $VAR(ey_t)$ | | $COV(by_t,ey_t)$ | | |
|---|---|---|---|---|---|
| | | | $einf_t$ | $rrf_t$ | $irp_t$ |
| $ey^{erp-sp}$ | 0.49 | $ey_t^{erp-sp}$ | 0.47 | 0.11 | 0.00 |
| | (0.33,0.59) | | (0.33,0.59) | (0.06,0.14) | $(-0.07,0.06)$ |
| $ey^{erp-re}$ | 0.26 | $ey^{erp-re}$ | 0.08 | 0.06 | 0.04 |
| | (0.17,0.46) | | (0.01,0.14) | $(-0.01,0.12)$ | $(-0.01,0.16)$ |

Panel C: Equity yields, expected inflation and subjective earnings expectations biases

Correlations

| | |
|---|---|
| $einf_t - bias_t$ | 0.53 |
| | (0.15,0.63) |
| $einf_t - ey_t$ | 0.84 |
| | (0.71,0.89) |
| $bias_t - ey_t$ | 0.34 |
| | $(-0.02,0.53)$ |

Fractional contributions to $einf_t - ey_t$ Covariance

| $ey_t^{\Delta d}$ | $ey_t^{rrf}$ | $ey^{erp-sp}$ | $ey^{erp-re}$ |
|---|---|---|---|
| 0.20 | 0.09 | 0.61 | 0.10 |
| (0.11,0.33) | (0.08,0.12) | (0.48,0.72) | (0.01,0.18) |

Results in this table are based on the latent VAR, $Y_t = \mu + AY_{t-1} + \Sigma\varepsilon_t$, where $Y_t = [einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x_t]'$ and $x_t = [ra_t, vr_t, \Delta ern_t, gern_t^s]'$, $\varepsilon \sim (0, I)$ and $irp_t$ and $erp_t$ are the unobserved variables, the inflation risk premium and the equity risk premium, respectively. The $Y_t$ system parameters are derived from VAR estimates on the observable vector $W_t = [einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t]'$ using the data and methodology described in Section 2 and the supplemental appendix. The endogenous variables are: $einf_t$ is survey-expected inflation, $rrf_t$ is the real short rate, $\Delta d_t$ is dividend growth, $ey_t$ is the equity yield, $by_t$ is the bond yield, $ra_t$ is risk aversion, $vr_t$ is survey macroeconomic uncertainty, $\Delta ern_t$ is earnings growth, $gern_t^s$ is survey expected growth. $ey_{t,}^{erp} ey_{t,}^{\Delta d} ey_{t,}^{rrf}$ are the equity yield components due to the equity risk premium, dividend growth, and the real short rate respectively. $bias_t$ is the difference between four-quarter earnings growth expectations under the VAR minus $gern.^s$

The procedure for decomposing $ey_t$ and $by_t$ into their component pieces (e.g. $ey_t^{\Delta d}$ for $ey_t$, and $rrf_t$ for $by_t$) is described in Section 2 as is the procedure for decomposing $ey_t^{erp}$ into parts spanned-by and orthogonal-to proxies of rational equity risk premiums. Bootstrapped 90 percent confidence intervals are reported in parentheses. $*$ denotes that the reported statistic has been multiplied by 100 for readability.

positively correlated with periods of low cash flow expectations, as the cash flow component of the equity yield is negatively related to cash flow projections. This in itself already suggests that above-average inflation in the US has occurred often at times of depressed earning (and dividend) expectations. This effect is of course closely related to the "proxy hypothesis" of Fama (1981) and Kaul (1987), and shows that while it definitely plays a role, its explanatory power is rather limited. Finally, expected inflation and the real rate are positively correlated, which contributes 7 percent to the comovement between the bond and equity yield. While this number is small, it is relatively precisely estimated. This result is inconsistent with the well-known Mundell–Tobin effect, which suggests a negative relation. However, our measures here are long-term (proxying for a 5–10 year horizon) and Ang et al. (2008) also find a positive correlation between expected inflation and long-term real rates.

The last row of the covariance decomposition matrix reveals that 76 percent of the comovement between equity yields and bond yields comes through the equity premium, a residual in the equity yield decomposition. While it is tempting to conclude that irrational forces are at work, the next panel suggests otherwise. In Panel B, we decompose the equity yield

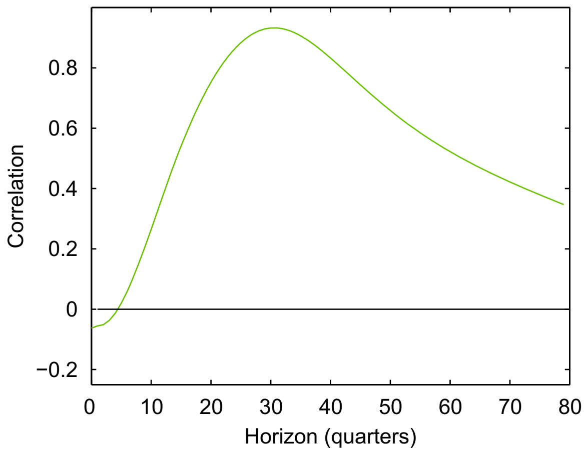


**Fig. 3.** Term structure of correlation between expected excess equity returns and expected inflation. This chart plots $corr(E_t[erp_{t+j}], einf_t)$ as a function of $j$=0,…80. Results in this table are based on the latent VAR, $Y_t=\mu+AY_{t-1}+\Sigma\varepsilon_t$, where $Y_t=[einf_t, rrf_t, \Delta d_t, erp_t, irp_t, x_t']'$ and $x_t=[ra_t, vr_t, \Delta ern_t, gern_t^s]'$, $\varepsilon\sim(0,I)$ and $irp_t$ and $erp_t$ are unobserved. The endogenous variables are: $einf_t$ is survey-expected inflation, $rrf_t$ is the real short rate, $\Delta d_t$ is dividend growth, $ey_t$ is the equity yield, $by_t$ is the bond yield, $ra_t$ is risk aversion, $vr_t$ is survey macroeconomic uncertainty, $\Delta ern_t$ is earnings growth, $gern_t^s$ is survey expected growth. $ey_{t,}^{erp} ey_{t,}^{\Delta d} ey_{t,}^{rrf}$ are the equity yield components due to the equity risk premium, dividend growth, and the real short rate respectively. $bias_t$ is the difference between four-quarter earnings growth expectations under the VAR minus $gern_t^s$. The $Y_t$ system parameters are derived from VAR estimates on the observable vector $W_t=[einf_t, rrf_t, \Delta d_t, ey_t, by_t, x_t']'$ using the data and methodology described in Section 2 and the supplemental appendix.

into a part spanned by risk aversion and uncertainty and an unspanned part. Note that the spanned part represents more than 65 percent of the total variation in the equity premium (49/(49+26)); within the spanned part, the contributions of risk aversion and uncertainty are not statistically different from one another, with risk aversion accounting for 42 percent and uncertainty for the remainder of the variation. More importantly, 76 percent of what the equity premium explains of the total $ey_t-by_t$ covariance comes from the spanned, rational part.[8] If we focus on $COV(ey_t^{erp}, einf_t)$, the expected inflation component, about 85 percent (47/55) can be ascribed to the rational component, $COV(ey_t^{erp-sp}, einf_t)$ with the rest, potentially, coming from money illusion.

Panel C reports the comovements among equity yields, expected inflation, and the subjective earnings bias. On the left side, the subjective earnings bias is seen to be positively correlated with both the equity yield and expected inflation. While a positive correlation with the equity yield suggests that subjective bias may indeed drive some variation in equity valuations, the positive correlation of the bias with inflation is contrary to the prediction of money illusion theories. For example, money illusion suggests that the earnings bias should be negative (positive) when expected inflation is relatively high (low). Still, equity yields *are* highly correlated with expected inflation; the correlation is 84 percent. On the right hand side of Panel C, we decompose this comovement because the Fed model puzzle essentially is due to the high correlation between expected inflation and equity premiums. The Panel shows that 9 percent of their comovement comes from the positive comovements of real rates and expected inflation, 20 percent of the comovement can be ascribed to the negative correlation between expected inflation and (rational) cash flow expectations, but a whopping 61 percent can be ascribed to the fact that risk aversion and uncertainty are high in times of high expected inflation. The unexplained residual is a paltry 10 percent, which severely limits the potential role of money illusion.

Given previous results in the literature, our findings are perhaps surprising. For example, Campbell and Vuolteenaho (2004, CV henceforth) perform a closely related VAR-based analysis and interpret their findings as clearly suggestive of money illusion. How can their results be so different from ours? There are four main reasons. First, CV treat equity cash flows as residuals. All unexplained variation is hence assigned to cash flow variation. In contrast, we attempt to measure cash flows directly and leave the equity premium as the residual component. Second, CV measure the equity risk premium with a variable due to Cohen et al. (2005) that may be subject to considerable measurement error and is not, to date, widely used in the literature. Third, CV work directly in terms of excess returns, and therefore ignore one potentially important rational source of common variation in the two yield variables: real rates. Our results in Table 2 indicate that they therefore "miss" about 20 percent of the comovement between equity and bond yields. Finally, subsequent research has found that CV's results are not robust to the post-war subsample used in this study (Joutz and Wei, 2007).

Finally, the positive correlation between the "equity premium piece" of the equity yield and expected inflation may also appear, at first glance, inconsistent with an older literature showing that expected equity returns and (expected) inflation are negatively correlated, see Fama and Schwert (1977) and Fama (1981). However, our results are entirely consistent with the literature. What we call the "equity premium" for short is the sum of the current equity premium and all future premiums necessary to discount future cash flows (see the definition of $ey_t^{erp}$ after Eq. (17)). Fig. 3 shows the correlations of

[8] Calculated as the sum of the first line in Panel B divided by the sum of the last line in Panel A (58/76).

the components of this sum with expected inflation. At lag 0, the correlation between expected inflation and the current equity premium is indeed negative, and this is the finding stressed in the extant literature. However, the correlation between expected inflation and expected future equity premiums quickly turns positive and the sum of all these components correlates positively with expected inflation. It is also interesting to note that the negative short-term correlation is driven by the part of the equity premium not spanned by $ra_t$ and $vr_t$, both of which correlate positively with expected inflation for our US sample (see Table A2 in the supplemental appendix). These findings suggest that the Fed model may work less well at medium and short horizons. This is indeed the case. When regressing equity yields on short, medium and long-term bond yields, only the long bond yield is significant.

## 4. International results

We first motivate why it can be useful to examine international data and comment on our data sources. Then, we demonstrate how the cross-sectional variation in the correlation between bond and equity yields actually confirms our main hypothesis: high correlations stem from the incidence of periods in which high inflation and recessions (which drive up risk premiums) coincide.

### *4.1. Motivation*

Our work analyzes one US based data set, with one history of inflation, bond yields and equity yields. Using this data set alone, it is hard to definitively exclude the money illusion story in favor of our story. International data offer an interesting out-of-sample test of our hypothesis. The essence of our argument is that the US experienced high correlations between equity yields and bond yields because higher inflation happened to occur during recessions, so that in recessions equity and bond premiums are both relatively high. In other words, the Fed model "works" in countries with a high incidence of stagflation.

Estrada (2009) shows that there is indeed substantial cross-sectional variation in the strength of the correlation between bond and equity yields across countries. He focuses on statistical problems in interpreting the correlations in a panel of international data. We now explore the possibility that "stagflation incidence" accounts for part of the cross-sectional variation in stock-bond yield correlations using data similar to the Estrada sample. Specifically, we collect four variables for 20 countries over the period from December 1987 to June 2005: dividend yields, ($ey_{i,t}$), bond yields ($by_{i,t}$), inflation rates, ($infl_{i,t}$), and recession dummy indicators ($recess_{i,t}$). The dividend yield data are provided by Thomson for each country's equity index. The measure is not perfectly available, but 97 percent of all possible country-months are populated. The bond yield is a long term local currency nominal bond yield from Thomson. The inflation rate is reported by the local governments. Where available, it is the continuously compounded change in the CPI index. If no such series is available for a particular country, the GDP deflator is used. If this variable is available only quarterly, we divide the quarterly inflation rate by three and use repeated values for months in that quarter. Finally, the recession indicator is published by the Economic Cycle Research Institute, which provides monthly indicator series for the incidence of recession. Where recession indicators are not available (eight countries and in 2005 for all countries), recessions are defined as two consecutive quarters of negative real GDP growth.

### *4.2. Cross-country analysis*

We start with a heuristic analysis of the cross-sectional association between "Fed model effect intensity" and "stagflation intensity." To capture the intensity of the Fed model effect, we compute the time series correlation between the dividend yield and the nominal long bond yield for each country. To measure the intensity of stagflation for a country, we similarly compute the time series correlation of the recession indicator with inflation for each country. Fig. 4 plots each country along these two dimensions. Although there are only 20 country observations, a positive relationship seems evident. In fact, the cross-sectional correlation between Fed model intensity and stagflation intensity on this plot is 0.50, and significant at the 5 percent level (not accounting for the sampling uncertainty in the time series correlations). Moreover, a cross sectional OLS regression of Fed model intensity on stagflation intensity produces a positive slope coefficient of 1.35 which is also significant at the 5 percent level (again, not accounting for the sampling uncertainty in the time series correlations). The significance of the slope coefficient is robust to the (sequential) exclusion of Japan and Austria, potential outliers. Hence, these results are supportive of a positive relationship. The relationship exists even though the US itself has not exhibited stagflation in the post-1987 sample while retaining a high $by_t - ey_t$ correlation. That the stock and bond yield correlation is actually negative in Japan, perhaps the best known "deflation-rich" country, is also a nice corroboration of our interpretation of the data.

We add more statistical formality to this analysis by estimating two sets of cross-sectional regressions with the cross-section of countries' stock-bond yield correlations as the dependent variable. The results for both sets of regressions are reported in Table 3. The first regression set (numbers on the left of the table) focuses on the incidence of stagflation, defined as the percent of observations where a recession occurs simultaneously with high inflation. Our cut off value for high inflation is 10 percent, but an analysis using an inflation level of 5 percent as the cut-off yields largely

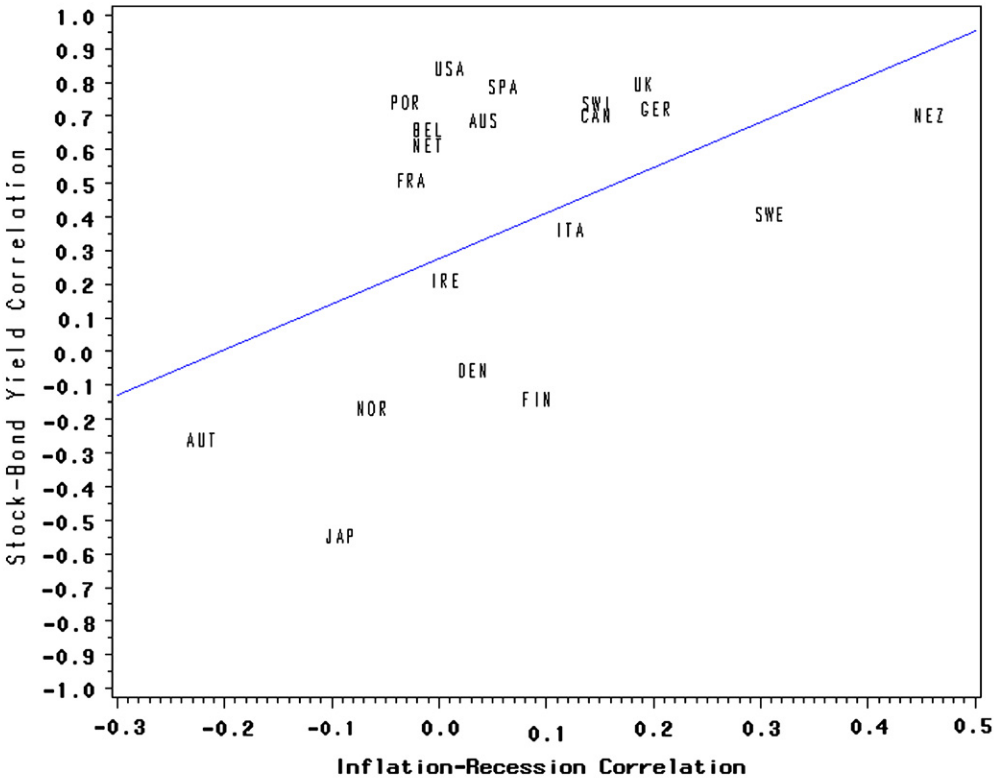


**Fig. 4.** Multi-country relationship between stagflation and the fed model. This figure plots countries in the panel data set along two dimensions: (1) the country specific time–series correlation between the dividend yield and the long term (locally risk free) nominal bond yield, and (2) the time series correlation between inflation and a recession indicator. The sample is monthly from December 1987 through June 2005. The slope of the regression line is 1.35 with an OLS standard error of 0.59. A regression (line not shown) estimated excluding the Japan (Austria) observation has a slope of 1.04 with an OLS standard error of 0.54 (1.10 with a standard error of 0.66).

similar results. Regression (3) shows that stagflation by itself has a huge effect on the equity–bond yield correlation: a country with 1 percent higher stagflation incidence than the average has a 21 percent point higher equity–bond yield correlation. Of course, the stagflation effect could be due to its separate components, recession or simply inflation. Regressions (1) and (2) show that the percent of high inflation months by itself does increase the equity yield–bond yield correlation whereas a high incidence of recessions actually reduces it, but the latter effect is not significant. Regression (4) includes all three dependent variables in one regression. This regression provides a nice test of our stagflation story versus just money illusion. If money illusion drives the correlation, the coefficient on inflation should be significant, but there is little reason for stagflation to have a particular effect on the bond–equity yield correlation. However, inflation has an insignificant effect on the correlation. The recession effect is still negative but not significant, and the stagflation effect is large and significantly different from zero. While the associated $t$-statistic is large, the regression suffers from three econometric problems. First, the sample is small (20 observations). Second, the regressors and regressands involve pre-estimated statistics. Third, the different observations arise from correlated time series. Therefore, we conduct a Monte Carlo analysis, described in detail in the supplemental appendix, and generate a small sample distribution for the $t$-statistics in the regressions. This Monte Carlo analysis uses the asymptotic variance–covariance matrix for estimating the independent and dependent variables in the regression to draw new regression variables and it imposes the null hypothesis of no cross-sectional dependence. Significant $t$-statistics according to the small sample distribution are indicated with asterisks. The stagflation coefficient remains significant when using the small sample distribution for the $t$-statistics.

The second set of regressions, replace “high inflation incidence” by average inflation, and “stagflation” by the interaction of inflation and the recession indicator. The univariate regression, Regression (5), reveals that countries with high average inflation do have significantly higher equity yield–bond yield correlations, but when this variable is added to a regression that includes the inflation–recession interaction, Regression (7), the direct effect of inflation disappears. The inflation–recession interaction comes in very significantly and the significance survives at the 5 percent level under the small sample distribution. The direct effect of the frequency of recessions continues to be negative but is insignificant.

**Table 3**
Cross-country results.

| Specification | $hinfl_i^{percent}$ | $recess_i^{percent}$ | $stag_i^{percent}$ | $\overline{infl}_i$ | $\overline{infl\_rec}$ | $R^2$ |
|---|---|---|---|---|---|---|
| (1) | 3.95<br>(1.10)* | | | | | 0.07 |
| (2) | | −0.40<br>(0.43) | | | | 0.01 |
| (3) | | | 21.37<br>(2.24)** | | | 0.23 |
| (4) | −0.68<br>(0.19) | −1.59<br>(1.70) | 30.52<br>(2.55)*** | | | 0.37 |
| (5) | | | | 3.06<br>(2.74)** | | 0.32 |
| (6) | | | | | 8.78<br>(3.38)*** | 0.41 |
| (7) | | 1.25<br>(0.62) | | −0.50<br>(0.37) | 7.93<br>(1.85)** | 0.52 |

This table presents results for cross-sectional regressions of the general form

$$corr_i(ey_t,by_t) = a + b\,hinfl_i^{percent} + c\,recess_i^{percent} + d\,stag_i^{percent} + u_i \quad (1)$$

and

$$corr_i(ey_t,by_t) = a + b\,\overline{infl_i} + c\,recess_i^{percent} + d\,\overline{infl\_rec} + u_i$$

where $by_t$ is the locally nominally risk free long bond yield for country i at time t and $ey_t$ is the dividend yield. The variable $corr_i(ey_t,by_t)$ is the time-series correlation between $ey_t$ and $by_t$ for country $i$. The variable $hinfl_i^{percent}$ denotes the percentage of observations during which the country exhibited high inflation, defined as 10 percent or more (annualized) inflation per month. The variable $recess_i^{percent}$ denotes the percentage of observations during which the country was in recession (the mean of the binary recession indicator variable $recess_{i,t}$). The variable $stag_i^{percent}$ denotes the percentage of observations during which the country exhibited stagflation, defined as the coincidence of high inflation and recession. The variable $\overline{infl_i}$ denotes the full-sample country-specific mean of inflation, $infl_{i,t}$. The variable $\overline{infl\_rec}$ denotes the country-specific time-series mean of the interaction, $infl_{i,t} \cdot recess_{i,t}$. Data are monthly from 1987–2005 for 20 countries. OLS coefficients and $t$-ratios (in parentheses) are reported. The superscripts *, ** and *** denote significance at the 10, 5, and 1 percent level. Significance is determined using corrections for the small sample and pre-estimation effects of the regressors and regressand utilizing a Monte-Carlo method detailed in the supplemental appendix.

## 5. Robustness checks

The first three sub-sections in this section describe a set of robustness exercises pertaining to our main results in Table 2. The final subsection focuses on the robustness of the international results. Table 4 presents our robustness results. For each alternative specification, we report two salient statistics: the percent contribution of the covariance between expected inflation and the equity premium to the total yield covariation, and the percent contribution of the covariance between expected inflation and the non-spanned, residual part of the equity premium, $erp_t^{re}$.

### 5.1. VAR specification

First, given the VAR specification tests reported earlier, we repeat the analysis using a VAR(2) data generating process. The results of Table 2 are essentially unchanged. Our second and third experiments focus on the fact that with a VAR of large dimension relative to the sample size, insignificant coefficients could affect the statistics of interest. Our bootstrapping procedure for calculating standard errors should address this issue to a large extent, but two exercises directly verify the robustness of the point estimates: zeroing-out any element of $A$ which has an OLS $t$-statistic less than one, and using a smaller VAR excluding the information variables, that is dropping $x_t$. Under both experiments, the results of Panel A of Table 2 are essentially unchanged.

Third, we repeat the analysis while explicitly accounting for the Jensen's inequality term that is implicitly included in our standard measurement of expected equity returns. Note that in a no-arbitrage, log-linear framework, expected real log returns obey:

$$erp_t = -COV_t[m_{t+1},r_{t+1}] - \frac{1}{2}VAR_t[r_{t+1}],$$

where $m_{t+1}$ is the real log stochastic discount factor. Implicitly, our measurement of $erp_t$ lumps together the two terms on the right-hand side, but only the first term would be considered a genuine risk premium. Our approach is to isolate the risk premium component by explicitly measuring the Jensen's inequality component, $-\frac{1}{2}VAR_t[r_{t+1}]$. Our conditional variance proxy is derived by fitting an ARMA(1,1) time series model to a realized volatility series, which is in turn obtained by adding up squared daily returns for each quarter in our sample. This variable is added to our main VAR specification. Finally, we use a modified version of Eq. (17) to identify the two components of $ey_{t:}^{erp}$ one due to the covariance term, and the other due to a Jensen's inequality correction. This procedure shows that the component of $ey_t$ due to the variance term

**Table 4**
US VAR robustness exercises.

| Percent contribution to $ey_t - by_t$ covariance under alternative specifications<br>Specification | $COV(einf_t, ey_t^{ferp})$ | $COV(einf_t, ey_t^{erp-re})$ |
|---|---|---|
| Main VAR | 0.61 | 0.10 |
| | (0.48,0.72) | (0.01,0.18) |
| VAR(2) | 0.58 | 0.06 |
| | (0.24,1.08) | (−0.04,0.25) |
| Small VAR | 0.47 | -NA- |
| | (0.19,1.05) | -NA- |
| Zeroed-out | 0.56 | 0.08 |
| | (0.22,1.14) | (−0.01,0.15) |
| Jensen's correction | 0.60 | 0.10 |
| | (0.15,0.87) | (0.00,0.40) |
| Alternative $\nu r_t^*$ measure | 0.30 | 0.13 |
| | (−0.03,0.69) | (0.00,0.75) |
| w/inflation uncertainty | 0.57 | 0.07 |
| | (0.18,1.11) | (−0.01,0.30) |
| Long-term inflation exp. | 0.47 | 0.08 |
| | (0.19,0.88) | (0.00,0.34) |
| Alternative real rate | 0.58 | 0.08 |
| | (0.15,1.08) | (−0.03,0.28) |
| Cash flow = earnings | 0.42 | 0.10 |
| | (−0.20,1.21) | (−0.20,1.21) |
| Cash flow = div+repo | 0.36 | 0.35 |
| | (−3.79,4.78) | (−1.29,2.37) |

This table reports two key statistics (and their confidence intervals) reported for our main specification in Table 1 under a variety of alternative VAR specifications. The "Main VAR" row simply reproduces the statistics of interest from Table 1: the percent contribution to total $ey_t - by_t$ covariance of $COV(einf_t, ey_t^{erp})$ and $COV(einf_t, ey_t^{erp-re})$ where $ey_t$ is the equity yield, $by_t$ is the bond yield, $einf_t$ is survey-based expected inflation, $ey_t^{erp}$ is the equity premium component due to the equity risk premium, and $ey_t^{erp-re}$ is the component of $ey_t^{erp}$ orthogonal to the risk factors, $ra_t$ and $vr_t$, risk aversion and macroeconomic uncertainty respectively. The "VAR(2)" specification expands the Main VAR to include two lags of all the dependent variables. The "Small VAR" specification drops the $x_t$) vector from the VAR list (without $x_t$, the $COV(einf_t, ey_t^{erp-re})$ contribution cannot be calculated). The "Zeroed-out" specification employs a two-step estimation procedure for our main VAR: first estimate the VAR by OLS, noting all elements of $A^W$ with OLS $t$-statistics less than 1. In the second step, re-estimate the VAR imposing that the low $t$-statistic coefficients are zero. The "Jensen correction" specification corrects our measure of $ey_t^{erp}$ for the component due to the conditional variance of returns. The "alternate $\nu r_t^*$" line replaces the measure of real uncertainty in the observable VAR to a measure filtered from actual consumption growth using the consumption growth model of Bansal and Yaron (2004) and a nonlinear Kalman filter. The "w/inflation uncertainty" specification adds our measure of inflation uncertainty, $\nu\pi_t$, to the information variable vector, $x_t$. The "long-term inflation expectations" specification replaces our usual four-quarter expected inflation measure with a longer-term survey-based inflation expectations measure (see supplemental data appendix). The "alternative real rate" specification assumes we can measure the inflation risk premium component of bond yields directly as proportional to inflation uncertainty, leaving the real rate as the residual component. The "cash flow = earnings" specification replaces the dividend yield and dividend growth in the Main VAR with earnings growth and the earnings–price ratio. The "cash flow = div + repo" specification adds repurchases to dividends before calculating dividend growth and the dividend yield.

is generally very small due to the relatively fast rate of mean reversion of $VAR_t[r_{t+1}]$.[9] None of our main results are impacted by instead measuring $ey_t^{erp}$ as the single component due to the covariance term.

A final robustness exercise uses an alternative economic uncertainty proxy that is directly derived from BY's article (see supplemental appendix). The contribution of the covariance between expected inflation and the equity yield decreases and the relative contribution of the covariance between expected inflation and the residual equity premium increases somewhat. However, this is mostly due to the limited ability of the BY-based uncertainty measure to help span the equity premium component of the equity yield.

### 5.2. Bond yield decomposition

Three exercises verify the robustness of our results to alternative bond yield decompositions. The first exercise adds an additional information variable to the VAR, a measure of inflation uncertainty based on SPF data (using a procedure similar to that which we used for real uncertainty). The second exercise employs a longer-term measure of survey-based inflation expectations (our standard measure looks ahead only four quarters) as the measure of expected inflation. The longer-term measure predicts inflation over 10 years but is not available early in the sample, so we must first filter its early values (see supplemental appendix for a description of this procedure). The third exercise uses a completely different measure of the real rate to measure the inflation risk premium directly—as proportional to inflation uncertainty. Specifically, the real rate is now the nominal rate minus long-term inflation expectations and a constant times inflation uncertainty. The residual is

[9] The quarterly autocorrelation of the fitted series is 0.45, implying that shocks to conditional variance decay by 95 percent within one year.

an alternative real rate measure. The constant of proportionality is chosen such that the unconditional mean of the real rate matches that of our standard measure from Ang et al. (2008).

### 5.3. Cash flow measurement

We consider two alternative measures of the cash flow from equity. Our first exercise uses earnings instead of dividends, both for constructing cash flow growth and calculating the equity yield. Investigating the earnings yield is of considerable interest because practitioners overwhelmingly focus on earnings as the unit of fundamental analysis for equity valuation. The results for earnings-based equity yields are largely consistent with our main results. (1) The stock–bond yield covariance is very high, (2) the majority of the comovement comes through the covariance of the equity yield with expected inflation, and (3) very little of the covariance involves the $ey_t^{erp-re}$ component of the equity yield. One difference from our main results is that the contribution of $COV(ey_t^{\Delta d}, einf_t)$ to the total $ey_t - by_t$ covariance is substantially larger when using earnings rather than dividends, accounting for 41 percent of the covariance versus just 12 percent under our baseline VAR as reported in Table 2. Hence, rather than the covariance between expected inflation and the equity risk premium being the main driver for the stock–bond yield covariance, it is now comovement between expected inflation and expected cash flow growth. This is consistent with Fama's (1981) proxy hypothesis. Nevertheless, even if this is the correct interpretation of the data, stagflation remains a critical ingredient: Inflation happens to occur at times of depressed earnings expectations. Note that our use of objective, not subjective, earnings forecasts, implies that this effect cannot be caused by money illusion.

Our second experiment adds repurchases to dividends in calculating cash flow, because repurchases have been an important channel by which companies have returned cash to shareholders in the past few decades, which could have important asset pricing implications (see Boudoukh et al., 2007). The correlation of the resulting equity yield measure with the bond yield remains positive but it is not statistically significant. This owes to the fact that repurchases have, on a quarterly basis, been extremely volatile, especially over the past few years. The point estimates of our main results are broadly similar to those presented in Table 1, but the estimates of all the $ey_t - by_t$ covariance components are very imprecise and none are individually statistically different from zero. While this is a disappointing result, it is likely similarly due to the excessive volatility of repurchases.

### 5.4. International results

For robustness to our use of dividends as the relevant equity cash flow in the international data, we also conduct the analysis using one year-ahead analyst-expected earnings in calculating the equity yield. This change does not affect the results of Table 2 very much. Finally, because the dependent variable in the cross-sectional regressions are correlations and thus limited to the interval $[-1,1]$, the OLS regressions are repeated using a transformation of the correlation, $\ln(1+corr.)/\ln(2-corr.)$, which effectively spreads the range of the dependent variable to $(-\infty, +\infty)$. The OLS *t*-statistics using this transformation are very similar to those reported in Table 2.

## 6. Conclusion

A careful re-examination of alternative explanations for the surprisingly high correlation between the "real" equity yields and nominal bond yields in US post-war data shows that the prevailing explanation, money illusion, actually has rather limited explanatory power. A large part of this covariation is due to the rather high incidence of stagflations in the US data. We postulate that in recessions economic uncertainty and risk aversion may increase leading to higher equity risk premiums, which, in turn, increase yields on stocks. If expected inflation happens to also be high in recessions, bond yields increase through their expected inflation and, potentially, their inflation risk premium components, and positive correlations emerge between equity and bond yields and inflation. We establish this result using a VAR methodology that uses measures of inflation expectations and two proxies for rational variation in risk premiums, one based on economic uncertainty, one based on the habit model formulated by Campbell and Cochrane (1999). Our confidence in these findings is bolstered by a cross-country analysis that demonstrates that "stagflation incidence" accounts for a significant fraction of the cross-sectional variation in equity–bond yield correlations.

Our findings have potentially important policy implications. If money illusion afflicts pricing in the stock market, inflation stabilization also helps prevent distortions and mis-pricing in the stock market. If money illusion does not affect the stock market, the Federal Reserve's inflation policy has no bearing on the equity market beyond its implications for real economic growth.

To conclude, we note that the 2008–2009 crisis period is consistent with our interpretation of the data. This period witnessed extremely low correlations between equity yields and bond yields. Given our main hypothesis, this is to be expected, as this period experienced a recession (and hence high equity premiums) but coupled with subdued inflation pressures and thus low expected inflation. More generally, one would expect that in periods where inflation is pro-cyclical, the correlation between bond and equity yields should be much lower. One such period, of course, is the Great Depression. While direct evidence on this is lacking, a careful reading of the literature uncovers a number of results consistent with this. Thomas and Zhang (2008) show that the Fed model does not work during the 1915-1960 period, in which stagflations were rare. Kaul (1987) shows that inflation was not negatively correlated with equity returns in the Great Depression years in either the US or Canada. In sum, both cross-sectional (our international study) and intertemporal out-of-sample evidence supports our interpretation of the data.

## Appendix. Supplementary data

Supplementary data and technical appendices associated with this article can be found in the online version at doi:10.1016/j.jmoneco.2010.02.004.

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