



A Synthesis of Equity Valuation Techniques and the Terminal Value Calculation for the Dividend Discount Model

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Abstract. This paper lays out alternative equity valuation models that involve forecasting for finite periods and shows how they are related to each other. It contrasts dividend discounting models, discounted cash flow models, and “residual income” models based on accrual accounting. It shows that some models that are apparently different yield the same valuation. It gives the general form of the terminal value calculation in these models and shows how this calculation serves to correct errors in the model. It also shows that all models can be interpreted as providing a particular specification of the terminal value for the dividend discount model. In so doing it shows how one calculates the terminal value for the dividend discount formula. The calculation involves weighting forecasted stocks and flows of value with weights determined by a parameter that can be discovered from pro forma analysis.

There are a variety of equity valuation techniques used in practice and discriminating among them is difficult. Many involve forecasting the future but they differ as to what is to be forecasted. Some forecast dividends, some forecast cash flows, some forecast earnings or residual income, and some forecast operating profit. Most techniques are based on a particular valuation model—the dividend discount model, the discounted cash flow model, and the residual income model, for example—and these models work for going concerns if the analyst forecasts an infinite stream. But in practice forecasts are made for a finite number of years and this truncation of the forecast horizon typically requires a “terminal value” or “continuing value” calculation at the horizon. Terminal values often have a significant effect on the valuation but their calculation is sometimes ad hoc or relies on doubtful assumptions. The variety of techniques and the variety of terminal value prescriptions is more than a little confusing and competing claims as to the superiority of particular techniques—by academics and vendors—are hard to sort out. As the various techniques yield the same valuation with infinite horizon forecasting, it is the calculation of the terminal value that is problematic.

This paper brings some order to finite-horizon valuation by formulating many of the popular techniques on a common basis. It presents a unifying framework in which each technique is shown to be a particular application of a general finite-horizon equity valuation model. The synthesis yields the following.

First, the generic terminal value calculation for all models is provided. All appropriate terminal value calculations are special cases of this general form and ad hoc calculations are identified as those that are inconsistent with it.

Second, differences between valuation techniques are identified. A number of equivalences are also established and models that are seemingly different are shown to be the same.

Third, the paper shows that all the common techniques can be recast as the dividend

discount model with a particular terminal value calculation. Accordingly the paper supplies the terminal value calculation for the dividend discount model. This is somewhat ironic because discounted cash flow and residual income techniques (for example) have been proposed to get over the difficulty of determining a terminal value for the dividend discount model. Dividend discounting, it is said, does not work for finite horizons so something more “fundamental” (like cash flow or earnings) must be forecasted instead of dividends. The analysis here shows that the substitution of other attributes for dividends amounts to calculating the terminal value for the dividend discount model. Indeed the dividend discount model is the umbrella model over the other models and they are compared in terms of their prescriptions for the terminal value for the dividend discount model.

The terminal value calculation amounts to forecasting future accounting stocks and flows and weighting them to yield one number. The weights are determined by a parameter that reflects the accounting principles for measuring stocks and flows. This parameter, along with the forecasted stocks and flows to be combined, is discovered from pro forma analysis.

The terminal value calculation serves to correct the error introduced by truncating the forecast horizon. This error arises of course because forecasts of payoffs beyond the horizon are omitted in the truncation. But the error is also due to forecasts of payoffs up to the horizon. If forecasts of payoffs to the horizon capture the value without error there is no need for a terminal value. If they do not, a terminal value is needed to correct the error. The error in the forecasts to the horizon depends on what is forecasted and how it is measured. These are issues of recognition and measurement familiar to accounting. So the paper characterizes forecasting and valuation as a matter of specifying pro forma accounting (for the future). Valuation errors are due to the rules for the pro forma accounting. The paper identifies the form of the accounting measurement error that gives rise to the terminal value calculation, the type of accounting error that can be tolerated in finite horizon valuation, and the type of error that cannot be tolerated. It identifies cases where the accounting is ideal such that a terminal value is not required and shows explicitly how accounting error is handled in the accounting for the terminal value.

Section 1 lays out the familiar dividend discount model and points out the reasons why this model, as usually formulated, is difficult to apply over finite horizons. Section 2 summarizes recognized approaches that forecast attributes other than dividends and unifies them in a general formulation. This formulation provides the generic terminal value calculation for these approaches. Section 3 then shows that these models are isomorphic to the finite-horizon dividend discount model and draws a number of implications from this. More detail on the insights from the paper is given in the conclusion in Section 4.

1. Dividend Discount Calculations of Equity Value

The valuation of equity is assumed to be based on expected dividend payoffs. Formally, intrinsic equity price at time t is

$$P_t = \sum_{\tau=1}^{\infty} \rho^{-\tau} E_t(\tilde{d}_{t+\tau}), \quad (1)$$

with the standard convergence condition for a finite price. The payoffs, $d_{t+\tau}$, are dividends at the end of each future period, $t + \tau$, and E_t indicates an expectation conditional upon information at time t . Firm subscripts are understood. The specification of (one plus) the discount rate (cost of equity capital), ρ , is not at issue here and it is represented as non-stochastic and flat. Varying stochastic discount rates and the form in Rubinstein (1976) (where risk is incorporated as an adjustment to expectations in the numerator) can be accommodated in this and subsequent expressions, as in Feltham and Ohlson (1994). Although ad hoc in the accommodation of risk, most practical valuation techniques apply a risk-adjusted discount rate to expectations, as in (1), and our aim is to examine techniques used in practice.

For going concerns this formula requires forecasting dividends “to infinity” but in practice finite forecast horizons are entertained. This clearly presents implementation problems. For a finite $t + T$,

$$P_t = \rho^{-T} \left[\sum_{\tau=1}^T \rho^{T-\tau} E_t(\tilde{d}_{t+\tau}) + E_t(\tilde{P}_{t+T}) \right], \quad (\text{NA})(1a)$$

which states the no arbitrage condition (NA) implied by (1). (The expected terminal value of dividends and the expected $t + T$ price within the outside brackets are referred to as the expected cum-dividend price at $t + T$, which is denoted as $E_t(\tilde{P}_{t+T}^C)$ in the paper.) This is not very helpful. First, circularity is involved because no direction is given for the determination of $E_t(\tilde{P}_{t+T})$ except the no arbitrage condition with respect to P_t , and P_t is the subject of investigation. Second, given the Miller and Modigliani (“M&M”) (1961) dividend irrelevance proposition, forecasted dividends from t to $t + T$ are irrelevant to P_t : equation (1a) holds for all payoff patterns to $t + T$ as changes in expected dividends just displace $E_t(\tilde{P}_{t+T})$ (and dividends subsequent to $t + T$) by the same present value amount. While payout tied to value generation is one feasible dividend policy, observation indicates that actual payouts are not typically so disciplined.

To avoid the circularity problem one requires a “terminal value” determination, TV_{t+T} , calculated independently of prices such that the calculated price,

$$\begin{aligned} P_t^T &\equiv \rho^{-T} \left[\sum_{\tau=1}^T \rho^{T-\tau} E_t(\tilde{d}_{t+\tau}) + E_t(T\tilde{V}_{t+T}) \right] \\ &= P_t \end{aligned} \quad (1b)$$

(Here, as elsewhere in the paper, “ \equiv ” reads “calculated as.”) By a comparison of (1a) and (1b) it is clear that the appropriate terminal value is a calculation of the expected $t + T$ price. If one accepts M&M and wishes to calculate a valuation that is payout irrelevant, the terminal value must be reduced by the terminal value of dividends at $t + T$, one for one, such that P_t^T is insensitive to it. The calculation of this terminal value is the focus of the paper.

2. Accounting Calculations of Equity Value

Most of the popular techniques do not attempt to calculate a terminal value for the dividend discount model. Rather they embrace other valuation models that substitute alternative attributes for dividends as the forecast target. These are claimed to be “more fundamental” because, it is said, they relate to value-generating activities within the firm rather than value-irrelevant distributions from the firm. Typically these techniques forecast future stocks and/or flows resulting from these activities and specify how these stocks and flows are to be recognized and measured. This amounts to specifying a pro forma accounting for stocks and flows, so a valuation technique is really a pro forma accounting system. Here we lay out the alternative techniques and the accounting that governs them. We represent a measured stock of value at time t by B_t (“book value”) and a measured flow of value (“earnings”) from $t - 1$ to t as X_t . As coding for the synthesis, we indicate techniques in the same class by the same number, and variants of the class by an alphabetic letter appended to that number. We indicate statements of accounting principles with capital letters.

2.1. Residual Earnings Calculations

It has long been recognized¹ that, if one tracks stocks and flows that are related to dividends in a “clean surplus” accounting system such that, for all $\tau > 0$,

$$E_t(\tilde{B}_{t+\tau}) = E_t(\tilde{B}_{t+\tau-1} + \tilde{X}_{t+\tau} - \tilde{d}_{t+\tau}), \quad (\text{CSR})$$

then

$$W_t^T \equiv B_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau} - (\rho - 1)\tilde{B}_{t+\tau-1}) \quad (2)$$

$$\rightarrow P_t \text{ in (1) as } T \rightarrow \infty \text{ provided that } \rho^{-T} E_t(\tilde{B}_{t+T}) \rightarrow 0 \text{ as } T \rightarrow \infty.$$

The forecast target, $X_{t+\tau} - (\rho - 1)B_{t+\tau-1}$ is typically referred to as residual income, residual earnings or abnormal earnings. The “clean surplus relation” (CSR) states that one accounts for successive stocks by measuring flows and subtracting net dividends, and it is implicit that dividends are not involved in the calculation of flows. The expectation operator serves to indicate that the accounting is in pro forma: values are calculated from forecasted accounting numbers.

The calculation of W_t^T in (2) is from forecasts of residual income for a specific forecast horizon, $t + T$. The calculation is equivalent to the following:

$$W_t^T = B_t + \rho^{-T} \sum_{\tau=1}^T \rho^{T-\tau} E_t(\tilde{X}_{t+\tau} - (\rho - 1)\tilde{B}_{t+\tau-1}) \quad (2a)$$

$$W_t^T = \rho^{-T} \left[\sum_{t=1}^T \rho^{T-\tau} E_t(\tilde{d}_{t+\tau}) + E_t(\tilde{B}_{t+T}) \right] \quad (2b)$$

$$W_t^T = \rho^{-T} \left[B_t + E_t \left(\sum_{\tau=1}^T \tilde{X}_{t+\tau}^C \right) \right] \quad (2c)$$

where, $E_t(\sum_{\tau=1}^T \tilde{X}_{t+\tau}^C) \equiv E_t(\sum_{\tau=1}^T \tilde{X}_{t+\tau} + \sum_{\tau=1}^T (\rho^{T-\tau} - 1) \tilde{d}_{t+\tau})$. Calculation (2a) just recognizes that the present value of a stream is equal to the present value of the terminal value of the stream. The forecast target in (2b) is cum-dividend book value at the horizon. The (2b) calculation says that adding the present value of forecasted residual income to current book value is the same as the present value of forecasted cum-dividend book value at the horizon, as recognized by Brief (1996). The forecast target in (2c) is the sum of earnings to the horizon plus earnings on any dividends paid out. We will refer to this as cum-dividend earnings to the horizon. The comparison of (2b) and (2c) says that current stocks plus forecasted future flows can always be represented as forecasted future cum-dividend stocks provided the accounting is clean surplus. Both (2b) and (2c) are derived from (2) by iterating out earnings, book values and dividends from the forecasted stream of residual income using the clean surplus relation.²

Comparing (2b) with (1b) it is clear that the W_t^T supplies a terminal value number for the dividend discount model. But by comparing (2b) with (1a) it is also clear that this terminal value can give an incorrect valuation, and

$$P_t = W_t^T + \rho^{-T} E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) \quad (3)$$

for all T . The last term, the present value of the expected premium at $t + T$, is the valuation error introduced by the truncation of the forecast horizon at $t + T$. Ohlson and Zhang (1997) analyze how this error changes as the horizon increases. W_t^T provides valuation without error if the expected premium at $t + T$ is zero, as is well recognized,³ and correspondingly the present value of expected residual earnings subsequent to $t + T$ is zero.

W_t^T is defined for all clean surplus accounting principles for identifying and measuring B and X , including random numbers. Calculations of W_t^T , for all specifications of X and B , converge as $T \rightarrow \infty$, and all converge to P_t in (1). But the horizon truncation can introduce error and equation (3) tells us that the valuation error is determined by the accounting principles for B_{t+T} . The infinite summation in the limit can be viewed (in the extreme) as necessary to iterate out errors in accounting principles for B_{t+T} in capturing value accumulation. For finite horizon analysis the problem of calculated value being sensitive to forecasted dividend payout (with many payout patterns possible) is replaced with the problem of calculated value being sensitive to accounting principles (with many accounting principles possible). Just as payout can be value irrelevant, so can clean-surplus accounting principles. Further, this calculation, as it stands, does not guarantee a valuation that is insensitive to forecasted payout. Clearly more structure is necessary on X (and thus B) to make the residual earnings formula amenable to finite horizon analysis.

Indicative of these points, Feltham and Ohlson (1995) show that the common discounted cash flow and economic profit valuation methods (that are contrasted in Copeland, Koller and Murrin (1995), for example) apply cash flow accounting and accrual accounting for operating activities as particular accounting systems for W_t^T . We summarize their formulation here, to build on it later. Set $B_t \equiv \text{FA}_t$ in (2), and so for all $t + \tau$, where FA is financial assets net of financial obligations (sometimes referred to as marketable

securities minus debt), which is negative for net debt. Reconcile successive stocks of financial assets to flows in all $t + \tau$ such that in expectation, applying CSR period by period, $FA_{t+\tau} = FA_{t+\tau-1} + C_{t+\tau} - I_{t+\tau} + i_{t+\tau} - d_{t+\tau}$. Accordingly, $X_{t+\tau} = C_{t+\tau} - I_{t+\tau} + i_{t+\tau}$, where $C - I$ is cash from operations minus cash investments ("free cash flow") and i is interest flow from financial assets. Then W_t^T is calculated as follows. As this is a particular case of W_t^T , we number the calculation in the (2) series but refer to it as W_t^T (DCF):

$$W_t^T \text{ (DCF)} \equiv FA_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{C}_{t+\tau} - \tilde{I}_{t+\tau} + \tilde{i}_{t+\tau} - (\rho - 1)F\tilde{A}_{t+\tau-1}), \quad (2d)$$

and if FA_t is measured at present value (market value) by recording interest such that

$$E_t(\tilde{i}_{t+\tau}) = (\rho_d - 1)E_t(F\tilde{A}_{t+\tau-1}), \quad (\text{EIM})$$

all τ , then

$$W_t^T \text{ (DCF)} = FA_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t(\tilde{C}_{t+\tau} - \tilde{I}_{t+\tau}), \quad (2e)$$

which is the standard discounted cash flow (DCF) formula where FA_t is usually negative (for net debt)⁴.

The issue of appropriate discount rates is not a concern here. But practice recognizes that expected payoffs must be discounted at a rate that reflects their risk. So not to offend the eye, we apply discount rates to streams that are standard in corporate finance. The rate, ρ_d , indicates one plus the expected return on financial assets (cost of capital for debt). In (2e) expected free cash flows from operations are discounted at one plus the cost of capital for operations, the so-called "weighted average cost of capital." EIM (the "effective interest method") requires accrual accounting for financial activities which yields the book value of financial assets at market value, for both current financial assets (at t) and expected future financial assets (at each $t + \tau$).

Alternatively, set $B_t \equiv FA_t + OA_t$, and so for all $t + \tau$, where OA_t is measured net operating assets, and reconcile successive stocks such that, in expectation, $FA_{t+\tau} + OA_{t+\tau} = FA_{t+\tau-1} + OA_{t+\tau-1} + OI_{t+\tau} + i_{t+\tau} - d_{t+\tau}$. Accordingly, $X_{t+\tau} = OI_{t+\tau} + i_{t+\tau}$ where $OI_{t+\tau} = C_{t+\tau} + oa_{t+\tau}$ is operating income and $oa_{t+\tau}$ is (new) operating accruals for the period.⁵ The W_t^T calculated with this accounting uses accrual accounting for operating activities and we refer to it as W_t^T (OA):

$$W_t^T \text{ (OA)} \equiv FA_t + OA_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(O\tilde{I}_{t+\tau} + \tilde{i}_{t+\tau} - (\rho - 1)(F\tilde{A}_{t+\tau-1} + O\tilde{A}_{t+\tau-1})), \quad (2f)$$

and given EIM,

$$W_t^T \text{ (OA)} = FA_t + OA_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t(O\tilde{I}_{t+\tau} - (\rho_W - 1)O\tilde{A}_{t+\tau-1}). \quad (2g)$$

The forecast target here is referred to as residual operating income, economic profit, or economic value added. Similar representations to (2g) can be made for any clean-surplus accounting principles where some net assets are marked to market: the book value term includes these assets but, as expected residual earnings on these assets is zero (they are expected to earn at the cost of capital), the residual income calculation need only involve earnings from those assets not marked to market.

Feltham and Ohlson (1995) also show that, if dividends are paid out of financial assets and do not affect the calculation of the flow, then, given EIM, W_t^T is insensitive to dividends for all accounting principles: $E_t(\tilde{B}_{t+T})$ in (2b) is reduced dollar for dollar by the $t + T$ value of any change in expected dividends, just like $E_t(\tilde{P}_{t+\tau})$ in (1a) is under M&M dividend irrelevance. Accordingly, the calculation of W_t^T like P_t , is unaffected. This accounting will be assumed for $E_t(\tilde{B}_{t+T})$.

For the DCF formula, the valuation error is given by setting $E_t(\tilde{B}_{t+T}) = E_t(F\tilde{A}_{t+T})$ in (3). If expected FA_{t+T} is negative (net debt), as is typical, the valuation error will be large. The DCF valuation error is equal to the present value of the projected value of operating assets at $t + T$ (as the financial assets are expected to be at market value by EIM). The error for the operating accrual model in (2g) is the present value of the projected value of the operating assets at $t + T$ minus their measured book value. The error for the DCF formula is zero only if operating assets are to be liquidated into financial assets at the horizon, whereas that for the operating activities model is zero if measured operating assets are projected to be at market value (that is, zero expected residual operating earnings are forecasted subsequent to $t + T$). In short, valuation errors and terminal value corrections are determined by the rules for the recognition and measurement of stocks of value. This can be interpreted as "measurement error in the accounting," that is, in the pro forma accounting indexed on $t + T$.

2.2. Capitalized Earnings Calculations (Ohlson (1995))

For brevity, denote residual income as $X_{t+\tau}^a \equiv X_{t+\tau} - (\rho - 1)B_{t+\tau-1}$. Given CSR, the calculation

$$V_t^T \equiv B_t + (\rho^T - 1)^{-1} \sum_{\tau=1}^T \rho^{T-\tau} E_t(\tilde{X}_{t+\tau}^a) \tag{4}$$

$$= B_t + \rho^T / (\rho^T - 1) \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) \tag{4a}$$

$\rightarrow W_t^T$ and P_t as $T \rightarrow \infty$.

Equivalently, by iterating out earnings and dividends from changes in book values using the clean surplus relation, V_t^T can be calculated as

$$V_t^T = (\rho^T - 1)^{-1} E_t \left(\sum_{\tau=1}^T \tilde{X}_{t+\tau}^C \right), \tag{4b}$$

$$= (\rho^T - 1)^{-1} [E_t(\tilde{B}_{t+T}^C) - B_t] \tag{4c}$$

where $E_t(\tilde{B}_{t+T}^C) \equiv E_t[\sum_{\tau=1}^T \rho^{T-\tau} \tilde{d}_{t+\tau} + \tilde{B}_{t+T}]$ is the expected cum-dividend book value at $t+T$ in (2b). If $E_t(\tilde{B}_{t+T})$ is reduced by the expected terminal value of dividends through the accounting for financial assets in EIM, V_t^T is also insensitive to expected dividends.

The forecast target for V_t^T is the same as that for W_t^T , so the two calculations differ in how the cost of capital is applied to forecasts. The W_t^T calculation in (2a) adds the present value of the terminal value of expected residual earnings to current book value, whereas V_t^T in (4) adds the capitalized value of the terminal value of expected residual earnings to current book value. Equivalently, V_t^T is calculated by capitalizing expected aggregated cum-dividend earnings. Comparing (4a) with (2), it is clear that $V_E^t = W_E^t$ for all $T < \infty$, if and only if $\sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) = 0$ and, as $\frac{\rho^T}{(\rho^T-1)} > 1$ for all $T < \infty$, then $W_t^T - V_t^T > 0$, for all $T < \infty$, if $\sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) < 0$ and $W_t^T - V_t^T < 0$ if $\sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) > 0$.

V_t^T , like W_t^T , is a calculation that varies over all clean surplus accounting principles. For given accounting principles,⁶

$$P_t = V_t^T + (\rho^T - 1)^{-1} [E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) - (P_t - B_t)] \quad (5)$$

for all T , and the capitalized change in expected premium from t to $t+T$ is the valuation error of V_t^T and terminal value correction required. Clearly, $P_t = V_t^T$ if and only if there is no expected change in premium to the horizon. This contrasts with the zero-error requirement for W_t^T of zero expected premium at the horizon. If there is no expected change in premium, then by substituting P_t for V_t^T in (4b),

$$E_t \left(\sum_{\tau=1}^T \tilde{X}_{t+\tau}^C \right) = (\rho^T - 1) P_t. \quad (5b)$$

That is, projected cum-dividend earnings are equal to the expected stock return and thus forecasted earnings can be considered "normal." Correspondingly, the terminal value correction can be expressed as $(\rho^T - 1)^{-1} [(\rho^T - 1) P_t - E_t(\sum_{\tau=1}^T \tilde{X}_{t+\tau}^C)]$, the capitalized difference between expected cum-dividend earnings and the expected stock return. Thus, whereas the valuation error for W_t^T reflects differences in forecasted market values and accounting values of (stocks of) equity, that for V_t^T reflects differences in projected changes in equity values (stock returns) and accounting measures of changes of value (earnings). Both errors are accounting measurement errors and we refer to the W_t^T error as measurement error in the forecasted stocks and the V_t^T error as measurement error in the forecasted flows (earnings).

2.3. A Complete Representation

For both W_t^T and V_t^T , calculated values differ over accounting principles and, for given accounting principles, the two calculations differ in the way forecasted accounting is transformed into price (by discounting residual earnings or capitalizing their terminal value). The applicability of the two approaches for finite horizon valuation without error is limited to accounting that produces expected zero premiums or expected zero change in premiums. If the accounting for book value is such that book values are expected always to be kept

low (through conservative accounting that expenses investment in R&D or brand assets, or “accelerated depreciation,” for example), the W_t^T calculation will not work. But V_t^T will not work either if premiums are expected to change. The following combines the two approaches in a general model which accommodates changing, non-zero premiums and in which W_t^T and V_t^T appear as special cases.

For any horizon, $t + T$, define K_S such that, for S periods subsequent to $t + T$,

$$E_t(\tilde{P}_{t+T+S}^{\text{CS}} - \tilde{B}_{t+T+S}^{\text{CS}}) = K_S E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}), \quad (6)$$

where the superscript “CS” indicates values cum-dividend from $t + T$ to $t + T + S$. K_S is the expected change or growth in the premium over the S periods after $t + T$ but, as the premium is the measurement error in the accounting for book value, it can also be interpreted as the expected change in this error. Increasing R&D expenditure, for example, increases the premium if the expenditure is expensed rather than capitalized in book value (holding all else constant). As $E_t(\tilde{P}_{t+T+S}^{\text{CS}} - \tilde{P}_{t+T}) = (\rho^S - 1)E_t(\tilde{P}_{t+T})$ by NA, then given CSR for accounting over the S periods, (6) implies

$$(\rho^S - 1)E_t(\tilde{P}_{t+T}) - E_t\left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}}\right) = (K_S - 1)E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}). \quad (7)$$

The left-hand side of (7) compares the expected stock return from $t + T$ to $t + T + S$ with expected earnings over that period. Thus K_S reflects accounting principles that determine measured earnings relative to stock returns. For $K_S = 1$, forecasted earnings for the S periods equal the expected S -period stock return. These earnings are identified as “normal” earnings (without measurement error) in (5b). This is the case where, for example, the expensing of R&D keeps book value below the value for the assets but there is no effect on earnings. This occurs when there is no change in R&D expenditures: earnings are unaffected by the accounting because the income statement expense is the same with capitalization and amortization as with expensing (and so with accelerated depreciation and no change in investment in plant assets). If $K_S \neq 1$, forecasted earnings are measured with a displacement from normal and this measurement error is given by K_S and the projected measurement error already in book value at $t + T$. This is the case of changing R&D expenditures: increasing R&D, for example, gives a higher expense with expensing than with capitalization and amortization so, accordingly, earnings are lower (all else constant). It is usual to think of $K_S \geq 1$ (accounting earnings as unbiased or biased downwards) but the case of $K_S < 1$ (accounting earnings biased upwards) also is captured. ($K_S < 0$ implies premiums flip-flopping from positive to negative which is probably an uninteresting case.)⁸ From (7),

$$E_t\left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}}\right) - (\rho^S - 1)E_t(\tilde{B}_{t+T}) = (\rho^S - K_S)[E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T})]. \quad (8)$$

The left-hand side of this equation is expected multi-period residual earnings.

From (8)

$$E_t(\tilde{P}_{t+T}) = (\rho^S - K_S)^{-1} E_t\left[\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}} - (K_S - 1)\tilde{B}_{t+T}\right] \quad (9)$$

and the terminal value correction to W_t^T in (3) is supplied:

$$\begin{aligned} P_t^G &\equiv B_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) \\ &\quad + \rho^{-T} \left[(\rho^S - K_S)^{-1} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS} - (K_S - 1) \tilde{B}_{t+T} \right) - E_t(\tilde{B}_{t+T}) \right] \\ &= P_t, \end{aligned} \quad (10)$$

where for a finite price it is required that $K_S < \rho^S$.

2.3.1. Inferring Measurement Error

It is clear that, in the absence of a determination of the measurement error parameter, K_S , the valuation in (10) cannot be calculated. As it is a feature of the accounting principles, the error might be assessed by reference to the actual accounting policies employed. However, it can be inferred from the expected evolution of future earnings and book values. Think of time subsequent to $t + T$ as blocks of S periods. If $t + T$ is identified such that for all $N > 0$,

$$E_t(\tilde{P}_{t+T+NS}^{CN} - \tilde{B}_{t+T+NS}^{CN}) = K_S E_t(\tilde{P}_{t+T+(N-1)S} - \tilde{B}_{t+T+(N-1)S}), \quad (11)$$

(where "CN" now indicates cum-dividends for the N th set of S periods), then K_S , a constant, represents measurement error *consistently applied*. By iterating (8) for $N > 1$,

$$\begin{aligned} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+(N-1)S+\tau}^{CN} - (\rho^S - 1) \tilde{B}_{t+T+(N-1)S} \right) \\ = K_S E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+(N-2)S+\tau}^{C(N-1)} - (\rho^S - 1) \tilde{B}_{t+T+(N-2)S} \right) \end{aligned}$$

for all N and so the S -period residual income is forecasted to grow at a constant rate determined by K_S . Thus K_S can be inferred from the projected growth rate of S -period residual earnings subsequent to $t + T$: measurement error "shows up" as a permanent difference between earnings and book value. The left-hand side of (8) can be restated as $(\rho^S - 1) E_t[\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS} / (\rho^S - 1) - \tilde{B}_{t+T}]$, that is, as the cost of capital applied to the difference between expected capitalized earnings and book value at $t + T$, so K_S can alternatively be inferred from the growth in this difference. Accordingly, P_t^G in (10) can be calculated.

This formulation characterizes equity valuation as involving three forecast periods, as depicted in Figure 1. The first period involves predicting accounting stocks and/or flows up to the "horizon," $t + T$, the second is the period over which flows are forecasted for the terminal value calculation, and the third is the period over which forecasts of flows

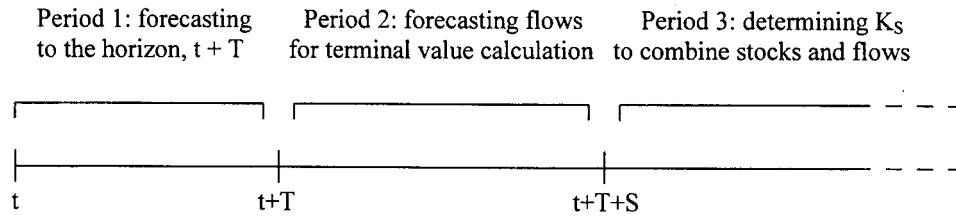


Figure 1. Time periods involved in calculating equity value.

relative to stocks reveal K_S . Clearly, the “horizon” is an elusive notion: for the forecasted accounting to be completely revealing of value, Periods 2 and 3 past the “horizon” are required. All forecasts for Period 1 can be restated as an expected cum-dividend stock at $t + T$, by (2b), and the terminal value calculated at this point combines this stock with forecasted flows over Period 2 as in (10). However, to combine these stocks and flows, one requires Period 3 forecasts to discover K_S which yields the weights for this combination. From this perspective, V_t^T in (4b) is viewed as a Period 2 calculation rather than Period 1, with a Period 1 application when $T = 0$, that is, Period 2 is the first forecast period (see below).

The weighting factor, K_S , has been given a measurement error interpretation. The issue in finite-horizon valuation is dealing with the error that the horizon truncation induces and the formulation shows how this error is accommodated. The formulation can be characterized as capturing two sources of accounting “measurement error” in current book value, $P_t - B_t$. The first is the measurement error in $P_t - B_t$ that is recognized in the residual earnings term in (10) over period 1. It is transitory and is corrected in expected book values by $t + T$. The second, $E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T})$ captured in the last term in (10), is that which is a persistent property of the accounting principles and which amounts to a permanent displacement between book value and price or between earnings and returns (value changes), or both. This is the distinction made by Beaver and Ryan (1995). They call the two sources of measurement error “delayed recognition” and “accounting conservatism” and attempt to extract them from the data. Either or both of these errors may be zero, as identified within the special cases of the general formulation that follows.

The common interpretation of K_S is as a growth factor. Indeed P_t^G in (10) is usually calculated (equivalently) as

$$\begin{aligned}
 P_t^G \equiv & B_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) \\
 & + \rho^{-T} \left[(\rho^S - K_S)^{-1} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS} - (\rho^S - 1) \tilde{B}_{t+T} \right) \right], \quad (10a)
 \end{aligned}$$

usually with $S = 1$. Here the terminal value involves capitalizing expected residual income at the horizon at a rate that reflects anticipated growth in residual income. The appeal is to a perpetuity growth concept. The point of the analysis here (besides providing unification) is to stress that growth is really an accounting construct: growth is due to the way flows are measured, and forecasted growth reveals the measurement.

2.4. Special Cases

Special cases of the general formation involve particular values for K_S , S , T , and expected $t + T$ premiums.

(a) $K_S = 1$ (expected premiums constant subsequent to $t + T$).

$$P_t^G \equiv B_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) + \rho^{-T} \left[(\rho^S - 1)^{-1} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}} \right) - E_t(\tilde{B}_{t+T}) \right] \quad (10b)$$

$$= P_t.$$

This is referred to as the “combination model” in Ou and Penman (1996) because it combines the residual earnings approach to $t + T$ and the capitalized earnings approach from $t + T$ to $t + T + S$. It has the property that projected cum-dividend earnings subsequent to $t + T$ are “normal,” that is, equal to the expected stock return. The expected earnings yield on expected P_{t+T} price is “normal,” by (7), and $E_t(\tilde{P}_{t+T}) = E_t(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}}) / (\rho^S - 1)$.

(b) $K_S = 1$; $S = 1$ (expected constant premiums each period subsequent to $t + T$).

$$P_t^G \equiv B_t + \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{X}_{t+\tau}^a) + \rho^{-T} \left(\frac{1}{(\rho - 1)} E_t(\tilde{X}_{t+T+1}) - E_t(\tilde{B}_{t+T}) \right) \quad (10c)$$

$$= P_t$$

and by NA,

$$E_t(\tilde{P}_{t+T+1} + \tilde{d}_{t+T+1}) = \frac{\rho}{(\rho - 1)} E_t(\tilde{X}_{t+T+1}).$$

The multiplier, $\frac{\rho}{(\rho - 1)}$, is the normal multiplier and the horizon is determined at the point where the P/E ratio is expected to be normal. This is because expected $t + T + 1$ earnings are normal: they are measured without error. This is familiar in prescriptions for DCF analysis. Substituting the measured book value and flows in the DCF analysis into (10c),

$$P_t = P_t^G(\text{DCF}) \equiv \text{FA}_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t[\tilde{C}_{t+\tau} - \tilde{I}_{t+\tau}]$$

$$+ \rho_W^{-T} [(\rho_W - 1)^{-1} E_t[(\tilde{C} - \tilde{I} + \tilde{i})_{t+T+1}] - E_t(F\tilde{A}_{t+T})] \quad (10d)$$

$$\begin{aligned}
&= FA_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t[(\tilde{C} - \tilde{I})_{t+\tau}] \\
&\quad + \rho_W^{-T} [(\rho_W - 1)^{-1} E_t(\tilde{C} - \tilde{I})_{t+T+1}] \quad (10e)
\end{aligned}$$

given the EIM condition for recording interest applied in $t + T + 1$. This is recognized in, for example, Rappaport (1986), Copeland, Koller and Murrin (1995) and Cornell (1993), where the last (terminal value) term is proposed by appeal to a perpetuity concept. But the calculation also implies that the free cash flow is expected to equal the stock return. Similarly, with accrual accounting for operating activities,

$$\begin{aligned}
P_t = P_t^G(\text{OA}) \equiv & FA_t + \text{OA}_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t(O\tilde{I}_{t+\tau} - (\rho_W - 1)O\tilde{A}_{t+\tau-1}) \\
& + \rho_W^{-T} [(\rho_W - 1)^{-1} E_t(O\tilde{I}_{t+T+1}) - E_t(O\tilde{A}_{t+T})] \quad (10f)
\end{aligned}$$

as recognized in Bernard (1994). Note that while (10f) targets operating profitability at the horizon, the DCF model in (10e) can be construed as targeting net financing activities at the horizon because, by the cash conservation equation, the free cash flow, $(C - I)_{t+T+1} = (d + F)_{t+T+1}$ where F is non-equity financing cash flows from financial assets.

(c) $W_t^T = P_t^G = P_t$ if for some S

$$E_t(\tilde{P}_{t+T+S} - \tilde{B}_{t+T+S}) = E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) = 0.$$

Then, from (8),

$$E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}} \right) = (\rho^S - 1) E_t(\tilde{B}_{t+T}),$$

that is, expected book value at $t + T$ is sufficient for projecting subsequent earnings given the cost of capital (and expected capitalized earnings at $t + T$ equals book value).⁹ Clearly $W_t^T = P_t^G = B_t = P_t$ is a corner case where the horizon is zero.

(d) $V_t^T = P_t^G = P_t$ if

$$E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) = P_t - B_t.$$

A corner case arises when $P_t - B_t = P_{t-1} - B_{t-1}$, and thus $X_t = (\rho - 1)P_{t-1}$ and $P_t + d_t = \frac{\rho}{(\rho-1)}X_t$. Here the horizon is zero, current earnings equal returns, and the current P/E ratio is normal.

3. Synthesis and the Terminal Value for the Dividend Discount Formula

The generalized representation unifies valuation techniques involving pro forma accounting for stocks and flows within the firm. The final synthesis reconciles this representation to the dividend discount model.

The “point of departure” is the standard Gordon textbook model which attempts to satisfy the terminal value calculation for the dividend discount model by capitalizing terminal dividends adjusted for an assumed growth rate. For a specified $t + T$, this model calculates

$$P_t^T \equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [E_t(\tilde{d}_{t+T+1})/(\rho - g_T)] \quad (12)$$

in satisfaction of (1b), where $g_T < \rho$ is (one plus) the projected constant growth rate in payout subsequent to $t + T$. This is clearly restricted to particular payout policies. Constant growth dividend policies are not uniformly or even typically observed and projected deviations from such a policy are unrelated to value under the M&M condition.

3.1. The Generic Terminal Value

Given the premium condition (6), $E_t(\tilde{P}_{t+T})$ is calculated according to (9). Thus,

$$\begin{aligned} P_t^T &\equiv \rho^{-T} \left[\sum_{\tau=1}^T \rho^{T-\tau} E_t(\tilde{d}_{t+\tau}) + (\rho^S - K_S)^{-1} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}} - (K_S - 1) \tilde{B}_{t+T} \right) \right] \quad (13) \\ &= P_t \end{aligned}$$

in satisfaction of (1b), and the terminal value for the dividend discount formula is provided: (13) is equal to

$$\begin{aligned} P_t^T &\equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} \left[(\rho^S - K_S)^{-1} E_t \left(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{\text{CS}} - (K_S - 1) \tilde{B}_{t+T} \right) \right] \quad (14) \\ &= P_t \end{aligned}$$

in contrast to the Gordon model. Further, P_t^G in (10) is equal to this calculation and thus (14) provides an umbrella over all finite-horizon stocks and flows accounting models that are special cases of P_t^G . These techniques are equivalent to discounting expected dividends to the horizon with a particular accounting for the terminal value.

This is of course suggested by the canceling calculations in P_t^G : in (10) and (10a)–(10f) the present value of current book value plus expected additions to book value through residual earnings to $t + T$ are calculated in the first two terms and this equals the present value of expected cum-dividend terminal book value, by equation (2b). This is the Period 1 calculation. But then the present value of this accumulation, ex-dividend, is subtracted in the terminal book value in the third term in (10) to remove a redundancy given the terminal value calculation. What remains is just the dividends in the cum-dividend book value. This effectively removes the accounting for Period 1 to get back to dividends. Exceptions are the cases where $W_t^T = P_t$ and $V_t^T = P_t$ where expected stocks at the Period 1 horizon or flows to the horizon produce valuation without error (and no terminal value is required). In these cases, the accounting to the horizon suffices. In all other cases (of accounting

producing valuation error), the Period 1 accounting is “backed out,” replaced by expected dividends to the horizon, and the accounting at the horizon based on Periods 2 and 3 is the determining calculation. We observed that, for infinite horizons, the infinite summation serves to iterate out the accounting to get back to dividends. Similarly, for finite horizons, the Period 1 calculations are subtracted out unless the accounting is ex ante without error for some finite $t + T$. In terms of the characterization of two sources of measurement error earlier, Period 1 measurement error is removed but the persistent error revealed from the analysis of Periods 2 and 3 is explicitly accommodated.

3.2. Special Cases and Some Equivalencies

The representation of all models as the dividend discount formula with different terminal values is demonstrated for a few cases. For $K_S = 1$ and $S = 1$,

$$\begin{aligned} P_t^T &\equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [(\rho - 1)^{-1} E_t(\tilde{X}_{t+T+1})] \\ &= P_t^G \text{ (in equation 10c)} \end{aligned} \tag{14a}$$

and

$$\begin{aligned} P_t^T &= \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [E_t(\tilde{B}_{t+T})] \\ &= W_t^T, \end{aligned} \tag{14b}$$

and both are equal to P_t given the premium condition for the relevant case. Calculation (14a) replaces the accounting up to $t + T$ in P_t^G with dividends, but equation (14b), which restates (2b), preserves it if the expected $t + T$ premium is zero. For DCF analysis (for $K_S = 1$ and $S = 1$),

$$P_t^T \text{ (DCF)} \equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [(\rho - 1)^{-1} E_t(\tilde{C} - \tilde{I} + \tilde{i})_{t+T+1}] \tag{14c}$$

equals the standard formulation in (10d) and, given EIM, this is equivalent to

$$P_t^T \text{ (DCF)} \equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [E_t(F\tilde{A}_{t+T}) + (\rho_w - 1)^{-1} E_t(\tilde{C} - \tilde{I})_{t+T+1}] \tag{14d}$$

and similarly so for $K_S \neq 1$ and $S > 1$. Similarly,

$$P_t^T \text{ (OA)} \equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [(\rho_w - 1)^{-1} E_t(O\tilde{I}_{t+T+1}) + E_t(F\tilde{A}_{t+T})] \tag{14e}$$

is equivalent to (10f) and is equal to P_t given the premium condition with respect to operating assets.

Expressing all techniques as cum-dividend terminal values also identifies equivalences and the difference between them. A reportedly common technique sets capitalized expected operating income as the terminal value calculation in DCF analysis (see Copeland, Koller and Murrin (1995), for example, where operating income is referred to as NOPLAT). The calculation is

$$P_t^T(\text{DCF}) \equiv \text{FA}_t + \sum_{\tau=1}^T \rho_W^{-\tau} E_t(\tilde{C} - \tilde{I})_{t+\tau} + \rho_W^{-T} [(\rho_W - 1)^{-1} E_t(O\tilde{I}_{t+T+1})]. \quad (14f)$$

But this is equal to

$$P_t^T(\text{DCF}) \equiv \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} [(\rho_W - 1)^{-1} E_t(O\tilde{I}_{t+T+1}) + E_t(F\tilde{A}_{t+T})] \quad (14g)$$

and so comes under the umbrella. And this is the same as $P_t^T(\text{OA})$ in (14e). (Similar equivalences can be stated for the “growth” case.) The difference between the (14g) version of DCF analysis and the free cash flow version in (14d) is the expected difference between expected $t + T + 1$ operating income and free cash flow. As $O\tilde{I}_{t+T+1} = C_{t+T+1} + oa_{t+T+1}$, this difference is expected $t + T + 1$ investment plus accruals. This also explains the difference between (14d) and (14e) and between (10e) and (10f). Hence forecasting free cash flow or earnings to the horizon is not the crux of the matter: one can just forecast dividends. Rather it is the treatment of anticipated investment and accruals at the horizon that differentiates the methods.

This formulation shows, more generally, that the specification of attributes to be forecasted to the horizon over Period 1 in Figure 1 is not the issue in equity valuation. Given the terminal value implied by the premium condition, these attributes simply reflect projected payout to the horizon which is the value irrelevant attribute that is nominally being avoided by substituting other attributes for dividends. The determining aspect is the terminal value calculation over Periods 2 and 3 and the various techniques adjust this cum-dividend for dividend payoffs in Period 1. The lengths of the horizons, T and $T + S$ for a particular technique, are not determined here. This is an empirical issue which Penman and Sougiannis (1998) investigate.

The formulation also reveals what’s at work in these techniques in comparison with the forecasting of dividends. For example, the DCF technique without a terminal value is equivalent to projecting dividends with expected net debt as a terminal value, by (14d). With the DCF terminal value calculation, it adds capitalized horizon free cash flow. By the cash conservation equation, $C_{t+\tau} - I_{t+\tau} = d_{t+\tau} + F_{t+\tau}$, this is an adjustment for projected net debt and equity financing beyond the horizon and thus the method is appropriate over finite horizons for constant projected financing flows beyond the horizon. In the case of a pure equity firm (zero expected net debt), the DCF calculation in (14d) is identical to the Gordon model in (12) with zero expected dividend growth (and can be modified for $K_S > 1$ to incorporate expected growth). Accrual accounting techniques replace capitalized operating flows for financing flows in the terminal value for the dividend discount formula,

by a comparison of (14e) with (14d). However, the specification of the terminal value in the so-called DCF technique in (14f) does the same thing and is equal to the accrual accounting valuation in (14e) and (10f). It effectively backs out the forecasted free cash flows to get to cum-dividend operating accrual income. This makes sense: free cash flow is equal to total financing flows so, if financing flows are irrelevant to value, one wants to substitute the value-operating aspects of the business for them. But the technique can hardly be called cash flow analysis.

3.3. Terminal Values as Weighted Stocks and Flows

The terminal value for the dividend discount model is a combination of forecasted flows subsequent to $t + T$ and the forecasted stock at $t + T$. Setting $S = 1$ for simplicity, the calculation in (14) can be made as

$$P_t^T = \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-T} E_t \left(\frac{1}{\rho - K_1} \tilde{X}_{t+T+1} - \frac{K_1 - 1}{\rho - K_1} \tilde{B}_{t+T} \right). \quad (15)$$

But, as $E_t(\tilde{B}_{t+T}) = E_t(\tilde{B}_{t+T+1} - \tilde{X}_{t+T+1} + \tilde{d}_{t+T+1})$ by CSR, then the calculation can be done by weighting contemporaneous stocks and flows at $t + T + 1$:

$$P_t^T = \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \frac{\rho^{-T}}{\rho - K_1} E_t [K_1 \tilde{X}_{t+T+1} + (1 - K_1)(\tilde{B}_{t+T+1} + \tilde{d}_{t+T+1})]. \quad (15a)$$

Here expected $t + T + 1$ earnings and cum-dividend book values are weighted by the accounting measurement parameter, K_1 , and then capitalized at the rate $\rho - K_1$. The weights sum to unity: the higher the measurement error, K_1 , in measuring value flows, the higher the forecasted flows have to be weighted and the less weight is given to book value.

Taking the present value of the terminal value at $t + T + 1$ rather than $t + T$,

$$P_t^T = \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-(T+1)} E_t \left[\frac{\rho K_1}{\rho - K_1} \tilde{X}_{t+T+1} + \frac{\rho(1 - K_1)}{\rho - K_1} (\tilde{B}_{t+T+1} + \tilde{d}_{t+T+1}) \right] \quad (15b)$$

$$= \sum_{\tau=1}^T \rho^{-\tau} E_t(\tilde{d}_{t+\tau}) + \rho^{-(T+1)} E_t [w\phi \tilde{X}_{t+T+1} + (1 - w)(\tilde{B}_{t+T+1} + \tilde{d}_{t+T+1})], \quad (15c)$$

where $\phi = \frac{\rho}{\rho - 1}$ is the multiplier for normal earnings. Now the weights, $w = \frac{K_1(\rho - 1)}{\rho - K_1}$ and $(1 - w) = \frac{\rho(1 - K_1)}{\rho - K_1}$, sum to unity.

This “convexification” says that one can think of the terminal value as applying a normal multiplier to expected $t + T + 1$ earnings and then weighting these earnings with expected $t + T + 1$ cum-dividend book values. The weights depend on K_1 , that is, the error in

earnings in measuring value flows. If earnings measures value changes without error, (that is, earnings are normal), $K_1 = 1$, the terminal value is given by capitalized earnings, and no weight is given to book value. In this case (15c) reduces to (14a) and is equivalent to (10c). If $K_1 > 1$, then $w > 1$: more weight is placed on the earnings (it's weighted up to compensate for the error) but value is subtracted by applying a complementary weight to expected $t + T + 1$ book value. If $K < 1$, w is less than one. $K = 0$ puts no weight on forecasted earnings and unity on book value so (15c) reduces to W_t^T in (2b) and (14b) with the horizon set to $t + T + 1$.

GAAP accounting is considered to be conservative for many firms so one expects $K_S > 1$ and high earnings relative to book values for these firms during the recent history when investments have been growing. It is of some interest then, that Penman (1998) documents empirically that, for firms with return on equity above what one would consider a normal cost of equity capital, the weight on earnings that combines earnings and book value into equity prices, is typically greater than one.

This convexification of earnings and book value is at the core of the Ohlson (1995) valuation model. In that model, weights, given by parameters assumed to be known at t , are applied to current (time t) earnings and book value, and other information is added to yield current price. But here, weights are applied to the pro forma future accounting numbers without other information. Thus the perspective here is a little different. It directs us to think of value as a weighted average of pro forma capitalized earnings and book value at a future point in time, not as current numbers. The accounting parameter, K_S , is not a given at t but is revealed (at t) through pro forma forecasting. One does not get a valuation until K_S is discovered and once K_S is discovered all else drops out except forecasted dividends and horizon stock and flows. Current price is the present value of expected cum-dividend future price obtained by applying K_S to expected earnings and book values at a future point in time. The convexification does not work prior to that point in time. Other information besides earnings and book value is needed. Indeed a special case of the Ohlson model, where no other information besides earnings and book values is needed for valuation, is a special case of (15c) with a horizon of zero. Ohlson and Zhang (1997) analyze how other information works its way into earnings and book values as the forecast horizon increases.

3.4. Dividend Displacement and the Terminal Value Calculation

For the finite horizon dividend discount model in (13) or P_t^G in (10) to be insensitive to payout, the calculated terminal value must be displaced by the dividends to $t + T$. Let $\delta(\cdot)$ represent the change in the accounting for (\cdot) as a result of a dollar change in $E_t(\sum_{\tau=1}^T \rho^{T-\tau} \tilde{d}_{t+\tau})$; thus it is required that $\delta E_t(T \tilde{V}_{t+T}) = -1$. By the assumed accounting for financial assets, $\delta E_t(\tilde{B}_{t+T}) = -1$ (Feltham and Ohlson (1995)) and this changes the generic terminal value in (13) by $(K_S - 1)/(\rho^S - K_S) = (\rho^S - 1)/(\rho^S - K_S) - 1$. Accordingly, $\delta E_t(T \tilde{V}_{t+T}) = -1$ requires $\delta E_t(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS}) = -(\rho^S - 1)$. As $E_t(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS}) = E_t(\tilde{B}_{t+T+S}^{CS} - \tilde{B}_{t+T})$ by CSR applied over Period 2, and, as $\delta E_t(\tilde{B}_{t+T+S}^{CS}) = -\rho^S$ by the accounting for financial assets, the requirement is satisfied.

4. Conclusion

This paper provides a synthesis of equity valuation techniques that forecasts stocks and flows and brings them under a unifying framework that highlights the differences between them. The paper outlines how the valuation task is structured and provides the generic terminal value calculation that adjusts for the error that finite horizon truncation introduces.

The paper shows that the valuation problem involves three forecast periods. The first is the period up to a horizon over which flows are forecasted. These flows, combined with current stocks, can always be represented by an accumulated stock at the horizon. The terminal value calculated at the end of this period is a combination of these stocks and flows forecasted over a second period subsequent to the horizon. Yet a third period is involved over which stocks and flows are forecasted to determine the weights that combine Period 1 stocks and Period 2 flows in the terminal value calculation.

The paper then shows that all finite horizon valuation models are special cases of an umbrella model that involves forecasting dividends with an appropriate terminal value calculation. In so doing it supplies the terminal valuation calculation for the finite-horizon dividend discount model. All models are equivalent to a finite-horizon dividend discount model and their calculus reduces to calculating its terminal value.

Accordingly, the specification of attributes to be forecasted to a horizon (in the first period) is not the important issue as, given the terminal value, they just reflect that period's projected dividends. Rather the calculation of the terminal value based on Periods 2 and 3 is the determining aspect. Indeed, except for the case where no terminal value is required, the substitution of other attributes for dividends, with a terminal value, involves canceling calculations that remove the attribute and replaces it with dividends. This is ironic because the various models used in practice are justified on the grounds that they substitute something more "fundamental" for dividends as the forecast attribute. A model is seen as suspect if too much weight is placed on the terminal value in the calculation of value. However, the inclusion of the appropriate terminal value substitutes dividends back in and puts all the weight on the terminal value. Indeed finite-horizon valuation is satisfied purely by a terminal value, cum-dividend.

The elimination of the forecast attribute through the terminal value calculation serves to eliminate the valuation error in the attribute. This error has been characterized in the paper as being introduced by accounting rules for recognizing and measuring the forecast attribute. The error eliminated is "transitory" error which "works its way out" at the forecast horizon. But persistent measurement error that arises from permanent biases in the accounting cannot be eliminated and the analysis shows how this is accommodated in the terminal value calculation. The analysis also shows that all techniques and their terminal value calculations can be validated by consistency with the finite-horizon dividend discount formula and the no-arbitrage condition it implies.

The unifying framework identifies just what is being added when other attributes are substituted for dividends. For example, the DCF technique which forecasts free cash flows to the horizon and capitalizes expected terminal free cash flows is equivalent to specifying expected net debt plus capitalized financing flows as the terminal value in the dividend discount formula. The residual income technique that capitalizes terminal residual income

is equivalent to capitalizing terminal income and adding the present value of dividends forecasted to the horizon. The framework also identifies certain equivalences in the techniques. DCF techniques that specify capitalized operating income as the terminal value are equivalent to techniques that forecast accrual earnings, and both are equivalent to the dividend discount formula with the terminal value as capitalized expected operating income less expected net debt. Correspondingly, differences in techniques are also identified. DCF and residual earnings methods are distinguished by the accounting for investment and operating accruals at the horizon, and forecasting free cash flow or residual income to the horizon is not an issue.

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Notes

1. In Preinreich (1938), Edwards and Bell (1961), Peasnell (1982), Brief and Lawson (1992) and Ohlson (1995), for example.
2. To derive (2b) from (2) it is necessary that $E_t(\tilde{B}_{t+T}) = B_t + \sum_{\tau=1}^T E_t(\tilde{X}_{t+\tau} - \tilde{d}_{t+\tau})$; that is, the accounting need only be clean surplus (in expectation) on average over the T years, rather than in each period as stated in CSR. And the equivalences of (2a)–(2c) to (2) hold under this less demanding condition.
3. In Brief and Lawson (1992), Bernard (1994), and Ohlson (1995), for example.
4. By the cash conservation equation, $d_t = C_t - I_t - F_t$ where F is non-equity financing flows, and so for all $t + \tau$ the DCF formula can also be derived by substituting $C - I - F$ for d in each $t + \tau$ in (1), with the EIM assumption for measuring financial assets and obligations.
5. As $FA_{t+\tau} = FA_{t+\tau-1} + C_{t+\tau} - I_{t+\tau} + i_{t+\tau} - d_{t+\tau}$, it follows that $OA_{t+\tau} = OA_{t+\tau-1} + I_{t+\tau} + oa_{t+\tau}$. That is, changes in operating assets are cash investments plus operating accruals.
- 6.

$$\begin{aligned} P_t - V_t^T &= P_t - (\rho^T - 1)^{-1} [E_t(\tilde{B}_{t+T}^C) - B_t] && \text{(by CSR)} \\ &= (\rho^T - 1)^{-1} [\rho^T P_t - P_t - E_t(\tilde{B}_{t+T}^C) + B_t] && \text{(by NA)} \\ &= (\rho^T - 1)^{-1} [E_t(\tilde{P}_{t+T}^C) - E_t(\tilde{B}_{t+T}^C) - (P_t - B_t)]. \end{aligned}$$

If dividends reduce both price and book value dollar-for-dollar, then $P_t - V_t^T = (\rho^T - 1)^{-1} [E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) - (P_t - B_t)]$. See Easton, Harris and Ohlson (1992). Dividends reduce book value dollar-for-dollar under current accounting principles and reduce price dollar-for-dollar under the M&M dividend displacement property.

7. If dividends do not affect premiums (as in note 6), the expression can be stated ex dividend. The many subscripts and superscripts (and the summation notation that follows) is necessary to give the general model. One can get the intuition without the generalization by setting $S = 1$ in one's mind, and this simplifies the notation. $S > 1$ accommodates business cycles: earnings may differ within years over the cycle but one forecasts the total earnings over the cycle.
8. Feltham and Ohlson (1995) model the effect of conservative accounting on premiums when investment is changing.
9. Constant zero premiums over some $S < \infty$ is a sufficient condition. But S can be very large. So the necessary condition for $W_t^T = P_t^G = P_t$ is $E_t(\sum_{\tau=1}^S \tilde{X}_{t+T+\tau}^{CS}) \rightarrow (\rho^S - 1)E_t(\tilde{B}_{t+T})$ as $S \rightarrow \infty$ and accordingly $E_t(\tilde{P}_{t+T} - \tilde{B}_{t+T}) = 0$ (but subsequent expected premiums can be non-zero).

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