Long Run Growth of Financial Data Technology

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October 11, 2019‡

Abstract

"Big data" financial technology raises concerns about market inefficiency. A common concern is that the technology might induce traders to extract others' information, rather than to produce information themselves. We allow agents to choose how much they learn about future asset values or about others' demands, and we explore how improvements in data processing shape these information choices, trading strategies and market outcomes. Our main insight is that unbiased technological change can explain a market-wide shift in data collection and trading strategies. However, in the long run, as data processing technology becomes increasingly advanced, both types of data continue to be processed. Two competing forces keep the data economy in balance: data resolves investment risk, but future data creates risk. The efficiency results that follow from these competing forces upend two pieces of common wisdom: our results offer a new take on what makes prices informative and whether trades typically deemed liquidity-providing actually make markets more resilient.

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‡We thank the editor, John Leahy, as well as 3 anonymous referees for their help and support. We also thank Daniel Andrei, Marios Angeletos, Markus Brunnermeier, Martin Eichenbaum, Pete Kyle, Sergio Rebelo, Steven Strongin, Robert Ulbricht, and Xavier Vives, seminar and conference participants at AFA, Barcelona GSE, Cornell, Fordham, Maryland, NYU, Princeton, Stanford, Yale and the SED conference, the NASDAQ DRP research day, and the LAEF conference on information in finance for comments. We thank the Global Markets Institute at Goldman Sachs for their financial support through the GMI Fellowship program. Any opinions, findings, or conclusions are those of the authors and do not reflect the views of Goldman Sachs or the GMI. We thank John Barry, Chase Coleman, Matias Covarrubias, Adam Lee, Ran Liu, Roxana Mihet, and Arnav Sood for their capable research assistance. JEL codes: G14, E2. Keywords: FinTech, big data, financial analysis, liquidity, information acquisition, growth.

In most sectors, technological progress boosts efficiency. But in finance, more efficient data processing and the new data-intensive trading strategies it spawned have been blamed for market volatility, illiquidity and inefficiency. One reason financial technology is suspect is that its rise has been accompanied by a shift in the nature of financial analysis and trading. Instead of "kicking the tires" of a firm, investigating its business model or forecasting its profitability, many traders today engage in statistical arbitrage: they search for "dumb money" or mine order flow data and develop algorithms to profit from patterns in others' trades. Why might investors choose one strategy over another and why are these incentives to process each type of data changing? Answering these questions requires a model. Just as past investment rates are unreliable forecasts for economies in transition, empirically extrapolating past financial trends is dubious in the midst of a technological transformation.

To make sense of current and future long-run trends requires a growth model of structural changes in the financial sector. Since much of the technological progress is in the realm of data processing, we use an information choice model to explore how unbiased technological progress changes what data investors choose to process, what investment strategies they adopt, and how the changing strategies alter financial market efficiency. Structural change in the financial sector arises because improvements in data processing trigger a shift in the type of data investors choose to process. Instead of data about firm fundamentals, firms choose to process more and more data about other investors' demand. Each data choice gives rise to an optimal trading strategy. The resulting shift in strategies is an abandonment of value investing and the rise of a strategy that is part statistical arbitrage, part retail market making, and part to extract what others know. Just like the shift from agriculture to industry, some of the data-processing shift we describe takes place because growing efficiency interacts with decreasing returns. But unlike physical production, information leaks out through equilibrium prices, producing externalities and a region of endogenous increasing returns that do not arise in standard growth models.

The consequences of this shift in strategy upend two pieces of common wisdom: first, the abandonment of fundamentals-based investing does not necessarily compromise financial market efficiency. Efficiency, as measured by price informativeness, continues to rise, even as fundamental data gathering falls. These results change the way we interpret empirical measurement. They support the interpretation of price informativeness as a proxy for total information, but not the idea that price informativeness measures information that is specifically about firm fundamentals. Our second surprise is that the price impact of an uninformed trade (liquidity) does not consistently fall. Even though demand data allows investors to identify uninformed trades, and even though investors use this information to "make markets" for demand-driven trades, market-wide liquidity may not improve.

A key theme of the paper and the force underlying the long-run results, including the stagnation of liquidity, is the following observation: data resolves risk, but it also creates it. First, data resolves risk by allowing investors to better forecast asset payoffs. If more of the payoff is predictable, the remaining risk is less. That force is present in most information choice models. The opposing force arises because we use a dynamic model. Most models of information choice have assets that are one-period lived; at the start of the period, someone buys assets that offer an exogenous payoff at the end. In our model, as in equity markets, assets are multi-period lived. The payoff of an asset depends on its future price. But the future price depends on what the future investors will know. If future investors have a large amount of data, the future price is more informative and thus more sensitive to future innovations in dividends or in demand. If future prices are more sensitive to future news, they are riskier today. A future price that is less forecastable today creates risk, which we call *future information risk*. This competition between static risk-reduction and future information risk-creation governs the long-run market liquidity and information choices.

To think about choices related to information and their equilibrium effects, a noisy rational expectations framework is an obvious starting point. We add three ingredients. First, we add a continuous choice between firm fundamental information and investor demand information. We model data processing in a way that draws on the information processing literatures in macroeconomics and finance.[1] However, the trade-off between processing data about trading demand and data about firm fundamentals is new to the literature. This trade-off is central to contemporary debate and essential to our main results. Second, we add long-run technological progress. It is straightforward to grow the feasible signal set. But doing so points this tool in a new direction, so that it answers a different set of questions. Third, we use long-lived assets, as in Wang (1993), because ignoring the fact that equity is a long-lived claim fundamentally changes our results. Because long-lived assets have future payoff shocks that are not learnable today, they ensure that uncertainty does not disappear and the equilibrium continues to exist, even when technology is advanced. Long-lived assets are also responsible for our long-run balanced growth path and the stagnation of liquidity. There are many aspects to the financial technology revolution and many details of modern trading strategies from which our analysis abstracts away. But building up a flexible equilibrium framework from well-understood building blocks, instead of a stylized, detail-oriented model, we give others the ability to adapt and use the framework to answer many long-run questions.

The key to the transition from a fundamental to a demand data strategy is understanding what makes each type of data valuable. Fundamental data is always valuable. It allows investors to predict future dividends and future prices. Demand data contains no information about any future cash flows. Yet it has value because it enables an investor to trade against demand shocks – sometimes referred to as searching for "dumb money." By buying when demand shocks are low and

[1]See e.g., Caplin, Leahy and Matejka (2016); Maćkowiak and Wiederholt (2012); Nimark (2008); Briggs et al. (2017); Kacperczyk, Nosal and Stevens (2015); Barlevy and Veronesi (2000); Goldstein, Ozdenoren and Yuan (2013); Blanchard, L'Huillier and Lorenzoni (2013); Fajgelbaum, Schaal and Taschereau-Dumouchel (2017); Angeletos and La'O (2014); Atkeson and Lucas (1992); Chien, Cole and Lustig (2016); Basetto and Galli (2017); Abis (2017).

selling when they are high, an investor can systematically buy low, sell high, and profit. This is the sense in which the demand-data trading strategy looks like market making for uninformed retail investors. Demand-data processors stand ready to trade against – make markets for – uninformed orders. The mathematics of the model suggest another, complementary interpretation of the rise in demand-based strategies. Demand shocks are the noise in prices. Knowing something about this noise allows investors to remove that known component and to reduce the noise in prices. Since prices summarize what other investors know, removing price noise is a way of extracting others' fundamental information. Seen in this light, the demand-driven trading strategy shares some of the hallmarks of automated trading strategies, largely based on order flow data, that are also designed to extract the information of other market participants.

Our main results in Section 2 describe the evolution of data processing in three phases. *Phase one: technology is poor and fundamental data processing dominates.* In this phase, fundamental data is preferred because demand data has little value. To see why, suppose that no investor has any fundamental information. In such an environment, all trades are uninformed. No signals are necessary to distinguish between informed and uninformed trades. As technology progresses and more trades are information-driven, identifying and trading against the remaining non-informational trades becomes more valuable. *Phase two: moderate technology generates increasing returns to demand data processing.* Most physical production as well as most of the information choices in financial markets exhibit decreasing returns, also called strategic substitutability. Returns decrease because acquiring the same information as others leads one to buy the same assets that others do, and these assets are more expensive because they are popular. Our increasing returns come from an externality that is specific to data: information leaks through the equilibrium price. When more investors process demand data, they extract more fundamental information from equilibrium prices and they trade on that information. More trading on fundamental information, even if extracted, makes the price more informative, which encourages more demand data processing to enable more information extraction from the equilibrium price. *Phase three: high technology restores balanced data processing growth.* As technology progresses, both types of data become more abundant. In the high-technology limit, both types grow in fixed proportion to each other. When information is abundant, the natural substitutability force in asset markets strengthens and overtakes complementarity.

The consequences of this shift in data analysis and trading strategies involve competing forces. Three economic forces – decreasing returns, increasing returns, and future information risk – appear whether technology is unbiased or biased, whether demand is persistent or not, and for most standard formulations of data constraints. We identify each force theoretically. However, if we want to learn which force is likely to dominate, we need to put some plausible numbers into the model. Section 3 calibrates the model to financial market data so that we can numerically explore the growth transition path and its consequences for market efficiency.

The market efficiency results call popular narratives into question. First, even as demand analysis crowds out fundamental analysis and reduces the discovery of information about the future asset value, price informativeness does not fall. The reason for this outcome is that demand information allows demand traders to extract fundamental information from prices. That makes the demand traders, and thus the average trader, better informed about future asset fundamentals. When the average trader is better informed, prices are more informative.

Second, even though demand traders systematically take the opposite side of uninformed trades, the rise of demand trading does not always enhance market liquidity. This is surprising because taking the opposite side of uninformed trades is often referred to as "providing liquidity." This is one of the strongest arguments that proponents of activities such as high-frequency trading make to defend their methods. But if by providing liquidity, we really mean reducing the price impact of an uninformed trade, then the rise of demand trading may not accomplish that end. The problem is not demand trading today, but the expectation of future informed trading of any kind – fundamental or demand – creating future information risk. So future data processing raises the risk of investing in assets today. More risk per share of asset today is what causes the sale of one of the asset's shares to have a larger effect on the price. Finally, the rise in demand-driven trading strategies, while it arises concurrent with worrying market trends, does not cause those trends. The rise in return uncertainty, and the stagnation of liquidity, emerge as concurrent trends with financial data technology, not demand analysis, as their common cause.

Finally, Section 4 explores suggestive evidence in support of the model. Section 5 concludes, offering ideas for future research.

Contribution to the existing literature Our model combines features from a few disparate literatures. Long run trends in finance are featured in Asriyan and Vanasco (2014); Biais, Foucault and Moinas (2015); Glode, Green and Lowery (2012); and Lowery and Landvoigt (2016), who model growth in fundamental analysis or an increase in its speed. Caplin, Dean and Leahy (2018) and Davila and Parlatore (2016) explore a decline in trading costs. The idea of long-run growth in information processing is supported by the rise in price informativeness documented by Bai, Philippon and Savov (2016).

A small, growing literature examines demand information in equilibrium models. In Yang and Ganguli (2009), agents can choose whether or not to purchase a fixed bundle of fundamental and demand information. In Yang and Zhu (2016) and Manzano and Vives (2010), the precision of fundamental and demand information is exogenous. Babus and Parlatore (2015) examine intermediaries who observe their customers' demands. Our demand signals also resemble Angeletos and La'O (2014)'s sentiment signals about other firms' production; Banerjee and Green (2015)'s signals about motives for trade; the signaling by He (2009)'s intermediaries; and the noise in government's market interventions in Brunnermeier, Sockin and Xiong (2017). But none of this research examines the choice central to this paper: whether to process more about asset payoffs or to analyze more demand data. Without that trade-off, previous studies cannot explore how trading strategies change as productivity improves. Furthermore, our research adds a long-lived asset to a style of model that has traditionally been static.[2] We do this because assets are not in fact static and assuming that they are reverses many of our results.

One interpretation of our demand information is that it is what high-frequency traders learn by observing order flow. Like high-frequency traders, our traders use data on asset demand to distinguish informed from uninformed trades, and they stand ready to trade against uninformed order flow. While our model has no frequency, making for a loose interpretation, it does contribute a perspective on this broad class of strategies. As such, it complements work by Du and Zhu (2017) and Crouzet, Dew-Becker and Nathanson (2016) on the theory side, as well as empirical work such as Hendershott, Jones and Menkveld (2011), which measures how fundamental and algorithmic trading affects liquidity. At the same time, if many high frequency trades are executed for the purpose of obscuring price information, our model does not capture this phenomenon. Such a practice could work in the opposite direction.

Another, more theoretical, interpretation of demand signals is that they make a public signal, the price, less conditionally correlated. Work by Myatt and Wallace (2012), Chahrour (2014), and Amador and Weill (2010) delves into similar choices between private, correlated and public information that arise in strategic settings.

1 Model

To explore growth and structural change in the financial economy, we use a noisy rational expectations model with three key ingredients: a choice between fundamental and demand data, long-lived assets, and unbiased technological progress in data processing. A key question is how to model structural change in trading activity – the way in which investors earn profits has changed. A hallmark of that change is the rise in information extraction from demand. In practice, demand-based trading takes many forms. It might take the form of high-frequency trading, where information about an imminent trade is used to trade before the new price is realized. It could be in mining tweets or Facebook posts to gauge sentiment. Extraction could take the form of "partnering," a practice where brokers sell their demand information (order flow) to hedge funds, who systematically trade against what are presumed to be uninformed traders.Finally, it may mean looking at price trends, often referred to as technical or statistical analysis, in order to discern what information others may be trading on. What all of these practices have in common is that they are not uncovering original information about the future payoff of an asset. Instead, they use public information, in conjunction with private analysis, to profit from what others already know (or don't know). We capture this general strategy, while abstracting from many of its details, by allowing

[2] Exceptions include 2- and 3-period models, such as Cespa and Vives (2012).

investors to observe a signal about the non-informational trades of other traders. This demand signal allows our traders to profit in three ways. 1) They can identify and then trade against uninformed order flow, 2) they can remove noise from the equilibrium price to uncover more of what others know, or 3) they can exploit the transitory nature of demand shocks to buy before the price rises and sell before it falls. These three strategies have an equivalent representation in the model and collectively cover many of the ways in which modern investment strategies profit from information technology.

A second significant modeling choice is our use of long-lived assets. In this literature, static models have proven very useful in explaining the many forces and trade-offs in a simple and transparent way. However, when the assumption of one-period-lived assets reverses the predictions of the more realistic dynamic model, the static assumption is no longer appropriate. Long-run growth means not only more data processing today, but even more tomorrow. In many instances, the increase today and the further increase tomorrow have competing effects. That competition is a central theme of this paper. Without the long-lived asset assumption, the long-run balanced growth and stagnating liquidity results would be overturned.

Finally, technological progress in its present form gives investors access over time to an increasingly larger set of feasible signals. While there are many possible frameworks that one might use to investigate financial growth, this one makes for a useful lens, because it explains many facts about the evolution of financial analysis, it can forecast future changes that empirical extrapolation alone would miss, and because it offers surprising, logical insights about the financial and real consequences of the structural change. One could go further and argue that some types of data have become relatively easier to collect over time. That may well be true. But changes in relative costs could explain any such pattern. We would not know what results came from relative cost changes and what comes from the fundamental economic forces created by technological change. Our simple problem is designed to elucidate economic forces, at the expense of many realistic features that could be added.

1.1 Setup

Investors At the start of each date t, a measure-one continuum of overlapping generations investors is born. Each investor i born at date t has constant absolute risk aversion utility over total, end of period t consumption c_{it+1}:

$$U(c_{it+1}) = -e^{-\rho c_{it+1}}, \tag{1}$$

where ρ is absolute risk aversion.

Each investor is endowed with an exogenous income that is e_{it} units of consumption goods. At the start of each period, investors decide how much of their income to eat now and how much to spend on a risky asset that pays off at the end of the period.

There is a single tradeable, risky asset.[3] Its supply is one unit per capita. It is a claim to an infinite stream of dividend payments $\{d_t\}$:

$$d_{t+1} = \mu + G(d_t - \mu) + y_{t+1}. \tag{2}$$

μ and $G < 1$ are known parameters. The innovation $y_{t+1} \sim N(0, \tau_0^{-1})$ is revealed and d_{t+1} is paid out at the end of each period t. While d_{t+1} and d_t both refer to dividends, only d_t is already realized at the start of time t.

In order to disentangle the static and dynamic results, we introduce parameter $\pi \in \{0, 1\}$. When $\pi = 1$, a time-t asset pays $p_{t+1} + d_{t+1}$, the future price of the long-lived asset plus its dividend. When $\pi = 0$, the asset is not long-lived. Its payoff is only the dividend, d_{t+1}. We call the $\pi = 0$ model the "static" model because current information choices do not depend on future or past choices. It is a repeated static problem with an information constraint that changes over time.

In the dynamic ($\pi = 1$) model, an investor born at date t collects dividends d_{t+1} per share and sells the asset at price p_{t+1} to the $t+1$ generation of investors. In both versions, investors combine the proceeds from risky assets with the endowment that is left $(e_{it} - q_{it}p_t)$ times the rate of time preference $r > 1$, and then consume all those resources. Cohort t consumption can only be realized in $t+1$, after dividends d_{t+1} are realized and, if $\pi = 1$, the assets are sold to the next cohort. Using c_{it+1} to denote the consumption of cohort t, which takes place in $t+1$, avoids the double subscript in $c_{t+1,t}$. Thus the cohort-t investor's budget constraint is

$$c_{it+1} = r(e_{it} - q_{it}p_t) + q_{it}(\pi p_{t+1} + d_{t+1}), \tag{3}$$

where q_{it} is the shares of the risky asset that investor i purchases at time t and d_{t+1} are the dividends paid out at the end of period t. Since we do not prohibit $c_t < 0$, all pledges to pay income for risky assets are riskless.

Demand shocks The economy is also populated by a unit measure of noise traders in each period. These traders trade for non-informational reasons.[4]Each noise trader sells x_{t+1} shares of the asset, where $x_{t+1} \sim N(0, \tau_x^{-1})$ is independent of other shocks in the model. x_{t+1} is revealed at the end of period t. For information to have value, prices must not perfectly aggregate the asset payoff information. This is our source of noise in prices. Equivalently, x_{t+1} could also be interpreted as sentiment. For now, we assume that x_{t+1} is independent over time. In Section 2.3, we discuss the possibility of autocorrelated x_{t+1}.

[3]We describe a market with a single risky asset because our main effects do not require multiple assets.

[4]In previous versions, we micro-founded heterogenous investor hedging demand that would rationalize this trading behavior.

Information Choice If we want to examine how the nature of financial analysis has changed over time, we need to have at least two types of analysis between which we can choose. Financial analysis in this model means signal acquisition. Our constraint on acquisition could represent the limited research time for uncovering new information. But it could also represent the time required to process and compute optimal trades based on information that is readily available from public sources.

Investors choose how much information to acquire or process about the end-of-period dividend innovation y_{t+1}, and also about the noisy demand shocks, x_{t+1}. We call $\eta_{fit} = y_{t+1} + \tilde{\epsilon}_{fit}$ a fundamental signal and $\eta_{xit} = x_{t+1} + \tilde{\epsilon}_{xit}$ a demand signal. What investors are choosing is the precision of these signals. In other words, if the signal errors are distributed $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{fit}^{-1})$ and $\tilde{\epsilon}_{xit} \sim N(0, \Omega_{xit}^{-1})$, then the precisions Ω_{fit} and Ω_{xit} are choice variables for investor i.

Next, we recursively define two information sets. The first is all the variables that are known to agent i at the end of period $t-1$. This information set is $\{\mathbb{I}_{t-1}, y_t, d_t, x_t\} \equiv \mathbb{I}_t^-$. This is what investors of generation t know when they choose what signals to acquire. The second information set is $\{\mathbb{I}_t^-, \eta_{fit}, \eta_{xit}, p_t\} \equiv \mathcal{I}_{it}$. This includes the two signals that the investor chooses to see and the information contained in the equilibrium prices. This is the information set the investor has when they make an investment decision. Let $\mathbb{I}_t = \cup_i \mathcal{I}_{it}$. The time-0 information set includes the entire sequence of information capacity: $\mathbb{I}_0 \supset \{K_t\}_{t=0}^{\infty}$.

When choosing information ($\Omega_{fit} \geq 0$ and $\Omega_{xit} \geq 0$), investors maximize

$$E[U(c_{it+1})|\mathbb{I}_t^-], \tag{4}$$

$$\text{s.t.} \qquad \Omega_{fit}^2 + \chi_x \Omega_{xit}^2 \leq K_t. \tag{5}$$

χ_x is a marginal processing cost parameter that allows us to consider demand data that is easier or harder to process than is fundamental data. Data constraint (5) represents the idea that getting increasingly precise information about a given variable is increasingly difficult.[5] But acquiring information about a different variable is a separate task, with a shadow cost that is additive.

The main force in the model is technological progress in information analysis. Specifically, we assume that K_t is a deterministic, increasing process.

Equilibrium An equilibrium is a sequence of investors' information choices $\{\Omega_{fit}\}$, $\{\Omega_{xit}\}$, prices $\{p_t\}$ and portfolio choices $\{q_{it}\}$, such that

1. Investors choose signal precisions Ω_{fit} and Ω_{xit} to maximize (4), taking the choices of other agents as given. This choice is subject to (5), $\Omega_{fit} \geq 0$ and $\Omega_{xit} \geq 0$.

2. Agents use Bayes' Law to combine prior information $\mathbb{I}_t^-$ with signals η_{fit}, η_{xit}, and p_t, in $\mathcal{I}_{it}$, and to update beliefs.

[5] See the online appendix for results with linear and entropy-based information constraints.

3. Investors choose their risky asset investment q_{it} to maximize $E[U(c_{it+1})|\mathcal{I}_{it}]$, taking the asset price and the actions of other agents as given, subject to the budget constraint (3).

4. At each date t, the risky asset price equates demand, minus demand shocks (sales) and one unit of supply:
$$\int_i q_{it} di - x_{t+1} = 1 \qquad \forall t. \tag{6}$$

1.2 Solving the Model

There are four main steps to solving the model.

Step 1: Solve for the optimal portfolios, given information sets. Each investor i at date t chooses a number of shares q_{it} of the risky asset to maximize the expected utility (1), subject to budget constraint (3). The first-order condition of that problem is

$$q_{it} = \frac{E[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - rp_t}{\rho Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}]}. \tag{7}$$

When using the term "investor," we do not include demand shocks (noise trades).

Step 2: Clear the asset market. Let $\bar{\mathcal{I}}_t$ denote the average investor's information set, with average signal realizations and average precision. Given the optimal investment choice, we can impose market clearing (6) and obtain a price function that is linear in past dividends d_t, the t-period dividend innovation y_{t+1}, and the demand shock x_{t+1}:

$$p_t = A_t + B(d_t - \mu) + C_t y_{t+1} + D_t x_{t+1}, \tag{8}$$

where A_t governs the price level, B is the time-invariant effect of past dividends, C_t governs the information content of prices about current dividend innovations (price information sensitivity), and D_t regulates the amount of demand noise in prices:

$$A_t = \frac{1}{r}\left[\pi A_{t+1} + \mu - \rho Var[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]\right], \tag{9}$$

$$B = \frac{G}{r - \pi G}, \tag{10}$$

$$C_t = \frac{1}{r - \pi G}\left(1 - \tau_0 Var[y_{t+1}|\bar{\mathcal{I}}_t]\right), \tag{11}$$

$$rD_t = -\rho Var[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] + \frac{r}{r - \pi G} Var[y_{t+1}|\bar{\mathcal{I}}_t]\frac{C_t}{D_t}\tau_x. \tag{12}$$

The posterior uncertainty about next-period dividend innovations is

$$Var[y_{t+1}|\bar{\mathcal{I}}_t] = (\tau_0 + \bar{\Omega}_{ft} + \bar{\Omega}_{pt})^{-1}, \tag{13}$$

where $\bar{\Omega}_{ft} = \int_i \Omega_{fit} di$ is the average fundamental signal precision and $\bar{\Omega}_{pt}$ is the average precision of the information about d_{t+1}, extracted jointly from prices and demand signals. The resulting uncertainty about the future payoff is

$$Var[\pi p_{t+1} + d_{t+1} | \bar{\mathcal{I}}_t] = \pi \left(C_{t+1}^2 \tau_0^{-1} + D_{t+1}^2 \tau_x^{-1} \right) + (1 + \pi B)^2 Var[y_{t+1} | \bar{\mathcal{I}}_t]. \tag{14}$$

Step 3: Compute the ex-ante expected utility. When choosing what information to observe, investors do not know what the signal realizations will be, nor do they know what the equilibrium price will be. The relevant information set for this information choice is $\mathbb{I}_t^-$.

We substitute optimal portfolio choice (7) and equilibrium price rule (8) into utility (1), and take the beginning of time-t expectation, $-E[\exp(-\rho c_{it+1}) | \mathbb{I}_t^-] = -E[E[\exp(-\rho c_{it+1}) | \eta_{fit}, \eta_{xit}, p_t] | \mathbb{I}_t^-]$, where the equality follows from the law of iterated expectations. Appendix A shows that the interim conditional expectation solution is similar to most CARA-normal models: $g_{it} \exp\{(1/2) w^2 Var[\pi p_{t+1} + d_{t+1} | \bar{\mathcal{I}}_t]^{-1}\}$, where g_{it} is a scaling factor, related to i's endowment, and w is a function of the equilibrium pricing coefficients and model parameters, all of which the investor knows or deduces from the environment.

The key feature of this solution is that the agent's choice variables, Ω_{fit} and Ω_{xit}, show up only through the conditional precision of payoffs, $Var[\pi p_{t+1} + d_{t+1} | \mathcal{I}_{it}]^{-1}$. The reason they only appear there is that the first variance term in asset demand, $Var[\pi p_{t+1} + d_{t+1} | \mathcal{I}_{it}]$, and p_t have ex-ante expected values that do not depend on the *precision* of any given investor's information choices. In other words, choosing to get more data of either type does not, by itself, lead one to believe that payoffs or prices will be particularly high or low. So, information choices amount to minimizing payoff variance $Var[\pi p_{t+1} + d_{t+1} | \mathcal{I}_{it}]$, subject to the data constraint. The payoff variance has terms that the investor takes as given, plus a term that depends on the dividend variance, $Var[y_{t+1} | \mathcal{I}_{it}]$.

So, the information choice problem boils down to this question: what information minimizes dividend uncertainty $Var[y_{t+1} | \mathcal{I}_{it}]$? According to Bayes' Law, $Var[y_{t+1} | \mathcal{I}_{it}]$ depends on the sum of fundamental precision Ω_{fit} and price information precision Ω_{pit} (eq. 13). Since price precision is $\Omega_{pit} = (C_t / D_t)^2 (\tau_x + \Omega_{xit})$, the expected utility is a deterministic, increasing function of the sum of Ω_{fit} and $(C_t / D_t)^2 \Omega_{xit}$.

Thus, the optimal information choices maximize the weighted sum of the fundamental and demand precisions:

$$\max_{\Omega_{fit}, \Omega_{xit}} \ \Omega_{fit} + \left(\frac{C_t}{D_t} \right)^2 \Omega_{xit} \tag{15}$$

$$\text{s.t. } \Omega_{fit}^2 + \chi_x \Omega_{xit}^2 \leq K_t, \quad \Omega_{fit} \geq 0, \text{ and } \Omega_{xit} \geq 0.$$

The reason we combine fundamental and demand information in this way is because of the linear price equation (8) and Bayes' law. This would be true in any information choice problem, where the objective is a decreasing function of the dividend or payoff uncertainty and prices take the form that they do in (8). Appendix A shows that the same information objectives arise with a different utility function, where investors prefer the *early resolution* of uncertainty.

Step 4: Solve for information choices. The first-order conditions yield

$$\Omega_{xit} = \frac{1}{\chi_x}\left(\frac{C_t}{D_t}\right)^2 \Omega_{fit}. \tag{16}$$

This solution implies that information choices are symmetric across agents. Therefore in what follows, we drop the i subscript to denote an agent's data processing choice. Moreover, the information set of the average investor is the same as each investor's information set, $\bar{\mathcal{I}}_t = \mathcal{I}_{it}$.

The information choices are a function of the pricing coefficients, like C and D, which are, in turn, functions of information choices. To determine the evolution of analysis and its effect on asset markets, we need to compute a fixed point to a highly non-linear set of equations. After substituting in the first-order conditions for Ω_{ft} and Ω_{xt}, we can write the problem as a recursive non-linear equation in one unknown.

Since this is an overlapping generations model, we expect there to be multiple equilibria. For some parameter values, there are multiple real solutions to this problem. While in some models, multiple equilibria can complicate predictions, in our model, they are not problematic for three reasons: 1) the calibrated model has a unique solution; 2) the theoretical results hold for any equilibrium; and 3) in the static model, there is a clear selection criterion. One equilibrium typically converges to the $\Omega_{xt} = 0$ solution, as demand data becomes scarce. The other has price coefficients that become infinite. The obvious choice is the solution with a continuous limit.

1.3 Interpreting Demand Data Trading

Why are demand signals useful? They do not predict future dividends or future prices. They only provide information about current demand. However, this information is valuable because it tells the investor something about the difference between price and expected asset value. One can see this by looking at the signal extracted from price. Price is a noisy signal about dividends. To extract the price signal, we subtract the expected value of all the terms besides the dividend and divide by the dividend coefficient C_t. The resulting signal extracted from prices is:

$$\frac{\left(p_t - A_t - B(d_t - \mu) - D_t E[x_{t+1}|\bar{\mathcal{I}}_t]\right)}{C_t} = y_{t+1} + \underbrace{\frac{D_t}{C_t}(x_{t+1} - E[x_{t+1}|\bar{\mathcal{I}}_t])}_{signal\ noise}. \tag{17}$$

Notice how demand shocks x_{t+1} are the noise in the price signal. So information about this demand reduces noises in the price signal. In this way, the demand signal can be used to better extract others' dividend information from the price. In this sense, demand analysis is information extraction.

Of course, real demand traders are not taking their orders then inverting an equilibrium pricing model to infer future dividends. But another way to interpret the demand trading strategy is that it identifies non-informational trades to trade against. In equation (17), when x_{t+1} is high, noise trades are mostly sales and, as $D_t < 0$, prices are low. Moreover, $(D_t/C_t) < 0$ implies that high x_{t+1} makes the expected dividend minus price high, which induces those with demand information to buy. Thus, demand trading amounts to finding the non-informational trades and then systematically taking the opposite side: buying when prices are low and selling when they are high for non-dividend-related reasons. This strategy of trading against uninformed trades is commonly referred to as trading against "dumb money." An alternative way of interpreting the choice between fundamental and demand data is that agents are choosing between decoding private or public signals. Fundamental signals have noise that is independent across agents. These are private. But although demand data's noise is independent, such data is used in conjunction with the price, which is a public signal. The resulting inference about shock y_{t+1}, conditional on the price and the x_{t+1} signal, is conditionally correlated across agents, just as a public signal would be.

The key to the main results that follow is that reducing the noise in x_{t+1} reduces the price noise variance in proportion to $(D_t/C_t)^2$. Put conversely, increasing the precision of information about x_{t+1} (the reciprocal of variance) increases the precision of the dividend information, in proportion to $(C_t/D_t)^2$. The long-run shifts are caused by the marginal rate of substitution of demand signal precision for fundamental signal precision, $(C_t/D_t)^2$, which changes as technology grows.

If we interpret demand trading as finding dumb money, it is easy to see why it would become more valuable over time. If there is very little information, then everyone is "dumb," and finding dumb money is pointless. But when traders are sufficiently informed, distinguishing dumb from smart money, before taking the other side of a trade, becomes essential.

1.4 Measuring Financial Market Efficiency

To study the effects of financial technology on market efficiency, we assess efficiency in two ways. One efficiency measure is price informativeness. The asset price is informative about the unknown future dividend innovation y_{t+1}. The coefficient C_t on the dividend innovation y_{t+1} in equilibrium price equation (8) governs the extent to which price reacts to a dividend innovation. The coefficient D_t governs the extent to which demand shocks add noise to the price. Therefore, C_t/D_t is a signal-to-noise ratio that we use to measure price informativeness. It corresponds closely to the price informativeness measure of Bai, Philippon and Savov (2016).

The other measure of market efficiency is liquidity. Liquidity is the price impact of an uninformed noise trade. That impact is the price coefficient D_t. Note that D_t is negative because it represents selling pressure; the reduced demand lowers the price. So, a more negative D_t represents a higher price impact and a less liquid market. Increasing (less negative) D_t indicates an improvement in liquidity.

1.5 Existence

One issue with the static ($\pi = 0$) model is that, for any set of parameters, if $K > \frac{\rho^2}{2}\sqrt{\chi_x}$, there is no solution to the model. Since one of our main points is to understand what happens as technology grows, this equilibrium non-existence at high levels of technology is particularly problematic. A key reason to use a model of long-lived assets is that, as long as $\tau_0 \tau_x \geq \left(\frac{4\rho}{r(r-G)}\right)^2$, equilibrium exists at every level of K (See eq. 104). Therefore in what follows, whenever we assume $\pi = 1$, we also assume that $\tau_0,\ \tau_x$ are sufficiently large that the existence condition holds. This allows us to explore information choices both when information is scarce and when it is abundant.

The reason equilibrium is preserved is that the unlearnable risk, introduced by future price fluctuations that cannot be known today, keeps prices from being too informative. Because the unlearnable risk grows as technology progresses, the asset never becomes nearly riskless and demand for it never explodes.

For the static results that follow, we ensure existence by assuming that whenever $\pi = 0$, information is not too abundant: $K \leq \frac{\rho^2}{2}\sqrt{\chi_x}$.

2 Main Results: A Long-Run Shift in Financial Analysis

This section explores the logical consequences of growth in information (data) processing technology, for financial analysis, trading strategies, and market efficiency. In order to understand what forces produce these results, we first explore the static trade-offs involved in processing either fundamental or demand data. In Section 2.1, we consider the effect of an incremental technological change in a setting where an asset's payoff is only its exogenous dividend. When the payoff is the sole exogenous dividend, it clarifies the trade-off between fundamental and demand data and the static forces by which the trade-off is shaped. When $\pi = 0$, future choices or outcomes have no bearing on today's decisions. This is obviously false: by its nature, equity is a long-lived claim. But this setting allows us to clearly derive forces also present in the dynamic model and to distinguish between static and dynamic forces.

The main results center around the model's dynamics. When assets are long-lived ($\pi = 1$, Section 2.2), future information risk arises. The risk posed by shocks that will be realized in the future governs the long-run market convergence. We find that as data technology becomes more and more productive, fundamental and demand data processing grow proportionately, price informativeness is high, and there are competing forces in liquidity. For asset prices, the presence of two types of information matters for the transition path. Demand data also changes the value of the price coefficients to which the economy converges in the long run. But, surprisingly, the presence and the growth of demand data does not qualitatively change the long-run price convergence.

2.1 Short Run Data Trade-offs

This section investigates the within-period trade-offs in our model. First, we explore what happens in the neighborhood near no information processing, $K \approx 0$. We show that all investors prefer to acquire only fundamental information in this region. Thus, at the start of the growth trajectory, investors primarily investigate firm fundamentals. Next, we prove that an increase in aggregate information processing increases the value of demand information relative to fundamental information. Fundamental information has diminishing relative returns. But in some regions, demand information has increasing returns. What do these phenomena mean for the evolution of analysis? The economy begins by doing fundamental analysis before rapidly shifting to demand analysis. We explore this mechanism, as well as its market efficiency effects, in the following propositions.

To understand why investors with little information capacity use it all on fundamental information, we start by thinking about what makes each type of information valuable. Fundamental information is valuable because it informs an investor about whether the asset is likely to have a high dividend payoff tomorrow. Since prices are linked to current dividends, this also predicts a high asset price tomorrow and thus a high return. Knowing this allows the investor to buy more of the asset when its return will be high and less of it when the return is likely to be low.

In contrast, demand information is not directly relevant to the future payoff or price. But one can still profit from trading on demand. An investor who knows that noisy demands are high will systematically profit by selling (buying) the asset when high (low) demand makes the price higher (lower) than the fundamental value, on average. In other words, demand signals allow one to trade against dumb money.

The next result proves that if the price has very little information embedded in it because information is scarce (K_t is low), then getting demand data to extract price information is not very valuable. In other words, if all trades are "dumb," then identifying the uninformed trades has no value.

Result 1 ***When information is scarce, demand analysis has zero marginal value (dynamic or static):***
As $K_t \to 0$, for $\pi = 0$ or 1, $dU_1/d\Omega_{xt} \to 0$.

The proof in Online Appendix B, which holds for the static and dynamic models ($\pi = 0$ or 1), establishes two key claims: 1) that when $K \approx 0$, there is no information in the price: $C_t = 0$ and 2) that the marginal rate of substitution of demand information for fundamental information is proportional to $(C_t/D_t)^2$. In particular, $dU_1/d\Omega_{xt} = (\frac{C}{D})^2 dU_1/d\Omega_{ft}$. Thus, when the price contains no information about future dividends ($C_t = 0$), then analyzing demand has no marginal value ($(C_t/D_t)^2 = 0$). Demand data is only valuable in conjunction with the current price, p_t, because it allows one to extract more information from the price. Demand data trading when $K_t = 0$ is like removing noise from a signal that has no information content. Put differently, when there is no fundamental information, the price perfectly reveals noise trading. There is no need to process data on noisy demand if it can be perfectly inferred from the price.

This result explains why analysts focus on fundamentals when financial analysis productivity is low. In contrast, when prices are highly informative, demand information is like gold because it allows one to exactly identify the price fluctuations that are not informative and are therefore profitable to trade on. The next results explain why demand analysis increases with productivity growth and why it may eventually start to crowd out fundamental analysis.

Next we turn to understanding how technological growth affects prices. Specifically, we examine the static effect on price informativeness, C_t/D_t (the signal-to-noise-ratio); the price information sensitivity, C_t; and (il)liquidity, D_t. Technology improvements affect the price coefficients through two channels: a change in fundamental analysis and a change in demand analysis. Using the chain rule, we can describe which portion of the total effect of the change in information technology, K_t, works through which channel.

Result 2 ***Price response to technological growth (static).*** *For* $\pi = 0$,

(a) $\frac{dC_t/|D_t|}{dK_t} > 0$
$\frac{\Omega_{ft}}{2K_t(2\Omega_{xt}C_t/D_t+\rho)} > 0$ *of this effect comes through changes in fundamental information* Ω_{ft}*;*
$-\frac{\rho C_t/D_t+\Omega_{ft}}{2K_t(2\Omega_{xt}C_t/D_t+\rho)} > 0$ *of this effect comes through changes in demand information* Ω_{xt}*.*

(b) $\frac{dC_t}{dK_t} > 0$,

(c) $\frac{d|D_t|}{dK_t} > 0$ *iff* $K_t < \bar{K}_D$, *where* $\bar{K}_D$ *is defined by equation* (82) *in Online Appendix B.*

For each type of analysis, there is a direct and an indirect effect of K_t. The direct effect is what would arise if we kept the signal-to-noise ratio, i.e, the marginal rate of transformation across the two types of analysis, constant. An additional indirect effect arises because the change in the type of analysis affects the signal-to-noise ratio, which then affects the information choices. However, using the envelope theorem, the marginal indirect effect is zero at the optimum. Therefore, the results above reflect only the direct effect. The proof of result 5 shows that our finding that more information today increases the concurrent price informativeness also carries over to the dynamic model ($\pi = 1$).

The concern about the deleterious effects of financial technology on market efficiency stems from the worry that technology will deter the research and discovery of new fundamental information. This concern is not unwarranted. Not only does more fundamental information encourage the extraction of information from demand, but once it starts, demand analysis feeds on itself. The next corollary shows that when $\pi = 0$, aggregate demand analysis increases an individual's incentive to learn about demand.

For most of our results, we use Ω_{xt} to indicate the demand of every investor, because all are symmetric. At this point, it is useful to distinguish between one particular investor's choice, $\boldsymbol{\Omega}_{xit}$, and the aggregate symmetric choice, Ω_{xt}.

Result 3 ***Complementarity in demand analysis (static).*** *For $\pi = 0$, $\frac{\partial \boldsymbol{\Omega}_{xit}}{\partial \Omega_{xt}} \geq 0$.*

Fundamental information, Ω_{ft}, exhibits strategic substitutability in information, just as in Grossman and Stiglitz (1980). But for demand information, the effect is the opposite. More precise average demand information (higher Ω_{xt}) can increase $(C_t/D_t)^2$, which is the marginal rate of the substitution of demand information for fundamental information. The rise in the relative value of demand data is what makes investors shift their data analysis from fundamental to demand when others do more demand analysis. That is complementarity. It holds in both the static model and the dynamic model, with conditions. (For a result with $\pi = 1$, see Online Appendix B).

Intuitively, higher signal-to-noise (more informative) prices encourage demand trading because the value of demand analysis comes from the ability to better extract the signal from prices. In this model (as in most information processing problems), it is easier to clear up relatively clear signals than it is very noisy ones. So the aggregate level of demand analysis improves the signal clarity of prices, which makes demand analysis more valuable.

Why does the price signal-to-noise ratio rise? From (11), we know that C_t is proportional to $1 - \tau_0 Var[y_{t+1}|\bar{\mathcal{I}}_t]$. As either type of information precision (Ω_{ft} or Ω_{xt}) improves, the uncertainty about the next period's dividend innovation $Var[y_{t+1}|\bar{\mathcal{I}}_t]$ declines and C_t increases. D_t is the coefficient on noise x_{t+1}. The price impact of uninformative trades $|D_t|$ may also increase with information (Result 2c). But $|D_t|$ does not rise at a rate faster than C_t. Thus, the ratio $C_t/|D_t|$, which is the signal-to-noise ratio of prices, and the marginal value of demand precision, increases with more information.

The final result of this section characterizes how different types of analysis change as there is technological progress, in a world with one-period-lived assets.

Result 4 ***Fundamental and demand analyses response to technological growth (static).*** *For $\pi = 0$,*

(a) fundamental analysis initially grows and then declines, $\frac{d\Omega_{ft}}{dK_t} > 0$ iff $K_t < \bar{K}_f = \frac{\sqrt{3}}{2}\bar{K}$,

(b) demand analysis is monotonically increasing, $\frac{d\Omega_{xt}}{dK_t} > 0$.

As technology improves, both types of information analysis initially grow. However, fundamental analysis experiences two competing forces. On one hand, more available capacity increases fundamental analysis. On the other hand, the higher marginal rate of substitution between demand and fundamental analyses (a higher signal-to-noise ratio) dampens the level of fundamental analysis. When there is not much information available, the first force dominates. Once the information processing capacity grows beyond a certain threshold, substitution towards demand analysis takes over and fundamental analysis falls. The online appendix shows how this intuition also carries over to the dynamic economy ($\pi = 1$). We characterize the conditions under which price informativeness, price information sensitivity, and liquidity improve in response to exogenous changes in data endowments (Result 8).

A rise in data processing efficiency is not the only force that can boost price informativeness. Section C in the online appendix also derives comparative statics. The results show that greater risk tolerance (lower ρ) or an increase in demand data processing efficiency (lower χ_x) both raise $C_t/|D_t|$. Trends in either variable could therefore also prompt a switch to demand-based trading strategies.

2.2 Dynamic Results

Next, we explore the model's dynamic forces. By assuming $\pi = 1$, the asset's payoff is $p_{t+1}+d_{t+1}$, as with a traditional equity payoff. This introduces one new concept, future information risk: knowing that tomorrow's investors will get more information makes it harder to predict what those investors will believe, which makes future demand and future prices more uncertain. Since the future price is part of the payoff to today's asset, future information risk makes this payoff more uncertain. Assets look riskier to investors. The result that future information creates risk is central to the main finding of long-run balanced data processing. Without long-lived assets, information learned tomorrow cannot affect the payoff risk today. Long-lived assets are integral to all the results that depend on future information risk, including our main result, the long run balanced growth of data processing.

Future information risk is the part of payoff risk $Var[p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]$ that comes from $\{K_{t+1}, \ldots, K_\infty\} > 0$. This risk arises from shocks to tomorrow's price that are unknowable today. But not all unknowable shocks come from future information. Tomorrow's demand shock will create price noise, regardless of whether or not others learn about it. Instead, future information grows the impact of unknowable price shocks. The next result shows that more learning tomorrow makes tomorrow's prices, p_{t+1}, more sensitive to shocks that are unlearnable today, and this additional risk makes today's prices less informative.

Result 5 Future Information Creates Risk, Reduces Informativeness (dynamic only) *If $\pi = 1$ and ρ is sufficiently low, then an increase in future information K_{t+1},*

(a) increases payoff risk $Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]$, and

(b) reduces current price informativeness $C_t/|D_t|$.

The proof in Online Appendix B goes further and makes it clear that what reduces informativeness is the fact that the information will be learned in the future. More information today (higher K_t) unambiguously increases price informativeness $C_t/|D_t|$ today.

Conceptually, future information creates risk because, if many investors will trade on precise $(t+1)$ information tomorrow, then tomorrow's price will be very sensitive to the next day's dividend information y_{t+2} and possibly to the next day's demand information x_{t+2}. But investors today do not know what will be learned tomorrow. Therefore, tomorrow's analysis makes tomorrow's price (p_{t+1}) more sensitive to shocks about which today's investors are uninformed. Because future information has no effect on today's dividend uncertainty, $Var[y_{t+1}|\bar{\mathcal{I}}_t]$, and because it raises future price uncertainty, the net effect of future information is to raise today's payoff variance. That is what creates risk today.

Mathematically, the relationship between tomorrow's price coefficients and future information risk is evident in the C_{t+1} and D_{t+1} coefficients in the formula for future information risk. Future information affects today's risk because the payoff variance is $Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B)^2 Var[y_{t+1}|\bar{\mathcal{I}}_t]$. The key terms are $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$. Future information affects the future price coefficients C_{t+1}^2 and D_{t+1}^2. We know that time-t information increases period-t information content C_t. Similarly, time $t+1$ information increases C_{t+1}. Future information may increase or decrease D_{t+1}. But as long as $C_{t+1}/|D_{t+1}|$ is large enough, the net effect of $t+1$ information is to increase $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$.

One reason future information risk is important is that it can reduce today's liquidity. It makes future price p_{t+1} more sensitive to future information and thus harder to forecast today. That raises the time-t asset payoff risk $Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]$. A riskier asset has a less liquid market. We can see this relationship in the formula for D_t (eq 12), where $Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]$ shows up in the first term. Thus, future information reduces today's liquidity.

Technology growth improves information today and then improves it again tomorrow. That means the static and dynamic effects are in competition.The net effect of the two is sometimes positive, sometimes negative. Price volatility does not become arbitrarily large. Just as the conditional variance of dividends converges to zero, price volatility converges to a positive, finite level. The point is that the net effect is not as clear-cut as what a static information model predicts. We learn that with long-lived assets, information technology efficiency and liquidity are not synonymous. In fact, because financial technology makes future prices more informative, it can also make markets function in a less liquid way.

The static result that demand analysis feeds on itself suggests that in the long run, it will completely crowd out fundamental analysis. But that does not happen. When demand precision (Ω_{xt}) is high, the conditions for Proposition 3 break down. The next result tells us that in the long run as information becomes abundant, growth in fundamental and demand analyses becomes balanced. This result for the long-lived asset contrasts with the static asset Result 4, where fundamental and demand analyses diverge.

Result 6 High-Information Limit (dynamic only) *If $\pi = 1$ and $K_t \to \infty$, both analysis choices Ω_{ft} and Ω_{xt} tend to ∞ such that* *(a)* Ω_{ft}/Ω_{xt} *does not converge to* 0*;*

(b) Ω_{ft}/Ω_{xt} *does not converge to* ∞*; and*

(c) if τ_0 *is sufficiently large, there exists an equilibrium where* Ω_{ft}/Ω_{xt} *converges to a finite, positive constant.*

(d) No perfect liquidity: There is no equilibrium for any date t *with* $D_t = 0$*.*

See Online Appendix B for the proof and an expression (104) for the lower bound on τ_0.

It is not surprising that fundamental analysis will not push demand analysis to zero (part (a)). We know that more fundamental analysis lowers the value of additional fundamental analysis and raises the value of demand analysis by increasing $C_t/|D_t|$. This is the force that prompts demand analysis to explode at lower levels of information K.

But what force restrains the growth of demand analysis? It's the same force that keeps liquidity in check: information today, competing with the risk of future information that will be learned tomorrow. The first-order condition tells us that the ratio of fundamental to demand analyses is proportional to the squared signal-to-noise ratio, $(C_t/D_t)^2$. If this ratio converges to a constant, the two types of analysis remain in fixed proportion. Recall from Result 5 that information acquired tomorrow reduces D_t. That is, D_t becomes more negative, but larger in absolute value. On the other hand, as data observed today becomes more abundant, the price information sensitivity (C_t) grows and liquidity improves – D_t falls in absolute value. As data processing grows, the upward force of current information and the downward force of future information bring C_t and D_t each to rest at a constant, finite limit. For C_t, this infinite-data limit is $\bar{C} = 1/(r - G)$. For D_t, the limit is $\bar{D} = [-r \pm \sqrt{r^2 - 4(\frac{\rho}{r-G})^2 \tau_0^{-1} \tau_x^{-1}}]/(2\rho\tau_x^{-1})$. In Online Appendix B, Lemma 4 explores the properties of the ratio $C_t/|D_t|$ in this limit. It shows formally that $C_t/|D_t|$ is bounded above by the inverse of future information risk. When assets are not long-lived, their payoffs are exogenous, future information risk is zero, and $C_t/|D_t|$ can grow without bound. Put differently, without a long-lived asset, the limit on $C_t/|D_t|$ is infinite. Data processing would not be balanced.[6]

2.3 Persistent Demand or Information about Future Events

A key to many of our results is that the growth of financial technology creates more and more future information risk. This risk arises because shocks that affect tomorrow's prices are not learnable today, which raises the question: what if information about future dividend or demand shocks was available today?

[6] Another way to introduce unlearnable risk is to simply have a static model with some portion of the payoff that cannot be learned. While this economy would have an equilibrium and analysis would converge, the convergence path and the ratio of analysis the economy converges to would differ. A version of the long-lived asset effect can also arise in a dynamic model with only fundamental analysis (see Cai (2016)).

As long as there is still some uncertainty and thus something to be learned in the future, future information will still create risk for returns today. Tomorrow's price would depend on information learned tomorrow about shocks that will materialize in $t+2$ or $t+3$. That new information, observed in $t+1$, will affect $t+1$ prices. The general point is this: new information is constantly arriving; it creates risk. Whether it is about tomorrow, the next day, or the far future, this yet-to-arrive information will have an uncertain impact on future prices.When information processing technology is poor, the poorly-processed future information has little price impact. Thus scarce future information poses little risk. When information processing improves, the risk of unknown future information grows.

Of course, if persistent demand was the source of future information, then signals about demand, x_{t+1}, would be payoff relevant. The x_{t+1} signal would be informative about x_{t+2}, which affects the price p_{t+1} and thus the payoff of a time t risky asset. In such a model, agents would attempt to distinguish the persistent and transitory components of demand. The persistent, payoff-relevant component would play the role of dividend information in this model. The transitory component of demand would play the role of the i.i.d. x_{t+1} shock.

A broader interpretation of the model is that there are three categories of activities: learning about persistent payoff-relevant shocks, learning about transitory price shocks, and future learning about any shocks that cannot be known today. Exactly what is in each of these categories is a matter of interpretation.

2.4 Price Informativeness, Liquidity, Welfare and Real Economic Output

Why are price informativeness and liquidity sensible measures of market efficiency? In this setting, all dividends are exogenous. Information has no effect on firms' real output. It only facilitates reallocation of these goods from one trader to another. In fact, the social optimum is achieved with full risk-sharing, which arises when there is no data and beliefs are therefore symmetric.

Does all this mean that data is bad for society? Not necessarily. One way to approach social welfare is to consider maximizing price informativeness to be a social objective. The question then becomes: are data choices socially efficient? It turns out that they do indeed maximize price informativeness.

Result 7 ***Social Efficiency (static).*** *For $\pi = 0$, the equilibrium attention allocation maximizes price informativeness $C_t/|D_t|$.*

The reason that social and private objectives are aligned is that the same sufficient statistic governs both. Individual data choices that maximize expected utility do so by maximizing the precision of information about dividend innovations: $Var[y_{t+1}|\mathcal{I}_t]^{-1}$. For the representative investor, price informativeness also depends on aggregate data choices, but only through $Var[y_{t+1}|\mathcal{I}_t]^{-1}$. Price informativeness is increasing in $Var[y_{t+1}|\mathcal{I}_t]^{-1}$. Thus the data choices that maximize individual utility maximize $Var[y_{t+1}|\mathcal{I}_t]^{-1}$, and therefore also price informativeness.

An alternative approach is to relate financial markets and the real economy. This model is stripped to its barest essentials to make its logic most transparent. For that reason, it has no link between financial and real economic outcomes. With that link added back in, liquid and information-rich financial markets can have real benefits.

Online appendices C.5 and C.6 sketch two models of real economic production where the amount produced by a firm depends on price information sensitivity or liquidity, as in David, Hopenhayn and Venkateswaran (2016). In the first model, a manager makes a costly effort to increase the future value of the firm and is compensated with equity. When the equity price is more informative, that means the price reacts more to effort and the associated output. That makes equity a better incentive for providing effort and raises firm output. In the second model, a firm needs to issue equity to raise funds for additional real investment. When markets are illiquid, issuing equity exerts strong downward pressure on the equity price. This reduces the firm's ability to raise revenue, reduces the size of the capital investment, and depresses output. While these models are just caricatures of well-known effects, they illustrate why the objects that are the central focus of analysis in this paper, price informativeness and liquidity, are of such interest.

3 Illustrating Financial Technology Growth: Numerical Example

Our main results reveal competing forces are work. Quantifying the model provides some understanding of which effect is likely to dominate. The equilibrium effects that we focus on are price informativeness and liquidity. A common concern is that as financial technology improves, the extraction of information from demand will crowd out original research, and in so doing, will reduce the informativeness of market prices. On the flip side, if technology allows investors to identify uninformed trades and take the other side of those trades, such activity is thought to improve market liquidity. While each argument has a grain of truth to it, countervailing equilibrium effects mean that neither conjecture is correct.

We begin by quantifying the forces that make demand information more valuable over time. Then we explore why the shift from information production to extraction does not harm price informativeness or improve liquidity. Finally, we ask whether the model contradicts the long-run trends in price volatility, decomposes demand and fundamental data effects, explores alternative parameter values, and considers the possibility of biased technological change.

3.1 Calibration

Our calibration strategy is to measure the growth of computer processor speed directly to discipline technology K and then to estimate our equilibrium price equation on the recent asset price and dividend data, assuming assets are long-lived. Then we use the time path of the price coefficients and the signal-to-noise ratio to calibrate the model parameters.

We describe the data and then the moments of the data and model that we match to identify the model parameters. Most of these moments come from estimating a version of our price equation (8) and choosing parameters that match the price coefficients in the model with the data. In the next section, we report the results.

Measuring Data Growth Investors can acquire information about asset payoffs y_{t+1} or demand x_{t+1} by processing digital data, which is coded in binary code. To calibrate the growth rate of data technology, we choose a growth rate for K, which implies a growth rate of bits processed equal to the growth rate of computer processor speed or cloud computing capacity.

To map the economic measure of data K into a binary string length, we use a concept from information theory called the Gaussian channel. In a Gaussian channel, all data processing is subject to noise (error).[7] The number of bits required to transmit a message is related to the channel's signal-to-noise ratio. Clearer signals can be transmitted through the channel, but they require more bits. The relationship between bits and signal precision for a Gaussian channel is $bits = 1/2ln(1 + \text{signal-to-noise})$ (Cover and Thomas (1991), theorem 10.1.1). The signal-to-noise is the ratio of signal precision to prior precision. In the notation of this model, if the prior precision is τ, the number of bits $\tilde{b}$ required to transmit Ω units of precision in a signal is $\tilde{b} = \frac{1}{2}ln\left(1 + \frac{\Omega}{\tau}\right)$.

If this is true both for fundamental precision Ω_{ft} and for demand precision Ω_{xt}, and presumably, each piece of data is transmitted separately, then the total number of bits processed b is the sum of fundamental and demand bits:

$$b = \frac{1}{2}ln\left(1 + \frac{\Omega_{ft}}{\tau_0}\right) + \frac{1}{2}ln\left(1 + \frac{\Omega_{xt}}{\tau_x}\right). \tag{18}$$

Using this mapping, we choose a growth rate of K_t, such that the equilibrium choices of Ω_{ft} and Ω_{xt} imply a growth rate of bits that matches the data. We explain the procedure we use to do so shortly. That path of K_t is

$$K_t = 0.00095 \times 2^{0.49(t-1)} \qquad \text{for t} = 1, \ldots, \text{T}. \tag{19}$$

The multiplier 0.00095 is simply a normalization to keep units from becoming too large. The choice of 0.49 in the exponent ensures that the (cumulative) average bit growth is close to 20% per year for the first 18 periods, because these periods correspond to the past observed data in our calibration, as explained later in the section. The 20% annual growth rate of bit processing reflects evidence from multiple sources. One source is hardware improvement: the speed of frontier processors has

[7] As Cover and Thomas (1991) explain, "The additive noise in such channels may be due to a variety of causes. However, by the central limit theorem, the cumulative effect of a large number of small random effects will be approximately normal, so the Gaussian assumption is valid in a large number of situations."

grown by 27% since the 1980s, and has more recently slowed to 20% growth per year (Hennessy and Patterson, 2009). Another fact that supports this rate of growth is the 19% growth rate of the workloads in data centers (22% for cloud data centers), where most banks process their data (Cisco, 2018).

Asset Data The data is obtained from Compustat and covers S&P500 firms over the 1985 - 2015 period. We construct a measure $ea_{i,t}$ for relevant firm i as earnings before interest and taxes (EBIT) divided by total assets. For each firm i, we fit an AR(1) model to the time series $ea_{i,t}$ and use the residual $ue_{i,t}$ as a measure of $y_{i,t}$. Finally, we construct a measure $lma_{i,t}$, which is the log of market capitalization over total assets.[8]

Our measure of dividends is the lagged value of $ea_{i,t}$ and therefore μ is estimated as the sample mean of $ea_{i,t-1}$ averaged over firms and dates. Then, to create the time series of price coefficients, we run the following cross-sectional regression for each year $t = 1, \ldots, T$,[9]

$$lma_{i,t} = \hat{A}_t + \hat{B}_t(ea_{i,t-1} - \mu) + \hat{C}_t ue_{i,t} + \hat{H}_{i,t} Y_{i,t} + \hat{\epsilon}_{i,t}, \tag{20}$$

where $Y_{i,t}$ is a collection of dummies at the SIC3 industry level.[10]

This first step results in a time series of estimated coefficients that are imperfect measures of A_t, B_t, and C_t in the model. In addition, the squared residuals $\hat{\epsilon}_{i,t}$ correspond to the model's $D_t^2 \tau_x^{-1}$. For $\hat{B}_t$, $\hat{C}_t$, and $1/N \sum_{i=1}^{N} \hat{\epsilon}_{i,t}^2$, we remove their high-frequency time-series fluctuations. To do this, we simply regress each one on a constant, a time trend and a squared time trend. That is, if the estimated coefficient is x_t, we estimate β_a, β_b, and β_c in $x_t = \beta_a + \beta_b t + \beta_c t^2 + \nu_t$. Then, we construct a smoothed series as $x_{t+1} = \hat{\beta}_a + \hat{\beta}_b t + \hat{\beta}_c t^2$, where $\hat{\beta}_a$, $\hat{\beta}_b$, and $\hat{\beta}_c$ are the estimates of β_a, β_b, and β_c. This procedure results in three series: $\tilde{B}$, $\tilde{C}$, and $\tilde{D}_t^2 \tau_x^{-1}$, which we use to calibrate to the average rate of the coefficient change in the last 30 (approximately) years.[11]

Estimation

We pick three parameters of the model directly. μ is set to match the average earnings, as described above. The riskless rate $r = 1.05$ is set to match a 5% annual net return. Risk aversion clearly matters for the level of the risky asset price, but it is not well identified. Doubling the variance and halving the risk aversion mostly just redefines the units of risk. In what follows, we set $\rho = 0.05$. In the online appendix, we explore other values to show that our results are robust to changes of this kind.

[8] That is, if M is market capitalization and A total assets, $\log(M/A)$.

[9] See online appendix for a discussion of the generated regressor.

[10] Notice that B_t makes a return here with a t subscript, even though we proved that B was a constant. That is because when we estimate price coefficients in each year, the B_t estimate is, of course, not constant. We could first average B_t over time and use the average. But using the B_t that is consistent with the C_t and D_t for that period t results in more precise parameter estimates.

[11] Allowing just a constant and time trend yields very similar results.

Four parameters, $\theta = (G, \tau_0, \tau_x, \chi_x)$, as well as the growth rate of K_t, remain to be estimated. The sequence of information capacities, K_t, is chosen to match the average bit growth rate, which depends on the equilibrium outcomes. We use the following procedure: Pick a growth rate for K_t and estimate θ as explained below. Given the θ estimate, compute the equilibrium and find the date that corresponds most closely to the most recent observations in the constructed calibration series (2015), $\hat{t}$.[12] Next calculate the corresponding average bit growth rate for period $(0, \hat{t})$. Iterate on this procedure until the 20% average target bit growth rate is hit.

Finally we estimate $\theta = (G, \tau_0, \tau_x, \chi_x)$. We describe four moments derived from the model. Using these four moment conditions, we estimate the parameter vector, θ, by the generalized method of moments. The theory constrains $\theta \in [0, 1) \times (0, \infty)^3$. Let $X \in \mathbb{R}^{(T-1)\times 5}$ be the sequences $\tilde{B}_t$, $\tilde{C}_t$, $\widetilde{D_t^2 \tau_x^{-1}}$, and K_t.[13] The first three moments below are the equilibrium solution for the price coefficients B_t, C_t, and D_t. They are simply re-arrangements of (44), (45), and (46). The fourth equation uses the information budget constraint and the pricing solutions to characterize the signal-to-noise ratio in prices C_t/D_t. It is a variation of equation (69).

$$g_{1,t}(\theta, r, \rho, X) := \frac{1}{r}(1 + B_{t+1})G - B_t, \tag{21}$$

$$g_{2,t}(\theta, r, \rho, X) := \frac{1}{r-G}\left(1 - \tau_0\left(\tau_0 + \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2 (\tau_x + {}^{\Omega_{ft}}/_{\chi_x}\left(\frac{C_t}{D_t}\right)^2)\right)^{-1}\right) - C_t, \tag{22}$$

$$\begin{aligned} g_{3,t}(\theta, r, \rho, X) := &\left(\frac{\tau_x}{r-G}\frac{C_t}{D_t} - \frac{\rho r}{(r-G)^2}\right)\left(\tau_0 + \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2 (\tau_x + {}^{\Omega_{ft}}/_{\chi_x}\left(\frac{C_t}{D_t}\right)^2)\right)^{-1} \\ &- \frac{\rho}{r}\left(C_{t+1}^2 \tau_0^{-1} + D_{t+1}^2 \tau_x^{-1}\right) - D_t, \end{aligned} \tag{23}$$

$$g_{4,t}(\theta, r, \rho, X) := \left(\frac{C_t}{D_t}\right)^3 Z_t \tau_x + \frac{C_t}{D_t}\left(\frac{\rho r}{r-G} + Z_t \tau_0\right) + (1 + \frac{C_t}{D_t} Z_t)\frac{K_t}{\Omega_{ft}}. \tag{24}$$

Z_t captures future information risk and the equilibrium demand for fundamental information Ω_{ft} comes from combining information capacity constraint (5) with first-order condition (16):

$$Z_t := \frac{\pi \rho}{r}(r - \pi G)(C_{t+1}^2 \tau_0^{-1} + D_{t+1}^2 \tau_x^{-1}), \tag{25}$$

$$\Omega_{ft} := \left(\frac{K_t \chi_x}{\chi_x + (C_t/D_t)^4}\right)^{1/2}. \tag{26}$$

According to the model, each of these four moments $g_{1,t}$ though $g_{4,t}$ should be zero at each date t. To estimate the four parameters $\theta = (G, \tau_0, \tau_x, \chi_x)$ from these four moment equations, we compute the actual value of $g_{1,t}$ though $g_{4,t}$ at each date t, for a candidate set of parameters θ. We average those values (errors) over time, square them, and sum over the four moments. Formally,

[12] Specifically, $t = 18$ minimises $(A_t - a)^2 + (C_t - c)^2 + (D_t - d)^2$ over the 150 simulated model periods, where $a = \tilde{A}_T$, $c = \tilde{C}_T$, and $d = -\sqrt{(\widetilde{D_T^2 \tau_x^{-1}} \times \tau_x)}$.

[13] The last observation has to be dropped, due to the presence of $t+1$ parameters in the time-t moment conditions.

let $\overline{g}(\theta)$ (a 4×1 vector) be the time-series mean of the $(T-1) \times 4$ matrix

$$g(\theta) \coloneqq \begin{bmatrix} g_{1,1}(\theta, r, \rho, X) & g_{2,1}(\theta, r, \rho, X) & g_{3,1}(\theta, r, \rho, X) & g_{4,1}(\theta, r, \rho, X) \\ \vdots & \vdots & \vdots & \vdots \\ g_{1,T-1}(\theta, r, \rho, X) & g_{2,T-1}(\theta, r, \rho, X) & g_{3,T-1}(\theta, r, \rho, X) & g_{4,T-1}(\theta, r, \rho, X) \end{bmatrix}' \tag{27}$$

and let I be the 4×4 identity matrix. The estimated parameter vector $\hat{\theta}$ solves

$$\hat{\theta} \coloneqq \underset{\theta \in [0,1) \times (0,\infty)^3}{\arg\min} \; \overline{g}(\theta)' I \overline{g}(\theta).$$

We optimize with constraints that τ_0, τ_x, and χ_x are positive and then check that the estimated value of G lies within its admissible range $[0, 1)$. This procedure produces the following parameter values:

Table 1: Parameters

ρ	r	μ	G	τ_0	τ_x	χ_x
0.05	1.02	0.04	0.98	79.76	20.08	20.98

Computation and equilibrium selection We use the estimated parameters to solve for the long-run stable pricing coefficients. Specifically, we set $\frac{C_T}{D_T}$ to the smaller solution to quadratic equation (103). Moreover, we choose $T = 150$ to be our long-run, and create path K_t according to equation (19). Then we solve the model backward: Knowing the time-t price coefficients allows us to numerically find the root of equation (67) to obtain $\frac{C_{t-1}}{D_{t-1}}$, from which we can solve for the information choices and the price function coefficients in period $t-1$.

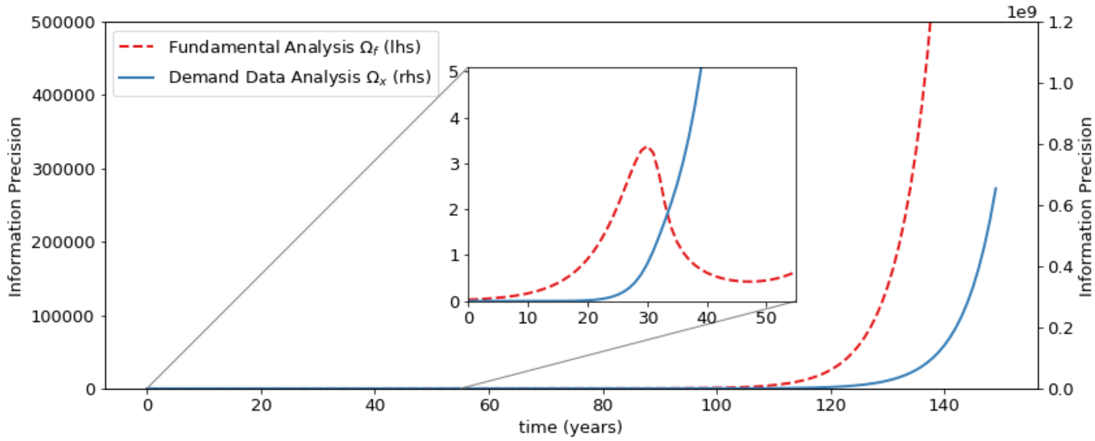
The non-linear equation in $\frac{C_t}{D_t}$ that characterizes the solution can have multiple solutions. As it happens, for the parameter values that we explore, given a C_{t+1} and D_{t+1}, this equation has only one real root. Moreover, the date that corresponds most closely to the most recent observations in our constructed calibration series, corresponding to data from 2015, is period 18.

3.2 Result: The Transition from Fundamental to Demand Analysis

With our calibrated model, we can now illustrate the three phases of information processing.

Figure 1 shows that initially demand analysis is scarce. Consistent with Result 1, we see that when the ability to process information is limited, almost all of that ability is allocated to processing fundamental information. Once fundamental information is sufficiently abundant, demand analysis takes off. Not only does demand processing surge, but it increases by so much that the amount of fundamental information declines, even though the total ability to process information

Figure 1: **Evolution of fundamental analysis and demand analysis.** What drives the change over time is an increase in total information processing K. Fundamental information is choice variable Ω_{ft}, scaled by fundamental variance τ_0^{-1}. Demand information is choice variable Ω_{xt}, scaled by non-fundamental demand variance τ_x^{-1}.



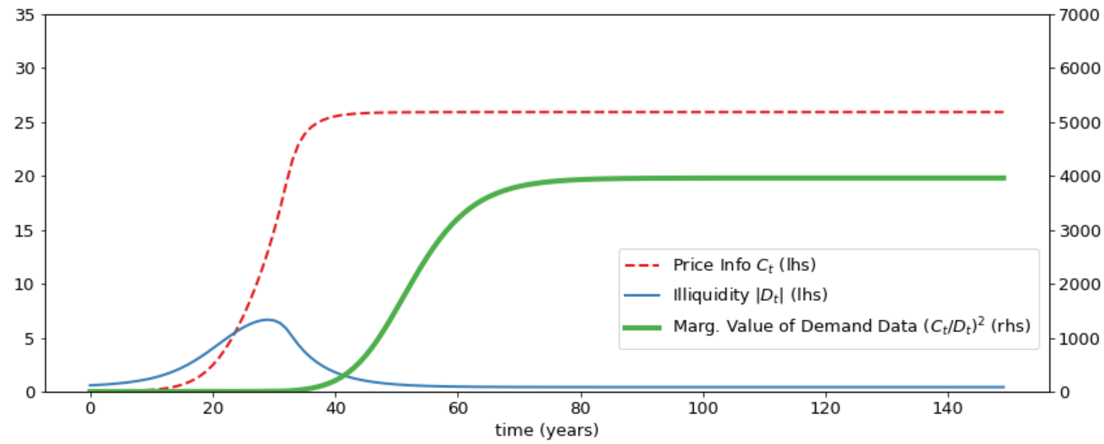
has improved. Once demand trading takes off, it quickly comes to dominate fundamentals-based trading. In the long run, the two types of information processing grow at the same rate (albeit at different levels).

3.3 Price Informativeness

Prices are informative to the extent that they reflect a change in future dividends relative to a change in aggregate supply. Our equilibrium price solution (8) reveals that the absolute value of this relative price impact, $|\frac{dp_t/dy_{t+1}}{dp_t/dx_{t+1}}|$ is $\frac{C_t}{|D_t|}$. As the productivity of a financial analysis rises, and more information is acquired and processed, the information sensitivity of price (C_t) rises, while the illiquidity ($|D_t|$) changes non-monotonically.

Our analysis shows that overall as technology improves, investors are able to better discern an asset's true value, and price informativeness rises. The thick solid line labeled $(C_t/D_t)^2$ in Figure 2 confirms that as financial analysis becomes more productive, informativeness rises. This change is driven primarily by the increase in the information sensitivity of prices C_t. The effect of a one-unit change in the dividend innovation, which is about 2 standard deviations, increases the price by between 0 and 8 units. Because the average price level is about 80, this 2 standard deviation shock to dividends produces a negligible price change for very low levels of technology and a 10% price rise when financial technology becomes more advanced.

Figure 2: **Price Information Sensitivity (C_t) Rises and the Price Impact of Trades ($|D_t|$) Stagnates.** C_t is the impact of future dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. This illiquidity measure is non-monotonic, despite the rise in market-making (demand driven) trades. $(C_t/D_t)^2$ is the marginal value of demand information relative to fundamental information. The x-axis is time in years.



3.4 Price Impact of Trades (Illiquidity)

A common metric of market liquidity is the sensitivity of an asset's price to a buy or sell order. In our model, price impact is the impact of a one-unit noise trade $(dp_t/d(-x_{t+1}))$.[14] Linear price solution (8) reveals that price impact is $dp_t/d(-x_{t+1}) = |D_t|$.

Looking at the thin line in Figure 2, we see that the price impact of noise trades, $|D_t|$, rises in the early periods when only Ω_{ft} is increasing. It then declines, but it stays positive as information becomes more abundant. Since 2015 corresponds to period 18, this suggests that we are close to the maximum illiquidity. But surprisingly, the changes are quite small. A noise trade that is the size of 1% of all outstanding asset shares would increase the price by $0.05 - 0.06$ units. Since the average price is 80, this amounts to a $0.6\% - 0.7\%$ (60 - 70 basis point) increase in the price. By exploring different parameters, we see that the dynamics of market liquidity can vary. But what is consistent is that the liquidity changes are small, compared to the price information changes.

Flat liquidity is the result of two competing forces. Recall from Section 2 that the liquidity of a risky asset is determined largely by the riskiness (uncertainty) of its payoff. The static force $(r/(r-G))Var[y_{t+1}|\bar{\mathcal{I}}_t](C_t/D_t)$, which reduces risk and the dynamic force $-\rho Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]$, which increases it, are nearly cancelling each other out.

Price and Return Volatility One might think that future information risk implies an unrealistically high future price or return volatility. It does not. Between the start of the simulation and

[14]We consider a noise trade because the impact of an information-based trade would reflect the fundamental (future dividend), which must have moved in order to change the information. We have already explored the question of how much a change in the fundamental changes the price. That is price information sensitivity.

period 18, which corresponds to 2015, the price variance rises by only 3.4%. (See Online Appendix C.7 for the complete volatility time series.) That is less than 0.2% annually. Such a minute increase in price volatility is sufficient to offset the elimination of most dividend uncertainty, because prices are so much larger in magnitude than dividends.

In the return space, the change is even less detectable. When we examine the volatility of the returns, we find no significant time trend in either the model or the data. To substantiate this claim, we simulate prices from the model, inflate them with a CPI price index, and then construct a return series: $(p_{t+1} + d_{t+1})/p_t$. We compare this model return to an empirical return series, derived from the monthly S& P 500 price index (1980-2015). For both the model and data, we estimate a GARCH model to calculate volatility for each month.[15] When we regress this GARCH-implied return variance σ_t on time (monthly), the average variance is 0.005, but the coefficient on time is zero out to 5 digits. Because of the large number of observations that we can simulate, this number is statistically different from zero. But its economic magnitude is trivial. Variance of the S& P 500 returns has a time trend of 0.00001. That coefficient is statistically indistinguishable from zero. In short, the return variance, in both the model and the data, appears stable over time.

3.5 Which Trends Come from Where?

The rise in informativeness and the stagnation of liquidity come from the combination of changes in fundamental and demand data, both today and in the future. To understand the source of these results, we turn off the growth in one of these types of data and see how the resulting price coefficients differ.[16] Online Appendix D reports the full set of results from each exercise, with figures.

In a first exercise, designed to isolate the effect of demand data, we set demand analysis to zero and keep fundamental analysis on the same trajectory that it follows in the full model. We also do a version of this exercise where all of the growing data capacity is allocated to fundamental analysis ($\Omega_f = \sqrt{K_t}$). Given these exogenous information sequences, we re-solve for the equilibrium pricing coefficients in the equilibrium of the financial market. We learn that demand data makes prices only slightly more sensitive to new information. The effect on the price sensitivity to information, C_t, is negligible throughout most of the path. Demand data's effect is small because by the time demand data becomes prevalent, C_t is nearly at its maximum level. In contrast, demand data governs market liquidity. Without it, the price impact at the end of the simulation would be twice as large as the end of the path, when demand data is abundant. Moreover, we do see demand data affect forecast accuracy, defined as the conditional precision of the future dividend. By the end of the path, nearly all the accuracy of the dividend forecasts is due to demand data.

[15] The dividends are imputed, as described in the calibration section. The equation we estimate is a TGARCH(1,1), which takes the form, $\sigma^2_{t|t-1} = \omega + (\alpha + \gamma \cdot 1_{(r_{t-1}<0)})r^2_{t-1} + \beta\sigma^2_{t-1|t-2}$. It allows for positive and negative returns to affect volatility differently. We estimate the coefficients by maximum likelihood.

[16] We thank our anonymous referees for suggesting these exercises.

In a second exercise, we do the reverse: We fix fundamental data at a low level ($\Omega_{ft} = 0.01$) and keep demand data on its previous equilibrium trajectory. We cannot set it to zero, because otherwise $C_t = 0$, mechanically for any level of demand analysis. When information is scarce and fundamental analysis is dominant, removing fundamental data significantly hurts price sensitivity to dividends (C_t), but it has little effect on liquidity or informativeness in the long run. As technology improves, demand analysis becomes dominant. Thus, while some fundamental information is essential, its growth has little quantitative effect on prices in the long run.

A third exercise isolates the effect of long-lived assets and risk from future prices. The most important role of long-lived assets is that they preserve equilibrium existence. When we switch to computing the static model by setting $\pi = 0$, we can only simulate the first few periods before the static solution ceases to exist. The price coefficients have similar dynamics. The key difference is that the dynamic model price is more sensitive to dividend innovations. The reason is that for a long-lived asset, dividends signal not only a rise in current dividends, but also a rise in expected future dividends. That has a larger effect on the expected asset value and thus on price.

3.6 What is Robust? What is Fragile?

These numerical results are simply examples. Some of the features they illustrate are robust to other parameter values, others are not. The online appendix explores results with different risk aversions, variances of dividends and demand shocks, and rates of time preference. What is consistent is that demand analysis always rises, price informativeness always rises, and the marginal value of demand information $(C/D)^2$ also always rises. Quantitatively, Ω_{xt} consistently surpasses Ω_{ft} once C_t/D_t crosses $\sqrt{\chi_x}$. (See Online Appendix E for details).

Unbalanced Technological Change We chose to model technological progress in a way that increases the potential precision of fundamental or demand information equally. We made this choice because otherwise we would not know which results come from the technological process and which arise from its imbalanced nature. But it is quite possible that technological progress has not been balanced. To explore this possibility, we consider a world where demand data processing efficiency grows more quickly.

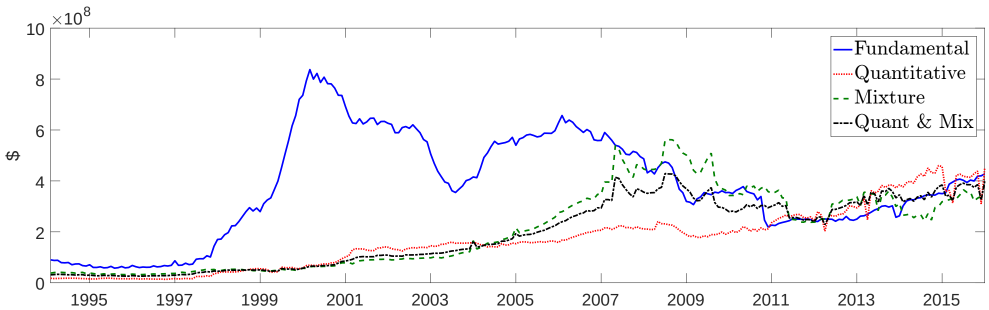
When demand analysis efficiency is low, investors analyze fundamental data, just like before. The reason is that no matter how efficient, demand data processing still has zero marginal value at zero data. But as demand analysis becomes relatively cheaper, fundamental analysis falls by more than before. In the long run, the marginal benefit of both types of analysis converges to a constant ratio, as before. (See Figure 18 in the online appendix.)

In short, most of our conclusions are unaltered. Liquidity is still flat. However, if we see stagnating market efficiency, this is consistent with a world where demand analysis efficiency is improving at a faster rate and is displacing fundamental analysis.

4 Suggestive Evidence

The shift from fundamental to demand analysis in our model should appear empirically as a change in investment strategies. Indeed, there is some evidence that funds have shifted their strategies in a way that is consistent with our predictions. In the TASS database, many hedge funds report that their fund has a "fundamental", "mixture," or "quantitative" strategy. Since 2000, the assets under management of fundamental funds, whether measured by fund or in total, has waned (Figure 3). Instead, other strategies, some based on market data, are surging.[17]

Figure 3: **Hedge Funds are Shifting Away from Fundamental Analysis.** Source: Lipper TASS. Data is monthly assets under management per fund, from 1994-2015. Database reports on 17,534 live and defunct funds.



A different indicator that points to the growing importance of demand data comes from the frequency of web searches. From 2004 to 2016, the frequency of Google searches for information about "order flow" has risen roughly three fold (Figure 20 in the online appendix). This is not an overall increase in attention to asset market information. In contrast, the frequency of searches for information about "fundamental analysis" fell by about one-half over the same time period.

In practice, much of the trade against order flow takes the form of algorithmic trading, for a couple of reasons. First, while firm fundamentals are slow-moving, demand can reverse rapidly. Therefore, mechanisms that allow traders to trade quickly are more valuable for fast-moving, demand-based strategies. Second, while fundamental information is more likely to be textual, partly qualitative, and varied in nature, demand is more consistently data-oriented and therefore more amenable to algorithmic analysis. Hendershott, Jones and Menkveld (2011) measure algorithmic trading and find that it has increased, but this increase occurred most rapidly between the start of 2001 and the end of 2005. Over this period, average trade size fell and algorithmic trading increased about seven fold, consistent with model's predictions for demand-based trading strategies.

[17]Source: Lipper TASS. The data are monthly from 1994-2015. The database reports on 17,534 live and defunct funds. Quantitative or "quant" strategies are defined in Investopedia as follows: "Quantitative trading techniques include high-frequency trading, algorithmic trading and statistical arbitrage. These techniques are rapid-fire and typically have short-term investment horizons. Many quantitative traders are more familiar with quantitative tools, such as moving averages and oscillators."

Asset price trends are also consistent with our predictions. Bai, Philippon and Savov (2016) measure a long-run rise in equity price informativeness. They measure price informativeness using a coefficient from a regression of future earnings (at the 1-year, 3-year, and 5-year horizons) on the current ratio of market value to book value. Over the 1960-2010 period, they find a 60% rise in the three-year price informativeness and an 80% rise in the five-year price informativeness, both of which are highly statistically significant. Similarly, studying liquidity over the last century, Jones (2002) finds a great deal of cyclical variation, but little trend in liquidity, as measured by bid-ask spreads.[18]

Our claim is not that our model is the primary explanation for these phenomena, or that we can match the timing or magnitude of the increases or decreases. We only wish to suggest that our predictions are not obviously at odds with long-run financial trends.

5 Conclusion

We explore the consequences of a simple deterministic increase in the productivity of information processing in the financial sector. We find that when the financial sector becomes more efficient at processing information, it changes the incentives to acquire information about future dividends (fundamentals) versus demand (non-fundamental shocks to price). Thus a simple rise in information processing productivity can explain the transformation of financial analysis from a sector that primarily investigates the fundamental profitability of firms to a sector that does a little fundamental analysis but that mostly concentrates on acquiring and processing client demand. This is consistent with suggestive evidence that the nature of financial analysis and its associated trading strategies have changed.

Many feared that this technological transformation would harm market efficiency, while others argued that markets are more liquid/efficient than ever before. Neither phenomenon is a logical consequence of information technology. Although fundamental analysis declines, price informativeness still rises. The reason is that even if many traders are extracting others' information, this still makes the average trader better informed and the price more informative. But the benefits of this technological transformation may also be overstated. The promise that traders would be standing ready to take the other side of uninformed trades would improve market liquidity is only half the story. What this narrative misses is that more informed traders in the future make prices react more strongly to new information, which makes future asset values riskier. This increase in risk makes traders move market prices by more and pushes market liquidity back down. The net effect

[18] Recent work by Koijen and Yogo (2016) measures a large fall in the price impact of institutional traders. This may not be inconsistent with our results, for two reasons. First, our liquidity measure is the price impact of a non-informational trade. That is not the same as the price impact of an institutional trader, who will often be trading on information. Second, in many cases, institutional traders have reduced their price impact by finding uninformed demand to trade against. To the extent that the reduced price impact reflects more market making and less direct trading on information, this reduced impact is consistent with our long-run demand analysis trend.

could go either way.

Of course, there are many other features one might want to add to this model to speak to other related trends in financial markets. One might make fundamental changes more persistent than demand innovations so that different styles of trade are associated with different trading volumes. Another possibility is to explore regions in this model where the equilibrium does not exist and use its non-existence as the basis for a theory of market breakdowns or freezes. Another extension might ask where demand signals come from. In practice, people observe demand data because they intermediate trades. Thus, the value of demand information might form the basis for a new theory of intermediation. In such a world, more trading might well generate more information for intermediaries and faster or stronger responses by market participants to changes in market conditions. Finally, one might regard this theory as a prescriptive theory of optimal investment, compare it to investment practice, and compute expected losses from sub-optimal information and portfolio choices. For example, a common practice now is to blend fundamental and demand trading by first selecting good fundamental investment opportunities and then using demand information to time the trade. One could construct such a strategy in this model, compare it to the optimal blend of trading strategies, and see if the optimal strategy performs better on market data.

While this project, with its one simple driving force, leaves many questions unanswered, it also provides a tractable foundation on which to build, to continue exploring how and why asset markets are evolving as financial technology improves.

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A Model Solution Details

A.1 Bayesian Updating

To form the conditional expectation, $E[f_{it}|\mathcal{I}_{it}]$, we need to use Bayes' law. But first, we need to know what signal investors extract from price, given their demand signal, η_x. We can rearrange linear price equation (8) to write a function of the price as the dividend innovation plus the mean zero noise: $\eta_{pit} = y_{t+1} + (D_t/C_t)(x_{t+1} - E[x_{t+1}|\eta_{xit}])$, where the price signal and the signal precision are

$$\eta_{pit} \equiv (p_t - A_t - B(d_t - \mu) - D_t\, E[x|\eta_{xit}])/C_t \tag{28}$$

$$\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt}) \tag{29}$$

For the simple case of an investor who learned nothing about demand ($E[x] = 0$), the information contained in prices is $(p_t - A_t - B(d_t - \mu))/C_t$, which is equal to $y_{t+1} + D_t/C_t x_{t+1}$. Since x_{t+1} is a mean-zero random variable, this is an unbiased signal of the asset dividend innovation, y_{t+1}. The variance of the signal noise is $Var[D/Cx] = (D/C)^2\tau_x^{-1}$. The price signal precision Ω_{pt} is the inverse of this variance.

But conditional on η_{xit}, x_{t+1} is typically not a mean-zero random variable. Instead, investors use Bayes' law to combine their prior, that $x_{t+1} = 0$, with precision τ_x with their demand signals: η_{xit} with precision Ω_{xit}. The posterior mean and variance are

$$E[x|\eta_{xit}] = \frac{\Omega_{xit}\eta_{xit}}{\tau_x + \Omega_{xit}} \tag{30}$$

$$V[x|\eta_{xit}] = \frac{1}{\tau_x + \Omega_{xit}} \tag{31}$$

Since that is equal to $y_{t+1} + D_t/C_t(x_{t+1} - E[x_{t+1}|\eta_{xit}])$, the variance of price signal noise is $(D_t/C_t)^2 Var[x_{t+1}|\eta_{xit}]$. In other words, the precision of the price signal for agent i (and therefore for every agent, since we are looking at symmetric information choice equilibria) is $\Omega_{pit} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xit})$.

Now, we can again use Bayes' law for normal variables to form beliefs about the asset payoff. We combine the prior μ, price/demand information η_{pit}, and fundamental signal η_{fit} into a posterior mean and variance:

$$E[y_{t+1}|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fit})^{-1}(\Omega_{pit}\eta_{pit} + \Omega_{fit}\eta_{fit}) \tag{32}$$

$$V[y_{t+1}|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fit})^{-1} \tag{33}$$

Average expectations and precisions: Next, we integrate over investors i to get the average conditional expectations. Begin by considering the average price information. The price information content is $\Omega_{pit} \equiv (C_t/D_t)^2(\tau_x+\Omega_{xit})$. In principle, this can vary across investors. But since all are ex-ante identical, they make identical information decisions. Thus, $\Omega_{pit} = \Omega_{pt}$ for all investors i. Since this precision is identical for all investors, we drop the i subscript in what follows. But the realized price signal still differs because signal realizations are heterogeneous. Since the signal precisions are the same for all agents, we can just integrate over signals to get the average signal: $\int \eta_{pit} di = (1/C_t)(p_t - A_t - B(d_t - \mu)) - (D_t/C_t)Var(x_{t+1}|\bar{\mathcal{I}}_t)\Omega_{xt}x_{t+1}$. Since $\Omega_{pt}^{-1} = (D_t/C_t)^2 Var(x_{t+1}|\bar{\mathcal{I}}_t)$, we can rewrite this as

$$\int \eta_{pit} di = \frac{1}{C_t}(p_t - A_t - B(d_t - \mu)) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}x_{t+1} \tag{34}$$

Next, let's define some conditional variance / precision terms to simplify notation. The first term, Ω_t, is the precision of the future price plus dividend (the asset payoff). It comes from taking variance of pricing equation (8). It turns out that variance Ω_t^{-1} can be decomposed into a sum of two terms. The first, $\hat{V}$, is the variance of the dividend innovation. This variance depends on information choices Ω_{ft} and Ω_{xt}. The other term, Z_t, depends on future information choices through the $t+1$ price coefficients.

$$\hat{V}_t \equiv Var(y_{t+1}|\bar{\mathcal{I}}_t) = (\tau_0 + \Omega_{ft} + \Omega_{pt})^{-1} = (\tau_0 + \Omega_{ft} + (C_t/D_t)^2(\tau_x + \Omega_{xt}))^{-1} \tag{35}$$

$$\Omega_t^{-1} \equiv Var[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] = \pi C_{t+1}^2 \tau_0^{-1} + \pi D_{t+1}^2 \tau_x^{-1} + (1+\pi B_{t+1})^2 \hat{V}_t \tag{36}$$

$$Z_t = \frac{\pi\rho}{r}(r-\pi G)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) \tag{37}$$

$$\Omega_t^{-1} = \frac{r}{\rho(r-\pi G)} Z_t + (\frac{r}{r-\pi G})^2 \hat{V}_t \tag{38}$$

Thus $Z_t = 0$ if $\pi = 0$.

The last equation, (38), shows the relationship between Ω, $\hat{V}$, and Z_t. This decomposition is helpful because we will repeatedly take derivatives where we take future choices (Z_t) as given and vary current information choices ($\hat{V}$).

Next, we can compute the average expectations

$$\int E[y_{t+1}|\bar{\mathcal{I}}_t]\, di = \hat{V}_t \left[\Omega_{ft} y_{t+1} + \Omega_{pt}\left(\frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}x_{t+1}\right)\right] \tag{39}$$

$$= \hat{V}_t \left[\Omega_{ft} y_{t+1} + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) - \frac{C_t}{D_t}\Omega_{xt}x_{t+1}\right] \tag{40}$$

$$\int E[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]\, di = A_{t+1} + (1+\pi B)E[d_{t+1}|\bar{\mathcal{I}}_t] = A_{t+1} + (1+\pi B)\left(\mu + G(d_t-\mu) + E[y_{t+1}|\bar{\mathcal{I}}_t]\right). \tag{41}$$

A.2 Solving for Equilibrium Prices

The price conjecture is

$$p_t = A_t + B_t(d_t - \mu) + C_t y_{t+1} + D_t x_{t+1} \tag{42}$$

We will solve for the prices for the general supply of asset, $\bar{x}$, although in the main text it is normalized to one unit.

The price coefficients solve the system of recursive equations.

$$A_t = \frac{1}{r}\Bigg[\pi A_{t+1} + \mu - \rho\bar{x}\Bigg(\pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} + (1+\pi B_{t+1})^2\Big(\tau_0 + (\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}} + \xi^2\big(\tau_x + \frac{\xi^2}{\chi_x}(\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}}\big)\Big)^{-1}\Bigg)\Bigg] \tag{43}$$

$$B_t = \frac{1}{r}(1+\pi B_{t+1})G = \frac{G}{r-\pi G} \tag{44}$$

$$C_t = \frac{1}{r-\pi G}\left(1 - \tau_0\Big(\tau_0 + (\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}} + \xi^2\big(\tau_x + \frac{\xi^2}{\chi_x}(\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}}\big)\Big)^{-1}\right) \tag{45}$$

$$D_t = \Big(\frac{\tau_x}{r-\pi G}\xi - \frac{\rho r}{(r-\pi G)^2}\Big)\Big(\tau_0 + (\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}} + \xi^4\big(\tau_x + \frac{\xi^2}{\chi_x}(\frac{K}{(1+\frac{1}{\chi_x}\xi^4)})^{\frac{1}{2}}\big)\Big)^{-1} - \pi\frac{\rho}{r}\Big(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\Big) \tag{46}$$

where $\xi = \frac{C_t}{D_t}$ denotes the date-t signal to noise ratio, which is the solution to equation (69). The high-K limit pricing coefficients are the fixed point of the above system.

The sequence of pricing coefficients is known at every date. Signals η_{fit} and η_{xit} are the same as before, except that their precisions, Ω_{ft} and Ω_{xt}, may change over time if that is the solution to the information choice problem.

The conditional expectation and variance of y_{t+1} (32) and (33) are the same, except that the Ω_{pt} term gets a t subscript now because $\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt})$. Likewise, the mean and the variance of x_{t+1} (30) and (31) are the same with a time-subscripted Ω_{xt}. Thus, the average signals are the same with t-subscripts:

$$\int \eta_{pit} di = \frac{1}{C_t}(p_t - A_t - B_t(d_t - \mu)) - \frac{D_t}{C_t} Var(x_{t+1}|\bar{\mathcal{I}}_t)\Omega_{xt} x_{t+1} \tag{47}$$

Since $\Omega_{pt}^{-1} = (D_t/C_t)^2 Var(x_{t+1}|\bar{\mathcal{I}}_t)$, we can rewrite this as

$$\int \eta_{pit} di = \frac{1}{C_t}(p_t - A_t - B_t(d_t - \mu)) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt} x_{t+1} \tag{48}$$

Solving for non-stationary equilibrium prices To solve for equilibrium prices, start from the portfolio first-order condition for investors (7) and equate total demand with total supply. The total risky asset demand (excluding noisy demand) is

$$\int q_{it} di = \frac{1}{\rho}\Omega_t \Bigg[\pi A_{t+1} + (1+\pi B_{t+1})\Bigg(\mu + G(d_t - \mu) + \hat{V}_t\Big[\Omega_{ft} y_{t+1} + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B_t(d_t - \mu)) - \frac{C_t}{D_t}\Omega_{xt} x_{t+1}\Big]\Bigg) - \pi B_{t+1}\mu - p_t r\Bigg]. \tag{49}$$

The market clearing condition equates the expression above to the residual asset supply $\bar{x} + x_{t+1}$. The model assumes the asset supply is 1. We use the notation $\bar{x}$ here for more generality because then we can apply the result to the model with issuance costs where asset supply is a choice variable. Rearranging the market clearing condition (just multiplying through by $\rho\Omega_t^{-1}$ and bringing the p terms to the left) yields

$$\begin{aligned}[r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]p_t = & -\rho\Omega_t^{-1}(\bar{x} + x_{t+1}) + \pi A_{t+1} + (1+\pi B_{t+1})(\mu + G(d_t - \mu)) + (1+\pi B_{t+1})\hat{V}_t\Omega_{ft} y_{t+1} \\ & - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}(A_t + B_t(d_t - \mu) - (1+\pi B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt} x_{t+1} - \pi B_{t+1}\mu \end{aligned} \tag{50}$$

Solve for p to get

$$\begin{aligned}&\left(r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}\right)(A_t + B_t(d_t - \mu) + C_t y_{t+1} + D_t x_{t+1}) \\ &= -\rho\Omega_t^{-1}(\bar{x} + x_{t+1}) + \pi A_{t+1} + (1+\pi B_{t+1})(\mu + G(d_t - \mu)) + (1+\pi B_{t+1})\hat{V}_t\Omega_{ft} y_{t+1} \\ &\quad - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}(A_t + B_t(d_t - \mu)) - (1+\pi B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt} x_{t+1} - \pi B_{t+1}\mu\end{aligned}$$

Multiply both sides by the first term on the left hand side and match the coefficients to get

$$A_t = [r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}\left(-\rho\Omega_t^{-1}\bar{x} + \pi A_{t+1} + (1+\pi B_{t+1})\mu - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t - \pi B_{t+1}\mu\right) \tag{51}$$

Multiplying both sides by the inverse term:

$$rA_t - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t = -\rho\Omega_t^{-1}\bar{x} + \pi A_{t+1} + (1+\pi B_{t+1})\mu - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t - \pi B_{t+1}\mu$$

and canceling the $(1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t$ term on both sides leaves

$$rA_t = -\rho\Omega_t^{-1}\bar{x} + \pi A_{t+1} + (1+\pi B_{t+1})\mu - \pi B_{t+1}\mu$$

$$
\begin{aligned}
A_t =& \frac{1}{r}\left[\pi(A_{t+1} - B_{t+1}\mu) + (1+\pi B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x}\right] \\
=& \frac{1}{r}\left[\pi A_{t+1} + \mu - \rho\Omega_t^{-1}\bar{x}\right]
\end{aligned} \tag{52}
$$

Risk Premium. The risk premium is defined as

$$
rp_t = \frac{\mathbb{E}[p_{t+1} + d_{t+1}]}{\mathbb{E}[p_t]} - r \tag{53}
$$

The risk premium can be written as

$$
rp_t = \frac{A_{t+1} + \mu}{A_t} - r = \frac{r(A_{t+1} + \mu)}{A_{t+1} + \mu - \rho\Omega_t^{-1}\bar{x}} - r = \frac{r\rho\bar{x}\Omega_t^{-1}}{A_{t+1} + \mu - \rho\Omega_t^{-1}\bar{x}}
$$

where the first equality takes the unconditional expectation, recognizing that $\mathbb{E}[d_t] = \mu$, and the second equation uses the derivation of A_t in equation (52). Note that if all the variance goes to zero, $\Omega_t^{-1} \to 0$, the risk premium also goes to zero.

Note that because, $\bar{x} = 1$ for the main model, in the main text, in equation (9) $\bar{x}$ is set to 1.

Matching coefficients on d_t yields:

$$
B_t = [r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}\left[(1+\pi B_{t+1})G - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t}\right] \tag{54}
$$

Multiplying on both sides by the inverse term

$$
rB_t - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}B_t = (1+\pi B_{t+1})G - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t} \tag{55}
$$

and canceling the last term on both sides yields

$$
B_t = \frac{1}{r}(1+\pi B_{t+1})G \tag{56}
$$

As long as r and G do not vary over time, a stationary solution for B exists. That stationary solution would be (10).

Next, collecting all the terms in y_{t+1}

$$
C_t = [r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}(1+\pi B_{t+1})\hat{V}_t\Omega_{ft} \tag{57}
$$

multiplying both sides by the first term inverse yields $rC_t - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt} = (1+\pi B_{t+1})\hat{V}_t\Omega_{ft}$. Then, dividing through by r and collecting terms in $\hat{V}(1+\pi B_{t+1})$ yields $C_t = (1/r)(1+\pi B_{t+1})\hat{V}_t(\Omega_{pt} + \Omega_{ft})$. Next, using the fact that $\hat{V}^{-1} = \tau_0 + \Omega_{pt} + \Omega_f$, we get $C_t = 1/r(1+\pi B_{t+1})(1-\tau_0\hat{V}_t)$. Of course the $\hat{V}$ term has C_t and D_t in it. If we use the stationary solution for B (if r and G do not vary), then we can simplify to get

$$
C_t = \frac{1}{r - \pi G}(1 - \tau_0\hat{V}_t). \tag{58}
$$

Finally, we collect terms in x_{t+1}.

$$
D_t = [r - (1+\pi B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}[-\rho\Omega_t^{-1} - (1+\pi B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}] \tag{59}
$$

multiply by the inverse term, and then use $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$ to get

$$
rD_t - (1+\pi B_{t+1})\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_{xt}) = -\rho\Omega_t^{-1} - (1+\pi B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt} \tag{60}
$$

Then, adding $(1+B)\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}$ to both sides, and substituting in B (stationary solution), we get

$$D_t = \frac{1}{r-\pi G}\hat{V}_t\tau_x\frac{C_t}{D_t} - \frac{\rho}{r}\Omega_t^{-1}$$
$$D_t = \frac{1}{r-\pi G}\left[\left(\tau_x\frac{C_t}{D_t} - \frac{r\rho}{r-\pi G}\right)\hat{V}_t - Z_t\right] \tag{61}$$

Of course, D_t still shows up quadratically, and also in $\hat{V}_t$. The future coefficient values C_{t+1} and D_{t+1} show up in Ω_t.

A.3 Solving Information Choices

Details of Step 3: Compute ex-ante expected utility. Note that the expected excess return $(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)$ depends on fundamental and supply signals, and on prices, all of which are unknown at the beginning of the period. Because asset prices are linear functions of normally distributed shocks, $E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr$ is normally distributed as well.

With $ElnE$ preferences, $(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)\Omega(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)$ is a non-central χ^2_x-distributed variable. Computing its mean yields a first term that depends on known endowment e_{it} and on terms that depend on information: $\rho\, r\, e_{it} + \rho E[q_{it}(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)|\mathbb{I}_t^-] - \frac{\rho^2}{2}E[q_{it}^2 Var[\pi p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]^{-1}|\mathbb{I}_t^-]$. As we argue in the main text, $Var[\pi p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]$ depends only on the posterior variance of the total payoff, Ω^{-1}. Ω_{ft} and Ω_{ft} do not enter separately.

With the expected utility, $(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)\Omega(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)$ is still a non-central χ^2_x-distributed variable. But the expected utility is the expectation of the exponential of this expression: $E[exp\{(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)\Omega(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr)\}|\mathbb{I}_t^-]$. The exponential of a chi-square distribution is a Wishart. Expected utility is the mean of this expression.

The end-of-period expectation of excess return is distributed $(E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr) \sim N((1-C)\mu - A, V_{ER})$ where V_{ER}, as in the $ElnE$ model, is an increasing function of payoff precision Ω, and does not contain terms in Ω_{ft} or Ω_{xt}, except through Ω. Using the formula for the mean of a Wishart (see Veldkamp (2011) textbook, Appendix Ch.7), we compute the ex-ante expected utility:

$$U = \frac{1}{2}Tr(\Omega V_{ER}) + \frac{1}{2}((1-C)\mu - A)'\Omega((1-C)\mu - A). \tag{62}$$

Since the precisions Ω_{ft} and Ω_{xt} only enter expected the utility through the posterior precision of payoffs Ω, the same is true for the exponential of this expression. Since the exponential function is a monotonic increasing function, we know that expected utility takes the form of an increasing function of Ω. As long as Ω is a sufficient statistic for the data choices in utility, investors' data choices that maximize Ω also maximize the expected utility.

Details of Step 4:

Solve for fundamental information choices. Note that in expected utility (15), choice variables Ω_{ft} and Ω_{xt} enter only through posterior variance Ω^{-1} and through $V[E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr|\mathbb{I}_t^-] = V[\pi p_{t+1}+d_{t+1}-p_tr|\mathbb{I}_t^-] - \Omega_t^{-1}$. Since there is a continuum of investors, and since $V[\pi p_{t+1}+d_{t+1}-p_tr|\mathbb{I}_t^-]$ and $E[E[\pi p_{t+1}+d_{t+1}|\mathcal{I}_{it}]-p_tr|\mathbb{I}_t^-]$ depend only on $t-1$ variables and parameters, and on aggregate information choices, each investor takes them as given. If the objective is to maximize an increasing function of Ω, then the information choices must maximize Ω as well.

Internet Appendix: Not for Publication

B Proofs

We start by proving a few preliminary lemmas that are useful in proving the main results. Throughout this appendix, as we will often treat the signal-to-noise ratio in prices as a single variable, we define

$$\xi_t \equiv \frac{C_t}{D_t} \tag{63}$$

Lemma 1 *If $\Omega_{ft} > 0$, then $C_t > 0$.*

Proof. Using equation (58), it suffices to show that $1/(r-G) > 0$ and $(1-\tau_0\hat{V}_t) > 0$. From the setup, we assumed that $r > 1$ and $G < 1$. By transitivity, $r > G$ and $r - G > 0$. For the second term, we need to prove equivalently that $\tau_0\hat{V}_t < 1$ and thus that $\tau_0 < \hat{V}_t^{-1}$. Recall from (35) that $\hat{V}^{-1} = \tau_0 + \Omega_{ft} + \Omega_{pt}$. Since Ω_{ft} and Ω_{pt} are defined as precisions, they must be non-negative. Furthermore, we supposed that $\Omega_{ft} > 0$. Thus, $\tau_0 < \hat{V}_t^{-1}$, which completes the proof. ■

Lemma 2 $D_t \leq 0$.

Proof. Start from equation (60) and substitute in (35). Moreover, let $\alpha \equiv \frac{\rho r}{r-G}$. Simplify to get:

$$\xi_t^3(Z_t\tau_x + Z_t\Omega_{xt}) + \xi_t^2(\Omega_{xt}) + \xi_t(\alpha + Z_t\tau_0 + Z_t\Omega_{ft}) + \Omega_{ft} = 0 \tag{64}$$

Then, use the budget constraint to express the first-order conditions as (16). One can solve for both Ω_{xt} and Ω_{ft} in terms of ξ_t:

$$\Omega_f = \Big(\frac{K_t}{(1+\frac{1}{\chi_x}\xi_t^4)}\Big)^{\frac{1}{2}} \tag{65}$$

$$\Omega_x = \Big(\frac{K_t}{\chi_x}\big(1 - \frac{1}{1+\frac{1}{\chi_x}\xi_t^4}\big)\Big)^{\frac{1}{2}} = \Big(\frac{K_t\frac{1}{\chi_x}}{\chi_x\big(1+\frac{1}{\chi_x}\xi_t^4\big)}\Big)^{\frac{1}{2}}\xi_t^2 = \frac{\xi_t^2}{\chi_x}\Big(\frac{K_t}{(1+\frac{1}{\chi_x}\xi_t^4)}\Big)^{\frac{1}{2}} \tag{66}$$

Substituting these into equation (64) fully determines ξ_t in terms of exogenous variables.

$$\xi_t\Big(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + \xi_t^2\Omega_{xt}(1+\xi_t Z_t) + \Omega_{ft}(1+\xi_t Z_t) = 0 \tag{67}$$

First note that

$$\Omega_{ft} + \xi_t^2\Omega_{xt} = -\frac{\xi_t(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0)}{(1+\xi_t Z_t)} \tag{68}$$

where the left hand side is the objective function. Therefore, we know the maximized value of the objective function solely as a function of $\xi_t = \frac{C}{D}$. Keep in mind that since we already imposed an optimality condition, this latter equation holds only at the optimum.

Substituting in for Ω_{ft} and Ω_{xt} from (65) and (66) yields an equation that implicitly defines ξ_t as a function of primitives, K_t and future equilibrium objects, embedded in Z_t.

$$\xi_t\Big(\xi_t^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + (1+\xi_t Z_t)(1+\frac{1}{\chi_x}\xi_t^4)\Big(\frac{K_t}{(1+\frac{1}{\chi_x}\xi_t^4)}\Big)^{\frac{1}{2}} = 0$$

$$\xi_t^3 Z_t\tau_x + \xi_t(\alpha + Z_t\tau_0) + (1+\xi_t Z_t)(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0 \tag{69}$$

The left hand side must equal zero for the economy to be in equilibrium. However, all the coefficients $K_t, , \chi_x, \tau_0$, and τ_x are assumed to be positive. Furthermore, Z_t is a variance. Inspection of (37) reveals that it must be strictly positive. Thus, the only way that the equilibrium condition can possibly be equal to zero is if $\xi_t < 0$. Recall that $\xi_t = C_t/D_t$. The previous lemma proved that $C_t > 0$. Therefore, it must be that $D_t < 0$. ■

The next lemma proves the following: If no one has information about future dividends, then no one's trade is based on information about such dividends, and thus the price cannot contain information about them. Since C_t is the price coefficient on future dividend information, $C_t = 0$ means that the price is uninformative. In short, the price cannot reflect information that no one knows.

Lemma 3 ***When information is scarce, price is uninformative:*** *As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j}, and D_{t+1}, $\forall j > 0$), the unique solution for price coefficient C_t is $C_t = 0$.*

Proof. *Step 1: As $\Omega_{ft} \to 0$, prove $C_t = 0$ is always a solution.*

Start with the equation for D_t (12). Substitute in for Ω using (38) and $1 + B = r/(r - G)$ and rewrite it as

$$D_t = \frac{1}{r-G}\hat{V}_t\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}\right] \tag{70}$$

Then, express C_t from (58) as $C_t = 1/(r - G)\hat{V}_t(\hat{V}_t^{-1} - \tau_0)$ and divide C_t by D_t, cancelling the $\hat{V}_t/(r - G)$ term in each to get

$$\frac{C_t}{D_t} = \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}} \tag{71}$$

If we substitute in $\hat{V}_t^{-1} = \tau_0 + \Omega_{pt} + \Omega_{ft}$ from (35) and then set $\Omega_{ft} = 0$, we get

$$\frac{C_t}{D_t} = \frac{\Omega_{pt}}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \Omega_{pt})} \tag{72}$$

Then, we use the solution for price information precision $\Omega_{pt} = (C/D)^2(\tau_x + \Omega_{xt})$ and multiply both sides by the denominator of the fraction to get

$$\frac{C_t}{D_t}\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt}))\right] = \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt}) \tag{73}$$

We can see right away that since both sides are multiplied by C/D, as $\Omega_{ft} \to 0$, for any given future price coefficients C_{t+1} and D_{t+1}, $C = 0$ is always a solution.

Step 2: Prove uniqueness.

Next, we investigate what other solutions are possible by dividing both sides by C/D:

$$\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt})) - \left(\frac{C_t}{D_t}\right)(\tau_x + \Omega_{xt}) = 0 \tag{74}$$

This is a quadratic equation in C/D. Using the quadratic formula, we find

$$\frac{C_t}{D_t} = \frac{\Omega_{xt} \pm \sqrt{\Omega_{xt}^2 - 4Z_t(\tau_x + \Omega_{xt})(\rho r/(r-G) + \tau_0 Z_t)}}{-2Z_t(\tau_x + \Omega_{xt})} \tag{75}$$

If we now take the limit as $\Omega_{xt} \to 0$, the term inside the square root becomes negative, as long as $r - G > 0$. Thus, there are no additional real roots when $\Omega_{xt} = 0$.

Similarly, if Ω_{xt} is not sufficiently large, (75) has no real roots, which proves that: as $\Omega_{ft} \to 0$, if we take C_{t+1} and D_{t+1} as given and Ω_{xt} is sufficiently small, then the unique solution for price coefficient C is $C = 0$. ■

Proof of Result 1. From lemma 3, we know that as $C_t = 0$. From the first-order condition for information (16), we see that the marginal utility of demand information relative to fundamental information (the marginal rate of substitution) is a positive constant times $(C_t/D_t)^2$. If $C_t = 0$, then $\partial U_{it}/\partial \Omega_{xit}$ is a positive constant times zero, which is zero. ■

Proof of Result 2.

(2a)

Part 1: $\frac{dC/|D|}{dK} > 0$. In the model where $\pi = 0$, a simpler set of equations characterize a solution. In this environment, we can show exactly how changes in parameters affect information choices and price coefficients. These static forces are also at play in the dynamic model. But there are additional dynamic forces that govern the model's long-run behavior.

Let $\xi = \frac{C}{D}$. With $\pi = 0$,

$$C = \frac{1}{r}(1 - \tau_0 \hat{V})$$
$$D = \frac{1}{r}(\tau_x \frac{C}{D} - \rho)\hat{V}$$

Divide and rearrange to get

$$\tau_x \xi^2 - \rho\xi = \hat{V}^{-1} - \tau_0$$

Substitute for $\hat{V}^{-1} = \tau_0 + \Omega_f + \xi^2(\tau_x + \Omega_x)$ and cancel terms on both sides. The following equations characterize the equilibrium of the static ($\pi = 0$) model:

$$\xi^2\Omega_x + \xi\rho + \Omega_f = 0 \tag{76}$$

which has two solutions

$$\xi = \frac{-\rho \pm \sqrt{\rho^2 - 4\Omega_f\Omega_x}}{2\Omega_x}. \tag{77}$$

We pick the larger solution (with +), because, when there is no demand information (for instance $\chi_x \to \infty$), the solution converges to the unique solution in the models where there is only fundamental information acquisition, $-\frac{\Omega_f}{\rho}$.

Thus,

$$\xi = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_f\Omega_x}}{2\Omega_x} \tag{78}$$

Now there are two extra equations to complete the model, budget constraint and investor FOC

$$\Omega_f^2 + \chi_x\Omega_x^2 = K$$
$$\frac{\Omega_x}{\Omega_f} = \frac{1}{\chi_x}\xi^2$$

which, using equation (78), implies

$$\Omega_f = \sqrt{\frac{K}{(1+\frac{\xi^4}{\chi_x})}} \tag{79}$$

$$\Omega_x = \frac{\xi^2}{\chi_x}\sqrt{\frac{K}{(1+\frac{\xi^4}{\chi_x})}} \tag{80}$$

Put this back into equation (78) to get the signal-to-noise ratio

$$\xi = -\frac{1}{\sqrt{2}}\sqrt{\frac{\rho^2\chi_x}{K} \pm \frac{\sqrt{-\chi_x\left(4K^2-\rho^4\chi_x\right)}}{K}}$$

Again, we pick the solution that is consistent with the limit $\chi_x \to \infty$, $\xi = -\frac{\sqrt{K}}{\rho}$

$$\xi = -\frac{1}{\sqrt{2}}\sqrt{\frac{\rho^2\chi_x}{K} - \frac{\sqrt{-\chi_x\left(4K^2-\rho^4\chi_x\right)}}{K}} = -\rho\sqrt{\frac{\chi_x}{2K}}\sqrt{1-\sqrt{1-4K^2/(\rho^4\chi_x)}}$$

which implies

$$\frac{d(\frac{C}{D})}{dK} = \frac{d\xi}{dK} = -\frac{\rho\sqrt{\chi_x}}{(2K)^{3/2}}\frac{\sqrt{1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}}{\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}} < 0 \tag{81}$$

which means $\frac{C}{|D|}$ is increasing, i.e, the signal-to-noise ratio improves as more information becomes available.

Part 2: $\frac{\partial C/|D|}{\partial\Omega_f}$ *and* $\frac{\partial C/|D|}{\partial\Omega_x}$. Let $\xi = \frac{C}{D}$ denote the equilibrium signal-to-noise ratio associated with total information capacity K_t, and for brevity suppress subscript t. We have

$$\begin{aligned}\frac{d\xi}{dK} &= \frac{\partial\xi}{\partial\Omega_f}\left(\frac{d\Omega_f}{dK}+\frac{\partial\Omega_f}{\partial\xi}\frac{d\xi}{dK}\right)+\frac{\partial\xi}{\partial\Omega_x}\left(\frac{d\Omega_x}{dK}+\frac{\partial\Omega_x}{\partial\xi}\frac{d\xi}{dK}\right)\\ &= \frac{\partial\xi}{\partial\Omega_f}\frac{d\Omega_f}{dK}+\frac{\partial\xi}{\partial\Omega_x}\frac{d\Omega_x}{dK}+\left(\frac{\partial\xi}{\partial\Omega_f}\frac{\partial\Omega_f}{\partial\xi}+\frac{\partial\xi}{\partial\Omega_x}\frac{\partial\Omega_x}{\partial\xi}\right)\frac{d\xi}{dK}\end{aligned}$$

The first term is the direct effect of the change in K on ξ through the change in fundamental analysis, the second term is the direct effect through the change in demand analysis, and the third term (in parentheses) is the indirect effect. We have

$$\begin{aligned}\frac{\partial\xi}{\partial\Omega_f}\frac{d\Omega_f}{dK} &= -\frac{\Omega_f}{2K(2\xi\Omega_x+\rho)}\\ \frac{\partial\xi}{\partial\Omega_x}\frac{d\Omega_x}{dK} &= \frac{\xi\rho+\Omega_f}{2K(2\xi\Omega_x+\rho)}\\ \left(\frac{\partial\xi}{\partial\Omega_f}\frac{\partial\Omega_f}{\partial\xi}+\frac{\partial\xi}{\partial\Omega_x}\frac{\partial\Omega_x}{\partial\xi}\right)\frac{d\xi}{dK} &= \frac{\Omega_f\left(\xi^4+\chi_x\right)+\xi\rho\chi_x}{\frac{K\Omega_x\left(\xi^4+\chi_x\right)(2\xi\Omega_x+\rho)\left(\rho^2\chi_x-2K\xi^2\right)}{\rho^2\chi_x}}\end{aligned}$$

Note that

$$\Omega_f\left(\xi^4+\chi_x\right)+\xi\rho\chi_x = \xi^4\Omega_f+\chi_x(\xi\rho+\Omega_f) = \xi^4\Omega_f-\xi^2\chi_x\Omega_x = \xi^2\left(\xi^2\Omega_f-\Omega_x\right) = 0$$

that is, the indirect effect is zero, consistent with what the envelope theorem implies. Thus we have the following decomposition

$$\frac{d\xi}{dK} = \frac{\partial \xi}{\partial \Omega_f}\frac{d\Omega_f}{dK} + \frac{\partial \xi}{\partial \Omega_x}\frac{d\Omega_x}{dK} = \frac{-\Omega_f}{2K(2\xi\Omega_x + \rho)} + \frac{\xi\rho + \Omega_f}{2K(2\xi\Omega_x + \rho)}.$$

From equation (76), $\xi\rho + \Omega_f < 0$, thus both effects have the same sign. Moreover, we have already proven in result 2 that $\frac{d\xi}{dK} < 0$, which in turn implies that both effects must be negative and $2\xi\Omega_{xt} + \rho > 0$. Thus the increase in either type of information acquisition, following an increase in capacity, improves the signal-to-noise ratio (i.e, $\frac{C}{D}$ increases in absolute value).

(2b)

Recall that with $\pi = 0$

$$C = \frac{1}{r}\left(1 - \frac{\tau_0}{\tau_0 + \Omega_f + \xi^2(\tau_x + \Omega_x)}\right) = \frac{1}{r}\left(1 - \tau_0\hat{V}\right)$$

Thus to prove $\frac{dC}{dK} > 0$, it is sufficient to show that $\frac{d\hat{V}}{dK} < 0$. Using the first-order condition, along with definition of $\hat{V}$ and that $\frac{d\xi^2}{dK} > 0$, we have that a sufficient condition for $\frac{d\hat{V}}{dK} < 0$ is $d\sqrt{\left(1 + \frac{\xi^4}{\chi_x}\right)K}/dK > 0$, which is true. Thus $\frac{dC}{dK} > 0$.

(2c)

Recall that with $\pi = 0$,

$$D = \frac{1}{r}\frac{\tau_x\xi - \rho}{\tau_0 + \Omega_f + \xi^2(\tau_x + \Omega_x)} = \frac{1}{r}(\tau_x\xi - \rho)\hat{V}.$$

Thus,

$$\frac{dD}{dK} = \frac{1}{r}\left[\tau_x\hat{V}\frac{d\xi}{dK} + (\tau_x\xi - \rho)\frac{d\hat{V}}{dK}\right].$$

The derivative is the sum of two terms. The first term is negative since $\frac{d\xi}{dK} < 0$, while the second term is positive since $\frac{d\hat{V}}{dK} < 0$, as argued in part (2b), and $(\tau_x\xi - \rho) < 0$. So we have to determine which one is larger.

To do so, substitute the closed form solutions into the above expression, and solve for $\bar{K}_D = \min\{\bar{K}, K_D\}$ such that

$$\frac{1}{r}\left[\tau_x\hat{V}\frac{d\xi}{dK} + (\tau_x\xi - \rho)\frac{d\hat{V}}{dK}\right]_{K=K_D} = 0. \tag{82}$$

The algebra is cumbersome, but it is straightforward to show that $0 < \bar{K}_D < \bar{K}$ is unique, and that $\frac{dD}{dK} < 0$ if and only if $K < \bar{K}_D$. To observe the latter point, note that when $K \to 0$, $\xi \to 0$, thus

$$\frac{dD}{dK} \Leftrightarrow \frac{d\xi}{dK} + \frac{\rho}{2\tau_0\sqrt{K}} < 0.$$

Substitute for $\frac{d\xi}{dK}$ from equation (81) and use L'Hopital rule to get that as $K \to 0$, the latter inequality holds.

■

Proof of Result 3. From the individual first-order condition (16), the only channel where aggregate information choices affect the individual choice is through the signal-to-noise ratio. More specifically for $\pi = 0$, one can solve for both the signal-to-noise ratio and individual information choices in closed form, as we did in the proof of result 2.

As $\xi_t < 0$, from equation (78) it is immediate that $\frac{\xi_t}{\Omega_{ft}} < 0$. Next, equation (80) implies

$$\frac{d\mathbf{\Omega}_{xit}}{d\xi_t} = \frac{2\xi_t\sqrt{\frac{k\chi_x}{\xi_t^4+\chi_x}}}{\xi_t^4+\chi_x} < 0,$$

which together implies $\frac{d\mathbf{\Omega}_{xit}}{d\Omega_{ft}} > 0$.

■

Proof of Result 4.

(4a)

Substitute the closed form for ξ into Ω_f and take the derivative to get[19]

$$\frac{d\Omega_f}{dK} = \frac{2\left(8K^4 + 3K^2\rho^4\chi_x\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right)\right)}{\rho^8\chi_x^2\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right)^2\sqrt{K\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}+1\right)}\sqrt{2-\frac{8K^2}{\rho^4\chi_x}}}$$

Each term in the denominator is positive. Thus for $\frac{d\Omega_x}{dK}$ to be positive, it must be that

$$8K^2 + 3\rho^4\chi_x\left(\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}-1\right) > 0.$$

Manipulating the latter equation, the necessary and sufficient condition is

$$K < \frac{\sqrt{3}}{4}\rho^2\sqrt{\chi_x} = \frac{\sqrt{3}}{2}\bar{K},$$

where $\bar{K} = \frac{\rho^2\sqrt{\chi_x}}{2}$, as is defined in the main text.

(4b)

From equation (80)

$$\Omega_x = \frac{1}{\chi_x}\sqrt{\frac{K}{\left(\frac{1}{\xi^4}+\frac{1}{\chi_x}\right)}}$$

Therefore, as $K\uparrow$, the numerator increases and the denominator falls (part a), thus $\frac{d\Omega_x}{dK} > 0$. ■

Proof of Result 5.

(5a)

Prove: $\frac{d\left(C_{t+1}^2\tau_0^{-1}+D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} > 0$, *keeping* K_{t+j}, $\forall\ j > 1$ *constant.* By differentiating (58) and (128), we can show that

$$\begin{aligned}\frac{dC_{t+1}}{dK_{t+1}} &= -\frac{\tau_0}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}}\\ \frac{dD_{t+1}}{dK_{t+1}} &= \frac{1}{r-G}\left[\tau_x\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}} + (\tau_x\xi_{t+1} - \frac{r\rho}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right]\end{aligned}$$

[19] For brevity, we suppress the t subscripts.

Also, recall that $\frac{d\xi_{t+1}}{dK_{t+1}} < 0$ and $\frac{d|\xi_{t+1}|}{dK_{t+1}} > 0$.

$$\begin{aligned}
\frac{d\left(C_{t+1}^2\tau_0^{-1}+D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} &= 2\left(\tau_0^{-1}C_{t+1}\frac{dC_{t+1}}{dK_{t+1}}+\tau_x^{-1}D_{t+1}\frac{dD_{t+1}}{dK_{t+1}}\right) \\
&= 2\left(-\tau_0^{-1}C_{t+1}\frac{\tau_0}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}}+\tau_x^{-1}D_{t+1}\frac{1}{r-G}\left[\tau_x\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}}+(\tau_x\xi_{t+1}-\frac{r\rho}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right]\right) \\
&= \frac{2D_{t+1}}{r-G}\left(-\xi_{t+1}\frac{d\hat{V}_{t+1}}{dK_{t+1}}+\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}}+(\xi_{t+1}-\frac{r\rho\tau_x^{-1}}{r-G})\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right) \\
&= \frac{2D_{t+1}}{r-G}\left(\hat{V}_{t+1}\frac{d\xi_{t+1}}{dK_{t+1}}-\frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right) \qquad (83)
\end{aligned}$$

The term outside the parentheses is negative. Inside the parentheses, the first term is negative, while the second term (with the minus sign) is positive. Therefore, we need to show that the first term is larger in magnitude. Next, move to computing $\frac{d\hat{V}_{t+1}}{dK_{t+1}}$. Rewrite

$$\begin{aligned}
\hat{V}_{t+1}^{-1} &= \tau_0+\Omega_{ft+1}+\xi_{t+1}^2(\tau_x+\Omega_{xt+1}) = \tau_0+\xi_{t+1}^2\tau_x+(\Omega_{ft+1}+\xi_{t+1}^2\Omega_{xt+1}) \\
&= \tau_0+\xi_{t+1}^2\tau_x-\frac{\xi_{t+1}(\xi_{t+1}^2Z_{t+1}\tau_x+\frac{\rho r}{r-G}+Z_{t+1}\tau_0)}{(1+\xi_{t+1}Z_{t+1})} \\
&= \tau_0+\frac{\xi_{t+1}(\xi_{t+1}\tau_x-\frac{\rho r}{r-G}-Z_{t+1}\tau_0)}{(1+\xi_{t+1}Z_{t+1})}
\end{aligned}$$

where the second line follows from equation (68) in the main text.

This also implies

$$\hat{V}_{t+1} = \frac{(1+\xi_{t+1}Z_{t+1})}{\tau_0(1+\xi_{t+1}Z_{t+1})+\xi_{t+1}(\xi_{t+1}\tau_x-\frac{\rho r}{r-G}-Z_{t+1}\tau_0)} = \frac{(1+\xi_{t+1}Z_{t+1})}{\tau_0-\xi_{t+1}\frac{\rho r}{r-G}+\xi_{t+1}^2\tau_x} \qquad (84)$$

Thus, we have

$$\frac{d\hat{V}_{t+1}}{dK_{t+1}} = \frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}$$

which reduces equation (83) to

$$\frac{d\left(C_{t+1}^2\tau_0^{-1}+D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} = \frac{2D_{t+1}}{(r-G)}\frac{d\xi_{t+1}}{dK_{t+1}}\left(\hat{V}_{t+1}-\frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\right) \qquad (85)$$

Next, we compute $\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}$:

$$\frac{d\hat{V}_{t+1}}{d\xi_{t+1}} = -\frac{d\hat{V}_{t+1}}{d|\xi_{t+1}|} = \hat{V}_{t+1}^2\frac{-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1})+\frac{\rho r}{r-G}+Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2} > 0 \qquad (86)$$

We use that to rewrite equation (85) as

$$\frac{d\left(C_{t+1}^2\tau_0^{-1}+D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} = \frac{2\hat{V}_{t+1}D_{t+1}}{(r-G)}\frac{d\xi_{t+1}}{dK_{t+1}}\left(1+\frac{r\rho\tau_x^{-1}}{r-G}\hat{V}_{t+1}\frac{\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1})-\frac{\rho r}{r-G}-Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2}\right)$$

The term outside the parentheses on the rhs is positive, so for the lhs to be positive, we need the term inside the parentheses to also be positive.

Since $D_{t+1} < 0$ and $r - G > 0$, it is sufficient that

$$\frac{d\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)}{dK_{t+1}} > 0 \iff \frac{d\xi_{t+1}}{dK_{t+1}}\left(\hat{V}_{t+1} - \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\right) < 0$$

$$\iff \frac{d\xi_{t+1}}{dK_{t+1}}\hat{V}_{t+1} < \frac{r\rho\tau_x^{-1}}{r-G}\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}$$

$$\overset{\frac{d\xi_{t+1}}{dK_{t+1}}<0,\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}>0}{\iff} \hat{V}_{t+1}\left|\frac{d\xi_{t+1}}{dK_{t+1}}\right| > \frac{r\rho\tau_x^{-1}}{r-G}\left|\frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}\right| = \frac{r\rho\tau_x^{-1}}{r-G}\left|\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right|.$$

Therefore, for the future information risk to be increasing in K_{t+1}, we need

$$\frac{\hat{V}_{t+1}\tau_x(r-G)}{r\rho}\left|\frac{d\xi_{t+1}}{dK_{t+1}}\right| > \left|\frac{d\hat{V}_{t+1}}{dK_{t+1}}\right|. \tag{87}$$

Notice that $\frac{d\hat{V}_{t+1}}{dK_{t+1}} = \frac{d\hat{V}_{t+1}}{d\xi_{t+1}}\frac{d\xi_{t+1}}{dK_{t+1}}$, with $\frac{d\hat{V}_{t+1}}{d\xi_{t+1}} > 0$ and $\frac{d\xi_{t+1}}{dK_{t+1}} < 0$. Plug them into equation (87) and the $\frac{d\xi_{t+1}}{dK_{t+1}}$ terms cancel, thus

$$\frac{\hat{V}_{t+1}\tau_x(r-G)}{r\rho} > \frac{d\hat{V}_{t+1}}{d\xi_{t+1}} = \hat{V}_{t+1}^2\frac{-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0}{(1+\xi_{t+1}Z_{t+1})^2}.$$

Cancelling $\hat{V}_{t+1}$ and rearranging, we have

$$\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})^2 > \hat{V}_{t+1}r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \tag{88}$$

We showed that $\hat{V}_{t+1} = \frac{1+\xi_{t+1}Z_{t+1}}{\tau_0 - \xi_{t+1}\frac{\rho r}{r-G} + \xi_{t+1}^2\tau_x}$. Substituting it in equation (88) leads to

$$\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})^2(\tau_0-\xi_{t+1}\frac{\rho r}{r-G}+\xi_{t+1}^2\tau_x) > (1+\xi_{t+1}Z_{t+1})r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1})+\frac{\rho r}{r-G}+Z_{t+1}\tau_0\Big). \tag{89}$$

From equation (80) in the online appendix, we can write $\Omega_f + \xi^2\Omega_x = -\frac{\xi(\xi^2 Z_{t+1}\tau_x + \alpha + Z_{t+1}\tau_0)}{1+\xi Z_{t+1}}$. The LHS is positive. For the RHS, we know that $\xi < 0$, and thus $-\xi$ is positive. The remaining term in the parentheses must be positive because it is a sum of variances, precisions, and squares. To make the signs of the LHS and the RHS match, we must have $1 + \xi_{t+1}Z_{t+1} > 0$. This helps reduce inequality (89):

$$\begin{aligned}
\tau_x(r-G)(1+\xi_{t+1}Z_{t+1})(\tau_0 - \xi_{t+1}\frac{\rho r}{r-G} + \xi_{t+1}^2\tau_x) &> r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \\
\iff \tau_x(1+\xi_{t+1}Z_{t+1})((\tau_0+\xi_{t+1}^2\tau_x)(r-G) - \xi_{t+1}\rho r) &> r\rho\Big(-\tau_x\xi_{t+1}(2+\xi_{t+1}Z_{t+1}) + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \quad (90) \\
\iff \tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) &> r\rho\Big(-\tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big). \quad (91)
\end{aligned}$$

Both the LHS and RHS are positive. For a sufficiently small ρ, this inequality will hold.

There are other ways to arrive at this result. From equation (90), we can find a sufficient condition, that is

$$\begin{aligned}
\tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) &> r\rho\Big(-\tau_x\xi_{t+1}(1+\xi_{t+1}Z_{t+1}) - \tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \\
&= \underbrace{r\rho Z_{t+1}\Big(-\tau_x\xi_{t+1}^2 + \tau_0\Big)}_{\gtrless 0} + \underbrace{r\rho\Big(-2\tau_x\xi_{t+1} + \frac{\rho r}{r-G}\Big)}_{>0}.
\end{aligned}$$

A sufficient condition for the above inequality to hold is

$$\begin{aligned}
& \tau_x(1+\xi_{t+1}Z_{t+1})(\tau_0+\xi_{t+1}^2\tau_x)(r-G) > r\rho\Big(-\tau_x\xi_{t+1}(1+\xi_{t+1}Z_{t+1}) - \tau_x\xi_{t+1} + \tfrac{\rho r}{r-G} + Z_{t+1}\tau_0\Big) \\
\iff & \tau_x(1+\xi_{t+1}Z_{t+1})\Big[\underbrace{(\tau_0+\xi_{t+1}^2\tau_x)(r-G)}_{>0} + \underbrace{r\rho\xi_{t+1}}_{<0}\Big] > \underbrace{r\rho\Big(-\tau_x\xi_{t+1} + \frac{\rho r}{r-G} + Z_{t+1}\tau_0\Big)}_{>0}.
\end{aligned}$$

For a sufficiently small ρ, the right side will be small and the negative term on the left will also be small, ensuring that the inequality will hold.

The conclusion is that, if the risk aversion is not too high ($\rho < \bar{\rho}$), then future information risk is increasing in K_{t+1}. The economic force is this: As K_{t+1} increases, $Var[y_{t+2} \mid \mathcal{I}_{t+1}]$ decreases, but C_{t+1} increases. The effect of decreasing $Var[y_{t+2} \mid \mathcal{I}_{t+1}]$ is mediated by risk aversion. If that's not too large, then the future information risk also increases.

(5b)

Prove that $\partial(C_t/|D|_t)/\partial K_{t+1} < 0$.

Future information K_{t+1} shows up through the variance term Z_t. Therefore, we begin by differentiating equation (69) with respect to Z_t.

$$\frac{\partial \xi_t}{\partial Z_t} = -\frac{\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right) + \chi_x \sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}}\left(\alpha + Z_t\left(3\xi_t^2\tau_x + \tau_0\right)\right)\right)}{\xi_t \chi_x \sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}}\left(\sqrt{K_t}\sqrt{\frac{\xi_t^4+\chi_x}{\chi_x}} + \xi_t^2\tau_x + \tau_0\right)} \tag{92}$$

First, we argue that the numerator is positive. Consider the first term,

$$\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right).$$

We will argue that this term is also always positive. As $K_t \to 0$ from above, this term converges to zero since ξ_t, C_{t+1} and D_{t+1} are all bounded. As $K_T \to \infty$, we have already shown that $\xi_t \to -\frac{1}{Z_t}$, thus this term converges to $\sqrt{K_t}\left(-\xi_t^3 - \frac{\chi_x}{\xi_t}\right) > 0$. Next, take the derivative of the above expression with respect to K_t (keeping Z_t constant):

$$\frac{\partial\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right)\right)}{\partial K_t} = \frac{1}{2\sqrt{K_t}}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right)$$

Thus this expression and its derivative always have the same sign. Now assume this expression is negative for some K_t. The derivative then has to be negative as well, which means that as K_t grows, the expression can never become positive again. However, we showed that as $K_t \to \infty$, this expression is positive, a contradiction.

Next, the denominator is negative, because all terms are positive, except ξ, which is negative. Thus $\frac{\partial \xi_t}{\partial Z_t} > 0$. In other words, price informativeness (signal-to-noise ratio $\frac{C_t}{|D_t|}$ falls) $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$ increases. Moreover, result 5 proves that $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}$ is increasing in K_{t+1} if ρ is not too high, which completes the proof. ∎

Re-proving $\partial(C_t/|D|_t)/\partial K_t > 0$ with long-lived assets. We begin by differentiating equation (69) with respect to K_t.

$$\frac{\partial \xi_t}{\partial K_t} = -\frac{2\sqrt{K_t}}{\left(\xi_t^4 + \chi_x\right)\left(1 + \xi_t Z_t\right)}\left(\sqrt{K_t}\left(\xi_t^3(3\xi_t Z_t + 2) + \chi_x Z_t\right) + \chi_x\sqrt{\frac{\xi_t^4 + \chi_x}{\chi_x}}\left(\alpha + Z_t\left(3\xi_t^2\tau_x + \tau_0\right)\right)\right) \tag{93}$$

Consider the first term, in front of the large parentheses. From Lemma 4, we know that $1 + \xi_t Z_t > 0$. Thus the ratio outside these parentheses is positive. Inside the parentheses, this is the same term as in the $\frac{\partial \xi_t}{\partial Z_t}$ expression above. We signed that positive. If the term in parentheses in (93) is positive and the term in front is also positive, then $\frac{\partial \xi_t}{\partial K_t} < 0$. In other words, price informativeness (the signal-to-noise ratio, $\frac{C_t}{|D_t|}$) rises as information becomes more abundant.

Lemma 4 ***Balanced data processing growth depends on future information risk and long-lived assets.*** *$|D_t| \geq \frac{\rho(r-G)}{r}\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)C_t$, with strict inequality if $K_t > 0$.*

Proof. Use equation (69) to write

$$(1+\xi Z_t)(1+\frac{1}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{1}{K_t})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0), \tag{94}$$

since we've proven that $\xi \leq 0$ (lemma 2). And we know from the structure of the optimization problem (linear objective subject to convex cost) that for any $K_t > 0$, $\Omega_{ft} > 0$, which implies that $C_t > 0$, and thus $\xi < 0$ with strict inequality. The other terms on the right side are strictly positive squares or positive constants, with a negative sign in front. Thus, the right hand side of equation (94) is positive. On the left, since $(1+\frac{1}{\chi_x}\xi^4)^{\frac{1}{2}}$ is a square root, and therefore positive, $(1+\xi Z_t)$ must be also positive for the equality to hold. $(1+\xi Z_t) > 0$ implies that $Z_t < -1/\xi$. Substitute for Z_t to get the result. This result puts a bound on how liquid the price can be. The liquidity is bounded by the product of price informativeness and un-learnable, future risk. ■

Proof of Result 6.

(6a) Ω_{ft}/Ω_{xt} does not converge to 0.

If Ω_{ft}/Ω_{xt} converges to 0, then by the first-order condition, it must be that $\xi_t \to \infty$. It is sufficient to show that $\xi_t \to \infty$ violates equation (69). Rearrange (69) to get

$$\left[\xi_t Z_t\Big(\xi_t^2\tau_x + (K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} + \tau_0\Big) + \xi_t\alpha\right] + (K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0 \tag{95}$$

The term in square brackets is negative and the term outside is positive. Assume $\xi_t \to \infty$. If Z_t does not go to zero, then the negative term grows faster and the equality cannot hold. So it must be that $Z_t \to 0$. That requires that both $C_{t+1} \to 0$ and $D_{t+1} \to 0$ (see (37)). In order for C_{t+1} to go to zero, $\hat{V} \to \tau_0^{-1}$. But since $\xi_t \to \infty$, from equation (35), $\hat{V} \to 0$, which is a contradiction.

(6b) As $K_t \to \infty$, Ω_{ft}/Ω_{xt} does not converge to ∞.

If Ω_{ft}/Ω_{xt} did converge to ∞ as $K_t \to \infty$, then by first-order condition (16), it would have to be that $\xi_t \to 0$. So it suffices to show that $\Omega_{ft}/\Omega_{xt} = \infty$ is inconsistent with $\xi_t = 0$, in equilibrium.

Start from the equilibrium condition (67), which must be zero in equilibrium. If $\xi_t \to 0$, then the first term goes to zero. The proof of lemma 4 proves, along the way, that $(1+\xi_t Z_t) > 0$. (Otherwise, (67) can never be zero because it is always negative.) Thus the second term $\Omega_{xt}\xi_t^2(1+\xi_t Z_t)$ must be non-negative.

The third term $\Omega_{ft}(1+\xi_t Z_t)$ also converges to ∞ because $\Omega_{ft} \to \infty$ and $(1+\xi_t Z_t) > 0$. How do we know that $\Omega_{ft} \to \infty$? In principle, Ω_{ft}/Ω_{xt} could become infinite either because Ω_{ft} became infinite or because Ω_{xt} goes to zero. But if Ω_{xt} goes to zero and Ω_{ft} is finite, then information processing constraint (3), which requires that the weighted sum of Ω_{ft} and Ω_{xt} be K_t, cannot be satisfied as $K_t \to \infty$.

Since one term of (67) becomes large and positive and the other two are non-negative in the limit, the sum of these three terms cannot equal zero. Therefore, $\Omega_{ft}/\Omega_{xt} \to \infty$ cannot be an equilibrium.

(6c) there exists an equilibrium where Ω_{ft}/Ω_{xt} converges to a constant.

By first-order condition (16), we know that Ω_{ft}/Ω_{xt} converges to a constant, if and only if ξ_t also converges to a constant. Thus, it suffices to show that there exists a constant ξ_t that is consistent with equilibrium, in the high-K limit.

Suppose ξ_t and Z_t are constant in the high-K limit. In equation (69) as $K_t \to \infty$, the last term goes to infinity, unless $\xi_t \to -\frac{1}{Z_t}$. If the last term goes to infinity and the others remain finite, this cannot be an equilibrium because equilibrium requires the left side of (69) to be zero. Therefore, for a constant solution to ξ_t, and thus for Ω_{ft}/Ω_{xt} and Z_t to exist, it must be that $\xi_t \to \frac{-1}{Z_t}$, at the correct rate

$$\xi_t^3 Z_t\tau_x + \xi_t(\alpha + Z_t\tau_0) + (1+\xi_t Z_t)(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}} = 0$$

$$\lim_{K_t\to\infty}\left[\xi_t-(-\frac{1}{Z_t})\right]=\frac{\frac{\alpha}{Z_t^2}\tau_x+\frac{\alpha}{Z_t}+\tau_0}{Z_t(\frac{1}{})^{\frac{1}{2}}(1+\frac{1}{\chi_x}\frac{1}{Z_t^4})^{\frac{1}{2}}}\frac{1}{\sqrt{K_t}}\to 0$$

The question that remains is whether ξ_t and Z_t are finite constants in the high-K limit, or whether one explodes and the other converges to zero.

Suppose $\xi_t=-\frac{1}{Z_t}$, which is constant ($\xi_t=\bar{\xi}$). $Z_t=\bar{Z}$ is then also constant. The rest of the proof checks to see if such a proposed constant- $\bar{\xi}$ solution is consistent with equilibrium. We do this by showing that ξ_t does not explode or contract as K_t increases. In other words, for $\xi_t=\frac{-1}{Z_t}$ to be stable and thus the ratio of fundamental to technical analyses to be stable, we need it to be that $\partial\xi_t/\partial K_t\to 0$, in other words, ξ_t and therefore Ω_{ft}/Ω_{xt} converges to a constant as $K_t\to\infty$.

Step 1: Derive $d\xi_t/dK_t$: Start from the equilibrium condition for ξ_t (69) and apply the implicit function theorem:

$$\Big(3Z_t\tau_x\xi_t^2+A+Z_t\tau_0\Big)d\xi_t+\frac{1}{2}(\frac{1}{K_t})^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}dK_t$$
$$+\left[\frac{1}{2}(\frac{K_t}{\chi_f})^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{-\frac{1}{2}}(4\frac{1}{\chi_x}\xi_t^3)+Z_t(K_t)^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}\right]d\xi_t=0$$

So we have

$$\frac{d\xi_t}{dK_t}=\frac{1}{2}(\frac{1}{K_t})^{\frac{1}{2}}\frac{-(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}}{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0+2\frac{1}{\chi_x}K_t^{\frac{1}{2}}(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{-\frac{1}{2}}\xi_t^3+Z_tK_t^{\frac{1}{2}}(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}}$$

Use equation (69) to write the numerator as

$$(1+\xi_tZ_t)(1+\frac{1}{\chi_x}\xi_t^4)^{\frac{1}{2}}=-(\frac{1}{K_t})^{\frac{1}{2}}\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)\tag{96}$$

Now use this to rewrite $\frac{d\xi_t}{dK_t}$ as

$$\frac{d\xi_t}{dK_t}=\frac{1}{2K_t}\frac{1}{\frac{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0}{\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)}-2\frac{1}{\chi_x}(1+\frac{1}{\chi_x}\xi_t^4)^{-1}\xi_t^3-\frac{Z_t}{(1+\xi_tZ_t)}}\tag{97}$$

Step 2: Show that $d\xi_t/dK_t\to 0$ as $K_t\to\infty$, as long as $X(\cdot)\not\to 0$

As $K_t\to\infty$, it is clear that $1/2K_t\to 0$. As long as the term that multiplies $1/2K_t$ stays finite, the product will converge to zero. Since the numerator is just 1, the second term will be finite, as long as the denominator does not go to zero. Define

$$X(\xi_t,Z_t)=\frac{3Z_t\tau_x\xi_t^2+A+Z_t\tau_0}{\xi_t(\xi_t^2Z_t\tau_x+A+Z_t\tau_0)}-2\frac{1}{\chi_x}(1+\frac{1}{\chi_x}\xi_t^4)^{-1}\xi_t^3-\frac{Z_t}{(1+\xi_tZ_t)}\tag{98}$$

which is the denominator of the second fraction on the rhs of equation (97). Then if $X\not\to 0$, $1/X$ is finite, then $1/2K_t*1/X$ goes to zero as K_t gets large. Thus, we get that $\partial\xi_t/\partial K_t\to 0$ as $K_t\to\infty$.

Step 3: $X(\cdot)\not\to 0$.

To complete the proof, we need to show that $\bar{\xi}=-\frac{1}{\bar{Z}}$, which satisfies equilibrium condition (103) as $K_t\to\infty$, does not cause $X(\cdot)=0$. We can check this directly: in equation (98), if $\xi_t=-\frac{1}{Z_t}$, the denominator of the last term becomes zero; so the last term becomes infinite. The only term in (98) with the opposite sign is the middle term, which is finite if $\xi=\frac{C}{D}$ is finite (the running assumption). If the last term of X tends to infinity and the only term of the opposite sign is finite, the sum cannot be 0. Thus, for $\bar{\xi}=-\frac{1}{\bar{Z}}$, which is the limit attained in the limit as $K_t\to\infty$, we have that $X(\bar{\xi})\neq 0$.

Step 4: As $K_t \to \infty$, if (104) holds, a real-valued, finite-ξ solution exists.

From equations (35-38), as $K_t \to \infty$ at least one of the two information choices goes to ∞, so with finite, non-zero $\frac{C}{D}$:

$$\lim_{K_t\to\infty} \hat{V}_t = 0 \tag{99}$$

$$\lim_{K_t\to\infty} \Omega_t^{-1} = \frac{r}{\rho(r-G)} Z_t = D_{t+1}^2(\xi_{t+1}^2\tau_0^{-1} + \tau_x^{-1}) \tag{100}$$

$$\lim_{K_t\to\infty} D_t = -\frac{\rho}{r}\Omega_t^{-1} = -\frac{1}{(r-G)} Z_t \tag{101}$$

A word of interpretation here: Equation (38), which defines Ω^{-1}, is the total future payoff risk. As $\hat{V} \to 0$, it means that the predictable part of this variance vanishes as information capacity gets large. Z_t, which is the unpredictable part, remains and governs liquidity, D_t.

Next, we solve (100) for D_{t+1}, then backdate the solution 1 period to get an expression for D_t. And we equate it to the expression for D_t in (101). This implies that $\lim_{K_t\to\infty} D = \bar{D}$ is constant and equal to both of the following expressions

$$\bar{D}^2 = \frac{-rZ_t}{\rho(r-G)\bar{\xi}(\bar{\xi}^2\tau_0^{-1} + \tau_x^{-1})} = \frac{Z_t}{(r-G)^2\bar{\xi}^2} \tag{102}$$

We can cancel Z_t on both sides, which delivers a quadratic equation in one unknown in $\bar{\xi}$:

$$\bar{\xi}^2\tau_0^{-1} + \frac{r(r-G)}{\rho}\bar{\xi} + \tau_x^{-1} = 0. \tag{103}$$

In order for $\bar{\xi}$ to exist, equation (103) requires the expression inside the square root term of the quadratic formula (often written as $(b^2 - 4ac)$) to not be negative. This imposes the parametric restriction

$$\left(\frac{r(r-G)}{\rho}\right)^2 - 4\tau_0^{-1}\tau_x^{-1} \geq 0,$$

or equivalently,

$$\tau_0\tau_x \geq \left(\frac{4\rho}{r(r-G)}\right)^2. \tag{104}$$

Rearranging this to put τ_0 on the left delivers $\tau_0 \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_x^{-1}\rho^2(r(r-G))^{-2}$. If we instead rearrange this to put τ_x on the left, we get $\tau_x \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_0^{-1}\rho^2(r(r-G))^{-2}$.

Thus if (104) holds, we have

$$\bar{\xi} = (r-G)\frac{-r \pm \sqrt{r^2 - 4(\frac{\rho}{r-G})^2\tau_0^{-1}\tau_x^{-1}}}{2\rho\tau_0^{-1}} \tag{105}$$

$$\bar{C} = \frac{1}{r-G} \tag{106}$$

$$\bar{D} = -\frac{r \pm \sqrt{r^2 - 4(\frac{\rho}{r-G})^2\tau_0^{-1}\tau_x^{-1}}}{2\rho\tau_x^{-1}} \tag{107}$$

Step 5: Balanced growth. Finally, use lemma 4 to prove the existence of balanced growth. The lemma shows that $C_t/|D_t| < \left(\rho\left((r-G)/r\right)\left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)\right)^{-1}$. The first term is just fixed parameters. The second term, $(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})$, is the variance of the part of tomorrow's price that depends on future shocks, x_{t+1} and y_{t+1}. This is the future information risk. It converges to a large, positive number as K_t grows. When information is abundant, high future information risk pushes $C_t/|D_t|$ down toward a constant.

In contrast, if demand analysis were to keep growing faster than fundamental analysis (Ω_{ft}/Ω_{xt} were to fall to zero), by first-order condition (16), $(C_t/D_t)^2$ would keep rising to infinity. But if $(C_t/D_t)^2$ is converging to infinity, then at some point, it must violate the inequality above because the right side of the inequality is decreasing over time. Thus, demand analysis cannot grow faster than fundamental analysis forever.

The only solution that reconciles the first-order condition with the equilibrium price coefficients is one where $(\Omega_{ft}/\Omega_{xt})$ stabilizes and converges to a constant. If fundamental analysis grows proportionately with demand analysis, then the rise in the amount of fundamental analysis makes prices more informative about dividends: C_t increases. Proportional growth in fundamental and demand analyses allows C_t to keep up with the rise in D_t, described above. Therefore, as information technology grows ($K_t \to \infty$), a stable C_t/D_t rationalizes information choices (Ω_{xt}, Ω_{ft}) that grow proportionately, so that Ω_{xt}/Ω_{ft} converges to a constant.

(6d) No perfect liquidity equilibrium, $D_t \neq 0$, $\forall\, t$.

Lemmas 1 and 2 prove that for any $\Omega_{ft}, \Omega_{xt} \geq 0$, $C \geq 0$, and $D_t \leq 0$. Moreover, from the structure of the optimization problem (the linear objective subject to convex cost), for any $K_t > 0$, $\Omega_{ft} > 0$, which implies $C_t > 0$. Since $C_t > 0$, if $D_t \to 0$, the first-order condition implies that Ω_{ft}/Ω_{xt} has to converge to zero. This directly violates equation (95) for any finite K_t, and part (6a) of the result shows that the same contradiction happens in the limit as $K_t \to \infty$. Thus there is no level of technological progress for which the market becomes perfectly liquid, $D_t = 0$.

■

Proof of Result 7.

For the static model, we want to evaluate the effect on price informativeness of reallocating attention from the supply shock to fundamental. Because we have to respect the budget constraint on attention allocation, we have that

$$\Omega_f = \sqrt{K - \chi_x \Omega_x^2}.$$

Thus

$$\frac{d\Omega_f}{d\Omega_x} = -\chi_x \frac{\Omega_x}{\Omega_f}$$

using, the F.O.C $\frac{\Omega_x}{\Omega_f} = \frac{\xi^2}{\chi_x}$, we get that in equilibrium $\frac{d\Omega_f}{d\Omega_x} = -\xi^2$.

We are going to calculate the effect of increasing one unit of Ω_x on $\xi \equiv C/D$, but while considering the decrease in Ω_f needed to achieve the increase. Again, our starting point is

$$\xi^2 \Omega_x + \xi\rho + \Omega_f = 0$$

Differentiating with respect to Ω_x, we have

$$2\xi \frac{d\xi}{d\Omega_x}\Omega_x + \xi^2 + \frac{d\xi}{d\Omega_x}\rho + \frac{d\Omega_f}{d\Omega_x} = 0$$

Replacing $\frac{d\Omega_f}{d\Omega_x} = -\xi^2$, we finally obtain

$$\frac{d\xi}{d\Omega_x}[2\xi\Omega_x + \rho] = 0$$

The term in brackets is 0 only if $\xi = \frac{-\rho}{2\Omega_x}$. In fact, we know that $\xi > \frac{-\rho}{2\Omega_x}$ because the solution of $\xi^2\Omega_x + \xi\rho + \Omega_f = 0$ that behaves as expected when $\chi_x \to \infty$ is

$$\xi = \frac{-\rho}{2\Omega_x} + \frac{\sqrt{\rho^2 - 4\Omega_f\Omega_x}}{2\Omega_x}$$

Thus, the only solution to the equation $\frac{d\xi}{d\Omega_x}[2\xi\Omega_x + \rho] = 0$ is $\frac{d\xi}{d\Omega_x} = 0$.

Second order condition: Of course, it could be that the equilibrium allocation minimizes the price informativeness. To show that this is a maximum, we also need to show that the second-order condition is negative.

Thus starting from

$$2\xi\frac{d\xi}{d\Omega_x}\Omega_x+\xi^2+\frac{d\xi}{d\Omega_x}\rho+\frac{d\Omega_f}{d\Omega_x}=0,$$

we group the terms, use $\frac{d\Omega_f}{d\Omega_x}=-\chi_x\frac{\Omega_x}{\Omega_f}$, and then differentiate a second time to get

$$\begin{aligned}\frac{d\xi}{d\Omega_x}(2\xi\Omega_x+\rho)&=-\xi^2+\chi_x\frac{\Omega_x}{\Omega_f}\\ \frac{d^2\xi}{d\Omega_x^2}(2\xi\Omega_x+\rho)+\frac{d\xi}{d\Omega_x}\frac{d(2\xi\Omega_x+\rho)}{d\Omega_x}&=-2\xi\frac{d\xi}{d\Omega_x}+\chi_x\left[\frac{1}{\Omega_f}-\frac{\Omega_x}{\Omega_f^2}\frac{d\Omega_f}{d\Omega_x}\right]\end{aligned}$$

Now we use $\frac{d\xi}{d\Omega_x}=0$, and $\frac{\Omega_x}{\Omega_f}=\frac{\xi^2}{\chi_x}$ to get

$$\begin{aligned}\frac{d^2\xi}{d\Omega_x^2}(2\xi\Omega_x+\rho)&=\frac{\chi_x}{\Omega_f}\left[1+\chi_x\left(\frac{\Omega_x}{\Omega_f}\right)^2\right]\\ \frac{d^2\xi}{d\Omega_x^2}&=\frac{\chi_x}{\Omega_f(2\xi\Omega_x+\rho)}\left(1+\frac{1}{\chi_x}\xi^4\right)>0\end{aligned}$$

While this is positive, it is positive in ξ, which is (C/D). Since $D<0$, this implies that the second derivative with respect to $C/|D|$ is positive. In other words, the efficient allocation minimizes (C/D), the negative signal-to-noise ratio. Since C/D is a negative number, minimizing it is maximizing the absolute value. Thus, the equilibrium information processing allocation maximizes the measure of price informativeness $C/|D|$.

■

Result 8 ***Information response to technological growth (dynamic).*** *For $\pi=1$,*

(a) If $\Omega_x<\tau_0+\Omega_f$ and $Var[p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]<\max\{\sqrt{3},\frac{1}{2}|C_t/D_t|\}$, then $\frac{\partial C/D}{\partial\Omega_f}<0$ and $\frac{\partial C/D}{\partial\Omega_x}\leq 0$.

(b) Both fundamental and demand analyses increase the price information sensitivity. If $r-G>0$ and $(\tau_x+\Omega_{xt})$ is sufficiently small, then $\partial C_t/\partial\Omega_{ft}>0$ and $\partial C_t/\partial\Omega_{xt}>0$.

(c) If demand is not too volatile, then both fundamental and demand analyses improve concurrent liquidity. If $\tau_x>\rho r/(r-G)$ and $D_t<0$, then $\partial D_t/\partial\Omega_{ft}>0$ and $\partial D_t/\partial\Omega_{xt}>0$.

Proof.

(8a)

The strategy for proving this result is to apply the implicit function theorem to the price coefficients that come from coefficient matching in the market-clearing equation. After equating supply and demand and matching all the coefficients on x_{t+1}, we arrive at (12). Rearranging that equation gives us the expression for C_t/D_t in (71). If we subtract the right side of (71) from the left, we are left with an expression that is equal to zero in equilibrium. We will name this expression F:

$$F=\frac{C_t}{D_t}-\frac{\hat{V}_t^{-1}-\tau_0}{\tau_x\frac{C_t}{D_t}-\frac{\rho r}{(r-G)}-Z_t\hat{V}_t^{-1}}$$

We compute $\frac{\partial C/D}{\partial\Omega_x}=-\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial\Omega_x}$ and $\frac{\partial C/D}{\partial\Omega_f}=-\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial\Omega_f}$. In particular, we have:

$$
\begin{aligned}
\frac{\partial F}{\partial C/D} &= 1-\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1} \\
&\quad +(\hat{V}^{-1}-\tau_0)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right) \\
&= 1-\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2} \\
&\quad \left[\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F}{\partial \Omega_f} &= -(1)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1}+(\hat{V}^{-1}-\tau_0)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}(-Z_t) \\
&= -\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-2}\left[\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)\right]
\end{aligned}
$$

We notice that $\frac{\partial F}{\partial \Omega_x}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}$ since

$$
\frac{\partial F}{\partial \Omega_x}=\frac{\partial F}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial \Omega_x}=\frac{\partial F}{\partial \hat{V}^{-1}}\left(\frac{C_t}{D_t}\right)^2\frac{\partial \hat{V}^{-1}}{\partial \Omega_f}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial F}{\partial \Omega_f}
$$

.

then:

$$
\frac{\partial C/D}{\partial \Omega_f}=\frac{\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)}{\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^2-\left[\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\right]} \tag{108}
$$

Part 1: If $\Omega_x<\tau_0+\Omega_f$ and $C/D>-Z_t/2$, then $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}\leq 0$.

The numerator of (108) is

$$
\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)=\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\tau_0<0
$$

The inequality holds since we have proven that $C_t/D_t<0$ and $r>G$.

In the denominator, however, not all the terms are negative. The denominator of (108), divided by $\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)+Z_t(\hat{V}^{-1}-\tau_0)$ is:

$$
\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)-\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)+(\hat{V}^{-1}-\tau_0)\left(\tau_x-Z_t\left(2\frac{C_t}{D_t}(\tau_x+\Omega_x)\right)\right)\left(\tau_x\frac{C_t}{D_t}-\frac{\rho r}{r-G}-Z_t\hat{V}^{-1}\right)^{-1} \tag{109}
$$

The only positive term is $-2\frac{C_t}{D_t}\Omega_x$. As a result, it is easy to see that if C/D is sufficiently close to zero, then $-2\frac{C_t}{D_t}\Omega_x<\frac{\rho r}{r-G}+Z_t(\tau_0+\Omega_f)$, so (109) is negative.

The numerator is thus negative. And if C/D is sufficiently close to zero, the denominator is positive, so $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}=\left(\frac{C_t}{D_t}\right)^2\frac{\partial C/D}{\partial \Omega_f}<0$ if $C/D<0$ and $\frac{\partial C/D}{\partial \Omega_x}=0$ if $C/D=0$.

Part 2: If $C/D<-\frac{2Z_t^{-1}}{3}$, then $\frac{\partial C/D}{\partial \Omega_f}<0$ and $\frac{\partial C/D}{\partial \Omega_x}\leq 0$.

To see this, we analyze whether, under these new conditions, inequality (109) holds. We have:

$$
\begin{aligned}
& -\frac{\rho r}{r-G} - Z_t(\tau_0 + \Omega_f) - 2\frac{C_t}{D_t}\Omega_x - 3Z_t\left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x) \\
= \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x
\end{aligned}
$$

So if $C/D < -\frac{2Z_t^{-1}}{3}$, we can prove the above claim:

$$
\begin{aligned}
= \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\
< \quad & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\
< \quad & 0
\end{aligned}
$$

Now, by combining the two previous claims, if $\Omega_x < \tau_0 + \Omega_f$ and $Z_t > \frac{1}{\sqrt{3}}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$. The condition $Z_t > \frac{1}{\sqrt{3}}$ implies that $\frac{-Z_t}{2} < -\frac{2Z_t^{-1}}{3}$, which in turn implies the result for the entire support of C/D.

(8b)

From (58), $C_t = \frac{1}{r-G}(1 - \tau_0\hat{V}_t)$.

From (35), $\hat{V}_t$ is defined as

$$
\hat{V} = [\tau_0 + \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_{xt})]^{-1} \tag{110}
$$

Notice that C_t shows up twice, once on the left side and once in $\hat{V}$. Therefore, we use the implicit function theorem to differentiate. If we define $F \equiv C_t - \frac{1}{r-G}(1 - \tau_0\hat{V})$, then $\partial F/\partial C_t = 1 + \frac{1}{r-G}\tau_0\partial\hat{V}/\partial C_t$. Since τ_x and Ω_{xt} are both precisions, both are positive. Therefore, $\partial\hat{V}^{-1}/\partial C_t = 2C_t/D_t^2(\tau_x + \Omega_{xt})$. This is positive, since we know that $C_t > 0$. That implies that the derivative of the inverse is $\partial\hat{V}/\partial C_t = -\hat{V}^2 2C_t/D_t^2(\tau_x + \Omega_{xt})$, which is negative. The $\partial F/\partial C_t$ term is therefore one plus a negative term. The result is positive, as long as the negative term is sufficiently small: $\frac{2}{r-G}\tau_0\hat{V}^2 C_t/D_t^2(\tau_x + \Omega_{xt}) < 1$. We can express this as an upper bound on $\tau_x + \Omega_{xt}$ by rearranging the inequality to read: $(\tau_x + \Omega_{xt}) < 1/2(r-G)\tau_0^{-2}\hat{V}^{-2}D_t^2/C_t$.

Next, we see that $\partial\hat{V}^{-1}/\partial\Omega_{ft} = 1$. Thus, $\partial\hat{V}/\partial\Omega_{ft} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{ft} < 0$.

Likewise, $\partial\hat{V}^{-1}/\partial\Omega_{xt} = (C_t/D_t)^2$. Since the square is always positive, $\partial\hat{V}/\partial\Omega_{xt} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{xt} < 0$.

Finally, the implicit function theorem states that $\partial C_t/\partial\Omega_{ft} = -(\partial F/\partial\Omega_{ft})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{ft} > 0$. Likewise, $\partial C_t/\partial\Omega_{xt} = -(\partial F/\partial\Omega_{xt})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{xt} > 0$.

(8c)

Part 1: $\partial D_t/\partial\Omega_{ft} > 0$.

From market clearing:

$$
D_t = [r - (1+B)\hat{V} + \Omega_p\frac{1}{C}]^{-1}[-\rho\Omega_t^{-1} - (1+B)\frac{C}{D}\hat{V}\Omega_x] \tag{111}
$$

Use $\Omega_p = (\frac{C}{D})^2(\Omega_x + \tau_x)$ to get $D_t r - (1+B)\hat{V}_t\frac{C}{D}(\tau_x) = -\rho\Omega_t^{-1}$. Then, use the stationary solution for B :

$1 + B = \frac{r}{r-G}$:

$$D_t - \frac{1}{r-G}\hat{V}_t \frac{C}{D}\tau_x = -\frac{\rho}{r}\Omega_t^{-1} \tag{112}$$

Then use (38) to substitute in for Ω_t^{-1}:

$$D_t = -\frac{1}{r-G}Z_t - \frac{r\rho}{(r-G)^2}\hat{V} + \frac{1}{r-G}\hat{V}_t \frac{C_t}{D_t}\tau_x \tag{113}$$

In the above, the RHS, less the last term, is the loading on X_{t+1}. The last term represents price feedback. We then define $F \equiv$ lhs of (113) $-$ rhs of (113). So that we can apply the implicit function theorem as $\partial D_t/\partial\Omega_f = -\frac{\partial F}{\partial\Omega_f}/\frac{\partial F}{\partial D_t}$. We begin by working out the denominator.

$$\frac{\partial F}{\partial D_t} = 1 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial D_t} - \frac{1}{r-G}\frac{\partial \hat{V} + \frac{C_t}{D_t}}{\partial D_t}\tau_x \tag{114}$$

$$\frac{\partial \hat{V}}{\partial D_t} = \frac{\partial \hat{V}}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial D_t} = -\hat{V}^2[-\frac{2C_t^2}{D_t^3}(\tau_x + \Omega_x)] = 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) \tag{115}$$

$$\frac{\partial \hat{V}\frac{C_t}{D_t}}{\partial D_t} = \frac{C_t}{D_t}\frac{\partial \hat{V}_t}{\partial D_t} + \hat{V}(-\frac{C}{D^2}) \tag{116}$$

$$= \frac{C}{D^2}\hat{V}[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{117}$$

$$\frac{\partial F}{\partial D_t} = 1 + \frac{r\rho}{(r-G)^2}\cdot 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{118}$$

$$\frac{\partial F}{\partial \Omega_f} = 0 - 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_t} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial \hat{V}}{\partial \Omega_t} \tag{119}$$

Recall the definition $\hat{V}_t \equiv [\tau_0 + \Omega_{ft} + \frac{C_t}{D_t}^2(\tau_x + \Omega_x)]^{-1}$. Differentiating $\hat{V}$, we get

$$\frac{\partial \hat{V}}{\partial \Omega_f} = \frac{\partial \hat{V}_t}{\partial \hat{V}_t^{-1}}\cdot\frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2\frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2 \tag{120}$$

Substituting this in to (119) yields

$$\frac{\partial F}{\partial \Omega_f} = \frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}] \tag{121}$$

Substituting in the derivative of $\hat{V}$, we get

$$\frac{\partial D_t}{\partial \Omega_f} = -\frac{\frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}]}{1\frac{2r\rho}{(r-G)^2}\frac{C^2}{D^3}\hat{V}_t^2(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C}{\rho}(\tau_x + \Omega_x) - 1]} \tag{122}$$

We observe that if $\frac{C_t}{D_t} < 0$ and $r > G$, then the numerator is positive (including the leading negative sign).

The denominator is also positive if the following expression is positive:

$$\frac{r-G}{\frac{C}{D^2}\hat{V}} + 2\rho\frac{r}{r-G}\frac{C_t}{D_t}\hat{V}_t(\tau_x+\Omega_x) - \tau_x\hat{V}_t[\frac{2C}{D}(\tau_x+\Omega_x-1)] > 0 \tag{123}$$

This is equivalent to

$$\frac{r-G}{\hat{V}_t}\frac{D^2}{C} + 2\hat{V}_t\frac{C_t}{D_t}(\tau_x+\Omega_x)[\frac{r\rho}{r-G} - \tau_x] + \tau_x\hat{V}_t > 0. \tag{124}$$

Lemma 2 proves that $D < 0$. That makes the middle term potentially negative. However, if $[\frac{r\rho}{r-G} - \tau_x] < 0$ as well, the product of this and D is positive, which means that the middle term is positive. That inequality can be rearranged as $\tau_x > \frac{r\rho}{r-G}$. Since the rest of the terms are squares and precisions, the rest of the expression is positive as well.

Thus if $\tau_x > \frac{r\rho}{r-G}$, then $\frac{\partial D_t}{\partial \Omega_t} > 0$.

Part 2: $\partial D_t/\partial\Omega_{xt} > 0$.

Begin with the implicit function theorem: $\partial D_t/\partial\Omega_x = -\frac{\partial F}{\partial \Omega_x}/\frac{\partial F}{\partial D_t}$. The previous proof already proved that if $\tau_x > \frac{r\rho}{r-G}$, the denominator is positive. All that remains is to sign the numerator.

$$\frac{\partial F}{\partial \Omega_x} = 0 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_x} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial \hat{V}}{\partial \Omega_x}$$

where $\partial\hat{V}/\partial\Omega_x = -\hat{V}^2(C^2)/(D^2)$. Substituting the partial of $\hat{V}$ into the partial of F yields

$$\frac{\partial F}{\partial \Omega_x} = \hat{V}^2\frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x).$$

Combining terms,

$$\frac{\partial D_t}{\partial \Omega_x} = -\frac{\hat{V}^2\frac{C^2}{D^2}(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x)}{\frac{\partial F}{\partial D_t}}$$

We know from lemmas 1 and 2 that $\frac{C_t}{D_t} < 0$. Since $r > G$, by assumption, $\partial F/\partial\Omega_x$ is negative (i.e., the $\frac{C^2}{D^2}$ factor does not change the sign). Applying the implicit function theorem tells us that $\partial D_t/\partial\Omega_{xt} > 0$.

■

Corollary 1 ***Complementarity in demand analysis (dynamic).*** *For* $\pi = 1$*, if* $\Omega_{xt} < \tau_0 + \Omega_{ft}$*, then* $\frac{\partial \boldsymbol{\Omega}_{xit}}{\partial \Omega_{xt}} \geq 0$.

Proof. With the exact same argument that we use in our proof of result 1, complementarity follows from the individual first condition whenever $|\frac{C}{D}|$ is increasing. ■

C Additional Results and Features of the Model

C.1 Comparative Statics: Risk Aversion and Demand Data Relative Cost

To explore the role of risk aversion and the possibility of unbalanced technological change, we take the comparative statics of the static ($\pi = 0$) version of the model with respect to absolute risk aversion ρ and the relative cost of demand data χ_x. We begin with risk aversion.

$$\frac{d(\frac{C}{D})}{d\rho} = \frac{\rho^4\chi_x\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)-2K^2}{2K^2\rho^3\left(\frac{\chi_x}{L}\right)^{3/2}\sqrt{2(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}})}\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}$$

The sign is the same as the sign of the numerator. Manipulate the numerator to get

$$\begin{aligned}
&\rho^4\chi_x\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)-2K^2>0\\
&1-\frac{2K^2}{\rho^4\chi_x}>\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\\
&1+\frac{4K^4}{\rho^8\chi_x^2}-\frac{4K^2}{\rho^4\chi_x}>1-\frac{4K^2}{\rho^4\chi_x}\\
&\frac{4K^4}{\rho^8\chi_x^2}>0
\end{aligned}$$

The interpretation is that, as agents become more risk averse, the signal-to-noise ratio of prices deteriorates.

Next, we explore changes in the shadow cost of demand data, without changing the ability to process fundamental data.

$$\frac{d(\frac{C}{D})}{d\chi_x} = \frac{\sqrt{\frac{\chi_x}{K}}\left(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}\right)}{\sqrt{2(1-\sqrt{1-\frac{4K^2}{\rho^4\chi_x}})}\sqrt{1-\frac{4K^2}{\rho^4\chi_x}}}>0$$

which means the signal-to-noise ratio of prices deteriorates (falls in absolute value) as the marginal cost of demand data increases.

C.2 Extension: Informed and Uninformed Investors

C.2.1 Bayesian Updating

Throughout this section, we denote informed investors by i and uninformed investors by i'. We use the same notation for any relevant aggregates. The analysis of an informed individual investor is identical to the baseline model. For uninformed investor i', the optimal quantity of asset demand has the same form, except $\Omega_{fi't}=\Omega_{xi't}=0$.

Next, we turn to the aggregation.

Average expectations and precisions: The price information content for an informed investor i is $\Omega_{pit}\equiv(C_t/D_t)^2(\tau_x+\Omega_{xit})$, and for an uninformed investor i' is $\Omega_{pi't}\equiv(C_t/D_t)^2\tau_x$. Since all investors within the same group are ex-ante identical, they make identical information decisions. Thus, $\Omega_{pit}=\Omega_{pt}$

($\Omega_{pi't}=\Omega'_{pt}$) for all informed (uninformed) investors i (i'). The realized price signal still differs because the signal realizations are heterogeneous. Thus for informed investors

$$\int\eta_{pit}di=\frac{1}{C_t}(p_t-A_t-B(d_t-\mu))-\frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}x_{t+1}$$

And for uninformed investors

$$\int\eta_{pi't}di=\frac{1}{C_t}(p_t-A_t-B(d_t-\mu))$$

Next, we add equivalent definitions of the conditional variance / precision terms that simplify notation for uninformed investors.

$$\begin{aligned}
\hat{V}_t' &= (\tau_0 + (C_t/D_t)^2 \tau_x)^{-1} \\
\Omega_t^{'-1} &= \pi C_{t+1}^2 \tau_0^{-1} + \pi D_{t+1}^2 \tau_x^{-1} + (1+\pi B_{t+1})^2 \hat{V}_t' \\
Z_t' &= \frac{\pi\rho}{r}(r-\pi G)(C_{t+1}^2 \tau_0^{-1} + D_{t+1}^2 \tau_x^{-1}) = Z_t \\
\Omega_t^{'-1} &= \frac{r}{\rho(r-\pi G)} Z_t' + (\frac{r}{r-\pi G})^2 \hat{V}_t'
\end{aligned}$$

Note that future information risk is the same for the two types of investors, since it is by definition unlearnable today.

Next, we can compute the average expectations

$$\begin{aligned}
\int E[y_{t+1}|\bar{\mathcal{I}}_{i't}]\, di' &= \hat{V}_t' \Omega_{pt}' \frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) = (1-\tau_0 \hat{V}_t') \frac{1}{C_t}(p_t - A_t - B(d_t-\mu)) \\
\int E[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_{i't}]\, di' &= A_t + (1+\pi B) E[d_{t+1}|\bar{\mathcal{I}}_t'] = A_t + (1+\pi B)\left(\mu + G(d_t-\mu) + E[y_{t+1}|\bar{\mathcal{I}}_t']\right).
\end{aligned}$$

C.2.2 Solving for Equilibrium Prices

The price conjecture is again

$$p_t = A_t + B_t(d_t-\mu) + C_t y_{t+1} + D_t x_{t+1} \tag{125}$$

We will solve for the prices for general supply of asset, $\bar{x}$, although in the main text it is normalized to one unit.

The average price signal in the economy is

$$\lambda \int \eta_{pi} di + (1-\lambda) \int \eta_{pi'} di = \frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu)) - \lambda \frac{C_t}{D_t} \Omega_{pt}^{-1} \Omega_{xt} x_{t+1}$$

where $\Omega_{pt}^{-1} = (D_t/C_t)^2 Var(x_{t+1}|\bar{\mathcal{I}}_t)$.

Solving for non-stationary equilibrium prices. To solve for equilibrium prices, we start from the portfolio first-order condition for investors (7) and equate total demand with total supply. The total risky asset demand (excluding noisy demand) is

$$\begin{aligned}
&\lambda \int q_{it} di + (1-\lambda) \int q_{i't} di' \\
&= \frac{\lambda}{\rho}\Omega_t \left[\pi A_{t+1} + (1+\pi B_{t+1})\left(\mu + G(d_t-\mu) + \hat{V}_t \left[\Omega_{ft} y_{t+1} + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu)) - \frac{C_t}{D_t}\Omega_{xt} x_{t+1}\right]\right) - \pi B_{t+1}\mu - p_t r\right] \\
&\quad + \frac{1-\lambda}{\rho}\Omega_t' \left[\pi A_{t+1} + (1+\pi B_{t+1})\left(\mu + G(d_t-\mu) + \hat{V}_t' \Omega_{pt}' \frac{1}{C_t}(p_t - A_t - B_t(d_t-\mu))\right) - \pi B_{t+1}\mu - p_t r\right].
\end{aligned}$$

The market clearing condition equates the expression above to the residual asset supply $\bar{x} + x_{t+1}$. To simplify notation, let

$$\begin{aligned}
\bar{\Omega}_t &= \lambda \Omega_t + (1-\lambda)\Omega_t' \\
\lambda_{It} &= \frac{\lambda \Omega_t}{\bar{\Omega}_t}, \quad \lambda_{Ut} = \frac{(1-\lambda)\Omega_t'}{\bar{\Omega}_t} = 1 - \lambda_{It}.
\end{aligned}$$

Matching the coefficients on $(d_t - \mu)$ yields:

$$B_t = \left[r - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t \Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{1}{C_t}\right]^{-1} \left[(1+\pi B_{t+1})G - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{B_t}{C_t}\right]$$

Multiplying on both sides by the inverse term

$$rB_t - (1+\pi B_{t+1})(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}')\frac{1}{C_t}B_t = (1+\pi B_{t+1})G - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{B_t}{C_t}$$

and canceling the last term on both sides yields

$$B_t = \frac{1}{r}(1+\pi B_{t+1})G \tag{126}$$

As long as r and G do not vary over time, a stationary solution for B exists. That stationary solution would be (10).

Next, collecting all the terms in y_{t+1}

$$\begin{aligned}
&\frac{\lambda}{\rho}\Omega_t\left[(1+\pi B_{t+1})\left(\hat{V}_t\left[\Omega_{ft}y_{t+1} + \Omega_{pt}y_{t+1}\right]\right) - C_t y_{t+1} r\right] + \frac{1-\lambda}{\rho}\Omega_t'\left[(1+\pi B_{t+1})\left(\hat{V}_t'\Omega_{pt}'y_{t+1}\right) - C_t y_{t+1} r\right] = 0\\
&\lambda\Omega_t(1+\pi B_{t+1})\hat{V}_t\left[\Omega_{ft} + \Omega_{pt}\right] + (1-\lambda)\Omega_t'(1+\pi B_{t+1})\hat{V}_t'\Omega_{pt}' = rC_t\bar{\Omega}_t\\
&\lambda_{It}\bar{\Omega}_t(1+\pi B_{t+1})(1-\tau 0\hat{V}_t) + \lambda_{Ut}\bar{\Omega}_t(1+\pi B_{t+1})(1-\tau_0\hat{V}_t') = rC_t\bar{\Omega}_t.
\end{aligned}$$

Thus, C_t simplifies to

$$C_t = \frac{1}{r-\pi G}\left(1 - \tau_0(\lambda_{It}\hat{V}_t + (1-\lambda_{It})\hat{V}_t')\right).$$

Similar to $\bar{\Omega}_t$, let

$$\bar{\hat{V}}_t = (\lambda_{It}\hat{V}_t + (1-\lambda_{It})\hat{V}_t'),$$

which in turn implies that

$$C_t = \frac{1}{r-\pi G}\left(1 - \tau_0\bar{\hat{V}}_t\right). \tag{127}$$

Finally, we collect the terms in x_{t+1}.

$$D_t = [r - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\Omega_{pt} + (1-\lambda_{It})\hat{V}_t'\Omega_{pt}'\right)\frac{1}{C_t}]^{-1}[-\rho\Omega_t^{-1} - (1+\pi B_{t+1})\lambda_{It}\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}]$$

We multiply by the inverse term, and then use $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$ and $\Omega_{pt}' = (C_t/D_t)^2\tau_x$ to get

$$rD_t - (1+\pi B_{t+1})\left(\lambda_{It}\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_{xt}) + (1-\lambda_{It})\hat{V}_t'\frac{C_t}{D_t}\tau_x\right) = -\rho\Omega_t^{-1} - (1+\pi B_{t+1})\lambda_{It}\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}$$

By substituting in B in the stationary solution and using the $\bar{\hat{V}}_t$, we get

$$\begin{aligned}
D_t &= \frac{1}{r-\pi G}\bar{\hat{V}}_t\tau_x\frac{C_t}{D_t} - \frac{\rho}{r}\bar{\Omega}_t^{-1}\\
D_t &= \frac{1}{r-\pi G}\left[\left(\tau_x\frac{C_t}{D_t} - \frac{r\rho}{r-\pi G}\right)\bar{\hat{V}}_t - Z_t\right]
\end{aligned} \tag{128}$$

Next we compute the expression for informed trader demand, q_t. Since A_{t+1}, B, C_{t+1} and D_{t+1} are non-random (conditional on $\bar{\mathcal{I}}_t$), y_{t+2} and x_{t+2} are independent of the elements of $\bar{\mathcal{I}}_t$ (and so $\mathbb{E}[z_{t+2}|\bar{\mathcal{I}}_t] = \mathbb{E}[z_{t+2}] = 0$ for

$z \in \{x, y\}$) it follows that:

$$\begin{aligned}\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] &= \pi A_{t+1} + (1+\pi B)\mathbb{E}[d_{t+1}|\mathcal{I}_{it}] - \pi B\mu + \pi C_{t+1}\mathbb{E}[y_{t+2}|\mathcal{I}_{it}] + \pi D_{t+1}\mathbb{E}[x_{t+2}|\mathcal{I}_{it}] \\ &= \pi A_{t+1} + (1+\pi B)\mathbb{E}[\mu + G(d_t - \mu) + y_{t+1}|\mathcal{I}_{it}] - \pi B\mu \\ &= \pi A_{t+1} + \mu + \pi B\mu + (1+\pi B)G(d_t-\mu) + (1+\pi B)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - \pi B\mu \\ &= \pi A_{t+1} + \mu + (1+\pi B)G(d_t-\mu) + (1+\pi B)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}].\end{aligned}$$

which implies that

$$\begin{aligned}\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - rp_t &= \mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - r\left(A_t + B(d_t-\mu) + C_t y_{t+1} + D_t x_{t+1}\right) \\ &= \big(\pi A_{t+1} + \mu - rA_t\big) + \big((1+\pi B)G - rB\big)(d_t-\mu) + \big(1+\pi B\big)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - rC_t y_{t+1} - rD_t x_{t+1}\end{aligned}$$

As a result, we obtain

$$\begin{aligned}\mathbb{E}[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] &= \pi A_{t+1} + \mu + (1+\pi B)G(d_t-\mu) + (1+\pi B)\frac{\Omega_{pit}\eta_{pt} + \Omega_{fit}\eta_{ft}}{\tau_0 + \Omega_{pit} + \Omega_{fit}} \\ &= \pi A_{t+1} + \mu + \frac{rG}{r-\pi G}(d_t-\mu) + \frac{r}{r-\pi G}\frac{\Omega_{pt}\eta_{pit} + \Omega_{ft}\eta_{fit}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\end{aligned}$$

where the second line uses symmetry in information choices. Since by Bayes' rule,

$$\mathbb{E}[x_{t+1}|\eta_{xit}] = \frac{\Omega_{xt}\eta_{xt}}{\tau_x + \Omega_{xt}}.$$

And

$$\begin{aligned}\Omega_t^{-1} := \text{Var}[\pi p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t] &= \pi(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) + (1+\pi B)^2\left(\tau_0 + \Omega_{ft} + \Omega_{pt}\right)^{-1} \\ &= \pi(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) + (1+\pi B)^2\left(\tau_0 + \Omega_{ft} + (C/D)^2(\tau_x + \Omega_{xt})\right)^{-1}.\end{aligned}$$

Next, we substitute the above expressions in q_t to obtain:

$$\begin{aligned}q_t =& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big) + \big((1+\pi B)G - rB\big)(d_t-\mu) + \big(1+\pi B\big)\mathbb{E}[y_{t+1}|\mathcal{I}_{it}] - rC_t y_{t+1} - rD_t x_{t+1}\right] \\ =& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big) + \frac{r}{r-\pi G}\frac{\Omega_{ft}(y_{t+1} + \tilde{\epsilon}_{fit}) + \Omega_{pt}\left(y_{t+1} + (D_t/C_t)(x_{t+1} - \mathbb{E}[x_{t+1}|\eta_{xit}])\right)}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - rC_t y_{t+1} - rD_t x_{t+1}\right] \\ =& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big)\right] + \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]y_{t+1} + \frac{\Omega_t}{\rho}\frac{r}{r-\pi G}\left[\frac{\Omega_{ft}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{fit} \\ &+ \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{D_t}{C_t}\frac{\Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\frac{\tau_x x_{t+1} - \Omega_x\tilde{\epsilon}_{xit}}{\tau_x + \Omega_{xt}} - D_t x_{t+1}\right] \\ =& \frac{\Omega_t}{\rho}\left[\big(\pi A_{t+1} + \mu - rA_t\big)\right] + \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]y_{t+1} + \frac{\Omega_t}{\rho}\frac{r}{r-\pi G}\left[\frac{\Omega_{ft}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{fit} \\ &+ \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\frac{\tau_x}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - D_t\right]x_{t+1} - \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\frac{\Omega_{xt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}}\right]\tilde{\epsilon}_{xit}\end{aligned}$$

where the last equality substitutes $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$.

Covariance between q_t and x_{t+1}. Note that the first term in q_t is a constant and does not appear in any covariance. Moreover, $y_{t+1} \sim \mathcal{N}(0, \tau_0^{-1})$ and iid, and $|G| < 1$, thus d_{t+1} is a (weakly) stationary AR(1) process and so $\mathbb{E}[d_{t+1}] = \mu < \infty$. Thus with $x_{t+1} \sim \mathcal{N}(0, \tau_x^{-1})$, we have $\mathbb{E}[x_{t+1}] = 0$ and so $\text{Cov}(q_t, x_{t+1}) = \mathbb{E}[q_t x_{t+1}]$. Lastly, because y_{t+1}, $\tilde{\epsilon}_{xit}$ and $\tilde{\epsilon}_{fit}$ are iid, they are independent of x_{t+1}, thus $\mathbb{E}[x_{t+1}y_{t+1}] = \mathbb{E}[x_{t+1}\tilde{\epsilon}_{xit}] = \mathbb{E}[x_{t+1}\tilde{\epsilon}_{fit}] = 0$.

Therefore,

$$\begin{aligned}
\mathrm{Cov}(q_t, x_{t+1}) = \mathbb{E}[q_t x_{t+1}] &= \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x \hat{V}_t - D_t\right]\tau_x^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x \hat{V}_t - \frac{1}{r-\pi G}\bar{\hat{V}}_t \tau_x \frac{C_t}{D_t} + \frac{\rho}{r}\bar{\Omega}_t^{-1}\right]\tau_x^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{\rho}{r}\bar{\Omega}_t^{-1} - \frac{1}{(r-\pi G)}\frac{C_t}{D_t}\tau_x (1-\lambda_{It})(\hat{V}_t' - \hat{V}_t)\right]\tau_x^{-1} \\
&= \frac{\Omega_t}{\lambda\Omega_t + (1-\lambda)\Omega_t'}\tau_x^{-1} - \frac{r\Omega_t}{\rho(r-\pi G)}\frac{C_t}{D_t}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t),
\end{aligned}$$

which is equation (138) in the main text.

Covariance between q_t and y_{t+1}. Since $\mathbb{E}[y_{t+1}] = 0$, $\mathrm{Cov}(q_t, y_{t+1}) = \mathbb{E}[q_t y_{t+1}]$. Additionally, as y_{t+1} is independent of x_{t+1}, ϵ_{xt+1}, and ϵ_{ft+1}, then using the same expression for q_t, we have:

$$\begin{aligned}
\mathrm{Cov}(q_t, y_{t+1}) = \mathbb{E}[q_t y_{t+1}] &= \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}\frac{\Omega_{ft} + \Omega_{pt}}{\tau_0 + \Omega_{pt} + \Omega_{ft}} - C_t\right]\tau_0^{-1} \\
&= \frac{r\Omega_t}{\rho}\left[\frac{1}{r-\pi G}(1-\tau_0\hat{V}_t) - \frac{1}{r-\pi G}\left(1-\tau_0\bar{\hat{V}}_t\right)\right]\tau_0^{-1} \\
&= \frac{r\Omega_t}{\rho(r-\pi G)}(\bar{\hat{V}}_t - \hat{V}_t) = \frac{r\Omega_t}{\rho(r-\pi G)}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) \\
&= \frac{r}{\rho(r-\pi G)}(\pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} + (\frac{r}{r-\pi G})^2\hat{V}_t)^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t),
\end{aligned}$$

which is equation (139) in the main text.

C.2.3 Static Economy, $\pi = 0$

In the static economy, where $\pi = 0$, we can further simplify equations (138) and (139). First, note that with $\pi = 0$, $\bar{\Omega}_t = \lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}$. Thus we have

$$\begin{aligned}
\mathrm{Cov}(q_t, x_{t+1}) &= \frac{\hat{V}_t^{-1}}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}\tau_x^{-1} - \frac{C_t}{D_t}\frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) \\
&= \frac{\tau_x^{-1}}{\lambda + (1-\lambda)\frac{\hat{V}_t'}{\hat{V}_t}} - \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1}) \\
&= \frac{\tau_x^{-1}}{\lambda + (1-\lambda)\frac{\tau_0 + (C/D)^2\tau_x}{\tau_0 + \Omega_{ft} + (C/D)^2(\tau_x + \Omega_{xt})}} - \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1})
\end{aligned} \tag{129}$$

and

$$\begin{aligned}
\mathrm{Cov}(q_t, y_{t+1}) &= \frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda_{It})(\hat{V}_t' - \hat{V}_t) = \frac{1}{\rho}\hat{V}_t^{-1}(1-\lambda)\frac{\hat{V}_t'^{-1}}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}(\hat{V}_t' - \hat{V}_t) \\
&= \frac{1-\lambda}{\rho}\frac{1}{\lambda\hat{V}_t^{-1} + (1-\lambda)\hat{V}_t'^{-1}}(\hat{V}_t^{-1} - \hat{V}_t'^{-1}) = \frac{1-\lambda_{It}}{\rho}\frac{\Omega_{ft} + (C/D)^2\Omega_{xt}}{\tau_0 + (C/D)^2\tau_x} \\
&= \frac{1-\lambda}{\rho}\frac{\Omega_{ft} + (C/D)^2\Omega_{xt}}{\tau_0 + (C/D)^2\tau_x + \lambda\left(\Omega_{ft} + (C/D)^2\Omega_{xt}\right)} = \frac{1-\lambda}{\rho}\left(\lambda + \frac{\tau_0 + (C/D)^2\tau_x}{\Omega_{ft} + (C/D)^2\Omega_{xt}}\right)^{-1}
\end{aligned} \tag{130}$$

We use equation (130) to compute the total effective precision acquired about innovation in dividends:

$$\Omega_{ft} + (\frac{C_t}{D_t})^2\Omega_{xt} = \frac{\rho \mathrm{Cov}(q_t, y_{t+1})}{1 - \lambda\left(1 + \rho \mathrm{Cov}(q_t, y_{t+1})\right)}\left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right), \tag{131}$$

and then we use that in equation (129) to infer the size of informed trading:

$$\begin{aligned}\lambda &= \frac{\left(1 - \tau_x \mathrm{Cov}(q_t, x_{t+1})\right)\left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right) + \mathrm{Cov}(q_t, y_{t+1})\left(1 - \tau_x \frac{C_t}{D_t}\left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right)\right)}{\tau_x \mathrm{Cov}(q_t, y_{t+1})\left(\mathrm{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1})\right)} \\ &= \frac{\mathrm{Cov}(q_t, y_{t+1}) + \left(\tau_0 + (\frac{C_t}{D_t})^2\tau_x\right)\left[1 - \tau_x\left(\mathrm{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1})\right)\right]}{\mathrm{Cov}(q_t, y_{t+1})\tau_x\left(\mathrm{Cov}(q_t, x_{t+1}) + \frac{C_t}{D_t}\mathrm{Cov}(q_t, y_{t+1})\right)}.\end{aligned} \tag{132}$$

Thus equations (131), (132), and (76) can be used to make the same inference for the static economy.

C.3 CRRA utility and heterogeneous risk aversion

Solving a CRRA portfolio problem with information choice is challenging because the equilibrium prices are no longer linear functions of the shocks. With two sources of information, this non-linearity implies that one of the signals no longer has normally-distributed signal noise about the asset fundamental. That makes combining the two sources of information analytically intractable.

At the same time, we can come very close to CRRA with state-dependent risk aversion in exponential utility. For example, suppose we set the absolute risk aversion to $\rho_{it} = [(\gamma-1)ln(C_{it}) + ln(\gamma-1)]/C_{it}$, where C_{it} denotes consumption at time t. In this case, the two utility functions would be identical: $exp(\rho_{it}C_{it}) = (\gamma-1)C_{it}^{\gamma-1}$. The problem with doing so is that risk aversion becomes a random variable that depends on asset payoffs, through C_{it}. But suppose we do a close approximation to this. Suppose we allow ρ_{it} to be a function of $E_t[C_{it}]$, where t denotes the beginning of the period t information set, prior to any information processing. This approximation implies that the utility is

$$U(C_{it}) \approx -exp\left[-\left((\gamma-1)ln(E_t[C_{it}]) + ln(\gamma-1)\right)\frac{C_{it}}{E_t[C_{it}]}\right]$$

We can then rewrite this log-linear approximation in a form that is like exponential utility $-\exp(-\rho_{it}C_{it})$, with a coefficient of absolute risk aversion

$$\rho_{it} \equiv [(\gamma-1)ln(E_t[C_{it}]) + ln(\gamma-1)]/E_t[C_{it}]. \tag{133}$$

This form of risk aversion introduces wealth effects on portfolio choices but preserves linearity in prices.

Each investor chooses a number of shares q of the risky asset to maximize (C.3) subject to budget constraint (3). The first-order condition of that problem is

$$q_{it} = \frac{E[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] - rp_t}{\rho_{it}Var[f_{it}|\mathcal{I}_{it}]} - h_{it}$$

Given this optimal investment choice, we can impose market clearing (6) and obtain a price function that is linear in asset payoffs and noisy demand shocks:

$$p^{CRRA} = A + B(d_t - \mu) + Cy + Dx$$

where A, B, C, and D are the same as before, except that in place of each homogeneous ρ is $\bar{\rho} \equiv (\int 1/\rho_i di)^{-1}$, which is the harmonic mean of investors' risk aversions, and captures aggregate wealth effects.

Of course, in this formulation, if investors' wealth grows over time, asset prices trend up. In that sense, the solution changes. However, it is still the case that the decision to learn about fundamental or demand data depends on $(C/D)^2$. But now wealth is an additional force that moves D_t over time. Because ρ^2 shows up in the numerator once and ρ shows up in the denominator, these effects largely cancel each other out. Quantitatively, the effect on D is small. But large changes in wealth can now have an effect on data choices.

C.4 A Linear or Entropy-Based Information Constraint

The reason we use a constraint that is convex in signal precision is that it produces an interior optimum. A constraint that is linear in signal precision or that takes the form of an entropy reduction produces information choices that are corner solutions. Such corner solutions have the same forces as those at work in our version of the model. The reason is that the main results – the substitutability of fundamental data, the complementarity of demand data and the interactions between the two information types – all arise from the marginal utility for information, not from the cost formulation. However, keeping track of corner solutions introduces some additional complexity. This subsection describes how one can work out that version of the one-period asset model ($\pi = 0$).

Step 1. Individual objective. This is still the same as before:

$$\max\ \Omega_f + (\frac{C}{D})^2 \Omega_x$$

subject to an mutual information (entropy - reduction) constraint, $H(x, \eta_x) + H(y, \eta_y) \leq K$ or a linear constraint,

$$\Omega_f + \chi_x \Omega_x = K$$

Step 2. Information aggregation and price coefficients given information choices. We follow very the derivations of Appendix A.4, which solves the model for two types of agents. There, one set of agents has both types of information. Here, one group specializes in fundamental data, and another in demand data (denoted by $'$ agents).

Let λ denote the fraction of agents specializing in fundamental data and $1 - \lambda$ in demand data. Moreover, let $\xi = \frac{C}{D}$. Using the same notation as before, in the static model, we have that for an individual fundamental specialist versus demand specialist:

$$\Omega_i = \hat{V}_i^{-1} = \tau_0 + \Omega_{fi} + \xi^2 \tau_x$$
$$\Omega'_i = \hat{V}_i'^{-1} = \tau_0 + \xi^2 (\tau_x + \Omega_{xi})$$

where the first equality in each line is true because the model is static. And since all fundamental agents are identical, and all demand agents are identical, we have $\Omega = \Omega_i$ and $\Omega' = \Omega'_i$. Thus, to aggregate,

$$\bar{\Omega} = \lambda\Omega + (1 - \lambda)\Omega'$$
$$\bar{\hat{V}}^{-1} = \frac{\lambda\Omega}{\bar{\Omega}} \hat{V}^{-1} + \frac{(1 - \lambda)\Omega'}{\bar{\Omega}} \hat{V}'^{-1} = \frac{1}{\bar{\Omega}}$$

The information aggregation is therefore very simple

$$\bar{\Omega} = \bar{\hat{V}}^{-1} = \tau_0 + \Omega_f + \xi^2 (\tau_x + \Omega_x)$$

where $\Omega_x = \frac{(1-\lambda)K}{\chi_x}$, and $\Omega_f = \lambda K = K - \chi_x \Omega_x$.

Step 3. Solving for optimal information choices. Let Ω_f (Ω_x) denote the total information of agents specializing in fundamental (demand) analysis. We have

$$\xi = \frac{C}{D} = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_f \Omega_x}}{2\Omega_x}$$

and, from the aggregation step,

$$\Omega_f = K - \chi_x \Omega_x$$

We know that when $K \to 0$, $\Omega_x \to 0$ and $\xi = -\frac{\Omega_f}{\rho}$ while $\xi \geq -1$. Thus if $K < \rho$, $\Omega_f = K$ and $\Omega_x = 0$.

Next assume $K > \rho$. For both types of information to be processed, it must be that $\xi = -1$. Let $\Omega_f = K - \chi_x \Omega_x$ and substitute into ξ equation above and solve

$$-1 = \frac{-\rho + \sqrt{\rho^2 - 4\Omega_x(K - \chi_x \Omega_x)}}{2\Omega_x}$$

This yields

$$\Omega_x = \frac{K - \rho}{\chi_x - 1}$$

Since $K > \rho$, it must be that $\chi_x > 1$ for $\Omega_x > 0$, a valid solution. Moreover, $\Omega_x < \frac{K}{\chi_x}$. Thus we must have

$$K < \rho \chi_x$$

As before, when information becomes abundant, no solution exists.

To summarize:

1. $K < \rho$: $\Omega_f = K$ and $\Omega_x = 0$.
2. $K > \rho$ while $K < \rho\chi_x$ and $\chi_x > 1$: $\Omega_f = \frac{\rho\chi_x - K}{\chi_x - 1}$ and $\Omega_x = \frac{K - \rho}{\chi_x - 1}$.
3. Otherwise there is no solution. Once $K > \rho$, it must be that $\chi_x > 1$, as otherwise there is no solution. Even when $\chi_x > 1$, as K becomes sufficiently large, $K > \rho\chi_x$, the solution ceases to exist.

C.5 The Real Economic Benefits of Price Information Sensitivity

We have argued that growth in financial technology has transformed the financial sector and affected financial market efficiency in unexpected ways. But why should we care about financial market efficiency? What are the consequences for real economic activity? There are many possible linkages between the financial and real sectors. In this section, we illustrate two possible channels through which changes in information sensitivity and price impact can alter the efficiency of real business investment.

Manager Incentive Effects The key friction in the first spillover model is that the manager's effort choice is unobserved by equity investors. The manager makes a costly effort only because he or she is compensated with equity. Managers only have an incentive to exert themselves if the value of their equity is responsive to their efforts. Because of this, the efficiency of a manager's effort choice depends on the asset price information sensitivity.

Of course, this friction reflects the fact that the wage is not an unconstrained optimal contract. The optimal compensation for the managers is to pay them for their effort directly, or to give them all the equity in their firm. We do not model the reasons why this contract is not feasible because it would distract from our main point. Our stylized sketch of a model is designed to show how commonly-used compensation contracts that tie wages to firm equity prices (e.g., options packages) also tie price information sensitivity to optimal effort.

Time is discrete and infinite. A single firm with profits d_{t+1} depends on a firm manager's labor choice l_t. Specifically, instead of the dividend process specified in Section 1, asset payoffs take the static form: $d_{t+1} = g(l_t) + y_{t+1}$, where g is increasing and concave and $y_{t+1} \sim N(0, \tau_0^{-1})$ is unknown at t. Because effort is unobserved, the manager's pay, w_t, is tied to the firm's equity price p_t: $w_t = \bar{w} + p_t$. However, effort is costly. We normalize the units of effort so that a unit of effort corresponds to a unit of utility cost. Insider trading laws prevent the manager from participating in the equity market. Thus the manager's objective is

$$U_m(l_t) = \bar{w} + p_t - l_t \tag{134}$$

The firm pays out all its profits each period as dividends to its shareholders. Firm equity purchased at time t is a claim to the present discounted stream of future profits $\{d_{t+1}, d_{t+2} \ldots\}$.

Investors' preferences, endowments, budget constraint, and information choice sets are the same as they were before. The demand data signals are defined as before. Fundamental analysis now generates signals of the form $\eta_{fit} = g(l_t) + y_{t+1} + \tilde{\epsilon}_{fit}$, where the signal noise is $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{ft})$. Investors choose the precision Ω_{ft} of this signal, as well as their demand signal Ω_{xt}. Equilibrium is defined as before, with the additional condition that the manager effort decision maximizes (134).

Solution As before, the asset market equilibrium has a linear equilibrium price:

$$p_t = A_t + C_t(g(l_t) + y_{t+1}) + D_t x_{t+1} \tag{135}$$

Notice that since dividends are not persistent, d_t is no longer relevant for the t price.

The firm manager chooses his effort to maximize (134). The first-order condition is $C_t g'(l_t) = 1$, which yields an equilibrium effort level $l_t = (g')^{-1}(1/C_t)$. Notice that the socially optimal level would set the marginal utility cost of effort equal to the marginal product, $g'(l_t) = 1$. When C_t is below one, managers under-provide effort relative to the social optimum because their stock compensation moves less than one-to-one with the true value of their firm.

Similar to before, the equilibrium level of price information sensitivity C is

$$C_t = \frac{1}{r}\left(1 - \tau_0 Var[g(l_t) + y_{t+1}|\bar{\mathcal{I}}_t]\right). \tag{136}$$

Thus, as more information is analyzed, dividend uncertainty ($Var[g(l_t) + y_{t+1}|\bar{\mathcal{I}}_t]$) falls, C_t rises and managers are better incentivized to exert optimal effort. While the model is stylized and the solution presented here is only a sketch, it is designed to clarify why trends in financial analysis matter for the real economy.

The most obvious limitation of the model is its single asset. One might wonder whether the effect would disappear if the asset's return was largely determined by aggregate risk, which is beyond the manager's control. However, if there were many assets, one would want to rewrite the compensation contract so that the manager gets rewarded for high firm-specific returns. This would look like benchmarked performance pay. If the contract focused on firm-specific performance, the resulting model would look similar to the single asset case here.

In short, this mechanism suggests that recent financial sector trends boost real economic efficiency. More data analysis – of either type – improves price information sensitivity, and thereby incentives. But this is only one possible mechanism that offers one possible conclusion. Our next example presents an alternative line of thought.

C.6 Real Economic Benefits of Liquidity

The second real spillover highlights a downside of financial technology growth. More information technology creates future information risk, which raises the risk of holding equity, raising the equity premium, and making capital more costly for firms. This enormously simplified mechanism is meant as a stand-in for a more nuanced relationship, such as that in Bigio (2015).

Suppose that a firm has a profitable investment opportunity and wants to issue new equity to raise capital for that investment. For every dollar of capital invested, the firm can produce an infinite stream of dividends, d_t. Dividends follow the same stochastic process as described in the original model. However, the firm needs funds to invest, which it raises those funds by issuing equity. The firm chooses the number of shares, $\bar{x}$, to maximize the total revenue raised (maximize output). Each share sells at price p, which is determined by the investment market equilibrium, minus the investment or issuance cost:

$$E[\bar{x}p - c(\bar{x})|\mathcal{I}_{it}]$$

The firm makes its choice conditional on the same prior information that all the investors have. But the firm does not condition on p. It does not take the price as given. Rather, the firm chooses $\bar{x}$, taking into account its impact on the equilibrium price. The change in issuance is permanent and unanticipated. The rest of the model is identical to the dynamic model in section 1.

Solution Given the new asset supply, $\bar{x}$, the asset market and information choice solutions to the problem are the same as before. But how the firm chooses $\bar{x}$ depends on how new issuances affect the asset price. When the firm issues new equity, all asset market participants are aware that new shares are coming online. Equity issuance permanently changes the known supply of the asset $\bar{x}$. Supply $\bar{x}$ enters the asset price in only one place in the equilibrium pricing formula, through A_t. Recall from (9) that

$$A_t = \frac{1}{r}\left[A_{t+1} + \frac{r\mu}{r-G} - \rho Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]\bar{x}\right]. \tag{137}$$

Taking A_{t+1} as given for the moment, $dA_t/d\bar{x} = -\rho Var[p_{t+1} + d_{t+1}|\bar{\mathcal{I}}_t]/r$.[20] In other words, the impact of a one-period change in the asset supply depends on the conditional variance (the uncertainty about) the future asset payoff, $p_{t+1} + d_{t+1}$. Recall from the discussion of the price impact of trades in Section 3.4 that in a dynamic model, more information analysis reduces dividend uncertainty but it can result in more uncertainty about future prices. These two effects largely offset each other.

When we simulate the calibrated model, we find a modest change in the payoff risk from these competing effects on the price impact of issuing new equity. To give the units of the price impact some meaning, the issuance cost is scaled by the average dividend payment so that it can be interpreted as the change in the price-dividend ratio from a one-unit change in equity supply. Thus a one-unit increase in issuance reduces the asset price by an amount equal to 4 months of dividends, on average.

We learn that technological progress in information analysis – of either type – initially makes asset payoffs slightly more uncertain, making it more costly to issue new equity. When we now take into account the fact that the increase in asset supply is permanent, the effect of issuance is amplified, relative to the one-period (fixed A_{t+1}) case. But when analysis becomes sufficiently productive, issuance costs decrease again, as the risk-reducing power of more precise information dominates.

Again, a key limitation of the model is its single asset. With multiple assets, one firm's issuance is a tiny change in the aggregate risk supply. But the change in the supply of firm-specific risk looks similar to this problem. If one were to evaluate this mechanism quantitatively, the magnitude would depend on how much the newly issued equity loads on the idiosyncratic versus the aggregate risk.

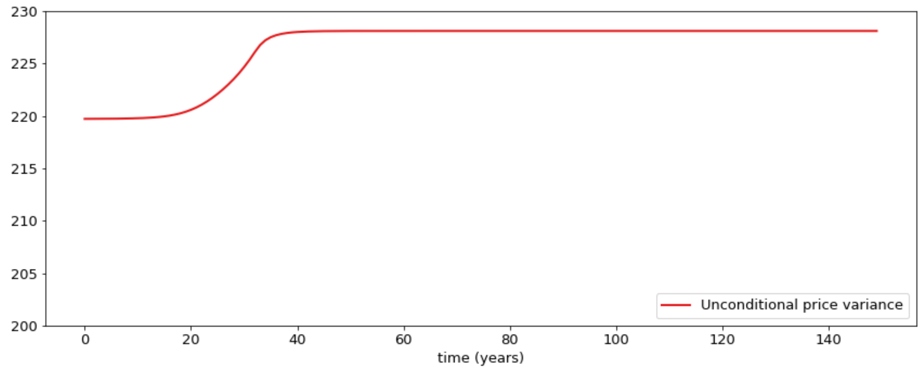
C.7 Price Volatility

One concern with the model is that the future information risk might manifest itself as an implausible rise in price volatility. Price volatility is $Var[p_t]$. Taking the unconditional variance of the model's pricing equation, we get

[20]In principle, a change in issuance $\bar{x}$ could change the payoff variance, $Var[p_{t+1}+d_{t+1}|\bar{\mathcal{I}}_t]$. However, in this setting, the conditional variance does not change because the information choices do not change. Information does not change because the marginal rate of transformation of fundamental and demand information depends on $(C_t/D_t)^2$, which is not dependent on $\bar{x}$. If there were multiple assets, issuance would affect information choices, as in Begenau, Farboodi and Veldkamp (2017).

$B_t^2 Var[d_t] + C_t^2 Var[y_{t+1}] + D_t^2 Var[x_{t+1}] = B_t^2 \tau_0^{-1}/(1-G^2) + C_t^2 \tau_0^{-1} + D_t^2 \tau_x^{-1}$. Figure 4 shows that the price volatility time series exhibits a modest increase. Because price is larger in magnitude than dividends are, a small increase in price volatility can offset a large decrease in dividend uncertainty.

Figure 4: **Price Volatility (model)** Price volatility is $Var[p_t] = \left(B^2/(1-G^2) + C_t^2\right)\tau_0^{-1} + D_t^2 \tau_x^{-1}$.



D Decomposing the Numerical Results

In this section, we explore what part of the results are attributable to the growth in fundamental information, what part to the growth in demand information, and what role future payoff risk plays.

D.1 Turning off Demand Data

For this next set of results, we turn off demand data by setting $\Omega_{xt} = 0$. We keep Ω_{ft}, on the same sequence that it was in the unconstrained model. Obviously, that is not an optimal choice for fundamental data in this setting because it leaves some data capacity unused. But it does allow for a clear comparison of results because it does not conflate the effects of less demand data with more or less fundamental data. In Figure 5, the amount of fundamental data analysis is exactly the same as in Figure 2. The only difference is that the results here have zero demand data analysis. In other words, we substitute $\Omega_{xt} = 0$ into pricing coefficient equations 11 and 12.

To highlight the differences between this no-demand-data version of the model and the original results, we plot each price coefficient as the difference from the level in the original model. Figure 6 reveals that the lack of demand data has only a tiny effect on C_t but a sizeable effect on D_t. Specifically, removing demand data makes the market significantly more illiquid. At high levels of data K_t, the price impact of a trade without demand data is nearly double what it would be with demand data. Note that the jump at the end of the plot is a relic of our calibration procedure. It arises because period 150 is assumed to be the steady state. Since the steady state is the same in both cases, the difference appears as zero. Nothing economically interesting occurs there.

Figure 7 plots how much of the precision in the investors' forecast of $p_{t+1} + d_{t+1}$ comes from their demand data. When K_t is low (left side), there is almost no demand data processing. So, readding the equilibrium amount of demand data adds almost nothing to forecast precision. When K_t gets high (on the right), almost 100% of the forecast precision comes from demand data. The forecasts without demand data have almost no precision.

Alternatively, we turn off demand data by setting $\Omega_{xt} = 0$ and re-optimize over Ω_{ft} given a path of K_t. Here, data capacity is used to its fullest. Trivially, we have $\Omega_{ft} = \sqrt{K_t}$. In the long-run, we see clearly that the shift in demand analysis has no effects on market efficiency, whether it is measured by the steady state value of C, D, or

Figure 5: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Demand Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.

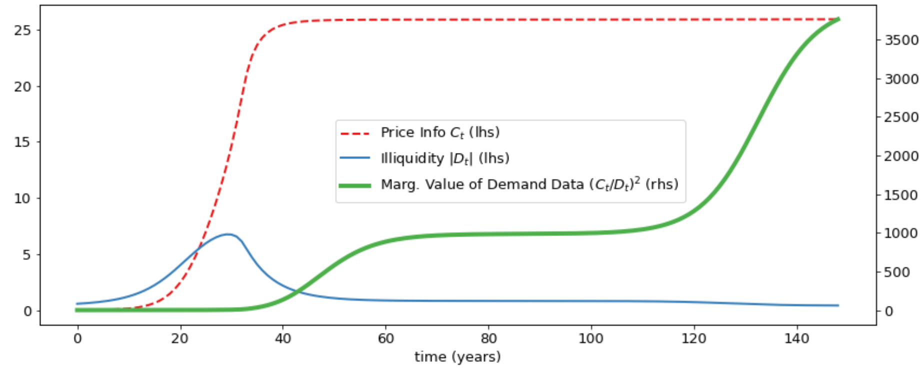
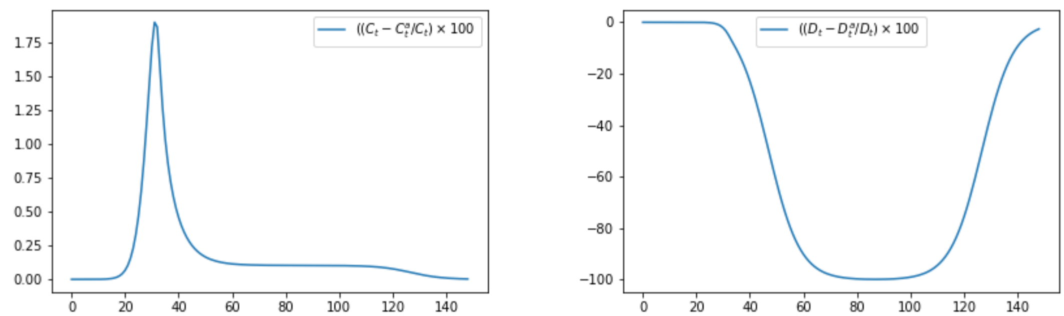


Figure 6: **Change in C_t and $|D_t|$ from Removing Demand Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ back to its optimal level. The x-axis is time.



C/D. This exercise also illustrates the effects of demand analysis on the transition path. Allowing demand data to adjust endogenously smooths out the bumps in the marginal value of demand information ($C/|D|$).

Similar to the no-demand-data version with the same sequence for Ω_{ft} as in the unconstrained model, we plot each price coefficient as a difference from the level in the original model. Figure 9 reveals that the lack of demand data has only a tiny effect on C_t, but it has a sizeable effect on D_t.

In this scenario, we repeat the exercise of computing and plotting how much of the precision in investors' forecasts of $p_{t+1} + d_{t+1}$ comes from their demand data. The result looks indistinguishable from Figure 7. When K_t is low (left side), there is almost no demand data processing. When K_t becomes high (on the right), the forecast precision comes almost entirely from demand data processing.

D.2 Turning off Fundamental Data Growth

Next, we perform the opposite exercise to see what effect fundamental information has on the results. We turn off fundamental data by setting $\Omega_{ft} = 0.01$. Unlike before, when we set the demand data precision exactly to zero, we cannot set the fundamental precision to zero. Doing so would trivially give us $C_t = 0$ no matter what. Instead, to see

Figure 7: **Additional Payoff Forecast Precision from Demand Data.** V^{-1} is $Var[d_t|\bar{\mathcal{I}}_t]^{-1}$ in the main model. V_a^{-1} is $Var[d_t|\bar{\mathcal{I}}_t]^{-1}$ in the model without demand analysis ($\Omega_{xt} = 0$). The vertical axis, $(V^{-1} - V_a^{-1})/V^{-1}$ represents the fraction of forecast precision due to demand analysis. The x-axis is time in years.

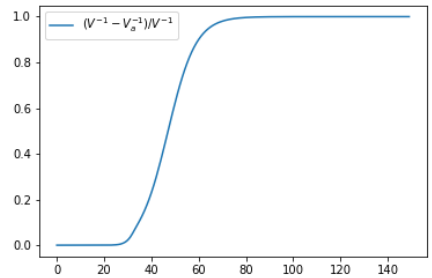
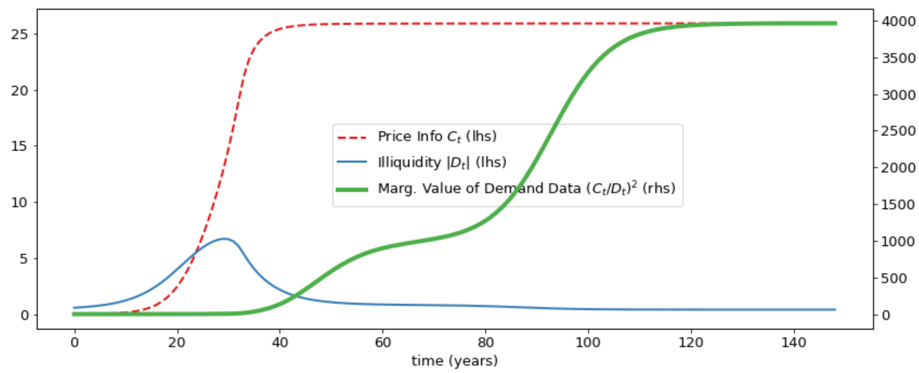


Figure 8: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Demand Data, Ω_{ft} Optimized.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.



the role of demand analysis, we hold the fundamental data precision at a small, exogenous amount. For Ω_{xt}, we keep it on the same sequence as it was in the unconstrained model. In Figure 10, the amount of demand data analysis is exactly the same as in Figure 2. The only difference is that the results here have zero fundamental data analysis. In other words, we substitute $\Omega_{ft} = 0$ into pricing coefficient equations 11 and 12.

To highlight the differences between this no-demand-data version of the model and the original results, we plot each price coefficient as the difference from the level in the original model. Figure 11 reveals that the lack of fundamental data has only a tiny effect on D_t but a sizeable effect on C_t. This is the reverse of the previous exercise. Specifically, removing fundamental data makes the market significantly less sensitive to dividend innovations. At low levels of data (K_t low), the price dividend sensitivities, with and nearly-without fundamental data are very different. At high levels of K_t, since most of the information comes from demand data anyway, the levels with and nearly-without fundamental data are almost the same.

D.3 Turning Off Dynamics

To see the role that long-lived assets play in the results, it is useful to remove all dynamic effects by setting $\pi = 0$ and seeing how the results change. We can only report results from the first few periods because, after that the

Figure 9: **Change in C_t and $|D_t|$ from Removing Demand Data and Optimizing over Ω_{ft}.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ and $\Omega_{ft} = \sqrt{K_t}$ back to its optimal level. The x-axis is time.

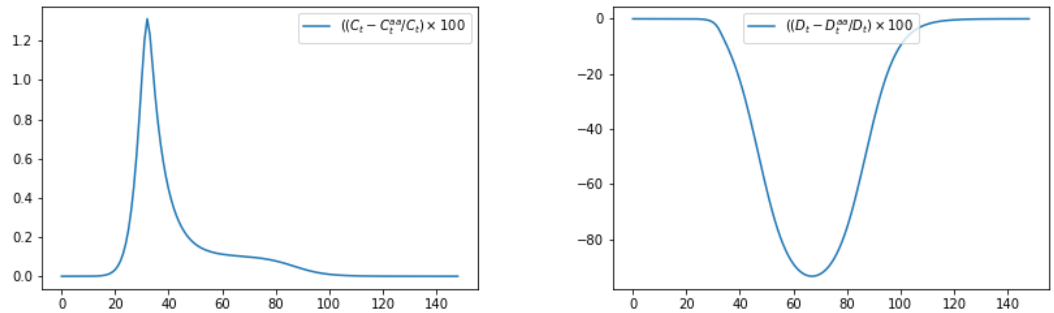
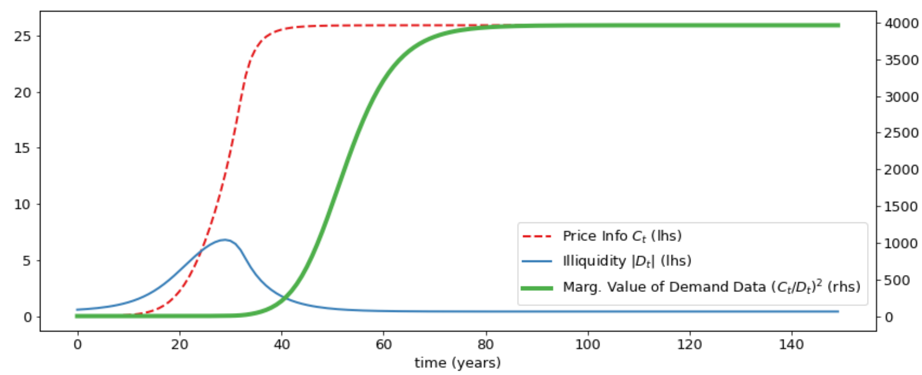


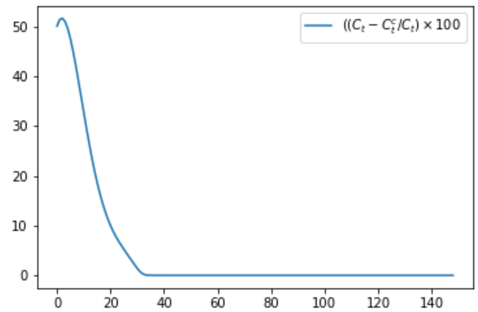
Figure 10: **Price Information Sensitivity (C_t) and the Price Impact of Trades ($|D_t|$) without Fundamental Data.** C_t is the impact of dividend innovations on price. ($|D_t|$) is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of demand information relative to fundamental information. The x-axis is time in years.

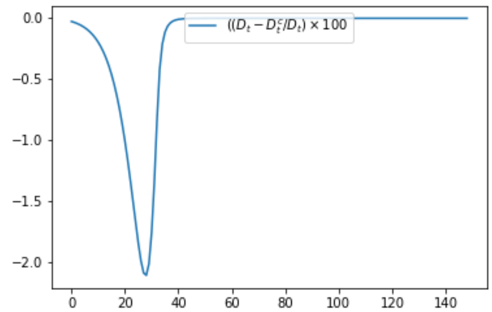


equilibrium no longer exists.

The main difference between the static and dynamic models is that the magnitudes are quite different. The static model features price sensitivity to fundamentals and demand shocks that are between four and ten times less than the same coefficients in the dynamic model. In the dynamic model, a whole stream of payoffs is affected by the dividend information observed today. A small change in a signal affects not only today's dividend estimate, but also tomorrow's and every future date's dividend. That cumulative effect moves the price by more. It also raises the effect of demand shocks because these shocks affect the price, which is used as a signal about future dividends. Because any signal about dividends, including a price signal, has more impact on price, and because demand shocks affect the price signal, demand shocks also have a larger impact on price.

Figure 11: **Change in C_t and $|D_t|$ from Removing Fundamental Data.** C_t is the impact of dividend innovations on price. $(|D_t|)$ is the price impact of a one-unit uninformed trade. These plots report the percentage change in the coefficient that would result from changing $\Omega_{xt} = 0$ back to its optimal level.





E Robustness of the Numerical Results

We want to investigate the effect of changing parameters on the predictions of the numerical model. First, we show how re-calibrating the model with different risk aversion affects the values of other calibrated parameters. Then we show how changes in risk aversion and other parameters have modest effects on the results. We first consider changes to the exogenous parameters: time preference, risk aversion, and the growth rate of the data technology. Then we consider altering the endogenous, calibrated parameters: dividend innovation variance, noise trade variance, and the relative cost of demand information.

Changes to fixed parameters

We consider a lower/higher time preference and risk aversion. Whenever a parameter is changed, all other parameters are re-calibrated to match that new value and the numerical model is simulated again.

Table 2 shows the original calibration alongside a higher and lower-risk aversion calibration to show how the other parameters adjust when risk aversion changes.

Figure 12 shows the model outcomes for various levels of risk aversion. Data demands are almost identical and market outcomes are qualitatively similar, particularly in the first 20 periods, which correspond to observed, past data. The reason that not much changes is that the other parameters adjust to the change in risk aversion. Figures 13 and 17 repeat the analogous robustness exercises for the rate of the time preference and the technological growth rate.

Table 2: Parameters

	lower risk av	original calibration	high risk av
G	0.98	0.98	0.98
μ	0.04	0.04	0.04
τ_0	80.08	79.76	80.08
τ_x	19.75	20.08	19.75
χ_x	21.13	20.98	21.12
r	1.02	1.02	1.02
ρ	0.0425	0.05	0.0575

Figure 12: Results with Different Risk Aversion. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top plots and measures C_t and $|D_t|$ on the bottom plots. The remaining lines are measured on the right side axis. Column 1 corresponds to $\rho = 0.0425$. Column 2 is the baseline calibration used in the paper, $\rho = 0.05$, and column 3 displays the paths with $\rho = 0.0575$.

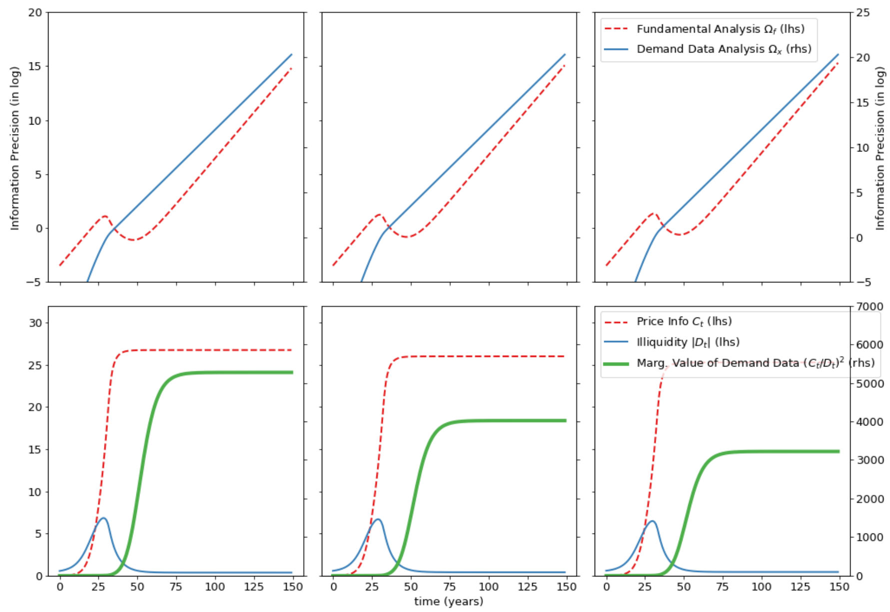
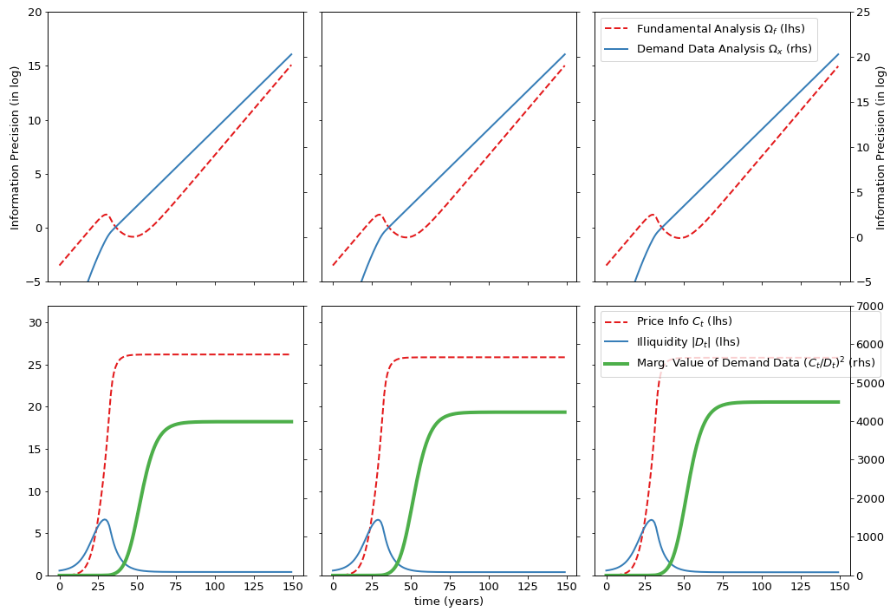


Figure 13: Results with Different Rates of Time Preference. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top plots and C_t and $|D_t|$ on the bottom plots. The remaining lines are measured on the right side axis. Column 1 displays the path with $r = 1.02$, column 2 is the baseline calibration used in the paper corresponding to $r = 1.05$, and column 3 displays the path with $r = 1.08$.



Changes to calibrated parameters

We consider lower/higher dividend shock variance, noise trade variance, the growth rate of the data capacity constraint, and the relative cost of demand information. As these parameters are determined jointly by the calibration, we cannot simply change them and re-calibrate as above. Rather, we calibrate to the baseline then change the parameter of interest for the experiment and then recover the model's terminal values associated with that new parameter of interest. It is important to note that when we make changes here, we do not re-calibrate the other parameters.

Figure 14: Results with Different Values of τ_0. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 2 is the baseline calibration used in the paper. Column 1 displays the path for $\tau_0^* = 0.75 \times \tau_0$ and column 3 for $\tau_0^* = 1.25 \times \tau_0$.

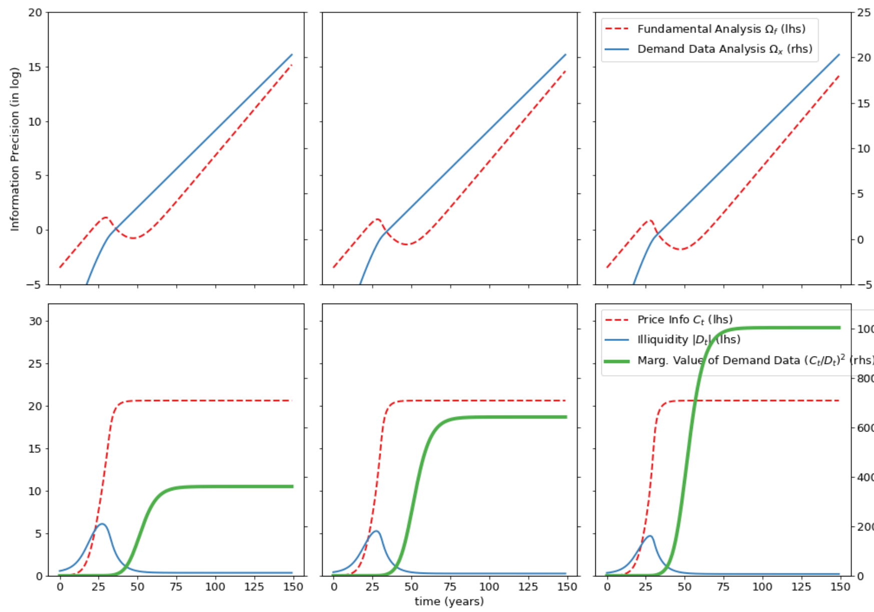


Figure 15: Results with Different Values of τ_x. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 2 is the baseline calibration used in the paper. Column 1 displays the path for $\tau_x^* = 0.75 \times \tau_x$ and column 3 for $\tau_x^* = 1.25 \times \tau_x$.

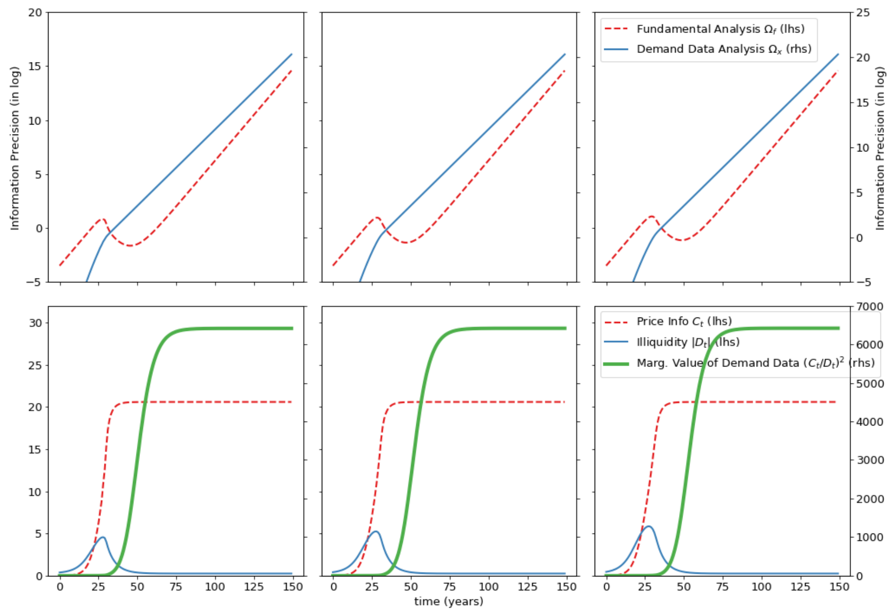


Figure 16: Results with Different Values of χ_x. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 1 is the baseline calibration used in the paper. Columns 2 and 3 display paths for $\chi_x^* = 0.5 \times \chi_x$ and $\chi_x^* = 2 \times \chi_x$, respectively.

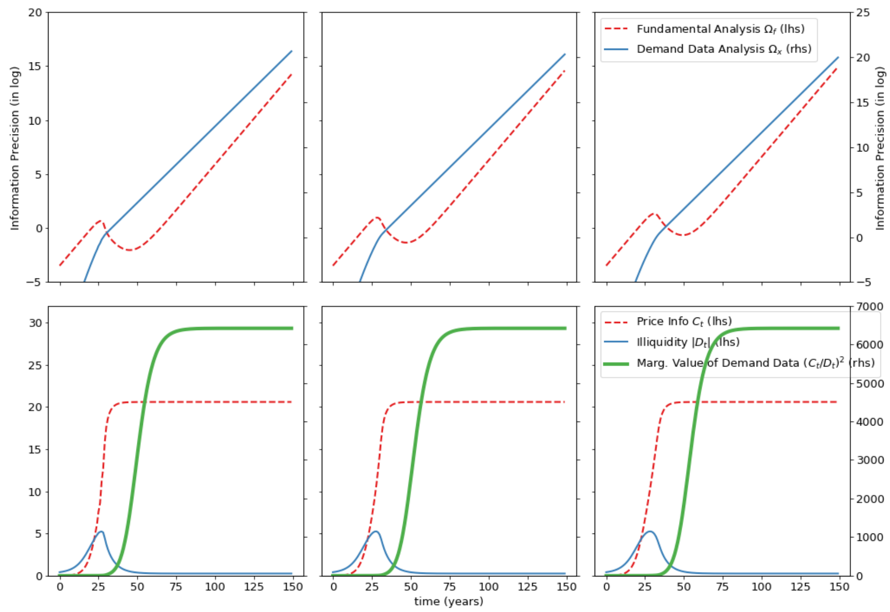


Figure 17: Results with Different Growth Rates of K_t. The first row is information acquisition and the second is the price coefficients. The left axis measures Ω_{ft} on the top 3 plots and C_t and $|D_t|$ on the bottom 3 plots. The remaining lines are measured on the right side axis. Column 1 has $\alpha = 0.47$, column 2 is the baseline specification in the paper with $\alpha = 0.49$, and column 3 has $\alpha = 0.51$.

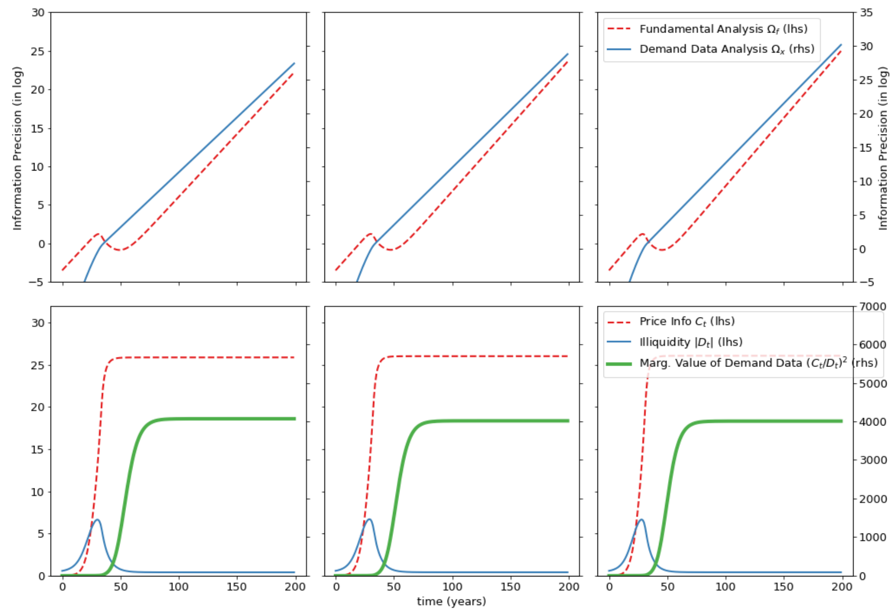
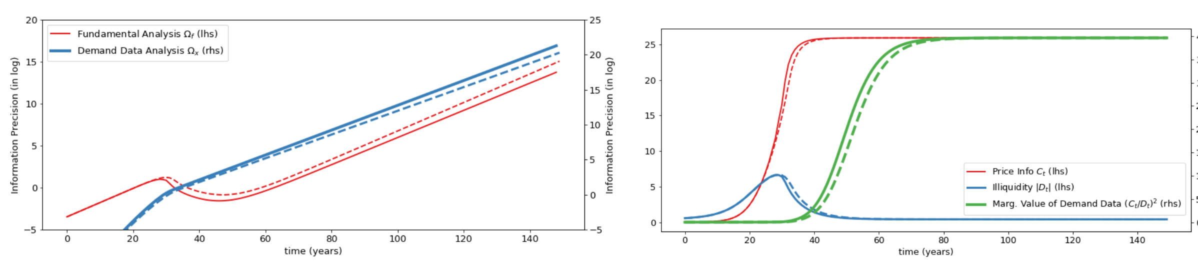


Figure 18: **Unbalanced Technological Progress: χ_x falls.** This figure shows the information choices (left plot) and market efficiency (right plot) with faster productivity growth in demand analysis. The left axis measures Ω_{ft} on left plot and C_t and $|D_t|$ on the right plots. The remaining lines are measured on the right side axis. The path for total information K_t is the same as before. But the marginal cost of demand analysis χ_x follows a path that is log linear: The points $ln(\chi_{xt})$ are evenly spaced between 0 and $ln(\chi_{xt}/10)$. The $x-$axis is time.



(a) Information Acquisition

(b) Price Coefficients

Generated Regressor Problem. In our numerical results, we run time series regression (2) asset by asset and take the regression residual as the innovation term in dividends (y_{t+1}). We then use this innovation term as a regressor in cross sectional regression (8) to get the price coefficients as well as the variance of the residual for each point of time. The variance of the residual corresponds to $D_t^2 \tau_x^{-1}$ in our model, and we use it as one of our moment conditions to calibrate our parameters. Since one of the regressors in the regression of (8) is generated inside the model, this can potentially lead to the generated regressor problem and contaminate the variance of the residual in regression (8).

This problem is unlikely to be quantitatively important in our setting because G_i is estimated quite precisely. Of the 171 firms we consider, 103 of them have G_i estimates that are significantly different from zero, at the 5% level, and 80 of them are significant at the 1% level. With such small standard errors and such large variance of the residuals, the generated regressor is clearly not the main source of the variance of the AR(1) residuals.

F Data Appendix

Hedge Fund Data: Lipper TASS Database The figure showing the shift over time in investment strategies is based on hedge fund data from Lipper. Lipper TASS provides performance data on over 7,500 actively reporting hedge funds and funds of hedge funds and also provides historical performance data on over 11,000 graveyard funds that have liquidated or that stopped reporting. In addition to performance data, data are also available on certain fund characteristics, such as the investment approach, management fees, redemption periods, minimum investment amounts and geographical focus. This database is accessible from Wharton Research Data Services (WRDS).

Though the database provides a comprehensive window into the hedge fund industry, data reporting standards are low. A large portion of the industry (representing about 42% of assets) simply does not report anything (Edelman, Fund and Hsieh, 2013). Reporting funds regularly report only performing assets (Bali, Brown and Caglayan, 2014). While any empirical analysis must be considered with caution, some interesting stylized facts about the current state and evolution of the hedge fund industry do exist in these data.

All hedge fund data is monthly and come from Lipper TASS. In total, the database reports on 17,534 live and defunct funds. Data are from 1994-2015, as no data was kept on defunct funds before 1994. A significant portion of this total consists of the same fund reported in different currencies and thus are not representative of independent fund strategies (Bali, Brown and Caglayan, 2014). Therefore, we limit the sample to only U.S.-based hedge funds and remove funds of funds. This limits the sample size to 10,305 funds. As the focus is to gain insight into the division between fundamental and quantitative strategies in the market, we further limit the sample to the 7093 funds that explicitly possess these characteristics, described below. Throughout the sample, funds are born and die regularly. At any point in time, there are never more than 3000 existing, qualifying funds. By the end of 2015, there were just over 1000 qualifying funds.

Lipper TASS records data on each fund's investment strategy. In total, there are 18 different classifications, most of which have qualities of both fundamental and quantitative analyses. As an example of a strategy that could be considered both, "Macro: Active Trading strategies utilize active trading methods, typically with high frequency position turnover or leverage; these may employ components of both Discretionary and Systematic Macro strategies." However, 4 strategy classifications explicitly denote a fund's strategy as being either fundamental or quantitative. They are:

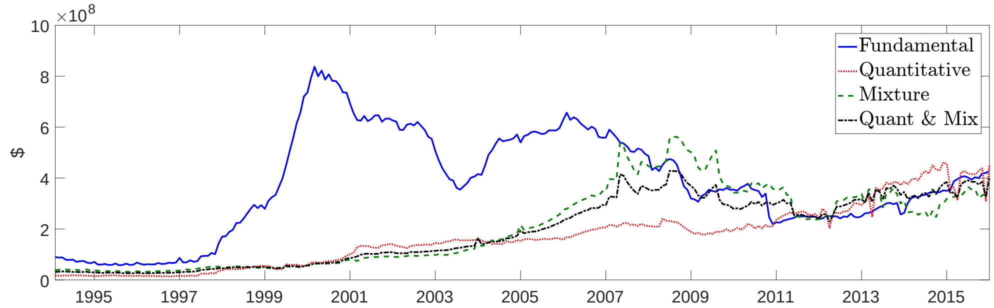
- Fundamental: This denotes that the fund's strategy is explicitly based on fundamental analysis.
- Discretionary: This denotes that the fund's strategy is based upon the discretion of the fund's manager(s).
- Technical: This denotes that the fund deploys a technical strategy.
- Systematic Quant: This denotes that funds deploy a technical/algorithmic strategy.

Using these classifications, it is possible to divide hedge fund strategy into three broad groups:

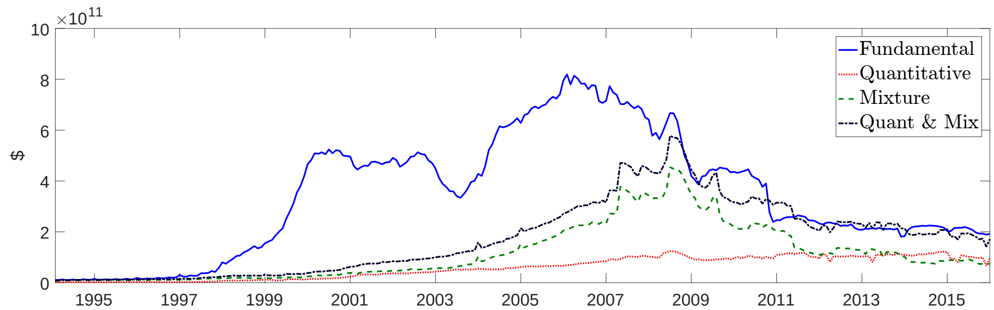
- Fundamental: Those funds with a strategy that is classified as fundamental and/or discretionary, and *not* technical and/or sytematic quant.
- Quantitative: Those funds with a strategy that is classified as technical and/or systematic quant, and *not* fundamental and/or discretionary.
- Mixture: Those funds with a strategy that is classified as having at least one of fundamental or discretionary and at least one of technical or systematic quant.

From 2000-2015, the assets under management (AUM) systematically shifted away from fundamental funds and towards those that deploy some sort of quantitative analysis in their investment approach. In mid-2000, the assets under management per fundamental fund was roughly 8 times the size of those in a quantitative or mixture fund, but by 2011 this had equalized, representing a true shift away from fundamental and towards quantitative analysis in the hedge fund industry.

Figure 19: **Hedge Funds are Shifting away from Fundamental Analysis.**
Source: Lipper TASS. The data are monthly from 1994-2015. The database reports on 17,534 live and defunct funds.



(a) Assets Under Management per Fund



(b) Total Assets Under management

Evidence for Growth of Algorithmic Trade Some suggestive evidence supports the notion that funds are shifting away from fundamental analysis. Figure 19 plots the share of funds that report their own style to fundamental. This share is falling, both as a share of funds and as a share of assets managed. Figure 20 presents very different sort of evidence that points in the same direction. It shows that the fraction of google searches for the term "fundamental analysis" has been falling, while searches for the term "order flow" have been rising.

G How To Test This Model

One of the benefits of a framework like this is that it can generate testable predictions to isolate technology's effects. This section lays out a new measurement strategy for using the model to infer information choices. The end of the section describes how to use these new information measures to test this model, or other related theories.

G.1 Testable Predictions

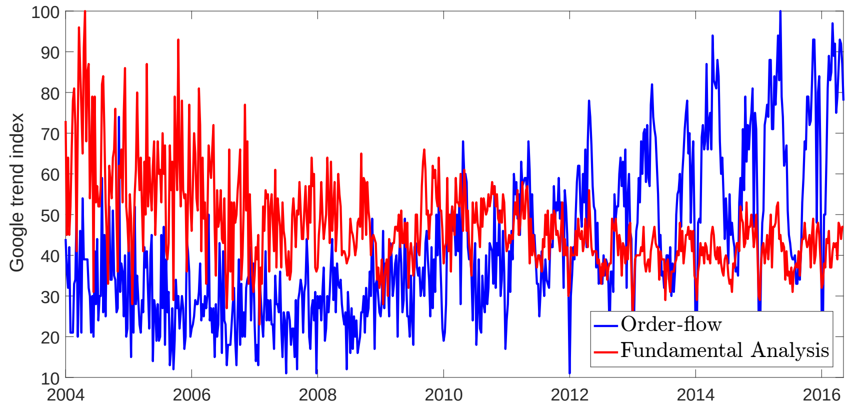
Two predictions are central to the main point of this paper. We first lay out these predictions and then describe how one might infer information choices in order to test them.

Prediction 1 ***Demand Data Grew, Relative to Fundamental Data***

The implied measure of Ω_{xt} has grown at a faster rate than the measure of Ω_{ft}.

The model calibration points to the current regime as one in which demand data is rising relative to fundamental data (Figure 1). With the implied data measures, this would be simple to test by constructing growth rates and

Figure 20: **Google Trends:** Fraction of Google searches involving "order flow" or "fundamental analysis." Source: Google trends. The data are the weekly fraction of searches involving these search terms. The series is normalized to make the highest data point equal to 100.



testing for differences in means. One could also examine whether demand data growth is speeding up, suggesting complementarity.

Prediction 2 ***Price Informativeness Predicts Demand Data Usage***

When prices are highly informative (large $C_t/|D_t|$), investors use more demand data (high Ω_{xt}).

The key insight of the information choice part of the model is that the marginal rate of substitution of demand for fundamental data is proportional to $(C_t/D_t)^2$. One could test whether, controlling for other factors, highly informative prices coincide with, or predict, demand data increases.

G.2 Extending the Model to Facilitate Empirical Testing

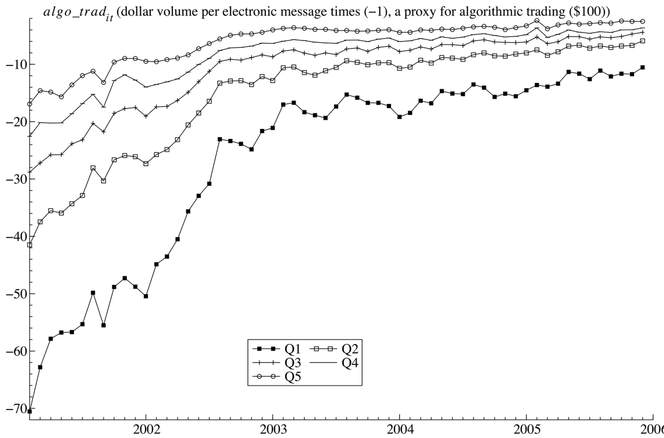
The key barrier to testing the predictions above is that one cannot observe investors' data choices. However, data choices do show up in portfolio choice. Data is valuable because it allows investors to trade in a way that is correlated with what they observe. They can buy when dividends are likely to be high or sell when the price appears high for non-fundamental reasons. These strategies are not feasible – not measurable in theory parlance – without observing the relevant data. If many investors systematically buy when payouts are going to be high, this would be conclusive evidence of information. But not all investors can buy at one time, as doing so violates the market clearing condition. In order to test this hypothesis, we need to consider a simple extension of the model to incorporate informed and uninformed traders.

We extend the model to include a measure λ of investors, who are endowed with capacity K to acquire information, and the complementary measure of investors who do not acquire information but who submit demand optimally based on their priors. Priors are common to all investors.

This extension facilitates testing because it allows informed investors' portfolios to react more to shocks about which they have data, and still have the market clear. That is crucial because our measures of fundamental information and demand information are based on the covariance of an informed investor's portfolio q_t with shocks $\tilde{x}_t$ and $\tilde{y}_t$. This extension resolves the tension because uninformed investors can hold less of an asset that informed investors demand more of.

The solution to this model, derived in Online Appendix C.2, is a simple variant of the original solution. We denote all variables corresponding to uninformed investors with a prime ($'$). The uninformed agents' portfolio takes

Figure 21: **Algorithmic Trading Growth** 2001-2006. Source: Hendershott, Jones and Menkveld (2011). Their proxy for algorithmic trading is the dollar volume of trade per electronic message. The rise is more pronounced for the largest market cap (Q1) stocks. Q1-Q5 are the 5 quintiles of NYSE stocks, ordered by size (market capitalization).



the same form as (7), except that the mean and variance are conditional on all information revealed in the last period and today's price.

The equilibrium price coefficients, adjusted for heterogeneous agents, are given by (127) and (128). To construct the portfolio covariances with shocks, we take the portfolio first-order condition (7) and then substitute in the definition of signals, equilibrium price (8), and conditional expectations and variances (32) and (33). Expressing q as a function of the shocks x_{t+1}, y_{t+1} and signal noise ($\tilde{\epsilon}_{fit}$, $\tilde{\epsilon}_{xit}$) allows us to compute

$$\begin{aligned}\mathrm{Cov}(q_{it}, x_{t+1}) = & \frac{Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}]}{\lambda Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{it}] + (1-\lambda)Var[\pi p_{t+1} + d_{t+1}|\mathcal{I}_{i,t-1}, p_t]}\tau_x^{-1} \\ & - \frac{r\Omega_{it}}{\rho(r-\pi G)}\frac{C_t}{D_t}(1-\lambda_{It})(Var[y_{t+1}|\mathcal{I}_{i,t-1}, p_t] - Var[y_{t+1}|\mathcal{I}_{it}]), \quad (138)\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(q_{it}, y_{t+1}) = & \frac{r}{\rho(r-\pi G)}(\pi C_{t+1}^2\tau_0^{-1} + \pi D_{t+1}^2\tau_x^{-1} \\ & + (\frac{r}{r-\pi G})^2 Var[y_{t+1}|\mathcal{I}_{it}])^{-1}(1-\lambda_{It})(Var[y_{t+1}|\mathcal{I}_{i,t-1}, p_t] - Var[y_{t+1}|\mathcal{I}_{it}]). \quad (139)\end{aligned}$$

These covariances depend on aggregate terms like C_t, D_t, as well as Ω_{it} and conditional variances, which depend on individual i's data precision, Ω_{fit} and Ω_{xit}.

G.3 Measuring Information

To test the model, one needs to measure the pricing coefficients in (127) and (128), as well as the covariances in (138) and (139), and combine them, in order to back out Ω_{fit} and Ω_{xit}. Then, determine whether data processing of each type is increasing or not.

To construct these measures, we first need to estimate the variance and persistence of dividends (τ_0^{-1} and G), riskless rate (r), variance of demand shocks (τ_x^{-1}), and a sequence of pricing equation coefficients. Section 3.1 details how we estimated these objects from publicly available financial data. Given these estimates and a decision about whether to use a static ($\pi = 0$) or dynamic ($\pi = 1$) framework, we can construct Z_t from (25), $Var[y_{t+1}|\bar{\mathcal{I}}_t]$ from (13), and $Var[y_{t+1}|\mathbb{I}_t^-, p_t] = (\tau_0 + \tau_x(C_t/D_t)^2)^{-1}$.

To compute the portfolio covariance with shocks requires a time series of the portfolio holdings of some informed investors. Mutual fund or hedge fund portfolio holdings might make a good informed data set.Then for each fund, we compute the covariance over the window of a year, or over the first and second halves of the sample. Backing out Ω_{ft} and Ω_{xt} then requires solving two equations, (138) and (139), for the two unknowns, Ω_{ft} and Ω_{xt}.

With multiple assets, a simple principal components analysis would allow a researcher to construct linear combinations of assets that are independent. For each independent asset or risk factor, one could follow the above procedure, to recover Ω_{ft} and Ω_{xt} data for that asset or risk (as in Kacperczyk, Van Nieuwerburgh and Veldkamp (2016)). One could use these measures and the model structure to answer many questions. For example, one could infer a series for χ_x, the relative shadow cost of processing demand versus fundamental data. That would inform the debate about the role of technological change in high frequency trading.

Cross-fund implied information might be interesting in relation to questions pertaining to the distribution of skill or financial income inequality. But for questions about the long-run trend, averaging the implied precisions (Ω_{ft}, Ω_{xt}) of various investors is consistent with the model: with heterogeneous information quality, the aggregates in the model are the same as they are for a representative agent who has information precision that is the average of all investors' precisions.

Of course, these measures depend on a model that is never entirely correct. However, the parts of the model used to derive these measures are the most standard parts. Specifically, (138) and (139) depend on the form of the first-order condition, which has a very standard form in portfolio problems. They also depend on the way in which the model assumes that agents form expectations and conditional variances, using Bayes law and extracting information from linear prices. But these measures do not depend on the information choice portion of the model. They do not assume that agents optimally allocate data. These measures infer what data must be present in order for agents to be making the portfolio choices that they make and for prices to reflect the information they contain. As such, they offer meaningful ways of testing this model, as well as others.