OPTIMAL PRICE AND INVENTORY ADJUSTMENT IN AN OPEN-ECONOMY MODEL OF THE BUSINESS CYCLE*

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This paper presents a macroeconomic model containing optimizing, inventory-holding firms that is consistent with a number of prominent empirical regularities concerning fluctuations in output, exchange rates, relative prices, and money. Prices are sticky, but they are not predetermined. Still, our model is consistent with exchange rate overshooting in the sense of Dornbusch. Typical sticky-price models allow a divergence between current production and current demand, but this divergence is never allowed to feed back into the model. Our optimal inventory adjustments reconcile divergences between current demand and production, and the inventory stock movements provide expected future dynamics.

I. INTRODUCTION

The purpose of this paper is to develop an open-economy model that can be used to interpret the observed fluctuations in output, inventories, prices, and exchange rates. We have constructed the model to be consistent with several of the empirical regularities that characterize fluctuations in these magnitudes discovered in studies of business and inventory cycles and in studies of the determination of prices and exchange rates in open economies.

At the center of our model is the optimization problem of domestic firms facing uncertain demand. The representative firm must set its price at the beginning of the period without knowledge of actual demand that occurs during the period. Although firms have less than full information about the current state of the economy, they do observe market clearing prices in asset markets, the government's preliminary announcement of the monetary aggregate, as well as prices being charged by other firms. Consequently, firms use this information to make inferences about what demand will actually occur, and they set their

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prices to maximize the expected present value of profits. Each period firms make two sequential decisions. First, they set their prices based on incomplete information. Second, after they have received orders for their products, they decide how much of the orders to meet out of current production and how much out of inventories.

Our model is consistent with two major empirical regularities discovered in studies of business cycles.[1] These two regularities are (i) changes in the money supply result in real output fluctuations and (ii) deviations of output from a "natural rate" of output show persistence. Fully perceived monetary shocks have no real effects in our model, but unperceived monetary shocks affect real variables in our framework because price-setting firms are unable to infer from asset prices the exact values of monetary disturbances and demand disturbances.

A controversial aspect of our model is that real effects of money shocks depend only on the part of those shocks that is *unperceived* rather than on the full *unexpected* shock. An unexpected shock is one that is unpredictable, based on *past* information. An unperceived shock is one that cannot be inferred from *current* information. It is the distinction between unperceived money and unexpected money that separates the monetary business cycle models of the Lucas [1973]–Barro [1977, 1980] type (the island models) from those of the Gray [1976]–Fischer [1977] type (the labor-contracting models).[2] The well-known empirical work of Barro [1977, 1978] examines only the effects of unexpected money. Since all unperceived money must also be unexpected, Barro's work does not clarify the type of monetary shock important for business cycles.

The research of Barro and Hercowitz [1980] and Boschen and Grossman [1982] does attempt to disentangle these two types of shocks. Both sets of authors find evidence that they interpret as being unfavorable to the hypothesis that the monetary portion of the U. S. business cycle is due entirely to unperceived money, and in Section V we discuss the relationship of these empirical tests to hypotheses that emerge from our model.

A desirable feature of our model is that the transmission

1. Lucas [1976] and Barro [1981] summarize the empirical regularities of business cycles.

2. The hypothesis that only unanticipated money has real effects also has been challenged by the empirical work of Makin [1982] and Mishkin [1982]. Other empirical research in support of the hypothesis is in Barro and Rush [1980], Leiderman [1980], and Wogin [1980].

mechanism from money to output does not rely on "price surprise" terms. Island models and labor contracting models, in contrast, do rely on such terms in structural aggregate supply equations. The importance of avoiding this channel has been emphasized by Barro [1981, p. 71], who notes, "Given the relatively minor role played by price surprises in the results of Sargent [1976] and Fair [1979], . . . , it appears that monetary influences on output involve channels that have yet to be isolated." Although additional econometric work may show that price surprise terms are an important transmission mechanism of monetary shocks, the response of output in our model is consistent with the current evidence.[3]

Among the major empirical regularities confronting theories of exchange rate determination are the following closely related facts: (i) exchange rates are more volatile than nominal goods prices in the sense that for one period ahead, exchange rates are harder to predict than are goods prices, and (ii) changes in countries' exchange rates are negatively correlated with changes in their terms of trade, that is, a currency depreciation tends to coincide with a deterioration in the terms of trade.[4] In light of these facts, the dominant model of exchange-rate determination has become the one developed by Dornbusch [1976].

Despite the large number of extensions of the basic Dornbusch framework, some awkward aspects remain. Two such aspects are (i) that constant output versions of the model such as Flood [1981] or Mussa [1982] allow a deviation between current demand and supply but never specify how the deviation feeds back into the economy; and (ii) the treatment of domestic prices of domestic goods, which are predetermined, and domestic prices of foreign goods, which may respond to current real and monetary disturbances.

In the present paper both of these awkward aspects are confronted. Because firms are using asset market information when setting prices, domestic prices in our model are correlated with current disturbances. But, since the agents do not see and are unable to infer exactly the values of the actual disturbances affecting asset prices, domestic price adjustment to a money supply

3. It is notable that not all business cycle models depend on price surprise terms. In particular, the simple Keynesian model, popular in undergraduate texts, postulates demand-determined output with all prices known to agents. Further, Grossman and Weiss [1982] and McCallum [1982] present models designed to avoid price surprise terms in aggregate supply.

4. These two regularities were documented by Flood [1981] and Stockman [1980], respectively.

disturbance, for example, is less than its full information counterpart becoming complete only with the resolution of uncertainty. Further, the firms in our model are engaged in pricing, production, and inventory management. Any deviation between current demand and current production is accommodated by a corresponding optimal inventory adjustment.

While our model was set up to match the impact effects in Dornbusch's model, our post-shock dynamics are quite different from his. The Dornbusch dynamics are driven by the slow adjustment of prices, and following a shock, price adjustment is direct to the new steady state. Our dynamics are driven by the slow process of inventory accumulation. A shock that induces an immediate reduction in inventories requires a future inventory accumulation to approach the steady state. Such an inventory cycle influences the behavior of the other endogenous variables.

The final empirical regularity from the inventory cycle literature that we impose on the model was reported by Feldstein and Auerbach [1976, p. 363]. It is that average absolute sales forecast errors for durable goods are typically nine times larger than average absolute changes in inventories. This fact suggests that production bridges part of the gap between actual sales and forecasts of sales, indicating that production responds to unanticipated demand as in our framework.[5]

Our investigation yields two major results. First, we provide a new channel for the effects of monetary disturbances on the real economy. Second, our model is able to match the impact effects of the Dornbusch [1976] model but provides alternative dynamics. In both cases the reason for the initial effect is a confusion by price setters concerning the true nature of disturbances impinging on asset markets, and the dynamics are due to optimal price and inventory management in the future.

Our analysis is presented in the next two sections. In Section II we develop the model by focusing first on the goods markets and second on the asset markets. Section III presents the full reduced-form solution of the model, and Section IV is an analysis of the dynamic responses of the endogenous variables to the exogenous stochastic shocks that drive the model. The consistency

5. Blinder [1981] discusses the importance of inventories and their correlation with output. Aggregate inventories include goods in process as well as finished goods which are the focus of our inventory analysis. Hence, we do not attempt to match Blinder's empirical regularities. See also note 7.

of the predictions of the model with the various stylized facts is discussed in Section V, which is followed by some concluding remarks.

II. The Open Economy Macro Model

This section presents an open economy macro model based in part on the decision problems of rational profit-maximizing firms. Our presentation is in two parts. In the first part we develop the equations of the goods markets that consist of demands for and supplies of the goods produced in a medium-sized open economy. The result of this part is a set of optimal decision rules governing pricing, inventory accumulation, and production. These decision rules are not reduced forms, however, since beliefs concerning currently unobservable disturbances are imbedded in the decision rules. The formation of these beliefs is based on information extracted from asset markets. In the second part of this section, we provide the asset market structure. We emphasize that the goods markets determine only relative prices, and the interaction of the goods and asset markets is required to determine nominal prices.

Our model is one in which some irrevocable decisions are made sequentially, and they are based on incomplete information. At the beginning of the period, agents choose their portfolios for the period, and firms choose their prices for the period. These decisions are based on identical incomplete information concerning the state of the aggregate economy. Later in the period, firms and agents discover the actual level of demand facing the firms. Given the prices posted at the beginning of the period, firms respond to the actual quantities demanded by choosing profit-maximizing levels of production and inventory accumulation.[6]

A. Demand in the Goods Markets

There are J firms in the economy, each facing a demand curve of the form,

$$D_t^j = (1/J)D_t - J\beta_4(R_t^j - \overline{R}_t), \qquad \beta_4 > 0, \tag{1}$$

where D_t is economy-wide demand, $D_t = \Sigma_{j=1}^J D_t^j$, and R_t^j is the

6. Our structure implicitly imposes a sufficiently high cost on firms to prohibit intraperiod price changes. We do not discuss an explicit model of uncertainty that would rationalize this structure.

relative price charged by firm j, which is equal to that firm's nominal price H_t^j divided by the price level $P_t, R_t^j = H_t^j/P_t$. The average economy-wide relative price is $\bar{R}_t = (1/J)\Sigma_{j=1}^J R_t^j$. Aggregate demand D_t, $\rho_0 - \rho_1\bar{R}_t + \rho_2 X_t$ and foreign demand $\rho_0^* - \rho_1^*\bar{R}_t$, where the ρ coefficients are positive parameters, and X_t is the level of real expenditure by domestic residents. Real expenditure is assumed to depend positively on real income $\bar{R}_t Y_t$, with the specification given by the following linearization:

$$X_t = \kappa_1\bar{R}_t + \kappa_2 Y_t + u_t, \qquad \kappa_1,\kappa_2 > 0, \tag{2}$$

where u_t is a white noise disturbance to the aggregate saving-spending decision.

B. Pricing, Production, and Inventory Holding

Firm j faces the demand curve given by (1). If it charges the average economy-wide relative price $R_t^j = \bar{R}_t$, then its demand is its share of economy-wide demand, $(1/J)D_t$. If the firm charges a higher (lower) relative price than the average, its demand is reduced (increased) by the amount $J\beta_4(R_t^j - \bar{R}_t)$. The larger is $J\beta_4$, the greater is the firm's sensitivity to deviations from the average relative price. In the limit $(J\beta_4 \rightarrow \infty)$ each firm would choose to charge the average price.

A firm produces output Y_t^j and holds inventories N_t^j, such that

$$Y_t^j = D_t^j + N_t^j - N_{t-1}^j \tag{3}$$

describes the law of motion for end-of-period inventories. Firms hold inventories to smooth production costs that are assumed to be an increasing convex function of the firm's output, and an increasing function of aggregate output $Y_t = \Sigma_{j=1}^J Y_t^j$. We choose a specific functional form for firm production costs, which is given by $\gamma_1 Y_t Y_t^j + (\gamma_2/2)(Y_t^j)^2$, $\gamma_1, \gamma_2 > 0$.[7] Holding inventories is also costly. We allow negative inventories, interpreting them as a backlog of unfilled orders as in Blinder and Fischer [1981]. Backlogged orders are costly to the firm because it must discount price

7. Anothe. cost structure that we investigated makes production costs $a_t^j Y_t^j + \gamma_1 Y_t Y_t^j + (\gamma_2/2)(Y_t^j)^2$, where a_t^j is a firm-specific white noise cost shock and $\Sigma_{j=1}^J a_t^j \equiv a_t$, the economy-wide cost disturbance. Introducing such shocks allows the model to reproduce a procyclical pattern of output and final goods inventories. Blinder [1981] documents the strong procyclical nature of total inventories that include goods in process. For brevity we have set $a_t^j = 0$ for all j and t in this presentation. More general results, with $a_t^j \neq 0$, are available in Flood and Hodrick [1984a].

to consumers to induce them to pay now and accept delivery in the future.[8] Inventory costs are incurred on beginning-of-period inventories in accord with the cost function $\delta_1 N_{t-1} N^j_{t-1} + (\delta_2/2)(N^j_{t-1})^2$, $\delta_1, \delta_2 > 0$, where $N_{t-1} = \Sigma^J_{j=1} N^j_{t-1}$ is the aggregate inventory level.

We think of our cost functions as tractable approximations of more complex behavior. Our functions are nonstandard in that Y_t appears in the representative firm's production costs, and N_{t-1} appears in inventory holding costs. The presence of Y_t is intended to capture the presumed positive association of economy-wide real wages and aggregate output. The presence of N_{t-1} is intended to capture the presumed positive association of the level of aggregate inventories and storage-space rents when N_{t-1} is positive, and the presumed positive association of backlogging costs and aggregate backlogs when N_{t-1} is negative.[9]

The firms' first stage contingency plans are found from the following maximization problem:

$$\text{(4)} \quad \max_{\{R^j_{t+i}\, N^j_{t+1}\}} E_t \left\{ \sum_{i=0}^{\infty} \left[D^j_{t+i} R^j_{t+i} - \gamma_1 Y_{t+i} Y^j_{t+i} - \frac{\gamma_2}{2} (Y^j_{t+i})^2 - \delta_1 N_{t+i-1} N^j_{t+i-1} - \frac{\delta_2}{2} (N^j_{t+i-1})^2 \right] \sigma^i \right\}, \quad j = 1, \ldots, J.$$

The firm's maximization problem is subject to an initial stock of inventories N^j_{t-1} and to the relationships (1) and (3).[10] The discount rate σ is a constant between zero and unity. The operator E_t denotes the mathematical expectation conditional on the information available to the firm at the beginning of period t. All firms have identical information sets, so the operator is not specific to the firm.

In finding the firm's optimal plans, we have assumed that J

8. Given the assumed cost structure, firms maximize profit by meeting all demand, possibly through backlogs. An alternative paradigm allows firms to stock-out as in Brunner, Cukierman, and Meltzer [1983].

9. The cost structure for inventories dictates that firms find backlogs to be the optimal steady-state inventory. Since both firm and aggregate inventories can be negative, the cost structure implies an incentive for each firm's inventories to have the opposite sign of average inventories; i.e., $\delta_1 N_{t-1} N^j_{t-1} = \delta_1 J \overline{N}^2_{t-1} + \delta_1 J \overline{N}_{t-1} (N^j_{t-1} - \overline{N}_{t-1})$, where $\overline{N}_{t-1} \equiv N_{t-1}/J$. Since a firm's costs increase with $(N^j_{t-1})^2$, the firm's choice of N^j_{t-1} remains well defined.

10. Early work in this type of model was done by Holt, Modigliani, Muth, and Simon [1960] and by Lovell [1961]. See Amihud and Mendelson [1982]; Blinder [1982]; and Brunner, Cukierman, and Meltzer [1983] for additional macroeconomic applications.

is sufficiently large that each firm takes the economy-wide variables, $\overline{R}_{t+i}$, N_{t+i}, and Y_{t+i}, as invariant to the firm's decisions. Such a strategy is exactly profit maximizing only when $J \to \infty$. There is nothing in our setup to preclude $J \to \infty$, and the reader may want to interpret our results in terms of this special case.[11]

The problem stated in (4) implies a pair of linear Euler equations for each of the J firms. Since our concern is with aggregate phenomena, we record here only the aggregate Euler equations, which are obtained by summing the firm-specific Euler equations. These aggregate Euler equations are

$$E_t\{D_{t+i} - J^2\beta_4\overline{R}_{t+i} + (J\gamma_1 + \gamma_2)J\beta_4 Y_{t+i}\} = 0, \tag{5a}$$

$$E_t\{Y_{t+i} - \sigma Y_{t+i+1} + \sigma\mu N_{t+i}\} = 0, \tag{5b}$$

where $\mu \equiv (\delta_1 J + \delta_2)/(\gamma_1 J + \gamma_2)$. Equations (5a) and (5b) are obtained by summing across all the firm-specific Euler equations resulting from differentiating (4) with respect to R^j_{t+i} and N^j_{t+1}, $j = 1, 2, \ldots, J$. We wish to solve (5a) and (5b) for aggregate contingency plans concerning $E_t\overline{R}_{t+i}$ and E_tN_{t+i}. Prices are set based on beginning-of-the-period information; therefore the planned value $E_t\overline{R}_t$ and the actual magnitude $\overline{R}_t$ will coincide.

Before solving (5a) and (5b), it is convenient to define some demand-associated parameters. Use the definition of D_t, the aggregate law of motion $N_t = Y_t + N_{t-1} - D_t$, and the expenditure function in (2) to obtain

$$D_t = \beta_0 - \beta_1\overline{R}_t + \beta_2(N_t - N_{t-1}) + \beta_3 w_t, \tag{6}$$

where $\beta_0 \equiv (\rho_0 + \rho_0^*)/(1 - \kappa_2\rho_2)$, $\beta_1 \equiv (\rho_1 + \rho_1^* - \rho_2\kappa_1)/(1 - \rho_2\kappa_2)$, $\beta_2 \equiv \rho_2\kappa_2/(1 - \kappa_2\rho_2)$, $\beta_3 \equiv 1/(1 - \kappa_2\rho_2)$, and $w_t \equiv \rho_2 u_t$. We assume that the marginal propensity to consume the home good is less than unity, $\rho_2\kappa_2 < 1$, and that $\rho_1 + \rho_1^* - \rho_2\kappa_1 > 0$, which means the Marshall-Lerner condition is satisfied. Hence, $\beta_i > 0$, $i = 1, 2, 3$. Equation (6) gives the aggregate demand function we use when solving (5a) and (5b).

Since w_t is a white noise disturbance, solutions for the Euler equations take the following form:

11. In a setup much like ours, Eichenbaum [1983] has shown that decision rules at the industry level of an unknown number J of firms acting as (i) perfect competitors, (ii), a J-plant monopolist, and (iii) Nash competitors are equivalent up to a term whose coefficient depends on J. Consequently, we expect the qualitative properties of our aggregate decision rules to be robust to a wide variety of industrial organizations.

(7a) $$\overline{R}_t = \pi_{R0} + \pi_{R1}N_{t-1} + \pi_{R2}E_t w_t$$

(7b) $$E_t N_t = \pi_{N0} + \pi_{N1}N_{t-1} + \pi_{N2}E_t w_t.$$

The values of the π coefficients in (7a) and (7b) are found by the method of the undetermined coefficients and are the following:

(8a) $$\pi_{N0} = \text{constant},$$

(8b) $$\pi_{N1} = \tfrac{1}{2}[A - (A^2 - 4/\sigma)^{1/2}], \quad 0 < \pi_{N1} < 1,$$

(8c) $$\pi_{N2} = [1/(\pi_{N1} - A)\sigma][J^2\beta_3\beta_4/(\beta_1 + J^2\beta_3\beta_4)], \quad \pi_{N2} < 0,$$

(8d) $$\pi_{R0} = \text{constant},$$

(8e) $$\pi_{R1} = (\Delta_2/\Delta_1)(\pi_{N1} - 1), \qquad \pi_{R1} < 0,$$

(8f) $$\pi_{R2} = (\Delta_2/\Delta_1)\pi_{N2} + (\Delta_3/\Delta_1), \qquad \pi_{R2} > 0,$$

where

$$A = 1 + (1/\sigma) + \mu\Delta_1/(\beta_1 + J^2\beta_3\beta_4),$$
$$\Delta_1 = (1 + J^2\gamma_1\beta_4 + J\gamma_2\beta_4)\beta_1 + J^2\beta_4,$$
$$\Delta_2 = (1 + J^2\gamma_1\beta_4 + J\gamma_2\beta_4)\beta_2 + J^2\gamma_1\beta_4 + J\gamma_2\beta_4, \text{ and}$$
$$\Delta_3 = (1 + J^2\gamma_1\beta_4 + J\gamma_2\beta_4)\beta_3.$$

Equation (7a) gives both the contingency plan and actual value of $\overline{R}_t$. However, equation (7b) does *not* give actual N_t, only its expected value, since actual inventories are not determined until the second stage of optimization when actual demand is revealed to the firms. Equations (7a) and (7b) make intuitive sense. Since $\pi_{R1} < 0$ and $\pi_{R2} > 0$, $\overline{R}_t$ responds negatively to beginning-of-period inventories and positively to expected demand disturbances. Since $0 < \pi_{N1} < 1$, expected inventories obey a stable autoregression; and since $\pi_{N2} < 0$, inventories are expected to fall in response to a positive demand disturbance.

After the firms set prices, they are confronted with actual demand. Although we assume that the firms may not alter their posted prices after they see demand, the firms can deviate from their contingency plans for inventory accumulation. Upon seeing demand, the firms respond with an optimal combination of current production and inventory change that satisfies demand. This is the second stage of optimization, and in this stage each firm takes as given its own price, the economy-wide average price, beginning-of-period inventory, and *actual demand* for the current period. The economy-wide Euler equations for this stage of the optimization are obtained in a manner similar to (5a) and (5b) except that R_t^j is now *not* a decision variable, and the information set relevant to the optimization now includes the actual value of demand at time t.

The inventory decision may be derived from the following aggregate Euler equation:

$$E_t'\{Y_t - \sigma Y_{t+1} + \sigma\mu N_t\} = 0, \tag{9}$$

where E_t' is the expectation operator conditional on full information for period t, which includes w_t.

Since actual inventories will differ from contingency plans only due to differences of w_t from $E_t w_t$, we express the solution for inventories as

$$N_t = \pi_{N0} + \pi_{N1} N_{t-1} + \pi_{N2} E_t w_t + \pi_{N3}(w_t - E_t w_t). \tag{10}$$

Using (9) and our previous results, we find that

$$\pi_{N3} = -\beta_3/\{\beta_3 + [\sigma(1 - \pi_{N1})(\beta_1 + J^2\beta_3\beta_4)/\Delta_1] + \sigma\mu\}, \tag{11}$$

where $-1 < \pi_{N3} < \pi_{N2} < 0$.

In (10) note that $\pi_{N3} < \pi_{N2}$ implies a stronger response of inventories to unexpected demand than to expected demand. Firms respond to expected demand shocks with their relative prices and an expected response in inventories and production. When actual demand occurs, the firm responds optimally given its set price. Consequently, the response of inventories and production to unexpected demand under the constraint of no price change is greater than the response to expected demand.

Aggregate output is given by $Y_t = D_t + N_t - N_{t-1}$. Using our previous results, we derive

$$Y_t = \pi_{Y0} + \pi_{Y1} N_{t-1} + \pi_{Y2} E_t w_t + \pi_{Y3}(w_t - E_t w_t), \tag{12}$$

where the coefficients are the following:

$$\pi_{Y0} = \text{constant}, \tag{13a}$$

$$\pi_{Y1} = (\pi_{N1} - 1)(\beta_1 + J^2\beta_3\beta_4)/\Delta_1, \qquad \pi_{Y1} < 0, \tag{13b}$$

$$\pi_{Y2} = (J^2\beta_3\beta_4/\Delta_1)[1 + 1/(\pi_{N1} - A)\sigma], \qquad \pi_{Y2} > 0, \tag{13c}$$

$$\pi_{Y3} = \beta_3(1 - \pi_{N3}), \qquad \pi_{Y3} > \pi_{Y2}. \tag{13d}$$

Because $\pi_{Y1} < 0$, larger beginning-of-period inventories result in lower output. Since $\pi_{Y2} > 0$, increased demand increases expected ouput. An increase in $E_t w_t$ produces an increase in $\overline{R}_t$ and a higher expected quantity demanded along the shifted demand curve. Firms plan on meeting this increase in demand partly out of current production and partly by drawing down current inventory stocks. Because $\pi_{Y3} > \pi_{Y2}$, unexpected demand has a larger output effect than does expected demand, since expected demand is reflected in increases in relative prices while unexpected demand is not.

This completes our development of the goods markets. We have not yet obtained reduced forms for relative prices, inventories, or output because our expressions for these magnitudes all contain the expectation of w_t. To determine this magnitude, agents use their knowledge of the entire economy, which consists of both the goods markets and the asset markets. We turn now to the development of the asset markets.

C. The Asset Markets

The economy is assumed to be one that is small in both the world securities markets, where all securities are perfect substitutes, and in the markets for foreign produced goods. The country is large, though, in the markets for domestically produced goods and for domestic money. Thus, foreign interest rates and foreign goods prices are exogenous to our economy. The principal equations describing the asset markets are the following:

$$\text{(14a)} \quad m_t - p_t = -\alpha_1 i_t + \alpha_2 \tilde{X}_t, \qquad \alpha_1, \alpha_2 > 0,$$

$$\text{(14b)} \quad i_t = i_t^* + E_t s_{t+1} - s_t,$$

$$\text{(14c)} \quad p_t = \theta \overline{h}_t + (1 - \theta)(\overline{h}_t^* + s_t), \qquad 0 < \theta < 1.$$

Equation (14a) expresses money market equilibrium and states that the real money supply $m_t - p_t$ equals real money demand $-\alpha_1 i_t + \alpha_2 \tilde{X}_t$. In (14a), m_t is the logarithm of the supply of nominal transactions balances, and p_t is the logarithm of the nominal price level. According to (14c), p_t is a weighted average of the logarithm of the average domestic currency price of domestic goods, $\overline{h}_t, = \ln[J^{-1} \Sigma_{j=1}^{J} H_t^j]$, and the average domestic currency price of imported goods, $\overline{h}_t^* + s_t$, where $\overline{h}_t^*$ is the logarithm of the average foreign currency price of imported goods and s_t is the logarithm of the exchange rate quoted as the home currency price of foreign currency. The relative price of home goods $\overline{R}_t$ is approximated with a first-order Taylor's series around $\overline{H}_0$ and P_0, and we normalize $\overline{H}_0/P_0 = 1$ giving $\overline{R}_t = \overline{h}_t - p_t + 1$.

Money demand is specified in the spirit of cash-in-advance models such as those of Clower [1967] and Lucas [1980]. The opportunity cost of holding cash balances in excess of planned expenditure is i_t, the level of the domestic rate of interest. According to (14b), i_t obeys the uncovered interest rate parity condition, with i_t^* being the level of the foreign interest rate.[12] The scale variable in money demand is the sum of agents' expected total expenditures, $\tilde{X}_t = \Sigma_{i=1}^K E_t^i X_t^i$, where K is the number of agents in the economy, E_t^i is agent i's expectation operator at the beginning of period t, and X_t^i is agent i's expenditure during period t on both goods. We assume that X_t^i obeys

$$(15) \qquad X_t^i = \frac{\kappa_1}{K}\overline{R}_t + \frac{\kappa_2}{K} Y_t + u_t^i, \qquad i = 1,2,\ldots,K,$$

where u_t^i is the individual's saving-expenditure disturbance at time t. We allow each agent to see his own u_t^i at the beginning of the period. However, we assume that u_t^i is composed of two uncorrelated white noise components, e_t^i and a_t^i, $u_t^i = e_t^i + a_t^i$. Further, we impose $\Sigma_{i=1}^K e_t^i = 0$. Thus, u_t^i contains an individual-specific component e_t^i and the individual's contribution to the aggregate disturbance $\Sigma_{i=1}^K a_t^i = u_t$. We also assume that the variance of e_t^i is sufficiently large compared with the variance of a_t^i that even though each agent sees his own expenditure disturbance u_t^i, he always thinks that disturbance to be dominated by the individual-specific component e_t^i. Hence, the agent cannot use his observation of u_t^i to form useful inferences concerning u_t or other aggregate disturbances. Thus, when $\tilde{X}_t$ is formed, one obtains

$$(16) \qquad \tilde{X}_t = \kappa_1\overline{R}_t + \kappa_2 E_t Y_t + u_t.$$

$\overline{R}_t$ appears in (16) because it is in an agent's information set. $E_t Y_t$ appears because $E_t^i Y_t = E_t Y_t$, since knowing u_t^i provides an agent with no aggregate information. The aggregate saving-expenditure disturbance u_t appears in (16) because $E_t^i u_t^i = u_t^i$ and by construction $\Sigma_{i=1}^K u_t^i = u_t$.

The logarithm of actual nominal transactions balances is assumed to follow a random walk $m_t = m_{t-1} + v_t$, where v_t is white

12. We view equation (14b) as a useful simplifying assumption that allows us to focus directly on production, exchange rates, and prices without complicating the theory with a model of a time-varying risk premium. The evidence in Hansen and Hodrick [1983] suggests that statistically significant risk premiums may characterize the relationship between forward exchange rates and the expected future spot rates. However, their evidence also suggests that if risk premiums exist, they are small in comparison to unexpected changes in exchange rates.

noise. At the beginning of the period agents do not know v_t, but it is assumed that they do know m_{t-1}. Also, in keeping with the practices of many countries, we assume that at the beginning of the period agents observe a preliminary noisy indicator of the nominal money supply, the "money number," $m_t^{\#} = m_t + z_t$. The three white noise disturbances, u_t, v_t, and z_t, are assumed to be mutually orthogonal, although we relax this assumption in Section V.

For simplicity of presentation, we complete the model by assuming that the average price of foreign goods is constant, $\bar{h}_t^* = \bar{h}^*$, and the foreign interest rate is also constant, $i_t^* = i^*$.

Prior to providing the explicit solution to the model, it is useful to summarize informally the working of the model. At the beginning of each period prices are set, and the exchange rate and interest rate are determined. However, at this stage agents do not know the actual values of the aggregate disturbances, v_t, w_t, and z_t. The agents see all prices, the exchange rate, the current money number, and both domestic and foreign interest rates. From these data the agents form inferences concerning the values of the disturbances. It is the inferred value of the demand disturbance that feeds into the pricing decision. After prices are set, the actual value of the demand disturbance and the other disturbances are revealed to the agents. Prices are sticky in that no recontracting is allowed at this stage. The firms then choose optimal production and inventory accumulation based on the actual quantity demanded, which is determined in part by the prices set under partial information and in part by the demand disturbance w_t.

III. THE SOLUTION

In this section we shall provide our model's reduced-form solutions for the level of output, inventories, the exchange rate, the average relative price of the domestic good, and the average nominal price of the domestic good. The first step required in obtaining a solution is to extract information from the clearing of the asset markets and from the money number to form agents' perceptions about current disturbances. Agents will be able to observe two signals of the three underlying aggregate disturbances.

Information and the Asset Markets

At the beginning of the period each agent has the information set I_t, which contains the values of $s_t, p_t, \bar{h}_t^*, i_t, i_t^*, R_t^j (j = 1, \ldots, J)$, $m_t^{\#}$, and full information concerning all variables dated $t - 1$ or earlier as well as complete information concerning the structure of the model. I_t does *not* contain separately the current disturbances, v_t, w_t, or z_t. Since agents' decisions at the beginning of the period in both price setting and in the asset markets depend on their perceptions of these disturbances, they will use the information in I_t to draw inferences about the disturbances.[13] We assume that $E_t v_t$, $E_t w_t$, and $E_t z_t$ are the linear least squares projections of the respective disturbances onto the information set I_t.

To find the values of these estimates, we isolate the new information concerning the disturbances that enters I_t at the beginning of the period as in Canzoneri, Henderson, and Rogoff [1983]. Two of the disturbances impinge directly on the asset markets, and it is from these markets that agents extract one signal concerning the disturbances. Substituting international capital market equilibrium, (14b), the expenditure relation (16), and the money supply process into money market equilibrium gives

$$(17) \quad m_{t-1} + v_t - p_t = -\alpha_1(i_t^* + E_t s_{t+1} - s_t) + \alpha_2(\kappa_1 \bar{R}_t + \kappa_2 E_t Y_t) + \alpha_3 w_t,$$

where $\alpha_3 \equiv \alpha_2/\rho_2$. Since I_t contains m_{t-1}, p_t, i_t^*, s_t, and $\bar{R}_t$ as well as the parameters α_1, α_2, and α_3, and because I_t is used to form $E_t s_{t+1}$ and $E_t Y_t$, equation (17) implies that I_t contains the following variable: $g_{1t} = v_t - \alpha_3 w_t$. The variable g_{1t} carries the asset markets' information concerning the underlying disturbances. The second signal is contained in the money number, $m_t^{\#} = m_t + z_t = m_{t-1} + v_t + z_t$. The beginning-of-period information set contains m_{t-1}, so the new information in $m_t^{\#}$ is $g_{2t} = v_t + z_t$. The variables g_{1t} and g_{2t} contain the current-period information about v_t, w_t, and z_t available to agents at the beginning of the period. Agents use these two pieces of information to form $E_t v_t$ and $E_t w_t$ as linear least squares projections of v_t and w_t onto g_{1t} and g_{2t}:

13. The information content of asset prices has been emphasized by Barro [1980], King [1982], and Grossman and Weiss [1982] in the context of business cycle models.

(18a) $$E_t v_t = \phi_{v1} g_{1t} + \phi_{v2} g_{2t}$$

(18b) $$E_t w_t = \phi_{w1} g_{1t} + \phi_{w2} g_{2t},$$

where

$$\phi_{v1} = \Delta^{-1}\sigma_v^2\sigma_z^2 > 0, \qquad \phi_{v2} = \Delta^{-1}\alpha_3^2\sigma_w^2\sigma_v^2 > 0,$$

$$\phi_{w1} = -\Delta^{-1}\alpha_3\sigma_w^2(\sigma_v^2 + \sigma_z^2) < 0, \; \phi_{w2} = \Delta^{-1}\alpha_3\sigma_w^2\sigma_v^2 > 0,$$

$$\text{and} \qquad \Delta = [\sigma_v^2\sigma_z^2 + \alpha_3^2\sigma_w^2\sigma_v^2 + \alpha_3^2\sigma_w^2\sigma_z^2].$$

Using these projections, we can derive the full reduced-form solution of the model. The reduced-form solutions for the real sector of the model, $\overline{R}_t$, N_t, and Y_t, can be found by substituting $E_t w_t$ in (18b) into (7a), (10), and (12). Reduced-form solutions for the exchange rate and the domestic price are found from the money market equilibrium in (17), the price index (14c), and from the approximation $\overline{R}_t = \overline{h}_t - p_t + 1$. Given the assumed time series properties of the exogenous stochastic processes and ignoring constant terms, reduced-form equations have the following form for $Q_t = N_t, Y_t, \overline{R}_t, s_t, \overline{h}_t$:

(19) $$Q_t = \lambda_{QN} N_{t-1} + \lambda_{Qm} m_{t-1} + \lambda_{Qv} v_t + \lambda_{Qw} w_t + \lambda_{Qz} z_t.$$

The algebraic signs of the λ coefficients of the full reduced form are recorded in Table I, and the actual values of the coefficients are listed in the Appendix. The dynamics of the model are described in the next section with the aid of Figure I.

TABLE I.

SIGNS OF REDUCED-FORM COEFFICIENTS ON STATE VARIABLES

Endogenous variable	N_{t-1}	m_{t-1}	v_t	w_t	z_t
N_t	$0 < \lambda_{NN} < 1$	$\lambda_{Nm} = 0$	$\lambda_{Nv} < 0$	$\lambda_{Nw} < 0$	$\lambda_{Nz} > 0$
Y_t	$\lambda_{YN} < 0$	$\lambda_{Ym} = 0$	$\lambda_{Yv} > 0$	$\lambda_{Yw} > 0$	$\lambda_{Yz} < 0$
$\overline{R}_t$	$\lambda_{RN} < 0$	$\lambda_{Rm} = 0$	$\lambda_{Rv} < 0$	$\lambda_{Rw} > 0$	$\lambda_{Rz} > 0$
s_t	$\lambda_{sN} > 0$	$\lambda_{sm} = 1$	$\lambda_{sv} > 0$	$\lambda_{sw} < 0$	$\lambda_{sz} \gtreqless 0$
$\overline{h}_t$	$\lambda_{hN} \gtreqless 0$	$\lambda_{hm} = 1$	$\lambda_{hv} \gtreqless 0$	$\lambda_{hw} \gtreqless 0$	$\lambda_{hz} \gtreqless 0$

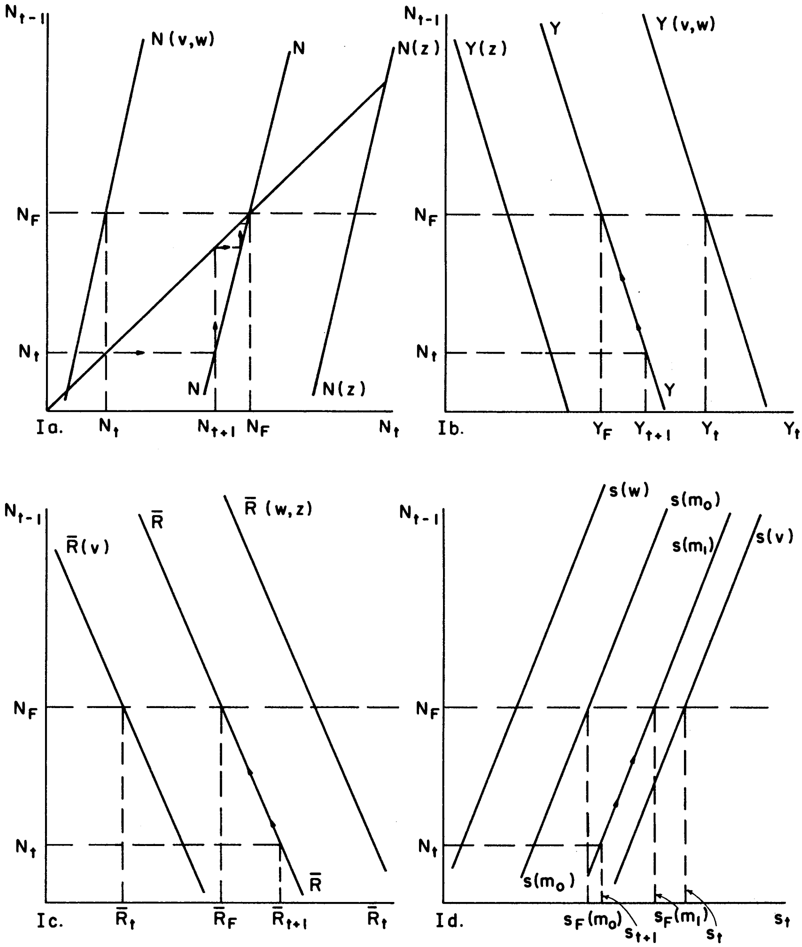

FIGURE I

IV. THE DYNAMICS OF THE MODEL

As the reduced-form equations (19) indicate, the beginning-of-period inventory stock N_{t-1}, the actual money supply from the previous period m_{t-1}, and the stochastic disturbances, v_t, w_t, and z_t, are the state variables of the system. We assume that actual money is known with a one-period lag. Therefore, the lagged nominal money stock does not influence the real sector of the model, and since the logarithm of the actual nominal money supply is assumed to follow a random walk, the exchange rate and domestic price change equiproportionately to known changes in m_{t-1}. Consequently, $\lambda_{Nm} = \lambda_{Ym} = \lambda_{Rm} = 0$, and $\lambda_{sm} = \lambda_{hm} = 1$.

The dynamic path of the economy is induced by innovations in the exogenous stochastic processes, the innovation in the actual money stock v_t, the domestic demand disturbance w_t, and the error in the money number z_t. These contemporaneously unobservable disturbances shock the system away from its steady state, which is labeled with an F subscript in Figure I. In that figure the NN, YY, $\overline{RR}$, and $s(m_0)s(m_0)$ loci indicate the values of N_t, Y_t, $\overline{R}_t$, and s_t that are consistent with any particular value of N_{t-1} given a level of money, $m_{t-1} = m_0$, and no new shocks to the system. Figure Ia demonstrates that when inventories are away from their steady state, they converge over time in a stable autoregression toward the full equilibrium N_F. As YY indicates, output is above Y_F when inventories are below N_F. Along the adjustment path firms set their relative prices higher when inventories are low as indicated by $\overline{RR}$, and for $N_{t-1} < N_F$ the exchange rate is expected to increase, as $s(m_0)s(m_0)$ indicates, as the economy moves toward full equilibrium. This is consistent with asset market equilibrium and with the expected fall in $\overline{R}$. We turn now to consideration of how the economy responds to the stochastic disturbances.

Consider the response of the economy to an unobservable stochastic increase in the money supply v_t, given that it begins in full equilibrium and given that $w_t = z_t = 0$. From (18b), notice that $E_t w_t = (\phi_{w1} + \phi_{w2})v_t < 0$ indicating that agents misperceive the increase in the money supply as a reduction in real goods demand. This occurs because the information provided by the equilibrium values of prices, the interest rate, and the exchange rate obtained by firms in observing g_{1t} is consistent with an increase in the money supply and with a reduction in expenditure. As (18a) indicates, combining g_{1t} with the information in the money number allows firms to infer that v_t has increased, but $E_t v_t = (\phi_{v1} + \phi_{v2})v_t$, which is positive and smaller than v_t. Since firms expect a fall in real demand, they lower average relative price to $\overline{R}_t$ in Figure Ic, which is the intersection of the locus $R(v)$ and N_F. At this point firms are anticipating an increase in inventories and a reduction in output along a shifted aggregate demand curve. When demand is actually realized, it occurs along an unshifted demand curve because we are discussing the influence of a monetary shock and are holding $w_t = 0$. Since firms have set low relative prices, demand is unexpectedly high. Firms respond with an optimal combination of increased production, at the intersection of the locus $Y(v,w)$ and N_F in Figure Ib, and inventory depletion, at the intersection of the locus $N(v,w)$ and N_F in Figure Ia. The domestic currency depreciates in response to the v_t shock for two reasons. First, to the extent that the monetary shock is perceived, all nominal prices including the exchange rate rise equiproportionately. Second, part of the deterioration in the terms of trade, the decrease in R_t, is accomplished by a depreciation of the currency. Therefore, the exchange rate rises. In Figure Id, the exchange rate is determined by the intersection of the locus $s(v)$ with N_F.

A fundamental insight of Dornbusch [1976] was that monetary shocks would cause exchange rate overshooting if goods prices were fixed and the money market was in equilibrium. In this model overshooting is not a necessary result, although it is more likely the smaller is α_1, the semi-elasticity of the demand for money with respect to the interest rate. To demonstrate this result, notice that λ_{sv}, the initial response of the exchange rate to a money shock, can be written as its full information response plus an additional term:

$$(20) \quad \lambda_{sv} = 1 + \left(\frac{1}{1+\alpha_1}\right)\left(\frac{\alpha_3\sigma_w^2\sigma_z^2}{\Delta}\right)\left\{\left[\left(\frac{\theta}{1-\theta} + \alpha_2\kappa_1\right)\pi_{R2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2}\right] - \alpha_1\alpha_3\right\}.$$

The exchange rate overshoots if the positive term in square brackets in (20) is larger than $\alpha_1\alpha_3$. Figure Id is drawn under that supposition.

After the initial response to the monetary shock, the economy adjusts over time back to its unconditional equilibrium, unless new stochastic disturbances alter its path. Inventories begin to accumulate as firms raise the average relative price of their product above its unconditional equilibrium value and produce output in excess of the quantity demanded. The exchange rate falls in period $t + 1$ to facilitate the improvement in the terms of trade. As Figure Id indicates, the currency is then expected to depreciate over time to its unconditional equilibrium value $s_F(m_1)$. Rather than approach its new steady state from above as in the Dornbusch [1976] framework, the approach is from below.

Now consider the influence of a positive shock to real demand that we normalize to have the same effect on inventories and output as the previously discussed money shock in Figure I. This shock is considered in isolation from other shocks; i.e., $v_t = z_t = 0$. When a positive but unobservable real shock occurs, $w_t > E_t w_t > 0$, since $E_t w_t = -\alpha_3 \phi_{w1} w_t$ and $0 < -\alpha_3 \phi_{w1} < 1$. Firms expect an increase in demand and raise their relative prices. In Figure Ic, $\overline{R}_t$ is given by the intersection of the locus $\overline{R}(w,z)$ and N_F. Firms expect to draw down inventories and to increase production, but they are surprised by the magnitude of actual demand. Output for period t occurs at the intersection of the locus $Y(v,w)$ and N_F in Figure Ib, and N_t is given by the intersection of N_F and the locus $N(v,w)$ in Figure Ia. The exchange rate falls as the currency appreciates for two reasons. First, the currency appreciates to facilitate the improvement in the terms of trade. Second, agents think that the supply of money has fallen, since $E_t v_t = -\alpha_3 \phi_{v1} w_t < 0$. Consequently, the exchange rate falls to reflect the perceived decrease in the money supply. The dynamic adjustment in period $t + 1$ and afterward is exactly as in the case of the positive monetary disturbance except that the exchange rate in period $t + 1, i = 1, 2, \ldots$, is given by the intersection of N_{t+i} and the locus $s(m_0)$ in Figure Id.

Next, consider the response of the economy to z_t, the reporting error between the logarithms of the measured nominal money supply and the actual money supply, given $v_t = w_t = 0$. The money number $m_t^{\#}$ is a source of "news" about the actual money supply. Knowing $m_t^{\#}$ allows agents to make inferences about the state of the economy. Frenkel [1981] has stressed the importance of such new information in modern asset theories of the exchange rate. If that news is measured with error, such as $m_t^{\#}$ is, then the noise in the news will be a fundamental determinant of all of the endogenous variables of the economy including the exchange rate.

Given the stochastic structure of the economy, agents misinterpret positive z_t disturbances as positive real demand disturbances, since $E_t w_t = \phi_{w2} z_t > 0$, and as positive money supply disturbances, since $E_t v_t = \phi_{v2} z_t > 0$. Firms expect an increase in demand, raise their relative prices, and expect to increase production and decumulate inventories. When actual demand is realized, it is lower than expected, and firms must cut back on production and increase inventories. Because the positive z_t is misinterpreted as a positive increase in the actual money supply, the effect of z_t on the exchange rate is ambiguous without further assumptions. Under the assumption that produces exchange rate overshooting with respect to a v disturbance, $\lambda_{sz} < 0$, since

$$(21)\quad \lambda_{sz} = \frac{-\alpha_3\sigma_w^2\sigma_v^2}{(1+\alpha_1)\Delta}\left[\left(\frac{\theta}{1-\theta}+\alpha_2\kappa_1\right)\pi_{R2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2} - \alpha_1\alpha_3\right].$$

The effects of the disturbances on the nominal price of the domestic good are also generally indeterminate in algebraic sign, which is why we have not discussed the effects of the shocks on this endogenous variable.

V. Consistency with Empirical Regularities

Several empirical regularities were mentioned in the introduction, and this section discusses the consistency of the implications of our model with these regularities.

The first regularity addressed is that nominal monetary disturbances must have persistent real effects. This is true in our model, since v_t affects all real variables and because the explicit modeling of inventories induces persistent dynamics. A potential criticism of the model is that the real effects of money are caused only by unperceived money.

Two empirical papers, one by Barro and Hercowitz [1980] hereafter B-H, and one by Boschen and Grossman [1982] hereafter B-G, address this issue. It is important to discuss the relationship between the present structure of our model and the regressions used to test hypotheses in B-H and B-G because the results of these studies provide some evidence against the hypothesis that unperceived money is the primary channel through which nominal money affects real variables.

For clarity of presentation, the stochastic processes, v_t, w_t, and z_t, were specified as being jointly orthogonal as well as being independently and identically distributed. It turns out that the predictions of our model in the jointly orthogonal case may be interpreted as being inconsistent with the evidence presented by B-G and B-H. Significant covariance between v_t and z_t in one case and z_t and w_t in the other, however, is enough to overturn the inconsistencies between the model and the data.

The two empirical propositions of B-G and B-H are the following: (i) the measurement error between actual money and reported money should have a significant effect on real output, and (ii) reported money, since it is fully perceived, should have no real effect. The first hypothesis is tested and rejected in B-H with U. S. annual average data from 1950–1975 and in B-G with U. S. quarterly average data for 1953–1978, while the second hypothesis is rejected in B-G.

The hypotheses are most easily discussed in terms of the reduced form for output, which may be written as

$$Y_t = \pi_{Y1}N_{t-1} + (\pi_{Y2} - \pi_{Y3})[\phi_{w1}(v_t - \alpha_3 w_t) + \phi_{w2}(v_t + z_t)] + \pi_{Y3}w_t, \tag{22}$$

where $(\pi_{Y2} - \pi_{Y3}) < 0$. Define $\tilde{Y}_t = Y_t - \pi_{Y1}N_{t-1}$, where $\tilde{Y}_t$ is the innovation in Y_t.[14] The first empirical proposition is that z_t, the noise in the news about the money supply, should have a significant coefficient in ordinary least squares regressions (OLS) of $\tilde{Y}_t$ on z_t. Consider the estimation of

$$\tilde{Y}_t = \beta_z z_t + \nu_{1t} \tag{23}$$

by OLS. The estimated parameter β_z is

$$\beta_z = \frac{E[z_t(\beta_z z_t + \nu_{1t})]}{E(z_t)^2} = \beta_z + \frac{E(z_t \nu_{1t})}{E(z_t)^2}. \tag{24}$$

If z_t and ν_{1t} are uncorrelated, β_z is an unbiased estimate of the true influence of z_t on $\tilde{Y}_t$. In the present form of our model the population parameter $\beta_z = (\pi_{Y2} - \pi_{Y3})\phi_{w2} < 0$. Also, $\nu_{1t} = (\pi_{Y2} - \pi_{Y3})(\phi_{w1} + \phi_{w2})v_t + [(\pi_{Y2} - \pi_{Y3})(-\alpha_3\phi_{w1}) + \pi_{Y3}]w_t$ which is orthogonal to z_t making OLS appropriate. Since B-G and B-H estimate β_z to be insignificantly different from zero, this specification of our model is suspect. Relaxing the restriction that v_t, w_t, and z_t are mutually orthogonal, though, implies that the OLS estimates of β_z given in (24) is not an unbiased estimate. A sufficient condition to bias the coefficient toward zero is a negative covariance between v_t and z_t. In a more complicated framework with a complete covariance matrix, presumably other combinations of covariances would bias the OLS estimate β_z toward zero as well.

Now consider the second empirical hypothesis of B-G. In OLS regressions of output on perceived money, the OLS estimate should be zero, but it is estimated to be significantly different from zero

14. The way we create Y_t is specific to our model and is not equivalent to the model-free methods used by B-H and B-G to create their measures of detrended output. This difference alone may make their results inapplicable to our model. We choose to proceed as if their empirical methods were applicable to our work.

by B-G. This is inconsistent with the present version of the model because OLS regression of $\tilde{Y}_t$ on $m_t^{\#} - m_{t-1}$ would produce a zero coefficient if the covariances between v_t and z_t with w_t are zero. Intuitively, $m_t^{\#} - m_{t-1} = v_t + z_t$ is uncorrelated with w_t, hence it provides no information about w_t and consequently cannot affect anything real in the model. Clearly, this would not be the case if the covariance of w_t with z_t or v_t was nonzero, in which case the money number would provide direct evidence about the shock to aggregate demand for the home good.[15]

The third empirical regularity that was mentioned in the Introduction was that changes in the exchange rate and changes in the relative price of the export good of the country were negatively correlated. Thus, depreciations of the currency and deteriorations of the term of trade tend to coincide. Let $C(A_t;B_t)$ be the unconditional covariance of two random variables A_t and B_t. Then, the formal requirement on the model is that $C(s_t - s_{t-1}; \overline{R}_t - \overline{R}_{t-1}) < 0$. This condition is satisfied for our model since

$$(25)\quad C(s_t - s_{t-1}; \overline{R}_t - \overline{R}_{t-1}) = 2\lambda_{sN}\lambda_{RN}(\lambda_{Nv}^2\sigma_v^2 + \lambda_{Nw}^2\sigma_w^2 + \lambda_{Nz}^2\sigma_z^2) + \lambda_{Rv}(2\lambda_{sv} - \lambda_{sm})\sigma_v^2 + 2\lambda_{sw}\lambda_{Rw}\sigma_w^2 + 2\lambda_{sz}\lambda_{Rz}\sigma_z^2.$$

Examination of the algebraic signs of the λ coefficients in Table I and imposition of the argument that $\lambda_{sz} < 0$ confirms this finding.

The fourth empirical regularity discussed in the introduction requires exchange rates to be more volatile than domestic price indexes where by volatility is meant one-step-ahead predictability. Let $V'_{t-1}(A_t) = E'_{t-1}(A_t - E'_{t-1}A_t)^2$ be the definition of volatility for any random variable A_t, and $E'_{t-1}(\cdot)$ denotes the mathematical expectation conditional on full information about variables dated $t - 1$ or earlier. Recognize that $p_t = [\theta/(1 - \theta)](\overline{R}_t - 1) + s_t + h_t^*$. Then, the volatility definition under our assumption of constant foreign prices implies that

15. Consider, for instance, the case in which w_t and z_t are correlated and v_t and z_t are correlated. In this case $\tilde{Y}_t = \beta'_m(m_t^{\#} - m_{t-1}) + \nu'_{2t}$, where $\beta'_m = (\pi_{Y2} - \pi_{Y3})\phi'_{w2}$, $\nu'_{2t} = (\pi_{Y2} - \pi_{Y3})\phi'_{w1}(v_t - \alpha_3 w_t) + \pi_{Y3}w_t$ and ϕ'_{w1} and ϕ'_{w2} are the new OLS regression coefficients on the linear prediction of w_t using g_{1t} and g_{2t}. An OLS regression of $\tilde{Y}_t$ on $m_t^{\#} - m_{t-1}$ produces the estimates $\beta'_m = [\sigma_{wz}/(\sigma_v^2 + \sigma_z^2 + 2\sigma_{vz})]\,\pi_{Y2}$. In this case, neither the true β'_m nor its OLS estimate is nonzero. Such correlations might arise, for example, if real shocks w_t altered the distribution of money across banks with different reporting requirements and with different reserve requirements. Further, in our model we have ignored the policy reactions of the monetary authority. In data, of course, such reactions, particularly interest rate policy, would be present and would confound the B-H and B-G interpretations.

$$(26)\quad V'_{t-1}\,(p_t) = \left(\frac{\theta}{1-\theta}\right)^2 V'_t - 1\,(\overline{R}_t) + 2\left(\frac{\theta}{1-\theta}\right) C'_{t-1}\,(\overline{R}_t;s_t) + V'_{t-1}(s),$$

which is smaller than $V'_{t-1}(s_t)$ when $|2C'_{t-1}(\overline{R}_t;s_t)| > [\theta/(1-\theta)]\, V'_{t-1}(\overline{R}_t)$, where $C'_{t-1}(\cdot)$ denotes the conditional covariance. For this condition to be true,

$$(27)\quad \left\{\lambda_{Rv}\left[\left(\frac{\theta}{1-\theta}\right)\lambda_{Rv} + 2\lambda_{sv}\right]\sigma_v^2 - \lambda_{Rw}\left[-\left(\frac{\theta}{1-\theta}\right)\lambda_{Rw} - 2\lambda_{sw}\right]\sigma_w^2 - \lambda_{Rz}\left[-\left(\frac{\theta}{1-\theta}\right)\lambda_{Rz} - 2\lambda_{sz}\right]\sigma_z^2 < 0.\right.$$

In (27) each term multiplying the terms in square brackets is negative. Hence, if each term in square brackets is positive, the condition is satisfied. A sufficient condition for each of the terms in square brackets to be positive is that $[2/(1+\alpha_1)] > 1$. This is only a sufficient condition and is not necessary. The point is that the model allows domestic prices of domestic goods to be determined within the period as opposed to assuming them to be predetermined variables, yet it remains consistent with the empirical regularity for at least some values of free parameters of the model.

VI. Concluding Remarks

Our model was constructed to be consistent with major empirical regularities discovered in studies of business cycles and those discovered in studies of prices and exchange rates. Unexpected monetary disturbances are not neutral in our model because price-setting agents do not observe money directly. They see only indicators of the underlying disturbances, and they tend to confuse positive (negative) monetary shocks with negative (positive) demand shocks. Business cycles are propagated through time via optimal inventory adjustment.

Prices in our model are set at the beginning of the period, prior to the revelation of actual values of the underlying disturbances. Thus, our prices are sticky in the sense that they do not respond as quickly to monetary disturbances as they would if pricing were based on full information. Our model is consistent with the observations that exchange rates are more difficult to predict than are commodity prices and that changes in countries' exchange rates and terms of trade are negatively correlated.

In presenting our results we worked with unrealistically simple time series processes governing the supply of money and the level of real expenditure. These processes were chosen for clarity of presentation, and none of the results we have emphasized concerning the effects of unperceived monetary disturbances on output depend on our choice of processes. These effects stem only from innovations to the money supply. Consequently, these results will be robust to any stationary time series process for the money supply. What will change with changes in the time series process for the money supply are the time series properties of nominal prices.

We recognize that much work remains to be done on our framework. In particular, the linkages between firm level outcomes and the levels of aggregate domestic expenditure and aggregate money demand ought to be incorporated into the maximizing framework. We conjecture however, that the crucial analytic feature of our model in this area, which is the correlation between the scale variable in money demand and the scale variable in goods demand, will appear in a wide variety of sensible specifications.[16]

APPENDIX

This Appendix records the actual values of the reduced-form parameters, the λ coefficients, whose algebraic signs were given in Table I. For a typical variable $Q_t = Y_t, N_t, \bar{R}_t, s_t, \bar{h}_t,$ the reduced-form equation is the following:

$$\text{(A1)} \quad Q_t = \lambda_{QN} N_{t-1} + \lambda_{Qm} m_{t-1} + \lambda_{Qv} v_t + \lambda_{Qw} w_t + \lambda_{Qz} z_t$$

<u>Coefficients in N_t equation</u>

$$\lambda_{NN} = \pi_{N1} = \tfrac{1}{2}[A - (A^2 - [4/\sigma])^{1/2}], \qquad 0 < \lambda_{NN} < 1$$

$$\lambda_{Nm} = 0$$

$$\lambda_{Nm} = (\pi_{N2} - \pi_{N3})(\phi_{w1} + \phi_{w2}) < 0$$

$$\lambda_{Nv} = (\pi_{N2} - \pi_{N3})(-\alpha_3 \phi_{w1}) + \pi_{N3} < 0$$

$$\lambda_{Nz} = (\pi_{N2} - \pi_{N2})\phi_{w2} > 0$$

16. In Flood and Hodrick [1984b] we exploit this correlation in a model with full flexible prices, where expenditure is determined by a stochastic permanent income model.

Coefficients in Y_t equation

$$\lambda_{YN} = \pi_{Y1} < 0$$
$$\lambda_{Ym} = 0$$
$$\lambda_{Yv} = (\pi_{Y2} - \pi_{Y3})(\phi_{w1} + \phi_{w2}) > 0$$
$$\lambda_{Yw} = (\pi_{Y2} - \pi_{Y3})(-\alpha_3\phi_{w1}) + \pi_{Y3} > 0$$
$$\lambda_{Yz} = (\pi_{Y2} - \pi_{Y3})\phi_{w2} < 0$$

Coefficients in $\overline{R}_t$ equation

$$\lambda_{RN} = \pi_{R1} < 0$$
$$\lambda_{Rm} = 0$$
$$\lambda_{Rv} = \pi_{R2}(\phi_{w1} + \phi_{w2}) < 0$$
$$\lambda_{Rw} = \pi_{R2}(-\alpha_3\phi_{w1}) > 0$$
$$\lambda_{Rz} = \pi_{R2}\phi_{w2} > 0$$

Coefficients in $\overline{h}_t$ equation

$$\lambda_{hN} = \left(\frac{1}{1-\theta}\right)\pi_{R1} + \lambda_{sN} \gtreqless 0$$

$$\lambda_{hm} = 1$$

$$\lambda_{hv} = \left(\frac{1}{1-\theta}\right)\pi_{R2}(\phi_{w1} + \phi_{w2}) + \lambda_{sv} \gtreqless 0$$

$$\lambda_{hw} = \left(\frac{1}{1-\theta}\right)\pi_{R2}(-\alpha_3\phi_{w1}) + \lambda_{sw} \gtreqless 0$$

$$\lambda_{hz} = \left(\frac{1}{1-\theta}\right)\pi_{R2}\phi_{w2} + \lambda_{sz} \gtreqless 0$$

Coefficients in s_t equation

$$\lambda_{sN} = \frac{1}{[1 + \alpha_1(1 - \pi_{N1})]}\left[-\left(\frac{\theta}{1-\theta} + \alpha_2\kappa_1\right)\pi_{R1} - \alpha_2\kappa_2\pi_{Y1}\right] > 0$$

$$\lambda_{sm} = 1$$

$$\lambda_{sv} = \left(\frac{1}{1+\alpha_1}\right)\left\{1 - (\phi_{w1} + \phi_{w2})\left[\left(\frac{\theta}{1-\theta} + \alpha_2\kappa_1\right)\pi_{R2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2}\right] + \alpha_1(\phi_{v1} + \phi_{v2})\right\} > 0$$

$$\lambda_{sw} = \left(\frac{-\alpha_3}{1+\alpha_1}\right)\left\{1 - \phi_{w1}\left[\left(\frac{\theta}{1-\theta} + \alpha_2\kappa_1\right)\pi_{R2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2}\right] + \alpha_1\phi_{v1}\right\} < 0$$

$$\lambda_{sz} = \left(\frac{1}{1+\alpha_1}\right)\left\{- \phi_{w2}\left(\frac{\phi}{1-\phi} + \alpha_2\kappa_1\right)\pi_{R2} - \alpha_1\lambda_{sN}\pi_{N2} + \alpha_2\kappa_2\pi_{Y2} + \alpha_1\phi_{v2}\right\} < 0$$

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