Internet Appendix to "Should the government be paying investment fees on $3 trillion of tax-deferred retirement assets?"

Mattia Landoni[*] and Stephen P. Zeldes[†]

This version: June 4, 2024

Abstract – This Internet Appendix contains supplementary materials. Section 1 examines how, in the partial equilibrium model in Section 2 of the paper, alternative arrangements regarding the deductibility of fees influence the incidence of the higher fees that occur under Traditional. Section 2 provides additional background information on fees. Section 3 calibrates the size of the implicit government account and total fees paid for six additional countries, similar to what is done in Section 4 of the main text for the US. Sections 4 and 5 contain a set of proofs and derivations for our general equilibrium model.

Keywords: asset management, fees, taxes, retirement savings
JEL: D14, G11, G23, G28, G51, H21, J26, J32

[*]Federal Reserve Bank of Boston

[†]Frank R. Lautenberg Professor of Economics and Public Policy, Graduate School of Business, Columbia University; and NBER

1 Extension: Accounting for the deductibility of fees

This section examines how, in the partial equilibrium model in Section 2 of the paper (holding constant both percentage fees f and tax rates), alternative arrangements regarding the deductibility of fees influence the incidence of the higher fees that occur under Traditional. We examine two related questions that arose often when explaining our argument. First, does Roth result in higher present-value government revenue in the model than Traditional simply because under Traditional fees are paid with pretax money (i.e., implicitly tax-deductible), whereas under Roth they are paid with after-tax money? Second, our argument assumes that fees are paid with money from within the retirement account ("account money"). Would our argument still hold if fees were paid with money from outside of the retirement account ("outside money")? We show that the lower present-value government revenue under Traditional is not solely due to the deductibility of fees, but also to the existence of additional assets. For this reason, even if a Roth account holder manages to pay fees using outside money and then deduct these fees as an investment expense, present-value government revenue is still higher under Roth. We also show that the government can make the individual fully responsible for fees, but doing so only shifts the burden of fees onto individuals. In short, the additional assets under Traditional result in greater fee revenue for asset managers; depending on the incidence of these fees, either the government or individuals or both prefer Roth.

1.1 Background on deductibility of fees

Under the U.S. tax code, some retirement account investment costs paid using "outside money" (i.e., money from an ordinary taxable account) are or have been tax-deductible. For an employer paying the expenses of a plan, "ordinary and necessary" plan-related expenses are deductible business expenses under U.S.C. 26 §162. For an individual owning an IRA, prior to the 2017 US tax reform, fees were deductible as "miscellaneous itemized deductions" under U.S.C. 26 §212 relating to expenses for production of income (Dold and Levine, 2011).

At the time of writing, the tax code does not explicitly discuss the treatment of IRA and retirement plan fees, leaving the matter in the hands of the Internal Revenue Service (IRS). The IRS has not issued detailed public guidance, but it has repeatedly upheld in private that the payment of certain retirement account fees using outside money is not considered a contribution to the account (and therefore it is presumably a deductible expense under Sections 162 or 212, although the IRS has not pronounced itself on deductibility).

In recent taxpayer guidance (Private Letter Ruling 201104061), the IRS appears to make a distinction between account-level fees and asset-level fees (without using this terminology). Account-level fees, such as wrap fees and advisory fees, are considered akin to overhead expenses, and therefore payable with outside money; whereas asset-level fees, such as brokerage commissions, are "intrinsic to the value" of the assets and should be capitalized in the asset values, in essence requiring that they be paid with account money. For instance, if the wrap fee covers brokerage commissions but does not depend on the number of trades, it is considered an account-level fee and it becomes payable with outside money.

Importantly, the IRS has never made a distinction between Traditional and Roth IRAs with regard to fee deductibility. In practice, however, there is a very relevant distinction. Assume for simplicity that contribution limits are not binding, and that the tax rate at the time of contributions is the same as the tax rate on distributions ($\tau_L = \tau_R = \tau$). Under these assumptions, regardless of account type, one dollar of fees could be paid with outside money, and then deducted as an expense, for a total after-tax cost of $1 - \tau$. An individual paying for fees with money from a Traditional account should be indifferent about the source of the fee payment, because one dollar of Traditional account money is worth only $1 - \tau$ in after-tax terms. In contrast, an individual paying one dollar of fees from a Roth account would prefer to use outside money because the account money is already after-tax. Under these assumptions, paying fees with outside money is only beneficial for Roth investors.[1]

1.2 The sources of the revenue difference between Roth and Traditional

The above discussion of fee deductibility raises important questions. Would alternative assumptions about the deductibility of fees shift some or all of the burden to individuals

[1] In reality, contributions to Traditional and Roth accounts are subject to limits. Since these limits are set at the same nominal amount (currently $6,000) for both accounts, the Traditional limit is more likely to be binding. If the limit is binding, the shadow cost of using Traditional account money to pay fees could be greater than the cost of using outside money, and therefore there are taxpayers for whom it is advantageous to pay Traditional fees with outside money regardless of deductibility considerations. Nonetheless, for taxpayers that currently use Traditional account money to pay fees, a switch to a system in which Roth is the only option and fees paid with outside money are deductible would create an incentive to use outside money.

(unlike in Table 2 of the paper, where all of the added fees under Traditional are borne by the government)? In particular, if the U.S. government were to switch to a Roth-only system, and fees were made deductible again, would part or all of the additional revenue of Roth be offset by individuals' increased fee deductions?

To examine these questions, we extend our results from Section 2 of the paper, obtained under the assumption that the level of fees (f) is not affected by the amount of assets under management. We decompose the difference in tax revenue between Traditional and Roth into two components: (i) fee deductibility and (ii) the sheer existence of additional assets. Assuming that all labor, retirement and investment income is taxed at the same flat rate ($\tau_L = \tau_R = \tau_I = \tau$), a fraction $1 - \tau$ of the difference in tax revenue is due to the fact that fees are implicitly deductible under Traditional and nondeductible under Roth. However, the remainder (a fraction τ) is due to the fact that under Traditional there are more assets and more fees are paid.

To see this, consider an individual allocating \$1 of pretax income to retirement savings at time 0. Depending on the taxation scheme, tax may be deferred or not, and therefore the initial account balance may be 1 or $1 - \tau$. The account grows at a rate r, and the asset manager charges fees proportional to the account balance at a rate f. At time 1, if the taxation scheme is Traditional, the government takes a fraction τ of the account balance, and the remainder is paid out to the individual.

Panel (a) of Table 1 shows the different present value outcomes in terms of retirement wealth, fee revenue and tax revenue under each of four account types. Because of the absence of frictions other than fees, the present value of these three quantities must sum to the initial contribution, i.e., to one. The four account types are obtained by combining two taxation schemes (Roth or Traditional) and two deductibility rules (deductible or nondeductible fees, represented as "Ded" and "NDed"). The first two accounts are $\text{Roth}_{\text{NDed}}$ (or simply "Roth", since fees in standard Roth accounts are non-deductible) and Trad_{Ded} (or simply "Trad", since fees in standard Traditional accounts are effectively deductible). The other two accounts are hypothetical. A fee-deductible Roth (Roth_{Ded}) is a Roth account in which the individual is able to get a deduction for fees paid. We assume that the future value of these deductions is added to the individual's retirement wealth.[2] A fee-nondeductible Traditional ($\text{Trad}_{\text{NDed}}$) is

[2]Alternative assumptions (e.g., that the value of the current-period deduction is immediately added to the account) yield the same qualitative result but with less-tractable expressions.

	Retirement wealth	Fee Revenue	Tax Revenue
a. Present value			
Roth	$(1-\tau)(1-f)^T$	$(1-\tau)\left(1-(1-f)^T\right)$	τ
Trad	$(1-f)^T(1-\tau)$	$1-(1-f)^T$	$\tau(1-f)^T$
Roth_{Ded}	$(1-\tau)\left[(1-f)^T(1-\tau)+\tau\right]$	$(1-\tau)\left(1-(1-f)^T\right)$	$\tau\left[\tau+(1-\tau)(1-f)^T\right]$
$\text{Trad}_{\text{NDed}}$	$(1-f)^T-\tau$	$1-(1-f)^T$	τ
b. Decomposition 1 (Practical)			
Roth_{Ded} – Roth	$\tau(1-\tau)\cdot\left(1-(1-f)^T\right)$	0	$-\tau(1-\tau)\left(1-(1-f)^T\right)$
Trad – Roth_{Ded}	$-\tau(1-\tau)\cdot\left(1-(1-f)^T\right)$	$\tau\left(1-(1-f)^T\right)$	$-\tau^2\left(1-(1-f)^T\right)$
Trad – Roth	0	$\tau\left(1-(1-f)^T\right)$	$-\tau\left(1-(1-f)^T\right)$
c. Decomposition 2 (Policy)			
$\text{Trad}_{\text{NDed}}$ – Roth	$-\tau\left(1-(1-f)^T\right)$	$\tau\left(1-(1-f)^T\right)$	0
Trad – $\text{Trad}_{\text{NDed}}$	$\tau\left(1-(1-f)^T\right)$	0	$-\tau\left(1-(1-f)^T\right)$
Trad – Roth	0	$\tau\left(1-(1-f)^T\right)$	$-\tau\left(1-(1-f)^T\right)$

Table 1: Present value of retirement wealth, fee revenue, and tax revenue under different combinations of taxation scheme and fee deductibility. Roth is an account with front-loaded taxation, such as a U.S. Roth IRA. Roth is an account with back-loaded taxation, such as a U.S. Traditional IRA. “Ded” and “NDed” indicate that fees are, respectively, deductible and nondeductible. Since in a Traditional account fees are implicitly deductible, there is no Trad_{Ded}, but just Trad. Similarly, since in a Roth account fees are implicitly nondeductible, there is no $\text{Roth}_{\text{NDed}}$, but just Roth.

a Traditional account in which the individual is taxed on the gross-of-fees balance $(1 + r)^T$, i.e., fees are explicitly made nondeductible.

Our goal is to understand the sources of present-value differences in fee revenue, retirement wealth, and tax revenue between account types. To do this, we calculate the difference between different account types with respect to these three quantities. The three differences must sum to zero.

Panel (b) of Table 1 decomposes the difference between Traditional and Roth based on the following identity:

$$\text{Trad} - \text{Roth} = (\text{Roth}_{\text{Ded}} - \text{Roth}) + (\text{Trad} - \text{Roth}_{\text{Ded}})\,.$$

The first term $(\text{Roth}_{\text{Ded}} - \text{Roth})$ is the effect of starting from a pure Roth and making fees deductible. Doing so increases retirement wealth at the expense of the government. The second term $(\text{Trad} - \text{Roth}_{\text{Ded}})$ is the effect of switching from a fee-deductible Roth to a Traditional. Doing so directly increases assets under management, and thus fees, this time at the expense of both the individual and the government.

This decomposition is of practical importance because, although a fee-deductible Roth account does not exist per se, as discussed above, individuals were able in the past to deduct some fees paid with outside money and might be able again in the future. A fee-deductible Roth represents the case in which individuals are able to deduct *all* fees. In this extreme case, a switch from Traditional to Roth fails to realize a fraction $(\text{Roth}_{\text{Ded}} - \text{Roth})\,/\,(\text{Trad} - \text{Roth}) = 1 - \tau$ of the expected improvement in tax revenue. Thus, a fraction $1 - \tau$ of the expected revenue benefit of switching from Traditional to Roth is attributable to the implicit nondeductibility of fees under Roth, and the remainder (τ) is attributable to the lower total fee revenue, i.e., to the existence of additional assets under Traditional.

Panel (c) of Table 1 decomposes the difference between Traditional and Roth based on a different identity:

$$\text{Trad} - \text{Roth} = (\text{Trad}_{\text{NDed}} - \text{Roth}) + (\text{Trad} - \text{Trad}_{\text{NDed}})\,.$$

The first term $(\text{Trad}_{\text{NDed}} - \text{Roth})$ is the effect of starting from Roth and switching to a fee-nondeductible Traditional. Doing so leaves tax revenue intact, but it still increases fee revenue by an amount $\tau\left(1 - (1 - f)^T\right)$ at the expense of retirement wealth. The second term $(\text{Trad} - \text{Trad}_{\text{NDed}})$ is the effect of going from a fee-nondeductible Traditional to a pure Traditional. Fee revenue is unvaried, but the burden of the excess fees is transferred from the government to the individual.

This decomposition is of policy importance because it shows that making the individual fully responsible for fees obviously solves the government's revenue problem, but only by shifting the burden of fees onto individuals. Thus, if fees are deductible, individuals are indifferent and the government prefers Roth; if fees are nondeductible, the government is indifferent and individuals prefers Roth.

1.3 Practical importance

Above we showed that a switch to Roth combined with a deduction allowance for Roth fees may cause individuals to start paying fees with money from outside the account. In this scenario, individuals would capture a fraction $1 - \tau$ of the total fee savings from the switch. Assuming $\tau = 25\%$, this would amount to a three-quarters reduction in the expected government savings. In reality, however, the extent to which individuals would be able to take advantage of this opportunity is limited by several factors.

For IRA owners, even when fees were deductible, there were significant limits to the deductibility of expenses incurred in the production of income under U.S.C. 26 §212. First, miscellaneous itemized deductions were subject to a floor of 2% of adjusted gross income. Second, they were a preference item for purposes of the alternative minimum tax. Third, they were itemized deductions and therefore worthless for taxpayers taking the standard deduction (Dold and Levine, 2011). Taken together, these restrictions would have prevented a large number of IRA owners from taking deductions for Roth fees.

For participants in employer-sponsored retirement plans, there is no obvious way to cover investment costs with outside money. Some employers already cover some of the costs, but it is not obvious that upon a switch to Roth employers would have an incentive to do more than they already do. According to a Deloitte study (Rosshirt et al., 2014), employers cover roughly one-tenth of account costs, or 6 bps—a small fraction of the total.

Finally, our estimated investment costs largely consists of trading costs and expense ratios of mutual funds and similar investment product, two types of expenses for which using outside money is usually impractical or impossible. IRA owners might be able to cover their explicit trading costs and advisory services with a wrap fee, which could be paid with outside money, but they would still be have to use account money for implicit trading costs such as bid-ask spreads, as well as for mutual fund expense ratios. Retirement plan participants are largely invested in mutual funds and other collective investment products, and many employers would find it difficult to cover more costs than they already do.

2 Further detail on investment fees

2.1 Additional detail on fees and costs

2.1.1 Distribution fees and complimentary advice provided with the account

Often, asset management accounts come with some level of complimentary advice. Part of the cost of these services is financed by "distribution fees" that are charged not by the account provider, but rather by the managers of the products available in the account. These fees are then rebated to the account provider in the same way that salespeople receive a commission from the manufacturer of the products they sell. For instance, in the case of mutual funds, there are "load" fees, e.g., one-time fees paid upon purchase or redemption of shares, as well as ongoing fees called variously "level load fees", "service fees", or "12b-1" fees that are included in the ongoing expense ratio.[3] 12b-1 fees cover two types of expense: distribution costs, i.e., commissions to the sales force (capped at 75 bps), and shareholder servicing costs, e.g., cost of providing internet access to fund filings, etc. (capped at 25 bps). All these fees are distribution fees and are rebated to the account provider.

For instance, with a 5% front load, an investor giving $100 to the broker is only investing $95. If the fund has 12b-1 fees in addition to loads, these fees will be levied every year upon the $95. The same fund may have multiple classes of shares. According to Morningstar's Glossary, "In a typical multi-class situation, the class A fund has a front-end load and either a 0.25% distribution fee or a 0.25% service fee. Class B shares usually have a contingent deferred sales charge and a corresponding 0.75% 12b-1 fee, plus a maximum 0.25% service fee. [...] Class C shares customarily charge a level load with the same fee structure found in a class B share."

An account provider may derive revenue from explicit account fees, ongoing distribution

[3] 12b-1 fees are so called after SEC Rule 12b-1 under the Investment Company Act of 1940. FINRA regulations from 1993 establish the caps on these fees. See SEC > Mutual Funds Fees and Expenses (https://www.sec.gov/answers/mffees.htm).

fees like 12b-1 fees, and one-time distribution fees like loads. In the presence of explicit account fees, investors typically have access to "no-load" funds, although no-load funds can still charge up to 0.25% of level load fees. Overall, advisory and distribution fees (excluding 12b-1 fees, which are already included in the net expense ratio) average about 50 bps (Bogle, 2014).

2.1.2 Advisory fees charged by mutual fund managers

A mutual fund's net expense ratio includes three types of fees. First, paperwork fees: custodial fees, legal fees, record-keeping fees, etc. These fees typically cover the cost of inevitable services provided by third parties unaffiliated with the mutual fund. Second, distribution and service fees, discussed above. Third, asset management advisory fees, i.e., the actual revenue of the money management company that sponsors the fund.

2.1.3 Trading costs

Trading costs include explicit commissions and implicit costs like bid-ask spreads and market impact. Quantifying trading costs is challenging. We are not aware of any peer-reviewed, asset-weighted estimates of trading costs for U.S. equity mutual funds, or of any estimates (asset-weighted or equal-weighted) for bond funds. Equal-weighted estimates are useful to discuss the average *fund*, but the government is interested in the average *dollar* invested in a fund, because a fraction of each dollar will eventually generate tax revenue. Often, equal-weighted estimates are driven by many inefficient small funds, whereas most of the dollars are in a few efficient, large funds. Here we report some equal-weighted estimates for completeness' sake.

Livingston and Zhou (2015) estimates that equal-weighted average explicit portfolio commissions alone are in the order of 18 bps. Wealthfront (2016) finds a very similar number (20 bps). The literature on implicit trading costs reports a wide range of estimates, perhaps because of the difficulty of quantifying these costs. Wermers (2000) estimates that commissions, transaction costs and cash drag due to liquidity cause a 230 bps wedge between the average equity mutual fund's returns and the return of the stocks they hold. Edelen et al. (2013) estimate average total trading costs of 144 bps using a sample of over 3,000 U.S. domestic equity funds. In this sample, implicit costs exceed the average expense ratio (119

	Performance (bps)		
Source	Net	Gross	Benchmark
Berk and van Binsbergen (2015)	-12	*	Investable Vanguard funds
Malkiel (2013)	-64		Large cap active vs. SP500 Index
Malkiel (2013)	-82		Bond funds vs. Barclays US Agg
Fama and French (2010)	-100	-5	3- and 4-factor benchmarks
Wermers (2000)	-100	130	Own stock holdings
Carhart (1997)	-102	**	1-, 3- and 4-factor benchmarks
Jensen (1968)	-40	~0	1-factor benchmark (CAPM)
Fama (1965)	-60	+20	Market

Table 2: **Estimates of average equity mutual fund underperformance.** "Net" and "Gross" refers to expenses. The definition of "expenses" is typically the expense ratio, but in the case of Wermers (2000) it includes everything including cash drag and trading costs (see text) — Footnotes:
[*] Underperformance with respect to the Vanguard benchmark, which charges fees of 18 bps
[**] 100 bps of expense ratio are associated with underperformance of 154 bps. Using our asset-weighted estimate of 66bps, $102 = 154 \times 66/100$.

bps).

2.2 Summary of the literature on performance of actively managed mutual funds

Measuring mutual fund performance is difficult. First, actual performance net of the benchmark has a large random component, and a reliable estimate of performance requires a long time series. Second, unlike direct estimates of fees, every benchmark-based estimate implies and depends on an asset-pricing model. As a result, the literature on mutual fund performance contains numerous estimates done using different methodologies and benchmarks, a few of which are summarized Table 2. Some of these are in the main paper, and the remainder are described next.

The literature begins with classics such as Fama (1965) and Jensen (1968). Both studies show no evidence of managers predictably beating the market on a net-of-fee basis; on average, mutual funds show a small underperformance with respect to the market benchmark. Consistent with market efficiency, this underperformance is of the same magnitude of fees and cash drag. More recently, Carhart (1997) compiles a mutual fund database that is comprehensive and free of survivorship bias, and uses it to replicate the basic result that there is no evidence of skilled or informed mutual fund managers.[4] Using four-factor and three-factor benchmarks, Carhart finds that there is manager-specific persistence in performance that is not explained by fees, but only for the worst-performing funds. He estimates that 100 bps of expense ratio are associated with a 154 bps underperformance with respect to the market. Wermers (2000) decomposes mutual fund returns into stock-picking talent, style, transaction costs, and expenses, concluding that mutual funds hold stocks that beat the market by 1.3%, but the funds' returns underperform the market by 1%. He attributes the large discrepancy to cash drag (0.7%) and expenses and transaction costs (1.6%).

Based on a CAPM benchmark, Fama and French (2010) estimate net-of-fees underperformance of about 1% per year. Malkiel (2013) compares several categories of funds with their indices, finding that active large-cap equity funds underperform the S&P 500 Index by 64 bps, and bond funds underperform the Barclay US Aggregate Bond Index by about 84 bps.

Some recent studies have focused on investable benchmarks. French (2008) estimates a broad measure of the annual cost of active management, including not only costs faced by individual investors but also costs faced by institutions and market-making gains by financial intermediaries over 1980-2006. The cost of active management is 0.67% of the aggregate value of the market, in addition to the approximately 0.10% cost of passive management. As a passive benchmark, French uses the Vanguard Total Stock Market Index. Berk and van Binsbergen (2015) compare active funds' dollar returns (as opposed to percent returns) against the relevant Vanguard benchmarks. They estimate a value weighted net alpha of -12 bps (not statistically different from zero) in addition to the fees on the Vanguard benchmark (18 bps), implying a total cost of active money management of at least 30 bps, not including the benchmark's implicit trading costs, and any account-level fees for recordkeeping and advice.

[4]Malkiel (1995) also addresses survivorship bias and extends the sample period of previous studies which claimed to find persistence in returns. Carhart also addresses those studies, explaining their findings as the result of momentum investing.

Country	Data Year	Retirement Assets $b	Retirement Assets % Deferred	Gov. Acct. τ_R	Gov. Acct. $b	Fees	τ_C	Subsidy $b	Subsidy % GDP
United States	2018	16,464	94%	20%	3,084	0.80%	21%	19.5	0.10%
Canada	2015	1,003	86%	15%	129	2.06%	15%	2.3	0.15%
United Kingdom	2015	950	32%	20%	41	1.45%	20%	0.7	0.02%
Netherlands	2015	108	100%	39%	41	1.41%	25%	0.4	0.06%
Switzerland	2015	945	100%	4.0%	38	1.29%	18%	0.4	0.06%
Australia	2015	1,797	55%	3.4%	34	1.10%	30%	0.3	0.02%
Japan	2015	112	100%	2.6%	3	1.47%	30%	0.0	0.00%

Table 3: **Estimated subsidy to the asset management industry in seven countries with the largest Traditional retirement assets.** Fees are the asset-weighted average of money market, equity and fixed-income mutual fund fees based on overall (not retirement-only) asset allocation in that country. For each country, τ_R (the tax rate on retirement income, and therefore the fraction of Traditional assets that implicitly belong to the government) is calculated as the average tax rate faced by a person earning the average retirement income with no other income. τ_C, the corporate tax rate, is simply the top statutory tax rate. Sources: see text.

3 International fee calibration

In Table 3, we carry out a back-of-the envelope calibration of the annual subsidy to asset managers for the seven countries with the largest dollar amounts of tax-deferred assets (the rows are ordered by the size in dollars of the implicit government account). As in the main text, the subsidy is calculated as

$$\text{Annual subsidy} = S^{Trad} \cdot \tau_R^{Trad} \cdot f \cdot (1 - \tau_C)\,. \tag{1}$$

The data covers the most recent available year as of 2019. We use data on all existing types of tax-advantaged retirement plans and their tax treatment from the Organization for Economic Cooperation and Development (OECD, 2015a,b), estimates of assets under management by plan type from various sources (national statistical office and country trade associations), average retirement income from national statistical offices, information on basic deductions, personal tax brackets, and the corporate tax rate from country tax authorities, and fee estimates from Morningstar (Alpert et al., 2013) and other sources.[5]

[5]Our estimate of τ_R is a rough lower bound, equal to the average tax rate faced by a person earning the

For consistency with our U.S. estimates, we exclude defined benefit (DB) pension plans from the calculation. With or without DB plans, the U.S. has the world's largest retirement assets, and therefore leads the list. However, other countries have substantial amounts of DB retirement assets (United Kingdom, Netherlands and Japan), and omitting DB leads to an important underestimate of the size of the implicit government account. In the case of United Kingdom and Netherlands, this underestimate meaningfully affects the estimated subsidy.

Each of the components of the subsidy has substantial variation across countries. For instance, although Switzerland, Australia and Japan have significant tax-deferred assets, the estimated subsidy is small simply because under current tax law retirement payouts are lightly taxed. Canada has the second-largest subsidy in dollar terms ($2.3 billion) and the largest as a fraction of GDP (0.15%), driven by the surprisingly large fees charged by Canadian funds (2.06%).

4 Proofs and derivations

In this section, we lay out a number of proofs and derivation for sections 3, 5, and 6 of the main text. Please note that this section is not meant to be read sequentially. Each subsection of this section is a stand-alone derivation.

4.1 Proof of the supply-side fee equivalence result

In Section 2 of the paper we have formalized a well-known individual equivalence result under the assumptions of constant tax rates and constant individual income across Roth and Traditional. Here, under the same assumptions, we prove the new equivalence result presented in Section 3 of the paper: if a linear pricing schedule is profit-maximizing for a firm under Roth, an equivalent schedule that results in the same variable and fixed percent fees is profit-maximizing under Traditional. The proof is general, and the result also holds if the variable or fixed component of fees is constrained to be zero.

Formally, define a linear pricing schedule $\Phi^{Roth} = \left(f_j^v, F_j\right)$ that is profit-maximizing for firm j under Roth, and an *equivalent* schedule $\Phi^{Trad} \equiv \left(f_j^v, K \cdot F_j\right)$ under Traditional.

average retirement income with no other income.

Note that we purposefully omit the "Roth" superscript for the Roth fee schedule. $K \equiv \left(1-\tau_S^{Roth}\right)/\left(1-\tau_S^{Trad}\right)$ is the increase in saving per unit of forgone consumption under Traditional over the same quantity under Roth. Note that in the paper we defined $K \equiv 1/\left(1-\tau_L\right)$, while here we prove a more general result. Here, τ_S^{Roth} is not constrained to be zero, so that this proof also holds when there is an implicit match (defined as $\tau_M \equiv (\tau_S-\tau_R)/(1-\tau_S)$) under Traditional and an equal implicit or explicit match under Roth.[6]

Given these definitions, we need to show that if Φ^{Roth} is optimal under Roth ($\left.\frac{\partial \pi_j}{\partial F_j}\right|_{Roth,\Phi^{Roth}} = 0$ and $\left.\frac{\partial \pi_j}{\partial f_j^v}\right|_{Roth,\Phi^{Roth}} = 0$), then Φ^{Trad} is optimal under Traditional ($\left.\frac{\partial \pi_j}{\partial F_j}\right|_{Trad,\Phi^{Trad}} = 0$ and $\left.\frac{\partial \pi_j}{\partial f_j^v}\right|_{Trad,\Phi^{Trad}} = 0$). If F or f_v are constrained to be zero under both Roth and Traditional, then we only need to show one of these two conditions ($\left.\frac{\partial \pi_j}{\partial F_j}\right|_{Roth,\Phi^{Roth}} = 0 \implies \left.\frac{\partial \pi_j}{\partial F_j}\right|_{Trad,\Phi^{Trad}} = 0$, if f_v is constrained, or $\left.\frac{\partial \pi_j}{\partial f_j^v}\right|_{Roth,\Phi^{Roth}} = 0 \implies \left.\frac{\partial \pi_j}{\partial f_j^v}\right|_{Trad,\Phi^{Trad}} = 0$ if F is constrained). Our proof consists of first conjecturing that the conditions hold and then verifying that there is no contradiction. We start by assuming an equilibrium under Roth with saving S^{Roth} and fee schedule Φ^{Roth}.

We assume equal individual incomes ($Y^{Trad} = Y^{Roth} = Y$), income tax rates ($\tau_L^{Trad} = \tau_L^{Roth}$), and match rates ($\tau_M^{Trad} = \tau_M^{Roth}$) across Roth and Traditional.[7] We also assume that every firm j has differentiating, nonprice attributes that affect individuals' utility, and individuals are heterogeneous in their preferences so that a smooth change in fees will result in a smooth change in revenue. Finally, we assume that the same firms exist under Roth and Traditional.[8] Individuals' utility is separable in consumption and firm nonprice attributes.

Next we conjecture that, under these assumptions, there is a Traditional equilibrium in

[6]Under Roth, τ_R^{Roth} is zero, so $\tau_M^{Roth} = \tau_S^{Roth}/(1-\tau_S^{Roth})$, or, equivalently, $\tau_S^{Roth} = \tau_M^{Roth}/(1+\tau_M^{Roth})$.

[7]Assuming equal income amounts to disregarding profits, and any impact of Traditional on aggregate profits. Similarly, assuming equal income tax rates amounts to disregarding the government's budget constraint. Note that a weaker assumption that after-tax lifetime resources are equal ($\left(Y+\Pi^{Trad}\right)\left(1-\tau_L^{Trad}\right) = \left(Y+\Pi^{Roth}\right)\left(1-\tau_L^{Roth}\right)$) would be sufficient. This weaker condition can endogenously arise in equilibrium if the larger assets under management under Traditional do not result in more real resources spent on asset management. One such example is the no-entry equilibrium with $c = 0$ in Section 5 in our paper.

[8]This is a stronger assumption than either no entry or an equal number of firms ($N^{Trad} = N^{Roth}$).

which

$$\begin{cases} S^{Trad} = K \cdot S^{Roth} \\ C_0^{Trad} = C_0^{Roth} \\ F_j^{Trad} = K \cdot F_j \quad \forall j \\ f_j^{v,Trad} = f_j^v \quad \forall j \end{cases}, \tag{2}$$

where the last two equations signify that firms use equivalent fee schedules as defined above, and every individual chooses the same firm as they would under Roth.

We begin by showing that, conditional on choosing the same firm j, if the individual faces equivalent fee schedules under Roth and Traditional that result in the same level of percent fees, the standard equivalence result for individuals holds regardless of their choice of firm j. For every j:

1. The individual's first-order condition (FOC) is the same under either system, because under our conjecture f_j^v is the same and we have assumed that τ_M is the same:

   $$u'_{C_{0,i}}(C_{0,i}) = \delta(1+r)\left(1 - f_j^v\right)(1+\tau_M)u'_{C_{1,i}}(C_{1,i}). \tag{3}$$

   Thus, if a $(C_{0,i},\ C_{1,i})$-pair satisfies the Euler equation under Roth, the same pair satisfies it under Traditional.

2. The individual's budget constraint is

   $$C_{1,i} = \left\{\left[Y(1-\tau_L) - C_{0,i}\right](1-f^v) - F(1-\tau_S)\right\}(1+r)(1+\tau_M), \tag{4}$$

   which is also the same under either system. Thus, for a given $C_{0,i,j}$, individual i can achieve the same $C_{1,i,j}$.

3. Thus, the same consumption plan is both optimal and feasible under both systems, and consumption under Traditional is the same as under Roth in both periods for every individual i:

   $$C_{t,i}^{Trad}\left(f_j^v, K \cdot F_j\right) = C_{t,i}^{Roth}\left(f_j^v, F_j\right),\ t \in \{0,1\}, \tag{5}$$

Then, for every given firm j, the individual obtains the same utility under either system,

$$u_i\left(C_{0,i,j}^{Trad}, C_{1,i,j}^{Trad}, j\right) = u_i\left(C_{0,i,j}^{Roth}, C_{1,i,j}^{Roth}, j\right) \quad \forall j.$$

Under the assumption that the same firms exist under Roth and Traditional, then, each individual chooses the same firm under either system because

$$j^* = \arg\max_{j\in J(i)} u_i\left(C_{0,i,j}, C_{1,i,j}, j\right),$$

where $J(i)$ is the subset of all firms that are actually available to individual i. Thus, equivalent fee schedules yield the same equilibrium market shares under either system:

$$q_j^{Trad}\left(f_j^v, K\cdot F_j\right) = q_j^{Roth}\left(f_j^v, F_j\right) \quad \forall j. \tag{6}$$

Finally, we verify that charging the same percent fees is optimal, i.e., that under our initial conjecture the firm's first-order conditions are still satisfied. We begin by examining the firm's FOC for the variable component of fees:

$$\frac{\partial \pi_j}{\partial f_j^v} = S_j\cdot q_j + \left[F_j + \left(f_j^v - c\right)S_j\right]\cdot\frac{\partial q_j}{\partial f_j^v} + \left(f_j^v - c\right)\cdot\frac{\partial S_j}{\partial f_j^v}\cdot q_j = 0. \tag{7}$$

Note that all three terms under Traditional are proportional by a factor K to their Roth equivalents:

1. Under our initial conjecture (2), $S_j^{Trad}\cdot q_j^{Trad} = K\cdot S_j^{Roth}\cdot q_j^{Roth}$.
2. By Eq. (6), $\left.\frac{\partial q_j}{\partial f_j^v}\right|_{Trad} = \left.\frac{\partial q_j}{\partial f_j^v}\right|_{Roth}$, and thus our conjecture (2) implies
$$\left[F_j^{Trad} + \left(f_j^v - c\right)S_j^{Trad}\right]\left.\frac{\partial q_j}{\partial f_j^v}\right|_{Trad} = K\cdot\left[F_j + \left(f_j^v - c\right)S_j^{Roth}\right]\left.\frac{\partial q_j}{\partial f_j^v}\right|_{Roth}.$$
3. By the individual-level equivalence result (5), if Roth and Traditional feature identical consumption for every pair of equivalent fee schedules, then the response of time-0 consumption to a change in variable fees is the same ($\left.\frac{\partial C_{0,i,j}}{\partial f_j^v}\right|_{Trad} = \left.\frac{\partial C_{0,i,j}}{\partial f_j^v}\right|_{Roth}$). Since a \$1 change in consumption causes an opposite change in saving, and since, by the definition of K, this change is K times larger under Traditional than under Roth, then the response of saving to a change in variable fees is K times larger under Traditional than under Roth ($\left.\frac{\partial S_{i,j}}{\partial f_j^v}\right|_{Trad} = K\cdot\left.\frac{\partial S_{i,j}}{\partial f_j^v}\right|_{Roth}$).

These statements imply that if the variable fee component f_j^v under Roth is optimal ($\left.\frac{\partial \pi}{\partial f^v}\right|_{Roth,\Phi^{Roth}} = 0$), then the same variable fee under Traditional is also optimal ($\left.\frac{\partial \pi}{\partial f^v}\right|_{Trad,\Phi^{Trad}} = K\times\left.\frac{\partial \pi}{\partial f^v}\right|_{Roth,\Phi^{Roth}} = 0$).

Analogous reasoning shows the optimality of $F_j^{Trad} = K\cdot F_j$. The firms' FOC for the fixed component of fees,

$$\frac{\partial \pi_j}{\partial F_j} = q_j + \left[F_j + \left(f_j^v - c\right) S_j\right] \cdot \frac{\partial q_j}{\partial F_j} + \left(f_j^v - c\right) \cdot \frac{\partial S_j}{\partial F_j} \cdot q_j = 0, \tag{8}$$

has three terms, and it is easy to show that all three terms are the same under Roth and Traditional:

1. Equation (6) directly implies $q_j^{Trad} = q_j^{Roth}$.
2. By the chain rule, (6) also implies $\left.\frac{\partial q_j}{\partial F_j}\right|_{Trad} = \frac{1}{K} \left.\frac{\partial q_j}{\partial F_j}\right|_{Roth}$. Then, (2) implies
$$\left[F_j^{Trad} + \left(f_j^v - c\right) S_j^{Trad}\right] \left.\frac{\partial q_j}{\partial F_j}\right|_{Trad} = \left[F_j + \left(f_j^v - c\right) S_j^{Roth}\right] \left.\frac{\partial q_j}{\partial F_j}\right|_{Roth}.$$
3. By the chain rule, (5) implies $\left.\frac{\partial C_{0,i,j}}{\partial F_j}\right|_{Trad} = \frac{1}{K} \left.\frac{\partial C_{0,i,j}}{\partial F_j}\right|_{Roth}$. However, since the effect of forgoing consumption on AUM is K times larger under Traditional, $\left.\frac{\partial S_{i,j}}{\partial F_j}\right|_{Trad} = K \cdot \frac{1}{K} \left.\frac{\partial S_{i,j}}{\partial F_j}\right|_{Roth} = \left.\frac{\partial S_{i,j}}{\partial F_j}\right|_{Roth}$.

So, finally, if $\left.\frac{\partial \pi}{\partial f^v}\right|_{Roth,\Phi^{Roth}} = 0$, then $\left.\frac{\partial \pi}{\partial F}\right|_{Trad,\Phi^{Trad}} = 0$, and the same fee structure is profit-maximizing.

As should be clear from the above proof, this result does not rely on the two-part fee structure we assumed. If the fixed component of fees F_j is set to zero (i.e., if firms are forced to charge only variable fees), Eq. (7) is unaffected (because $F_j^{Trad} = K \cdot F_j = 0$ is still true) and Eq. (8) no longer applies. Similarly, if the variable component of fees is forced to be zero, Eq. (8) is unaffected and Eq. (7) no longer applies.

4.2 Equilibrium fees with Salop and log utility

In this section we derive the expression for fees shown in Section 5.2 of the paper, where we do not yet impose (i) that the aggregate profits received by individuals (Π) are equal to aggregate firm profits or (ii) that taxes balance the government's budget. As discussed in Section 5.1 of the paper, we assume that individuals are uniformly distributed over a circle of circumference 1 and the N firms are evenly distributed around the circle. This subsection closely follows the solution of Salop (1979) and Tirole (1988, Ch. 7).

In this setting, there exists a symmetric equilibrium such that all firms charge the same fees, i.e., $F_j = F$ and $f_j^v = f^v$ $\forall j \in \{1, 2, \ldots N\}$. The equilibrium fee structure (F, f^v) is such that no firm has an incentive to switch to a different structure given that every other firm is also charging fees with the same structure (F, f^v).

Consider the situation of the marginal investor i living between firms j and $j+1$ who is indifferent between the two firms. For this marginal investor, the distance from the two adjacent firms is such that

$$\ln C_{0,i,j} + \delta \ln C_{1,i,j} - \gamma \ln d_{i,j} = \ln C_{0,i,j+1} + \delta \ln C_{1,i,j+1} - \gamma \ln d_{i,j+1}, \tag{9}$$

where $C_{t,i,j}$ is i's time-t consumption conditional on choosing firm j.

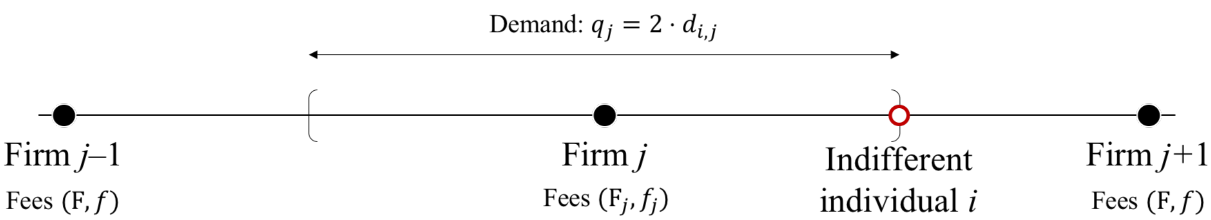

Figure 1: Geometric intuition for the calculation of firm-level demand.

Firm j chooses $\left(F_j, f_j^v\right)$ taking all other firms' fees as given, and therefore $\left(F_{j+1}, f_{j+1}^v\right)$ is simply the equilibrium fee structure, (F, f^v) . Moreover, since the distance between firms is $1/N$, the distance of the investor from firm $j+1$ is

$$d_{i,j+1} = d_{j,j+1} - d_{i,j} = \frac{1}{N} - d_{i,j}. \tag{10}$$

Thus, (9) simplifies

$$C_{0,i,j} + \delta \ln C_{1,i,j} - \gamma \ln d_{i,j} = \ln C_{0,i} + \delta \ln C_{1,i} - \gamma \ln \left(\frac{1}{N} - d_{i,j} \right). \tag{11}$$

Finally, as shown in Figure 1, the demand faced by firm j is equal to twice the number of individuals living between the firm and the indifferent individual, because there are individuals living to the left and to the right of the firm. Thus, solving (11) for $d_{i,j}$, we obtain the equilibrium demand function faced by firm j:

$$q_j = 2d_{i,j} = \frac{2}{N} \frac{\tilde{C}_j}{1 + \tilde{C}_j} \text{ where } \tilde{C}_j \equiv \left(\frac{C_{0,i,j}}{C_{0,i}} \right)^{\frac{1}{\gamma}} \cdot \left(\frac{C_{1,i,j}}{C_{1,i}} \right)^{\frac{\delta}{\gamma}}. \tag{12}$$

The optimal level of fees is found by deriving the first-order conditions of the firm's objective function with respect to F_j and f_j^v.

$$\frac{\partial \pi_j}{\partial f_j^v} = S_j \cdot q_j + \left[F_j + \left(f_j^v - c\right) S_j\right] \cdot \frac{\partial q_j}{\partial f_j^v} + \left(f_j^v - c\right) \cdot \frac{\partial S_j}{\partial f_j^v} \cdot q_j = 0. \tag{13}$$

$$\frac{\partial \pi_j}{\partial F_j} = q_j + \left[F_j + \left(f_j^v - c\right) S_j\right] \cdot \frac{\partial q_j}{\partial F_j} + \left(f_j^v - c\right) \frac{\partial S_j}{\partial F_j} \cdot q_j = 0. \tag{14}$$

To solve, we impose a symmetric equilibrium in which $F_j = F$ and $f_j^v = f^v$ (and $q_j = 1/N$). We examine the general case with fixed and variable fees, and the two special cases with only fixed or only variable fees.

4.2.1 General case: F, $f^v > 0$

This is the main case examined in Section 5.2 of the paper. Solving (13) and (14) requires first finding an explicit expression for S_j. Maximizing utility with respect to C_0 gives

$$\max_{C_0} U = \ln C_{0,i} + \delta \ln C_{1,i} - \gamma \ln d_{i,j}$$

subject to the individual's intertemporal budget constraint

$$C_{1,i} = \left\{\left[(Y + \Pi)(1 - \tau_L) - C_{0,i}\right]\left(1 - f_j^v\right) - F_j (1 - \tau_S)\right\}(1 + r)(1 + \tau_M), \tag{15}$$

and defining $\tilde{Y} = (Y + \Pi)(1 - \tau_L) / (1 - \tau_S)$ ("saveable" lifetime resources), so that

$$C_{1,i} = \left\{\left[\tilde{Y}(1 - \tau_S) - C_{0,i}\right]\left(1 - f_j^v\right) - F_j (1 - \tau_S)\right\}(1 + r)(1 + \tau_M),$$

the F.O.C. implies

$$C_0^* = \frac{1}{1 + \delta}\left[(Y + \Pi)(1 - \tau_L) - \frac{F_j}{1 - f_j^v}(1 - \tau_S)\right].$$

Then, note that by definition,

$$S_i = \frac{(Y + \Pi)(1 - \tau_L) - C_{0,i}}{1 - \tau_S}$$

so that

$$S_i^* = \frac{\delta}{1 + \delta}(Y + \Pi)\frac{1 - \tau_L}{1 - \tau_S} + \frac{1}{1 + \delta} \cdot \frac{F_j}{1 - f_j^v} \equiv \frac{\delta}{1 + \delta}\tilde{Y} + \frac{1}{1 + \delta} \cdot \frac{F_j}{1 - f_j^v} \tag{16}$$

Thus, saving is a convex combination of lifetime resources and total fixed fees (grossed up for variable fees). This second term means that a fraction $1/(1 + \delta)$ of fixed costs F gets financed by giving up time-0 consumption and therefore added here. The remainder is financed by giving up time-1 consumption.

Next we assume, as in the main text, that firms take into account the fact that if they raise their fees, people save less. We impose a symmetric equilibrium as discussed above so that $q_j \overset{eq}{=} 1/N$, $S_j \overset{eq}{=} S$, $F_j \overset{eq}{=} F$, $f_j^v \overset{eq}{=} f^v$, $C_{t,i,j} = C_{t,i}$ for all i, j, t. We now find an explicit expression for all the implicit terms of Eqs. (13) and (14).

Equation (16) implies (switching from i to j subscripts to indicate that this is now the firm's perspective)

$$\frac{\partial S_j}{\partial f_j^v} = \frac{1}{1+\delta} \cdot \frac{F_j}{\left(1-f_j^v\right)^2} \overset{eq}{=} \frac{1}{1+\delta} \cdot \frac{F}{\left(1-f^v\right)^2}. \tag{17}$$

$$\frac{\partial S_j}{\partial F_j} = \frac{1}{1+\delta} \cdot \frac{1}{1-f_j^v} \overset{eq}{=} \frac{1}{1+\delta} \cdot \frac{1}{1-f^v}, \tag{18}$$

Moreover, Eq. (12) together with (16) implies

$$\frac{\partial q_j}{\partial f_j^v} \overset{eq}{=} -\frac{1}{2N} \cdot \frac{S_j}{\tilde{Y}} \cdot \frac{1+\delta}{\gamma\left(1-f^v-F/\tilde{Y}\right)}, \tag{19}$$

$$\frac{\partial q_j}{\partial F_j} \overset{eq}{=} -\frac{1}{2N} \cdot \frac{1}{\tilde{Y}} \cdot \frac{1+\delta}{\gamma\left(1-f^v-F/\tilde{Y}\right)}. \tag{20}$$

We now substitute expressions (17)–(20) into (13) and (14), and solve to obtain

$$f^v = c, \tag{21}$$

$$F = \frac{2\gamma}{1+2\gamma+\delta}\left(1-c\right)\left(Y+\Pi\right)\frac{1-\tau_L}{1-\tau_S}. \tag{22}$$

Note that N does not appear in these expressions. This is a consequence of our logarithmic utility assumption, and it is discussed in footnote 38 of the paper. Fixed cost ϕ also does not appear in these expressions, as ϕ only affects the entry decision but does not appear in the firm's first-order condition.

4.2.2 Only fixed fees ($F > 0$, $f^v = 0$)

In this case, (14) becomes the only optimality condition and simplifies to:

$$\frac{\partial \pi_j}{\partial F_j} = q_j + \left(F_j - cS_j\right) \cdot \frac{\partial q_j}{\partial F_j} - c\frac{\partial S_j}{\partial F_j} \cdot q_j = 0. \tag{23}$$

Solving, we obtain

$$F = \frac{2\gamma + c\delta \frac{1+\delta}{1+\delta-c}}{1+2\gamma+\delta} (Y + \Pi) \frac{1-\tau_L}{1-\tau_S}. \tag{24}$$

4.2.3 Only variable fees ($F = 0$, $f^v > 0$)

In this case, (13) becomes the only optimality condition and simplifies to:

$$\frac{\partial \pi_j}{\partial f_j^v} = S_j \cdot q_j + \left(f_j^v - c\right) S_j \cdot \frac{\partial q_j}{\partial f_j} + \left(f_j^v - c\right) \cdot \frac{\partial S_j}{\partial f_j} \cdot q_j = 0. \tag{25}$$

Solving, we obtain

$$f^v = \frac{2\gamma + c\delta}{2\gamma + \delta} (> c). \tag{26}$$

4.3 Aggregate AUM, consumption, saving, and profits

We now show that, as discussed in Section 5.3 of the main text, Traditional has higher AUM and higher profits than Roth. Since $C_{0,i}$, $C_{1,i}$, and S_i are the same for all individuals, the corresponding aggregate values ($C_0 \equiv \int_0^1 C_{0,i} di$, $C_1 \equiv \int_0^1 C_{1,i} di$, $S \equiv \int_0^1 S_i di$) are equal to the individual values. Solving yields

$$C_0 = \frac{1}{1+\delta+2\gamma} (Y + \Pi) (1 - \tau_L), \tag{27}$$

$$C_1 = \frac{\delta}{1+\delta+2\gamma} (Y + \Pi) (1 - \tau_L) (1 - c) (1 + r) (1 + \tau_M), \tag{28}$$

$$S = \frac{2\gamma+\delta}{1+\delta+2\gamma} (Y + \Pi) \frac{1-\tau_L}{1-\tau_S}. \tag{29}$$

Moreover, (29) implies

$$\frac{S^{Trad}}{S^{Roth}} = \frac{Y^{Trad} + \Pi^{Trad}}{Y^{Roth} + \Pi^{Roth}} \cdot \frac{1}{1 - \tau_L^{Roth}}. \tag{30}$$

We now impose that profits received by individuals are equal to aggregate firm profits ($\Pi = \sum_j \pi_j$). Combining Eqs. (21)–(29) and solving yields equations for aggregate fees, profits, and saving:

$$F_j = F = \frac{Y - \phi N}{\frac{1+\delta+2\gamma}{2\gamma} \cdot \frac{1}{1-c} \cdot \frac{1-\tau_S}{1-\tau_L} - 1}, \tag{31}$$

$$\Pi = \sum_j \pi_j = F - \phi N = \frac{Y - \phi N}{1 - \frac{2\gamma}{1+\delta+2\gamma} (1-c) \frac{1-\tau_L}{1-\tau_S}} - Y, \tag{32}$$

$$S = \frac{(2\gamma + \delta)(Y - \phi N)}{(1 + \delta + 2\gamma)\frac{1-\tau_S}{1-\tau_L} - 2\gamma(1-c)}. \tag{33}$$

It is easy to see from (31)–(33) that Traditional has higher assets under management, higher total variable fees ($f^v \cdot S$), a higher fixed fee, and higher profits than Roth. Note that, if $(1 - \tau_S^{Trad})/(1 - \tau_L^{Trad}) = 1$ and $\tau_S^{Roth} = 0$, for given N this result holds as long as $\tau_L^{Roth} > 0$, and it does not depend on an assumption that $\tau_L^{Trad} = \tau_L^{Roth}$.

4.4 Tax policy

We now close the model by assuming that the government is subject to budget constraints and derive closed-form expressions for tax rates under Roth and Traditional. These expressions show, as discussed in Section 5.3 of the main text, that taxes under Traditional are higher than taxes under Roth.

The government spends an exogenously given amount G. To satisfy its time-0 budget constraint, the government borrows an amount B at the market interest rate r to cover its deficit, so that

$$B = G + S \cdot \tau_S - (Y + \Pi)\tau_L. \tag{34}$$

To satisfy its time-1 budget constraint, the government taxes retirement income at a rate τ_R:

$$B(1 + r) = [S(1 - f^v) - F](1 + r)\tau_R, \tag{35}$$

where f^v and F are the equilibrium values. Putting these together yields the government's single (intertemporal) budget constraint:

$$G = (Y + \Pi)\tau_L - (F/S + f^v) \cdot (\tau_R \cdot S) - S(\tau_S - \tau_R), \tag{36}$$

where G is government expenditure and the right-hand side is the present value of tax revenue.

Assuming for simplicity $\tau_M^{Trad} = 0$ (i.e., $\tau_R^{Trad} = \tau_L^{Trad}$), and using the equations for optimal fees (21) and (22), optimal saving (33), and the expression for aggregate profits (32), we obtain:

$$\tau_L^{Roth} = \frac{(G/Y)(1 + \delta + 2c\gamma)}{(1 + \delta + 2\gamma)(1 - N \cdot \phi/Y) - 2(1 - c)\gamma \cdot G/Y} \tag{37}$$

$$\tau_L^{Trad} = \frac{(G/Y)(1+\delta+2c\gamma)}{[1+\delta(1-c)](1-N\cdot\phi/Y)} \tag{38}$$

Thus, *for a given* N, Traditional always results in a higher tax rate ($\tau_L^{Trad} > \tau_L^{Roth}$).[9]

4.5 N and L with endogenous entry

Next, we allow for entry as discussed in Section 5.4 of the paper and solve for the new equilibrium with endogenous number of firms N. We assume that N can be noninteger, and therefore firms enter the market until equilibrium profits π_j equal zero for all firms j.[10] Since π_j is the same for all firms j, we can set Eq. (32) to zero and solve for N, obtaining

$$N = \frac{Y}{\phi}\cdot\frac{2\gamma(1-c)}{1+\delta+2\gamma}\cdot\frac{1-\tau_L}{1-\tau_S}. \tag{39}$$

The share of employment in the asset management industry (L) is found by substituting (39) and (33) into the definition of labor supply L:

$$L \equiv \frac{cS+\phi N}{Y} = \frac{c\delta+2\gamma}{1+\delta+2\gamma}\cdot\frac{1-\tau_L}{1-\tau_S}. \tag{40}$$

Continuing to assume $\tau_M^{Trad}=0$, and substituting (39) into (38), we obtain[11]

$$\tau_L^{Trad} = \tau_R^{Trad} = {}^{G}\!/_{Y}\left(1+\frac{c\delta+2\gamma}{1+\delta(1-c)}\right) > {}^{G}\!/_{Y} = \tau_L^{Roth}. \tag{41}$$

Note that since tax rates do not depend on ϕ, total expenditure within each system on the fixed costs of asset management $=\phi\cdot N$ are independent of the size of the fixed cost ϕ, because an increase in ϕ causes an offsetting decrease in N.

4.5.1 Entry and cost structure

Here we examine how different cost structures affect fees and entry in equilibrium. Our results so far allow for both variable and fixed costs. A fixed-cost-only scenario is simply obtained by setting $c=0$. Examining a variable-cost-only scenario is more complex.

[9] Technically, this requires that $Y-\phi N-G>0$, i.e., that exogenous claims on total resources leave enough resources for consumption to be positive.

[10] Requiring N to be integer can lead to a situation in which N firms make positive profits, and $N+1$ firms make negative profits. This possibility, which could have important consequences if N were small, is analyzed by Mankiw and Whinston (1986).

[11] Note that the constraint $\tau_L^{Trad} < 100\%$ implies a limit on the public expenditure share of output ${}^{G}\!/_{Y}$ that is tighter than Roth. The higher γ, the higher the fees individuals are willing to pay, the larger the government's transfer to asset managers, and the fewer resources available for public expenditure.

- One approach is to let $\phi \to 0^+$. In this case, it is clear from Eq. (39) that the total resources devoted to asset management fixed cost, ϕN, are constant. As ϕ vanishes, the number of firms approaches infinity while profits remain zero. Fees remain the same throughout because, as discussed in Section 4.2.1, they do not depend on ϕ or (because of our logarithmic specification) on N.

- A second approach is to *set* $\phi = 0$. In this case, the equilibrium fee structure described by Eqs. (13) and (14) leads to positive equilibrium profits.[12] The reason is that, once again, fees do not depend on N. Entry does not eliminate firm profitability, and continued firm profitability continues to induce entry. Individual firm profits vanish as the number of firms goes to infinity, but aggregate profits remain positive.

- A third approach is to set $\phi = 0$ *and* enforce zero profits, then deduce the fee structure from Eq. (13) together with the zero-profit condition, which yields

$$F = 0 \text{ and } f^v = c, \tag{42}$$

  if variable fees are permitted, and otherwise

$$F = cS = \frac{\delta c}{1 + \delta - c} Y \frac{1 - \tau_L}{1 - \tau_S} \text{ and } f^v = 0. \tag{43}$$

Table 4 summarizes all relevant combinations of fee structure (based on the analysis of Section 4.2) and cost structure (based on the analysis in this section).

4.6 Proof that $\partial\Pi/\partial N < 0$, $\partial S/\partial N < 0$ and $\partial F/\partial N < 0$ for given N

Here we formally show that aggregate profits, fixed fees, and AUM are all decreasing in the number of firms. For Traditional, the proof is trivial. Eqs. (31)–(33) above are of the form $a - bN$ because tax rates simplify out. For Roth,

- $\partial\Pi/\partial N < 0$: substituting $\tau_L = G/(Y + \Pi)$ and $\tau_S = 0$ in Eq. (32), we obtain

$$\Pi = \frac{Y - \phi N}{1 - \tilde{K}\left(1 - \frac{G}{Y+\Pi}\right)} - Y \text{ with } \tilde{K} \equiv \frac{2\gamma}{1 + 2\gamma + \delta}(1 - c) \in (0, 1) \tag{44}$$

  and further simplified to

$$\Pi = \frac{(Y - G)\tilde{K} - \phi N}{1 - \tilde{K}}, \tag{45}$$

[12] A similar outcome is shown in Section 5 of Mankiw and Whinston (1986).

Costs	Fees		
	Both fixed and variable	Fixed only ($f^v = 0$)	Variable only ($F = 0$)
Both fixed and variable	1. Main result: $f^v = c$. Too many firms due to business stealing.	2. $f^v < c$. Too many firms due to oversaving *and* business stealing.	3. $f^v > c$. Undersaving *vs.* business stealing. Uncertain result.
Fixed only ($c = 0$)	4. $f^v = c = 0$. Special case of #1.	5. $f^v = c = 0$. Special case of #1.	6. $f^v > c$. Special case of #3.
Variable only, letting $\phi \to 0^+$: same as cases #1–6 above.			
Variable only, setting $\phi = 0$ and...			
...allowing equilibrium fees, positive profits ($\Pi > 0$)	7. $f^v = c$ and $F > 0$. Special case of #1.	8. $f^v < c$, special case of #2.	9. $f^v > c$. Special case of #3.
...setting fees to enforce zero profits ($\Pi = 0$)	10. $f^v = c$, $F = 0$.	11. $f^v < c$, F from eq. (43).	12. $f^v = c$. Special case of #1.

Table 4: Equilibrium fees under different assumptions for cost and fee structures.

a decreasing function of N.

- $\partial F / \partial N < 0$ follows by observing that

$$\Pi = F - \phi N$$

and thus

$$F = \Pi + \phi N = \frac{(Y - G)\tilde{K} - \tilde{K}\phi N}{1 - \tilde{K}}$$

- $\partial S / \partial N < 0$ follows trivially as F and S are proportional.

To build intuition for these results, begin by considering the case in which profits received by individuals (Π) are independent of aggregate firm profits. As N increases, firms charge each customer the same fixed fees (F), but the market share of each firm shrinks, so firm-level profits become lower, while new entrants still make positive profits. Aggregate profits shrink too because aggregate revenues remain the same, while aggregate fixed costs rise due to the increase in the number of firms. When we require profit passthrough ($\Pi = \sum_{j=1}^{N} \pi_j$), an increase in N further reduce individuals' resources by reducing aggregate profits; in turn, individuals save less and are charged lower fixed fees.

4.7 Derivation of the planner's simplified objective function

Here we derive the simplified planner objective function discussed in Section 6.1 of the main text. The planner maximizes

$$U = \max_{\{C_{0,i}\},\{C_{1,i}\},N} \int_0^1 \ln C_{0,i} + \delta \ln C_{1,i} - \gamma \ln d_{i,j} \, di. \tag{46}$$

Note that it is optimal for the planner to give equal consumption to all individuals ($C_{0,i} = C_0$ and $C_{1,i} = C_1$) because the utility function is concave and separable in its arguments. Then, using the assumption that the N firms are located equidistantly along the circle, the planner's objective function simplifies to

$$U = \max_{C_0,N} \ln C_0 + \delta \ln C_1 - \gamma \cdot 2N \int_0^{1/(2N)} \ln i \, di, \tag{47}$$

or

$$U = \max_{C_0,N} \ln C_0 + \delta \ln C_1 + \gamma \ln N - \gamma \left(1 + \ln 2\right). \tag{48}$$

4.8 Planner solution

We now solve the planner's optimization problem as discussed in Section 6.1 of the main text. There are two first-order conditions. The first is

$$U'(C_0^*) = \delta \left(1 + r\right)\left(1 - c\right) U'(C_1^*), \tag{49}$$

and the second one is

$$\phi \frac{1}{1-c} U'(C_0^*) = U'(N). \tag{50}$$

Combining and simplifying the first-order conditions yields the following optimal quantities:

$$C_0^* = \frac{1}{1+\delta+\gamma} \left(Y - G\right), \tag{51}$$

$$C_1^* = \frac{\delta}{1+\delta+\gamma} \left(Y - G\right)\left(1 - c\right)\left(1 + r\right), \tag{52}$$

$$N^{*}=\frac{\gamma}{1+\delta+\gamma}\left(Y-G\right)\left(1-c\right)\frac{1}{\phi}. \tag{53}$$

Note that the distaste for distance enters into the formulae for optimal consumption in each period, because it affects the optimal number of firms, and thus the aggregate resources available for consumption. Note also that $N^{*}\phi$, the total allocation to the fixed costs of asset management, does not depend on the size of the fixed cost ϕ.

4.9 Welfare analysis

In this subsection we show that, as discussed in Section 6.2 of the main text, in our model $U^{Roth}\geq U^{Trad}$. We show this result both under the assumption of arbitrary N constant across Roth and Traditional and under the assumption of free entry and endogenous N determined by market competition.

4.9.1 N given

With N given, notice that

$$U^{Roth}\geq U^{Trad}\iff e^{U^{Roth}}/e^{U^{Trad}}\geq 1 \tag{54}$$

Simplifying, we obtain

$$\frac{\left(1+\delta\left(1-c\right)\right)\left(1-{}^{N\phi}\!/_{Y}-{}^{G}\!/_{Y}\right)}{\left(1+\delta\left(1-c\right)\right)\left(1-{}^{N\phi}\!/_{Y}\right)-{}^{G}\!/_{Y}\left(1+\delta+2c\gamma\right)}\geq 1. \tag{55}$$

Since the left-hand side expression is the ratio of two exponentials, both its numerator and denominator must be positive. Then, (55) simplifies to

$$1+\delta\left(1-c\right)\leq 1+\delta+2c\gamma, \tag{56}$$

which always holds, and holds with equality when $c=0$.

4.9.2 Endogenous N

Given the optimal N, we have

$$U^{Roth}-U^{Trad}=\left(1+\gamma+\delta\right)\ln\left(1-{}^{G}\!/_{Y}\right)-\left(1+\delta\right)\ln\left(1-{}^{G}\!/_{Y}\cdot\frac{1+2\gamma+\delta}{1+\delta\left(1-c\right)}\right). \tag{57}$$

The argument of the logarithm in the second term can in principle be negative. However, note that

$$ {}^{G}\!/_{Y} \cdot \frac{1+2\gamma+\delta}{1+\delta\,(1-c)} = \tau_L^{Trad}, \tag{58} $$

i.e., for the argument to be positive it is sufficient that the tax rate under Traditional be less than 100%. If $\gamma = 0$ and $c = 0$, this simplifies to the trivial condition ${}^{G}\!/_{Y} < 1$, i.e., public expenditure cannot be more than output. However, in our model we allow $\gamma > 0$ and $c \geq 0$, and thus $\tau_L^{Trad} < 1$ is a stricter condition than ${}^{G}\!/_{Y} < 1$. If γ or c are large enough, under this policy there are so many firms in equilibrium that society cannot afford both asset management and public expenditure at current levels. We thus simply assume that $\tau_L^{Trad} < 1$ to rule out this pathological case.

Rearranging, we obtain

$$ U^{Roth} - U^{Trad} > 0 \iff 1 - {}^{G}\!/_{Y} > \left(1 - {}^{G}\!/_{Y} \cdot \frac{1+2\gamma+\delta}{1+\delta\,(1-c)}\right)^{\frac{1+\delta}{1+\gamma+\delta}}. \tag{59} $$

To show that this inequality always holds, set $x \equiv \frac{1+\delta}{1+\gamma+\delta}$. Then,

$$ 1 - {}^{G}\!/_{Y} > \left(1 - {}^{G}\!/_{Y} \cdot \frac{1}{x}\right)^{x} > \left(1 - {}^{G}\!/_{Y} \cdot \frac{1+2\gamma+\delta}{1+\delta\,(1-c)}\right)^{x} \tag{60} $$

where the first inequality is true for every $0 < {}^{G}\!/_{Y} < x < 1$ and the second one is true because $x > (1+\delta\,(1-c))\,/\,(1+2\gamma+\delta)$.

4.10 Welfare decomposition

To better understand the role of fixed and variable costs, we decompose the welfare effect of switching from Traditional to Roth into two parts:

$$ U^{Roth} - U^{Trad} = \left(U^{Trad}\big|_{N=N^{Roth}} - U^{Trad}\right) + \left(U^{Roth} - U^{Trad}\big|_{N=N^{Roth}}\right), \tag{61} $$

where $U^{Trad}\big|_{N}$ is conditional aggregate welfare, i.e., the sum of all individual utilities in a market equilibrium under the Traditional scheme given an exogenous N, and therefore $U^{Trad}\big|_{N=N^{Roth}}$ is conditional aggregate welfare evaluated at $N = N^{Roth}$. This decomposition is then used in our welfare calibration of Section 6.3 in the main text. Starting from a market equilibrium under Traditional, the first term on the right-hand side is the *net* effect from decreasing the number of firms to the Roth level, which can be further decomposed into two terms: the welfare gain from lower fixed costs, and the welfare loss from the increase in average distance between investors and their chosen firm. The second term is the welfare effect in the model of switching to Roth while leaving $N = N^{Roth}$. This second step causes assets under management (and hence variable costs) to decrease, but leaves the number of firms (and hence fixed costs) unchanged, and therefore we define this term as the gain from lower variable costs.

Substituting the appropriate equilibrium values into (61) and simplifying, we obtain

$$\underbrace{U^{Roth} - U^{Trad}}_{\substack{\text{Effect of}\\ \text{switch to Roth}}} = \underbrace{(1+\delta)\ln\frac{1 - {}^{G}/_{Y}}{1 - {}^{G}/_{Y}\cdot(\tilde{A}-\tilde{B})}}_{\substack{\text{Gain from lower}\\ \text{variable costs}}} + \underbrace{(1+\delta)\ln\frac{1 - {}^{G}/_{Y}\cdot(\tilde{A}-\tilde{B})}{1 - {}^{G}/_{Y}\cdot\tilde{A}}}_{\substack{\text{Gain from lower}\\ \text{fixed costs}}} + \underbrace{\gamma\ln\left(1 - {}^{G}/_{Y}\right)}_{\substack{\text{Loss from}\\ \text{fewer firms}}}, \tag{62}$$

where $\tilde{A} = (1+\delta+2\gamma)/[1+\delta(1-c)]$ and $\tilde{B} = 2\gamma(1-c)/(1+\delta+2c\gamma)$.[13]

4.11 Welfare calibration

Here we provide quantitative detail regarding the three calibration exercises discussed in Section 6.3 of the main text.

In our first exercise, we quantify the welfare gain in the model of a switch from Traditional to Roth expressed as a percent of retirement consumption. Specifically, we define α as the fraction of retirement consumption that could be taken away under Roth such that aggregate utility would be the same as under Traditional.[14] In other words, we solve for α such that

$$\ln C_0^{Trad} + \delta\ln C_1^{Trad} - \gamma\ln d_{i,j}^{Trad} = \ln C_0^{Roth} + \delta\ln\left[C_1^{Roth}(1-\alpha)\right] - \gamma\ln d_{i,j}^{Roth}. \tag{63}$$

The right hand side can be rewritten as $U^{Roth} + \delta\ln(1-\alpha)$, which implies we want to solve for α such that $\delta\ln(1-\alpha) = U^{Trad} - U^{Roth}$. This yields

$$-\ln(1-\alpha) = \frac{1}{\delta}\left(U^{Roth} - U^{Trad}\right) = \frac{\ln\left(1-\tau_L^{Roth}\right)^{1+\delta+\gamma} - \ln\left(1-\tau_L^{Trad}\right)^{1+\delta}}{\delta}. \tag{64}$$

[13]Note that each term of (62) is equal to zero when $G = 0$. This is simply because when $G = 0$, $\tau_L = 0$, and thus there is no implicit government account under Traditional and no effect of a switch to Roth.

[14]A switch would result in higher tax rates for all individuals and an increase in the number of firms. If firms change positions on the circle and individuals do not, most individuals would find themselves at a lower distance from the closest firm but some would be farther. In that case, to make every individual indifferent, a customized (and possibly negative) fraction of consumption α_i would have to be taken away from each. However, individuals' positions can be rearranged so that each individuals' distance from the closest firm is a constant fraction of the original distance, making $\alpha_i = \alpha$ for all individuals.

Since $-\ln(1-\alpha) \approx \alpha$, the right-hand side of Eq. (64) represents the consumption-equivalent welfare gain of Roth in the model. Since our approximate expression for α is simply $(U^{Roth} - U^{Trad})/\delta$, the welfare gain decomposition of Eq. (62) can also be applied to α.

To calibrate α, we require estimates of δ, G/Y, c, and γ. We set $G/Y = 20\%$, a rough estimate of federal public expenditure as a fraction of domestic output,[15] and $\delta = 0.64$, an annual 1.5% real discount rate over a horizon of $T = 30$ years. We set total percentage fees $f = f^v + F/S = 21.4\%$, corresponding to the value of 77 bps/year that we estimated in Section 4 of the main text over the same horizon. Under this assumption on f, and given δ, the fee equation $f = (2\gamma + c\delta)/(2\gamma + \delta)$ implies a relation between c and γ. We examine a range of values for c such that c/f varies between 0% and 100%, and we let γ vary accordingly.[16]

In our second exercise, we scale α by the overall tax expenditure on retirement savings accounts under Roth.[17] We define tax expenditure in the model (TX) as the additional revenue (expressed in future value at time 1) that the government would receive if it eliminated the tax break on retirement saving, so that the returns on saving were all taxable, but everything else (first period consumption, number of firms, etc) stayed the same. We then express this quantity as a percent of retirement consumption ($TX\%$). Using the definitions of f, F, and S, we obtain $TX\% = r\tau_I/\left(1 + r\left(1 - \tau_I\right)\right)$, where τ_I is the tax rate on investment income generated on taxable accounts.[18] To calibrate $TX\%$, we set $r = 56\%$, a 1.5% real return for $T = 30$ years, and $\tau_I = 39.1\%$, a rough approximation of the long-run effect of personal taxes on real returns,[19] obtaining $TX\% = 16.4\%$, i.e., 16.4% of retirement wealth

[15] This figure is based on Federal Net Outlays as Percent of Gross Domestic Product (`https://fred.stlouisfed.org/series/FYONGDA188S`). The average of this series since 1946, included, is roughly 20%. 20% is also the typical value since the mid-1970s and until 2019.

[16] As fixed costs vanish ($c/f \to 1$), γ must go to zero to keep $f = 78$ bps. In this case, the number of firms N goes to zero as well. Algebraically, the model allows for $N = 0$. Conceptually, it is difficult to imagine individuals paying fees and society incurring (variable) costs in a world with zero firms. One can loosely interpret this scenario as N as being a large number under normal conditions and becoming very small as $\gamma \to 0$.

[17] No adjustment is needed to this measure because tax expenditure is expressed in the same units as unadjusted α (percent of retirement consumption).

[18] In practice, taxation is based on nominal investment income. Since all variables in the model are real, τ_I is the effective tax rate on real returns, which depends on the level of inflation.

[19] We assume the annual tax on all capital income to be $\tau_I^{Annual} = 20\%$. We define nominal returns as $R = (1 + r)(1 + i) - 1$, where $i = 56\%$ based on a 1.5% inflation rate for $T = 30$ years. Since interest and dividends are taxed every year, whereas capital gains are taxed once when the asset is sold, the long-run effect is different. For interest and dividend income, we define the effective tax rate to be $\tau_I^{Div/Int} =$

of an individual who has saved in a retirement saving account is the result of not having paid taxes on returns.

In our third and last exercise, we start from a zero-profit world with Traditional accounts and assume that, upon a switch to Roth, τ_L remains the same. That is, we assume that the tax rate on labor income under Roth is $G/Y \{1 + (c\delta + 2\gamma) / [1 + \delta (1 - c)]\}$ as in Eq. (41). In this case, leaving everything else the same, the government will have a budget surplus under Roth. We assume that the government uses this surplus to provide an explicit match $\tau_M^{Roth} > 0$ to those who save in a Roth account (e.g., if $\tau_M^{Roth} = 5\%$, for every \$100 contributed, the government adds an extra \$5 into the account). Note that, for Roth, \$1 of forgone consumption results in a \$1 account contribution, so the match rate τ_M can be equivalently expressed as either a percentage of forgone consumption or a percentage of account contribution. This match is[20]

$$\tau_M^{Roth} = \frac{2\gamma + \delta c}{2\gamma + \delta} \cdot \frac{\tau_L^{Trad}}{1 - \tau_L^{Trad}} > 0. \tag{65}$$

A fraction of this match compensates individuals for the lower number of firms upon the switch:

$$\tau_M^{Roth,\,Comp} = \left(1 - \tau_L^{Trad}\right)^{-\gamma/(\delta+\gamma)} - 1. \tag{66}$$

The remainder of the match generates a net improvement in welfare in the model.

4.12 Alternative specifications of costs and fees

So far we have discussed welfare when firms face both fixed and variable costs and are free to set both fixed and variable fees (with zero variable costs a special case), and showed that firms choose to set variable fees equal to marginal costs ($f^v = c$). In Section 6.2 of the paper, we consider the possibility that firms are instead restricted to charge either variable

$1 - [1 + R(1 - \tau_I^{Annual})]^T / (1 + R)^T$, and for capital gains we define the effective tax rate to be $\tau_I^{Gain} = \tau_I^{Annual}[(1 + R)^T - 1]/(1 + R)^T$. The effective long-run tax rate on investment income is then a weighted average of the two rates, $\tau_I = w \cdot \tau_I^{Gain} + (1 - w) \cdot \tau_I^{Div/Int}$, where $w = 48\%$ is calibrated based on Internal Revenue Service Statistics of Income data using average fractions of interest, dividend and capital gain income from 1990 to 2016.

[20] To find τ_M^{Roth} we solve for the τ_S^{Roth} that balances the government's budget constraint. Under Roth ($\tau_R = 0$) and with entry ($\Pi = 0$), the government's budget constraint is $G = Y\tau_L - S\tau_S$. Eqs. (33) and (76) imply $S = Y\,(2\gamma + \delta) / (1 + \delta + 2\gamma) \cdot (1 - \tau_L) / (1 - \tau_S)$. We solve for τ_S and then substitute in the expression for τ_L from (41) which yields $\tau_S^{Roth} = (2\gamma + c\delta) / \left[(G/Y)^{-1} (1 + \delta + 2\gamma)^{-1} (2\gamma + \delta) (1 + \delta (1 - c)) - \delta (1 - c)\right]$ and then use the existing definition $\tau_M \equiv (\tau_S - \tau_R) / (1 - \tau_S)$.

fees or fixed fees, but not both. Above in Section 4.2 we have solved the model under these restrictions and other possible combinations of fixed and variable costs and fixed and variable fees.

The business-stealing effect exists independent of fee and cost structure, and in each case it steers society in the model towards an overly large asset management sector ($N^{Roth} > N^*$). However, if there is a mismatch between the fee structure and the cost structure, another friction arises. For example, if there are variable costs ($c > 0$) but fees are restricted to be fixed-only ($f^v = 0 < c$), individuals do not internalize the cost of managing assets in their saving decision, so that the Euler equation is now different from the planner's ($1-f^v \neq 1-c$), i.e., the optimal C_1/C_0 under Roth is higher than what the planner would choose. The additional fixed fees F that firms charge to cover their variable costs imply that, compared to the baseline scenario with $f^v = c$, initial consumption is lower ($C^{Roth}_{0,f^v=0} < C^{Roth}_{0,f^v=c}$) and saving is larger ($S^{Roth}_{f^v=0} > S^{Roth}_{f^v=c}$), so that $N^{Roth}_{f^v=0} > N^{Roth}_{f^v=c}$ and $U^{Roth}_{f^v=0} < U^{Roth}_{f^v=c}$. A switch from Roth to Traditional in this scenario will, as in the baseline scenario, increase the equilibrium number of firms and variable costs cS, and cause a welfare loss in the model.

In contrast, if there are fixed costs ($\phi > 0$) but fees are restricted to be variable-only ($F = 0$), firms must set $f^v > c$ in order to cover their fixed costs. Individuals face marginal fees higher than the marginal cost of managing assets, so that the optimal C_1/C_0 under Roth is *lower* than what the planner would choose. The additional resources released by setting $F = 0$ imply that compared to the baseline scenario with $F > 0$, initial consumption is higher ($C^{Roth}_{0,F=0} > C^{Roth}_{0,F>0}$) and saving is less ($S^{Roth}_{F=0} < S^{Roth}_{F>0}$), starving the asset management sector of assets and offsetting the business-stealing effect, so that the net effect on the number of firms and welfare is unclear ($N^{Roth}_{F=0} \gtrless N^{Roth}_{F>0}$ and $U^{Roth}_{F=0} \gtrless U^{Roth}_{F>0}$). If the decrease in saving is large enough that $N^{Roth}_{F=0} < N^* < N^{Roth}_{F>0}$, Traditional in the model may yield higher welfare than Roth by increasing aggregate saving and thereby the number of firms. However, under reasonable calibrations, a switch from Traditional to Roth in the model is still welfare-enhancing.[21] The intuition is that Traditional does not create a bigger saving

[21] By "reasonable calibrations" we mean the following. Even assuming no variable costs ($c = 0$, the most favorable scenario for Traditional), maximized utility under Roth is higher than under Traditional as long as γ is smaller than a certain threshold. If, as in Section 6.3 of the paper, we assume $\delta = 0.55$ and $G/Y = 0.2$, the threshold for γ is roughly 0.57, implying that about 40% of total resources in the economy are devoted to asset management. In general, for reasonable choices of G/Y (between 0.1 and 0.5), the threshold for γ is of the same order of magnitude of δ, implying that individuals in the model are willing to devote as many resources to asset management services as they do to retirement consumption itself. We find this

subsidy than Roth in our model, and therefore it does not solve the undersaving problem; it merely exacerbates the individuals' price insensitivity because it subsidizes fees while leaving the individual's consumption/saving tradeoff intact.

4.13 Optimal Taxation

4.13.1 Optimal taxation under Traditional

Can the government in the model exercise its one degree of freedom under Traditional to obtain the same outcomes as the planner, or at least better outcomes than under Roth? Here we show that the tax policy used in the paper ($\tau_R = \tau_L$) could be improved upon unless $c = 0$. However, the improvement is not enough in the model to make Traditional better than Roth. These findings are discussed in Section 6.2, footnote 42 of the main text.

In a market equilibrium under Traditional, maximized utility simplifies to

$$U = \kappa + \ln(1 - \tau_L) + \delta \ln(1 - \tau_R) \tag{67}$$

for some constant κ that does not depend on government policy. Thus the government chooses τ_L and τ_R to maximize (67) subject to its budget constraint

$$\tau_R = \frac{{}^{G}\!/\!_{(Y+\Pi)}\,(1 + 2\gamma + \delta) - \tau_L}{\delta\,(1 - c)}, \tag{68}$$

which yields the following tax rate path:

$$\tau_L^* = {}^{G}\!/\!_{(Y+\Pi)}\frac{1 + 2\gamma + \delta}{1 + \delta} + \frac{c}{1 + \delta} \cdot \delta, \tag{69}$$

and

$$\tau_R^* = \left({}^{G}\!/\!_{(Y+\Pi)}\frac{1 + 2\gamma + \delta}{1 + \delta} + \frac{c}{1 + \delta} \right) \cdot \frac{1}{1 - c}. \tag{70}$$

This path can be upward or downward sloping, depending on the specific values of $G/(Y + \Pi)$ (or equivalently G/Y when $\Pi = 0$), δ, γ, and c. However, even with these constrained-optimal tax rates, utility in the model cannot be higher under Traditional than under Roth. To see this, recall the expressions for consumption and number of asset management firms.

unreasonable, and we therefore conclude that γ should be less than this threshold, implying that a switch from Traditional to Roth in the model is welfare-enhancing. This result is independent of the size of firm-level fixed costs ϕ.

The planner chooses:

$$C_0^* = \frac{1}{1+\delta+\gamma}\left(Y-G\right), \tag{71}$$

$$C_1^* = \frac{\delta}{1+\delta+\gamma}\left(Y-G\right)\left(1-c\right)\left(1+r\right), \tag{72}$$

$$N^* = \frac{\gamma}{1+\delta+\gamma}\left(Y-G\right)\left(1-c\right)\frac{1}{\phi}. \tag{73}$$

And the market equilibrium quantities are (assuming zero profits):

$$C_{0,i}^{Mkt} = \frac{1}{1+2\gamma+\delta}Y\left(1-\tau_L\right), \tag{74}$$

$$C_{1,i}^{Mkt} = \frac{\delta}{1+2\gamma+\delta}Y\left(1-\tau_L\right)\left(1-c\right)\left(1+r\right)\left(1+\tau_M\right), \tag{75}$$

$$N^{Mkt} = \frac{1}{\phi}\cdot\frac{2\gamma}{1+2\gamma+\delta}\left(1-c\right)Y\frac{1-\tau_L}{1-\tau_S}. \tag{76}$$

Because under Traditional $\tau_S = \tau_L$, N^{Trad} *does not depend on tax rates at all,* and therefore $N^{Trad} > N^{Roth} > N^*$ regardless of τ_L^{Trad}. In turn, a higher N implies $\tau_L^{Trad} > \tau_L^{Roth} = G/Y$ for the government budget constraint to be satisfied, which in turn implies fewer resources available for consumption. Moreover, since in equilibrium $f^v = c$, the intertemporal consumption choice in the model is not distorted under Roth. With fewer resources and no distortions to correct, even with the best possible tax rates, Traditional in the model cannot be as good as Roth.

4.13.2 Comparing Roth to Taxable; Optimal taxation with TTE accounts

In order to compare Taxable with Roth, we derive the optimal tax rate on investment income in our model. If this rate is zero (or negative), Roth in the model is optimal (or constrained-optimal).

Aggregate utility under a taxable (TTE) system is

$$U^{TTE} = \ln C_0^{TTE} + \delta \ln C_1^{TTE} + \gamma \ln N^{TTE} + \gamma\left(1+\ln 2\right) \tag{77}$$

Using the results from the paper with a slight modification (using the aftertax return

$r(1-\tau_I)$ instead of just r), and setting $\tau_S = 0$, we obtain

$$C_0 = \frac{1}{1+\delta+2\gamma} Y(1-\tau_L), \tag{78}$$

$$C_1 = \frac{\delta}{1+\delta+2\gamma} Y(1-\tau_L)(1-c)(1+r(1-\tau_I)), \tag{79}$$

$$N = \frac{2\gamma}{1+\delta+2\gamma} \frac{Y}{\phi}(1-c)(1-\tau_L). \tag{80}$$

Substituting in and rearranging we obtain utility as a function of tax rates:

$$U^{TTE} = \tilde{K} + (1+\delta+\gamma)\ln(1-\tau_L) + \delta \ln(1+r(1-\tau_I)), \tag{81}$$

where $\tilde{K}$ is a constant that does not depend on tax rates. Next, we use the government's budget constraint to pin down τ_L as a function of τ_I, and solve for the τ_I that maximizes welfare.

Taxes on returns are collected at time 1. To compute tax revenue, consider that the final account balance in retirement is $(S(1-f^v)-F)(1+r)$ or, succinctly, $S(1-f)(1+r)$, where

$$f \equiv f^v + F/S = \frac{2\gamma + c\delta}{\delta+2\gamma} \tag{82}$$

and

$$S = \frac{\delta+2\gamma}{1+\delta+2\gamma} Y(1-\tau_L). \tag{83}$$

We assume fees are nondeductible, reflecting the current U.S. tax environment.[22] Then, the tax basis of the investment is $S(1-f)$ and the tax revenue is $S(1-f)r\tau_I$.

The government's intertemporal budget constraint is

$$G = Y\tau_L + S(1-f)\frac{r}{1+r}\tau_I. \tag{84}$$

Note that

$$\begin{aligned} S(1-f) &= \frac{\delta+2\gamma}{1+\delta+2\gamma} Y(1-\tau_L)\left(1-\frac{2\gamma+c\delta}{\delta+2\gamma}\right) \\ &= \frac{\delta}{1+\delta+2\gamma}(1-c)Y(1-\tau_L) \equiv s\cdot Y(1-\tau_L), \end{aligned} \tag{85}$$

where $s \in (0,1)$, a constant, is defined for notational convenience. Substituting this expres-

[22]Prior to 2018, in the U.S., investment management fees and financial planning fees were deductible if they exceeded 2% of AGI. For a broader discussion, see Section 1 of this Internet Appendix.

sion into the budget constraint, and rearranging, we obtain $1-\tau_L$ as a function of τ_I:

$$1-\tau_L=\frac{1-{}^{G}\!/\!{}_{Y}}{1-s\frac{r}{1+r}\tau_I}. \tag{86}$$

Then, the utility function becomes

$$U^{TTE}=\tilde{K}_2-(1+\delta+\gamma)\ln\left(1-s\frac{r}{1+r}\tau_I\right)+\delta\ln\left(1+r\left(1-\tau_I\right)\right) \tag{87}$$

where $\tilde{K_2}$ is another constant that does not depend on tax rates. The first-order condition is:

$$\frac{\partial U^{TTE}}{\partial\tau_I}=-\left(1+\delta+\gamma\right)\frac{-s\frac{r}{1+r}}{1-s\frac{r}{1+r}\tau_I}+\delta\frac{-r}{1+r\left(1-\tau_I\right)}=0 \tag{88}$$

Simplifying, we obtain

$$\tau_I^*=-\frac{1}{1-c}\cdot\frac{\gamma+c\left(1+\delta+\gamma\right)}{1+\gamma}\cdot\frac{1+r}{r}<0 \tag{89}$$

Thus, in this case, $\tau_I^*<0$ which means Roth in the model is better than Taxable, and negative tax rates on investment returns in the model would be even more welfare-enhancing. The intuition is the following:

- First, note that with logarithmic utility, regardless of the after-tax rate of return $r\left(1-\tau_I\right)$, dollar saving S and number of firms N are the same.

- Second, a negative τ_I means higher τ_L to balance the budget. A higher τ_L means lower S and lower N, which is good, because the equilibrium with $\tau_I=0$ results in too many firms. Thus, at $\tau_I=0$, $\partial U^{TTE}/\partial\tau_I<0$.

- Finally, as τ_I moves away from zero, the Euler equation gets more and more distorted. At some point the damage from the distortion balances out the benefit from fewer firms, and an optimum is reached.

The calibrated optimal τ_I^* is somewhere between -1% and -3% based on $\gamma=0.005$ to 0.2, $r=150\%$ and $c=20\%$.

5 Advanced results beyond logarithmic utility

In this section, we solve the two-period model in Section 5 of the paper, but using a more general constant relative risk-aversion (CRRA) utility specification. We also derive some results that hold with any concave increasing utility function. Note that, throughout this section, we apply the $Trad$ or $Roth$ superscripts to whole expressions to indicate Traditional and Roth, respectively. Any expressions without superscripts hold regardless of the system. We maintain the same notation as in the rest of the paper, e.g., C_0 and C_1 indicate consumption at times 0 and 1, respectively, S indicates saving, etc.

5.1 Basic expressions for C and S under power utility

We assume a power utility specification with coefficient ρ for consumption at times 0 and 1 as well as for distance disutility. The utility function of individual i is thus

$$u_i(C_{0,i}, C_{1,i}, d_{i,j}) = \frac{C_{0,i}^{1-\rho}}{1-\rho} + \delta \frac{C_{1,i}^{1-\rho}}{1-\rho} - \gamma \frac{d_{i,j}^{1-\rho}}{1-\rho}. \tag{90}$$

We omit the i subscripts henceforth to lighten the notation. As in Section 5.2 of the paper, the basic budget constraints are

$$S = \frac{(Y+\Pi)(1-\tau_L) - C_0}{1-\tau_S} \tag{91}$$

and

$$C_1 = \left[S\left(1-f^v\right) - F\right]\left(1+r\right)\left(1+\tau_M\right)\left(1-\tau_S\right). \tag{92}$$

Notice further that the Euler equation requires

$$[\delta(1+r)(1-f^v)(1-\tau_M)]^{\frac{1}{\rho}} C_0 = C_1. \tag{93}$$

Throughout this section, let

$$\bar{\delta} \equiv \frac{[\delta(1+r)(1-f^v)(1-\tau_M)]^{\frac{1}{\rho}}}{(1+r)(1-f^v)(1-\tau_M)}. \tag{94}$$

By combining eqs. (92) and (93), we then obtain

$$C_0 = \frac{(Y+\Pi)(1-\tau_L) - (1-\tau_S)F/(1-f^v)}{1+\bar{\delta}}. \tag{95}$$

Substituting this expression for C_0 into Eqs. (92) and (91), we obtain

$$\begin{aligned} C_1 &= \left[\frac{(Y+\Pi)(1-\tau_L) - (\frac{(Y+\Pi)(1-\tau_L) - \frac{(1-\tau_S)F}{1-f^v}}{1+\bar{\delta}})}{1-\tau_S}(1-f^v) - F\right]\left(1+r\right)\left(1+\tau_M\right)\left(1-\tau_S\right) \\ &= \frac{\bar{\delta}}{1+\bar{\delta}}[(Y+\Pi)(1-\tau_L)(1-f^v) - F(1-\tau_s)](1+r)(1+\tau_M) \end{aligned} \tag{96}$$

and

$$S = \frac{(Y+\Pi)(1-\tau_L) - \frac{(Y+\Pi)(1-\tau_L) - \frac{(1-\tau_S)F}{1-f^v}}{1+\bar{\delta}}}{1-\tau_S}$$

$$= \frac{\frac{\bar{\delta}}{1+\bar{\delta}}(Y+\Pi)(1-\tau_L) + \frac{(1-\tau_S)F}{(1-f^v)(1+\bar{\delta})}}{1-\tau_S}. \tag{97}$$

5.2 Basic assumptions

Throughout this section we assume that certain quantities are fixed and constant across regimes. Namely, as in Section 3 of the paper, we assume:

- No entry (fixed N) and the same firms exist under Roth and Traditional
- No government budget constraint (fixed τ_L)
- Constant lifetime resources (fixed Y and fixed Π, ignoring the constraint that $\Pi = \sum \pi_j$)

Finally, we also assume there is no match ($\tau_M = 0$).

5.3 A basic "scaling" result

Under our basic assumptions, $\bar{\delta}$ is constant across regimes. If fee schedules are equivalent as defined in Section 3 of the paper ($[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}$ as well as $f^{v,Trad} = f^{v,Roth}$), then eqs. (95), (96), and (97) immediately imply

$$C_0^{Trad} = C_0^{Roth}, \tag{98}$$

$$C_1^{Trad} = C_1^{Roth}, \tag{99}$$

$$[(1-\tau_S)S]^{Trad} = [(1-\tau_S)S]^{Roth}, \tag{100}$$

and therefore

$$[\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth}. \tag{101}$$

In sections 5.4 and 5.5, we prove that the optimal fees for firms actually satisfy $[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}$ (and $f^{v,Trad} = f^{v,Roth}$) under our basic assumptions and power utility, which therefore implies scaling of both F and S by $(1-\tau_S)$ and therefore $[\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth}$ as shown above.

5.4 Derivation of optimal F under power utility

Continuing to use our basic assumptions from Section 5.2, we now want to derive the F and f^v chosen by the firm. Our derivation follows the same pattern used for logarithmic utility, although for brevity we simply assume that the variable component of fees (f^v) equals variable costs (c) under both regimes.[23] To solve for the fixed component (F), we substitute $f^v = c$ into FOC (8) and simplify to

$$q_j + F_j \cdot \frac{\partial q_j}{\partial F_j} = 0 \iff F_j = -\frac{q_j}{\frac{\partial q_j}{\partial F_j}}, \tag{102}$$

Which in a symmetric equilibrium becomes

$$F = -\frac{\frac{1}{N}}{\frac{\partial q_j}{\partial F_j}}. \tag{103}$$

We obtain q_j by noting that in equilibrium it equals $2d_{i,j}$ for the marginal investor i who's indifferent between two neighboring funds. Investor i's indifference condition is represented by

$$\frac{C_{0,i,j}^{1-\rho}}{1-\rho} + \delta\frac{C_{1,i,j}^{1-\rho}}{1-\rho} - \gamma\frac{d_{i,j}^{1-\rho}}{1-\rho} = \frac{C_{0,i}^{1-\rho}}{1-\rho} + \delta\frac{C_{1,i}^{1-\rho}}{1-\rho} - \gamma\frac{(\frac{1}{N} - d_{i,j})^{1-\rho}}{1-\rho} \tag{104}$$

or

$$d_{ij}^{1-\rho} - (\frac{1}{N} - d_{i,j})^{1-\rho} = \frac{1}{\gamma}[C_{0,i,j}^{1-\rho} - C_{0,i}^{1-\rho}] + \frac{\delta}{\gamma}[C_{1,i,j}^{1-\rho} - C_{1,i}^{1-\rho}], \tag{105}$$

This expression does not allow for a closed-form solution for $d_{i,j}$, so we use the implicit function theorem, which (using eqs. (95) and (96) for $C_{0,i,j}$ and $C_{1,i,j}$) yields

$$\begin{aligned}\frac{\partial q_j}{\partial F_j} &= 2\frac{\partial d_{i,j}}{\partial F_j}\\ &= -\frac{(1+\bar{\delta})^{\rho-1}(1-\tau_S)[(Y+\Pi)(1-\tau_L)(1-f^v)-(1-\tau_S)F]^{-\rho}[(1-f^v)^{\rho-1}+\delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}}{\gamma(2N)^\rho},\end{aligned} \tag{106}$$

[23]As discussed in Section 5.2 of the paper, this is a common result that does not depend on the utility function (see, e.g., Oi, 1971).

Combined with (103), we find

$$F = -\frac{\frac{1}{N}}{\frac{\partial q_j}{\partial F_j}}$$

$$\iff F = \frac{1}{1-\tau_S} \frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S)F]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{107}$$

With $\rho = 1$ and $f^v = c$, this yields (22) as expected. With generic ρ, however, we cannot solve for F explicitly.

5.5 Supply-side equivalence result under power utility

Here we derive explicit equations that demonstrate, for the special case of CRRA utility, the basic supply-side equivalence result in Section 4.1. We continue to use our basic assumptions from Section 5.2. Notice that expression (107) immediately implies

$$(1-\tau_S)F = \frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S)F]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{108}$$

Now suppose by contradiction that $[(1-\tau_S)F]^{Roth} \neq [(1-\tau_S)F]^{Trad}$ (Without loss of generality, let us set $[(1-\tau_S)F]^{Roth} > [(1-\tau_S)F]^{Trad}$.) Then, we would have

$$\frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - [(1-\tau_S)F]^{Roth}]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} \tag{109}$$

$$< \frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - [(1-\tau_S)F]^{Trad}]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}, \tag{110}$$

a contradiction. Hence, we have $[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}$. By Section 5.3, we can therefore conclude that

$$C_0^{Trad} = C_0^{Roth}, \tag{111}$$

$$C_1^{Trad} = C_1^{Roth}, \tag{112}$$

$$[(1-\tau_S)S]^{Trad} = [(1-\tau_S)S]^{Roth}, \tag{113}$$

and therefore

$$[\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth} \tag{114}$$

holds. Since $f^v = c$, this shows (consistent with our results in Section 3 of the main paper) that percent fees are the same under both regimes given our basic assumptions from Section 5.2 and power utility.

5.6 Does $\frac{F}{S}$ depend on τ_L?

Next, we allow τ_L to differ between regimes, but we keep all other basic assumptions from Section 5.2. Under these assumptions, we show that our fee equivalence does not generally hold when tax rates are allowed to vary across regimes.

Notice that by (97), i.e.

$$S = \frac{\frac{\bar{\delta}}{1+\bar{\delta}}(Y+\Pi)(1-\tau_L) + \frac{(1-\tau_S)F}{(1-f^v)(1+\bar{\delta})}}{1-\tau_S}, \tag{115}$$

we obtain

$$\frac{S}{F} = \frac{\frac{\bar{\delta}}{1+\bar{\delta}}\frac{Y+\Pi}{F}(1-\tau_L) + \frac{(1-\tau_S)}{(1-f^v)(1+\bar{\delta})}}{1-\tau_S}. \tag{116}$$

From the last expression, we can deduce that $\frac{S}{F}$ is independent of τ_L and τ_S (and therefore equal across regimes) iff $F = a \cdot \frac{1-\tau_L}{1-\tau_S}$ for some a constant across regimes. To see why, consider that if F has this form, we have

$$\begin{aligned} \frac{S}{F} &= \frac{\frac{\bar{\delta}}{1+\bar{\delta}}\frac{Y+\Pi}{a}\frac{1-\tau_S}{1-\tau_L}(1-\tau_L) + \frac{(1-\tau_S)}{(1-f^v)(1+\bar{\delta})}}{1-\tau_S} && (117) \\ &= \frac{\bar{\delta}}{1+\bar{\delta}}\frac{Y+\Pi}{a} + \frac{1}{(1-f^v)(1+\bar{\delta})}, && (118) \end{aligned}$$

which is constant across systems. If F is not of this form, we either have that a is not constant across regimes, or that τ_L and/or τ_S do not cancel out. As can be seen in eq. (22), the closed-form solution of F for the log case (i.e. $\rho = 1$) fulfills this requirement.

Now consider the following for the case of arbitrary ρ for which we have the implicit Eq. (107),

$$F = \frac{1}{1-\tau_S}\frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S)F]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{119}$$

Suppose that $F = a \cdot \frac{1-\tau_L}{1-\tau_S}$ for some a constant across regimes holds. This implies

$$\begin{aligned} a \cdot \frac{1-\tau_L}{1-\tau_S} &= \frac{1}{1-\tau_S}\frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S) \cdot a \cdot \frac{1-\tau_L}{1-\tau_S}]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} && (120) \\ \iff a &= \frac{1}{(1-\tau_L)^{1-\rho}}\frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-f^v) - a]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} && (121) \end{aligned}$$

To simplify this expression, let's denote all terms constant across regimes in the expression above by B, C, and D, obtaining

$$a = \frac{1}{(1-\tau_L)^{1-\rho}} \frac{B[C-a]^\rho}{D}, \tag{122}$$

where

$$B = \gamma \cdot 2^\rho N^{\rho-1}, \tag{123}$$

$$C = (Y+\Pi)(1-f^v), \text{ and} \tag{124}$$

$$D = (1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]. \tag{125}$$

Since B, C, and D are constant across regimes, it is immediate to see that a changes across regimes, since it depends on τ_L (provided $\rho \neq 1$). Hence, we have reached a contradiction, proving that F/S depends on τ_L iff $\rho \neq 1$. (For reference, divide eq. (22) by eq. (29) to find that F/S is independent of taxes and therefore constant across systems in the log case.)

5.7 Proof of constant F/S (with $c = 0$ and fixed N)

In this section, we continue to assume that $\tau_M = 0$ and that $f^{v,Trad} = f^{v,Roth} = c$ in equilibrium. Under the further assumption that $c = 0$, we show that our fee equivalence does hold for all values of ρ, as long as there are no variable costs.

First, notice that by (107),

$$(1-\tau_S)F = \frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S)F]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{126}$$

By our assumptions, $\bar{\delta}$ is constant across regimes, and thus the only two non-constant terms in (126) are $(1-\tau_S)F$ and $(Y+\Pi)(1-\tau_L)$, so that

$$[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth} \implies [(Y+\Pi)(1-\tau_L)]^{Trad} = [(Y+\Pi)(1-\tau_L)]^{Roth}. \tag{127}$$

Consider similarly that by our expression for saving (97),

$$(1-\tau_S)S = \frac{\bar{\delta}}{1+\bar{\delta}}(Y+\Pi)(1-\tau_L) + \frac{(1-\tau_S)F}{(1-f^v)(1+\bar{\delta})}. \tag{128}$$

Here, since $[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}$ implies $[(Y+\Pi)(1-\tau_L)]^{Trad} = [(Y+\Pi)(1-\tau_L)]^{Roth}$), it also implies

$$[(1-\tau_S)S]^{Trad} = [(1-\tau_S)S]^{Roth}. \tag{129}$$

(Notice that by Eq. (95), this also implies $C_0^{Trad} = C_0^{Roth}$. Furthermore, notice that this statement of $[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth} \implies [(1-\tau_S)S]^{Trad} = [(1-\tau_S)S]^{Roth}$ is different from that in section 5.3 in that Π and τ_L are no longer exogenous.) Hence, we can conclude that

$$[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth} \implies [\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth} \tag{130}$$

(Notice that this is not an iff statement.)

To check if $[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}$ holds in equilibrium, consider once again the expression for saving (97),

$$S = \frac{\frac{\bar{\delta}}{1+\bar{\delta}}(Y+\Pi)(1-\tau_L) + \frac{(1-\tau_S)F}{(1-f^v)(1+\bar{\delta})}}{1-\tau_S}, \tag{131}$$

as well as the tax rates that satisfy the government budget constraint

$$\tau_L^{Roth} = \frac{G}{Y+F-\phi N} \tag{132}$$

$$\tau_L^{Trad} = \frac{G}{Y-\phi N - f^v S} \tag{133}$$

$$= \frac{G}{Y-\phi N - f^v[\frac{\bar{\delta}}{1+\bar{\delta}}(Y+F-\phi N) + \frac{F}{(1-f^v)(1+\bar{\delta})}]}. \tag{134}$$

Substituting these tax rates into the implicit equation for F (107),

$$F = \frac{1}{1-\tau_S}\frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y+\Pi)(1-\tau_L)(1-f^v) - (1-\tau_S)F]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}, \tag{135}$$

we obtain

$$F^{Roth} = \frac{\gamma \cdot 2^\rho N^{\rho-1}[(Y-\phi N - G)(1-f^v) - Ff^v]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} \tag{136}$$

$$F^{Trad} = \frac{\gamma \cdot 2^\rho N^{\rho-1}(1-\tau_L)^{\rho-1}[(Y-\phi N)(1-f^v) - Ff^v]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} \tag{137}$$

$$= \frac{\gamma \cdot 2^\rho N^{\rho-1}(1 - \frac{G}{Y-\phi N - f^v[\frac{\bar{\delta}}{1+\bar{\delta}}(Y+F-\phi N) + \frac{F}{(1-f^v)(1+\bar{\delta})}]})^{\rho-1}[(Y-\phi N)(1-f^v) - Ff^v]^\rho}{(1+\bar{\delta})^{\rho-1}[(1-f^v)^{\rho-1} + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{138}$$

Now let $c = f^v = 0$. Then, we have

$$F^{Roth} = \frac{\gamma \cdot 2^\rho N^{\rho-1}[Y-\phi N - G]^\rho}{(1+\bar{\delta})^{\rho-1}[1 + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]} \tag{139}$$

$$F^{Trad} = \frac{\gamma \cdot 2^\rho N^{\rho-1}(Y-\phi N - G)^{\rho-1}[Y-\phi N]}{(1+\bar{\delta})^{\rho-1}[1 + \delta[\bar{\delta}(1+r)(1+\tau_M)]^{1-\rho}]}. \tag{140}$$

By $1-\tau_L^{Trad} = \frac{Y-\phi N-G}{Y-\phi N}$, we immediately find

$$(1-\tau_L^{Trad})F^{Trad} = F^{Roth}, \tag{141}$$

or equivalently,

$$[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}. \tag{142}$$

Hence, for $c = 0$ and arbitrary ρ, we have shown that

$$[(1-\tau_S)F]^{Trad} = [(1-\tau_S)F]^{Roth}, \tag{143}$$

which, by the logic above, implies

$$[(1-\tau_S)S]^{Trad} = [(1-\tau_S)S]^{Roth}, \tag{144}$$

$$C_0^{Trad} = C_0^{Roth}, \tag{145}$$

and

$$[\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth}. \tag{146}$$

5.8 Willingness to pay and willingness to travel

Define λ as the distance an individual is willing to travel for one more unit of consumption. Formally,

$$\frac{\partial u}{\partial C_{0,i}} + \lambda \frac{\partial u}{\partial d_{i,j}} = 0. \tag{147}$$

Under our assumption of power utility, (147) becomes

$$C_{0,i}^{-\rho} - \lambda\gamma d_{i,j}^{-\rho} = 0 \iff \lambda = \frac{C_{0,i}^{-\rho}}{\gamma d_{i,j}^{-\rho}}.$$

Further define

$$\Lambda_{i,j} \equiv \frac{\frac{1}{\lambda}}{C_{0,i,j}} = \frac{\gamma d_{i,j}^{-\rho}}{C_{0,i,j}^{1-\rho}} \tag{148}$$

as the the *percentage* of initial consumption that an individual would give up to save a mile of travel. Now notice that since $d_{i,j}$ is constant across Traditional and Roth under our assumption that the same firms exist under either system, Λ_i is constant across systems unless $C_{0,i}^{1-\rho}$ differs. Obviously, $C_{0,i}^{1-\rho}$ is constant across systems if $\rho = 1$ regardless of $C_{0,i}^{Trad}$ and $C_{0,i}^{Roth}$. If $\rho \neq 1$, however, we have

$$\Lambda_{i,j}^{Trad} = \Lambda_{i,j}^{Roth} \iff C_{0,i,j}^{Trad} = C_{0,i,j}^{Roth} \quad \forall j. \tag{149}$$

Hence, we also have

$$\Lambda_i^{Trad} = \Lambda_i^{Roth} \iff C_{0,i}^{Trad} = C_{0,i}^{Roth} \tag{150}$$

in equilibrium. Notice that we have $C_{0,i}^{Trad} = C_{0,i}^{Roth}$ as well as $[\frac{F}{S}]^{Trad} = [\frac{F}{S}]^{Roth}$ for $\rho \neq 1$ in both section 5.5 and 5.7.

References

Alpert, B.N., Rekenthaler, J., Suh, S., 2013. Global Fund Investor Experience 2013 Report. Technical Report. Morningstar Fund Research.

Berk, J., van Binsbergen, J., 2015. Measuring skill in the mutual fund industry. Journal of Financial Economics 118, 1–20.

Bogle, J.C., 2014. The arithmetic of 'all-in' investment expenses. Financial Analysts Journal 70, 13–21.

Carhart, M.M., 1997. On persistence in mutual fund performance. Journal of Finance 52, 57–82.

Dold, E.T., Levine, D.N., 2011. Tax treatment on the payment of IRA fees gets a new but familiar private letter ruling. Taxes—The Tax Magazine , 17–20.

Edelen, R., Evans, R., Kadlec, G., 2013. Shedding light on "invisible" costs: Trading costs and mutual fund performance. Financial Analysts Journal 69, 33–44.

Fama, E.F., 1965. The behavior of stock market prices. Journal of Business 38, 34–105.

Fama, E.F., French, K.R., 2010. Luck versus skill in the cross-section of mutual fund returns. The Journal of Finance 65, 1915–1947.

French, K.R., 2008. Presidential address: The cost of investing. The Journal of Finance 53, 1537–1573.

Jensen, M.C., 1968. Problems in selection of security portfolios. The Journal of Finance 23, 389–416.

Livingston, M., Zhou, L., 2015. Brokerage commissions and mutual fund performance. Journal of Financial Research 38, 283–303.

Malkiel, B.G., 1995. Returns from investing in equity mutual funds 1971 to 1991. Journal of Finance 50, 549–572.

Malkiel, B.G., 2013. Asset management fees and the growth of finance. Journal of Economic Perspectives 27, 97–108.

Mankiw, N.G., Whinston, M.D., 1986. Free entry and social inefficiency. RAND Journal of Economics 17, 48–58.

OECD, 2015a. Stocktaking of the tax treatment of funded private pension plans in OECD and EU countries.

OECD, 2015b. The tax treatment of funded private pension plans - OECD and EU country profiles.

Oi, W.Y., 1971. A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly. Quarterly Journal of Economics 85, 77–96.

Rosshirt, D.E., Parker, S.A., Pitts, D.A., 2014. Inside the structure of defined contribution/401(k) plan fees, 2013: A study assessing the mechanics of the 'all-in' fee. Deloitte Consulting LLP.

Salop, S.C., 1979. Monopolistic competition with outside goods. Bell Journal of Economics 10, 141–156.

Tirole, J., 1988. The Theory of Industrial Organization. MIT Press.

Wealthfront, 2016. Mutual fund fees. Available at https://web.archive.org/web/20221206131126/https://www.wealthfront.com/research/mutual-fund-fees.

Wermers, R., 2000. Mutual fund performance: An empirical decomposition into stock-picking talent, style, transaction costs, and expenses. Journal of Finance 55, 1655–1703.