How do Regimes Affect Asset Allocation?*

Andrew Ang

Geert Bekaert

Columbia University and NBER

First Version: 12 April, 2002

This Version: 30 April, 2002

JEL Classification: F30, G11, G12

Keywords: market timing, tactical asset allocation, regime switching, international diversification

*We thank Cam Harvey for providing data and Theo Nijman for helpful comments on the research proposal. This research is funded by a grant from INQUIRE Europe. We thank seminar participants at a joint INQUIRE Europe - INQUIRE UK meeting. Andrew Ang: aa610@columbia.edu; Geert Bekaert: gb241@columbia.edu

Abstract

Everyone who has studied international equity returns has noticed the episodes of high volatility and unusually high correlations coinciding with a bear market. We develop quantitative models of asset returns that match these patterns in the data and use them in two quantitative asset allocation analyses. First, we show that the presence of regimes with different correlations and expected returns is difficult to exploit with within a global asset allocation framework focussed on equities. The benefits of international diversification dominate the costs of ignoring the regimes. Nevertheless, for all-equity portfolios, a regime-switching strategy out-performs static strategies out-of-sample. Second, we show that substantial value can be added when the investor chooses between cash, bonds and equity investments. When a persistent bear market hits, the investor switches primarily to cash. This desire for market timing is enhanced because the bear market regimes tend to coincide with periods of relatively high interest rates.

1 Introduction

It has been known for some time that international equity returns are more highly correlated with each other in bear markets than in normal times (see Erb, Harvey and Viskanta, 1994; Campbell, Koedijk and Kofman, 2002). Longin and Solnik (2001) recently formally establish the statistical significance of this asymmetric correlation phenomenon.[1] Whereas standard models of time-varying volatility (such as GARCH models) fail to capture this salient feature of international equity return data, recent work by Ang and Bekaert (2002a) shows that asymmetric correlations are well captured by a regime-switching (RS) model.

RS models build on the seminal work by Hamilton (1989). In its simplest form, a regime-switching model allows the data to be drawn from two or more possible distributions ("regimes"), where the transition from one regime to another is driven by the realization of a discrete variable (the regime), which follows a Markov chain process. That is, at each point of time, there is a certain probability that the process will stay in the same regime next period. Alternatively, it might transition to another regime next period. These transition probabilities may be constant or they may depend on other variables. Ang and Bekaert (2002a) estimate a number of such models on equity returns from the US, Germany and the UK and find that international equity returns appear to be characterized by two regimes: a normal regime and a bear market regime where stock market returns are on average lower and much more volatile than in normal times. Importantly, in the bear market regime, the correlations between various returns are higher than in the normal regime.

Regime switching behavior is not restricted to equity returns: Gray (1996), Bekaert, Hodrick and Marshall (2001) and Ang and Bekaert (2002b and c), among others, find strong evidence of regimes in US and international short-term interest rate data. Short rates are characterized by high persistence and low volatility at low levels, but lower persistence and much higher volatility at higher levels. Again, RS models perfectly capture these features of the data. It also appears that the interest rate and equity regimes are somewhat correlated in time and may be related to the stage of the business cycle.

Surprisingly, quantitative asset allocation research appears to have largely ignored these salient features of international equity return and interest rate data. The presence of asymmetric correlations in equity returns has so far primarily raised a debate on whether they cast doubt on the benefits of international diversification, in that these benefits are not forthcoming when you need them the most. However, the presence of regimes should be exploitable in an active asset

[1] This asymmetric correlation phenomenon is not restricted to international portfolios, see Ang and Chen (2002).

allocation program. The optimal equity portfolio in the high volatility regime is likely to be very different (for example more home biased) than the optimal portfolio in the normal regime. When bonds and T-bills are considered, optimally exploiting regime switching may lead to portfolio shifts into bonds or cash when a bear market regime is expected. In this article, we illustrate concretely how the presence of regimes can be incorporated into two asset allocation programs, a global asset allocation setting (with 6 equity markets, and potentially cash) and a market timing setting for US cash, bonds and equity.

There exists some related previous work. Clarke and de Silva (1998) show how the existence of two "states" (their terminology) affects mean variance asset allocation, but the article is silent about how the return characteristics in the two states may be extracted from the data. Ramchand and Susmel (1998) estimate a number of regime switching models on international equity return data, but do not explore how the regimes affect portfolio composition. Das and Uppal (2001) model jumps in correlation using a continous time jump model and investigate the implications for asset allocation. However, these jumps are only transitory and cannot fully capture the persistent nature of bear markets. Our work here builds mostly on the framework developed in Ang and Bekaert (2002a), who investigate optimal asset allocation when returns follow various regime switching processes. Their article restricts attention to returns from the US, UK and Germany.

It is important to realize that we did not try to mine the data or attempted to estimate a large number of different models. Instead, we proposed two general forms of models, estimated these models together with a number of simpler nested models and performed asset allocation. We also made a number of modelling choices that simplified the application of our models to asset allocation, even though they may adversely affect the performance of the models. It is our opionion that somewhat more complex RS models may substantially outperform the models we discuss here. In the conclusions, we reflect on the most fruitful extensions to our framework.

2 Data

In our first application, we focus on a universe of developed equity markets for a US based investor. Apart from North-America (Canada and the US), we consider the UK and Japan as two large markets, the euro-bloc (which we split into two, large and small markets) and the Pacific ex-Japan. Table 1 details the countries involved. All data are from the MSCI and the sample period dates back to February 1975 and runs till the end of 2000. Table 2 reports some simple characteristics of the equity data. We measure all returns in dollars. These return properties may play a large role in mean-variance based asset allocations. For example, it is immediately apparent that the use of historical data may lead to relatively large weights to Europe small, because it has witnessed relatively high returns with relatively low volatility, primarily because these small European markets represent a very diversified portfolio of economic exposures. In actual asset allocation programs, constraints could be imposed to keep expected returns closer to returns consistent with for example a market capitalization weighted benchmark (see Black and Litterman, 1992).

In our second application, we restrict attention to US returns, allowing for the US investor to time between cash (one month T-bills), 10-year bonds and the US stock market. Here we have a large sample available starting in January 1952. The second panel in Table 2 shows that bonds earned a premium of about 1.34% over the sample period with a standard deviation of about 9%, whereas stocks earned a 7.37% premium with a standard deviation of 14.33%.

3 Regime-Switching Beta Model

3.1 Description of the Model

To illustrate how regime switching models work, we use the simplest possible model. We hypothesize that there is only one world regime, which drives all other markets. The equation for the world equity return, in excess of the US T-bill rate), is:

$$y_t^w = \mu^w(s_t) + \sigma^w(s_t)\epsilon_t^w, \tag{1}$$

Here $\mu^w(s_t)$ denotes the expected return and $\sigma^w(s_t)$ the conditional volatility. Both can take on different values depending on the realization of the regime variable, s_t. We assume that the regime can only take on two values, 1 and 2. The transition between the two regimes is governed by a transition probability matrix, which can be characterized by two transition probabilities:

$$\begin{aligned} P &= p(s_t = 1|s_{t-1} = 1) \\ Q &= p(s_t = 2|s_{t-1} = 2). \end{aligned} \tag{2}$$

If the portfolio manager knows the regime, the expected excess return for the world market next period will be either:

$$e_1^w = P\mu^w(s_{t+1} = 1) + (1 - P)\mu^w(s_{t+1} = 2), \tag{3}$$

when the regime realization today is $s_t = 1$, or

$$e_2^w = (1-Q)\mu^w(s_{t+1}=1) + Q\mu^w(s_{t+1}=2) \tag{4}$$

when the regime realization today is $s_t = 2$.

The econometrician does not know the regime and he will try to infer from the data in what regime we are at each point by constructing the regime probability, which is the probability that tomorrow's regime is the first regime given current and past information. In this simple model, his information set consists simply of the international equity returns data. Regime probabilities play a critical role in the estimation of RS models, which uses maximum likelihood techniques (see Hamilton, 1994; Gray, 1996; and Ang and Bekaert, 2002b) or Bayesian techniques (see Albert and Chib, 1993).

The model also embeds time-varying volatility for the world market return, which consists of two components. For example, if the regime today is the first regime, the conditional variance for the world market excess return is given by:

$$\begin{aligned}\Sigma_1^w &= P(\sigma^w(s_{t+1}=1))^2 + (1-P)(\sigma^w(s_{t+1}=2))^2 \\ &\quad +P(1-P)[\mu^w(s_{t+1}=2) - \mu^w(s_{t+1}=1)]^2.\end{aligned} \tag{5}$$

Similarly, if $s_t = 2$, then the conditional variance is:

$$\begin{aligned}\Sigma_2^w &= (1-Q)(\sigma^w(s_{t+1}=1))^2 + Q(\sigma^w(s_{t+1}=2))^2 \\ &\quad +Q(1-Q)[\mu^w(s_{t+1}=2) - \mu^w(s_{t+1}=1)]^2.\end{aligned} \tag{6}$$

The first component in these equations is simply a weighted average of the conditional variances in the two regimes; the second component is a jump component that arises because the conditional mean is different across regimes.

The economic mechanism behind a world market regime is likely the world business cycle. Stock markets may be characterized by larger uncertainty and lower returns when a global recession is anticipated, as was the case in 2001. It may be that there are also country specific regimes, but we will not allow regime switching in idiosyncratic country behavior.[2]

Instead, we model the individual country excess returns, y_{t+1}^j, using a CAPM-inspired beta model:

$$y_{t+1}^j = (1-\beta^j)\mu^z + \beta^j\mu^w(s_{t+1}) + \beta^j\sigma^w(s_{t+1})\epsilon_{t+1}^w + \bar{\sigma}^j\epsilon_{t+1}^j. \tag{7}$$

[2] Ang and Bekaert (2002a) reject the presence of a country specific regime in the UK in a US-UK model, but it is conceivable that there are country specific regimes for other countries.

This model is very parsimonious and only requires the estimation of the world market process, the μ^z parameter and one beta and an idiosyncratic volatility per country. In a static world without time-variation in interest rates and regime switches, the model would be a CAPM model, where the μ^z constant admits a flatter Security Market Line, for which there is plenty of empirical evidence (see Black, Jensen and Scholes, 1972 for an early example). Theoretically, such a term can be motivated by the presence of differential borrowing and lending rates and is basically a version of Black's (1972) zero-beta CAPM.

With regime switches, this simple model captures time-variation in expected returns, volatilities and correlations, all driven by the world regime variable. There are good reasons for keeping the model simple, because additional flexibility comes at the considerable cost of parameter proliferation that the data cannot handle. The current model has only 19 parameters (for 6 equity classes). More general models (accommodating regime switching μ^z, regime switching betas, and/or regime switching idiosyncratic volatilities) fit the data better but some estimations are ill-behaved and it was often difficult to make inferences about the regime in such models.

Table 3 contains the actual estimation results for the model in equations (1)-(7). The first regime is a normal, quiet regime, where world excess returns are expected to yield 0.90% per month (10.8% annualized), with 2.81% (9.73% annualized) volatility. However, there is also a volatile regime (5.04% innovation variance) with a lower mean, namely 0.13%. The latter parameter is not significantly different from zero. The parameter μ^z is also not significantly positive but it is larger than the expected excess equity return in the low volatility regime. Consequently, the expected returns for high beta assets are lower than for low beta assets in this regime (the security market line has negative slope). The betas are estimated very precisely and their magnitudes seem economically appealing. The only surprise is that Japan, which has a rather low average return in the data, nevertheless gets assigned a high beta. However, Japan has the highest volatility of all the equity returns we consider (see Table 3), which the model fits through a high beta and a high idiosyncratic volatility (the highest idiosyncratic volatility across all markets). Moreover, the $(1 - \beta^j)\mu^z$ term decreases the expected return of Japan, whereas it increases the expected return of most other countries. Table 4 (Panel A) shows that the expected return for Japan implied by our model is the highest of all markets in the normal regime, but by far the lowest in the bear market regime. The lowest idiosyncratic volatility is observed for North America followed by Europe small, the two markets with the lowest overall volatility in the data.

Figure 1 shows the cumulative (total) returns on the 6 markets over the sample period and the ex-ante and smoothed regime probabilities. The former is the probability that the regime next month is the low volatility world market regime given current information, the latter is the probability that the regime next month is the low volatility regime given all of the information present in the data sample. Notable high volatile bear markets are the early 1980's, the period right after the October 87 crash, the early 1990's and a period in 1999. Overall, the stable (or unconditional) probability of the normal regime is 53%.

One feature of our model we have not emphasized yet, is that it is expressed in terms of simple excess returns. In other work (Ang and Bekaert, 2002a), we expressed the model, perhaps more reasonably, in logarithmic terms. Logarithmic returns constrain actual returns to be bounded at -100% but complicate the link between the RS model and expected returns used in the asset allocation optimization. Simple excess returns also provide a tighter link with a standard CAPM model.

3.2 Asset Allocation

Even in this simple model, the first and second moments vary through time. Consequently, investors with different horizons may hold different portfolios. However, Brandt (1999) and Ang and Bekaert (2002a), among others, show that the differences across these portfolios are not large and we ignore them in the present paper. Instead, we use a simple mean-variance optimization with monthly rebalancing, consistent with the data frequency.

We have discussed the conditional expected return and conditional variance of the world market return above. To derive the expected returns and covariance matrix for the returns on the available assets, we must introduce additional notation. Let the variance covariance matrix of our 6 risky securities, conditional on today's regime, be denoted by $\Sigma_i = \Sigma(s_t = i)$, (with i denoting the current regime) and let the vector of excess returns likewise be denoted by $e_i = e(s_t = i)$.

Since the mean of the world excess return switches between regimes, the expected excess return of country j is given by $(1 - \beta^j)\mu^z + \beta^j e_i^w$ for the current regime i, where e_i^w are given in equations (3) and (4). Let

$$\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}$$

for N countries. Hence, the expected return vector is given by:

$$e_i = (1 - \beta)\mu^z + \beta e_i^w.$$

Expected returns differ across individual equity indices only through their different betas with respect to the world market. In Table 4, Panel A, we report the implied expected excess returns for the six markets. Because the betas are close to 1, the expected returns are close to one another in the normal regime. In the bear market regime, expected excess returns are dramatically lower and there is more dispersion, with the UK and Japan now having the lowest expected excess returns. In this regime, the zero beta excess return is higher than the excess return on the world market, causing the high beta countries to have lower expected returns.

The variance covariance matrix has three components. First, there is an idiosyncratic part that we capture in a matrix V, where V is a matrix of zeros with $(\bar{\sigma}^j)^2$ along the diagonal. Second, the differences in systematic risk across the different markets and the correlations are completely driven by the variance of the world market and the betas as in any factor model. However, because the world market variance next period depends on the realization of the regime, we have two possible variance matrices for the unexpected returns next period:

$$\Omega_i = (\beta\beta')(\sigma^w(s_{t+1} = i))^2 + V, \qquad i = 1, 2 \tag{8}$$

Third, the actual covariance matrix today takes into account the regime structure, in that it depends on the realization of the current regime and it adds a jump component to the conditional variance matrix, which arises because the conditional means change from one regime to the other. As a consequence, the conditional covariance matrices can be written as:

$$\begin{aligned} \Sigma_1 &= P\Omega_1 + (1-P)\Omega_2 + P(1-P)(e_1 - e_2)(e_1 - e_2)' \\ \Sigma_2 &= (1-Q)\Omega_1 + Q\Omega_2 + Q(1-Q)(e_1 - e_2)(e_1 - e_2)', \end{aligned} \tag{9}$$

where the subscripts indicate the current regime.

It is straightforward to show that this model structure implies that the correlations implied by Ω_1, the normal regime, will be lower than the correlations implied by Ω_2, the high volatility regime. Ignoring the jump terms, this property will be inherited by Σ_1 and Σ_2 as long as the regimes are persistent (P and Q are greater than 0.5). Table 4, Panel A confirms that this is indeed the case in the data. In fact, the correlations in regime 2 are on average some 20 percent higher.

The standard optimal mean-variance portfolio vector is given by:

$$w_i = \frac{1}{\gamma}\Sigma_i^{-1} e_i, \tag{10}$$

where γ is the investor's risk aversion.

There are a number of ways we could implement mean-variance optimization. The first issue is that we have to specify the risk free rate. We choose to implement a nominal return framework, where each month the risk free rate is known and taken to be the 1-month T-bill. Hence, the risk free rate varies over time as we implement the allocation program. The tangency portfolio, the 100% equity portfolio, will hence move over time as well. For a particular interest rate however, there are only two optimal all equity portfolios the investor would choose in this simple example, one for each regime. An obvious extension of the framework is to add state dependence by using predictor variables for equity returns. Our second application illustrates this possibility.

A second issue is that mean-variance portfolios, based on historical data, may be quite unbalanced (see Green and Hollifield, 1992; and Black and Litterman, 1992). Practical asset allocation programs therefore impose constraints (short-sell constraints for example) or keep asset allocations close to the market capitalization weights, by partially using reverse-engineered expected returns. Although it is possible to do this in our application, we choose to not impose constraints at all, but we also show the mean-variance asset allocations based on purely historical moments.

Panel B of Table 4 shows the tangency portfolios (in regime 1 and regime 2) at an interest rate of 7.67%, the sample average. In the normal regime, the portfolio invests 42% of wealth in the US portfolio, which is not too far from the average relative market capitalization over the sample period. The European and Pacific indices are over-weighted relative to their market capitalizations, but the UK and Japanese markets are underweighted. There is even a small short position for the Japanese market. The high volatility of the UK and Japanese markets is the main culprit. Our model also slightly over-estimates the correlation between North America and Japanese returns relative to the data, which may explain the short positions in Japan. In regime 2, the investor switches resolutely towards the less volatile markets, which includes North-America. This does not mean the portfolio is now home biased because the investor also invests more heavily in the European markets, allocating more than 50% of wealth to Europe small. The short position in Japan is now quite substantial, exceeding 50%. When we consider the optimal allocation using unconditional moments (implied from the regime-switching model), the optimal portfolio is nicely in between the two regime-dependent portfolios, still underweighting the UK and Japan relative to market capitalization weights.

It is important to note that these portfolios may never be observed during an actual implementation of the asset allocation program because the magnitude of the interest rate may also be correlated with the realization of the regime, an issue we address more explicitly in the second application.

Figure 2 shows the essence of what taking regime switching into account adds to standard mean-variance optimization. The solid line represents the frontier using the unconditional moments, ignoring regime switches. The other frontiers are the ones applicable in the two regimes. The frontier near the top represents the normal regime. The risk-return trade-off is generally better here, because the investor takes into account that, given the regime is persistent, the likelihood of a bear market regime with high volatility next period is small. At the average interest rate of 7.67%, the Sharpe ratio available along the capital allocation line (the line emanating from the risk free rate on the vertical axis tangent to the frontier) is 0.871. In the bear market regime, the risk-return trade-off worsens and the investor selects a very different portfolio, only realizing a Sharpe ratio of 0.268 with the tangency portfolio. When we average the moments in the two regimes, we obtain an unconditional frontier implied from the RS Beta Model. The best possible Sharpe ratio for this frontier is 0.505. Note that the world market portfolio (using average market capitalization weights) is inefficient; it is inside the unconditional frontier.

Theoretically, the presence of two regimes and two frontiers means that the regime-switching investment opportunity set dominates the investment opportunity set offered by one frontier (see also Clarke and de Silva, 1998). In particular, in regime 1, if an investor held the unconditional tangency portfolio, averaging across the regimes, this would yield a Sharpe ratio of only 0.619. The investor could improve this trade-off to 0.871 holding the risk-free asset and the optimal tangency portfolio for this low variance regime. In regime 2, the unconditional tangency portfolio yields a Sharpe ratio of only 0.129, which could be improved to 0.268 holding the optimal tangency portfolio for the high variance regime.

However, to concretely implement the optimal asset allocation strategy, the parameters must be estimated from the available data. Because of sampling error and the possibility of model mis-specification, RS models may imperfectly match the true state-dependent moments, and out-performance for the RS model is no longer guaranteed. We show the results of two strategies, both starting with 1 dollar. The first strategy is an "in-sample" strategy assuming that we know the parameters estimated from the full sample and starts trading in Feb 1975. The regime strategy simply requires the risk-free rate and the realization of the regime. For the first, we simply take the available one-month Treasury bill. For the regime realization, the investor infers the regime probability from the current information. In particular, she computes: $Pr(s_t|I_t)$, which can be easily computed as a by-product of the optimization of the RS model. When this probability is larger than 1/2, the investor classifies the regime as 1, otherwise she classifies it as 2.

In Panel A of Table 5, we show results for risk aversion parameters (γ) ranging from 2 to 10. The non-regime dependent allocations (static mean-variance using moments estimated from data) realize slightly lower returns, but do so at considerably lower risk than the regime-dependent allocations. Hence, the regime-dependent model leads to lower in-sample Sharpe ratios than unconditional portfolios.

In Panel B of Table 5, we focus on all-equity portfolios both in-sample over the whole sample period, and over an out-sample, over the last 15 years. In the out-of-sample analysis, the model is estimated up to time t, and the regime-dependent and non-regime dependent weights are computed using information available only up to time t. The model is re-estimated every month. The non-regime dependent strategy uses means and covariances estimated from data up to time t. Our performance criterion is again the ex-post Sharpe ratio realized by the various strategies. Over the in-sample, the regime-dependent model outperforms holding the market portfolio, but is in turn outperformed by the non-regime dependent portfolio. A major reason for the under-performance of the world market portfolio is the presence of a relatively large Japanese equity allocation in the world market. The regime-dependent portfolios probably underperform because they are too unbalanced, relative to the non-regime dependent portfolios.

Over the out-sample, from Jan 1985 to Dec 2000, the regime-dependent strategy's Sharpe Ratio is 1.07, more than double the out-sample world market portfolio Sharpe Ratio (0.52). This is also higher than the non-regime dependent Sharpe Ratio (0.90). The regime-dependent strategy does so well because over this sample period the US market records very large returns and Japan performs very poorly. In fact, the US Sharpe ratio over the period is 0.65! In the normal regime, the all-equity portfolio for the regime-switching model has a very large weight on North America (see Panel B of Table 4). In the bear market regime, the regime-switching strategy has a very large short position in Japanese equities.

Figure 3 shows how wealth cumulates over time in these strategies. The large North American and the short Japanese positions imply that both the regime-dependent and the non-regime dependent strategies out-perform the world market and the North American market consistently. Nevertheless, the out-performance is particularly striking for the last 5 years. It is also over the last 5 years that the RS strategy is particularly successful in out-performing the non-regime dependent strategy.

Given that this example is highly stylized, and our results intimately linked to a perhaps special historical period, we do not want to claim that the success of the RS strategy shown here is a good indicator for future success. The important conclusion to draw is that RS strategies have the potential to out-perform because they set up a defensive portfolio in the bear market regime that hedges against high correlations and low returns. This portfolio need not be completely home biased, and in our example still involves substantial net international positions. It is likely that in any practical implementation of a RS model, which relies less on historical moments, or is based on a different sample period, the optimal portfolios will be even more internationally diversified. In the recent work by Ang and Bekaert (2002a), this is actually the case.

4 Regime Switching Market Timing Model

4.1 Description of the Model

Ang and Bekaert (2001) point out that there is significant predictive power in short rates for future equity premiums. In particular, when short rates are low, subsequent equity returns tend to be high. Hence, when a bear market regime is expected, the optimal asset allocation response may be to switch to a safe asset or a bond. The model we explore in this section considers asset allocation among three assets, cash, a 10 year (constant maturity) bond and an equity index (all for the US). We formulate the model in excess returns. We use r_t to denote the risk free rate (the nominal T-bill rate), r_t^b as the excess bond return and r_t^e as the excess return on US equity. The Market Timing Model is given by:

$$\begin{aligned} r_t &= \mu_r(s_t) + \rho(s_t) r_{t-1} + \epsilon_t^1 \\ r_t^b &= \mu_b(s_t) + \beta^b(s_t) r_{t-1} + \epsilon_t^2 \\ r_t^e &= \mu_e(s_t) + \beta^e(s_t) r_{t-1} + \epsilon_t^3, \end{aligned} \tag{11}$$

where $\epsilon_t = (\epsilon_t^1\ \epsilon_t^2\ \epsilon_t^3)' \sim$ N(0,$\Lambda(s_t)$).

The short rate follows an autoregressive process, but the constant term and the autoregressive parameter depend on the regime. Countless articles in the term structure literature (for example, see Gray, 1996; and Ang and Bekaert, 2002b) have demonstrated that the data support such a model, where one regime captures normal times in which interest rates are highly persistent and not too variable, and another regime captures times of very variable, higher interest rates which revert quickly to lower rates. The excess bond and excess equity returns follow the same structure. The conditional mean depends on the lagged short rate with the dependence varying with the regime. However, this dependence is weak, and we fail to reject the null that the betas are constant across regimes. The variance covariance matrix is parameterized as $\Lambda(s_t) = R(s_t)' R(s_t)$, with R a lower triangular matrix. We can rewrite this model as:

$$Y_t = \mu(s_t) + \Phi(s_t) Y_{t-1} + \epsilon_t \tag{12}$$

where $Y_t = (r_t \, r_t^b \, r_t^e)$, $\mu(s_t) = (\mu^r(s_t) \, \mu^b(s_t) \, \mu^e(s_t))'$ and

$$\Phi(s_t) = \begin{pmatrix} \rho(s_t) & 0 & 0 \\ \beta^b(s_t) & 0 & 0 \\ \beta^e(s_t) & 0 & 0 \end{pmatrix}.$$

Clearly, a good many models are a special case of (12). The model accommodates time variation in expected returns through two channels: regime changes and interest rate variation. The predictive power of nominal interest rates for equity premiums has a long tradition in finance going back to at least Fama and Schwert (1979). Recently, Ang and Bekaert (2001) show that the short rate is a robust predictor of excess equity returns in 5 countries and is more significant and robust that dividend or earnings yields.

Our specification of transition probabilities gives us another channel for predictability of a non-linear kind. We specify the transition probabilities to be a function of the short rate:

$$\begin{aligned} P_{t-1} &\equiv p(s_t = 1 | s_{t-1} = 1, I_{t-1}) = \frac{\exp(a_1 + b_1 r_{t-1})}{1 + \exp(a_1 + b_1 r_{t-1})} \\ Q_{t-1} &\equiv p(s_t = 2 | s_{t-1} = 2, I_{t-1}) = \frac{\exp(a_2 + b_2 r_{t-1})}{1 + \exp(a_2 + b_2 r_{t-1})}. \end{aligned} \tag{13}$$

Consequently, the short rate helps predict transitions in the regime, providing an additional channel for time-variation in expected returns. We call the full model, Model I. Because of the presence of non-linear predictability through the transition probabilities, we also estimate a much simpler model (Model II) where μ_r and μ_b are constant across regimes and $\beta_b(s_t) = \beta_e(s_t) = 0$.

The full estimation results for Models I and II are reported in Table 6. Let's start with the parameters for the conditional variance. Clearly, the second regime is a high volatility regime. The standard deviations implied for the interest rate shock are 0.02% per month in the first regime versus 0.09% per month in the second regime; for shocks to the excess bond return the relative numbers are 6.07% and 13.79% per annum and for shocks to the excess equity return the regime-dependent standard deviations are 11.74% and 19.14% per annum. Consistent with the previous empirical literature, the interest rate is much more mean reverting in the first regime, where the short rate is nearly a random walk (ρ is 0.99 in the first regime versus 0.94 in the second regime). The drift of the short rate is higher in the second regime, which has the implication that interest rate realizations in the second regime correspond to large, rapidly mean-reverting, volatile interest rates. The constants and the betas in the excess return regressions are estimated with much error. The negative equity betas reflect the fact that low interest rates are typically associated with high expected excess equity returns.

The final set of parameters are the transition probability parameters and they are reported first in Table 6. The coefficient b_1 measures the dependence of the probability of staying in regime 1 on the interest rate and this dependence is very significantly negative. As interest rates rise, the probability of transitioning into the second high volatility and bear market regime becomes higher. Similarly, as interest rates move higher while in the second regime, the probability of staying in that regime increases. This coefficient is only borderline significant. However, when we put both coefficients equal to 0, the resulting model is strongly rejected by a likelihood ratio test. Hence, non-linear predictability is an important feature of the data.

Since the conditional mean parameters are estimated with little precision, we consider the much simpler Model II, where we set $\beta_b(s_t) = \beta_e(s_t) = 0$ and we restrict the bond and equity return μ's to be the same across regimes. The mean for the bond excess return now becomes positive (but still estimated with a lot of error), whereas the mean for the equity returns is an average of the estimates of the μ's in the two regimes in Model I. A formal likelihood ratio test does not reject this model relative to the more intricate model. In this model, it is also the case that a model with constant transition probabilities is very strongly rejected. Whatever the predictability present in the data, it appears it is best captured by the non-linear predictability entering through the transition probabilities.

Note that there is only one regime variable in this model, and it is likely driven by regimes in short rates. An interesting question is whether regimes in short rates and equity returns are the same or different. It is actually possible to test this conjecture but we defer this to future work. If interest rate and equity return regimes are both driven by business cycle variation, they are likely positively correlated. Figure 4 (second graph) shows the ex-ante and smoothed regime probabilities for Model I. Note how the famous Fed experiment with monetary targeting in 1979-1983 is clearly marked as a high volatility regime. Overall, the stationary probability of the normal regime, $Pr(s_t = 1)$, implied by Model I is 0.6851, for Model II it is 0.7014. These probabilities were determined by simulating a very long sample from the respective models. The third graph in Figure 4 shows simultaneously the ex-ante regime probabilities from the interest rate model and from the RS Beta model. The correlation is not perfect but it is clearly positive.

4.2 Asset Allocation

We follow the same mean-variance strategy as in Section 3.2. The optimal asset allocation vector is a function of the expected excess returns on the two risky assets, the bond and equity and their covariance matrix. In this model, if the realization of the regime variable next period at time $t+1$ is known, the means are given by:

$$\begin{aligned} \mu_{1t} &= \begin{pmatrix} \mu^b(s_{t+1}=1) \\ \mu^e(s_{t+1}=1) \end{pmatrix} + \begin{pmatrix} \beta^b(s_{t+1}=1) \\ \beta^e(s_{t+1}=1) \end{pmatrix} r_t \\ \mu_{2t} &= \begin{pmatrix} \mu^b(s_{t+1}=2) \\ \mu^e(s_{t+1}=2) \end{pmatrix} + \begin{pmatrix} \beta^b(s_{t+1}=2) \\ \beta^e(s_{t+1}=2) \end{pmatrix} r_t. \end{aligned} \tag{14}$$

However, the expected excess return today depends on the current regime and the probability of transitioning to either a μ_{1t} or μ_{2t} realization:

$$\begin{aligned} e_{1t} &= P_t\mu_{1t} + (1-P_t)\mu_{2t} \\ e_{2t} &= (1-Q_t)\mu_{1t} + Q_t\mu_{2t}. \end{aligned} \tag{15}$$

The conditional covariances (conditional on the current regime) can be computed using the same reasoning as in Section 3.2. Defining $\Upsilon_i = \Upsilon(s_{t+1}=i)$, as the lower 2×2 matrix of $\Lambda(s_{t+1}=i)$, then the conditional variance (conditional on the current regime) matrices for bond and equity returns $\Sigma_i = \Sigma(s_t = i)$ are given by:

$$\begin{aligned} \Sigma_1 &= P_t\Upsilon_1 + (1-P_t)\Upsilon_2 + P_t(1-P_t)(\mu_{1t}-\mu_{2t})(\mu_{1t}-\mu_{2t})' \\ \Sigma_2 &= (1-Q_t)\Upsilon_1 + Q_t\Upsilon_2 + Q_t(1-Q_t)(\mu_{1t}-\mu_{2t})(\mu_{1t}-\mu_{2t})'. \end{aligned} \tag{16}$$

To obtain intuition on the asset allocation weights for this model, Figure 5 graphs the optimal asset allocations to bonds and stocks (which will add to 1 minus the weight assigned to the risk free asset) as a function of the short rate (risk free rate) level at the estimated parameters. We set the risk aversion level to $\gamma = 5$. Let us first focus on Model I (which may be over-parameterized) in the top panel.

In regime 1, there is a very clear and almost monotone pattern. At low interest rates, the investor essentially shorts bonds and invests most of her wealth in stocks. This is because the equity premium at low interest rates is positive and large but the bond premium is negative. As the interest rate increases the bond premium increases (β_b is positive) and the equity premium decreases (β^e is negative). Moreover, the probability that the regime will transition to the second regime becomes higher and in the second regime, equities are relatively much less attractive. When interest rates continue to increase, the probability of transitioning to second regime is very high and the investor starts to hedge against the bear market regime by investing in bonds. At interest rates higher than 7%, the investor invests in bonds and cash and shorts the equity market. For Model I in regime 2, equities are much less attractive, but they are more attractive the lower the risk free rate. This is because the lower the risk free rate is, the less negative is the within-regime equity premium and the higher the probability of transitioning back into the normal regime. Consequently, the asset allocation pattern is similar to that observed for regime 1, but it is less extreme. Note that for an empirically relevant range of interest rates (between 3% and 6%), the short positions are not that large.

The bottom panel of Figure 5 focuses on the more parsimonious Model II. In this model, the bond and equity premium are positive and identical within regimes, but of course volatility is much larger in the second regime. In regime 1, if interest rates are low enough, the investor will borrow at the risk free rate and invest a small fraction of her portfolio in bonds and more than 100% in equities. As interest rates rise, equities become less attractive as the probability of switching to the high variance regime increases. Bonds also become less attractive and because the bond premium is very small, it quickly becomes optimal to short bonds. In the second regime, the investor always shorts bonds, but the investment in equities is never higher than 80%. The main hedge for volatility clearly is the risk-free asset, not a bond investment.

Because the interest rate is so important in this model, the optimal asset allocation varies substantially over time with different realizations of the interest rate. Figure 6 shows optimal asset allocation weights for all three assets across time for the full-sample. It is assumed that the investor uses the moments implied by the full sample estimation. The over-parameterization of Model I implies that in reality there are quite large confidence bands around the asset allocation weights shown in the top panel of Figure 6. The asset allocations implied by Model I are much more extreme than those implied by Model II, but the equity allocations typically move in similar directions over time. Note that at the 1987 crash, the investor is heavily invested in equity. After the crash the investor shifts this equity portion into risk-free holdings.

What performance is associated with the asset weights shown in Figure 6? The answer is in Table 7. We show mean returns, volatility and Sharpe ratios for following the optimal regime-dependent strategies for both models and compare it with a strategy that simply uses unconditional moments. Not surprisingly, the Model I strategy, featuring highly levered portfolios, yields higher returns but also much more volatile returns than the Model II strategy, which in turn delivers higher average returns but also higher volatility than a non-regime dependent strategy. This is true for all risk aversion levels. Nevertheless, the two regime-dependent strategies yield Sharpe ratios far superior to the Sharpe ratio resulting from the non-regime dependent strategy. (The in-sample Sharpe Ratios for Model I, Model II and the non-regime dependent strategies are 0.77, 0.65 and 0.51, respectively.)

This superior performance may of course be due to look ahead bias, since we base our asset allocation on full sample moments. In an actual asset allocation context, the parameters have to be estimated with the available data. Therefore we also consider an out-of-sample exercise as in Section 3.2, starting in 1985. The results are reported in Table 8. Not surprisingly, the Sharpe ratios drop for all strategies and become quite low for highly risk averse people. Now, the more parsimonious Model II is the best performing model in terms of Sharpe ratios, and it appears to add value relative to unconditional portfolios even out-of-sample. Figure 7 shows that the superior performance is not due to a few isolated months in the sample, but that the last 5 years do play an important role in giving the regime-dependent strategies an edge. During these years, both the Model I and Model II strategies allocate more money to equity and benefit handsomely from the US bull market. However, the Model I positions are more leveraged and although they have higher returns, they also have much higher volatility. In fact, for Model I with $\gamma = 2$, the investor actually goes bankrupt in the 87 crash, as the investor's leveraged equity position at this time exhausts her wealth.

5 Conclusion

There is much evidence in the academic literature that both expected returns and volatility vary through time, probably driven by the business cycle and monetary policy changes. Moreover, in high volatility environments across the world, equity returns become more highly correlated and do not perform very well. If this is true, active portfolio management should be able to exploit these regime changes to add value. In this article, we show how this can be done formally. Our results are meant to be illustrative. On the one hand, we exaggerate the performance of the models, because we do not take transaction costs into account. On the other hand, we greatly undersell the potential of regime switching models, because we did not at all try to estimate the best possible model, or do an extensive model search.

There is a long list of extensions that can be accommodated in the framework and are likely to improve performance. First, equity portfolio allocation programs typically are compensated based on tracking error relative to an index. Therefore, most active management start from expected returns according to a Black-Litterman (1992) or similar approach (reverse engineered from the benchmark) and only deviate minimally from the benchmark towards the predictions of a proprietary model. Instead, we have used only historical data. However, it is straightforward to build a Black-Litterman (1992) type equilibrium into the regime-switching model and into the estimation of the parameters.

Second, in international asset allocation, it is often the case that the equity benchmarks are hedged against currency risk. Ang and Bekaert (2002a) show that the regime switching beta model can be adjusted to allow both currency hedged and non-hedged returns. In that case, the asset allocation model yields the optimal currency hedge ratio.

Third, we have assumed that there is only one regime variable. However, it would be interesting to test whether there are country specific regimes, and whether the regimes in short rates and equity returns are less than perfectly correlated.

Finally, in the optimization we have only focused on first and second moments, but many investors prefer positive skewness and dislike kurtosis. Regime switching models have non-trivial higher order moments, because they can be interpreted as a model of time-varying mixture of normals. For investors with preferences involving higher order moments of returns, RS models are a viable alternative to consider.

Despite this long agenda for future research, our current results point to two robust conclusions. First, whereas it is possible to add value in all equity portfolios, the presence of a bear market high correlation regime does not negate the benefits of international diversification. Although it is likely that portfolios in that regime are more home-biased, they typically still involve significant international exposures. Second, it may be most valuable to consider regime switching models in tactical asset allocation programs that allow switching to a risk-free asset.

References

[1] Albert, J. H., and S. Chib, 1993, "Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markove Mean and Variance Shifts," *Journal of Business and Economic Statistics*, 11, 1, 1-15.

[2] Ang, A., and G. Bekaert, 2001, "Stock Return Predictability: Is it There?" working paper, Columbia Business School.

[3] Ang, A., and G. Bekaert, 2002a, "International Asset Allocation with Regime Shifts," forthcoming *Review of Financial Studies*

[4] Ang, A., and G. Bekaert, 2002b, "Regime Switches in Interest Rates," *Journal of Business and Economic Statistics*, 20, 2, 163-182.

[5] Ang, A., and G. Bekaert, 2002c, "Short Rate Nonlinearities and Regime Switches," *Journal of Economic Dynamics and Control*, 26, 7-8, 1243-1274.

[6] Ang, A., and J. Chen, 2002, "Asymmetric Correlations of Equity Portfolios," *Journal of Financial Economics*, 63, 3, 443-494.

[7] Bekaert, G., R. Hodrick, and D. Marshall, 2001, "Peso Problem Explanations for Term Structure Anomalies," *Journal of Monetary Economics*, 48, 241-270.

[8] Black, F., 1972, "Capital Market Equilibrium with Restricted Borrowing," *Journal of Business*, 45, 444-454.

[9] Black, F., M. Jensen and M. Scholes, 1972, "The Capital Asset Pricing Model: Some Empirical Tests," in *Studies in the Theory of Capital Markets*, Jensen. M., ed., Praeger, New York.

[10] Black, F., and R. Litterman, 1992, "Global Portfolio Optimization," *Financial Analysts Journal*, Sept, 28-43.

[11] Brandt, M. W., 1999, "Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach," *Journal of Finance*, 54, 5, 1609-1646.

[12] Campbell, R., K. Koedijk, and P. Kofman, 2002, "Increased Correlation in Bear Markets," *Financial Analyst Journal*, Jan-Feb, 87-94.

[13] Clarke, R. G., and H. de Silva, 1998, "State-Dependent Asset Allocation," *Journal of Portfolio Management*, 24, 2, 57-64.

[14] Das, S. R., and R. Uppal, 2001, "Systemic Risk and Portfolio Choice," working paper, London Business School.

[15] Erb, C. B., C. R. Harvey, and T. E. Viskanta, 1994, "Forecasting International Equity Correlations," *Financial Analysts Journal*, Nov-Dec, 32-45.

[16] Fama, E., and G. W. Schwert, 1977, "Asset Returns and Inflation," *Journal of Financial Economics*, 5, 115-146.

[17] Gray, S. F., 1996, "Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process," *Journal of Financial Economics*, 42, 27-62.

[18] Green, R., and B. Hollifield, 1992, "When Will Mean-Variance Efficient Portfolios be Well Diversified?" *Journal of Finance*, 47, 5, 1785-1809.

[19] Hamilton, J. D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, 57, 357-384.

[20] Longin, F., and B. Solnik, 2001, "Correlation Structure of International Equity Markets During Extremely Volatile Periods," *Journal of Finance*, 56, 2, 649-676.

[21] Ramchand, L., and R. Susmel, 1998, "Cross Correlations Across Major International Markets," *Journal of Empirical Finance*, 5, 397-416.

Table 1: Composition of International Returns

North America	UK	Japan	Europe large	Europe small	Pacific ex-Japan
Canada			France	Austria	Australia
US			Germany	Belgium	New Zealand
			Italy	Denmark	Singapore
				Finland	
				Ireland	
				Netherlands	
				Norway	
				Spain	
				Sweden	
				Switzerland	

The table lists the country composition of the geographic returns. Within each geographic region, we construct monthly returns, value-weighted in US dollars.

Table 2: Sample Moments

Panel A: International Excess Returns

	World	North America	UK	Japan	Europe large	Europe small	Pacific ex-Japan
mean	6.55	7.51	9.90	6.34	6.80	7.29	5.01
stdev	13.94	14.82	21.22	23.27	18.30	15.59	21.50
beta		0.88	1.03	1.21	0.90	0.89	0.92

Correlation Matrix:

	World	N Amer	UK	Japan	Eur lg	Eur sm	Pacific
N Amer	0.83						
UK	0.68	0.51					
Japan	0.72	0.30	0.40				
Eur lg	0.68	0.45	0.54	0.45			
Eur sm	0.80	0.59	0.64	0.51	0.82		
Pacific	0.59	0.53	0.50	0.36	0.39	0.53	

Panel B: US Returns

	Short Rate	Excess Bond	Excess Stock
mean	5.09	1.34	7.37
stdev	2.75	9.00	14.33
auto	0.97	0.07	0.01

The table reports summary statistics mean and standard deviation (stdev) for the international returns (panel A) and in the US (panel B). In Panel A, international returns are expressed as simple returns at a monthly frequency in percentages and are annualized by multiplying the mean (standard deviation) by 12 ($\sqrt{12}$). International returns are denominated in US dollars, are from MSCI and are in excess of the US 1-month T-bill return. Within each geographical area, the country returns are value-weighted by their market capitalization in US dollars to form the geographic area returns. The row labeled beta is the full-sample beta for each country's excess return with the world market excess return. The sample period for the international returns is Feb 1975 to Dec 2000. In Panel B, the US returns are monthly, from Ibbotson. The short rate is the 1 month T-bill return, the excess bond return uses Ibbotson's 10 year government bond holding period return, and the excess stock return uses total returns (capital gain and dividends) on the S&P500 index. Means are annualized by multiplying by 12. For the short rate, the standard deviation is the standard deviation of the annualized monthly yield, while the volatility of excess bond and stock returns are annualized by multiplying by $\sqrt{12}$. Auto denotes the monthly autocorrelation. The sample period for the US returns is Jan 1952 to Dec 2000.

Table 3: Regime-Switching Beta Model Parameter Estimates

Transition Probabilities and μ_z

	P	Q	μ_z
Estimate	0.8917	0.8692	0.74
Std error	0.0741	0.1330	0.68

World Market

	μ_1	μ_2	σ_1	σ_2
Estimate	0.90	0.13	2.81	5.04
Std error	0.32	0.62	0.44	0.55

Country Betas β

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.88	1.03	1.21	0.90	0.89	0.92
Std error	0.03	0.06	0.07	0.05	0.04	0.07

Idiosyncratic Volatilities $\bar{\sigma}$

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	2.40	4.50	4.62	3.87	2.72	4.99
Std error	0.09	0.18	0.19	0.16	0.11	0.20

All parameters are monthly and are expressed in percentages, except for the transition probabilities P and Q. The stable probability $p(s_t = 1) = 0.5285$.

Table 4: Regime-Dependent Beta Model Asset Allocation

Panel A: Regime-Dependent Means and Covariances

Regime-Dependent Excess Returns

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
$s_t = 1$	9.64	9.76	9.90	9.65	9.65	9.67
$s_t = 2$	3.47	2.54	1.42	3.36	3.39	3.22

Regime-Dependent Covariances/Correlations

Regime 1						
N Amer	1.35	[0.44]	[0.48]	[0.45]	[0.54]	[0.38]
UK	0.90	3.08	[0.37]	[0.35]	[0.42]	[0.29]
Japan	1.06	1.25	3.60	[0.38]	[0.46]	[0.32]
Eur lg	0.79	0.92	1.08	2.30	[0.43]	[0.30]
Eur sm	0.78	0.91	1.07	0.80	1.53	[0.36]
Pac	0.81	0.94	1.11	0.82	0.82	3.33
Regime 2						
N Amer	2.37	[0.64]	[0.68]	[0.65]	[0.73]	[0.58]
UK	2.10	4.49	[0.58]	[0.55]	[0.63]	[0.49]
Japan	2.47	2.89	5.53	[0.58]	[0.66]	[0.52]
Eur lg	1.83	2.14	2.52	3.36	[0.63]	[0.49]
Eur sm	1.82	2.13	2.50	1.85	2.58	[0.56]
Pac	1.88	2.20	2.58	1.91	1.90	4.45

Panel B: Tangency Portfolio Weights

	N Amer	UK	Japan	Eur lg	Eur sm	Pac
$s_t = 1$	0.42	0.06	-0.01	0.15	0.31	0.08
$s_t = 2$	0.79	-0.14	-0.55	0.25	0.54	0.10
Unconditional	0.52	0.04	-0.16	0.18	0.37	0.09
Ave Mkt Cap	0.50	0.09	0.22	0.08	0.08	0.02

We report the regime-dependent means and covariances of excess returns implied by the estimates of the Regime-Switching Beta Model in Table 3. Panel A reports the regime-dependent excess return means and covariances, where we list correlations in the upper-right triangular matrix in square brackets. All numbers are listed in percentages, and are annualized. Panel B reports the mean variance efficient (MVE) (tangency) portfolios, computed using an interest rate of 7.67%, which is the average 1-month T-bill rate over the sample). The Ave Mkt Cap denotes the average market capitalization, averaged across the sample.

Table 5: Portfolio Allocation with the Regime-Switching Beta Model

Panel A: In-Sample Results

Regime-Dependent Allocations

	Risk Aversion				
	2	3	4	5	10
mean ret	26.77	17.22	14.03	12.44	11.49
stdev ret	43.56	21.73	14.46	10.83	8.65
Sharpe Ratio	0.44	0.44	0.44	0.44	0.44

Non-Regime Dependent Allocations

	Risk Aversion				
	2	3	4	5	10
mean ret	25.10	16.39	13.48	12.03	11.15
stdev ret	29.43	14.69	9.78	7.34	5.88
Sharpe Ratio	0.59	0.59	0.59	0.59	0.59

Panel B: All-Equity Portfolios

	In-Sample				Out-of-Sample			
	World Market	N America	Regime Dependent	Non-Regime Dependent	World Market	N America	Regime Dependent	Non-Regime Dependent
mean ret	14.22	15.18	15.43	16.33	13.73	15.84	21.46	20.04
stdev ret	13.85	14.75	15.42	14.60	14.86	15.21	14.51	15.67
Sharpe Ratio	0.47	0.51	0.50	0.59	0.52	0.65	1.07	0.90

The top panel gives mean and standard deviation of total portfolio returns using optimal (regime-dependent) for an all-equity portfolio and portfolios formed by different risk aversion levels over the full sample. We use the actual 1 month T-bill yield at time t as the risk-free asset. In Panel B, we look at all-equity portfolio holdings both in-sample and on an out-sample of the last 180 months (Jan 1985 to Dec 2000). Over the out-sample, the model is estimated up to time t, and the regime-dependent and non regime-dependent weights are computed using information available only up to time t. The model is re-estimated every month. The non-regime dependent strategy estimates means and covariances from data up to time t. The Non-Regime Dependent Allocations are computed with static one-period mean-variance utility, using the returns up to time t. The columns labeled 'World Market' and 'US' refer to returns on holding a 100% world market and 100% US portfolio, respectively. All returns are annualized and are reported in percentages.

Table 6: Market-Timing Model Parameters

	Model I		Model II	
	Regime 1	Regime 2	Regime 1	Regime 2
a	4.73	-0.93	4.67	-0.83
	(0.87)	(1.08)	(0.84)	(1.24)
b	-6.25	3.46	-6.06	3.43
	(1.85)	(1.93)	(1.83)	(2.04)
μ^r	0.01	0.03	0.01	0.04
	(0.00)	(0.02)	(0.00)	(0.02)
μ^b	-0.20	0.45	0.07	
	(0.23)	(0.89)	(0.09)	
μ^e	1.49	1.23	0.68	
	(0.42)	(1.26)	(0.16)	
ρ	0.99	0.94	0.99	0.94
	(0.01)	(0.03)	(0.01)	(0.03)
β^b	0.74	-0.34		
	(0.68)	(1.29)		
β^e	-2.16	-1.54		
	(1.21)	(1.80)		
R_{rr}	0.02	0.09	0.03	0.09
	(0.00)	(0.01)	(0.00)	(0.01)
R_{br}	0.11	-0.26	0.11	-0.26
	(0.10)	(0.31)	(0.10)	(0.31)
R_{bb}	1.75	3.97	1.75	3.97
	(0.09)	(0.25)	(0.09)	(0.25)
R_{er}	0.31	-1.06	0.31	-1.07
	(0.19)	(0.43)	(0.19)	(0.43)
R_{eb}	0.63	1.76	0.61	1.73
	(0.19)	(0.42)	(0.19)	(0.43)
R_{ee}	3.32	5.13	3.33	5.16
	(0.13)	(0.32)	(0.13)	(0.33)

All parameters are monthly and are expressed in percentages, except for the transition probability parameters a_i and b_i. Standard errors are reported in parentheses. We report the Cholesky decomposition of $\Lambda(s_t) = R(s_t)'R(s_t)$, where superscripts indicate the matrix elements corresponding to short rates (r), long-term bonds (b) and equity returns (e).

Table 7: In-Sample Portfolio Allocation with the Market-Timing Model

Model I Regime-Dependent Allocations

Risk Aversion	2	3	4	5	10
mean ret	35.30	25.23	20.20	17.18	11.13
stdev ret	39.06	26.00	19.47	15.56	7.74
Sharpe Ratio	0.77	0.77	0.77	0.77	0.77

Model II Regime-Dependent Allocations

Risk Aversion	2	3	4	5	10
mean ret	24.52	18.05	14.81	12.87	8.98
stdev ret	29.79	19.89	14.85	11.87	5.91
Sharpe Ratio	0.65	0.65	0.65	0.65	0.65

Non Regime-Dependent Allocations

Risk Aversion	2	3	4	5	10
mean ret	18.34	13.92	11.72	10.39	7.74
stdev ret	25.65	17.07	12.79	10.22	5.10
Sharpe Ratio	0.51	0.51	0.51	0.51	0.51

We present the mean, standard deviation and Sharpe Ratios of in-sample returns following Model I, Model II and a naive non-regime dependent strategy. All returns are annualized and are reported in percentages.

Table 8: Out-of-Sample Portfolio Allocation Back-Testing with the Market Timing Model

Model I Regime-Dependent Allocations

Risk Aversion	2	3	4	5	10
mean ret	34.93	24.12	18.71	15.46	8.98
stdev ret	59.42	39.61	29.71	23.77	11.89
Sharpe Ratio	0.51	0.48	0.46	0.43	0.32

Model II Regime-Dependent Allocations

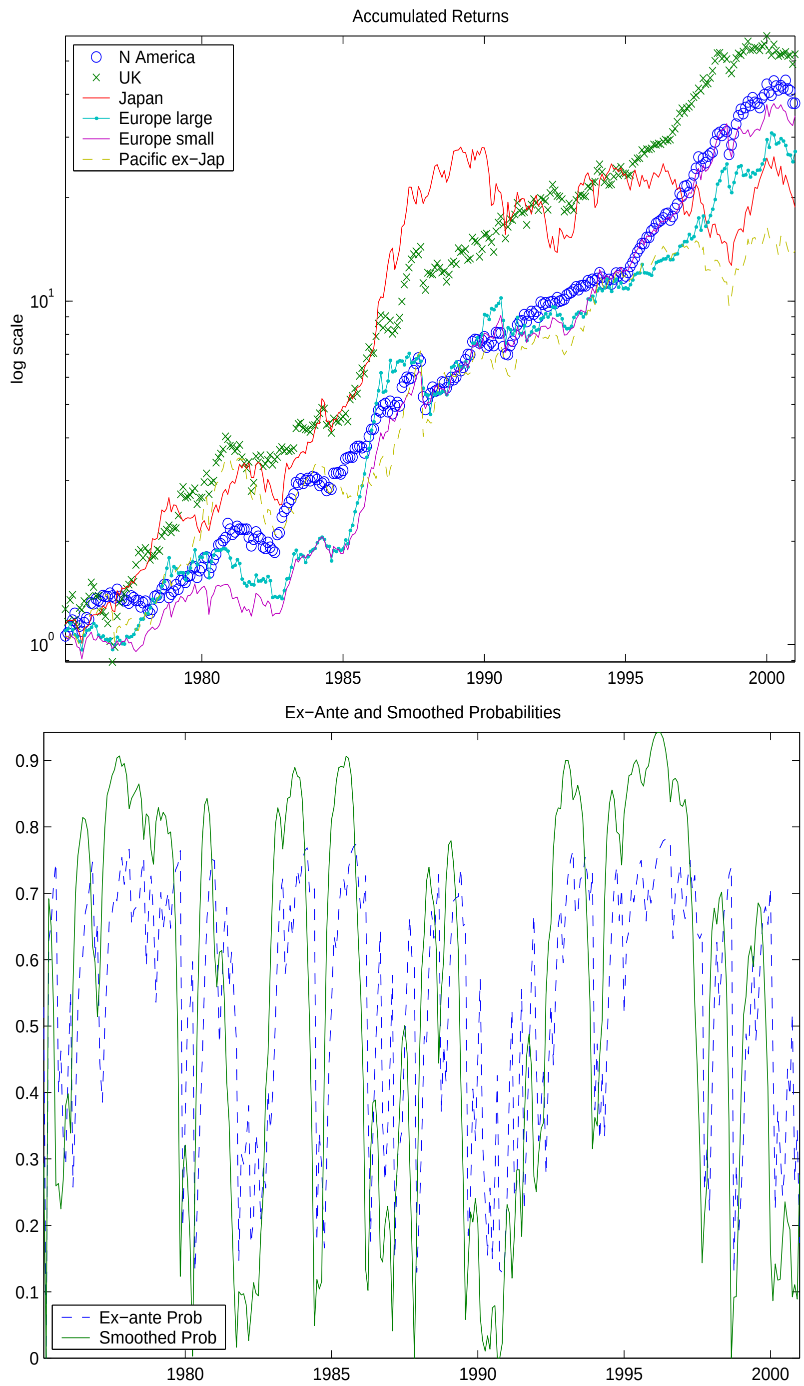
Risk Aversion	2	3	4	5	10
mean ret	25.29	17.69	13.89	11.61	7.05
stdev ret	34.53	23.02	17.27	13.82	6.91
Sharpe Ratio	0.58	0.54	0.50	0.47	0.27

Non Regime-Dependent Allocations

Risk Aversion	2	3	4	5	10
mean ret	17.65	12.60	10.07	8.55	5.52
stdev ret	26.25	17.50	13.13	10.50	5.26
Sharpe Ratio	0.48	0.42	0.37	0.32	0.07

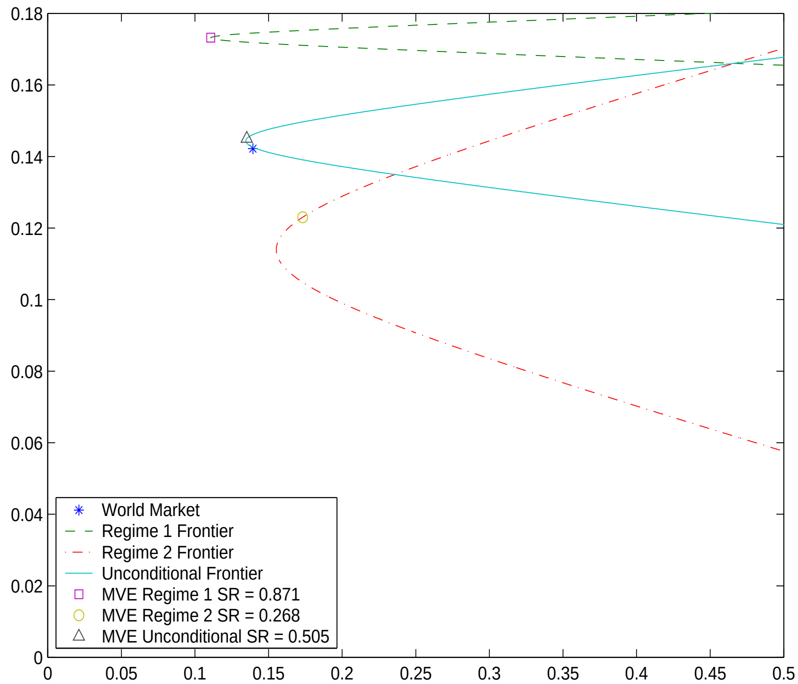
We present the mean, standard deviation and Sharpe ratios of out-of-sample returns following Model I, Model II and a naive non-regime dependent strategy over an out-sample of the last 15 years (Jan 1985 to Dec 2000) are used. Over the out-sample, the model is estimated up to time t, and the regime-dependent and non regime-dependent weights are computed using information available only up to time t. The model is re-estimated every month. The non-regime dependent strategy estimates means and covariances from data up to time t. All returns are annualized and are reported in percentages.

Figure 1: Ex-Ante and Smoothed Proabilities of the Beta Model



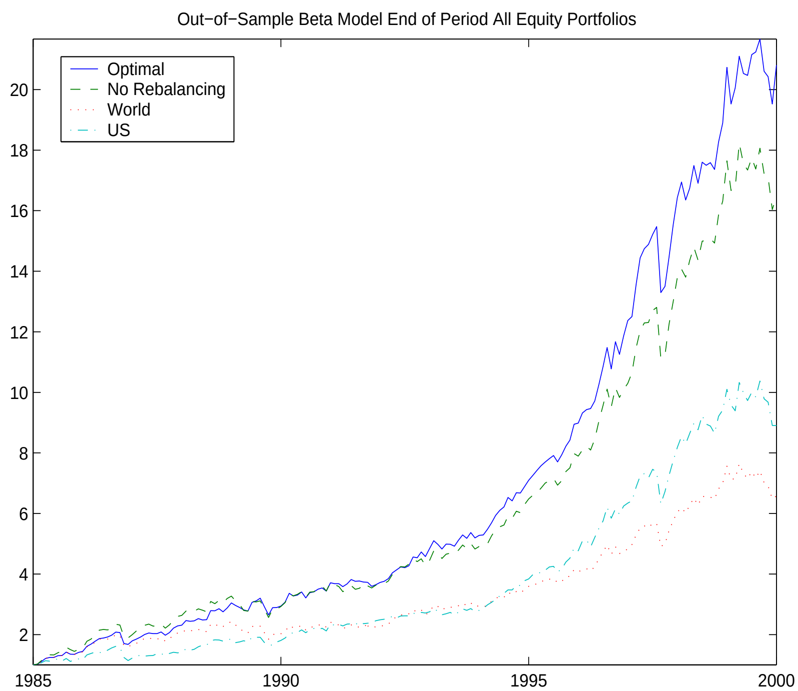
The top plot shows the accumulated total returns of \$1 at Jan 1975, through the same until Dec 2000 of each of the geographic regions. The bottom plot shows the ex-ante probabilities $p(s_t = 1|I_{t-1})$ and the smoothed probabilities $p(s_t = 1|I_T)$ of being in the first regime, where the first regime is the world low variance regime.

Figure 2: Mean-Standard Deviation Frontiers of the Regime-Switching Beta Model



We plot the mean-variance frontier of regime 1 (the world low variance regime), regime 2 (high variance regime), and the unconditional mean-variance frontier, which averages across the two regimes. The mean variance efficient (tangengy) MVE portfolios for each frontier are also marked. The mean and variance have been annualized by multiplying by 12. In computing the MVE portfolios, we assume an annualized risk-free return of 7.67%, the average 1-month T-bill rate over the sample. We also mark the position of the World Market as an asterix.

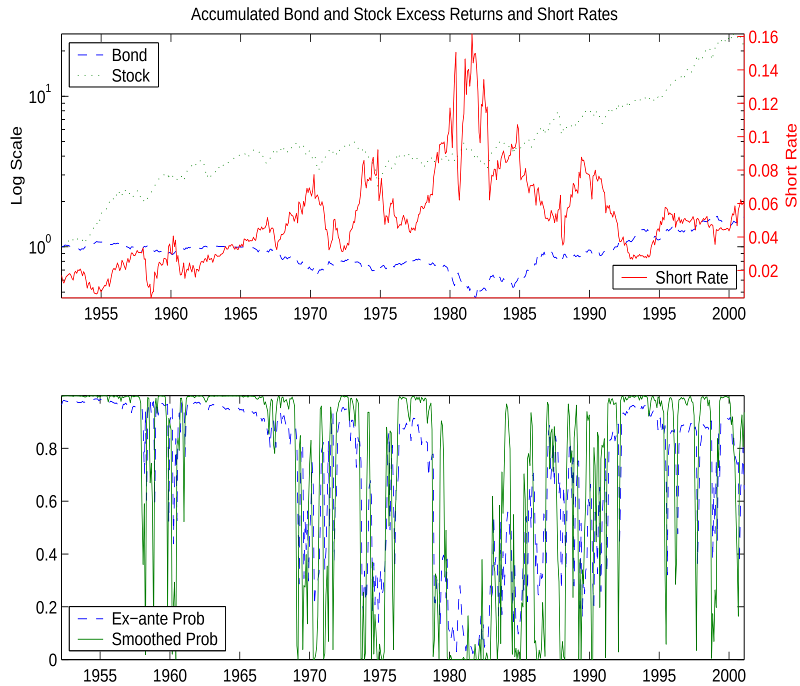
Figure 3: Out-of-Sample Wealth for the Market Timing Model



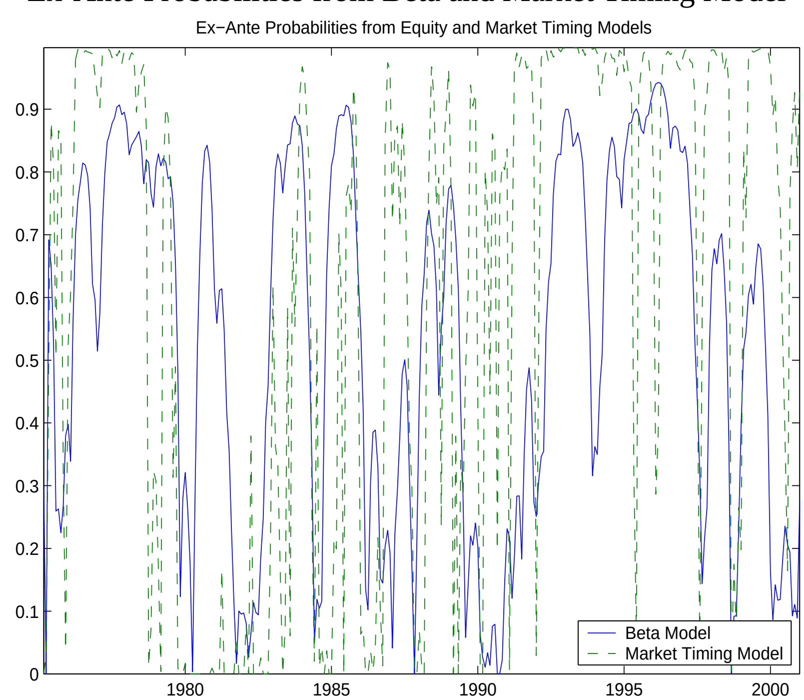
We show the out-of-sample wealth for the value of $1 at Jan 1985 for the Regime-Switching Beta Model, contrasted with a static mean-variance strategy, and the returns for the world and US portfolios.

Figure 4: Ex-Ante and Smoothed Proabilities of the Market-Timing Model

Market Timing Model

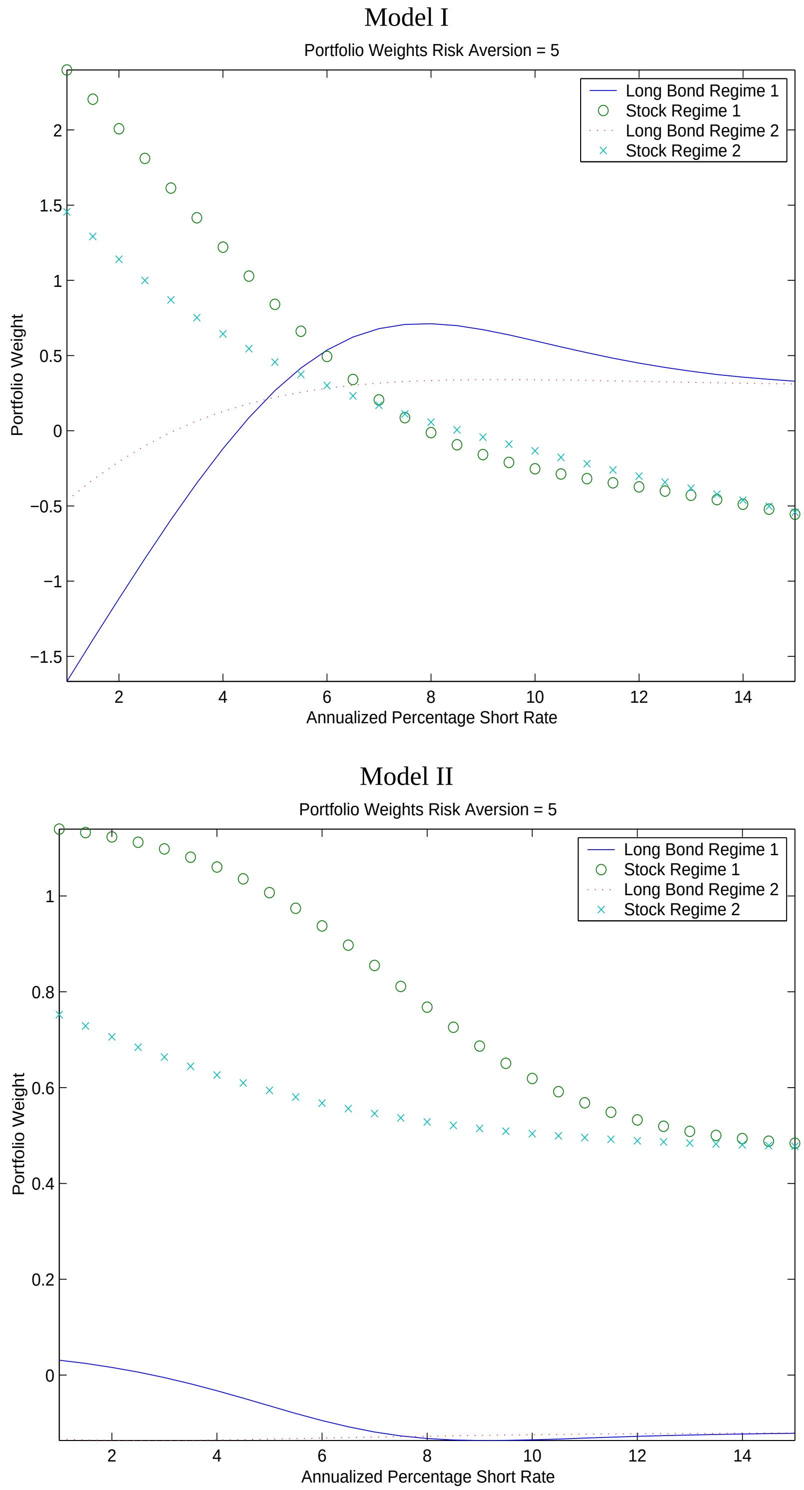


Ex-Ante Probabilities from Beta and Market Timing Model



The top panel shows the accumulated returns of $1 at Jan 1952, through the same until Dec 2000 of excess bond and stock returns, together with the monthly (annualized) risk-free rate in the top plot, and the ex-ante probabilities $p(s_t = 1|I_{t-1})$ and the smoothed probabilities $p(s_t = 1|I_T)$ of being in the first regime, where the first regime is the low variance regime in the bottom plot. The bottom panel compares the smoothed probabilities of the Regime-Switching Beta Model with the smoothed probabilities of the Market-Timing Model. The correlation between the ex-ante probabilities of the two models is 0.36.

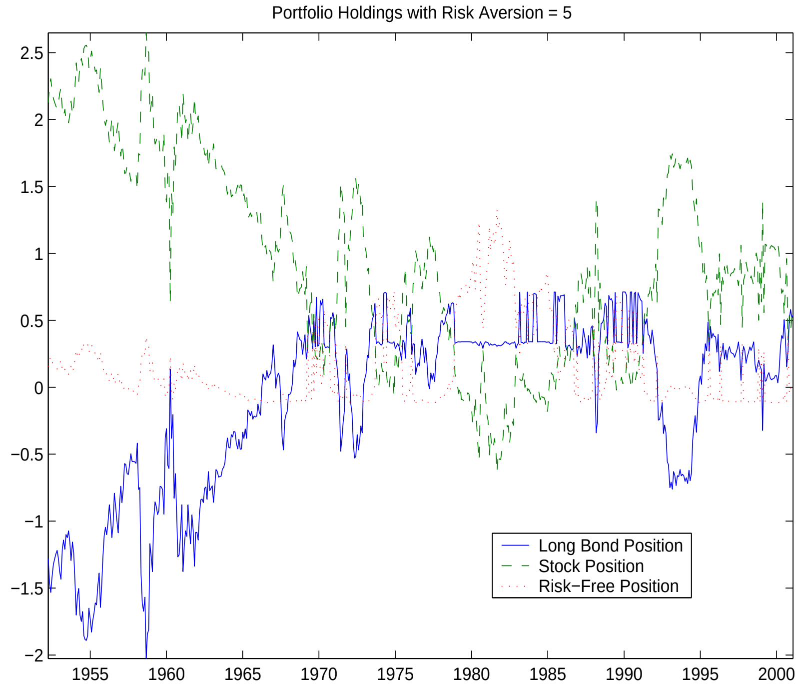
Figure 5: Asset Allocation of the Market-Timing Model as a Function of the Short Rate



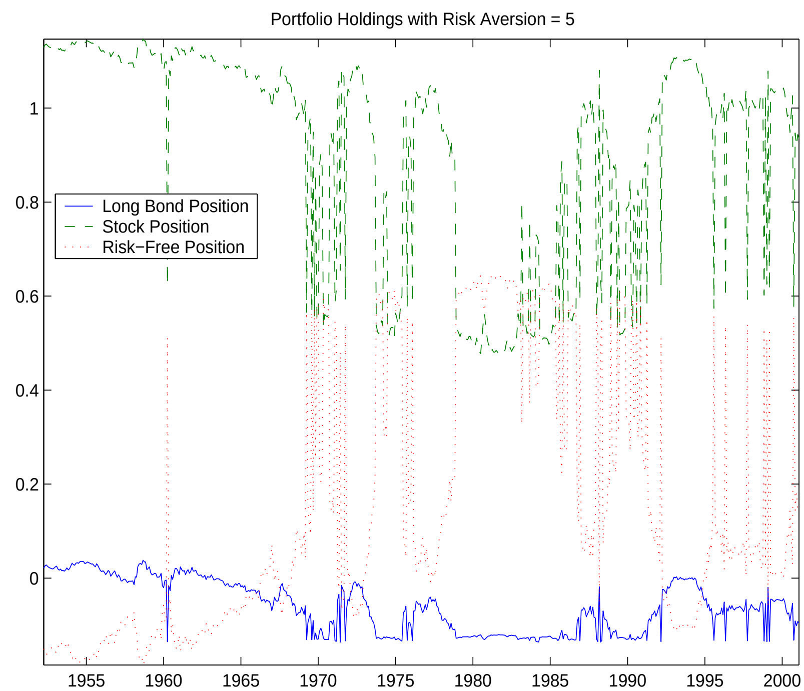
The figure plots the position in bonds and stocks as a function of the short rate for Model I (top plot) and Model II (bottom plot).

Figure 6: Asset Allocation of the Market Timing Model Across Time

Model I

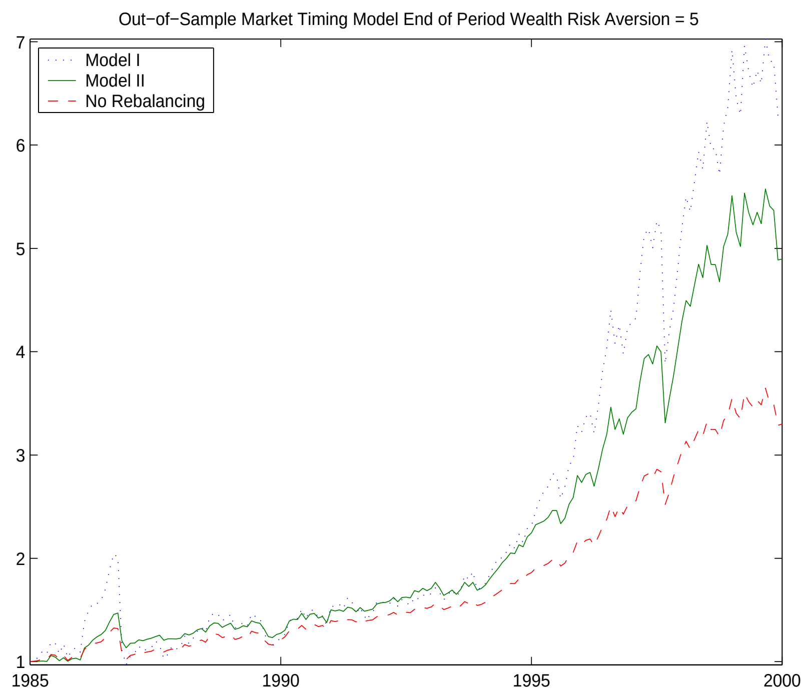


Model II



We show the position in bonds, stocks and the risk-free asset across time for the full-sample for Model I (top plot) and Model II (bottom plot).

Figure 7: Out-of-Sample Wealth for the Market Timing Model



We show the out-of-sample wealth for the value of $1 at Jan 1985 for the Market Timing Model I, Market Timing Model II, and the static mean-variance strategy.