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Comparative statics of monopoly pricing

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Summary. When consumers' willingness-to-pay increases by a uniform amount, the change in the resulting monopoly price is generally indeterminate. Our analysis identifies sufficient conditions on the underlying demand curve which predict both the sign and the magnitude of the resulting price change.

Keywords and Phrases: Monopoly pricing, Comparative statics, Log-concavity.

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1 Introduction

This note seeks to fill a small but significant gap in the literature on monopoly pricing. We ask how an expansion of market demand affects the resulting monopoly price. Specifically, if consumers' (aggregate) willingness-to-pay increases uniformly by some amount, will the monopoly price increase, and, if so, by how much? It is readily verified that in case of a linear demand curve a uniform shift induces a price increase at the rate of one half of the size of the shift. On the other hand, the resulting monopoly price will decrease when such a demand shift is applied to a constant elasticity demand curve.[1] Our analysis identifies conditions on the underlying demand curve which predict both the sign and the magnitude of the resulting price change.

Our results are applicable to a range of issues in the industrial organization literature. In particular, these include changes in consumer preferences, a reduction of the monopolist's cost, or a lower excise tax for the good in question.

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[1] Quirmbach (1988) notes that in general the price effect of a positive demand change is ambiguous. See also Mas-Colell, Whinston and Green (1995, p.429).

Our results are also relevant for understanding investment incentives in vertically related firms.[2] When a downstream firm undertakes specific investments, it may increase its willingness-to-pay for an intermediate product supplied by an upstream firm. Yet, if the upstream firm has monopoly power, the investment incentives of the downstream firm are partly driven by the anticipated change in the price for the intermediate product.

2 The result

Consider the one-period monopoly problem with constant unit variable cost:

$$\max_p \{(p - c) \cdot D(p)\} . \tag{1}$$

The demand curve $D(\cdot)$ is assumed to be strictly decreasing and twice differentiable on some interval $[0, \overline{p}]$. We assume that the pricing problem in (1) has a solution p_0 in the interior of $[c, \overline{p}]$. Suppose now that consumers' willingness-to-pay increases by a constant Δ. If $P(q)$ denotes the original willingness-to-pay (i.e., $P(\cdot)$ is the inverse of $D(\cdot)$), then the resulting willingness-to-pay is $P(q)+\Delta$. Equivalently, the market demand curve becomes $D(p - \Delta)$ for $p \geq \Delta$, and the new pricing problem becomes

$$\max_p \{(p - c) \cdot D(p - \Delta)\} . \tag{2}$$

It will be technically convenient to restrict attention to values of Δ for which $0 \leq \Delta \leq c$ and to assume that, for all Δ, any solution, $p(\Delta)$, to the pricing problem in (2) is in the interior of $[c, \overline{p}]$.[3] In order to identify changes in the monopoly price as demand shifts upward, we consider arbitrary parameter values Δ_0 and Δ_1, such that $\Delta_1 > \Delta_0 \geq 0$.

Theorem. *For any values $\Delta_1 > \Delta_0 \geq 0$, suppose that $p(\Delta_1)$ and $p(\Delta_0)$, respectively, solve the monopoly pricing problem in (2). Then:*

(i) $p(\Delta_1) < p(\Delta_0) + (\Delta_1 - \Delta_0)$,
(ii) $p(\Delta_1) > p(\Delta_0)$ *if $P(\cdot)$ is log-concave,*[4]
(iii) $p(\Delta_1) \left\{ \begin{matrix} \geq \\ = \\ < \end{matrix} \right\} p(\Delta_0) + \frac{1}{2}(\Delta_1 - \Delta_0)$ *if* $P''(\cdot) \left\{ \begin{matrix} \leq \\ = \\ > \end{matrix} \right\} 0.$

Before giving the proof, we briefly discuss and interpret the results. Part (i) is known from earlier observations in the industrial organization literature, e.g. Tirole (1988, pp.66-67). A positive shift in demand (or a reduction in the monopolist's unit cost) must result in a larger quantity supplied to the market,

[2] See, for example, Williamson (1985), Hart and Moore (1988), Edlin and Reichelstein (1996), Baldenius, Reichelstein, and Sahay (1999).

[3] We do not require the pricing problem to have a unique solution.

[4] Log-concavity of $P(\cdot)$ is equivalent to $(P'(\cdot)/P(\cdot))$ being decreasing.

i.e., $q(\Delta_1) > q(\Delta_0)$. Since $p(\Delta_1) \equiv P(q(\Delta_1)) + \Delta_1$, it must be that $p(\Delta_1) - \Delta_1 < p(\Delta_0) - \Delta_0$. The sufficient condition in part (ii) can be restated as requiring the "relative" price elasticity of demand $\frac{\varepsilon(q)}{q}$, with $\varepsilon(q) \equiv -P'(q) \div \frac{P(q)}{q}$, to be increasing in q. Obviously, this condition is met for linear $P(q)$, but not for a constant elasticity curve.

Part (iii) provides bounds for the price change in terms of the second derivative of $P(\cdot)$. Setting $\Delta_0 = 0$ and $\Delta_1 = \Delta$, the result says that $p(\Delta) \geq p(0) + \frac{1}{2} \cdot \Delta$ if $P''(\cdot) \leq 0$, with the reverse inequality if $P''(\cdot) \geq 0$. If one makes the additional assumption that $p(\Delta)$ is differentiable, then the theorem can be restated as follows: part (i) says that $p'(\Delta) < 1$, while part (ii) states that $p'(\Delta) > 0$ if $P(\cdot)$ is log-concave, and according to part (iii) we have $p'(\Delta) \geq \frac{1}{2}$ if $P(\cdot)$ is concave, while $p'(\Delta) \leq \frac{1}{2}$ if $P(\cdot)$ is convex.[5]

Part (iii) of the theorem also speaks to a situation where demand remains unchanged but the monopolist's costs fall by Δ. The resulting price is then $P(q(\Delta))$ and since $P(q(\Delta)) = p(\Delta) - \Delta$, we find that the monopolist will pass on at least (no more than) half of the reduction in cost to consumers provided $P(\cdot)$ is concave (convex).

For another interpretation, suppose that demand and production cost remain unchanged, but Δ reflects lower payments to third parties, e.g., a lower excise tax on the good in question or reduced sales commissions (which the firms pay in proportion to the sales quantity). In the original situation, consumers pay $\tilde{P}(q_0)$, the government (or salespeople) receives a tax of $t \cdot q_0$, leaving the firm with a net-revenue of $p_0 \equiv \tilde{P}(q_0) - t$ per unit of sales. When the excise tax is lowered by \$ Δ, we conclude that the firm's unit revenue will increase provided that $\frac{\tilde{P}'(\cdot)}{\tilde{P}(\cdot) - t}$ is decreasing in q. Consumers will definitely pay a lower price since

$$\tilde{P}(q_0) - \tilde{P}(q(\Delta)) = p_0 + t - [p(\Delta) + (t - \Delta)] > 0,$$

by part (i). Again, concavity (convexity) of $P(\cdot)$ (or $\tilde{P}(\cdot)$) is a sufficient condition for this price decrease to be at least (at most) one half of the tax cut.

Appendix: Proof of the Theorem

(i) Let $q(\Delta_1)$ and $q(\Delta_0)$ denote optimal monopoly quantities for the two problems. Thus,

$$q(\Delta_1) \in \underset{q}{\operatorname{argmax}} \{(P(q) + \Delta_1 - c) \cdot q\} \ .$$

and

$$q(\Delta_0) \in \underset{q}{\operatorname{argmax}} \{(P(q) + \Delta_0 - c) \cdot q\}$$

A standard "revealed preference" argument shows that $q(\Delta_1) \geq q(\Delta_0)$. Given interior pricing solutions, it also follows from the first-order conditions that

[5] $p(\Delta)$ will be differentiable if one assumes that the pricing problem in (2) is strictly concave in p for all Δ (the Implicit Function Theorem then ensures that the unique maximizer is differentiable in Δ). We note that, with this additional assumption, the proof of part (iii) below can be shortened somewhat by simply differentiating the first-order conditions.

$q(\Delta_1) \neq q(\Delta_0)$. By definition, $p(\Delta_1) = P(q(\Delta_1)) + \Delta_1$ and $p(\Delta_0) = P(q(\Delta_0)) + \Delta_0$. Thus, $p(\Delta_1) - p(\Delta_0) = P(q(\Delta_1)) - P(q(\Delta_0)) + \Delta_1 - \Delta_0 < \Delta_1 - \Delta_0$.

(ii) We show that if $P(\cdot)$ is log-concave, then the function

$$\Gamma(p, \Delta) \equiv (p - c) \cdot D(p - \Delta)$$

has strictly increasing differences, i.e., $\partial^2 \Gamma / (\partial p \partial \Delta) > 0$ for all (p, Δ). As observed in Edlin and Shannon (1998), strictly increasing differences are a sufficient condition for $p(\Delta_1) > p(\Delta_0)$. We find that

$$\frac{\partial^2 \Gamma}{\partial p \partial \Delta} = -D''(p - \Delta) \cdot (p - c) - D'(p - \Delta) \ .$$

Clearly, $\partial^2 \Gamma / (\partial p \partial \Delta) > 0$ if $D''(p - \Delta) \leq 0$. Suppose that for some (p, Δ), $D''(p - \Delta) > 0$. In that case:

$$-D''(p - \Delta) \cdot (p - c) - D'(p - \Delta) \geq -D''(p - \Delta) \cdot (p - \Delta) - D'(p - \Delta), \quad (3)$$

since $\Delta \leq c$. By definition, $P(D(p - \Delta)) \equiv p - \Delta$, and therefore

$$D'(\cdot) \cdot P'(D(\cdot)) = 1,$$

and

$$D''(\cdot) \cdot P'(D(\cdot)) = -\frac{P''(D(\cdot))}{[P'(D(\cdot))]^2}. \quad (4)$$

Substitution into (3) then shows that the right-hand side of (3) is positive if and only if

$$[P'(D(p - \Delta))]^2 > P(D(p - \Delta)) \cdot P''(D(p - \Delta)),$$

which will be satisfied if $(P'(\cdot)/P(\cdot))$ is decreasing everywhere, or, equivalently, if $P(\cdot)$ is log-concave.

(iii) Suppose that $P''(\cdot) \leq 0$ and yet $p(\Delta_1) < p(\Delta_0) + \frac{1}{2}(\Delta_1 - \Delta_0)$. We will derive a contradiction. From (4) we know that $P''(\cdot) \leq 0$ implies $D''(\cdot) \leq 0$ since $P(\cdot)$ is strictly decreasing. The first-order condition for the optimality of $p(\Delta)$ is:

$$D'(p(\Delta_1) - \Delta_1) \cdot (p(\Delta_1) - c) + D(p(\Delta_1) - \Delta_1) = 0. \quad (5)$$

At the same time:

$$D'(p(\Delta_0) - \Delta_0) \cdot (p(\Delta_0) - c) + D(p(\Delta_0) - \Delta_0) = 0.$$

Combining these equations yields

$$\int_0^{p(\Delta_1)-c} D'(p(\Delta_1) - \Delta_1) du = \int_{p(\Delta_1)-\Delta_1}^{p(\Delta_0)-\Delta_0} D'(u) du + \int_0^{p(\Delta_0)-c} D'(p(\Delta_0) - \Delta_0) du. \quad (6)$$

By hypothesis, $p(\Delta_1) < p(\Delta_0) + \frac{1}{2}(\Delta_1 - \Delta_0)$, and therefore:

$$p(\Delta_1) - c < (p(\Delta_0) - \Delta_0) - (p(\Delta_1) - \Delta_1) + p(\Delta_0) - c \ .$$

We recall from part (i) that $p(\Delta_1) \leq p(\Delta_0) + (\Delta_1 - \Delta_0)$. Since $D'(\cdot) < 0$ we obtain the following bound for the left-hand side of (6):

$$\int_0^{p(\Delta_1)-c} D'(p(\Delta_1) - \Delta_1)du \quad > \quad \int_{p(\Delta_1)-\Delta_1}^{p(\Delta_0)-\Delta_0} D'(p(\Delta_1) - \Delta_1)du + \int_0^{p(\Delta_0)-c} D'(p(\Delta_1) - \Delta_1)du. \quad (7)$$

Finally, $D'(\cdot)$ is decreasing because $D''(\cdot) \leq 0$, implying that:

$$D'(p(\Delta_1) - \Delta_1) \geq D'(u) \quad \text{for} \quad u \in (p(\Delta_1) - \Delta_1, p(\Delta_0) - \Delta_0).$$

and

$$D'(p(\Delta_1) - \Delta_1) \geq D'(p(\Delta_0) - \Delta_0) \ .$$

Substituting these inequalities into the right-hand side of (7), we obtain a contradiction with equality (6), thereby completing the proof of the Theorem.

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