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RATIONAL PONZI GAMES*

BY STEPHEN A. O'CONNELL AND STEPHEN P. ZELDES¹

1. INTRODUCTION

What are feasible paths of debt for a government that borrows either internally or externally? The question is suggested by recent concerns about the international debt crisis and high federal budget deficits in the U.S. In this paper, we analyze the benchmark case in which all market participants have perfect foresight, so that only risk-free lending is done. We study the conditions under which the borrower's opportunities include strategies with positive net present value.

The strategies we investigate are perfect foresight versions of the "Ponzi schemes" discussed by Minsky (1982) and Kindleberger (1978), where individuals or companies pay out funds to some parties by borrowing funds from others.² Since the perfect foresight assumption rules out schemes based on imperfect information (e.g., swindles) or irrationality of lenders (e.g., fallacies of composition), we are asking under what circumstances these Ponzi games can continue indefinitely. When, in other words, is it feasible for a government to incur debt and never pay back any principal or interest? We call such a policy, where all principal repayments and interest are forever "rolled over," i.e., financed by issuing new debt, a "rational Ponzi game."

Examples of rational Ponzi games are not hard to find in the growth theory literature. The Diamond (1965) overlapping generations model, for example, has steady states in which the interest rate is below the growth rate of the labor force and government debt per capita is positive. In these steady states, the total stock of government debt is increasing at a rate higher than the interest rate. New debt finances all of the interest payments on the existing debt plus some additional transfers to the young. This means that the government can cut current taxes by a small amount without ever raising future taxes to finance the increased interest payments. Our aim in this paper is to uncover the general conditions that make Ponzi games feasible and to apply these results in a variety of growth contexts. We make the following observations.

First, the feasibility of a rational Ponzi game depends on some key characteristics of the economy whose agents are going to hold the debt. For the case of external debt, this means that the characteristics of the borrower's economy are irrelevant to the feasibility of perpetual debt rollover. With regard to the Third World debt situation, it follows that the feasibility of perpetual rollover of debt

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¹ We are grateful for useful discussions with Stanley Fischer, Jean Tirole, and members of the Macro Lunch Group at the University of Pennsylvania, and for helpful comments from two anonymous referees. Any errors are ours.

² For an interesting discussion of the life and times of Charles Ponzi, see Russell (1973).

depends on the performance of the economy of the lenders—in particular, on the relationships between real interest rates, population growth rates, and growth in per-capita income—and not that of the borrowing countries.

Second, for a Ponzi game to work, agents in the lending economy at all points of time must be willing to hold the outstanding stock of debt. This means that the debt must have a holding period return equal to the rate of interest, that the outstanding stock of debt must grow at the rate of interest, and that the aggregate desired wealth of the economy must be growing at least as fast, asymptotically, as the rate of interest.

When the lender acts as an infinitely lived representative consumer, a necessary condition for his optimal plan is that his wealth not grow faster than the rate of interest. This is sufficient to rule out his participation as a lender in a rational Ponzi game, regardless of the relationship between the interest rate and the rate of growth of income or population.

When we relax the representative consumer assumption, it becomes more difficult to rule out Ponzi games. We consider two types of economies. An essential characteristic of each is that over the course of time there are an infinite number of decision makers. One economy consists of a growing number of infinitely lived agents, and the other consists of an infinite sequence of two period lived agents. In either case, agents care only about themselves, and not about other family members. There are conditions under which rational Ponzi games can exist in either of these economies (the feasibility conditions in the endowment economy versions of each of these models are quite similar). It is, therefore, not the length of the horizons that is relevant, but rather whether there is an infinite number of agents as opposed to a finite number of (effective) agents who care only about lifetime consumption.

The issues raised in this paper have appeared in a variety of contexts in the growth theory and monetary theory literatures. Of particular relevance is the literature on asset price bubbles. A bubble is defined as the difference between the price of an asset and the present discounted value of the payments associated with that asset (its fundamental). Intrinsically useless fiat money, for example, constitutes a bubble whenever it has a positive price in terms of goods, since it has a fundamental of zero. In fact, the questions of whether fiat money can be valued, whether bubbles can exist on assets, whether the government can independently choose the steady state deficit and rate of monetary expansion, and whether a government can run a rational Ponzi game all turn out to be basically the same question. Our analysis draws on the existing literature in these areas and especially on Tirole's (1985) study of rational asset price bubbles in the overlapping generations model.³

In characterizing the relationship between Ponzi games and asset price bubbles, we show that any monetary equilibrium can be replicated by a Ponzi game equilibrium with finitely lived debt, and derive a strong irrelevancy result for

³ See also McCallum's (1984) paper, which analyzes feasible government debt paths in a version of the Sidrauski model. Diamond (1965), Sargent and Wallace (1981) and Darby (1984) analyze closely related issues of steady state government finance.

open market operations between money and Ponzi game debt. We also show, however, that there exist some monetary equilibria that cannot be replicated by a Ponzi game equilibrium with positive coupon consols. In addition, the payments on debt contracts reduce the indeterminacy inherent in monetary equilibria. Governments, in other words, can select among the range of possible equilibria by issuing finitely lived debt rather than “money.” Some indeterminacy remains in the price of infinitely lived debt, since the price can be greater than the present discounted value (PDV) of the coupon payments. The distinction between bubble and fundamental is not relevant here, however. Identical real equilibria exist where consols are priced at their fundamental and where they are priced above their fundamental (i.e., with bubbles). The reason the bubble/fundamental distinction is not relevant here is that for the case of rational Ponzi games, the entire debt is acting like a bubble.

Finally, we note that when borrowers are running rational Ponzi schemes, this does not imply that lenders are in any sense losing out. In the models we study in this paper, rational Ponzi games are only feasible when the economy is in a dynamically Pareto inefficient equilibrium. The introduction of perpetually rolled over debt will never make the lending economy worse off and will in general make it better off relative to a world in which no Ponzi game is run.

The paper is organized as follows. In the next section, we set out notation and give a rigorous definition of a rational Ponzi game. In Section 3, we study the optimal borrowing problem for an individual agent and present the basic result that Ponzi games can only exist in economies with an infinite number of agents over time. The application of this result to various growth models is immediate, and we do this in Section 4. In Section 5, we analyze the relationship between Ponzi games, asset price bubbles, and valued fiat money. In Section 6 we use the results of the previous sections to analyze LDC borrowing. Section 7 concludes the paper.

2. NOTATION AND A DEFINITION

Consider an agent who is a price taker in the loan market (i.e., who takes interest rates as parametric) and who is formulating a plan for borrowing and lending over the possibly infinite future. Any plan will imply a sequence of net cash inflows Z_t from other agents. We will assume for most of the paper that all contracts are perfectly enforceable, so that promised payments and actual payments coincide. We define the agent’s net indebtedness at the end of period t , D_t , as the present value of the net cash flows already received as the result of past credit market transactions.⁴

⁴ One could alternatively define net indebtedness as the present value of current and *future* net cash outflows on existing loan contracts held by the individual. Both definitions imply that net indebtedness, like any other form of financial wealth, satisfies the recursion: $D_t = (1 + r_t)D_{t-1} + Z_t$. The two definitions are equivalent whenever all loan contracts held by the individual yield present value zero over the life of the contract. This need not be the case; in certain cases there may be a bubble on the individual’s debt. We discuss this possibility with respect to government debt in Section 5.

Viewed from period zero, the present value of net indebtedness in any period T equals the present value of the stream of prospective cash inflows for the borrower between 0 and T :

$$(1) \quad \Gamma(T)D_T = \sum_{s=1}^T \Gamma(s)Z_s$$

where r_t is the real interest rate between periods $t - 1$ and t (identical for all assets under perfect foresight), and $\Gamma(s)$ is the discount factor applicable in period 0 to income received in period s :

$$(2) \quad \Gamma(s) \equiv \prod_{j=1}^s (1 + r_j)^{-1}.$$

The term $\Gamma(T)D_T$ will play a crucial role in this paper. In an infinite horizon setting, a common approach in the literature (e.g., Blanchard 1985) is to impose the constraint that this present value be non-positive as t approaches infinity:⁵

$$(3) \quad \lim_{T \rightarrow \infty} \Gamma(T)D_T \leq 0.$$

When the interest rate is constant over time, this takes the familiar form

$$(3a) \quad \lim_{T \rightarrow \infty} (1 + r)^{-T}D_T \leq 0.$$

For a sovereign borrower, the implication of (3) is that debts must be paid off by future savings, either in the form of non-interest government budget surpluses or, in the absence of a government/private sector distinction, non-interest current account surpluses. Condition (3) is usually viewed as being imposed by lenders to rule out Ponzi games. We will adopt the implied definition of rational Ponzi games as borrowing strategies in which the limit in (3), i.e., the present value of all cash flows associated with a borrowing strategy, is strictly positive. More formally,

DEFINITION 1. A *rational Ponzi game* is a sequence of loan market transactions with positive net present value to the borrower in the following sense: there exists an $\varepsilon > 0$ and a T' such that $\Gamma(T)D_T \geq \varepsilon$ for all $T \geq T'$. If the limit of $\Gamma(T)D_T$ exists, then a rational Ponzi game is a strategy such that $\lim_{T \rightarrow \infty} \Gamma(T)D_T > 0$.

By engaging in a Ponzi game, the borrower is able to extract positive resources (in present value terms) from the lender(s). The classic form of this scheme consists of borrowing money (issuing short or long term debt) and financing all promised payments of principal or interest by issuing new debt. In other words,

⁵ The limit may not exist; we treat this problem more carefully in Definition 1. In addition, condition (3) is usually imposed with equality; we argue below that the correct condition is the weak inequality.

in each period promised cash outflows (interest, coupon payments, etc.) are offset by other cash inflows (new borrowing).⁶

Our definition of rational Ponzi games intentionally leaves aside the issue of how individual loan contracts are priced. For most of the paper we will be concerned with Ponzi game equilibria in which contracts are priced at the present value of the stream of promised payments. In many cases, however, Ponzi game equilibria also exist in which the contracts are priced above their fundamental. In fact, even if all promised payments are made through net transfers from the borrower to lenders (rather than through new borrowing), a Ponzi game exists whenever a borrower can issue debt priced above its fundamental. We discuss the issues of price indeterminacies and multiple equilibria in Section 5.

3. THE TRANSVERSALITY CONDITION AND THE NUMBER OF TRADERS

Under what circumstances will lenders agree to be part of a rational Ponzi game? The basic result we establish in this section is that Ponzi games require the participation of an infinity of agents. This follows from work by Cass (1972) and Tirole (1982, 1985). The result stems from the fact that no single agent will wish to form the lending side of a rational Ponzi game; to do so would involve a sacrifice of consumption with no offsetting benefit. The same argument can be shown to apply to the joint behavior of any finite set of agents. This means that in an economy with a finite number of agents, each agent faces a constraint of the form (3) as the result of optimal behavior by all other agents. With an infinite number of agents over time, by contrast, there always exists a new set of agents with whom to trade, and this may make it possible for debt to be rolled over perpetually.

This argument is formalized in Proposition 1 and 2 below. We begin by making two assumptions to dispose of uninteresting cases.

ASSUMPTION 1. (Finiteness) There are a finite number of individual decision makers in the credit market in each period.

Assumption 1 forces us to focus on Ponzi games that occur over time, i.e., that exploit the potential infinity of traders due to the birth of new agents.

ASSUMPTION 2. (Nonsatiation) The utility of individuals depends only on consumption, and individuals prefer more consumption to less in each period.

Assumption 2 leads to a familiar definition of consumption efficiency (see Cass 1972), which we apply here specifically to borrowing/lending strategies:

DEFINITION 2. A borrowing/lending strategy is *consumption efficient* if it is impossible for the agent, leaving the remainder of his portfolio strategy unchanged, to rearrange loan market transactions so as to raise consumption in any

⁶ But note that the definition also includes the case in which some payments are made, as long as the present value of net payments is less than the amount initially received by issuing debt.

period without lowering it in some other period(s). Otherwise the strategy is *consumption inefficient*.

By nonsatiation, a rational agent will never choose a consumption inefficient consumption plan. More formally, we have the following proposition:

PROPOSITION 1. (*The Transversality Condition*) Under Assumption 2, the following condition is necessary for optimality of an individual's borrowing/lending strategy:

$$(4) \quad \limsup_{T \rightarrow \infty} \Gamma(T)D_T \geq 0.$$

The proposition, which holds regardless of the agent's horizon⁷ is a restatement of Cass's (1972) condition for consumption inefficiency in a one-good economy with a linear technology. Cass's condition applies directly to our case, since an individual taking interest rates as parametric effectively faces a linear technology for transferring consumption intertemporally.⁸

The intuition behind Proposition 1 is straightforward. Violation of (4) implies that the present value of debt is bounded above by some strictly negative number $-\varepsilon$ from some period t' onwards (i.e., the present value of *assets* is bounded *below* by some $\varepsilon > 0$ from t' onwards). If this were true, the agent could consume an extra amount ε in period t' , leave all consumption beyond t' unchanged, and still have assets nonnegative for all t . Since such a plan is feasible and by nonsatiation is preferred to the original plan, the original plan cannot be optimal.

What prevents an individual from running a rational Ponzi game, i.e., from choosing a strategy in which the limit in (4) is strictly *positive*? Since equation (4) must hold for all participants in the loan market (currently alive or not), we have the result:

PROPOSITION 2. *Rational Ponzi games do not exist in a credit market with a finite number of participants over time.*

The proof follows from Tirole (1982). The intuition is straightforward. Consider any possible equilibrium sequence of interest rates. As can be seen by Definition 1, Ponzi games involve borrowing an amount that, after a point at least, grows at least as fast as the inverse of the discount factor. The existence of a rational Ponzi game means that some (finite or infinite) group of lenders is allowing its total lending to grow at least as fast as the inverse discount factor in equilibrium. But by Proposition 1, no single agent will allow his own wealth to grow as fast as the inverse discount factor, because this would be a consumption inefficient strategy.

⁷ For a finitely lived agent, the condition is simply $D_T \geq 0$, or equivalently $\Gamma(T)D_T \geq 0$, where T is the agent's terminal date.

⁸ Cass's theorem implies that a strategy is inefficient if the sequence $\{\Gamma(T)D_T\}$ of present values of debt is bounded above by any strictly negative number for all $T \geq T'$. Efficiency therefore requires that the sequence have at least one subsequence with a nonnegative limit point. Equivalently, the least upper bound of the set of limit points of subsequences—the *lim sup*—must be nonnegative for consumption efficiency.

By simple addition, this implies that the total net wealth of any finite number of agents must grow more slowly than the inverse discount factor. It follows that lending side of the Ponzi game must be composed of an infinite number of agents entering the economy over time.^{9,10} Each individual continues to satisfy his transversality condition, but the economy of lenders as a whole does not satisfy any aggregate transversality condition.

The analysis in this section makes it clear that the standard “transversality condition” one often encounters in the literature,

$$(5) \quad \lim_{T \rightarrow \infty} \Gamma(T)D_T = 0,$$

is really a combination of two things: (1) a genuine transversality condition stating that nonnegativity of the limit in (5) is necessary for optimality, (i.e. the individual *chooses* not to be on the *lending* side of a Ponzi game), and (2) a restriction on the individual’s budget set stating that the limit must be non-positive (i.e. *equilibrium rules out* individuals being on the *borrowing* side of a Ponzi game).¹¹ The second condition holds when the number of potential loan market participants is finite. Together, the two conditions yield (5).

4. INFINITE HORIZONS

Proposition 2 motivates what lies behind the examples of Ponzi schemes that we present in this section and the next. The key characteristic we focus on is whether the behavior of a possibly infinite group of potential lenders can be consolidated into the behavior of a finite set of decision makers over time whose preferences satisfy Assumption 2. If so, Proposition 2 applies to the consolidated group of agents, and rational Ponzi games can be ruled out. The simplest example of this is the infinitely lived representative consumer model of optimal growth theory. If consolidation does not occur—as for example in Samuelson’s (1958)

⁹ This possibility was suggested by Shell (1971), who pointed out that the form of the lifetime budget constraint is not obvious for an infinitely lived agent in a world with an infinity of traders.

¹⁰ Although such a scheme could conceivably work in finite time in an economy with an infinite number of agents (and also an infinite endowment), we have ruled out this uninteresting case by Assumption 1. The following example (similar to Gamow’s infinite hotel problem described in Shell 1971), although Assumption 1 rules it out, may provide some helpful intuition on how rational Ponzi games work (the intergenerational schemes we discuss later are isomorphic):

Consider a countably infinite current population, each with an identical positive endowment (the aggregate endowment is therefore infinite). Now array the population side by side to the right of a particularly clever first individual. Let everyone pass one unit of endowment to the person on his left, except for the first individual, who gives up nothing. Consumption of the first individual goes up by one unit, and everyone else’s consumption is unchanged. The first individual has successfully run a rational Ponzi game.

¹¹ The transversality condition (TVC) is necessary for optimality for an individual taking interest rates as parametric, and also, therefore, for a central planner facing a linear technology. The issue is more complicated, however, when we consider the planner’s problem in an economy with a neoclassical technology. There, failure of the TVC does not necessarily imply the existence of Pareto dominating consumption plans, because any attempt to run down the capital stock will change the equilibrium interest rate sequence. In certain knife-edge cases, such as the Golden Rule equilibrium ($r = n$), the TVC fails but consumption efficiency nonetheless holds.

overlapping generations model without intergenerational altruism, or in the infinite horizon model without intergenerational altruism that we present below—the feasibility of rational Ponzi games that extract resources depends on whether the economy is in a Pareto inefficient equilibrium in the absence of Ponzi games or asset price bubbles (Tirole 1985). We begin with the infinite horizon, stationary population case.

4.1. *Stationary Population.* Consider an economy with a finite number of infinitely lived agents. Population growth is zero; the economy is composed of the same group of agents each period. Since Proposition 2 applies directly to this economy, we can conclude immediately that rational Ponzi games are ruled out.¹²

4.2. *Growing Population.* Consider now an economy consisting of a growing number of infinitely lived agents.¹³ Let $L_t = (1 + n)L_{t-1}$ be the number of agents alive at t , so that nL_{t-1} children are born in period t . Now suppose that there is a central planner who maximizes a welfare functional defined over the path of aggregate consumption. If the welfare functional shows nonsatiation in aggregate consumption in all periods, Proposition 1 holds for the economy as a whole, and it follows that no outsider can run a rational Ponzi game trading solely with individuals in this economy. The same result emerges if we view the economy as consisting of a finite number of initial “dynasties” i.e., families growing at rate n . If the utility function of the initial parents shows nonsatiation and is defined solely over the (total or per-capita) consumption of the family, and if we endow the initial parents with dictatorial power over the consumption of their children, we have reduced an economy with an infinite number of potential lenders over time to one with a finite set of participants whose preferences satisfy Assumption 2. Rational Ponzi games are then ruled out by Proposition 2.

Alternatively, however, suppose that the economy is decentralized and that there is no intergenerational altruism. New agents are born into the economy and fend for themselves. The key point here is that while each individual will satisfy his own transversality condition, this will not suffice to rule out rational Ponzi games. Even though each individual's wealth will not be growing faster than the inverse discount factor, population growth may make it possible for aggregate desired wealth to grow at the rate of interest or faster. Ponzi games are therefore feasible in an economy with infinite-horizon agents. We illustrate this with the following example.

¹² McCallum (1984) studies a model such as this in which Ponzi games are ruled out and Ricardian Equivalence holds with respect to debt that does *not* constitute a Ponzi game. In this setting, he shows that there is no limit on how high debt can go in any period in the finite future. This is because individuals willingly hold all of the debt in order to pay the future taxes. However, this does not mean that the interest and principal can be *perpetually* rolled over, because in such a circumstance no one would hold the debt.

¹³ Since completing an earlier draft of this paper, independent work by Weil (1986) has been brought to our attention. Weil shows the possible dynamic inefficiency of infinite horizon models with population growth and neoclassical technology.

Consider an economy with a constant returns to scale storage technology yielding return $r > 0$, with each infinitely lived agent receiving an initial endowment of goods e_0 and no further endowments.¹⁴ We assume that individuals have time separable utility functions with rate of time preference $\delta = r$, and that the population grows at rate $n > 0$. We also assume that individuals act as if they cannot run rational Ponzi games, i.e., they optimize subject to the constraint that the limit of the PDV of wealth is nonnegative. This latter assumption is required for the optimization problems of individuals to be well defined. These agents take the interest rate as parametric and would like, given nonsatiation in consumption, to use Ponzi schemes to run up “unbounded” terminal indebtedness (cf. equation (5)).¹⁵

The optimal consumption strategy under these assumptions is to consume the annuity value of the initial endowment: $c_t = re_0$ for all t . This yields a steady state equilibrium with storage per capita equal to e_0 . Aggregate wealth will be growing at rate n , however. If $n \geq r$ in the economy without debt, then the competitive equilibrium is consumption inefficient, and a rational Ponzi game can exist in equilibrium. A small amount of foreign (or domestic) bonds could be issued in the current period, crowding out an equal amount of storage. Agents holding the original debt would be willing to roll over the principal, and the interest payments could be raised by selling new debt to the newcomers.

Consumption inefficient equilibria and rational Ponzi games can also exist in more general infinite horizon economies with population growth.¹⁶ These results mean that it is perfectly possible for rational Ponzi games to exist in economies with infinitely lived agents, as long as there are an infinite number of independent decision makers over time. This also means that rational bubbles and intrinsically useless money can exist in these economies. It is the infinity of agents, not the length of individuals’ horizons—birth rather than death—that opens up the possibility of dynamic inefficiency and, thus, of rational Ponzi games and related phenomena.¹⁷

¹⁴ We restrict attention to economies with a constant returns to scale storage technology to avoid distinguishing between the interest rate before and after the introduction of debt. The two rates will turn out to be the same in the storage economy as long as the amount of initial debt issued does not exceed initial storage.

¹⁵ When Ponzi games are feasible, in other words, we assume that only governments can run them. See footnote 17 below.

¹⁶ If endowments grow at rate g , for example, and the time preference rate is not necessarily equal to the interest rate, a rational Ponzi game can exist with time separable constant relative risk aversion (CRRA) utility ($u = c^{1-A}/(1-A)$) if and only if $x < r \leq n + g$ and $n > 0$, where $(1+x) \equiv ((1+r)/(1+\delta))^{1/A}$. An appendix containing this result is in an earlier unpublished version of this paper (O’Connell and Zeldes 1986). Note that $r \leq n + g$ (although not $n > 0$) is also the condition for the existence of rational Ponzi games in a two period overlapping generations model with CRRA utility.

¹⁷ We have skirted a difficult issue by allowing only governments (domestic or foreign) to run rational Ponzi games. Theoretically, in a world in which rational Ponzi games can exist, *any* infinitely lived agent can issue debt and perpetually roll it over. (In principle, even a finitely lived agent can issue consols with zero coupon, i.e., fiat money.) All that is required is that other agents believe that new generations will be willing to purchase this agent’s debt. An analysis of the strategic issue of who gets to run a rational Ponzi game is beyond the scope of this paper.

4.3. *Rational Ponzi Games and Intergenerational Altruism.* Rational Ponzi games are clearly feasible in the standard overlapping generations model without intergenerational altruism. In a celebrated paper, however, Barro (1974) argued that if altruistically motivated bequests from parents to children were positive, these resource transfers would serve to reduce the behavior of the infinite sequence of households to that of a single infinitely lived dynasty. By our earlier arguments, this "consolidation" would appear to rule out rational Ponzi games even though such phenomena would be possible in the absence of altruism.

The bequest economy shows the importance of Assumption 2, that lenders care only about their consumption. As Gale (1983, pp. 55–61) has shown, Barro's period-by-period altruism is consistent with dynastic preferences that value not only the consumption of all generations, but also the limiting value of the discounted aggregate bequest. In a companion to this paper (O'Connell and Zeldes 1987), we show that rational Ponzi games cannot be ruled out with these preferences. The intuition is simple: consumption efficiency is no longer a necessary condition for optimality if preferences do not satisfy Assumption (2). The dynasty's taste for the terminal bequest may lead it to reduce consumption in all periods in order to accumulate assets to be passed on from generation to generation.

5. FINITE HORIZONS: BUBBLES, MONEY, PONZI GAMES, AND MULTIPLE EQUILIBRIA

In this section, we investigate the relationships between Ponzi games, bubbles, and fiat money using Diamond's (1965) neoclassical overlapping generations model. Tirole (1985) has used the model to demonstrate that asset price bubbles and intrinsically useless fiat money serve identical roles when agents have finite horizons, and that the conditions for fiat money to be valued are the same as the conditions for the existence of bubbles. The main point of this section is that Ponzi games play basically the same role as asset price bubbles or intrinsically useless fiat money. We show that what is generally relevant for characterizing the equilibrium is the *sum* of the initial values of bubbles, money, and debt, as opposed to each separate value. This implies a strong irrelevancy result: open market operations between Ponzi game debt and money will generally leave the set of real and nominal equilibria unchanged.

We then go on, however, to explain an important difference between Ponzi games and bubbles. It is well known that multiple equilibria can exist in models in which bubbles are feasible. Given an initial quantity of debt, however, the payment stream associated with debt implies a floor and sometimes a ceiling on the value of the outstanding debt. These added conditions restrict the set of equilibria with Ponzi games to a subset (sometimes empty) of those implied by Tirole's (1985) analysis of bubbles and money. In some cases, the set of equilibria is reduced to a single equilibrium that can be chosen by the government. These extra restrictions explain differences between models with money that serves strictly as a store of value and models with debt (e.g., Dornbusch 1985).

At the end of this section, we briefly describe the correspondence between money and debt when the quantity of money is falling. A falling money stock is

equivalent to a debt whose coupons are paid not by issuing new debt, but instead by raising future taxes. We show that there can be situations where we can get monetary equilibria (with a falling money stock) but not Ponzi game equilibria.

Finally, we describe why the “bubble”/“fundamental” distinction is not economically relevant for the case of infinitely lived debt that constitutes a rational Ponzi game.

5.1. Why Ponzi Games and Bubbles are Basically the Same. Consider a government that runs a rational Ponzi game by issuing consols with constant real coupon R , and then selling additional (identical) consols in all subsequent periods to finance all required coupon payments. Rational Ponzi games can also be run with consols with smaller coupons (and correspondingly greater price appreciation). Given this, the basic similarity between rational Ponzi games and fiat money stems simply from the fact that intrinsically useless fiat money is formally identical to an infinitely lived government consol with a coupon of zero. To establish the point that perpetually rolled over debt plays the same role in an economy as bubbles and fiat money, we show that there exist real equilibria with Ponzi games that mimic most real equilibria with bubbles and/or fiat money.

To begin, assume no government debt. Tirole (1985) shows that if the asymptotic interest rate in the “bubbleless” Diamond economy (the economy in which fiat money is valued at zero and there are no other bubbles) is less than the population growth rate—i.e., if the bubbleless economy is dynamically inefficient—then there also exists a continuum of equilibria with money and/or bubbles, indexed by B_0 ($0 < B_0 \leq B_{0 \max}$), in which the total initial value of money and bubbles is equal to B_0 .¹⁸ If B_0 is equal to $B_{0 \max}$, then the equilibrium is dynamically efficient, and the per capita bubble is constant over time. If B_0 is less than $B_{0 \max}$, the equilibrium is inefficient and the per capita bubble shrinks over time toward zero, (although it never actually hits zero).¹⁹

To extend this to include government debt, define X_0^M as the initial nominal

¹⁸ The intuition is the following. Introducing a bubble (or Ponzi game) must raise the asymptotic interest rate (this requires an assumption about preferences and technology; see Diamond 1965, Tirole 1985, or Weil 1987), and the change in the interest rate will be continuous in the initial size of the bubble. Therefore if $r < n$ in the bubbleless economy, there exist initial bubbles which would imply $r < n$ in the bubbly economy. The bubble would grow at rate r and the desired aggregate wealth by n (assuming a steady state), and therefore a bubble and/or rational Ponzi game would be feasible.

If endowments (or wages) are increasing over time (e.g., due to technological progress), then aggregate wealth will be growing faster than rate n , and therefore Ponzi games will be feasible for a wider range of initial r . As an example, take a storage/endowment economy with endowments growing at rate g . If saving of the young is proportional to lifetime wealth (as it would be if agents have CRRA utility functions), then the condition for the feasibility of rational Ponzi games would be $r \leq n + g$. If saving is log linear in wealth, but not proportional, the condition would be $r \leq (n + \alpha g)$, where α is the elasticity of saving with respect to the endowment. In other words, the rate of per capita income growth need not enter with a coefficient of one in the feasibility condition.

¹⁹ Tirole (1985) shows that bubbles can be ruled out if there exists an asset whose rents grow at the rate of growth of output. The reason is that the existence of such an asset rules out dynamically inefficient equilibria: the asset’s fundamental (and thus its price) is infinite unless the interest rate exceeds the rate of growth of output. McCallum (1986) argues that land is such an asset. The

quantity of money, P_0^M as the price of a unit of money in terms of goods, and $B_0^M = X_0^M \cdot P_0^M$ as the initial value of the fiat money or bubble. Similarly, define X_0^D , P_0^D , and $B_0^D \equiv X_0^D \cdot P_0^D$ as the initial quantity, price and value of the government debt. Government bonds yield payoffs in terms of goods, and we consider both finitely and infinitely lived bonds. For simplicity, the finitely lived bond will be a one period pure discount bond that is issued at time t , pays one good at time $t + 1$, and disappears at the end of period $t + 1$ if not redeemed. The infinitely lived bond is a consol that pays R goods per period forever. Define B_0 now as the sum of the initial value of debt and bubble/money ($B_0 \equiv B_0^D + B_0^M$). This brings us to the following proposition:

PROPOSITION 3. *Let X_0^D be the initial quantity of government debt that is to be perpetually rolled over. The set of equilibria is indexed by B_0 and consists of any choices of B_0^M and $B_0^D \equiv X_0^D \cdot P_0^D$ satisfying:*

$$(6) \quad 0 \leq B_0 \leq B_{0 \max}$$

$$(7) \quad P_0^D \begin{cases} \geq \text{PDV (cash flows) for infinitely lived assets} \\ = \text{PDV (cash flows) for finitely lived assets,} \end{cases}$$

where the upper limit $B_{0 \max}$ in (6) is identical to the limit in Tirole (1985).

The proof of the proposition is in the Appendix. Two observations are worth making here. First, the key step in the proof is to show that when all payments associated with debt are made by issuing new debt at the current market price, (and no *extra* new debt is issued), the value of the total debt outstanding ($X_t^D \cdot P_t^D$) must rise at the rate of interest. This is true no matter what type of debt is used and whether or not the debt is priced at its fundamental. Second, while finitely lived debt must be priced at its fundamental, infinitely lived debt can be priced at or above its fundamental. This feasibility of bubbles on infinitely lived debt opens up the possibility of multiple equilibria for a given initial quantity of debt.²⁰

Proposition 3 tells us that in an economy in which there are equilibria with (intrinsically useless) valued fiat money, rational Ponzi games are feasible and can be used to replicate *any* monetary equilibrium. The proposition also makes it

intuition is the following: if land is a factor in fixed supply in a Cobb-Douglas production function, the rent on land (the marginal product of land) must grow exactly at the rate of growth of output. We should point out, however, that not all economies in which land is productive exhibit this property. For example, if land enters the production function additively separably from the other inputs, then the rent on land can grow more slowly than output. In this case, the interest rate could be less than the growth rate of output with a finite price of land, and bubbles (and rational Ponzi games) cannot be ruled out.

²⁰ Tirole (1985) emphasizes that the durability of the asset is a basic condition for a price bubble. We assume that the finitely lived bond disappears in order to ensure that it does not continue to circulate forever even though there are no additional promised payments associated with it. Our assumption that bond payments are specified in terms of goods serves a similar purpose of terminating the effective maturity of finitely lived bonds. If payments were made in terms of new bonds, for example, then even finitely lived bonds would give rise to claims infinitely far into the future. Bubbles could then exist on these finitely lived assets.

clear that in any equilibrium with both rational Ponzi game debt and fiat money, only the *sum* of the value of the initial money stock (or bubble) and Ponzi game is relevant for the equilibrium. This leads directly to a strong irrelevancy result on government open market operations between money and (Ponzi game) bonds.

PROPOSITION 4. *Take any equilibrium in a model with valued fiat money. If the government issues new money and uses it to purchase outstanding Ponzi game debt, or issues new (finitely lived) debt and uses it to purchase outstanding money, there exists a new equilibrium with exactly the same real allocation and price path as the initial equilibrium.*²¹

The proof of Proposition 4 is in the Appendix.²² Note that while the irrelevancy holds for Ponzi game debt, it will not in general hold for debt that is *not* perpetually rolled over. Proposition 4 highlights the incompleteness of these models as models of money: the asset labeled “money” here performs exactly the same role as real government bonds.

5.2. *Differences between Ponzi Game Equilibria and Bubble Equilibria.* Up to this point, we have stressed the basic similarity between bubbles and Ponzi games. We now investigate some subtle differences between the two. We do this by comparing economies with only money to economies in which rational Ponzi games exist, but money (and asset bubbles) do not.

In a world with only money, there exist a continuum of equilibria (indexed by B_0), and there is no way to determine which equilibrium the economy will choose. In a world with Ponzi games and no money, condition 7 restricts the set of equilibria to a subset of the feasible equilibria with money. With finitely lived debt, in fact, the government can select a single equilibrium from the set of feasible equilibria with money.

PROPOSITION 5. *Any monetary equilibrium can be replicated by a Ponzi game equilibrium with finitely lived debt contracts. There is a unique initial quantity (X_0^P) of debt that will replicate a given monetary equilibrium.*

Proposition 5 (proved in the Appendix) follows from the fact that finitely lived bonds must be priced at their fundamental. Given this, the government can set the initial value of the Ponzi game uniquely by choosing the initial quantity of bonds.

In the case of infinitely lived debt, in contrast, we cannot tie down the equilib-

²¹ We qualify part of the proposition by referring to finitely lived Ponzi game debt. The reason (as seen in Proposition 6) is that there may be some cases where Ponzi games cannot be run with positive coupon consols, and introducing these consols to a model that did not previously include them drives out money and changes the real equilibrium.

²² Wallace (1980, 1981) proved an irrelevance result regarding exchanges by the government between physical storage with stochastic real returns and money, when the difference in government proceeds in subsequent periods is rebated to the public. His result followed because individuals see through the government veil in the same way that individuals see through the corporate veil in the Modigliani-Miller theorem. Our result is simpler, and follows directly from the observation that money and Ponzi game debt are perfect substitutes in individuals' portfolios in our model.

rium exactly, and in general, there still remains a multiplicity of equilibria with consols. But a difference with money remains: given an initial quantity of consols, we can now put a floor on the value of the Ponzi game, because the bonds must be priced at or above their fundamental. This rules out equilibria with low values of B_0 . In addition, by choosing a large enough quantity of consols, the government can raise the floor up to $B_{0 \max}$ and thereby choose the efficient equilibrium, ruling out other equilibria that could not be ruled out if the government just issued money.

PROPOSITION 6. *In any monetary equilibrium, let $\{\Gamma^*(t)\}$ be the equilibrium sequence of discount factors, $B_0^{M^*}$ the initial value of the money stock and $P_0^{DF^*}$ the fundamental (at the interest rate sequence in the monetary equilibrium) associated with a consol paying R . Then*

- (i) *If $\sum_{t=1}^{\infty} \Gamma^*(t) < \infty$ (which implies a finite fundamental for the consol), then one can construct an identical Ponzi game equilibrium for any finite coupon $R > 0$. In fact, for any coupon, there exists a range of initial quantities of debt ($0 < X_0^D \leq B_0^{M^*}/P_0^{DF^*}$) that can support this equilibrium.*
- (ii) *If $\sum_{t=1}^{\infty} \Gamma^*(t) = \infty$, then there does not exist a Ponzi game equilibrium with positive coupon consols.*

An example of the exception in the second part of Proposition 6 is the simplest "Samuelson case" of the pure exchange overlapping generations model with no population growth (Gale 1973). This economy has a stationary monetary equilibrium (the Golden Rule equilibrium) with $r = 0$. This equilibrium can not be replicated with positive coupon consols that are rolled over forever, because the consols in such an equilibrium would have to have infinite value. However, if the population growth rate were positive, then the Golden Rule equilibrium could be achieved with a Ponzi game equilibrium with positive coupon consols.²³

5.3. Extensions and Related Issues. In this section, we make three additional points. First, the existence of valued fiat money or an alternative asset with a bubble does not rule out Ponzi games, and it does not imply that the introduction of Ponzi games has to crowd out these other stores of value. In particular, in an economy with fiat money initially valued somewhere between 0 and $B_{0 \max}$, there is room for the government to issue new debt of value not exceeding $B_{0 \max} - B_0$, and never raise taxes, either implicitly (by causing an unanticipated decrease in the real value of outstanding fiat money) or explicitly.²⁴

Second, while rational Ponzi games can be used to replicate equilibria with

²³ Dornbusch (1985) uses just such an economy. If he were instead to allow for positive population growth, then consols would not in general be priced at their fundamental, and there would exist equilibria in this economy with different stocks of bonds but identical total values of bonds outstanding. Changes in endowments (and therefore asset demands) need not produce changes in interest rate sequences when the value of bonds is indeterminate. These possibilities are absent in Dornbusch's paper.

²⁴ Hamilton and Michener (1986) rule out this possibility by assuming that the economy is always at $B_{0 \max}$.

growing nominal money stocks, the analog to a contracting money stock is national debt that does not constitute a Ponzi game. The intuition is the following. Suppose that the government regularly collects lump sum taxes and uses the proceeds to buy back money, such that the nominal money stock shrinks at rate μ . (Wallace 1980 shows that there exist monetary equilibria in this case even if $r > n$, as long as $r - \mu \leq n$.) The analogy with debt is if the government were to pay off μ percent of outstanding debt each period with taxes, and roll over the rest. In this case, however, the present value of the taxes collected is independent of μ and exactly equals the value of the initial debt. This implies $\lim_{T \rightarrow \infty} \Gamma(T)B_T^D = 0$, so that the debt is not a Ponzi game.

Third, the distinction between fundamentals and bubbles does not have economic significance when debt is perpetually rolled over. In particular, any Ponzi game equilibrium with consols priced above their fundamental can be replicated exactly using a larger initial number of consols, each priced at its fundamental. When debt is perpetually rolled over, the coupon payments that enter into the calculation of the fundamental are in fact nothing more than new bubbles from a macro perspective.²⁵

6. INTERNATIONAL LENDING

The debt servicing difficulties of developing countries (LDCs) have attracted a good deal of attention from economists and policymakers. In this section, we use our results to discuss the feasibility of perpetual rollover of Third World debt.

Up to this point in the paper, we have assumed that all contracts are honored. Niehans (1985) points out that this assumption presupposes a legal enforcement structure or other source of default costs that may be largely absent in international credit markets. When default costs are zero and debt claims are unenforceable, Niehans argues, a country (under perfect foresight) will repudiate all existing debts as soon as the present value of net required payments to lenders is positive for all future horizons.²⁶ This shrinks the set of borrowing strategies to rational Ponzi games. In other words, the only way that international borrowing can exist in this world is if neither interest nor principal will ever be repaid.

In discussing the policy implications of his work, Niehans offers "strong advice for the lending banks: do not acquire unenforceable claims unless the borrower's rate of economic growth exceeds the rate of interest" (1985, p. 76). The implied condition, that the ratio of a country's external debt to its GNP not grow without bound, is often encountered in studies of government finance that emphasize steady states (Diamond 1965; Anderson, Ando, and Enzler 1984). When rational Ponzi games are ruled out, the condition has some plausibility whether or not the economy is in a steady state. Our main point in this section, however,

²⁵ This point is closely related to Tirole's (1985, p. 1091) discussion of bubble accounting.

²⁶ In other words, borrowers calculate present values of future cash flows for all possible horizons and plan to default in the period for which the present value, viewed from today, is at a maximum. This rule implies repudiation today if and only if there is no remaining horizon for which the PDV of cash flows is positive.

is that conditions in the borrower's economy are irrelevant to the feasibility of Ponzi game equilibria. As our analysis has shown, the feasibility of perpetual rollover of debt depends entirely on conditions in the *lenders'* economy.²⁷ To ensure the existence of an equilibrium in which U.S. lenders forever hold Argentinian debt that is growing at rate r , we need the growth rate of the U.S. (not the Argentine) economy to exceed r .

While creditworthiness indicators cannot help determine whether or not Ponzi game equilibria exist, they may nonetheless have a role to play even when borrowing is never repaid. There are two reasons for this. First, creditworthiness indicators may serve as focal points in sustaining the expectations of current lenders about the behavior of future lenders. Whenever a Ponzi game equilibrium exists, there also exists a perfect foresight equilibrium in which there is no Ponzi game because each generation believes that future generations will not continue lending. Niehans' statement can therefore be interpreted as proposing a convention that might serve to sustain the expectations—and the lending—of lenders. However, in a world in which loan contracts are completely unenforceable, conventions of this sort are no different than sunspots: they can affect equilibria merely because agents believe they do.

A second interesting role for creditworthiness indicators emerges if we examine the case in which loan contracts *are* enforceable. Suppose that conditions in the lenders' economy are such that Ponzi games are feasible. Now suppose that lenders know that they can recover the full value of debt service from the borrower if necessary. It follows that the borrower's debt must be valued at least at the present value of these payments, and therefore that the non-Ponzi game equilibrium is ruled out. In other words, the ability to repay debt ensures in this case that the country will never have to carry through and repay!

7. CONCLUSION

Since we summarized the results in the introduction, we will not do so again here. Instead, we briefly discuss some questions that have been left unanswered in the paper.

A government that can run a Ponzi game can cut taxes today without ever having to raise them in the future. Such a policy represents an increase in net wealth for the private sector, and one might therefore expect Ricardian equivalence to fail with respect to tax cuts financed by Ponzi games. We address this issue in a companion paper (O'Connell and Zeldes 1987) and find, surprisingly, that Ricardian equivalence is generally (although not always) unaffected by whether or not future taxes are increased.

A second issue is what happens if more than one individual or government tries to run a rational Ponzi game. The issues involved here also appear in

²⁷ Cohen (1985) proposes as a measure of solvency the constant fraction of a country's exports that must be devoted to debt service forever in order to repay current debt. This "index of solvency" is a decreasing function of the difference between the growth rate of exports in the borrower's economy and the interest rate. While this index is appropriate when debt must be repaid it is irrelevant to the question of whether lenders will in fact *require* repayment.

models of fiat money, where they are resolved by assuming that the government has a legal monopoly on the creation of currency. This assumption is unsatisfactory, however, in a world where government obligations (e.g., sovereign debt of the LDCs) cross national boundaries easily. If the U.S. government can run a rational Ponzi game in the U.S., what prevents the government of Argentina or Nigeria from running its own Ponzi game in the U.S.?

A final set of issues relates to the empirical relevance of rational Ponzi games. We often observe situations where the risk-free real interest rate is below the sum of the population growth rate and the growth of per capital income. For Ponzi games to exist, however, this condition must continue to hold infinitely far into the future. How informative is the analysis in this paper in a world in which the future is intrinsically uncertain? To answer this requires that we drop the perfect foresight assumption and model uncertainty explicitly. Weil (1987) has examined the conditions under which stochastic bubbles can exist in an otherwise non-stochastic real economy. This analysis applies, with minor modifications, to Ponzi games with infinitely lived debt. With finitely lived debt, however, the value of the debt is tied down and, therefore, these issues do not arise. A natural extension would be to introduce some source of uncertainty into the real economy of the lenders (e.g., uncertain horizon, or shocks to the storage technology, income, or population growth rates). One result we anticipate is an inability to roll over debt completely in some states of the world, so that conditions in the borrower's economy would become relevant to the feasibility of rational Ponzi games.

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APPENDIX

PROOF OF PROPOSITIONS 3–6

PROOF OF PROPOSITION 3. Under perfect foresight, arbitrage brings the one period holding return on all assets into equality, implying

$$(A1) \quad (P_{t+1} + R)/P_t = 1 + r_{t+1},$$

where P_{t+1} is the price of the government bond at time $t + 1$ (after the time $t + 1$ coupon is paid) and r_{t+1} is the equilibrium interest rate between t and $t + 1$ in the economy with the government debt (recall that introduction of the debt can change the interest rate).

For a one period discount bond, the coupon is one, and P_{t+1} is zero since the bond disappears at the end of $t + 1$. Equation (A1) therefore implies that the price at t is determinate and given by:

$$(A2) \quad P_t = 1/(1 + r_{t+1}).$$

For a consol, equation (A1) gives a difference equation for the price sequence $\{P_t\}$. There are multiple solutions, all but one of which are "bubble" price paths for the asset. Free disposal of the asset, however, rules out negative bubbles,

because a negative bubble would eventually imply a negative price of the asset (Tirole 1985). The price of a consol must therefore be at least as great as its fundamental, P_t^F :

$$(A3) \quad P_t \geq \sum_{s=1}^{\infty} [\Gamma(t+s)/\Gamma(t)]R \equiv P_t^F.$$

Equations (A2) and (A3) give us condition (7) in Proposition 3.

Define X_t as the quantity of bonds outstanding at time t . Let the government roll over all interest and principal on the bonds by issuing new bonds. Since one period bonds pay a single unit of goods each, the quantity X_{t+1} of new bonds must satisfy $X_{t+1} = X_t \cdot 1/P_{t+1}$. In the case of consols, the increment in bonds must be sufficient to pay the coupon R on existing bonds; i.e., $(X_{t+1} - X_t) \cdot P_{t+1} = X_t R$. Using these equations for bond quantities, and equations (A1) and (A2) for prices, it follows that in both the discount bond and consol case, the total value of bonds satisfies

$$(A4) \quad P_{t+1}X_{t+1} = (1 + r_{t+1})P_tX_t.$$

The aggregate Ponzi game therefore grows at the rate of interest, independent of R , P , and the maturity of the bonds.

Tirole's (1985) asset bubbles have only two economically relevant characteristics: (1) they grow at the rate of interest, and (2) they bring no net output into the economy. These two properties, plus the restrictions in condition (7), fully characterize rational Ponzi games. Therefore, for the case where both bubbles and rational Ponzi games exist simultaneously, the set of equilibria is characterized by Tirole's conditions applied to the *sum* of the initial bubble and initial Ponzi game, plus equation (7). Q.E.D.

For the proofs of Propositions 4, 5, and 6, variables denoted by * are equilibrium values in the monetary economy.

PROOF OF PROPOSITION 4. From Proposition 3, equilibria are indexed by the value of $B_0 = B_0^D + B_0^M$. Since any existing debt must have finite value and any new debt is assumed to be finitely lived and therefore must be of finite value, trading an equal value of money for Ponzi game debt (or vice-versa) will leave B_0 unchanged at the original sequence of interest rates. Therefore the original interest rate sequence continues to constitute an equilibrium. Q.E.D.

PROOF OF PROPOSITION 5. For one period debt, $P_0^D = 1/(1 + r_1) < \infty$, by equation (A2). For debt of maturity $T < \infty$, $P_{T+1}^D = 0$, and equation (A1) therefore implies that the bond price P_0 must exactly equal the fundamental. Choose $X_0^D = B_0^*/P_0^D$. This implies $X_0^D \cdot P_0^D = B_0^D = B_0^*$. Q.E.D.

PROOF OF PROPOSITION 6, PART (i). $P_0^{DF*} = \sum_{t=1}^{\infty} \Gamma^*(t)R < \infty$. Choose X_0 such that $0 < X_0 \leq B_0^*/P_0^{DF*}$, so $0 < X_0 P_0^{DF*} \leq B_0^*$. Therefore, there exists a $P_0 \geq P_0^{DF*}$ such that $X_0^D P_0^D = B_0^*$. Since $P_0 \geq P_0^{DF*}$, it is feasible. Q.E.D.

PROOF OF PROPOSITION 6, PART (ii). For any $R > 0$, $P_0^{DF*} = \infty$, and $P_0^D \geq P_0^{DF*}$. If positive coupon consols existed in this economy, they would be infinitely priced, regardless of the coupon size. Therefore there is no quantity of consols X_0^D small enough such that the initial value of consols $X_0^D \cdot P_0^D$ could be equal to B_0^* .

Q.E.D.

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