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MITIGATING DISASTER RISKS IN THE AGE OF CLIMATE CHANGE

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ABSTRACT

Emissions control cannot address the consequences of global warming for weather disasters until decades later. We model regional-level mitigation or adaptation, which reduces disaster risks to capital in the interim. Mitigation depends on belief regarding the adverse consequences of global warming. Pessimism jumps with a disaster and slowly reverts in the absence of arrivals. Mitigation spending by firms is less than first-best because of externalities. We prove that capital taxes to fund public mitigation, which requires collective action, restores first-best. We apply our model to country-level mitigation of major tropical cyclones, using GDP growth damages, government flood-control budgets, and climate-model projections of increasing cyclone frequency. For a typical country exposed to cyclones, a disaster arrival not only damages its capital stock, but elevates perceived risks, and as a result has persistent effects on taxes, the mix of private and public mitigation, growth, and welfare.

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1   Introduction

Global costs of weather-related disasters have increased sharply in recent decades (see, e.g., Bouwer et al., 2007). While this trend increase is partly due to economic growth and exposure of physical capital (Pielke et al., 2008; Bouwer, 2011), recent climate research is increasingly confident in linking climate change to more frequent or severe natural disasters (National Academy of Sciences, 2016). For instance, climate models point to increased frequency and damage from hurricanes that make landfall (Kossin et al., 2020). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams, 2016). Emissions control and carbon taxes, which have been the main focus of research using integrated assessment models (Nordhaus, 2017; Golosov, Hassler, Krusell, and Tsyvinski, 2014), will only impact such losses decades down the road to the extent they are even implemented globally.

At the same time, willingness to pay to avoid weather disasters are likely to be large given household risk preferences and permanence of such shocks. Hence, mitigation of natural disaster risks at the regional level, be it adaptations to flooding from tropical cyclones or damage from wildfires, may need to play a major role going forward. In contrast to global mitigation of disaster risks via decarbonization, such regional-level strategies have thus far been relatively under-emphasized both in climate change research and practice (Bouwer et al., 2007). Among key questions are what determines regional-level mitigation, how valuable is it for social welfare, and what are the tax, growth and asset pricing implications?

To answer these questions, we start by introducing costly mitigation into a continuous-time stochastic general-equilibrium model with disasters along the lines emphasized by Rietz (1988) and Barro (2006). Output is determined by an AK growth function augmented with capital adjustment costs and disaster shocks following a Poisson process as in Pindyck and Wang (2013). Convex adjustment costs to capital (e.g., Hayashi, 1982) make capital stock illiquid and hence give rise to rents for installed capital and the value of capital (Tobin's average q). Households are endowed with one of the two widely-used non-expected risk preferences: recursive utility proposed by Epstein and Zin (1989), which separates risk aversion from the elasticity of intertemporal substitution,[1] or Campbell and Cochrane (1999), which uses external

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[1]Recent work in the context of valuing emissions curtailment points to the importance of using such risk preferences in generating a high social cost of carbon (see, e.g., Jensen and Traeger (2014), Bansal, Ochoa, and Kiku (2017), Cai and Lontzek (2019), Daniel, Litterman, and Wagner (2019), and Barnett, Brock, and Hansen (2020)).

habit preferences to generate state-dependent risk aversion.

Absent mitigation spending today, the percentage losses of capital stock due to jump arrivals follow a Pareto distribution and are i.i.d. across arrivals (Gabaix, 2009).[2] There are two mitigation technologies that ameliorate damages conditional on an arrival. The first type reduces the *exposure* of a firm's capital to a draw from the conditional damage distribution. The second type changes the conditional damage *distribution* by reducing the fat-tailedness of damages in the sense of first-order stochastic dominance. How much the disaster distribution changes (e.g., how much of the fat-tailedness of damages is reduced) depends on the *aggregate* mitigation spending in the region and benefits all firms in the region. Our mitigation technologies are motivated by best practices for dealing with disasters such as tropical cyclones, where mitigation is a mix of private efforts such as temporary barriers like sandbags to protect buildings (corresponding to the first type) and public or government efforts such as an early warning system and infrastructure maintenance and preparedness (corresponding to the second type).[3]

Since mitigation strategies naturally depend on perceived risks, a defining aspect of costly mitigation in the age of climate change is that it depends on households learning about the consequences of global warming for disasters based on past arrivals. Each new disaster brings additional evidence that will result in belief updating regarding the consequences. For instance, scientific consensus on the impact of global warming on the frequency of hurricanes changed markedly in 2005, when a record number of hurricanes including Katrina made landfall (Emanuel, 2005). Recent weather disasters have moved public opinion on the consequences of climate change (see, e.g., the Yale Climate Opinion Maps website at https://climatecommunication.yale.edu/visualizations-data/ycom-us/.)

Our model thus also features households learning from natural disaster arrivals about whether the disaster arrival rate is high or low (i.e., what we refer to as a bad (B) versus good (G) state). The B state corresponds to more frequent arrivals due to global warming, while the G state corresponds to no or mild effects of climate change. An important feature of our learning model is that "bad" news (an unexpected arrival) leads to a discontinuous jump (worsening) of belief, as a disaster arrival is a discrete event also serving as a discrete

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[2]For instance, the literature on weather disasters points to persistent declines in growth and productivity due to destruction of physical capital (Dell, Jones, and Olken, 2014). Of course, weather disasters are related to extreme temperature and precipitation.

[3]These mitigation technologies are in line with existing work on the value of protective investments (Kousky, Luttmer, and Zeckhauser, 2006; Anbarci, Escaleras, and Register, 2005; Smith et al., 2006).

signal.[4] Absent any arrivals, belief drifts gradually towards the G state, as no news is good news when it comes to no arrival of disasters. Such a model is consistent with uncertainty regarding disaster arrival rates that will be resolved over time and the importance of modeling uncertainty of climate models more generally (Barnett, Brock, and Hansen, 2020).

The planner's first-best solution is characterized by an endogenously derived non-linear ordinary differential equation in the case of Epstein and Zin (1989) preferences for a welfare measure (proportional to the certainty equivalent wealth) together with first-order conditions for investment and the two types of mitigation spending that depend on household belief regarding disaster arrivals. The boundary conditions are given by solutions when the household belief is permanently in the G or B state.[5] In our model with Campbell and Cochrane (1999) external habit preferences, the solution is characterized by a non-linear partial differential equation (PDE) system with household belief and the surplus consumption ratio as state variables.

The planner's first-best solution features an optimal mix of both types of mitigation spending. However, firms only spend on exposure mitigation and nothing on distribution mitigation in laissez faire market economies. The reason is externalities: whereas firms benefit directly from their spending on exposure mitigation (e.g., placing sandbags around their plants to control flooding from tropical cyclones), the benefits of distribution mitigation depend on aggregate spending and are shared by all. Since firms do not internalize the benefits of aggregate risk mitigation, they underspend on total mitigation in market economies. We prove that an optimal tax on capital to fund government spending on distribution mitigation restores the first-best solution while still maintaining a balanced budget.

Our model can be applied to different weather disasters. We use it to value mitigation that reduces the damages of tropical cyclones globally. Tropical cyclones include hurricanes, typhoons, cyclones, and tropical storms.[6] Hsiang and Jina (2014) estimate that nearly 35% of the global population is seriously affected, making them one of the most broadly relevant forms of disasters in addition to being one of the most costly. Building on their work, we

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[4]Our model generates time-varying disaster arrival rates via learning (also see e.g., Wachter and Zhu, 2019). Learning about a model parameter (e.g., in Colin-Dufresne, Johannes, and Lochstoer, 2016) and disaster models with time-varying arrival rates (as in Gabaix, 2012; Gourio, 2012; and Wachter, 2013) have been shown to be quantitively important to simultaneously explain business cycles and asset price fluctuations.

[5]The solutions for the two special cases (at the boundaries) generalize the solution for the model in Pindyck and Wang (2013), which originally examined the general-equilibrium effects of disasters in a continuous-time production model with Poisson arrivals of disasters, by allowing for mitigation.

[6]They are referred to as tropical storms or hurricanes in Atlantic, typhoons in the Pacific, and cyclones in Indian Ocean.

gather historical data on major cyclone strikes. For a sample of 109 countries over the period of 1960-2010, we estimate that a typical country in our sample is exposed to one major cyclone strike every 7.4 years (implying a cyclone arrival rate of 0.135 per annum) and a reduction of GDP growth rate by around 1% per disaster.

To discipline our rational learning model, we use data from the most recent survey of leading climate model projections (Knutson et al., 2020) on the increased frequency of major tropical cyclones. These projections are for a world where global temperatures are 2° Celsius higher than pre-industrial levels, which is expected within this century absent major decarbonization. The median projection of the arrival rate is a 13% increase from the pre-industrial level and the most pessimistic projection is a 125% increase. Using these moments, we calibrate the annual frequency of major tropical cyclones caused by global warming to be 0.304 in the bad state, which is more than double that of the arrival rate pre-climate change. The prior that the economy is in the bad state is set at 0.104 so as to match the median projection of a 13% increase of the disaster arrival rate from the pre-climate change level.

We then gather data on government flood control budgets for a subset of these countries that are highly exposed to tropical cyclones. The median budget is around 0.1% of a country's capital stock. Studies indicate that private mitigation efforts also plays a supporting role along side government efforts (Genovese and Thaler, 2020). We use these moments to discipline our calibration of the optimal spending to mitigate the disaster risk.

For our baseline analysis, we focus on Epstein and Zin (1989) household preferences, choosing parameters along the lines of the long-run risk literature following Bansal and Yaron (2004). For a typical country exposed to major tropical cyclones, we find that a disaster arrival has large quantitative and persistent effects on beliefs. An arrival immediately leads to a jump in belief from a prior of 0.104 to 0.207 (i.e., a doubling of belief that the economy is in the B state). Since belief drifts smoothly toward the optimistic scenario absent jumps and remains persistently high for a while even after a disaster arrival, there is also an increase of mitigation spending (i.e., the annual tax rate) of 10%. A comparison of the first-best solution and the market economy solution shows that disaster distribution mitigation plays a more significant role when the economy becomes riskier with more frequent arrivals.

Our model provides a learning-based explanation for why disaster arrivals are followed by persistently low economic growth for a number of years (Hsiang and Jina, 2014). Such an effect is absent from disaster models with no learning such as in Pindyck and Wang (2013),

where there are only direct effects of capital destruction but no indirect effects to economic growth. The tax and mitigation implications along with the asset pricing implications are also new to the literature and can be tested using event studies. For instance, damages conditional on arrival are higher when an economy has few prior arrivals and long inter-arrival times since perceived risks and mitigation spending or preparedness are low as a result.[7]

We also use our model to calculate two sets of willingness-to-pay (WTP) for flood risk mitigation: (1) the difference between welfare in the first-best economy and the economy without any mitigation and (2) the difference between welfare in the market economy with private mitigation and the economy without any mitigation. The first-best economy WTP is high, around 17% at the prior belief that the economy is in the bad state is 0.104, and then rises to nearly 19% with an arrival of a major cyclone. The market-economy WTP is close to the first-best economy WTP when perceived risks are low. The wedge between the first-best economy WTP and the market-economy WTP gives the welfare gains from having an optimal tax. But this wedge increases with perceived risks.[8]

We show that these conclusions regarding mitigation are robust to two important changes to our model. The first is a generalized belief updating process that allows the underlying state to switch between the good and bad states. The second is having different risk preferences. However, the implications for the behavior of consumption and investment after a disaster arrival can differ depending on risk preferences.

2   Model

In this section, we develop a model in which there is an externality when it comes to the mitigation of disaster risks in a market economy. Time is continuous and the horizon is infinite. There is a continuum of identical firms and households, both with a unit measure.

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[7]See Hong, Karolyi, and Scheinkman (2020) for a review of recent findings on weather disasters and climate risks including the impact of sea-level rise on coastal property prices. Beliefs of the risks are shown to play a role (Bakkensen and Barrage, 2017).

[8]The caveat to these calculations is that traditional willingness-to-pay calculations to avoid disasters as in our model is sensitive to modeling of multiple disasters and when disasters affect both consumption and loss of life (Martin and Pindyck, 2015).

2.1 Firm's and Households' Optimization Problems

Firm production. A firm produces output, Y_t, using its capital stock, K_t, the sole factor of production. Specifically, Y_t is proportional to its contemporaneous capital stock K_t:

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity. This is a version of the AK model but importantly augmented with capital adjustment costs as we show later.

Firm investment, capital accumulation, and arrival of jumps (disasters). Let I_t denote firm investment. The firm's capital stock K_t evolves as:[9]

$$dK_t = I_{t-}dt + \sigma K_{t-}d\mathcal{W}_t - N_{t-}K_{t-}(1-Z)d\mathcal{J}_t \,. \tag{2}$$

The second term captures continuous diffusive shocks to capital, where $\mathcal{W}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility. This term is the standard source of shocks for AK models in macroeconomics and sometimes is interpreted as stochastic depreciation shocks. The last term in (2) captures the loss to the firm's capital from a stochastic arrival of a disaster.

The process $\mathcal{J}_t$ in (2) is a Poisson process where each jump arrives at a constant but unobservable rate, which we denote by λ. We will return to discuss the details for the arrival rate λ. There is no limit to the number of these jump shocks.[10] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t. The N_{t-} process is chosen by the firm to mitigate its exposures to disasters, which we introduce later.

Distribution of damage $(1-Z)$**.** Let Z denote the stochastic recovery fraction of the capital stock conditional on a jump arrival absent firm mitigation spending, which corresponds to $N_{t-} = 1$. Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for Z, respectively. While the firm takes the distribution of Z as given, the society as a whole can spend resources to influence the distribution of Z by

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[9]This capital accumulation technology has been widely used in macro and finance. For example, see Barro (2006) and Pindyck and Wang (2013).

[10]Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

making disasters less damaging to the economy. We introduce the determinants of $\Xi(Z)$ at the aggregate level in Section 2.4.

Without mitigating disaster exposure (which implies $N_{t-} = 1$), upon a disaster arrival at t ($d\mathcal{J}_t = 1$), a stochastic fraction $(1 - Z) \in (0,1)$ of the firm's capital stock K_{t-} is permanently destroyed at t and hence the surviving capital stock is $K_t = ZK_{t-}$. (For example, if the firm incurred no disaster exposure mitigation spending at $t-$ and a shock arrived at t destroying 15 percent of capital stock, we would have $Z = 0.85$.) Naturally, anticipating damages caused by these disasters, the firm has incentives to *ex-ante* mitigate its exposures to disaster shocks by spending resources (e.g., sandbags to keep a building from flooding during a tropical cyclone.)

Firm's disaster exposure mitigation. Let X_{t-}^e denote the firm's disaster exposure mitigation spending, where the superscript e refers to "exposure" at $t-$. With this spending at $t-$, should a disaster arrive at t, the firm decreases its capital loss from $(1 - Z)K_{t-}$ to $N_{t-}(1 - Z)K_{t-}$, where $N_{t-} \in [0,1]$ depends on X_{t-}^e. This effect of mitigation spending on capital stock dynamics is captured by the N_{t-} term in (2). Let $x_{t-}^e = X_{t-}^e/K_{t-}$ denote the firm's scaled disaster *exposure* mitigation spending.

To preserve our model's homogeneity property, we assume that N_{t-} is a function of x_{t-}^e:

$$N_{t-} = N(x_{t-}^e). \tag{3}$$

Equations (2) and (3) imply that if we double X_{t-}^e and capital stock K_{t-} simultaneously, the benefit from reducing disaster damages (in units of goods) also doubles. To see why, observe that $N_{t-} = N(x_{t-}^e)$ is unchanged with the simultaneous doubling of X_{t-}^e and K_{t-} but the amount of loss reduced by mitigation, is doubled since K_{t-} has doubled.

We require $N'(x^e) \leq 0$ as mitigation spending reduces damages. Additionally, the marginal effect of spending on reducing damages is decreasing in x^e, which implies $N''(x^e) \geq 0$. Finally, by definition, $N(0) = 1$, as no mitigation spending ($x^e = 0$) no damage reduction.

Capital adjustment costs and firm's objective. Following the q theory of investment (e.g., Hayashi, 1982 and Abel and Eberly, 1994), we assume that when investing $I_t dt$, the firm also incurs capital adjustment costs, which we denote by $\Phi_t dt$. That is, the total cost of investment per unit of time is $(I_t + \Phi_t)$ including both capital purchase and adjustment costs. Let CF_t denote the firm's cash flow/dividend payout. Then, a firm's payout is given by:

$$CF_t = Y_t - (I_t + \Phi_t) - X_t^e. \tag{4}$$

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Let $cf_t = CF_t/K_t$ denote the scaled cash flow and $i_t = I_t/K_t$ denote the investment-capital ratio. Next, we specify the capital adjustment cost function. Following Hayashi (1982), we assume that $\Phi(I,K)$ is homogeneous with degree one in I and K by writing:

$$\Phi(I,K) = \phi(i)K \ , \tag{5}$$

where $\phi(i)$ is increasing and convex.[11]

The representative firm chooses investment I and the disaster exposure mitigation spending X^e to maximize its risk-adjusted present value of future cash flows by solving:[12]

$$\max_{I,X^e} \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_t}{\mathbb{M}_0}\left(Y_t - (I_t + \Phi_t) - X_t^e\right)dt\right] \ , \tag{6}$$

where $\mathbb{M}$ is the equilibrium stochastic discount factor (SDF) that the firm takes as given. Let Q_0 denote the firm's value at $t=0$, the solution for (6). Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's average q, the ratio between the firm's value (Q_0) and the replacement cost of capital (K_0), exceeds one.[13]

Households' preferences. Next, we turn to the household's side. We consider two widely used preferences in the macro finance literature: Epstein-Zin (1989) recursive utility and Campbell-Cochrane (1999) external habit preferences. We first work with the Duffie and Epstein (1992) continuous-time version of Epstein and Zin (1989) and Weil (1990). Then we re-solve our model with external habit formation in Section 8. The life-time utility of our representative consumer's Duffie-Epstein-Zin homothetic recursive preferences is given by:

$$V_0 = \mathbb{E}\left[\int_0^\infty f(C_t, V_t)dt\right] \ , \tag{7}$$

where $f(C,V)$ is known as the normalized aggregator given by

$$f(C,V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^\omega}{((1-\gamma)V)^{\omega-1}} \tag{8}$$

and $\omega = (1-\psi^{-1})/(1-\gamma)$. Here ρ is the rate of time preference, ψ is the elasticity of intertemporal substitution (EIS), γ is the coefficient of relative risk aversion. Unlike expected

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[11]Homogeneous adjustment cost functions are analytically tractable and have been widely used in the q theory of investment literature. Hayashi (1982) showed that with homogeneous adjustment costs and perfect capital markets, marginal and average q are equal.

[12]Financial markets are perfectly competitive and complete. While the firm can hold financial positions (e.g., DIS contracts in net zero supply), these financial hedging transactions generate zero NPV for the firm. Therefore, financial hedging policies are indeterminate, a version of the Modigliani-Miller financing irrelevant result. The firm can thus ignore financial contracts without loss of generality.

[13]In Barro (2006), he also analyzes an endogenous AK growth model with disaster risks but without capital adjustment costs in a discrete-time setting. Therefore, Tobin's average q in his model is always one.

utility, recursive preferences as defined by (7) and (8) disentangle risk aversion from the EIS.[14] An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi^{-1}}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also on V. This more flexible recursive utility is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature. We show that in our model, the EIS parameter plays an important role as well.

2.2   Bayesian Belief Updating about the Disaster Arrival Frequency

Next, we turn to the disaster arrival process. Since the arrival rate while constant is unobservable to the agent, an arrival of a disaster not only directly destroys capital stock, but also serves as a signal from which households and firms update their beliefs about λ.

While the true disaster arrival rate λ is constant, households and firms do not have complete information about this true value of λ. What the households and firms know at time 0 is that the true value of λ is either λ_G or λ_B with $\lambda_B > \lambda_G$. If the true λ is λ_B rather than λ_G, capital stock is more likely to be hit by a disaster (i.e., a negative jump). We refer to the low-arrival-rate and high-arrival-rate scenarios as the good (G) state and the bad (B) state, respectively. Additionally, all agents are endowed with the same prior belief π_{0-} that the true λ is λ_B. That is, our model features incomplete, symmetric information. All agents have the same information sets, share the same prior, and use the same Bayes rule to update beliefs.

The agent learns about the true value of the unobservable constant λ over time.[15] Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$. That is,

$$\pi_t = \mathbb{P}_t(\lambda = \lambda_B), \tag{10}$$

where $\mathbb{P}_t(\,\cdot\,)$ is the conditional probability operator at t. The expected jump arrival rate at t,

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[14]If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C, V) = \frac{\rho\, C^{1-\gamma}}{1 - \gamma} - \rho\, V. \tag{9}$$

[15]In Section OA of the Online Appendix, we generalize our model to a setting where the unobservable disaster arrival rate λ is stochastic and follows a two-state continuous-time Markov chain.

denoted by λ_t, is given by

$$\lambda_t = \mathbb{E}_t(\lambda) = \lambda(\pi_t) = \lambda_B \pi_t + \lambda_G(1 - \pi_t), \tag{11}$$

which is a weighted average of λ_B and λ_G. A higher value of π_t corresponds to a belief that the economy is more likely in State B where the jump arrival rate is $\lambda_B > \lambda_G$.

What leads the agent's belief to worsen (increasing π) is jump arrivals. What leads the belief to revise favorably is no jump arrivals. In this sense, no-jump news is good news. Mathematically, the agent updates belief by following the Bayes rule:[16]

$$d\pi_t = \sigma_\pi(\pi_{t-}) \left(d\mathcal{J}_t - \lambda_{t-} dt \right), \tag{12}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B \pi + \lambda_G(1-\pi)} > 0. \tag{13}$$

Here, signals come from $\mathcal{J}_t$. Because $\mathbb{E}_{t-}[d\mathcal{J}_t] = \lambda_{t-} dt$, (12) implies that π is a martingale.

When a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t = \pi_t^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where

$$\pi_t^{\mathcal{J}} = \pi_{t-} + \sigma_\pi(\pi_{t-}) = \frac{\pi_{t-} \lambda_B}{\lambda(\pi_{t-})} > \pi_{t-}. \tag{14}$$

Note that the percentage change of belief in response to the arrival of a jump, $\pi_t^{\mathcal{J}}/\pi_{t-}$, which equals $\lambda_B/\lambda(\pi_{t-})$, decreases with the prior π. That is, a disaster arrival causes a larger *percentage* increase of belief if the agent is more optimistic (i.e., with a lower prior π_{t-}).

If there is no arrival over time interval dt, the household becomes more optimistic. Mathematically, if $d\mathcal{J}_t = 0$, we have

$$\frac{d\pi_t}{dt} = \mu_\pi(\pi_{t-}) dt = \pi_{t-}(1 - \pi_{t-})(\lambda_G - \lambda_B), \tag{15}$$

where $\mu_\pi(\pi_{t-}) = -\sigma_\pi(\pi_{t-})\lambda(\pi_{t-})$. Equation (15) is a logistic differential equation. Conditional on no jump during a time interval (s, t), i.e., $dJ_v = 0$ for $s < v \le t$, we obtain the following closed-form logistic function for π_t by integrating $\{\pi_v; v \in (s, t)\}$ from s to t:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B - \lambda_G)(t-s)}}{1 + \pi_s(e^{-(\lambda_B - \lambda_G)(t-s)} - 1)}. \tag{16}$$

In Figure 1, we plot a simulated path for π starting from $\pi_{0-} = 0.1$. It shows that absent a jump arrival, belief becomes more optimistic and π_t decreases deterministically between two consecutive jumps following the logistic function given in (16). Once a jump arrives at t, the belief worsens moving upward to $\pi_t^{\mathcal{J}}$ given in (14) by a discrete amount $\sigma_\pi(\pi_{t-})$.

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[16]See Theorem 19.6 in Liptser and Shiryaev (2001).
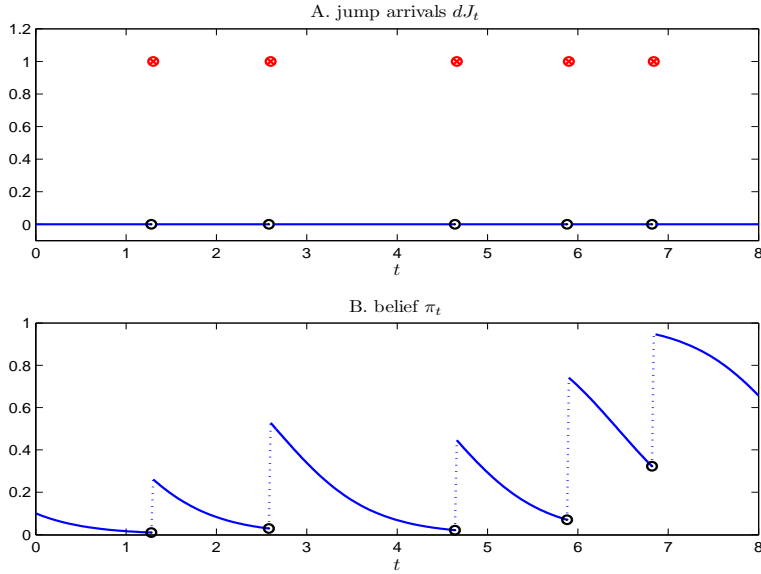
A. jump arrivals dJ_t

B. belief π_t

Figure 1: This figure simulates a path for jump arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.1$. The belief decreases deterministically in the absence of jumps but discretely increases upward upon a jump arrival.

2.3   Competitive Market Structure and Equilibrium

Next, we turn to the competitive market structure and define market equilibrium. Financial markets are dynamically complete. Without loss of generality, it is sufficient to assume that the following financial securities exist at all time t: (i) a risk-free asset thats pays interest at the rate of r_t and (ii) the aggregate equity market.[17]

Let $\{\mathbf{Q}_t\}$ denote the equilibrium ex-dividend aggregate stock market value and $\{\mathbf{D}_t\}$ denote the aggregate dividends, respectively. The cum-dividend return is given by

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \mu_{\mathbf{Q}}(\pi_{t-})dt + \sigma d\mathcal{W}_t + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)d\mathcal{J}_t, \tag{17}$$

where $\mu_{\mathbf{Q}}(\pi)$ is the expected stock market return (leaving aside the jump effect) to be determined later. We later verify that the diffusion volatility of the stock market return equals σ, the same as the diffusion volatility in capital accumulation process given in (2).

Competitive equilibrium.   We define the recursive competitive equilibrium as follows: (a.) Taking the equilibrium risk-free rate r and the aggregate stock market return given in (17) as

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[17]For markets to be dynamically complete, we also need actuarially fair diffusion and jump hedging contracts for each possible Z as in Pindyck and Wang (2013). The net demand is zero for all hedging contracts. As introducing these hedging contracts do not alter the equilibrium outcomes, for expositional simplicity, we omit these hedging contracts and refer readers to Pindyck and Wang (2013) for related detailed analysis.

given, the representative household chooses consumption C and allocation to the aggregate stock market H to maximize lifetime utility given by (7)-(8);[18] (b.) Taking the equilibrium SDF $\{\mathbb{M}_t; t \geq 0\}$ as given, the representative firm chooses investment I and the disaster exposure mitigation spending X^e to maximize its market value given in (6); (c.) The interest rate r, the stock market return process (17), and the SDF $\{\mathbb{M}_t; t \geq 0\}$ are consistent with the households' and firms' optimal decisions and all markets clear in equilibrium.

2.4 Source of Externality: Technology Reducing Tail Risk of the Damage Distribution $\Xi(Z)$ for All Firms

Next, we introduce a mitigation technology, which reduces the tail risk of the aggregate disaster distribution $\Xi(Z)$. Adding this feature into the model is important as it captures a key real-world relevant aspect of aggregate risk mitigation, namely externalities. We assume that this new technology curtails the left tail risk of the aggregate distribution for the fractional loss $(1 - Z)$ by making larger fractional damages to capital stock $(1 - Z)$ less likely.

To ease exposition of economic insights, we distinguish aggregate variables from micro level variables in notations. Throughout this paper, we use **boldfaced** letters to refer to aggregate variables. We assume that only the *aggregate* spending made at $t-$ can curtail left-tail large downside risks at t if a jump arrives at t. The idea is that changing the distribution of Z for all firms is very costly and requires a spending that is at the order of a fraction of the aggregate capital stock $\mathbf{K}$. Let $\mathbf{X}^d_{t-}$ denote the *aggregate* spending on this distribution-tail-curtailing technology, where the superscript d refers to the notion that this spending is to make the *distribution* of fractional loss $(1 - Z)$ less damaging. Let $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ denote the scaled aggregate distribution mitigation spending. Since aggregate risk reduction is a public good, no firm has incentives to spend on this new technology. This is the reason why market fail.

Specifically, by spending on disaster distribution (public) mitigation, we change the distribution of the post-jump fractional recovery Z from $\Xi(Z)$ to $\Xi(Z; \mathbf{x}^d_{t-})$. While simultaneously doubling the aggregate disaster distribution mitigation spending $\mathbf{X}^d_{t-}$ and the aggregate capital stock $\mathbf{K}_{t-}$ does not change the distribution $\Xi(Z; \mathbf{x}^d_{t-})$, as the ratio $\mathbf{x}^d_{t-} = \mathbf{X}^d_{t-}/\mathbf{K}_{t-}$ remains unchanged, doing so doubles the benefit of this public spending (i.e., the total reduction of damages) *in levels* as the benefit is proportional to $\mathbf{K}_{t-}(1 - Z)$ at the aggregate level.[19]

---

[18]Since each household is infinitesimally small and has no impact on any aggregate variables, there is no incentive to spend on mitigation. We provide additional discussions later in the paper.

[19]This is similar to the homogeneity assumption for disaster distribution (private) mitigation spending X^e_{t-}.

In contrast to the exposure mitigation technology, which works at the firm level, this disaster distribution mitigation technology operates at the aggregate level and hence features an externality since its effectiveness depends on collective contributions of all firms.

We have completed the description of our market economy model. Before solving it in Section 4, we first analyze three versions of the planner's problems with varying degrees of access to the mitigation technologies. The solutions for these planner's models serve as important benchmarks for our analysis.

3   Planner's Solutions

The social planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, the aggregate disaster distribution mitigation spending $\mathbf{X}^d$, and the aggregate disaster exposure mitigation spending $\mathbf{X}^e$ to maximize the representative household's utility given in (7)-(8) subject to the representative firm's production/capital accumulation technology, the disaster exposure and distribution mitigation technologies, and the aggregate resource constraint described in Section 2. We report planner's solution for three cases: 1.) the first-best case where the planner has access to both types of mitigation technologies; 2.) the case where the planner only has access to the disaster exposure mitigation technology introduced in Section 2.1; and 3.) the case where the planner has access to neither mitigation technology.

3.1   Planner's First-Best Solution with Mitigation Technology

With access to both types of mitigation technologies, the planner attains the first-best outcome. To save on notation, we drop the subscript fb in this subsection until the end when we summarize the main results.

Dynamic programming.   Let $V(\mathbf{K}, \pi)$ denote the representative household's value function. The Hamilton-Jacobi-Bellman (HJB) equation for the planner is:

$$
\begin{aligned}
0 \;=\; \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \mathbf{x}^d} \; & f(\mathbf{C}, V) + \mathbf{I} V_\mathbf{K}(\mathbf{K}, \pi) + \mu_\pi(\pi) V_\pi(\mathbf{K}, \pi) + \frac{1}{2}\sigma^2 \mathbf{K}^2 V_{\mathbf{K}\mathbf{K}}(\mathbf{K}, \pi) \\
& + \lambda(\pi) \mathbb{E}^{\mathbf{x}^d} \left[V\left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - V(\mathbf{K}, \pi) \right] \;,
\end{aligned} \tag{18}
$$

where $\pi^{\mathcal{J}}$ is the post-jump belief given in (14), $\mathbf{K}^{\mathcal{J}}$ is the post-jump capital stock given by

$$
\mathbf{K}^{\mathcal{J}} = (1 - N(\mathbf{x}^e)(1 - Z)) \,\mathbf{K} \,, \tag{19}
$$

13

$\mu_\pi(\pi)$ is the expected change of belief absent jumps given in (15), $\lambda(\pi)$ is the jump arrival rate given in (11), and $\mathbb{E}^{\mathbf{x}^d}[\cdot]$ is the expectation operator with respect to the pdf $\xi(Z; \mathbf{x}^d)$ for the recovery fraction Z for a given level of scaled disaster distribution mitigation spending $\mathbf{x}^d$.

The first term on the right side of (18) is the household's normalized aggregator (Duffie and Epstein, 1992); the second term captures how investment $\mathbf{I}$ affects $V(\mathbf{K}, \pi)$; the third term reflects how belief updating (in the absence of jumps) impacts $V(\mathbf{K}, \pi)$; and the fourth term captures the effect of capital-stock diffusion shocks on $V(\mathbf{K}, \pi)$. It is worth noting that as the signals in our learning model are discrete (jump arrivals), there is no diffusion volatility induced quadratic variation term involving $V_{\pi\pi}$ in the HJB equation (18).

Direct (value destroying) versus learning effects. Finally, the last term (on the second line) of (18) captures the effect of jumps on the expected change in $V(\mathbf{K}, \pi)$. This term captures rich economic forces and warrants additional explanations. When a jump arrives at t $(d\mathcal{J}_t = 1)$, capital falls from $\mathbf{K}_{t-}$ at time $t-$ to $(1-Z)\mathbf{K}_{t-}$ absent exposure mitigation spending. By spending $\mathbf{x}^e_{t-}$ to mitigate the exposure, the planner reduces the capital loss from $(1-Z)\mathbf{K}_{t-}$ by $N(\mathbf{x}^e_{t-})(1-Z)\mathbf{K}_{t-}$, so that the post-jump capital is $\mathbf{K}^{\mathcal{J}}_t = (1 - N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$ at t.

In sum, a jump triggers two effects on $V(\mathbf{K}, \pi)$. First, there is a direct capital destruction effect. As a jump arrival lowers capital stock from $\mathbf{K}_{t-}$ to $\mathbf{K}^{\mathcal{J}}_t = (1 - N(\mathbf{x}^e_{t-})(1-Z))\mathbf{K}_{t-}$, the value function decreases from $V(\mathbf{K}_{t-}, \pi_{t-})$ to $V(\mathbf{K}^{\mathcal{J}}_t, \pi_{t-})$ even if we ignore the agent's belief updating due to learning. Second, there is a learning (belief-updating) effect. As a jump arrival also cause the belief to increase from π_{t-} to $\pi^{\mathcal{J}}_t$ given in (14), the agent becomes more pessimistic causing the value function to further decrease from $V(\mathbf{K}^{\mathcal{J}}_t, \pi_{t-})$ to $V(\mathbf{K}^{\mathcal{J}}_t, \pi^{\mathcal{J}}_t)$. These two effects reinforce each other over time leading to potentially significant losses.

The planner chooses consumption $\mathbf{C}$, investment $\mathbf{I}$, two types of scaled mitigation spendings, $\mathbf{x}^d$ and $\mathbf{x}^e$, to maximize recursive utility given in (7)-(8) by setting the sum of all the five terms on the right side of (18) to zero, implied by the optimality argument underpinning the HJB equation for recursive utility (see Duffie and Epstein, 1992). Because of the resource constraint, it is sufficient to focus on $\mathbf{I}$, $\mathbf{x}^d$ and $\mathbf{x}^e$ as control variables.

First-order conditions for investment and two types of mitigation spending. The first-order condition (FOC) for investment $\mathbf{I}$ is

$$(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K}))f_{\mathbf{C}}(\mathbf{C}, V) = V_{\mathbf{K}}(\mathbf{K}, \pi) \ . \tag{20}$$

The right side of (20), $V_{\mathbf{K}}(\mathbf{K}, \pi)$, is the marginal (utility) benefit of accumulating capital stock. The left side of (20) is the marginal cost of accumulating capital, which is given by the product of forgone marginal utility of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ and the marginal cost of accumulating capital, $(1 + \Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K}))$. Because of capital adjustment costs, increasing $\mathbf{K}$ by one unit requires incurring investment costs more than one unit, which explains the marginal adjustment cost $\Phi_{\mathbf{I}}(\mathbf{I}, \mathbf{K})$. Because of non-separability of preferences, $f_{\mathbf{C}}(\mathbf{C}, V)$ depends on not just consumption $\mathbf{C}$ but also the continuation utility V.

The FOC for the scaled aggregate disaster distribution mitigation spending, $\mathbf{x}^d \geq 0$, is

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{\mathbf{K}} \lambda(\pi) \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d)}{\partial \mathbf{x}^d} V \left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}} \right) \right] dZ , \tag{21}$$

if the solution is positive, $\mathbf{x}^d > 0$.[20] The planner optimally chooses $\mathbf{x}^d$ to equate the marginal cost of mitigation, which is the forgone marginal (utility) benefit of consumption $f_{\mathbf{C}}(\mathbf{C}, V)$ given on the left side of (21), with the marginal benefit of mitigation given on the right side of (21).[21] By spending $\mathbf{x}^d$ per unit of capital to make the distribution of Z less damaging, the planner changes the pdf $\xi(Z; \mathbf{x}^d)$ for the fractional capital recovery, Z, from $\xi(Z; 0)$ to $\xi(Z; \mathbf{x}^d)$. The FOC for the scaled aggregate disaster exposure mitigation spending $\mathbf{x}^e$ is

$$f_{\mathbf{C}}(\mathbf{C}, V) = -\lambda(\pi) N'(\mathbf{x}^e) \mathbb{E}^{\mathbf{x}^d} \left[(1 - Z) V_{\mathbf{K}} \left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}} \right) \right] , \tag{22}$$

if the solution is strictly positive, $\mathbf{x}^e > 0$.[22] The planner optimally chooses $\mathbf{x}^e$ to equate the marginal benefit of reducing the disaster exposure with the marginal cost of doing so. By spending $\mathbf{x}_{t-}^e$ per unit of capital, the planner reduces the post-jump fractional capital loss from $(1 - Z)\mathbf{K}_{t-}$ to $\mathbf{K}_{t-} - \mathbf{K}_t^{\mathcal{J}} = N(\mathbf{x}_{t-}^e)(1 - Z)\mathbf{K}_{t-}$.

Using the homogeneity property to simplify the solution. Our model has the following homogeneity property. If we double capital stock $\mathbf{K}$, it is optimal for the planner to simultaneously double its policies including the two types of mitigation spendings $\mathbf{X}^d$ and $\mathbf{X}^e$, investment $\mathbf{I}$, and consumption $\mathbf{C}$ at all time. As a result, the value function $V(\mathbf{K}, \pi)$ is homogeneous with degree $(1 - \gamma)$ in $\mathbf{K}$. We can write $V_{fb}(\mathbf{K}, \pi)$ as follows:

$$V_{fb}(\mathbf{K}, \pi) = \frac{1}{1 - \gamma} \left(b_{fb}(\pi) \mathbf{K} \right)^{1-\gamma} , \tag{23}$$

---

[20]Otherwise, $\mathbf{x}^d = 0$ as mitigation in reality cannot be negative. When do we see $\mathbf{x}^d = 0$? One scenario is when the mitigation technology is very inefficient. Technically in this case the agent may want to choose negative mitigation spending (shorting the mitigation spending in a sense) if negative mitigation spending were feasible, as doing so allows the agent to boost investment or consumption, which can be welfare enhancing.

[21]We verify that the second-order condition (SOC) $\lambda(\pi) \int_0^1 \left[\frac{\partial^2 \xi(Z; \mathbf{x}^d)}{\partial (\mathbf{x}^d)^2} V \left(\mathbf{K}^{\mathcal{J}}, \pi^{\mathcal{J}} \right) \right] dZ < 0$ is satisfied.

[22]Otherwise, $\mathbf{x}^e = 0$ since mitigation cannot be negative.

where $b_{fb}(\pi)$ is a welfare measure proportional to certainty equivalent wealth under first best to be determined as part of the solution. Using the FOCs (20), (21), (22) and substituting the value function $V(\mathbf{K}, \pi)$ given in (23) into the HJB equation (18), and simplifying the equations, we obtain the following four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$:

$$
\begin{aligned}
0 = {} & \frac{\rho}{1 - \psi^{-1}} \left[\left[\frac{b(\pi)}{\rho(1 + \phi'(\mathbf{i}(\pi)))} \right]^{1-\psi} - 1 \right] + \mathbf{i}(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi) \frac{b'(\pi)}{b(\pi)} \\
& + \frac{\lambda(\pi)}{1 - \gamma} \left[\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d(\pi)} ((1 - N(\mathbf{x}^e(\pi))(1 - Z))^{1-\gamma}) - 1 \right],
\end{aligned} \tag{24}
$$

$$
b(\pi) = [A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)]^{1/(1-\psi)} [\rho(1 + \phi'(\mathbf{i}(\pi)))]^{-\psi/(1-\psi)}, \tag{25}
$$

$$
\frac{1}{1 + \phi'(\mathbf{i}(\pi))} = \lambda(\pi) \left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right]^{1-\gamma} N'(\mathbf{x}^e(\pi)) \mathbb{E}^{\mathbf{x}^d(\pi)} \left[(Z - 1)(1 - N(\mathbf{x}^e(\pi))(1 - Z))^{-\gamma} \right], \tag{26}
$$

$$
\frac{1}{1 + \phi'(\mathbf{i}(\pi))} = \frac{\lambda(\pi)}{1 - \gamma} \left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right]^{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d(\pi))}{\partial \mathbf{x}^d} (1 - N(\mathbf{x}^e(\pi))(1 - Z))^{1-\gamma} \right] dZ. \tag{27}
$$

We derive the system of ODEs (24)-(27) in Appendix A.1.

Next, we provide the boundary conditions at $\pi = 0$ and $\pi = 1$ and discuss the intuition. As we show, the model at the two boundaries map to the full model in Pindyck and Wang (2013), but generalized to allow for mitigation spending. When $\pi = 0$, the economy is permanently in state G. Therefore there is no learning and the solution boils down to solving the four unknowns, $b(0)$, $\mathbf{i}(0)$, $\mathbf{x}^d(0)$, and $\mathbf{x}^e(0)$, via the following four-equation system:

$$
-\frac{\left[\frac{b(0)}{\rho(1 + \phi'(\mathbf{i}(0)))} \right]^{1-\psi} - 1}{1 - \psi^{-1}} \rho = \mathbf{i}(0) - \frac{\gamma\sigma^2}{2} + \frac{\lambda_G \left[\mathbb{E}^{\mathbf{x}^d(0)} ((1 - N(\mathbf{x}^e(0))(1 - Z))^{1-\gamma}) - 1 \right]}{1 - \gamma}, \tag{28}
$$

$$
b(0) [\rho(1 + \phi'(\mathbf{i}(0)))]^{\psi/(1-\psi)} = [A - \mathbf{i}(0) - \phi(\mathbf{i}(0)) - \mathbf{x}^d(0) - \mathbf{x}^e(0)]^{1/(1-\psi)}, \tag{29}
$$

$$
\frac{1}{1 + \phi'(\mathbf{i}(0))} = \lambda_G N'(\mathbf{x}^e(0)) \mathbb{E}^{\mathbf{x}^d(0)} \left[(Z - 1)(1 - N(\mathbf{x}^e(0))(1 - Z))^{-\gamma} \right], \tag{30}
$$

$$
\frac{1}{1 + \phi'(\mathbf{i}(0))} = \frac{\lambda_G}{1 - \gamma} \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d(0))}{\partial \mathbf{x}^d} (1 - N(\mathbf{x}^e(0))(1 - Z))^{1-\gamma} \right] dZ. \tag{31}
$$

Once π reaches zero at time t (i.e., $\pi_t = 0$), $\mathbf{i}$, $\mathbf{x}^d$, $\mathbf{x}^e$, $\mathbf{c}$, and welfare measure b all remain constant at all time $s \geq t$. By applying essentially the same analysis to the other boundary at $\pi = 1$, i.e., when the economy reaches state B, we solve for the four unknowns, $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$ and $\mathbf{x}^e(1)$, via (A.4)-(A.7), another four-equation system in Appendix A.1.

Next, we summarize our model's solution for the entire belief region $\pi \in [0, 1]$.

Proposition 1 *The first-best solution is given by the value function (23) and the quartet policy rules, $b_{fb}(\pi)$, $\mathbf{i}_{fb}(\pi)$, $\mathbf{x}_{fb}^d(\pi)$, and $\mathbf{x}_{fb}^e(\pi)$, where $0 \leq \pi \leq 1$, via the four-equation ODE system (24)-(27) with the conditions (28)-(31) for $\pi = 0$ and (A.4)-(A.7) for $\pi = 1$.*

See Appendix A.1 for a proof.

3.2 The Case with Exposure Mitigation Technology Only

To connect to the competitive market equilibrium solution (reported in Section 4), it is useful to analyze the planner's solution for the case where there is disaster exposure mitigation technology but no disaster distribution mitigation technology. That is, we shut down the distribution mitigation technology described in Section 2.4 in this version of our planner's model. For value functions and policy rules, we use the hat ^ to differentiate from the other ones. Using the same argument as that for the planner's first-best solution, we know that the value function, $\widehat{V}(\mathbf{K}, \pi)$, is homogeneous with degree $(1 - \gamma)$ in $\mathbf{K}$:

$$\widehat{V}(\mathbf{K}, \pi) = \frac{1}{1 - \gamma}\big(\widehat{b}(\pi)\mathbf{K}\big)^{1-\gamma}, \tag{32}$$

but with a different welfare measure, $\widehat{b}(\pi)$ (also proportional to the certainty equivalent wealth). Setting $\widehat{\mathbf{x}}^d(\pi) = 0$ for all values of π, we obtain the solution for $\widehat{b}(\pi)$ together with the optimal investment $\widehat{\mathbf{i}}(\pi)$ and the disaster exposure mitigation spending $\widehat{\mathbf{x}}^e(\pi)$.

Proposition 2 *Without disaster risk mitigation technology ($\widehat{\mathbf{x}}^d(\pi) = 0$), the planner's solution is given by the value function (32), where $\widehat{b}(\pi)$, $\widehat{\mathbf{i}}(\pi)$, and $\widehat{\mathbf{x}}^e(\pi)$ solve (24), (25), and (26) together with the boundary conditions (28), (29), and (30) and (A.4), (A.5), and (A.6).*

In Section 4, we show that the market economy model is equivalent to this version of the planner's model. Next, we turn to the case where the planner has access to neither mitigation technology.

3.3 The Case with Neither Types of Mitigation Technology

To quantify the value of mitigation technology for the society, it is useful to summarize the planner's solution when we shut down both mitigation technologies. For value functions and policy rules, we use the underline _ to differentiate from the other ones. Using the same argument as for the two cases we just analyzed, we know that the planner's value function,

$\underline{V}(\mathbf{K}, \pi)$, is homogeneous with degree $(1-\gamma)$ in $\mathbf{K}$:

$$\underline{V}(\mathbf{K}, \pi) = \frac{1}{1-\gamma}\left(\underline{b}(\pi)\mathbf{K}\right)^{1-\gamma}, \tag{33}$$

where $\underline{b}(\pi)$ is a welfare measure (also proportional to the certainty equivalent wealth) but different from $b(\pi)$ and $\widehat{b}(\pi)$ just analyzed. The next proposition summarizes the key results for $\underline{b}(\pi)$ and the optimal investment-capital ratio $\underline{\mathbf{i}}(\pi)$ in the setting where $\underline{\mathbf{x}}^d(\pi) = \underline{\mathbf{x}}^e(\pi) = 0$.

Proposition 3 *With access to neither types of disaster risk mitigation technology ($\underline{\mathbf{x}}^d(\pi) = \underline{\mathbf{x}}^e(\pi) = 0$), the planner's solution is given by the value function (33), where $\underline{b}(\pi)$ and $\underline{\mathbf{i}}(\pi)$ jointly solve (24)-(25) together with the boundary conditions (28)-(29) and (A.4)-(A.5).*

4 Competitive Equilibrium Solution

While the planner's (first-best) public mitigation spending is strictly positive, no firms have incentives to mitigate aggregate risk distribution in a market economy. Moreover, we show that the market solution is equivalent to the planner's solution for the case where only the disaster exposure mitigation technology is available (given in Section 3.2.)

4.1 Firm's Optimization Problem

When making its decisions, the firm takes the equilibrium risk-free rate r_t and the market price of (diffusion and jump) risks as given. Formally, the firm maximizes its market value given by (6) taking the following equilibrium SDF $\mathbb{M}_t$ as given:

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r_{t-}dt - \gamma\sigma d\mathcal{W}_t + (\eta_t - 1)\left(d\mathcal{J}_t - \lambda(\pi_{t-})dt\right), \tag{34}$$

where the equilibrium risk-free rate is a function of belief (π_{t-}): $r_{t-} = r(\pi_{t-})$ and the equilibrium market price of jump risk η_t depends on belief π_{t-} and the realized value of Z, i.e., $\eta_t = \eta(\pi_{t-}; Z)$.[23] The first term on the right side of (34) states that the equilibrium drift of $d\mathbb{M}_t/\mathbb{M}_{t-}$ has to equal $-r_{t-}dt$, a standard asset-pricing result (Duffie, 2001). The second term on the right side of (34) is the diffusion martingale and $\gamma\sigma$ is the equilibrium market price of diffusion risk as in Pindyck and Wang (2013) and verified later. As $\lambda(\pi_{t-})dt = \mathbb{E}_{t-}(d\mathcal{J}_t)$, the last term in (34) is a jump martingale and implies that when a jump arrives at t, the SDF changes discretely from $\mathbb{M}_{t-}$ to $\mathbb{M}_t^{\mathcal{J}}$ by a multiple of stochastic η_t:

$$\frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \eta_t. \tag{35}$$

---

[23]We provide equilibrium solutions for $r(\pi_{t-})$ and $\eta(\pi_{t-}; Z)$ in Section 6 and Subsections 4.3, respectively.

Applying the Ito's Lemma to firm value $Q(K_t, \pi_t) = q(\pi_t)K_t$ given in (6) and using (34), we obtain the following HJB equation for the firm's Tobins q, $q(\pi)$, (see Appendix B.1):

$$r(\pi)q(\pi) = \max_{i,\, x^e,\, x^d} \; A - i - \phi(i) - x^e - x^d + iq(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma^2 q(\pi)$$
$$+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z)\left(q(\pi^{\mathcal{J}})(1 - N(x^e)(1 - Z)) - q(\pi)\right)\right] . \qquad (36)$$

The expectation operator in (36) takes the aggregate disaster mitigation spending in the economy, $\mathbf{x}^d$, as given. In addition to the preceding HJB equation, we also have two FOCs.

First, (36) implies that $x^d = 0$, as a firm is infinitesimal and hence mitigating aggregate disaster distribution brings no benefit but only cost to itself.[24] Note that when choosing its x^d, the firm takes the aggregate $\mathbf{x}^d$ which determines the expectation in (36) as given. Of course in equilibrium, $\mathbf{X}^d$ equals the sum of all X^d chosen by firms and households.[25] Second, unlike x^d, (36) implies a rather different FOC for the firm's exposure mitigation spending x^e:

$$1 = -\lambda(\pi)q(\pi^{\mathcal{J}})N'(x^e)\mathbb{E}^{\mathbf{x}^d}\left[(1 - Z)\eta(\pi; Z)\right] . \qquad (37)$$

By spending a dollar at the margin on exposure risk mitigation, the firm reduces the destruction of its capital stock by $-(1 - Z)N'(x^e) > 0$ units should a jump arrive. Upon a jump arrival, the gross percentage change of SDF is $\mathbb{M}_t^{\mathcal{J}}/\mathbb{M}_{t-} = \eta(\pi_{t-}; Z)$ and the Tobin's q jumps from $q(\pi)$ to $q(\pi^{\mathcal{J}})$. To obtain the marginal benefit of spending on exposure mitigation X^e, we multiply the marginal reduction of capital stock destruction caused by a jump arrival, $-(1 - Z)N'(x^e) > 0$, by $\lambda(\pi)q(\pi^{\mathcal{J}})\eta(\pi; Z)$, and then integrate over all possible values of Z. We then obtain the expected marginal value of mitigating the disaster exposure, the right side of (37), which equals the unit marginal cost of mitigating the exposure on the left side.

In contrast, the firm does not spend on the disaster distribution mitigation technology, as doing so yields no private payoff. The FOC for investment implied by (36) is:

$$q(\pi) = 1 + \phi'(i(\pi)), \qquad (38)$$

which is the standard investment optimality condition that equates the marginal q to the marginal cost of investing $1 + \phi'(i(\pi))$ (the homogeneity property) as in Hayashi (1982).

---

[24]To be precise, since the firm's FOC for x^d only has marginal cost but no marginal benefit, the FOC cannot hold with equality and hence the corner solution $x^d = 0$ is optimal.

[25]Using the law of large numbers, in equilibrium the aggregate mitigation spending in a laissez-faire economy equals the sum of all mitigation *distribution* disaster spendings by households and firms, i.e., $\mathbf{X}^d = \int X^d d\nu_f + \int X^d d\nu_c$, where ν_f is the unit measure of firms and ν_c is the unit measure of households/consumers.

4.2 Household's Optimization Problem

The household maximizes utility taking the risk-free rate process $r(\pi_t)$ and the stock market return process, given by (17), as given. We show that the household's value function $J_t = J(W_t, \pi_t)$ takes the form of:

$$J(W, \pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}, \tag{39}$$

where $u(\pi)$ is a welfare measure that will be endogenously determined.

The household chooses consumption C, allocation to the stock market H, disaster exposure mitigation spending X^e, and distribution mitigation spending X^d to maximize utility. The HJB equation for the household in our decentralized market setting is given by

$$0 = \max_{C, H, X^e, X^d} \quad f(C, J) + \mu_\pi(\pi)J_\pi + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W, \pi)\right]\xi(Z; \mathbf{x}^d)dZ$$
$$+ \left[r(\pi)W + (\mu_\mathbf{Q}(\pi) - r(\pi))H - C\right]J_W + \frac{\sigma^2 H^2 J_{WW}}{2}, \tag{40}$$

where $\mu_\mathbf{Q}(\pi)$ is defined in (17), $\pi^{\mathcal{J}}$ is the post-jump belief given in (14), and $W^{\mathcal{J}}$ is the post-jump wealth given by

$$W_t^{\mathcal{J}} = W_{t-} + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)H_{t-}. \tag{41}$$

Households spend nothing on disaster distribution mitigation ($X^d = 0$), as doing so yields no benefit. Each household is infinitesimally small and cannot possibly change the aggregate disaster distribution $\Xi(Z; \mathbf{x}^d)$, which only depends on aggregate $\mathbf{x}^d$. Since neither households nor firms have incentives to spend on public mitigation, in equilibrium the aggregate $\mathbf{x}^d$ is zero. Similarly, since households have no power to influence the stock market, they have no incentives to spend on exposure mitigation either, $X^e = 0$.

Now we turn to the post-jump wealth given in (41). The second term in (41) is the change of the portfolio's market value upon the arrival of a disaster, where $\mathbf{Q}_t^{\mathcal{J}}$ is the post-jump aggregate stock market value at t. The homogeneity property implies that $\mathbf{Q}_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ where $\mathbf{q}(\pi_t)$ is the Tobin's q for the aggregate capital stock $\mathbf{K}$ and equals the firm's average q: $\mathbf{q}(\pi_t) = q(\pi_t)$ in equilibrium. When a jump arrives, the aggregate stock market value changes from $\mathbf{Q}_{t-}$ to $\mathbf{Q}_t^{\mathcal{J}}$ where:

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}}{\mathbf{q}(\pi_{t-})\mathbf{K}_{t-}} = \frac{\mathbf{q}(\pi_t^{\mathcal{J}})}{\mathbf{q}(\pi_{t-})}(1 - N(\mathbf{x}_{t-}^e)(1 - Z)). \tag{42}$$

Equation (42) states that the aggregate stock market value changes from $\mathbf{Q}_{t-} = \mathbf{q}(\pi_{t-})\mathbf{K}_{t-}$ to $\mathbf{Q}_t^{\mathcal{J}} = \mathbf{q}(\pi_t^{\mathcal{J}})\mathbf{K}_t^{\mathcal{J}}$ as a jump arrives for two reasons: 1.) capital stock decreases from $\mathbf{K}_{t-}$ to $\mathbf{K}_t^{\mathcal{J}} = [1 - N(\mathbf{x}_{t-}^e)(1-Z)]\mathbf{K}_{t-}$ by a fraction of $N(\mathbf{x}_{t-}^e)(1-Z)$ and 2.) the aggregate Tobin's q changes from $\mathbf{q}(\pi_{t-})$ to $\mathbf{q}(\pi_t^{\mathcal{J}})$, where $\pi_t^{\mathcal{J}} = \pi_{t-}\lambda_B/\lambda(\pi_{t-})$ is given in (14). For brevity, we drop the time subscripts when it does not cause confusion. That is, we write $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q} = \mathbf{Q}_t^{\mathcal{J}}/\mathbf{Q}_{t-}$.

Substituting (39) into the HJB equation (40), we obtain the following consumption rule:

$$C(\pi) = \rho^\psi u(\pi)^{1-\psi} W. \tag{43}$$

Consumption is linear in wealth with a marginal propensity to consume of $\rho^\psi u(\pi)^{1-\psi}$ that depends on π. The market portfolio allocation H is given by

$$H = -\frac{\mu_\mathbf{Q}(\pi) - r(\pi)}{\sigma^2}\frac{J_W(W,\pi)}{J_{WW}(W,\pi)} + \frac{\lambda(\pi)}{\sigma^2}\mathbb{E}^{\mathbf{x}^d}\left[\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\frac{J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)}{J_{WW}(W,\pi)}\right]. \tag{44}$$

The first term in (44) is the standard Merton's mean-variance demand (absent jumps) and the second term in (44) captures the intertemporal hedging demand due to the agent's belief updating caused by a jump arrival.

4.3 Market Equilibrium and Solution

In equilibrium, all of the household's wealth is invested in the stock market, $W_t = H_t = \mathbf{Q}_t$. Additionally, the equilibrium aggregate disaster exposure mitigation spending in a laissez-faire economy, $\mathbf{X}^e$, equals the sum of all disaster *exposure* mitigation spendings by firms.[26] As neither households nor firms have incentives to spend on X^d, the aggregate disaster distribution mitigation spending is zero: $\mathbf{X}^d = 0$.

In Appendix B.3, we simplify the household's HJB equation as:

$$0 = \frac{\psi^{-1}\rho^\psi u(\pi)^{1-\psi} - \rho}{1 - \psi^{-1}} + \mu_\mathbf{Q}(\pi) + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} + \frac{\lambda(\pi)}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta(\pi; Z, \mathbf{x}^e)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right) - 1\right], \tag{45}$$

where $\eta(\pi; Z, \mathbf{x}^e)$ is the ratio of the pre-jump and the post-jump SDF in equilibrium:

$$\eta(\pi; Z, \mathbf{x}^e) = \frac{\mathbb{M}_t^{\mathcal{J}}}{\mathbb{M}_{t-}} = \frac{J_W(\mathbf{Q}^{\mathcal{J}}, \pi^{\mathcal{J}})}{J_W(\mathbf{Q}, \pi)} \tag{46}$$

---

[26]Since households contribute nothing to disaster exposure mitigation spending, $\int X^e d\nu_c = 0$, where ν_c is the unit measure of households/consumers. Again, using the law of large numbers, the aggregate *exposure* mitigation spending is given by $\mathbf{X}^e = \int X^e d\nu_f + \int X^e d\nu_c = \int X^e d\nu_f$, where ν_f is the unit measure of firms.

and $\mu_{\mathbf{Q}}(\pi)$ defined in (17) is given by[27]

$$\mu_{\mathbf{Q}}(\pi) = r(\pi) + \gamma\sigma^2 + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)\left(1 - \frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right)\right] \tag{47}$$

$$= \frac{c(\pi)}{q(\pi)} + i(\pi) + \mu_\pi(\pi)\frac{q'(\pi)}{q(\pi)}. \tag{48}$$

The second equality in (46) states that $\eta(\pi; Z, \mathbf{x}^e)$ also equals the ratio of the household's post-jump marginal value of wealth $J_W(\mathbf{Q}^{\mathcal{J}}, \pi^{\mathcal{J}})$ and the pre-jump marginal value of wealth $J_W(\mathbf{Q}, \pi)$. In equilibrium both the household's pre-jump and post-wealth wealth areis in the stock market and hence $W = \mathbf{Q}$ and $W^{\mathcal{J}} = \mathbf{Q}^{\mathcal{J}}$. Using the homogeneity property, we write $\eta(\pi; Z, \mathbf{x}^e)$ as:

$$\eta(\pi; Z, \mathbf{x}^e) = \left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\right)^{-\gamma}(1 - N(\mathbf{x}^e)(1 - Z))^{-\gamma}. \tag{49}$$

Using (42) for $\mathbf{Q}_t^{\mathcal{J}}/\mathbf{Q}_{t-}$ and (49) for $\eta(\pi; Z, \mathbf{x}^e)$, we obtain the ODE (45) for the welfare measure $u(\pi)$ with belief π being the only state variable.

In sum, the model solution is given by 1.) the ODE (45) for $u(\pi)$ and the FOCs (43)-(44) for households and 2.) the ODE (36) for $q(\pi)$ and the FOCs (37)-(38) for firms. Next, we show that this solution of our market model is the same as that of a planner's problem, where the planner has no access to the disaster mitigation technology ($\mathbf{x}^d(\pi) = 0$.) That is, rather than solving for $u(\pi)$ and $q(\pi)$ in our market economy, it is equivalent to solve for $b(\pi)$ and optimal policies in the planner's economy and interpret them as the market economy solution.

Proposition 4 *The competitive equilibrium solution is the same as the planner's solution (summarized in Proposition 2) where there is no mitigation technology to change the distribution of the recovery fraction Z ($\mathbf{x}^d(\pi) = 0$).*

See Appendix B.3 for proof. Note that this proposition states that the Welfare Theorem applies when there is no distribution mitigation technology.

5 Taxation, First-Best, and Welfare

In this section, we show that introducing optimal capital taxation into our competitive market economy of Section 2 changes the market economy solution given in Section 4 to the one implied by the planner's first-best solution given in Section 3.1.

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[27]We use the FOC given in (44) and the equilibrium condition $H_t = W_t$ to obtain (47). Substituting the resource constraint $c(\pi) = A - i(\pi) - \phi(i(\pi)) - \mathbf{x}^e(\pi)$ into the ODE (36) for $q(\pi)$, we obtain (48).

5.1   Firm and Household Optimization under Capital Taxation

The government taxes the firm's capital stock K_t at a rate of $\tau_t = \mathbf{x}^d_{fb,t}$, where $\mathbf{x}^d_{fb,t}$ is the first-best mitigation spending to change the distribution of Z, obtained in Section 3.1. Then, the government spends $\mathbf{X}^d_t = \tau_t K_t$ to reduce the tail risk of the disaster distribution.[28] We make the dependence of the tax rate τ_t on π_t explicit by writing $\tau_t = \tau(\pi_t) = \mathbf{x}^d_{fb,t} = \mathbf{x}^d_{fb}(\pi_t)$.

Facing a capital tax rate of $\tau(\pi_t)$, each firm solves the following problem:

$$\max_{I, X^e, X^d} \mathbb{E}\left[\int_0^\infty \left(\frac{\mathbb{M}_t}{\mathbb{M}_0}\left[(A - \tau(\pi_t))K_t - I_t - \Phi_t - X^e_t - X^d_t\right]\right) dt\right], \tag{50}$$

taking the equilibrium SDF $\mathbb{M}_t$ as given. First, the firm has no incentive to spend on disaster distribution mitigation, again as doing so is costly but yields no benefit for the firm. Thus, $X^d = 0$. The tax makes the firm behave as if its productivity is lowered from A to $A - \tau(\pi_t)$. Applying the Ito's Lemma to firm value $Q(K_t, \pi_t) = q(\pi_t)K_t$ given in (6) and using (34), we obtain the following HJB equation for $q(\pi_t)$:

$$r(\pi)q(\pi) = \max_{i,\, x^e}\ A - \tau(\pi) - i - \phi(i) - x^e + i(\pi)q(\pi) + \mu_\pi(\pi)q'(\pi) - \gamma\sigma^2 q(\pi)$$
$$+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)\left(q(\pi^\mathcal{J})(1 - N(x^e)(1 - Z)) - q(\pi)\right)\right]. \tag{51}$$

Note that the tax rate $\tau(\pi)$ appears in (51). The FOCs for i and x^e are given by (37) and (38), respectively, the same as in the no-tax competitive-market economy model of Section 4.

The household's problem is the same as in Section 4. That is, the HJB equation (45) for $u(\pi)$ and the FOCs (43) for consumption and (44) for the stock market portfolio allocation characterize the household's problem. Next, we prove that with optimal capital taxation the competitive-market economy yields the first-best solution.

5.2   Optimal Taxation in Markets Restores First-Best

In this section, we show that the household's value function in the competitive economy with optimal taxes is the same as the value function under the first-best. As the household's value function in a market economy depends on wealth W while the planner's value function depends on $\mathbf{K}$, we use the equilibrium result $W_t = \mathbf{q}(\pi_t)\mathbf{K}_t$ to write the household's value function as $J(W_t, \pi_t) = J(\mathbf{q}(\pi_t)\mathbf{K}_t, \pi_t)$ in the market economy with taxation. Therefore, the value functions in the two economies are equal, $V(\mathbf{K}_t, \pi_t) = J(W_t, \pi_t)$, if and only if $b(\pi)$ in the first-best economy equals the product $u(\pi)\mathbf{q}(\pi)$ in the competitive economy with taxes.

---

[28]Equivalently the government can impose via a tax on sales $Y_t = AK_t$ at the firm level.

Specifically, we show the following results: (1.) the first-order conditions for $\mathbf{i}(\pi)$ and $\mathbf{x}^e(\pi)$ in the competitive economy with an optimal tax rate (set at the planner's first best distribution mitigation $\mathbf{x}^d_{fb,t}$) are the same as the corresponding first-order conditions in the planner's economy; (2.) the implied ODE for $u(\pi)\mathbf{q}(\pi)$ in the competitive market economy is the same as the ODE (24) for $b(\pi)$ in the planner's economy; (3.) all the boundary conditions at $\pi = 0$ and $\pi = 1$ in the two economies are the same. Below is a step-by-step proof.

First, combining the equilibrium aggregate investment FOC, $\mathbf{q}(\pi) = 1+\phi'(\mathbf{i}(\pi))$, implied by (38) with the optimal scaled consumption rule $\mathbf{c}(\pi) = \rho^\psi u(\pi)^{1-\psi}\mathbf{q}(\pi) = (\rho\mathbf{q}(\pi))^\psi [u(\pi)\mathbf{q}(\pi)]^{1-\psi}$, implied by (43) and $W = \mathbf{q}(\pi)\mathbf{K}$, we obtain the following expression for consumption:

$$\mathbf{c}(\pi) = [\rho(1 + \phi'(\mathbf{i}(\pi)))]^\psi [u(\pi)\mathbf{q}(\pi)]^{1-\psi} . \tag{52}$$

Using the goods market clear condition $\mathbf{c}(\pi) = A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)$ and the conjecture $b(\pi) = u(\pi)\mathbf{q}(\pi)$, we obtain the following expression:

$$b(\pi) = [A - \tau(\pi) - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e(\pi)]^{1/(1-\psi)} [\rho(1 + \phi'(\mathbf{i}(\pi)))]^{-\psi/(1-\psi)} , \tag{53}$$

which is the same as the investment FOC, given in (25), for the planner's problem, provided that the capital tax rate equals $\mathbf{x}^d_{fb}(\pi)$: $\tau(\pi) = \mathbf{x}^d_{fb}(\pi)$. Note that (53) summarizes both the consumer's and the firm's optimization FOCs in the market economy with optimal taxes.

Second, substituting (49) for η into the disaster exposure mitigation (x^e) FOC (37) in the competitive market economy, we obtain

$$1 = -\lambda(\pi)q(\pi^{\mathcal{J}})N'(x^e)\mathbb{E}^{\mathbf{x}^d}\left[(1 - Z)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)}\right)^{-\gamma}(1 - N(\mathbf{x}^e)(1 - Z))^{-\gamma}\right] . \tag{54}$$

Using the investment FOC $q(\pi) = 1 + \phi'(i(\pi))$, the equilibrium conditions, $q(\pi) = \mathbf{q}(\pi)$, $i(\pi) = \mathbf{i}(\pi)$, and the conjecture $b(\pi) = u(\pi)\mathbf{q}(\pi)$ between the two economies, we obtain

$$1 = -N'(\mathbf{x}^e(\pi))\lambda(\pi)(1 + \phi'(\mathbf{i}(\pi)))\left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right]^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi)}\left[(1 - Z)(1 - N(\mathbf{x}^e(\pi))(1 - Z))^{-\gamma}\right], \tag{55}$$

which is the same as the planner's FOC (26) for $\mathbf{x}^e$. So far, we have verified that the FOCs for investment and exposure mitigation spending in the two economies are the same.

Third, substituting (48) into (45) and using the consumption rule $c(\pi) = \rho^\psi u(\pi)^{1-\psi}q(\pi)$

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implied by the FOC (43), we may rewrite the ODE (45) for the household's $u(\pi)$ as

$$0 = \frac{\rho^\psi u(\pi)^{1-\psi} - \rho}{1 - \psi^{-1}} + i(\pi) + \mu_\pi(\pi)\frac{q'(\pi)}{q(\pi)} + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2}$$
$$+ \frac{\lambda(\pi)}{1 - \gamma}\left[\mathbb{E}^{\mathbf{x}^d}\left(\eta(\pi; Z, \mathbf{x}^e)\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\right) - 1\right]$$
$$= \frac{\rho^\psi u(\pi)^{1-\psi} - \rho}{1 - \psi^{-1}} + i(\pi) + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{q'(\pi)}{q(\pi)}\right) - \frac{\gamma\sigma^2}{2}$$
$$+ \frac{\lambda(\pi)}{1 - \gamma}\left[\left(\frac{u\left(\pi^{\mathcal{J}}\right)\mathbf{q}\left(\pi^{\mathcal{J}}\right)}{u(\pi)\mathbf{q}(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma}) - 1\right]. \tag{56}$$

We obtain (56) by using $\eta(\pi; Z, \mathbf{x}^e)$ given in (49) and $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ given in (42).

Finally, using the conjecture $b(\pi) = u(\pi)\mathbf{q}(\pi) = u(\pi)(1 + \phi'(\mathbf{i}(\pi)))$, we may simplify the ODE (56) and obtain the following ODE for $b(\pi) = u(\pi)\mathbf{q}(\pi)$:

$$0 = \frac{\rho}{1 - \psi^{-1}}\left[\left[\frac{b(\pi)}{\rho(1 + \phi'(\mathbf{i}(\pi)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi) + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} - \frac{\gamma\sigma^2}{2}$$
$$+ \frac{\lambda(\pi)}{1 - \gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma}) - 1\right], \tag{57}$$

which is the same as the ODE (24) for $b(\pi)$ in the planner's (first-best) economy.

Finally, applying the same arguments as the above to the boundaries at $\pi = 0$ and $\pi = 1$, we can show that the two economies have the same FOCs and, moreover, $b(0)$ and $b(1)$ in the planner's economy equal to $u(0)\mathbf{q}(0)$ and $u(1)\mathbf{q}(1)$ in the market economy with optimal taxation, respectively.

In sum, we have verified that setting the optimal capital tax at $\tau(\pi) = \mathbf{x}_{fb}^d(\pi)$ in the market economy yields the same resource allocation as in the planner's first-best economy. The next proposition summarizes this result.

Proposition 5 *In a competitive market economy, household consumption, corporate investment, and disaster risk exposure mitigation attain the first-best levels in Section 3.1, provided that the government sets the capital tax rate $\tau(\pi_t)$ to the first-best level $\mathbf{x}_{fb}^d(\pi_t)$ for all firms and then spends 100% of the tax proceeds each period to mitigate the tail risk of the disaster distribution, i.e., $\tau(\pi_t) = \mathbf{x}_{fb}^d(\pi_t)$. The government balances its budget period by period.*

6   Asset Prices

We next compute the equilibrium asset prices for the various economies that we have analyzed:
1.) market economy with optimal taxation (which yields the same outcome as the planner's

first-best economy); 2.) laissez faire market economy with both mitigation technology; 3.) competitive markets with neither mitigation technology.

The next proposition summarizes the equilibrium asset-pricing implications for a given market economy type. The proofs are in Appendices A.2 and B.3.

Proposition 6 *Tobin's average q for the aggregate capital stock is* $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$, *where* $\mathbf{i}(\pi)$ *is the optimal investment-capital ratio. The equilibrium risk-free rate,* $r(\pi)$, *is given by*

$$
r(\pi) = \rho + \psi^{-1}\mathbf{i}(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}\right]\mu_\pi(\pi)
$$
$$
- \lambda(\pi)\left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi;Z,\mathbf{x}^e)) - 1\right] - \lambda(\pi)\frac{\psi^{-1}-\gamma}{1-\gamma}\left[1 - \mathbb{E}^{\mathbf{x}^d}\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}}\eta(\pi;Z,\mathbf{x}^e)\right)\right], \quad (58)
$$

where $\eta(\pi;Z,\mathbf{x}^e)$ *given in (49) is the (gross) percentage change of the marginal value of wealth (SDF),* $\mathbb{M}$, *upon a jump arrival and* $\mathbf{Q}^{\mathcal{J}}/\mathbf{Q}$ *is the jump-triggered (gross) percentage change of the stock market value given in (42). The stock market risk premium,* $rp(\pi)$, *is*

$$
rp(\pi) = \gamma\sigma^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[(\eta(\pi;Z,\mathbf{x}^e) - 1)\left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1\right)\right]. \quad (59)
$$

Out of the six terms in (58), the first three terms are the standard contributing factors to the interest rate in AK models with diffusion shocks. The fourth term captures the effect of belief updating. The fifth term describes how the jump-induced expected change of the marginal value of wealth ($\mathbb{M}^{\mathcal{J}}/\mathbb{M}$) contributes to the risk-free rate. The sixth term captures the additional effect of jumps on the equilibrium risk-free rate due to the household's recursive (non-separable) Epstein-Zin preferences rather than expected utility.[29]

There are two terms for the stock market risk premium given in (59). The first term is the standard diffusion risk premium as in Lucas (1978).[30] The second term captures the jump risk premium, which equals the expectation over the product of the (net) percentage change of marginal value of wealth ($\mathbb{M}$), given by $(\eta(\pi;Z,\mathbf{x}^e)-1)$, and the (net) percentage change of the stock market value given in (42), both of which are caused by jump arrivals. A downward jump causes the household's marginal utility to increase ($\eta(\pi;Z,\mathbf{x}^e) = \mathbb{M}^{\mathcal{J}}/\mathbb{M} \geq 1$). Therefore, if the stock market valuation decreases upon a jump arrival, ($\mathbf{Q}^{\mathcal{J}} < \mathbf{Q}$), the jump risk premium (the second term) is then positive.

---

[29]To be precise, for recursive utility, $f_{CV} \neq 0$, which means that the SDF $\mathbb{M}_t$ is not additively separable. This non-separability makes jumps to have an additional intertemporal effect on the risk-free rate. Note that for expected utility where $\gamma = \psi^{-1}$, this term disappears.

[30]While both Lucas (1978) and Bansal and Yaron (2004) are discrete-time models, but the insight for diffusion risk premium is the same in discrete-time and continuous-time models. Diffusion shocks in continuous-time formulations correspond to Gaussian shocks in discrete time formulations.

7 Application to Tropical Cyclones and Flood Control

We apply our model to flood control spendings that mitigate the damages of tropical cyclones on economic growth and welfare.

7.1 Distributional and Functional Form Specifications

As in Barro (2006) and Pindyck and Wang (2013), we assume that the cdf of the recovery fraction Z from a cyclone strike is given by a power law over $Z \in (0, 1)$:

$$\Xi(Z; \mathbf{x}^d) = Z^{\beta(\mathbf{x}^d)} , \tag{60}$$

where $\beta(\mathbf{x}^d)$ is the exponent function that depends on scaled disaster distribution mitigation $\mathbf{x}^d$. To ensure that our model is well defined, we require $\beta(\mathbf{x}^d) > \gamma - 1$. Conditional on a jump arrival, the expected fractional capital loss for a firm is given by

$$\ell(\pi) = N(x^e)(1 - \mathbb{E}^{\mathbf{x}^d}(Z)) = \frac{N(x^e)}{\beta(\mathbf{x}^d) + 1} . \tag{61}$$

The larger the value of $\beta(\cdot)$, the smaller the expected fractional loss $\mathbb{E}^{\mathbf{x}^d}(1 - Z)$ even absent the firm's disaster exposure mitigation x^e. To capture the benefit of public mitigation, we assume that $\beta(\mathbf{x}^d)$ is increasing in $\mathbf{x}^d$, $\beta'(\mathbf{x}^d) > 0$. The benefit of public disaster distribution mitigation $\mathbf{x}^d$ is to increase the capital stock recovery (upon the arrival of a disaster) in the sense of first-order stochastic dominance, i.e., $\Xi(Z; \mathbf{x}_a^d) \leq \Xi(Z; \mathbf{x}_b^d)$ for $Z < 1$ if $\mathbf{x}_a^d > \mathbf{x}_b^d$.

Let $g_t = g(\pi_t)$ denote a firm's expected growth rate including the jump effect. The homogeneity property implies that growth is independent of the capital stock $\mathbf{K}$ and hence

$$g(\pi) = i(\pi) - \lambda(\pi)\ell(\pi) = i(\pi) - \frac{\lambda(\pi)N(x^e)}{\beta(\mathbf{x}^d(\pi)) + 1} . \tag{62}$$

In addition, we specify the firm's exposure mitigation technology $N(x^e)$ as follows:

$$N(x^e) = 1 - (x^e)^\alpha , \tag{63}$$

where $0 < \alpha < 1$. That is, the more exposure mitigation spending the smaller the damage (the lower the level of $N(x^e)$). Additionally, the marginal benefit of x^e on reducing damages diminishes. Moreover, we use the following linear specification for $\beta(\mathbf{x}^d)$ which governs the public disaster distribution mitigation technology:

$$\beta(\mathbf{x}^d) = \beta_0 + \beta_1 \mathbf{x}^d , \tag{64}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for the c.d.f. for the fractional recovery Z in the absence of mitigation. The coefficient β_1 is a key parameter in our model and measures the efficiency of the aggregate disaster distribution mitigation technology.

Finally, we use the widely used quadratic adjustment cost function (e.g., Hayashi, 1982):

$$\phi(i) = \frac{\theta i^2}{2} , \tag{65}$$

where the parameter θ measures how costly it is to adjust capital.

7.2 Calibration and Parameter Choices

We use our calibration to highlight the importance of learning and both (public and private) mitigation spendings for welfare analysis for a typical country exposed to tropical cyclones.

λ_G, λ_B **and** π_0. Our model better maps into major hurricanes that make landfall. To this end we gather data on major cyclones that made landfall. Our sample contains annual observations for the real GDP per capita growth rate and cyclone landfalls across 109 countries from 1960 to 2010, with 5,410 county-year observations in total. These are the same set of countries as in Hsiang and Jina (2014) excluding Taiwan for which there is no GDP data from the World Bank Development Indicator. Note that we cannot use Hsiang and Jina (2014) findings since they focus on estimating the marginal effect of windspeed on GDP growth damage.

We define an indicator variable, Landfall$_{i,t}$, that equals one if a country i experienced at least one cyclone landfall in year t. We refer to Landfall$_{i,t}$ as our (cyclone) landfall disaster exposure measure, where Landfall$_{i,t} = 1$ iff the cyclone scale is "tropical storm" and above.

We assign the 109 countries into 4 regions: North Atlantic (including North America, the Caribbean, and West Europe) West Pacific (including Oceania), North India (including North India, Middle East, North Africa, and Central Europe), and South Atlantic (including Latin America and Sub-Saharan Africa).

Table 1 shows the summary statistics of cyclone landfalls in the sample for each region.

As a typical country in our sample is exposed to one major cyclone strike every 7.4 years, we set the arrival rate at $\lambda_G = 1/7.4 = 0.135$ per annum, which is representative of a world without global warming ($\pi = 0$). To calibrate λ_B, we use data from the most recent authoritative survey of climate model projections for the increased frequency of major cyclones (Knutson et al., 2020). This survey covers the projections of about 50 models for the change

Table 1: Summary statistics of cyclone landfall

Region	(1) Total # of country-year obs.	(2) Total # of cyclone landfall obs.	(3) Freq. of landfalls = (2)/(1): Disaster arrival rate estimate λ
North Atlantic	1,587	229	0.144
West Pacific	638	326	0.511
North India	719	75	0.104
South Atlantic	2,466	99	0.040
Global	5,410	729	0.135

in frequency of major tropical cyclones assuming the world is $2^o c$ higher than in the pre-industrial era, which is expected around the end of this century assuming business as usual. The median projection is a 13% increase relative to λ_G and the most pessimistic projection is 2.25 times of λ_G. We thus obtain $\lambda_B = 2.25\lambda_G = 0.304$ and a prior belief of $\pi_0 = 0.104$ so that the expected arrival rate $\mathbb{E}(\lambda) = \pi_0\lambda_B + (1 - \pi_0)\lambda_G = 0.153$, which is a 13% increase from $\lambda_G = 0.135$. While there is heterogeneity across regions, we focus our analysis on the global mean primarily because climate models' projections by region are quite uncertain.

β_0, β_1 **and** α. To calibrate the three mitigation technology parameters, we use the following three moments from the data. First, we estimate the impact of a major cyclone making landfall on GDP growth. Table 2 reports the estimation results for each region and also for the world (with all countries pooled together). The dependent variable is the per capita GDP growth rate. The independent variable is the Landfall indicator. The panel regression has country fixed effects, year fixed effects, and country-specific quadratic time trends. A landfall disaster reduces the annual growth rate by 0.61%, 0.29%, 0.88%, and 2.75% in North Atlantic, West Pacific, North India, and South Atlantic respectively, and by 0.77% in the global sample.

Table 2: Baseline model estimation results

	Dependent variable is the growth rate of real GDP per capita				
	(1) North Atlantic	(2) West Pacific	(3) North India	(4) South Atlantic	(5) Global
Landfall	-0.0061*	-0.0029*	-0.0088***	-0.0275***	-0.0077***
	(-2.01)	(-1.94)	(-3.35)	(-3.69)	(-4.29)
Country FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Time trends	Yes	Yes	Yes	Yes	Yes

Second, we gather data on government flood control budgets focusing on countries in the West Pacific (including Oceania). We are able to obtain through various sources 72 country-

year observations of government flood control budget for a cross section of eight countries.[31] The mean annual government flood control budget is around 0.10% (8.5 bps) of capital stock with a standard deviation of 5 bps across country-years.

In sum, we calibrate β_0, β_1 and α in a world without global warming under the assumption that countries are optimally mitigating cyclone arrivals with a prior of 0.104. We use the following three moments. First, the optimal public mitigation equals 0.1% of the capital stock, $\mathbf{x}^d(0.104) = 0.1\%$. Second, studies indicate that private mitigation also plays an important role and on par with that of public mitigation. So we set the optimal private mitigation spending of around 0.04% of the capital stock, $\mathbf{x}^e(0.104) = 0.04\%$. Third, for the reduction of the expected annual GDP growth rate from the arrival of a major cyclone, the estimates range quite a bit from a low of 0.29% to a large damage of 2.75% per annum depending on the region. We use the mid-range of these estimates and set $N(\mathbf{x}^e)\mathbb{E}^{\mathbf{x}^d}(1 - Z) = 1.2\%$. We use these moments from data to discipline our model on the optimal mitigation.

EIS ψ. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero (e.g., Hall, 1988) to values as high as two. Bansal and Yaron (2004) show that an EIS larger than one is necessary to generate plausible equilibrium asset pricing predictions in long-run risk (LRR) settings. We choose $\psi = 1.5$ following the LRR literature.

Table 3: PARAMETER VALUES

Parameters	Symbol	Value
jump arrival rate in State G	λ_G	0.135
power law exponent absent mitigation	β_0	16
mitigation technology parameter	β_1	$10,000$
mitigation technology parameter	α	0.05
jump arrival rate in State B	λ_B	0.304
prior of being in State B	π_0	0.104
elasticity of intertemporal substitution	ψ	1.5
time rate of preference	ρ	5%
productivity	A	15%
quadratic adjustment cost parameter	θ	15
coefficient of relative risk aversion	γ	3.5
capital diffusion volatility	σ	14%

All parameter values, whenever applicable, are continuously compounded and annualized.

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[31]West Pacific countries include China, Japan, Korea, and the Philippines. Oceania countries include Australia, Indonesia, New Zealand and Papua New Guinea.

Other parameters. We calibrate the adjustment cost parameter θ along with the time rate of preference ρ, risk aversion γ, diffusion volatility σ, and productivity A by targeting five key moments for state G. These include the annual (real) risk-free rate of 2.50%, the expected annual stock market risk premium of 6.98%, the annual stock market return volatility of $\sqrt{0.0206} = 14.14\%$, the expected growth rate of 4.83%, and Tobin's q of 1.75 (e.g., in line with Eberly, Rebelo and Vincent, 2012), when the prior is 0.104. Doing so yields the following parameter values: $\sigma = 14\%$, $\theta = 15$, $\gamma = 3.5$, $A = 15\%$, and $\rho = 5\%$. These parameter values are broadly in line with those used in the literature.[32]

We report the values for all the twelve parameters in Table 3.
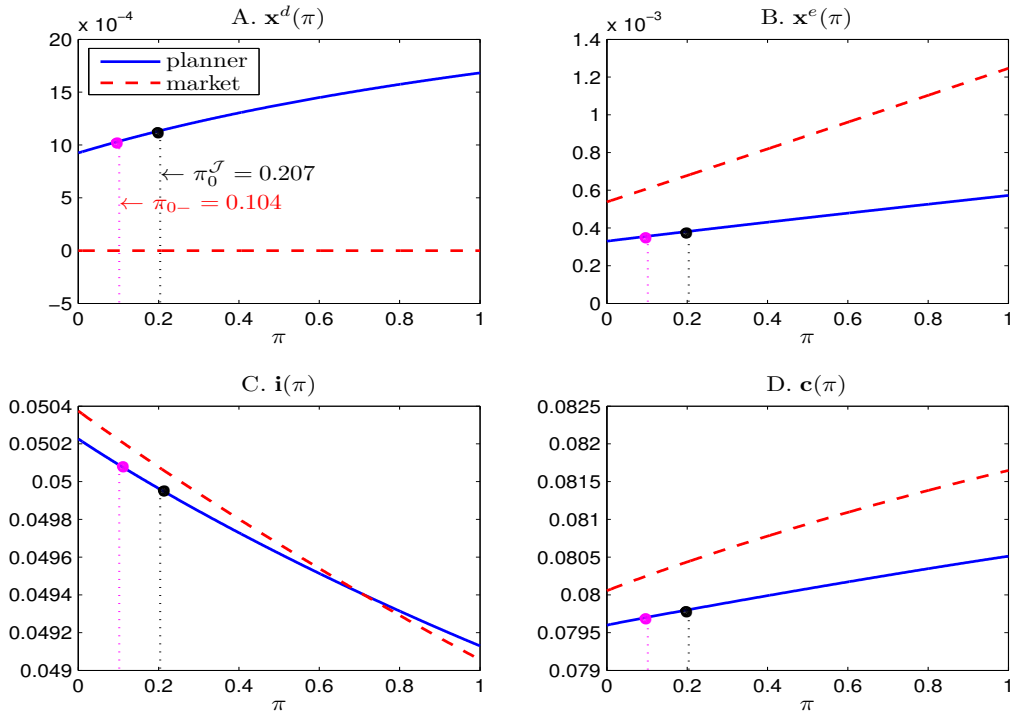
7.3 Impact of Major Cyclone Arrival

Figure 2: This figure compares the planner's first-best solution (solid blue lines) with the market economy solution (dashed red lines). The parameters values are given in Table 3.

Taxes, private versus public mitigation, investment, and consumption. First consider the first-best solution (solid blue lines) in Figure 2. The pink solid dots map to the

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[32]As an example, while using a different calibration strategy (for example, they do not target the capital adjustment costs), Barro and Jin (2011) also report the calibrated coefficient of relative risk aversion in their paper is about three and time rate of preference around 5%. Our estimates are also close to those in Pindyck and Wang (2013), even though they use a different set of moments for the disaster arrival rate and the damage function.

results at the prior of $\pi_{0-} = 0.104$, imputed from the distribution of climate model projections surveyed by Knutson et al. (2020). The black solid dots correspond to the results at the post-jump belief of $\pi_0^{\mathcal{J}} = 0.207$ as a cyclone arrival changes π immediately from the prior of π_{0-} to the posterior of $\pi_0^{\mathcal{J}}$ by a discrete size of $\sigma_\pi(\pi_{0-}) = 0.103$.

Panel A of Figure 2 shows that the first-best scaled public mitigation $\mathbf{x}^d$ increases from 0.1% to about 0.11% per annum. The market solution without taxes features no public mitigation spending regardless of beliefs due to externalities.

Panel B shows that private mitigation $\mathbf{x}^e$ in the market economy is higher than in the planner's first-best economy at all levels of π. The marginal benefit of private mitigation is higher in the market economy than in the planner's economy, as the former has no public mitigation and firms thus optimally spend more on private mitigation to manage aggregate climate risk. The total mitigation spending, $\mathbf{x}^e + \mathbf{x}^d$, is lower in the market economy than in the first-best solution. That is, aggregate risk mitigation is under-provided in a laissez faire economy.

Panel C shows that investment $\mathbf{i}$ decreases as belief worsens in both economies. The rate at which $\mathbf{i}$ decreases with π in the first-best solution is slower (less steep) than in the market economy. Similarly, Panel D shows that the rate at which $\mathbf{c}$ increases with π in the first-best solution is also slower (less steep) than in the market economy.

Suppose that we live in an economy with neither mitigation technology (status quo) and then can gain access to both types of mitigation technologies by paying a cost. How much would we be willing to pay for such a transition in a laissez faire market economy versus in the planner's economy? To answer this question, we introduce the following willingness to pay (WTP) metrics following Pindyck and Wang (2013).

Welfare gains from having access to mitigation technologies. Let $\zeta_p(\pi)$ denote the fraction of capital stock that the society is willing to give up in a market economy with neither mitigation technology[33] for an option to transition to the planner's first-best economy with both mitigation technologies. To make the society indifferent between these two choices, the following condition has to hold:

$$V_{fb}((1 - \zeta_p(\pi))\mathbf{K}, \pi) = \underline{V}(\mathbf{K}, \pi) \, , \tag{66}$$

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[33]The market economy here yields the same first-best outcome as a planner's economy, as there is no externality nor other frictions and the welfare theorems hold.

where $\underline{V}(\mathbf{K}, \pi)$ given in (33) is the value function under the status quo in the economy with neither mitigation technology (see Proposition 3) and V_{fb} given in (23) is the planner's value function in the first-best economy with access to both mitigation technologies (see Proposition 1). Note that the capital stock level appearing in $V_{fb}(\,\cdot\,, \pi)$ is $(1 - \zeta_p(\pi))\mathbf{K}$ adjusted for the WTP calculation (as a $\zeta_p(\pi)$ fraction of $\mathbf{K}$ is given up).

Similarly, let $\zeta_m(\pi)$ denote the fraction of capital stock that society is willing to give up under the status quo in the market economy with neither mitigation technology for an option to transitioning into the market economy, which has access to both types of the mitigation technologies (analyzed in Section 4.) We define $\zeta_m(\pi)$ by using

$$\widehat{V}((1 - \zeta_m(\pi))\mathbf{K}, \pi) = \underline{V}(\mathbf{K}, \pi) \ , \tag{67}$$

where $\widehat{V}$ is the value function as a function of $\mathbf{K}$ and π in the market economy with both mitigation technologies given in (32) (see Propositions 2 and 4). It is important to note that although both technologies are available in the market economy, only the private mitigation technology has demand in equilibrium. Therefore, the WTP ζ_m only captures the society's WTP for the disaster exposure mitigation technology.

By using the homogeneity properties of the value functions, V_{fb}, $\widehat{V}$, and $\underline{V}$ for the first-best, market economy (with access to both mitigation technologies), and the market economy (with access to neither mitigation technology), we obtain

$$\zeta_p(\pi) = 1 - \frac{\underline{b}(\pi)}{b_{fb}(\pi)} \quad \text{and} \quad \zeta_m(\pi) = 1 - \frac{\underline{b}(\pi)}{\widehat{b}(\pi)} > 0 \ . \tag{68}$$

The result $\zeta_p(\pi) > \zeta_m(\pi)$ follows from $b_{fb}(\pi) > \widehat{b}(\pi) > \underline{b}(\pi)$. The WTP $\zeta_m(\pi)$ measures the welfare enhancement solely due to optimal private mitigation regulation/spending, as neither firms nor households incur any public mitigation spending in a market economy: $(\mathbf{x}^d = 0)$.

Therefore, the WTP wedge $\zeta_p(\pi) - \zeta_m(\pi)$ measures the additional welfare gain due to having access to the public mitigation technology in an economy that already has access to the private mitigation technology.

In Panel A of Figure 3, we plot the society's WTP $\zeta_m(\pi)$ for a market economy which uses the private mitigation technology (the dashed red line) and the WTP $\zeta_p(\pi)$ for the planner's economy (or the market economy with optimal capital taxation and hence access to both mitigation technologies) (the solid blue line).[34] Naturally, more pessimistic beliefs lead to a

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[34]We can decompose the WTPs into the risk premium and timing premium components by building on the
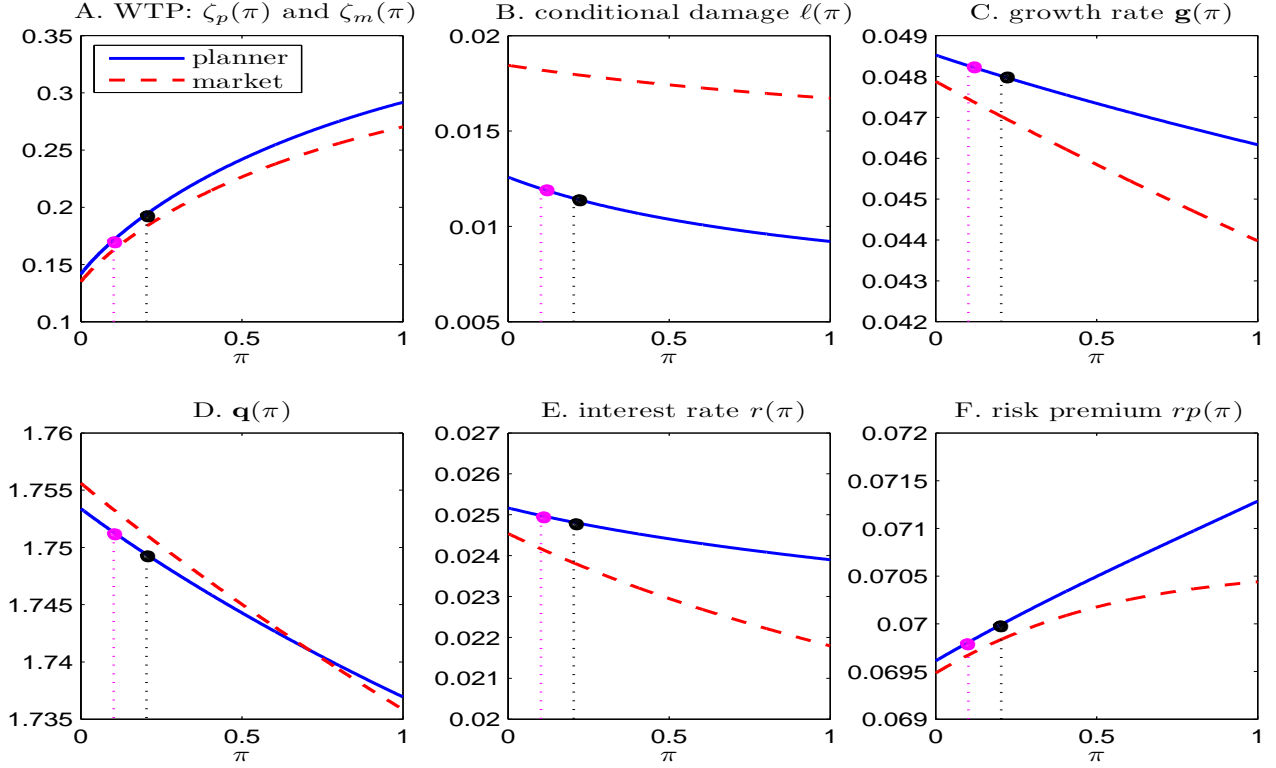
33

Figure 3: This figure compares the planner's first-best solution (solid blue lines) with the market economy solution (dashed red lines). The parameters values are given in Table 3.

higher use of mitigation technology for both economies. For the first-best solution, the WTP increases from 17.2% (the pink dot) to 19.5% (the black dot), which is around 13% increase.

Conditional damage $\ell(\pi)$ and the expected growth rate $\mathbf{g}(\pi)$. In Panel B of Figure 3, we corroborate the benefit of using the mitigation technology by showing that the conditional damage $\ell(\pi)$ decreases as belief worsens and private/public mitigation spendings increase.

In the market economy, the firm only pays for private mitigation x^e, which increases with π. Therefore, in the aggregate economy we have $\mathbf{x}^e = x^e$ and $\mathbf{x}^d = 0$. As a result, the conditional damage for the aggregate economy is $\ell(\pi) = N(\mathbf{x}^e(\pi))/(\beta_0 + 1)$ decreases with π. In the first-best economy, there are both private and public mitigation spendings. Recall that the total disaster risk reduction is larger in the first-best economy than in the market economy explaining why the solid blue line for $\ell(\pi)$ in the planner's economy is lower than the dashed red line for $\ell(\pi)$ in the market economy. Because of larger risk mitigation and smaller

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idea and extending the procedure proposed by Epstein, Farhi, and Strzalecki (2014). We show that for our calibrated baseline where $\gamma = 3 > 1/\psi = 1/1.5$, while the timing premium is also important, the risk premium component is the major contributor to the total WTP.

conditional damage $\ell(\pi)$ in the first-best economy than in the market economy, the expected growth rate $\mathbf{g}(\pi)$ is higher in the former than the latter economy (Panel C).

Tobin's q, interest rate, and risk premium. Finally, in Panels D, E, and F of Figure 3, we plot Tobin's average $\mathbf{q}$, the interest rate $r(\pi)$, and risk premium $rp(\pi)$ for both the first-best and market economies. We show that in both economies the average $\mathbf{q}$ decreases as belief worsens tracking the investment-capital ratio $\mathbf{i}(\pi)$. Note that the interest rate and risk premium are relatively insensitive to the change of belief.

7.4 Comparative Statics

In Online Appendix OD, we conduct comparative static analyses with respect to four key parameters: the EIS ψ, the disaster arrival rate λ_B in state B, the time rate of preference ρ, and the coefficient of relative risk aversion γ.

First, we compare the results for three values of the EIS ψ: 0.286 (the expected utility case as $\gamma = 3.5$), 1 and 1.5 (our baseline). Our main mitigation findings are robust across these three parameter values. The main difference lies in valuation ratios, e.g., the price-dividend ratio. When EIS $\psi = 1$, the price-dividend ratio, $\mathbf{q}/\mathbf{c}$, equals $1/\rho$, the inverse of the time rate of preference, for all levels of π, which is known in the asset pricing literature, e.g., Weil (1990) and Wachter (2013). When ψ is greater (less) than one, this $\mathbf{q}/\mathbf{c}$ ratio decreases (increases) with π. That is, equity valuation ratios react negatively to bad (e.g., disaster arrival) news consistent with the reason why the long-run risk literature chooses $\psi > 1$.

Second, we compare the results for three values of λ_B: 0.15, 0.3 (baseline) and 0.6 per annum. The effects of an economically meaningful change of λ are quantitatively significant. For example, doubling the arrival rate of disaster in state B from $\lambda_B = 0.3$ (our baseline) to $\lambda_B = 0.6$ roughly doubles the WTP for mitigation.

Third, we compare the results for three widely used values of the time rate of preference ρ: 3%, 4%, and 5% (our baseline). Decreasing ρ (e.g., from 5% to 4%) has a large quantitative effect on mitigation spendings and willingness-to-pay for mitigation. Decreasing ρ from 5% (our baseline) to 4% makes the agents allocate more resources for mitigation and investment to finance future consumption. At the prior belief of 0.104, this change nearly doubles the WTP from our baseline.

Finally, we compare the results for three values of the coefficient of relative risk aversion γ: 1.5, 3.5, and 10. We find that the quantitative effects of changing γ from 3.5 to 10 are

large. For example, when $\pi = 0.104$ as we increase γ from 3.5 to 10, the WTP ζ_p increases from 17.2% to 29.4%, the disaster distribution mitigation spending $\mathbf{x}^d$ increases from 0.1% to 0.13%, and the disaster exposure mitigation spending $\mathbf{x}^e$ decreases from 0.04% to 0.03%. The effects of changing risk aversion become even larger as belief worsens.

7.5 Generalized Learning Model with Stochastic Arrival Rate λ

The disaster arrival rate in our baseline model of Section 2, while unobservable, is constant.[35] In Appendix OA, we generalize our baseline model to allow for the unobservable disaster arrival rate to be stochastic, by using a two-state Markov Chain (see, e.g., Wachter and Zhu (2019, 2021)).[36] We show that our main quantitative results and conclusions continue to hold in the generalized model where the transition probability between state G and B is small.

8 External Habit Model

In this section, we replace the Epstein-Zin recursive utility used in our baseline model of Section 2 with another widely-used risk preference—the external habit model proposed by Campbell and Cochrane (1999).[37] For brevity, we focus on the planner's solution.

8.1 Model

The representative agent has a non-expected utility over consumption $\{C_t; t \geq 0\}$ relative to a stochastic habit process $\{\mathcal{H}_t; t \geq 0\}$ (Campbell and Cochrane, 1999) given by:

$$\mathbb{E}\left(\int_0^\infty \rho e^{-\rho t} U(C_t, \mathcal{H}_t) dt\right), \tag{69}$$

where $\rho > 0$ is the time rate of preference, $U(C, \mathcal{H}) = \frac{(C-\mathcal{H})^{1-\gamma}}{1-\gamma}$, and $\gamma > 0$ is a curvature parameter. It is convenient to work with S_t, the surplus consumption ratio at t defined as

$$S_t = \frac{C_t - \mathcal{H}_t}{C_t}. \tag{70}$$

Let s_t be its natural logarithm: $s_t = \ln(S_t)$. As in Campbell and Cochrane (1999) and this literature, we assume that s_t follows a mean-reverting process with stochastic volatility:

$$ds_t = (1-\kappa)(\bar{s} - s_t)dt + \delta(s_t)\sigma d\mathcal{W}_t, \tag{71}$$

---

[35]Collin-Dufresne, Johannes, and Lochstoer (2016) develop equilibrium asset pricing models where the agent learns about the parameter to address standard macro asset-pricing challenges.

[36]Ghaderi, Kilic, and Seo (2022) develop a Bayesian learning model that builds on Wachter (2013).

[37]An alternative to the external habit model analyzed in this section is to specify an internal habit formation model as in Jermann (1998). Due to space constraints, we leave the internal habit formation model out.

where $\overline{s} > 0$ is the steady-state value of s_t and κ measures the degree of persistence.[38] The function $\delta(s_t)$ in (71) is the same sensitivity function as the one in Campbell and Cochrane (1999) and stated in Section OB of the Online Appendix. The production side of the economy and the learning model are the same as in our baseline model of Section 2.

Planner's solution. The (log) surplus consumption ratio $\{s_t; t \geq 0\}$ acting as the exogenous preferences shock is the new state variable. Let $V(\mathbf{K}, \pi, s)$ denote the household's value function. The following HJB equation characterizes the planner's optimal resource allocation:

$$
\rho V = \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \, \mathbf{x}^d} \; \rho \frac{(\mathbf{C}e^s)^{1-\gamma}}{1-\gamma} + \mathbf{I}V_{\mathbf{K}} + \mu_\pi(\pi)V_\pi + (1-\kappa)(\overline{s} - s)V_s + \frac{\sigma^2 \mathbf{K}^2 V_{\mathbf{KK}}}{2} + \frac{1}{2}\sigma^2 \delta(s)^2 V_{ss}
$$
$$
+ \sigma^2 \delta(s)\mathbf{K}V_{\mathbf{K}s} + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \pi^{\mathcal{J}}, s\right) - V(\mathbf{K}, \pi, s) \right]. \quad (72)
$$

Unlike in our baseline model with the Epstein-Zin utility, the agent now not only takes into account the evolution of s (via the drift term involving V_s and the quadratic-variation term involving V_{ss}), but also has incentives to hedge against shocks to the surplus consumption ratio (via the quadratic-covariation term involving $V_{\mathbf{K}s}$).

We show that the value function $V(\mathbf{K}, \pi, s)$ is homogeneous with degree $(1 - \gamma)$ in $\mathbf{K}$:

$$
V(\mathbf{K}, \pi, s) = \frac{1}{1-\gamma}\left(b(\pi, s)\mathbf{K}\right)^{1-\gamma}, \quad (73)
$$

where $b(\pi, s)$ is a measure of welfare proportional to the certainty equivalent wealth under optimality. (To ease comparison, we still use b as the function for the welfare measure here but with the understanding that the b function for external habit model depends on both π and s and differs from the b function for our baseline Epstein-Zin model.)

Importantly, unlike the welfare measure ($b(\pi)$) in our baseline planner's model of Section 3, $b(\pi, s)$ in our external habit model depends on not only belief π but also the (log) surplus consumption ratio s. In Online Appendix OB, we provide details summarizing how we obtain the PDE for $b(\pi, s)$ together with optimal policies and boundary conditions. Our external habit model is technically more challenging than our baseline model with Epstein-Zin utility, as the external habit becomes an additional state variable in addition to capital stock and belief.[39]

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[38]We write $1 - \kappa$ as the rate of mean reversion as in Campbell and Cochrane (2015). The higher the value of κ, the more persistent the s_t process. The $\kappa = 1$ special case corresponds to a unit-root process.

[39]Because of the homogeneity property of the Epstein-Zin utility, only capital stock and belief are state variables after simplifying the model solution.

8.2 Quantitative Results

We calibrate our external habit model by targeting the same moments as we do for our baseline model whenever feasible. We highlight two main results. First, the quantitative implications on mitigation spendings and welfare in our external habit model are similar to those in our baseline model with Epstein-Zin preferences. Second, the two models generate opposite predictions on how investment **i** and Tobin's average **q** respond as belief becomes more pessimistic (π increases). While **i** and **q** increase with π in our habit model, the opposite holds in our baseline Epstein-Zin model. To see why, note that replacing the Epstein-Zin utility with the external Campbell-Cochrane habit model induces two key changes: (1) introducing pure external habit and (2) dropping the EIS ψ parameter. While the first force significantly raises risk aversion, the latter makes the agent much less willing to substitute consumption over time in response to changes in the interest rate, especially when endogenous risk aversion is high (which implies a very low EIS).[40]

9 Conclusion

We develop a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate potential future damages. Mitigation—by curtailing aggregate risk and insuring sustainable growth—is undersupplied relative to the first-best planner's solution in competitive markets due to externalities. The planner's solution can be implemented via a capital tax and mitigation subsidy scheme. Our model provides an integrated assessment of the cost and benefit of mitigation efforts such as flood protection via an aggregate risk management rationale. It also delivers a number of new results for the literature on the adaptation of the economy to global warming. While the literature on local fiscal multipliers (Leduc and Wilson (2013) and Ramey (2011)) is a cautionary tale on the limits of general equilibrium analysis such ours in understanding the impact of government spending, our analysis suggests it is possible to arrive at robust conclusions on the effectiveness of public and private mitigation investments for reducing the conditional damage of disasters and improving social welfare.

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[40]From the long-run risk literature and the comparative static analysis for our baseline Epstein-Zin model with respect to ψ in Section OD of the Online Appendix, we know that an EIS (lower than one) causes the valuation ratios, e.g., the price-dividend ratio, to go up in response to bad news. Our habit model inherits this property, which explains the key differences between the two utility models.

References

Abatzoglou, J.T. and Williams, A.P., 2016. Impact of anthropogenic climate change on wildfire across western US forests. *Proceedings of the National Academy of Sciences*, 113(42), pp.11770-11775.

Abel, A.B. and Eberly, J.C., 1994. A unified model of investment under uncertainty. *American Economic Review*, 84, pp.1369-1384.

Anbarci, N., Escaleras, M. and Register, C.A., 2005. Earthquake fatalities: the interaction of nature and political economy. *Journal of Public Economics*, 89(9-10), pp.1907-1933.

Bakkensen, L.A. and Barrage, L., 2017. Flood risk belief heterogeneity and coastal home price dynamics: Going under water? (No. w23854). National Bureau of Economic Research.

Bansal, R. and Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), pp.1481-1509.

Bansal, R., Ochoa, M. and Kiku, D., 2017. Climate change and growth risks (No. w23009). National Bureau of Economic Research.

Barnett, M., Brock, W. and Hansen, L.P., 2020. Pricing uncertainty induced by climate change. *Review of Financial Studies*, 33(3), pp.1024-1066.

Barro, R.J., 2006. Rare Disasters and Asset Markets in the Twentieth Century. *Quarterly Journal of Economics*, 121, pp.823-866.

Barro, R.J. and Jin, T., 2011. On the size distribution of macroeconomic disasters. *Econometrica*, 79(5), pp.1567-1589.

Bouwer, L.M., Crompton, R.P., Faust, E., Happe, P. and Pielke Jr, R.A., 2007. Confronting disaster losses. *Science-New York then Washington*, 318(5851), p.753.

Bouwer, L.M., 2011. Have disaster losses increased due to anthropogenic climate change?. *Bulletin of the American Meteorological Society*, 92(1), pp.39-46.

Cai, Y. and Lontzek, T.S., 2019. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6), pp.2684-2734.

Campbell, J.Y. and Cochrane, J.H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2), pp.205-251.

Campbell, J.Y. and Cochrane, J.H., 2015. The Fragile Benefits of Endowment Destruction. *Journal of Political Economy*, 123(5), pp.1214-1226.

Collin-Dufresne, P., Johannes, M. and Lochstoer, L.A., 2016. Parameter learning in general equilibrium: The asset pricing implications. *American Economic Review*, 106(3), pp.664-698.

Daniel, K.D., Litterman, R.B. and Wagner, G., 2019. Declining CO2 price paths. *Proceedings of the National Academy of Sciences*, 116(42), pp.20886-20891.

Dell, M., Jones, B.F. and Olken, B.A., 2014. What do we learn from the weather? The new climate-economy literature. *Journal of Economic Literature*, 52(3), pp.740-98.

Duffie, D., 2001. Dynamic asset pricing theory, Third edition, *Princeton University Press*.

Duffie, D. and Epstein, L.G., 1992. Stochastic differential utility. *Econometrica*, 60(2), pp.353-394. Investment and value: A neoclassical benchmark, Working paper, Northwestern University.

Eberly, J., Rebelo, S. and Vincent, N., 2012. What explains the lagged-investment effect?. Journal of Monetary Economics, 59(4), pp.370-380.

Epstein, L.G., Farhi, E. and Strzalecki, T., 2014. How much would you pay to resolve long-run risk?. *American Economic Review*, 104(9), pp.2680-97.

Epstein, L.G. and Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption. *Econometrica*, 57(4), pp.937-969.

Emanuel, K., 2005. Increasing destructiveness of tropical cyclones over the past 30 years. *Nature*, 436(7051), pp.686-688.

Gabaix, X., 2009. Power laws in economics and finance. *Annual Review Economics*, 1(1), pp.255-294.

Gabaix, X., 2012. Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly Journal of Economics*, 127(2), pp.645-700.

Genovese, E. and Thaler, T., 2020. The benefits of flood mitigation strategies: effectiveness of integrated protection measures. *AIMS Geosciences*, 6(4), pp.459-472.

Ghaderi, M., Kilic, M. and Seo, S.B., 2022. Learning, Slowly Unfolding Disasters, and Asset Prices. *Journal of Financial Economics,* 143, pp.527-549.

Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1), pp.41-88.

Gourio, F., 2012. Disaster risk and business cycles. *American Economic Review*, 102(6), pp.2734-2766.

Hall, R.E., 1988. Intertemporal substitution in consumption. *Journal of Political Economy*, 96(2), pp.339-357.

Hayashi, F., 1982. Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica*, 50: 215-224.

Hong, H., Karolyi, G.A. and Scheinkman, J.A., 2020. Climate finance. *Review of Financial Studies*, 33(3), pp.1011-1023.

Hsiang, S.M. and Jina, A.S., 2014. The causal effect of environmental catastrophe on long-run economic growth: Evidence from 6,700 cyclones (No. w20352). National Bureau of Economic Research.

Jensen, S. and Traeger, C.P., 2014. Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69, pp.104-125.

Jermann, U. J., 1998. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2), pp.257-275.

Knutson, T., Camargo, S.J., Chan, J.C., Emanuel, K., Ho, C.H., Kossin, J., Mohapatra, M., Satoh, M., Sugi, M., Walsh, K. and Wu, L., 2020. Tropical cyclones and climate change assessment: Part II: Projected response to anthropogenic warming. *Bulletin of the American Meteorological Society*, 101(3), pp.E303-E322.

Kossin, J.P., Knapp, K.R., Olander, T.L. and Velden, C.S., 2020. Global increase in major tropical cyclone exceedance probability over the past four decades. *Proceedings of the National Academy of Sciences*, 117(22), pp.11975-11980.

Kousky, C., Luttmer, E.F. and Zeckhauser, R.J., 2006. Private investment and government protection. *Journal of Risk and Uncertainty*, 33(1-2), pp.73-100.

Leduc, S. and Wilson, D., 2013. Roads to prosperity or bridges to nowhere? Theory and evidence on the impact of public infrastructure investment. *NBER Macroeconomics Annual*, 27(1), pp.89-142.

Liptser, R.S. and Shiryaev, A.N., 2001. Statistics of random processes II: Applications. New York: Springer-Verlag, 2ed.

Lucas Jr, Robert E., 1978. Asset prices in an exchange economy. Econometrica, 46(6), pp.1429-1445.

Martin, I.W. and Pindyck, R.S., 2015. Averting catastrophes: The strange economics of Scylla and Charybdis. *American Economic Review*, 105(10), pp.2947-2985.

Merton, R.C., 1971. Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, 3(4), pp.373-413.

National Academy of Sciences (2016), *Attribution of extreme weather events in the context of climate change*. Washington, D.C.: The National Academies Press.

Nordhaus, W.D., 2017. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7), pp.1518-1523.

Pielke Jr, R.A., Gratz, J., Landsea, C.W., Collins, D., Saunders, M.A. and Musulin, R., 2008. Normalized hurricane damage in the United States: 1900-2005. *Natural Hazards Review*, 9(1), pp.29-42.

Pindyck, R.S. and Wang, N., 2013. The economic and policy consequences of catastrophes. *American Economic Journal: Economic Policy*, 5(4), pp.306-339.

Ramey, V.A., 2011. Can government purchases stimulate the economy?. *Journal of Economic Literature*, 49(3), pp.673-85.

Rietz, T. A., 1988. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1), pp.117-131.

Smith, V.K., Carbone, J.C., Pope, J.C., Hallstrom, D.G. and Darden, M.E., 2006. Adjusting to natural disasters. *Journal of Risk and Uncertainty*, 33(1), pp.37-54.

Wachter, J.A., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility?. *Journal of Finance*, 68(3), pp.987-1035.

Wachter, J.A. and Zhu, Y., 2019. Learning with rare disasters. Wharton School Working Paper.

Wachter, J.A. and Zhu, Y., 2021. A model of two days: discrete news and asset prices. Wharton School Working Paper.

Weil, P., 1990. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, 105(1), pp.29-42.

Appendices

A   Proof for Planner's First-Best Economy in Section 3

A.1   Planner's Resource Allocation

Substituting the value function (23) into the FOC (20) for investment, the FOC (21) for the aggregate disaster distribution mitigation spending, and the FOC (22) for the aggregate disaster exposure mitigation spending, we obtain:

$$b(\pi) = \mathbf{c}(\pi)^{1/(1-\psi)} \left[\rho(1 + \phi'(\mathbf{i}(\pi)))\right]^{-\psi/(1-\psi)} , \tag{A.1}$$

$$\rho \mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \frac{\lambda(\pi)}{1-\gamma} \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right)^{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d)}{\partial \mathbf{x}^d}(1 - N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}\right] dZ , \tag{A.2}$$

$$\rho \mathbf{c}(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \lambda(\pi) \left[\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right]^{1-\gamma} N'(\mathbf{x}^e(\pi)) \mathbb{E}^{\mathbf{x}^d(\pi)} \left[(Z-1)(1 - N(\mathbf{x}^e(\pi))(1-Z))^{-\gamma}\right] , \tag{A.3}$$

where the post-jump $\pi^{\mathcal{J}}$ is given in (14) as a function of the pre-jump π. Substituting the resource constraint, $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^d(\pi) - \mathbf{x}^e(\pi)$, into (A.1), we obtain (25). Substituting (A.1) into (A.3), we obtain (26) and substituting (A.1) into (A.2), we obtain (27). Finally, substituting the value function (23) and the FOC (25) into the HJB equation (18), we obtain the ODE (24).

At $\pi = 1$, we have the following four equations that characterize $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$, and $\mathbf{x}^e(1)$:

$$0 = \frac{\left(\frac{b(1)}{\rho(1+\phi'(\mathbf{i}(1)))}\right)^{1-\psi} - 1}{1 - \psi^{-1}} \rho + \mathbf{i}(1) - \frac{\gamma\sigma^2}{2} + \frac{\lambda_B \left[\mathbb{E}^{\mathbf{x}^d(1)}((1 - N(\mathbf{x}^e(1))(1-Z))^{1-\gamma}) - 1\right]}{1 - \gamma} , \tag{A.4}$$

$$b(1) = [A - \mathbf{i}(1) - \phi(\mathbf{i}(1)) - \mathbf{x}^d(1) - \mathbf{x}^e(1)]^{1/(1-\psi)} \left[\rho(1 + \phi'(\mathbf{i}(1)))\right]^{-\psi/(1-\psi)} , \tag{A.5}$$

$$1 = \lambda_B(1 + \phi'(\mathbf{i}(1))) N'(\mathbf{x}^e(1)) \mathbb{E}^{\mathbf{x}^d(1)} \left[(Z-1)(1 - N(\mathbf{x}^e(1))(1-Z))^{-\gamma}\right] , \tag{A.6}$$

$$1 = \frac{\lambda_B(1 + \phi'(\mathbf{i}(1)))}{1 - \gamma} \int_0^1 \left[\frac{\partial \xi(Z; \mathbf{x}^d(1))}{\partial \mathbf{x}^d}(1 - N(\mathbf{x}^e(1))(1-Z))^{1-\gamma}\right] dZ . \tag{A.7}$$

Solving (A.4)-(A.7) yields $b(1)$, $\mathbf{i}(1)$, $\mathbf{x}^d(1)$, and $\mathbf{x}^e(1)$. Similarly, we obtain the four equations, (28)-(31), for the left boundary, $\pi = 0$. Solving these four equations yields $b(0)$, $\mathbf{i}(0)$, $\mathbf{x}^d(0)$, and $\mathbf{x}^e(0)$. In sum, we now have fully characterized the model solution summarized in Proposition 1.

A.2   Asset Pricing Implications of the Planner's Problem

Duffie and Epstein (1992) show that the SDF $\{\mathbb{M}_t : t \geq 0\}$ implied by the planner's solution is given by:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(\mathbf{C}_s, V_s) \, ds\right] f_{\mathbf{C}}(\mathbf{C}_t, V_t) . \tag{A.8}$$

Using the FOC for investment (20), the value function (23), and the resource constraint, we obtain:

$$f_{\mathbf{C}}(\mathbf{C}, V) = \frac{1}{1 + \phi'(\mathbf{i}(\pi))} b(\pi)^{1-\gamma} \mathbf{K}^{-\gamma} \tag{A.9}$$

and

$$f_V(\mathbf{C}, V) = \frac{\rho}{1 - \psi^{-1}} \left[\frac{(1 - \omega)\mathbf{C}^{1-\psi^{-1}}}{((1 - \gamma))^{\omega - 1}} V^{-\omega} - (1 - \gamma) \right] = -\epsilon(\pi), \qquad (A.10)$$

where

$$\epsilon(\pi) = -\frac{\rho(1 - \gamma)}{1 - \psi^{-1}} \left[\left(\frac{\mathbf{c}(\pi)}{b(\pi)} \right)^{1-\psi^{-1}} \left(\frac{\psi^{-1} - \gamma}{1 - \gamma} \right) - 1 \right]. \qquad (A.11)$$

Using the equilibrium relation between $b(\pi)$ and $\mathbf{c}(\pi)$, we simplify (A.11) as:

$$\epsilon(\pi) = \rho + \left(\psi^{-1} - \gamma \right) \left[\mathbf{i}(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi) \frac{b'(\pi)}{b(\pi)} \right]$$

$$+ \left(\psi^{-1} - \gamma \right) \left[\frac{\lambda(\pi)}{1 - \gamma} \left(\left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d} \left[(1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma} \right] - 1 \right) \right], \qquad (A.12)$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given in (14) as a function of the pre-jump belief π. Note that for expected utility where $\psi = 1/\gamma$, we have $\epsilon(\pi) = \rho$. Using Ito's Lemma and the optimal allocation, we obtain

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -\epsilon(\pi)dt - \gamma \left[\mathbf{i}(\pi)dt + \sigma d\mathcal{W}_t \right] + \frac{\gamma(\gamma+1)}{2}\sigma^2 dt + \left((1 - \gamma)\frac{b'(\pi)}{b(\pi)} - \frac{\mathbf{i}'(\pi)\phi''(\mathbf{i}(\pi))}{1 + \phi'(\mathbf{i}(\pi))} \right) \mu_\pi(\pi)dt$$

$$+ \left[\frac{1 + \phi'(\mathbf{i}(\pi))}{1 + \phi'(\mathbf{i}(\pi^{\mathcal{J}}))} \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} (1 - N(\mathbf{x}^e)(1 - Z))^{-\gamma} - 1 \right] d\mathcal{J}_t. \qquad (A.13)$$

As the expected percentage change of $\mathbb{M}_t$ equals $-r_t$ per unit of time (Duffie, 2001), we obtain the following expression for the interest rate:

$$r(\pi) = \rho + \psi^{-1}\mathbf{i}(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1 - \psi^{-1})\frac{b'(\pi)}{b(\pi)} - \frac{\mathbf{i}'(\pi)\phi''(\mathbf{i}(\pi))}{1 + \phi'(\mathbf{i}(\pi))} \right] \mu_\pi(\pi)$$

$$- \lambda(\pi) \left[\frac{1 + \phi'(\mathbf{i}(\pi))}{1 + \phi'(\mathbf{i}(\pi^{\mathcal{J}}))} \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))^{-\gamma}) - 1 \right]$$

$$- \lambda(\pi) \left[\frac{\psi^{-1} - \gamma}{1 - \gamma} \left(1 - \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))^{1-\gamma}) \right) \right]. \qquad (A.14)$$

Since $\mathbf{D}_t = \mathbf{C}_t$ in equilibrium and $\mathbb{M}_{t-}\mathbf{D}_{t-}dt + d(\mathbb{M}_t\mathbf{Q}_t)$ is a martingale under the physical measure (Duffie, 2001), using Ito's Lemma, setting its drift to zero, and simplifying the expression, we obtain

$$\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} = \rho - (1 - \psi^{-1}) \left[\mathbf{i}(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \right]$$

$$+ \lambda(\pi)\omega \left[1 - \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d} \left[1 - N(\mathbf{x}^e)(1 - Z) \right]^{1-\gamma} \right], \qquad (A.15)$$

where $\omega = (1 - \psi^{-1})/(1 - \gamma)$. We obtain Tobin's average q from (A.15). For the special case with $\psi = 1$ and any risk aversion $\gamma > 0$, the dividend yield (and equivalently the consumption-wealth ratio) is $\mathbf{c}(\pi)/\mathbf{q}(\pi) = \rho$.

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B Proof for Market Equilibrium Solution in Section 4

B.1 Firm Value Maximization

First, using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t, \pi_t)$:

$$dQ_t = \left(IQ_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi \right) dt + \sigma K Q_K d\mathcal{W}_t$$
$$+ \left(Q((1 - N(x^e)(1 - Z))K, \pi^\mathcal{J}) - Q(K, \pi) \right) d\mathcal{J}_t. \tag{B.16}$$

No arbitrage implies that the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X_{t-}^e - X_{t-}^d)dt + d(\mathbb{M}_t Q_t)$ is zero, where the SDF is given by (34). By applying Ito's Lemma to this martingale, we obtain

$$0 = \max_{I, x^e, x^d} \mathbb{M}(AK - I - \Phi(I, K) - x^e K - x^d K) + \mathbb{M}\left(Q_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi \right)$$
$$+ Q\left[-r(\pi) - \lambda(\pi)\left(\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1 \right) \right] \mathbb{M} - \mathbb{M}\gamma\sigma^2 K Q_K$$
$$+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi; Z, \mathbf{x}^e)Q((1 - N(x^e)(1 - Z))K, \pi^\mathcal{J}) - Q(K, \pi) \right] \mathbb{M}. \tag{B.17}$$

And then by using the homogeneity property $Q(K, \pi) = q(\pi)K$, we obtain the simplified HJB equation (36). Simplifying the FOC for the exposure mitigation spending implied by (B.17), we obtain (37). Similarly, simplifying the investment FOC implied by (B.17), we obtain (38).

B.2 Household's Optimization Problem

We conjecture and verify that the cum-dividend return of the aggregate asset market is given by

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \mu_\mathbf{Q}(\pi_{t-})dt + \sigma d\mathcal{W}_t + \left(\frac{\mathbf{Q}_t^\mathcal{J}}{\mathbf{Q}_{t-}} - 1 \right) d\mathcal{J}_t, \tag{B.18}$$

where $\mu_\mathbf{Q}(\pi)$ is the expected cum-dividend return (leaving aside the jump effect), defined in (17), to be determined in equilibrium. In (B.18), the diffusion volatility in equilibrium equals σ, the same parameter for the capital accumulation process given in (2). The representative household accumulates wealth as:[41]

$$dW_t = r(\pi_{t-})W_{t-}dt + (\mu_\mathbf{Q}(\pi_{t-}) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt + \left(\frac{\mathbf{Q}_t^\mathcal{J}}{\mathbf{Q}_{t-}} - 1 \right) H_{t-}d\mathcal{J}_t. \tag{B.19}$$

By using the W process given in (B.19), we obtain the HJB equation (40) for the household's value function. The FOCs for consumption C and the market portfolio allocation H are given by

$$f_C(C, J) = J_W(W, \pi) \tag{B.20}$$

$$\sigma^2 H J_{WW}(W, \pi) = -(\mu_\mathbf{Q}(\pi) - r(\pi))J_W(W, \pi) + \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\left(1 - \frac{\mathbf{Q}^\mathcal{J}}{\mathbf{Q}} \right) J_W\left(W^\mathcal{J}, \pi^\mathcal{J} \right) \right]. \tag{B.21}$$

Subsituting (39) into (B.20), we obtain the optimal consumption rule given by (43). Simplifying the FOC for H given by (B.21), we obtain (44).

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[41]The first four terms in (B.19) are standard as in the classic portfolio-choice problem with no insurance or disasters (Merton, 1971). The last term is the loss of the household's wealth from her portfolio's exposure to the market portfolio. (We leave out the disaster insurance demand as they net out to zero in equilibrium and do not change the equilibrium analysis.) Pindyck and Wang (2013) provide a detailed description of their dynamically complete markets setting (with various diffusion and stage-contingent actuarially fair jump hedging contracts.). Our dynamically complete markets setting builds on Pindyck and Wang (2013).

B.3 Market Equilibrium

First, the firm's disaster exposure mitigation spending is positive and equals the aggregate exposure mitigation spending: $x^e = \mathbf{x}^e > 0$. Second, in equilibrium, the household invests all wealth in the market portfolio and holds no risk-free asset, $H = W$ and $W = \mathbf{Q}$. Simplifying the FOCs, (43) and (44), and using the value function (39), we obtain:

$$c(\pi) = \rho^\psi u(\pi)^{1-\psi} \mathbf{q}(\pi), \tag{B.22}$$

$$\mu_{\mathbf{Q}}(\pi) = r(\pi) + \gamma\sigma^2$$
$$+ \lambda(\pi) \left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e)) \right]. \tag{B.23}$$

Then substituting (39) into the HJB equation (40), we obtain (45). Using these equilibrium conditions, we simplify the HJB equation (40) as follows:

$$0 = \frac{1}{1 - \psi^{-1}} \left(\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - \rho \right) + \left(\mu_{\mathbf{Q}}(\pi) - \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} \right) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi) \frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)}$$
$$+ \frac{\lambda(\pi)}{1 - \gamma} \left[\frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z)\eta(\pi; Z, \mathbf{x}^e))) - 1 \right]. \tag{B.24}$$

Third, by substituting $\mathbf{c}(\pi) = A - \mathbf{i}(\pi) - \phi(\mathbf{i}(\pi)) - \mathbf{x}^e$ into (36), we obtain

$$0 = \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} - r(\pi) + \mathbf{i}(\pi) + \mu_\pi(\pi) \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} - \gamma\sigma^2$$
$$- \lambda(\pi) \left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e)) \right]. \tag{B.25}$$

By using the homogeneity property and comparing (B.18) and (B.16), we obtain

$$\mu_{\mathbf{Q}}(\pi) = \frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} + \mathbf{i}(\pi) + \mu_\pi(\pi) \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)}. \tag{B.26}$$

Then substituting (B.26) into (B.24), we obtain

$$\frac{\mathbf{c}(\pi)}{\mathbf{q}(\pi)} = \rho - (1 - \psi^{-1}) \left[\mathbf{i}(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi) \left(\frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} \right) \right]$$
$$+ \lambda(\pi) \left(\frac{1 - \psi^{-1}}{1 - \gamma} \right) \left[1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e)) \right]. \tag{B.27}$$

Substituting (B.27) into (B.25), we obtain the following expression for the equilibrium risk-free rate:

$$r(\pi) = \rho + \psi^{-1}\mathbf{i}(\pi) - \frac{\gamma(\psi^{-1} + 1)\sigma^2}{2} - \left[(1 - \psi^{-1}) \left(\frac{\mathbf{u}'(\pi)}{\mathbf{u}(\pi)} + \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} \right) - \frac{\mathbf{q}'(\pi)}{\mathbf{q}(\pi)} \right] \mu_\pi(\pi)$$
$$- \lambda(\pi) \left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi; Z, \mathbf{x}^e)) - 1 \right]$$
$$- \lambda(\pi) \left[\frac{\psi^{-1} - \gamma}{1 - \gamma} \left(1 - \frac{\mathbf{q}(\pi^{\mathcal{J}})}{\mathbf{q}(\pi)} \mathbb{E}^{\mathbf{x}^d}((1 - N(\mathbf{x}^e)(1 - Z))\eta(\pi; Z, \mathbf{x}^e)) \right) \right]. \tag{B.28}$$

Using (B.18) and (B.23), we obtain the following expression for the market risk premium $rp(\pi)$:

$$rp(\pi) = \mu_{\mathbf{Q}}(\pi) + \lambda(\pi) \left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1 \right) - r(\pi) = \gamma\sigma^2 - \lambda(\pi)\mathbb{E}^{\mathbf{x}^d} \left[(\eta(\pi; Z, \mathbf{x}^e) - 1) \left(\frac{\mathbf{Q}^{\mathcal{J}}}{\mathbf{Q}} - 1 \right) \right], \tag{B.29}$$

which implies (59).

In sum, we have derived the equilibrium resource allocation and the asset pricing implications summarized in Proposition 4 and Proposition 6.

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Online Appendices

OA   Model with Stochastic Disaster Arrival Rate

The disaster arrival rate in our baseline model of Section 2, while unobservable, is constant. In this section, we generalize the baseline model to allow for the unobservable disaster arrival rate to be stochastic. We assume that the disaster arrival rate follows a two-state continuous-time Markov chain taking two possible values, λ_G in state G and $\lambda_B > \lambda_G$ in state B. Let φ_G denote the transition rate from state G to state B and φ_B denote the transition rate from state B to state G. That is, over a small time period Δt, the transition probability from the G state to the B state is $\varphi_G \Delta t$ and similarly the transition probability from the B state to the G state is $\varphi_B \Delta t$. Our baseline unobservable constant λ model of Section 2 is a special case of this model with $\varphi_G = \varphi_B = 0$.

OA.1   Model

As in our baseline model, let π_t denote the conditional probability that the economy is in state B. The belief process $\{\pi_t\}$ evolves as:

$$d\pi_t = \mathbb{E}_{t-}[d\pi_t] + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) , \tag{OA.1}$$

where $\sigma_\pi(\pi)$ is given by (13) and $\lambda_{t-} = \lambda_B \pi_{t-} + \lambda_G(1 - \pi_{t-})$ is the expected disaster arrival rate at $t-$ given in (11). Note that the second term is a martingale by construction. Since the economy follows a two-state Markov china, the expected change of belief is given by

$$\mathbb{E}_{t-}[d\pi_t] = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt . \tag{OA.2}$$

We can thus rewrite (OA.1) as follows:

$$d\pi_t = (\varphi_G - (\varphi_B + \varphi_G)\pi_{t-})dt + \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) . \tag{OA.3}$$

Equation (OA.3) implies that π_t in our generalized model is no longer a martingale. This is in sharp contrast with our baseline model (with constant arrival rate), where belief π_t given in (12) is a martingale.

Rewriting the drift term in (OA.3), we see that the expected change of belief π_t in our generalized learning model is given by the difference between $\varphi_G(1 - \pi_{t-})$, which is the transition rate out of state G, φ_G, multiplied by $1 - \pi_{t-}$, the conditional probability in state G, and $\varphi_B \pi_t$, which is the transition rate out of state B, φ_B, multiplied by π_{t-}, the conditional probability in state B.[1]

We note that the jump martingale term (the second term in (OA.3)) in our generalized model is the same as in the belief updating process (12) for our baseline model. As a result, when a disaster strikes at t, the belief immediately increases from the pre-jump level π_{t-} to $\pi_t = \pi^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where $\pi^{\mathcal{J}}$ is given by (14), the same as in our baseline model with unobservable constant arrival rate λ.

---

[1]As a result, when $\pi_t = 0$ (in the G state for sure), the drift of belief π_t is exactly φ_G, the arrival rate from the G to the B state. Similarly by symmetry, when $\pi_t = 1$ (in the B state for sure), the drift is exactly $-\varphi_B$.

Taking these results together, absent jump arrivals (i.e., $d\mathcal{J}_t = 0$), we obtain the following expression for the rate at which belief changes, $\widehat{\mu}_\pi(\pi_{t-}) = d\pi_t/dt$:

$$\widehat{\mu}_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi) - \sigma_\pi(\pi)\lambda(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi) + \pi(1-\pi)(\lambda_G - \lambda_B) \quad \text{(OA.4)}$$

Changing the unobservable λ from a constant to a stochastic process (two-state Markov chain) does not change the belief updating upon the immediate arrival of a jump. However, belief updating conditional on no jump arrival is qualitatively different from the baseline case with unobservable constant arrival rate λ.

Next, we calculate the posterior belief π_t at t conditional on no jump arrival over the time interval (s, t), i.e., $dJ_v = 0$ for $s < v \leq t$. Using (OA.3) and integrating $\{\pi_v; v \in (s, t)\}$ from s to t conditional on no jump over the interval (s, t), we obtain the following function:

$$\pi_t = \pi_s - \frac{2(\delta_0 \pi_s^2 + \delta_1 \pi_s + \delta_2)(e^{-\sqrt{\delta_1^2 - 4\delta_0\delta_2}(t-s)} - 1)}{(\sqrt{\delta_1^2 - 4\delta_0\delta_2} + \delta_1 + 2\delta_0\pi_s)(e^{-\sqrt{\delta_1^2 - 4\delta_0\delta_2}(t-s)} - 1) + 2\sqrt{\delta_1^2 - 4\delta_0\delta_2}}, \quad \text{(OA.5)}$$

where

$$\delta_0 = -(\lambda_G - \lambda_B), \quad \delta_1 = \lambda_G - \lambda_B - (\varphi_G + \varphi_B), \quad \delta_2 = \varphi_G. \quad \text{(OA.6)}$$

For our baseline mode ($\varphi_G = \varphi_B = 0$), π_t in (OA.5) can be simplified to (16).
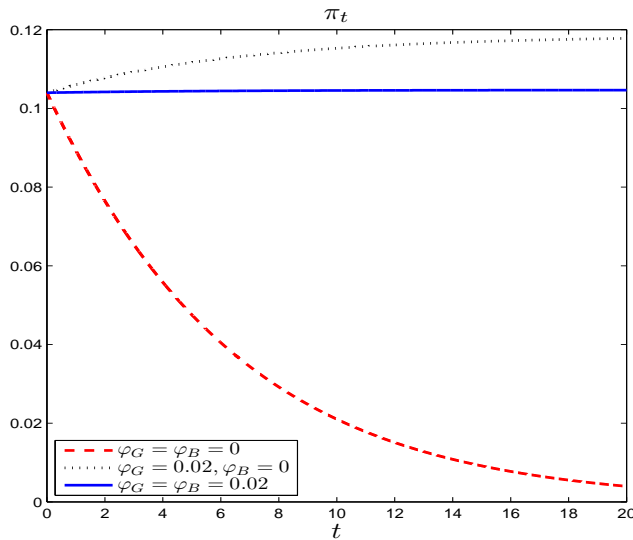

Figure O-1: This figure plots the time series of π_t absent jumps in our generalized model, where the jump arrival rate, λ, is unobservable and stochastic taking two possible values ($\lambda_G = 0.135$ and $\lambda_B = 0.304$) with a prior of $\pi_0 = 0.104$ that the current value of λ is λ_B. Our baseline model with constant unobservable λ corresponds to $\varphi_G = \varphi_B = 0$ (the dashed red line).

In Figure O-1, we plot the belief process $\{\pi_t : t \in (0, 20)\}$ conditional on no jump arrival over the period $(0, t)$, which means $d\mathcal{J}_v = 0$ where $v \in (0, t)$, for three cases: 1.) the stationary case with $\varphi_G = \varphi_B = 2\%$ (the solid blue line); 2.) the case with $\varphi_G = 2\%$ and $\varphi_B = 0\%$, where the economy is eventually absorbed at the B state, (the dotted black line); and 3.)

the baseline constant λ case with $\varphi_G = \varphi_B = 0\%$ (the dashed red line). The prior is set at $\pi_0 = 0.104$ for all three cases.

First, for the two cases with stochastic λ, π_t increases with t even absent jump arrivals. For example, the solid blue line (for the $\varphi_G = \varphi_B = 2\%$ case) shows that π_t slowly increases to 0.105 in twenty years absent jump arrivals. For the other case where the B state is absorbing ($\varphi_B = 0$), π_t increases to 0.118 at $t = 20$ absent jumps (the dotted black line.) The belief dynamics for these two cases with stochastic λ are different from our constant unobservable λ model (the dashed red line), which shows that π decreases over time and the agent becomes more optimistic (the no-news-is-good-news result). This difference is due to the prediction that in our baseline model, belief change is unpredictable (as belief is a martingale), while in the stochastic λ model, there is a force of mean reversion. So long as the transition rates φ_G and φ_B are small (which is the practically relevant case), our baseline model (with constant unobservable λ) and the stochastic unobservable λ model generate similar quantitative predictions. It is for reasons of parsimony that we use the constant λ model for our quantitative analysis in the paper.

OA.2 Solution

Using the belief process $\{\pi_t\}$ given in (OA.3), we obtain the following HJB equation for the planner's allocation problem:

$$
0 = \max_{\mathbf{C}, \mathbf{I}, \mathbf{x}^e \mathbf{x}^d} f(\mathbf{C}, V) + \mathbf{I} V_{\mathbf{K}}(\mathbf{K}, \pi) + \widehat{\mu}_\pi(\pi) V_\pi(\mathbf{K}, \pi) + \frac{1}{2}\sigma^2 \mathbf{K}^2 V_{\mathbf{KK}}(\mathbf{K}, \pi)
$$
$$
+ \lambda(\pi) \mathbb{E}^{\mathbf{x}^d}\left[V\left((1 - N(\mathbf{x}^e)(1-Z))\mathbf{K}, \pi^{\mathcal{J}}\right) - V(\mathbf{K}, \pi)\right] , \qquad \text{(OA.7)}
$$

where $\widehat{\mu}_\pi(\pi)$ is given in (OA.4). The FOCs for aggregate investment $\mathbf{I}$, (scaled) aggregate disaster distribution mitigation spending $\mathbf{x}^d$, and (scaled) aggregate disaster exposure mitigation spending $\mathbf{x}^e$ are the same as those for our baseline model (with constant unobservable λ), which are given in (20), (21), and (22), respectively.

Substituting the value function $V(\mathbf{K}, \pi)$ given in (23) and its derivatives into the HJB equation (OA.7), using the three FOCs ((20), (21), and (22)), and simplifying these equations, we obtain the four-equation ODE system for $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$, given in

$$
0 = \frac{\rho}{1 - \psi^{-1}}\left[\left[\frac{b(\pi)}{\rho(1 + \phi'(\mathbf{i}(\pi)))}\right]^{1-\psi} - 1\right] + \mathbf{i}(\pi) - \frac{\gamma\sigma^2}{2} + \widehat{\mu}_\pi(\pi)\frac{b'(\pi)}{b(\pi)}
$$
$$
+ \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d(\pi)}((1 - N(\mathbf{x}^e(\pi))(1-Z))^{1-\gamma}) - 1\right]. \qquad \text{(OA.8)}
$$

and (25)-(27) for $\pi \in (0, 1)$. The key difference between (OA.8) and the ODE (24) for $b(\pi)$ in our baseline model (with constant but unobservable λ) is that the drift of π absent jumps, $\widehat{\mu}_\pi(\pi)$ given in (OA.4), appears in (OA.8) while $\mu_\pi(\pi)$ given in (15) appears in the ODE (24).[2] The other three equations for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ for our stochastic λ model are (25), (26), and (27), the same as those for our baseline model of Section 2.

---

[2] The wedge $\widehat{\mu}_\pi(\pi) - \mu_\pi(\pi) = (\varphi_G - (\varphi_B + \varphi_G)\pi)$ precisely captures the effect of stochastic transition between the G and B states.

Next, we turn to the boundary conditions at $\pi = 0$ and $\pi = 1$. At $\pi = 0$, we have

$$
\begin{aligned}
0 &= \frac{\rho}{1 - \psi^{-1}} \left[\left[\frac{b(0)}{\rho(1 + \phi'(\mathbf{i}(0)))} \right]^{1-\psi} - 1 \right] + \mathbf{i}(0) - \frac{\gamma \sigma^2}{2} + \frac{\varphi_G b'(0)}{b(0)} \\
&\quad + \frac{\lambda_G \left[\mathbb{E}^{\mathbf{x}^d(0)} ((1 - N(\mathbf{x}^e(0))(1 - Z))^{1-\gamma}) - 1 \right]}{1 - \gamma} .
\end{aligned}
\tag{OA.9}
$$

Compared with the boundary condition (28) for $b(\pi)$ at $\pi = 0$ in our baseline model of Section 2, we have a new term $\frac{\varphi_G b'(0)}{b(0)}$ on the right side of (OA.9). This is because when $\pi_t = 0$, while the state at t is G for sure, it stochastically transitions out of G to B at the rate of φ_G. Our baseline model of Section 2 is a special case with $\varphi_G = 0$. The other three boundary conditions at $\pi = 0$ for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ in our stochastic λ model are (29), (30), and (31), the same as those for our baseline model of Section 2.[3]

Similarly, at $\pi = 1$, we have

$$
\begin{aligned}
0 &= \frac{\rho}{1 - \psi^{-1}} \left[\left[\frac{b(1)}{\rho(1 + \phi'(\mathbf{i}(1)))} \right]^{1-\psi} - 1 \right] + \mathbf{i}(1) - \frac{\gamma \sigma^2}{2} - \frac{\varphi_B b'(1)}{b(1)} \\
&\quad + \frac{\lambda_B \left[\mathbb{E}^{\mathbf{x}^d(1)} ((1 - N(\mathbf{x}^e(1))(1 - Z))^{1-\gamma}) - 1 \right]}{1 - \gamma} ,
\end{aligned}
\tag{OA.10}
$$

where the term $-\frac{\varphi_B b'(1)}{b(1)}$ describes the stochastic transition into G from B. All other terms are the same as in (A.4), the corresponding condition for our baseline model of Section 2. The other three boundary conditions at $\pi = 1$ for $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$ and $\mathbf{x}^e(\pi)$ in our stochastic λ model are (A.5), (A.6), and (A.7), the same as those for our baseline model of Section 2.[4]

Next, we summarize the solution for our generalized learning model.

Proposition 7 *The first-best solution for our generalized learning model is given by the value function (23) and the quartet policy rules, $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^d(\pi)$, and $\mathbf{x}^e(\pi)$, where $0 \leq \pi \leq 1$, via the four-equation ODE system ((OA.8), (25), (26), and (27)) with the four conditions ((OA.9), (29), (30), and (31)) for $\pi = 0$, and ((OA.10), (A.5), (A.6), and (A.7)) for $\pi = 1$.*

OA.3 Quantitative Analysis

Next, we analyze the solutions for our generalized model with stochastic unobservable λ. For the stochastic λ model, we set both the transition rate from state G to B (φ_G) and that from state B to G (φ_B) to 2%, i.e., $\varphi_G = \varphi_B = 1/50 = 2\%$, which imply an average duration of 50 years for both G and B states. In the long run, the economy is in either state G or B with equal (50%) probability.

To ease exposition and facilitate comparison with our baseline (constant unobservable λ) model of Section 2, we use the same values for all the other parameters as in our baseline model. Additionally, we focus on the belief transition from a prior of $\pi_0 = 0.104$ to a posterior of

---

[3]Note that when $\pi = 0$, we also have $\pi^{\mathcal{J}} = 0$. This is why the last term in (OA.9) does not involve $b(\cdot)$ while the last term in (OA.8) for $\pi \in (0, 1)$ does.
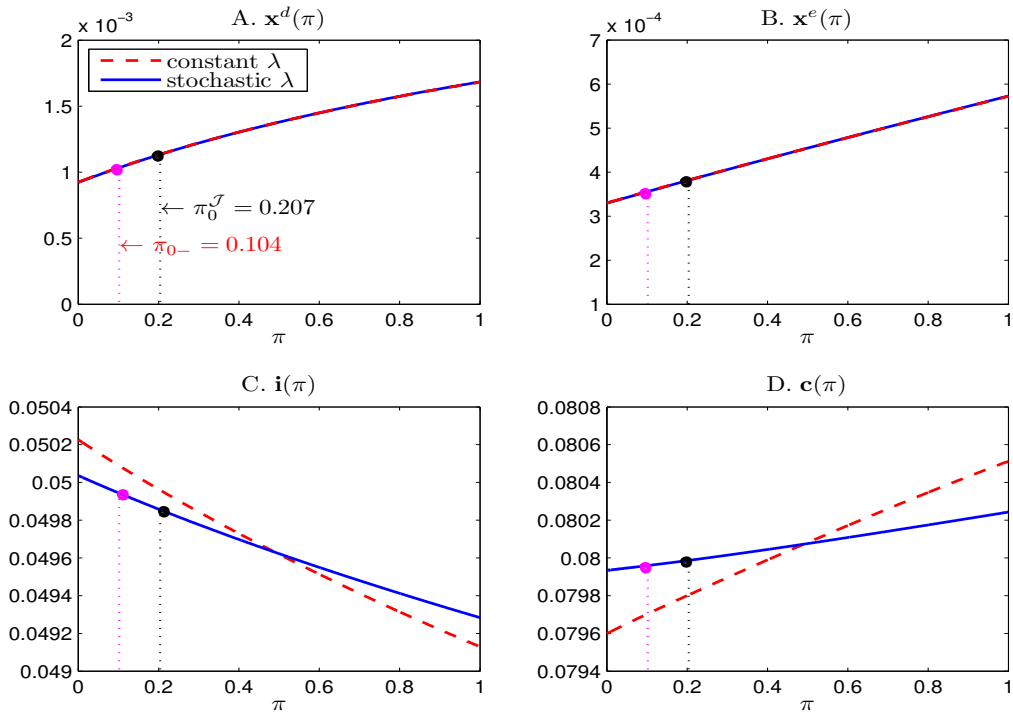
[4]As for the $\pi = 0$ case, when $\pi = 1$, we also have $\pi^{\mathcal{J}} = 1$. This is why the last term in (OA.10) does not involve $b(\cdot)$ while the last term in (OA.8) for $\pi \in (0, 1)$ does.
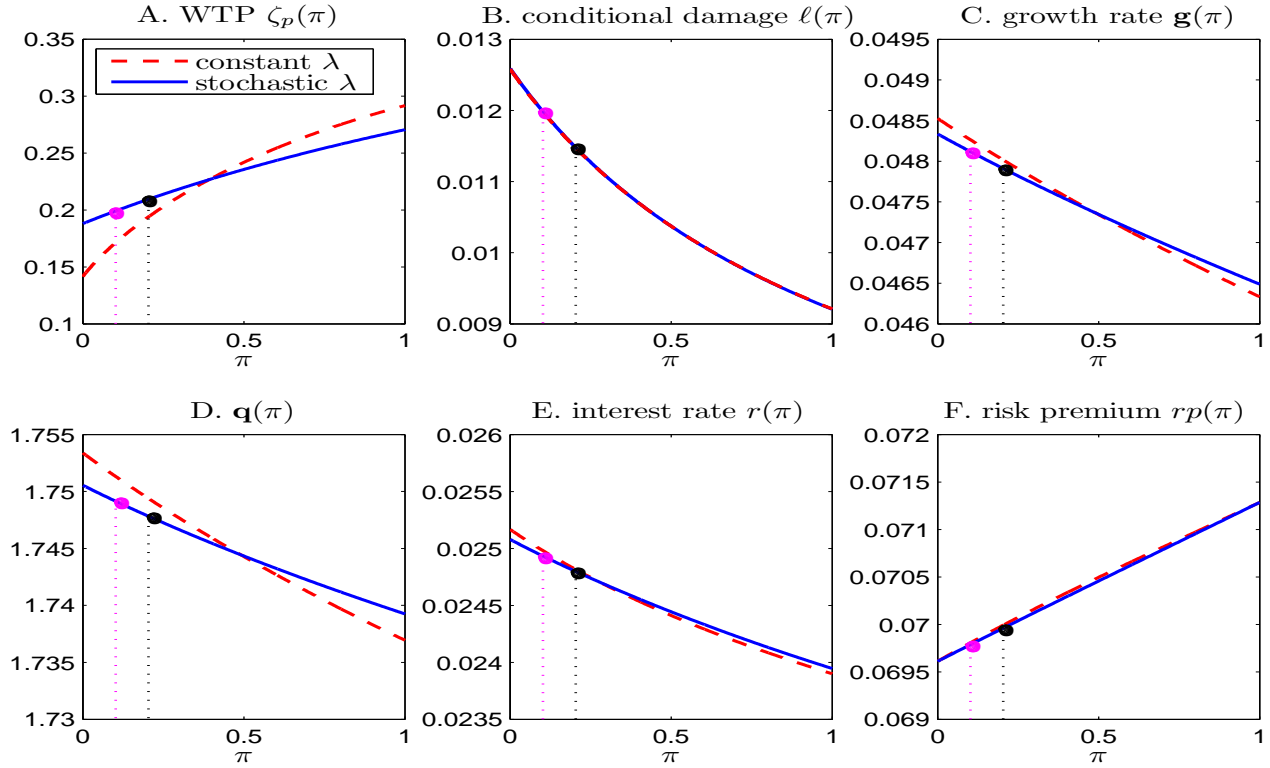
Figure O-2: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

$\pi^{\mathcal{J}} = 0.207$ upon an immediate jump arrival. Note that belief jumps by the same magnitude, $\sigma_\pi(\pi_0) = 0.103$, in both constant and stochastic λ models.

In Figure O-2, we plot (scaled) public mitigation $\mathbf{x}^d(\pi)$ (Panel A), (scaled) private mitigation $\mathbf{x}^e(\pi)$ (Panel B), investment-capital ratio $\mathbf{i}(\pi)$ (Panel C), and consumption-capital $\mathbf{c}(\pi)$ (Panel D) as functions of belief π for the planner's first-best solutions: the solid blue lines are for the stochastic λ model and the dashed red lines are for the baseline constant λ model.

Panels A and B show that for both public mitigation $\mathbf{x}^d(\pi)$ and private exposure mitigation $\mathbf{x}^e(\pi)$, there are essentially no differences between the two economies at all levels of π. Quantitatively, the differences for investment and consumption are of very small (second- and third-order effects, as we can see from the scale for the vertical axes in Panels C and D.) This is because the transition of λ occurs once every fifty years on average.

Note that investment and consumption are even flatter (less responsive to changes of belief) in the stochastic λ model than in the constant λ model. Figure O-3 corroborates the belief mean reversion effect on welfare, growth, and valuation by showing that the welfare measure, the WTP $\zeta_p(\pi)$ (Panel A), the expected growth rate $\mathbf{g}(\pi)$ (Panel C), Tobin's average q, and the risk-free rate $r(\pi)$ are all smoother (flatter) as functions of π in the stochastic λ model than in the constant λ model.

The intuition is as follows. As belief mean reversion in the stochastic λ model, the agent is less optimistic in the low-π state but also less pessimistic in the high-π state, in the stochastic λ model, i.e., compared with the constant λ model. As a result, the planner reduces both consumption and investment in response to changes of belief (so that the planner better smoothes investment/consumption across states and over time.)

In sum, our analysis shows that for plausible values of slow belief mean reversion, the quantitative results of our learning model (with stochastic λ) are similar to those of our learning model (with constant λ). For example, there is no visual difference for conditional damages across the two models as mitigation spending differences are essentially zero as we see in Figure O-2. We also confirm the intuition that belief mean reversion reduces the impact of learning on welfare, valuation and policy rules.

Figure O-3: This figure compares two learning models: the constant λ and the stochastic λ models. The transition rates are $\varphi_G = \varphi_B = 0.02$ for the stochastic λ model (solid blue lines). The transition rates are $\varphi_G = \varphi_B = 0$ for our baseline (constant λ) model (dashed red lines).

OB   External Habit Model

We now solve the model with external habit (Campbell-Cochrane) preferences of Section 8, then calibrate it, and provide a quantitative analysis.

In (71), $\delta(s_t)$ is the sensitivity function proportional to the conditional volatility of ds_t in response to $d\mathcal{W}_t$, which we assume is given by the following square-root function:

$$\delta(s) = \frac{1}{\overline{S}}\sqrt{1 - 2(s - \overline{s})} - 1, \qquad s \leq s_{\max} \tag{OA.11}$$

and $\delta(s) = 0$ for $s > s_{\max}$, where $s_{\max} = \overline{s} + \frac{1 - \overline{S}^2}{2}$ and $\overline{S} = e^{\overline{s}}$.[5]

---

[5]Additionally, we set $\overline{S} = \sigma\sqrt{\frac{\gamma}{1-\kappa}}$ as in Campbell and Cochrane (1999).

OB.1 Solution

Using the surplus consumption ratio process $\{s_t\}$ given in (71) and the external habit utility function given in (69), we obtain the HJB equation (72) for the planner's resource allocation problem. Substituting the value function given in (73) into the HJB equation (72), we obtain

$$
\begin{aligned}
0 = \max_{\mathbf{c,i,x^e\,x^d}} \frac{\rho}{1-\gamma} & \left[\left(\frac{\mathbf{c}(\pi,s)e^s}{b(\pi,s)} \right)^{1-\gamma} - 1 \right] + \mathbf{i}(\pi,s) + \mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)} + (1-\kappa)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& -\frac{\gamma\sigma^2}{2} + \frac{\sigma^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2} \right) + (1-\gamma)\sigma^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d}((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}) - 1 \right].
\end{aligned}
\tag{OA.12}
$$

Using the resource constraint $\mathbf{c} = A - \mathbf{i} - \phi(\mathbf{i}) - \mathbf{x}^d - \mathbf{x}^e$ to simplify the FOC for investment $\mathbf{i}$, we obtain the ODE system for $b(\pi,s)$, $\mathbf{i}(\pi,s)$, $\mathbf{x}^e(\pi,s)$ and $\mathbf{x}^d(\pi,s)$ in the region where $\pi \in (0,1)$ and $s \in (-\infty, s_{\max})$:

$$
\begin{aligned}
0 = \frac{\rho}{1-\gamma} & \left[\left(\frac{b(\pi,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(\pi,s)))} \right)^{1-\gamma^{-1}} - 1 \right] + \mathbf{i}(\pi,s) + (1-\kappa)(\overline{s}-s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\mu_\pi(\pi)\frac{b_\pi(\pi,s)}{b(\pi,s)} - \frac{\gamma\sigma^2}{2} + \frac{\sigma^2\delta(s)^2}{2}\left(\frac{b_{ss}(\pi,s)}{b(\pi,s)} - \gamma\frac{(b_s(\pi,s))^2}{b(\pi,s)^2} \right) + (1-\gamma)\sigma^2\delta(s)\frac{b_s(\pi,s)}{b(\pi,s)} \\
& +\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)} \right)^{1-\gamma} \mathbb{E}^{\mathbf{x}^d(\pi,s)}((1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma}) - 1 \right],
\end{aligned}
\tag{OA.13}
$$

$$
b(\pi,s) = [A - \mathbf{i}(\pi,s) - \phi(\mathbf{i}(\pi,s)) - \mathbf{x}^d(\pi,s) - \mathbf{x}^e(\pi,s)]^{\gamma/(\gamma-1)} \left[\rho\mathbf{q}(\pi,s)\right]^{1/(1-\gamma)} e^s,
\tag{OA.14}
$$

$$
\frac{1}{\mathbf{q}(\pi,s)} = \lambda(\pi)\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)} \right]^{1-\gamma} N'(\mathbf{x}^e(\pi,s))\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left[(Z-1)(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{-\gamma} \right],
\tag{OA.15}
$$

$$
\frac{1}{\mathbf{q}(\pi,s)} = \frac{\lambda(\pi)}{1-\gamma}\left[\frac{b\left(\pi^{\mathcal{J}},s\right)}{b(\pi,s)} \right]^{1-\gamma} \int_0^1 \left[\frac{\partial\xi(Z;\mathbf{x}^d(\pi,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(\pi,s))(1-Z))^{1-\gamma} \right] dZ,
\tag{OA.16}
$$

where $\mathbf{q}(\pi,s)$ is given by

$$
\mathbf{q}(\pi,s) = 1 + \phi'(\mathbf{i}(\pi,s)).
\tag{OA.17}
$$

Using the resource constraint $\mathbf{c} = A - \mathbf{i} - \phi(\mathbf{i}) - \mathbf{x}^d - \mathbf{x}^e$ to simplify the FOCs for the two types of mitigation spending, $\mathbf{x}^e$ and $\mathbf{x}^d$, we obtain the optimal exposure mitigation and distribution mitigation spending rules, (OA.15) and (OA.16) for $\mathbf{x}^e$ and $\mathbf{x}^d$, respectively.

Since $\pi = 0$ is an absorbing state, we have the following boundary conditions at $\pi = 0$:

$$0 = \frac{\rho}{1-\gamma}\left[\left(\frac{b(0,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(0,s)))}\right)^{1-\gamma^{-1}} - 1\right] + \mathbf{i}(0,s) + (1-\kappa)(\overline{s}-s)\frac{b_s(0,s)}{b(0,s)}$$
$$+\frac{\sigma^2\delta(s)^2}{2}\left(\frac{b_{ss}(0,s)}{b(0,s)} - \gamma\frac{(b_s(0,s))^2}{b(0,s)^2}\right) + (1-\gamma)\sigma^2\delta(s)\frac{b_s(0,s)}{b(0,s)}$$
$$+\frac{\lambda_G}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(\pi,s)}((1-N(\mathbf{x}^e(0,s))(1-Z))^{1-\gamma}) - 1\right], \tag{OA.18}$$

$$b(0,s) = [A - \mathbf{i}(0,s) - \phi(\mathbf{i}(0,s)) - \mathbf{x}^d(0,s) - \mathbf{x}^e(0,s)]^{\gamma/(\gamma-1)}[\rho\mathbf{q}(0,s)]^{1/(1-\gamma)}e^s, \tag{OA.19}$$

$$\frac{1}{\mathbf{q}(0,s)} = \lambda_G N'(\mathbf{x}^e(0,s))\mathbb{E}^{\mathbf{x}^d(\pi,s)}\left[(Z-1)(1-N(\mathbf{x}^e(0,s))(1-Z))^{-\gamma}\right], \tag{OA.20}$$

$$\frac{1}{\mathbf{q}(0,s)} = \frac{\lambda_G}{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(0,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(0,s))(1-Z))^{1-\gamma}\right]dZ, \tag{OA.21}$$

where $\mathbf{q}(0,s) = 1 + \phi'(\mathbf{i}(0,s))$.

Similarly, at the $\pi = 1$ absorbing state, we have the following boundary conditions:

$$0 = \frac{\rho}{1-\gamma}\left[\left(\frac{b(1,s)e^{-s}}{\rho(1+\phi'(\mathbf{i}(1,s)))}\right)^{1-\gamma^{-1}} - 1\right] + \mathbf{i}(1,s) + (1-\kappa)(\overline{s}-s)\frac{b_s(1,s)}{b(1,s)}$$
$$+\frac{\sigma^2\delta(s)^2}{2}\left(\frac{b_{ss}(1,s)}{b(1,s)} - \gamma\frac{(b_s(1,s))^2}{b(1,s)^2}\right) + (1-\gamma)\sigma^2\delta(s)\frac{b_s(1,s)}{b(1,s)}$$
$$+\frac{\lambda_B}{1-\gamma}\left[\mathbb{E}^{\mathbf{x}^d(1,s)}((1-N(\mathbf{x}^e(1,s))(1-Z))^{1-\gamma}) - 1\right], \tag{OA.22}$$

$$b(1,s) = [A - \mathbf{i}(1,s) - \phi(\mathbf{i}(1,s)) - \mathbf{x}^d(1,s) - \mathbf{x}^e(1,s)]^{\gamma/(\gamma-1)}[\rho\mathbf{q}(1,s)]^{1/(1-\gamma)}e^s, \tag{OA.23}$$

$$\frac{1}{\mathbf{q}(1,s)} = \lambda_B N'(\mathbf{x}^e(1,s))\mathbb{E}^{\mathbf{x}^d(1,s)}\left[(Z-1)(1-N(\mathbf{x}^e(1,s))(1-Z))^{-\gamma}\right], \tag{OA.24}$$

$$\frac{1}{\mathbf{q}(1,s)} = \frac{\lambda_B}{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;\mathbf{x}^d(1,s))}{\partial\mathbf{x}^d}(1-N(\mathbf{x}^e(1,s))(1-Z))^{1-\gamma}\right]dZ, \tag{OA.25}$$

where $\mathbf{q}(1,s) = 1 + \phi'(\mathbf{i}(1,s))$.

At $s = s_{\max}$, we have the following boundary condition:

$$0 = \frac{\rho}{1-\gamma}\left[\left(\frac{b(\pi,s_{\max})e^{-s_{\max}}}{\rho(1+\phi'(\mathbf{i}(\pi,s_{\max})))}\right)^{1-\gamma^{-1}} - 1\right] + \mathbf{i}(\pi,s_{\max}) + (1-\kappa)(\overline{s}-s_{\max})\frac{b_s(\pi,s_{\max})}{b(\pi,s_{\max})}$$
$$-\frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b_\pi(\pi,s_{\max})}{b(\pi,s_{\max})}$$
$$+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}},s_{\max}\right)}{b(\pi,s_{\max})}\right)^{1-\gamma}\mathbb{E}^{\mathbf{x}^d(\pi,s_{\max})}((1-N(\mathbf{x}^e(\pi,s_{\max}))(1-Z))^{1-\gamma}) - 1\right].$$
$$\tag{OA.26}$$

Additionally, $\mathbf{i}(\pi,s_{\max})$, $\mathbf{x}^e(\pi,s_{\max})$ and $\mathbf{x}^d(\pi,s_{\max})$, satisfy (OA.14)- (OA.16) at $s = s_{\max}$.[6]

We summarize our model's solution in the following proposition.

---

[6]Note that as $s \to -\infty$ is not reachable in equilibrium, we can ignore the corresponding boundary conditions for our numerical analysis.

Proposition 8 *The first-best solution for our external habit model is given by the value function (73) and the quartet policy rules, $b(\pi, s)$, $\mathbf{i}(\pi, s)$, $\mathbf{x}^d(\pi, s)$, and $\mathbf{x}^e(\pi, s)$, where $0 \leq \pi \leq 1$ and $-\infty < s \leq s_{\max}$, via the four-equation ODE system (OA.13), (OA.14), (OA.15) and (OA.16), together with the boundary conditions (OA.18)-(OA.21) for $\pi = 0$, (OA.22)-(OA.25) for $\pi = 1$, (OA.26) and (OA.14)-(OA.16) for $s = s_{\max}$.*

Next, we use the equilibrium resource allocation to derive our model's asset pricing implications.

OB.2 Asset Pricing Implications

Using the planner's solution, we can also infer the SDF process for the equilibrium outcome (under optimal taxation which supports the first-best equilibrium outcome) by applying the Ito's Lemma to $\mathbb{M}_t$ given below

$$\mathbb{M}_t = e^{-\rho t} \frac{U_C(C_t, \mathcal{H}_t)}{U_C(C_0, \mathcal{H}_0)} = e^{-\rho t} \left(\frac{C_t S_t}{C_0 S_0} \right)^{-\gamma} . \tag{OA.27}$$

We then use the no-arbitrage restriction for the SDF to obtain the equilibrium risk-free rate, the market price of risk, and the stock market risk premium.

Using (OA.27), we obtain the following expression for the logarithmic SDF, $\ln(\mathbb{M}_t)$:

$$\ln(\mathbb{M}_t) = -\rho t - \gamma \left(\ln(C_t) + \ln(S_t) - \ln(C_0) - \ln(S_0) \right) . \tag{OA.28}$$

Then using Ito's lemma, we obtain

$$
\begin{aligned}
\frac{d\mathbb{M}_t}{\mathbb{M}_t} ={}& -\rho dt - \gamma \left(\mathbf{i}(\pi, s) - \frac{\sigma^2}{2} \right) dt + (1 - \kappa)(\bar{s} - s_t) \left((1 - \gamma) \frac{b_s(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - 1 \right) dt \\
&+ \mu_\pi(\pi) \left((1 - \gamma) \frac{b_\pi(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_\pi(\pi, s)}{\mathbf{q}(\pi, s)} \right) dt - \left[(1 - \gamma) \frac{b_s(\pi, s)^2}{b(\pi, s)^2} - \frac{\mathbf{q}_s(\pi, s)^2}{\mathbf{q}(\pi, s)^2} \right] \frac{(\sigma \delta(s))^2}{2} dt \\
&+ \frac{\sigma_\mathbb{M}(\pi, s)^2}{2} dt - \sigma_\mathbb{M}(\pi, s) d\mathcal{W}_t + \left[\eta(\pi, s; Z, \mathbf{x}^e) - 1 \right] d\mathcal{J}_t ,
\end{aligned} \tag{OA.29}
$$

where

$$\eta(\pi, s; Z, \mathbf{x}^e) = \frac{\mathbf{q}(\pi, s)}{\mathbf{q}(\pi^{\mathcal{J}}, s)} \left(\frac{b\left(\pi^{\mathcal{J}}, s\right)}{b(\pi, s)} \right)^{1-\gamma} (1 - N(\mathbf{x}^e(\pi, s))(1 - Z))^{-\gamma}, \tag{OA.30}$$

and

$$\sigma_\mathbb{M}(\pi, s) = \left[\left(1 + \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - (1 - \gamma) \frac{b_s(\pi, s)}{b(\pi, s)} \right) \delta(s) + \gamma \right] \sigma . \tag{OA.31}$$

Using the equilibrium restriction that the drift of $\frac{d\mathbb{M}_t}{\mathbb{M}_t}$ equals $-r_{t-}dt$, we obtain the following expression for the equilibrium risk-free rate:

$$
\begin{aligned}
r(\pi, s) ={}& \rho + \gamma \left(\mathbf{i}(\pi, s) - \frac{\sigma^2}{2} \right) - (1 - \kappa)(\bar{s} - s_t) \left((1 - \gamma) \frac{b_s(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_s(\pi, s)}{\mathbf{q}(\pi, s)} - 1 \right) \\
&- \mu_\pi(\pi) \left((1 - \gamma) \frac{b_\pi(\pi, s)}{b(\pi, s)} - \frac{\mathbf{q}_\pi(\pi, s)}{\mathbf{q}(\pi, s)} \right) + \left[(1 - \gamma) \frac{b_s(\pi, s)^2}{b(\pi, s)^2} - \frac{\mathbf{q}_s(\pi, s)^2}{\mathbf{q}(\pi, s)^2} \right] \frac{(\sigma \delta(s))^2}{2} \\
&- \frac{\sigma_\mathbb{M}(\pi, s)^2}{2} - \lambda(\pi) \left[\mathbb{E}^{\mathbf{x}^d}(\eta(\pi, s; Z, \mathbf{x}^e)) - 1 \right] .
\end{aligned} \tag{OA.32}
$$

Using the equilibrium SDF, we may calculate firm value, $Q(K, \pi, s)$ by using

$$Q(K_t, \pi_t, s_t) = \int_t^\infty \frac{\mathbb{M}_v}{\mathbb{M}_t}(AK_v - I_v - \Phi(I_v, K_v) - X_v^e)dv. \tag{OA.33}$$

Applying the Ito's Lemma to firm value $Q(K, \pi, s) = q(\pi, s)K$ and using (OA.29), we obtain the following PDE for $q(\pi, s)$:

$$r(\pi, s)q(\pi, s) = \max_{i,\, x^e} \; A - i - \phi(i) - x^e + (i - \sigma_{\mathbb{M}}(\pi, s)\sigma)q(\pi, s) + \mu_\pi(\pi)q_\pi(\pi, s)$$

$$+ \left[(1 - \kappa)(\bar{s} - s) + \delta(s)\sigma^2 - \sigma_{\mathbb{M}}(\pi, s)\delta(s)\sigma\right] q_s(\pi, s) + \frac{\sigma^2\delta(s)^2}{2}q_{ss}(\pi, s)$$

$$+ \lambda(\pi)\mathbb{E}^{\mathbf{x}^d}\left[\eta(\pi, s; Z, \mathbf{x}^e)\left(q(\pi^{\mathcal{J}}, s)(1 - N(x^e)(1 - Z)) - q(\pi, s)\right)\right]. \tag{OA.34}$$

The cum-dividend return dR_t over the period dt is given by

$$
\begin{aligned}
dR_t &= \frac{(AK_{t-} - I_t - \Phi(I_{t-}, K_{t-}) - X_{t-}^e)dt}{Q_{t-}} + \frac{dQ_t}{Q_{t-}} \\
&= \frac{A - i_{t-} - \phi(i_{t-}) - x_{t-}^e}{q(\pi_{t-}, s_{t-})}dt + \frac{dq(\pi_t, s_t)}{q(\pi_{t-}, s_{t-})} + \frac{dK_t}{K_{t-}} + \frac{< dq(\pi_t, s_t), dK_t >}{q(\pi_{t-}, s_{t-})K_{t-}} \\
&= \frac{A - i_{t-} - \phi(i_{t-}) - x_{t-}^e + (1 - \kappa)(\bar{s} - s_{t-})q_s(\pi_{t-}, s_{t-}) + \frac{\sigma^2\delta(s_{t-})^2}{2}q_{ss}(\pi_{t-}, s_{t-})}{q(\pi_{t-}, s_{t-})}dt \\
&\quad + \frac{\mu_\pi(\pi_{t-})q_\pi(\pi_{t-}, s_{t-})}{q(\pi_{t-}, s_{t-})}dt + i_{t-}dt + \frac{q_s(\pi_{t-}, s_{t-})\delta(s_{t-})}{q(\pi_{t-}, s_{t-})}\sigma^2 dt + \left[\frac{q_s(\pi_{t-}, s_{t-})\delta(s_{t-})}{q(\pi_{t-}, s_{t-})} + 1\right]\sigma d\mathcal{W}_t \\
&\quad + \left[\frac{((1 - N(x_{t-}^e)(1 - Z))q\left(\pi_t^{\mathcal{J}}, s_{t-}\right)}{q(\pi_{t-}, s_{t-})} - 1\right]d\mathcal{J}_t \\
&= \left[r(\pi_{t-}, s_{t-}) + \sigma_{\mathbb{M}}(\pi_{t-}, s_{t-})\left(\sigma + \delta(s_{t-})\sigma\frac{q_s(\pi_{t-}, s_{t-})}{q(\pi_{t-}, s_{t-})}\right)\right]dt \\
&\quad - \lambda(\pi)\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-}, s_{t-}; Z, \mathbf{x}_{t-}^e)\left(\frac{(1 - N(x_{t-}^e)(1 - Z))q\left(\pi_t^{\mathcal{J}}, s_{t-}\right)}{q(\pi_{t-}, s_{t-})} - 1\right)\right]dt \\
&\quad + \left[\frac{q_s(\pi_{t-}, s_{t-})\delta(s_{t-})}{q(\pi_{t-}, s_{t-})} + 1\right]\sigma d\mathcal{W}_t + \left[\frac{(1 - N(x_{t-}^e)(1 - Z))q\left(\pi_t^{\mathcal{J}}, s_{t-}\right)}{q(\pi_{t-}, s_{t-})} - 1\right]d\mathcal{J}_t. \tag{OA.35}
\end{aligned}
$$

Finally, using the equilibrium conditions $q(\pi, s) = \mathbf{q}(\pi, s)$ and $x^e(\pi, s) = \mathbf{x}^e(\pi, s)$, we write

$$\frac{d\mathbf{Q}_t + \mathbf{D}_{t-}dt}{\mathbf{Q}_{t-}} = \left(\mu_{\mathbf{Q}}(\pi_{t-}, s_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right)dt \tag{OA.36}$$

$$+ \left[\frac{\mathbf{q}_s(\pi_{t-}, s_{t-})\delta(s_{t-})}{\mathbf{q}(\pi_{t-}, s_{t-})} + 1\right]\sigma d\mathcal{W}_t + \left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)(d\mathcal{J}_t - \lambda(\pi_{t-})dt),$$

where

$$\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} = \frac{(1 - N(\mathbf{x}_{t-}^e)(1 - Z))\mathbf{q}\left(\pi_t^{\mathcal{J}}, s_{t-}\right)}{\mathbf{q}(\pi_{t-}, s_{t-})}, \tag{OA.37}$$

and

$$
\begin{aligned}
\mu_{\mathbf{Q}}(\pi_{t-}, s_{t-}) &= r(\pi_{t-}, s_{t-}) + \sigma_{\mathbb{M}}(\pi_{t-}, s_{t-})\left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-}, s_{t-})}{\mathbf{q}(\pi_{t-}, s_{t-})}\right)\sigma \\
&\quad + \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\eta(\pi_{t-}, s_{t-}; Z, \mathbf{x}_{t-}^e)\left(1 - \frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}}\right)\right].
\end{aligned}
\tag{OA.38}
$$

The market risk premium is

$$
\begin{aligned}
rp(\pi_{t-}, s_{t-}) &= \mu_{\mathbf{Q}}(\pi_{t-}, s_{t-}) + \lambda(\pi_{t-})\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right) - r(\pi_{t-}, s_{t-}) \\
&= \sigma_{\mathbb{M}}(\pi_{t-}, s_{t-})\left(1 + \delta(s_{t-})\frac{\mathbf{q}_s(\pi_{t-}, s_{t-})}{\mathbf{q}(\pi_{t-}, s_{t-})}\right)\sigma \\
&\quad - \lambda(\pi_{t-})\mathbb{E}^{\mathbf{x}_{t-}^d}\left[\left(\eta(\pi_{t-}, s_{t-}; Z, \mathbf{x}_{t-}^e) - 1\right)\left(\frac{\mathbf{Q}_t^{\mathcal{J}}}{\mathbf{Q}_{t-}} - 1\right)\right].
\end{aligned}
\tag{OA.39}
$$

Next, we calibrate the model and provide a quantitative analysis.

Table 4: PARAMETER VALUES FOR EXTERNAL HABIT MODEL

Parameters	Symbol	Value
jump arrival rate in State G	λ_G	0.135
power law exponent absent mitigation	β_0	14
mitigation technology parameter	β_1	10,000
mitigation technology parameter	α	0.05
jump arrival rate in State B	λ_B	0.304
prior of being in State B	π_0	0.104
surplus consumption parameter	κ	0.9
time rate of preference	ρ	4%
productivity	A	15%
quadratic adjustment cost parameter	θ	11
coefficient of relative risk aversion	γ	3
capital diffusion volatility	σ	14%

All parameter values, whenever applicable, are continuously compounded and annualized.

OB.3   Quantitative Analysis

Calibration. We first calibrate our model with the Campbell-Cochrane external habit model to match the key global warming and macro moments.

The key new parameter for the external habit model is the (log) surplus consumption parameter κ. We set the persistence parameter for external habit at $\kappa = 0.9$ per annum as in Campbell and Cochrane (1999). We calibrate β_0, β_1 and α in a world with no or low global warming risk, i.e., under the assumption that countries are optimally mitigating cyclone arrivals with belief $\pi_0 = 0.104$. We use the following three moments at the steady state level

of the surplus consumption ratio $\overline{S}$:[7] 1.) the optimal public mitigation of 0.1% of the capital stock, $\mathbf{x}^d(0.104) = 0.1\%$; 2.) the optimal private mitigation of 0.04% of the capital stock, $\mathbf{x}^e(0.104) = 0.04\%$; and 3.) a reduction of the expected annual GDP growth rate by 1.3% per annum caused by the arrival of a major cyclone, $N(\mathbf{x}^e)\mathbb{E}\mathbf{x}^d(1 - Z) = 1.3\%$.

As in our baseline model with Epstein-Zin utility, we calibrate the adjustment cost parameter θ along with the time rate of preference ρ, risk aversion γ, diffusion volatility σ, and productivity A by targeting five key moments for state G. These include the annual (real) risk-free rate of 2.5%, the expected annual stock market risk premium of 7%, the annual stock market return volatility of $\sqrt{0.0206} = 14\%$, the expected growth rate of 4.4%, and Tobin's q of 1.5 (e.g., in line with Eberly, Rebelo and Vincent, 2012), when the prior is $\pi_0 = 0.104$. The resulting parameter values are $\sigma = 14\%$, $\theta = 11$, $\gamma = 3$, $A = 15\%$, and $\rho = 4\%$. These parameter values are in line with those used in the literature. Moreover, these calibrated parameter values are close to those in our baseline calibration with Epstein-Zin utility, even though the building blocks of the two models differ significantly.

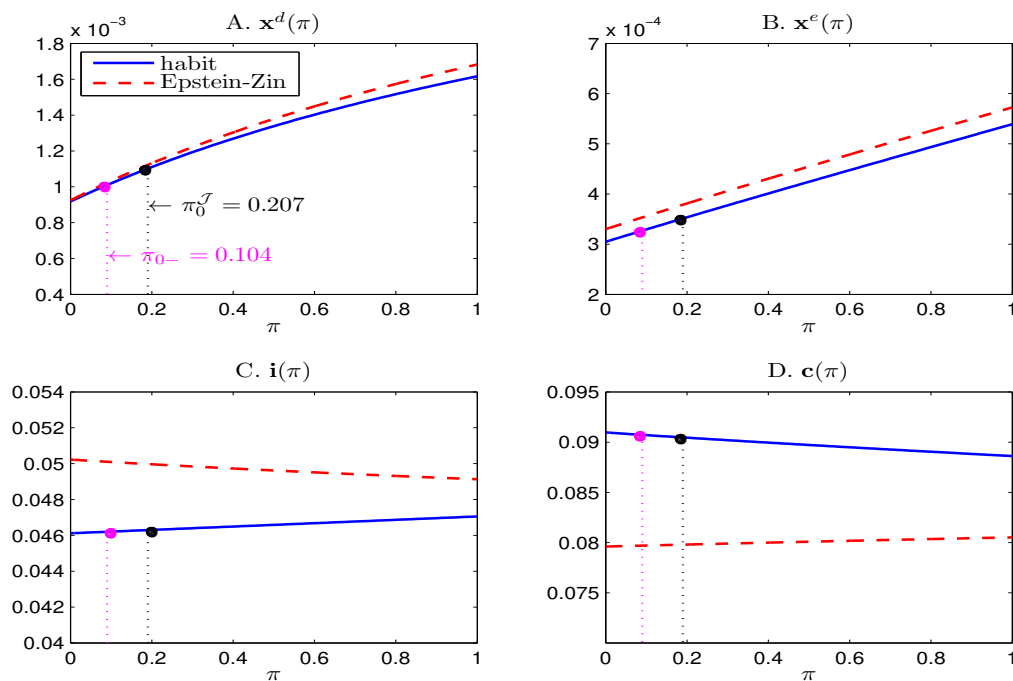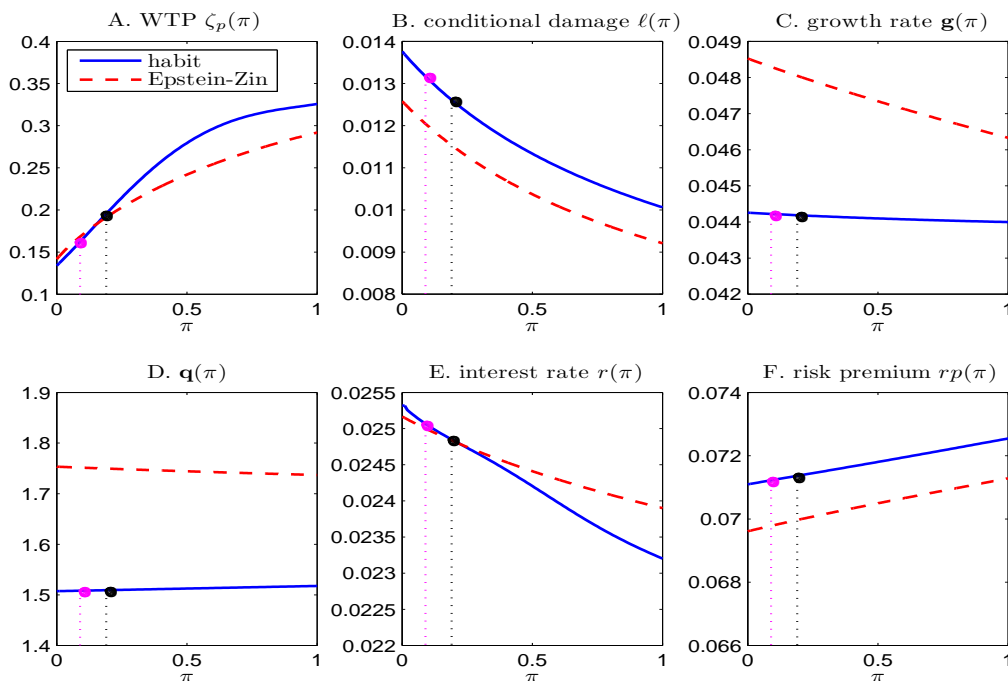We report the values for all the twelve parameters in Table 4.

Figure O-4: This figure compares the first-best planner's model solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility. The parameter values for our baseline (Epstein-Zin) model are summarized in Table 3 and those for the external habit (Campbell-Cochrane) model are summarized in Table 4.

OB.4   Quantitative Results

In Figures O-4 and O-5, we compare the external habit model at the steady state where $S = \overline{S} = 0.857$ with the Epstein-Zin recursive utility model. Recall that both models are

---

[7]The steady-state value of S is $\overline{S} = 0.857$ and $S_{\max} = 0.979$.

recalibrated to match climate change and macro finance moments at the belief level of $\pi = 10.4\%$. Panel A of Figure O-4 shows that the distribution mitigation $\mathbf{x}^d(\pi)$ policies for the two (different utility) models are quite close to each other. Similarly, Panel B of Figure O-4 shows that the exposure mitigation $\mathbf{x}^e(\pi)$ policies for the two models are also quite close. These two findings suggest that our main results on how changes of belief impact disaster distribution and exposure mitigation spendings are reasonably robust to preference specifications. This is encouraging as our key results are not sensitive to the choices of our preferences.

Figure O-5: This figure compares the first-best planner's model solutions for the external habit model (solid blue lines) and the baseline model with Epstein-Zin recursive utility. The parameter values for our baseline (Epstein-Zin) model are summarized in Table 3 and those for the external habit (Campbell-Cochrane) model are summarized in Table 4.

Panel C of Figure O-4 shows that the investment-capital ratio is a bit higher with Epstein-Zin preferences than with external habit at the steady state where $S = \overline{S} = 0.857$. Panel D of Figure O-4 shows that the consumption-capital ratio is a bit lower with Epstein-Zin preferences than with external habit, which is expected as the sum of total mitigation spending, investment, and consumption is the same and equals the productivity A in the two models. Nonetheless, the quantitative differences between the two models in terms of consumption and investment are of the second order. Again, this is good news as our results seem robust to changing preferences assumptions.

It is interesting to note that while $\mathbf{i}(\pi)$ decreases with π for the Epstein-Zin utility model, $\mathbf{i}(\pi)$ increases with π in the external habit model. This difference is caused by the long-run risk force in the Epstein-Zin utility specification, where the EIS $\psi > 1$. To generate the prediction that worsening belief (increasing π) lowers Tobin's q and equivalently investment (as investment increases with Tobin's q), we require $\psi > 1$.

The external habit model differs from the baseline Epstein-Zin utility model in two ways. First, risk aversion is significantly enhanced by and also varies with external habit. Second,

the EIS implied by our external habit model also generates a time-varying elasticity of in-tertemporal substitution (EIS). As risk aversion increases with habit stock, the EIS decreases. This is why our model predicts investment (and hence Tobin's q) increases with belief.

Figure O-5 reports the WTP, conditional damage $\ell(\pi)$, the expected growth rate $\mathbf{g}(\pi)$, Tobin's average $q(\pi)$, the risk-free rate $r(\pi)$, and the market risk premium $rp(\pi)$. While there are some differences, we see that these two models, calibrated to match key moments, generate quantitatively similar results.

In sum, these findings are encouraging when it comes to interpreting our key results.
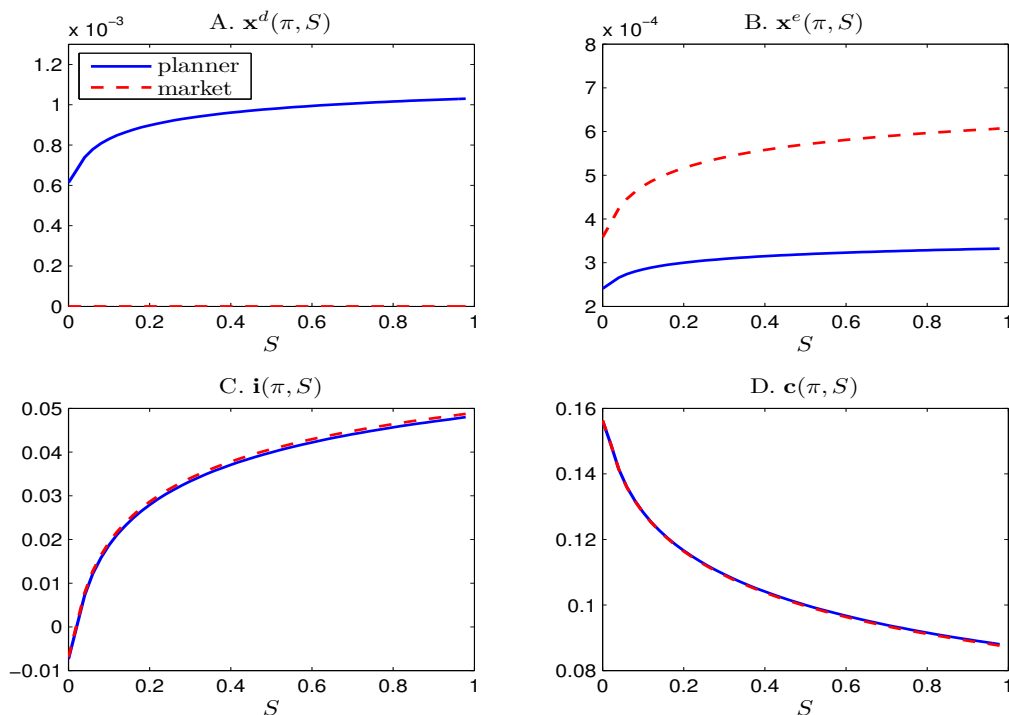
Figure O-6: This figure plots the optimal policies for the planner's first-best economy (solid blue lines) and the market economy (dashed red lines) as functions of surplus consumption ratio S, for the external habit (Campbell-Cochrane) model, where $\pi = 0.104$.

In Figure O-6, we focus on the external habit utility model by comparing two formulations: the planner's first-best economy (solid blue lines) with the market economy solution (dashed red lines). We plot the two mitigation spending, investment, and consumption policies for varying levels of S, for a given belief $\pi = 0.104$.

Panel A of Figure O-6 shows that there is no public mitigation in a competitive market economy for the same externality argument as in our baseline model with Epstein-Zin utility. This panel also shows that $\mathbf{x}^d$ increases as the surplus consumption ratio increases. Similarly, both the exposure mitigation spending and investment increase with S (Panels B and C). The intuition for these results is as follows. As we increase S, the marginal utility of consumption (and SDF $\mathbb{M}_t$) decrease, which causes $\mathbf{c}$ to decrease with S (see Panel D). Additionally, the marginal value of investment and that of mitigation (for both types) increase, which causes $\mathbf{x}^d$, $\mathbf{x}^e$, and $\mathbf{i}$ to increase with S as shown in Panels A, B, and C.

Finally, we note that the private mitigation spending $\mathbf{x}^e$ is larger for the market economy than for the planner's economy. This is because the marginal benefit of private mitigation

is higher in the market economy as there is no public mitigation. In contrast, as the public mitigation spending $\mathbf{x}^d$ is positive and significant under the planner's economy, the additional value of private mitigation spending in the planner's economy is much smaller and hence $\mathbf{x}^e$ is much smaller under the planner's economy than under the market economy (a substitution effect.)

In sum, we show that time-varying risk aversion induced by external habit influences optimal mitigation policies, but the general results that we obtain from our baseline model with Epstein-Zin utility remains valid in our external habit model.

OC   Details on Numerical Analysis

In this appendix, we offer a detailed discussion of numerical analysis used in our paper. We proceed as follows. First, we compare the technical differences between our baseline model of Section 2, where the unobservable disaster arrival rate λ is constant, and Pindyck and Wang (2013). Second, we discuss the additional technical complication in our generalized learning model of Section OA, where the disaster arrival rate λ is stochastic and unobservable. Third, we discuss how our model with external habit of Section 8 further brings technical complications to our analysis.

OC.1   Comparing Baseline Model with Pindyck and Wang (2013)

Recall that Pindyck and Wang (2013), henceforth PW (2013), is a jump diffusion model with stochastic capital recovery, Epstein-Zin recursive utility, and capital adjustment costs, but features no learning and no mitigation. As a result, the state variable in PW (2013) is capital stock K. Then using the homogeneity property, PW (2013) show that their model solution can be further simplified. To be precise, to solve the PW (2013) model, one first solves a simple nonlinear equation for the optimal constant investment-capital ratio i^* (given by equation (12) in their paper), then calculates a welfare measure (proportional to certainty equivalent wealth) b by substituting i^* into equation (11) in their paper, and finally obtain equilibrium asset pricing implications and conduct willingness-to-pay (WTP) calculations. That is, there is no differential equation or even coupled nonlinear equations involved. Therefore, in terms of numerical solution, PW (2013) is very simple. The PW (2013) model is purposefully designed with parsimony and transparency to highlight the key features of disasters in mind.

The economics and technical details for our baseline model (with constant unobservable disaster arrival rate λ) are inevitably more involved than PW (2013), as we need to incorporate learning and two types of mitigation into PW (2013). As a result, there are two state variables in our baseline model: belief π and capital stock K. After using the homogeneity property, we still need to deal with a numerical problem that has one more dimension than PW (2013). Specifically, this one-dimensional problem involves a system of ordinary differential equations (ODEs). To obtain solutions for four unknown functions, $b(\pi)$, $\mathbf{i}(\pi)$, $\mathbf{x}^e(\pi)$, and $\mathbf{x}^d(\pi)$, we need to solve the ODE system of four inter-connected nonlinear differential equations subject to various boundary conditions. It is worth noting that this ODE system is more difficult to work with than some ODEs that we see in various economics and finance applications, e.g., the ODEs appearing in dynamic contracting, e.g., DeMarzo and Sannikov (2006) and Sannikov (2008), are easier to work with.

It is also worth emphasizing that our model has both jumps and diffusion shocks. Jumps

further complicate our numerical analysis. For diffusion models, finite difference methods only require local information, as discretizing a second-order ODE (of diffusion models) calls for analyzing tridiagonal matrix. In contrast, as belief may jump in our model, to solve the model at a given level of π, we also need to take into account the nonlocal effect of jump on value function and policy rules.

As a reference to the technical difficulty of our ODE system, our baseline model's technical difficulty is at least at par with the technical difficulty level of Brunnermeier and Sannikov (2014), which is a diffusion model and hence analyzing tridiagonal matrices is sufficient when solving the coupled ODEs in that paper. In terms of numerical analysis, jumps effectively increase the difficulty of our numerical analysis by increasing the dimension of our problem by "0.5 dimension."

To solve the interconnected ODE system, we also need a set of four interconnected non-linear equations for the boundary $\pi = 0$ and similarly another set of four interconnected nonlinear equations for the boundary $\pi = 1$. Since both boundaries in our baseline model are absorbing, they are relatively easy to work with but are technically still more involved than the full PW (2013) model. This is because for our boundary conditions at $\pi = 0$ and $\pi = 1$, we solve for four unknowns simultaneously while in PW (2013), we only need to sequentially solve one unknown using one nonlinear equation.

OC.2   Additional Difficulties in Stochastic λ Model of Section OA

In our generalized learning model where the arrival rate is stochastic and unobservable, while we still characterize the solution with four interconnected ODEs in the interior belief region where $\pi \in (0, 1)$, the boundary conditions are more complicated posing additional technical and numerical challenges. To be precise, with stochastic transitions between the G and B states, i.e., $\varphi_G > 0$ and/or $\varphi_B > 0$, the two belief boundaries, $\pi = 0$ and $\pi = 1$, are no longer absorbing. Therefore, we can no longer first solve the four nonlinear equation system to pin down the values of welfare b and policy functions ($\mathbf{i}$, $\mathbf{x}^e$, and $\mathbf{x}^d$) at each boundary. To be precise, consider the boundary $\pi = 0$, the term $\frac{\varphi_G b'(0)}{b(0)}$ in the ODE (OA.9) is no longer zero. Indeed, to solve for $b(0)$, we need information about $b'(0)$, which depends on the solution in the interior region $\pi \in (0, 1)$.

In sum, the interconnected ODE system in the $\pi \in (0, 1)$ region and the nonlinear equation systems at the boundaries, $\pi = 0$ and $\pi = 1$, are interdependent, as summarized in Proposition 7. This interdependence between the interior region and the boundary conditions further complicate our numerical analysis. We can no longer solve the ODE by first solving the boundary values and then focus on the ODE system for the interior region as we do for our baseline model with constant unobservable λ.

Despite these challenges, we are able to obtain very high precision for our numerical solution.

OC.3   Additional Difficulties of External Habit Model of Section 8

Replacing Epstein-Zin recursive utility with Campbell-Cochrane external habit model invites a new state variable and inevitably we face an optimization problem with three state variables: (log) surplus consumption ratio s being the new state variable in addition to belief π and

capital stock K. As we have shown in Section OB, using the homogeneity property, we can simplify our model to a two-dimensional problem, which yields an interconnected partial differential equation (PDE) system.

The interconnected PDE system in the $\pi \in (0, 1)$ region and the nonlinear equation systems at the two belief boundaries, $\pi = 0$ and $\pi = 1$, as well as the boundary conditions, (OA.26) and (OA.14)-(OA.16) for $s = s_{max}$ have to be solved jointly. Proposition 8 summarizes the entire PDE system with 4 interconnected PDEs in the interior region with 12 nonlinear (differential) equations for the boundaries. This system is numerically quite challenging.

Moreover, we note that as the boundary $s = s_{\max}$ is not absorbing, the value function $b(\pi, s)$ at $s = s_{\max}$ depends on $b_s(\pi, s_{\max})$ and other equilibrium objects, which have to be solved jointly with the PDEs in the interior region where $s \in (-\infty, s_{\max})$. This further complicates our numerical analysis.

In sum, compared with our Epstein-Zin-utility-based models which require us to solve interconnected ODE system, Campbell-Cochrane external-habit-based model is technically much more challenging, as we have to solve an involved interconnected PDE problem described above.
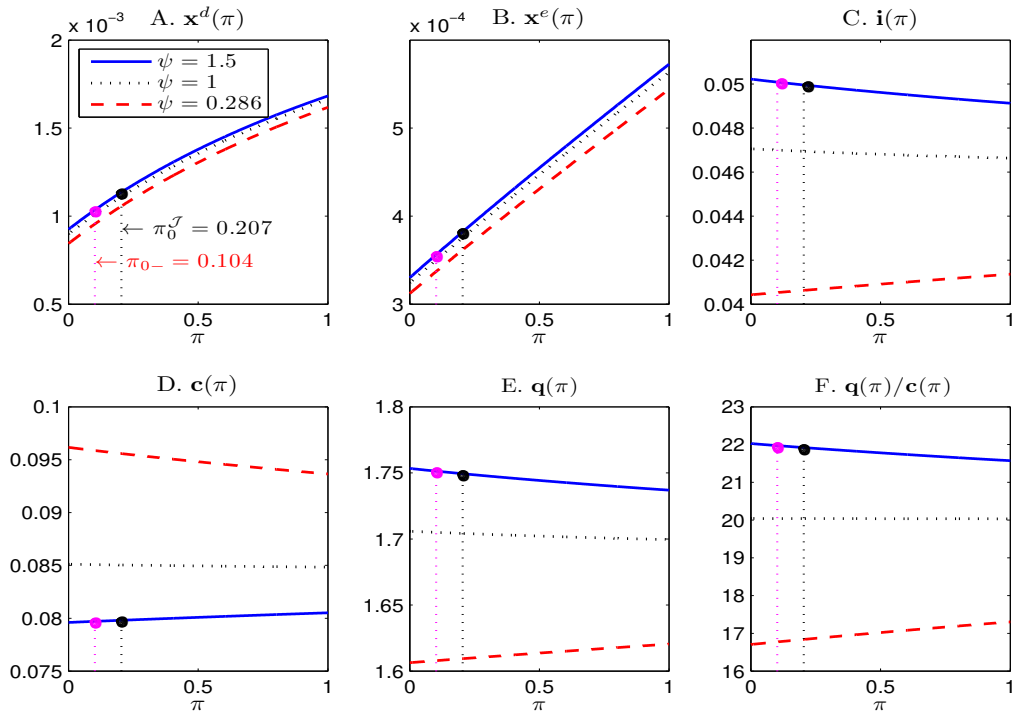

Figure O-7: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.286, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

Summary. In this online appendix, we have summarized the numerical challenges for the various models developed in this paper. The technical difficulties for our numerical solution are substantial and our numerical solution is significantly different from the ones in the literature.

OD   Comparative Statics

OD.1   Elasticity of Intertemporal Substitution ψ

In Figure O-7, we plot the first-best solutions for three levels of the EIS ψ: $\gamma = 0.286, 1, 1.5$. Panels A and B show that the larger the EIS ψ the higher both public mitigation $\mathbf{x}^d$ and private mitigation $\mathbf{x}^e$ spendings. Quantitatively, these differences are not very large. Panel C shows that the larger the EIS ψ the higher the investment-capital ratio $\mathbf{i}(\pi)$. Panel D shows that the higher the EIS ψ the lower the consumption-capital ratio $\mathbf{c}(\pi)$, as $\mathbf{c} = A - (\mathbf{i} + \mathbf{x}^d + \mathbf{x}^e)$. Panel E shows that the larger the EIS ψ the higher Tobin's average $\mathbf{q}(\pi)$. This follows directly from the comparative static result of changing ψ on $\mathbf{i}$ (Panel C), as Tobin's q is increasing with $\mathbf{i}$: $\mathbf{q}(\pi) = 1 + \phi'(\mathbf{i}(\pi))$. Panel F shows that the larger the EIS ψ the higher the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$, which follows from the comparative effects shown in Panels D and E.

The intuition for these results is as follows. The higher the EIS ψ, the more willing the agent is to substituting consumption over time. As a result, the agent spends more on mitigation and also invests more for the future.
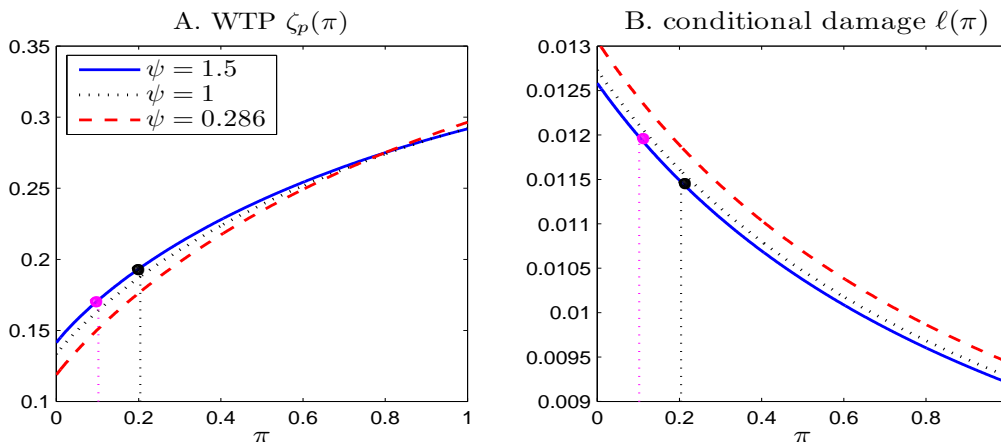

A. WTP $\zeta_p(\pi)$ B. conditional damage $\ell(\pi)$

Figure O-8:   This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.286, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

Additionally, we show that whether the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases or decreases when disaster arrives (which increases (worsens) belief π) crucially depends on whether the EIS ψ is larger or smaller than one. In our baseline case where $\psi = 1.5 > 1$, the equilibrium price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ decreases when a disaster arrives (i.e., when π increases). This result is consistent with Bansal and Yaron (2004) and the subsequent long-run risk literature, who show that the price-dividend ratio decreases in response to a negative growth shock when the EIS parameter ψ is set to be larger than one. Unlike Bansal and Yaron's pure exchange economy, our model features production and hence we need to compute the endogenous dividend $\mathbf{c}$ together with value of capital, Tobin's $\mathbf{q}$, in order to obtain the price-dividend ratio. However, we obtain the same results for the effect of EIS on the price-dividend ratio.

For the unity EIS ($\psi = 1$) Epstein-Zin utility case, which is a generalized version of expected logarithmic utility (with a flexible choice of risk aversion parameter γ), the wealth and the substitution effects exactly offset each other. As a result, the equilibrium price-

dividend ratio remains constant, i.e., $\mathbf{q}(\pi)/\mathbf{c}(\pi) = 1/\rho = 20$ at all levels of π (See the dotted line in Panel F.) Finally, with $\psi = 1/\gamma = 0.286 < 1$, the wealth effect is stronger than the substitution effect. For this case, as belief worsens (increases), the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{c}(\pi)$ increases, which is empirically counterfactual. This is one reason (among others) why Epstein-Zin utility with an EIS larger than one ($\psi > 1$) is a more appealing utility specification than commonly used expected utility for asset pricing.

In Figure O-8, we plot the WTPs for three levels of the EIS (Panel A). Note that the WTP curves for the three levels of ψ are reasonably close. In Panel B, the higher the EIS ψ, the lower the conditional damages $\ell(\pi)$. This is because the agent with a higher EIS mitigates more as we show in Panels A and B of Figure O-7. As a result, the higher EIS the lower the conditional damages $\ell(\pi)$.
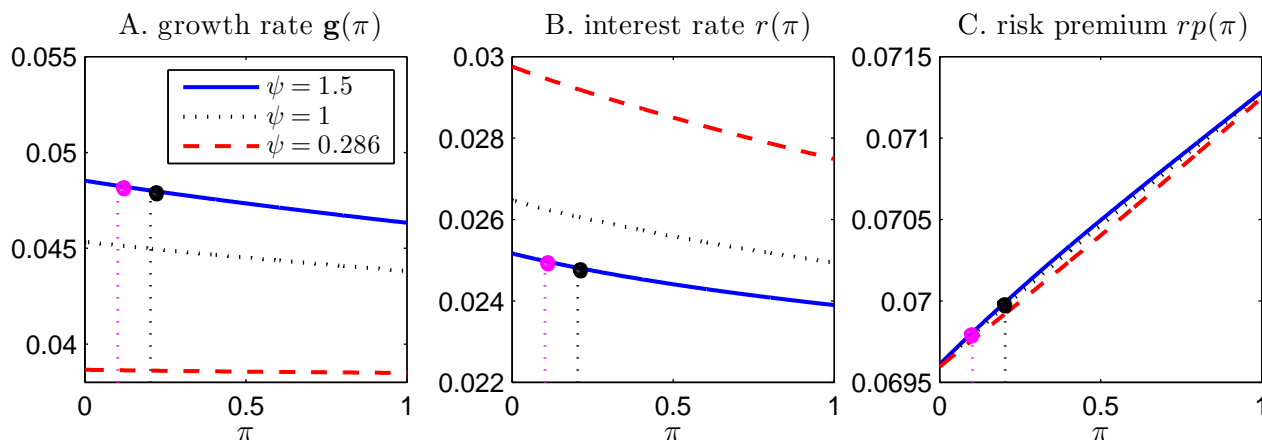

Figure O-9: This figure plots the planner's first-best solution for three values of the EIS ψ: $1/\gamma = 0.286, 1, 1.5$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

Figure O-9 of Panel A shows that the higher the EIS ψ, the higher the expected growth rate $\mathbf{g}(\pi)$. This result follows from 1.) the higher the EIS the higher investment result (as shown in Panel C in Figure O-7) and 2.) the higher the EIS the smaller damage $\ell(\pi)$ (as shown in Panel B of Figure O-8.)

Panel B of Figure O-9 shows that the higher the EIS the more willing the agent is to substituting consumption over time and in equilibrium the lower the interest rate. Note that the effect of the EIS on risk premium $rp(\pi)$ is negligible (Panel C.)

OD.2 Disaster Arrival Rate λ_B in State B

In Figure O-10, we plot the first-best solutions for three levels of the disaster arrival rate in state B: $\lambda_B = 0.15, 0.3, 0.6$. Panel A shows that the higher the disaster arrival rate λ_B in state B, the higher the public mitigation spending $\mathbf{x}^d$. Moreover, the more pessimistic the agent's belief the stronger this effect. Note that the wedge between the lines for two different levels of λ widens as π increases.

Panel B shows that increasing the arrival rate λ_B has a highly nonlinear effect on the private mitigation spending $\mathbf{x}^e$. Increasing λ_B from 0.15 to 0.3 significantly increases the
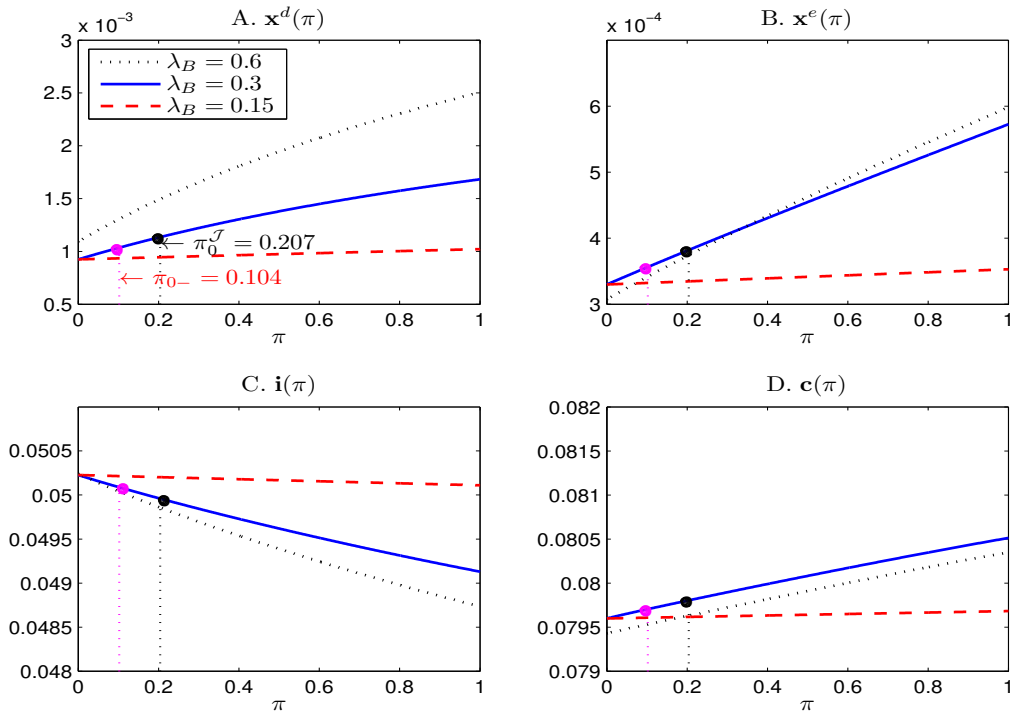
Figure O-10: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.15, 0.3, 0.6$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

mitigation spending (for sufficiently large values of π.) However, further increasing λ_B from 0.3 to 0.6 has limited effects on the private mitigation spending $\mathbf{x}^e$.

Panel C shows that as λ_B increases, investment falls. The higher the belief level π (the more pessimistic the agent) the larger the impact of λ_B on $\mathbf{i}$. Panel D shows that the impact of λ_B on consumption $\mathbf{c}(\pi)$ is ambiguous due to the general equilibrium effect.

In Figure O-11, we show that λ_B has a large effect on the WTP ζ_p (Panel A). For example, as a jump arrival changes the belief from prior $\pi_{0-} = 0.104$ to posterior $\pi_0^{\mathcal{J}} = 0.207$, the WTP increases from 26.8% to 33.3% when $\lambda_B = 0.6$. In contrast, when $\lambda_B = 0.15$, the WTP barely changes from 14.3% to 14.5% in response to the same jump arrival. Panel B shows that the higher the arrival rate λ_B the smaller the conditional damage $\ell(\pi)$. This is intuitive as mitigation spending is higher when λ_B is larger. However, as investment is lower when λ_B is larger, the impact of λ_B on the growth rate $\mathbf{g}(\pi)$ is minimal as the two channels (investment and conditional damage) offset each other (Panel C). Panel D shows that the higher the arrival rate λ_B the lower Tobin's $\mathbf{q}$, tracking the impact of λ_B on $\mathbf{i}$ as $\mathbf{q} = 1 + \theta\mathbf{i}$. Panel E shows that the quantitative effect of λ_B on the risk-free rate r is moderate at best and Panel F shows that the effect of λ_B on the market risk premium rp is very small.

OD.3 Time Rate of Preference ρ

In our baseline calculation, we set the time rate of preference ρ at 5% per annum, a commonly used value. Next, we compare our baseline model results with two other economies with lower discount rates: $\rho = 3\%$ and $\rho = 4\%$.
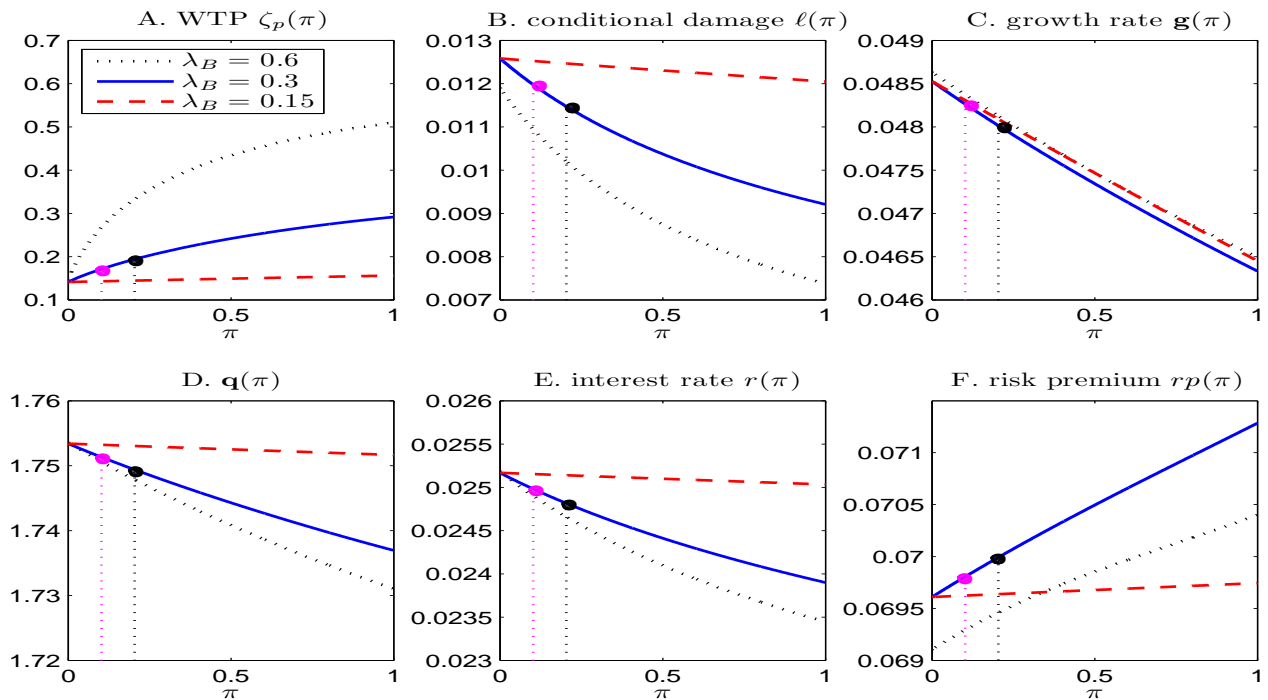
Figure O-11: This figure plots the planner's first-best solution for three values of the annual disaster arrival rate λ_B: $0.15, 0.3, 0.6$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.
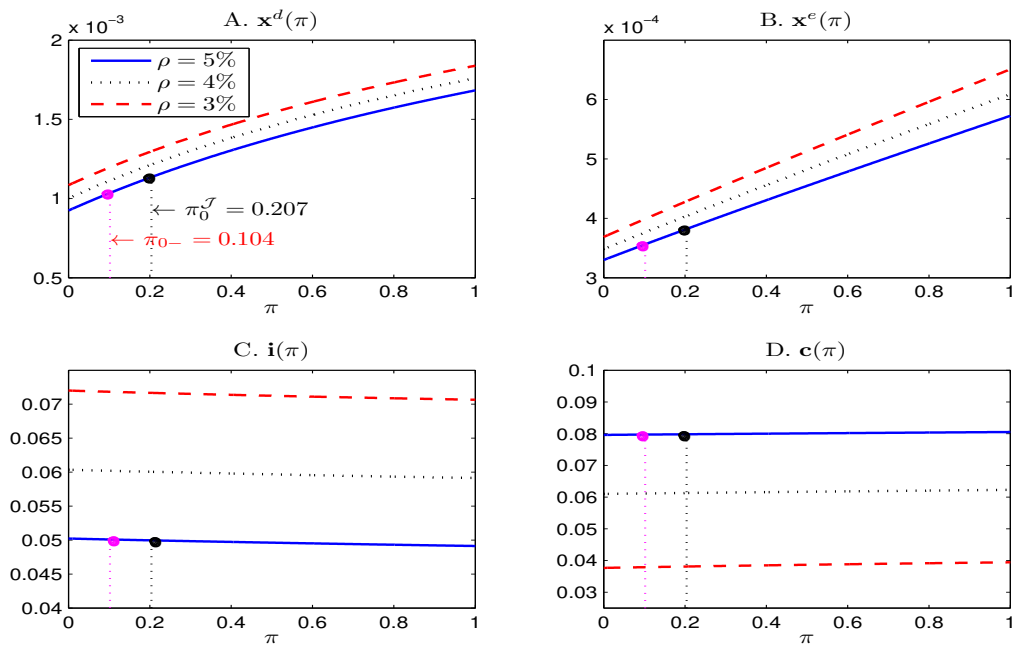

Figure O-12: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $3\%, 4\%, 5\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.
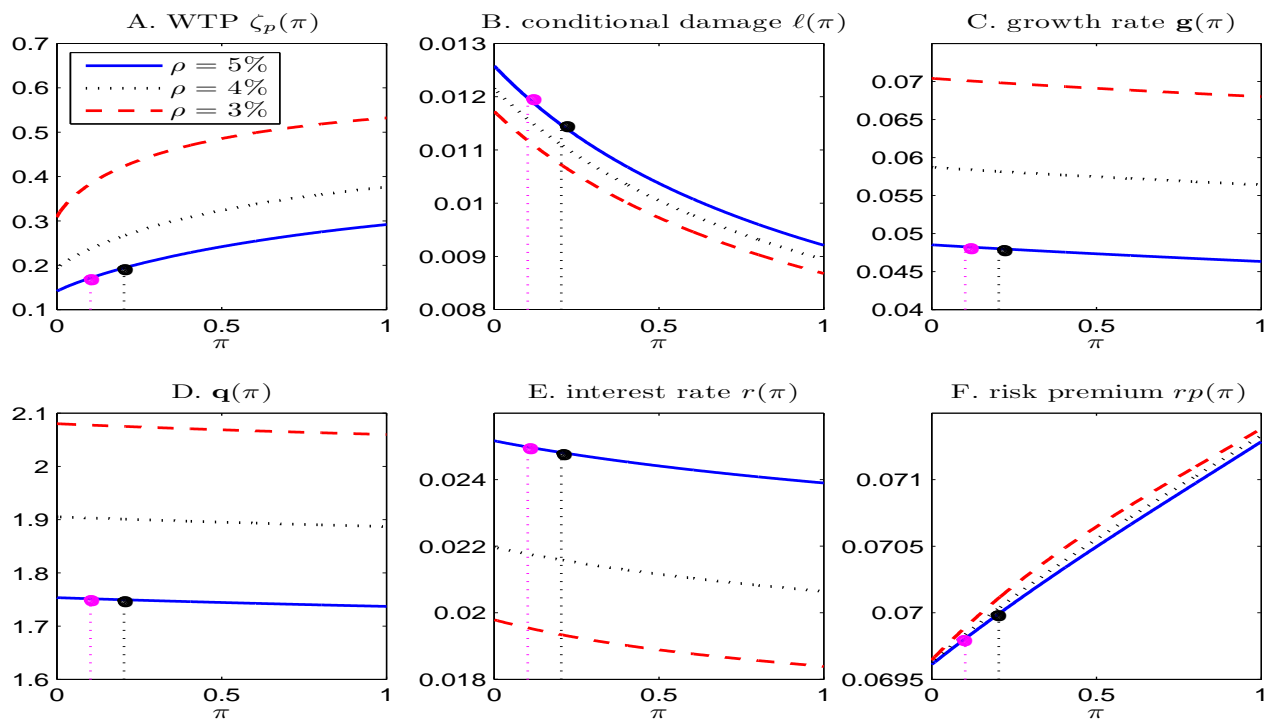
Figure O-13: This figure plots the planner's first-best solution for three values of the annual time rate of preference ρ: $3\%, 4\%, 5\%$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

Panels A and B of Figure O-12 show that the higher the time rate of preference ρ, the less the planner spends on both types of mitigation spendings, $\mathbf{x}^d$ and $\mathbf{x}^e$. Similarly, Panel C of Figure O-12 shows that the higher the time rate of preference ρ, the less the planner invests and Panel D shows that the higher the time rate of preference ρ the more the agent consumes. The quantitative effects on consumption are large. For example increasing ρ from 4% to 5% roughly increases consumption $\mathbf{c}$ from 6% to 8% per annum.

In Figure O-13, we show that the quantitative effects of the time rate of preference ρ on the WTP is large (Panel A). For example, as a jump arrival changes the belief from prior $\pi_{0-} = 0.104$ to posterior $\pi_0^{\mathcal{J}} = 0.207$, the WTP increases from 23.7% to 26.7% when $\rho = 4\%$, and increases from 38.4% to 42.3% when $\rho = 3\%$.

The higher the time rate of preference ρ the higher the conditional damage $\ell(\pi)$ (Panel B) and the lower the Tobin's $\mathbf{q}$ (Panel D) as the agent is less patient and puts a smaller weight on the future. Since these two forces push towards the same direction, the higher the discount rate ρ the lower growth rate $\mathbf{g}$ (Panel C).

Finally, Panel E shows that the quantitative effect of ρ on the risk-free rate r is moderate at best and Panel F shows that the effect of ρ on the market risk premium rp is very small.

OD.4 Coefficient of Relative Risk Aversion γ

In our baseline calculation, we set the coefficient of relative risk aversion γ at 3.5, which is within the range of widely used values (e.g., 2 to 5). Next, we compare our baseline model results to two other economies with $\gamma = 1.5$ and $\gamma = 10$.
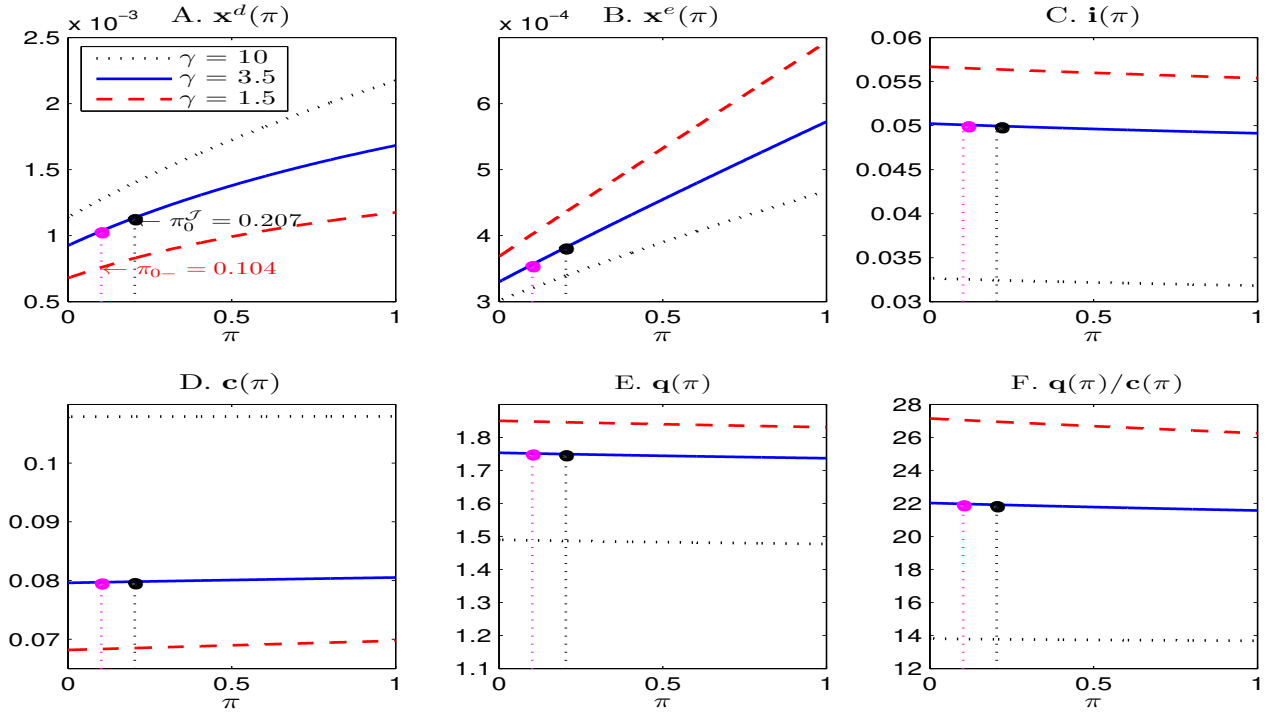

Figure O-14: This figure plots the planner's first-best solution for three values of coefficient of relative risk aversion γ: $1.5, 3.5, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

Panel A of Figure O-14 shows that the higher the coefficient of relative risk aversion γ, the more the planner spends on distribution mitigation $\mathbf{x}^d$ and the less the planner spends on exposure mitigation $\mathbf{x}^e$. The higher the coefficient of relative risk aversion γ the less the planner invests (Panel C), the more the agent consumes (Panel D) the lower Tobin's average $\mathbf{q}$ (Panel E) and the lower the price-dividend ratio $\mathbf{q}(\pi)/\mathbf{i}(\pi)$ (Panel F).

In Figure O-15, we show that the quantitative effects of increasing risk aversion from $\gamma = 3.5$ to $\gamma = 10$ on the WTP is large (Panel A). For example, when $\pi = 0.104$ as we increase γ from 3.5 to 10, the WTP ζ_p increases from 17.2% to 29.4%, the disaster distribution mitigation spending $\mathbf{x}^d$ increases from 0.1% to 0.13%, and the disaster exposure mitigation spending $\mathbf{x}^e$ decreases from 0.04% to 0.03%. The effects of changing risk aversion become even larger as belief worsens.

The higher the coefficient of relative risk aversion γ the lower the conditional damage $\ell(\pi)$ (Panel B of Figure O-15) but the lower the growth rate $\mathbf{g}(\pi)$ (Panel A of Figure O-16). This is because a more risk-averse agent mitigates more but invests less. Quantitatively, the negative effect of increasing γ via investment on growth dominates the positive effect of increasing γ via mitigation. As a result, the net effect of increasing γ on growth is negative.

Finally, Panels B and C of Figure O-16 show that the quantitative effects of γ on the
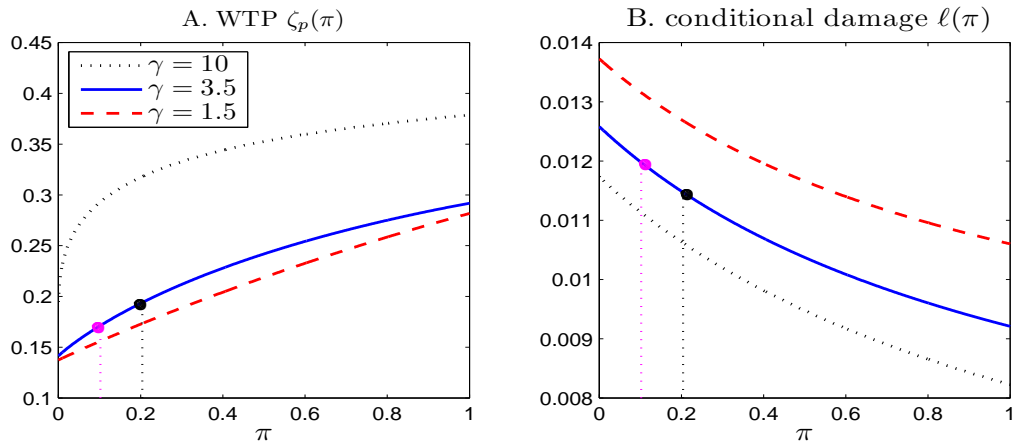
Figure O-15: This figure plots the planner's first-best solution for three values of the coefficient of relative risk aversion γ: $1.5, 3.5, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

risk-free rate r and the market risk premium rp are very large, as we expect (in line with standard asset pricing results.)
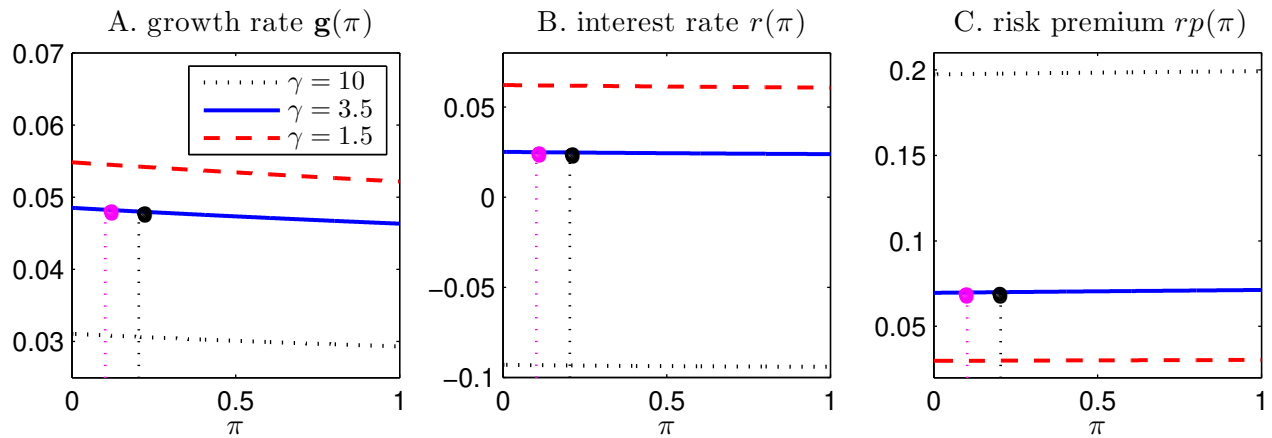

Figure O-16: This figure plots the planner's first-best solution for three values of the coefficient of relative risk aversion γ: $1.5, 3.5, 10$ for our baseline learning model (with Epstein-Zin utility). The other parameter values are given in Table 3.

References

Bansal, R. and Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), pp.1481-1509.

Brunnermeier, M.K. and Sannikov, Y., 2014. A macroeconomic model with a financial sector. *American Economic Review*, 104(2), pp.379-421.

Campbell, J.Y. and Cochrane, J.H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107(2), pp.205-251.

DeMarzo, P.M. and Sannikov, Y., 2006. Optimal security design and dynamic capital structure in a continuous–time agency model. *Journal of Finance*, 61, pp.2681-2724.

Sannikov, Y., 2008. A continuous-time version of the principal-agent problem. *The Review of Economic Studies*, 75, pp.957-984.

Pindyck, R.S. and Wang, N., 2013. The economic and policy consequences of catastrophes. *American Economic Journal: Economic Policy*, 5(4), pp.306-339.