International Asset Allocation With Regime Shifts

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Correlations between international equity market returns tend to increase in highly volatile bear markets, which has led some to doubt the benefits of international diversification. This article solves the dynamic portfolio choice problem of a U.S. investor faced with a time-varying investment opportunity set modeled using a regime-switching process which may be characterized by correlations and volatilities that increase in bad times. International diversification is still valuable with regime changes and currency hedging imparts further benefit. The costs of ignoring the regimes are small for all-equity portfolios but increase when a conditionally risk-free asset can be held.

In standard international portfolio choice models such as Sercu (1980) and Solnik (1974a), agents optimally hold the world market portfolio and a series of hedge portfolios to hedge against real exchange rate risk. From the perspective of these models, investors across the world display strongly home-biased asset choices. One popular argument often heard to rationalize the "home bias puzzle" relies on the asymmetric correlation behavior of international equity returns. A number of empirical studies document that correlations between international equity returns are higher during bear markets than during bull markets.[1] If the diversification benefits from international investing are not forthcoming at the time that investors need them the most (when their home market experiences a downturn), the strong case for international investing may have to be reconsidered.

Our goal is to formally evaluate this claim. To quantify the effect of these asymmetric correlations on optimal portfolio choice, we need a dynamic asset allocation model that accommodates time-varying correlations and volatilities. In the standard portfolio choice models and their empirical applications [French and Poterba (1991), Tesar and Werner (1995)], correlations

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[1] See, among others, Erb, Harvey, and Viskanta (1994), King, Sentana, and Wadhwani (1994), Longin and Solnik (1995, 2001), De Santis and Gerard (1997), and Das and Uppal (2001).

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and volatilities are constant. More specifically, our contribution consists of four parts.

First, we formulate a data-generating process (DGP) for international equity returns that reproduces the asymmetric correlation phenomenon. The asymmetric exceedance correlations documented by Longin and Solnik (2001) constitute the empirical benchmark we set for our model. We show that a regime-switching (RS) model reproduces the asymmetric exceedance correlations, whereas standard models, such as multivariate normal or asymmetric GARCH models, do not.

Second, we numerically solve and develop intuition on the dynamic asset allocation problem in the presence of regime switches for investors with constant relative risk aversion (CRRA) preferences. Here our contribution extends beyond international finance. There has recently been a resurgence of interest in dynamic portfolio problems where investment opportunity sets change over time.[2] In most of these articles, time variation in expected returns characterizes the changes in the investment opportunity set and the time variation is captured by a linear function of the state variables. In contrast, expected returns, volatilities, and correlations vary with the regime, rather than with state variables, in our benchmark model. Moreover, we combine predictability by state variables with regime switches in our DGPs.

Third, we investigate the portfolio choice of the investor for a number of different RS DGPs, horizons, and preference parameters. To characterize uncertainty in the portfolio allocations resulting from uncertainty in the parameters of the DGP, Kandel and Stambaugh (1996) and Barberis (2000) use a Bayesian setting, and Brandt (1999) estimates portfolio weights using a Euler equation approach and instruments. Instead, we characterize uncertainty in the portfolio choices from a classical econometric perspective, using the delta method. Our approach allows us to formally test for the presence of intertemporal hedging demands (the difference between the investor's one period ahead and long-horizon portfolio choice), for the presence of regime-dependent asset allocation for investors with different horizons, and for the statistical significance of international diversification.

Finally, we investigate the economic significance of our results and the claim in the initial paragraph. We attempt to quantify and contrast the utility cost (using the certainty equivalent notion) of (a) not being internationally diversified and (b) ignoring the occurrences of periods of higher volatility with higher correlations across all countries. It is quite conceivable that long-horizon investors need not worry about an occasional episode of high correlation, either because the effect on utility is minor or because they can temporarily rebalance away from international stocks, if these states of the world are somewhat predictable. In the latter case, their safe haven may be

[2] See Brennan, Schwartz, and Lagnado (1997), Balduzzi and Lynch (1999), Campbell and Viceira (1999, 2001), Liu (1999), and Barberis (2000). All these articles work in a domestic setting.

U.S. stocks or it may be cash. As a by-product of one of our setups, we put an economic value on the ability to hedge foreign exchange rate risk. In most models, we preclude this ability.

Our work is most closely related to Das and Uppal (2001), who consider portfolio selection when perfectly correlated jumps across countries affect international equity returns with constant short rates. Our RS DGPs produce a "normal" regime with low correlations, low volatilities, and a "bear" regime with higher correlations, higher volatilities, and lower conditional means. However, both regimes are persistent and such persistence cannot be captured by transitory jumps independent of equity returns. Furthermore, we consider the effect of regime changes on portfolio choice when short rates are time varying and predict returns, and we examine currency hedging demands.

To make the analysis tractable, we leave out many aspects of international asset allocation that may be important but may blur the focus of the article. Examples include transaction costs, inflation risk, cross-country informational differences, and human capital and labor. In line with recent studies on dynamic portfolio choice, our framework is a partial equilibrium setup with an exogenous return-generating process. Hence we ignore international equilibrium considerations.

The outline of the article is as follows. We start by formulating the general asset allocation problem in Section 1, and show how to numerically solve the problem with regime switching. We also demonstrate how to calculate tests of statistical significance and economic costs associated with taking nonoptimal portfolio strategies in our framework. In Section 2 we present a benchmark RS model which we use as our base case with all-equity portfolios. In Section 3 we introduce a conditionally risk-free asset under two scenarios. First, we examine the benchmark model with a constant risk-free rate. Second, we enrich the model by allowing the short rate to switch regimes and predict asset returns. In Section 4 we examine the benefits of currency hedging in the presence of regimes. Section 5 concludes.

1. Asset Allocation with Changes in Regimes

1.1 The general problem

Consider the following asset allocation problem. A U.S. investor facing a T-month horizon who rebalances her portfolio over N assets every month maximizes her expected end of period utility. The problem can be stated more formally as

$$\max_{\alpha_0, \ldots, \alpha_{T-1}} \mathrm{E}_0[U(W_T)], \tag{1}$$

subject to the constraint that the portfolio weights at time t must sum to 1, $\alpha_t'\mathbf{1} = 1$, where W_T is end of period wealth and $\alpha_0, \ldots, \alpha_{T-1}$ are the portfolio weights at time 0 (with T periods left), . . . , to time $T-1$ (with 1 period left).

There are no costs for short-selling or rebalancing. Wealth next period, W_{t+1}, is given by $W_{t+1} = R_{t+1}(\alpha_t)W_t$. The gross return on the portfolio, $R_{t+1}(\alpha_t)$, is

$$R_{t+1}(\alpha_t) = \sum_{j=1}^{N} \exp(y_{t+1}^j)\alpha_t^j \equiv \exp(y_{t+1})'\alpha_t, \tag{2}$$

where y_{t+1}^j is the logarithmic return on asset j in dollars (USD) from time t to $t+1$ and α_t^j is the proportion of the jth asset in the investor's portfolio at time t. We use CRRA, or isoelastic, utility:

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \tag{3}$$

with γ the investor's coefficient of risk aversion.

We concentrate on the investment problem of the U.S. investor and ignore intermediate consumption (or the investor is assumed to consume end of period wealth W_T). In effect, we take the savings decision to be exogenously specified. We choose the CRRA family of utility as it is a standard benchmark and enables comparison to earlier literature. In common with most empirical dynamic asset allocation articles in the literature, this approach does not address market equilibrium, so the investor is not necessarily the representative agent in the U.S. economy. We also do not consider the asset allocation problem faced by foreign agents.[3]

Using dynamic programming, we obtain the portfolio weights at each time t, for horizon $T-t$, by maximizing the (scaled) indirect utility:

$$\alpha_t^* = \arg\max_{\alpha_t} \mathrm{E}_t\left[Q_{t+1,T} W_{t+1}^{1-\gamma}\right], \tag{4}$$

where

$$Q_{t+1,T} = \mathrm{E}_{t+1}\left[(R_T(\alpha_{T-1}^*)\cdots R_{t+2}(\alpha_{t+1}^*))^{1-\gamma}\right], \tag{5}$$

and $Q_{T,T} = 1$. The first-order conditions (FOCs) of the investor's problem are

$$\begin{aligned} &\mathrm{E}_t\left[Q_{t+1,T} R_{t+1}^{-\gamma}(\alpha_t) \begin{pmatrix} (\exp(y_{t+1}^1) - \exp(y_{t+1}^N) \\ (\exp(y_{t+1}^2) - \exp(y_{t+1}^N) \\ \vdots \\ (\exp(y_{t+1}^{N-1}) - \exp(y_{t+1}^N)) \end{pmatrix} \right] \\ &\equiv \mathrm{E}_t\left[Q_{t+1,T} R_{t+1}^{-\gamma}(\alpha_t)\lambda_{t+1}\right] = 0, \end{aligned} \tag{6}$$

where λ_{t+1} is the vector of returns of assets 1 to $N-1$ in excess of asset N. We work both with all equity international portfolios, where the Nth asset

[3] For equilibrium approaches, see, among others, Solnik (1974a), Adler and Dumas (1983), and Dumas (1992).

is the return on U.S. equity, and also with the case of an investable risk-free asset, where the Nth asset is a one-period risk-free bond. The optimal portfolio weights α_t^* solve Equation (6). Note that α_t has $N-1$ degrees of freedom, as the weight in the Nth asset makes the portfolio weights sum to 1.

1.2 Introducing regime switching

Up to this point, no specific DGP has been assumed for the asset returns y_{t+1} and the setup of the problem is entirely general. In the special case of y_{t+1} i.i.d. across time, Samuelson (1969) shows that for CRRA utility the portfolio weights are constant ($\alpha_t^* = \alpha^*$, $\forall\, t$), and the T horizon problem becomes equivalent to solving the myopic $T = 1$ one-period problem in Equation (1). When returns are not i.i.d., then the portfolio weights can be broken down into a myopic and a hedging component [Merton (1971)]. The myopic component is the solution from solving the one-period problem. The hedging component results from the investor's desire to hedge against unfavorable changes in the investment opportunity set.

Suppose we introduce regimes $s_t = 1, \ldots, K$ into the DGP. At each time $t+1$, y_{t+1} is drawn from a different distribution, depending on which regime s_{t+1} is prevailing at time $t+1$. Following Hamilton (1989), the regimes s_t follow a Markov chain where the transition probabilities of going from regime i at time t to regime j at time $t+1$ are denoted by $p_{ij,\,t} = p(s_{t+1} = j | s_t = i, \mathcal{I}_t)$. Let $f(y_{t+1}|s_{t+1}) \equiv f(y_{t+1}|s_{t+1}; \mathcal{I}_t)$ denote the probability density function of y_{t+1} conditional on regime s_{t+1}. In our benchmark RS model, $f(y_{t+1}|s_{t+1})$ is a multivariate normal distribution and transition probabilities are constant ($p_{ij,\,t} = p_{ij}$). Conditional on s_t, the distribution of y_{t+1} is a mixture of normals. That is, the probability density function of y_{t+1} conditional on s_t, $g(y_{t+1}|s_t; \mathcal{I}_t) \equiv g(y_{t+1}|s_t)$, is given by

$$g(y_{t+1}|s_t = i; \mathcal{I}_t) = \sum_{j=1}^{K} p_{ij,\,t} \cdot f(y_{t+1}|s_{t+1} = j).$$

This allows the distribution to capture fat tails, stochastic persistent volatility, and other properties of equity returns.[4]

Assume that the regimes are known by the agent at time t.[5] With K regimes the random variable $Q_{t+1,\,T} = Q_{t+1,\,T}(s_{t+1})$ in Equation (5) may take on one of K values, one for each regime $s_{t+1} = 1, \ldots, K$. The optimal portfolio weights now become functions of the regime at time t, $\alpha_t^* = \alpha_t^*(s_t)$. Moreover, the investor wants to hedge herself against future regime switches.

[4] Liu (1999) and Chacko and Viceira (1999) analyze the effect of stochastic volatility on asset allocation, but not in the context of regime-switching models.

[5] If this assumption is weakened, the problem becomes considerably more difficult. All possible sample paths must be considered, so the state space increases exponentially, as agents must update their probabilities of being in a particular state at each time in a Bayesian fashion.

These intertemporal hedging demands cause portfolio weights for different horizons, $\alpha^*_\tau(s_t)$ for $t < \tau \leq T-1$, to differ from current portfolio weights $\alpha^*_t(s_t)$. Hence, even without instrument predictability of y_{t+1}, the asset allocation implications of regime switching are potentially important.

Under the alternative assumption where investors are uncertain about the regimes, the effects of regime-switching would be weaker since the regime-dependent solutions would deviate less from the i.i.d. solution. In this sense, the assumption of observable regimes is a worst-case scenario: if there are weak effects when the agents perfectly observe the regimes, the effects will be even smaller when learning about the regimes is introduced. If, however, there are strong regime effects when the regimes are observable, we cannot conclusively say anything about the regime effects when the agents are uncertain about the regimes.

For switching multivariate normal distributions the FOCs in Equation (6) do not have a closed-form solution. To our knowledge, the current state of analytical tools in continuous time also does not permit a solution for both state-dependent conditional means and covariances. Following Tauchen and Hussey (1991), we obtain a numerical solution to Equation (6) by quadrature. An M-point quadrature rule for the function $h(u)$, $u \in \mathbb{R}^n$, over the probability density $f(u)$ is a set of points $\{u_k\}$, $k = 1 \ldots M$, and corresponding weights $\{w_k\}$ such that

$$\int h(u) f(u) du \doteq \sum_{k=1}^{M} h(u_k) w_k.$$

For example, for the asset returns y_{t+1} at time $t+1$ in regime $s_{t+1} = j$, we use an M_j quadrature rule with points $\{y_{jk,t+1}\}$, $k = 1 \ldots M_j$ and corresponding weights $\{w_{jk,t+1}\}$. Using quadrature to determine $y_{jk,t+1}$ and $w_{jk,t+1}$ yields very accurate approximations for Gaussian i.i.d. distributed returns y_{t+1}. Balduzzi and Lynch (1999) and Campbell and Viceira (1999) note that as few as five quadrature points suffice.

Consider the one-period problem at $T-1$. For $s_{T-1} = i$ the FOCs are approximated by

$$\begin{aligned} &\mathrm{E}_{T-1}\left[R_T^{-\gamma}(\alpha_{i,T-1})\lambda_T | s_{T-1} = i\right] \\ &\quad = \sum_{j=1}^{K} p_{ij,T-1} \mathrm{E}\left[R_T^{-\gamma}(\alpha_{i,T-1})\lambda_T | s_T = j\right] \\ &\quad \doteq \sum_{j=1}^{K} p_{ij,T-1} \left(\sum_{k=1}^{M_j} (\exp(y_{jk,T})' \alpha_{i,T-1})^{-\gamma} \lambda_{jk,T}\, w_{jk,T} \right) = 0, \end{aligned} \tag{7}$$

where $\alpha_{i,T-1} \equiv \alpha_{T-1}(s_{T-1} = i)$ and

$$y_{jk,T} = \begin{pmatrix} y_{jk,T}^1 \\ y_{jk,T}^2 \\ \vdots \\ y_{jk,T}^N \end{pmatrix} \quad \text{and} \quad \lambda_{jk,T} = \begin{pmatrix} \exp(y_{jk,T}^1) - \exp(y_{jk,T}^N) \\ \exp(y_{jk,T}^2) - \exp(y_{jk,T}^N) \\ \vdots \\ \exp(y_{jk,T}^{N-1}) - \exp(y_{jk,T}^N) \end{pmatrix}.$$

The optimal portfolio weights $\alpha_{i,T-1}^*$ are the solution to Equation (7), which can be obtained by a nonlinear root solver. Since λ_T is an $N-1$ vector, Equation (7) describes a system of $N-1$ nonlinear equations in $N-1$ unknowns, the first $N-1$ elements of $\alpha_{i,T-1}$. Each regime $s_{T-1} = i$ has a different set of optimal portfolio weights $\alpha_{i,T-1}^*$.[6]

Note that the term in brackets in Equation (7) represents the normal FOC for CRRA utility conditional on being in regime $s_T = j$ at horizon T. Introducing regimes into the asset allocation problem makes Equation (7) a linear combination of FOC for each different regime, where the weights are the transition probabilities known at time t. This makes the asset allocation solution very tractable for switching multivariate normal returns.

To start the dynamic programming algorithm, define the scalar $Q_{i,T-1,T} \equiv Q_{T-1,T}(s_{T-1} = i)$ as

$$\begin{aligned} Q_{i,T-1,T} &= \mathrm{E}_{T-1}\left[R_T^{1-\gamma}(\alpha_{T-1}^*)|s_{T-1} = i\right] \\ &\doteq \sum_{j=1}^{K} p_{ij,T-1}\left(\sum_{k=1}^{M_j}(\exp(y_{jk,T})'\alpha_{i,T-1}^*)^{1-\gamma} w_{jk,T}\right). \end{aligned} \tag{8}$$

For K regimes, we have only K state variables, $Q_{i,t+1,T}$, which must be tracked at each horizon $T-t$, making the problem computationally very tractable. We solve the $T-2$ problem for each regime $s_{T-2} = i$ by finding the roots of

$$\begin{aligned} &\mathrm{E}_{T-2}[Q_{T-1,T}R_{T-1}^{-\gamma}(\alpha_{i,T-2})\lambda_{T-1}|s_{T-2}=i] \\ &\quad \doteq \sum_{j=1}^{K} p_{ij,T-2}\left(\sum_{k=1}^{M_j} Q_{j,T-1,T}(\exp(y_{jk,T-1})'\alpha_{i,T-2})^{-\gamma}\lambda_{jk,T-1}w_{jk,T-1}\right) = 0. \end{aligned} \tag{9}$$

We continue this process for $t = T-3$ onto $t = 0$.

When the return distributions of the assets depend on instruments z_t at time t, the distribution of the returns is a function of both the regime and the realization of the instrument at time t. In this case, the probability density function of y_{t+1} conditional on s_{t+1} becomes $f(y_{t+1}|s_{t+1}, z_t) \equiv f(y_{t+1}|s_{t+1}; \mathcal{I}_t)$.

[6] For portfolios which are not leveraged (so wealth is always positive), an interior solution to Equation (7) is guaranteed by concavity. In our solutions we do not impose any constraints on short sales.

The probability density function of y_{t+1} conditional on s_t, $g(y_{t+1}|s_t, z_t) \equiv g(y_{t+1}|s_t; \mathcal{I}_t)$, is found by integrating over all possible values for $s_{t+1} = j$:

$$g(y_{t+1}|s_t = i, z_t) = \sum_{j=1}^{K} p_{ij,t} \cdot f(y_{t+1}|s_{t+1} = j, z_t).$$

In this case we need to track the regime variable s_t and the realizations of the predictors z_t. To do this we construct a discrete Markov chain in each regime to approximate $f(y_{t+1}|s_{t+1}, z_t)$ and the distribution of z_t. These are then combined to approximate $g(y_{t+1}|s_t, z_t)$. In this setting the portfolio weights now become a function both of the current regime s_t and the instruments z_t, so $\alpha_t^* = \alpha_t^*(s_t, z_t)$. The scaled indirect utility $Q_{t+1,T}$ also becomes a function of both the regime and the predictor variables, $Q_{t+1,T} = Q_{t+1,T}(s_{t+1}, z_{t+1})$. Appendix B provides further details on the quadrature methods we employ in this case.

1.3 How important is regime switching?

Introducing regimes into the asset allocation problem has the potential to cause investors to wildly alter their portfolio allocations across regimes, and to induce intertemporal hedging demands, making the investor facing a T-period horizon hold substantially different portfolio weights from the myopic investor. We wish to test statistically and economically whether these effects are large under RS when realistic RS DGPs have been fitted to real data. These tests are more than interesting empirical exercises: if the asset allocations are similar across regimes, then in practice investors may not go to the trouble of rebalancing, especially if transactions costs are high. If intertemporal hedging demands are small, then investors may lose very little in solving a simple one-period problem at all horizons rather than solving the rather more complex dynamic problem. If there is a bad regime where international equity returns provide fewer diversification benefits, investing overseas may not be of benefit for investors.

1.3.1 Statistical tests. To formulate statistical tests we must derive standard errors for the portfolio weights. Suppose that the parameters of the RS process, $\hat{\theta}$, possess an asymptotic distribution $N(\theta_0, \Omega)$, where θ_0 is the vector of the true population parameters. The portfolio weights $\alpha_t^*(s_t)$ are implicitly defined by the FOCs in Equation (6). We suppress the dependence on $s_t = 1, \ldots, K$. Denote these FOCs for period t, horizon $T - t$, as $\Phi_t(\theta, \alpha)$, where $\Phi_t : \theta \times \alpha \to \mathbb{R}^{N-1}$.[7]

[7] In the case of regime switching and predictability, $\alpha_t^* = \alpha_t^*(s_t, z_t)$ and the FOCs become an implicit function dependent on z_t, that is, $G = G_{t,z_t}(\theta, \alpha)$. However, the analysis with a predictive variable is similar to the case presented here.

The FOCs implicitly define α_t^* as the solution to $\Phi_t(\hat{\theta}, \alpha_t^*) = 0$. Let α_{t0}^* satisfy $\Phi_t(\theta_0, \alpha_{t0}^*) = 0$, so α_{t0}^* are the portfolio weights at the population parameters. Assume the determinant

$$\det\left(\left. \frac{\partial \Phi_t}{\partial \alpha} \right|_{(\theta=\theta_0,\, \alpha=\alpha_{t0}^*)} \right) \neq 0. \tag{10}$$

The implicit function theorem guarantees the existence of a function φ such that $\Phi_t(\theta_0, \varphi(\theta_0)) = 0$, where

$$D = \left. \frac{\partial \varphi}{\partial \theta} \right|_{\theta=\theta_0} = \left\{ -\left(\frac{\partial \Phi_t}{\partial \alpha} \right)^{-1} \frac{\partial \Phi_t}{\partial \theta} \right\}_{(\theta=\theta_0,\, \alpha=\alpha_{t0}^*)} \tag{11}$$

is well defined. We apply the standard delta-method to obtain the asymptotic distribution of α_t^* as

$$\alpha_t^* \overset{a}{\sim} N(\varphi(\theta_0), D\Omega D'). \tag{12}$$

In practice, we compute numerical gradients as follows. For the estimated parameter vector $\hat{\theta}$ we solve the FOC to find the optimal portfolio weights $\hat{\alpha}_t^*$. We change the ith parameter in $\hat{\theta}$ by $\epsilon = 0.0001$ and recompute the new portfolio weights $\hat{\alpha}_t^{*\epsilon}$. The ith column of D is given by $(\hat{\alpha}_t^{*\epsilon} - \hat{\alpha}_t^*)/\epsilon$.

We focus on three main tests. First, we test for the significance of international diversification by testing whether the U.S. weight in regime s_t is significantly different from one. This test is important given that the results in Britten-Jones (1999) suggest that the evidence for international diversification may be statistically insignificant. Second, for a given t, we test if the portfolio weights for $s_t = i$ and $s_t = j$ are statistically different from each other, or from i.i.d. portfolio weights without regime switching. This is a test of regime effects. Finally, to test intertemporal hedging demands for horizon T, we may define an implicit function $\Phi = (\Phi_1' \; \Phi_T')'$ which stacks the FOCs for the myopic problem and the horizon T problem. This allows a test of hedging demands where first-period portfolio weights are equal to horizon T portfolio weights: $\alpha_0(s_t) = \alpha_{T-1}(s_t)$.

1.3.2 Economic significance. We wish to calculate the utility loss or monetary compensation required for an investor to use nonoptimal weights $\{\alpha^+\}$ instead of the optimal weights $\{\alpha^*\}$ for our RS DGP. For example, an investor may have to use nonoptimal weights, as she may not be allowed by external constraints to use forward derivatives to hedge currency risk, or even invest internationally. Similarly, an investor may use portfolio weights, thinking returns are i.i.d. when in fact the true DGP has regimes. We would like to compute the economic loss that results from holding these nonoptimal portfolios instead of using the optimal one.

We find the amount of wealth $\bar{w}$ required to compensate an investor for using $\{\alpha^+\}$ in place of $\{\alpha^*\}$ for a T-period horizon. Formally, this is given by the value of $\bar{w}$ which solves

$$\mathrm{E}_0[U(W_T^*|W_0 = 1)] = \mathrm{E}_0[U(W_T^+|W_0 = \bar{w})]. \tag{13}$$

Since CRRA utility is homogeneous in initial wealth and since $\mathrm{E}[U(W_T^\dagger|W_0 = 1)] = Q_{0,T}^\dagger/(1-\gamma)$ for $\dagger = *, +$, it follows that

$$\bar{w} = \left(\frac{Q_{0,T}^*}{Q_{0,T}^+}\right)^{\frac{1}{1-\gamma}}. \tag{14}$$

We express the compensation required in cents per dollar of wealth $w = 100 \times (\bar{w} - 1)$. That is, w is the actual monetary payment a risk-averse investor must receive in order to put \$1 of her wealth in the suboptimal portfolio rather than the optimal one. Equivalently, w is the percentage increase in the certainty equivalent from moving from strategy $\{\alpha^+\}$ to the optimal strategy $\{\alpha^*\}$. Campbell and Viceira (1999, 2001) and Kandel and Stambaugh (1996), among others, use changes in certainty equivalents in the context of asset allocation analysis.

2. Asset Allocation Under Regime Shifts

This section examines asset allocation with the set of equity returns $y_{t+1} = (y_{t+1}^{us}, y_{t+1}^{uk})'$ and $y_{t+1} = (y_{t+1}^{us}, y_{t+1}^{uk}, y_{t+1}^{ger})'$, where us, uk, ger denote unhedged U.S., U.K., and German equity returns, respectively. Appendix A contains a description of the data. In this section we concentrate on a benchmark no predictor model with regime shifts and examine asset allocation implications for all equity portfolios. Sections 3 and 4 examine more complex data-generating processes, including the introduction of a risk-free asset.

2.1 The benchmark no predictor model

Our benchmark model can be written as

$$y_{t+1} = \mu(s_{t+1}) + \Sigma^{\frac{1}{2}}(s_{t+1})\epsilon_{t+1}, \tag{15}$$

where the regimes s_{t+1} follow a two-state Markov chain with transition matrix

$$\begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}$$

and the transition probabilities, $P = p(s_{t+1} = 1|s_t = 1; \mathcal{I}_t)$ and $Q = p(s_{t+1} = 2|s_t = 2; \mathcal{I}_t)$, are constant.[8]

[8] To estimate our RS models we use the Bayesian algorithm of Hamilton (1989) and Gray (1996).

For the United States–United Kingdom, we estimate two other RS models which nest Equation (15) to test for robustness. Equation (15), which we denote as benchmark model I, assumes that the regimes in each country are perfectly correlated. To investigate if the United Kingdom undergoes regime switches different from the United States we introduce an extension, benchmark model II, with two regime variables s_t^{us} and s_t^{uk}. Model II has the feature that the regimes in the United States and United Kingdom do not have to be perfectly correlated. Generally there would be $2^2 = 4$ regimes for the bivariate system for two regimes of each country implying a 4×4 probability transition matrix. To preserve parsimony we assume that conditional on the U.S. regime, the U.K. process is a simple mixture of normals. That is, we let

$$\begin{aligned} p\left(s_{t+1}^{us} = 1 | s_t^{us} = 1\right) &= P \qquad p(s_{t+1}^{uk} = 1 | s_{t+1}^{us} = 1) = \alpha \\ p\left(s_{t+1}^{us} = 2 | s_t^{us} = 2\right) &= Q \qquad p(s_{t+1}^{uk} = 2 | s_{t+1}^{us} = 2) = \beta \end{aligned} \tag{16}$$

This parameterization implies that the U.S. transition probabilities P and Q are still the driving variables of the system and allows the U.S. and U.K. regimes to be dissimilar with only two additional parameters. Further, the correlation of the United States and United Kingdom depends only on the U.S. regime.

Finally we test for the presence of RS ARCH effects in benchmark model III for the United States–United Kingdom, by specifying the covariance $\Sigma(s_{t+1})$ as

$$\begin{aligned} \Sigma(s_{t+1}) &= C(s_{t+1})' C(s_{t+1}) + B(s_{t+1})' u_t u_t' B(s_{t+1}) \\ u_t &= y_t - \mathrm{E}_{t-1}(y_t) \\ \mathrm{E}_{t-1}(y_t) &= \sum_{j=1}^{2} p(s_t = j | \mathcal{I}_{t-1}) \mu(s_t = j), \end{aligned} \tag{17}$$

where $p(s_t = j | \mathcal{I}_{t-1})$ are ex ante probabilities. This model is estimated following a special case in Gray (1996). Hamilton and Susmel (1994) and Ramchand and Susmel (1998) estimate related models.

2.2 Estimation results

2.2.1 Parameter estimates. We report estimation results for the U.S.–U.K. and U.S.–U.K.–Germany systems in Tables 1 and 2. We turn first to the U.S.–U.K. system in Table 1. Likelihood ratio tests of model I versus model II and C fail to reject (p-values of 0.9950 and 0.9853, respectively). Moreover, the parameters α and β in Equation (16) are estimated to be one. This lends support to the simple, but parsimonious DGP of the benchmark model: the United States and United Kingdom face the same regime shifts and the stochastic volatility generated by the benchmark RS model suffices to capture the time variation in monthly equity return volatilities.

Table 1
U.S.–U.K. benchmark models

		Model I Basic model		Model I Restricted $\mu_1 = \mu_2$		Model II Imperfect correlation			Model III RS ARCH	
		Estimate	Std. error	Estimate	Std. error	Estimate	Std. error		Estimate	Std. error
	P	0.8552	0.0691	0.8546	0.0698	0.8556	0.0690	P	0.8555	0.0702
	Q	0.9804	0.0108	0.9818	0.0100	0.9804	0.0107	Q	0.9808	0.0108
U.S.	μ_1	−1.2881	1.1874	1.1613	0.2198	−1.2880	1.1902	μ_1^{us}	−0.6439	1.5659
	μ_2	1.2829	0.2287	$=\mu_1$		1.2828	0.8629	μ_2^{us}	1.3668	0.2353
	σ_1	7.0376	0.8629	7.5064	0.9515	7.0374	0.8629	μ_1^{uk}	−1.3287	2.5763
	σ_2	3.7689	0.1677	3.7917	0.1654	3.7691	0.1677	μ_2^{uk}	1.3341	0.3191
U.K.	α					1.0000	0.0023	$C_1[1,1]$	4.3372	1.5229
	β					1.0000	0.0005	$C_1[1,2]$	1.5160	2.9402
	μ_1	−0.6921	2.2627	1.2488	0.3090	−0.7253	2.2696	$C_1[2,2]$	11.3426	4.4303
	μ_2	1.3040	0.3141	$=\mu_1$		1.3043	0.3141	$C_2[1,1]$	3.6269	0.1712
	σ_1	13.7177	1.7558	14.0748	1.8432	13.7184	1.7560	$C_2[1,2]$	0.9876	0.1532
	σ_2	5.2194	0.2376	5.2470	0.2409	5.2197	0.2375	$C_2[2,2]$	5.0538	0.2560
	ρ_1	0.6097	0.1022	0.6181	0.1032	0.6096	0.1022	$B_1[1,1]$	−1.2763	0.7584
	ρ_2	0.4455	0.0496	0.4480	0.0491	0.4455	0.0496	$B_1[1,2]$	−1.5202	1.9194
								$B_1[2,1]$	0.3915	0.2907
								$B_1[2,2]$	0.7403	0.7284
								$B_2[1,1]$	0.0839	0.1773
								$B_2[1,2]$	0.2426	0.2565
								$B_2[2,1]$	0.0591	0.1601
								$B_2[2,2]$	−0.0658	0.1775
	RCM	10.77		10.50		10.11			11.74	
	log likelihood	−1992.31		−1994.46		−1992.30			−1990.46	

Basic model I: Wald tests of parameter equality

	U.S.	U.K.	Joint
Means $\mu_1 = \mu_2$	0.0351	0.3858	0.0975
Volatilities $\sigma_1 = \sigma_2$	0.0002	0.0000	0.0000
U.S.–U.K. correlation $\rho_1 = \rho_2$	0.1556		

U.S., U.K. refer to monthly equity returns with the subscripts indicating which regime. RCM refers to the Ang and Bekaert (2002) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the smoothed regime probability $p(s_t = 1|\mathcal{I}_T)$. Lower RCM values denote better regime classification. The basic model I is a simple bivariate RS model in Equation (15). The restricted $\mu_1 = \mu_2$ model sets the conditional mean constant across regimes. Model II uses transition probabilities specified in Equation (16). Model III, the RS ARCH model, parameterizes the conditional volatility as in Equation (17). The $A[i, j]$ notation refers to the element in row i, column j of matrix A. A likelihood ratio test for the basic model A versus the restricted $\mu_1 = \mu$ model I gives a p-value of 0.1165. A likelihood ratio test of model I versus model II produces a p-value of 0.9950. A likelihood ratio test of model I versus model III produces a p-value of 0.9853. Wald tests show p-values of parameter equality for each country across regimes $s_t = 1, 2$. Subscripts denote the regime.

In Tables 1 and 2 we find the following pattern in international equity returns. In one regime the equity returns have a lower conditional mean, much higher volatility, and are more highly correlated. We shall refer to this regime as "regime 1." In the second regime, equity returns have higher conditional means, lower volatility, and are less correlated. The RS models imply that volatility and correlations increase together simultaneously, a phenomenon also documented empirically by Karolyi and Stulz (1996). The strongest differentiating effect across the regimes for both systems is volatility. We reject the equality of volatilities across regimes at a 0.01% significance level, as is

Table 2
U.S.–U.K.–German benchmark models

		Basic model		Restricted $\mu_1 = \mu_2$	
		Estimate	Std. error	Estimate	Std. error
	P	0.8305	0.0760	0.8375	0.0714
	Q	0.9444	0.0269	0.9503	0.0258
U.S.	μ_1	−0.1751	0.7966	1.1467	0.2177
	μ_2	1.3546	0.2399	$= \mu_1$	
	σ_1	6.2463	0.6185	6.4124	0.6490
	σ_2	3.4655	0.1879	3.5086	0.1909
U.K.	μ_1	0.8124	1.3480	1.1412	0.3143
	μ_2	1.1492	0.3476	$= \mu_1$	
	σ_1	10.9400	1.1577	11.0689	1.1928
	σ_2	4.7864	0.2736	4.8285	0.2716
Germany	μ_1	0.3473	1.2073	1.0863	0.3040
	μ_2	1.1667	0.3735	$= \mu_1$	
	σ_1	8.3056	0.7395	8.3744	0.7670
	σ_2	4.7819	0.3206	4.8250	0.3131
	$\rho_1(us, uk)$	0.5994	0.0751	0.5996	0.0778
	$\rho_2(us, uk)$	0.4056	0.0607	0.4024	0.2669
	$\rho_1(us, ger)$	0.4540	0.1009	0.4627	0.1050
	$\rho_2(us, ger)$	0.2620	0.0742	0.2669	0.0726
	$\rho_1(uk, ger)$	0.4523	0.0917	0.4522	0.0940
	$\rho_2(uk, ger)$	0.4261	0.0622	0.4285	0.0609
	RCM	24.44		24.54	
	log likelihood	−3011.36		−3013.52	

Basic model: Wald tests for parameter equality

	U.S.	U.K.	Germany	Joint
Means $\mu_1 = \mu_2$	0.0747	0.8180	0.5559	0.2285
Volatilities $\sigma_1 = \sigma_2$	0.0000	0.0000	0.0000	0.0000
	U.S.–U.K.	U.S.–Germany	U.K.–Germany	Joint
Correlations $\rho_1 = \rho_2$	0.0586	0.1709	0.8246	0.2340

The basic model is a RS trivariate normal model of U.S., U.K., German monthly equity returns as in Equation (15). The restricted $\mu_1 = \mu_2$ model imposes the same conditional means across regimes. RCM refers to the Ang and Bekaert (2002) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the smoothed regime probability $p(s_t = 1|\mathcal{I}_T)$. Lower RCM values denote better regime classification. A likelihood ratio test of $\mu_1 = \mu_2$ in the basic model produces a p-value of 0.2289. Wald tests list p-values of parameter equality across regimes $s_t = 1, 2$. Subscripts denote the regime.

true in previous studies by Hamilton and Lin (1996) and Ramchand and Susmel (1998). The evidence of different correlations across the regimes is not particularly strong. The U.S.–U.K. correlations are borderline significantly different in the U.S.–U.K.–Germany model, but the p-value for the Wald equality test is 15% in the U.S.–U.K. system. We fail to reject that correlations for the United Kingdom and Germany are constant across regimes.

In Table 1, model I is estimated both with unconstrained regime-dependent means, and with means imposed equal across the regimes. We denote this second case as $\mu_1 = \mu_2$, where this notation is taken to mean $\mu^j(s_t = 1) = \mu^j(s_t = 2)$ for each country j. In both these estimations, all other parameter estimates are very similar. Model A with $\mu_1 = \mu_2$ implies that the expected duration of the first regime is 6.9 months, while the expected duration of the second regime is 4.25 years. The stable probabilities implied by the transition probability matrix are 0.1194 and 0.8806 for regimes 1 and 2, respectively. It is well known that conditional means are hard to estimate. With regime switches, as far fewer observations are inferred to belong to regime 1, estimates of the conditional mean in that regime are hard to pin down, leading to large standard errors. A likelihood ratio test of unconstrained versus constrained means across regimes fails to reject with a p-value of 0.1165. Table 1 also reports the Ang and Bekaert (2002) regime classification measure (RCM), which improves slightly when this restriction is imposed.[9] For this reason our analysis in Section 2.3 of models with unconstrained means must be carefully interpreted because the poor precision of the estimates of the conditional means affects the asset allocation inference.

Table 2 shows that we fail to reject the constraint of equal means across regimes for the U.S.–U.K.–Germany system (p-value 0.2289). Similar to the U.S.–U.K. system, regime classification improves slightly when this restriction is imposed. The conditional mean of the United Kingdom in the first regime changes sign, compared to the U.S.–U.K. system, but the change is well within one standard error, as the standard errors are large.

2.2.2 Reproducing Longin–Solnik (2001) exceedance correlations. In this section we demonstrate that despite the poor statistical significance levels for the difference in correlations across regimes, the RS models still pick up the higher correlations during extreme downturn events. To this end, we repeat Longin and Solnik's (2001) analysis of exceedance correlations in Figure 1. Consider observations $\{(y_1, y_2)\}$ drawn from a bivariate variable $Y = (y_1, y_2)$. Suppose the exceedance level θ is positive (negative). We take observations where values of y_1 and y_2 are greater (or less) than $\theta\%$ of their empirical means. That is, we select the subset of observations $\{(y_1\, y_2)|y_1 \geq (1+\theta)\bar{y}_1 \text{ and } y_2 \geq (1+\theta)\bar{y}_2\}$ for $\theta \geq 0$ and $\{(y_1\, y_2)|y_1 \leq (1+\theta)\bar{y}_1 \text{ and } y_2 \leq (1+\theta)\bar{y}_2\}$ for $\theta \leq 0$, where $\bar{y}_j$ is the mean of y_j. The correlation of this subset of points is termed the exceedance correlation.

The solid line in Figure 1 shows that the exceedance correlations of U.S.–U.K. returns in the data exhibit a pronounced asymmetric pattern, with negative exceedance correlations higher than positive exceedance correlations. The other three lines in Figure 1 represent the exceedance correlations computed on simulated samples of 100,000 observations from three models. First, the dotted-dashed line are exceedance correlations implied by a bivariate normal distribution calibrated to the data. It clearly cannot reproduce the Longin–Solnik exceedance correlations implied by the data, since a normal distribution implies symmetric exceedance correlations. Furthermore, for a normal distribution, the correlation conditional on exceedances tends to zero

[9] The RCM is given by $\text{RCM} = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the smoothed regime probability $p(s_t = 1|\mathcal{I}_T)$. Lower RCM values indicate better regime classification.

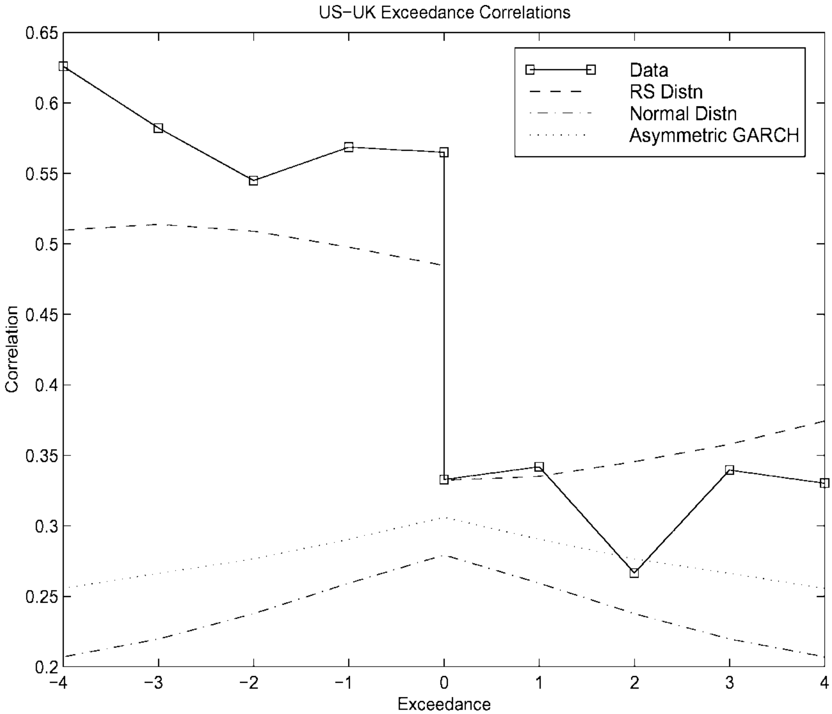


Figure 1
Calculates correlations of the U.S.–U.K., conditioning on exceedances θ. U.K. returns are in USD. We represent exceedances in percentages away from the empirical mean, so for an exceedance $\theta = +2$, we calculate the correlation conditional on observations greater than three times the U.S. mean, and three times the mean of the U.K. For $\theta = -2$, we calculate the correlation conditional on observations less than -1 times the US mean, and -1 times the mean of the U.K. The implied exceedance correlations from the U.S.–U.K. benchmark RS model are shown in dashed lines, and the correlations from the data represented by squares. The exceedance correlation for a normal distribution and an asymmetric GARCH model calibrated to the data are drawn in dotted-dashed and dotted lines, respectively.

as $\theta \to \pm\infty$. Second, the dotted line shows exceedance correlations from an asymmetric bivariate GARCH model calibrated to the data.[10] This model also fails to match the empirical exceedance correlation asymmetry. Finally, the dashed line represents the benchmark U.S.–U.K. RS model which captures a large part of the increasing correlations conditional on large negative returns.

The strong performance of the RS model in reproducing the Longin–Solnik figure derives from its ability to account for both persistence in conditional means and second moments. A draw from regime 1 this period (where conditional means are low, and correlations and volatility are high) makes a bad

[10] The asymmetric bivariate GARCH model we estimate on U.S.–U.K. returns is $y_{t+1} = \mu + \epsilon_{t+1}$, $\epsilon_{t+1} \sim N(0, H_{t+1})$, $H_{t+1} = C'C + A'H_tA' + B'\epsilon_t\epsilon_t'B + L'\eta_t\eta_t'L$, and $\eta_t = \epsilon_t \odot \mathbf{1}_{(\epsilon_t<0)}$, with A, B, C, and L lower triangular matrices, $\mathbf{1}_{(\epsilon_t<0)}$ a vector of one's or zero's depending on whether the individual elements of ϵ_t are negative, and $\odot$ represents element-by-element multiplication. In estimation, the parameters in L are significant. Kroner and Ng (1998) and Bekaert and Wu (2000) estimate similar models.

draw for the next period more likely. The GARCH model fails to reproduce the Longin–Solnik figure because it only captures persistence in second moments. Ang and Chen (2001) show that a model which combines normally distributed returns with transitory negative jumps, as in Das and Uppal (2001), also fails to reproduce the Longin–Solnik figure.

2.2.3 Interpretation of the regime-switching process as a momentum process. The benchmark models estimated in Tables 1 and 2 can be further interpreted using the framework in Samuelson (1991). Samuelson works with two assets, cash and a risky asset. The risky asset follows a Markov chain where the returns can be "low" or "high." He defines a "rebound" process, or mean-reverting process, as having a transition matrix which has a higher probability of transitioning to the alternative state than staying in the current state. Samuelson shows that with a rebound process, risk-averse investors *increase* their exposure to the risky asset as the horizon increases. That is, under rebound, long-horizon investors are more tolerant of risky assets than short-horizon investors.

Our setting is the opposite of a rebound process. Our transition matrix for the model with $\mu_1 = \mu_2$ is

$$\begin{pmatrix} \begin{matrix} 0.8546 \\ (0.0698) \end{matrix} & 0.1454 \\ & \\ 0.0182 & \begin{matrix} 0.9818 \\ (0.0100) \end{matrix} \end{pmatrix}, \tag{18}$$

with standard errors in parentheses. Samuelson calls such a process a "momentum" process: it is more likely to continue in the same state rather than transition to the other state. Under a momentum process, risk-averse investors want to *decrease* their exposure to risky assets as horizon increases. Intuitively, long-run volatility is smaller under a rebound process than under a momentum process (with the same short-run volatility).

The persistence of the regimes implies that we should see investors preferring *fewer* risky assets with longer horizons. In our benchmark model with $\mu_1 = \mu_2$ the safer asset is U.S. equity. For the United States and United Kingdom, Table 1 shows that the covariance matrix for monthly returns in regime 1 is

$$\Sigma_1 = \begin{pmatrix} \begin{matrix} 7.5064^2 \\ (0.9515) \end{matrix} & \begin{matrix} 0.6181 \times 7.5064 \times 14.0748 \\ (0.1032) \times (0.9515) \times (1.8432) \end{matrix} \\ & \\ \begin{matrix} 0.6181 \times 7.5064 \times 14.0748 \\ (0.1032) \times (0.9515) \times (1.8432) \end{matrix} & \begin{matrix} 14.0748^2 \\ (1.8432) \end{matrix} \end{pmatrix}, \tag{19}$$

with standard errors in parentheses, and the covariance matrix for regime 2 is

$$\Sigma_2 = \begin{pmatrix} \begin{matrix} 3.7917^2 \\ (0.1654) \end{matrix} & \begin{matrix} 0.4480 \times 3.7917 \times 5.2470 \\ (0.0491) \times (0.1654) \times (0.2409) \end{matrix} \\ & \\ \begin{matrix} 0.4480 \times 3.7917 \times 5.2470 \\ (0.0491) \times (0.1654) \times (0.2409) \end{matrix} & \begin{matrix} 5.2470^2 \\ (0.2409) \end{matrix} \end{pmatrix}. \tag{20}$$

In the first regime, the much lower volatility of the United States ($\sigma_1^{us} = 7.50$) versus the United Kingdom ($\sigma_1^{uk} = 14.07$) makes the United States relatively more attractive to risk-averse investors at the expense of international holdings. With only time-varying correlations and volatility, we should expect risk-averse investors to increase their holdings of U.S. equity, the safer asset, as the horizon increases.[11] The next section analyzes the statistical and economic significance of this effect.

2.3 Asset allocation results

We attempt to answer the following questions raised in the Introduction: Are there still benefits of international diversification in regimes of global financial turbulence? How do these regimes affect asset allocations? How costly is it to ignore regime switching? How large are the intertemporal hedging demands induced by regime switching? We defer two important questions to Sections 3 and 4, where we consider richer models. First, are our results sensitive to the absence of a conditional risk-free asset? With a risk-free asset, the high volatility regime may induce a shift out of all equity markets rather than out of riskier foreign equities. Second, how does currency hedging contribute to the benefits of international diversification in the presence of regime switches?

To address these questions in the context of our benchmark model, we report asset allocation results in Table 3 (the U.S. and U.K. model) and Table 4 (the U.S., U.K., and German model). Economic costs for no international diversification, ignoring regime switching, and myopia are reported in Table 5. We generally tabulate results for risk aversion levels of $\gamma = 5$ and 10.

2.3.1 Portfolio weights. Table 3 shows that for the U.S.–U.K. model, the proportion held in the United States is higher in regime 1, the high volatility, high correlation bear regime, than in regime 2. The table reports U.S. weights in all equity portfolios, so the U.K. weight is 1 minus the U.S. weight. The standard errors for the portfolio weights for the basic model in Table 3 are large. This results partly from the large standard errors in estimating the

[11] In the case where $\mu_1 \neq \mu_2$, both the effects of the conditional covariances and the conditional means play a role in determining the "safe" asset.

Table 3
Benchmark U.S.–U.K. model: weight of the United States in all-equity portfolios

	Risk aversion $\gamma = 5$				Risk aversion $\gamma = 10$			
	Basic model		Restricted $\mu_1 = \mu_2$		Basic model		Restricted $\mu_1 = \mu_2$	
Horizon	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
U.S. weight								
1	0.8587	0.7171	0.9348	0.6726	0.9652	0.7666	0.9999	0.7405
	(0.3662)	(0.2238)	(0.0977)	(0.2230)	(0.1739)	(0.1139)	(0.1072)	(0.1169)
12	0.8609	0.7297	0.9362	0.6769	0.9697	0.8585	1.0048	0.7769
	(0.1919)	(0.2067)	(0.0997)	(0.2203)	(0.1773)	(0.1229)	(0.1010)	(0.1063)
36	0.8614	0.7352	0.9365	0.6779	0.9699	0.8744	1.0057	0.7954
	(0.3645)	(0.2022)	(0.0989)	(0.2198)	(0.1754)	(0.1242)	(0.1013)	(0.1050)
60	0.8614	0.7356	0.9365	0.6779	0.9699	0.8744	1.0057	0.7965
	(0.2495)	(0.2224)	(0.1006)	(0.2192)	(0.1767)	(0.1240)	(0.1012)	(0.1049)
i.i.d. weights	0.7642		0.7642		0.8275		0.8275	
Tests for no international diversification								
1	0.5870	0.2074	0.5044	0.1417	0.8412	0.0403	0.9997	0.0306
12	0.6377	0.1867	0.4836	0.1431	0.8651	0.2456	0.9625	0.0359
36	0.6495	0.2285	0.4920	0.1416	0.8644	0.3128	0.9554	0.0513
60	0.5702	0.2138	0.5334	0.1434	0.8649	0.3117	0.9551	0.0524
Tests for equality with i.i.d. weights								
1	0.7964	0.8334	0.0808	0.6813	0.4287	0.5926	0.1076	0.4569
12	0.6142	0.8677	0.0845	0.6921	0.4224	0.8009	0.0793	0.6344
36	0.7897	0.8862	0.0815	0.6948	0.4168	0.7055	0.0786	0.7597
60	0.6968	0.8978	0.0867	0.6941	0.4202	0.7050	0.0782	0.7677
Tests for regime equality								
1	0.7465		0.1448		0.2977		0.0028	
12	0.6087		0.1630		0.3141		0.0446	
36	0.7337		0.1691		0.3085		0.0493	
60	0.6181		0.1952		0.3116		0.0496	
Joint	0.9844		0.2237		0.8047		0.0102	
Intertemporal hedging demand tests								
12	0.9932	0.9736	0.9260	0.9804	0.2971	0.3877	0.9605	0.6707
36	0.8701	0.9609	0.2091	0.2334	0.2091	0.2334	0.9533	0.5377
60	0.9848	0.9547	0.9619	0.9757	0.2358	0.2333	0.9529	0.5294

Asset allocation weights for the United States from the benchmark U.S.–U.K. model. The coefficient of risk aversion γ is set at either 5 or 10. Standard errors are given in parentheses. The table shows weights for an all-equity portfolio (so U.K. weight is $1 -$ U.S. weight). All reported values for the statistical tests are p-values. The test for no international diversification tests whether the U.S. weight is equal to 1. Tests for equality with i.i.d. weights test if the portfolio weights in each regime are equal to the i.i.d. weights. The regime equality test is a Wald test for equality of the U.S. portfolio weights across regimes. The intertemporal hedging demand test is a Wald test to test if the horizon T portfolio weights are different from the myopic portfolio weights within each regime state.

regime-dependent means. To mitigate this, we consider a restricted version of each model designed to limit sampling error in the means by restricting the means across regimes to be equal. As we show in Tables 1 and 2, there is little evidence against these models and they offer better regime classification. Constraining the conditional means to be equal across regimes also allows a sharper focus on the effect of time-varying covariances.

U.S. equity is the "safer" asset because of its lower volatility in the first regime compared to U.K. equity. The top panel of Figure 2 shows portfolio weights for the United States and United Kingdom as a function of risk aversion. Risk-averse investors choose to hold more U.S. equity at the expense of U.K. equity during both regimes, but hold even more U.S. equity

Table 4
Benchmark U.S.–U.K.–German model: weight of the United States and United Kingdom in all-equity portfolio

	Basic model				Restricted $\mu_1 = \mu_2$			
	Regime 1		Regime 2		Regime 1		Regime 2	
Horizon	U.S.	U.K.	U.S.	U.K.	U.S.	U.K.	U.S.	U.K.
Portfolio weights								
1	0.3714	0.2400	0.7379	0.0574	0.6836	0.0341	0.6144	0.1590
	(0.3752)	(0.2775)	(0.3085)	(0.2734)	(0.1551)	(0.0990)	(0.2703)	(0.2591)
12	0.3649	0.2426	0.7249	0.0658	0.6839	0.0337	0.6153	0.1572
	(0.3799)	(0.2801)	(0.2983)	(0.2636)	(0.1532)	(0.0978)	(0.2716)	(0.2601)
36	0.3645	0.2427	0.7238	0.0665	0.6839	0.0336	0.6154	0.1570
	(0.3805)	(0.2800)	(0.3071)	(0.2701)	(0.1533)	(0.0990)	(0.2701)	(0.2579)
60	0.3644	0.2427	0.7199	0.0682	0.6839	0.0336	0.6154	0.1570
	(0.4458)	(0.5058)	(1.5688)	(1.2708)	(0.1585)	(0.0969)	(0.2697)	(0.1585)
i.i.d. weights	U.S. = 0.5889, U.K. = 0.1449				U.S. = 0.6491, U.K. = 0.0800			
Tests of international diversification								
		$s_t = 1$	$s_t = 2$			$s_t = 1$	$s_t = 2$	
1		0.3224	0.0168			0.0000	0.0267	
12		0.3367	0.0151			0.0000	0.0265	
36		0.3381	0.0184			0.0000	0.0222	
60		0.4137	0.6463			0.0000	0.0243	
Tests for equality with i.i.d. weights								
1	0.5621	0.7316	0.6291	0.7491	0.5216	0.3974	0.9185	0.7405
12	0.5555	0.7273	0.6483	0.7642	0.6265	0.4994	0.9673	0.8038
36	0.5553	0.7267	0.6605	0.7718	0.5288	0.2879	0.9717	0.8237
60	0.6146	0.8466	0.9335	0.9519	0.5534	0.3900	0.9691	0.8148
Tests for regime equality								
	U.S.	U.K.	Joint U.S.–U.K.		U.S.	U.K.	Joint U.S.–U.K.	
1	0.4625	0.6204	0.7570		0.6681	0.5127	0.8064	
12	0.4431	0.6126	0.7363		0.6643	0.5057	0.8009	
36	0.4865	0.6293	0.7832		0.6974	0.5225	0.8151	
60	0.7964	0.8876	0.9554		0.6695	0.5220	0.8141	
Joint	0.7713	0.9817			0.9925	0.9649		
Intertemporal hedging demands								
12	0.3915	0.5339	0.8728	0.8435	0.9774	0.9733	0.9040	0.8717
36	0.4224	0.5611	0.9080	0.9246	0.9948	0.9793	0.6989	0.6102
60	0.9675	0.9934	0.9913	0.9936	0.9862	0.9500	0.8622	0.8075

Asset allocation weights for the United States and United Kingdom from the benchmark U.S.–U.K.–German model with the coefficient of risk aversion γ fixed at 5. The cases of unrestricted means (basic model) and means imposed equal across regimes for each country ($\mu_1 = \mu_2$) are shown. Standard errors are given in parentheses. The table shows weights for an all equity portfolio (so German weight is $1 - \text{U.S.} - \text{U.K.}$ weight). All reported values for the statistical tests are p-values. The test for no international diversification is a test of the U.K. and German weights being equal to 0, where s_t denotes the regime. Tests for equality with i.i.d. weights test if the portfolio weights in each regime are equal to the i.i.d. weights. The regime equality is a Wald test for equality of the portfolio weights across regimes. The intertemporal hedging demand test is a Wald test to test if the horizon T portfolio weights are equal to the myopic portfolio weights within each regime state.

in the bear regime. Hence it is no surprise that we only reject the optimality of a 100% U.S. portfolio in the case of a normal regime. For $\gamma = 10$ in the normal regime, we reject a nondiversified portfolio in the $\mu_1 = \mu_2$ case at all horizons, and for the one-month horizon in the unconstrained means case. For $\gamma = 5$, we fail to reject in both cases.

Table 3 also lists i.i.d. weights, which are portfolio weights using a multivariate normal distribution without regimes as the DGP. These portfolio weights lie in-between the regime-dependent weights and give a reasonable

Table 5
Economic costs under the benchmark model: all equity portfolios

		U.S.–U.K. model				U.S.–U.K.–German model			
		$\gamma=5$		$\gamma=10$		$\gamma=5$		$\gamma=10$	
	T	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$
Costs of no international diversification									
Basic model	1	0.05	0.05	0.01	0.07	0.38	0.04	0.38	0.10
	12	0.65	0.62	0.14	0.37	3.03	1.41	4.06	2.75
	36	1.94	1.90	0.31	0.44	7.56	5.72	12.53	11.08
	60	3.23	3.19	0.48	0.61	12.20	10.29	21.72	20.15
Restricted	1	0.01	0.07	0.00	0.09	0.12	0.07	0.22	0.14
$\mu_1=\mu_2$	12	0.44	0.78	0.26	0.80	1.19	0.97	2.35	1.90
	36	1.83	2.24	0.90	1.51	3.31	3.06	6.84	6.32
	60	3.29	3.70	1.47	2.07	5.45	5.20	11.51	10.96
Costs of ignoring regime switching									
Basic model	1	0.02	0.00	0.10	0.00	0.04	0.01	0.01	0.00
	12	0.22	0.05	1.26	0.65	0.38	0.21	0.12	0.07
	36	0.55	0.32	4.04	3.55	0.95	0.75	0.36	0.30
	60	0.87	0.63	6.92	6.41	1.51	1.31	0.59	0.53
Restricted	1	0.08	0.01	0.16	0.01	0.02	0.00	0.03	0.00
$\mu_1=\mu_2$	12	0.58	0.13	1.65	0.44	0.14	0.05	0.26	0.11
	36	1.09	0.54	4.84	3.16	0.29	0.20	0.66	0.48
	60	1.53	0.97	8.20	6.46	0.44	0.35	1.05	0.87
Costs of using myopic strategies									
Basic model	12	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
	36	0.00	0.00	0.03	0.14	0.00	0.00	0.03	0.06
	60	0.00	0.01	0.07	0.18	0.00	0.00	0.07	0.11
Restricted	12	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
$\mu_1=\mu_2$	36	0.00	0.00	0.03	0.06	0.00	0.00	0.00	0.00
	60	0.00	0.00	0.07	0.11	0.00	0.00	0.01	0.01

The table presents the "cents per dollar" compensation required for an investor to hold nonoptimal strategies. The first panel lists costs to hold only U.S. equity (so the portfolio weight is 1 on U.S. equity and zero on all other assets) instead of the optimal weights. The second panel presents costs to ignore regime-switching use Samuelson's (1969) myopic optimal portfolio weights in an i.i.d. multivariate normal setting with CRRA utility instead of the optimal portfolio weights. The last panel presents costs required for an investor to use the myopic one-month horizon weights for all horizons instead of the optimal weights. The regime is denoted by s_t.

approximation of the optimal weights in each regime. For example, for an investor with $\gamma=5$, the weight held in the United States in an i.i.d. setting is 0.76, whereas the same investor would hold 0.93 (0.67) in the United States in regime 1 (2) under the restricted model with $\mu_1=\mu_2$. Wald tests fail to reject that the regime-dependent weights are significantly different from the i.i.d. portfolio weights at the 5% level. This implies that the i.i.d. portfolio weights may serve as good proxies for both regime-dependent weights. Turning to tests for regime equality, for the restricted $\mu_1=\mu_2$ model with $\gamma=10$ we reject that portfolio weights are equal across regimes at the 5% and sometimes 1% level. However, when the $\mu_1=\mu_2$ restriction is relaxed, significant tests no longer occur because of the large standard errors associated with the means of the basic model.

Portfolio holdings of U.S. equity increase as the horizon increases, although the increase is small, in line with Samuelson (1991)'s intuition. After 3 years the portfolio weights converge to a constant. The convergence

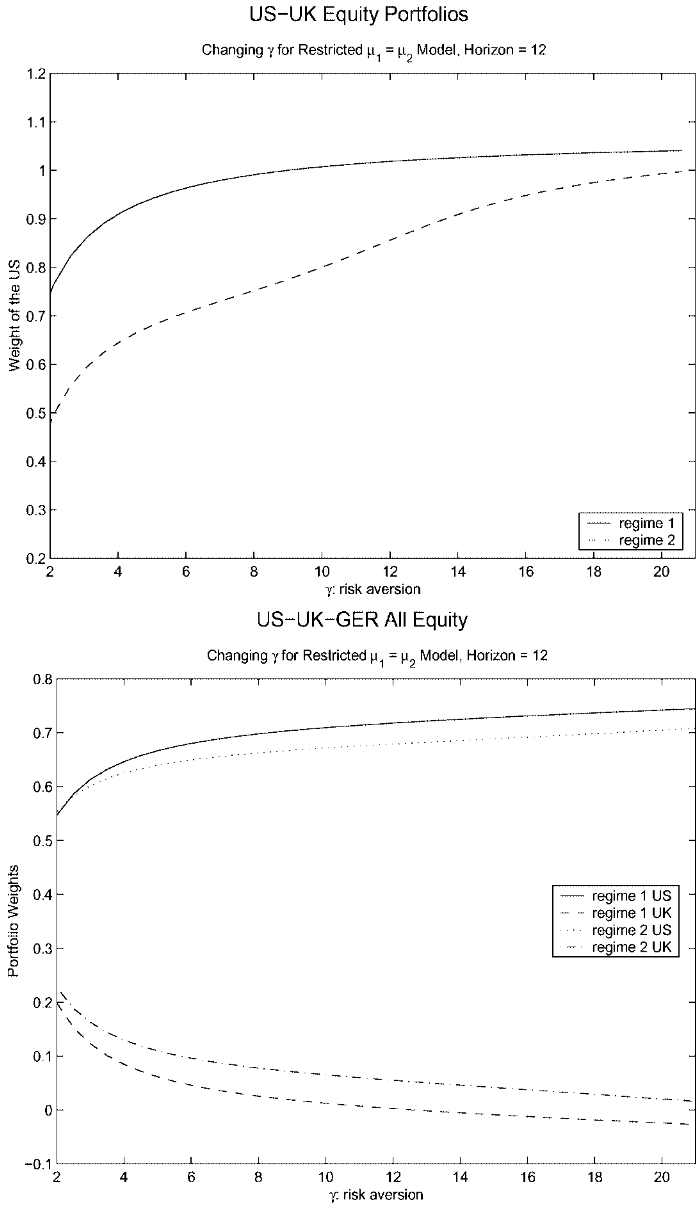


Figure 2
Plots the portfolio weights as the risk aversion γ changes at a fixed 12-month horizon. The top panel gives the weights of the U.S. in regime 1 and regime 2 for the restricted $\mu_1 = \mu_2$ benchmark U.S.–U.K. model. The portfolio is all-equity, so the U.K. weight is 1 minus the U.S. weight. The bottom panel shows the restricted $\mu_1 = \mu_2$ benchmark U.S.–U.K.–German model. The German weight is 1 minus the sum of the U.S. and U.K. weights.

is even faster than in Brandt (1999), who finds convergence after 15 years, in a setting with instrument predictability and rebalancing at intervals greater than 1 month. Not surprisingly, with only regime changes and monthly rebalancing, horizon effects become even smaller. The last panel of Table 3 reports tests of intertemporal hedging demands which have large p-values. Brandt (1999) also fails to reject myopia in his nonparametric estimation of domestic asset allocation weights.

Table 4 reports portfolio weights for the U.S.–U.K.–German system. For the basic model with unconstrained means, investors hold less U.S. equity in the bear regime, even though U.S. equity is less volatile in that regime. The reason for this surprising result is that the negative mean return estimated for the United States in this regime outweighs the volatility and correlation effects. In the restricted $\mu_1 = \mu_2$ estimation, standard errors on the portfolio weights are much smaller and U.S. equity again becomes a safer asset in regime 1. However, Table 4 also shows that both U.S. and German holdings increase at the expense of U.K. equity in regime 1. Portfolio weights as a function of γ are shown in Figure 2 for the $\mu_1 = \mu_2$ model. The more risk averse the investor, the greater the proportion of the U.S. equity held in both regimes.

In the restricted $\mu_1 = \mu_2$ model for the U.S.–U.K.–German system, Table 4 shows we strongly reject the null of no international diversification in both regimes. Even in the basic model with large standard errors around the conditional means, we reject that a pure U.S. portfolio is optimal in the normal regime with $\gamma = 5$. The differences in weights across regimes are quite substantial for all countries. Nevertheless, the standard errors are often large and we fail to reject the null that portfolio weights equal the i.i.d. portfolio weights, or that they are constant across regimes. Like in Table 3, hedging demands are statistically insignificant.

2.3.2 Economic costs. Table 5 presents the "cents per dollar" compensation required for an investor with an all-equity portfolio to hold nonoptimal portfolio weights. The first panel lists the costs of not diversifying internationally, the middle panel lists the costs of ignoring regime switching and holding i.i.d. portfolio weights, and the last panel lists the costs of using myopic strategies. We focus our discussion on results of models with μ_1 imposed equal to μ_2. We turn first to the costs of no international diversification.

In the U.S.–U.K. system, the compensation required for an investor to hold only U.S. equity starts out very small but, as expected, grows with the horizon. At the one-year horizon, the compensation still does not reach one cent. In the U.S.–U.K.–German system, an investor with a horizon of one year and risk aversion of 5 needs to be compensated 1.19 (0.97 cents) in regime 1 (2) to hold no U.K. or German equity under the benchmark model. For $\gamma = 10$, this compensation roughly doubles. The addition of Germany brings considerable economic benefit for international diversification, especially at long horizons where costs exceed 10 cents for $\gamma = 10$. This is because both U.S. and German holdings increase at the expense of U.K. equity in regime 1 (see Table 4).

We might expect that as correlations are higher in regime 1, the costs of no international diversification in that regime would be less than in regime 2. This is only true for the U.S.–U.K. system but not for the U.S.–U.K.–German system, because the increase in German holdings in the optimal portfolio in regime 1 is greater than the increase in U.S. holdings, making diversification more valuable in this regime. Figure 3 shows that even for the U.S.–U.K. system, the benefits of diversification for regime 1 may be greater than for regime 2 for small γ. The bottom panel of Figure 3 shows that because of the benefits of holding Germany in regime 1, the costs of no international diversification are uniformly higher in regime 1 than in regime 2. This demonstrates that increasing correlations a priori does not make international diversification less valuable. The results are qualitatively the same for the case $\mu_1 \neq \mu_2$, but the costs of not diversifying internationally are generally much larger.

We now focus on the middle and last panels of Table 5. In the absence of predictability, there are two implications of regime switching for portfolio weights: (a) portfolio weights become regime dependent, and (b) portfolio weights become horizon dependent, since regime switching generates intertemporal hedging demands. The middle panel of Table 5 addresses the former implication, and the last panel of Table 5 addresses the latter.

The economic costs of ignoring regimes range from fairly small to substantial at high levels of risk aversion. For example, for a one-year horizon, investors with $\gamma = 5$ in the benchmark U.S.–U.K.–German model lose only 0.14 (0.05) cents for ignoring regime switching in regime 1 (2). When investors ignore regimes, the i.i.d. weights they hold are reasonable approximations to the optimal weights, especially the weights in regime 2, the longest duration regime. Note that the cost of ignoring regimes is higher in regime 1 than regime 2. This is in accordance with intuition, since in the normal regime, conditional means and variances are closer to their unconditional counterparts than they are in regime 1. The markedly different behavior in regime 1, which may persist for several periods, makes the costs of ignoring regimes higher in this regime.

In Figure 3 we plot the costs of ignoring regime switching for the benchmark model as a function of γ. The plots confirm that the cost of ignoring regimes is higher in regime 1 for all levels of risk aversion and this is robust across the benchmark U.S.–U.K. and U.S.–U.K.–German models. Figure 3 also contrasts the costs of not diversifying internationally with the costs of ignoring regime switching. For the U.S.–U.K. system, the costs of failing to diversify internationally dominate the costs of ignoring regimes only at low levels of risk aversion. However, in the U.S.–U.K.–German system, they

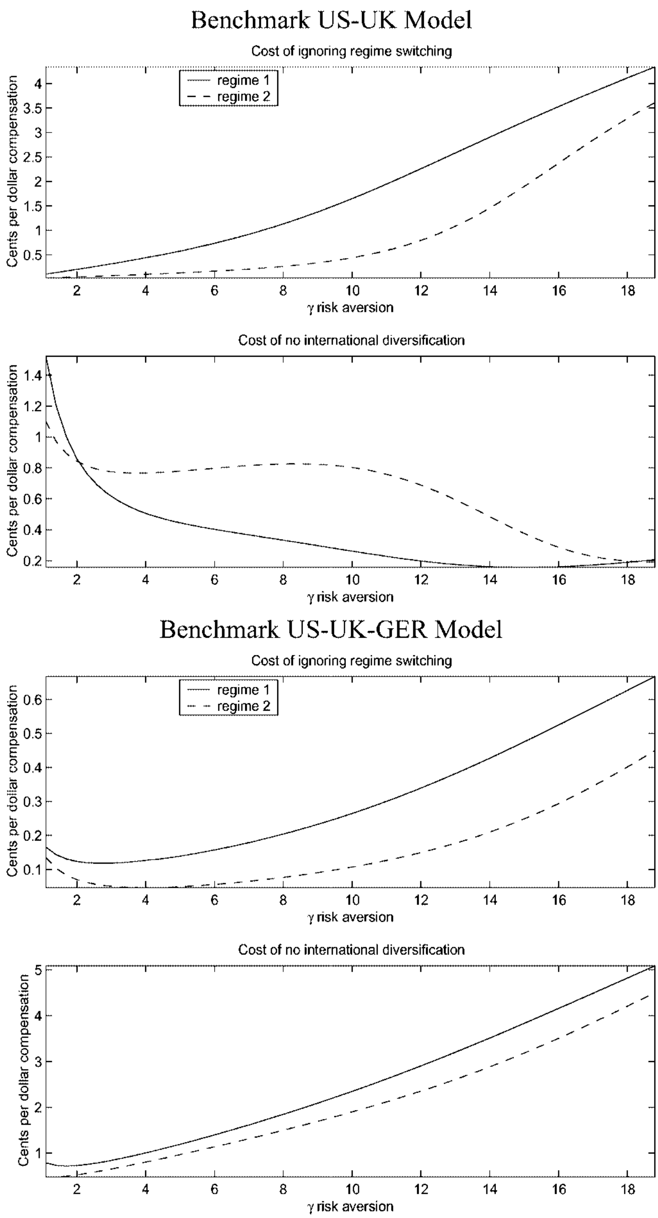


Figure 3
Plots the "cents per dollar" compensation required for ignoring regime switching [holding Samuelson (1969) i.i.d. portfolio weights] and not being internationally diversified (holding only the U.S.) as the risk aversion γ changes. We fix the horizon at 12 months. The top panel shows the benchmark U.S.–U.K. model, and the bottom panel the benchmark U.S.–U.K.–German model. We restrict $\mu_1 = \mu_2$ and consider all equity portfolios.

dominate for all γ. This is because for the U.S.–U.K. system, the optimal portfolio for regime 1 becomes the domestic U.S. equity portfolio when γ is high, whereas in the U.S.–U.K.–German system, positive German equity holdings remain optimal in the first regime. Table 5 demonstrates that the same results hold for the original model with unrestricted μ_1 and μ_2.

The final panel of Table 5 lists the compensation required for an investor to hold myopic portfolio weights instead of the optimal T-horizon weights. The numbers are astoundingly small for all models. This evidence suggests that investors lose almost nothing by solving a myopic problem at each horizon rather than solving the more complex dynamic programming problem for longer horizons.

2.4 Robustness experiments

In this section we conduct several experiments to determine the robustness of our results. In Section 2.4.1 we check the sensitivity of our results to the specification of the conditional means. In Section 2.4.2 we gain further intuition on optimal asset allocation under regime changes by examining how optimal portfolio weights change as a function of one changing parameter in the RS benchmark U.S.–U.K. model. In Section 2.4.3 we investigate whether our conclusions about the costs of ignoring RS and the benefits of international diversification remain robust to alternative parameter values for the DGP.

2.4.1 Regime-dependent conditional means. One disappointing aspect of our RS model estimation is that we fail to find strong evidence that highly volatile periods coincide with bear markets. Although the point statistics suggest this relationship, the standard errors on the conditional means in regime 1 are large. This in turn may dampen the potential asset allocation effects of the high-volatility regime. In order to examine this further, we reestimate the basic benchmark models, constraining the conditional means to be equal across countries, but different across regimes. These models are not rejected in favor of the alternative of unconstrained means [p-value = 0.8415 (0.4884) for the U.S.–U.K. (U.S.–U.K.–Germany) model]. In these models, the means in each regime (equal across countries) are also not significantly different (p-value = 0.1422 (0.1927) for the U.S.–U.K. (U.S.–U.K.–Germany) model). The quality of the regime classification measured by the Ang and Bekaert (2002) RCM statistic is largely unchanged for the U.S.–U.K.–Germany model, but is much worse for the U.S.–U.K. model. The resulting portfolio weights are largely unchanged, with almost the same economic costs and significance levels for the statistical tests. Consequently our focus on time-varying covariances seems justified.

2.4.2 Changing parameters in the benchmark U.S.–U.K. model. Figure 4 shows the effect on the portfolio weights of changing various parameter values. The baseline case is the unconstrained μ case. We alter one parameter while holding all the others constant and hold the horizon fixed at $T = 12$ months. From the top plot going downward in Figure 4 we show the effect of altering the transition probability $P = p(s_t = 1|s_{t-1} = 1)$ of staying in the first regime conditional on being in the first regime, the correlation ρ_1 of the U.S.–U.K. model in regime 1, the conditional mean μ_1^{us} of the United States in regime 1, and the volatility σ_1^{us} of the United States in regime 1.

The plots are very intuitive. As P increases, holdings of the safer U.S. asset increase in both regimes as the expected duration of regime 1 increases. The largest difference between the regime-dependent weights is at values around $P = 0.5$ (the sample estimate is $\hat{P} = 0.8552$). As ρ_1 increases the diversification benefits of holding U.K. equity decrease. Note that it is only for $\rho_1 > 0.8$ that the weights in each regime become substantially different. Our estimated $\hat{\rho}_1 = 0.6181$ is far less than this. As μ_1^{us} increases, the U.S. asset becomes even more attractive relative to the U.K. asset. (The sample estimate is $\hat{\mu}_1^{us} = -1.2881$.) Finally, as the U.S. σ_1^{us} increases beyond the sample estimate of $\hat{\sigma}_1^{us} = 7.0376$, the U.S. asset becomes less "safe" and the proportion allocated to the U.K. asset increases. For values of σ_1^{us} greater than 9, the portfolio weights in each regime are almost identical. Overall, Figure 4 suggests that among the parameters affecting the conditional distribution of returns in regime 1, the biggest effects on the regime-dependent weights come from conditional correlations and the relative difference in means.

2.4.3 Asymptotic distributions of economic costs. The previous section conveys intuition on which parameters have the largest effect on regime-dependent optimal asset allocation, but does not tell us whether our main conclusions are affected by these different parameters. Here we recompute the economic costs of no international diversification, the economic costs of ignoring RS, and the economic costs of myopic strategies for 1000 alternative parameter values drawn randomly from the asymptotic normal parameter distributions implied by the estimation. We take the sample estimates to be "population values" and use the estimations where the conditional means are constrained to be equal across regimes.

Table 6 reports some characteristics of the resulting empirical distributions for a risk aversion of $\gamma = 5$ and for horizons $T = 1$, 12, 36, and 60 months. The economic cost distributions have means which are larger than their sample values in Table 5. The median values of the economic costs are much closer to the sample values. This is because the economic cost computations are nonlinear transformations, which result in economic costs that are skewed to the right. In particular, the costs of not diversifying internationally are far more right skewed than the costs of holding i.i.d. weights. This means that

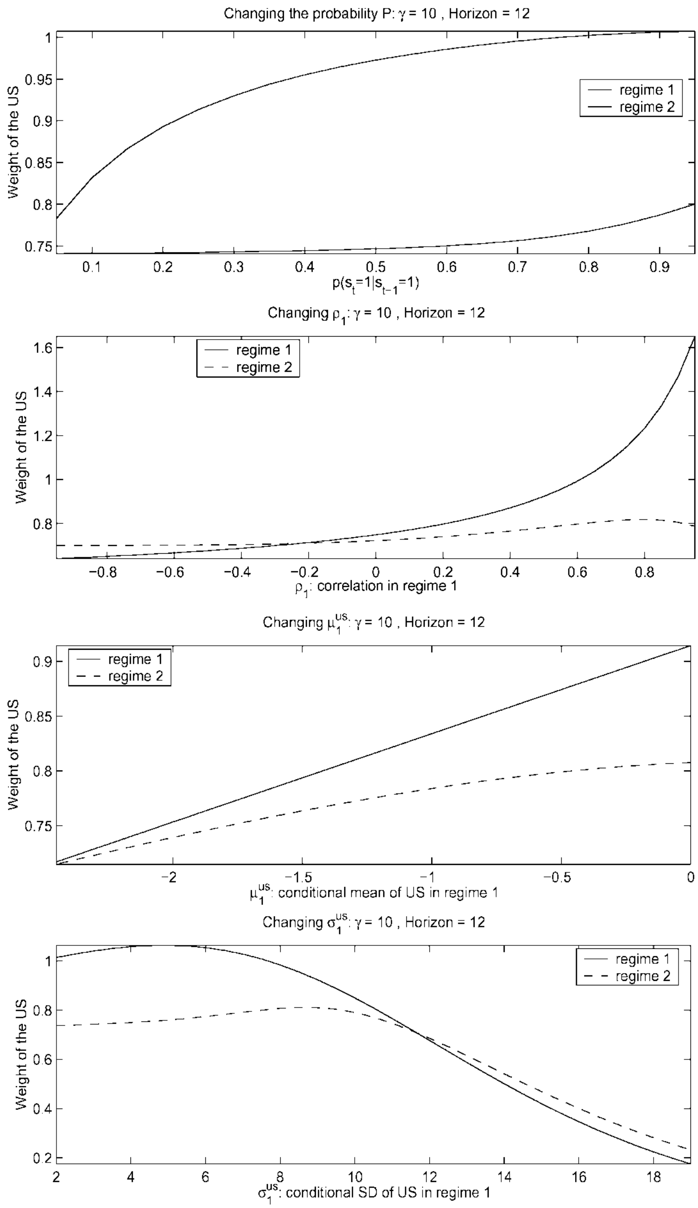


Figure 4
Plots the weight of the U.S. in the U.S.–U.K. benchmark model as a function of various parameters. We fix $\gamma = 10$ and the horizon at 12 months. The top panel plots the weights of the U.S. in regime 1 and 2 for changing $P = p(s_t = 1|s_{t-1} = 1)$ and ρ_1, the correlation between the U.S. and U.K. in regime 1. In the bottom panel, the conditional mean and standard deviation of the U.S. in regime 1 (μ_1 and σ_1, respectively) are altered. All other parameters are held fixed at the estimated values for the benchmark U.S.–U.K. model with unrestricted μ.

Table 6
Asymptotic distributions of economic costs

		Benchmark U.S.–U.K. model						Benchmark U.S.–U.K.–German model					
		No diversification		i.i.d. weights		Myopic weights		No diversification		i.i.d. weights		Myopic weights	
Horizon		$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$
$T=1$	Data	0.01	0.07	0.08	0.01			0.12	0.07	0.02	0.01		
	Mean	0.04	0.10	0.13	0.01			0.17	0.15	0.04	0.01		
	Stdev	0.05	0.11	0.16	0.01			0.12	0.13	0.05	0.01		
Quantiles	5%	0.00	0.00	0.00	0.00			0.02	0.01	0.00	0.00		
	50%	0.02	0.07	0.07	0.00			0.14	0.11	0.02	0.00		
	95%	0.16	0.32	0.46	0.03			0.38	0.41	0.14	0.03		
$T=12$	Data	0.44	0.78	0.58	0.01	0.00	0.00	1.19	0.97	0.14	0.05	0.00	0.00
	Mean	0.82	1.17	0.91	0.20	0.00	0.00	1.92	1.83	0.29	0.14	0.00	0.00
	Stdev	0.80	1.21	1.17	0.24	0.00	0.00	1.39	1.50	0.39	0.16	0.00	0.00
Quantiles	5%	0.04	0.03	0.01	0.00	0.00	0.00	0.30	0.24	0.01	0.01	0.00	0.00
	50%	0.55	0.78	0.49	0.12	0.00	0.00	1.59	1.43	0.16	0.08	0.00	0.00
	95%	2.45	3.61	3.27	0.71	0.00	0.00	4.47	4.86	1.01	0.45	0.00	0.00
$T=36$	Data	1.83	2.24	1.09	0.54	0.00	0.00	3.31	3.06	0.29	0.20	0.00	0.00
	Mean	2.96	3.42	1.82	0.86	0.00	0.00	5.81	5.71	0.68	0.49	0.00	0.00
	Stdev	2.98	3.57	2.51	1.09	0.00	0.00	4.52	4.67	0.86	0.58	0.00	0.00
Quantiles	5%	0.13	0.09	0.02	0.01	0.00	0.00	0.85	0.80	0.03	0.02	0.00	0.00
	50%	1.92	2.20	0.99	0.51	0.00	0.00	4.61	4.45	0.39	0.29	0.00	0.00
	95%	8.73	10.38	6.19	2.90	0.00	0.00	14.81	15.19	2.33	1.69	0.00	0.00
$T=60$	Data	3.29	3.70	1.53	0.97	0.00	0.00	5.45	5.20	0.44	0.35	0.00	0.00
	Mean	5.25	5.74	2.62	1.60	0.00	0.00	9.94	9.83	1.04	0.86	0.00	0.00
	Stdev	5.42	6.09	3.74	2.15	0.01	0.01	8.05	8.22	1.31	1.03	0.00	0.00
Quantiles	5%	0.20	0.16	0.02	0.02	0.00	0.00	1.40	1.36	0.05	0.04	0.00	0.00
	50%	3.34	3.64	1.40	0.92	0.00	0.00	7.82	7.57	0.60	0.51	0.00	0.00
	95%	16.06	17.77	8.76	5.61	0.01	0.01	25.88	26.48	3.62	2.89	0.00	0.01

We draw parameter values for the benchmark U.S.–U.K. and U.S.–U.K.–German models with means constrained across regimes from their asymptotic distribution. Economic costs of no international diversification (holding only the United States), ignoring regime switching (using i.i.d. weights from a multivariate normal distribution with the same implied unconditional mean and covariance as the simulated parameter values), and using myopic weights are calculated. The exercise is repeated 1000 times and the risk aversion is set at $\gamma = 5$. The table lists "cents per dollar" of wealth compensations. The numbers reported on the "data" line repeat the sample estimates of the economic costs in Table 5. The regime is denoted by s_t.

if we draw a particular set of realistic parameter values, we may likely find costs for not diversifying internationally that are substantially larger than the sample values. For example, for $T = 60$ for the U.S.–U.K.–German model, the cost of no international diversification is 26 cents at the 95th percentile, whereas the sample estimate was about 10 cents.

For the U.S.–U.K. model, for $T = 1$ and 12 months, the costs of ignoring regimes are slightly higher than the costs of no international diversification in the high-correlation regime, but for the longer horizons, failing to diversify internationally is much more costly than ignoring regime switching. In the case of the U.S.–U.K.–Germany model, failing to hold overseas equity is always more costly than using i.i.d. weights. For $T = 12$ months, the 95% tail estimate of the cost of no diversification is 4.47 cents (4.86 cents) in regime 1 (2), while the cost of ignoring RS is 1.01 cents (0.45 cents) in regime 1 (2). Finally, Table 6 confirms that the costs of using myopic weights are effectively zero.

3. Introducing a Risk-Free Asset

Section 2 considered the impact of regime-dependent asset allocation under the simplest possible model with all-equity portfolios. In this section we analyze international asset allocation with a risk-free asset. We consider two cases. First, in Section 3.1 we will assume the existence of a one-period risk-free bond with a constant interest rate and examine asset allocation with the benchmark models. We will work with an annualized continuously compounded rate of 5%. This is the standard benchmark setup in domestic dynamic asset allocation studies such as Balduzzi and Lynch (1999) and Kandel and Stambaugh (1996). With the introduction of a conditionally risk-free asset, the high-correlation, high-volatility regime is likely to induce a dramatic shift to cash, which may make the costs of ignoring regime switching much larger. Furthermore, Balduzzi and Lynch (1999) show that changes in the cash/equity proportion may also be important for intertemporal hedging.

Second, in Section 3.2 we analyze portfolio holdings under the case where the short rate process is time varying and regime dependent. In our setting the short rate nonlinearly predicts equity returns by entering the transition probabilities of the Markov process. This case produces an interesting dynamic since the predictor is itself the return on an investable asset. Although much of the literature focuses on the dividend yield as a predictor, we are unlikely to lose much predictive power, since dividend yields have no forecasting power when the 1990s are added to the sample [see Goyal and Welch (1999) and Ang and Bekaert (2001)].

3.1 Portfolio allocation with constant short rates

3.1.1 Portfolio weights. Table 7 presents equity weights with a risk-free asset for the U.S.–U.K. and U.S.–U.K.–Germany benchmark model with

Table 7
Benchmark models: equity weights with a constant risk-free asset

	U.S.–U.K. model				U.S.–U.K.–German model					
	Regime 1		Regime 2		Regime 1			Regime 2		
	U.S.	U.K.	U.S.	U.K.	U.S.	U.K.	German	U.S.	U.K.	German
Portfolio weights										
1	0.2785	0.1039	0.8621	0.4175	0.3437	0.0625	0.1589	0.8937	0.1967	0.3553
	(0.1410)	(0.0564)	(0.3362)	(0.2513)	(0.1901)	(0.0866)	(0.1218)	(0.3682)	(0.2651)	(0.2682)
12	0.2766	0.1038	0.8613	0.4173	0.3416	0.0625	0.1583	0.8890	0.1939	0.3339
	(0.1398)	(0.0559)	(0.3946)	(0.2685)	(0.1889)	(0.0861)	(0.1212)	(0.3648)	(0.2626)	(0.2666)
36	0.2762	0.1037	0.8612	0.4173	0.3414	0.0625	0.1582	0.8886	0.1936	0.3338
	(0.1392)	(0.0556)	(0.3526)	(0.2427)	(0.1888)	(0.0625)	(0.1211)	(0.3645)	(0.2624)	(0.2664)
60	0.2762	0.1037	0.8612	0.4173	0.3411	0.0623	0.1587	0.8793	0.1979	0.3338
	(0.1398)	(0.0560)	(0.3482)	(0.2551)	(0.1870)	(0.0861)	(0.1207)	(0.3504)	(0.2600)	(0.2619)
i.i.d. weights:	U.S.		U.K.		U.S.		U.K.		Germany	
	0.5267		0.2067		0.4695		0.1427		0.2204	
Tests of international diversification										
	$s_t = 1$	$s_t = 2$				$s_t = 1$	$s_t = 2$			
1	0.0654	0.0967				0.9846	0.9682			
12	0.0632	0.1202				0.9847	0.9686			
36	0.0620	0.0855				0.9847	0.9686			
60	0.0638	0.1019				0.9848	0.9689			
Tests for equality with i.i.d. weights										
1	0.0783	0.0681	0.3185	0.4015	0.5079	0.3544	0.6139	0.2493	0.8385	0.4727
12	0.0737	0.0654	0.3965	0.4329	0.4982	0.3518	0.6084	0.2502	0.8455	0.4732
36	0.0720	0.0641	0.3428	0.3855	0.4974	0.3516	0.6080	0.2503	0.8461	0.4732
60	0.0732	0.0659	0.3368	0.4090	0.4922	0.3505	0.6096	0.2422	0.8320	0.4656

	U.S.	U.K.	Joint U.S.–U.K.			U.S.	U.K.	Germany	Joint U.S.–U.K.–Germany	
Tests for regime equality										
1	0.0142	0.1261	0.0034			0.0099	0.4775	0.2774	0.0128	
12	0.0708	0.1579	0.0509			0.0093	0.4816	0.2757	0.0106	
36	0.0183	0.1138	0.0073			0.0093	0.4820	0.2756	0.0104	
60	0.0147	0.1337	0.0005			0.0069	0.4627	0.2656	0.0091	
Joint	0.1468	0.5108				0.0000	0.0000	0.0001		
Intertemporal hedging demands										
12	0.2572	0.8930	0.9962	0.9940	0.2453	0.9599	0.4194	0.3418	0.4256	0.5321
36	0.2792	0.9238	0.9813	0.9921	0.2454	0.9597	0.4195	0.3429	0.4251	0.5326
60	0.2662	0.8324	0.9910	0.9921	0.4596	0.8428	0.8869	0.4314	0.8361	0.8147

The table shows asset allocation weights for the benchmark U.S.–U.K. and U.S.–U.K.–German systems with $\mu_1 = \mu_2$ and γ fixed at 5 with an investable constant risk-free asset. Standard errors are given in parentheses. We list weights for U.S., U.K., and German equity, with the remainder of the portfolio in a risk-free asset paying an annualized continuously compounded rate of 5%. All reported values for the statistical tests are p-values. The test for no international diversification is a test of whether the U.K. and German weights are equal to 0, where s_t denotes the regime. Tests for equality with i.i.d. weights test if the portfolio weights in each regime are equal to the i.i.d. weights. The regime equality test is a Wald test for equality of the portfolio weights across regimes. The intertemporal hedging demand test is a Wald test to test if the horizon T portfolio weights are different from the myopic portfolio weights within each regime.

$\mu_1 = \mu_2$ imposed. Since portfolio weights sum to 1, the remainder of the portfolio is held in the risk-free asset, which has an annualized return of 5% continuously compounded. The table lists portfolio weights for a risk aversion level of 5. Table 7 shows that for $\gamma = 5$, leveraging occurs in regime 2, and a dramatic shift back to cash occurs in the bear regime. For example, for the U.S.–U.K. model, the investor holds 86% U.S. (42% U.K.) equity in normal periods, but only 28% U.S. (10% U.K.) equity in regime 1.

In Table 7, standard errors around the portfolio weights are smaller in regime 1 than in regime 2. This is because a much greater amount of the portfolio is held in cash in regime 1, and the cash return is known and constant. This drives the borderline rejection of the null hypothesis of no international diversification in regime 1 for the U.S.–U.K. model (p-value = 0.06), while in regime 2 p-values are almost twice as large. For the U.S.–U.K.–Germany system we fail to reject the hypothesis that a position in only U.S. cash or equity is optimal.

Although the i.i.d. portfolio weights are still weighted averages of regime-dependent portfolio weights, they are now more dissimilar to the portfolio weights in regime 2 than they were under the all-equity portfolios of the benchmark model (Tables 3 and 4). Compared to Tables 3 and 4, the p-values of the tests for equality with the i.i.d. weights are lower in Table 7, yielding a rejection at around the 7% level in regime 1 for the U.S.–U.K. system. We now reject for both the U.S.–U.K. and U.S.–U.K.–German systems that portfolio weights are equal across regimes. Before, this was only true for $\gamma = 10$ for the U.S.–U.K. system. This evidence suggests that the costs for ignoring the regimes may be substantially higher when risk-free holdings are allowed.

In this system, since cash is the safe asset, the equity portfolio weights decrease as the horizon increases, because of the Samuelson (1991) "momentum" effect. Like the case of the all-equity portfolios in Section 2, this effect is small and statistically insignificant as the bottom panel of Table 7 shows by reporting p-values for tests of intertemporal hedging demands. In the presence of a constant risk-free investment Balduzzi and Lynch (1999) and others find much larger intertemporal hedging demands than those found here. This is because our benchmark models do not have a highly correlated predictor like the dividend yield driving our asset allocations. The case of the short-rate predicting asset returns is examined below.

3.1.2 Economic costs. The economic costs of following nonoptimal strategies for the benchmark model are presented in Table 8. For $\gamma = 5$, the costs of no international diversification are comparable in magnitude to the costs with all-equity portfolios in Table 5. In the U.S.–U.K.–Germany model, an investor with a one-year horizon must be compensated 0.94 (1.26) cents in regime 1 (2). This compares to costs of 1.19 (0.97) cents in regime 1 (2) in Table 5 without a risk-free asset. The costs of not diversifying internationally

Table 8
Economic costs under the benchmark model with a constant risk-free asset

	U.S.–U.K. model				U.S.–U.K.–German model			
	$\gamma = 5$		$\gamma = 10$		$\gamma = 5$		$\gamma = 10$	
T	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$	$s_t = 1$	$s_t = 2$
Costs of no international diversification								
1	0.03	0.08	0.01	0.04	0.05	0.12	0.02	0.06
12	0.64	0.91	0.32	0.46	0.94	1.26	0.47	0.63
36	2.36	2.68	1.18	1.34	3.30	3.64	1.64	1.81
60	4.14	4.46	2.05	2.21	5.72	6.07	2.83	3.00
Costs of ignoring regime switching								
1	0.19	0.08	0.10	0.04	0.07	0.10	0.04	0.05
12	1.71	1.04	0.85	0.52	1.04	1.16	0.52	0.58
36	4.12	3.30	2.05	1.65	3.32	3.44	1.65	1.71
60	6.48	5.64	3.21	2.80	5.66	5.78	2.79	2.85
Costs of using myopic strategies								
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The table presents "cents per dollar" compensation required for an investor to hold nonoptimal strategies, or in other words the cost of the nonoptimal strategy. The first panel lists costs to hold only U.S. assets (so the portfolio weight is zero on overseas equity, nonzero on the risk-free asset and U.S. equity) instead of the optimal weights. The second panel presents costs of ignoring regime-switching and uses Samuelson's (1969) myopic optimal portfolio weights in an i.i.d. multivariate normal setting with CRRA utility instead of the optimal portfolio weights. The last panel presents costs faced by an investor using the myopic 1-month horizon weights for all horizons instead of the optimal weights. The regime is denoted by s_t.

remain substantial in the presence of investable riskless bonds, but they do decrease as γ increases, since the riskless asset becomes more attractive.

Table 8 shows that the costs of ignoring regimes are now dramatically higher than in the all-equity case, and of comparable magnitude to the costs of no international diversification. For the U.S.–U.K.–German system, an investor with a risk aversion level of 5 and a one-year horizon must be compensated 1.04 (1.16) cents in regime 1 (2) for holding an i.i.d. portfolio instead of the optimal regime-dependent portfolios. These costs are much higher than in the all-equity case for two reasons. First, the risk-free asset provides a sure return at all times, which is especially valuable in the down regime. Second, portfolio weights differ more across the regimes and the i.i.d. portfolio weights are less accurate approximations of the regime-dependent portfolio weights.

Finally, the bottom panel of Table 8 shows the economic cost of using a myopic strategy. As in the all-equity case, the cost of myopia is negligible, because of the small and insignificant hedging demands.

3.2 Portfolio allocation with regime-switching short rates

3.2.1 Description of the short-rate model. To analyze the effect of time-varying short rates, we incorporate the U.S. short rate as an additional state variable in the regime-switching process. In this model r_t is the driving variable predicting the asset returns. We work with two systems, the first with

U.S. and U.K. excess returns, and the second with U.S., U.K., and German excess returns. We denote excess returns for country j as $\tilde{y}^j_{t+1} = y^j_{t+1} - r_t$, for $j =$ U.K., U.K., Germany.

Excess returns follow

$$\tilde{y}^j_{t+1} = \mu^j(s_{t+1}) + \sigma^j(s_{t+1})u^j_{t+1}. \tag{21}$$

We also examine regime-dependent predictability in the conditional mean with the formula.

$$\tilde{y}^j_{t+1} = \mu^j(s_{t+1}) + \xi^j(s_{t+1})r_t + \sigma^j(s_{t+1})u^j_{t+1}. \tag{22}$$

We use a regime-switching discretized square root process [Cox, Ingersoll, and Ross (1985)] to model r_t:

$$r_{t+1} = c(s_{t+1}) + \phi(s_{t+1})r_t + v(s_{t+1})\sqrt{r_t}u^r_{t+1}. \tag{23}$$

The normally distributed error terms $\{u^j_{t+1}\}j =$ U.S., U.K., Germany, and u^r_{t+1} are correlated in each regime.

To illustrate the heteroscedasticity of the covariance matrix $\Omega(s_{t+1})$, consider the U.S.–U.K. system, where the covariance matrix of $(\tilde{y}^{us}_{t+1}, \tilde{y}^{uk}_{t+1}, r_{t+1})'$ is

$$\begin{pmatrix} (\sigma^{us}(s^*))^2 & \rho_{us,uk}(s^*)\sigma^{us}(s^*)\sigma^{uk}(s^*) & \rho_{r,us}(s^*)\sigma^{us}(s^*)v(s^*)\sqrt{r_t} \\ \rho_{us,uk}(s^*)\sigma^{us}(s^*)\sigma^{uk}(s^*) & (\sigma^{uk}(s^*))^2 & \rho_{r,uk}(s^*)\sigma^{uk}(s^*)v(s^*)\sqrt{r_t} \\ \rho_{r,us}(s^*)\sigma^{us}(s^*)v(s^*)\sqrt{r_t} & \rho_{r,uk}(s^*)\sigma^{uk}(s^*)v(s^*)\sqrt{r_t} & v^2(s^*)r_t \end{pmatrix},$$

where $s^* = s_{t+1}$; $\rho_{r,\,us}(s^*)$, $\rho_{r,\,uk}(s^*)$, and $\rho_{us,\,uk}(s^*)$ are the regime-dependent correlations of the short rate and U.S. equity, short rate and U.K. equity, and U.S. and U.K. equity, respectively.

To complete the model we specify the transition probabilities for $s_t = 1, 2$ as logistic functions of the short rate:

$$p(s_{t+1} = i | s_t = i; \mathcal{I}_t) = \frac{\exp(a_i + b_i r_t)}{1 + \exp(a_i + b_i r_t)}. \tag{24}$$

3.2.2 Estimation results. Table 9 reports parameter estimates and test statistics for the U.S–U.K. short-rate system.[12] Here we summarize the main findings. First, a likelihood ratio test for $b_i = 0$ in Equation (24) has a p-value of 0.0065. In particular, b_2 is negative and highly significant. Hence in regime 2, as the short rate increases a transition to the first regime becomes increasingly likely. Consequently we focus on state-dependent transition probabilities.

[12] Parameter estimates for the U.S.–U.K.–Germany short-rate system are available upon request, but are qualitatively similar to the U.S.–U.K. system.

Table 9
U.S.–U.K. short-rate model parameters

	Basic model				Restricted $\mu_1 = \mu_2$			
	$s_t = 1$		$s_t = 2$		$s_t = 1$		$s_t = 2$	
	Parameter	Std. error	Parameter	Std. error	Parameter	Std. error	Parameter	Std. error
Regime probability coefficients								
a	1.4239	1.6568	6.8465	1.7330	1.4921	1.6075	6.8574	1.6632
b	0.4033	1.8480	−5.0766	1.8696	0.2793	1.7792	−5.0521	1.7962
Short-rate coefficients								
c	0.0743	0.0417	0.0047	0.0051	0.0702	0.0428	0.0050	0.0050
ϕ	0.9158	0.0477	0.9939	0.0094	0.9078	0.0492	0.9937	0.0093
v	0.1294	0.0123	0.0362	0.0021	0.1314	0.0126	0.0364	0.0020
U.S. equity coefficients								
μ	−1.0583	0.8050	0.7260	0.2365	0.5860	0.2228	$= \mu_1$	
σ	6.3966	0.6038	3.5677	0.1683	6.6597	0.6294	3.5753	0.1673
$\rho_{r,us}$	−0.3409	0.1091	−0.1897	0.0642	−0.3537	0.1122	−0.1887	0.0640
$\rho_{us,uk}$	0.5958	0.0813	0.4371	0.0539	0.6189	0.0788	0.4371	0.0535
U.K. equity coefficients								
μ	−1.5589	1.3493	0.9010	0.3567	0.7452	0.3410	$= \mu_1$	
σ	10.7179	1.0082	5.4275	0.2667	11.0749	1.0441	5.4307	0.2650
$\rho_{r,uk}$	−0.2756	0.1146	−0.0282	0.0660	−0.2891	0.1171	−0.0296	0.0653
RCM	12.10				11.90			
log likelihood	−1283.38				−1285.71			

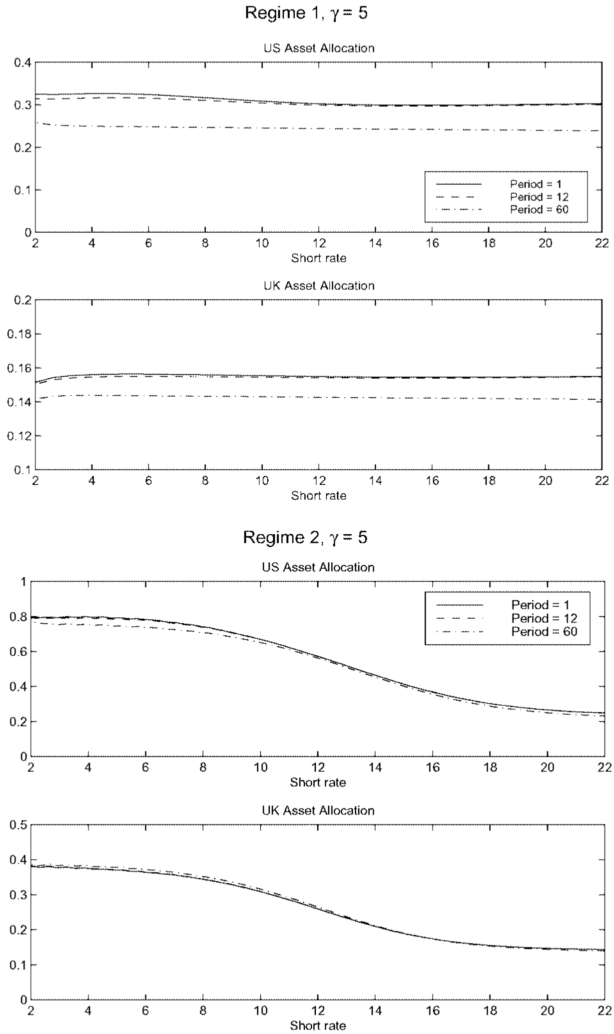
The basic model estimates unconstrained conditional means for equity. The restricted model sets μ_i to be constant across regimes for each country. U.S. and U.K. refer to returns in USD of U.S. and U.K. equity in excess of the U.S. short rate. RCM refers to the Ang–Bekaert (2002) regime classification measure $RCM = 400 * \frac{1}{T}\sum_{t=1}^{T} p_t(1-p_t)$, where p_t is the smoothed regime probability $p(s_t = 1|\mathcal{I}_T)$. Lower RCM values denote better regime classification. A likelihood test for the restricted $\mu_1 = \mu_2$ model versus the basic model produces a p-value of 0.0973. A likelihood test for constant probabilities ($b_i = 0$) in the basic model yields a p-value of 0.0065. The regime is denoted by s_t.

Second, we test whether $\xi^j(s_t) = 0$ in Equation (22) and fail to reject this hypothesis with a p-value of 0.9145. We impose the restriction of no predictability in the conditional mean, which improves efficiency considerably, and label this model the basic short-rate model in Table 9.

Third, we test whether the conditional means for the United States and United Kingdom are equal across regimes. That is, we test if $\mu^j(s_t = 1) = \mu^j(s_t = 2)$ for $j =$ U.S., U.K. We label this case $\mu_1 = \mu_2$, using the same notation as in the benchmark model. We fail to reject this hypothesis, with the likelihood ratio test yielding a p-value of 0.0973. Hence we impose this restriction as well. Note that the resulting model exhibits nonlinear predictability through the transition probabilities rather than linear predictability through the conditional mean.

The behavior of short rates and equity returns across the regimes is characterized as follows. Similar to what Gray (1996) finds, in the first regime, short rates have high conditional means with lower autocorrelation (higher mean reversion) and high conditional volatility. In the normal regime, interest rates are lower and behave like a unit root process. Since b_2 is negative, as the short rate increases in normal periods, a transition to the first regime becomes increasingly likely. In regime 1, equity returns are much more volatile and

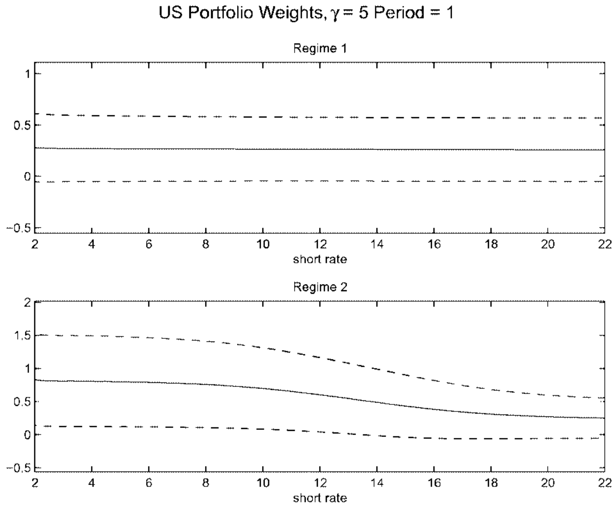
Panel A



more highly correlated across countries. However, in this regime, short rates and equity returns are more negatively correlated than in regime 2. This means that two effects increase the attractiveness of cash for investors in this regime. First, interest rates are higher in this regime; second, shocks to equity and short rates are more negatively correlated in bear markets.

3.2.3 Portfolio weights. Figure 5 presents portfolio weights as a function of the short rate and regime for the U.S.–U.K. system. Panel A shows the asset allocation weights of various horizons for U.S. and U.K. equity in

Panel B



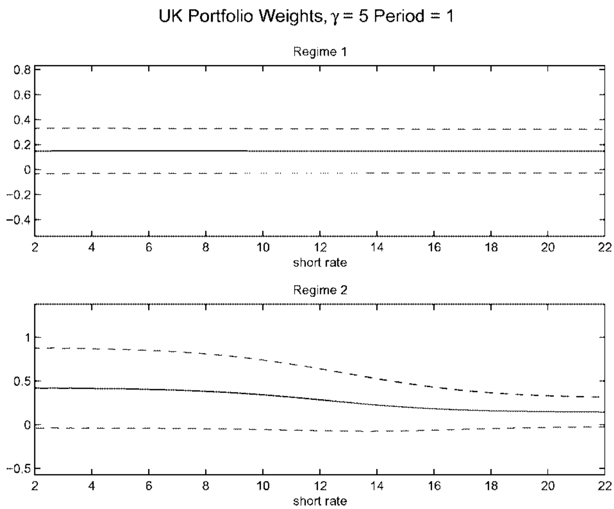


Figure 5
Plots the optimal U.S. and U.K. equity allocations as a function of the short rate for $\gamma = 5$. Portfolio weights sum to 1, so the remainder of the portfolio is held in the conditionally risk-free asset. In panel A we show the weights of the U.S. and U.K. equity in regime 1 for various horizons (left graph) and the weights of the U.S. and U.K. equity in regime 2 for various horizons (right graph). In panel B we show myopic (1 month) portfolio weights for the U.S. (left graph) and U.K. (right graph) with 95% standard error bounds. Parameter estimates are from the restricted $\mu_1 = \mu_2$ U.S.–U.K. short-rate model.

regime 1 and 2 (and the remainder of the portfolio is held in cash). The figures show that the hedging demand is small, and is only visible for the first regime. In regime 2, as the short rate increases, investors hold less equity, but in regime 1 there is almost no effect of the short rate on the portfolio allocations. This is driven by the nonlinear predictability in the transition probability coefficients. The portfolio holdings in regime 1 are flat because the excess returns are constant and no significant short-rate predictability (b_1 is small) drives the transitions from this regime. In the second regime, b_2 is highly significant and negative. As the short rate increases, a transition to regime 1 becomes increasingly likely. As the first regime has much higher equity volatility, investors seek to hold less equity to mitigate the higher risk. Note that equity holdings for a $\gamma = 5$ investor are leveraged in the normal regime.

Panel B of Figure 5 depicts the myopic (1 month) weights with confidence bands. Both the U.S. and U.K. portfolio weights are not significantly different from zero in regime 1. In regime 2, the bands tighten as short rates increase and optimal equity holdings decrease. Nevertheless, we only reject zero equity holdings for the United States at short rates lower than 15%.

Figure 6 shows portfolio weights of the U.S.–U.K.–German short-rate system, with $\mu_1 = \mu_2$ at a 1-month horizon. Since intertemporal hedging effects are very small, the portfolio weights for all horizons look very similar to the 1-month weights. The portfolio weights mimic the patterns of the U.S.–U.K. short-rate system in Figure 5, but with one additional feature. In regime 2, as the short rate increases the equity proportions decrease, but the decrease is not proportional across the equity markets. In the normal regime, at low short-rate levels, more U.K. equity is held than German equity, but for high short-rate levels, the amount of U.K. equity decreases faster than for Germany, so relatively more German stocks are held. This is because Germany is preferred relative to the United Kingdom in the first regime and at high interest rates a transition to the first regime is more likely.

3.2.4 Economic costs. Table 10 presents economic compensation in "cents per dollar" for the short-rate model. We present results for both the U.S.–U.K. and U.S.–U.K.–German systems with a risk-aversion level of 5. To determine the costs of no international diversification we must first solve a constrained optimization problem where investors are permitted to hold only cash and U.S. equity. This cost is not small: at a 12-month horizon, for the U.S.–U.K. (U.S.–U.K.–German) system this cost is 1.04 (3.39) cents at $r_t = 5.1\%$ in the normal regime. In the bear regime, most of the portfolio is held in cash in the U.S.–U.K. system so the cost of no overseas investment is lower. However, for the U.S.–U.K.–German short-rate model, the costs of not diversifying internationally in regime 1 are still considerable. At a 12-month horizon at $r_t = 5.1\%$ the cost is 3.33 cents, because the optimal portfolio in this regime has a relatively large amount of German equity (see Figure 6).

To determine the costs of ignoring regime switching, we first estimate and discretize a one-regime version of the short-rate model and determine portfolio weights for this model. Table 10 shows that similar to the case of the constant risk-free asset, the costs of ignoring regimes are substantial and

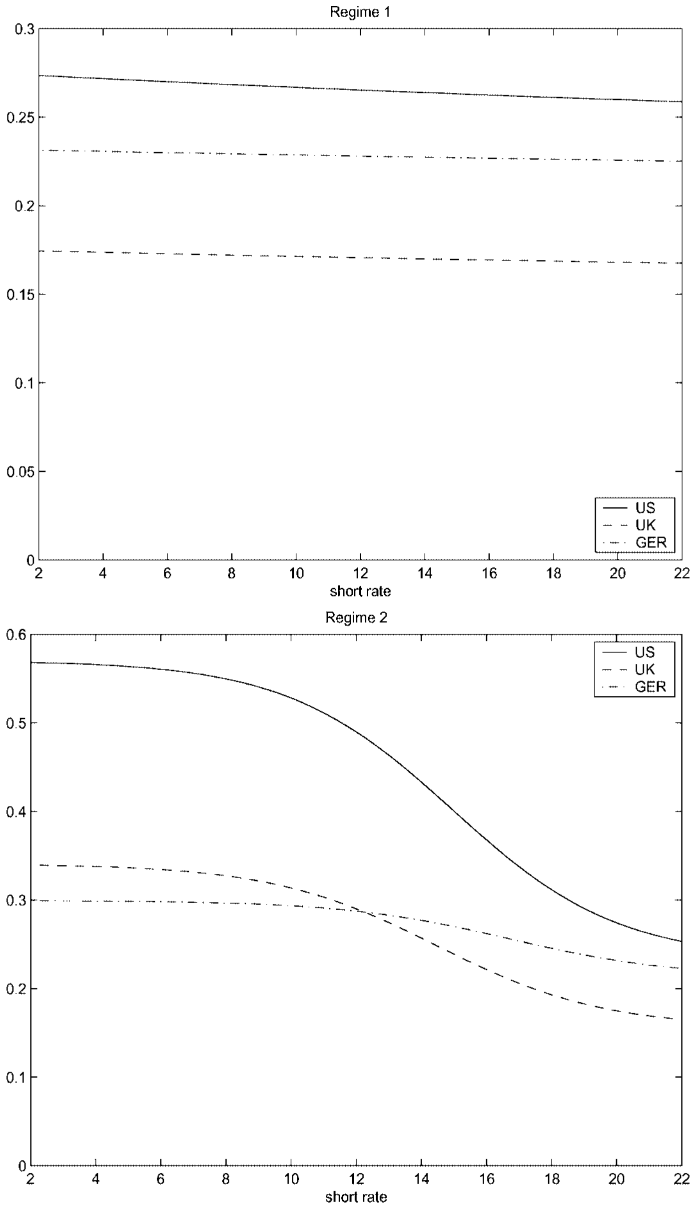


Figure 6
Plots the portfolio weights of U.S., U.K., and Germany in the short-rate model with $\mu_1 = \mu_2$. We fix $\gamma = 5$ and the horizon at one month. The top panel plots the weights of the U.S., U.K., and Germany in regime 1. The bottom panel plots the weights in regime 2. Portfolio weights sum to 1, so the remainder of the portfolio is held in the conditionally risk-free asset.

Table 10
Economic costs under the short-rate model

	$s_t = 1$			$s_t = 2$		
T	$r = 5.1\%$	$r = 9.9\%$	$r = 14.8\%$	$r = 5.1\%$	$r = 10.1\%$	$r = 15.1\%$
			U.S.–U.K. system			
Costs of not diversifying internationally						
1	0.03	0.03	0.03	0.09	0.07	0.04
12	0.73	0.61	0.50	1.04	0.74	0.48
36	2.60	2.19	1.91	3.03	2.24	1.84
60	4.48	3.91	3.57	4.95	3.91	3.48
Costs of ignoring regime switching						
1	0.07	0.07	0.07	0.29	0.24	0.12
12	3.01	1.79	0.32	3.31	2.66	0.82
36	10.88	6.97	1.52	9.54	8.38	2.19
60	18.35	13.01	4.94	15.94	14.90	5.63
Costs of using myopic strategies						
12	0.00	0.00	0.00	0.00	0.00	0.00
36	0.01	0.01	0.01	0.01	0.01	0.01
60	0.02	0.02	0.02	0.02	0.02	0.02
			U.S.–U.K.–German system			
Costs of not diversifying internationally						
1	0.27	0.27	0.27	0.30	0.30	0.27
12	3.33	3.33	3.33	3.39	3.38	3.35
36	10.34	10.34	10.34	10.41	10.40	10.37
60	17.84	17.84	17.83	17.90	17.89	17.86
Costs of ignoring regime switching						
1	0.16	0.16	0.16	0.28	0.26	0.20
12	2.72	2.72	2.72	2.71	2.69	2.67
36	8.47	8.47	8.47	8.46	8.44	8.42
60	14.54	14.54	14.55	14.53	14.51	14.49
Costs of using myopic strategies						
12	0.00	0.00	0.00	0.00	0.00	0.00
36	0.00	0.00	0.00	0.00	0.00	0.00
60	0.01	0.01	0.00	0.00	0.00	0.00

The table presents "cents per dollar" compensation required to accept nonoptimal portfolios for the short-rate model where we impose $\mu_1 = \mu_2$ for excess equity returns. We set $\gamma = 5$. The cost of no international diversification refers to the compensation required to hold only U.S. equity and cash. For this we need to solve a restricted optimization with zero weight on overseas assets. To calculate the cost of ignoring regime switching we first estimate a one-regime version of the short-rate model and calculate the implied portfolio weights. We then calculate the compensation required to hold these portfolio weights instead of the optimal regime-dependent weights. The cost of myopia refers to the compensation required to use one-month horizon portfolio weights instead of optimal T-horizon weights. The short rate r refers to annualized continuously compounded values. The regime is denoted by s_t.

are larger than the costs of not diversifying internationally in the U.S.–U.K. system. At a one-year horizon the costs of ignoring regime switching are 3.31 (2.71) cents for the U.S.–U.K. (U.S.–U.K.–German) system in regime 2 at $r_t = 5.1\%$. These costs are high for several reasons. First, the conditionally risk-free asset is particularly attractive in the bear market regime because interest rates are on average higher than normal, and shocks to short rates and equity are more negatively correlated. Second, the one-regime portfolio weights do not depend on the short rate (since excess returns are constant) and optimal portfolio weights in the second regime are decreasing functions of the short rate. This means the one-regime portfolio weights are not very good approximations for the regime-dependent portfolio weights over low and high interest rate levels in the normal regime. Finally, Table 10 presents the economic compensation required for myopic strategies. Like the all-equity portfolios and the constant risk-free asset case, the cost of myopia is negligible.

4. Currency Hedging

One of the largely unresolved questions in international finance is the question of how much currency risk should be hedged [Solnik (1998)]. Having demonstrated that there are still significant benefits to diversifying internationally in the presence of regimes with all-equity portfolios and with an investable conditionally risk-free asset, we now address the question of the benefits of currency hedging under an RS DGP. To quantify the role of currency hedging we increase the asset space to include hedged equity investments. We achieve parsimony by imposing restrictions linking the conditional means and variances. To focus on the benefits of international diversification we work with all-equity portfolios, so that the influence of a risk-free asset will not bias results. We describe the DGP, which we call the regime-switching beta model, in Section 4.1, and we discuss the estimation results in Section 4.2. Section 4.3 examines the benefits of currency hedging.

4.1 Description of the regime-switching beta model

One problem with the benchmark and short-rate models is their lack of parsimony. Expanding the models to multiple assets is difficult, since any new asset leads to $4+2(N-1)$ new parameters (two new means, two volatilities plus regime-dependent covariance terms), where N is the number of existing assets. One way to deliver parsimony is to build on the large literature on international CAPMs [see Solnik (1974a) and Adler and Dumas (1983)]. In these models, the expected return on every asset depends on its beta relative to the world market and on currency risk factors.[13] In contrast, our beta model precludes the pricing of currency risk, but both our betas with respect to the world market return and the idiosyncratic volatilities are allowed to change with the regime. We apply this model to both hedged and unhedged international equity returns, treating hedged and unhedged instruments as separate assets.[14] We consider both U.S.–U.K. and U.S.–U.K.–German models.

Denote excess returns for country j by $\tilde{y}^j_{t+1} = y^j_{t+1} - r_t$ for $j =$ U.S., U.K., Germany. Let β^j denote the factor loading of asset j on the conditional mean of the excess world portfolio return $\tilde{y}^w_{t+1} = y^w_{t+1} - r_t$, where r_t is the U.S. short rate. The factor loading for asset j is given by

$$\beta^j(s_{t+1}) = \frac{\operatorname{cov}(\tilde{y}^j_{t+1}, \tilde{y}^w_{t+1} | s_{t+1})}{(\sigma^w(s_{t+1}))^2}, \tag{25}$$

[13] Dumas and Solnik (1995) find that exchange rate risk is priced in international equity markets.

[14] See Appendix A for a description of the construction of hedged and unhedged returns.

where $\sigma^w(s_{t+1})$ denotes the regime-dependent volatility of the world portfolio.

Excess returns follow

$$\begin{aligned}\tilde{y}^w_{t+1} &= \mu^w(s_{t+1}) + \sigma^w(s_{t+1})\epsilon^w_{t+1} \\ \tilde{y}^j_{t+1} &= \beta^j(s_{t+1})\mu^w(s_{t+1}) + \beta^j(s_{t+1})\sigma^w(s_{t+1})\epsilon^w_{t+1} + \sigma^j(s_{t+1})\epsilon^j_{t+1},\end{aligned} \tag{26}$$

where ϵ^w_{t+1} and ϵ^j_{t+1} are uncorrelated i.i.d. $N(0, 1)$ variables. As in any CAPM-type model, higher betas (covariances) imply higher risk premiums, but the beta's are regime dependent. Moreover, since we assume that the asset-specific idiosyncratic shocks are uncorrelated, the model is very parsimonious: the introduction of an extra asset means only four additional parameters to estimate, fewer if some of the parameters are imposed to be equal across regimes.

Finally, to complete the model we specify a constant transition probability structure over two regimes $s_t = 1, 2$ with $P = p(s_{t+1} = 1|s_t = 1; \mathcal{I}_t)$ and $Q = p(s_{t+1} = 2|s_t = 2; \mathcal{I}_t)$.

4.2 Estimation results

We now qualitatively describe the estimation results of the RS beta models.[15] Like the benchmark and the short-rate models, pinning down estimates of the conditional mean across regimes is hard. Using a likelihood ratio test we fail to reject the hypothesis that the world mean μ^w is equal across regimes (p-value 0.0644 (0.2435) for the U.S.–U.K. (U.S.–U.K.–German) system). Hence we work with a model with $\mu^w(s_t = 1)$ equal to $\mu^w(s_t = 2)$. We denote this restriction as $\mu^w_1 = \mu^w_2$. Likewise, using a joint Wald test, we do not reject the hypothesis that correlations of international equity returns are equal across regimes (p-value 0.2340 (0.6825) for the U.S.–U.K. (U.S.–U.K.–German)). In common with the benchmark and short-rate models, volatility effects across the regimes are extremely strong, and a likelihood ratio test of equal volatility across regimes rejects with a p-value close to zero.

The higher volatility in the first regime is driven by three parameters. In this regime, world volatility is higher, the βs are higher and the idiosyncratic volatilities are higher than in regime 2. It is never possible to reject that the βs are significantly different from 1 in the first regime, but they are often significantly less than 1 in the second regime, which is more influenced by the idiosyncratic shocks.

The difference between the unhedged and hedged excess equity returns in the RS beta models is the currency return cr_{t+1}, which is the excess return from investing in the foreign money market and is given by $cr_{t+1} = e_{t+1} + r^*_t - r_t$, where r^*_t is the short rate in the foreign country and e_{t+1} is the logarithmic exchange rate change. Its expected value, the currency

[15] Parameter estimates for the beta models are available upon request.

premium $E_t(cr_{t+1})$, is the topic of a large empirical and theoretical literature. Our model implies that conditional on the regime, the currency premium is constant, but the actual premium varies over time with the regime probability. Since the βs of the unhedged returns are larger than the βs of the hedged returns, we estimate the currency premiums to be positive in both regimes; hence U.S. investors are always compensated for taking foreign exchange risk. The unconditional premium is approximately 1.5–2% per annum for both the pound and the deutschemark.

4.3 Benefits of currency hedging

Table 11 shows the asset allocation weights for the RS beta models. Like the simple RS models, the proportion of U.S. equity is larger in the first regime. The foreign equity positions are total positions of both unhedged and hedged equity. We also list the proportion of the portfolio covered by a forward contract position, which is the negative of the proportion of hedged foreign equity. In the RS beta models, short positions in the forward contracts hedge the currency risk of the foreign equity position. These positions are statistically significant. The tables also list hedge ratios, which are the value of the short forward position as a proportion of the foreign equity holdings. Our models produce hedge ratios of about 50%, which are fairly similar across regimes.

Confirming previous evidence in Glen and Jorion (1993), being able to hedge currency risk imparts further benefit to international diversification. In Table 12, the economic compensation required to not diversify internationally under the RS beta models is higher than under the pure unhedged benchmark RS models in Table 5. In this model, no international diversification refers to holding neither hedged nor unhedged foreign equity. To obtain a measure of the benefits of currency hedging, we compute the optimal portfolios under the restriction that only unhedged equity investments are available.

The economic compensation required for holding such portfolios is listed in the second panel of Table 12. This shows that the costs of not using currency hedging, like the costs of not internationally diversifying, are relatively large. For a one-year horizon with $\gamma = 5$, around 70 basis points are required to not engage in currency hedging. Comparing the two panels in Table 12, currency hedging contributes about half of the total benefit of no international diversification under the RS beta models.

5. Conclusion

Ever since Solnik (1974b) demonstrated the considerable benefits of international diversification, the academic community has proposed equity portfolios that are more tilted toward international securities than most investors hold. Recently some have sought to rationalize this reluctance to hold international

Table 11
Currency hedging regime-switching beta models: asset allocation weights

U.S.–U.K. beta model with $\mu_1^w = \mu_2^w$

	$s_t = 1$			$s_t = 2$		
Horizon	U.S.	U.K. forward	Hedge ratio	U.S.	U.K. forward	Hedge ratio
1	0.7766	−0.1164	0.5211	0.4874	−0.2565	0.5004
	(0.0980)	(0.0611)		(0.0395)	(0.0497)	
12	0.7775	−0.1160	0.5211	0.4886	−0.2560	0.5006
	(0.1095)	(0.0642)		(0.0411)	(0.0478)	
36	0.7775	−0.1160	0.5211	0.4886	−0.2560	0.5006
	(0.1095)	(0.0642)		(0.0411)	(0.0478)	

U.S.–U.K.–German Beta Model with $\mu_1^w = \mu_2^w$

	$s_t = 1$						$s_t = 2$					
Horizon	U.S.	U.K.	U.K. forward	Hedge ratio U.K.	Germany forward	Hedge ratio Germany	U.S.	U.K.	U.K. forward	Hedge ratio U.K.	Germany forward	Hedge ratio Germany
1	0.3989	0.2826	−0.1497	0.5298	−0.1667	0.5235	0.3721	0.3721	−0.1502	0.4036	−0.1056	0.4129
	(0.0621)	(0.0559)	(0.0318)		(0.0377)		(0.0592)	(0.0595)	(0.0245)		(0.0813)	
12	0.3990	0.2824	−0.1497	0.5302	−0.1669	0.5237	0.3722	0.3716	−0.1502	0.4042	−0.1060	0.4137
	(0.0622)	(0.0559)	(0.0318)		(0.0376)		(0.0591)	(0.0595)	(0.0245)		(0.0811)	
36	0.3990	0.2824	−0.1497	0.5302	−0.1669	0.5237	0.3722	0.3716	−0.1502	0.4042	−0.1060	0.4137
	(0.0622)	(0.0559)	(0.0318)		(0.0376)		(0.0591)	(0.0595)	(0.0245)		(0.0811)	

Asset allocation weights for the RS beta models with the risk-free rate fixed at 6%. The coefficient of risk aversion γ is fixed at 5. Standard errors are given in parentheses. The table shows weights for an all equity portfolio (so for the U.S.–U.K. model, U.K. weight is 1 − U.S. and for the U.S.–U.K.–German model, German weight is 1 − U.S. − U.K. weight). The weights on foreign equity represent both unhedged and hedged equity holdings. We show weights for positions in pound forward contracts (denoted by U.K. forward) and deutschmark forward contracts (denoted by Germany forward). The forward position is the negative of the hedged equity proportion. The hedge ratio is the value of the short forward position as a proportion of foreign equity holdings. The regime is denoted by s_t.

Table 12
Economic costs of the currency hedging beta models

	U.S.–U.K. model				U.S.–U.K.–German model			
	$\gamma=5$		$\gamma=10$		$\gamma=5$		$\gamma=10$	
T	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$	$s_t=1$	$s_t=2$
Cost of not diversifying internationally								
1	0.04	0.09	0.01	0.14	0.17	0.10	0.25	0.19
12	0.74	0.97	0.71	1.36	1.53	1.37	2.65	2.50
36	2.58	2.83	2.84	3.55	4.43	4.26	7.97	7.81
60	4.46	4.72	5.04	5.76	7.42	7.24	13.56	13.40
Costs of not currency hedging								
1	0.02	0.03	0.00	0.08	0.06	0.06	0.11	0.10
12	0.26	0.32	0.38	0.47	0.72	0.74	1.27	1.23
36	0.88	0.95	1.50	1.87	2.22	2.23	3.82	3.77
60	1.50	1.57	2.64	3.02	3.73	3.74	6.42	6.38

The first panel presents the compensation in "cents per dollar" required for an investor to hold only U.S. equity. The second panel presents the compensation required for an investor to only hold U.S. and unhedged foreign equity instead of the optimal weights. In this case we solve an optimal asset allocation problem with holdings restricted only to U.S. and unhedged foreign equity and find the compensation required to hold these weights instead of the optimal weights, which allow currency hedging. We impose $\mu_1^w=\mu_2^w$. The regime is denoted by s_t.

equities by appealing to the existence of a high-correlation bear regime in international equity markets.

The main conclusion of this article is that the existence of a high-volatility bear market regime does not negate the benefits of international diversification. To establish this result, we introduce regime switching into a dynamic international asset allocation setting. We investigate a U.S. investor with constant relative risk aversion (CRRA) utility who maximizes end-of-period wealth and dynamically rebalances in response to regime switches.

We estimate regime-switching models on U.S., U.K., and German equity and find evidence of a high-volatility, high-correlation regime which tends to coincide with a bear market. However, the evidence on higher volatility is much stronger than the evidence on higher correlation and lower means. Within this setting we establish three main results.

First, there are always relatively large benefits to international diversification, although statistically we do not always reject the optimality of home-biased portfolios. This conclusion is robust across a number of different settings from regime-switching multivariate normals to a model where short rates predict equities through their effect on the regime transition probabilities. In our U.S.–U.K.–German system, the cost of not diversifying over a one-year horizon varies between 0.94 and 3.39 cents per dollar for a risk aversion coefficient equal to 5. We demonstrate that when currency hedging is allowed, the ability to hedge accounts for half the total benefit of international diversification.

Second, the costs of ignoring regime switching may be small or large depending on the presence of a conditionally risk-free asset. The high volatility regime mostly induces a switch toward the lower volatility assets, which are cash (if available), U.S. equity, and also German equity if available. Hence there are some cases in which the high-volatility regime features more internationally diversified portfolios than the normal regime. However, in the all-equity three-country system, it only costs an investor with a risk aversion coefficient of 5 between 0.21 and 0.38 cents per dollar over a one-year horizon to ignore regime switches. Asset allocations that are optimal under an i.i.d. data-generating process diversify risk well in both regimes. These results are similar to results reported in Das and Uppal (2001) for a model with transitory correlated jumps.

When a conditionally risk-free asset is introduced, ignoring regime switching becomes much more costly and of a similar order of magnitude as ignoring the investment opportunities in overseas equities. When the short rate switches regimes and predicts equity returns, cash becomes very valuable in the bear market regime, because in this regime interest rates tend to be on average higher and equity returns more negatively correlated with the short rate. This leads to very dissimilar asset allocations across the two regimes. The cost of ignoring regime switches in the three-country system now jumps to about 2.70 cents per dollar for an investor with a risk aversion level of 5 at a one-year horizon.

Third, in common with the nonparametric results obtained by domestic dynamic allocation studies such as Brandt (1999), we find that the intertemporal hedging demands under regime switches are economically negligible and statistically insignificant. This result holds even with a conditionally risk-free asset and when the short rate predicts equity returns. Investors have little to lose by acting myopically instead of solving a more complex dynamic programming problem for horizons greater than one period.

Our results remain premised on our assumptions, which include CRRA preferences, the absence of transactions costs, limited investment opportunities, and full knowledge on the part of the investors of the data-generating process. With transaction costs, or learning about the regime, it is less likely to be worthwhile for investors to change their allocations when the regime changes. However, different utility functions, for example, first-order risk aversion [Epstein and Zin (2002)], could potentially cause regime switching to have much larger effects than in the traditional CRRA utility case. Agents endowed with such preferences dislike outcomes below the certainty equivalent. Hence a switch toward a high-volatility, high-correlation bear market regime might induce a much larger "flight to safety" effect than with CRRA preferences. Such preferences can be treated in the dynamic programming framework considered in this article, as shown by Ang, Bekaert, and Liu (2001). Finally, in this article, only three developed equity markets with cash comprise the international investment opportunity set. Given the presence of multiple multinational companies in these particular stock markets, it is likely that our analysis significantly underestimates the potential diversification benefits if the investment opportunity set is expanded to include other developed and emerging markets.

Appendix A: Data

Our core dataset consists of equity total return (price plus dividend) indices from Morgan Stanley Capital International (MSCI) for the United States, United Kingdom, and Germany. The short rate is the U.S. one-month EURO rate. Our sample period is from January 1970 to December 1997, for a total of 335 monthly return observations. In the short-rate models, our sample period is from January 1972 to December 1997. The focus on the United States, United Kingdom, and Germany arises from our desire to select the major equity markets that can be considered to be reasonably integrated during our sample period. This is definitely the case for the U.S. and U.K. markets, which on 31 July 1998 represented 49.4% and 10.5% of total market capitalization, respectively, in the world MSCI index. Bekaert and Harvey (1995) find that they cannot reject that Germany is fully integrated with the Unites States during our sample period. Since Japan underwent a gradual liberalization process in the 1980s we exclude it from our analysis [see Gultekin, Gultekin, and Penati (1989)]. Adding Germany brings the total market capitalization represented to 65.5%. We use dollar-denominated monthly returns in our empirical work. The returns show insignificant autocorrelations. Unconditionally correlations are positive and range from 36% for the United States and Germany to 51% for the United States and the United Kingdom. A full list of sample statistics is given in the NBER working paper version of this article.

The RS beta models of currency hedging use excess returns over the one-month U.S. EURO rate from January 1975 to July 1997. Our hedged returns are constructed using logarithmic returns. We define excess unhedged foreign equity returns as $\tilde{y}^{uh}_{t+1} = y^{USD}_{t+1} - r_t$, where y^{USD}_{t+1} are returns in U.S. dollars and r_t is the continuously compounded U.S. short rate. The excess hedged foreign equity return is defined as $\tilde{y}^{h}_{t+1} = y^{LC}_{t+1} - r^*_t$, where y^{LC}_{t+1} are returns in local currency and r^*_t is the foreign short rate (the continuously compounded one-month foreign EURO rate).

Appendix B: Markov Discretization Under Regimes and Predictability

Under the case of regime switching and predictability, we follow Tauchen and Hussey (1991) by calibrating an approximating Markov chain to the RS DGP. We will discuss the calibration of the short-rate model to the U.S.–U.K. system, as the extension to the U.S.–U.K.–German system is straightforward. We first fit a discrete Markov chain to the predictor instrument r_t, which follows

$$r_{t+1} = c(s_{t+1}) + \phi(s_{t+1}) r_t + v(s_{t+1}) \sqrt{r_t}\, u^r_{t+1}, \tag{B1}$$

with $u^r_{t+1} \sim N(0, 1)$. The transition probabilities are state dependent:

$$p(s_{t+1} = i | s_t = i; \mathcal{I}_t) = \frac{\exp(a_i + b_i r_t)}{1 + \exp(a_i + b_i r_t)}. \tag{B2}$$

We first fit a Markov chain to short rates for regime 1, then to regime 2, and then combine the chain using the transition probabilities. From hereon, we use the word "state" to refer to the discrete states of the Markov chain, which approximate the continuous distribution in each "regime state," or "regime." The equity return shocks are correlated with the short rate, but the short-rate states are the only driving variables in the system. We will show how to easily incorporate equity without expanding the number of states beyond those needed to approximate the distribution of r_t.

The idea behind Markov discretization is to choose points $\{r_i\}$ and a transition matrix Π which approximate the distribution of r_t. We choose $\{r_i\}$ from the unconditional distribution of r_t. We can then find the transition probabilities p_{ij} from r_i to r_j by evaluating the conditional density of r_j [which is normal from Equation (B1)] and then normalizing the densities so that they sum to unity, that is,

$$\sum_j p_{ij} = 1. \tag{B3}$$

Any highly persistent process such as short rates requires a lot of states for reasonable accuracy. When a square root process is introduced, the asymmetry of the distribution and the requirement that the states be nonnegative introduce further difficulties.

To aid us in picking an appropriate grid for r_t in each regime, we first simulate a sample of length 200,000 from Equations (B1) and (B2), with an initial presample of length 10,000 to remove the effects of starting values. During the simulation we record the associated regime with each interest rate. We record the minimum and maximum simulated points in each regime. For regime 1, which is the less persistent, higher conditional mean regime, we take a grid over points 2.5% higher (lower) than the simulated maximum (minimum). For regime 2, the "normal regime" with very low mean reversion, the persistence leads us to take a grid starting close to zero to 2.5% higher than the simulated maximum. We use 50 points for regime 1 and 100 points for regime 2 to take the stronger persistence in this regime into account. We also employ a strategy of "oversampling" from the overlapping range of the regimes to more accurately adjust for the transition process across regimes. We place 95% (90%) of the points in regime 1 (2) in the overlap.

Let $\{r_i^k\}$ denote the states in regime k. We create the following partial transition matrices by the method outlined above: from $\{r_i^1\}$ to $\{r_i^1\}$, from $\{r_i^1\}$ to $\{r_i^2\}$, from $\{r_i^2\}$ to $\{r_i^1\}$, and from $\{r_i^2\}$ to $\{r_i^2\}$. Denote these by $\Pi_{j\to k}$ for $j, k = 1, 2$. The rows of each $\Pi_{j\to k}$ will sum to 1. The total states for the Markov chain consist of $\{\{r_i^1\}\{r_i^2\}\}$.

Denote $P_{jk}(r) = p(s_t = k | s_{t-1} = j, r_{t-1} = r)$, which is given by Equation (B2). To mix the $\Pi_{j\to k}$ matrices to obtain Π for each r_i^k, we calculate $P_{jk}(r_i^k)$ and then weight the appropriate row of each $\Pi_{j\to k}$ to combine into Π. For example, for a state in the first regime, r_i^1, we calculate $P_{11}(r_i^1)$ and $P_{12}(r_i^1)$. Then the appropriate row in Π corresponding to r_i will consist of $P_{11}(r_i^1)$ times the appropriate row corresponding to $\Pi_{1\to 1}$, and $P_{12}(r_i^1)$ times the appropriate row corresponding to $\Pi_{1\to 2}$.

This Markov chain is an accurate approximation of the RS process in Equations (B1) and (B2). In particular, when a sample of 100,000 is simulated from the Markov chain and the RS process reestimated, all the parameters are well within one standard error of the original parameters. Also, the first two moments of the chain match the population moments of the RS process to two to three significant digits.

The Markov chain for r_t now consists of the states $\{r_i\}$ with a transition matrix Π, which is 150×150. To introduce equity into the chain we introduce the triplets $\{(r_i, y_i^1, y_i^2)\}$, where y_i^j are the equity points for country j. We choose the points $\{y_i^j\}$ approximating country j by Gaussian–Hermite weights for the conditional normal distribution for each regime. In our setup, the equity returns for country j are given by

$$y_{t+1}^j = \mu^j(s_{t+1}) + \sigma^j(s_{t+1}) u_{t+1}^j, \tag{B4}$$

where cross-correlations between u_{t+1}^j, $j = 1, 2$ and u_{t+1}^r are state dependent. In a given regime, a Cholesky decomposition can be used to make a transformation from the uncorrelated normal errors $(u_1\, u_2\, u_3)'$ into the correlated errors $(e_1\, e_2\, e_3)'$.

Note that in this formulation only the short rate is the driving process, and it is the only variable we need to track at each time t. To accomodate the equity states we can expand Π column-wise. We choose three states per equity, making an effective transition matrix of 150×1350 where the rows sum to 1. (Note, a full 1350×1350 transition matrix could also be constructed, but the nine rows corresponding to a particular r_i would be exactly the same.) Each short-rate state is associated with nine possible equity states. The only modification we need in the method outlined above is to construct new partial transition matrices, so $\Pi_{1\to 1}$ becomes 50×450, $\Pi_{1\to 2}$ becomes 50×900, $\Pi_{2\to 1}$ becomes 150×450, and $\Pi_{2\to 2}$ becomes 100×900. These partial transition matrices can be mixed in the same manner as outlined before.

We find that there is a systematic downward bias when the implied moments conditional on the regime and the unconditional moments are calculated from the Markov chain. This results from the regime-dependent distributions not being exactly unconditionally normally distributed in each regime from the presence of the square root term in the volatility of r_t, so Gaussian–Hermite weights will not be optimal in this setting. We make a further adjustment of scaling the volatility of the U.S.(U.K.) by 4% (5%) upwards. Our final Markov chain matches means, variances, and correlations to two to three significant digits.

When we solve the FOCs in Equation (6) we find that strong persistence in r_t causes some instability at very low ($<1.5\%$) and very high ($>28\%$) interest rates. In these ranges the portfolio weights are not as smooth as the plots that appear in Figure 5. At very high interest rates, the portfolio weights also start rapidly increasing for regime 2. These do not affect any solutions in the middle range. The inaccuracies arise because at the end of the chains, the Markov chain must effectively truncate the conditional distributions on the left (right) at low (high) interest rates. With experimentation we found that the inaccuracies at the end of the chain decrease as the persistence decreases.

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